Author/s:
Nesic, D

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A simple controllability test for generalized Hammerstein models

Dragan Nešić[1]
Center for Systems Engineering and Applied Mechanics (CESAME),
Batiment Euler, Avenue G. Lemaitre, 4,
1348 LOUVAIN-LA-NEUVE, Belgium
`nesic@auto.ucl.ac.be`

Abstract

Simple necessary and sufficient conditions for dead-beat and complete controllability for a class of discrete-time generalized Hammerstein systems are derived. Due to the mild nonlinear structure of the considered systems, only linear algebra is used in the controllability test. A similar result is then proved for continuous-time generalized Hammerstein systems.

1 Introduction

Controllability is one of the fundamental notions in control theory which shows our ability to achieve a desired operating regime of a system in finite time by means of actuators. Hence, it is one of the most important properties of a controlled system, since it uncovers some fundamental limitations to the systems' performance. Therefore, controllability tests are important tools in the analysis of control systems. Linear controllability problem has been throughly studied and understood [5]. On the other hand, there is still no universal and unified method for the investigation of nonlinear systems controllability.

The purpose of this paper is to present several simple controllability tests for a class of generalized Hammerstein systems. Generalized Hammerstein systems may arise from identification techniques of the so called block oriented models [3, 4]. They represent a subclass of the class of input-output polynomial systems, very often referred to as NARMAX (nonlinear ARMAX) [3, 4]. Generalized Hammerstein systems can be regarded as a parallel connection of a simple Hammerstein system whose input nonlinearity is quadratic and a linear system, see Figure 1. Examples of this class of systems can be found in [6], where the model of a cement mill is identified as a generalized Hammerstein system, and in [1], where the model for the cooling water circulation of a thermal power plant was also identified in this form.

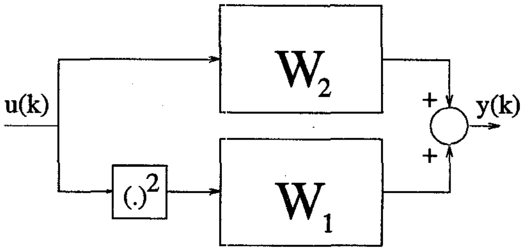

Figure 1: Block diagram of a generalized Hammerstein system

In this paper, we present necessary and sufficient conditions for dead-beat/complete controllability of discrete and continuous-time generalized Hammerstein systems. We exploit the result on controllability of discrete and continuous-time linear systems with positive controls [2, 10] in the proof of our main result. The obtained dead-beat controllability tests are surprisingly simple and very easy to use.

The paper is organized as follows. In Section 2 we present definitions, the class of systems that we consider and some results that are needed in the sequel. The main result is stated and proved in Section 3. Section 4 contains an analogous controllability result for continuous time generalized Hammerstein systems. The ideas and results are illustrated by some examples given in Section 5.

2 Preliminaries

We start by considering discrete-time generalized Hammerstein systems of the form [3, 4]:

$$\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \end{pmatrix} = \begin{pmatrix} F_1 & 0 \\ 0 & F_2 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} + \begin{pmatrix} g_1 \\ 0 \end{pmatrix} u(k) + \begin{pmatrix} 0 \\ g_2 \end{pmatrix} u^2(k)$$

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$$
\begin{aligned}
y(k) &= \begin{pmatrix} c_1^T & c_2^T \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} + \\
&\qquad d_0 + d_1 u(k) + d_2 u^2(k) \qquad (1)
\end{aligned}
$$

where $x(k) = (x_1(k) \quad x_2(k))^T \in \mathbb{R}^n$ is a state of the system at time k and $u(k) \in \mathbb{R}$ is the control at time k. We also have $F_1 \in \mathbb{R}^{n_1 \times n_1}$, $F_2 \in \mathbb{R}^{n_2 \times n_2}$, $g_1 \in \mathbb{R}^{n_1 \times 1}$, $g_2 \in \mathbb{R}^{n_2 \times 1}$, $x_1(k) \in \mathbb{R}^{n_1}$ and $x_2(k) \in \mathbb{R}^{n_2}$.

