

Minerva Access is the Institutional Repository of The University of Melbourne

Author/s:

Kaliyar, AB; Behera, P; Mohanty, GB; Gaur, V; Adachi, I; Ahn, JK; Aihara, H; Al Said, S; Asner, DM; Aulchenko, V; Aushev, T; Ayad, R; Babu, V; Badhrees, I; Bahinipati, S; Bakich, AM; Bansal, V; Beleño, C; Bhardwaj, V; Bilka, T; Biswal, J; Bobrov, A; Bozek, A; Bračko, M; Cao, L; Červenkov, D; Chekelian, V; Chen, A; Cheon, BG; Chilikin, K; Cho, HE; Cho, K; Choi, SK; Choi, Y; Choudhury, S; Cinabro, D; Cunliffe, S; Dash, N; Di Carlo, S; Dingfelder, J; Doležal, Z; Dong, TV; Drásal, Z; Eidelman, S; Epifanov, D; Fast, JE; Ferber, T; Frey, A; Fulsom, BG; Garg, R; Gabyshev, N; Garmash, A; Gelb, M; Giri, A; Goldenzweig, P; Golob, B; Greenwald, D; Grzymkowska, O; Haba, J; Hara, T; Hayasaka, K; Hayashii, H; Hou, WS; Hsu, CL; Iijima, T; Inami, K; Ishikawa, A; Itoh, R; Iwasaki, M; Iwasaki, Y; Jacobs, WW; Jeon, HB; Jia, S; Joffe, D; Joo, KK; Kahn, J; Karyan, G; Kawasaki, T; Kichimi, H; Kiesling, C; Kim, CH; Kim, DY; Kim, HJ; Kim, SH; Kimmel, TD; Kinoshita, K; Kodyš, P; Korpar, S; Kotchetkov, D; Križan, P; Kroeger, R; Krokovny, P; Kuhr, T; Kulasiri, R; Kumar, R; Kuzmin, A; Kwon, YJ; Lee, IS; Lee, JK; Lee, JY

Title:

Measurements of branching fraction and direct CP asymmetry in $B^\pm \rightarrow KS_0KS_0K^\pm$ and a search for $B^\pm \rightarrow KS_0KS_0\pi^\pm$

Date:

2019-02-01

Citation:

Kaliyar, A. B., Behera, P., Mohanty, G. B., Gaur, V., Adachi, I., Ahn, J. K., Aihara, H., Al Said, S., Asner, D. M., Aulchenko, V., Aushev, T., Ayad, R., Babu, V., Badhrees, I., Bahinipati, S., Bakich, A. M., Bansal, V., Beleño, C., Bhardwaj, V., ... Lee, J. Y. (2019). Measurements of branching fraction and direct CP asymmetry in $B^\pm \rightarrow KS_0KS_0K^\pm$ and a search for $B^\pm \rightarrow KS_0KS_0\pi^\pm$. *Physical Review D*, 99 (3), <https://doi.org/10.1103/PhysRevD.99.031102>.

Persistent Link:

<https://hdl.handle.net/11343/290336>

License:

CC BY

Measurements of branching fraction and direct CP asymmetry in $B^\pm \rightarrow K_S^0 K_S^0 K^\pm$ and a search for $B^\pm \rightarrow K_S^0 K_S^0 \pi^\pm$

A. B. Kaliyar,²⁷ P. Behera,²⁷ G. B. Mohanty,⁸² V. Gaur,⁹⁰ I. Adachi,^{19,15} J. K. Ahn,⁴⁰ H. Aihara,⁸⁶ S. Al Said,^{81,37} D. M. Asner,⁴ V. Aulchenko,^{5,67} T. Aushev,⁵⁶ R. Ayad,⁸¹ V. Babu,⁸² I. Badhrees,^{81,36} S. Bahinipati,²⁴ A. M. Bakich,⁸⁰ V. Bansal,⁶⁹ C. Beleño,¹⁴ V. Bhardwaj,²³ T. Bilka,⁶ J. Biswal,³³ A. Bobrov,^{5,67} A. Bozek,⁶⁴ M. Bračko,^{50,33} L. Cao,³⁴ D. Červenkov,⁶ V. Chekelian,⁵¹ A. Chen,⁶¹ B. G. Cheon,¹⁷ K. Chilikin,⁴⁴ H. E. Cho,¹⁷ K. Cho,³⁹ S.-K. Choi,¹⁶ Y. Choi,⁷⁹ S. Choudhury,²⁶ D. Cinabro,⁹¹ S. Cunliffe,⁹ N. Dash,²⁴ S. Di Carlo,⁴² J. Dingfelder,³ Z. Doležal,⁶ T. V. Dong,^{19,15} Z. Drásal,⁶ S. Eidelman,^{5,67,44} D. Epifanov,^{5,67} J. E. Fast,⁶⁹ T. Ferber,⁹ A. Frey,¹⁴ B. G. Fulsom,⁶⁹ R. Garg,⁷⁰ N. Gabyshev,^{5,67} A. Garmash,^{5,67} M. Gelb,³⁴ A. Giri,²⁶ P. Goldenzweig,³⁴ B. Golob,^{46,33} D. Greenwald,⁸³ O. Grzymkowska,⁶⁴ J. Haba,^{19,15} T. Hara,^{19,15} K. Hayasaka,⁶⁶ H. Hayashii,⁶⁰ W.-S. Hou,⁶³ C.-L. Hsu,⁸⁰ T. Iijima,^{58,57} K. Inami,⁵⁷ A. Ishikawa,⁸⁵ R. Itoh,^{19,15} M. Iwasaki,⁶⁸ Y. Iwasaki,¹⁹ W. W. Jacobs,²⁸ H. B. Jeon,⁴¹ S. Jia,² D. Joffe,³⁵ K. K. Joo,⁷ J. Kahn,⁴⁷ G. Karyan,⁹ T. Kawasaki,³⁸ H. Kichimi,¹⁹ C. Kiesling,⁵¹ C. H. Kim,¹⁷ D. Y. Kim,⁷⁸ H. J. Kim,⁴¹ S. H. Kim,¹⁷ T. D. Kimmel,⁹⁰ K. Kinoshita,⁸ P. Kodyš,⁶ S. Korpar,^{50,33} D. Kotchetkov,¹⁸ P. Križan,^{46,33} R. Kroeger,⁵³ P. Krokovny,^{5,67} T. Kuhr,⁴⁷ R. Kulasiri,³⁵ R. Kumar,⁷³ A. Kuzmin,^{5,67} Y.-J. Kwon,⁹³ I. S. Lee,¹⁷ J. K. Lee,⁷⁶ J. Y. Lee,⁷⁶ S. C. Lee,⁴¹ D. Levit,⁸³ C. H. Li,⁴⁵ L. K. Li,²⁹ Y. B. Li,⁷¹ L. Li Gioi,⁵¹ J. Libby,²⁷ T. Luo,¹² J. MacNaughton,⁵⁴ T. Matsuda,⁵⁴ D. Matvienko,^{5,67,44} M. Merola,^{31,59} K. Miyabayashi,⁶⁰ H. Miyata,⁶⁶ R. Mizuk,^{44,55,56} S. Mohanty,^{82,89} T. Mori,⁵⁷ M. Nakao,^{19,15} K. J. Nath,²⁵ Z. Natkaniec,⁶⁴ M. Nayak,^{91,19} N. K. Nisar,⁷² S. Nishida,^{19,15} K. Nishimura,¹⁸ K. Ogawa,⁶⁶ S. Ogawa,⁸⁴ H. Ono,^{65,66} Y. Onuki,⁸⁶ W. Ostrowicz,⁶⁴ G. Pakhlova,^{44,56} B. Pal,⁴ S. Pardi,³¹ S. Patra,²³ S. Paul,⁸³ T. K. Pedlar,⁴⁸ R. Pestotnik,³³ L. E. Piilonen,⁹⁰ V. Popov,^{44,56} K. Prasanth,⁸² E. Prencipe,²¹ A. Rabusov,⁸³ P. K. Resmi,²⁷ M. Ritter,⁴⁷ A. Rostomyan,⁹ G. Russo,³¹ D. Sahoo,⁸² Y. Sakai,^{19,15} M. Salehi,^{49,47} S. Sandilya,⁸ T. Sanuki,⁸⁵ V. Savinov,⁷² O. Schneider,⁴³ G. Schnell,^{1,22} J. Schueler,¹⁸ C. Schwanda,³⁰ A. J. Schwartz,⁸ Y. Seino,⁶⁶ K. Senyo,⁹² M. E. Sevier,⁵² C. P. Shen,² T.-A. Shibata,⁸⁷ J.-G. Shiu,⁶³ F. Simon,⁵¹ E. Solovieva,^{44,56} M. Starič,³³ Z. S. Stottler,⁹⁰ M. Sumihama,¹³ T. Sumiyoshi,⁸⁸ W. Sutcliffe,³⁴ M. Takizawa,^{77,20,74} K. Tanida,³² Y. Tao,¹¹ F. Tenchini,⁹ K. Trabelsi,⁴² M. Uchida,⁸⁷ T. Uglov,^{44,56} Y. Unno,¹⁷ S. Uno,^{19,15} P. Urquijo,⁵² Y. Usov,^{5,67} R. Van Tonder,³⁴ G. Varner,¹⁸ K. E. Varvell,⁸⁰ A. Vossen,¹⁰ E. Waheed,⁵² B. Wang,⁸ C. H. Wang,⁶² M.-Z. Wang,⁶³ P. Wang,²⁹ X. L. Wang,¹² E. Won,⁴⁰ S. B. Yang,⁴⁰ H. Ye,⁹ J. Yelton,¹¹ J. H. Yin,²⁹ Y. Yusa,⁶⁶ Z. P. Zhang,⁷⁵ V. Zhilich,^{5,67} and V. Zhukova⁴⁴

