**Advances in techniques to formulate the watertight concept for cadastre**

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# Abstract

The world's rising urban density expansion has resulted in a proliferation of attempts to efficiently use space and a higher level of spatial complexity in metropolitan areas. 3D geospatial data models are increasingly being embraced to facilitate communicating the spatial dimensions of complex-built environments in different applications. For example, the use of 3D models in land administration systems has been recognised as a good approach for communicating the spatial complexity of legal spaces within multi-storey buildings.

The spatial extent of legal space, to which rights, restrictions and responsibilities relate in a 3D digital cadastre needs to be accurately defined and geometrically closed; watertight. Therefore, this study aims to address the challenges regarding checking the closure of diverse 3D legal spaces and engage several techniques to formulate the watertight concept for cadastre. The research's methodology is built on 3D polyhedral surface using a half-edge data structure. A primitive check is employed to assess the spatial consistency of lower-dimensional primitives of 3D objects. Subsequently, advanced checks ensure the closure of volumetric legal spaces represented by 2-manifold and non-2-manifold data models. The paper concludes that by adopting the proposed approaches, the internal spatial consistency of legal spaces in urban land administration will be certified.

**KEY WORDS: Validation, Internal spatial consistency, Watertight, Geometry, 2-manifold, 3D Digital Cadastre, Land Administration System**

# 1. Introduction

Realisation and interpretation of the real world in the past were only possible by projecting 3D features onto 2D maps. This simple approach is still practical in many applications. For example, land registration systems in Australia use 2D subdivision plans to delineate the spatial dimensions of legal and physical spaces in high-rise buildings (Atazadeh et al., 2017). However, communicating spatial and legal information during the lifecycle of a building is no longer restricted to 2D-based design and subdivision plans (Rajabifard et al., 2018). 3D geospatial data models now make it possible to realise and interpret 3D complex features in the built environment. For example, using 3D geospatial data models in the land administration system is now considered a potentially worthwhile approach for managing complex, vertically stratified ownership arrangements (Atazadeh et al., 2016). Leveraging 3D geospatial data models is a common strategy employed in the Architecture, Engineering and Construction (AEC) industry (Barzegar et al., 2019), principally to communicate complex architectural and structural information within multi-storey buildings. Visibility and shadow analyses, 3D graphics and CAD, positioning and navigation, real estate and facility management, 3D city models, estimation of noise propagation and energy modelling are some of the applications in which 3D models are used (Biljecki et al., 2015).

The internal spatial consistency principles assure 3D geospatial data is correctly and accurately defined based on the standards in each application (Asghari et al., 2019). The level of accuracy and details of 3D geospatial data differ from one application to another and are tailored to each application's requirements. For instance, if a 3D model is used for visualisation, the demands about that model's quality or validity can decrease markedly. In legally significant applications such as 2D/3D cadastre, however, a very accurate representation of 2D/3D parcels is critical, and the geometric object should represent a continuous representation of the real world (Herring, 2005). According to Victoria's Surveying Regulations 2015 (Victorian Consolidated Regulations, 2015), a 2D parcel must be closed. Similarly, 3D parcels (volumetric legal spaces) must be closed (watertight) to prevent any potential geometrical and topological errors in their definition emerging (Shojaei et al., 2017). Hence, checking the geometric closure of 3D geospatial data is one of the fundamental principles that must guarantee the internal spatial consistency of 3D cadastral objects (3D parcels).

In the design phase of a building subdivision process, surveyors use 2D design-based architectural maps to create subdivision plans. However, with the widespread use of 3D models such as BIM among architects (Shojaei et al., 2015), it is more likely that in future, surveyors utilise these 3D architectural models for cadastral purposes (Atazadeh, 2017). The range of designs as well as various types of legal interests defined in multi-storey buildings leads to a wide range of regular and irregular geometries (odd shapes) in digital built environments emerging. Checking the internal spatial consistency of these geometries can be done through a data validation process. The geometry closure checks in this research are classified into two levels; primitive and advanced checks and they are applied to 3D models designed for cadastral purposes as well as other applications. While some studies have explored this concept in GIS (Ledoux, 2013, Gröger and Plümer, 2011, Gröger and Plümer, 2012, Ledoux, 2018, Biljecki et al., 2016), what has been less studied is the comprehensive formulation for checking the closure of 3D objects considering the legal aspects in 3D cadastre. Figure 1 illustrates the structure of geometry closure formulation. 3D closure is often referred to as watertightness and a geometrically closed object is recognised as a watertight object. These terminologies – although not thought of as formal terms in mathematics and spatial and legal definitions - should nonetheless be used to leverage these terminologies in cadastre.

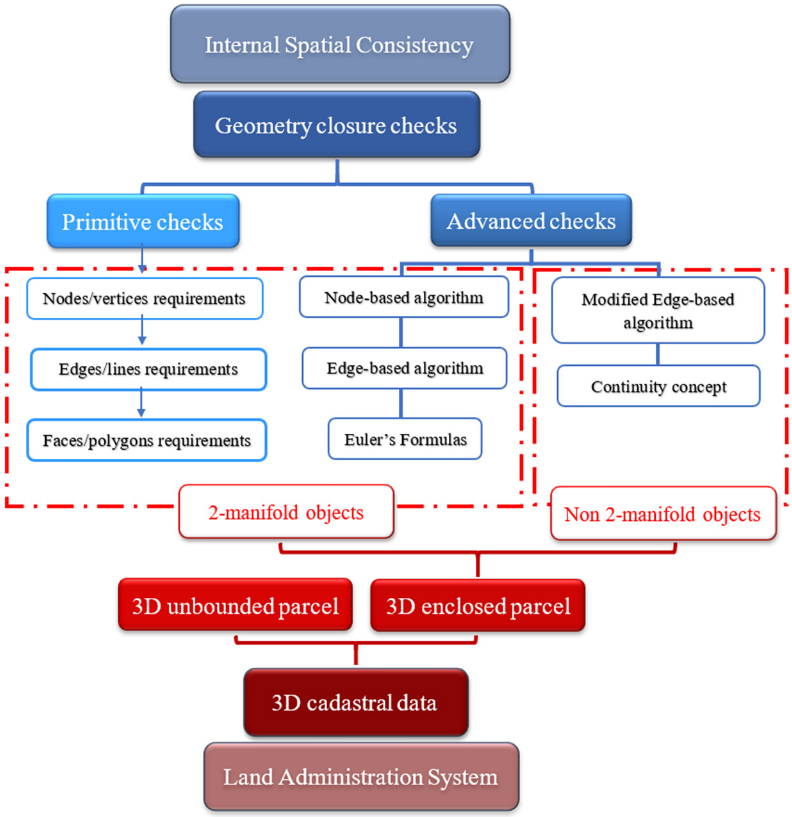



*Figure 1 A snapshot of 3D geometry closure (watertight concept) formulation structure*

This study aims to address the challenges involved in checking the closure of diverse 3D legal spaces represented by 3D geospatial models in cadastre. It also tries to consider several techniques to formulate a watertight concept for 3D digital cadastre. Consequently, the research methodology includes four main stages as follows:

1. Definitions and geometry representations of 3D cadastral objects: since watertightness is applied to 3D cadastral objects, prior to proposing the techniques and principles to formulate the watertight concept for cadastre, a formal definition of cadastral object should be given. Different geometry representations of 3D enclosed parcel also need to be investigated.
2. Proposing primitive checks: the basic requirements for checking the watertightness of cadastral objects are discussed. Several internal spatial consistency principles as primitive checks should be met before checking the geometry closure of legal spaces.
3. Proposing advanced checks: to check the watertightness of 3D enclosed parcels, proposed and discussed here are several algorithms including empirical and conceptual approaches for checking the watertightness of diverse legal spaces with different geometry representations.
4. Developing test scenarios: to test the algorithms, different scenarios were designed to showcase how the geometry closure of 3D digital data associated with various complex legal spaces of a real multi-storey building can be checked through primitive and advanced checks.

The next section reviews the importance of the watertight concept in different applications and an overview of essential concepts and terminologies to classify cadastral data definition and representations. Following the primitive checks for checking the spatial consistency of solid's lower dimensionality in Section 3, the advanced techniques for checking the closure of 3D enclosed parcels will be discussed, while various scenarios will be implemented and tested in Section 4. Section 5 examines the outputs of advanced checks, and each one's implication for the validation of cadastral objects (3D parcels). Finally, the paper concludes with the recommended approach and suggested directions for future research.

# 2. Background

## 2.1 The watertight concept in varied applications

The watertight concept is terminology used increasingly where 3D data serves different purposes. "Constructed or fitted so tightly as to be impervious to water" is a natural language definition for 'watertight' in an English dictionary. Watertight compartments were first coined by the Chinese in the shipbuilding industry to ensure that if one part of a ship is leaking, the ship itself will not sink (Needham, 1971). Thus, doors on a ship should be watertight to prevent flowing water inside the vessel. The watertight concept is now being employed in a wide range of applications such as construction and automotive industries, 3D printing, modelling, and electronic devices. For example, one of the main criteria for a printable 3D model is being watertight. In 3D modelling, a volumetric primitive in 3D space represented by solids must fulfil the requirements of watertightness to support various calculations and analysis concerning 3D objects.

The watertight concept is an essential concept in the applications of 3D geospatial data models and plays a decisive role in 3D digital cadastre. From the 3D cadastral perspective, the volumetric legal spaces in a 3D environment: firstly, need to be mutually exclusive; and secondly, need to exhaustively partition the extent of the domain, i.e. no gaps are allowed (Ying et al., 2015a). A 3D object without any gaps is considered watertight, which refers to a 3D object able to hold water with no holes, cracks, or missing faces (Karki et al., 2010). The watertight concept in 3D cadastre is a generalisation of a 2D misclosure concept. The 2D misclosure concept ensures that 2D parcels are geometrically closed. Similarly, the watertight concept precludes any errors in the 3D geometrical representation of legal spaces (Shojaei et al., 2017). Checking watertightness (i.e. checking 3D closure) is one of the fundamental principles to ensure that representations of 3D regions are internally consistent (Karki et al., 2010).

