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Observer Design for Sampled-Data Nonlinear Systems via Approximate Discrete-Time Models

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Abstract

Since exact discrete-time models are often unavailable for nonlinear sampled-data systems, the most realistic approach for observer design is to employ approximate discrete-time models. This paper investigates under what conditions, and in what sense, such an approximate design achieves convergence for the unknown exact discrete-time model. We first present examples which show that designs via approximate discrete-time models may indeed lead to instability when implemented on the exact model. We then present conditions for approximate designs that guarantee robustness for the exact discrete-time model.

1 Introduction

The study of discrete-time observers is important for two reasons: First, continuous-time designs are often implemented digitally and, second, for classes of systems, such as those in [4], observer design may be easier for discrete-time models than for continuous-time models. The main drawback of the existing discrete-time observer theory, however, is that the availability of exact discrete-time models is assumed, which is usually unrealistic. A more practical approach is to employ an approximate discrete-time model, and to study how robust such an approximate design would be when implemented on the exact model.

This paper reports initial results from our investigation of this robustness problem. We first present examples which illustrate that observer design via approximate models may fail to pro-

duce a stable observer for the exact discrete-time model. This non-robustness is due to excessive overshoot or slow convergence rate in the approximate design, which, when coupled with the mismatch between the exact and the approximate models, leads to instability. We then present a set of Lyapunov conditions for the approximate design, which guarantee convergence for the exact model in a semiglobal practical sense. Thus, our results give guidelines for sampled-data observer design when only an approximate model is available. They parallel recent results in [6, 5] for sampled-data control design based on approximate discrete-time models. However, observer design problems pose additional difficulties, which cannot be addressed with a direct application of results from control design. We also emphasize that our primary goal in this paper is to study observer convergence properties independently of a feedback design task, because the interest in observers is not restricted to output-feedback control. Studies that address output-feedback stabilization of sampled-data systems include [2], [3], and [5].

The paper is organized as follows: Section 2 gives the preliminary definitions and the problem formulation. Section 3 presents examples of non-robust designs, which give clues of why observers based on approximate models may give rise to instability for the exact model. Following these clues, Section 4 derives a set of sufficient conditions for the approximate design, which ensure its robustness when applied to the exact model. Section 5 presents a robust design example which conforms to the conditions derived in Section 4.

Finally, Section 6 gives the conclusions and summarizes our ongoing research in extending these results.

2 Preliminaries and Problem Statement

We consider the system

$$\dot{x} = f(x, u) \quad (1)$$

$$y = h(x), \quad (2)$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^m, y \in \mathbb{R}^p$, and $f(x, u)$ is locally Lipschitz. Given a sampling period $T > 0$, we assume that the control u is constant during sampling intervals $[kT, (k+1)T)$ and the output y is measured at sampling instants kT ; that is, $y(k) := y(kT)$. The family of exact discrete-time models of (1)-(2) is:

$$\begin{aligned} x(k+1) &= F_T^e(x(k), u(k)) \\ y(k) &= h(x(k)), \end{aligned} \quad (3)$$

where $F_T^e(x, u)$ is the solution of (1) at time T starting at x and with the constant input u . This model is well-defined when the continuous model (1) does not exhibit finite-escape time. Even when there is finite escape time, (3) is valid on compact sets which can be rendered arbitrarily large by reducing T .

To compute (3) we need a closed-form solution for the initial value problem

$$\dot{x} = f(x, u(k)), \quad x_0 = x(k) \quad (4)$$

over one sampling interval $[kT, (k+1)T)$, which is impossible to obtain in general. It is realistic, however, to assume that a family of approximate discrete-time models is available:

$$\begin{aligned} x(k+1) &= F_T^a(x(k), u(k)) \\ y(k) &= h(x(k)). \end{aligned} \quad (5)$$

Throughout the paper we assume that this approximate model is *consistent* with the exact model, as defined in [6, 5] with motivation from numerical analysis literature [7]:

Definition 1 *The family $F_T^a(x, u)$ is said to be consistent with $F_T^e(x, u)$ if, for each compact set $\Omega \subset \mathbb{R}^n \times \mathbb{R}^m$, there exists a class- $\mathcal{K}$ function $\rho(\cdot)$ and a constant $T_0 > 0$ such that, for all $(x, u) \in \Omega$ and $T \in (0, T_0]$,*

$$|F_T^e(x, u) - F_T^a(x, u)| \leq T\rho(T). \quad (6)$$

This definition means that the difference between F_T^e and F_T^a converges to zero faster than linearly as $T \rightarrow 0$. It is not necessary to know the exact model $F_T^e(x, u)$ to verify this consistency property. Indeed, as shown in [6, Lemma 1], if $F_T^a(x, u)$ is consistent with the Euler approximation

$$F_T^{\text{Euler}}(x, u) := x + Tf(x, u), \quad (7)$$

then, under mild assumptions on $f(x, u)$, it is also consistent with the unknown exact model $F_T^e(x, u)$.