(Belle Collaboration)

¹University of the Basque Country UPV/EHU, 48080 Bilbao

²Beihang University, Beijing 100191

³University of Bonn, 53115 Bonn

⁴Brookhaven National Laboratory, Upton, New York 11973

⁵Budker Institute of Nuclear Physics SB RAS, Novosibirsk 630090

⁶Faculty of Mathematics and Physics, Charles University, 121 16 Prague

⁷Chonnam National University, Kwangju 660-701

⁸University of Cincinnati, Cincinnati, Ohio 45221

⁹Deutsches Elektronen-Synchrotron, 22607 Hamburg

¹⁰Duke University, Durham, North Carolina 27708

¹¹University of Florida, Gainesville, Florida 32611

¹²Key Laboratory of Nuclear Physics and Ion-beam Application (MOE) and Institute of Modern Physics, Fudan University, Shanghai 200443

¹³Gifu University, Gifu 501-1193

¹⁴II. Physikalisches Institut, Georg-August-Universität Göttingen, 37073 Göttingen

¹⁵SOKENDAI (The Graduate University for Advanced Studies), Hayama 240-0193

¹⁶Gyeongsang National University, Chinju 660-701

¹⁷Hanyang University, Seoul 133-791

¹⁸University of Hawaii, Honolulu, Hawaii 96822

¹⁹High Energy Accelerator Research Organization (KEK), Tsukuba 305-0801

²⁰J-PARC Branch, KEK Theory Center, High Energy Accelerator Research Organization (KEK), Tsukuba 305-0801

²¹Forschungszentrum Jülich, 52425 Jülich

²²IKERBASQUE, Basque Foundation for Science, 48013 Bilbao

²³Indian Institute of Science Education and Research Mohali, SAS Nagar, 140306

- ²⁴Indian Institute of Technology Bhubaneswar, Satya Nagar 751007
- ²⁵Indian Institute of Technology Guwahati, Assam 781039
- ²⁶Indian Institute of Technology Hyderabad, Telangana 502285
- ²⁷Indian Institute of Technology Madras, Chennai 600036
- ²⁸Indiana University, Bloomington, Indiana 47408
- ²⁹Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049
- ³⁰Institute of High Energy Physics, Vienna 1050
- ³¹INFN—Sezione di Napoli, 80126 Napoli
- ³²Advanced Science Research Center, Japan Atomic Energy Agency, Naka 319-1195
- ³³J. Stefan Institute, 1000 Ljubljana
- ³⁴Institut für Experimentelle Teilchenphysik, Karlsruher Institut für Technologie, 76131 Karlsruhe
- ³⁵Kennesaw State University, Kennesaw, Georgia 30144
- ³⁶King Abdulaziz City for Science and Technology, Riyadh 11442
- ³⁷Department of Physics, Faculty of Science, King Abdulaziz University, Jeddah 21589
- ³⁸Kitasato University, Sagami-hara 252-0373
- ³⁹Korea Institute of Science and Technology Information, Daejeon 305-806
- ⁴⁰Korea University, Seoul 136-713
- ⁴¹Kyungpook National University, Daegu 702-701
- ⁴²LAL, Université Paris-Sud, CNRS/IN2P3, Université Paris-Saclay, Orsay
- ⁴³École Polytechnique Fédérale de Lausanne (EPFL), Lausanne 1015
- ⁴⁴P. N. Lebedev Physical Institute of the Russian Academy of Sciences, Moscow 119991
- ⁴⁵Liaoning Normal University, Dalian 116029
- ⁴⁶Faculty of Mathematics and Physics, University of Ljubljana, 1000 Ljubljana
- ⁴⁷Ludwig Maximilians University, 80539 Munich
- ⁴⁸Luther College, Decorah, Iowa 52101
- ⁴⁹University of Malaya, 50603 Kuala Lumpur
- ⁵⁰University of Maribor, 2000 Maribor
- ⁵¹Max-Planck-Institut für Physik, 80805 München
- ⁵²School of Physics, University of Melbourne, Victoria 3010
- ⁵³University of Mississippi, University, Mississippi 38677
- ⁵⁴University of Miyazaki, Miyazaki 889-2192
- ⁵⁵Moscow Physical Engineering Institute, Moscow 115409
- ⁵⁶Moscow Institute of Physics and Technology, Moscow Region 141700
- ⁵⁷Graduate School of Science, Nagoya University, Nagoya 464-8602
- ⁵⁸Kobayashi-Maskawa Institute, Nagoya University, Nagoya 464-8602
- ⁵⁹Università di Napoli Federico II, 80055 Napoli
- ⁶⁰Nara Women's University, Nara 630-8506
- ⁶¹National Central University, Chung-li 32054
- ⁶²National United University, Miao Li 36003
- ⁶³Department of Physics, National Taiwan University, Taipei 10617
- ⁶⁴H. Niewodniczanski Institute of Nuclear Physics, Krakow 31-342
- ⁶⁵Nippon Dental University, Niigata 951-8580
- ⁶⁶Niigata University, Niigata 950-2181
- ⁶⁷Novosibirsk State University, Novosibirsk 630090
- ⁶⁸Osaka City University, Osaka 558-8585
- ⁶⁹Pacific Northwest National Laboratory, Richland, Washington 99352
- ⁷⁰Panjab University, Chandigarh 160014
- ⁷¹Peking University, Beijing 100871
- ⁷²University of Pittsburgh, Pittsburgh, Pennsylvania 15260
- ⁷³Punjab Agricultural University, Ludhiana 141004
- ⁷⁴Theoretical Research Division, Nishina Center, RIKEN, Saitama 351-0198
- ⁷⁵University of Science and Technology of China, Hefei 230026
- ⁷⁶Seoul National University, Seoul 151-742
- ⁷⁷Showa Pharmaceutical University, Tokyo 194-8543
- ⁷⁸Soongsil University, Seoul 156-743
- ⁷⁹Sungkyunkwan University, Suwon 440-746
- ⁸⁰School of Physics, University of Sydney, New South Wales 2006
- ⁸¹Department of Physics, Faculty of Science, University of Tabuk, Tabuk 71451
- ⁸²Tata Institute of Fundamental Research, Mumbai 400005
- ⁸³Department of Physics, Technische Universität München, 85748 Garching

