

UNIVERSITY OF MELBOURNE

DOCTORAL THESIS

**Towards modelling the downstream
development of a turbulent boundary layer
following a rough-to-smooth step change**

Author: Mogeng Li

ORCID: 0000-0002-9875-6468

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Abstract

Turbulent boundary layers are important phenomena, affecting the performance of numerous systems in engineering and nature. At most practical Reynolds numbers, these flows occur over rough surfaces and this surface roughness is often distributed heterogeneously (such as biofouling on a ship hull). Understanding the behaviour of the turbulent boundary layer over this type of roughness is essential for improving the estimation of the drag penalty in full-scale systems. In this work, a turbulent boundary layer over a rough-to-smooth change in the surface condition is examined experimentally. A comprehensive dataset is acquired, which allows us to study the effect of the step height, friction Reynolds number and viscous-scaled equivalent sandgrain roughness height separately. Multiple experimental techniques are employed, including hotwire anemometry and high-magnification particle image velocimetry. The recovering wall-shear stress on the smooth surface is measured directly using oil-film interferometry.

Early works of Antonia and Luxton (*J. Fluid Mech.*, 53:737–757, 1972) questioned the reliability of standard smooth-wall methods to measure the wall-shear stress immediately downstream of a rough-to-smooth transition, and subsequent studies show significant disagreement depending on the approach used to determine the wall-shear stress downstream. Here we address this by utilising a collection of experimental databases that have access to both ‘direct’ and ‘indirect’ measures of the wall-shear stress to understand the recovery to equilibrium conditions to the new surface. We present evidence that any estimate of the wall-shear stress from the mean velocity profile in the buffer region or further away from the wall tends to underestimate its magnitude in the near vicinity of the rough-to-smooth transition. This is likely to be partly responsible for the large scatter of recovery lengths to equilibrium conditions reported in the literature. Our results also reveal that the smaller energetic scales in the near-wall region recover to an equilibrium state associated with the new wall conditions within one boundary layer thickness downstream of the transition, while the larger energetic scales exhibit an over-energised state for many boundary layer thicknesses downstream of the transition. Based on these observations, an alternative approach to estimating the wall-shear stress from the premultiplied energy spectrum is proposed.

For surfaces with heterogeneous roughness, there can be a change of virtual origin alongside the variation of roughness heights, an extreme case of which is the flow over a forward- or backward-facing step, where no roughness exists but only a height difference is present. In the present study, cases with various step heights between the rough and smooth surfaces behave similarly beyond two boundary layer thicknesses downstream of the roughness transition, and the higher order statistics seem to recover within approximately the same fetch as required for the recovery of the mean velocity profiles, suggesting that the influence of the step height is limited to the vicinity of the roughness transition with no far-field effect. A comparison of the experimental results with a direct numerical simulation of an open-channel

flow shows a qualitatively similar effect on the mean velocity profile, and the change in the virtual origin Δd is found to be a useful length scale in normalising the recovery length.

Dimensional analysis shows that the upstream condition of a turbulent boundary layer over a rough-to-smooth change in the surface condition can be fully described by two nondimensional numbers: $Re_{\tau 0}$ (friction Reynolds number) and k_{s0}^+ (viscous-scaled equivalent sandgrain roughness height). Therefore, to investigate the dependence of the flow on each parameter separately, we acquire a unique and comprehensive dataset which takes two cuts in the parameter space: one at a fixed k_{s0}^+ with varying $Re_{\tau 0}$, and the other at a fixed $Re_{\tau 0}$ with varying k_{s0}^+ . With the aid of these data, a blending model of the mean velocity profile is developed. The modelled mean velocity profile approaches an equilibrium smooth-wall velocity profile in the near-wall region, and it asymptotes to the upstream rough-wall profile above the IBL (internal boundary layer). The blending model is then incorporated into the original Elliott's model (*Trans. Am. Geophys. Union*, 39:1048–1054, 1958) together with further refinements. The refined Elliott's model leads to a better agreement with the experimental data compared to the original one, permitting an improved prediction of the evolution of a developing turbulent boundary layer downstream of a step change in surface roughness.

Declaration of Authorship

This is to certify that:

- the thesis comprises only my original work towards the PhD,
- due acknowledgement has been made in the text to all other material used,
- the thesis is fewer than 100, 000 words in length, exclusive of tables, maps, bibliographies and appendices.

Mogeng Li

21/02/2020

Preface

Publications

- M. LI; C. M. DE SILVA; R. BAIDYA; A. ROUHI; D. CHUNG; I. MARUSIC; AND N. HUTCHINS 2019 Recovery of the wall-shear stress to equilibrium flow conditions after a rough-to-smooth step-change in turbulent boundary layers. *J. Fluid Mech.* **872**, 472-491.
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To: Dr. Ahmad Zafari

Our story started together with my PhD. Now with the latter coming to an end, I am getting more excited to explore this world with you.

To: Jinghua Qu and Tiqiang Li

For giving me life and the best days of my life.

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List of Abbreviations

AHR	A veraged H ull R oughness
DNS	D irect N umerical S imulation
EL	E quilibrium L ayer
FOV	F ield O f V iew
HRNBLWT	H igh R eynolds N umber B oundary L ayer W ind T unnel
IBL	I nternal B oundary L ayer
IBM	I mmersed B oundary M ethod
LES	L arge- E ddy S imulation
OFI	O il- F ilm I nterferometry
PIV	P article I mage V elocimetry
PTV	P article T racking V elocimetry
RANS	R eynolds- A veraged N avier- S tokes equations
TNTI	T urbulent/ N on- T urbulent I nterface

List of Symbols

Latin letters

A	Constant
A_0	Multiplying constant in the power-law fit to the δ_i versus $\hat{x}$ trend
A'_{FR}	Additive constant in Nikuradse's (1950) fully-rough relation. $A'_{FR} = 8.5$.
B	Additive constant in Clauser chart method, equals to 4.17 unless stated otherwise
b_0	Exponent in the power-law fit to the δ_i versus $\hat{x}$ trend
$\overline{C_F}$	Drag coefficient of the entire flat plate, defined as $\overline{C_F} \equiv \frac{1}{L} \int_0^L C_f(x) dx$
C_f	Skin-friction velocity, defined as $C_f \equiv \tau_w / \frac{1}{2} \rho U_\infty^2$
C_{fe}	Equilibrium skin-friction coefficient
C_{fo}	Skin-friction velocity at the last measuring station in each dataset
C_p	Pressure coefficient, defined as $C_p = (p(x) - p_{ref}) / \frac{1}{2} \rho U_\infty^2$, where p_{ref} is the static pressure at the reference point
E	Blending function
h	Channel half height, or height of an open channel
K	Streamwise acceleration parameter, defined as $K \equiv \frac{\nu}{U_\infty^2} \frac{\partial U}{\partial x}$
k_p	Peak-to-trough distance of the roughness
k_{rms}	Root-mean-squared roughness height
k_s	Equivalent sandgrain roughness
L	Length of a flat plate
l_m	Mixing length, defined as $l_m \equiv \frac{\tau^{1/2}}{\partial U / \partial z}$
l_ϵ	Dissipation length, defined as $l_m \equiv \frac{\tau^{3/2}}{\epsilon}$
M	Magnitude of the roughness change, defined as $M \equiv \ln(z_{02}/z_{01})$
$\overline{q^2}$	Turbulent kinetic energy, defined as $\overline{q^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$
Re_θ	Reynolds number based on momentum thickness, defined as $Re_\theta \equiv U_\infty \theta / \nu$
Re_τ	Friction Reynolds number, defined as $Re_\tau \equiv \delta_{99} U_\tau / \nu$
T	Time period of velocity fluctuations
T_s	Hotwire sampling time
U	Mean streamwise velocity
U_i	Mean streamwise velocity at $z = \delta_i$
U_∞	Freestream velocity
V	Mean spanwise velocity
W	Mean wall-normal velocity
$\mathcal{W}$	Wake function
x	Streamwise location
x_0	Streamwise location of the roughness transition

$\hat{x}$	Fetch on the downstream surface, $\hat{x} = x - x_0$
y	Spanwise location
z	Wall-normal location
z_{01}	Roughness length of the upstream surface
z_{02}	Roughness length of the downstream surface

Greek letters

α	Shear-stress-to-turbulent-kinetic-energy ratio, defined as $\alpha = -\overline{uw}/\overline{q^2}$
Δd	Step change of the virtual origin
ΔH	Height difference between the roughness crest and the smooth wall
Δt	Hotwire sampling interval
ΔU^+	Hama function
$\Delta \delta_i^+$	A constant shift between $\delta_i U_{\tau 0}/\nu$ and $\delta_{i,log} U_{\tau 0}/\nu$
δ_*	Displacement thickness
δ_{99}	Boundary layer thickness defined as the wall-normal location where $U = 0.99U_\infty$
δ_c	Jones et al.'s (2001) boundary layer thickness
δ_e	Equilibrium layer thickness
δ_i	Internal boundary layer thickness
$\delta_{i,log}$	IBL thickness defined as the intersection between the equilibrium smooth-wall profile with the upstream profile
δ_s	Vertical displacement of a streamline
ϵ	Relative error in the estimated C_f ; dissipation rate
ε	Distance between the roughness crest and the virtual origin
η	Non-dimensional wall-normal location, defined as $\eta \equiv z/\delta_i$ or $\eta \equiv z/\delta_c$ depending on the context
κ	Kármán constant, equals to 0.384 unless stated otherwise
λ	Blending function used by Chamorro and Porté-Agel (2009)
μ	Mean location of the assumed fluctuating interface in the blending model
ν	Kinematic viscosity
ω	Angular frequency, $\omega = 2\pi/T$
ϕ	Non-dimensional wind shear, defined as $\phi \equiv \frac{\kappa z}{\tau^{1/2}} \frac{\partial U}{\partial z}$
ϕ_{uu}	Energy spectrum of the streamwise velocity fluctuation
ψ	Streamfunction
ρ	Fluid density
σ	Standard deviation of the assumed fluctuating interface location in the blending model
θ	Momentum thickness
τ	Total shear stress
τ_w	Wall shear stress
$(\cdot)^+$	Viscous normalisation
$(\cdot)^{*+}$	Viscous normalisation of the upstream rough-wall profile to represent the outer layer after a rough-to-smooth transition

$\widetilde{(\cdot)}$	Instantaneous velocity or spanwise variation of C_f
$(\cdot)_1$	Quantities of the upstream surface
$(\cdot)_2$	Quantities of the downstream surface
$(\cdot)_0$	Quantities in the immediate upstream of the roughness transition
$(\cdot)_{\text{out}}$	(Friction velocity or skin-friction coefficient) determined by assuming an outer-layer similarity
$(\cdot)_R$	Equilibrium rough-wall quantities
$(\cdot)_S$	Equilibrium smooth-wall quantities

Chapter 1

Introduction

1.1 Motivation and aims

Surface roughness with heterogeneity is present in wall-bounded turbulent flows in a variety of conditions. Examples include the patchiness of biofouling on the hull of a ship or the changes in the surface roughness conditions that occur at the interface between forest and grasslands. Understanding the flow response to such a change in the surface condition is beneficial to various real-world applications: for the shipping industry, it will allow us to have a better estimate of the drag penalty introduced by the patchy biofouling roughness and make more informed cleaning schedules. When setting up a weather station (to survey, for example, a new possible site for a wind farm), it is desirable to choose a location at a sufficient distance away from the change in surface condition to avoid the ‘leading-edge effects’. The height of the tower should also be fully contained in the region where the flow is in equilibrium with the new surface condition. These will require solid knowledge of the downstream development of the region where the flow is modified by the local surface condition. Furthermore, the prediction of various physical processes will be facilitated, such as the dispersion of air pollutants between dense urban and open rural areas, the development of a tropical cyclone as it approaches the land, and also the migration of sand dunes under the wind shear (Jerolmack et al., 2012). Though such heterogeneity can occur in a wide range of configurations, one simple distillation of this problem is to consider a sudden transition of the surface condition from rough to smooth occurring in the streamwise direction, as examined in the seminal work of Antonia and Luxton (1972). We aim at capturing the entire relaxation process until the flow has established a new equilibrium with the downstream surface, with a special focus on the rapid flow response immediately downstream of the roughness transition. Here, ‘equilibrium’ refers to a state where the flow statistics, especially the streamwise mean velocity and turbulence intensity profiles, behave in a similar fashion (local similarity, based on the kinematic viscosity and local value of the friction velocity at the wall) as those of a boundary layer at the same Reynolds number developing on a uniformly smooth surface without any roughness transition. It is different from the criteria used by Townsend (1976) that the production and dissipation of the turbulent energy are large compared to advection in a near-wall layer. We also investigate the recovery of the skin friction coefficient C_f downstream of the roughness transition. When C_f is indistinguishable from that of a canonical smooth-wall turbulent boundary layer at the same Reynolds number, we also consider that

C_f has reached a fully recovered state in the sense that it has recovered to the equilibrium C_f versus Reynolds number trajectory despite a different developing history.

In the existing literature, flows over such heterogeneous roughness have received less attention compared to its homogeneous counterpart. Although it is meaningful to fully understand the flow behaviour on each surface before investigating the transition in between, studying the flow over heterogeneous roughness directly can provide more physical insight into the flow, which may not be achievable under canonical conditions. For instance, the setup of flow over a rough-to-smooth transition allows us to examine the interaction between the large- and small-scale structures when the former is over-energised compared to that in a canonical boundary layer.

In addition, most of the high-Reynolds number datasets in the literature are obtained from field measurements, where the incoming flow conditions and the surface roughness are more difficult to manipulate. The laboratory experiments, on the other hand, can produce high-fidelity datasets but often have a Reynolds number much lower than the industrial or meteorological applications as limited by the size of the working section. Therefore, in this study, we aim to acquire a high-quality experimental dataset in the unique High Reynolds Number Boundary Layer Wind Tunnel (HRNBLWT) in the University of Melbourne, to achieve a better understanding of the flow response to a rough-to-smooth change in the surface condition, and also to use this knowledge to develop a predictive model.

1.2 Outline of the thesis

We start with reviewing the relevant literature in §2. An extensive dataset covering a wide range of flow conditions is formed by collating data from the existing literature. We also reprocess the dataset to investigate the scaling laws of the flow recovery, and also to evaluate the performance of the predictive models in the literature.

An overview of the experimental campaign is presented in §3. Two sets of measurements are planned in this study, focusing on the effect of change in the virtual origin accompanying the roughness transition and the effect of upstream rough-wall flow conditions (the Reynolds number of the roughness and the oncoming boundary layer) at high Reynolds numbers. A supplementary DNS dataset is also included for comparison. In this section, we also provide details of the wind tunnels, surface roughness and measuring techniques.

A comparison of the wall-shear stress results obtained via ‘direct’ and ‘indirect’ techniques is presented in §4. The discrepancy between the results from different techniques highlight the necessity of a direct measurement of the wall-shear stress for flows in such non-equilibrium states after a rough-to-smooth transition. We also present a novel surrogate method to estimate U_τ from the pre-multiplied energy spectrum when direct measurements are not available. Results of the two sets of measurements, focusing on the effect of the change in virtual origin and the effect of the upstream rough-wall flow conditions are analysed in §5 and §6, respectively. We further examine Elliott’s (1958) classical model with

these data in §7, and with the knowledge gained, an empirical model of the mean velocity profile is developed in §8. Finally, in §9, we explore possibilities to further refine and improve Elliott's original model.

In this study, we will be using x , y and z to denote streamwise, spanwise and wall-normal directions. The corresponding mean velocity components are U , V and W . The instantaneous and fluctuating velocities are denoted by $\widetilde{(\cdot)}$ and the lower case, respectively. x_0 is the streamwise location of the surface transition and $\hat{x} \equiv x - x_0$ is the distance downstream of the transition. We use subscripts $(\cdot)_1$ and $(\cdot)_2$ to distinguish the friction velocity U_τ and roughness length z_0 of the upstream and downstream surfaces. We also use subscript $(\cdot)_0$ to denote the flow quantities obtained just prior to the roughness transition. For instance, $U_{\tau 0} = \lim_{\hat{x} \rightarrow 0^-} U_{\tau 1}(\hat{x})$, where $U_{\tau 1}(\hat{x})$ represents the friction velocity on the upstream surface.

Chapter 2

Literature review

2.1 The internal boundary layer

Here we consider a simplified two-dimensional case of a roughness change in the streamwise direction, as depicted in figure 2.1, where upstream of the transition, a turbulent boundary layer in equilibrium with the local roughness length z_{01} has developed. Following the transition, the new downstream roughness z_{02} initially modifies the near-wall region, which then gradually propagates towards the interior of the flow with increasing distance downstream of the transition. The layer that separates the modified near-wall region (which ‘sees’ the new smooth-wall condition) from the unaffected oncoming flow, further away from the wall (which ‘remembers’ the rough-wall condition) is generally referred to as the internal boundary layer (IBL) with a thickness denoted by δ_i . A large portion of the flow within the IBL is not completely in equilibrium with the new wall condition (Antonia and Luxton, 1971b). The equilibrium with the downstream surface firstly establishes close to the wall, and then similar to the IBL, propagates towards the interior of the flow. The layer where the flow is in equilibrium with the new wall condition is referred to as the equilibrium layer (Garratt, 1990; Savelyev and Taylor, 2005) with thickness δ_e . In most cases, the majority of the flow within the IBL is still not in equilibrium with the local wall condition, and a general consensus is δ_e takes up about 5% of δ_i defined based on the shear stress profile adjustment for a flow over rough-to-smooth change (see Rao et al., 1974; Shir, 1972). The term ‘transition layer’ is used by some researchers to refer to the region between EL and IBL.

The definition of the IBL does not indicate an unambiguous method to calculate δ_i from a measured or simulated flow field. As a matter of fact, δ_i determined based on different

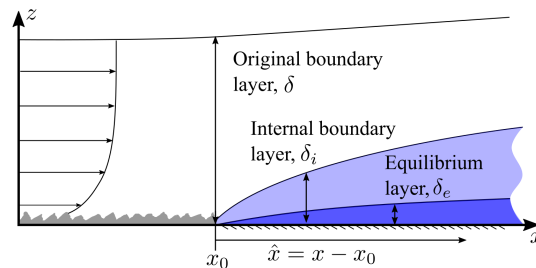


FIGURE 2.1: Schematic of a turbulent boundary layer flow over a rough-to-smooth change in the surface condition. The roughness transition occurs at x_0 , and $\hat{x} = x - x_0$ denotes the fetch downstream of the transition.

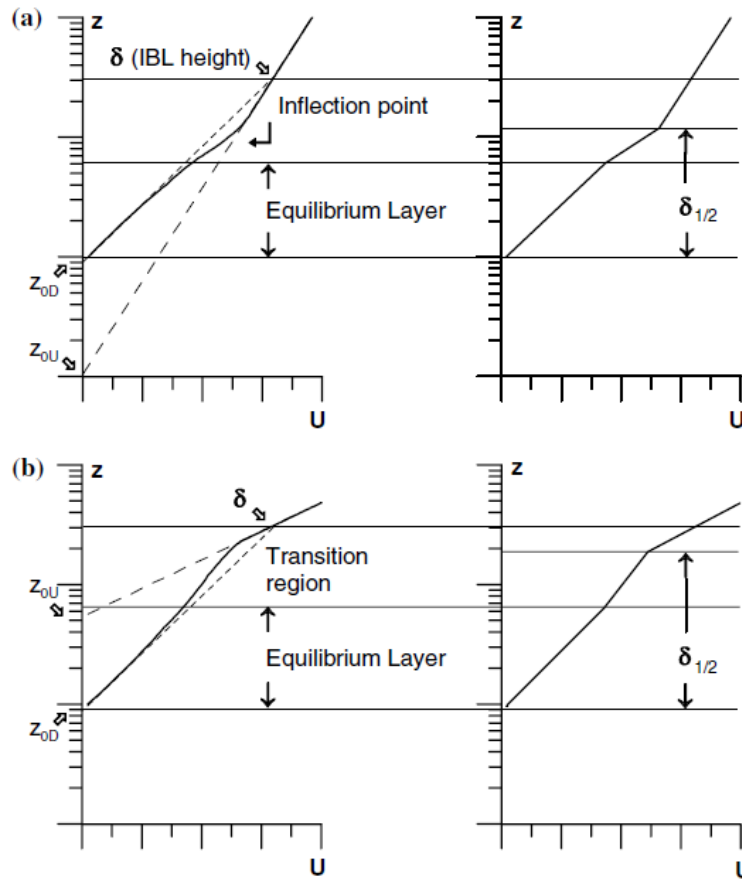


FIGURE 2.2: Sketches of the wind speed profile (left panel) at some distance from a leading edge and possible definitions of the IBL height near the inflection point of the profile (right panel); smooth-to-rough (a) and rough-to-smooth (b) transitions. The roughness length of the upstream and downstream surfaces is denoted by z_{0U} and z_{0D} , respectively. δ is the intersection of the upstream and downstream equilibrium log-law velocity profiles, and $\delta_{1/2}$ is the wall-normal location where the mean velocity profile deviates from the upstream reference. The figure is reproduced from Savelyev and Taylor (2005).

quantities (i.e. mean streamwise velocity, velocity gradient, Reynolds shear stress, etc.) of the same flow field can vary significantly (Shir, 1972; Bou-Zeid et al., 2004; Rouhi et al., 2019b). In the majority of field observations and some laboratory experiments, only the averaged streamwise velocity is available, therefore in this chapter we focus on calculating δ_i using the mean velocity profile only, although comparison will be drawn with δ_i determined based on other flow statistics when applicable.

In the simplified model composited of two intersecting logarithmic velocity profiles (Elliott, 1958), the IBL is equivalent to the EL with no transition layer in between, and the location of the IBL coincides with the inflection point in the mean velocity profile. However, in a more realistic picture of the recovering flow, the intersection of the new log law at the wall with the existing log law in the outer layer is above the inflection point, due to a blending of the profiles, as shown by the sketches in figure 2.2.

2.2 Experimental and numerical studies

2.2.1 A compilation of the existing data

Numerical simulations focusing on the effect of heterogeneous roughness first appeared in the meteorology community in 1970s. Early studies mainly adopted one-equation (Peterson, 1969; Shir, 1972) or two-equation models of RANS (Rao et al., 1974) in the context of surface layers at infinite Reynolds numbers, while more recent work has included LES (Glendening and Lin, 2002; Saito and Pullin, 2014; Yang, 2016; Eisma et al., 2018) and DNS (Lee, 2015; Ismail et al., 2018b,a; Rouhi et al., 2019b) to study laboratory-scale turbulent boundary layers or channel flows.

Similar to the history of numerical studies, experiments related to the heterogeneous roughness were initially conducted by meteorologists in field measurements. One of the most extensive datasets in this period was obtained by Bradley (1968), who considered both smooth-to-rough and rough-to-smooth transitions with measurements of the mean stream-wise velocity profiles at various fetches downstream of the surface transition and also wall-shear stress measured by a floating element drag balance. Antonia and Luxton (1972) realised the discrepancy between the skin-friction coefficients obtained via different conventional techniques (Clauser chart and preston tube) that rely on some assumptions of the mean velocity profile. They concluded that in such highly non-equilibrium flow conditions, direct methods are required to measure the local wall-shear stress accurately.

We collect both experimental and numerical datasets reported in the previous studies, focusing on the trends of IBL growth and wall-shear stress recovery. This is an expansion of the data collection of Walmsley (1989), with the inclusion of more recent datasets and also the wall-shear stress results. A summary of the sources of datasets in the compilation and their detailed flow conditions are presented in table 2.2. The compilation includes both rough-to-smooth and smooth-to-rough configurations, and in many cases, the ‘smooth’ patch has a non-zero roughness height which is smaller than that of the roughness patch preceding it or upstream (i.e. it is not hydraulically smooth). The effect of these two different types of ‘smooth’ patch is unclear, however, here we include both in the collection for completeness. When computing the flow parameters not directly available in the original source, we use $\kappa = 0.41$ and $B = 5.0$ to be consistent with most of the datasets included. A unique symbol is assigned to each dataset. Note that in some (mainly numerical) studies, a large number of cases with different parameters are reported in the same study. The corresponding flow conditions are all listed in the table, but in these cases we only include a few selected cases (indicated by the symbol) in the compiled dataset for a fair representation of the literature.

The magnitude of the roughness change between the two surfaces is usually quantified by some form of the ratio between the two roughness lengths. In this study, we choose the definition of $M \equiv \ln(z_{02}/z_{01})$. The roughness length of a hydraulically smooth surface is evaluated as

$$z_0 = \frac{\nu}{U_\tau} e^{-\kappa B}, \quad (2.1)$$

Reference	Case code	Symbol	Type	$Re_{\tau 0}$	k_{s0}^+	$z_{01}(\text{mm})$	$z_{02}(\text{mm})$	M	Method to find δ_i	Method to find $U_{\tau 2}$
Kutzbach (1961)		⚡	A	-	-	0.1	12.2	4.80	not specified	
Blackadar et al. (1967)		×	A	-	-	35	10	-1.25	U merges with up- stream profile	
Bradley (1968)	tarmac-spikes	⚡		-	≈ 0	0.02	2.5	4.83		floating element
	spikes-tarmac	⚡	A	-	2300	2.5	0.02	-4.83	log-intersect in U	balance
	grass-tarmac	⊗		-	4500	5	0.02	-5.52		-
Antonia and Luxton (1971a)		⚡	L	8.0×10^2	≈ 0	0.012	0.38	3.46	‘knee’ point in U vs. $z^{1/2}$	
Antonia and Luxton (1971b)	lowRe	⚡	L	7.3×10^2	≈ 0	0.013	0.38	3.38	‘knee’ point in U	pressure taps on
	highRe	⚡		1.2×10^3	≈ 0	0.008	0.38	3.86	vs. $z^{1/2}\dagger$	the rod roughness $\dagger$
Antonia and Luxton (1972)	lowRe	□	L	3.4×10^3	850	0.55	0.011	-3.91	U merges with	preston
	highRe	⊠		6.1×10^3	1560	0.55	0.006	-4.52	adjacent profile	tube $\dagger$
Peterson et al. (1979)		□	A	-	-	0.7	6	2.15	U merges with up- stream profile	
Pendergrass and Arya (1984)		⚡	L	7.9×10^3	23	0.068	0.710	2.35	99% of $\overline{u^2}$ before transition $\dagger$	

Table 2.2 – Continued on next page

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Reference	Case code	Symbol	Type	$Re_{\tau 0}$	k_{s0}^+	$z_{01}(\text{mm})$	$z_{02}(\text{mm})$	M	Method to find δ_i	Method to find $U_{\tau 2}$
Cherrywood	I	*	A	-	-	219	4.4	-3.91	not specified	
	II	⚡		-	-	293	0.68	-6.07		
Bergström et al. (1988)		⊗	A	-	-	0.25	40	5.08	U merges with up-stream profile	
Jegade and Foken (1999)	SR	⊞	A	-	-	8	32	1.39	log-intersect in U	
	RS	⊕		-	-	53	14	-1.33		
Cheng and Castro (2002)		⚡	L	5.4×10^3	147	0.13	1.11	2.14	99% of U before transition	
Chamorro and Porté-Agel (2009)		⊙	L	1.5×10^4	598	0.5	0.006	-4.42	not specified	near-wall hotwire
Efros and Krogstad (2011)		⚡	L	4.2×10^3	≈ 0	0.003	0.57	5.14	‘knee’ point in $\overline{u^2}$	floating element balance
Hanson and Gana- pathisubramani (2016)	grit	○	L	2.3×10^3	209	0.17	0.005	-3.4	‘knee’ point in U vs. $z^{1/2}$	preston tube
	mesh	⊗		3.5×10^3	1300	0.85	0.005	-5.1		

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Reference	Case code	Symbol	Type	$Re_{\tau 0}$	k_{s0}^+	$z_{01}(\text{mm})$	$z_{02}(\text{mm})$	M	Method to find δ_i	Method to find $U_{\tau 2}$
Bou-Zeid et al. (2004)	A2	Δ	S_C	-	-	250	25	-2.30	where $\partial U / \partial z = \langle \partial U / \partial z \rangle_x^\dagger$	wall-modelled LES
		Δ		-	-	25	250	2.30		
	B2	-		-	1000	100	-2.30			
		-		-	100	1000	2.30			
	C2	-		-	1000	10	-4.61			
		-		-	10	1000	4.61			
	D2	-		-	100	1	-4.61			
		-		-	1	100	4.61			
Saito and Pullin (2014)	SRS.1.3	$\triangleleft$	S_C	2.1×10^4	21	$0.6\nu / U_{\tau R}$	$0.12\nu / U_{\tau S}$	-1.40	where $\overline{\partial u^2} / \partial x = 0$	wall-modelled LES
				1.7×10^4	≈ 0	$0.12\nu / U_{\tau S}$	$0.6\nu / U_{\tau R}$	1.40		
	SRS.2.3			2.2×10^5	219	$6.7\nu / U_{\tau R}$	$0.12\nu / U_{\tau S}$	-3.68		
				1.5×10^5	≈ 0	$0.12\nu / U_{\tau S}$	$6.7\nu / U_{\tau R}$	3.68		
	SRS.3.3			2.5×10^6	2458	$75.3\nu / U_{\tau R}$	$0.12\nu / U_{\tau S}$	-5.91		
				1.5×10^6	≈ 0	$0.12\nu / U_{\tau S}$	$75.3\nu / U_{\tau R}$	5.91		
	SRS.3.1			2.3×10^6	225	$6.9\nu / U_{\tau R}$	$0.12\nu / U_{\tau S}$	-3.73		
				1.7×10^6	≈ 0	$0.12\nu / U_{\tau S}$	$6.9\nu / U_{\tau R}$	3.73		
	SRS.3.2			2.4×10^6	1193	$36.6\nu / U_{\tau R}$	$0.12\nu / U_{\tau S}$	-5.29		
				1.6×10^6	≈ 0	$0.12\nu / U_{\tau S}$	$36.6\nu / U_{\tau R}$	5.29		

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Table 2.2 – Continued from previous page

Reference	Case code	Symbol	Type	$Re_{\tau 0}$	k_{s0}^+	$z_{01}(\text{mm})$	$z_{02}(\text{mm})$	M	Method to find δ_i	Method to find $U_{\tau 2}$
Lee (2015)		∇	S_B	7.5×10^2	≈ 0	$0.017\theta_{in}$	$0.4\theta_{in}$	3.18	99% of U before transition	DNS
Ismail et al. (2018b)	A			5.0×10^2	343	$10.5\nu/U_{\tau R}$	$0.12\nu/U_{\tau S}$	-3.75	U intersects rough-wall profile †	DNS
	B		S_C	2.2×10^3	1540	$47.2\nu/U_{\tau R}$	$0.12\nu/U_{\tau S}$	-5.11		
	C			2.1×10^3	1105	$33.9\nu/U_{\tau R}$	$0.12\nu/U_{\tau S}$	-4.78		
	D	$\diamond$		2.4×10^3	2105	$64.5\nu/U_{\tau R}$	$0.12\nu/U_{\tau S}$	-5.34		
Ismail et al. (2018a)	A			1.6×10^3	338	$10.4\nu/U_{\tau R}$	$0.12\nu/U_{\tau S}$	-3.84		DNS
	B		S_C	1.5×10^3	233	$7.1\nu/U_{\tau R}$	$0.12\nu/U_{\tau S}$	-3.50		
	C			2.2×10^3	1570	$48.1\nu/U_{\tau R}$	$0.12\nu/U_{\tau S}$	-5.17		
Sridhar (2018)	R0			1.8×10^4	115	$3.5\nu/U_{\tau R}$	$0.12\nu/U_{\tau S}$	-3.03		wall-modelled LES
	R1			4.8×10^4	115	$3.5\nu/U_{\tau R}$	$0.12\nu/U_{\tau S}$	-3.07		
	R2	$\triangleright$		8.8×10^4	115	$3.5\nu/U_{\tau R}$	$0.12\nu/U_{\tau S}$	-3.09		
	R1L			4.8×10^4	56	$1.7\nu/U_{\tau R}$	$0.12\nu/U_{\tau S}$	-		
	R1H		S_B	4.8×10^4	324	$9.9\nu/U_{\tau R}$	$0.12\nu/U_{\tau S}$	-		
	S0			2.8×10^4	≈ 0	$0.12\nu/U_{\tau S}$	$4.0\nu/U_{\tau R}$	3.51		
	S1			2.8×10^4	≈ 0	$0.12\nu/U_{\tau S}$	$4.0\nu/U_{\tau R}$	3.51		
	S2	∇		1.3×10^5	≈ 0	$0.12\nu/U_{\tau S}$	$4.0\nu/U_{\tau R}$	3.51		
	S1L			8.4×10^4	≈ 0	$0.12\nu/U_{\tau S}$	$1.5\nu/U_{\tau R}$	2.53		
	S1H			8.4×10^4	≈ 0	$0.12\nu/U_{\tau S}$	$9.3\nu/U_{\tau R}$	4.35		

Table 2.2 – Continued on next page

Table 2.2 – Continued from previous page

Reference	Case code	Symbol	Type	$Re_{\tau 0}$	k_{s0}^+	$z_{01}(\text{mm})$	$z_{02}(\text{mm})$	M	Method to find δ_i	Method to find $U_{\tau 2}$
Rouhi et al. (2019b)		$\blacktriangle$	S_C	7.2×10^2	165	$5.1\nu/U_{\tau R}$	$0.12\nu/U_{\tau S}$	-3.28	log-intersect in $U^\dagger$	DNS
		$\blacktriangle$		4.4×10^2	≈ 0	$0.12\nu/U_{\tau S}$	$5.1\nu/U_{\tau R}$	3.28		
Rouhi et al. (2019a)	step-up		S_C	7.3×10^2	165	$5.1\nu/U_{\tau R}$	$0.12\nu/U_{\tau S}$	-3.28	log-intersect in U	DNS
				4.7×10^2	≈ 0	$0.12\nu/U_{\tau S}$	$5.1\nu/U_{\tau R}$	3.28		
	step-down			6.4×10^2	165	$5.1\nu/U_{\tau R}$	$0.12\nu/U_{\tau S}$	-3.28		
				4.0×10^2	≈ 0	$0.12\nu/U_{\tau S}$	$5.1\nu/U_{\tau R}$	3.28		
	smaller k_s			7.1×10^2	106	$3.2\nu/U_{\tau R}$	$0.12\nu/U_{\tau S}$	-2.81		
				4.7×10^2	≈ 0	$0.12\nu/U_{\tau S}$	$3.2\nu/U_{\tau R}$	2.81		
Chu and Bergstrom (2020)		$\blacktriangleleft$	S_B	2.0×10^3	≈ 0	0.0017	0.034	2.99	‘knee-point’ in $\overline{u^2}^\dagger$	near-wall RANS

TABLE 2.2: Summary of IBL data collected from the literature. In the ‘Type’ column, ‘L’ stands for laboratory experiments, ‘A’ stands for field experiments in atmospheric boundary layers, and ‘S’ stands for numerical simulations. All atmospheric and laboratory measurements are conducted in boundary layers, while both channel and boundary layer configurations present in numerical simulations, which are distinguished by the subscript: ‘B’ is for boundary layer and ‘C’ is for channel flow. In some channel flow simulations, a periodic boundary condition in streamwise is applied and the flow will go through both rough-to-smooth and smooth-to-rough transitions, as indicated by the two entries with one case code. $Re_{\tau 0}$ and k_{s0}^+ are evaluated in the upstream of the step change. Note that k_s and related quantities are not documented in the original paper of Antonia and Luxton (1972), and here we present the estimated values using the correlation in Jiménez (2004) with the given dimensions of the rod roughness elements. When multiple methods were employed to find δ_i or $U_{\tau 2}$ in one study, we only include the data from one of them and document the method in the table. These cases will be marked by a ‘†’ symbol.

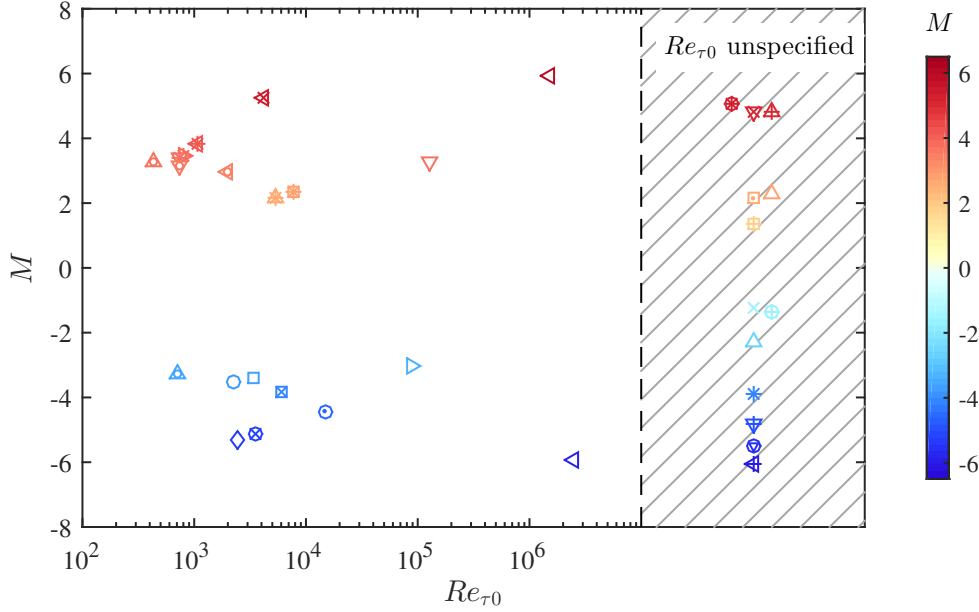


FIGURE 2.3: The magnitude of roughness change M versus the Reynolds number before the roughness transition $Re_{\tau 0}$ of cases summarised in table 2.2. The symbols are colour-coded according to their M , with red for a positive M (smooth-to-rough change) and blue for a negative M (rough-to-smooth change). A colorbar is also shown on the right of the figure. The hatched region in the figure isolates cases where $Re_{\tau 0}$ is not documented. The location of these symbols on the abscissa does not indicate $Re_{\tau 0}$, although these datasets are obtained from field measurements or numerical simulations of deep surface layers where the Reynolds numbers are supposed to be high.

where U_τ is the friction velocity where the flow has reached an equilibrium with the surface. By including a logarithmic function in the definition, the absolute value of M gives a good representation of the relative magnitude of the roughness change and the sign of M indicates the direction: a smooth-to-rough change leads to a positive M while a rough-to-smooth change leads to a negative one. Figure 2.3 shows the magnitude of roughness change M versus $Re_{\tau 0}$, the friction Reynolds number just prior to the roughness transition (where $Re_{\tau 0} \equiv \delta_0 U_{\tau 0} / \nu$, $U_{\tau 0}$ is the friction velocity just upstream of the transition, δ_0 is the boundary layer thickness at the transition and ν is the kinematic viscosity of the fluid). Cases with $Re_{\tau 0}$ unspecified (usually data obtained in a deep surface layer from a field measurement or simulated at infinite Reynolds numbers) are shown in the hatched area on the right side of the figure. The colour of each symbol is assigned according to its M value. We use red for a positive M (smooth-to-rough change) and blue for a negative one (rough-to-smooth change), with a darker shade indicating a higher absolute value of M (a more aggressive change in roughness). As shown in the figure, our collection of the data covers a range of $Re_{\tau 0}$ over 3 decades, from approximately 400 to 2×10^6 . For both rough-to-smooth and smooth-to-rough changes, $|M|$ is usually between 2 to 6, which means that the ratio between the roughness lengths of the (relatively) rough and the (relatively) smooth surfaces ranges between around 7 to 400.