For the approximate model (5), we design a one-parameter family of observers (depending on T) of the form

$$\hat{x}(k+1) = F_T^a(\hat{x}(k), u(k)) + \ell_T(\hat{x}(k), u(k), y(k)), \quad (8)$$

and analyze under what conditions, and in what sense, this design guarantees convergence when applied to the exact model (3). Due to the mismatch of the exact and approximate models, the observer error system is now driven by the plant trajectories $x(t)$ and controls $u(t)$, which act as disturbance inputs. When these inputs are bounded, we want the observer to guarantee *semiglobal practical* asymptotic convergence. “Practical” means that the observer error converges to a small set, the size of which depends on T . Likewise, “semiglobal” means that its region of attraction can be rendered as large as desired, again, by reducing T :

Definition 2 *We say that (8) is a bounded input - semiglobal practical asymptotic convergence (BISPAC) observer for the system (3), if for any real numbers $D > d > 0$, and compact sets $\mathcal{X} \subset \mathbb{R}^n, \mathcal{U} \subset \mathbb{R}^m$, we can find a class- $\mathcal{KL}$ function $\beta(\cdot, \cdot)$ which is independent of T , and a $T^* > 0$ such that, for all $T \in (0, T^*]$,*

$$|\hat{x}(0) - x(0)| \leq D \text{ and } x(k) \in \mathcal{X}, u(k) \in \mathcal{U}, \forall k \geq 0,$$

imply

$$|\hat{x}(k) - x(k)| \leq \beta(|\hat{x}(0) - x(0)|, kT) + d. \quad (9)$$

As illustrated with examples in Section 3, unless the observer design for the approximate model meets several criteria, it may not ensure BISPAC for the exact model. We thus address the following problem:

If we design an observer (8) for the approximate discrete-time model (5), under what conditions will this observer be BISPAC for the exact model (3) ?

In the examples of Section 3, either the overshoot or the convergence rate of the approximate design is not uniform in the sampling period T . In Section 4 we derive a set of Lyapunov conditions that ensure a T -independent convergence estimate for the approximate design, and guarantee BISPAC for the exact model.

3 Examples of Non-Robust Designs

In the following examples, the observer designs are based on approximate models, and result in instability for the exact models:

Example 1 Consider the quadruple chain of integrators

$$\dot{x}_1 = x_2; \quad \dot{x}_2 = x_3; \quad \dot{x}_3 = x_4; \quad \dot{x}_4 = u, \quad (10)$$

where the output $y = x_1$ is sampled at times $t = kT, k \in \mathbb{Z}$. For the Euler approximate model

$$\begin{aligned} x(k+1) &= A_T^a x(k) + B_T^a u(k) \\ y(k) &= Cx(k), \end{aligned}$$

$$A_T^a = \begin{pmatrix} 1 & T & 0 & 0 \\ 0 & 1 & T & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad B_T^a = \begin{pmatrix} 0 \\ 0 \\ 0 \\ T \end{pmatrix},$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix},$$

we design a family of *dead-beat* observers

$$\hat{x}(k+1) = A_T^a \hat{x}(k) + B_T^a u(k) - L_T[y(k) - C\hat{x}(k)], \quad (11)$$

in which the injection matrix

$$L_T = \begin{pmatrix} -4 & -\frac{6}{T} & -\frac{4}{T^2} & -\frac{1}{T^3} \end{pmatrix}'$$

places the eigenvalues of $A_T^a + L_TC$ at the origin for all $T > 0$. However, for the exact model,

$$A_T^e = \begin{pmatrix} 1 & T & \frac{T^2}{2!} & \frac{T^3}{3!} \\ 0 & 1 & T & \frac{T^2}{2!} \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