⁸⁴Toho University, Funabashi 274-8510⁸⁵Department of Physics, Tohoku University, Sendai 980-8578⁸⁶Department of Physics, University of Tokyo, Tokyo 113-0033⁸⁷Tokyo Institute of Technology, Tokyo 152-8550⁸⁸Tokyo Metropolitan University, Tokyo 192-0397⁸⁹Utkal University, Bhubaneswar 751004⁹⁰Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061⁹¹Wayne State University, Detroit, Michigan 48202⁹²Yamagata University, Yamagata 990-8560⁹³Yonsei University, Seoul 120-749

(Received 27 December 2018; published 20 February 2019)

We study charmless hadronic decays of charged B mesons to the final states $K_S^0 K_S^0 K^\pm$ and $K_S^0 K_S^0 \pi^\pm$ using a 711 fb^{-1} data sample that contains $772 \times 10^6 B\bar{B}$ pairs and was collected at the $\Upsilon(4S)$ resonance with the Belle detector at the KEKB asymmetric-energy e^+e^- collider. For $B^\pm \rightarrow K_S^0 K_S^0 K^\pm$, the measured branching fraction and direct CP asymmetry are $[10.42 \pm 0.43(\text{stat}) \pm 0.22(\text{syst})] \times 10^{-6}$ and $[+1.6 \pm 3.9(\text{stat}) \pm 0.9(\text{syst})]\%$, respectively. In the absence of a statistically significant signal for $B^\pm \rightarrow K_S^0 K_S^0 \pi^\pm$, we obtain a 90% confidence-level upper limit on its branching fraction as 8.7×10^{-7} .

DOI: 10.1103/PhysRevD.99.031102

Charged B -meson decays to the three-body charmless hadronic final states $K_S^0 K_S^0 K^\pm$ and $K_S^0 K_S^0 \pi^\pm$ mainly proceed via $b \rightarrow s$ and $b \rightarrow d$ loop transitions, respectively. Figure 1 shows Feynman diagrams of the dominant amplitudes that contribute to these decays. These flavor-changing neutral current transitions, being suppressed in the standard model (SM), are interesting, as they could be sensitive to possible non-SM contributions [1].

Further motivation, especially to study the contributions of various quasi-two-body resonances to inclusive CP asymmetry, comes from the recent results on $B^\pm \rightarrow K^+ K^- K^\pm$, $K^+ K^- \pi^\pm$ and other such three-body decays [2–4]. LHCb has found large asymmetries localized in phase space in $B^\pm \rightarrow K^+ K^- \pi^\pm$ decays [3]. Recently, Belle has also reported strong evidence for large CP asymmetry at the low $K^+ K^-$ invariant mass region of $B^\pm \rightarrow K^+ K^- \pi^\pm$ [4]. The fact that the $K\bar{K}$ system of $B^\pm \rightarrow K_S^0 K_S^0 h^\pm$ ($h = K, \pi$), in contrast to that of $B^\pm \rightarrow K^+ K^- h^\pm$, cannot form a vector resonance (Bose symmetry) may shed light on the source of large CP violation in the latter decays.

The three-body decay $B^+ \rightarrow K_S^0 K_S^0 K^+$ [5] was observed by Belle [6] and subsequently studied by BABAR [7]. Belle measured the decay branching fraction as $(13.4 \pm 1.9 \pm 1.5) \times 10^{-6}$ based on a data sample of 70 fb^{-1} [6], and BABAR reported a branching fraction of $(10.6 \pm 0.5 \pm 0.3) \times 10^{-6}$ and a CP asymmetry of $(+4_{-5}^{+4} \pm 2)\%$ using

426 fb^{-1} of data [7]. The quoted uncertainties are statistical and systematic, respectively.

The decay $B^+ \rightarrow K_S^0 K_S^0 \pi^+$ is suppressed by the squared ratio of CKM matrix [8] elements $|V_{td}/V_{ts}|^2 (= 0.046)$ with respect to $B^+ \rightarrow K_S^0 K_S^0 K^+$, and has not yet been observed. The most restrictive limit at 90% confidence level on its branching fraction, $\mathcal{B}(B^+ \rightarrow K_S^0 K_S^0 \pi^+) < 5.1 \times 10^{-7}$, comes from BABAR [9].

