ASSOCIATIONS BETWEEN THE ONTOGENESIS OF CONFIDENCE AND INCLINATION TO EXPLORE UNFAMILIAR MATHEMATICAL PROBLEMS

Gaye Williams

Deakin University

This video-stimulated post-lesson interview study of students displaying confidence in mathematics examines the nature of confidence theoretically by linking it to Seligman's (1995) indicators of optimism. It also explores the activity of confident students empirically; examining their inclination to explore unfamiliar challenging mathematics problems. Findings include associations between student inclination to explore challenging mathematics problems, and the ontogenesis of their confidence. These findings have implications for the teaching of mathematics: 'a transmissive teaching approach' was associated with an absence of the inclination to explore.

INTRODUCTION

The Melbourne Declaration of Educational Goals for Young Australians http://www.mceecdya.edu.au/mceecdya/melbourne_declaration,25979.html and the Australian Mathematics Curriculum (see http://www.australiancurriculum.edu.au/ Mathematics/Rationale) require the development of creative, innovative and resourceful problem solvers. Confidence has previously been linked to mathematical performance (see for example, Fennema & Sherman, 1976). Such links have been stronger between students' performances on multiple-choice questions than performance on open-ended mathematical tasks (Pajares & Miller, 1997). It is time to take a closer look at confidence as a construct to find whether and how confidence is exhibited differently, and whether only some types of confidence are associated with an inclination to explore unfamiliar challenging problems. Williams (2012) identified confident students who do perform well on open-ended problem solving tasks, and confident students who do not. The confident student who was not inclined to explore performed well on tests requiring recall of rules and procedures. This paper examines the confidence and the problem solving activity of five high performing final year elementary students. The research questions are: Are there differences in the nature of the confidence these students exhibit? And if so, are their associations between the type of confidence exhibited and student inclination to explore? This focus is important because problem-solving activity can 'deepen' mathematical understandings (Cobb, Wood, Yackel, & McNeal, 1992; Williams, 2005).

THEORETICALLY FRAMING THIS STUDY

'Optimism' is an orientation to 'failures' and 'successes' (Seligman, 1995). Optimistic children perceive failure as 'temporary' (able to be overcome), 'specific' (to the situation at hand), and 'external' (can be associated with factors beyond their control). They perceive successes as personal (achieved through their own effort), permanent

2013. In Lindmeier, A. M. & Heinze, A. (Eds.). *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4, pp. 393-400. Kiel, Germany: PME.

(able to be achieved again), and pervasive (internalized as characteristics of self: 'I did this, I am good at this').

With regard to optimistic activity specific to mathematical problem-solving successes ('optimistic problem-solving activity'), 'failure' is taken to be 'not knowing', and 'success' as 'finding out'. An optimistic problem solver perceives *not knowing* as temporary [Failure as Temporary] and able to be overcome through personal effort [Success as Personal] associated with looking into the situation of failure, and identifying what they can change [Failure as Specific] and what is outside their control [Failure as External], to help decide what to vary to increase their likelihood of success [Failure as Specific]. They perceive they can achieve such successes again [Success as 'Permanent'] because they internalize their successes as a characteristic of self [Success as 'Pervasive'].

'Confidence' has previously been defined as the "degree to which a person feels certain of her or his ability to learn and perform well in mathematics" (Hart, 1999, p. 243) [Success as Permanent], and related to a personal characteristic: "… one's ability to learn and to perform well on mathematical tasks" (Fennema & Sherman, 1976, p. 326) [Success as Pervasive]. When considered from the perspective of optimistic indicators, confidence is thus consistent with perceptions Success as Permanent and Success as Pervasive.

As optimistic students are inclined to explore unfamiliar mathematical ideas (Williams, 2005), and a confident student possesses the pair of optimistic indicators (Success as Permanent, Success as Pervasive), this study explores the combinations of other indicators of optimism possessed by confident students to see whether the combination of optimistic characteristics possessed provides insights into differences in the nature of confidence.