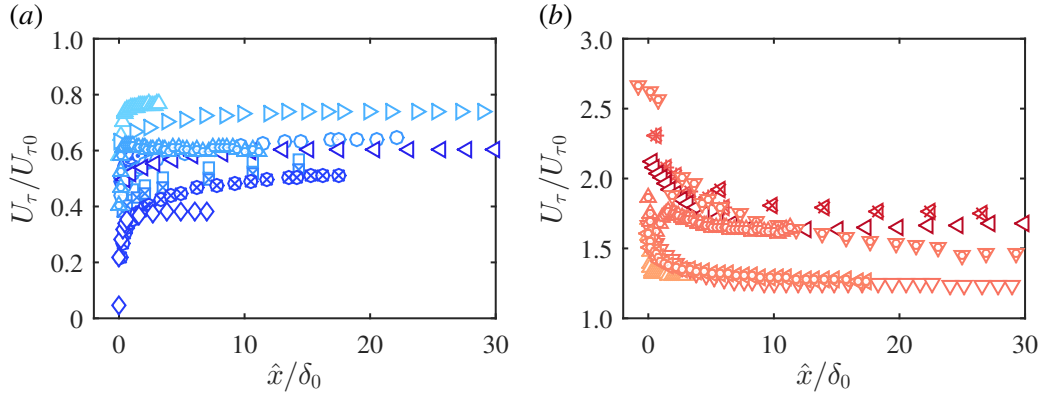


FIGURE 2.4: Recovering friction velocity U_τ normalised by $U_{\tau 0}$, the friction velocity immediately upstream of the roughness transition, plotted against $\hat{x}/\delta_0$, the downstream fetch normalised by the boundary layer thickness at the transition. (a) rough-to-smooth and (b) smooth-to-rough. The colour of each symbol represents M , and the references are listed in table 2.2.

2.2.2 Wall-shear stress

It is generally agreed that downstream of a rough-to-smooth transition, the wall-shear stress undershoots its corresponding value on a homogeneous downstream roughness at matched Reynolds number, while an overshoot is expected after a smooth-to-rough change (see figure 2.4). The fetch required for the wall-shear stress to reach a full recovery after a rough-to-smooth change has been reported to be in the order of $1\delta_0$ to $10\delta_0$. When normalised by δ_0 , the recovery length has been observed to increase slightly with both Reynolds number and roughness length for both smooth-to-rough and rough-to-smooth changes in high-Reynolds-number LES studies for both channel and boundary layers (Saito and Pullin, 2014; Sridhar, 2018), while the existence of a consistent trend when comparing data from various studies is not clear. Both Ismail et al. (2018b) and Rouhi et al. (2019b) conducted DNS in channel flows at relatively lower Reynolds numbers with fully resolved roughness geometry, and the wall-shear stress in their studies is shown to approach a plateau within $1\delta_0$ downstream, while a recovery length of $8.3\delta_0$ is reported in a simulation of a turbulent boundary layer with similar conditions (Lee, 2015). One difficulty hindering the clarification of the recovery length trend is that the recovery of the wall-shear stress slows down farther away from the roughness transition as it approaches the expected downstream value, therefore a small uncertainty in the wall-shear stress data can lead to a large difference in the recovery length.

In addition, the non-equilibrium behaviour of the mean flow within the IBL also questions the applicability of some conventional methods to estimate wall-shear stress with an assumed mean velocity profile. The wall-shear stress obtained from the Clauser chart method and the Reynolds shear stress in the logarithmic region has been shown to significantly deviate from the results of more direct measurements (such as the mean velocity gradient in the viscous sublayer) immediately downstream of a rough-to-smooth change (Loureiro et al., 2010). Curiously, although their Preston tube result agrees with direct measurements to within 10%, it exhibits a clear overshoot with the difference increasing further downstream of the transition, despite the re-establishment of an equilibrium layer which would presumably be beneficial

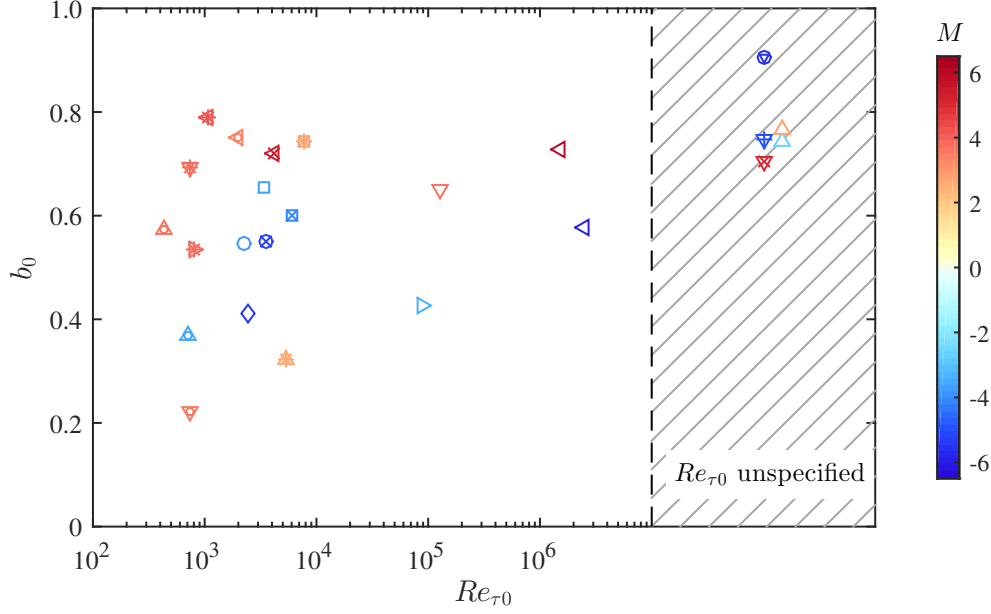


FIGURE 2.5: Values of the power-law exponent b_0 for δ_i from the previous studies listed in table 2.2. The legends are the same as in figure 2.3.

to the accuracy of the Preston tube method. Loureiro et al. (2010) only compared the various methods of determining skin friction has only been made within approximately one boundary layer thickness downstream of the roughness transition. Questions remain regarding the required fetch beyond which the conventional indirect methods can be accurately applied. These questions will be addressed in detail in §4.

An upstream effect of the roughness change has also been reported, mainly in numerical studies: Glendening and Lin (2002) observed a light decrease in the surface momentum flux ($\overline{uv}$ and $\overline{uw}$) before a smooth-to-rough change and an increase before a rough-to-smooth one, while both Saito and Pullin (2014) and Sridhar (2018) observed an increase in the friction velocity before a smooth-to-rough change. Before a rough-to-smooth change, the friction velocity shows a decrease in Saito and Pullin (2014) while the change is negligible in Sridhar (2018). Understanding the upstream effect of the roughness transition is meaningful to the selection of a reference streamwise location to represent the upstream flow condition unaffected by the change of surface condition, especially in experiments where usually only one reference profile is obtained.

2.2.3 General scaling of the IBL

Alongside the extensive datasets generated by field/laboratory experiments and numerical simulations, there have also been considerable efforts to interpret the trends associated with them. One of the most representative quantities of the flow recovery process, and probably the most investigated one is the IBL height δ_i . It is usually extracted from the measured profiles of the mean velocity or other quantities at a series of streamwise locations, with the δ_i versus $\hat{x}$ data typically fitted to a power law $\delta_i \propto \hat{x}^{b_0}$ to compress the recovery trend into one single parameter, the power-law exponent b_0 . Figure 2.5 shows b_0 computed from

the data collected in table 2.2, plotted against Re_{τ_0} , the friction Reynolds number before the roughness transition. The symbol colours indicate M , with red for a smooth-to-rough change and blue for a rough-to-smooth one. A range of b_0 from 0.2 to 0.8 has been observed in both cases, while smooth-to-rough cases in general tend to have a slightly higher b_0 . The scatter of the results is the largest at small Re_{τ_0} , while the scatter reduces and b_0 asymptote to 0.8 at higher Re_{τ_0} .

In the literature, a consensus on the definition of δ_i (especially the detailed method to extract it from the measured profiles of the mean velocity or other quantities) has not been reached. δ_i has been determined by a number of methods, including the wall-normal height where the recovering mean velocity profile merges with its upstream reference (Pendergrass and Arya, 1984), the inflection point in the U versus $z^{1/2}$ plot (Antonia and Luxton, 1971b) and the ‘kink’ in the turbulence intensity profile (Efros and Krogstad, 2011), to name a few. Rouhi et al. (2019b) compiled the δ_i definitions adopted by past studies and examined their effect on the resulting trends. By extracting δ_i using different definitions from the same DNS dataset in their study, they observed a scatter of ± 0.1 in b_0 for both rough-to-smooth and smooth-to-rough cases. This seems insufficient to explain the full range of scatter from 0.2 to 0.8 as found in figure 2.5. To further investigate the growth trend of the IBL, we take a step back from the fitted power law and present δ_i versus $\hat{x}$ in figure 2.6.

Various choices of length scale have been suggested in the literature to non-dimensionalise the evolution of δ_i . Schofield (1981) used z_{02} as the length scale. Wood (1982) concluded by performing a fit to the existing data that the length scale relevant to the growth of the IBL is the roughness length of the rougher surface, i.e. $\max(z_{01}, z_{02})$. Jackson (1976) used an alternative length scale, $z_r \equiv \sqrt{(z_{01}^2 + z_{02}^2)/2}$, which reduces to $\sqrt{\frac{1}{2}} \max(z_{01}, z_{02})$ when the roughness levels of the two surfaces are very different ($|M|$ is large). It should be noted that z_0 is a dynamic roughness length that cannot be measured directly, but must instead be ascertained from the effect that the roughness has on the flow (typically from a shift ΔU^+ in the measured mean profile). If we assume an error of ± 1 in the Hama roughness function ΔU^+ , this could easily amount to a factor 2 potential error in large roughness lengths, which would of course greatly influence any attempts to normalise δ_i data using this length scale. In the studies of channel flows or turbulent boundary layers where the boundary layer thickness is well defined, it is a common practice to normalise the results by δ_0 , the boundary layer thickness at the roughness transition.

In figure 2.6, we compare the normalised δ_i versus $\hat{x}$ results with the length scale chosen as z_{01} , z_{02} , $\max(z_{01}, z_{02})$ and δ_0 . Two straight lines, corresponding to $\delta_i \propto \hat{x}^{0.8}$ (solid) and $\delta_i \propto \hat{x}^{0.4}$ (dashed) are included in each plot. These are the representative power laws reported in the literature. The aim of this comparison is (1) to evaluate the performance of various length scales to collapse the data and (2) to provide further insight into the growth rate of δ_i over a wider range of fetch by collating the results from different sources. When using z_{01} as the length scale, as shown in figure 2.6(a), the smooth-to-rough results (shown by red symbols) are more concentrated in the top right quarter of the figure. This is expected as z_{01} , the roughness length of the upstream surface, is usually small in these cases. When normalised by z_{02} in (b), the rough-to-smooth cases (shown by blue symbols) are concentrated in the

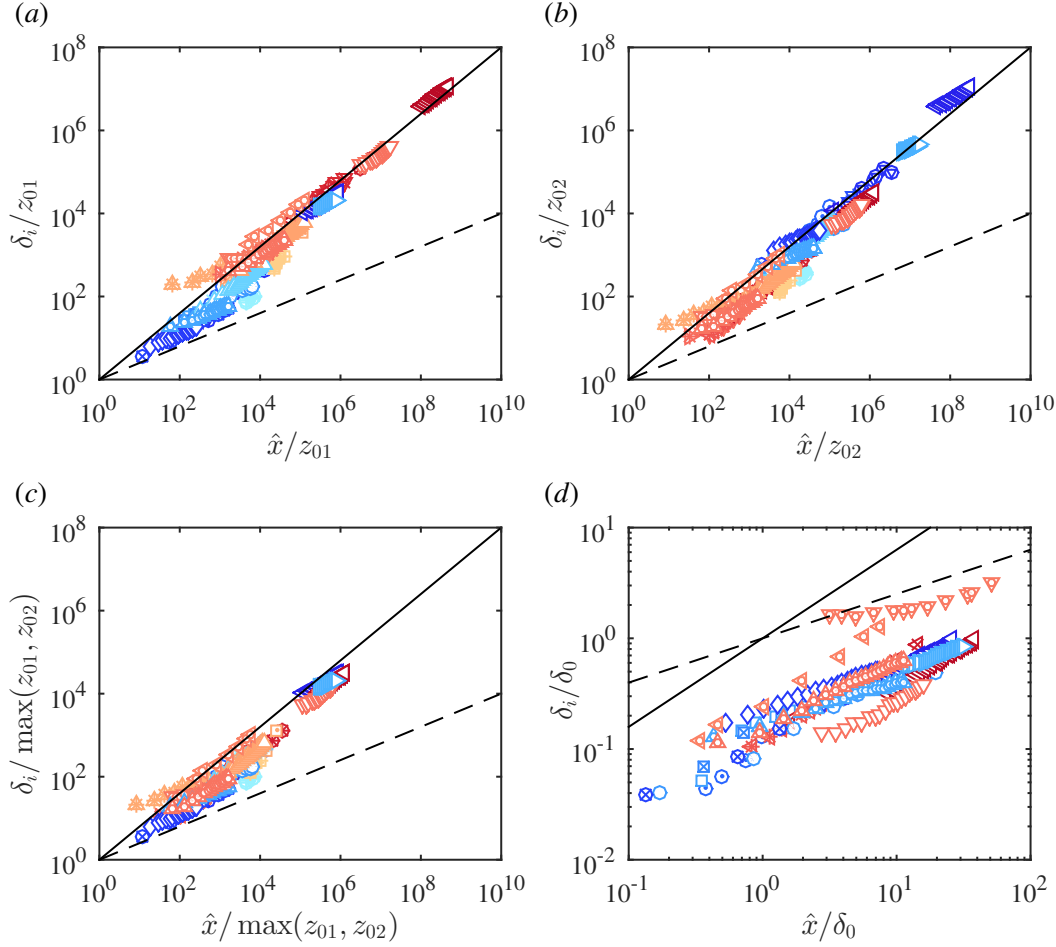


FIGURE 2.6: IBL height δ_i versus the downstream fetch $\hat{x}$ normalised by (a) z_{01} , the roughness length of the upstream surface; (b) z_{02} , the roughness length of the downstream surface; (c) $\max(z_{01}, z_{02})$, the roughness length of the rougher surface and (d) δ_0 , the boundary layer thickness at the roughness transition. The colour of each symbol represents M , and the references are listed in table 2.2. Note that the axis limits in (d) is different from the other three plots. The solid line shows $\delta_i \propto \hat{x}^{0.8}$, and the dashed line shows $\delta_i \propto \hat{x}^{0.4}$. Note that these two reference lines are only included as a comparison with the slope (power-law exponent) exhibited by the data points. One should not interpret into the relative magnitude between the line and the symbols.

top right quarter for the same reason. When the roughness length of the rougher surface (i.e. $\max(z_{01}, z_{02})$) is used, as shown in (c), the rough-to-smooth and smooth-to-rough cases appear to cover the same range of fetch. In both (a) and (b), all datasets appear to line up on a straight line, which has a slope closely following that of the solid black line ($\delta_i \propto \hat{x}^{0.8}$). In the near field (small $\hat{x}/z_{01}$), a few datasets (Cheng and Castro (2002), Ismail et al. (2018b) and Rouhi et al. (2019b), for example) are observed to depart from the power law of $\delta_i \propto \hat{x}^{0.8}$ and they have a slope closer to $\delta_i \propto \hat{x}^{0.4}$ instead. This agrees with the power-law exponent results summarised in figure 2.5. To summarise, the compilation of data suggests stronger evidence preferring a power law with the exponent of 0.8 with a wider range of data, although this observation does not oppose the lower exponent values reported previously. We must re-emphasise that it is not guaranteed that the δ_i versus $\hat{x}$ trend can be fully described by a single power law, and as predicted by some analytical models (see Panofsky and Dutton (1984), for

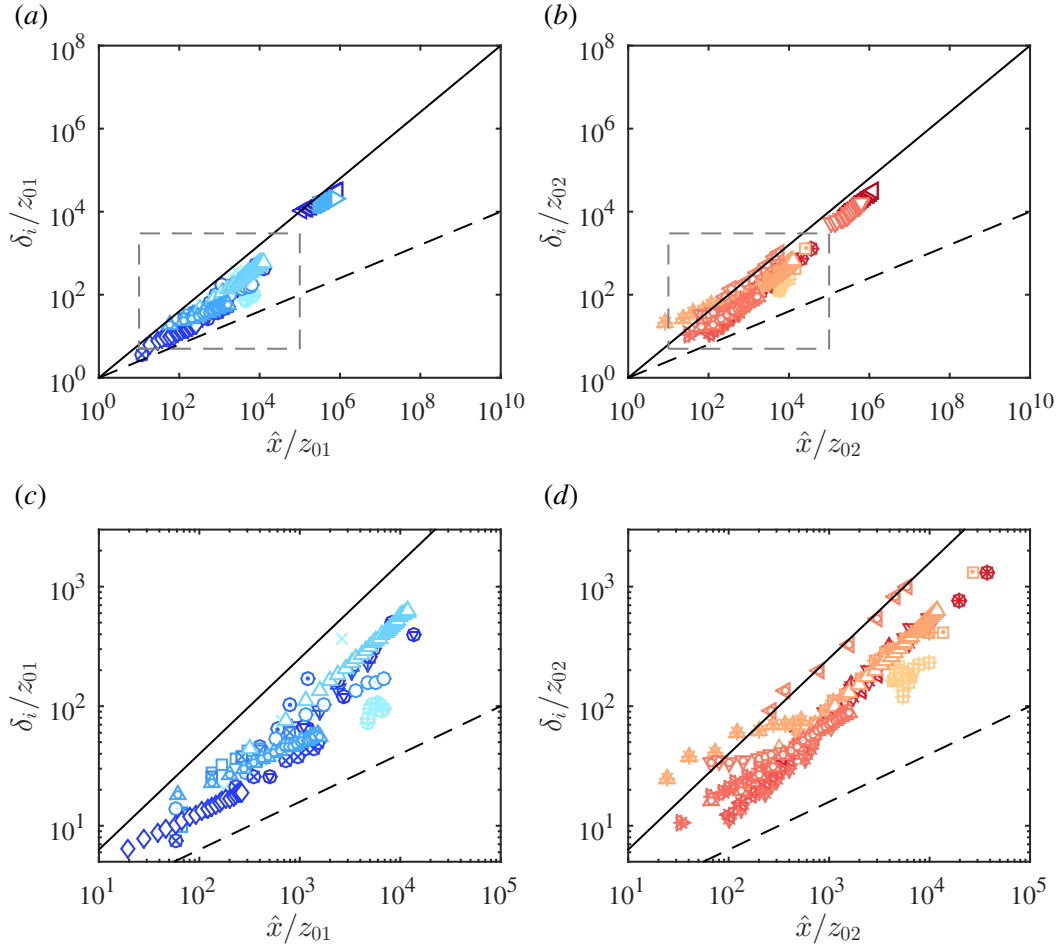


FIGURE 2.7: Downstream evolution of the IBL height δ_i for (a) rough-to-smooth and (b) smooth-to-rough cases. The length scale is chosen as the roughness length of the rougher surface, i.e. $\max(z_{01}, z_{02})$. Panels (c) and (d) are zoomed views of the boxed region in (a) and (b), respectively. The colour of each symbol represents M , and the references are listed in table 2.2.

example), the initial growth of δ_i is slow at a small fetch before it converges to the $\delta_i \propto \hat{x}^{0.8}$ trend downstream, or it can be affected locally by the step change of virtual origin associated with the roughness transition (Jackson, 1976), therefore any attempts to fit a power law to data limited to small fetches may be hampered by these factors.

Figure 2.6(d) presents a subset of the data where δ_0 is available as a length scale. This excludes most datasets from field measurements. Note that the axis limits in (d) are different from the other three panels. The scatter between datasets (compared to the overall range of the data) is higher than the normalisation by roughness lengths. In general, one can comment from figure 2.6(d) that the power-law exponent of 0.4 appears to be favoured over 0.8 to describe the overall trend, although within the scatter of data there are clearly a few examples (Antonia and Luxton, 1971b; Efros and Krogstad, 2011; Saito and Pullin, 2014; Chu and Bergstrom, 2020) that have an exponent that appears much closer to 0.8.

The dependence of the IBL growth on M is explored by presenting the rough-to-smooth ($M < 0$, shown by blue) and smooth-to-rough ($M > 0$, shown by red) cases separately in figure 2.7. We use the roughness length of the rougher surface as the length scale, i.e. z_{01} in

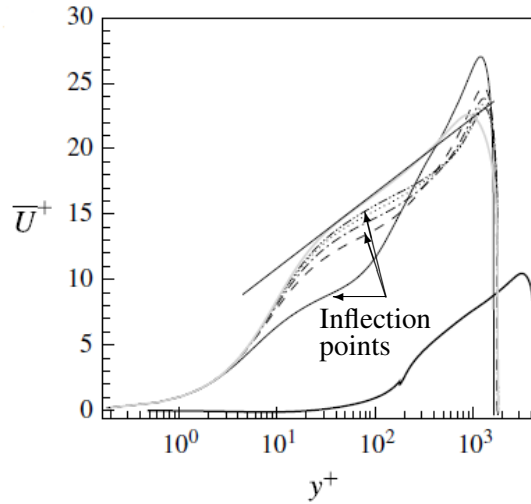


FIGURE 2.8: Mean streamwise velocity for case B (transition from square rib roughness to smooth wall) in the developing, smooth-wall regime scaled from the bottom wall and in inner coordinates. — in the fully developed rough-wall regime, — at $\hat{x}/\delta = 0.42$, - - - - at $\hat{x}/\delta = 2.08$, - - - - at $\hat{x}/\delta = 3.75$, ... at $\hat{x}/\delta = 5.42$, - - - - at $\hat{x}/\delta = 7.08$, — from fully developed smooth-wall channel flow. The figure and caption are reproduced from Ismail et al. (2018b).

rough-to-smooth cases and z_{02} in smooth-to-rough ones. The reference lines are at the same locations in both plots.

The high Reynolds number (large fetch) end of both cases is dominated by the LES datasets from Caltech group (Saito and Pullin, 2014; Sridhar, 2018). δ_i is slightly higher in rough-to-smooth cases than smooth-to-rough ones, although the power-law exponent (the slope) shows the opposite. A zoomed view of the lower Reynolds number (smaller fetch) part of the figure is shown in figures 2.7(c) and (d). Overall, $\delta_i / \max(z_{01}, z_{02})$ appears to be slightly higher in the smooth-to-rough cases, and the slope is more uniform and closer to 0.8, especially at a large fetch. However, it is difficult to draw conclusions regarding the effect of M on the IBL growth, since the difference between cases is usually very small and within the uncertainty of the data.

2.2.4 Mean velocity distribution

Similar to the behaviour of the wall-shear stress as detailed in the previous section, the mean streamwise velocity also goes through an acceleration (corresponding to the increase in the recovering τ_w) after a rough-to-smooth change, and a deceleration (corresponding to the decrease in the recovering τ_w) after a smooth-to-rough one (Antonia and Luxton, 1971b, 1972; Hanson and Ganapathisubramani, 2016; Rouhi et al., 2019b; Sridhar, 2018). Close to the step change, the acceleration and deceleration are the strongest and confined to the near-wall region, while the strength decreases, and the affected region widens farther downstream. In channel flows, a change in the sign of the streamwise flow acceleration is observed in the

wall normal direction: while the flow is accelerating in the near-wall region after a rough-to-smooth change, the velocity in the outer layer decelerates as dictated by the mass conservation in the channel (Saito and Pullin, 2014; Ismail et al., 2018b; Rouhi et al., 2019b). In a nominally zero pressure gradient turbulent boundary layer, on the other hand, the streamwise variation of the mean velocity appears to be negligible in experiments (Antonia and Luxton, 1971b, 1972; Hanson and Ganapathisubramani, 2016).

Another important feature in the recovering mean velocity profile is the ‘inflection point’ (Peterson, 1969; Savelyev and Taylor, 2005). As the recovering mean velocity profile deviates from its local equilibrium profile farther away from the wall, it leans towards the upstream reference, i.e. after a rough-to-smooth transition, the recovering mean velocity will undershoot the local equilibrium smooth-wall reference in the IBL, while it will overshoot a local equilibrium rough-wall reference after a smooth-to-rough change. Conceptually this is reflected in the sketches in figure 2.2 labelled as ‘transition region’, and evidence supporting the existence of the inflection point has been shown by the numerical results of Ismail et al. (2018b) (reproduced here as figure 2.8 with the locations of inflection points highlighted by arrows) and Rouhi et al. (2019b). This inflection point can be responsible for the underestimation of the local wall-shear stress after a rough-to-smooth change (to be detailed in §4).

2.2.5 Turbulent stresses

Downstream of a rough-to-smooth change, the streamwise turbulence intensity $\overline{u^2}$ is observed to fall below the rough-wall profile close to the wall, while it remains little modified in the outer layer (Antonia and Luxton, 1972; Hanson and Ganapathisubramani, 2016; Ismail et al., 2018b; Rouhi et al., 2019b). Similarly, downstream of a smooth-to-rough transition, $\overline{u^2}$ is observed to exceed the upstream smooth-wall profile. The region where $\overline{u^2}$ is different from its upstream value grows with an increasing fetch on the downstream surface, and there is evidence that the thickness of this region is strongly related to the IBL height (Hanson and Ganapathisubramani, 2016; Rouhi et al., 2019b). In fact, one method to determine the IBL height as proposed by Efros and Krogstad (2011) is to fit two straight lines to the inner and outer distribution of $\overline{u^2}$ and define the edge of the IBL as the intersecting point. The recovery of the wall-normal turbulence intensity $\overline{w^2}$ and Reynolds shear stress $\overline{uw}$ has also been observed to follow a similar trend.

It has been well established that the self-sustaining near-wall cycles are associated with the inner-peak that occurs at $z^+ \approx 15$ on a smooth wall (Jiménez and Pinelli, 1999), and their destruction by roughness elements is responsible for the absence of the inner-peak in fully rough flows (Grass, 1971; Schultz and Flack, 2007). When normalised by the freestream velocity, the magnitude of the inner-peak on a smooth wall is found to increase with the fetch after a rough-to-smooth change (Hanson and Ganapathisubramani, 2016), and decrease after a smooth-to-rough one (Antonia and Luxton, 1971b). This indicates the re-establishment of the near-wall cycle when the flow encounters a smooth surface where the disturbances from the roughness elements are removed, and conversely, the destruction of it when the flow goes through a smooth-to-rough change. When using the local recovering friction velocity

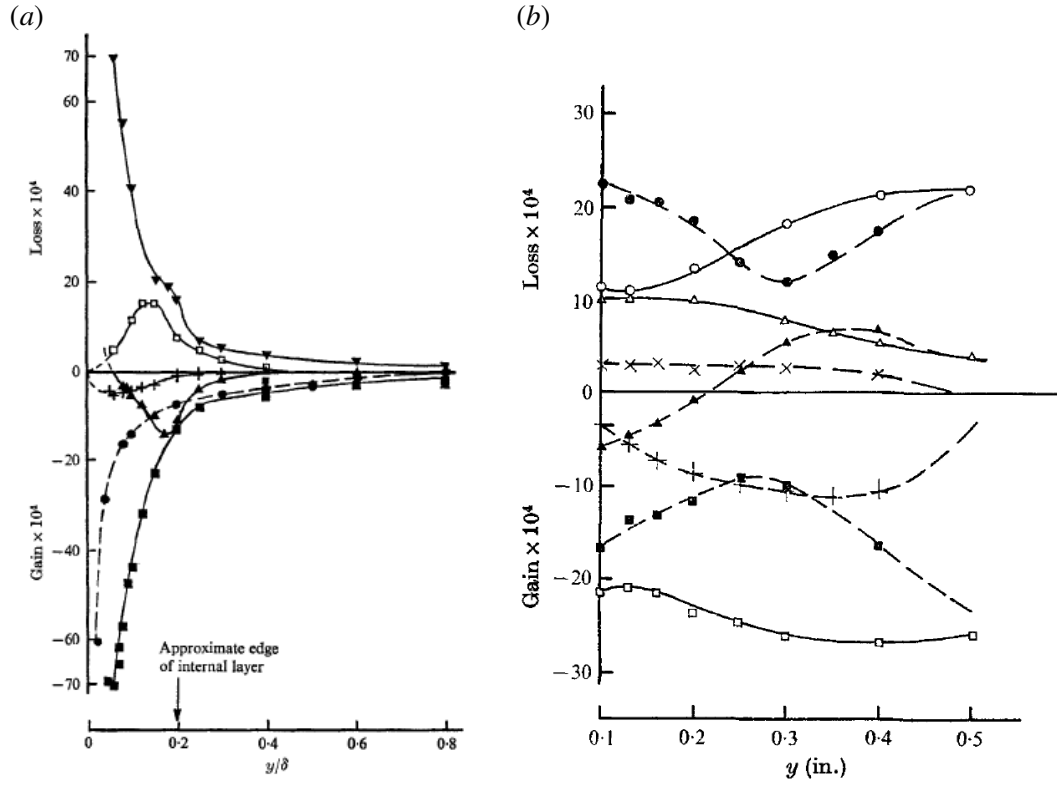


FIGURE 2.9: (a) Smooth-to-rough: turbulence energy balance at $\hat{x} = 4$ in. $U_\infty \delta_0 / \nu = 1.9 \times 10^4$. $\blacksquare$, production by $(\delta U_\infty^3) \overline{uw} \partial U / \partial z$; $\blacktriangle$, diffusion by $(\delta U_\infty^3) \partial(\frac{1}{2} \overline{q^2 w}) / \partial z$ only; $\square$, advection $(\delta U_\infty^3) [U \partial(\frac{1}{2} \overline{q^2}) / \partial x + W \partial(\frac{1}{2} \overline{q^2}) / \partial z]$; $+$, production by $(\delta U_\infty^3) (\overline{u^2} - \overline{w^2}) \partial U / \partial x$; $\blacktriangledown$, dissipation by difference; $\bullet$, Klebanoff(1955) smooth wall production at $U_\infty \delta / \nu = 7.8 \times 10^4$. (b) Rough-to-smooth: turbulence energy balance at $\hat{x} = 3$ in. $U_\infty \delta_0 / \nu = 2.6 \times 10^4$ and comparison with energy balance on rough wall at $\hat{x} = -1 \frac{7}{16}$ in., $U_\infty \delta_0 / \nu = 2.6 \times 10^4$. $\square$, production by $(\delta U_\infty^3) \overline{uw} \partial U / \partial z$; $\triangle$, diffusion by $(\delta U_\infty^3) \partial(\frac{1}{2} \overline{q^2 w}) / \partial z$ only; $\times$, production by $(\delta U_\infty^3) (\overline{u^2} - \overline{w^2}) \partial U / \partial x$; $+$, advection by $(\delta U_\infty^3) [U \partial(\frac{1}{2} \overline{q^2}) / \partial x + W \partial(\frac{1}{2} \overline{q^2}) / \partial z]$; $\circ$, dissipation by difference. Open symbols and solid curves are for $\hat{x} = -1 \frac{7}{16}$ in. Closed symbols and broken curves are for $\hat{x} = 3$ in. Both the advection and the production by the normal stresses are negligibly small at $\hat{x} = -1 \frac{7}{16}$ in. Figures adapted from Antonia and Luxton (1971b) and Antonia and Luxton (1972), respectively.

as the velocity scale instead, a decreasing trend is observed in the peak magnitude after a rough-to-smooth change (Hanson and Ganapathisubramani, 2016; Ismail et al., 2018b).

The shear-stress-to-turbulent-kinetic-energy ratio, $\alpha \equiv -\overline{uw} / \overline{q^2}$, is usually taken as a constant in the IBL in one-equation RANS models (Peterson, 1969; Shir, 1972). However, both experimentally and numerically, it has been shown that α is higher than its equilibrium value of the corresponding surface after a smooth-to-rough change, while lower after a rough-to-smooth change (Antonia and Luxton, 1971b, 1972; Pendergrass and Arya, 1984; Rao et al., 1974).

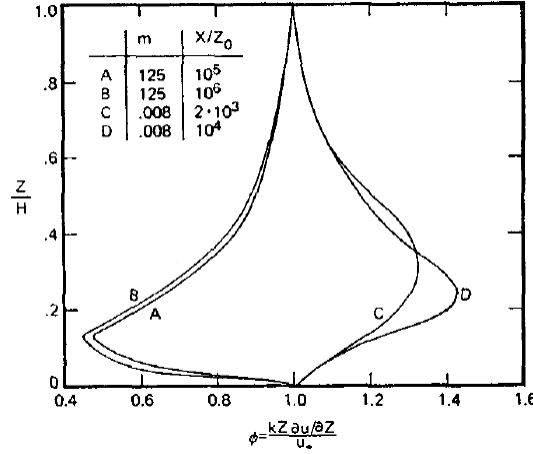


FIGURE 2.10: Nondimensional wind shear distribution within stress boundary layer. H is defined as the wall-normal location where the local shear stress is within 1% of the upstream value. $m \equiv z_{01}/z_{02}$ and z_0 in the figure is z_{02} following the current notation. The figure is reproduced from Shir (1972).

2.2.6 Turbulent energy budget

A simplified form of the turbulent energy equation for a two-dimensional incompressible mean flow is (Townsend, 1976):

$$\underbrace{\left(U \frac{\partial}{\partial x} + W \frac{\partial}{\partial z} \right) \frac{1}{2} \overline{q^2}}_{\text{advection}} + \underbrace{\overline{uw} \frac{\partial U}{\partial z}}_{\text{production}} + \underbrace{\frac{\partial}{\partial z} (\overline{pw} + \frac{1}{2} \overline{q^2 w})}_{\text{diffusion}} + \underbrace{\epsilon}_{\text{dissipation}} = 0, \quad (2.2)$$

where $\overline{q^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$. An equilibrium condition is achieved close to the wall where the local production and dissipation are so large that the flow is almost solely determined by the shear-stress distribution in the layer and not affected by the conditions outside (Townsend, 1961). In this scenario, the advection and diffusion terms will be negligible and the energy budget equation (2.2) reduces to a balance between the production and dissipation:

$$\overline{uw} \frac{\partial U}{\partial z} + \epsilon = 0. \quad (2.3)$$

The mixing length is defined as

$$l_m \equiv \frac{\tau^{1/2}}{\partial U / \partial z} \quad (2.4)$$

and the dissipation length is

$$l_\epsilon \equiv \frac{\tau^{3/2}}{\epsilon}, \quad (2.5)$$

where $\tau(z)$ is the shear stress. In the logarithmic region of an equilibrium boundary layer, (2.3) is satisfied. l_m and l_ϵ are identical and can be approximated by

$$l = \kappa z, \quad (2.6)$$

leading to a logarithmic velocity profile.

The turbulence energy budget after a roughness transition reproduced from Antonia and Luxton (1971b) and Antonia and Luxton (1972) is shown in figure 2.9. Downstream of a rough-to-smooth change, the dissipation term is observed to have an excess over production, with a gain in (negative) advection transporting the energy towards the wall (Antonia and Luxton, 1972; Mulhearn, 1978; Ismail et al., 2018b,a). The magnitude of advection is comparable with production, and it is mainly contributed by the enhanced streamwise gradient of $\overline{q^2}$ (i.e. $U \frac{\partial \overline{q^2}/2}{\partial x}$ term) immediately downstream of the rough-to-smooth change. Following a smooth-to-rough change, the production is different from the distribution of an equilibrium smooth wall but in close agreement with the local dissipation, which may suggest that a new length scale is imposed in the rapidly growing IBL (Antonia and Luxton, 1971b). In general, a non-zero advection term is observed in the IBL while the production and dissipation are still comparable in magnitude while different from their distribution in a turbulent boundary layer in equilibrium with the local wall condition. The two length scales, l_m and l_ϵ have similar behaviour as a result of the close agreement between production and dissipation terms, and they are observed to be lower than κz downstream of a smooth-to-rough change, while higher for a rough-to-smooth one (Antonia and Luxton, 1971b, 1972; Rao et al., 1974; Glendening and Lin, 2002).

Another commonly examined quantity, non-dimensional wind shear ($\phi \equiv \frac{\kappa z}{\tau^{1/2}} \frac{\partial U}{\partial z}$), can be related to the mixing length as $\phi = \kappa z / l_m$. It is essentially a measure of how different l_m is from (2.6). As shown in figure 2.10, ϕ is found to deviate from unity (Blackadar et al., 1967; Peterson, 1969; Shir, 1972; Rao et al., 1974; Mulhearn, 1978; Pendergrass and Arya, 1984), suggesting that a logarithmic velocity profile as assumed by Elliott (1958) may not be realistic in the IBL. ϕ equals to 1 at the wall and the edge of the IBL, but it is greater than 1 within the IBL for smooth-to-rough transitions and less than 1 for rough-to-smooth ones, indicating that the mean velocity first follows the local equilibrium profile closely in the near-wall region, and then transitions to the upstream profile within the IBL via some non-equilibrium blending profiles that might give rise to an inflection point (see figure 2.2), where the velocity gradient reaches a local maximum for a smooth-to-rough change and a minimum for a rough-to-smooth one. It also makes the velocity profile at a higher wall-normal location closer to the upstream velocity, resulting in a lower δ_i defined based on $\partial U / \partial x = 0$ than the shear stress (Shir, 1972; Bou-Zeid et al., 2004).

2.2.7 Correlation length scales

The correlation coefficient of two signals, s_1 and s_2 can be defined as (Krogstad and Antonia, 1994)

$$R_{s_1 s_2} = \frac{\overline{s_1(\mathbf{x}, t) s_2(\mathbf{x} + \Delta \mathbf{x}, t + \Delta t)}}{\left(\overline{s_1^2(\mathbf{x})} \cdot \overline{s_2^2(\mathbf{x} + \Delta \mathbf{x})} \right)^{1/2}} \quad (2.7)$$

where s_1 and s_2 represent velocity fluctuations u , v or w . $\mathbf{x}$ denotes the location of the reference probe and $\mathbf{x} + \Delta \mathbf{x}$ is the location of the other probe. t is the reference time and Δt is the time delay. When $s_1 = s_2$, equation (2.7) reduces to a two-point space-time auto-correlation of the signal. Additionally with $\Delta \mathbf{x} = 0$, (2.7) further reduces to a temporal auto-correlation.

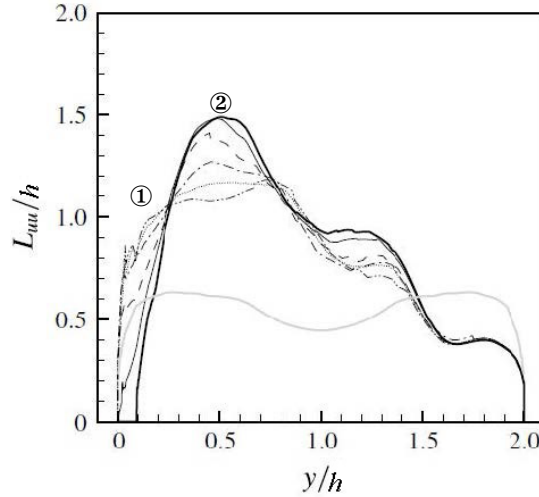


FIGURE 2.11: Variation of the streamwise integral length scale, L_{uu} , in the vicinity of step change in roughness for case B (transition from square rib roughness to smooth wall) in the wall-normal direction. — at $\hat{x}/h = -0.55$, — at $\hat{x}/h = -0.21$, - - - at $\hat{x}/h = 0.49$, - - - at $\hat{x}/h = 1.18$, ····· at $\hat{x}/h = 1.88$, - - - at $\hat{x}/h = 2.57$, — from fully developed smooth-wall channel flow. The figure and caption are reproduced from Ismail et al. (2018b).