We present an improved measurement of the branching fraction and direct CP asymmetry of the decay $B^+ \rightarrow K_S^0 K_S^0 K^+$ as well as a search for $B^+ \rightarrow K_S^0 K_S^0 \pi^+$ using a data sample of 711 fb^{-1} , which contains $772 \times 10^6 B\bar{B}$ pairs and was recorded near the $\Upsilon(4S)$ resonance with the Belle detector [10] at the KEKB e^+e^- collider [11]. The direct CP asymmetry is defined as

$$\mathcal{A}_{CP} = \frac{N(B^- \rightarrow K_S^0 K_S^0 h^-) - N(B^+ \rightarrow K_S^0 K_S^0 h^+)}{N(B^- \rightarrow K_S^0 K_S^0 h^-) + N(B^+ \rightarrow K_S^0 K_S^0 h^+)}, \quad (1)$$

where N is the obtained signal yield for the corresponding mode. The detector components relevant for our study are a silicon vertex detector (SVD), a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov

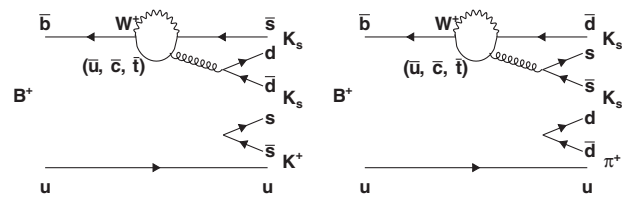


FIG. 1. Feynman diagrams of the dominant amplitudes that contribute to the decays $B^\pm \rightarrow K_S^0 K_S^0 K^\pm$ (left) and $B^\pm \rightarrow K_S^0 K_S^0 \pi^\pm$ (right).

counters (ACC), and a barrel-like arrangement of time-of-flight scintillation counters (TOF); all located inside a 1.5 T solenoidal magnetic field.

To reconstruct $B^+ \rightarrow K_S^0 K_S^0 h^+$ candidates, we begin by identifying charged kaons and pions. A kaon or pion candidate track must have a minimum transverse momentum of 100 MeV/c in the lab frame, and a distance of closest approach with respect to the interaction point (IP) of less than 0.2 cm in the transverse $r - \phi$ plane and less than 5.0 cm along the z axis. Here, the z axis is defined opposite the e^+ beam. Charged tracks are identified as kaons or pions based on a likelihood ratio $\mathcal{R}_{K/\pi} = \mathcal{L}_K / (\mathcal{L}_K + \mathcal{L}_\pi)$, where $\mathcal{L}_K$ and $\mathcal{L}_\pi$ are the individual likelihoods for kaons and pions, respectively, calculated with information from the CDC, ACC, and TOF. Tracks with $\mathcal{R}_{K/\pi} > 0.6$ are identified as kaons, while those with $\mathcal{R}_{K/\pi} < 0.4$ are identified as pions. The efficiency for kaon (pion) identification is 86% (91%), with a pion (kaon) misidentification rate of 9% (14%).

The K_S^0 candidates are reconstructed from pairs of oppositely charged tracks, both assumed to be pions, and are further subject to a selection [12] based on a neural network [13]. The network uses the following input variables: the K_S^0 momentum in the lab frame, the distance along the z axis between the two track helices at their closest approach, the K_S^0 flight length in the $r - \phi$ plane, the angle between the K_S^0 momentum and the vector joining the IP to the K_S^0 decay vertex, the angle between the pion momentum and the lab frame direction in the K_S^0 rest frame, the distances of closest approach in the $r - \phi$ plane between the IP and the two pion helices, the number of hits in the CDC for each pion track, and the presence or absence of hits in the SVD for each pion track. We require that the reconstructed invariant mass be between 491 and 505 MeV/c², corresponding to $\pm 3\sigma$ around the nominal K_S^0 mass [14], with σ denoting the experimental resolution.

We identify B -meson candidates using two kinematic variables: the beam-energy constrained mass, $M_{bc} = \sqrt{E_{\text{beam}}^2/c^4 - |\sum_i \vec{p}_i/c|^2}$, and the energy difference, $\Delta E = \sum_i E_i - E_{\text{beam}}$, where E_{beam} is the beam energy, and $\vec{p}_i$ and E_i are the momentum and energy of the i th daughter of the reconstructed B candidate, all calculated in the center-of-mass frame. For each B candidate, we perform a fit constraining its daughters to come from a common vertex, whose position is consistent with the IP profile. Events with $5.271 \text{ GeV}/c^2 < M_{bc} < 5.287 \text{ GeV}/c^2$ and $-0.10 \text{ GeV} < \Delta E < 0.15 \text{ GeV}$ are retained for further analysis. The M_{bc} requirement corresponds approximately to a $\pm 3\sigma$ window around the nominal B^+ mass [14]. We apply a looser (-6σ , $+9\sigma$) requirement on ΔE , as it is later used to extract the signal yield.

The average number of B candidates per event is 1.1 (1.5) for $B^+ \rightarrow K_S^0 K_S^0 K^+$ ($K_S^0 K_S^0 \pi^+$). In the case of multiple candidates, we choose the one with the minimum

χ^2 value for the aforementioned vertex fit. This criterion selects the correct B -meson candidate in 75% and 63% of Monte Carlo (MC) events having more than one candidate in $B^+ \rightarrow K_S^0 K_S^0 K^+$ and $B^+ \rightarrow K_S^0 K_S^0 \pi^+$, respectively.

The dominant background arises from the $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) continuum process. We use observables based on event topology to suppress it. The event shape in the c.m. frame is expected to be spherical for $B\bar{B}$ events, whereas continuum events are jetlike. We employ a neural network based on NeuroBayes [13] to separate signal from background using the following six input variables: a Fisher discriminant formed from 16 modified Fox-Wolfram moments [15], the cosine of the angle between the B momentum and the z axis, the cosine of the angle between the B thrust and the z axis, the cosine of the angle between the thrust axis of the B candidate and that of the rest of the event, the ratio of the second- to the zeroth-order Fox-Wolfram moments, and the vertex separation along the z axis between the B candidate and the remaining tracks. The first five quantities are calculated in the c.m. frame. The neural network training is performed with simulated signal and $q\bar{q}$ samples each containing 30 000 events after all selection requirements. Using MC events that are independent of the ones used for training, we verify that the network is not overtrained. Signal and background samples are generated with the EvtGen program [16]; for signal we assume a uniform decay in phase space. A GEANT-based [17] simulation is used to model the detector response.

We require the neural network output (C_{NB}) to be greater than -0.2 to substantially reduce the continuum background. For both decays, the relative signal efficiency due to this requirement is approximately 91%, and the achieved continuum suppression is close to 84%. The remainder of the C_{NB} distribution strongly peaks near 1.0 for signal, making it challenging to model it analytically. However, its transformed variable

$$C'_{\text{NB}} = \ln \left[\frac{C_{\text{NB}} - C_{\text{NB,min}}}{C_{\text{NB,max}} - C_{\text{NB}}} \right], \quad (2)$$

where $C_{\text{NB,min}} = -0.2$ and $C_{\text{NB,max}} \simeq 1.0$, can be parametrized by one or more Gaussian functions. We use C'_{NB} as a fit variable along with ΔE .