Watertight and watertightness are the terms referring to a 3D object, so before formally defining and formulating these terminologies in cadastre, a formal definition of 3D objects in cadastre is necessary. Generally, legal spaces in cadastre are variously defined and spatially represented in different ways. In the next section, firstly a formal definition for 3D cadastral objects is given. Then, different approaches for defining and geometrically representing the spatial extent of the legal spaces are explained to investigate how geometry closure applies to them.

## 2.2 Definitions and geometry representations of 3D cadastral objects

### 2.2.1 3D cadastral objects (Definition)

In this paper, we follow the definition of 3D cadastral objects as offered by Thompson and van Oosterom (2011b) and Kazar et al. (2008). Intuitively, the 3D cadastral objects (3D parcels) being defined here, is the minimal unit of space for the definition of property rights and is a generalisation of the known 2D land parcel. A 3D parcel is defined by the FIG joint commission 3 and 7 Working Group on 3D Cadastres (http://www.gdmc.nl/3DCadastres/) as the spatial unit against which one or more unique and homogeneous rights, restrictions and responsibilities are associated to the whole entity, as included in the Land Administration System (Thompson and van Oosterom, 2011b). The extent of a 3D parcel to which RRRs refer in 3D cadastre is defined in two different ways, these being “3D enclosed parcel” and “3D unbounded parcel” (Asghari et al., 2019). The term “3D enclosed parcel” refers to a volume of legal space defined by a set of faces defining the outer boundary and a number of closed sets of faces defining 3D cavities (Figure 2d). This term, “3D enclosed parcel”, also reminds us of a definition of 3D property given by Paulsson (2007): “the real property that is legally delimited both vertically and horizontally”. Each 3D enclosed parcel can be defined either as private ownership (lot) or as public ownership (common property). These are the two most common and widespread forms of ownership throughout the world, in countries such as Australia, Canada and South America and are respectively known as the condominium ownership type and the condominium user right type in some parts of Europe, for example Denmark, Germany and Sweden (Paulsson, 2011). It is worth noting here that the spatial extent of legal spaces either represents private ownership or public ownership. There is also a difference between the physical representation of objects and the legal space associated with this object (Stoter et al., 2013, Asghari et al., 2019). A 3D cadastral model typically incorporates two basic types of spatial objects: physical and legal (Atazadeh et al., 2016a). However, the watertight concept may not be applied to physical representation. For example, the physical extent of a balcony is not necessarily watertight, while its legal extent must be geometrically closed (Asghari et al., 2019).

The term “3D unbounded parcel” is used when the conventional 2D land parcel has no top and/or bottom defined (Figure 2a, b and c). A 3D parcel, when unbounded, is typically defined by vertical faces while missing top and/or bottom face(s) (Thompson and van Oosterom, 2011b). Many land registration systems use 3D unbounded parcels where depth and height limitations do not apply. This means that the unit owner owns the immediate airspace above their unit and the land down into the ground. However, in a densely built-up area such as city centres where depth and height limitations do apply, it is rare to see a 3D unbounded parcel. Since the 3D unbounded parcels are not represented by closed geometry, the concept of watertight does not apply to this type of 3D parcel.

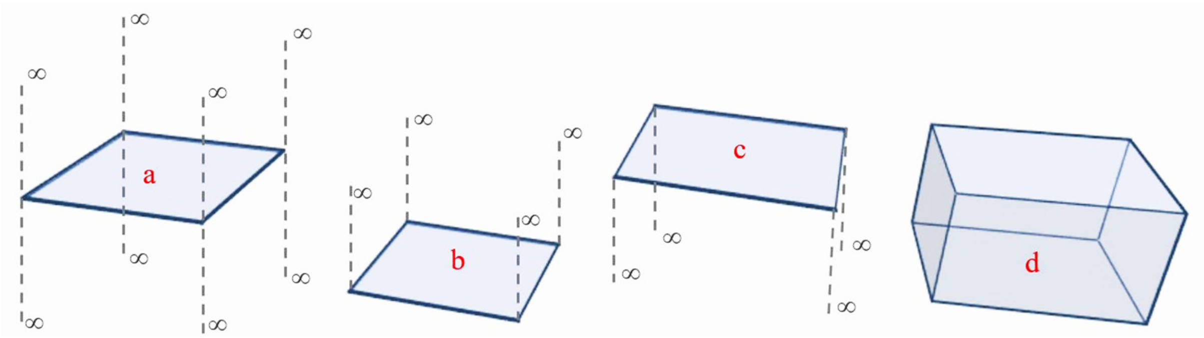



*Figure 2 Various 3D representations of legal spaces by 3D unbounded parcels (a, b and c) and 3D enclosed parcel (d) (Source: Asghari et al., 2019)*

### 2.2.2 3D cadastral objects (Geometry representations)

Many approaches and software dealing with 3D geometry draw on the concept that the boundary of a 3D space should be a 2-manifold one (Gröger and Plümer, 2011). They all follow the definition of solids provided by ISO190107. ISO190107 solids are defined as simple solids whose shells are not allowed to touch (i.e. they have to be 2-manifold). A 2-manifold is a topological space, where each point has a neighbourhood which is topologically equivalent to an open two-dimensional disk. The 2-manifold concept includes a set of conditions simplifying 3D modelling of the built environment. In a 2-manifold data model, each edge is shared by exactly two faces and is connected to precisely two vertices. Incident faces of each vertex should form one 'umbrella' and it is assumed that at least three edges are incidental to each vertex. This concept plays a critical role in geometrical and topological modelling (Mäntylä, 1988) which avoids self-intersections, holes or gaps in the surface of 3D objects.

The spatial extent of legal space, whether as private ownerships or as common properties, is prevalently defined by physical elements such as vertical walls and horizontal ceiling and floor in multi-storey buildings and therefore, they are mostly represented by 2-manifold geometries. However, due to the combination of ownership types happening together in a multi-storey building, sometimes, a portion of private ownership space (lot) is excluded by public ownership (common property space) which creates situations that cannot be handled by 2-manifold geometries (Figure 3). Conversely, a variety of building designs in architecture leads to constructing a wide range of odd objects with irregular geometries (Figure 4) in the building construction industry. Since the legal spaces are typically defined by referring to physical objects, this increases the possibility of experiencing new cadastral situations with irregular or non-2-manifold objects.

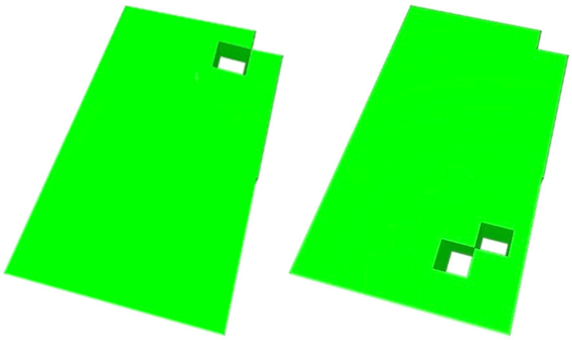


*Figure 3 Legal spaces represented as non-2-manifold objects*

As Van Oosterom (2013) argued, applying the watertight concept to 3D enclosed parcels presents essential challenges. The involved geometries may be non-2-manifold (self-touching in edge or vertex), and some legislative frameworks allow for non-linear (curved) primitives in cadastres. Several researchers including Kazar et al. (2008), Verbree and Si (2008), Thompson and van Oosterom (2011b), Ying et al. (2015a) and Ying et al. (2015b) observed that the ISO190107 solids are not sufficient for 3D cadastral applications and to comply with appropriate and complete 3D cadastre, adequate 3D geometries are required (van Oosterom et al., 2018). Therefore, to support a more extensive range of scenarios seen in the real cadastre, in this research both 2-manifold and non-2-manifold characteristics of cadastral objects are considered. Furthermore, empirical and conceptual methods are proposed to investigate their geometry closure.

A possible approach to dealing with non-manifold objects in GIS and many other fields is decomposition. Decomposition is a method for decomposing non-manifold objects into simpler parts (manifold objects), splitting an object at those elements (vertices, edges, faces) where singularities occur (Rossignac and Cardoze, 1999, Desaulniers and Stewart, 1992, Guéziec et

al., 1998, Guéziec et al., 1999). However, utilising this approach in cadastre has triggered considerable debate on two different aspects: legal (law) and geometrical (modelling and implementation). The latter is a widespread controversy in GIS and other fields as well.
From a modelling and implementation standpoint decomposition introduces some challenges. A decomposition should not cause arbitrary "cuts" through manifold parts. Under these assumptions, a decomposition into manifold components is possible, but generally only for 2D complexes (De Floriani et al., 2003). However, for (n)D objects where $n \geq 3$, a decomposition into manifold components may need to introduce arbitrary cuts through the object. The arbitrary cuts may change the properties of a 3D object by changing the underlying geometries. Although some studies addressed this issue, it is not always convenient, and even not always possible, to remove all singularities from components. In addition, for polyhedral solids, each non-manifold edge of N with 2k incident faces will be replicated k times in any manifold model M of that family. Furthermore, some non-manifold vertices of N must also be replicated in M, possibly several times (Rossignac and Cardoze, 1999). This occupies too much redundant storage space. Storage requirement can arise as a crucial problem in models in which topological data is more dominant than geometric data, such as tessellated or mesh models (Lee and Lee, 2001). On the other hand, since the integrity of cadastral objects must be preserved after decomposition, then consolidating the decomposed objects and finding a method that preserves the integrity well can be another challenge.
From the law perspective, it is still debated whether a licensed surveyor or a cadastral expert in land registry is allowed to split a subdivision plan (in 2D or 3D) into different parts. The rationale behind this is that a 3D enclosed parcel (legal space) with a homogeneous RRRs is the most atomic component of a cadastral system which might not be able to be decomposed (Kalantari et al., 2008). Hence, two approaches are proposed to investigate the geometry closure of non-2-manifold objects as one united spatial object.
It is worth stating here that not all the non-2-manifold scenarios are valid geometries for cadastre. Some pre-conditions are required for non-2-manifold objects for them to be acceptable as cadastral objects:

- **Pre-condition 1:** The interior of a solid representing the legal space should be connected:

Following the definition of a 3D parcel (Kazar et al., 2008, Thompson and van Oosterom, 2011b), a shell (the boundary of a solid) can self-touch (i.e. a solid can be non-2-manifold), if the interior of the solid stays connected and makes a connected legal space (Figure 4).