A length scale representing the size of the coherent structures can be readily defined from the correlation coefficient by thresholding it at a small number (Krogstad and Antonia, 1994; Lee, 2015) or by integrating the correlation coefficient from $\Delta \mathbf{x} = 0$ to the first zero-crossing point (Antonia and Luxton, 1971b; Ismail et al., 2018b). We start with examining the streamwise correlation length scale on homogeneous rough and smooth surfaces in the literature. Figure 2.11 shows both rough-wall (thick black line) and smooth-wall (thick grey line) limits of the streamwise integral length in addition to the results after a rough-to-smooth change. When normalised by the channel half height h , the streamwise length scale is greater in the near-wall region (marked by ① in the figure) in a smooth-wall channel flow compared to a rough-wall one, while smaller close to the core (marked by ②). Nakagawa and Hanratty (2001) studied a channel flow using PIV with one side homogeneously roughened by a wavy surface, reporting a streamwise correlation length similar to that of a smooth-wall channel flow at $z_{ref} \approx 0.23h$. Sabot et al. (1977) also reported similar streamwise correlation lengths obtained from a rough and a smooth pipe. Krogstad and Antonia (1994), however, showed that the streamwise correlation length of a smooth-wall turbulent boundary is more than twice higher than that measured on a surface roughened by a mesh screen. This discrepancy has been partly attributed to the ambiguity of convection velocity (i.e. Taylor’s frozen turbulence hypothesis) used when converting from hotwire time series (Nakagawa and Hanratty, 2001; Jiménez, 2004; Squire et al., 2017).

Regardless, after a smooth-to-rough change, both streamwise (Antonia and Luxton, 1971b) and spanwise (Lee, 2015) length scales of the turbulent structures in the logarithmic region are observed to decrease in the vicinity of the step change. Antonia and Luxton (1971b) associated the initial reduction in the streamwise correlation length with the upper limit of the IBL, and Lee (2015) further related the decrease in the spanwise length scale to the initial

breakdown of the structures as they encounter the first roughness element after a smooth-to-rough transition. Both studies reported an increase in the correlation length scale further downstream, which is attributed to the enhanced merging of vortex packets on the rough surface by Lee (2015). An increase in the length scale is also reported immediately downstream of a rough-to-smooth change (Ismail et al., 2018b,a). Both streamwise and spanwise length scales in the near-wall region of the transitional regime (where the flow is recovering to a new equilibrium with the smooth wall) appears to be higher than their fully developed counterparts, as a consequence of the occasional interruption of large-scale structures developed on the rough wall.

2.3 Analytical modelling

2.3.1 Evolution models

In the seminal study of Elliott (1958), a theoretical model predicting the flow recovery from a change of surface roughness is established through a control volume analysis with an assumed form of the mean velocity profile

$$U(z) = \begin{cases} \frac{U_{\tau 1}}{\kappa} \ln(z/z_{01}), & z > \delta_i \\ \frac{U_{\tau 2}}{\kappa} \ln(z/z_{02}), & z < \delta_i. \end{cases} \quad (2.8)$$

The stress profile can subsequently be deduced from the log law as a step function:

$$\tau(z) = \begin{cases} \rho U_{\tau 1}^2, & z > \delta_i \\ \rho U_{\tau 2}^2, & z < \delta_i \end{cases} \quad (2.9)$$

A schematic of the mean velocity and shear stress profiles can be found in figure 7.1. The upstream friction velocity $U_{\tau 1}$ and the roughness lengths in both upstream and downstream are given. The only two unknowns in the system are δ_i , the IBL height and $U_{\tau 2}$, the friction velocity downstream. Using the continuity of the mean velocity profile at δ_i , it arises naturally that

$$\frac{U_{\tau 1}}{\kappa} \ln(\delta_i/z_{01}) = \frac{U_{\tau 2}}{\kappa} \ln(\delta_i/z_{02}). \quad (2.10)$$

A second equation to close the system is obtained from the momentum equation of the internal layer:

$$\frac{d}{dx} \int_{z_{02}}^{\delta_i} U^2 dz - U(\delta_i) \frac{d}{dx} \int_{z_{02}}^{\delta_i} U dz = U_{\tau 2}^2 - U_{\tau 1}^2 \quad (2.11)$$

The approximated solution can be expressed as

$$\frac{\delta_i(\hat{x})}{z_{02}} = (0.75 - 0.03M) \left(\frac{\hat{x}}{z_{02}} \right)^{0.8}, \quad (2.12)$$

where $M \equiv \ln(z_{02}/z_{01})$. A more accurate solution can be obtained numerically through an evolution of the mean velocity profile in the streamwise direction.

The non-physical discontinuity in the shear stress profile (in equation (2.9), see also figure 7.1b) in Elliott's model can be circumvented by assuming a linear distribution of the shear stress across the internal layer, resulting in a shear stress profile as

$$\tau(z) = \begin{cases} \rho U_{\tau 1}^2, & z > \delta_i \\ \rho \left[U_{\tau 2} + (U_{\tau 1} - U_{\tau 2}) \frac{z}{\delta_i} \right]^2, & z < \delta_i \end{cases} \quad (2.13)$$

as proposed by Panofsky and Townsend (1964). The mean velocity profile is integrated using the relation $\partial U / \partial z = (\tau / \rho)^{1/2} / \kappa z$ as

$$U(z) = \begin{cases} \frac{U_{\tau 1}}{\kappa} \ln(z/z_{01}), & z > \delta_i \\ \frac{U_{\tau 2}}{\kappa} \left[\ln(z/z_{02}) + \frac{U_{\tau 1} - U_{\tau 2}}{U_{\tau 2}} \frac{z}{\delta_i} \right], & z < \delta_i \end{cases} \quad (2.14)$$

Similar to equation (2.10), the continuity of the velocity across the interface can be enforced by making the two parts of (2.14) equivalent at $z = \delta_i$,

$$\frac{U_{\tau 1}}{\kappa} \ln(\delta_i/z_{01}) = \frac{U_{\tau 2}}{\kappa} \ln(\delta_i/z_{02}) + \frac{1}{\kappa} (U_{\tau 1} - U_{\tau 2}). \quad (2.15)$$

So far in both mean velocity profile models described, the flow above the IBL remains unmodified from the upstream of the step change. That is to say, if we denote the upstream velocity profile as $U_1(z)$, then $U(z) = U_1(z)$ will hold for all $z > \delta_i$. Another way of modelling the flow above the IBL is to assume that the mean velocity remains constant along

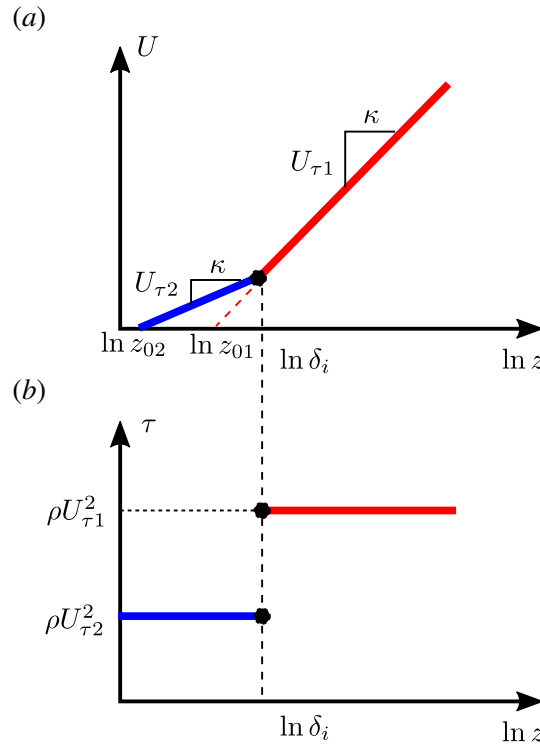


FIGURE 2.12: Schematic of (a) the mean velocity profile, equation (2.8) and (b) inferred shear stress profile, equation (2.9) as assumed by Elliott (1958). The velocity matching condition (2.10) is illustrated by the intersection of two log-law profiles at $z = \delta_i$.

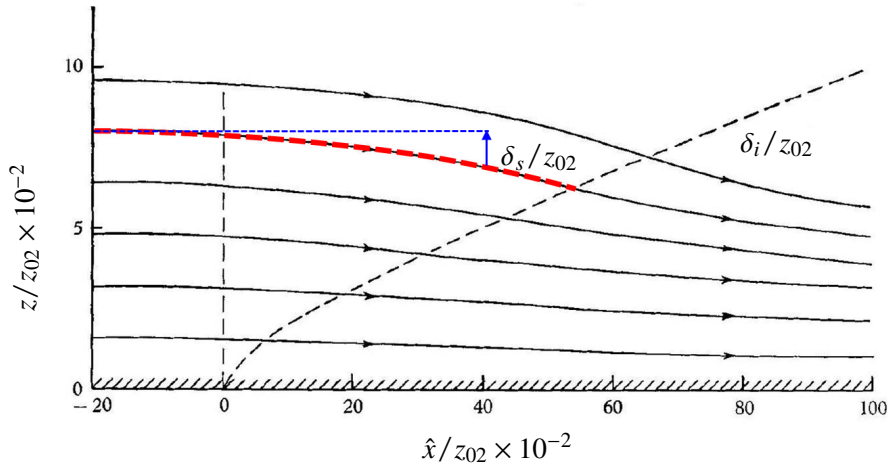


FIGURE 2.13: Schematic showing the streamlines of the mean flow in relation to the IBL after a rough-to-smooth change. The figure is reproduced from Townsend (1965b) with an extra sketch and adjusted labels for consistency with notation in this thesis.

a streamline above the IBL. As illustrated in figure 2.13, the mean velocity along the red dashed line is assumed to be constant before it intersects with the growing IBL. We define a vertical displacement of the streamline $\delta_s(\hat{x}, z)$, as illustrated by the blue arrow in the figure. Thus, the mean velocity relation will be restated as $U(z + \delta_s) = U_1(z)$ and the unknown δ_s can be evaluated through the mass conservation in a streamtube bounded by the wall and a streamline starting at z before the roughness transition:

$$\int_0^z U_1(z') dz' = \int_0^{z+\delta_s} U(z') dz'. \quad (2.16)$$

An example of the streamline displacement is shown in figure 2.13, where streamlines are closer to the wall downstream, indicating a flow acceleration resulted from a rough-to-smooth change. The streamline displacement remains non-zero farther away from the IBL. Townsend (1965b,a, 1966) proposed the following relation

$$U = \underbrace{\frac{U_{\tau 1}}{\kappa} \ln(z/z_{01})}_{\text{upstream profile}} - \underbrace{\frac{U_{\tau 1} \delta_s}{\kappa z}}_{\text{streamline displacement}} + \underbrace{\frac{U_0}{\kappa} f(\eta)}_{\text{acceleration}}, \quad (2.17)$$

which decomposed the change in the mean velocity into contributions from the streamline displacement (second term) and flow acceleration along the streamline (third term), and explored the possibility of a self-similar solution in the latter. Here, U_0 is a scale of the velocity change to be determined and $\eta \equiv z/\delta_i$ is the wall-normal location non-dimensionalised by δ_i .

Very close to the wall, the mean velocity profile is assumed to approach equilibrium with the new wall condition:

$$U = \frac{U_{\tau 2}}{\kappa} \ln(z/z_{02}). \quad (2.18)$$

For the decomposition of the recovering velocity profile (2.17) to satisfy this boundary condition, it requires

$$f(\eta) \rightarrow \ln(\eta) + C \quad \text{when } \eta \rightarrow 0, \quad (2.19)$$

and also

$$U_{\tau 2} = U_0 \left(1 + \frac{1}{\ln(\delta_i/z_{01}) - C_0} \right) + U_{\tau 1} \quad (2.20)$$

$$U_{\tau 1} M = U_0 \left[\frac{\ln(\delta_i/z_{02}) - C + 1}{\ln(\delta_i/z_{01}) - C_0} + \ln(\delta_i/z_{02}) - C \right],^1 \quad (2.21)$$

which are obtained by substituting (2.19) into (2.17), and then matching the coefficients of the logarithmic and constant terms of equations (2.17) and (2.18). C and C_0 are constants.

The change in the shear stress profile is also assumed to have a self-preserving form:

$$\frac{\tau}{\rho} = U_{\tau 1}^2 + 2U_0 U_{\tau 1} F(\eta). \quad (2.22)$$

In the limit of large $\ln(\delta_i/z_{02})$, the growth of δ_i can be determined directly from the coefficients of the momentum equation to ensure the existence of a self-preserving solution, without any knowledge of the exact form of $f(\eta)$ in the acceleration term:

$$\frac{d\delta_i}{dx} \ln(\delta_i/z_{02}) = 2\kappa^2. \quad (2.23)$$

The momentum equation will also give a relationship between the two self-preserving functions:

$$-\eta f' = F'. \quad (2.24)$$

In order to close the system and solve for $f(\eta)$ and $F(\eta)$, Townsend (1965b) assumed a ‘mixing-length’ transfer relation between the shear stress and mean velocity profile

$$\frac{\partial U}{\partial z} = \frac{\sqrt{\tau/\rho}}{\kappa z}, \quad (2.25)$$

which leads to a second equation of self-preserving functions:

$$f' = \eta^{-1} F. \quad (2.26)$$

Combining (2.24) and (2.26) and using the boundary condition $F(0) = 1$, the two self-preserving functions are found to be

$$F(\eta) = e^{-\eta}, \quad f(\eta) = Ei(-\eta) \quad (2.27)$$

where $Ei(\eta) \equiv \int_{-\infty}^{\eta} \frac{e^x}{x} dx$ is the exponential integral.

It has also been shown that the mean velocity profiles constructed by Elliott (1958) and Panofsky and Townsend (1964) can be incorporated into the Townsend’s self-preserving framework with streamline displacement. Essentially, a different form of $f(\eta)$ is used and C

¹There appears to be a mistake in the original equation (2.4) in Townsend (1965a), which reads $U_0[\ln(\delta_i/z_{02}) - C + 1 + (\ln(\delta_i/z_{01}) - C_0)^{-1}] - U_{\tau 1} M = 0$. This error also propagates to the approximation of this equation at large δ_i/z_{02} in (4.4) in Townsend (1965b) and (2.13) in Townsend (1966).

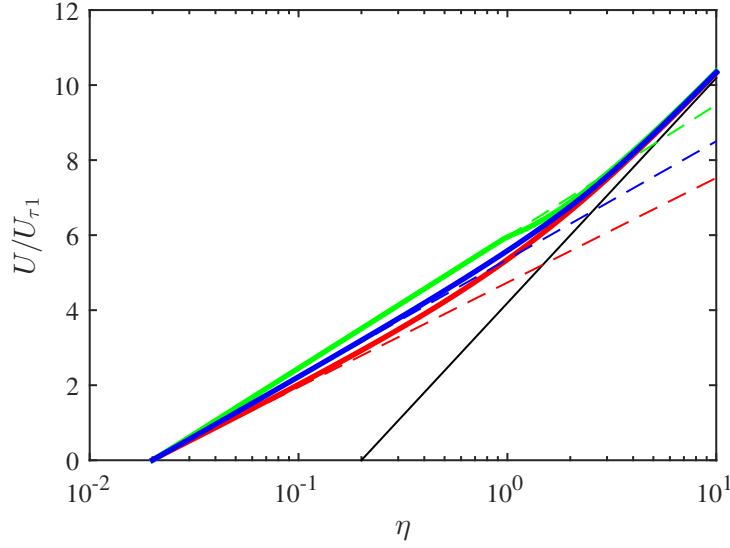


FIGURE 2.14: Mean velocity profiles assumed by various models. The black line is a reference profile on the upstream surface, while green, blue and red colours correspond to Elliott's, Panofsky & Townsend's and Townsend's model. The coloured dashed lines show the equilibrium logarithmic profile with the downstream $U_{\tau 2}$ of each model.

and C_0 will be determined accordingly once $f(\eta)$ is chosen. For Elliott's profile

$$f(\eta) = \begin{cases} \ln(\eta), & \eta < 1 \\ 0, & \eta > 1. \end{cases} \quad (2.28)$$

For Panofsky and Townsend's profile

$$f(\eta) = \begin{cases} \ln(\eta/2) + (1 - \eta/2), & \eta < 2 \\ 0, & \eta > 2. \end{cases} \quad (2.29)$$

Note that with the expression of Elliott's or Panofsky & Townsend's, the 'mixing-length' relation (2.26) will no longer hold.

	$\delta_i = 0.05\text{m}$	$\delta_i = 0.2\text{m}$	$\delta_i = 0.5\text{m}$
Elliott	0.59	0.60	0.65
Panofsky & Townsend	0.53	0.58	0.63
Townsend	0.46	0.55	0.61

TABLE 2.3: Normalised friction velocity $U_{\tau 2}/U_{\tau 1}$ for three mean velocity profile models at various δ_i .

As an illustration, the three mean velocity profiles are compared in figure 2.14 at matched conditions: the roughness lengths are $z_{01} = 10 \times 10^{-3}$ m and $z_{02} = 1 \times 10^{-3}$ m, with $U_{\tau 1} = 1\text{ms}^{-1}$ and $\delta_i = 0.05\text{m}$ for all models. Streamline displacement is also considered in all models. The local friction velocity $U_{\tau 2}$ is first evaluated using (2.21) and (2.20), and the results are tabulated in table 2.3. Elliott's model leads to the highest $U_{\tau 2}/U_{\tau 1}$, therefore the highest mean velocity profile, while Panofsky & Townsend's and Townsend's model produce the second highest and the lowest $U_{\tau 2}/U_{\tau 1}$. As shown in the figure, all velocity

profiles approach to the equilibrium reference (dashed lines) close to the wall as dictated by the boundary condition (2.18). The velocity distribution above the IBL is higher than the reference profile upstream as a result of the streamline displacement. It is worthwhile to mention that while Elliott's model follows a logarithmic profile within the IBL, both Panofsky & Townsend's and Townsend's models are slightly above than the reference equilibrium logarithmic law. As discussed earlier in §2.2.4, the non-equilibrium behaviour in the IBL after a rough-to-smooth transition is usually manifested as an undershoot in the mean velocity profile. Although both Panofsky & Townsend's and Townsend's models attempt to account for such non-equilibrium effects, the resulting mean velocity profiles are higher instead of lower than the equilibrium smooth-wall reference. One possible explanation is that, both models use a 'mixing-length' relation (2.25) to relate the shear stress and mean velocity profile. The applicability of this relation to a highly non-equilibrium boundary layer flow has been questioned by the deviation from unity of the non-dimensional wind shear (see §2.2.6).

Chamorro and Porté-Agel (2009) proposed a semi-empirical blending model of the recovering mean velocity profile without further incorporating the streamwise evolution. They considered that after a rough-to-smooth transition, the mean velocity profile asymptotes to the new smooth-wall log law at the wall and the upstream rough-wall log law at the edge of the IBL, which can be combined into one expression

$$U = (1 - \lambda) \frac{U_{\tau 2}}{\kappa} \ln\left(\frac{z}{z_{02}}\right) + \lambda \frac{U_{\tau 1}}{\kappa} \ln\left(\frac{z}{z_{01}}\right) \quad (2.30)$$

with the limits of λ given by

$$\lambda(z) = \begin{cases} 1, & z \geq \delta_i \\ 0, & z \ll \delta_i. \end{cases} \quad (2.31)$$

The two limiting cases are linked by a gradual transition in between, which is approximated by a log-linear blending function

$$\lambda = \frac{\log(z/z_{01})}{\log(\delta_i/z_{01})}. \quad (2.32)$$

This model does not assume a constant or linear distribution of the shear stress in the IBL (as did Elliott (1958) and Panofsky and Townsend (1964)). Instead, it relies on the empirical observation that the mean velocity profile gradually transitions from the local smooth-wall log law to the upstream rough-wall one as z/δ_i increases. It has been shown to have a good performance in capturing the mean velocity profile and local wall-shear stress close to the roughness transition in the same study. This model is further discussed in §8.

2.3.2 Diffusion analogy models

Another family of theoretical models are established based on the diffusion analogy, which was first introduced by Miyake (1965). In this approach, the region affected by the downstream wall condition is considered to propagate in the same manner as a passive contaminant, and the growth rate of δ_i is proportional to the vertical diffusion intensity $w_{rms} =$

$(\overline{w^2})^{1/2}$, i.e.

$$\frac{d\delta_i}{dt} = Aw_{rms}, \quad (2.33)$$

where A is a constant. Using the chain rule, the LHS of the equation can be rewritten as $d\delta_i/dt = (\partial\delta_i/\partial x)(dx/dt)$ for a steady state ($\partial\delta_i/\partial t = 0$). Assuming $dx/dt = U(\delta_i)$, which is the mean velocity at the interface, the equation above can be rewritten as

$$U(\delta_i) \frac{d\delta_i}{dx} = Aw_{rms}. \quad (2.34)$$

The velocity at the interface $U(\delta_i)$ is usually modelled using a logarithmic profile, and the w_{rms} term is further related to the friction velocity (upstream or downstream) through an assumed proportionality in neutrally stratified flow, $w_{rms} = CU_\tau$. Panofsky and Dutton (1984) assumed a logarithmic profile within the IBL, which leads to $U(\delta_i) = \frac{U_{\tau 2}}{\kappa} \ln(\delta_i/z_{02})$. They also assumed $A = 1$ and $w_{rms} = 1.25U_{\tau 2}$. This assumption is approximately valid in the log region, although a suppression up to 15% in the peak value of w_{rms}^+ on a transitionally rough surface has been reported recently (Morrill-Winter et al., 2017).

Equation (2.34) can be integrated with an initial condition of $\delta_i|_{\hat{x}=0} = z_{02}$, resulting in the final expression of IBL growth as

$$1.25\kappa \frac{\hat{x}}{z_{02}} = \frac{\delta_i}{z_{02}} \left[\ln\left(\frac{\delta_i}{z_{02}}\right) - 1 \right] + 1. \quad (2.35)$$

This expression has the same form as the relation obtained by Miyake (1965):

$$1.73\kappa \frac{\hat{x}}{z_{02}} = \frac{\delta_i}{z_{02}} \left[\ln\left(\frac{\delta_i}{z_{02}}\right) - 1 \right] - \frac{\delta_{i0}}{z_{02}} \left[\ln\left(\frac{\delta_{i0}}{z_{02}}\right) - 1 \right], \quad (2.36)$$

except that a different coefficient (1.73 instead of 1.25) and a more general initial condition of $\delta_i|_{\hat{x}=0} = \delta_{i0}$ is used. Except immediately after the roughness transition, the initial condition is insignificant as typically $O(\hat{x}/z_{02}) \gg 1$. Jackson (1976) further modified the model to account for the effect of the change in the virtual origin.

Bou-Zeid et al. (2004) considered an effective logarithmic profile averaging across the alternating roughness patches in an open channel, which leads to $U(\delta_i) = \frac{U_{\tau 0}}{\kappa} \ln(z/z_{0e})$ at the interface (where z_{0e} is the roughness length of the effective profile and $U_{\tau 0}$ is the global friction velocity dictated by the imposed pressure gradient of the open channel). They approximated w_{rms} by $U_{\tau 0}$ with $A = 0.85$ based on their LES data. Omitting the initial condition term, equation (2.34) can be integrated as

$$0.8\kappa \frac{\hat{x}}{z_{0e}} = \frac{\delta_i}{z_{0e}} \left[\ln\left(\frac{\delta_i}{z_{0e}}\right) - 1 \right]. \quad (2.37)$$

Note that δ_i is defined as the location where $\partial\langle u \rangle_{t,y}/\partial z = \partial\langle u \rangle_{t,x,y}/\partial z$, which is thinner than the conventional definition of the intersection of two log laws considering a transition region with finite thickness.

To avoid prescribing the velocity profile or the shear stress behaviour within the IBL, Savelyev and Taylor (2001) used the upstream velocity profile, i.e. $U(\delta_i) = \frac{U_{\tau 1}}{\kappa} \ln(\delta_i/z_{01})$,

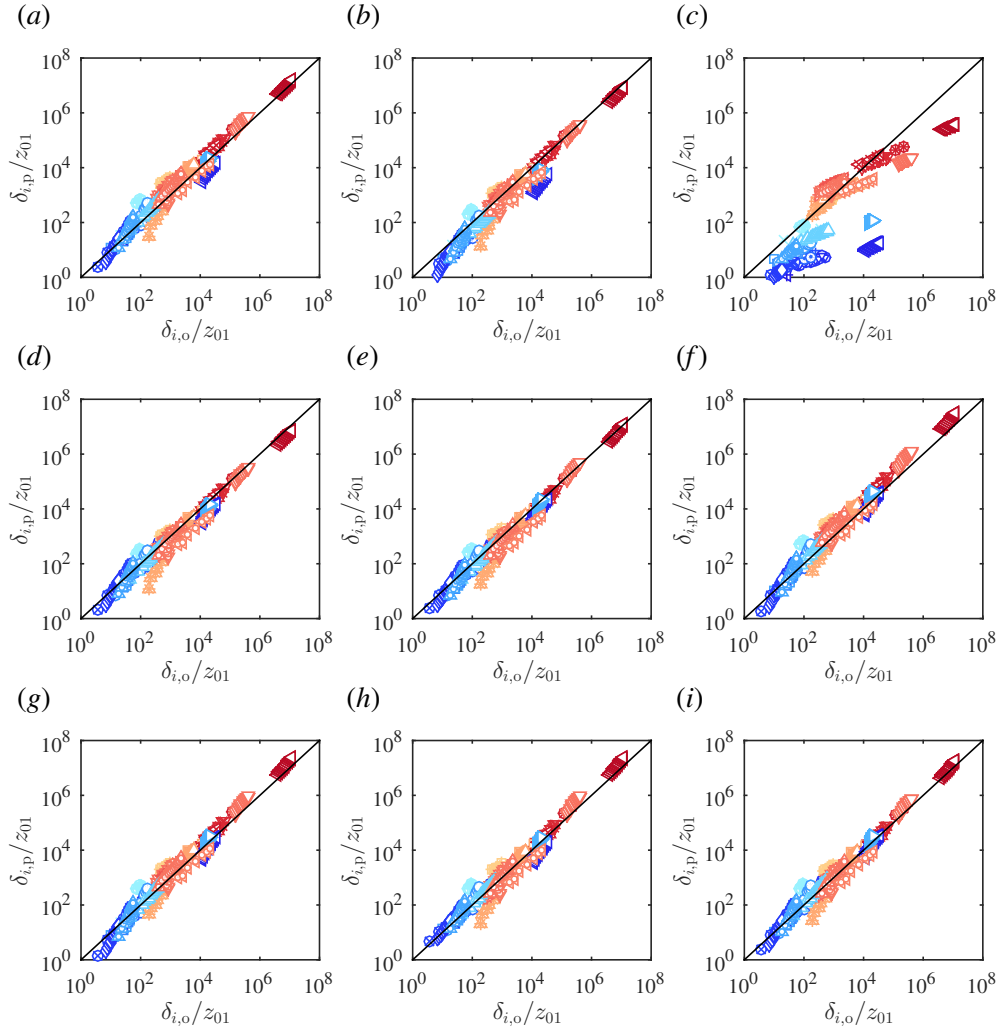


FIGURE 2.15: The predicted $\delta_{i,p}$ versus observed $\delta_{i,o}$ in logarithmic scale. The solid black line is $\delta_{i,p} = \delta_{i,o}$. Subfigures (a-h) correspond to the references listed in table 2.4.

and subsequently $w_{rms} = 1.25U_{\tau 1}$, instead of the downstream representation used by Panofsky and Dutton (1984). They obtained a linear relation of $A = 1.0 + 0.1M$ by fitting to a collection of field and laboratory datasets. The empirically fitted coefficients were later revised and extended to non-neutral stability conditions by considering the mean vertical velocity component due to mass conservation constraints at the roughness transition (Savelyev and Taylor, 2005).

With the diffusion analogy approach, usually only δ_i is predicted directly from the model. Assumptions of the mean velocity profile within the IBL are required to calculate the corresponding $U_{\tau 2}$.

2.4 Evaluating the performance of predictive models in the literature

The performance of a subset of the predictive models for δ_i compiled by Savelyev and Taylor (2005) is examined with the aid of the data collected in table 2.2. These models under

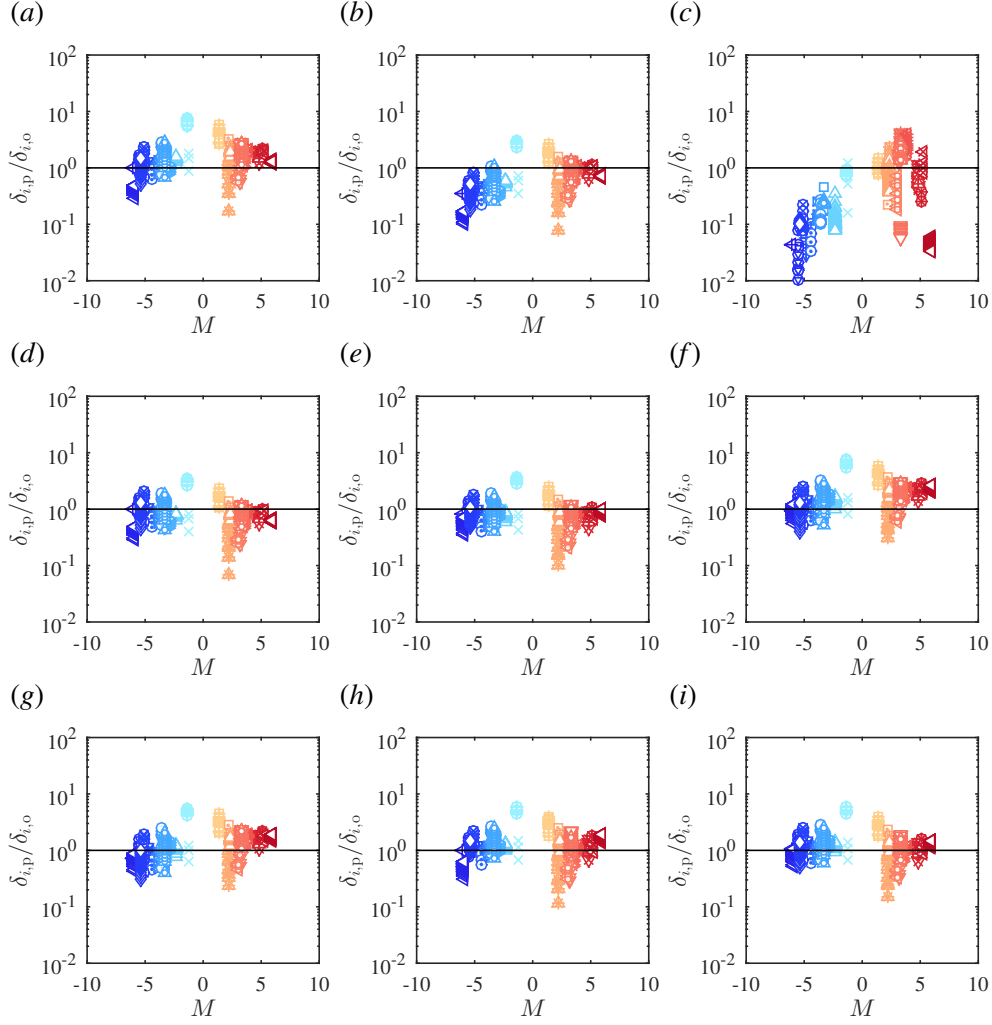


FIGURE 2.16: The ratio between predicted ($\delta_{i,p}$) and observed internal layer thickness ($\delta_{i,o}$) in logarithmic scale. Subfigures (a-h) correspond to the references listed in table 2.4.

examination are listed in table 2.4. Figure 2.15 shows the observed ($\delta_{i,o}$) and predicted ($\delta_{i,p}$) IBL. The ratio of $\delta_{i,p}/\delta_{i,o}$ is shown in figure 2.16 on a logarithmic scale as a representation of the relative error in the prediction. The first four models, i.e. Elliott (1958), Wood (1982), Pendergrass and Arya (1984) and Cheng and Castro (2002) are power-law type models with various coefficients. Note that for Elliott's model, we use the approximated solution (2.12) instead of the full streamwise evolution. Overall, Elliott's model tends to overestimate δ_i in smooth-to-rough cases (the majority of red symbols are above $\delta_{i,p}/\delta_{i,o} = 1$ in figure 2.16a). Pendergrass and Arya's model was obtained by fitting to an experimental dataset with a rough-to-rougher transition. As shown in figures 2.15(b) and 2.16(b), it underestimates δ_i in rough-to-smooth cases, especially at large $|M|$ values. Similarly, Cheng and Castro's model was also based on a power-law fit to an experimental dataset with a rough-to-rougher transition. The power-law exponent is 0.33, very different from the value 0.8 as used in other power-law type models. It does not capture the growth trend of δ_i in the majority of the collected cases. While the roughness length of the downstream surface (z_{02}) is used in the first three power-law models, Wood's model incorporates a length scale of $\max(z_{01}, z_{02})$

Formula	Reference	Relative error (%)	Correlation coefficient
(a) $\frac{\delta_i}{z_{02}} = (0.75 - 0.03M) \left(\frac{\hat{x}}{z_{02}} \right)^{0.8}$	Elliott (1958)	55	0.89
(b) $\frac{\delta_i}{z_{02}} = 0.32 \left(\frac{\delta_i}{z_{02}} \right)^{0.8}$	Pendergrass and Arya (1984)	40	0.93
(e) $\frac{\delta_i}{z_{02}} = 10.56 \left(\frac{\delta_i}{z_{02}} \right)^{0.33}$	Cheng and Castro (2002)	89	0.78
(d) $\frac{\delta_i}{\max(z_{01}, z_{02})} = 0.28 \left(\frac{\hat{x}}{\max(z_{01}, z_{02})} \right)^{0.8}$	Wood (1982)	30	0.90
(e) $\kappa^2 \hat{x} = \delta_i \frac{\ln \frac{\delta_i}{z_{02}} \left(\ln \frac{\delta_i}{z_{02}} + M \right)}{2 \ln \frac{\delta_i}{z_{02}} + M}$	Townsend (1966)	22	0.90
(f) $1.73\kappa \frac{\hat{x}}{z_{02}} = \frac{\delta_i}{z_{02}} \left(\ln \frac{\delta_i}{z_{02}} - 1 \right) - \frac{\delta_i(0)}{z_{02}} \left(\ln \frac{\delta_i(0)}{z_{02}} - 1 \right)$	Miyake (1965) ²	86	0.94
(g) $1.25\kappa \frac{\hat{x}}{z_{02}} = \frac{\delta_i}{z_{02}} \left(\ln \frac{\delta_i}{z_{02}} \right) + 1$	Panofsky and Dutton (1984)	49	0.94
(h) $\delta_i \left(\ln \frac{\delta_i}{z_{01}} - 1 \right) = 1.25\kappa(1 + 0.1M)\hat{x}$	Savelyev and Taylor (2001)	39	0.91
(i) $\delta_i \left(\ln \frac{\delta_i}{\sqrt{z_{01} z_{02}}} - 1 \right) = 0.5\hat{x}$	Savelyev and Taylor (2005)	33	0.88

TABLE 2.4: Predictive models and their accuracy evaluated by laboratory, field and numerical datasets. The relative error is defined as $\sqrt{\text{median} \left[\left(\frac{\delta_{i,p} - \delta_{i,o}}{\delta_{i,o}} \right)^2 \right]} \times 100\%$.

²The initial condition is chosen as $\delta_i(0) = 0$.

and it has better performance for both rough-to-smooth and smooth-to-rough cases. Panel (e) shows the results from Townsend’s (1966) model, which is developed based on a self-similar velocity acceleration profile, as discussed previously in §2.3.1. It gives reasonable prediction for both $M > 0$ and $M < 0$ cases. The last four formulae are based on the ‘diffusion analogy’ (see §2.3.2) with various assumptions regarding the detailed expression of w_{rms} . In panel (f), Miyake’s expression overpredicts δ_i for most of smooth-to-rough cases. In the far field, Panofsky and Dutton’s model is only different from Miyake’s expression by a constant multiplier of approximately 0.7. Therefore, panels (f) and (g) have very similar behaviour, including a slight increase in the ratio $\delta_{i,p}/\delta_{i,o}$ with an increasing M . Finally, both models of Savelyev and Taylor have good performance, although the revised version (Savelyev and Taylor, 2005) in panel (i) shows a slight advantage over the earlier one in (h).

By testing the performance of several predictive models with the extensive data in the literature, we conclude that so far there is no ‘perfect’ model that can accurately replicate all the measured data trends. As summarised in table 2.4, the relative error in these predictions are over 20%, and it can be much larger for models obtained by fitting to one particular dataset. On the other hand, one should also be aware of the error and uncertainties associated with these data, differences resulting from the definition of δ_i adopted by each study, and also the flow conditions that are not accounted for in the model. For instance, data from the field measurements can be contaminated by the unsteady incoming flow, non-neutral conditions, non-uniformity of roughness in each patch which is supposed to be homogeneous, to name a few. In addition, there is evidence that a change in the virtual origin can affect the flow recovery behaviour in the near field (Yeh and Nickerson, 1970; Rouhi et al., 2019a), yet it is not modelled in any of the formulae discussed above. The present work aims at addressing these issues by presenting a high-quality laboratory dataset with friction Reynolds numbers $Re_{\tau 0}$ up to 21000, and also developing a predictive model with the aid of these data.

Chapter 3

The measurement campaign

3.1 Facilities

In this study, the experimental measurements are carried out in two wind tunnel facilities in the Walter Basset Aerodynamics Laboratory at the University of Melbourne. The first set of the experiments, focusing on the step effect at the rough-to-smooth transition in surface conditions, are performed in an open-return boundary layer wind tunnel, while the second set which is designed to examine the Reynolds number dependence are conducted in the High Reynolds Number Boundary Layer Wind Tunnel (HRNBLWT). In both facilities, x is the distance downstream of the inlet to the working section, x_0 is the streamwise location of the surface transition and $\hat{x} \equiv x - x_0$ is the distance downstream of the transition.

3.1.1 Open-return boundary layer wind tunnel

The first set of experiments focusing on the step effect are carried out in the open-return blower boundary layer wind tunnel. This facility has been used extensively to study smooth-walled turbulent boundary layers (Marusic and Perry, 1995; Harun et al., 2013), and more recently, rough-walled measurements have been conducted by Nugroho et al. (2013) and Kevin et al. (2015). It is equipped with a settling chamber, followed by flow conditioning and then a contraction section with an area ratio of 8.9:1. The $6.7 \times 0.94 \times 0.38 \text{ m}^3$ working section of this facility has an adjustable perspex roof to enable fine pressure gradient adjustment. Further, the test surface (bottom wall) of the working section can be lowered to ease the interchange of the surface for different types of experiments. The turbulent boundary layer is tripped by a strip of P40 sandpaper at the inlet of the working section, and then develops on the tunnel floor. For the present work, the first 3.7 m of the tunnel surface is covered by P16 grit sandpaper (manufactured by Synteko Pty Ltd.), while the remaining streamwise length (1.9 m) is a smooth surface (see figure 3.1). Details of the roughness parameters are obtained by scanning a $60 \text{ mm} \times 60 \text{ mm}$ section of the sandpaper using an in-house built laser scanner. Colour contours of the roughness topography is shown in the inset of figure 3.1. The root-mean-squared roughness height (k_{rms}) of this surface is 0.387 mm, and the maximum roughness height between the crest and trough is $k_p \approx 2 \text{ mm}$, which is equivalently 2% of the boundary layer thickness δ_0 at the surface transition.