The background due to charmed B decays, mediated via the dominant $b \rightarrow c$ transition, is studied with an MC sample. The resulting ΔE and M_{bc} distributions are found to peak in the signal region for both $B^+ \rightarrow K_S^0 K_S^0 K^+$ and $B^+ \rightarrow K_S^0 K_S^0 \pi^+$ decays. For $B^+ \rightarrow K_S^0 K_S^0 K^+$, the peaking background predominantly stems from $B^+ \rightarrow D^0 K^+$ with $D^0 \rightarrow K_S^0 K_S^0$ and from $B^+ \rightarrow \chi_{c0}(1P) K^+$ with $\chi_{c0}(1P) \rightarrow K_S^0 K_S^0$. To suppress these backgrounds, we exclude candidates for which $M_{K_S^0 K_S^0}$ lies in the range $[1.85, 1.88] \text{ GeV}/c^2$ or $[3.38, 3.45] \text{ GeV}/c^2$, corresponding to a $\pm 3\sigma$ window around the nominal D^0 or $\chi_{c0}(1P)$ mass [14], respectively.

In the case of $B^+ \rightarrow K_S^0 K_S^0 \pi^+$, the peaking background largely arises from $B^+ \rightarrow D^0 \pi^+$ with $D^0 \rightarrow K_S^0 K_S^0$. To suppress it, we exclude candidates for which $M_{K_S^0 K_S^0}$ lies in the aforementioned D^0 mass window. The relative loss of signal efficiency due to these charm vetoes is 3% (1%) for $B^+ \rightarrow K_S^0 K_S^0 K^+$ ($K_S^0 K_S^0 \pi^+$).

A few background modes contribute in the M_{bc} signal region, but having their ΔE peak shifted from zero to the positive side for $B^+ \rightarrow K_S^0 K_S^0 K^+$ or to the negative side for $B^+ \rightarrow K_S^0 K_S^0 \pi^+$. To identify these so-called “feed-across” backgrounds, mostly arising due to $K - \pi$ misidentification, we use a $B\bar{B}$ MC sample in which one of the B mesons decays via $b \rightarrow u, d, s$ transitions, along with the charmed $B\bar{B}$ sample. For $B^+ \rightarrow K_S^0 K_S^0 \pi^+$, the feed-across background includes contributions from $B^+ \rightarrow K_S^0 K_S^0 K^+$ as well as $B^+ \rightarrow D^0 K^+$ and $B^+ \rightarrow \chi_{c0}(1P)K^+$ that survive the D^0 and $\chi_{c0}(1P)$ vetoes. For $B^+ \rightarrow K_S^0 K_S^0 K^+$, it comes entirely from $B^+ \rightarrow K_S^0 K_S^0 \pi^+$. All other events coming from neither the signal, the continuum, nor the feed-across components form the so-called “combinatorial” $B\bar{B}$ background.

After all selection requirements, the efficiencies for correctly reconstructed signal events are 24% for $B^+ \rightarrow K_S^0 K_S^0 K^+$ and 26% for $B^+ \rightarrow K_S^0 K_S^0 \pi^+$. The fractions of misreconstructed signal events for which one of the daughter particles comes from the other B -meson decay are 0.5% for $B^+ \rightarrow K_S^0 K_S^0 K^+$ and 1.1% for $B^+ \rightarrow K_S^0 K_S^0 \pi^+$. We consider these events as part of the signal.

The signal yield and $\mathcal{A}_{CP}$ are obtained with an unbinned extended maximum likelihood fit to the two-dimensional distribution of ΔE and C'_{NB} . The extended likelihood function is

$$\mathcal{L} = \frac{e^{-\sum_j n_j}}{N!} \prod_i \left[\sum_j n_j \mathcal{P}_j^i \right], \quad (3)$$

where

$$\mathcal{P}_j^i \equiv \frac{1}{2} (1 - q^i \mathcal{A}_{CP,j}) \times \mathcal{P}_j(\Delta E^i) \times \mathcal{P}_j(C'_{NB}^i). \quad (4)$$

Here, N is the total number of events, i is the event index, and n_j is the yield of the event category j ($j \equiv$ signal, $q\bar{q}$,

TABLE I. List of PDFs used to model the ΔE and C'_{NB} distributions for various event categories for $B^+ \rightarrow K_S^0 K_S^0 K^+$. “G,” “AG,” and “Poly1” denote Gaussian, asymmetric Gaussian, and first-order polynomial, respectively.

Event category	ΔE	C'_{NB}
Signal	3 G	G + AG
Continuum $q\bar{q}$	Poly1	2 G
Combinatorial $B\bar{B}$	Poly1	2 G
Feed-across	G + Poly1	G

combinatorial, and feed-across). $\mathcal{P}_j$ and $\mathcal{A}_{CP,j}$ are the probability density function (PDF) and the direct CP asymmetry corresponding to the category j , and q^i is the electric charge of the B candidate in event i . As the correlation between ΔE and C'_{NB} is small (the linear correlation coefficient ranges from 0.5% to 7.0%), the product of two individual PDFs is a good approximation for the total PDF. We apply a tight requirement on M_{bc} instead

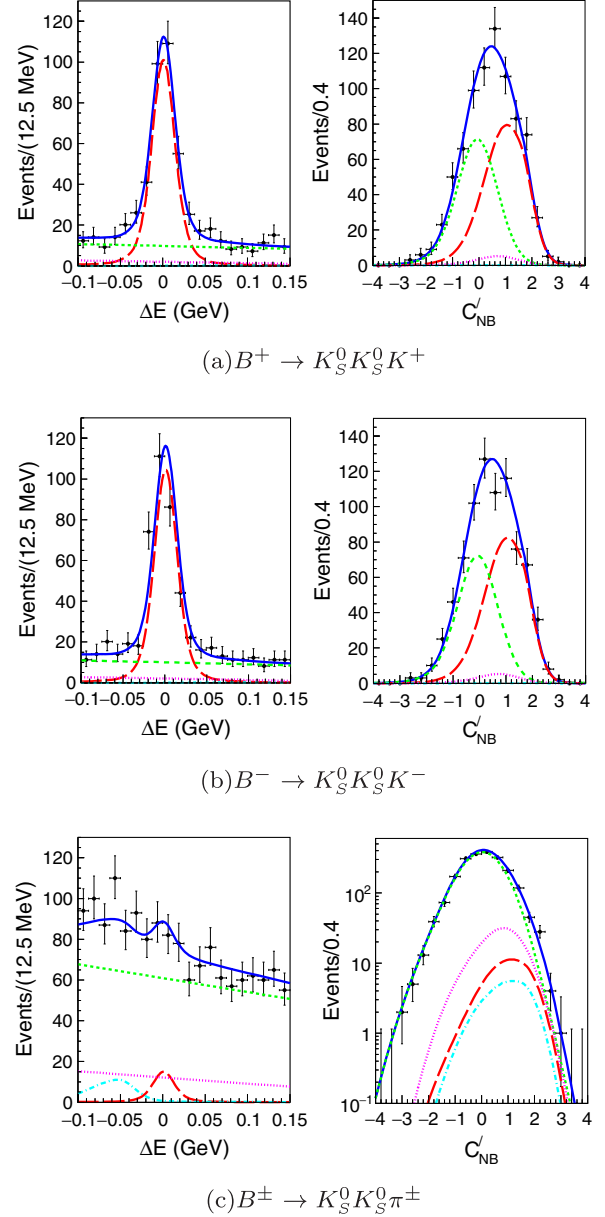


FIG. 2. Projections of the two-dimensional simultaneous fit to ΔE for $C'_{NB} > 0.0$ and C'_{NB} for $|\Delta E| < 50$ MeV. Black points with error bars are the data, solid blue curves are the total PDF, long-dashed red curves are the signal, dashed green curves are the continuum background, dotted magenta curves are the combinatorial $B\bar{B}$ background, and dash-dotted cyan curves are the feed-across background.