RESEARCH DESIGN

The study is part of a broader study of the role of optimism in collaborative problem solving and whether building optimism leads to increased problem-solving capacity. Data selected for this smaller study relates to five students (Patrick, Eliza, Sam, Aisha, and Hank. Williams 2007 and 2008 include some of the data used here for Sam. Eliza, and Patrick but the data is used for a different purpose in the present study. Hank and Aisha have been included as two more confident students who were not inclined to explore. All five of these students were high achieving students on their usual mathematics tests in class, with Hank, Sam, and Aisha in general achieving higher performances on these tests than Eliza and Patrick. The students were in various Grade 6 classes in the same school. Their usual mathematics tests for these students, like the mathematics tests for students in many Australian schools, were predominantly tests about recalling rules and procedures rather than about using the mathematics they have learnt to undertake unfamiliar challenging problems. This following section describes the students selected, one of the problem solving task they undertook ('How Many Boxes'), the pedagogical approach employed, and the data collection instruments utilized including rationale for why they were appropriate for collecting the data to answer the research questions herein. The students selected were all confident but they differed in whether or not they were inclined to explore.

Students undertook three complex problem-solving tasks each year (six eighty-minute sessions) for one to three years within the broader study. Tasks were accessible through a variety of representations and levels of mathematical sophistication to give groups opportunities to idiosyncratically discover and explore complexities just beyond their present understandings. Priority was given to the selection of evidence within one task, the How Many Boxes Task for this study to limit the amount of space required for task description, and decrease the amount of information the reader needed to become familiar with to consider the data. A summary of the How Many Boxes Task is included in Figure 1.

Task Introduction: the features of rectangular prisms were discussed and these shapes were identified as the ‘boxes’ in this task.

Part 1: Groups were asked how many different solid boxes they could find that each contained 24 ‘little’ cubes (cubic centimetres but the term was not provided at that stage). As they worked with this task, they were asked questions like: How many can you make? How do you know that you have got them all? Can you make a mathematical argument for how you know you have got them all?

Part 2: Groups were told they were to participate in a game where each group was trying to be first to find the dimensions of a box given the number of cubes within. Each group could ask a Yes/No question and all groups would have access to the questions and answers before beginning to find the dimensions of the box. Students were given five minutes to brainstorm the types of questions they might ask and during this time they had access to twenty-four little cubic centimetre blocks.

Figure 1. Summary of How Many Boxes Task

The pedagogical approach employed, ‘Engaged to Learn’, was developed by the researcher (author), informed by her research (Williams, 2005), and her teaching (Williams, 2002). As teacher, the RT and classroom teacher (T) team-taught with the RT as primary implementer of the task. Students worked in small groups composed by RT advised by T (3-4 students) (see Williams, 2008). Students gave brief reports to the class at 5-10 minute intervals. The order in which the groups reported was decided by RT and T. RT and T did not affirm pathways taken nor ideas presented but rather asked questions to stimulate further thinking. For more information about the teaching and learning approach, see Williams (2007).

DATA COLLECTION TECHNIQUES

Four video cameras captured the activity of each group in class during their problem solving sessions. After each session, the worksheets and artefacts produced by each group were collected and used as additional stimuli during individual post-lesson video-stimulated student interviews undertaken individually with four students after each lesson. Video-stimulated interviews increase the validity of student reconstructive reports through focus on memory traces related to specific activity that occurred (Ericsson & Simon, 1980). In their interviews, students had simultaneous access to video of their group and the reporting sessions. They identified and discussed parts of the lesson that were important to them and reconstructed their thinking and feelings during those parts of the lesson. These interviews informed the analysis of lesson video by helping to locate the parts in the lesson where new ideas were developed and the processes through which this occurred, and providing information about how students learnt mathematics, and how they perceived themselves as learners. Students also answered questions designed specifically to provide dialogue to identify optimistic or non-optimistic perceptions. For example, the questions: “How do you think you are going in maths, and how do you decide?” “How do you learn something like that [mathematics associated with the problem solving task]?” and “Does anyone help you with maths at home/outside school? tended to elicit information about whether students perceived Success as Personal or External. Students displayed indicators of Success as Personal where they perceived learning as predominantly associated with personal effort in reorganisation and synthesis of previously developed ideas to create new mathematical ideas. Indicators of Success as Personal were also displayed when students primarily evaluated their mathematical performance internally rather than through external sources like test results, or teacher or parent evaluations. In contrast, where students relied primarily on external judgments of their performance and described the way they learnt as occurring through ‘taking in’ and repeating of knowledge from external sources, they displayed indicators of Success as External. These questions also elicited information about how students perceived their performances in relation to future performances, and in terms of characteristics of self. For example: “I am really good at maths because I always get high marks on tests” indicates Success as Pervasive “I am really good” and Success as Permanent “I always get”. Questions like “Can you tell me what you were thinking about there? And, how did you work that out?” can elicit data about how a student altered variables to increase the likelihood of success [Failure as Specific]. Indicators of optimism or lack thereof were also displayed when students discussed how others might consider them when they ‘got something wrong’ (e.g., “they might all think I am an idiot”) [Failure as Pervasive], or “they would know that calculation is tricky” [Failure as Specific]. Optimistic or non-optimistic indicators are not always evident in responses to a particular question, and the probes following each question depend upon the student’s response. Thus indicators of optimism can be found in various parts of the interview transcript depending on the responses students give, and the probes the researcher is able to introduce.