We denote a control sequence as

$$\mathcal{U}_t = \{u(0), u(1), \ldots, u(t-1)\}, \; u(k) \in \mathbb{R}.$$

The state that is reached from $x(0)$ at time t by applying the control sequence $\mathcal{U}_t$ is denoted as $x(t, x(0), \mathcal{U}_t)$. We need the following definitions:

Definition 1 *The system (1) is dead-beat controllable if* $\forall x(0) \in \mathbb{R}^n$ *there exist finite* $t = t(x(0))$ *and a control sequence* $\mathcal{U}_t$ *such that* $x(t, x(0), \mathcal{U}_t) = 0$.

Definition 2 *The system (1) is completely controllable if* $\forall x(0)$, $x^* \in \mathbb{R}^n$ *there exist an integer* $H = H(x(0), x^*)$ *and a finite control sequence* $\mathcal{U}_H = \{u(0), u(1), \ldots, u(H-1)\}$ *such that the system is transferred from the state* $x(0)$ *to the state* x^* *under the action of the sequence* $\mathcal{U}_H$, *that is* $x(H, x(0), \mathcal{U}_H) = x^*$.

The following theorems play a crucial role for dead-beat controllability of generalized Hammerstein systems.

Theorem 1 *[2] The system*

$$x(k+1) = Ax(k) + bu(k) \qquad (2)$$

with $u \in [0, +\infty[$ *is completely controllable on* $\mathbb{R}^n$ *if and only if*

1. *rank*$[b \; Ab \; \ldots \; A^{n-1}b] = n$,
2. *the matrix* A *has no real positive or zero eigenvalues*

By slightly modifying Theorem 1, we obtain conditions for dead-beat controllability [7].

Theorem 2 *The system (2) with* $u \in [0, +\infty[$ *is dead-beat controllable on* $\mathbb{R}^n$ *if and only if*

1. *rank*$[\lambda I - A : b] = n$, $\forall \lambda \neq 0$, $\lambda \in \mathbb{C}$,
2. *the matrix* A *has no real positive eigenvalues*

Notice that the conditions of Theorem 1 are stronger than the conditions of Theorem 2. Indeed, in Theorem 2 the matrix A is allowed to have zero eigenvalues (Condition 2) and moreover zero modes do not have to be controllable (Condition 1).

The following theorem is a consequence of Theorem 1. Its proof is contained in the proof of Theorem 1 in [2].

Theorem 3 *If the second condition of Theorem 1 is satisfied, there exists a polynomial* $C(\zeta) = \sum_{i=0}^{N} c_i \zeta^i = 0$, $c_N = 1$ *with* $c_i > 0, \forall i = 0, 1, \ldots, N-1$ *such that the following matrix equation holds:*

$$C(A) = 0$$

In other words, is A has no real positive or zero eigenvalues, there exists a monic polynomial with positive coefficients which is divisible by the characteristic polynomial of the matrix A.

3 Controllability test

In this section we present necessary and sufficient conditions for dead-beat and complete controllability of discrete-time generalized Hammerstein systems. It is shown that the system (1) is dead-beat (completely) controllable if and only if its subsystems

$$x_1(k+1) = F_1 x_1(k) + g_1 u(k) \qquad (3)$$

and

$$x_2(k+1) = F_2 x_2(k) + g_2 u^2(k) \qquad (4)$$

are dead-beat (completely) controllable:

Theorem 4 *The system (1) is dead-beat controllable if and only if the following conditions are satisfied:*

1. *rank*$[I\lambda - F_1 : g_1] = n_1, \forall \lambda \neq 0, \lambda \in \mathbb{C}$
2. *rank*$[I\lambda - F_2 : g_2] = n_2, \forall \lambda \neq 0, \lambda \in \mathbb{C}$
3. F_2 *has no positive real eigenvalues*

Comment 1 *The first condition of Theorem 4 means that the subsystem (3) is dead-beat controllable. The second and third conditions of Theorem 4 represent necessary and sufficient conditions for dead-beat controllability of the subsystem (4). It is obvious that dead-beat controllability does not require zero modes to be controllable.*

If we consider odd monomial nonlinearities, an extra condition should be added to have complete controllability result, which is analyzed in [9]. Also, in the same reference the results are extended to include some other polynomial nonlinearities. However, if the nonlinearity is of even order, as in this paper, the complete controllability of subsystems always implies complete controllability of the overall system.