TABLE II. Efficiency, differential branching fraction, and $\mathcal{A}_{CP}$ in each $M_{K_S^0 K_S^0}$ bin for $B^+ \rightarrow K_S^0 K_S^0 K^+$.

$M_{K_S^0 K_S^0}$ (GeV/c ²)	Efficiency (%)	$d\mathcal{B}/dM \times 10^{-6}$ (c ² /GeV)	$\mathcal{A}_{CP}$ (%)
1.0–1.1	24.0 ± 0.4	$10.40 \pm 1.24 \pm 0.38$	$-3.9 \pm 10.9 \pm 0.9$
1.1–1.3	23.4 ± 0.2	$8.60 \pm 0.85 \pm 0.32$	$-0.1 \pm 9.3 \pm 0.9$
1.3–1.6	22.9 ± 0.1	$10.23 \pm 0.73 \pm 0.38$	$+6.6 \pm 6.9 \pm 0.9$
1.6–2.0	21.8 ± 0.1	$3.93 \pm 0.43 \pm 0.15$	$+16.1 \pm 10.3 \pm 0.9$
2.0–2.3	24.1 ± 0.1	$3.90 \pm 0.47 \pm 0.15$	$-3.3 \pm 11.3 \pm 0.9$
2.3–2.7	25.2 ± 0.1	$2.45 \pm 0.33 \pm 0.09$	$-5.7 \pm 12.2 \pm 1.0$
2.7–5.0	26.3 ± 0.0	$0.35 \pm 0.07 \pm 0.01$	$-31.9 \pm 19.7 \pm 1.2$

of including it as a fit variable, since it exhibits a large correlation with ΔE for the signal and feed-across background. We choose ΔE over M_{bc} in the fit because the former is a better variable to distinguish signal from feed-across background. To account for crossfeed between the two channels, they are fitted simultaneously, with the $B^+ \rightarrow K_S^0 K_S^0 K^+$ branching fraction in the correctly reconstructed sample determining the normalization of the crossfeed in the $B^+ \rightarrow K_S^0 K_S^0 \pi^+$ fit region, and vice versa.

Table I lists the PDFs used to model the ΔE and C'_{NB} distributions for various event categories for $B^+ \rightarrow K_S^0 K_S^0 K^+$. For $B^+ \rightarrow K_S^0 K_S^0 \pi^+$, we use the same PDF shapes except for the feed-across background component, where we add an asymmetric Gaussian function to the PDFs in Table I to accurately describe ΔE and C'_{NB} distributions. The free parameters in the fit are the continuum background yields and the branching fractions of $B^+ \rightarrow K_S^0 K_S^0 K^+$ and $B^+ \rightarrow K_S^0 K_S^0 \pi^+$, and the signal $\mathcal{A}_{CP}$ for $B^+ \rightarrow K_S^0 K_S^0 K^+$. In addition, the following PDF shape parameters of the continuum background are floated in the fit for both $B^+ \rightarrow K_S^0 K_S^0 K^+$ and $K_S^0 K_S^0 \pi^+$: the slope of the first-order polynomial used for ΔE and the mean and width of the dominant Gaussian component used to model C'_{NB} . The combinatorial $B\bar{B}$ yields are fixed to the MC values due to their correlation with the continuum yields. This is because C'_{NB} is the only variable that offers some discrimination between the two background categories. To improve the overall fit stability, $\mathcal{A}_{CP}$ for all components but for the $B^+ \rightarrow K_S^0 K_S^0 K^+$ signal are fixed to zero. The other PDF shape parameters for signal and background components are fixed to the corresponding MC expectations for both decays. We correct the signal ΔE and C'_{NB} PDF shapes for possible data-MC differences, according to the values obtained with a control sample of $B^+ \rightarrow \bar{D}^0 \pi^+$ with $\bar{D}^0 \rightarrow K_S^0 \pi^+ \pi^-$. The same correction factors are also applied for the feed-across background component of $B^+ \rightarrow K_S^0 K_S^0 \pi^+$.

We determine the branching fraction as

$$\mathcal{B}(B^+ \rightarrow K_S^0 K_S^0 h^+) = \frac{n_{\text{sig}}}{\epsilon \times N_{B\bar{B}} \times [\mathcal{B}(K_S^0 \rightarrow \pi^+ \pi^-)]^2}, \quad (5)$$

where n_{sig} , ϵ , and $N_{B\bar{B}}$ are the total signal yield, average detection efficiency, and number of $B\bar{B}$ pairs, respectively. Figure 2 shows signal-enhanced ΔE and C'_{NB} projections of the separate fit to B^+ and B^- samples for $B^+ \rightarrow K_S^0 K_S^0 K^+$ and of the charge-combined fit for $B^+ \rightarrow K_S^0 K_S^0 \pi^+$. For $B^+ \rightarrow K_S^0 K_S^0 \pi^+$, we fit a total of 5103 candidate events to obtain a branching fraction of

$$\mathcal{B}(B^+ \rightarrow K_S^0 K_S^0 \pi^+) = (6.5 \pm 2.6 \pm 0.4) \times 10^{-7}, \quad (6)$$

where the first uncertainty is statistical and the second is systematic (described below). Its signal significance is estimated as $\sqrt{-2 \ln(\mathcal{L}_0/\mathcal{L}_{\text{max}})}$, where $\mathcal{L}_0$ and $\mathcal{L}_{\text{max}}$ are the likelihood values for the fit with the branching fraction fixed to zero and for the best-fit case, respectively. Including systematic uncertainties by convolving the likelihood with a Gaussian function of width equal to the systematic uncertainty, we determine the significance to be 2.5 standard deviations. In view of the significance being less than 3 standard deviations, we set an upper limit on the branching fraction of $B^+ \rightarrow K_S^0 K_S^0 \pi^+$. We integrate the convolved likelihood over the branching fraction to obtain the upper limit of 8.7×10^{-7} at 90% confidence level. This limit is similar to that of *BABAR* [9].

For $B^+ \rightarrow K_S^0 K_S^0 K^+$, we perform the fit for 2709 candidate events in seven unequal bins of $M_{K_S^0 K_S^0}$ to decipher contributions from possible quasi-two-body

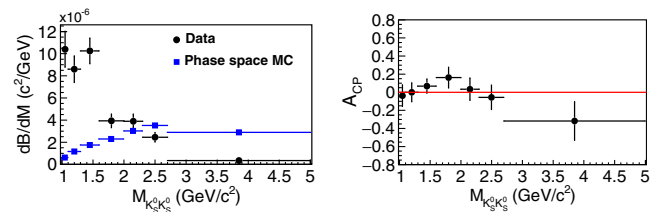


FIG. 3. Differential branching fraction (left) and $\mathcal{A}_{CP}$ (right) as functions of $M_{K_S^0 K_S^0}$ for $B^+ \rightarrow K_S^0 K_S^0 K^+$. Black points with error bars are the results from the two-dimensional fits to data and include systematic uncertainties. Blue squares in the left plot show the expectation from a phase-space MC sample, and the red line in the right plot indicates a zero CP asymmetry.