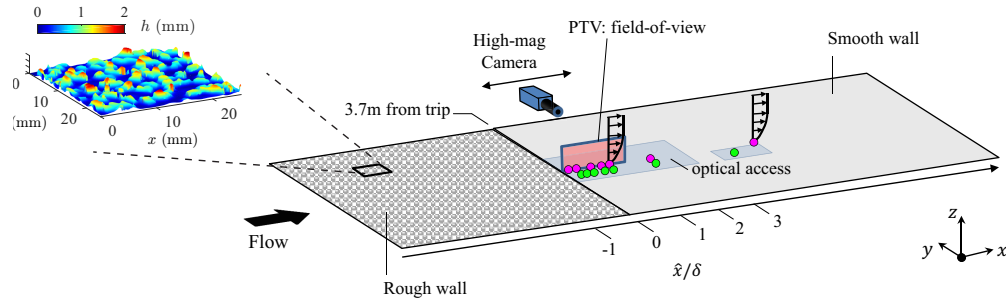


FIGURE 3.1: Overview of the experimental campaign in the open-return boundary layer wind tunnel facility at $Re_{\tau 0} \approx 4100$. The $\bullet$ symbols correspond to the locations of the hotwire wall-normal profiles, and the $\bullet$ symbols correspond to the locations of OFI measurements. The colour contours in the inset illustrate the topography of the P16 grit sandpaper which is employed as the rough-walled surface.

3.1.2 HRNBLWT

The high Reynolds number experiments are carried out in the HRNBLWT. The flow is generated by an axial fan driven by a 200kW D.C. motor, then passes through a heat exchanger, a settling chamber with a honeycomb flow straightener and six fine mesh screens, and a contraction section with an area ratio of 6.2:1 before entering the test section. The test section has a cross-sectional area of 1.89 m (width) $\times$ 0.92 m (height) and a length of 27 m, allowing the streamwise development of a thick boundary layer. More details of the facility are available in Nickels et al. (2005), Kulandaivelu (2012) and Marusic et al. (2015).

A photo of the sandpaper layout in the wind tunnel is shown in figure 3.2(a). An upstream portion of the tunnel floor in the test section is covered by a sheet of P24 grit sandpaper (SP40F, Awuko Abrasives) from the inlet to the location of x_0 , while the remaining length is a smooth aluminium surface. The sandpaper sheet spans the entire width of the tunnel floor, and is installed in a similar fashion as detailed in Squire et al. (2016b). Figure 3.3 is a schematic showing the installation of the sandpaper. The sandpaper sheet is affixed to the

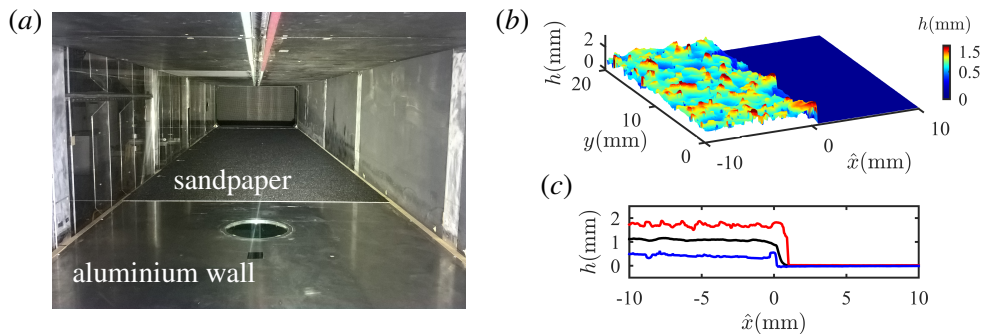


FIGURE 3.2: (a) Photo showing the inside of the HRNBLWT. The camera is facing the inlet of the working section. (b) The surface elevation at the rough-to-smooth transition measured using an in-house built laser scanner. The black line in (c) is the spanwise average of the surface elevation, and the red and blue lines are the maximum and minimum of the surface elevation along each spanwise line.

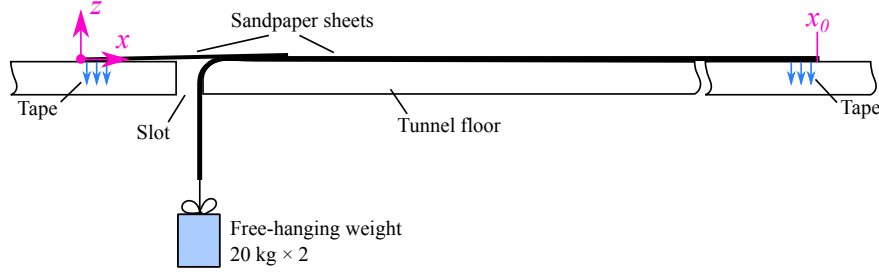


FIGURE 3.3: Schematic showing the installation of the sandpaper sheet.

tunnel floor at x_0 using five spanwise strips of double-sided tape. It is then fed through a slot on the tunnel floor at the inlet which spans the width of the tunnel and tensioned by two free-hanging weights. This helps remove undulations and allow for expansion or contraction of the sandpaper sheet. The slot is covered by a piece of the same type of sandpaper to avoid air leakage.

To characterise the roughness parameters, a $60 \text{ mm} \times 60 \text{ mm}$ patch of the rough-to-smooth transition is scanned using an in-house built laser scanner. The resulting surface topography is shown in figure 3.2(b-c). The maximum peak-to-trough roughness height is $k_p \approx 1.2 \text{ mm}$, and the smooth wall is approximately 1.8 mm below the roughness crest, i.e. the step height between the roughness crest and the smooth surface downstream is $\Delta H \approx -1.8 \text{ mm}$.

The overview of the experimental setup of all cases is depicted in figure 3.4. The main traversing system of the HRNBLWT is an in-house designed and built sting, as shown in the magnified view in figure 3.4 and also detailed in Kulandaivelu (2012). The system can be traversed, via computer control, in the streamwise and wall-normal directions. The cross-section of the sting resembles a symmetrical NACA0016 aerofoil. This design reduces the blockage effect in the upstream and the vibrations related to vortex shedding. At the bottom of the sting, a retractable pneumatic foot (see inset (b) in figure 3.4) holds the sting firmly against the tunnel floor during measurements to further reduce the flow induced vibrations. The pressure valve controlling the release and retraction of the pneumatic foot in the original design has recently been upgraded to a proportional one (Festo VEAB with a pressure range of 0–6 bar), allowing a more gradual movement of the pneumatic foot. A pitot-static tube is mounted on the sting at $z = 0.525 \text{ m}$, monitoring the freestream velocity. The pressure difference of the pitot-static tube is converted to a voltage signal by an MKS Baratron 10 Torr capacity pressure transducer. The sting system also houses a thermistor probe and an atmospheric pressure barometer, which enable the calculation of ν , the kinematic viscosity of air, using perfect gas law and Sutherland's law (Sutherland, 1893). The wall-normal traverse uses a servo-motor system with a Renishaw RGH24 optical linear encoder (④ in figure 3.4b) feedback. The encoder has a resolution of $1 \text{ } \mu\text{m}$.

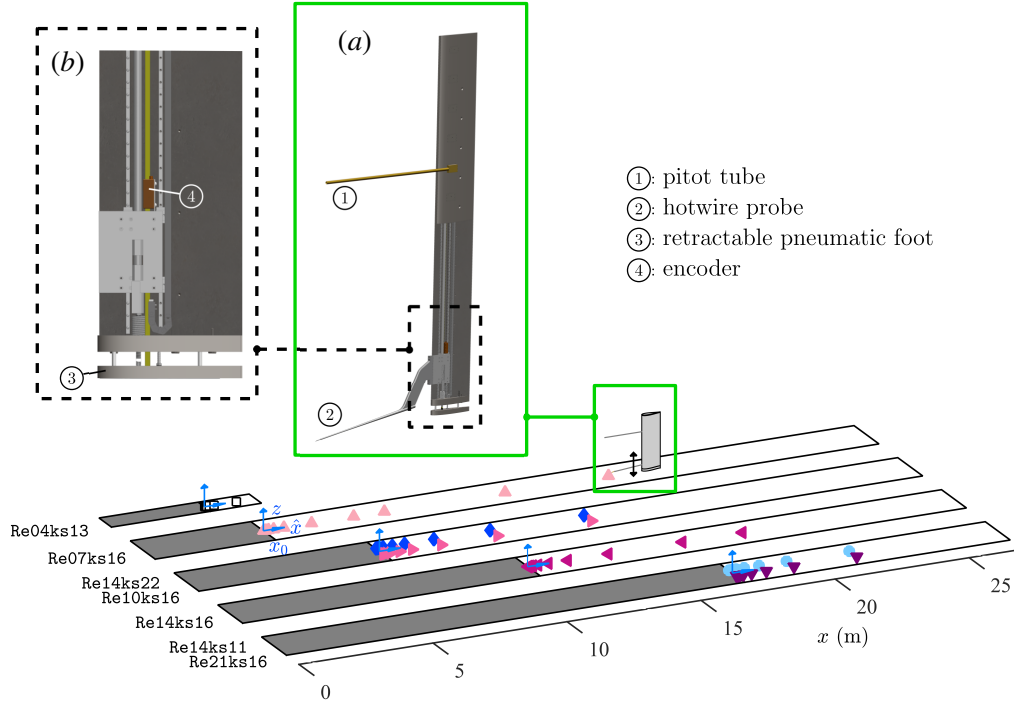


FIGURE 3.4: Overview of the experimental setup in HRNBLWT. The flow is going from the left to the right. The grey shaded surface represents the sandpaper, and the white colour represents the smooth wall. Streamwise locations where a wall-normal hotwire profile is acquired in each case are shown by the corresponding symbols. Please refer to §3.2 for the details of the case naming convention and the experiment plan. Note that as Re04ks13 (originally case H0 in the first part of experiments) is conducted in the other wind tunnel, the length and width of the working section are different from the rest. Inset (a) shows a zoomed view of the sting where the probes and traversing system are installed, and inset (b) shows a further close-up of the interior of the sting.

3.2 Experiments

Two sets of experiments are planned in the present study. All experiments in the first set are carried out in the open-return boundary layer wind tunnel at nominally the same flow condition, focusing on the effect of the step height change accompanying the roughness transition. The measurements in the second set are performed in the HRNBLWT at higher Reynolds numbers. These are designed to separately investigate the effect of the roughness Reynolds number k_{s0}^+ and the friction Reynolds number $Re_{\tau 0}$ on the flow recovery by holding one constant and varying the other. Details of each set of the experiments are given below.

3.2.1 Effect of step height

The step height variation was experimentally achieved by shimming the smooth surface up with vinyl sheets which have a nominal thickness of 1 mm. Additional measurement of the height difference between the roughness crest and the smooth wall is performed after each setup, and the results are summarised in table 3.1. These four cases cover various scenarios where the smooth surface is below the roughness canopy or above the roughness crest, and the case with matched virtual origin is expected to be represented by H2.

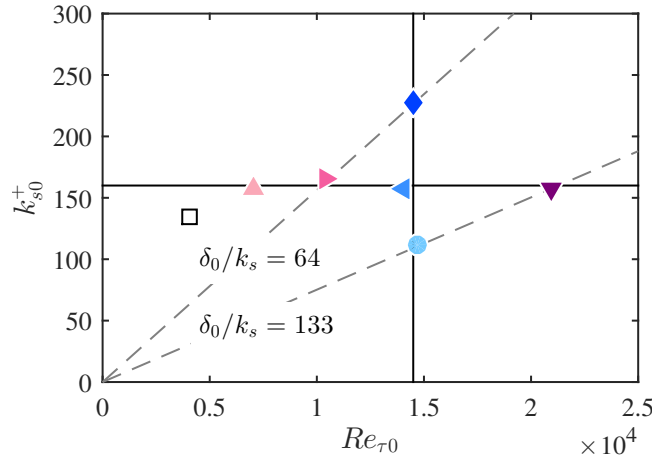


FIGURE 3.5: Flow conditions ($Re_{\tau 0}$ and k_{s0}^+) immediately upstream of the roughness transition of all cases. All symbols are defined in table 3.2. The horizontal line is at $k_{s0}^+ = 160$, and the vertical line is at $Re_{\tau 0} = 14500$. The two dashed lines show the cases with matched δ_0/k_s of 64 and 133.

For all step height cases examined, the equivalent sandgrain roughness is $k_s \approx 2.8$ mm, yielding a roughness Reynolds number of $k_{s0}^+ \equiv k_{s0} U_\tau / \nu \approx 134$ at x_0 . All measurements are acquired at a nominal freestream velocity of $U_\infty \approx 15 \text{ ms}^{-1}$. The friction Reynolds number ($Re_\tau \equiv \delta_{99} U_\tau / \nu$) immediately upstream of the transition location over the rough surface is $Re_{\tau 0} \approx 4100$. Here δ_{99} is the boundary layer thickness (defined as the wall-normal location where the mean velocity reaches $0.99 U_\infty$) and U_τ is the mean friction velocity. The subscript ‘0’ refers to conditions at the location of the rough-to-smooth transition.

3.2.2 Effect of $Re_{\tau 0}$ and k_{s0}^+

A turbulent boundary layer in the fully-rough regime can be described by the dimensionless parameter group: Re_τ and k_s^+ . Two groups of wind tunnel experiments are designed

Case	ΔH (mm)	Sketch
H0	-2.85	
H1	-1.94	
H2	-0.98	
H3	0.06	

TABLE 3.1: Illustrations of the step height ΔH (between the roughness crest and the smooth wall) of the four cases. Negative sign indicates the smooth wall is below the roughness crest. The flow is from left to right on the sketch, and the blue dashed line marks the location of the roughness crest.

	Case	Sym.	$Re_{\tau 0}$	k_{s0}^+	x_0 (m)	U_∞ (m s ⁻¹)	δ_0 (m)	l^+	Δt^+	$\bar{T}_s$
Group I	Re04ks13	□	4100	134	3.7	15.0	0.09	18	0.60	26.2
	Re07ks16	▲	7100	158	4.5	21.5	0.11	22	0.61	23.7
	Re10ks16	▶	10 400	165	7.2	22.5	0.15	24	0.64	26.4
	Re21ks16	▼	21 000	157	17.1	24.3	0.32	24	0.70	22.7
Group II	Re14ks16	◀ (◀)	14 000	157	11.1	23.3	0.22	23	0.68	22.7
	Re14ks11	●	14 700	111	17.1	17.0	0.32	18	0.35	21.4
	Re14ks22	◆	14 500	228	7.2	31.0	0.15	20	1.26	24.3

TABLE 3.2: Summary of the experimental cases. The friction velocity $U_{\tau 0}$ employed in calculating $Re_{\tau 0}$ and k_{s0}^+ is obtained over the rough fetch just upstream of the rough-to-smooth transition. The viscous-scaled hotwire sensor length l^+ and the sampling interval Δt^+ are calculated using the friction velocity at the most downstream measurement location on the smooth surface. $\bar{T}_s \equiv T_s U_\infty / \delta_{99} \times 10^{-3}$, where T_s is the hotwire sampling time. Note that case Re14ks16 is shared between *Group I* and *Group II*, therefore its symbol can take either pink or blue colour in the corresponding group.

to examine the effect of $Re_{\tau 0}$ and k_{s0}^+ on the flow recovery behaviour separately. *Group I* consists of measurements with varying $Re_{\tau 0}$ while holding k_{s0}^+ constant, while *Group II* measurements with varying k_{s0}^+ while holding $Re_{\tau 0}$ constant. The same type of sandpaper (P24 grit) is used in all cases, which ensures a constant k_s , while x_0 , the downstream location of the roughness transition is varied. For *Group I* measurements, the freestream velocity U_∞ is adjusted to account for the gradual decrease of the skin-friction coefficient $C_f (\equiv \tau_w / \frac{1}{2} \rho U_\infty^2)$ with Reynolds number, to maintain an approximately constant $U_{\tau 0}$ at the rough surface immediately upstream of the roughness transition. This will guarantee a nominally constant k_{s0}^+ for all cases. The variation of $Re_{\tau 0}$ is primarily achieved by varying the x_0 location of the transition. For *Group II* measurements, U_∞ is adjusted to account for the growth of δ_{99} with x_0 and maintain a constant $Re_{\tau 0}$. This will subsequently lead to a variation in k_{s0}^+ . The aforementioned variation of flow parameters in each group is demonstrated in figure 3.5.

Figure 3.4 shows a schematic of all the experiment cases, and parameters of the measurements are summarised in table 3.2. Each case is assigned a code in the format of Re xx ks yy , where $xx \approx Re_{\tau 0} / 1000$, and $yy \approx k_{s0}^+ / 10$. All cases in *Group I* are represented by symbols with different shades of magenta, while cases in *Group II* are represented by blue symbols. Note that case Re14ks16 is shared between the two groups and the symbol will be coloured correspondingly in each. The shading of symbols darkens with increasing $Re_{\tau 0}$ in *Group I* and k_{s0}^+ in *Group II*. Case Re04ks13 is originally H1 in the study of step height effect in the other wind tunnel, and it is included here only for comparison when applicable. The stream-wise locations where hotwire profiles are obtained are shown by the corresponding symbols in the main plot of figure 3.4.

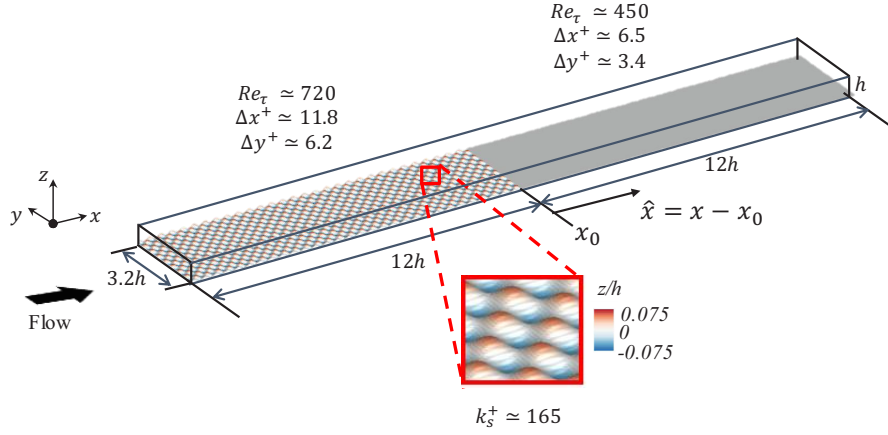


FIGURE 3.6: The computational domain for the DNS database (Rouhi et al., 2019b). The bottom surface is coloured according to the surface elevation relative to the location of the smooth wall plane featuring case D1. The inset shows a magnified view of the ‘egg carton’ roughness employed. The reported Re_τ on each patch corresponds to the recovered region and the wall parallel resolutions Δx^+ and Δy^+ are normalised by the asymptotic values of U_τ on each patch.

3.2.3 A reference numerical dataset

To complement the experimental data acquired in this study, we also present an open-channel DNS dataset with similar streamwise roughness transitions and make comparisons when appropriate. It is important to note that the pressure gradient configuration for a surface transition could be quite different in a turbulent boundary layer (where the layer thickness can grow and the freestream velocity is constant) than in a channel of fixed height.

The DNS was performed using a well-validated fourth-order, finite-difference code (Chung et al., 2014, 2015) with an Immersed Boundary Method (IBM) used to implement the roughness (Scotti, 2006; Rouhi et al., 2019b). The open-channel computational domain for the present simulations span $24h \times 3.2h \times h$ in the streamwise, spanwise and wall-normal directions, as shown in figure 4.6. Periodic boundary conditions are applied in the streamwise and spanwise directions and a free-slip condition is employed at the top boundary. For $0 < x < 12h$ the bottom wall of the channel is a no-slip rough boundary, which then has an abrupt transition to a smooth wall no-slip boundary for $12h < x < 24h$. The rough patches are composed of an ‘egg carton’ roughness (Chan et al., 2015) with a roughness height of $0.056h$ and a roughness wavelength of $0.40h$, where h corresponds to the channel height. The flow at the end of the rough patch is in the fully rough regime with an equivalent sandgrain roughness of $k_{s0}^+ \approx 165$. The driving pressure-gradient is set such that the global Reynolds number is maintained at $Re_\tau = hU_{\tau_o}/\nu = 590$, where U_{τ_o} is the global friction velocity. The flow is fully resolved down to the roughness elements (approximately 24 grid cells per roughness wavelength in the streamwise direction and 48 in the spanwise direction) with no modelling assumptions. Three step height cases are simulated, namely D0, D1 and D2, with the smooth wall aligned with the troughs, midplane and crests of the roughness, respectively. This leads to a step height of $\Delta H/k_p = -1, -0.5$ and 0 for these three cases. Further details

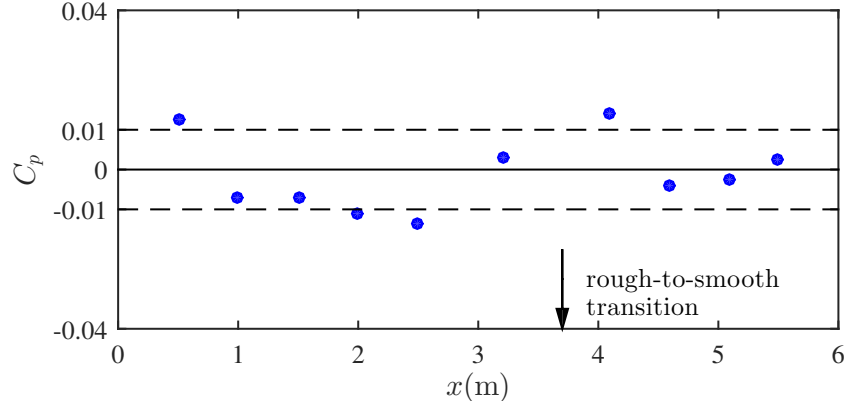


FIGURE 3.7: The pressure coefficient C_p in the open-return wind tunnel at a freestream velocity of $U_\infty \approx 15 \text{ m s}^{-1}$.

on the DNS database can be found in Rouhi et al. (2019b,a).

3.3 Pressure gradient

Similar to many measurements of a canonical smooth-wall boundary layer (Chauhan et al., 2009), here we also aim at achieving a zero pressure gradient over the working section, which facilitates the comparison with the former, and also allows us to focus on the inertial-driven process without complications of the pressure gradient effect. Detailed examination of turbulent boundary layers over a rough-to-smooth change subjected to a streamwise pressure gradient can be found in Schofield (1975) and could likely be an extension of the present work.

In the open-return boundary layer wind tunnel, the distribution of pressure coefficient $C_p(x) \equiv (p(x) - p_{ref}) / \frac{1}{2} \rho U_\infty^2$ over the working section is obtained using static pressure taps mounted on the tunnel roof, and $C_p = 0 \pm 0.01$ is achieved in most areas with no distinguishable localised pressure gradient observed in the vicinity of the rough-to-smooth change (see figure 3.7). The range of C_p variation is comparable with other zero-pressure gradient studies conducted in the same facility (see Harun et al., 2013; Nugroho, 2015), thus the pressure gradient effect over the current rough-to-smooth surface can be considered negligible.

In the HRNBLWT experiments, the distribution of the static pressure along the length of the tunnel is acquired by sampling the variation of the freestream velocity along the centre-line. The static pressure coefficient C_p is defined as

$$C_p(x) \equiv 1 - \left(\frac{U_\infty(x)}{U_{\text{ref}}} \right)^2, \quad (3.1)$$

where U_{ref} is obtained from an NPL (National Physical Laboratory) pitot-static tube mounted at $x = 0.7 \text{ m}$, and $U_\infty(x)$ from another pitot-static tube mounted on the sting and traversed along the test section. The two pitot-static tubes are sampled simultaneously to account for any variation of the tunnel speed with time.

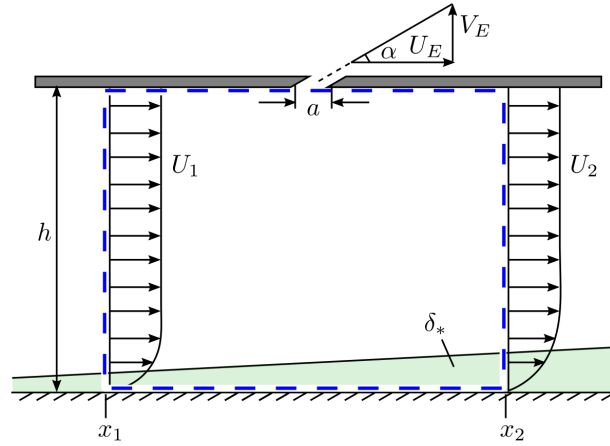


FIGURE 3.8: Schematic of the streamwise wall-normal plane of the tunnel, highlighting the bleeding slot between roof panels. The flow is from left to right.

To achieve a zero streamwise pressure gradient, instead of increasing the cross-section area as in many other facilities, the growth of the boundary layer thickness is accounted for bleeding air from slots on the roof of the working section (Kulandaivelu, 2012). The width a of the bleeding slot can be determined as detailed below.

Consider a control volume as shown by the blue rectangle in figure 3.8, where x_1 and x_2 are the mid-points of two neighbouring panels. The flow is assumed to be under zero pressure gradient conditions, therefore the freestream velocity $U_1 = U_2 \equiv U_\infty$. The air bleeds out at an angle α from the slot, with U_E , the streamwise component of the exit velocity assumed to be U_∞ , and the wall-normal component V_E will subsequently be $U_E \tan \alpha$. The boundary layer formed under the roof is neglected. δ_* and θ denote displacement thickness and momentum thickness, respectively.

Using mass conservation

$$\rho V_E a = \rho U_\infty (\delta_{*2} - \delta_{*1}), \quad (3.2)$$

the width of the bleeding slot can be expressed as

$$a = (\delta_{*2} - \delta_{*1}) \cot \alpha, \quad (3.3)$$

which implies that to maintain a zero pressure gradient, the slot size a should be increased for a rough surface with a higher growth rate of δ_* compared to a smooth one.

In practice, the 22 independent roof panels have been adjusted to ensure a zero pressure gradient for a smooth surface. An alternative approach is adopted to bypass the tedious process of readjusting these panels for the rough-to-smooth measurements and revert the change for other smooth-wall experiments afterwards. This involves selectively sealing some grids on the mesh at the tunnel exit to increase the back pressure (hereinafter referred to as ‘global adjustment’) and sealing a slot on the roof to create a localised effect to account for the sudden change in the surface condition (hereinafter referred to as ‘local adjustment’).

Take the case where $x_0 = 11.1$ m as an example, figure 3.9 shows the contribution to the

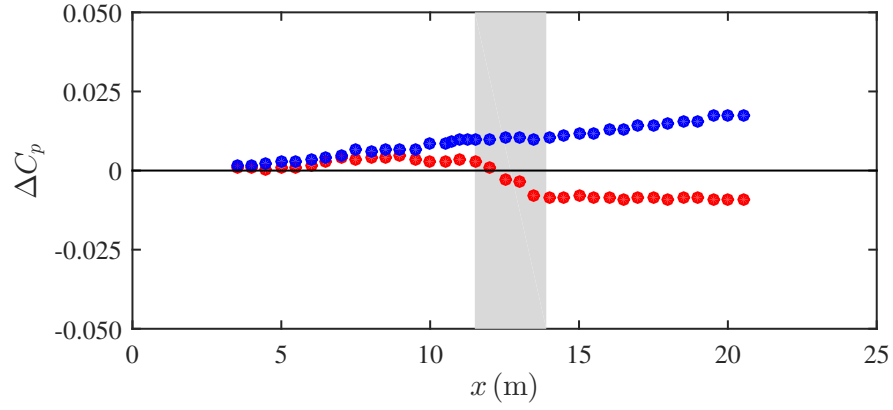


FIGURE 3.9: Contribution to the pressure coefficient change ΔC_p from local (●) and global (●) adjustments. The rough-to-smooth transition is at $x_0 = 11.1$ m, and the freestream velocity is $U_\infty \approx 20 \text{ ms}^{-1}$. For the local adjustment, two slots at $x = 12.1$ m and 13.3 m are sealed by tape, as shown by the shadowed region.

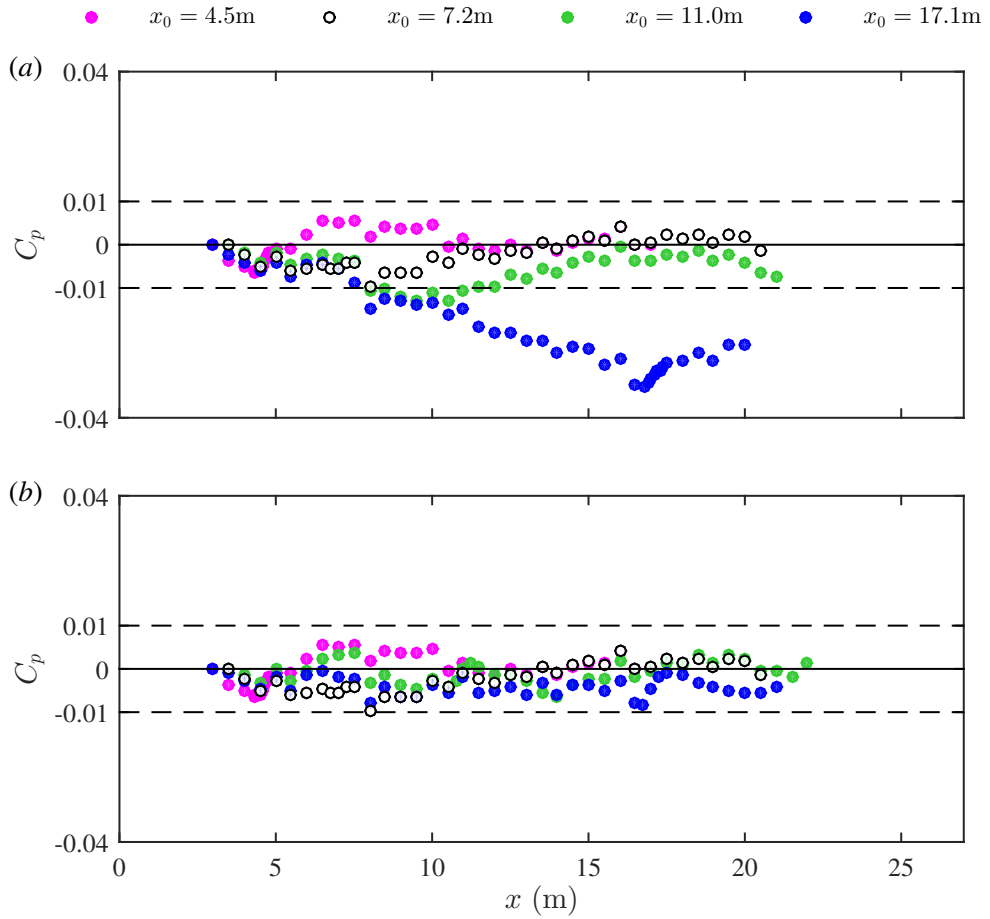


FIGURE 3.10: Pressure coefficient C_p (a) before and (b) after adjusting the bleeding slots on the roof and the wire mesh at the tunnel exit. All C_p measurements are performed at a freestream velocity of $U_\infty \approx 20 \text{ m s}^{-1}$, close to the designed condition of the majority of the experimental cases.

pressure coefficient change ΔC_p from the global and local adjustment separately. Partially sealing the mesh pressurises the tunnel and increases the bleeding in all slots, resulting in

an adverse pressure gradient along the streamwise direction. Closing a bleeding slot, on the other hand, is observed to cause a slight increase in C_p upstream followed by a sudden drop immediately downstream. A zero pressure gradient condition can eventually be achieved for the surface with a rough-to-smooth change by combining these two adjustments. The pressure coefficients of all cases before and after adjustments are summarised in figure 3.10. Note that for the first two cases with a relatively small sandpaper coverage ($x_0 = 4.5$ m and $x_0 = 7.2$ m), no extra adjustment is necessary. For all cases examined, following appropriate adjustment, a nominal zero pressure gradient condition ($C_p = 0 \pm 0.01$) is achieved and the effect of streamwise pressure gradient on the development of the boundary layer is negligible (see figure 3.10b).

3.4 Hotwire anemometry

A conventional single-wire hotwire probe is operated by an in-house Melbourne University Constant Temperature Anemometer (MUCTA). The hotwire signal is low-pass filtered using an analogue filter Frequency Devices 9002, and then sampled using a Data Translation DT9836 data acquisition board at 30 kHz in the open-return boundary layer wind tunnel and 50 kHz in the HRNBLWT. The cut-off frequency in the low-pass filter is chosen as half of the sampling frequency according to the Nyquist-Shannon sampling theorem. The sampling parameters are summarised in table 3.2. The viscous-scaled sampling interval Δt^+ is less than 2 for all cases, which is safely below the threshold of $\Delta t^+ \approx 3$ to ensure a temporally well resolved signal (Hutchins et al., 2009). The hotwire sampling time T_s is more than 20000 boundary layer turn-over times (δ_{99}/U_∞) to achieve a good convergence of the statistics. Calibration is performed following an *in-situ* procedure before and after each measurement. Thereafter, any drift is corrected by an intermediate single point re-calibration (ISPR) method discussed in Talluru et al. (2014), where the hotwire voltage is periodically monitored in the freestream. The uncertainty in U and $\overline{u^2}$ is usually within 1% and 3%, respectively (Yavuzkurt, 1984). The method of calibration drift correction proposed by Talluru et al. (2014) employed here offers further improvements.

Velocity profiles are obtained by traversing a single-normal hotwire probe over 40 logarithmically spaced wall-normal locations from $z/\delta_{99} \approx 0.001$ to 2. The sensing element of this probe has a diameter of $d = 2.5 \mu\text{m}$ and a length-to-diameter ratio of 200. Note that in case Re14ks22 where U_τ is the highest among all cases, a wire with a smaller diameter ($d = 1.5 \mu\text{m}$) is used to maintain a similar spatial resolution in wall units (Hutchins et al., 2009). Wall-normal boundary layer profiles are measured at over 10 logarithmically spaced streamwise locations downstream of the rough-to-smooth transition, from $\hat{x} = 12$ mm to $x = 21$ m, as shown in the main plot of figure 3.4. A reference profile above the rough surface is also acquired at $\hat{x} = -0.1$ m in each case. In order to accurately measure the wall location, a magnetic linear encoder Renishaw LM13 with a resolution of $5 \mu\text{m}$ is incorporated into the hotwire traversing system in the open-return boundary layer wind tunnel, and an optical linear encoder Renishaw RGH24 with a resolution of $1 \mu\text{m}$ is used in the HRNBLWT. The initial offset is determined using a wall-normal traversing microscope equipped

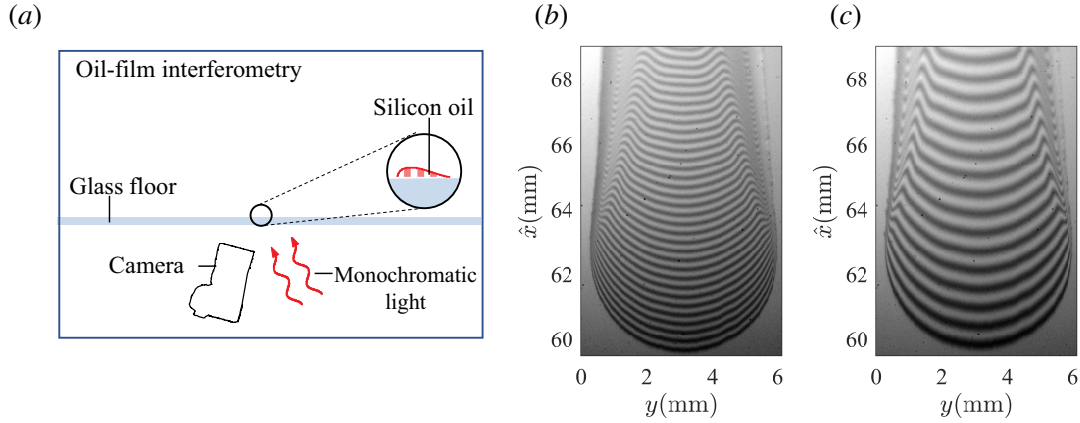


FIGURE 3.11: (a) Illustration of the OFI experimental setup. (b) and (c) are the first and last OFI image in the sequence obtained at $\hat{x} \approx 60\text{mm}$. The time interval between these two images is 495 s. Both images have been converted from pixel space to real space using a calibration process similar to that used in the PTV experiments.

with a digital micrometer (CDI BG3600) with a resolution of $1\text{ }\mu\text{m}$.

3.5 Oil-film interferometry

The wall-shear stress τ_w is directly measured using oil film interferometry (OFI), which is one of the few methods available for a direct measurement at the surface with the required streamwise resolution to capture the rapid evolution following the rough-to-smooth transition. The technique measures the thinning rate of an oil film as it is being acted upon by the shear near the wall, which in turn permits an accurate measure of the mean wall-shear stress. The thickness of a typical oil film is in the order of micrometres, which can be measured by the fringe pattern from the interference of light reflected from the top and from the bottom of the oil film. Moreover, the technique utilises inexpensive equipment, which includes a consumer camera and monochromatic light source (Zanoun et al., 2003; Fernholz et al., 1996).

The experimental configuration is illustrated in figure 3.11(a), where a droplet (or a line) of silicone oil is placed on a clear glass surface. In the open-return boundary layer wind tunnel, the oil film is illuminated using a monochromatic light source from a sodium lamp and the interference pattern is captured using a Nikon D800 DSLR camera equipped with a 105mm AF Micro NIKKOR lens and a Sigma APO EX DG 2 $\times$ teleconverter, which is mounted underneath the working section. Silicone oil with nominal viscosities of 50 cSt and 100 cSt is used in this study. Prior to the OFI measurements, the viscosity ν_{oil} of the oil is calibrated as a function of temperature T_{oil} using a constant temperature water bath and an Ubbelohde viscometer to obtain an exponential relationship, where $\nu_{oil} = Ae^{\alpha T_{oil}}$ (Ng et al., 2007). The calibration curves are shown in figure 3.12. The FOV of the OFI measurements is calibrated with the same calibration target with a 2.5 mm dot spacing, providing a precise conversion from the image to the real space.