the eigenvalues of $A_T^e + L_TC$ are, for all T ,

$$-0.5897 \pm 1.6865i; 0.5897 \pm 0.1334i,$$

in which the first two are outside the unit circle. This means that, in the observer error $e := \hat{x} - x$ dynamics

$$\begin{aligned} e(k+1) &= (A_T^a + L_TC)e(k) + (A_T^a - A_T^e)x(k) \\ &\quad + (B_T^a - B_T^e)u(k), \end{aligned} \quad (12)$$

the ℓ_∞ -gain from x to e is $\gamma_{xe} \geq 1$ for all values of T . (Otherwise, substituting $x = -e$ ($\gamma_{ex} = 1$) and $u = 0$ in (12), we would infer from the small-gain property $\gamma_{ex}\gamma_{xe} < 1$ that the resulting system $e(k+1) = (A_T^e + L_TC)e(k)$ is asymptotically stable, which contradicts our computation of eigenvalues above.) Because the ℓ_∞ -gain cannot be reduced arbitrarily by reducing T , we cannot assign d arbitrarily small in (9), even when $x(k)$ is bounded. Thus, the approximate design (11) does not guarantee BISPAC for the exact model.

The reason why this dead-beat design is non-robust for the exact model is because, when T is reduced, it attempts to achieve faster convergence at the cost of larger overshoots. The combination of this “peaking” in the transients, and the mismatch between the exact and approximate discrete-time models, leads to instability of $A_T^e + L_TC$. In the next example, the approximate design is non-peaking, but the convergence rate is slower for smaller T , which again leads to instability of $A_T^e + L_TC$.

Example 2 We consider the system:

$$\begin{aligned} \dot{x}_1 &= x_1 + x_2 \\ \dot{x}_2 &= x_2 + u \\ y &= x_1 \end{aligned}$$

and, again, design a Luenberger observer based on the Euler approximation

$$A_T^a = \begin{pmatrix} 1+T & T \\ 0 & 1+T \end{pmatrix}.$$

The injection matrix

$$L_T = \begin{pmatrix} -2T - 2T^3 \\ -T(1+T^2)^2 \end{pmatrix}$$

places both eigenvalues of $A_T^a + L_TC$ at $1 - T^3$, which is inside the unit circle. However, for the exact model

$$A_T^e = \begin{bmatrix} e^T & Te^T \\ 0 & e^T \end{bmatrix},$$

the eigenvalues of $A_T^e + L_TC$ are complex and located at

$$\{e^T - T(1+T^2)\} \pm \{T(1+T^2)\sqrt{e^T - 1}\}i,$$

which are outside the unit circle for all $T > 0$. Then, arguments similar to those at the end of Example 1 show that this design is not BISPAC for the exact model.

4 Main Result

We now derive conditions for the approximate design which guarantee that it achieves BISPAC for the exact model. From (3) and (8), the observer error

$$e := \hat{x} - x \quad (13)$$

satisfies

$$e(k+1) = F_T^a(\hat{x}(k), u(k)) + \ell_T(\hat{x}(k), u(k), y(k)) - F_T^e(x(k), u(k)). \quad (14)$$

Adding and subtracting $F_T^a(x(k), u(k))$, and defining

$$E_T(e, x, u) := F_T^a(\hat{x}, u) + \ell_T(\hat{x}, u, y) - F_T^a(x, u), \quad (15)$$

we rewrite (14) as

$$e(k+1) = E_T(e(k), x(k), u(k)) + F_T^a(x(k), u(k)) - F_T^e(x(k), u(k)), \quad (16)$$

where $E_T(e(k), x(k), u(k))$ represents the nominal observer error model of the approximate design, and $F_T^a(x(k), u(k)) - F_T^e(x(k), u(k))$ is the mismatch between the approximate and exact plant models. In Theorem 1 below, we guarantee robustness of the approximate design to this mismatch with the Lyapunov conditions (i)-(iii). In particular, conditions (ii) and (iii) guarantee, in contrast to the non-robust designs of the previous section, that we can find T -independent estimates for the overshoot and convergence rate in the approximate design.