TABLE III. Systematic uncertainties in the branching fraction of $B^+ \rightarrow K_S^0 K_S^0 \pi^+$.

Source	Relative uncertainty in $\mathcal{B}$ (%)
Tracking	0.35
Particle identification	0.80
Number of $B\bar{B}$ pairs	1.37
Continuum suppression	0.34
Requirement on M_{bc}	0.03
K_S^0 reconstruction	3.22
Fit bias	1.86
Signal PDF	1.30
Combinatorial $B\bar{B}$ PDF	+1.31, -1.98
Feed-across PDF	+3.57, -4.10
Fixed background yield	+2.63, -2.27
Fixed background $\mathcal{A}_{CP}$	0.50
Total	+6.30, -6.67

resonances. The efficiency, differential branching fraction, and $\mathcal{A}_{CP}$ thus obtained are listed in Table II. Figure 3 shows the differential branching fraction and $\mathcal{A}_{CP}$ plotted as a function of $M_{K_S^0 K_S^0}$. We observe an excess of events around 1.5 GeV/ c^2 beyond the expectation of a phase-space MC

sample. No significant evidence for CP asymmetry is found in any of the bins. Upon inspection, no peaking structure beyond kinematic reflection is seen in the $M_{K_S^0 K^+}$ distribution. We calculate the branching fraction by integrating the differential branching fraction over the entire $M_{K_S^0 K_S^0}$ range:

$$\mathcal{B}(B^+ \rightarrow K_S^0 K_S^0 K^+) = (10.42 \pm 0.43 \pm 0.22) \times 10^{-6}, \quad (7)$$

where the first uncertainty is statistical and the second is systematic. The $\mathcal{A}_{CP}$ over the full $M_{K_S^0 K_S^0}$ range is

$$\mathcal{A}_{CP}(B^+ \rightarrow K_S^0 K_S^0 K^+) = (+1.6 \pm 3.9 \pm 0.9)\%. \quad (8)$$

This is obtained by weighting the $\mathcal{A}_{CP}$ value in each bin with the obtained branching fraction in that bin. As the statistical uncertainties are bin independent, their total contribution is a quadratic sum. For the systematic uncertainties, the contributions from the bin-correlated sources are linearly added, and those from the bin-uncorrelated sources are added in quadrature. The results agree with BABAR [7], which reported an $\mathcal{A}_{CP}$ consistent with zero

TABLE IV. Systematic uncertainties in the differential branching fraction and $\mathcal{A}_{CP}$ in $M_{K_S^0 K_S^0}$ bins for $B^+ \rightarrow K_S^0 K_S^0 K^+$. “†” indicates that the uncertainty is independent of $M_{K_S^0 K_S^0}$, with the listed value being applicable for all the bins. An ellipsis indicates a value below 0.05% in $d\mathcal{B}/dM$ and below 0.001% in $\mathcal{A}_{CP}$.

$M_{K_S^0 K_S^0}$ (GeV/ c^2)	1.0–1.1	1.1–1.3	1.3–1.6	1.6–2.0	2.0–2.3	2.3–2.7	2.7–5.0
Source	Relative uncertainty in $d\mathcal{B}/dM$ (%)						
Tracking†				0.35			
Particle identification†				0.80			
Number of $B\bar{B}$ pairs†				1.37			
Continuum suppression†				0.34			
Requirement on M_{bc} †				0.03			
K_S^0 reconstruction†				3.22			
Fit bias†				0.53			
Signal PDF	+0.33 -0.27	+0.63 -0.48	+0.46 -0.44	+0.22 -0.63	+0.52 -0.38	0.67	1.10
Combinatorial $B\bar{B}$ PDF	0.09	+0.08 -0.13	0.12	+0.17 -0.21	+0.26 -0.34	0.40	0.40
Feed-across PDF	...	...	...	...	...	...	...
Fixed background yield	...	0.10	0.10	0.23	...	0.11	0.60
Fixed background $\mathcal{A}_{CP}$	...	...	...	0.20	0.10	...	0.13
Total	±3.68	±3.72	±3.69	±3.73	±3.72	±3.75	±3.89
$M_{K_S^0 K_S^0}$ (GeV/ c^2)	1.0–1.1	1.1–1.3	1.3–1.6	1.6–2.0	2.0–2.3	2.3–2.7	2.7–5.0
Source	Absolute uncertainty in $\mathcal{A}_{CP}$						
Signal PDF	0.001	0.002	0.001	0.002	0.001	0.001	0.004
Combinatorial $B\bar{B}$ PDF	0.001	0.001	0.001	...	0.001	0.002	0.001
Feed-across PDF	...	...	...	...	...	...	...
Fixed background yield	...	...	0.001	0.001	0.001	0.001	0.004
Fixed background $\mathcal{A}_{CP}$	...	...	0.001	0.001	0.001	0.002	0.006
Detector bias†				0.009			
Total	±0.009	±0.009	±0.009	±0.009	±0.009	±0.010	±0.012

as well as the presence of quasi-two-body resonances $f_0(980)$, $f_0(1500)$, and $f'_2(1525)$ in the low $M_{K_S^0 K_S^0}$ region.

Major sources of systematic uncertainty in the branching fractions are similar for both $B^+ \rightarrow K_S^0 K_S^0 K^+$ and $K_S^0 K_S^0 \pi^+$ decays. These are listed along with their contributions in Tables III and IV. We use partially reconstructed $D^{*+} \rightarrow D^0 \pi^+$ with $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decays to assign the systematic uncertainty due to charged-track reconstruction (0.35% per track). The $D^{*+} \rightarrow D^0 \pi^+$ with $D^0 \rightarrow K^- \pi^+$ sample is used to determine the systematic uncertainty due to particle identification. The uncertainty due to the number of $B\bar{B}$ pairs is 1.37%. The uncertainties due to continuum suppression and M_{bc} requirements are estimated with the control sample of $B^+ \rightarrow \bar{D}^0 \pi^+$ with $\bar{D}^0 \rightarrow K_S^0 \pi^- \pi^+$. The uncertainty arising due to K_S^0 reconstruction is estimated from $D^0 \rightarrow K_S^0 K_S^0$ decays [18]. A potential fit bias is checked by performing an ensemble test comprising 1000 pseudoexperiments in which signal events are drawn from the corresponding MC sample and background events are generated according to their PDF shapes. The uncertainties due to signal PDF shape are estimated by varying the correction factors by $\pm 1\sigma$ of their statistical uncertainty. Similarly, the uncertainties due to background PDF shape are calculated by varying all fixed parameters by $\pm 1\sigma$. We evaluate the uncertainty due to fixed background yields by varying them up and down by 20% of their MC values. The uncertainty due to fixed background $\mathcal{A}_{CP}$ is estimated by varying the $\mathcal{A}_{CP}$ values up and down by one unit of their statistical uncertainties. As for a possible systematics due to efficiency variation across the Dalitz plot in the $B^+ \rightarrow K_S^0 K_S^0 \pi^+$ channel, we find its impact to be negligible.