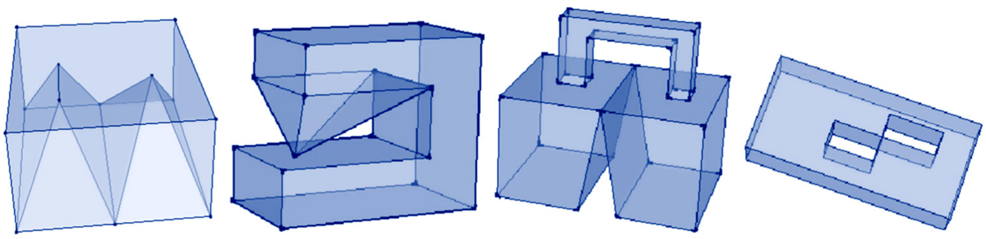


*Figure 4 3D enclosed parcels with connected legal space. Source: Ying et al. (2015a)*

- **Pre-condition 1:** The orientation of a solid should be valid:

The inside and outside of a non-2-manifold object should be differentiable (i.e. surfaces should be oriented to decide between the interior and exterior of a solid). Otherwise, it would not be

permitted as a cadastral object. The issue regarding a dangling face due to its arbitrary orientation can also be aligned with an orientation problem. That is why the dangling faces are not allowed in cadastre (Figure 15c).
In addition to the non-2-manifold geometries for the representation of 3D parcels, there could be a need for further specific geometries: curved surfaces (boundaries) according to the architectural design. Curved surfaces are architecturally designed and constructed in the real world and can be seen in cadastral plans (Karki et al., 2013). The curves can be represented approximately by merging straight segments. In a 3D polyhedral model, the surface of a 3D model is tessellated with simple geometric shapes (e.g. triangles, quadrangles, or more complex polygons). However, if a 3D parcel has curved surfaces in its boundary, the plan must carry that as a definition. Therefore, it is not acceptable to approximate curves with straight lines and planar faces (Thompson et al., 2016). A plan must contain intelligence that the boundary is curved with the definition of that curvature. As opposed to this approximate method, there are several methods that can represent freeform curves and surfaces. Bézier, B-spline and NURBS methods are among the most commonly used in practice. Although Bézier and B-splines are widely used representations, the currently most popular and powerful method for representing freeform curves and surfaces is the NURBS method. The advantage of this method is that more types of 3D shapes can be registered where the law and regulations do not enforce restrictions on the geometry types such as Queensland (FIG, 2017). 3D objects can be generated by accurate Non-Uniform Rational Basis Spline (NURBS) surfaces. NURBS geometries are a mathematically precise representation of curves and surfaces and as such produce a more accurate and smoother 3D model than the polyhedral surface modelling method. As illustrated in Figure 5, the shape of the surface is defined by several control points and the degree of that surface in each one of the two directions (u- and v-directions). NURBS have been included in many geometric standards, and are supported by many mainstream CAD systems (Zlatanova et al., 2006). Zlatanova et al. (2006) showed that NURBS is a very general representation of freeform shapes and demonstrated that appropriate data types for efficient management of freeform surfaces could be created at the DBMS level.

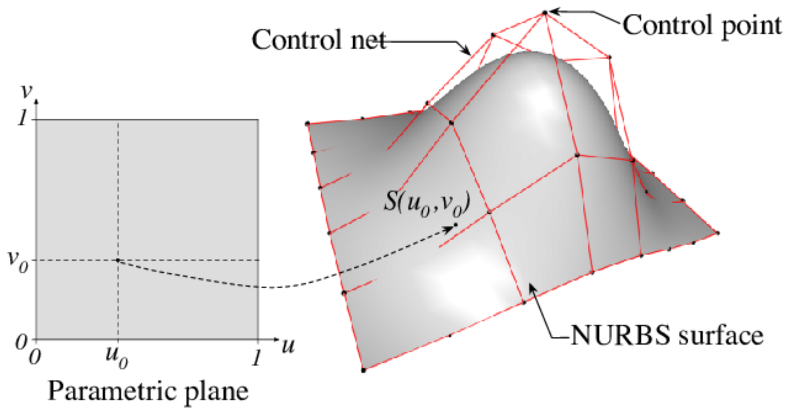



*Figure 5 NURBS concept. Source: Estratat and La Greca (2006)*

## 3. Primitive checks

There is a taxonomy of data models that represents 3D objects existing in the built environment and there is a dichotomy between geometric and topological models (Koussa and Koehl, 2009). Solid modelling is the most advanced method of geometric modelling in three dimensions. The representation and modelling of 3D enclosed parcels is handled with geometrical representation as “solid” in computer environment. A solid model is a complete representation of a polyhedron able to support a wide range of calculations and analysis of 3D cadastral data (Ying et al., 2011). The schemes for solid modelling include CSG (Construction Solid Geometry) and B-rep (Boundary Representation) (Guo et al., 2012). Both CSG and B-rep are relevant solid models for 3D cadastral applications (Atazadeh, 2017). Although the construction of 3D solids could be rather complicated in B-rep, additional adjacency information and spatial analysis compared to CSG could be well-supported.

A B-rep solid is constructed hierarchically by well-known topological primitives including node (0D), edge (1D), face (2D) to form a body (3D), which represent a spatial occupation of the 3D enclosed parcel (Ying et al., 2011). Terms such as "solid", "body" and "polyhedron" are interchangeably used in the literature. However, we mostly use the term "solid" in this paper. The topological information per se is not enough to describe a 3D object, and geometrical information must also be associated with each topological primitive. The geometric coordinates are only recorded in node, and other topological primitives (Edge, Face, and body) describe the relationships by using the references. The significant difference between validating single geometries and validating topological structures is the relationship between the 3D primitives (Brugman et al., 2011). Below, Table 1 presents the topological and geometrical definitions for lower-dimensional primitives.

| **Topological definitions** | **Geometrical definitions** |
|---|---|
| **Node:** The end of an edge is represented by a node. | **Point:** A vertex is the point where two straight rays or line segment meet. A point is a geometric object that can be attributed by a location (coordinates). |
| **Edge:** A collection of half-edges connecting the same pair of nodes consist an edge.<br>**Half-edge:** A half-edge is a straight-line segment connecting two separate nodes and participating in the definition of exactly one face. | **Line:** A line is defined as a line of points that extends infinitely in two directions. It has one dimension, length. |
| **Face:** A face is bounded by a simple cycle of half-edges defining the outer boundary. Conventionally, the half-edges are placed in anti-clockwise direction, viewed from outside. | **Polygon:** A polygon is a surface area with a planar linear ring as outer boundary and one or several inner rings. |

*Table 1 The topological and geometrical definitions for lower-dimensional primitives of a 3D solid*

Some steps need to be taken before checking the closure of 3D enclosed parcels. The process of checking the closure of 3D enclosed parcels can start with assessing the internal spatial consistency of its lower dimensionality primitives. This process also considers the cadastral examination requirements. This method's logic is a 3D enclosed parcel needing to meet some spatial and cadastral requirements regarding its lower dimensionality primitives. The objective is to make sure it is ready for advanced geometry closure check.

## 3.1 Nodes/vertices requirements

There is a distinction between a "node" and a "vertex" in mathematics. Vertices exist to describe where specific points are, while nodes exist to describe the topological structure of a feature. Node is the lowest dimension of a 3D object represented by topological data structure.

The methods for checking the validity of nodes or vertices rely on the standards and regulations applied to the model generation method. The consistency of a model can be affected by the accuracy of coordinates determining the position of a point in a reference system. A 3D geospatial data model is usually defined in a local or global 3D reference coordinate system. However, we might need a consistent coordinate system to connect different cadastral datasets together.

From a cadastral perspective, surveying and mapping include some challenges regarding the measurement. For example, measuring the bearing and distance between two features that are too close to each other is challenging as it may cause spurious results. Therefore, a threshold (an ε value) can be assigned by the user for different scenarios. For example, no two vertices must be closer than ε apart (Figure 6a) and no vertex is within ε of a face unless it is part of the definition of that face (Figure 6b) (Thompson and Van Oosterom, 2011a). This threshold can also be specified where features can be snapped. The threshold defines how close features must be before snapping takes place. Snapping allows two features to move so that the coordinates coincide. Snapping is a useful way of cleaning up and aligning features. Several geometry errors, for example spikes, self-intersections, and duplicate vertices can be handled through snapping.

On the other hand, from the design and modelling standpoint, two vertices can be too closely positioned with a very minimum distance. Arithmetic with floating-point types is not exact, due to roundoff error. This might lead to many complications and problems when implementing geometric algorithms and cause the algorithm to make an incorrect decision and fail to behave correctly (Adams, 2013). For example, due to roundoff error, an algorithm may incorrectly decide that a 3D object is watertight when it is literally not. In such cases, special techniques (e.g. arbitrary-precision arithmetic) must be employed to ensure the algorithms as well as the codes behave correctly. The calculation can be done as if no roundoff error occurred. The disadvantage of using exact predicates (relative to inexact ones) is greater than computational cost. For this reason, an exact calculation should only be used when required.