Theorem 1 Suppose that the approximate model (5) is consistent with the exact model (3) as in Definition 1, and the observer design (8) for the approximate model has a Lyapunov function $V_T(e)$ satisfying the following conditions:

(i) For every compact set $\mathcal{Z} \subset \mathbb{R}^n$, there exist constants $T_1 > 0$ and $M > 0$, such that, for all $T \in (0, T_1]$ and $\xi, \zeta \in \mathcal{Z}$,

$$|V_T(\xi) - V_T(\zeta)| \leq M|\xi - \zeta|. \quad (17)$$

(ii) There exist class- $\mathcal{K}_\infty$ functions $\alpha_1(\cdot)$ and $\alpha_2(\cdot)$, and a $T_2 > 0$ such that, for all $e \in \mathbb{R}^n$ and $T \in (0, T_2]$,

$$\alpha_1(|e|) \leq V_T(e) \leq \alpha_2(|e|). \quad (18)$$

(iii) There exist class- $\mathcal{K}_\infty$ functions $\alpha_3(\cdot)$ and $\rho_0(\cdot)$, nondecreasing functions $\gamma_0(\cdot)$, $\gamma_1(\cdot)$, $\gamma_2(\cdot)$, and a $T_3 > 0$, such that, for all $T \in (0, T_3]$,

$$\frac{V_T(E_T(e, x, u)) - V_T(e)}{T} \leq -\alpha_3(|e|) + \rho_0(T)[\gamma_0(|e|) + \gamma_1(|x|) + \gamma_2(|u|)]. \quad (19)$$

Then, the observer (8) is BISPAC for the exact model (3), as in Definition 2. $\square$

Proof: We first obtain, from (16) and (19), the inequality

$$\begin{aligned} \frac{V_T(e(k+1)) - V_T(e(k))}{T} &\leq -\alpha_3(|e(k)|) \\ &\quad + \rho_0(T)[\gamma_0(|e(k)|) + \gamma_1(|x(k)|) + \gamma_2(|u(k)|)] \\ &\quad + \frac{V_T(\xi) - V_T(\zeta)}{T}, \end{aligned}$$

where

$$\xi := E_T(e(k), x(k), u(k)) + F_T^a(x(k), u(k)) - F_T^e(x(k), u(k)) \quad (20)$$

$$\zeta := E_T(e(k), x(k), u(k)). \quad (21)$$

We next conclude from (17) and the consistency property (6) that, for any given compact set of e , x and u , we can find a constant M and a class- $\mathcal{K}$ function $\rho(\cdot)$ such that

$$\begin{aligned} \frac{V_T(\xi) - V_T(\zeta)}{T} &\leq M \frac{|\xi - \zeta|}{T} = \\ M \frac{|F_T^a(x(k), u(k)) - F_T^e(x(k), u(k))|}{T} &\leq M\rho(T). \end{aligned} \quad (22)$$

Because, M and $\rho(T)$ are larger for larger compact sets, we can find nondecreasing functions $\delta_0(\cdot)$, $\delta_1(\cdot)$, $\delta_2(\cdot)$, and a class- $\mathcal{K}$ function $\tilde{\rho}(\cdot)$, such that

$$M\rho(T) \leq \tilde{\rho}(T)[\delta_0(|e|) + \delta_1(|x|) + \delta_2(|u|)]. \quad (23)$$

Finally, by substituting (22) and (23) in (20), we conclude that

$$\begin{aligned} \frac{V_T(e(k+1)) - V_T(e(k))}{T} &\leq -\alpha_3(|e(k)|) \\ &\quad + \tilde{\rho}_0(T)[\tilde{\gamma}_0(|e(k)|) + \tilde{\gamma}_1(|x(k)|) + \tilde{\gamma}_2(|u(k)|)], \end{aligned} \quad (24)$$

where

$$\begin{aligned} \tilde{\rho}_0(T) &:= \max\{\rho_0(T), \tilde{\rho}(T)\} \\ \tilde{\gamma}_i(\cdot) &:= \gamma_i(\cdot) + \delta_i(\cdot), \quad i = 0, 1, 2. \end{aligned}$$

Because $\alpha_3(\cdot)$ and $\tilde{\gamma}_0(\cdot)$ are nondecreasing functions, and $x(k)$ and $u(k)$ belong to compact sets $\mathcal{X}$ and $\mathcal{U}$, we can show that for any pair of real numbers $R > r > 0$, there exists $T^{**} > 0$, which is smaller than T_0 , T_1 , T_2 and T_3 , and satisfies

$$\begin{aligned} r \leq V_T(e(k)) \leq R &\Rightarrow \\ \tilde{\rho}_0(T^{**})[\tilde{\gamma}_0(|e(k)|) + \tilde{\gamma}_1(|x(k)|) + \tilde{\gamma}_2(|u(k)|)] &\leq \frac{1}{2}\alpha_3(|e(k)|). \end{aligned}$$