For each OFI database, 100 images are captured with a time interval of five seconds between images. Figure 3.11(b) and (c) are the first and last images from one of such image

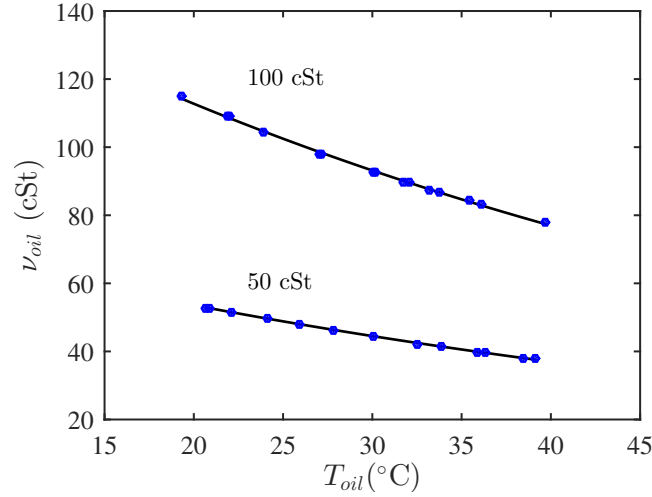


FIGURE 3.12: The variation of oil viscosity with temperature. Blue dots are the original data points and black lines are fitted curves.

sequence acquired at $\hat{x} \approx 60$ mm. For the present measurements, the resolution ranges from approximately 100 pixels per wavelength to 200 pixels per wavelength, from the first to the last image, respectively, to ensure that all the fringe patterns are fully resolved. The image sequences are then processed using an FFT based algorithm (Ng et al., 2007) to extract the fringe spacing of the interferograms. More specifically, the signal is zero-padded to improve the dominant frequency estimation. Thereafter, a linear trend is fitted to the extracted fringe spacing of the interferograms versus time to evaluate the wall shear stress, τ_w . Note that this process is performed independently for every line of pixels in y for the entire spanwise extent of the oil film, which is between 10–20 mm for each repeated measurement. Regions contaminated by dust are removed during processing. The resulting spanwise profile of the skin-friction coefficient is denoted as $\tilde{C}_f(\hat{x}, y) = \tau_w(\hat{x}, y) / \frac{1}{2}\rho U_{\infty}^2$ (U_{∞} is the freestream velocity), and $C_f(\hat{x})$ is the spanwise average of $\tilde{C}_f(\hat{x}, y)$. It should be noted that by performing FFT across the entire oil-film in the FOV, C_f is assumed to be constant in the streamwise direction within the FOV in each measurement. In reality we would generally expect C_f to vary by less than 1.5% over a typical 1 cm streamwise extent of the FOV, when $\hat{x}/\delta_0 > 0.3$ downstream of a rough-to-smooth transition.

In the HRNBLWT, a 1.4 m long glass insert has been installed in the tunnel floor at $x = 5$ m, providing optical access from the underside of the tunnel for $\hat{x}/\delta_0 < 7$ in case Re07ks16. An Imalent DX80 LED torch is used as the light source in place of the sodium lamp, and a 532 nm bandpass filter with a bandwidth of 10 nm is attached to the camera lens to obtain monochromatic fringe patterns. The interference fringes are recorded using a Nikon D810 DSLR camera with a Tamron 180 mm macro lens. This configuration is similar to the approach described above, and a well-resolved fringe pattern with approximately 50 pixels per wavelength can be achieved. For the remaining measurements, a glass insert on the centreline of the working section ceiling provides optical access from above. To improve the fringe quality, the tunnel floor is covered by a piece of black mylar film with a thickness less than 40 μm (equivalent to 2 wall units). The same illumination and imaging system as in

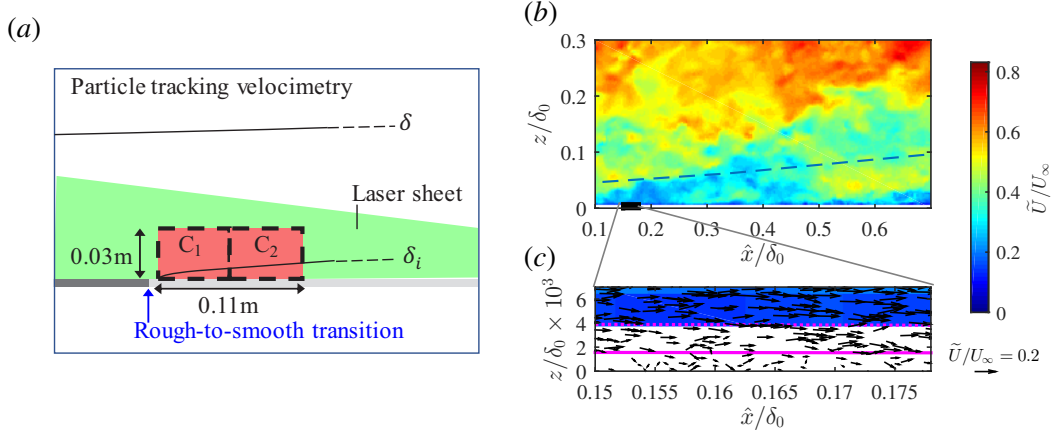


FIGURE 3.13: (a) illustrates the particle tracking velocimetry, where C_1 and C_2 correspond to the field-of-views captured non-simultaneously using a scientific double-frame camera with a vertical laser sheet that is projected upstream through the working section. (b) is an instantaneous field of the streamwise velocity component, $\tilde{U}$ from the PIV pass. The dashed line marks the location of δ_i . (c) is the enlarged view of the rectangular region in (b). The colour contour is the same as in (b), and the arrows show the velocity vectors from the PTV pass. The solid and dotted magenta lines indicate $z^+ = 4$ and $z^+ = 10$, respectively.

the previous configuration is used, but with a reduced resolution of approximately 30 pixels per wavelength due to the 1 m stand-off distance between the camera and oil film. Both OFI configurations (imaging from underneath and above) have been compared at $\hat{x}/\delta_0 = 4$ for case Re07ks16, and are shown to give the same result to within 1%.

3.6 Particle tracking velocimetry

In order to resolve the near-wall region, additional high magnification Particle Tracking Velocimetry (PTV) measurements are performed immediately after the rough-to-smooth transition. Unlike the hotwire anemometry and OFI detailed in the previous sections, these measurements are only performed in the open-return boundary layer wind tunnel for all step height cases (detailed in table 3.1). The magnified field of view targeted at the near wall region is ideally suited for this analysis, as the database provides access to well-resolved velocity signals within the viscous sublayer region of the flow ($z^+ \lesssim 4$). A summary of the parameters of the PTV databases is detailed in table 3.3. The imaging system consisted of a 14-bit PCO 4000 PIV camera (4008×2672 pixels) equipped with a Tamron 180mm Macro lens and a Sigma APO EX DG $2\times$ teleconverter. This configuration which is illustrated in figure 3.1 and figure 3.13(a) provides a FOV of approximately $5.5 \text{ cm} \times 3 \text{ cm}$ ($0.62\delta_0 \times 0.34\delta_0$) per camera with a pixel size of $\approx 12 \text{ }\mu\text{m}$ ($0.33\nu/U_\tau$). We note, the high-magnification optics limited the wall-normal extent of the FOV to $0.5\delta_0$. However, in order to capture a larger streamwise extent ($\approx 1\delta_0$), the camera is mounted on a linear traverse, which enables non-simultaneous measurements further downstream of the rough-to-smooth transition. For the present study, two measurement locations are captured as shown in figure 3.13(a) that span the range $0 \lesssim \hat{x}/\delta_0 \lesssim 1.2$.

TABLE 3.3: Summary of parameters for the PTV experiments.

General parameters	
Flow medium	Air
Seeding	Polyamide particles
Particle size	$\approx 1 - 2 \mu\text{m}$
Acquisition frequency	1 Hz
Laser sheet thickness	$\approx 1 \text{ mm } (\sim 30 \frac{v}{U_\tau})$
Flow parameters	
Reynolds number	$Re_\tau \approx 3500$
Freestream velocity	$U_\infty \approx 15 \text{ m s}^{-1}$
PIV/PTV parameters	
Number of images	600
Sensor resolution	$4008 \times 2672 \text{ pixel}^2$
Image pixel size	$\approx 12 \mu\text{m}/\text{pixel } (\sim 0.4 \frac{v}{U_\tau} / \text{pixel})$
Field of view ($x \times z$)	$\approx 0.05 \text{ m} \times 0.03 \text{ m}$
Interrogation size (PIV pass)	$128 \times 8 \text{ pixel}^2 (\sim 46 \frac{v}{U_\tau} \times 3 \frac{v}{U_\tau})$
Overlap (PIV pass)	75%
Interrogation size (PTV pass)	$4 \times 4 \text{ pixel}^2 (\sim 1.4 \frac{v}{U_\tau} \times 1.4 \frac{v}{U_\tau})$

Seeding is introduced into the wind tunnel upstream of the fan and the facility's flow conditioning and settling chamber, which provides a homogeneous seeding density across the test section. The illumination for the experiments is provided by an Innolas Spitlight Compact 400 Nd:YAG system that generates a light sheet on a streamwise wall-normal plane, which is projected upstream through the working section (see figure 3.13a). The mirror in the flow required for this configuration is 2.5 m (approximately $30\delta_0$) downstream of the surface transition. Hotwire surveys indicate that the presence of the mirror in the flow has no upstream effect on the flow at the measurement location.

Because multiple camera positions are used in the present experiments to construct a non-simultaneous wider streamwise FOV, a calibration procedure is essential to stitch the time-averaged statistics from the different camera positions together and also to account for image distortions. In the present experiments, we employ a calibration target that spans the entire extent of the FOV, which has been proven to work well for multi-camera large-FOV experiments (de Silva et al., 2012, 2014). For the present experiments the calibration target is composed of a grid of dots with a dot diameter of 1 mm and a spacing between dots of 2.5 mm in the x and z direction. A third order polynomial surface is fitted to the pixel location of the dots, which enables us to apply a pixel-to-real-space conversion by mapping the pixel positions to real space coordinates within the wind tunnel.

The experimental image-pairs are processed using an in-house PIV/PTV package developed at the University of Melbourne (de Silva et al., 2014). For the present study, the images are first pre-processed using a conventional PIV cross-correlation algorithm, with multi-grid (Willert, 1997) and window deformation (Scarano, 2001). Thereafter, a particle tracking pass is executed to track individual particles based on an initial estimate from the PIV pass (Cowen and Monismith, 1997). For the PIV processing, vector validation is performed using a median filter (Westerweel and Scarano, 2005) and to validate the particle tracks in PTV, thresholds

are applied to the correlation coefficient and the size of the detected particles (Adrian and Westerweel, 2011). Figure 3.13(b) shows an instantaneous velocity field obtained from the PIV pass, and figure 3.13(c) is a magnified view of the boxed region in (b), with vectors showing the instantaneous velocity from the PTV pass overlayed on top. The interrogation window size used in the PTV pass is approximately 1.4 wall units, much smaller than that in the PIV pass (see table 3.3), therefore it can resolve the velocity closer to the wall. As shown in figure 3.13(c), the PIV velocity data only extends to $z^+ = 10$ (dotted magenta line), while PTV pass can resolve the velocity even below $z^+ = 4$ (solid magenta line). Further, the wall-normal location of the PTV database is refined to subpixel accuracy by correlating the near wall particles and their reflections on a frame-by-frame basis.

Chapter 4

Recovery of the wall-shear stress

This chapter examines recovery of the wall-shear stress of a turbulent boundary layer that has undergone a sudden transition from a rough to a smooth surface. Early works of Antonia and Luxton (1972) questioned the reliability of standard smooth-wall methods to measure wall-shear stress in such conditions, and subsequent studies show significant disagreement depending on the approach used to determine the wall-shear stress downstream. Here we address this by utilising a collection of experimental databases at $Re_{\tau 0} \approx 4100$ that have access to both ‘direct’ and ‘indirect’ measures of the wall-shear stress to understand the recovery to equilibrium conditions to the new surface. Our results reveal that the viscous region ($z^+ \lesssim 4$) recovers almost immediately to an equilibrium state with the new wall conditions, however, the buffer region and beyond takes several boundary layer thicknesses before recovering to equilibrium conditions, which is longer than previously thought. A unique direct numerical simulation database of a wall-bounded flow with a rough-to-smooth wall transition is employed to confirm these findings. In doing so, we present evidence that any estimate of the wall-shear stress from the mean velocity profile in the buffer region or further away from the wall tends to underestimate its magnitude in the near vicinity of the rough-to-smooth transition, and this is likely to be partly responsible for the large scatter of recovery lengths reported in the literature. Our results also reveal that the smaller energetic scales in the near-wall region recover to an equilibrium state associated with the new wall conditions within one boundary layer thickness downstream of the transition, while the larger energetic scales exhibit an over-energised state for several boundary layer thicknesses downstream of the transition. Based on these observations, an alternative approach to estimating the wall-shear stress from the premultiplied energy spectrum is proposed.

^{||}This chapter presents the results of M. LI; C. M. DE SILVA; R. BAIDYA; A. ROUHI; D. CHUNG; I. MARUSIC; AND N. HUTCHINS 2019 Recovery of the wall-shear stress to equilibrium flow conditions after a rough-to-smooth step-change in turbulent boundary layers. *J. Fluid Mech.* **872**, 472-491.

4.1 Introduction

Although the streamwise rough-to-smooth heterogeneity has been studied extensively over the past few decades (Bradley, 1968; Antonia and Luxton, 1972; Shir, 1972; Rao et al., 1974; Chamorro and Porté-Agel, 2009; Hanson and Ganapathisubramani, 2016), to date, the recovery to equilibrium conditions of the new surface following such a transition is far from understood. For example, determining the local wall-shear stress τ_w after the transition (and subsequently the friction velocity $U_{\tau 2}$) have been hampered by reliability issues. Antonia and Luxton (1972) used three different techniques to determine τ_w following a rough-to-smooth transition (Clauser chart, Preston tube, and the momentum integral equation) noting that, ‘... none of the standard smooth-wall methods of obtaining skin friction from the mean profile is reliable for some distance downstream from the roughness change’.

The scatter in the recovery of the wall-shear stress is highlighted in figure 4.1, which collates the downstream skin-friction coefficient, C_f , from a collection of experimental and numerical databases (see table 4.1 for a summary of key parameters of each database). This leads to an uncertainty in drag over $0 < \hat{x}/\delta_0 < 10$, defined as $C_{D10} \equiv \int_0^{10} C_f d(\hat{x}/\delta_0)$, of up to 40%. In part, the disagreement between the databases is due to differences in Reynolds numbers, flow geometry (internal versus external wall-bounded flows) and surface conditions (the magnitude and type of the roughness change at $x = x_0$), however, here we demonstrate that the method of skin-friction coefficient measurement can introduce systematic variation in the reported C_f recovery. The colours of the symbols in figure 4.1 broadly distinguish the data based on the skin-friction coefficient measurement technique, with the black symbols showing methods operating in the log-region, the grey symbols indicating the buffer layer, and the green symbols showing measurements from the viscous sublayer (OFI, near-wall gradient, DNS etc). We conjecture that when τ_w is estimated directly at the wall or from the viscous region, a higher C_f , or a faster recovery is observed in general. Conversely, when

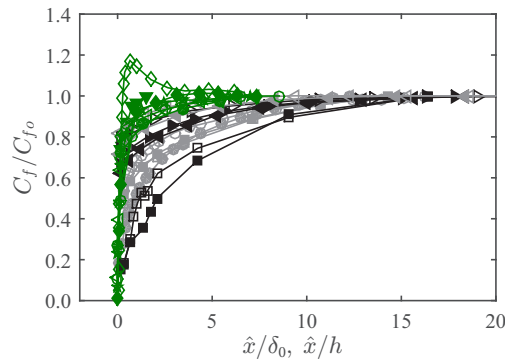


FIGURE 4.1: Recovery of skin friction coefficient C_f recovery downstream of a rough-to-smooth transition from a range of numerical and experimental databases. Details of each dataset are summarised in table 4.1. The colours of the symbols indicate the region where C_f is determined: green represents C_f measured within the viscous sublayer or directly at the wall, grey represents buffer layer and black the logarithmic layer. Results are normalised by C_{f0} , which corresponds to the most downstream reported C_f measurement from each dataset.

Reference	Symbol	Technique	Region	Geometry	Roughness	$Re_{\tau 0}$	k_{s0}^+	$\delta_0/k_p, h/k_p$	$\Delta H/k_p$
Hanson and Ganapathisubramani (2016)	○ ●	Preston tube	buffer	boundary layer	P16 grit mesh	2.3×10^3 3.5×10^3	209 1300	29 19	N/A
Antonia and Luxton (1972)	□	Clauser chart	log	boundary layer	square ribs	3.4×10^3	-	22	0
	■	chart	log			6.1×10^3	-	22	0
	□	Preston tube	buffer			3.4×10^3	-	22	0
	■	tube	buffer			6.1×10^3	-	22	0
Saito and Pullin (2014)	◁	wall-modelled LES	buffer	full channel (both sides roughened)	modelled	2.1×10^4	21	N/A	0
	◄		buffer			2.2×10^5	219		0
	◄		log			2.5×10^6	2458		0
	▷		log			2.3×10^6	225		0
	►		log			2.4×10^6	1193		0
Ismail et al. (2018b)	◇	DNS	wall	full channel (one side roughened)	square ribs	5.0×10^2	343	12	-1
	◁					2.2×10^3	1540	12	-1
	★					2.1×10^3	1105	16	-1
	◆					2.4×10^3	2105	9.6	-1
Ismail et al. (2018a)	○	DNS	wall	full channel (one side roughened)	3D cubes	1.6×10^3	338	12	-1
	●					1.5×10^3	233	12	-1
Chamorro and Porté-Agel (2009)	▼	near-wall hotwire	viscous	boundary layer	mesh	1.5×10^4	479	133	-0.5

TABLE 4.1: Summary of published data on wall-shear stress recovery in wall-bounded flows downstream of a rough-to-smooth transition. $Re_{\tau 0}$ and k_{s0}^+ correspond to the friction Reynolds number and the equivalent sandgrain roughness Reynolds number at the rough-to-smooth transition, and k_p is the maximum roughness height between the crest and trough. ΔH is the height increment from the roughness crest to the smooth surface downstream. $\Delta H = 0$ implies that the smooth wall is aligned with the roughness crest, whereas $\Delta H < 0$ indicates that the smooth wall is below the roughness crest.

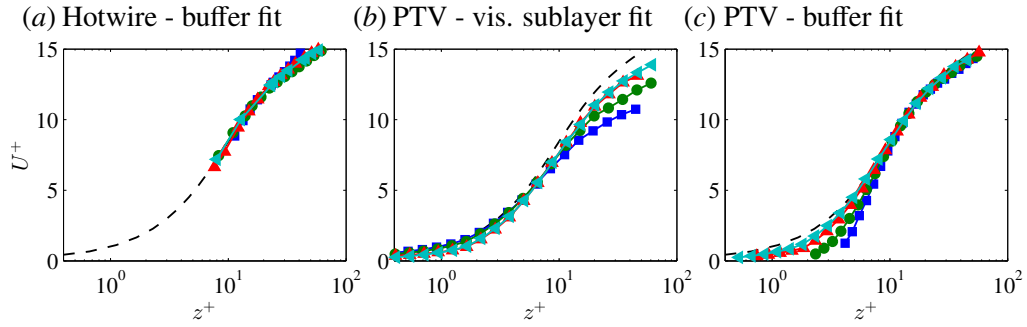


FIGURE 4.2: Mean streamwise velocity statistics from hotwire and PTV experimental data at $Re_{\tau_0} \approx 4100$. Normalisation is by friction velocity, U_{τ_2} , estimated from a fit to the (a,c) buffer and (b) viscous sublayer regions, and $\hat{x}$ corresponds to the streamwise distance from the rough-to-smooth transition. The black dashed line corresponds to a reference smooth-walled boundary layer DNS database at $Re_{\tau} \approx 2500$ (Sillero et al., 2013) and the $\blacksquare$, $\bullet$, $\blacktriangle$ and $\blacktriangleleft$ symbols correspond to results at $\hat{x}/\delta_0 = 0.11, 0.34, 0.68$ and 1.0 , respectively.

inferred from velocity signals further away from the wall, lower C_f and longer recovery lengths are observed. It is worth noting that similar challenges can arise when estimating τ_w in numerical studies. For example, many works bypass the expense of simulating changing surface conditions directly (using DNS) through the use of Reynolds-Averaged Navier Stokes (Rao et al., 1974) or Wall-Modelled Large-Eddy Simulations (Bou-Zeid et al., 2004; Saito and Pullin, 2014; Silva Lopes et al., 2015). In both approaches, the near-wall turbulence (below the logarithmic region) is inferred from modelling assumptions, which may not be applicable for flows in non-equilibrium conditions. Here we use both experimental and numerical databases to provide evidence for the reasoning above and explain why different methods of measuring C_f can explain some of the discrepancies in the literature.

4.2 Experimental databases

Details of the flow facility and surface roughness are presented in §3. The dataset employed in this chapter is case Re04ks13-H0 (unless stated otherwise), the experimental conditions of which are summarised in tables 3.2 and 3.1. Wall-normal hotwire profiles are taken at $\hat{x} = 10, 30, 60, 90, 180, 360$ and 1190 mm, corresponding to $\hat{x}/\delta_0 = 0.11, 0.34, 0.68, 1.0, 2.0, 4.1$ and 13.4 . To complement the hotwire databases with near-wall information, high magnification PTV measurements are performed immediately downstream of the rough-to-smooth transition with a field of view of $1.2\delta_0 \times 0.3\delta_0$. Finally, the wall-shear stress, τ_w , is measured directly using oil film interferometry (OFI) at streamwise locations where hotwire profiles are obtained. More detailed descriptions of these measuring techniques are also included in §3.

4.3 Experimental results

Figure 4.2(a) shows the hotwire measured mean streamwise velocity profiles U at various locations downstream of the rough-to-smooth transition. Due to an inability to make hotwire measurements in the viscous sublayer, the friction velocity $U_{\tau_2}(\hat{x})$ for this figure has initially been estimated from a least squares fit in the buffer region ($10 \lesssim z^+ \lesssim 30$) to a reference smooth-walled DNS profile from Sillero et al. (2013). We note, due to uncertainty associated with the precise wall-normal location of the hotwire measurements, a wall-normal shift is included as a free parameter in the fit (the wall correction returned by the fit is typically within 0.35mm). As dictated by the fit, the profiles in figure 4.2(a) exhibit an excellent collapse in the buffer region ($10 \lesssim z^+ \lesssim 30$) to the canonical case (dashed line). Critically, however, the quality of agreement in the near-wall region cannot be assessed due to the lack of near-wall data from the hotwire measurements.

To overcome this shortcoming, figure 4.2(b) presents U from the PTV database, where a more direct estimate of τ_w (hence U_{τ_2}) is accessible as we are able to compute U_{τ_2} using a least squares fit in the viscous sublayer ($z^+ \lesssim 4$) following

$$U_{\tau_2} = \sqrt{\frac{\tau_w}{\rho}} = \sqrt{\nu \frac{\partial U}{\partial z}}. \quad (4.1)$$

Scaled in this way, the PTV data must exhibit collapse in the near wall region, and figure 4.2(b) shows a growing departure from the reference smooth wall profile with increasing z in the buffer region ($10 \lesssim z^+ \lesssim 30$), demonstrating that U_{τ_2} (and hence τ_w) estimated from the buffer region and viscous sublayer region differ substantially. It should be noted that U_{τ_2} obtained from the near-wall gradient of the PTV data is believed to represent the correct estimate and matches very closely the value measured by oil-film interferometry (to be detailed further in figure 4.4). If we ascribe greater confidence to the PTV measured U_{τ_2} , the most likely interpretation here is that the buffer region is yet to recover to an equilibrium state to the new surface conditions, and consequently underestimates the wall-shear stress immediately downstream of a rough-to-smooth transition.

To further illustrate this behaviour, figure 4.2(c) shows U from the PTV database normalised by a U_{τ_2} estimated from the buffer region (following the same procedure as applied to the hotwire data in figure 4.2a). Despite the subpixel accuracy in the wall location for the PTV data, a free parameter accounting for the wall-normal shift is also included in the fit to fully replicate the hotwire procedure. The results reveal a lack of agreement in the near wall region below $z^+ \lesssim 10$, which confirms that the collapse observed in figure 4.2(a,c) in the buffer region ($10 \lesssim z^+ \lesssim 30$) appears to be an artefact of an erroneous U_{τ_2} . It should also be noted that the estimated internal boundary layer thickness δ_i is $O(100)$ viscous units above the buffer region beyond $\hat{x}/\delta_0 > 0.25$ for the present database. Therefore, our findings confirm that a substantial part of the internal layer remains in a non-equilibrium state with the local wall condition (see also Rao et al., 1974; Antonia and Luxton, 1972; Shir, 1972).

In the present set of experiments, a Preston tube is not tested as a method to measure wall-shear stress. However, since the typical diameter of a Preston tube is $O(1)$ mm which

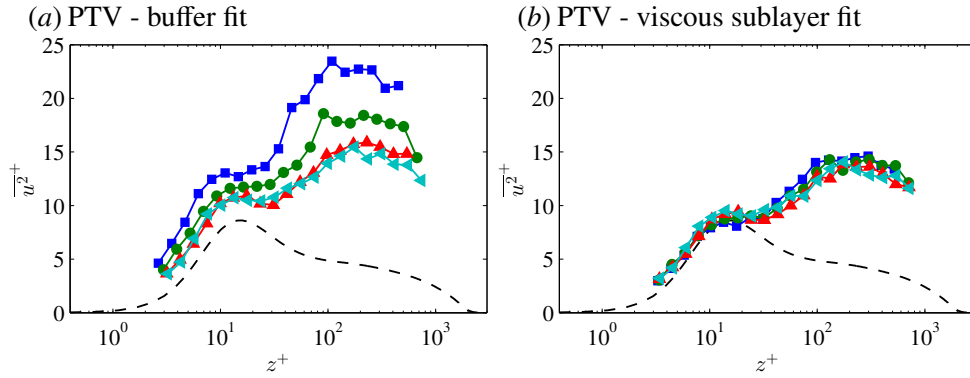


FIGURE 4.3: Streamwise turbulence intensity, $\overline{u^2}^+$ from the PTV experimental data at $Re_{\tau 0} \approx 4100$. Normalisation is by friction velocity, $U_{\tau 2}$ estimated from a fit to the (a) buffer region and (b) viscous sublayer. The black dashed line corresponds to a reference smooth walled boundary layer DNS database at $Re_{\tau} \approx 2500$ (Sillero et al., 2013) and the $\blacksquare$ -, $\bullet$ -, $\blacktriangle$ - and $\blacktriangleleft$ - symbols correspond to results at $\hat{x}/\delta_0 = 0.11, 0.34, 0.68$ and 1.0, respectively.

for this flow is approximately 30 wall units, we would expect similar errors from this device to those observed for the buffer layer fit shown in figure 4.2(a,c). In short, calibration of Preston tubes are conducted under equilibrium smooth-walled conditions (see Patel, 1965), and hence we would expect measurements with such a device to be compromised in the non-equilibrium buffer region flows occurring immediately downstream of a change in surface roughness.

Figure 4.3 presents the streamwise turbulence intensities, $\overline{u^2}^+$, from the PTV database, where 4.3(a) and 4.3(b) are normalised by $U_{\tau 2}$ estimated from the buffer and viscous sublayer regions, respectively. The results show that the $\overline{u^2}^+$ profile appears to be significantly altered by the inaccurate estimate of τ_w (hence $U_{\tau 2}$) based on a buffer fit immediately after the rough-to-smooth transition. For example, both profiles exhibit an energetic site which develops at a wall-normal location in close proximity to the ‘inner-peak’ reported in equilibrium smooth-walled boundary layers (Smits et al., 2011). However, 4.3(a) reveals a sharp reduction in the magnitude of this energetic site with increasing $\hat{x}$, while 4.3(b) exhibits only a subtle reduction in magnitude. As a consequence, these shortcomings could compromise any attempts at establishing the appropriate scaling or modelling of the flow behaviour after a sudden change in surface conditions. We note similar behaviour for the inner-scaled wall-normal turbulence intensity, $\overline{w^2}^+$, which is not reproduced here for brevity.

4.3.1 Skin-friction coefficient

Figure 4.4 compiles the skin-friction coefficient, C_f , downstream of the rough-to-smooth surface transition for all the current experimental databases. For PTV and OFI, C_f is measured directly from the near-wall velocity gradient deep in the viscous sublayer, while the hotwire databases use either a buffer fit in the range $10 < z^+ < 30$ or a Clauser fit (Clauser, 1954) in the expected log region. The Musker profile (Musker, 1979) and composite velocity profile (Chauhan et al., 2009) instead of the DNS data are also employed as the reference profile in

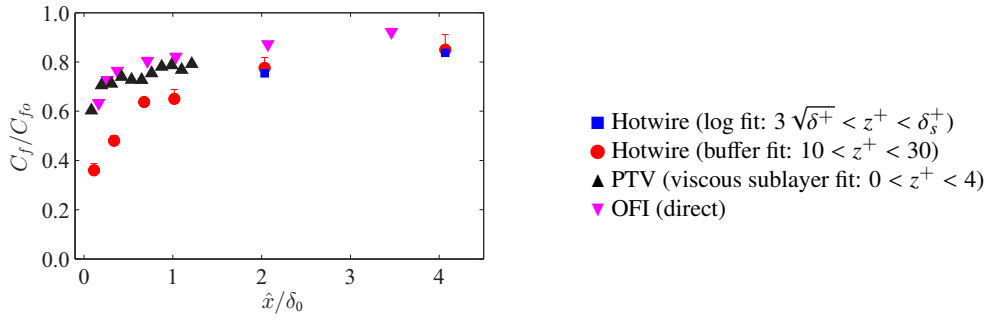


FIGURE 4.4: Skin-friction coefficient, C_f , estimates from the hotwire, OFI and PTV experimental data at $Re_{\tau 0} \approx 4100$. Normalisation is by C_{f0} , which correspond to the last measured magnitude of C_f from the OFI database at $\hat{x}/\delta_0 = 13.4$.

the buffer region fit, and the scatter in the resulting C_f is usually within 5%, as shown by the error bars in figure 4.4. Note that this error is associated with the choice of the reference profile and this error is also present in fully equilibrium smooth-wall boundary layers, therefore the fitted results should be interpreted with caution in general. For the Clauser fit we use constants $\kappa = 0.384$ and $B = 4.17$ in the range $3\sqrt{\delta_{99}^+} < z^+ < \delta_s^+$ (blue symbols). Here, the assumed upper limit of the logarithmic region δ_s^+ is defined as $\min(0.15\delta_{99}^+, 0.6\delta_i^+)$, where δ_{99}^+ is the local viscous-scaled boundary layer thickness, and δ_i is the IBL thickness, defined as the ‘knee-point’ in the $\overline{u^2}$ profile following Efros and Krogstad (2011) and Saito and Pullin (2014). We prefer this method to identify δ_i as the distinction associated with the roughness change is more pronounced in $\overline{u^2}$ compared to U and less subject to small uncertainties in the measurement, resulting in a more robust estimation of δ_i . The fit range is chosen in accordance with Marusic et al. (2013), with an extra constraint to the upper limit as $0.6\delta_i^+$, as a different U_τ is expected above the IBL (Elliott, 1958). The coefficient 0.6 is empirically chosen to eliminate any ‘kink’ in the mean velocity profile related to the internal boundary layer. Clauser fit results are not shown for $\hat{x}/\delta_0 < 2$ as δ_i^+ is small immediately downstream of the surface transition, thus there is an insufficient number of data points to perform the fit. Note that by performing a Clauser fit we do not imply the existence of a fully recovered log region immediately downstream of a rough-to-smooth transition. Our intention here is to demonstrate that, similar to the buffer fit as discussed in §4.3, if one takes the mean velocity data downstream of a rough-to-smooth transition and uses this to compute $U_{\tau 2}$ via a Clauser fit, an erroneous $U_{\tau 2}$ will result.

In addition to the error originated from the method to estimate C_f , the localised effect of the roughness geometry can also lead to a spanwise variation of the flow immediately downstream of the roughness transition. A measure of the spanwise variation of $\tilde{C}_f(\hat{x}, y)$ from the OFI measurements of case H1 is shown in figure 4.5(a). At $\hat{x}/\delta_0 = 0.2$, $\tilde{C}_f$ varies by more than 50% about its mean value. Typical low and high skin-friction regions are identified at $y_1/\delta_0 = -0.045$ (blue dashed line) and $y_2/\delta_0 = 0.011$ (red solid line), respectively. The variation of $\tilde{C}_f$ appears to be influenced by the distribution of the roughness elements immediately prior to the rough-to-smooth transition. Similar spanwise variation of the time-averaged velocity field has also been observed immediately downstream of a

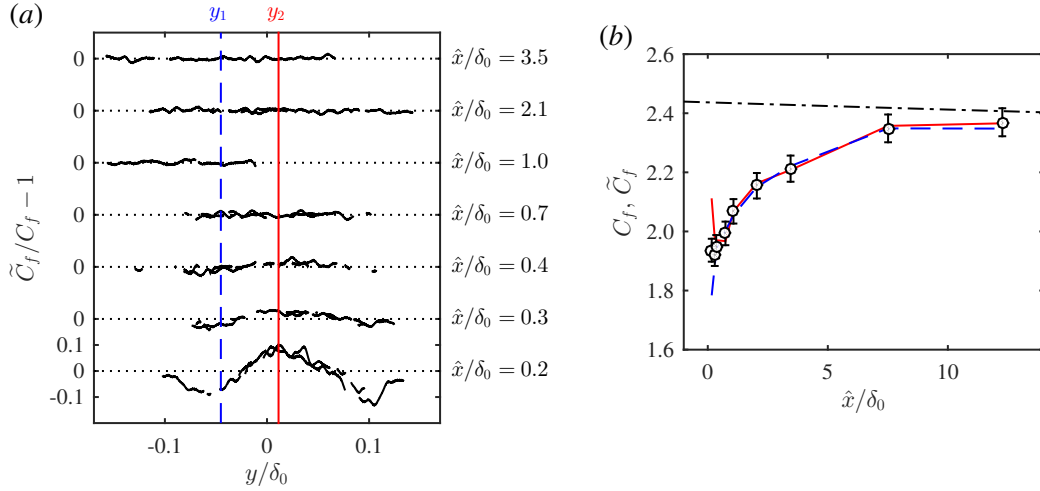


FIGURE 4.5: (a) Relative spanwise variation of the skin-friction coefficient, $\tilde{C}_f/C_f - 1$. Regions contaminated by dust are not shown in the figure. $y = 0$ is approximately the centreline of the tunnel, and the vertical dashed blue line and solid red line indicate regions of low skin-friction at $y_1/\delta_0 = -0.045$ and high skin-friction at $y_2/\delta_0 = 0.011$, respectively. (b) The spanwise averaged skin-friction coefficient $C_f(\hat{x})$ measured by OFI ($\circ$ symbols). The dashed blue and solid red lines show $\tilde{C}_f$ obtained at fixed spanwise locations, $y_1/\delta_0 = -0.045$ and $y_2/\delta_0 = 0.011$, respectively (vertical lines in panel a). The dot-dashed line corresponds to C_f of a canonical smooth-wall boundary layer calculated using the relation in (Baars et al., 2016b). A total uncertainty of $\pm 2\%$ in $\tilde{C}_f$ contributed by the oil viscosity ($\pm 0.5\%$) and the pitot tube ($\pm 1\%$) is shown by the error bars.

rough-wall backward-facing step (Wu et al., 2013). We also note this variation remains in the same spanwise location further downstream, while the amplitude of the variation diminishes rapidly within a short fetch. More specifically, $\tilde{C}_f$ is nominally constant for all y locations measured beyond $\hat{x}/\delta_0 = 0.4$, which is equivalent to $\hat{x}/k_p = 15$ in this study.

The recovery of the spanwise averaged C_f as a function of $\hat{x}$ is shown in figure 4.5(b). The overall trend of C_f exhibits an undershoot immediately downstream of the rough-to-smooth step change, and a gradual recovery towards the value of an equilibrium smooth-wall boundary layer as shown by the dot-dashed line calculated using the relation in (Baars et al., 2016b). The blue and red lines are the $\tilde{C}_f$ obtained along a streamwise line at fixed spanwise locations y_1 and y_2 (as shown in figure 4.5a), respectively. These results highlight that immediately downstream of the roughness transition, a strong variation in the recovery of the skin-friction coefficient may occur depending on the chosen spanwise location, thus measurements at a single spanwise location in this region must be interpreted with caution.

Despite of these aforementioned errors, if we define a recovery length L as the downstream fetch where the local C_f reaches, for example, 80% of the full-recovery value C_{f0} , then $L = 0.8\delta_0$ for the OFI and PTV C_f values, whereas $L = 2\delta_0$ for the buffer fit results. These results confirm that even beyond $\hat{x}/\delta_0 > 0.4$ downstream of the transition the magnitude of C_f is lower and exhibits a more gradual recovery as a function of $\hat{x}$ when estimated away from the wall (buffer and Clauser fits), as compared to estimates from closer to the wall (viscous sublayer) or at the wall (OFI). These discrepancies are likely to play a significant role in the wide range of recovery trends reported for C_f in past works (see figure 4.1). In

general, and to within experimental error, the OFI and PTV determined wall-shear stress are in close agreement. Evidence supporting this conclusion can be found in the literature, for instance, after a rough-to-smooth transition, C_f measured by a Preston tube is higher than the results from a log-region fit (Antonia and Luxton, 1972). Preston tube and log-region fit results are also reported to be lower than that of laser-oil flow interferometry behind a backward facing step (Jovic and Driver, 1995). These observations will be revisited in §4.4, where comparisons will be drawn to a direct numerical simulation of a rough-to-smooth surface change in a wall-bounded flow.

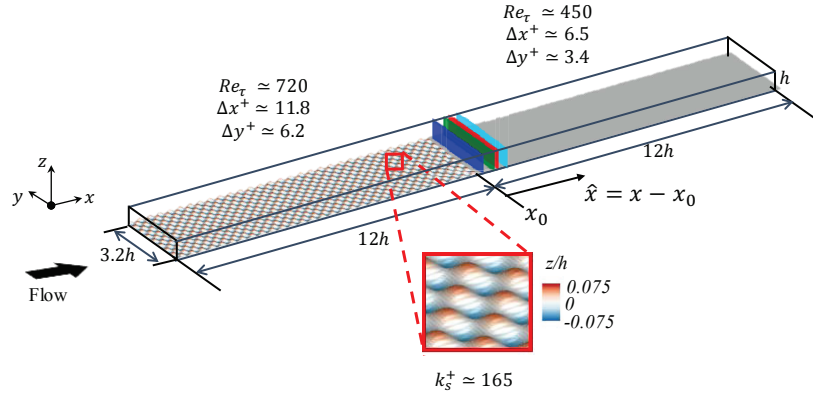
4.4 Numerical experiment of a rough-to-smooth transition

The results presented in the preceding discussions have highlighted that the accuracy of most ‘indirect’ experimental techniques for estimating τ_w will be compromised in non-equilibrium conditions which persist in the near-wall and buffer region of the internal layer for surprisingly large distances downstream of the surface transition. To complement the experiment, a DNS database was generated and analysed to test for this behaviour.

The DNS was performed on an open-channel computational domain that spans $24h \times 3.2h \times h$ in the streamwise, spanwise and wall-normal directions, as shown in figure 4.6(a). Case D1, where the midheight between the roughness crests and troughs is aligned with the smooth wall, is presented in this chapter to compare with the experimental results. The wall-shear stress τ_w over the smooth surface (and hence U_τ) is computed from the gradient of the streamwise mean flow at the grid point closest to the wall (see Eq. 4.1), which is located below $z^+ < 0.5$ for the present case. More details of the simulation are described in §3.2.3.