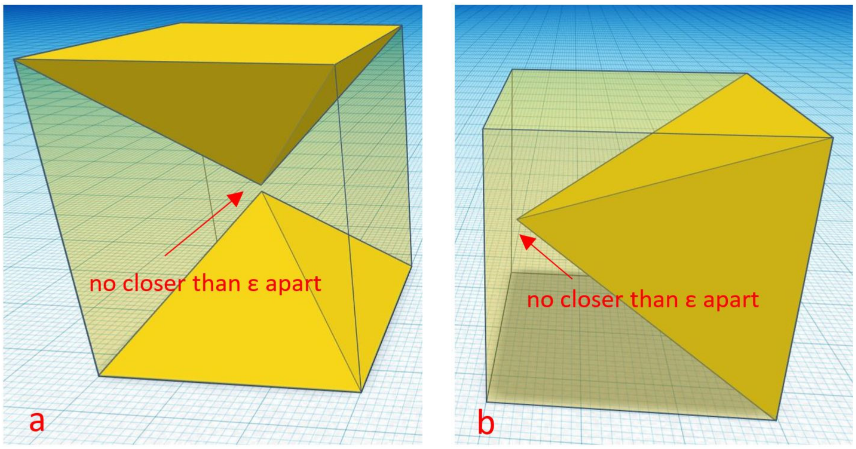



*Figure 6 Measurement challenges of concise lengths. Source: Thompson and Van Oosterom (2011a)*

## 3.2 Edges/boundaries requirements

In a 3D model represented by 2-manifold data structure, each edge is created by precisely two vertices, and each face consists of at least three edges. Hence, after ensuring the validity of vertices that make edges, it is now essential to check the validity of edges which make faces.

Each half-edge consists of two vertices, but the direction of the half-edge is important. The orientation of each half-edge should follow the orientation of adjacent faces (Figure 8b). Two edges may touch each other only at their shared start and end vertices, so other vertices of intersection or touching are not allowed (i.e. no self-intersection). If we consider edge as segments of a boundary of a parcel, a rare situation may arise in which two edges intersect with each other out of the start and end vertices. If this is the case, the measurement for an intersected vertex is required (coordinates or bearing and distance) (Figure 7).

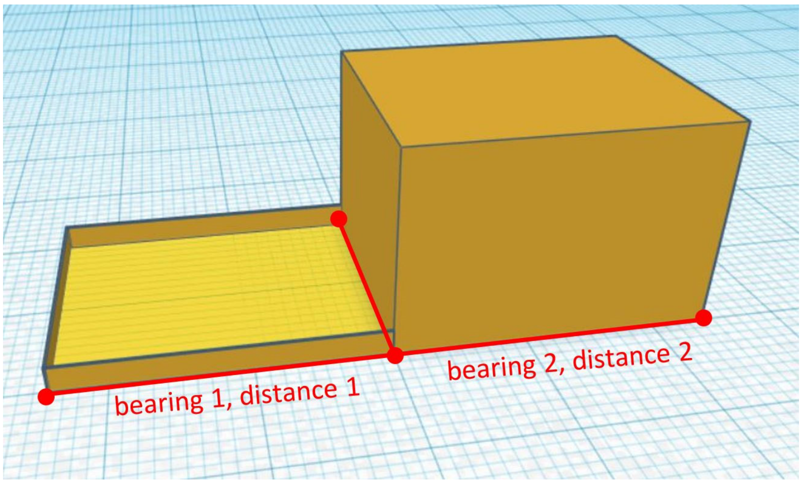



*Figure 7 Mesurments for intersection out of the start and end point*

Curves in cadastral objects also need to be managed in a cadastral and mapping system. A curved boundary may be used where it coincides with a curved feature such as a canal wall. Different methods can be used to define the curved boundaries. One approximate method is dividing any curved boundary into several straight lines. More straight lines improve the accuracy of this method. However, increasing number of straight segments may also be limited to a specific number due to data size limit. This method is not an accurate one for defining the curves and is only an approximate of curves. However, in the concept of cadastre, a precise definition of legal extent is essential to arrive at unambiguous and definitive data. Hence, more accurate methods are necessary to define the curve lines and surfaces.

Another approach for modelling curve boundaries is defining curvature by appropriate implicit functions. Where a curved boundary is observed during the surveying and mapping phases, the boundary must be defined by sufficient measurements. For example, a circular boundary must be defined by:

- tangent points of the curve;
- bearing and distance of the chord between the tangent points;
- radius; and
- arc length.

The parametric functions such as NURBS have several advantages over implicit functions. The key benefit of NURBS is that it is not an approximation of a smooth shape created by small straight-line segments and compared to implicit functions, they have more degrees of freedom to model shapes. The amount of information required for a NURBS representation of a piece of geometry is much smaller than the amount of information required by common faceted approximations. NURBS curves are defined by four parameters: control points, knot vector, degree, and weights. The accuracy of a curve can be controlled by changing these four parameters.

The validation approach for NURBS is derived from a mathematical description of the curves and surfaces. The process checks all required parameters such as coordinates for control points, degree and weight values and knot vector. The relationships between parameters should meet the following conditions (Zlatanova et al., 2006):

- Degree >1
- Number of control points >3
- Degree = number of knots – number of control points - 1
- Each weight value > 0
- Number of weight values = number of control points

## 3.3 Faces/polygons requirements

There are some requirements for checking the spatial consistency of faces/polygons of a cadastral data. These are explained in more detail below.

**The planarity of the faces** is necessary for defining an unambiguous extent of the legal space. A face can only be nonplanar if the number of its edges exceeds three. Since every three vertices create a plane (i.e. face), the method checks if each remaining vertex is coplanar with the first three vertices. We have implemented this method in our programs to check the planarity of faces which consist of a solid. Planarity of faces can also be checked by estimating the distance of each vertex to a fitted plane interpolated by a least-squares method. The face cannot be flat if one of the distances is greater than a threshold. According to Wagner et al. (2015), a tolerance ε of 0.01m can be used as deviation for a vertex from the plane.

**The vertices duplication** also checks the sequence of vertices making a face. A face must not have duplicate vertices, except at the start and end vertices. So, it is important to check that the first and last vertex of a surface are identical. This is a vertex-based topological check for the closure of a face. At least 3 vertices are needed to represent an area and the area must be greater than a threshold. Therefore, collinearity also needs to be checked for all vertices.

**The orientation** defines the outside and inside of a 3D object. A closed 3D object may be faulty with flipped faces (Figure 8a). The order of the vertices defining a face defines its orientation by the right-hand rule. The outer face is usually defined counter-clockwise. If the orientations of all neighbouring faces conform to each other, a closed shell separates outside from inside (Figure 8b). We have improved the primitive checks by adding an orientation check to our program. This analysis checks if the 3D object is correctly outward oriented.

A 3D object can topologically be a valid and watertight object while geometrically, suffer from **self-intersection** error. We have improved the checks by adding a self-intersection analysis to our program, mainly to observe if there is any self-intersection in the dataset. The implemented method also gives us the number of pairs of faces intersected.

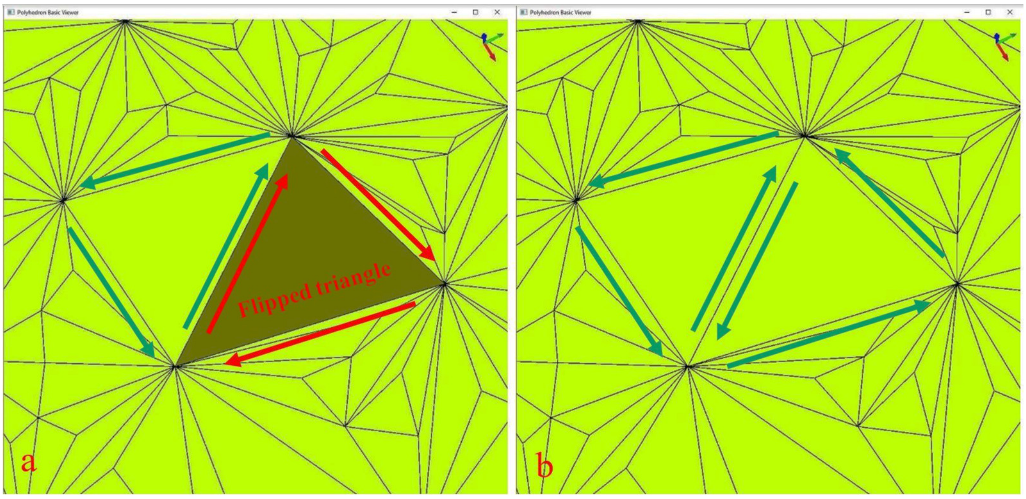



*Figure 8 Orientation status; a) Invalid oriented triangle and half edges, b) Valid orientation of triangles and half edges*

# 4. Advanced geometry closure checks

A wide range of cadastral objects can be modelled by solids (polyhedron in another terminology). A solid as the basis for 3D geometry is delimited by its outer shell, and one or more inner shells represent the cavity inside the solid. A solid with one or more than one cavity makes a polyhedron non-simple. A tetrahedron consisting of four triangles (polygons) is the simplest 3D solid. Hence, a 3D enclosed parcel defined by valid solids has at least 4 polygons situated in different planes (Alam et al., 2014). This basic condition is necessary but not sufficient for the closure of a 3D cadastral object. It is generally assumed that solids are 2-manifold objects. This assumption helps us to simplify the real world to model, visualise, and validate in 3D. However, the checks of geometry closure need to consider both 2-manifold and non-2-manifold objects if more scenarios in the real world are to be supported.

## 4.1 Checking the closure of 2-manifold cadastral data

### 4.1.1 Node-based algorithm

In a 3D solid with the half-edge data structure, the degree of a vertex $v_1$, corresponds to the number of edges connected to that vertex. The in-degree of vertex $v_1$ refers to the number of edges incident to that vertex. Based on a node-based algorithm, a 2-manifold object will be watertight if the number of faces which are incidental to a vertex equals to the in-degree value for that vertex.