This means that, for all $T \in (0, T^{**}]$,

$$\begin{aligned} r \leq V_T(e(k)) \leq R &\Rightarrow \\ \frac{V_T(e(k+1)) - V_T(e(k))}{T} &\leq -\frac{1}{2}\alpha_3(|e(k)|) \\ &\leq -\frac{1}{2}\alpha_3(\alpha_2^{-1}(V_T(e(k)))). \end{aligned}$$

We further note from (24) and the boundedness of $x(k)$ and $u(k)$ that, if $V_T(e(k)) \leq r$, then $V_T(e(k+1)) \leq r + \epsilon(T)$, for some class- $\mathcal{K}$ function $\epsilon(\cdot)$. It then follows from the arguments in the proof of [6, Theorem 2] that there exists a class- $\mathcal{KL}$ function $\beta_1(\cdot, \cdot)$, such that

$$\begin{aligned} V_T(e(0)) \leq R &\Rightarrow \\ V_T(e(k)) &\leq \max\{\beta_1(V_T(e(0)), kT), r + \epsilon(T)\}, \end{aligned}$$

$\forall k \geq 0$. This implies, from (18), that

$$\begin{aligned} |e(0)| &\leq \alpha_2^{-1}(R) \Rightarrow \\ |e(k)| &\leq \beta(|e(0)|, kT) + \alpha_1^{-1}(r + \epsilon(T)), \end{aligned} \quad (25)$$

where

$$\beta(s, \tau) := \alpha_1^{-1}(\beta_1(\alpha_2(s), \tau)). \quad (26)$$

Thus, given any real numbers $D > d > 0$ as in Definition 2, we can select $R = \alpha_2(D)$, $r = \frac{1}{2}\alpha_1(d)$, and T^* small enough that $\epsilon(T^*) \leq \frac{1}{2}\alpha_1(d)$ and $T^* \leq T^{**}$, and verify from (25) that (8) is BISPAC for the exact model (3). $\square$

5 A Robust Design Example

Theorem 1 is also applicable to reduced-order observers, when e is interpreted as the difference between the unmeasured components of x , and their observer estimates. We now design such a reduced-order observer for the van der Pol oscillator

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 + x_2 - \frac{1}{3}x_2^3 \end{aligned} \quad (27)$$

from sampled measurements of the output $y = x_1$. To exploit the monotone increasing property of the nonlinearity x_2^3 as in the continuous-time design procedure of [1], we use the Euler approximation

$$\begin{aligned} x_1(k+1) &= x_1(k) + Tx_2(k) \\ x_2(k+1) &= x_2(k) \\ &\quad + T[-x_1(k) + x_2(k) - \frac{1}{3}x_2(k)^3] \end{aligned} \quad (28)$$

and define the new variable

$$\chi := x_2 + \ell y, \quad (29)$$

where the parameter ℓ is yet to be selected. From (28), this variable is governed by

$$\begin{aligned} \chi(k+1) &= \\ &(\ell - T)y(k) + [1 + (\ell + 1)T]x_2(k) - \frac{T}{3}x_2(k)^3 \\ &= (\ell - T)y(k) + [1 + (\ell + 1)T](\chi(k) - \ell y(k)) \\ &\quad - \frac{T}{3}(\chi(k) - \ell y(k))^3, \end{aligned} \quad (30)$$

which means that, with the observer

$$\begin{aligned} \hat{\chi}(k+1) &= (\ell - T)y(k) - \frac{T}{3}(\hat{\chi}(k) - \ell y(k))^3 \\ &\quad + [1 + (\ell + 1)T](\hat{\chi}(k) - \ell y(k)) \\ \hat{x}_2 &= \hat{\chi} - \ell y, \end{aligned} \quad \begin{aligned} (31) \\ (32) \end{aligned}$$

the error variable $e = \hat{x}_2 - x_2 = \hat{\chi} - \chi$ satisfies

$$\begin{aligned} e(k+1) &= [1 + (\ell + 1)T]e(k) \\ &\quad - \frac{T}{3}[(e(k) + \chi(k) - \ell y(k))^3 - (\chi(k) - \ell y(k))^3]. \end{aligned} \quad (33)$$