Comment 2 *Notice that if there are some zero eigenvalues of* F_1 *or* F_2, *we can find a nonsingular transformation* T *such that*

$$\bar{F}_i = T^{-1} F_i T = \begin{pmatrix} D_{11}^i & 0 \\ 0 & D_{22}^i \end{pmatrix}, \; \bar{g}_i = T^{-1} g_i, \;\; i = 1, 2$$

and D^i_{22} *is a nil-potent matrix. Assume that the degree of nil-potency of* D^i_{22} *is* d_i*. Consider the state at step* $k+1 \geq d_i$*:*

$$x_i(k+1) = \bar{F}_i^k x_i(0) + \sum_{l=0}^{k-1} \bar{F}_i^{k-l-1} \bar{g}_i u(l),\ i = 1,2$$

If we apply $u(l) = 0, l = k-d_i, k-d_i+1, \ldots, k$*, we have that* $x_i(k+1) = (\hat{x}^T \quad 0)^T$*,* $i = 1,2$ *irrespective of the control sequence* $u(l)$*,* $l = 0,1,\ldots,k-d_i-1$*. Thus, there is no loss of generality if we concentrate just on situations when*

$$rank[F_i - I\lambda : g_i] = n_i, \forall \lambda \in \mathbb{C},\ i = 1,2$$

In other words, we assume that

1. *rank*$[g_1 : F_1 g_1 : \ldots : F_1^{n_1-1} g_1] = n_1$
2. *rank*$[g_2 : F_2 g_2 : \ldots : F_2^{n_2-1} g_2] = n_2$
3. F_2 *has no zero or positive real eigenvalues*

Proof [8]:
Necessity: Suppose that at least one of the conditions of Theorem 4 is violated. This implies that at least one of the subsystems (3) or (4) is not dead-beat controllable. Without loss of generality suppose that the subsystem (3) is not dead-beat controllable. From the definition of dead-beat controllability it follows that there exists an initial state $x_1^*(0) \in \mathbb{R}^{n_1}$ for the subsystem (3) that can not be driven to the origin in finite time. This implies that any initial states of the overall system (1) which is given by $(x^*(0))^T = ((x_1^*(0))^T \quad z^T)^T, z \in \mathbb{R}^{n_2}$ can not be driven to the origin in finite time. Consequently the overall system (1) is not dead-beat controllable by definition.

Sufficiency: In order to prove sufficiency we will consider special sequences of controls which can transfer any initial state of (1) to the origin if the conditions of theorem are satisfied.

Since the last two of the conditions in Comment 2 guarantee that the subsystems (4) is completely controllable, it is possible to find a sequence of controls $U_P = \{u(0), u(1), \ldots, u(P-1)\}$ which yields $x_2(P) = 0$ and $x_1(P) \in \mathbb{R}^{n_1}$. As a result, we assume without loss of generality that $x(0) = (x_1^T(0) \quad 0)^T$.

Since F_2 has no positive or zero eigenvalues (see Comment 2), according to Theorem 3 the matrix F_2 satisfies a polynomial equation with real positive coefficients:

$$C(F_2) = \sum_{i=0}^{i=N} c_i F_2^i = 0, \quad c_i > 0,\ \forall i = 0,1,\ldots,N. \quad (5)$$

Consider now the following sequence of controls:

$$\begin{aligned}
u(0) &= \pm\sqrt{c_N}v(0) \\
u(1) &= \pm\sqrt{c_{N-1}}v(0) \\
u(2) &= \pm\sqrt{c_{N-2}}v(0) \\
&\ldots \\
u(N) &= \pm\sqrt{c_0}v(0) \\
u(N+1) &= \pm\sqrt{c_N}v(1) \\
&\ldots \\
u((N+1)n_1-1) &= \pm\sqrt{c_0}v(n_1-1) \qquad (6)
\end{aligned}$$

It is obvious that because of (5) the state of the subsystem (4) $x_2(k)$ is zeroed every $N+1$ steps irrespective of the values $v(k) \in \mathbb{R}$, $k = 0,1,\ldots,n_1-1$. That is, $\forall v(k) \in \mathbb{R}$ we have that $x_2(N+1) = x_2(2(N+1)) = \ldots = x_2(n_1(N+1)) = 0$.