A boundary representation (B) of a 3D solid (S) can be defined as a triple $B_S = (V_S, E_S, F_S)$, in which S refers to the 3D solid and $V_S$, $E_S$, and $F_S$ denote the set of vertices, edges and faces of S, respectively. In Figure 9, the number of vertices, edges and faces of the square pyramid are given as num (Vs) = 5, num (Es) = 8 and num (Fs) = 5, respectively. The in-degree of vertex $v_1$, $\deg_{in}(v_1)$, corresponds to 4, so the number of faces that must be incident to this vertex must also be identical to 4 to make a watertight object.

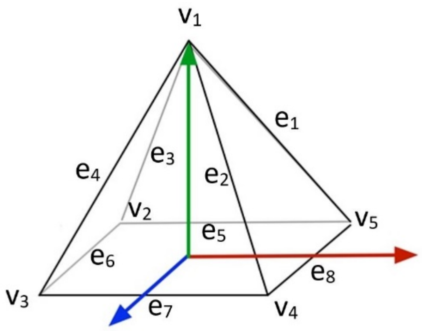



*Figure 9 A square pyramid using B-rep representation method*

### 4.1.2 Edge-based algorithm

In a 3D polyhedral model using a half-edge data structure, the surface of the 3D object is tessellated with simple geometric shapes (e.g. triangles, quadrangles, or more complex polygons). In the half-edge data structure, the vertices, edges (or half-edge) and faces of a 3D solid should establish good topological connectivity. Vertices represent points in space. Edges are straight line segments between two endpoints. Faces are defined by the circular sequence of half-edges along their boundary. The half-edges along the boundary of a hole are called border half-edges and have no incident face. An edge is a border edge if one of its half-edges is a border half-edge. A 3D object is watertight (i.e. closed) if it contains no border half-edges. As can be seen in Figure 10, a hole consists of a loop of border half-edges (represented by yellow lines) belonging to one face, as opposed to non-border half-edges shared by two faces.

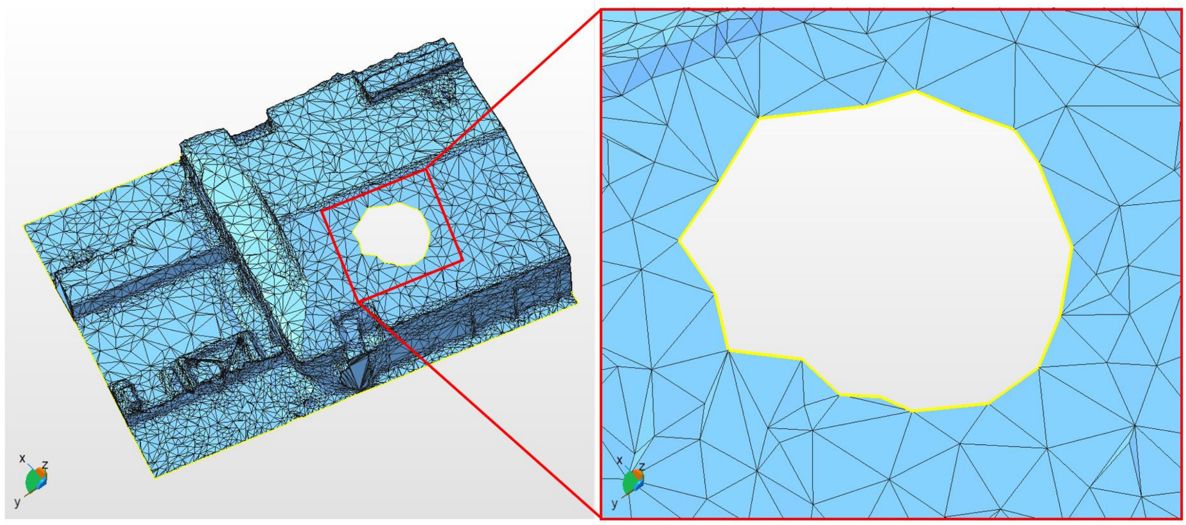


*Figure 10 Detected hole on a triangulated mesh model*

### 4.1.3 Euler's Formulas

The closure of a 3D enclosed parcel represented by 2-manifold geometry can also be checked by a consistency check using Euler's formula (Ericson, 2004). It is known that orientable 2-manifold models are closed under Euler operations (Mäntylä, 1988). The closure of a 3D enclosed parcel represented by simple polyhedron (i.e. solid without inner shell(s)) follows the conventional Euler formula and its derived formulas. The Euler formula defines the following quantitative relationship among the number of vertices (V), faces (F) and edges (E) in B-rep solid models.

$$V + F - E = 0$$

The relationships between primitives of a closed simple polyhedron made by triangles, quads and arbitrary convex faces follows the formulas summarised in Table 2.

| **For a closed simple polyhedron consisting of a number of …** | **The number of vertices, edges and faces relate as …** |
|---|---|
| Triangles (T) | $2E = 3T$, $T = 2V - 4$, $E = 3V - 6$ |
| Quads (Q) | $2E = 4Q$, $Q = V - 2$, $E = 2V - 4$ |
| Triangles (T) and Quads (Q) | $2E = 3T + 4Q$, $T = 2V - 2Q - 4$ |
| Arbitrary convex faces (F) | $2E \geq 3F$, $F \leq 2V-4$, $E \leq 3V-6$ |

*Table 2 Euler's derived formula for a closed mesh (Ericson, 2004).*

The Eular-Poincaré formula can help us to manage deformed models (non-simple polyhedron). It defines the following quantitative relationship among the number of vertices (V), faces (F), edges (E), inner loops on the faces (L), shells (S), and genus of shells (G) in B-rep solid models represented by non-simple polyhedrons.

$$V + F - E - L - 2(S - G) = 0$$

Figure 11 illustrates two solids with one genus in each (i.e. two non-simple polyhedrons). However, the one on the left-hand side has one inner loop on its front face which makes it non-watertight. Therefore, we have

$$V + F - E - L - 2(S - G) = 32 + 29 - 62 - 0 - 2(1 - 1) = -1$$

The value of the Euler-Poincaré formula is zero for the watertight one (Figure 11b) as shown below.

$$V + F - E - L - 2(S - G) = 32 + 32 - 64 - 0 - 2(1 - 1) = 0$$

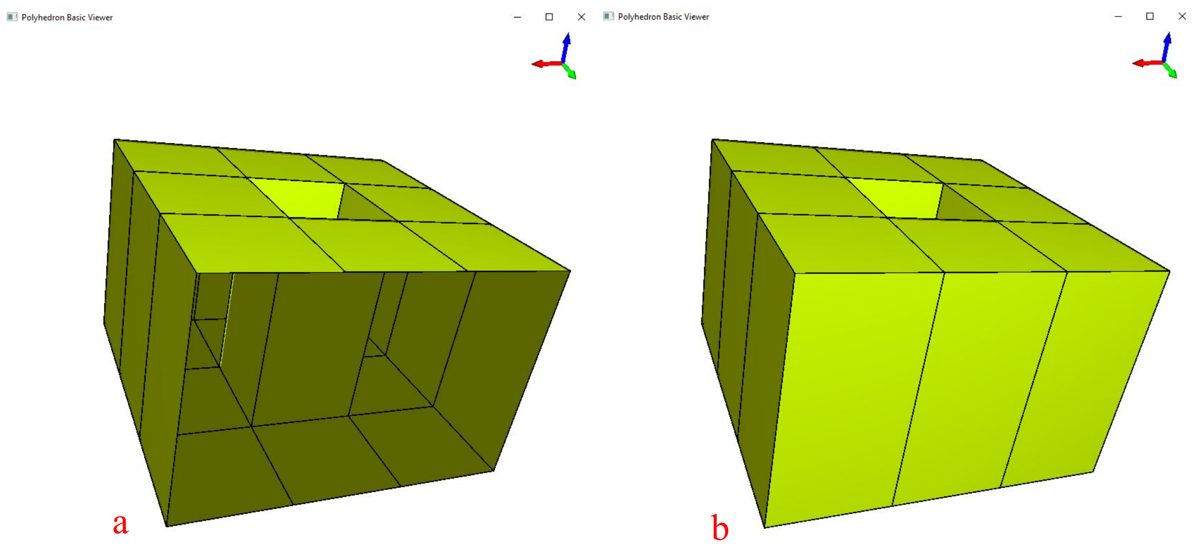



*Figure 11 a) A non-watertight non-simple polyhedron b) A watertight non-simple polyhedron*

### 4.1.4 Implementation and testing different scenarios

In this section, three methods for checking the closure of 3D cadastral data represented by 3D polyhedral models are implemented with C++ programming language using Computational Geometrical Algorithms Library (CGAL). The Computational Geometry Algorithms Library (CGAL) is a very powerful open-source C++ library for geometric computation. The data structure beneath is a half-edge data structure, which restricts the class of representable surfaces to orientable 2-manifolds (Kettner, 2020). A 3D polyhedral model has been known as a potential data model for spatially representation of cadastral data (Stoter, 2004, Arens et al., 2005, Zlatanova, 2000). A real cadastral dataset serves as our case study (Figure 12a). Since watertightness is not necessary for the physical object, only legal spaces of this 4-level building were extracted and tested through the algorithms. This dataset includes 50 legal spaces representing both private ownership and common property (Figure 12b).