RESULTS

In Table 1, data drawn from interviews and lesson activity has been synthesised to develop a summary of how each student perceived learning occurred, and illustrate the nature of their responses to the ‘box’ task. A small selection of illustrations of the data is then presented to illustrate the type of analysis undertaken.

Student	What is Learning	Excerpts of Activity During Box Task
Sam	Listen to teacher, read text book, search on internet	Knew volume formula initially; generated many relevant sets of three factors; realised these were the box dimensions; at end of task, still did not know why multiplying these dimensions gave number of cubes in box.
Hank	No interview in Grade 6. T confirmed learnt from external sources.	Recognised factors of 24 pattern after two boxes were made. Generated as many sets of three factors of 24 as he could by considering only the numerical pattern. Disregarded other group members, and the RT when they asked 'why?' the pattern existed. By the end of the task, still had not worked out why that particular pattern helped give the number of little cubes in the box.
Aisha	Listen to a teacher, family member, and expert student, explaining bit by bit.	Generated two sets of two factors of 24 fast. When another member asked for explanation, she gave only the procedure. Listened intently to other groups linking their 3 factor solutions to the structure not just the dimensions of the box. Excitedly identified that there were an infinite number of possibilities using fractional and decimal triplets (with product 24) began to produce them but not link to box.
Patrick	Think about what others are still trying to work out, and about mistakes other groups make.	Reflected on why Sam had not recognised a member of his group was reporting on a 3x3x3 cubic box rather than a box containing 24 cubes. Wondered if they had missed the middle cube. Used ideas his group developed about layers of cubes in a box to solve a problem encountered by a group who had made a box with 24 cubes when they had intended to make one with 12 cubes within it.
Eliza	Think hard, puzzle it out, if 'stuck' ask parents to ask questions not give answer.	Relied heavily on building with cubes until she began to work out what was happening. Provided lateral contribution when group did not have sufficient cubes to make the box they wanted to explore: drew grids of layers of cubes they could use to finish making the block stack and work out how many were needed.

Table 1. Students, how they perceive learning, and responses to parts of Box Task

Table 1 shows that Sam, Hank, and Aisha perceived learning to involve the assistance of external resources or experts that provide information about relevant rules and procedures. These three students' performances with the 'box' task were consistent with this. They were unable to develop new ideas so remained within what they knew because there was not an 'expert' to rely on. Each used numerical procedures in ways they had previously used them, and generated long lists of examples of the number pattern they identified. They focused on the numerical representations completely (Aisha, Sam) or also linked the numbers generated to the rule previously known to identify the dimensions of the box (Hank). In class Hank was sure he had finished and that the RT just did not understand what he knew:

RT: [to Hank] … does the number pattern, which you beautifully explained yesterday, fit with (pause) those actual cubes in that box- … I don't just mean length width and height (pause) why when you multiply those together (pause) do you get (pause) the total number [of little cubes] (pause) in that box [RT leaves group] …