Figure 4.6(b) presents the inner-normalised streamwise mean velocity U^+ from the DNS database at various locations downstream of the rough-to-smooth transition. In the viscous sublayer ($z^+ \lesssim 4$) the results exhibit good agreement with the reference smooth-walled profile (dashed line, taken from del Alamo et al. (2004) at $Re_\tau = 934$). However, in the same manner as observed previously for the correctly scaled PTV experiments, the buffer region and beyond exhibits poor agreement. Note that the mean flow recovers to the equilibrium state monotonically in the experiments as shown in figure 4.2(b), whereas in the simulation the mean velocity profile at $\hat{x}/h = 0.11$ (the blue curve) overshoots the other three profiles from more downstream locations (particularly in the outer part of the flow). This discrepancy of the flow behaviour in the vicinity of the roughness transition can be attributed to the high roughness blockage ratio in the simulations ($\delta_0/k_p \approx 45$ in the experiment versus $h/k_p \approx 9$ in the simulation). These results confirm that the buffer region requires a surprisingly long recovery length downstream of a rough-to-smooth transition to reach an equilibrium state that reflects the new smooth wall condition. As a consequence, any estimate of τ_w (hence $U_{\tau 2}$) obtained from the buffer region or above will be compromised. The extent of this discrepancy is highlighted by plotting the skin-friction coefficient C_f calculated from various methods, downstream of the rough-to-smooth transition for the DNS data. These results are presented in figure 4.7. The blue dotted curve shows the case where τ_w is estimated from the buffer region (fit in the range $10 \lesssim z^+ \lesssim 30$). This case exhibits a much longer and more gradual C_f

(a)



(b)

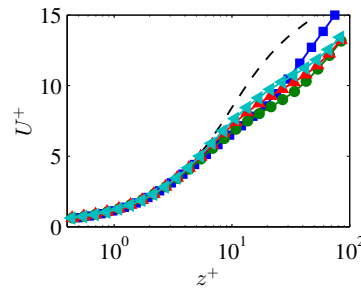


FIGURE 4.6: (a) The computational domain for the DNS database. The bottom surface is coloured according to the surface elevation relative to the location of the smooth wall plane. The inset shows a magnified view of the “egg carton” roughness employed. The reported Re_τ on each patch corresponds to the recovered region and the wall parallel resolutions Δx^+ and Δy^+ are normalised by the asymptotic values of U_τ on each patch. (b) Streamwise mean velocity, U , normalised by the local $U_{\tau 2}$, from the DNS database at $Re_\tau \approx 590$. The black dashed line corresponds to a reference DNS channel flow database at $Re_\tau = 934$ (del Alamo et al., 2004). The $\blacksquare$, $\bullet$, $\blacktriangle$ and $\blacklozenge$ symbols correspond to $\hat{x}/h = 0.11, 0.34, 0.68$ and 1.0 respectively, as shown by the wall-normal planes in (a), where $\hat{x}$ corresponds to the streamwise distance from the rough-to-smooth transition.

recovery as compared to the direct measure from the DNS (red dashed curve, obtained from the velocity gradient at the first off-wall grid cell). On the other hand, when C_f is estimated from the viscous sublayer region ($z^+ \lesssim 4$, the green curve of figure 4.7(a)), we observe closer agreement to the direct measure from the DNS database. This is promising for experiments, where the viscous sublayer is certainly more accessible to measurements than the gradient at the wall (e.g. the PTV measurements presented previously). For the present case, the error between the viscous sublayer fit and the wall gradient measure drops from approximately 5% to 1% as $\hat{x}/h$ increases from 0 to 2. These observations from the DNS data in figure 4.7 reconfirm the broad trends of C_f recovery for the various τ_w estimation techniques observed from the experiments (figure 4.4) and past works (figure 4.1), thus providing an explanation for some of the scatter observed. It is noted from a comparison of figure 4.7 with figure 4.4, that the buffer layer computed C_f recovery following the rough-to-smooth transition in the DNS is quite different to that from the experiments, with the DNS indicating a slower recovery. This suggests that the DNS retains non-equilibrium effects in the buffer layer to a greater distance downstream of the rough-to-smooth transition than the experiments. It is noted that

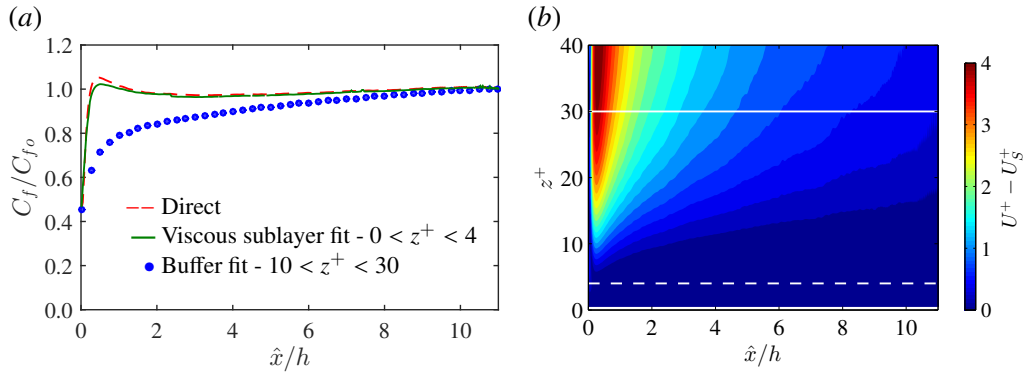


FIGURE 4.7: (a) Skin-friction coefficient, C_f , estimated from the DNS database at $Re_{\tau 0} \approx 590$. Normalisation is by C_{f0} , which corresponds to the last measured magnitude of C_f for each case. (b) Colour contours of the difference in the mean streamwise velocity ($U_d^+ = U^+ - U_S^+$) immediately downstream of a rough-to-smooth transition relative to a reference smooth-walled open-channel flow, U_S^+ , at a comparable Re_τ . Results are computed from DNS data where $U_{\tau 2}$ can be directly estimated from the velocity gradient at the wall. The white dashed and solid lines correspond to the upper limit of the viscous sublayer and buffer region, respectively.

the DNS is at a much lower Reynolds number, with a much greater k_p/δ_0 and is an open channel geometry, all of which would likely affect the recovery. Regardless, in the context of this study the overarching message is clear from both experiments and DNS: estimates of C_f made further from the wall (in the buffer or log region) can be surprisingly inaccurate, even at several boundary layer thicknesses downstream of the transition.

We additionally note that the DNS data exhibit an overshoot of C_f immediately downstream of the rough-to-smooth transition which is notably absent in the experiments (figure 4.4). This might be related to the difference of the step height ΔH at the roughness transition, as a greater down step is present in the experiment ($\Delta H/k_p = -1.5$) compared to the simulation ($\Delta H/k_p = -0.5$). Another possible factor for this behaviour might be associated with the lower Re_τ of the DNS database or the difference in geometry (this hypothesis is explored further in appendix 4.A). Further, the results from the DNS database also appear to exhibit a much slower recovery of the buffer region to the new wall conditions as a function of h (or δ_0) when compared to the experiments. This observation may be associated with the significantly larger k_p/δ_0 ratio in the DNS databases compared to the experiments ($k_s/h \approx 0.2$ for the DNS, compared to $k_s/\delta_0 \approx 0.04$ for the experiments). In any case, the broader trends from the DNS database confirm that any estimation of τ_w made using the data above the viscous sublayer region is compromised for several δ_0 downstream of a rough-to-smooth transition.

In order to quantify the rate of recovery of a wall-bounded flow to equilibrium conditions downstream of a rough-to-smooth surface change, figure 4.7(b) presents colour contours of the difference in the streamwise mean flow, U_d^+ , after the rough-to-smooth transition relative to a fully developed smooth-walled flow, U_S^+ , from a DNS database in the present study at matched Re_τ . In this case, a direct measure of the velocity gradient at the wall and hence τ_w is available from both databases. For simplicity, comparisons are drawn for the same

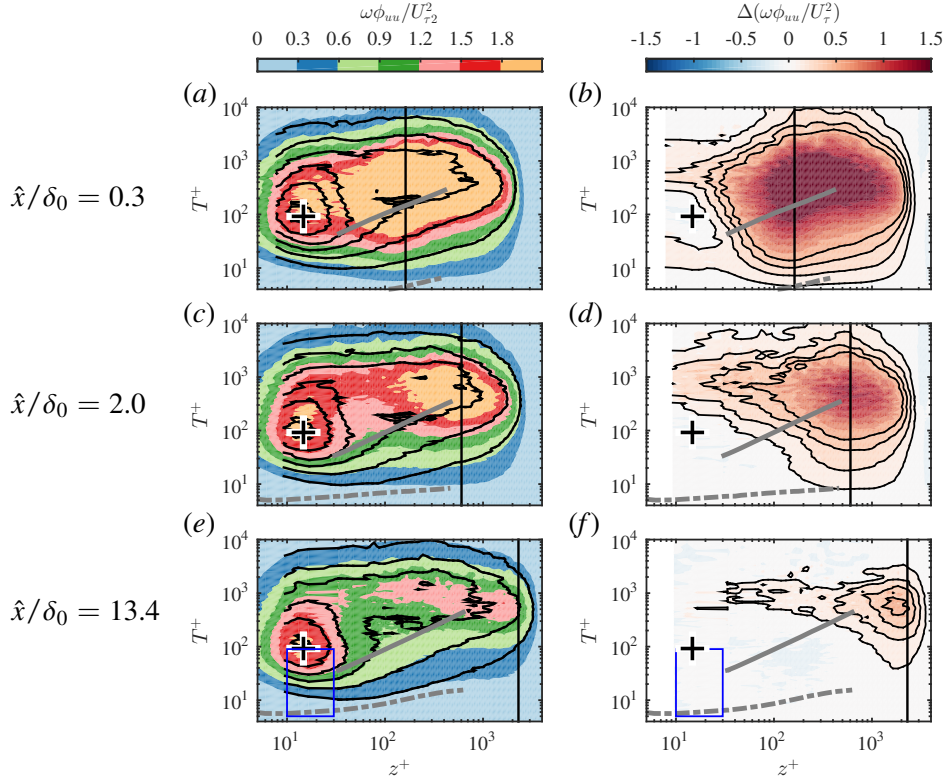


FIGURE 4.8: Premultiplied energy spectra $\omega\phi_{uu}/U_\tau^2$ at (a) $\hat{x}/\delta_0 = 0.3$, (c) 2.0 and (e) 13.4. The coloured contour is the rough-to-smooth case, and the black contour lines are from a reference smooth-wall experiment at matched $Re_{\tau 0}$, with contour levels of $\omega\phi_{uu}/U_\tau^2 = 0, 0.3, 0.6, 0.9, 1.2, 1.5, 1.8$. (b, d) and (f) are the difference between the rough-to-smooth case and the reference smooth case $\Delta(\omega\phi_{uu}/U_\tau^2)$ at streamwise locations corresponding to the left column. The four black contour lines indicate 0.15, 0.30, 0.45 and 0.60. The vertical black lines in all the figures represent the location of the IBL estimated from the turbulence intensity profile following Efros and Krogstad (2011) and Saito and Pullin (2014). The black cross is at $z^+ = 15$ and $T^+ = 95$ (equivalent to $\lambda_x \approx 1000$). The thick grey solid lines represent the time period of Townsend's attached eddies and the thick grey dot-dashed lines represent T_η^+ , the Kolmogorov time scale. The blue box in (e,f) is the same as that in figure 4.9(a), showing the region where the spectrum fit is performed.

flow geometry (open-channel flow) to avoid the spatial growth of a turbulent boundary layer. The colour contours of U_d^+ indicate an almost immediate recovery in the viscous sublayer region ($z^+ \lesssim 4$ indicated by the horizontal dashed line) to an equilibrium state of a smooth-walled channel flow, while further from the wall, in the buffer region and beyond, larger discrepancies are present throughout the range $0 < \hat{x}/h < 5$. These results confirm that (for a channel flow, and consistent with boundary layers) only beyond $\hat{x}/h \gtrsim 5$, can we reliably employ diagnostic tools that operate in the buffer region to estimate τ_w .

4.5 Premultiplied energy spectrum

From the preceding discussions, it is clear that the boundary layer gradually recovers to an equilibrium state of the new wall conditions after several boundary layer thicknesses downstream of the transition, yet it is not obvious which scales are responsible for this slow recovery. To provide a clearer picture of the recovery scale by scale, premultiplied energy spectra $\omega\phi_{uu}/U_\tau^2$ are shown in figure 4.8, where $\omega = 2\pi/T$ is the angular frequency, T is the time period (corresponding to wavelength in the spatial domain), ϕ_{uu} is the energy spectrum of the streamwise velocity fluctuation ($\int_0^\infty \phi_{uu} d\omega = \overline{u^2}$), and U_τ is the friction velocity measured from the OFI experiments (see §4.3). The spectrograms presented are computed from hotwire time series data. Further, since the flow is heterogeneous in x , we refrain from converting the spectrum from temporal to the spatial domain, which has been shown to have limited accuracy in rough-walled flows (Squire et al., 2017). The colour contour maps in the left column are computed from the rough-to-smooth cases, while the solid contour lines represent a smooth-wall reference, which is obtained by interpolating the spectrum reported by Marusic et al. (2015) to the same Re_τ as the corresponding rough-to-smooth case. The length of the hotwire filament in viscous units is $l^+ \approx 17$ for the present rough-to-smooth case at $\hat{x}/\delta_0 = 13.4$, while $l^+ \approx 24$ in the smooth-wall reference. This difference in the sensor spatial resolution indicates that the inner-site of the spectrum in the latter is more attenuated than the former by approximately 5% (Chin et al., 2011). The results reveal clear evidence that the rough-wall structures are present beyond the IBL and are over-energised relative to the local U_τ . In addition, there are signs that even within the IBL, the large-scale motions are over energised, providing further evidence that the IBL has not returned to equilibrium conditions. These results also show better agreement at smaller scales ($T^+ < 90$), particularly in the near wall region at larger $\hat{x}$. A similar observation has also been made by Ismail et al. (2018b) following a transition from transverse square ribs to a smooth wall in a channel flow DNS.

To further elucidate this behaviour, the right column shows the difference between the rough-to-smooth spectrum and the reference smooth-walled spectrum, defined as,

$$\Delta(\omega\phi_{uu}/U_\tau^2) \equiv (\omega\phi_{uu}/U_\tau^2)_{R \rightarrow S} - (\omega\phi_{uu}/U_\tau^2)_S. \quad (4.2)$$

These difference plots confirm that the energy distribution of the smaller scales recovers first, while the larger scales remain over-energised, reflecting the upstream rough wall condition. Interestingly, these over-energised large scales are not just restricted to the region above the IBL, but retain a footprint deep into the buffer region. These results suggest that the near-wall region recovering over the smooth surface will be subjected to a heightened degree of superposition and modulation from the over-energised large-scale events which retain the rough-wall upstream history (Mathis et al., 2009).

The largest T^+ where $\Delta(\omega\phi_{uu}/U_\tau^2)$ is negligible roughly coincides with the inner site (marked by the cross) of the energy spectra in the left column. Its wall-normal extent is initially limited to the buffer region ($10 < z^+ < 30$) at $\hat{x}/\delta_0 = 0.3$, and gradually expands further away from the wall with the growth of the IBL.

We may also compare the size of the recovered small-scale motions with some other common scales in wall-bounded flows. The smallest energy containing turbulent motions are characterised by the Kolmogorov time scale (Tennekes and Lumley, 1972; Pope, 2000)

$$T_\eta^+ = (\nu/\epsilon)^{1/2}, \quad (4.3)$$

where the dissipation rate ϵ is estimated from the streamwise energy spectrum as

$$\epsilon = 15\nu \int_0^\infty \frac{\omega^2}{U^2} \phi_{uu} d\frac{\omega}{U}. \quad (4.4)$$

T_η^+ is shown in figure 4.8 by the thick grey dot-dashed line. The small-scale end of the spectrum where the energy distribution has reached a close agreement with the smooth-wall reference contains the entire dissipation range except where $z \gtrsim \delta_i$, and it covers a substantial range of, if not all, scales in the inertial subrange.

The attached eddy hypothesis proposed by Townsend (1976) states that the velocity field of wall-bounded flows consist of a range of self-similar eddies extending all the way to the wall. With an aspect ratio of $\lambda_x/z \approx 14$ found experimentally (Baidya et al., 2019), the time period corresponding to an attached eddy with a height of z can be estimated as

$$T_{AE}^+ = \frac{14z^+}{U^+}. \quad (4.5)$$

Considering that at a given wall-normal location, the energy-containing attached eddies range from a height of $O(z)$ to a height of $O(\delta)$, (4.5) defines the smallest attached eddy that can be found at this wall-normal location. In figure 4.8, T_{AE}^+ is shown by thick grey solid lines. Strictly speaking, Townsend's attached eddy hypothesis is only applicable to inviscid, asymptotically-high Reynolds number wall-bounded flows (Marusic and Monty, 2019), whereas in practice, the inertial-dominant logarithmic region can be relatively well approximated. Close to the roughness transition (e.g. $\hat{x}/\delta_0 = 0.3$), the recovered small-scale energy distribution only exists in the buffer region and below, therefore it is not very meaningful to compare the size of the recovered small-scale motions and the attached eddies. As the IBL grows further downstream, the small-scale end of the energy spectrum begins to recover in the logarithmic region.

4.5.1 An alternative method to estimate $U_{\tau 2}$

The energy spectrum has revealed that the smaller energetic scales in the near-wall region recover to an equilibrium with the new smooth wall in a fetch less than one boundary thickness. Based on this observation, we propose an alternative method to estimate $U_{\tau 2}$ for the flow downstream of a rough-to-smooth transition when no direct measurement at the wall or within the viscous sublayer is available. The essence of this method is to minimise the difference of the energy spectrum of the small scales in the near-wall region between the rough-to-smooth case and a smooth-wall reference dataset by adjusting the velocity scale $U_{\tau 2}$ for the rough-to-smooth case. There is some precedent for this approach in the literature for smooth

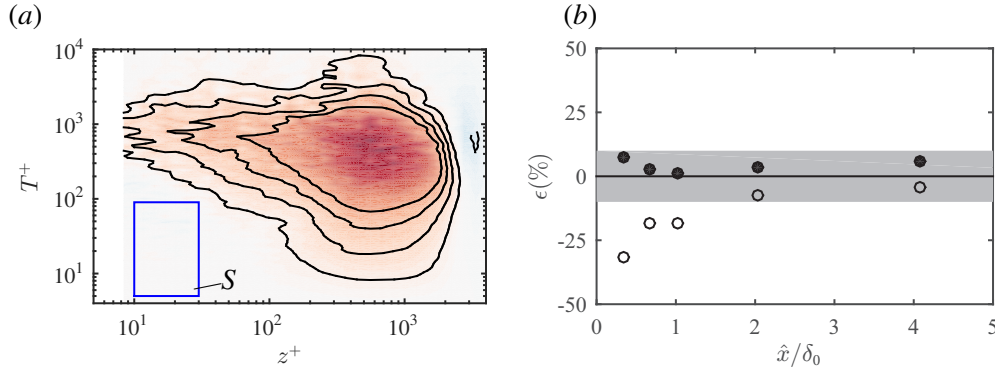


FIGURE 4.9: (a) Difference in premultiplied spectrum $\omega\phi_{uu}/U_{\tau 2}^2$ between the rough-to-smooth case at $\hat{x}/\delta_0 = 2$ with estimated $U_{\tau 2}$ ($U_{\tau 2}$ is adjusted such that the integral of the difference across the blue rectangle region S is minimum) and the smooth-wall reference. Contour levels are the same as in figure 4.8(d). (b) $\epsilon = (C_f|_M - C_f|_{OFI})/C_f|_{OFI}$, where M stands for buffer region fit (open symbols) or the spectrum fit (solid symbols). ϵ is the error relative to the OFI results. The shaded band covers -10% to 10% on the vertical axis. Note that the data points at $\hat{x}/\delta_0 = 0.11$ are not shown in the figure as they fall beyond the current axis limit.

wall canonical wall-bounded turbulent flows. Hutchins et al. (2009) have shown that over a range of Reynolds numbers, the energy in small scales appears to collapse on to a universal distribution when scaled by the local U_τ . Ganapathisubramani (2018) has shown that this universality is also persistent under the influence of freestream turbulence, and Monty et al. (2009) have observed small-scale universality between pipe, channel and boundary layer geometries. All of these cases suggest small-scale universality in the near-wall region, even in situations where we expect there to be large differences in the footprinting of the large-scales onto the near-wall region.

For the present case, a rectangular region S (marked in blue on figure 4.9(a)) of energetic scales in the near-wall region is chosen that is bounded by the limits $10 < z^+ < 30$ and $5 < T^+ < 90$. These bounds are chosen empirically based on the vanishing $\Delta(\omega\phi_{uu}/U_\tau^2)$ observed in this region from figure 4.8. As discussed in §4.5, this region contains only the near-wall cycle and dissipative viscous scales which would be expected to have much shorter lifetimes, and hence recover to a local equilibrium with the new surface condition over a much shorter fetch downstream of the transition than the larger scales that are centred further from the surface. The limit $T^+ < 90$ and $z^+ < 30$ ensures that we are only assuming a universality for the small-scale near-wall turbulence, which has been validated over a range of Reynolds numbers for a canonical smooth-wall boundary layer at least (Hutchins and Marusic, 2007a,b; Mathis et al., 2009). More specifically, the upper bound $T^+ = 90$ can be considered as a conservative threshold below which the linear coherence spectrum between the near-wall and logarithmic regions vanishes (Baars et al., 2016a), implying that the spectrum in this region receives little influence from the large-scale structures residing above. Note that the aforementioned threshold is derived from a canonical smooth-wall turbulent

boundary layer, the applicability of which to a turbulent boundary layer after a rough-to-smooth change remains to be verified with experimental evidence in future works. The difference between the viscous scaled energy spectra for the rough-to-smooth and the smooth case $\Delta(\omega\phi_{uu}/U_\tau^2)$, is then minimised across this region by varying $U_{\tau 2}$ for the rough-to-smooth case. Figure 4.9(a) shows an example where $U_{\tau 2}$ has been optimised in this manner, to give the minimum $\Delta(\omega\phi_{uu}/U_\tau^2)$ within the rectangular region S . To test the efficacy of this method of determining $U_{\tau 2}$, figure 4.9(b) presents the relative error ϵ in C_f obtained using the spectrum fit (solid symbols) and buffer region fit (open symbols) as compared to the OFI results. The results show that the spectrum fit, although still subject to error, provides a better estimate of $U_{\tau 2}$ immediately downstream of the rough-to-smooth transition compared to methods that purely rely on the mean streamwise velocity in the buffer region. We note, the precise nature of dependence of the bounds of S on $Re_{\tau 0}$, k_{s0}^+ and other flow parameters remain to be examined by performing more experiments covering a broader range of conditions in future works.

4.6 Summary and conclusions

In this chapter, we present a systematic study on estimating the wall-shear stress, τ_w , after a sudden change in surface conditions from a rough-to-smooth wall. To this end, a unique collection of experimental and numerical databases is examined offering access to both ‘direct’ and ‘indirect’ measures of τ_w .

1. Our experimental results reveal that the mean flow within the buffer region (defined as $10 < z^+ < 30$) only recovers to an equilibrium state with the new local smooth-wall conditions after approximately 5 boundary layer thicknesses downstream of the rough-to-smooth transition. Based on these findings, ‘indirect’ techniques that only have access to velocity information above the viscous sublayer are shown to consistently underestimate the magnitude of τ_w immediately downstream of a rough-to-smooth transition. This discrepancy, in turn, can give the erroneous impression of a longer recovery length of C_f to the new wall conditions and is likely to be responsible for the wide range of recovery trends reported for C_f following a rough-to-smooth transition. More specifically, the further from the wall that C_f is inferred from the velocity profile, the greater the underestimation of C_f will be, and the greater the recovery length will be.
2. To complement the experimental databases and to further confirm our findings, a DNS database with comparable flow conditions and the access to a direct measure of τ_w is employed. These data lead to similar conclusions, indicating that in the range of $0 < \hat{x}/h < 5$, an accurate estimate of the wall-shear stress can only be obtained in the viscous region ($z^+ \lesssim 4$). More specifically, diagnostic tools that operate in the buffer region are likely to provide a reliable estimate of the wall-shear stress only beyond $\hat{x}/h \gtrsim 5$.

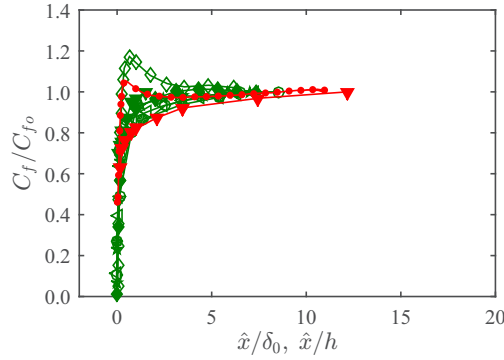


FIGURE 4.10: Revisit of the C_f data from literature as shown in figure 4.1. Only the datasets with a direct measurement of the wall-shear stress are shown, and symbols are the same as in table 4.1. OFI and DNS results from the current study are represented by $\blacktriangledown$ and $\bullet$, respectively.

3. Through an analysis of the energy spectra we observe that the smaller energetic scales ($T^+ < 90$) in the buffer region adjust to the new wall condition over a relatively short recovery length ($\hat{x}/\delta_0 \lesssim 1$). Conversely, the large-scale motions ($T^+ > 90$), which are over-energised (relative to the new smooth wall boundary condition) retain a strong footprint in the IBL, extending deep into the buffer region. Based on the observation that the small scales attain a universal form over relatively short recovery distances, an alternative approach to estimate the wall-shear stress from the premultiplied energy spectra is proposed when no direct measurement of the wall-shear stress is available. The results reveal improved performance relative to more conventional techniques that are based purely on the mean velocity profile in the buffer region.

4.A Appendix: C_f data in the literature with a direct measure of τ_w

In this paper, we have highlighted that the scatter for the recovery of C_f after a rough-to-smooth transition appears to be partly due to the measurement techniques employed. However, the recovery of C_f can be affected by a number of factors (see §4.1) including the Reynolds number, flow geometry (boundary layer or channel and pipe) and the roughness geometry. In order to examine some of these additional factors, figure 4.10 presents a subset of the datasets previously shown in figure 4.1 and table 4.1 that have access to a direct measure of the wall-shear stress. Consequently, we are limited to comparing data from the present study and the boundary layer data of Chamorro and Porté-Agel (2009) and the channel flow data of Ismail et al. (2018a,b). Although limited by available data, figure 4.10 suggests the overshoot in the current DNS database and the lowest Re database from Ismail et al. (2018b) ($\diamond$) might be a low Reynolds number effect. Certainly, the higher Re data from Ismail et al. (2018b) ($\blacklozenge$, $\blacktriangleleft$ and $\blackstar$) do not exhibit this overshoot, suggesting that the overshoot is not geometry related. Further data are required to confirm this tentative observation. In addition,

the boundary layer data shown in figure 4.10 (OFI in the present study and near-wall hotwire in Chamorro and Porté-Agel (2009)) reveal a substantial difference in the recovery length. However, from table 4.1 it is noted that the rough-to-smooth case of Chamorro and Porté-Agel (2009) had a much higher Re_{τ_0} and k_{s0}^+ than the current experimental study, which may suggest further influencing factors. Furthermore, datasets with a direct measure of τ_w are dominated by DNS studies, and they are mostly conducted with a channel configuration at low Reynolds numbers with high k_s/h values. These tendencies may also bias the comparison. To answer these questions future works over a wide range of Re , roughness parameters and flow geometries with a direct measure of wall-shear stress are necessary.

Chapter 5

Effects of the change in virtual origin

For surfaces with heterogeneous roughness, there can be a change of virtual origin alongside the variation of roughness heights, an extreme case of which is the flow over a forward/backward-facing step, where no roughness exists but only the height difference is present. The shift of the virtual origin can potentially have an effect of the flow evolution, but it is often set at zero in numerical studies and less well documented in most experimental studies. Perhaps the cleanest case to study the recovery over is two surfaces with matching virtual origin locations (streamwise variation of roughness height only). However, cases with a combination of roughness height and virtual origin changes are more common in reality, for instance, a flow over a change of terrain from a prairie to a forest is usually accompanied by a ‘step-up’ of the virtual origin. In this section, the effect of the change in virtual origin is investigated. With the aid of a hotwire dataset covering the entire $13\delta_0$ of the streamwise recovery and the high-resolution PTV dataset near the rough-to-smooth transition, we are able to observe the effect of step heights on both time-averaged and fluctuating quantities. For the present study, cases with various step heights behave similarly beyond $2\delta_0$ downstream of the rough-to-smooth transition, and the higher order statistics seem to recover within approximately the same fetch as required for the recovery of the mean velocity profiles. The internal boundary layer thickness appears to be slightly smaller where there is a relatively large down-step. Finally, the results are compared with a direct numerical simulation of an open-channel flow, which shows qualitatively similar effect of the step height.

5.1 Flow parameters of the experimental and numerical datasets

On a smooth surface, the virtual origin locates at $z = 0$ with little ambiguity. On a rough surface, there are several possible definitions of the virtual origin, including the mean height of momentum absorption by the surface (Thom, 1971; Jackson, 1981); the vertical shift required to collapse the velocity profile in the logarithmic region (see Perry and Li, 1990, for example); the wall normal location where $U^+ = 0$ and the vertical adjustment required to collapse the total shear-stress profile outside the roughness sublayer (Chan et al., 2015). These methods to determine the virtual origin can give various results, but they usually fall between the peak and trough of the roughness. In flows where δ/k_p is large, the choice of the virtual origin makes little difference to the velocity or stress profiles, as the profile will only be shifted in the wall-normal direction by a small amount compared to the boundary layer thickness. In this study, we assume that the virtual origin is located at the midpoint between the roughness peak and trough, i.e. $\varepsilon = -k_p/2$, where the virtual origin and k_p is the maximum peak-to-trough roughness height, the absolute value of ε is the distance between the roughness crest and the negative sign of ε indicates that the virtual origin is located below the roughness crest. The step change of the virtual origin is denoted by Δd . In practice, ΔH , the height difference between the roughness crest and the smooth wall, can be measured more easily without *a priori* knowledge of the roughness height. After acquiring ε , Δd can be determined subsequently from the relation $\Delta d + \varepsilon = \Delta H$, as shown by the schematic in figure 5.1. Note that both ΔH and Δd are directional: a positive ΔH means that the roughness crest is above the smooth wall while a negative ΔH means that the roughness crest is below the smooth wall, and a positive Δd indicates a ‘step-up’ while a negative one indicates a ‘step-down’. Both ΔH and Δd are negative for the schematic in figure 5.1.

A set of four experiment cases are designed where the smooth wall is located above the roughness peak (H3), below the roughness valley (H0) and in between the roughness canopy (H1 and H2). The flow field in the vicinity of the step change is captured by a near-wall high-magnification PIV/PTV measurement with a field-of-view of $1.1\delta_0 \times 0.3\delta_0$ (streamwise $\times$ wall normal). Hotwire profiles are acquired at streamwise locations of $\hat{x}/\delta_0 = 0.1\text{--}13.4$, and the wall-shear stress at each streamwise location is measured directly using OFI. A detailed summary of the experimental campaign is presented in §3.2.1, and the key flow parameters are summarised in table 5.1. We also include three open-channel DNS cases with various virtual origin steps (numerical method summarised in §3.2.3, see also Rouhi et al. (2019b,a) for details) to compare with the experimental dataset when appropriate in this chapter.

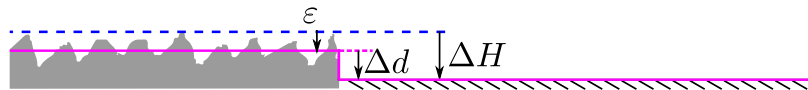


FIGURE 5.1: Schematic of a rough-to-smooth transition accompanied by a change in the virtual origin. The flow goes from the left to the right of the figure. The blue dashed line represents the location of the roughness crest, and the magenta solid line shows the virtual origin.

Case	$\Delta H/(\delta_0, h)$	$\varepsilon/(\delta_0, h)$	$\Delta d/(\delta_0, h)$	$Re_{\tau 0}$	k_{s0}^+
H0	-0.032	-0.011	-0.021	4100	134
H1	-0.022	-0.011	-0.011	4100	134
H2	-0.011	-0.011	0	4100	134
H3	0.001	-0.011	0.010	4100	134
D0	-0.112	-0.056	-0.056	680	160
D1	-0.056	-0.056	0	680	160
D2	0	0.056	-0.056	680	160

TABLE 5.1: Summary of the experimental (the first 4 cases) and numerical (the last 3 cases) datasets included for comparison in this study.

5.2 Skin-friction coefficient

The streamwise recovery of the skin-friction coefficient $C_f \equiv \tau_w / \frac{1}{2} \rho U_\infty^2$ as measured by OFI for all 4 cases are shown in figure 5.2. In the range of step heights examined, C_f decreases with increasingly negative ΔH , i.e. a larger step down from the roughness to the smooth wall results in an initially lower C_f over the smooth wall. The difference caused by ΔH diminishes downstream. Beyond $\hat{x}/\delta_0 = 4$, the difference between the 4 cases is negligible and C_f shows a slow and steady recovery. Eventually in the last measuring station at $\hat{x}/\delta_0 = 13.4$, C_f appears to have a good agreement with the canonical smooth-wall boundary layer relation (Baars et al., 2016b), as shown by the dashed black curve. The decrease of C_f

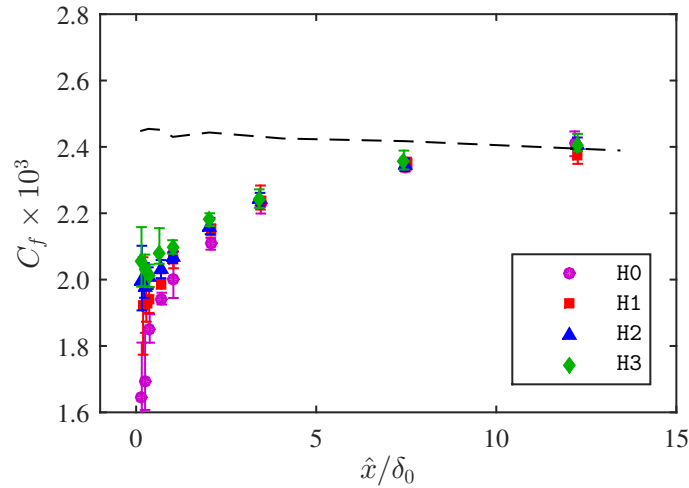


FIGURE 5.2: Skin-friction coefficient C_f as measured by OFI for all 4 step height cases. The symbol shows the median of the data (including multiple lines in the oil droplet image along the spanwise direction and repeat runs) at each streamwise location, and the errorbars show the $\pm 10\%$ bounds. Note that the data points at the first downstream location, $\hat{x}/\delta_0 = 0.1$, are not shown due to the low repeatability of the measurement. The dashed black line is the estimation of C_f for a canonical smooth-wall boundary layer at matched Re_θ (Baars et al., 2016b).

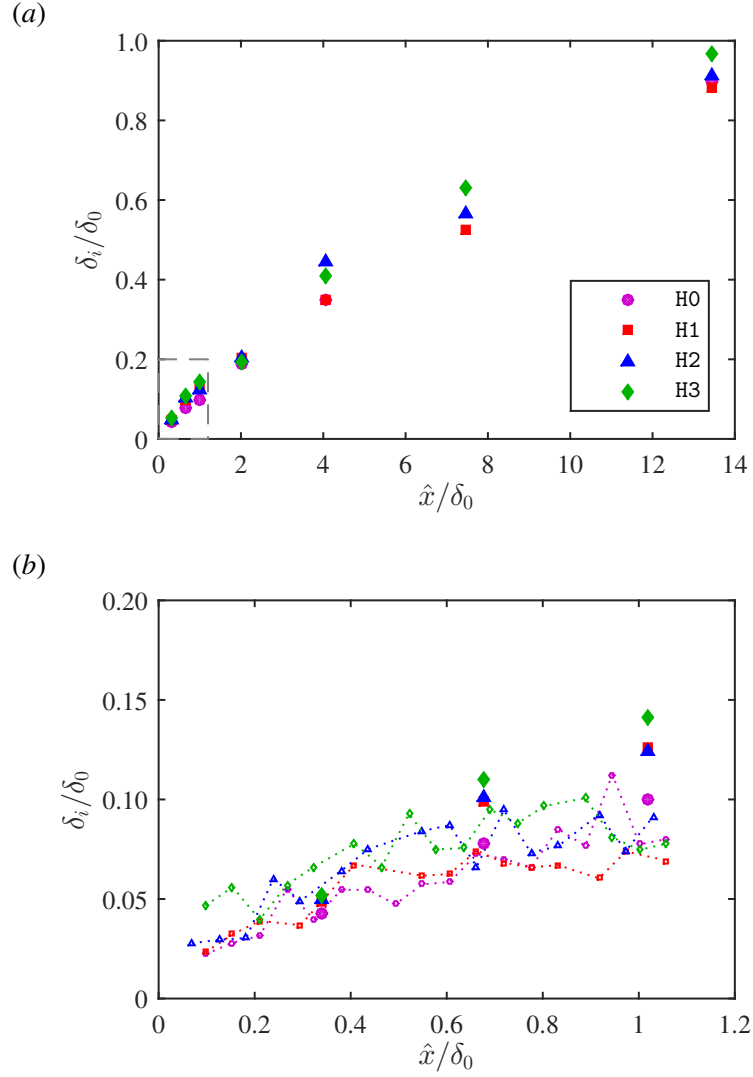


FIGURE 5.3: IBL thickness δ_i for all 4 step height cases versus $\hat{x}$, fetch on the smooth wall, both normalised by δ_0 . The solid symbols in (a) are calculated from the hotwire $\overline{u^2}$ profiles. (b) shows a magnified view of the boxed region in (a), where the solid symbols are the same as in (a) while the smaller empty symbols are the IBL thicknesses determined by the log-law intersection approach using the PTV velocity field.

downstream (effectively with an increasing Re_x) is not captured in the current measurement as the streamwise evolution is limited by the length of the tunnel working section. Thus, it is difficult to determine the extent of the overshoot and the trajectory of the measured C_f downstream of the overshoot. More detailed discussion regarding the far-field C_f behaviour can be found in §6.4.

5.3 Internal boundary layer thickness

The IBL thickness is estimated from the streamwise turbulence intensity profiles measured by hotwire following the same approach as detailed in §6. This method defines the edge of the IBL as the wall-normal location where $\overline{u^2}/U_\infty^2$ differs from its upstream neighbouring

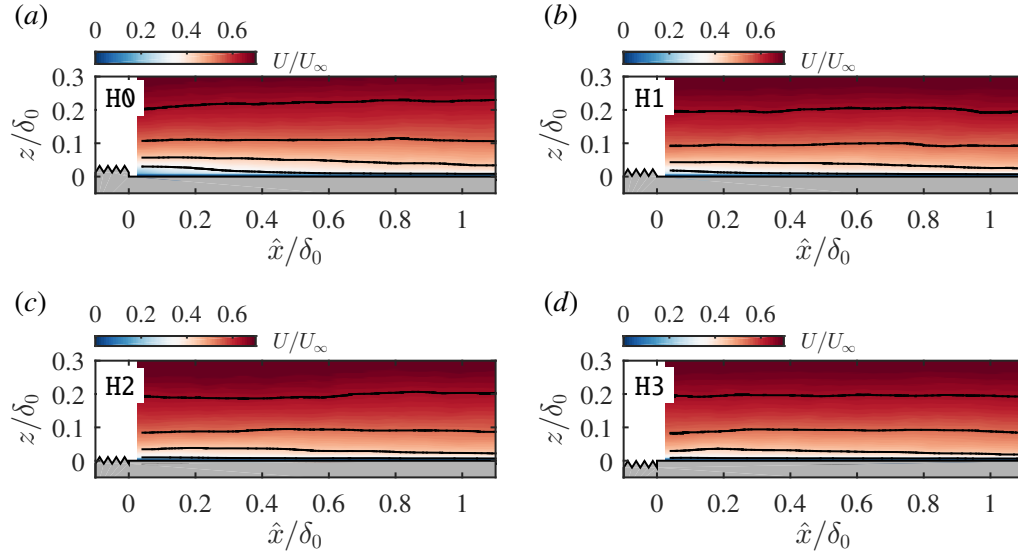


FIGURE 5.4: Streamwise mean velocity U/U_∞ in $x-z$ plane as measured by PTV. Panels (a-d) correspond to cases H0-H3 with the relative location of the rough wall and smooth wall shown by the grey silhouette. The contours of U/U_∞ increase from 0.35 to 0.65 with a step of 0.1.

profile by more than a nominated (small) threshold. As shown in figure 5.3(a), δ_i of all 4 step height cases have a similar trend of growth. The scatter between cases are relatively higher downstream, where the difference between the flow within and above the IBL is less distinct, therefore the estimation of δ_i is subjected to a higher level of uncertainty.