Hence, we now consider if it is possible to zero the state of the subsystem (3) $x_1(n_1(N+1))$ by using $v(k)$, $k = 0,1,\ldots,n_1-1$ if we start from any initial state $x_1(0) \in \mathbb{R}^{n_1}$. It is important to emphasize that the sign of control $u(k)$ and the values $v(k)$ in (6) can be arbitrarily assigned and it is this additional degree of freedom that we are exploiting in the proof.

We have:

$$\begin{aligned}
x_1((N+1)n_1) &= \sum_{i=0}^{(N+1)n_1-1} F_1^{(N+1)n_1-1-i} g_1 u(i) \\
&\quad + F_1^{(N+1)n_1} x_1(0) \qquad (7)
\end{aligned}$$

The control sequence (6) is now substituted in (7) and we want to specify the existence of appropriate signs and values $v(k), k = 0,1,\ldots,n_1-1$ such that:

$$\sum_{i=0}^{(N+1)n_1-1} F_1^{(N+1)n_1-1-i} g_1 u(i) = -F_1^{(N+1)n_1} x_1(0) \qquad (8)$$

We introduce the following vector functions:

$$\begin{aligned}
L_0 &= \sum_{i=0}^{i=N} F_1^{N-i} g_1 \delta_{0,i} \\
L_1 &= F_1^{N+1} \sum_{i=0}^{i=N} F_1^{N-i} g_1 \delta_{1,i} \\
&\ldots \\
L_{n_1-1} &= F_1^{(n_1-1)(N+1)-1} \sum_{i=0}^{i=N} F_1^{N-i} g_1 \delta_{n_1-1,i} \quad (9)
\end{aligned}$$

where $\delta_{k,i} = \pm\sqrt{c_{N-i}}$, $\forall k = 0,1,\ldots,n_1-1, i = 0,1,\ldots,N$. We can rewrite the equation (8) as follows:

$$-F_1^{(N+1)n_1} x_1(0) = [L_0 : L_1 : \ldots : L_{n_1-1}] \begin{pmatrix} v(0) \\ v(1) \\ \ldots \\ v(n_1-1) \end{pmatrix} \quad (10)$$

If there exists a sequence of controls of the form (6) such that the matrix $[L_0 : L_1 : \ldots : L_{n_1-1}]$ is nonsingular then the system (1) is dead-beat controllable.

Because of non-singularity of F_2 there exists at least one $\delta_{k,i} > 0$. Non-singularity of matrices F_1 and F_2 and controllability of the pair (F_1, g_1) causes the vectors L_k to have entries which are linear functions of $\delta_{k,i}$, $i = 0, 1, \ldots, N$. As a result, the determinant of $[L_0 : L_1 : \ldots : L_{n_1-1}]$ is a multi-linear function of $\delta_{k,i}$, which we denote as $p(\delta_{k,i})$.

For any scalar valued affine function $l(y) = ay + b$, $a, b \in \mathbb{R}$, $a \neq 0$ in a scalar variable y, we have that if $l(y) = 0$ then $l(-y) \neq 0$. This observation is exploited to select $\delta_{k,i}$ such that $p(\delta_{k,i}) \neq 0$.

Let us consider a multi-linear function with three $\delta_{k,i} \neq 0$, which we relabel as $\delta_1, \delta_2, \delta_3$. It is easy to check that any such function can be written in the following form:

$$((K_1\delta_1+L_1)\delta_2+(K_2\delta_1+L_2))\delta_3+(K_3\delta_1+L_3)\delta_2+K_4\delta_1+L_4 \qquad (11)$$

where $K_i, L_i \in \mathbb{R}$.