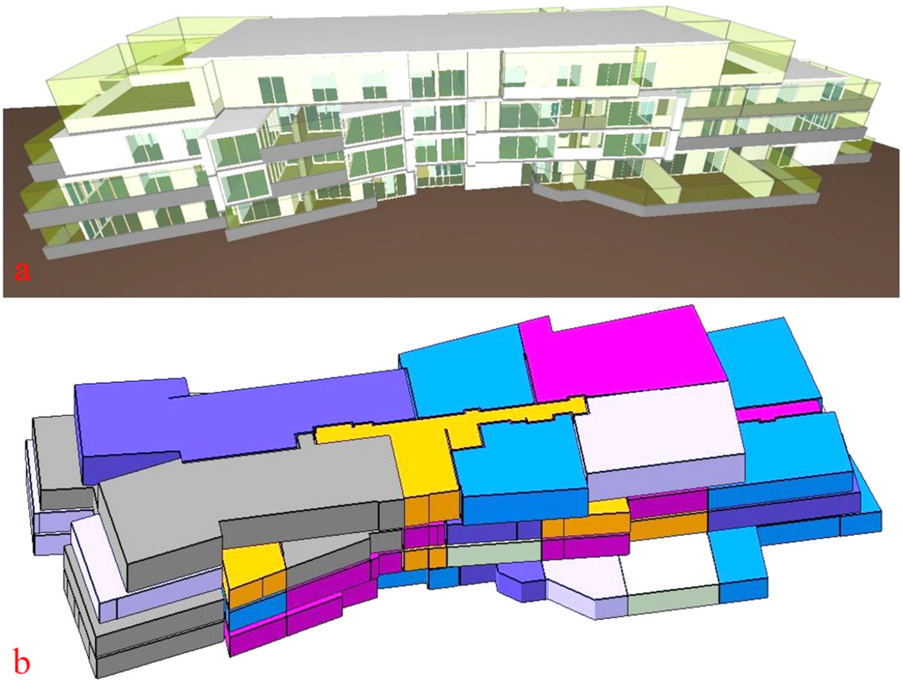
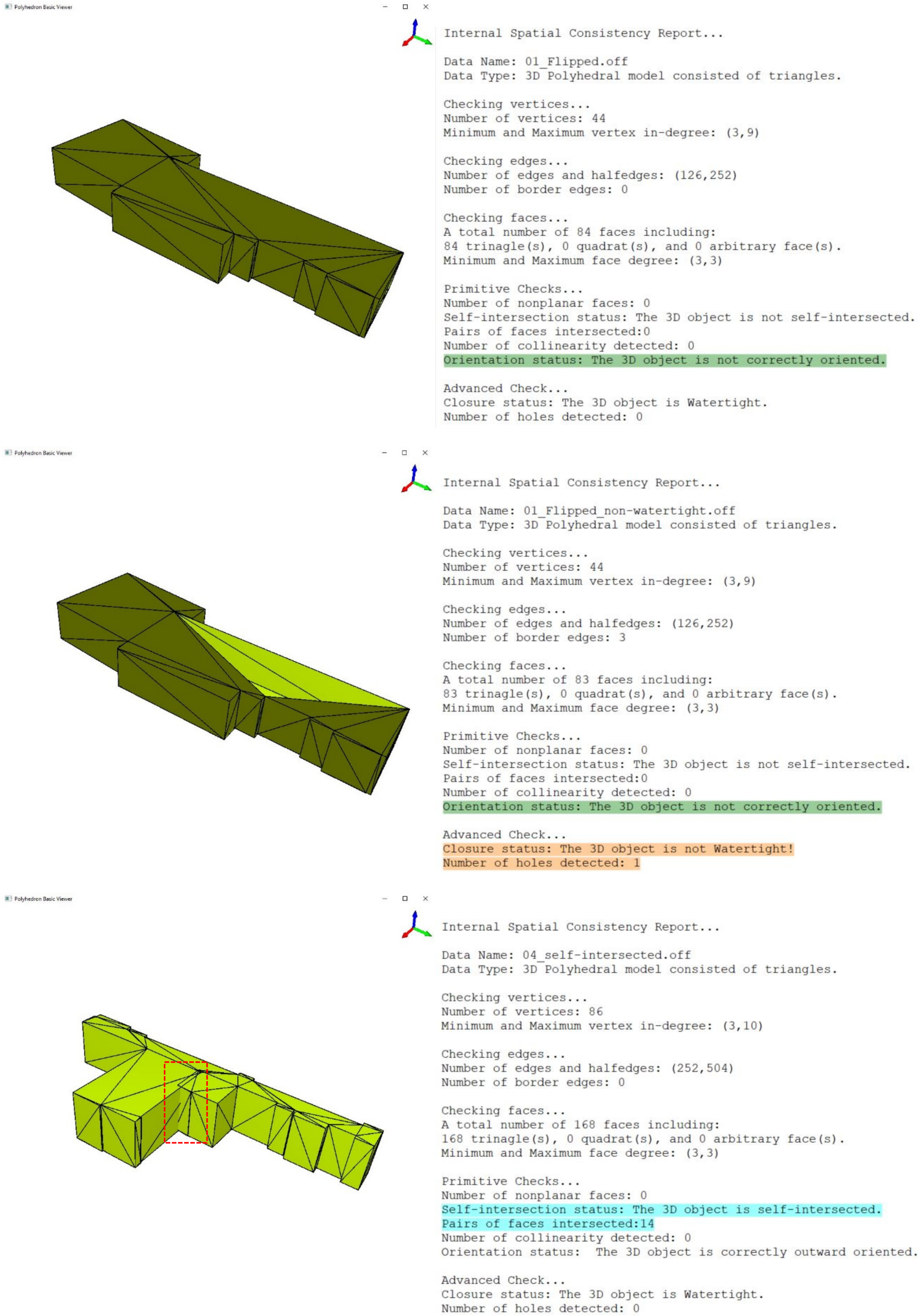


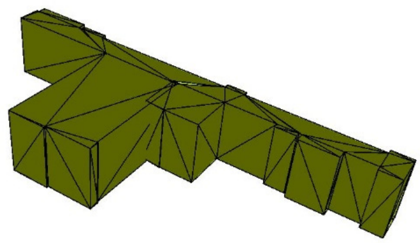

*Figure 12 A cadastral dataset; a) Legal and physical model. Source: Atazadeh et al. (2016b), b) A purely legal model*

To check the functionality and validity of the algorithms, we went through different scenarios by manipulating a set of 3D objects in the dataset with one or more errors. The primitive and advanced checks have been implemented and tested within each scenario to examine the requirements of spatial consistency. We have examined all the legal spaces in our dataset to check whether the algorithm is able to detect the errors. The program implemented in C++ using CGAL provides some general information about each object and investigates the primitive checks such as planarity, self-intersection, orientation, collinearity, and point duplications. The program then uses one of three proposed methods in previous sections to investigate the advanced geometry closure check and generates some information about the number of border half-edges, holes and pairs of faces intersected in the whole dataset. The results of primitive and advanced checks as well as the general information for each object are all saved into a text file. The developed codes and the dataset used for testing different scenarios are available at https://github.com/aliiasgharii/GeometryClosureChecks.

The internal spatial consistency reports including primitive and advanced checks show that the developed program can correctly detect the errors within each dataset (Figure 13).

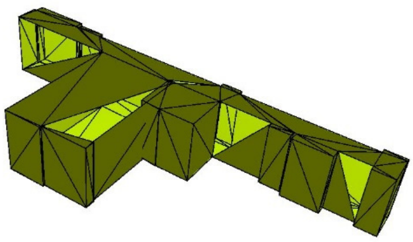


Polyhedron Basic Viewer



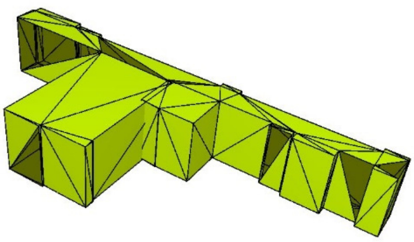
```
Internal Spatial Consistency Report...

Data Name: 04_self-intersected_Flipped.off
Data Type: 3D Polyhedral model consisted of triangles.

Checking vertices...
Number of vertices: 86
Minimum and Maximum vertex in-degree: (3,10)

Checking edges...
Number of edges and halfedges: (252,504)
Number of border edges: 0

Checking faces...
A total number of 168 faces including:
168 trinagle(s), 0 quadrat(s), and 0 arbitrary face(s).
Minimum and Maximum face degree: (3,3)

Primitive Checks...
Number of nonplanar faces: 0
Self-intersection status: The 3D object is self-intersected.
Pairs of faces intersected:14
Number of collinearity detected: 0
Orientation status: The 3D object is not correctly oriented.

Advanced Check...
Closure status: The 3D object is Watertight.
Number of holes detected: 0
```

Polyhedron Basic Viewer



```
Internal Spatial Consistency Report...

Data Name: 4_self-intersected_Flipped_non-watertight.off
Data Type: 3D Polyhedral model consisted of triangles.

Checking vertices...
Number of vertices: 86
Minimum and Maximum vertex in-degree: (3,10)

Checking edges...
Number of edges and halfedges: (252,504)
Number of border edges: 12

Checking faces...
A total number of 164 faces including:
164 trinagle(s), 0 quadrat(s), and 0 arbitrary face(s).
Minimum and Maximum face degree: (3,3)

Primitive Checks...
Number of nonplanar faces: 0
Self-intersection status: The 3D object is self-intersected.
Pairs of faces intersected:11
Number of collinearity detected: 0
Orientation status: The 3D object is not correctly oriented.

Advanced Check...
Closure status: The 3D object is not Watertight!
Number of holes detected: 4
```

Polyhedron Basic Viewer



```
Internal Spatial Consistency Report...

Data Name: 04_self-intersected_non-watertight.off
Data Type: 3D Polyhedral model consisted of triangles.

Checking vertices...
Number of vertices: 86
Minimum and Maximum vertex in-degree: (3,10)

Checking edges...
Number of edges and halfedges: (252,504)
Number of border edges: 21

Checking faces...
A total number of 161 faces including:
161 trinagle(s), 0 quadrat(s), and 0 arbitrary face(s).
Minimum and Maximum face degree: (3,3)

Primitive Checks...
Number of nonplanar faces: 0
Self-intersection status: The 3D object is self-intersected.
Pairs of faces intersected:13
Number of collinearity detected: 0
Orientation status:  The 3D object is correctly outward oriented.

Advanced Check...
Closure status: The 3D object is not Watertight!
Number of holes detected: 7
```