To design ℓ we note that the Lyapunov function

$$V_T(e) = \frac{1}{2}e^2 \quad (34)$$

satisfies assumptions (i) and (ii) of Theorem 1, and, along the trajectories of the approximate error model (33),

$$\begin{aligned} \frac{V_T(e(k+1)) - V_T(e(k))}{T} &= (1 + \ell)e(k)^2 \\ &\quad - \frac{e(k)}{3}[(e(k) + \chi(k) - \ell y(k))^3 - (\chi(k) - \ell y(k))^3] \\ &\quad + T\varphi(e(k), \chi(k), y(k)), \end{aligned} \quad (35)$$

where the function φ contains all terms that depend on T . Because the cubic nonlinearity is monotone increasing, the second term in the right-hand side of (35) satisfies

$$\begin{aligned} e(k) [(e(k) + \chi(k) - \ell y(k))^3 - (\chi(k) - \ell y(k))^3] \\ \geq 0. \end{aligned} \quad (36)$$

Next, with the choice $\ell = -2$, the first term is

$$(1 + \ell)e(k)^2 = -e(k)^2, \quad (37)$$

which is negative-definite. Finally, because φ is continuous, we can find nondecreasing functions $\gamma_i(\cdot)$, $i = 0, 1$, such that

$$\varphi(e, \chi, y) \leq \gamma_0(|e|) + \gamma_1(|x|). \quad (38)$$

Substituting (36), (37) and (38) in (35), we get

$$\frac{V_T(e(k+1)) - V_T(e(k))}{T} = -e(k)^2 + T[\gamma_0(|e(k)|) + \gamma_1(|x(k)|)]$$

as in assumption (iii) of Theorem 1. Because the trajectories of the van der Pol oscillator are bounded, we conclude that our observer (31)-(32) with $\ell = -2$ achieves semiglobal practical convergence for the unknown exact model. This is illustrated with numerical simulations in Figure 1 below, where the residual error between the solid trajectories and the dashed observer estimates becomes smaller as the sampling period T is reduced.

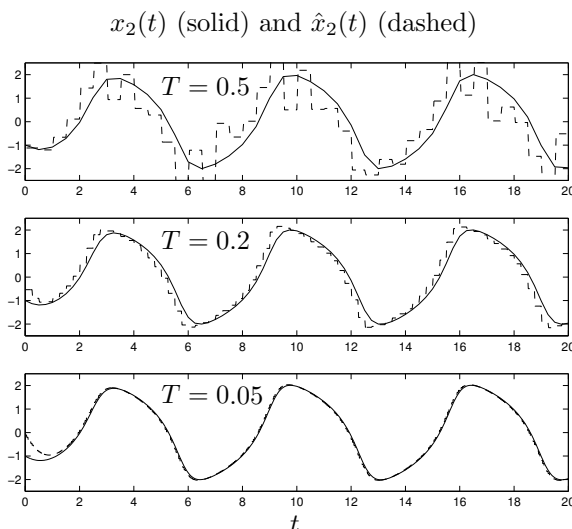


Figure 1: Simulation results for $x_2(t)$ (solid) from the van der Pol oscillator (27), and $\hat{x}_2(t)$ (dashed) from the observer (31)-(32). The sampling period is $T = 0.5$ sec. for the top plot, $T = 0.2$ sec. for the middle plot, and $T = 0.05$ sec. for the bottom plot.

6 Conclusions and Future Work

We have given a framework for semiglobal practical asymptotic observer design for sampled-data nonlinear systems via their approximate discrete-time models. This framework specifies conditions on the approximate model, continuous-time model, and the observer, guaranteeing that the same observer will perform well on the exact discrete-time model. We are currently pursuing several extensions of our results. Among

them is the possibility of refining the approximate models with a modelling parameter, independent of the sampling period T . This parameter can be thought of as the integration period in numerical schemes for ordinary differential equations, and would allow the designer to reduce the size of the residual observer error without modifying the sampling period. Another important research direction is to study discretization of continuous-time observers (emulation) for digital implementation. With Lyapunov conditions for the continuous-time design, similar to those derived in this paper for discrete-time design, and with an appropriate *consistency* property of the discretization, it would again be possible to achieve semiglobal practical observer convergence.

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