If $K_1 \neq 0$, we can render $K_1\delta_1 + L_1 \neq 0$ by an appropriate choice of δ_1. Moreover, with this choice of δ_1 we can render $(K_1\delta_1 + L_1)\delta_2 + (K_2\delta_1 + L_2)$ non zero by choosing δ_2 and finally the whole expression can be made non zero by a choice of δ_3. If $K_1 = 0$ but if $L_1 \neq 0$ we can do the same, etc. By induction, we show that there is no combination of $\delta_i = \pm\sqrt{c_i}$ which renders (11) non zero only if $K_i, L_i = 0, i = 1, 2, 3, 4$ or F_2 is singular (that is, $\delta_i = 0, i = 1, 2, 3$). Since we assumed that $\delta_i \neq 0$, it follows that either F_1 is singular or the pair (F_1, g_1) is not controllable (e.g. $g_1 = 0$). Contradiction completes the proof. The argument can be carried out for a multi-linear function in any number of variables $\delta_{i,k}$ and hence conditions of Theorem 4 are sufficient for dead-beat controllability. Q.E.D.

We note that the result holds for complete controllability and we can state:

Theorem 5 *The system (1) is completely controllable if and only if its subsystems (3) and (4) are both completely controllable.*

The only point that is different from the dead-beat argument is the possible existence of zero modes. However, notice that if the subsystem (4) is completely controllable, F_2 is necessarily non-singular. So the only situation that needs to be addressed is when F_1 is singular and both subsystems are completely controllable. We omit the proof but remark that recently we generalized the result to arbitrary degree of the nonlinearity - including the odd case. In this situation, the zero modes play a very subtle role which shows the difference between the dead-beat and complete controllability results.

Comment 3 *Note the difference between the statements of Theorems 4 and 5 and the result on complete (dead-beat) controllability of parallel linear connections. Denote the sets of poles of linear blocks $W_1(z)$ and $W_2(z)$ as $\mathcal{P}_1$ and $\mathcal{P}_2$. In [5] we can find the result which states that the parallel connection of linear systems $W_1(z)$ and $W_2(z)$ is completely (dead-beat) controllable if and only if both subsystems are completely (dead-beat) controllable and $\mathcal{P}_1 \cap \mathcal{P}_2 = \emptyset$ ($\nexists s \in \mathbb{C}$, $s \neq 0, s \in \mathcal{P}_1 \cap \mathcal{P}_2$). Hence, besides controllability of the subsystems we need an extra condition on the sets of poles of the subsystems. We can interpret our result by saying that the square nonlinearity, which appears in generalized Hammerstein systems, destroys this last condition on the poles. More surprisingly, no extra conditions on sets of poles of subsystems are needed for generalized Hammerstein systems, which is an interesting observation.*

4 Continuous-time case

In this section we show that the results on dead-beat controllability of discrete-time systems can be used to prove that Theorem 4 holds for continuous-time generalized Hammerstein systems of the following form:

$$\begin{aligned} S_1 : \dot{x}_1(t) &= A_1 x_1(t) + b_1 u(t) \\ S_2 : \dot{x}_2(t) &= A_2 x_2(t) + b_2 u^2(t) \end{aligned} \qquad (12)$$

where $x_i \in \mathbb{R}^{n_i}, i = 1, 2, \ldots, m, u(t) \in \mathbb{R}$ and matrices F_i, g_i have the appropriate dimensions. We prove the main result of this section by showing that a continuous-time generalized system (12), whose subsystems are controllable, can be transformed into a discrete-time generalized Hammerstein system (1), whose subsystems are controllable, by using piecewise constant controls (sampler and zero order hold).

Before we state the main result of this section we need to state some preliminaries. We denote the control function restricted to the time interval $[t_0, t_f]$ as $u_{[t_0,t_f]}$. The state of the system (12) at time T, which emanates from the initial state $x(0)$ under the control $u_{[0,T]}$ is denoted as $x(T, x(0), u_{[0,T]})$.