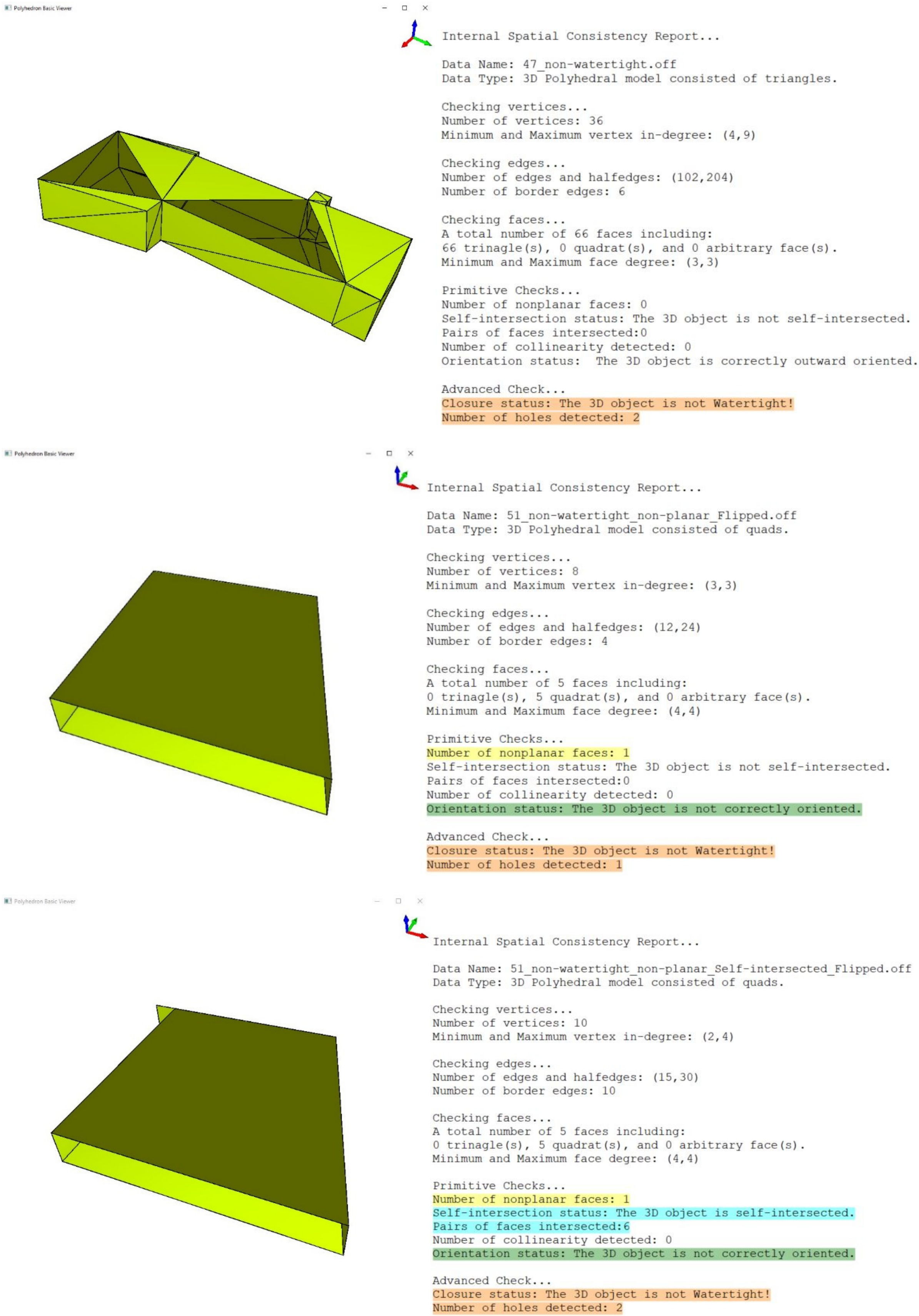



*Figure 13 Primitive and advanced checks verification by testing different scenarios*

Euler’s formula, and the formulas mentioned in Table 2 can all be used if a cadastral data is represented by a simple polyhedron model. A simple polyhedron model does not have any genus and is equivalent to a solid without any inner shell(s). Our dataset, however, contains legal spaces represented by non-simple polyhedrons (i.e. solids with one or more than one genus). One of these objects represented in Figure 14a was separately tested by each advanced method to evaluate the closure status. This examination revealed that Euler’s formula can only serve as necessary but not sufficient validity conditions since a deformed model with genus could meet the restrictions of the formulas. As can be seen in Figure 14, in contrast with Euler’s formula (Figure 14b), two other methods (Node and Edge-based) (Figure 14c) could correctly detect the closure status of this legal space represented by non-simple polyhedron.

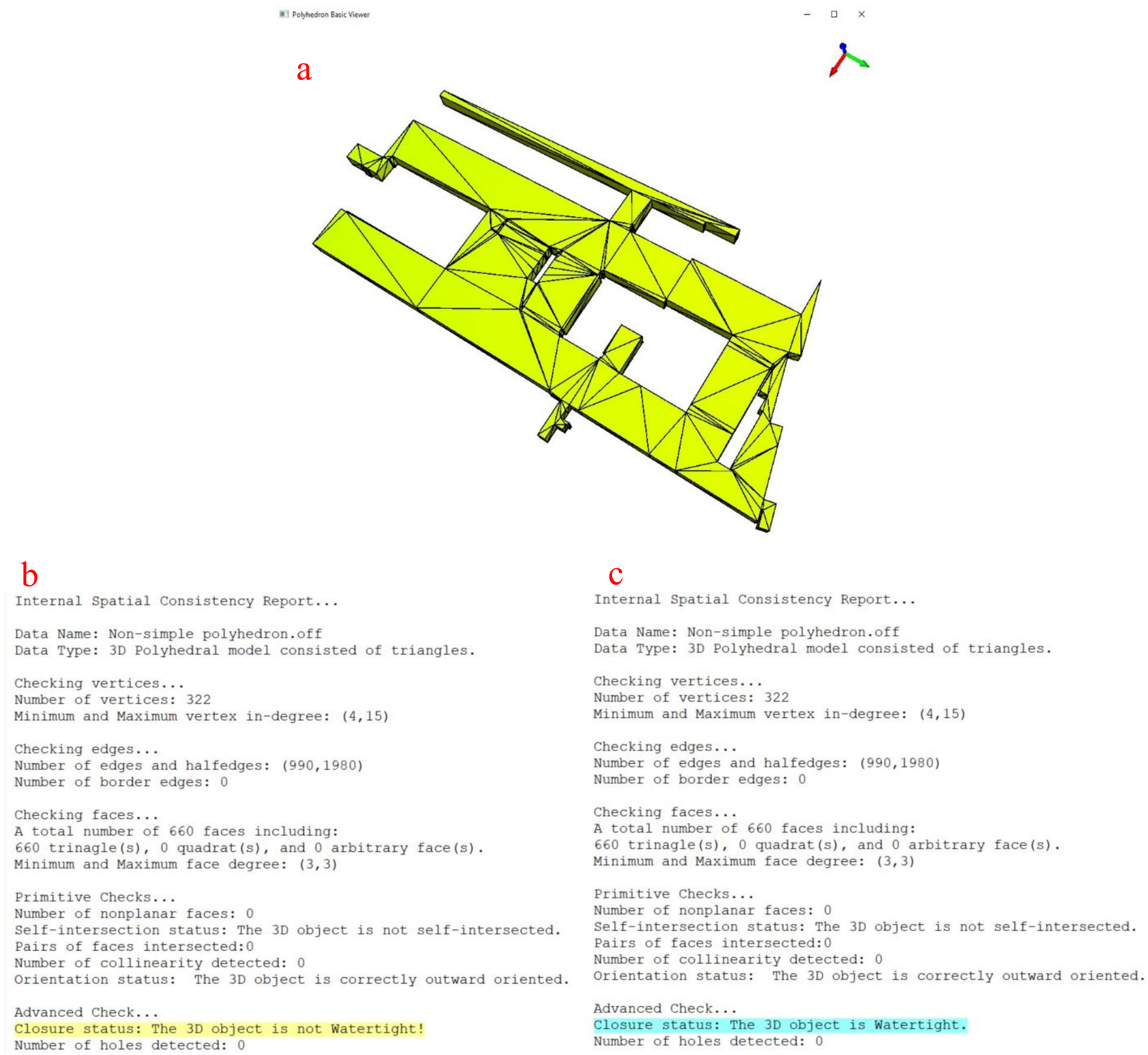



*Figure 14 a) A watertight Legal space represented by non-simple polyhedron, b) Closure status by Euler’s Formula, c) Closure status by Node-based and Edge-based algorithm*

From the implementation perspective, it would be hard to find the number of genera in a 3D object. Therefore, checking the closure of these kinds of cadastral objects including non-simple polyhedron requires node-based and edge-based algorithms. Euler’s formula, however, can still help to check the watertightness where the only accessible information of 3D object is quantitative information of vertices, edges and faces and there is no other topological and geometrical information.

## 4.2 Checking the closure of non-2-manifold objects

The non-manifold object can be considered as one single object. Following this notion, new algorithms and methods need to be conceptualised and developed to assist with checking the geometry closure of non-2-manifold cadastral data. In this section, two new algorithms are discussed.