Systematic uncertainties in $\mathcal{A}_{CP}$ are listed in Table IV. The systematic uncertainties due to the PDF modeling, fixed background yields, and $\mathcal{A}_{CP}$ are estimated with the same procedure as for the branching fraction. Uncertainties due to the intrinsic detector bias on charged particle detection are evaluated with the samples of $D^+ \rightarrow \phi \pi^+$ and $D_s^+ \rightarrow \phi \pi^+$ in conjunction with $D^0 \rightarrow K^- \pi^+$ [19]. The total systematic uncertainty is calculated by summing all individual contributions in quadrature.

In summary, we have reported measurements of the charmless three-body decays $B^+ \rightarrow K_S^0 K_S^0 K^+$ and $B^+ \rightarrow K_S^0 K_S^0 \pi^+$ using the full $\Upsilon(4S)$ data sample collected with the Belle detector. We perform a two-dimensional simultaneous fit to extract the signal yields of both decays. For $B^+ \rightarrow K_S^0 K_S^0 \pi^+$, a 90% confidence-level upper limit is set on the branching fraction at 8.7×10^{-7} . We measure the branching fraction and $\mathcal{A}_{CP}$ of $B^+ \rightarrow K_S^0 K_S^0 K^+$ to be $\mathcal{B}(B^+ \rightarrow K_S^0 K_S^0 K^+) = (10.42 \pm 0.43 \pm 0.22) \times 10^{-6}$ and $\mathcal{A}_{CP}(B^+ \rightarrow K_S^0 K_S^0 K^+) = (+1.6 \pm 3.9 \pm 0.9)\%$. These

results supersede Belle's earlier measurements [6] and are consistent with those of BABAR [7,9].

We thank the KEKB group for the excellent operation of the accelerator, the KEK cryogenics group for the efficient operation of the solenoid, the KEK computer group and the Pacific Northwest National Laboratory (PNNL) Environmental Molecular Sciences Laboratory (EMSL) computing group for strong computing support, and the National Institute of Informatics and Science Information NETwork 5 (SINET5) for valuable network support. We acknowledge support from the Ministry of Education, Culture, Sports, Science, and Technology (MEXT) of Japan, the Japan Society for the Promotion of Science (JSPS), and the Tau-Lepton Physics Research Center of Nagoya University; the Australian Research Council including Grants No. DP180102629, No. DP170102389, No. DP170102204, No. DP150103061, and No. FT130100303; the Austrian Science Fund under Grant No. P 26794-N20; the National Natural Science Foundation of China under Contracts No. 11435013, No. 11475187, No. 11521505, No. 11575017, No. 11675166, and No. 11705209; the Key Research Program of Frontier Sciences, Chinese Academy of Sciences (CAS) under Grant No. QYZDJ-SSW-SLH011; the CAS Center for Excellence in Particle Physics (CCEPP); the Shanghai Pujiang Program under Grant No. 18PJ1401000; the Ministry of Education, Youth and Sports of the Czech Republic under Contract No. LTT17020; the Carl Zeiss Foundation, the Deutsche Forschungsgemeinschaft, the Excellence Cluster Universe, and the VolkswagenStiftung; the Department of Science and Technology of India; the Istituto Nazionale di Fisica Nucleare of Italy; National Research Foundation (NRF) of Korea under Grants No. 2015H1A2A1033649, No. 2016R1D1A1B01010135, No. 2016K1A3A7A09005 603, No. 2016R1D1A1B02012900, No. 2018R1A2B3003 643, No. 2018R1A6A1A06024970, and No. 2018R1D1A1B07047294; the Radiation Science Research Institute, Foreign Large-size Research Facility Application Supporting project; the Global Science Experimental Data Hub Center of the Korea Institute of Science and Technology Information and KREONET/GLORIAD; the Polish Ministry of Science and Higher Education and the National Science Center; the Grant of the Russian Federation government, Agreement No. 14.W03.31.0026; the Slovenian Research Agency; Ikerbasque, Basque Foundation for Science, Spain; the Swiss National Science Foundation; the Ministry of Education and the Ministry of Science and Technology of Taiwan; and the United States Department of Energy and the National Science Foundation.

- [1] A. Bevan *et al.*, *Eur. Phys. J. C* **74**, 3026 (2014).
- [2] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. D* **90**, 112004 (2014).
- [3] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **112**, 011801 (2014).
- [4] C.-L. Hsu *et al.* (Belle Collaboration), *Phys. Rev. D* **96**, 031101(R) (2017).
- [5] Inclusion of charge-conjugate reactions are implicit unless stated otherwise.
- [6] A. Garmash *et al.* (Belle Collaboration), *Phys. Rev. D* **69**, 012001 (2004).
- [7] J. P. Lees *et al.* (BABAR Collaboration), *Phys. Rev. D* **85**, 112010 (2012).
- [8] N. Cabibbo, *Phys. Rev. Lett.* **10**, 531 (1963); M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).
- [9] B. Aubert *et al.* (BABAR Collaboration), *Phys. Rev. D* **79**, 051101(R) (2009).
- [10] A. Abashian *et al.* (Belle Collaboration), *Nucl. Instrum. Methods Phys. Res., Sect. A* **479**, 117 (2002); also see the detector section in J. Brodzicka *et al.*, *Prog. Theor. Exp. Phys.* **2012**, 04D001 (2012).
- [11] S. Kurokawa and E. Kikutani, *Nucl. Instrum. Methods Phys. Res., Sect. A* **499**, 1 (2003), and other papers included in this volume; T. Abe *et al.*, *Prog. Theor. Exp. Phys.* **2013**, 03A001 (2013) and following articles up to 03A011.
- [12] H. Nakano, Ph.D. thesis, Tohoku University, 2014, Chap. 4, unpublished, https://tohoku.repo.nii.ac.jp/?action=pages_view_main&active_action=repository_view_main_item_detail&item_id=70563&item_no=1&page_id=33&block_id=38.
- [13] M. Feindt and U. Kerzel, *Nucl. Instrum. Methods Phys. Res., Sect. A* **559**, 190 (2006).
- [14] M. Tanabashi *et al.* (Particle Data Group), *Phys. Rev. D* **98**, 030001 (2018).
- [15] S. H. Lee *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **91**, 261801 (2003).
- [16] D. J. Lange, *Nucl. Instrum. Methods Phys. Res., Sect. A* **462**, 152 (2001).
- [17] R. Brun *et al.*, CERN Report No. DD/EE/81-1, 1984.
- [18] N. Dash *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **119**, 171801 (2017).
- [19] M. Starič *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **108**, 071801 (2012).