Definition 3 *A continuous-time system Σ is completely controllable if given any states $x(0), x^*$, there exists $T \in \mathbb{R}$ and control $u_{[0,T]}$ such that $x^* = x(T, x(0), u_{[0,T]})$.*

The following result was proved in [10]:

Theorem 6 *Consider a continuous-time linear system with positive controls:*

$$\dot{x}(t) = Ax(t) + bu(t), \qquad (13)$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}$, $u(t) \geq 0$, $\forall t$. The system (13) is completely controllable if and only if:

1. *$rank[\lambda I - A : b] = n, \forall \lambda \in \mathbb{C}$*
2. *A has no real eigenvalues*

An interesting consequence of Theorem 6 is that only continuous-time linear systems of even order may be completely controllable with positive controls. For instance, a scalar continuous-time system can never be completely controllable with positive controls!

The main result of this section is given below:

Theorem 7 *The system (12) is completely controllable if and only if both its subsystems $S_i, i = 1, 2$ are completely controllable.*

Proof of Theorem 7: Necessity of the proof is trivial and we concentrate only on the sufficiency.

Assume that the control signal is piecewise constant, that is:

$$u(t) = u(k) = const., \ \forall t \in [kh, (k+1)h[, \ h > 0, \ k \in \mathbb{N}$$

In other words, we assume that a zero order hold and sampler are used. The particular structure of the system (12) allows us to obtain a discrete-time model of the system in the same manner as for the linear systems. Indeed, the discrete-time model of the system (12) with the assumption of zero order hold, is given by (1), where

$$F_i = e^{A_i h}, \quad g_i = \int_0^h e^{A_i s} b_i ds \tag{14}$$

If we can find a sampling period h such that for all controllable subsystems S_i of (12), we obtain that all the subsystems of the system (1) with (14) are controllable, the proof of Theorem 7 follows immediately from Theorem 4.

Both subsystems $S_i, i = 1, 2$ in (12) are controllable. Consider the subsystem S_1. Denote the eigenvalues of the matrix A_1 as $\sigma_t = \rho_t^1 + j\omega_t^1$. From [5][pg. 174-175] it follows that since the pair (A_1, b_1) is controllable, the pair (F_1, g_1), which is computed using (14), will be controllable if and only if whenever

$$\rho_i^1 - \rho_l^1 = 0 \text{ then } h \neq \frac{2k\pi}{\omega_i^1 - \omega_l^1}, \ \ i, l \in \{1, 2, \ldots, n_1\}, k \in \mathbb{N} \tag{15}$$

Hence only countably many values of h are critical. That is, the discrete-time subsystem S_1 may not be controllable only for the values of h defined in (15).

On the other hand, if we consider the subsystem S_2, we need besides the controllability condition of the pair (F_2, g_2) also that F_2 has no positive real eigenvalues. Consequently, we need the following condition to preserve controllability of the (F_2, g_2) pair:

$$\rho_i^2 - \rho_l^2 = 0 \text{ then } h \neq \frac{2k\pi}{\omega_i^2 - \omega_l^2}, \ \ i, l \in \{1, 2, \ldots, n_2\}, k \in \mathbb{N} \tag{16}$$

Notice that if the matrix A_2 had any real eigenvalues (the continuous-time subsystem S_2 is not controllable), then for any sampling period $h > 0$, the matrix F_2 defined by (14) would have a real positive eigenvalue, and hence the discretized subsystem S_2 is also not controllable. Since the matrix A_2 has only complex eigenvalues $\sigma_t^2 = \rho_t^2 + j\omega_t^2, \omega_t^2 \neq 0, \forall t$, if the sampling period is chosen so that:

$$h \neq \frac{2k\pi}{\omega_t^2}, \ \forall t \in \{1, 2, \ldots, n_2\}, \ k \in \mathbb{N} \tag{17}$$

then F_2 has no positive real eigenvalues. In summary, the critical values of h are given by conditions (15), (16) and (17) and therefore it is always possible to choose $h > 0$ so that all subsystems of the discretized system (1) with (14) are controllable. Notice also that F_2 obtained using (14) are all non-singular. The proof of Theorem 7 follows from Theorem 4. Q.E.D.