### 4.2.1 Modified edge-based algorithm

A modified version of the edge-based algorithm can be employed to check the closure of valid non-2-manifold cadastral data. This method assures that every edge of each face incident to an even number of faces. Figure 15 illustrates a series of non-2-manifold objects. The red highlighted edge in Figure 15a is connected to 3 faces; F1, F2 and F3 which is not an even number. As can be seen, this object has missed one of its faces and is not closed. In contrast to this case, 4 faces are incidental to the highlighted edge in Figure 15b which makes this object watertight. This scenario is also visible in all valid non-2-manifold cases shown in Figure 3 and Figure 4. Each edge in each case has met the even number of incident faces which shows this method correctly detects the geometry closure status of non-2-manifold cadastral objects. The topological structure is used to check if the number of faces incident to each edge is even. In such non-2-manifold objects, the two conditions for valid cadastral object are met. However, the 3D object in Figure 15c does not provide an orientable object because the dangling face can have arbitrary orientations. Consequently, this object is not a valid cadastral object unless the dangling part is removed.

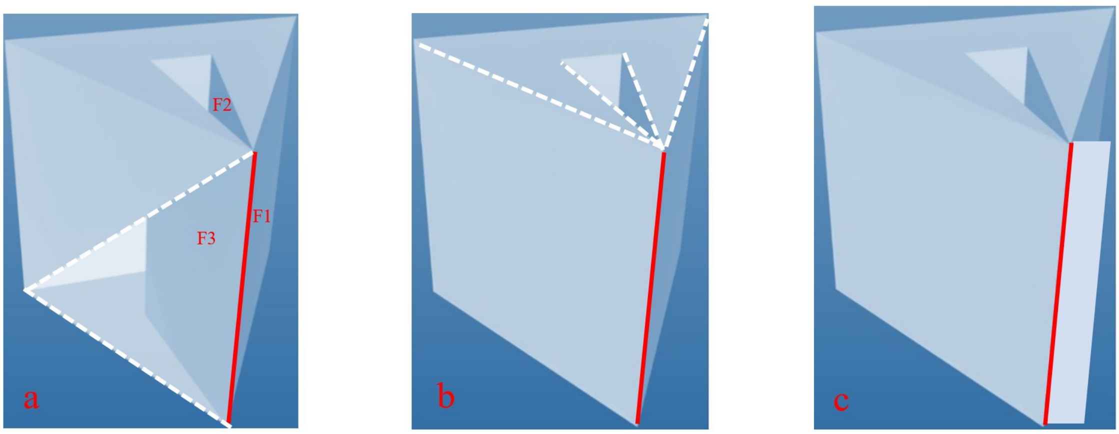



*Figure 15 a) An invalid non-2-manifold cadastral object missing one of its faces b) A valid non-2-manifold cadastral object c) An invalid non-2-manifold cadastral object with one dangling face*

### 4.2.2 Continuity concept

The Continuity concept was introduced for the first time during the 1960s by Hammer and Tyson (1963). A 3D watertight object is enclosed by a continuous 2D surface in every point of its surface. This 2D surface can be defined by a mathematical equation of two variables. For example, an equation of two variables can define a spherical surface as follows (Figure 16).

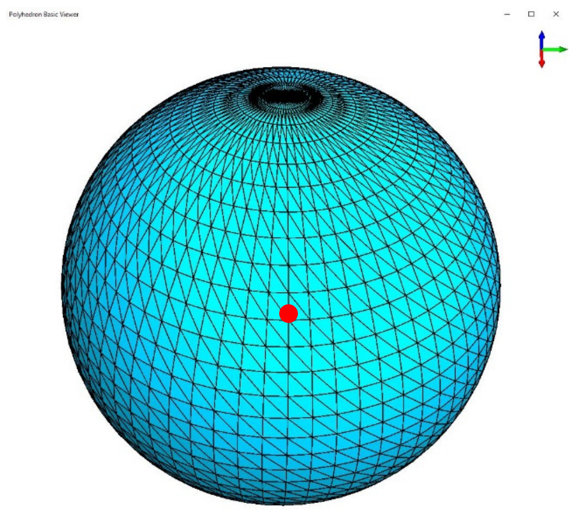



*Figure 16 A continuous 3D surface with the equation of* $z = \sqrt{1 - x^2 - y^2}$

Stated informally, a function of a single variable is continuous if its graph is an unbroken curve without jumps or holes. To extend this idea to functions of two variables, imagine that the graph of z = f (x, y) is moulded from a thin sheet of clay that has been hollowed or pinched into peaks and valleys. We will regard "f" as being continuous if the clay surface has no tears or holes. The precise definition of continuity at a point for functions of two variables is similar to that for functions of one variable. The limit and the value of the function should be the same at the point. The formal definition of Continuity (two variables) is defined as follows:

Let $f: D \subseteq R^2 \rightarrow R$ be defined for all (x, y) in some open region $D \subseteq R^2$ which contains a fixed ordered pair $(x_0, y_0)$. We will say that the function f (x, y) is continuous at the point $(x_0, y_0)$ if and only if given any real number $\varepsilon > 0$, there is a real number $\delta > 0$ (usually depends on ε) such that $|f(x, y) - f(x_0, y_0)| < \varepsilon$ whenever $(x, y) \in D$ and $\sqrt{(x - x_0)^2 + (y - y_0)^2} < \delta$.

and we will write:

$$\lim_{(x,y)\rightarrow(x0,y0)} f(x, y) = f(x0, y0)$$

to express the fact that $f(x_0, y_0)$ is the limit of f (x, y) as (x, y) approaches $(x_0, y_0)$ and the f (x, y) is continuous at $(x_0, y_0)$. In addition, if a function is continuous at each point in its domain D, then we say that the function is continuous on D. If f is continuous at every point in the xy-plane, then we say that f is continuous everywhere. If we consider this concept for every single 3D object that can be represented by a two-variable function, then this method can help us to check the closure of any non-2-manifold 3D cadastral objects even with dangling faces.

# 5. Discussion

The quality of cadastral data that underpins a land administration system guarantees the integrity of that system. There are many principles that can help to check the internal spatial consistency of 3D cadastral data (Asghari et al., 2019). The watertight concept is one of these principles which examines whether a 3D object is geometrically closed. Checking the closure of 3D objects needs some primitive principles. These principles examine the minimum requirements for spatial consistency of cadastral data including basic closure principles. However, the closure of a 3D enclosed parcel is not guaranteed by only applying the primitive checks. Primitive checks can either be a process during the data creation (Ying et al., 2015b) and modelling or can be applied as a process after data creation. The primitive checks are necessary but not sufficient conditions to ensure closure of a 3D enclosed parcel. However, advanced checks can guarantee the closure of a 3D enclosed parcel, but only when the primitive checks are passed. The primitive checks investigate the minimum requirements of a solid's lower dimensionality. A combination of primitive and advanced checks was implemented in C++ using computational geometry algorithms, the aim being to ensure internal spatial consistency of 3D cadastral data.

The advanced geometry closure checks provide a wide range of algorithms to check the closure of 3D objects. However, some of these algorithms are only applicable on a specific data model, 2-manifold model, for instance. The Euler's formula consistency check can be helpful to evaluate the watertightness where the only accessible information is quantitative information concerning the solid's lower dimensionality (i.e. vertices, edges, and faces) and there is no other topological and geometrical information. Unfortunately, this method is restricted to checking the closure of only solids without an inner shell (simple polyhedron). For cadastral data with 2-manifold data structure, the most certain algorithms are node-based and edge-based ones. As opposed to Euler's formula, these approaches analyse the geometrical and topological data structure of 3D objects and can check the closure of legal spaces represented by non-simple polyhedron. These algorithms ensure that a 3D parcel represented by 2-manifold data structure is geometrically closed or watertight. However, these approaches are only appropriate for cadastral objects with 2-manifold data structure while non-2-manifold objects exist in the real world.

The challenges and issues regarding the decomposition of non-2-manifold objects into simpler parts were discussed from the law as well as the modelling and implementation perspectives. Therefore, considering two conditions for non-2-manifold objects, connectivity and orientability, we proposed two new approaches for geometry closure check. These two methods do consider the non-2-manifold object as a unique and single one. The first method is a modified version of the edge-based algorithm that uses a trusted rule for checking the closure of non-2-manifold cadastral data. The investigation of this method on cases represented in Figure 3, Figure 4, and Figure 15b confirmed that this method can be trusted. The second method is a conceptual approach based on the concept of continuity in multivariate equations which is a well-known mathematical concept. This approach does also consider the curved surfaces which is frequently seen in cadastral data. The hypothetical prerequisite for applying this method is to make sure each 3D object can be defined by a two-variable equation.

# 6. Conclusion

The precise definition of 3D spatial extent of legal space to which rights, restrictions and responsibilities refer in land administration systems is very essential. The internal spatial consistency principles provide appropriate criteria and requirements for a better and more precise definition. Among the principles, checking the closure of various cadastral objects plays a decisive role in cadastre. This paper aimed to address the potential challenges around formulating the watertight concept in 3D digital cadastre and employed several techniques to examine the closure of diverse 3D cadastral objects. In cadastre, the extent of legal space in 3D is guaranteed only if accurately defined by a 3D closed, watertight geometry. In this study, advanced techniques were proposed to formulate the watertight concept for 3D digital cadastre. The primitive checks including checking the planarity, collinearity, self-intersection, orientation, and point duplication are pre-requirements before applying the advanced checks for checking the closure of 3D parcels. The geometry of 3D objects in built environments and specifically in cadastre are very diverse and classifying them into 2-manifold and non-2-manifold is an approach developed to manage this diversity. The advanced techniques in this paper support both 2-manifold and non-2-manifold models with irregular geometries in the real world.

This study tried to formulate an important concept and vital principle of the internal spatial consistency of 3D cadastral data. However, there are still steps to be taken when implementing some of the techniques recommended in this paper to check the closure of non-2-manifold cadastral data. The methods for applying the concept of continuity and NURBS will be developed in an appropriate environment for cadastral purposes. The next direction of this research is to investigate how the concepts developed in this paper, can be applied to a new cadastral data model having semantical information, BIM and IFC data model, for instance. The authors also consider that the decomposition approach is a potential method for dealing with non-2-manifold cadastral object, and it will be the subject of their future research work.

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## Conflicts of Interest

The authors declare no conflict of interest.

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