We also consider an alternative method that defines δ_i as the intersection of the two log laws fitted to the lower and upper parts of the mean velocity profile (Rouhi et al., 2019b). We apply this method to the PTV velocity field and compare the data with the turbulence intensity based approach in figure 5.3(b). δ_i defined by the two approaches have a satisfactory agreement with each other, and they are both observed to increase slightly from case H0 to H3. A similar dependence on the step height in the channel DNS dataset was also reported by Rouhi et al. (2019a).

5.4 Mean velocities

The streamwise and wall-normal component of the mean velocity, U and W , as captured by PTV for all 4 cases are shown in figure 5.4 and 5.5, respectively. The lowest U among all 4 cases is observed in H0 immediately downstream of the step change close to the wall, accompanied by a downwash (negative W). Flow reversal (negative U) is not observed in all cases. Case H0 has the most aggressive backward-facing step and its reattachment length is estimated (with extrapolation) to be around 4 step heights, i.e. $0.13\delta_0$ (Nadge and Govardhan, 2014), which falls within the PTV FOV. Note that due to the porosity of the roughness, the actual step height is smaller than ΔH and varies in spanwise, which might be responsible for the absence of flow reversals in the PTV data. As the smooth wall moves upward relative to the rough wall upstream, the streamwise velocity increases, while the magnitude of the

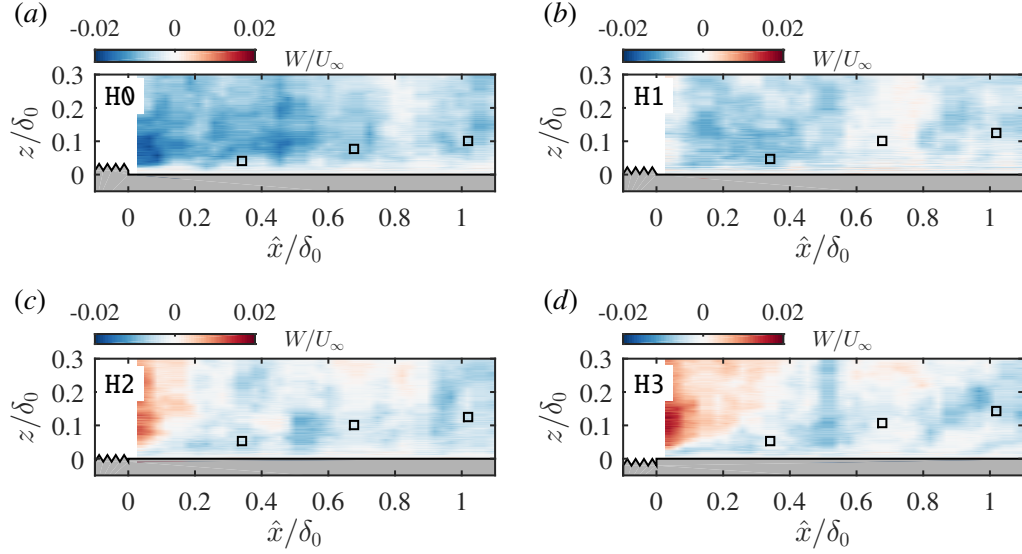


FIGURE 5.5: Wall-normal mean velocity W/U_∞ in $x-z$ plane as measured by PTV. Panels (a-d) correspond to cases H0-H3 with the relative location of the rough wall and smooth wall shown by the grey silhouette. The empty black squares show the location of the IBL.

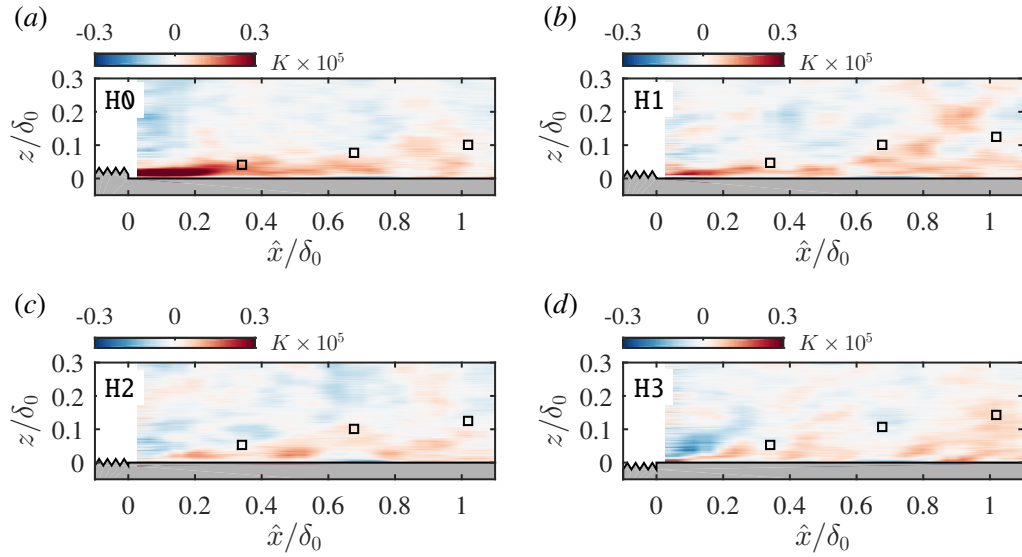


FIGURE 5.6: Acceleration of U in the streamwise direction. The acceleration parameter is defined as $K \equiv \frac{\nu}{U_\infty^2} \frac{\partial U}{\partial x}$. Panels (a-d) correspond to cases H0-H3 with the relative location of the rough wall and smooth wall shown by the grey silhouette. The empty black squares show the location of the IBL.

downwash decreases in H1, and its direction subsequently reverses in H2 and H3 such that upwash accompanies the transition in these cases. A shear layer forms when the flow from the upstream roughness impinges on the elevated downstream smooth surface in H2 and H3, resembling the flow structure of a forward-facing step (Largeau and Moriniere, 2007; Hattori and Nagano, 2010). W is found to be the strongest in the vicinity of the step change, and the streamwise extent where a non-zero W presents increases with the step-change of the virtual origin. Case H0 appears to have the longest streamwise fetch where W is non-zero, extending

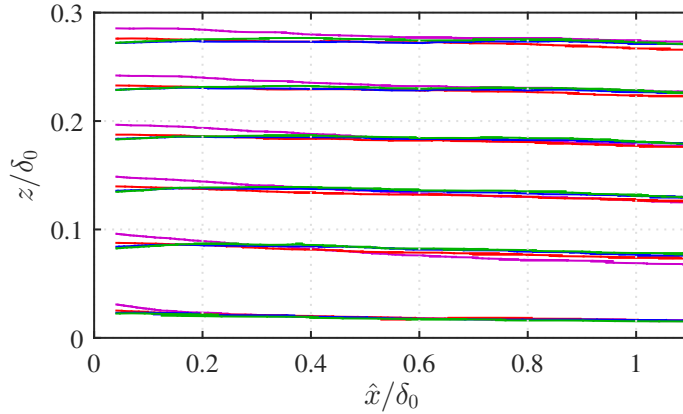


FIGURE 5.7: Contours of the streamfunction $\psi(x, z)$ computed from the mean velocity field obtained by PTV. From bottom to top, the contours of $\psi(x, z)$ increase from 0.01 to 0.21 with a step of 0.04. Line colours indicate various step-height cases, following the same convention as the legend in figure 5.2.

to almost $0.8\delta_0$ downstream of the rough-to-smooth transition. For the remaining three cases, non-zero W seems to be limited within $\hat{x}/\delta_0 < 0.5$. It is also noted that the downwash and upwash are not confined within the IBL. As a matter of fact, these non-zero W regions close to the roughness step-change penetrate the entire IBL and extend to the top of the PTV FOV ($z/\delta_0 \approx 0.3$).

The streamwise acceleration of U is investigated by examining the acceleration parameter $K \equiv \frac{\nu}{U_\infty^2} \frac{\partial U}{\partial x}$ in figure 5.6. The gradient $\frac{\partial U}{\partial x}$ is computed from the mean velocity field using a least-squares approach. A positive K is observed in the IBL for all 4 cases, and its magnitude decreases from H0 to H3. K is nominally zero above the IBL, while a localised negative K appears at $\hat{x}/\delta_0 \lesssim 0.2$ and $z/\delta_0 \lesssim 0.1$ in H3. The deceleration here is likely to be related to the slowing down of the flow after it impinges on the upstanding smooth surface.

5.4.1 Mean velocity along streamlines

Assuming a two-dimensional flow field, the streamfunction can be computed from the mean velocity field following

$$\psi(x, z) = \int^x U(x_1, z) dx_1 - \int^z W(x, z_1) dz_1 + C. \quad (5.1)$$

Here, the constant C is determined such that $\psi(0, 0) = 0$. Streamlines can be visualised by contour lines of ψ , as shown in figure 5.7. In a canonical turbulent boundary layer developing on a homogeneous surface without interruption, the W component is positive and the U component decreases slowly in streamwise, leading to a streamline with a small upward displacement. In the case of a rough-to-smooth change, however, all streamlines show a downward displacement instead due to the streamwise acceleration of the recovering flow in the IBL. The magnitude of the streamline displacement decreases from H0 to H3.

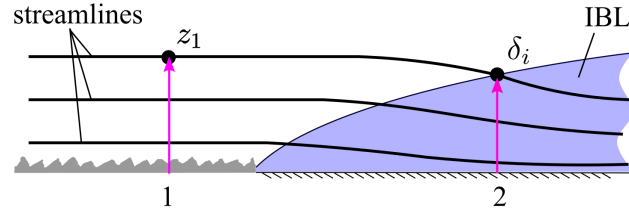


FIGURE 5.8: Sketch showing the streamline displacement over a rough-to-smooth transition. The flow is going from left to right in the figure. For a streamline that intersects the upper limit of the IBL at the downstream location 2, its wall-normal location in the upstream of the transition is denoted as z_1 .

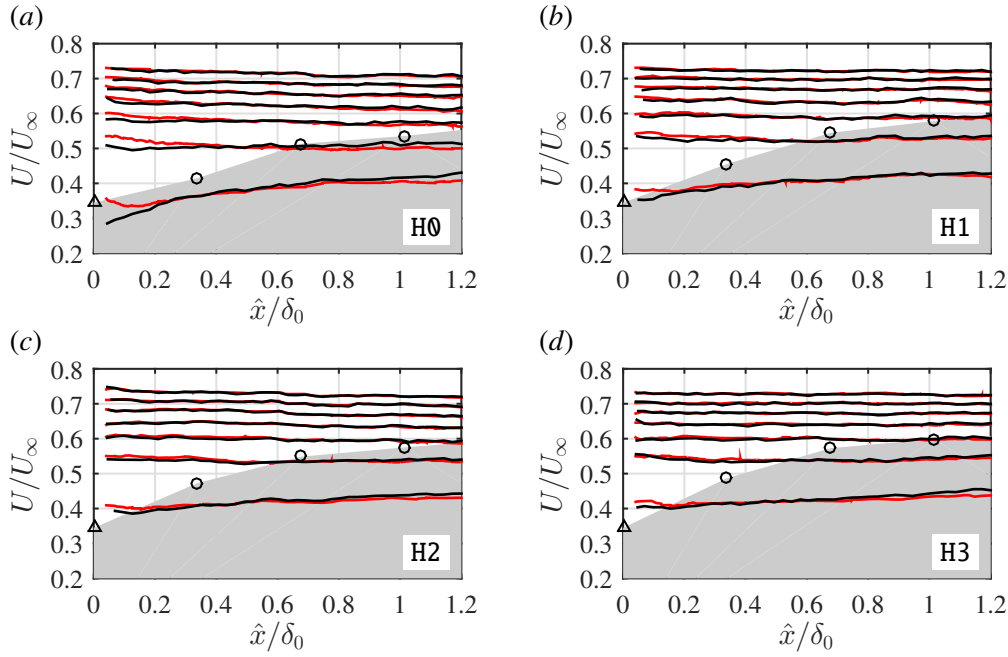


FIGURE 5.9: Normalised mean streamwise velocity along a streamline (red line), i.e. $U(z_s)/U_\infty$, and normalised mean streamwise velocity along a horizontal line located at the average height of the corresponding streamline (black line), i.e. $U(\bar{z}_s)/U_\infty$, where $(\bar{\cdot})$ denotes a spatial average across the entire field of view. The streamlines are contour lines of ψ , with a contour level increasing from 0.01 to 0.25 with a step of 0.04. The shaded area represents the region within the IBL, with the upper limit at each streamwise location estimated by $U(\delta_i)/U_\infty$ (shown by the circles). The upper limit of the shaded region at $\hat{x}/\delta_0 = 0$ is approximated by the mean streamwise velocity measured at the roughness crest. The four panels (a-d) correspond to cases H0-H3, respectively.

In contrast to Elliott's assumption that the streamwise velocity profile above the IBL remains unmodified compared to the equilibrium profile upstream, Panofsky and Townsend (1964) and Townsend (1965b) considered that the mean streamwise velocity is constant along a streamline above the IBL. Therefore, at a given streamwise location, $\hat{x}$, the matching condition of the mean velocity at the edge of the IBL is modified from $U(\hat{x}, \delta_i) = U_0(\delta_i)$ (used by Elliott) to $U(\hat{x}, \delta_i) = U_0(z_1)$, where U and U_0 are the mean velocity profiles in the upstream and downstream of the roughness transition, respectively. The streamline passing through δ_i at $\hat{x}$ has a wall-normal location of z_1 in the upstream (see the sketch in figure 5.8). Neglecting the wall-normal velocity and considering the mass conservation at two cross sections of

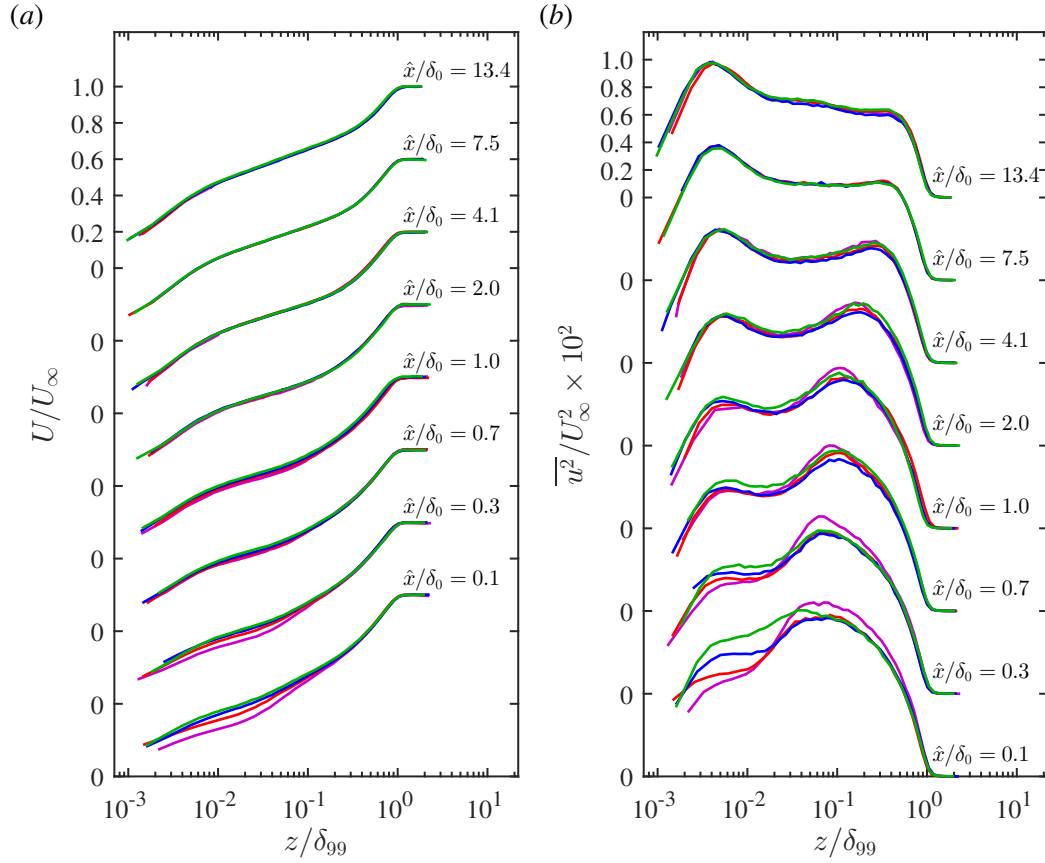


FIGURE 5.10: Outer-normalised profiles of (a) U/U_∞ and (b) $\overline{u^2}/U_\infty^2$. Note that the starting point of the vertical axis has been shifted for different streamwise locations for clearer presentation. Line colours indicate various step-height cases, following the same convention as the legend in figure 5.2.

the streamtube bounded by the wall at the streamline that passes through $(\hat{x}, \delta_i)$, z_1 can be evaluated from

$$\int_0^{z_1} U_1(z') dz' = \int_0^{\delta_i} U_2(z') dz'. \quad (5.2)$$

In a rough-to-smooth transition, a slightly higher $U(\hat{x}, \delta_i)$ is expected following Panofsky and Townsend's approach as a result of the downward streamline displacement.

The adequacy of these two assumptions are examined with the aid of the PTV velocity field. The mean streamwise velocity along a streamline is shown by red in figure 5.9, while the velocity at a constant wall-normal location (the average wall-normal height of the corresponding streamline) is shown by black. With an increasing fetch downstream of the roughness transition, the mean velocity along a streamline (red curves) decreases above the IBL and increases within the IBL. Similar observation has been made by Mulhearn (1978) using a hotwire dataset. Above the IBL, if Panofsky and Townsend's assumption is correct, then the red lines should be horizontal; if Elliott's assumption is correct, then the black lines should be horizontal. As shown in the figure, the red and black lines are in close agreement above the IBL beyond $\hat{x}/\delta_0 > 0.4$, confirming the conclusion made by Panofsky and Townsend (1964) from an analytical calculation of the streamline displacement that the difference between the two is negligible when δ_i is large. For case H0, the mean streamwise velocity computed

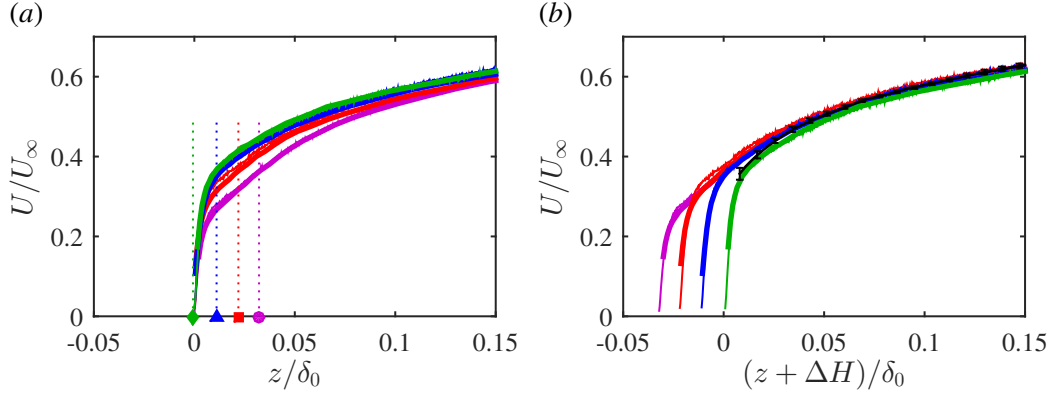


FIGURE 5.11: Streamwise mean velocity profile U/U_∞ at $\hat{x}/\delta_0 = 0.1$ plotted against the wall-normal position z/δ_0 on a linear axis. The datum is fixed at the roughness crest in (b), while it is located on the smooth wall in (a) and the location of the roughness crest is marked by the dotted line of the corresponding colour and symbol. Line colours indicate various step-height cases, following the same convention as the legend in figure 5.2. The thick coloured lines are taken from the hotwire dataset and the thin coloured lines are interpolated to the matched streamwise location in the PTV dataset. The black line in (b) is an averaged hotwire profile obtained on the upstream rough surface, with the error bars showing the scatter in three repeating measurements at slightly different locations.

along a streamline (red) shows a more prominent decrease close to the step change compared to its constant-wall-normal-position counterpart (black), indicating that Elliott's assumption is a better approximation of the flow within the range of roughness transition (i.e. rough-to-smooth) and step heights investigated here. Elliott's matching condition will be adhered to later on (§9) when modelling the flow.

5.5 Recovery of the streamwise velocity statistics

The hotwire profiles of U/U_∞ and $\overline{u^2}/U_\infty^2$ normalised by δ_{99} and U_∞ for all 4 cases and downstream locations ranging from $\hat{x}/\delta_0 = 0.1$ to $\hat{x}/\delta_0 = 13.4$ are summarised in figure 5.10. U/U_∞ increases from H0 to H3, confirming the observation in figure 5.4. The disparity between cases is the largest at $\hat{x}/\delta_0 = 0.1$, and it recovers quickly, in a fetch of approximately δ_0 . Curiously, at $\hat{x}/\delta_0 = 0.1$, the difference between cases extends above $z/\delta_{99} \approx 0.1$, which is approximately 3 times of the maximum step height change ($\Delta H|_{H3} - \Delta H|_{H0}$) and 30 times of the IBL height at this streamwise location. To further investigate this observation, we show the mean streamwise velocity profiles at $\hat{x}/\delta_0 = 0.1$ on a linear axis for all cases in figure 5.11. PTV data interpolated to the matched streamwise location is also included in the figure to complement the near-wall information, and a good agreement between the two datasets is observed. Figure 5.11(a) is essentially the first downstream location in figure 5.10(a) shown on a linear axis, while in figure 5.11(b) a common datum fixed on the roughness crest rather than the smooth wall is used instead. Note that the velocity profiles against $z + \Delta d$ would be almost identical as figure 5.11(b), except for a small constant shift (ϵ) in the horizontal axis. Comparing figure 5.11(b) with (a), it is apparent that a datum relatively fixed to the rough wall results in a better collapse between cases, with all lines merging at approximately $0.03\delta_0$

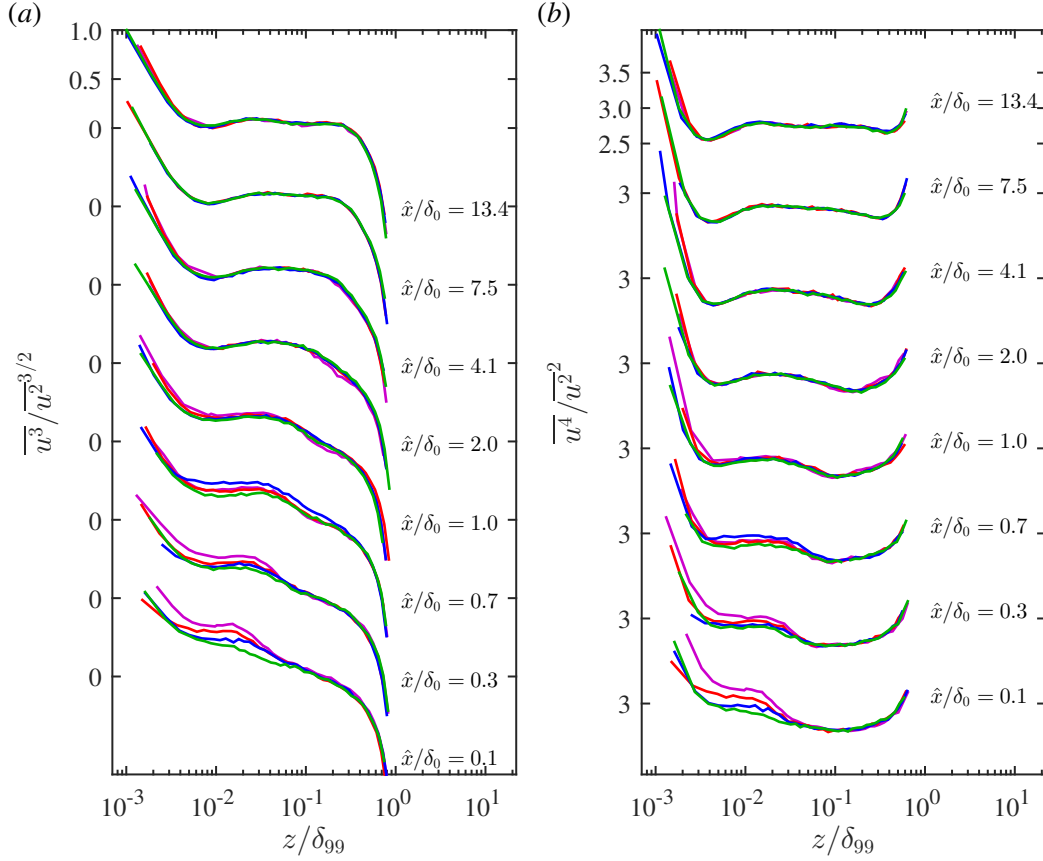


FIGURE 5.12: Outer-normalised profiles of (a) skewness and (b) kurtosis of the streamwise velocity fluctuation. Note that for clearer presentation, the starting point of the vertical axis has been shifted for different streamwise locations for clearer presentation and both skewness and kurtosis data are truncated beyond $z/\delta_{99} = 0.65$. Line colours indicate various step-height cases, following the same convention as the legend in figure 5.2.

compared to $\gtrsim 0.1\delta_0$ in (a). As discussed in the previous section, the streamwise acceleration of U is almost negligible above the IBL, thus it is fair to assume that U still remains very similar to the velocity profile on the upstream rough wall. Therefore, using a common datum fixed on the roughness crest gives a better collapse of the mean velocity profiles immediately downstream of the rough-to-smooth transition. On a side note, figure 5.11(a) shows that the mean velocity (therefore the gradient) increases from case H0 to H3 in the vicinity of the wall, confirming the increasing trend of C_f with an increasing ΔH close to the roughness transition in figure 5.2.

The magnitude of the ‘inner-peak’ in $\overline{u^2}$ under the outer scaling also increases from H0 to H3, while the two cases with a more aggressive step change (greater $|\Delta d|$), H0 and H3, show a higher outer peak which persists until approximately $2\delta_0$ downstream of the step change. The higher order statistics, skewness $\overline{u^3}/\overline{u^2}^{3/2}$ and kurtosis $\overline{u^4}/\overline{u^2}^2$ from all streamwise locations are summarised in figure 5.12. Both skewness and kurtosis decrease from H0 to H3, suggesting a more skewed (a longer tail on the positive fluctuations) distribution with a sharper peak in case H0. As the skewness bears a strong resemblance with the single-point amplitude modulation coefficient (Mathis et al., 2011b), a skewness closer to 1 indicates a stronger modulation effect (a higher correlation between the sign of the local large-scale motions and

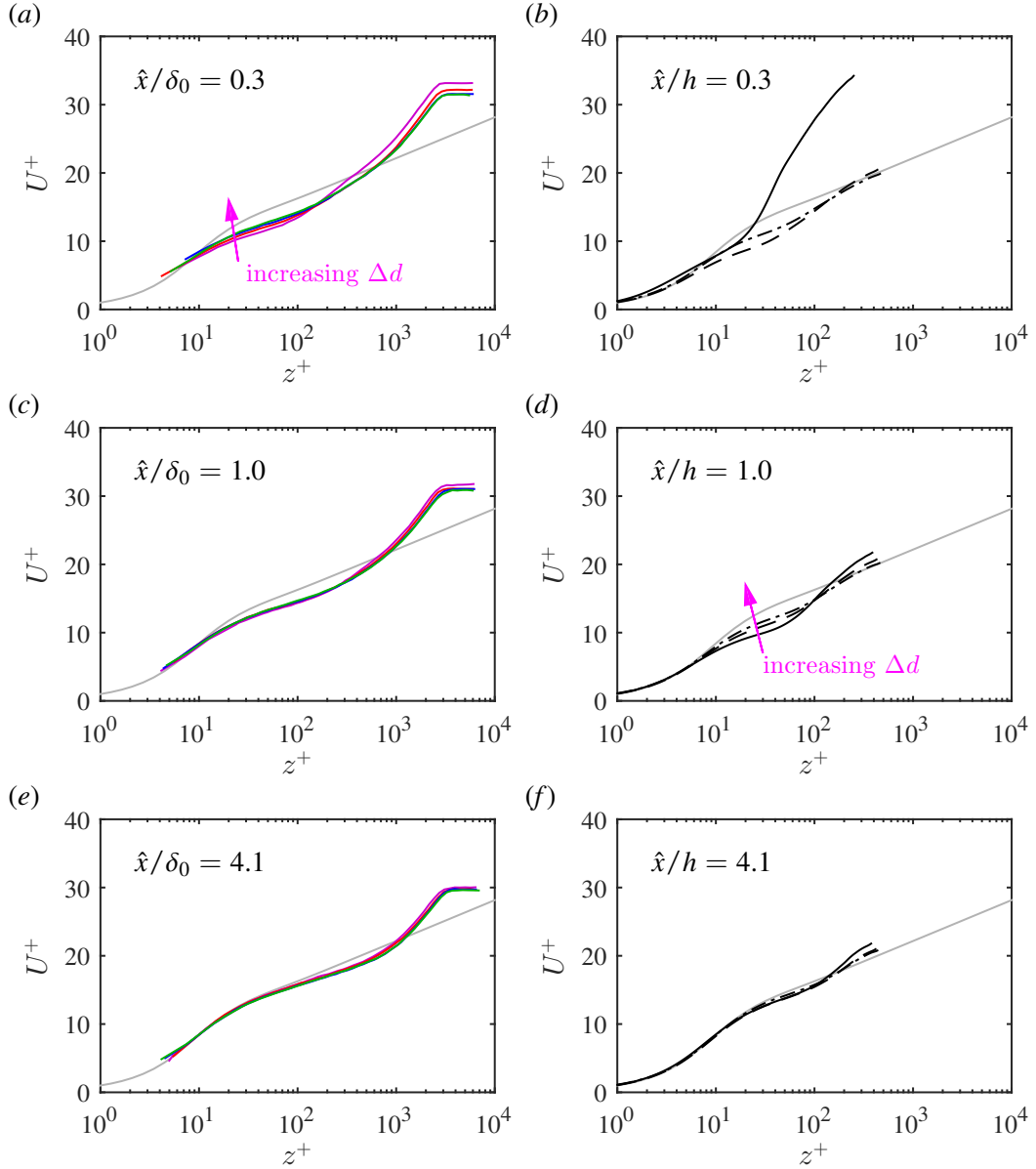


FIGURE 5.13: Inner-normalised mean streamwise velocity profiles. The left column shows the turbulent boundary layer experimental data, while the right column shows the open-channel DNS data. Colours green, blue and red represent D0, D1 and D2. The light grey line in each panel represents a Musker profile (§9.A) of a canonical smooth-wall turbulent boundary layer as reference. From top to bottom, the outer-normalised fetch $\hat{x}/\delta_0$, $\hat{x}/h = 0.3, 1.0$ and 4.1 .

the amplitude of the small-scale fluctuations) in $H0$. Similar to U and $\overline{u^2}$ profiles, the difference in higher order statistics between the 4 cases is also the largest in the first measuring location after the step change, and it gradually decreases downstream. The effect of different virtual origins is observed to diminish within $1\delta_0$ downstream of the step change. Comparing figure 5.10 and 5.12, there is no apparent trend that a longer fetch is required in a higher order moment for the effect of virtual origin to diminish. This observation is similar to that in the study of Marusic et al. (2015), where the effect of different tripping conditions of a turbulent boundary layer is also found to vanish within almost the same fetch for various orders of u

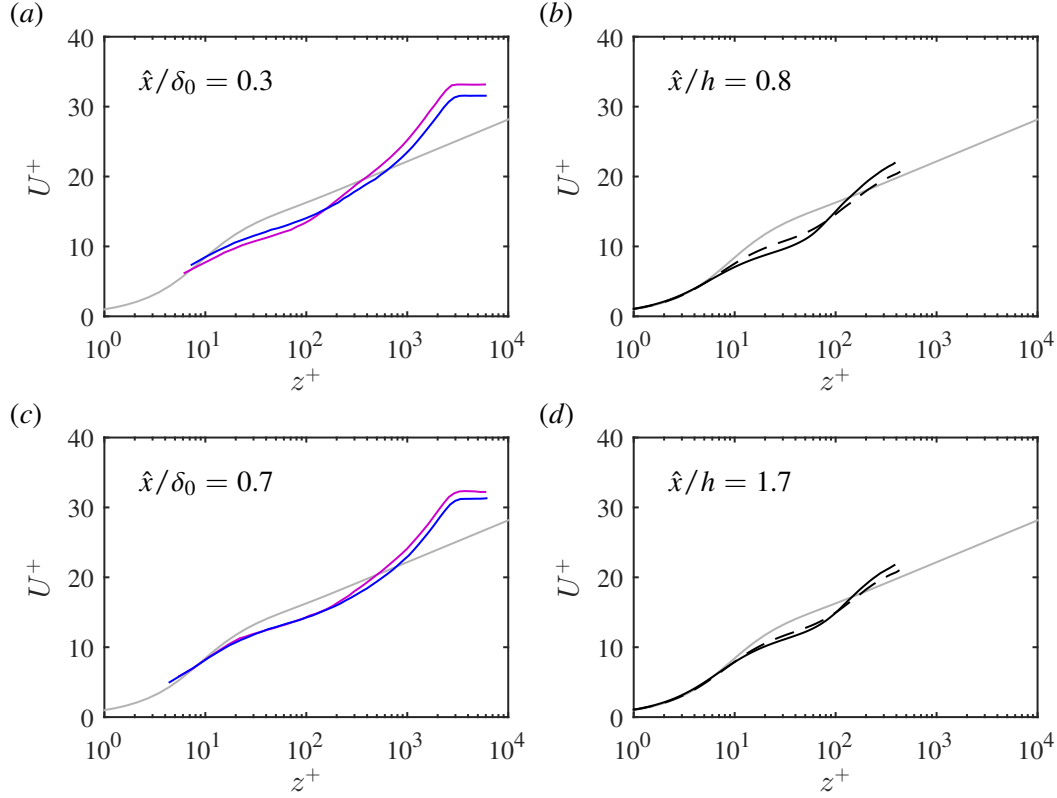


FIGURE 5.14: Inner-normalised mean streamwise velocity profiles at matched streamwise fetch. The top and bottom rows are at $\hat{x}/|\Delta d| = 15$ and 30, respectively.

moments.

5.6 Comparison of experimental and DNS results

Here we compare the experimental results with a DNS study of a similar problem. These two datasets have different external geometries (turbulent boundary layer versus open channel) and different k_s/δ ratios, but even so, it is useful to find out if there is any similar trend between the two and to explore the possibility to characterise the step effect.

The mean velocity profiles normalised by the local $U_{\tau 2}$ at a series of streamwise locations are shown in figure 5.13. The recovery of the mean velocity profile from various step heights in the experimental and numerical datasets are qualitatively similar, despite the difference in the fetch required for a full recovery to be achieved. At $0.3\delta_0$ downstream of the step change, the inner-scaled mean velocity increases with an increasing Δd in the experimental data, as shown in figure 5.13(a). Such a dependence on Δd quickly diminishes downstream, where profiles of all 4 cases collapse in the two streamwise locations of $\hat{x}/\delta_0 = 1.0$ and 4.1 shown subsequently in figures 5.13(c,e). A similar Δd trend is observed in the DNS data in all 3 streamwise locations shown, although in figure 5.13(b) case D0 overshoots the other two curves due to the extremely low $U_{\tau 2}$ close to the reattachment point. Comparing the profiles from both datasets at the same $\hat{x}/\delta_0$ or $\hat{x}/h$, it is revealed that the recovery is much slower in the DNS dataset. We may also consider the case with matched virtual origin locations

(i.e. $\Delta d = 0$) as the baseline case and study the fetch required for the difference between the baseline case and a case with a non-zero Δd to diminish. In this way, $|\Delta d|$ can be used as the length scale to normalise the fetch in each dataset. This method bears some resemblance to the normalisation by the step height in flows past a forward/backward-facing step. Here the baseline cases are chosen as H2 and D1, and the cases with non-zero Δd to compare to are chosen as H0 and D0, resulting in $|\Delta d|/\delta_0 = 0.023$ and $|\Delta d|/h = 0.056$, respectively. The mean velocity profiles are shown in figure 5.14, where $\hat{x}/|\Delta d| = 15$ (top row) and 30 (bottom row) for both datasets. The difference between the mean velocity profiles is more comparable between the two datasets at matched $\hat{x}/|\Delta d|$, although the numerical dataset still shows a slower recovery compared to the experiment: at $\hat{x}/|\Delta d| = 30$ for instance, the difference between profiles is not distinguishable in the experimental dataset, while D0 is still observed to have a higher U^+ than the baseline case below the IBL in the numerical dataset. The slower recovery observed in the numerical dataset can possibly be attributed to the difference in the Reynolds number or external/internal geometries.

5.7 Summary and conclusion

A systematic study on the step height effect in a rough-to-smooth transition is presented in this chapter. A range of step heights is covered, with the downstream smooth wall starting from below the roughness canopy to slightly above the roughness crest. The main conclusions drawn from this study are listed below:

1. The difference between cases with various step heights is the largest close to the roughness transition, and it gradually diminishes in a fetch of $\hat{x}/\delta_0 = 2 - 4$. This applies to the skin-friction coefficient C_f , IBL height δ_i , and also various orders of the fluctuating streamwise velocity moments.
2. In the near field, both C_f and δ_i are observed to decrease with a larger step down. This can be linked to the observation that at a certain wall height (relative to the smooth wall), the local mean velocity decreases with an increasing downward step.
3. In the near field, the step is observed to affect the streamwise acceleration of the streamwise mean velocity in the near-wall region. It has also been shown to modify the behaviour of the local wall-normal velocity component: a downwash is observed to accompany a step-down of the virtual origin (negative Δd), while an upwash is observed to accompany an step-up of the virtual origin (positive Δd).
4. With the aid of the mean velocity field with both streamwise and wall-normal components, two main matching conditions of the velocity at δ_i used in literature are tested here. The first approach is to assume a ‘frozen’ upstream profile above the IBL, while the second one is to assume that the mean streamwise velocity remains constant along a streamline above the IBL. The first approach is shown to be more realistic and it is also favoured because of its simplicity, while both approaches produce similar results.