5 An illustrative example

Consider the system:

$$\begin{aligned} S_1: \ x_1(k+1) &= F_1 x_1(k) + g_1 u(k) \\ S_2: \ x_2(k+1) &= F_2 x_2(k) + g_2 u^2(k) \end{aligned} \tag{18}$$

where:

$$F_1 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}; F_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; g_1 = g_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{19}$$

Since the following holds:

1. $\det[g_1 : F_1 g_1] = -1 \neq 0$,
2. $\det[g_2 : F_2 g_2] = -1 \neq 0$,
3. eigenvalues of F_2 are $\pm i, i = \sqrt{-1}$.

the system (18) is dead-beat controllable. Moreover, the system is also completely controllable.

Indeed, we check this statement using the method exploited in the proof of Theorem 4. Since the subsystem S_2 is dead-beat controllable, we can without loss of generality assume that $x_2(0) = 0$. Notice that the matrix F_2 satisfies the following equation $F_2^3 + F_2^2 + F_2 + I = 0$. Assume now that we apply the following control sequence: $u(0) = v(0), u(1) = -v(0), u(2) = -v(0), u(3) = -v(0), u(4) = v(1), u(5) = v(1), u(6) = v(1), u(7) = -v(1)$, with $x_2(0) = 0$. Then we have:

$$x_1(8) \ = \ F_1^8 x_1(0) + [(F_1^7 - F_1^6 - F_1^5 - F_1^4)g_1 :$$

$$
\begin{aligned}
& (F_1^3 + F_1^2 + F_1 - I)g_1] \begin{pmatrix} v(0) \\ v(1) \end{pmatrix} \\
x_2(8) \quad = \quad & F_2^8 \underbrace{x_2(0)}_{=0} + [\underbrace{(F_2^7 + F_1^6 + F_1^5 + F_1^4)}_{=0} g_1 : \\
& \underbrace{(F_1^3 + F_1^2 + F_1 + I)}_{=0} g_1] \begin{pmatrix} v^2(0) \\ v^2(1) \end{pmatrix} \qquad (20)
\end{aligned}
$$

and therefore we have that $x_2(0) = 0$ for any $v(0), v(1) \in \mathbb{R}$. Moreover, with the chosen control sequence, we can see that the state of the first subsystem can be arbitrarily assigned since the matrix

$$[(F_1^7 - F_1^6 - F_1^5 - F_1^4)g_1 : (F_1^3 + F_1^2 + F_1 + I)g_1] = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$$

is non-singular.

6 Conclusions

Necessary and sufficient conditions for dead-beat and complete controllability of discrete and continuous-time generalized Hammerstein systems are presented. The conditions are very easy to check and they resemble the linear systems controllability conditions.

References

[1] W. Bamberger and R. Isermann, "Adaptive on-line steady-state optimization of slow dynamic processes", *Automatica*, vol. 14, pp. 223-230, 1978.

[2] M. E. Evans and D. N. P. Murthy, "Controllability of discrete-time systems with positive controls", *IEEE Trans. Autom. Control*, vol. 22, pp. 942-945, 1977.

[3] R. Haber and L. Keviczky, "Identification of nonlinear dynamic systems", in: N. S. Rajbman, Ed., *Identification and system parameter estimation*: North-Holland, pp. 79-127, 1978.

[4] R. Haber and H. Unbehauen, "Structure identification of nonlinear dynamic systems-a survey of input/output approaches", *Automatica*, vol. 26, pp. 651-677, 1990.

[5] T. Kailath, *Linear systems.* Prentice Hall: New Jersey, 1980.

[6] L. Keviczky, "Nonlinear dynamic identification of a cement mill to be optimized", in: N. S. Rajbman, Ed., *Identification and system parameter estimation*: North-Holland, pp. 1185-1195, 1978.

[7] D. Nešić and I. M. Y. Mareels, "Dead-beat control of simple Hammerstein models", to appear in *IEEE Trans. Autom. Contr.*, 1997.

[8] D. Nešić, "A note on dead-beat controllability of generalized Hammerstein models", *Syst. Contr. Lett.*, vol. 29, pp. 223-231, 1997.

[9] D. Nešić, "Controllability for a class of parallelly connected polynomial systems", submitted for publication, 1997.

[10] S. H. Saperstone, "Global controllability of linear systems with positive controls", *SIAM J. Contr.*, vol. 11, pp. 417-423, 1973.