Hank [to group, fast and soft] factors are numbers that are multiple … you can multiply factors to get the number

The RT's comment (in transcript above) is intended to elicit thinking from Hank about the actual structure of the cubes in the boxes. Hank's response is a cyclical argument that draws on the definition of 'factor'. He is communicating that the dimensions must be multiplied to get 24 because they are factors of 24. Even though asked as part of the class, and in his group, on many occasions, Hank gave no thought to why it was that product of the dimensions gave the number of cubes in the box. Neither Aisha, or Sam, or Hank was able to link the numerical work they generated to the structure of the cubes in the boxes. This is illustrated with Hank's response to the RT's hint about the cross-section of the box whose dimensions they were finding Part 2 of the task:

1. RT … the cross section to it [the hidden box] has nine little squares in it
2. Hank [in his group writes/draws on page] … *hu* … *that's weird*!

The term 'cross section' had been explained earlier, and some groups were interpreting and using this clue. Hank's exclamations and his subsequent non-activity showed he was unable to use the information given. He was unaware of the structure of the cross section.

Patrick and Eliza on the other hand each described learning as an active process where they were making sense of ideas that became evident to them during their work with the task. In doing so they continually learnt more about the structure of the little cubes in the 'box'. The following comment made by Eliza in her interview captures some of her activity: "When I try to do things *in my mind* it is hard for me to figure it out 'til I really know how so the blocks help me to learn how to figure it out in my mind". Eliza illustrates the active nature of her engagement with the task and that she perceives *not knowing* as temporary and *finding out* more as something she can work out how to do. Later, before responding, she pauses and reflects on how she decides how she is going in maths:

> I know it's *not* because (pause) I get things right … I think it's because I … contribut- [hesitant] (pause) … rather than (pause) just agree and disagree … I actually (pause) say what I think (pause) …

Eliza further demonstrates her faith in her ability to think, and her ability to assess how well she is going with her mathematics internal interrogation (contributions she makes; that she is thinking deeply about ideas discussed) [Success as Personal]. Eliza contributed in several ways to the group's construction of new knowledge. It was her idea to use drawings on paper to represent more cubes. She also changed the orientation of the box (base 2x4) to produce four-layers-of-eight blocks to construct the box quickly. Eliza developed an understanding of the structure of the cubes in the 'box' to the extent of 'seeing' layers but not yet 'seeing' the base as an array.

DISCUSSION AND CONCLUSIONS

These cases illustrate associations between the ways students developed confidence and their inclination to explore new (to the student) mathematical ideas. Although Sam, Aisha, and Hank were confident of their ability to do mathematics, this confidence developed as a result of their demonstrated ability to reproduce rules and procedures they had been taught, and praise by others in relation to this ability. Although these three students possessed confidence [Success as Permanent, Success as Pervasive] they did not possess persistence [Failure as Temporary, Success from Personal Effort]. They possessed 'disabling confidence' in relation to mathematical problem solving.

Patrick and Eliza on the other hand possessed 'enabling confidence'—confidence developed through successes with overcoming mathematical challenges through puzzling about ideas and constructing new knowledge during the process and knowing they had the ability to work out more through such processes in the future [Success as Permanent, Pervasive, and Personal; Failure as Temporary]. They also demonstrated the optimistic characteristic Failure as Specific through their looking in to situations and varying what they did to increase their chances of success. This raises questions about whether possession of confidence and persistence is always accompanied by the optimistic characteristic Failure as Specific.

The ontogenesis of confidence thus differed for those who were and were not inclined to explore. This fits with Pajares and Miller's (1997) finding of the stronger associations found between high mathematical performances and confidence when multiple-choice items (rather than open ended tasks) were used to assess performance. This stronger association between multiple-choice test items and confidence is consistent with the predominance of pedagogies likely to develop disabling confidence in our schools. To conform to the expectations of curriculum documents requiring creative mathematical thinking be developed, we need to decrease the use of 'transmission' pedagogies in our schools.

Acknowledgements

This research was funded by ARC Grant DP0986955 with APD Fellowship hosted by the International Centre for Classroom Research at the University of Melbourne. Special thanks to the ICCR technical team for the high quality of the data collected.

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