5. A comparison of the experimental results in this study with a DNS dataset shows similar flow recovery trends, suggesting that the step height in virtual origin Δd may be a better length scale than δ_0 for normalising the fetch for the comparison between different flow conditions.

Chapter 6

Effects of $Re_{\tau 0}$ and k_{s0}^+ on the flow recovery

This chapter presents the results from the measurement campaign in the HRNBLWT. Dimensional analysis shows that the upstream condition of a turbulent boundary layer over a rough-to-smooth change in the surface condition can be fully described by two nondimensional numbers: $Re_{\tau 0}$ and k_{s0}^+ . Therefore, to investigate the dependence of the flow on each parameter separately, we acquire a unique and comprehensive dataset which takes two cuts in the parameter space: one at a fixed k_{s0}^+ and a varying $Re_{\tau 0}$, and the other at a fixed $Re_{\tau 0}$ and a varying k_{s0}^+ . This is achieved using various combinations of the sandpaper length and the freestream velocity. We first confirm that the measured upstream flow conditions behave as intended, then we examine whether there is a trend with $Re_{\tau 0}$ or k_{s0}^+ in various flow statistics including both time-averaged and fluctuating quantities. We found that for all cases, the skin-friction coefficient, C_f , recovers within approximately $20\delta_0$ downstream of the rough-to-smooth change, while a complete recovery of the 1D energy spectrum occurs at a fetch between $40 - 80\delta_0$. We also define C_{fe} , which is the skin-friction coefficient expected at the same Reynolds number using the smooth-wall relationship, and show that when normalised by C_{fe} , the recovery trend of C_f can be described by a single curve for all cases investigated.

^{||}This chapter is partially adapted from the results of M. LI; C. M. DE SILVA; R. BAIDYA; D. CHUNG; I. MARUSIC; AND N. HUTCHINS 2019 Experimental study of a turbulent boundary layer with a rough-to-smooth change in surface conditions at high Reynolds numbers. *Proceedings of the 11th Int. Symp. Turb. Shear Flow Phenom.*

6.1 Chapter outline

In this chapter, we present and analyse the dataset obtained from the measurement campaign in HRNBLWT. The structure of this chapter is arranged as follows: we first present the details of the current dataset and legends in §6.2, and then we examine the initial flow conditions on the upstream rough surface in §6.3, to ensure that a nominally constant k_{s0}^+ and the desired range of $Re_{\tau 0}$ are achieved in *Group I*, and a nominally constant $Re_{\tau 0}$ and the desired range of k_{s0}^+ are achieved in *Group II*. We present the recovery of the skin-friction coefficient, C_f , in §6.4, and the downstream development of the IBL height, δ_i , in §6.5. The effect of k_{s0}^+ and $Re_{\tau 0}$ on both C_f and δ_i are studied in these sections, and comparison with the prediction of Elliott’s model is made when possible. Next, we study the streamwise evolution of the mean velocity and turbulence intensity profiles in §6.6 and §6.7 with the focus on case Re07ks16 (which has the longest fetch on the downstream smooth wall), while results from other datasets are qualitatively similar. Finally, we take a further look into the turbulence energy of the recovering flow and examine the pre-multiplied energy spectra in §6.9.

6.2 Details of the experimental campaign

A detailed description of the facility and the surface roughness is provided in §3.1. In this section, we will introduce the system of legends that will be adhered to in this and the following chapters to denote the experimental cases. In order to study the effect of $Re_{\tau 0}$ and k_{s0}^+ separately, we designed two groups of experimental cases, the key parameters of which are summarised in table 6.1. *Group I* consists of cases Re07ks16, Re10ks16, Re14ks16 and Re21ks16, in which a nominally constant $k_{s0}^+ \approx 160$ is maintained while $Re_{\tau 0}$ increases from 7100 to 21000. *Group II* consists of cases Re14ks11, Re14ks16 and Re14ks22, in which a nominally constant $Re_{\tau 0} \approx 14000$ is maintained while k_{s0}^+ increases from 111 to 228. A unique symbol is assigned to each case, as shown in table 6.1. We use different shades of pink for cases in *Group I* and different shades of blue for cases in *Group II*, with a darker shade indicating either a higher $Re_{\tau 0}$ or k_{s0}^+ . Note that case Re14ks16 is at the ‘crossing

	Case	Sym.	$Re_{\tau 0}$	k_{s0}^+	x_0 (m)	U_∞ (m s ⁻¹)	δ_0 (m)
<i>Group I</i>	Re04ks13	□	4100	134	3.7	15.0	0.09
	Re07ks16	▲	7100	158	4.5	21.5	0.11
	Re10ks16	▶	10 400	165	7.2	22.5	0.15
	Re21ks16	▼	21 000	157	17.1	24.3	0.32
	Re14ks16	◀ (◀)	14 000	157	11.1	23.3	0.22
<i>Group II</i>	Re14ks11	●	14 700	111	17.1	17.0	0.32
	Re14ks22	◆	14 500	228	7.2	31.0	0.15

TABLE 6.1: Summary of the experimental cases. The friction velocity $U_{\tau 0}$ employed in calculating $Re_{\tau 0}$ and k_{s0}^+ is obtained over the rough fetch just upstream of the rough-to-smooth transition. Note that case Re14ks16 is shared between *Group I* and *Group II*, therefore its symbol can take either pink or blue colour in the corresponding group.

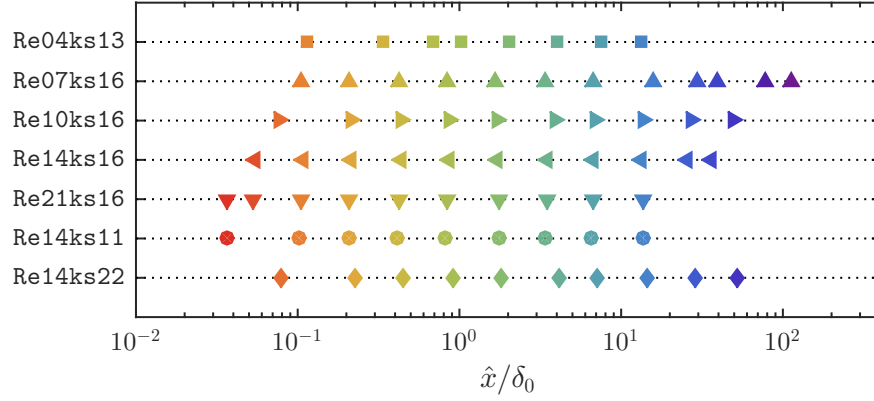


FIGURE 6.1: Downstream fetches where hotwire measurements are performed for all cases. The shape of each symbol indicates the case, and it follows table 3.2, while the colour reflects the magnitude of $\hat{x}/\delta_0$.

point’ in the parameter space so it is shared between the two groups, therefore its symbol can take either pink or blue when plotted as a part of the corresponding group. Case Re04ks13 is originally H1 in the study of the step height effect as detailed in §5. Its roughness Reynolds number $k_{s0}^+ = 134$ is slightly lower than the nominal value in *Group I*, but it can serve as a good addition to the low-Reynolds-number end of the parameter space. It is included here only for comparison when applicable.

As detailed in §3.2.2, the dataset is composed of hotwire profiles at a series of logarithmically spaced streamwise locations ranging from $0.03\delta_0$ to $120\delta_0$, with the wall-shear stress at each measurement station measured directly using OFI technique. The streamwise locations where the hotwire profiles are obtained are summarised in figure 6.1. The colour of each symbol is determined by its corresponding $\hat{x}/\delta_0$. This colour scheme will be used in the study of the streamwise evolution of the flow where a series of profiles at various streamwise locations are shown in one figure.

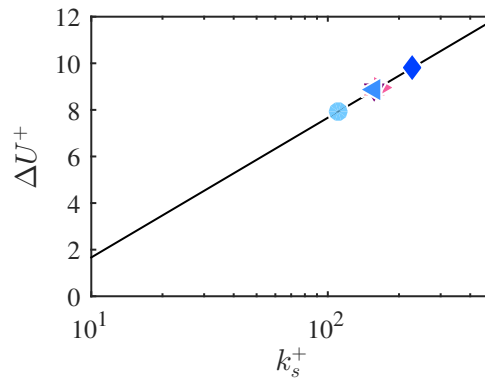


FIGURE 6.2: Hama function, ΔU^+ , as a function of the inner-scaled equivalent sand grain roughness k_s^+ for all cases. All symbols are defined in table 3.2, and the solid line is equation (6.1), Nikuradse’s fully-rough relationship.

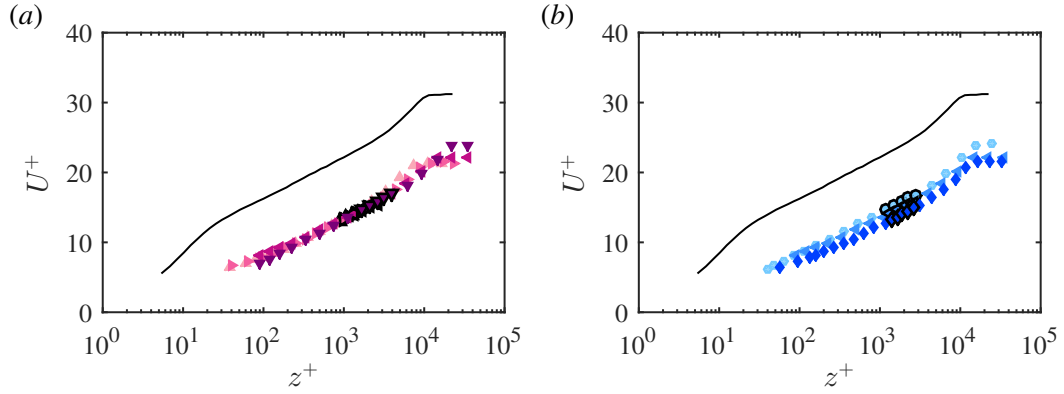


FIGURE 6.3: Inner-normalised mean streamwise velocity profiles at $\hat{x} = -0.1$ m from (a) *Group I* and (b) *Group II*. The black line in both figures is a smooth-wall reference profile with $Re_{\tau} = 1.0 \times 10^4$ acquired in the same facility using hotwire anemometry (Marusic et al., 2015). Symbols with a black outline represent the extent of the inertial range. These data points are employed in the fit to obtain $U_{\tau 0}$. Only every second data point is shown for clarity except in the inertial range.

6.3 Initial flow conditions on the rough surface

The streamwise evolution of the turbulent boundary layer starts from the initial flow condition on the rough surface. Once a rough-wall profile is specified, the flow recovery downstream on a smooth surface can be fully determined. A wall-normal hotwire profile is obtained at $\hat{x} = -0.1$ m in each case to document the initial condition. For the hotwire measurements on the rough surface, $z = 0$ is located at the roughness crest following the method of measuring the initial offset as detailed by Squire et al. (2016b). The virtual origin of the roughness is assumed to be at the midpoint between the crest and trough, therefore a wall positioning correction of $\varepsilon = -k_p/2 = -0.6$ mm is used.

Nikuradse's (1950) equivalent sand grain roughness k_s of the surface is obtained from the highest Reynolds number case Re21ks16 following the procedures below: firstly, U_{τ} is estimated by enforcing an outer-layer similarity (Townsend, 1976) in the mean velocity deficit profile for $z/\delta_{99} > 0.3$. The Hama function, ΔU^+ , which is the vertical shift between the rough-wall profile and the logarithmic law $U^+ = \frac{1}{\kappa} \ln(z^+) + B$, is then determined by minimising the least-squares error in the inertial range with the upper and lower bounds given in Mehdi et al. (2013) and Squire et al. (2016b). The constants in the logarithmic law are chosen as $\kappa = 0.384$ and $B = 4.17$ (Nagib et al., 2007). Finally, k_s^+ is computed from ΔU^+ assuming Nikuradse's fully-rough relationship

$$\Delta U^+ = \frac{1}{\kappa} \ln(k_s^+) + B - A'_{FR}, \quad (6.1)$$

where $A'_{FR} = 8.5$ (Nikuradse, 1950). The equivalent sand grain roughness is found to be $k_s \approx 2.43$ mm. This k_s value is assumed for all other cases using the same sandpaper, and U_{τ} is obtained by enforcing the fully-rough relationship (6.1) and minimising the least-squares error between the inner-normalised mean velocity profile and the fully-rough logarithmic law in the inertial range. The applicability of (6.1) can be justified as all the rough-wall

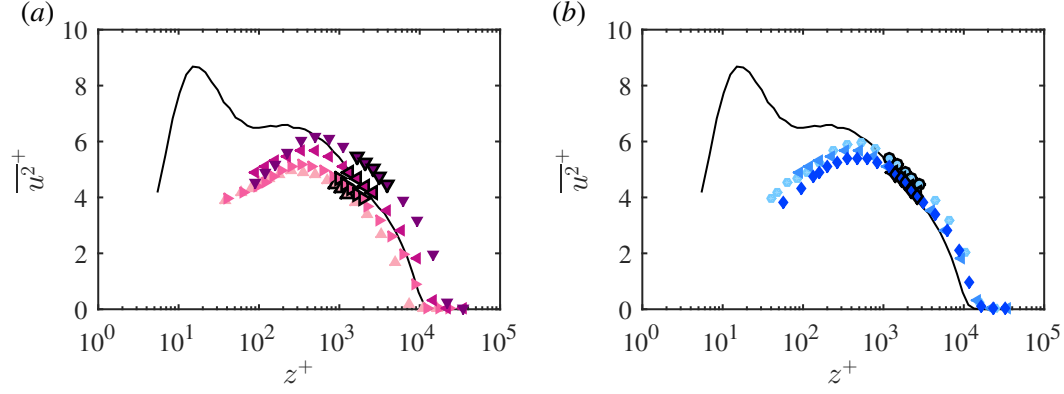


FIGURE 6.4: Inner-normalised streamwise turbulence intensity profiles at $\hat{x} = -0.1$ m from (a) *Group I* and (b) *Group II*. Legends are the same as in figure 6.3.

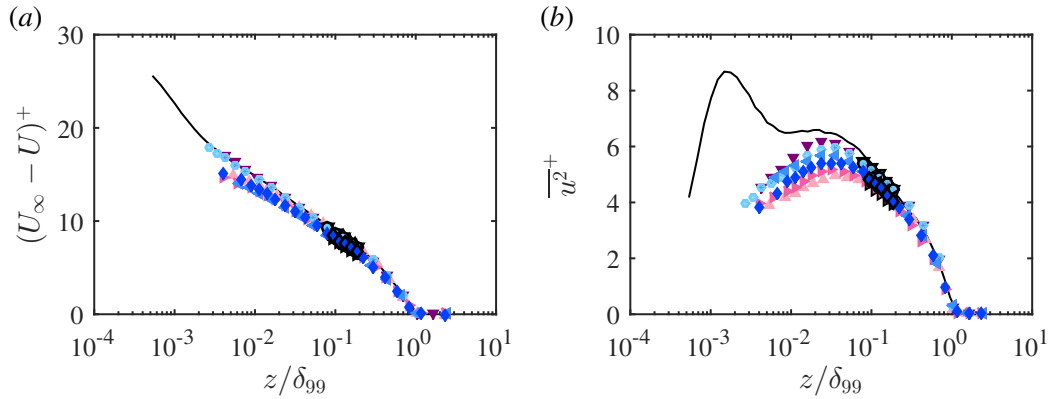


FIGURE 6.5: (a) Mean streamwise velocity deficit and (b) streamwise turbulence intensity versus outer-normalised wall-normal distance for both *Group I* and *Group II*. Legends are the same as in figure 6.3.

measurements are planned in the fully-rough regime with $k_s^+ > 100$. Note that this method to determine U_τ from the measured mean velocity profile is similar to the approach described by Squire et al. (2016b). It is essentially a more informed modified Clauser chart method with k_s (therefore a relationship between ΔU^+ and U_τ) prescribed and it does not invoke any assumption of the outer-layer similarity. ΔU^+ and k_s^+ for all cases are shown in figure 6.2. In *Group II* where the effect of k_s^+ is studied, k_s^+ varies from 111 to 228, corresponding to a range of ΔU^+ from 7.9 to 9.8.

The inner-normalised mean streamwise velocity and turbulence intensity profiles of *Group I* and *Group II* are shown in figures 6.3 and 6.4. As expected, a good collapse of the data in the logarithmic region in figure 6.3(a) indicates that ΔU^+ (therefore k_s^+) in these cases are closely matched, whereas the vertical shifts in the velocity profile in figure 6.3(b) correspond to the k_s^+ trend as planned. Similarly the Reynolds number trend can be observed in figure 6.3(a) and figure 6.4(a). Townsend's outer-layer similarity hypothesis (Townsend, 1976) appears to be satisfied as shown by the outer-normalised profiles for both groups in figure 6.5. Overall, these results demonstrate that the rough-wall profiles in the upstream of the roughness transition exhibit similar behaviours as a turbulent boundary layer developed on a homogeneous rough surface in a quasi-equilibrium state.

6.4 Skin-friction coefficient

Figure 6.6 shows the evolution of C_f following the rough-to-smooth transition. The skin-friction coefficient $C_f \equiv \tau_w / (\frac{1}{2}\rho U_\infty^2)$ over the smooth surface is obtained from the OFI measurements (see §3.5). For the reference profile on the rough wall, C_f is calculated using the method detailed in §6.3. As shown in figure 6.6(a,b), C_f undershoots the expected smooth-wall value (shown by the solid lines) immediately downstream of the roughness transition in all cases for both *Group I* and *Group II*. The expected skin-friction coefficient, C_{fe} , is defined as the skin-friction coefficient that a smooth-wall turbulent boundary layer at the same Reynolds number (based on the momentum thickness) would have. C_{fe} is estimated using an empirical relation obtained from drag balance measurements of a smooth-wall turbulent boundary layer in the same wind tunnel facility (Baars et al., 2016b):

$$C_{fe} = 2 (\ln(Re_\theta) / 0.38 + 3.7)^{-2}, \quad (6.2)$$

where $Re_\theta \equiv U_\infty \theta / \nu$, and θ is the momentum thickness computed locally by integrating the measured mean velocity profile. If the flow has fully recovered to the smooth-wall condition, then C_f should be equal to C_{fe} . Therefore, it can serve as an indication of the flow recovery. Note that case Re04ks13 is included in figure 6.6(a) for comparison despite a slight difference in k_{s0}^+ . Overall, C_f for both rough and smooth surfaces decreases with Re_{τ_0} . Such a Reynolds number dependence is expected and is similar to that observed for a turbulent boundary layer on a homogeneous surface (e.g. Nagib et al., 2007). For *Group II* measurements on the other hand, C_f is observed to increase with k_{s0}^+ on the rough surface only (as shown by the solid black symbols in figure 6.6b), while a collapse appears in both measured and expected smooth-wall value.

Figure 6.6(c,d) show the recovering C_f normalised by C_{fe} . C_f is approximately 60%—80% of C_{fe} immediately downstream of the rough-to-smooth transition for all cases, followed by a quick recovery within $20\delta_0$. The data points overshoot $C_f/C_{fe} = 1$ (the black horizontal line) slightly and then reach a plateau at $C_f/C_{fe} = 1.03$. This 3% difference is possibly related to the uncertainty in the data and the empirical relationship (6.2) employed. Regardless, there seems to be little difference between cases in both *Group I* and *Group II* in terms of the C_f recovery behaviour when scaled by δ_0 and C_{fe} in the far field. After C_f reaches its maximum at $\hat{x}/\delta_0 \approx 20$ in figure 6.6(a,b), it decreases gradually farther downstream as dictated by the increasing Reynolds number of the flow. When normalised by C_{fe} as shown in figure 6.6(c,d), to within the experimental uncertainty, beyond $\hat{x}/\delta_0 \approx 20$, C_f evolves as if the flow were fully in equilibrium with the smooth wall.

The scatter in C_f close to the roughness transition between cases appears to be more prominent in *Group II* compared to *Group I* as revealed by the magnified view in figure 6.6(e,f). As the step height between the roughness crest and the smooth surface $\Delta H = -1.8$ mm is the same for all cases in both groups, this trend in C_f might be dependent on the inner-normalised step height ΔH^+ , which increases with k_{s0}^+ in the current implementation. It has also been shown by OFI measurements using a spanwise line of silicone oil at a comparable Reynolds number that the local distribution of roughness elements can have an effect

on C_f in the range of $\hat{x}/\delta_0 < 0.4$ (see §4.3). This might also be responsible for the variation in C_f here as the local roughness geometry at the centreline of the tunnel floor (where the OFI measurements are performed) varies between cases where a different piece of sandpaper is used.

The normalised C_f/C_{fe} appears to follow an approximate log-linear relationship with the fetch, as shown in figure 6.7. An empirical description of this observation can be obtained by fitting a straight line through the data in the range of $\hat{x}/\delta_0 > 0.4$ and $C_f/C_{fe} < 1$, with

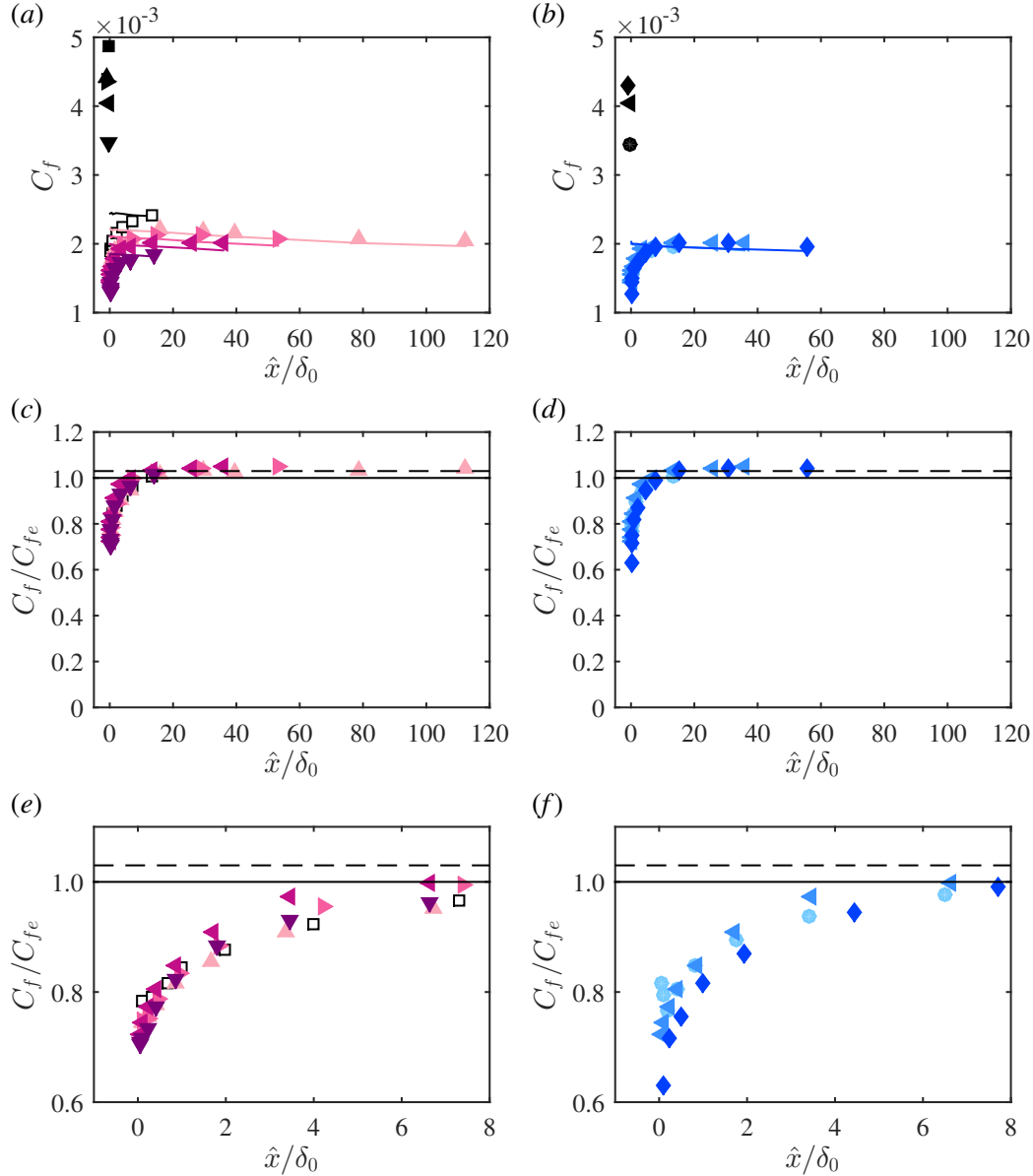


FIGURE 6.6: (a) and (b) are skin-friction coefficient C_f versus the fetch $\hat{x}$ over the downstream smooth surface scaled by δ_0 for *Group I* and *Group II*, respectively. The coloured symbols represent OFI measurements on the smooth wall, and the black symbols are obtained from the reference profile over the rough surface. The solid lines are C_{fe} at every streamwise location. (c) and (d) are C_f normalised by C_{fe} for *Group I* and *Group II*, respectively. The solid horizontal line is $C_f/C_{fe} = 1$, and the dashed line is $C_f/C_{fe} = 1.03$. (e) and (f) show a magnified view of the region immediately downstream of the transition.

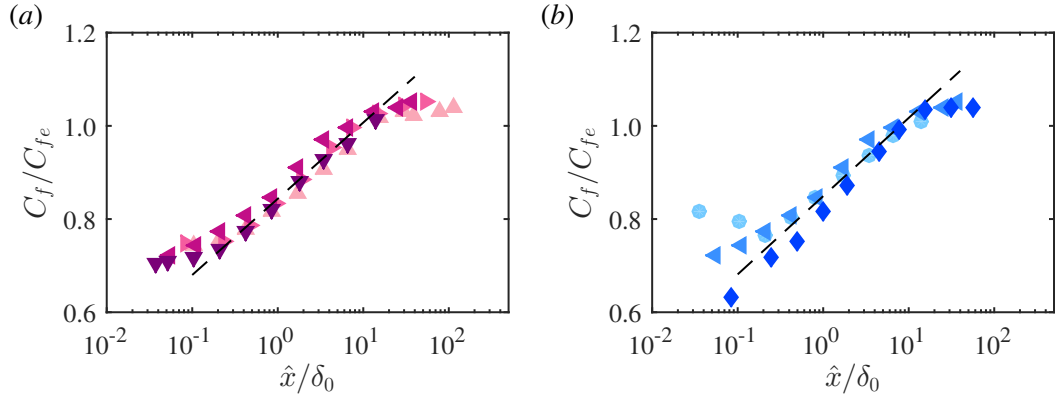


FIGURE 6.7: Normalised skin-friction coefficient C_f/C_{fe} plotted against $\hat{x}/\delta_0$ on a logarithmic scale for (a) *Group I* and (b) *Group II*. The black dashed line is a log-linear fit to all the data points in each figure in the range of $\hat{x}/\delta_0 > 0.4$ and $C_f/C_{fe} < 1$.

the lower limit in $\hat{x}/\delta_0$ selected to avoid the scatter as discussed above. The fitted line has an expression of

$$\frac{C_f}{C_{fe}} = a_1 \ln\left(\frac{\hat{x}}{\delta_0}\right) + a_2, \quad (6.3)$$

with coefficients $a_1 = 0.071$ and $a_2 = 0.84$ for data in *Group I*, and $a_1 = 0.073$, $a_2 = 0.85$ for data in *Group II* (although the collapse for *Group II* is notably worse). The proximity of the coefficients again demonstrates that for all cases examined, little $Re_{\tau 0}$ or k_{s0}^+ dependence is observed in the C_f recovery behaviour when normalised by C_{fe} and δ_0 .

6.5 Internal boundary layer

The extent of flow recovery can be quantified by the growth of the IBL. The IBL height δ_i at each streamwise location is calculated based on the difference between the $\overline{u^2}/U_\infty^2$ profile at the current location and the neighbouring upstream measurement location, i.e. δ_i is defined as the wall-normal location where $\partial(\overline{u^2}/U_\infty^2)/\partial x \rightarrow 0$. It is well-known that in a canonical boundary layer, $\overline{u^2}$ exhibits an outer-layer similarity only when normalised by U_τ^2 , and a dependence on Reynolds number presents if the velocity scale U_∞ is used instead. However, the adequacy of this approach can be justified considering that the largest change of Reynolds number between the neighbouring profiles used to compute δ_i is usually within 10%, and the difference in the Reynolds number is negligible close to the roughness transition. The majority of the measurements are concentrated in this region owing to the logarithmic streamwise spacing employed for these measurements. It has been shown that δ_i determined from the turbulence intensity profile is comparable with the results from the more conventional methods based on the mean velocity profiles (Pendergrass and Arya, 1984; Rouhi et al., 2019b; Li et al., 2019b). Here we favour the turbulence intensity approach as the distinction associated with the roughness change is more pronounced in $\overline{u^2}$ compared to U and less subject to small uncertainties in the measurement, resulting in a more robust estimation of δ_i .

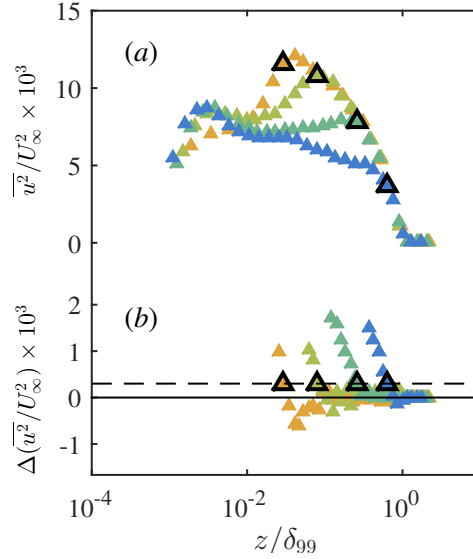


FIGURE 6.8: (a) Streamwise turbulence intensity of case Re07ks16 normalised by U_∞ , plotted against the outer-scaled wall-normal location z/δ_{99} . (b) The difference between two consecutive (in x) $\bar{u}^2/U_\infty^2$ profiles close to the edge of the IBL, plotted on the same abscissa. In both figures, shading of the symbols indicates the fetch with the colour scheme detailed in figure 6.1. The four profiles in this figure are obtained at $\hat{x}/\delta_0 = 0.2, 0.8, 3.4$ and 15.7 . Note that turbulence intensity profiles are shown only at every second streamwise location measured for clarity. The symbols with thick black outlines represent the location of the edge of the IBL. The solid horizontal line in (b) is at $\Delta(\bar{u}^2/U_\infty^2) = 0$, and the dashed line shows the threshold $\Delta(\bar{u}^2/U_\infty^2) = 3 \times 10^{-4}$.

Figure 6.8 illustrates the process of extracting δ_i from the outer-scaled turbulence intensity profiles. Figure 6.8(a) shows a good collapse in the outer layer with no appreciable Reynolds number trend, and the decrease in the turbulence intensity related to the internal layer growth is much more pronounced in comparison. Figure 6.8(b) shows the difference between two consecutive outer-scaled turbulence intensity profiles $\Delta(\bar{u}^2/U_\infty^2)$. We use this difference to extract the IBL height δ_i . In practice, a threshold of $\Delta(\bar{u}^2/U_\infty^2) = 3 \times 10^{-4}$ rather than 0 is selected to account for the noise in measurements and also the weak Reynolds number trend (this threshold is illustrated by the black dashed line in figure 6.8b).

Using the method described above, δ_i at various streamwise locations is calculated for all cases and presented in figure 6.9. Both δ_i and $\hat{x}$ are normalised by δ_0 , the boundary layer thickness at the rough-to-smooth transition. All data points collapse on to a straight line in logarithmic scale with no distinguishable Reynolds number trend. A power-law fit

$$\delta_i/\delta_{99} = A_0(\hat{x}/\delta_0)^{b_0} \quad (6.4)$$

results in coefficients of $A_0 = 0.097$ and $b_0 = 0.77$ for *Group I*, and $A_0 = 0.094$ and $b_0 = 0.75$ for *Group II*. This agrees closely with the observations of Bradley (1968) and Mulhearn (1978), where δ_i is defined as the ‘merging point’ in the mean velocity profile. The growth appears to be more aggressive than $\delta_i \propto \hat{x}^{0.43}$ as reported by Antonia and Luxton (1972). The major differences are: (1) δ_i is defined as the inflection point in the U versus $z^{1/2}$ plot and (2) the upstream surface is roughened by 2D square ribs instead

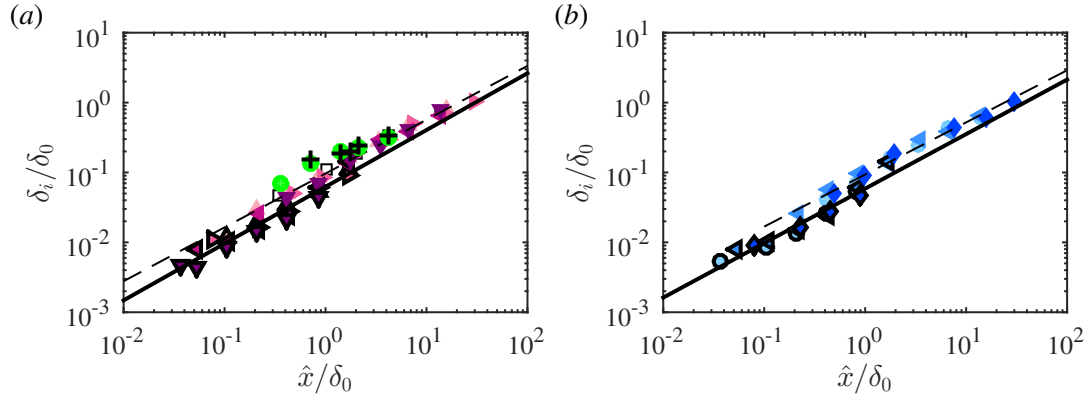


FIGURE 6.9: IBL thickness δ_i versus the fetch $\hat{x}$ over the downstream smooth surface, both normalised by δ_0 , the boundary layer thickness at the roughness transition (coloured symbols). The symbols with a thick black outline are δ_i calculated using the same definition as in Antonia and Luxton (1972). The thin dashed line is equation (6.4), a power-law fit through δ_i calculated from the variance profile using the present method, and the thick solid line is a power-law fit through δ_i calculated using the definition of Antonia and Luxton (1972). For comparison, δ_i reported in the original study of Antonia and Luxton (1972) is shown in (a), where the black pluses and green dots represent δ_i determined as the inflection point in the U versus $z^{1/2}$ profile and the conventional merging point in the neighbouring mean velocity profiles, respectively.

of sandpaper. In order to further clarify the underlying reason behind the disparity between these two results, we present δ_i estimated from the present dataset using the same definition as in Antonia and Luxton (1972). The resulting δ_i is shown in figure 6.9 by symbols with the corresponding shape and colour but with a thick black outline. Note that we only show the data points where $\delta_i/\delta_{99} < 0.15$ is satisfied to eliminate the contamination from the wake profile in this method. A power-law fit through the data points in each group is also shown in the figure by a thick solid line. The fitting coefficients are $A_0 = 0.062$ and $b_0 = 0.81$ for *Group I*, and $A_0 = 0.059$ and $b_0 = 0.78$ for *Group II*. δ_i calculated using Antonia and Luxton's definition appears to be lower than the results obtained by thresholding the variance profile. This is expected as the former definition returns the 'mid-point' of a dispersed or fluctuating IBL, while the current definition is more likely to pick up its upper limit. The exponent from the power-law fits of both *Group I* and *Group II* are approximately 0.8, which is very close to the result using our current definition. A careful examination of the work of Antonia and Luxton (1972) reveals that the δ_i data originally reported in their figure 5 are obtained using the 'merging point' definition in the mean velocity profile, while the U versus $z^{1/2}$ definition leads to almost identical results (see figure 6 in their article). The δ_i data extracted with these two definitions are reproduced here in figure 6.9(a) for comparison. The proximity of the two results confirms the conclusion in their preceding study (Antonia and Luxton, 1971b) that for a given boundary layer profile, the extraction method of δ_i usually makes little difference in the resulting IBL growth rate. Therefore, we may conclude that the difference in the power-law exponent is more likely to be related to the difference in the upstream roughness. A similar exponent, $\delta_i \propto \hat{x}^{0.41}$ has also been reported in a DNS study of a channel flow with the same type of 2D rod roughness upstream (Ismail et al., 2018b). Another possible factor responsible for the large disparity in the power-law exponents between studies

is that the exponent itself is a very sensitive quantity to assess. The streamwise extent where the fit is performed, the noise and scatter in the data points, and the step height effect at small fetches (see §5) can all affect the resulting power-law exponent. As shown in figure 6.9(a), the δ_i versus $\hat{x}$ trend in (Antonia and Luxton, 1972) is very similar to that in the present study. Note that in (Antonia and Luxton, 1972), the data point at $\hat{x}/\delta_0 \approx 0.4$ ($\hat{x} = 1\text{in}$) is omitted from the fit. A fit through all available data points will potentially bring the exponent closer to 0.8. More detailed discussion regarding the power-law exponent can be found in §2.2.3.

Based on the fitted power law, the downstream fetch required for IBL to reach the local δ_{99} is estimated to be $\hat{x}/\delta_0 = 26.5$. However, we would like to re-emphasise that here we adopt the definition of IBL as the region where the flow is modified by the new wall condition, and the flow inside IBL has been shown to be in a non-equilibrium state (see Antonia and Luxton, 1971b; Rouhi et al., 2019b). This implies that even when $\delta_i \rightarrow \delta_{99}$ (when the internal layer has grown to the full layer height), the boundary layer may still not be in equilibrium with the new wall condition (see for example figure 6.10b). A complete recovery of the flow to quasi-equilibrium is expected at a longer fetch.

6.6 Mean streamwise velocity

We utilise the hotwire data to further investigate the recovery of the mean flow statistics. We will primarily present the results of case Re07ks16 as it permits the greatest streamwise development downstream of the roughness transition (hence demonstrate the asymptotic trends more clearly), however, very similar trends are observed in other cases and are not shown here for brevity. Mean streamwise velocity profiles normalised by the local $U_{\tau 2}$ over the smooth wall measured using OFI are shown in figure 6.10(a). The black line in the figure is a Musker profile following the expression of Chauhan et al. (2009), representing a reference at a quasi-equilibrium state with the smooth surface. The measured mean velocity profile overshoots the reference in the wake region, and it decreases with the downstream fetch as dictated by the increasing C_f (see figure 6.6), approaching the equilibrium reference profile.

The locations of the IBL in each profile as determined in §6.5 are shown by the symbols with thick black outlines. The measured mean velocity profiles exhibit a good collapse with the equilibrium reference at $z^+ < 10$, however, there is a general lack of agreement in the buffer region and beyond, despite the fact that this region is well below the edge of the IBL. This confirms that the flow within the IBL is not in equilibrium with the local wall condition (Antonia and Luxton, 1972; Rouhi et al., 2019b; Li et al., 2019b). The deviation from the equilibrium Musker profile is better demonstrated by showing the velocity difference in figure 6.10(b). Note that only data points (and IBLs) below the wake region are shown in the figure, as the Musker profile does not capture the deviation from the logarithmic profile in the wake region. The difference between the measured profile and the equilibrium logarithmic law still persists even beyond $\hat{x}/\delta_0 > 10$ in the logarithmic region (see the light blue symbols in figure 6.10b). The undershoot of the mean velocity in the buffer and logarithmic regions is later modelled by blending a rough-wall and a smooth-wall profile in §8.

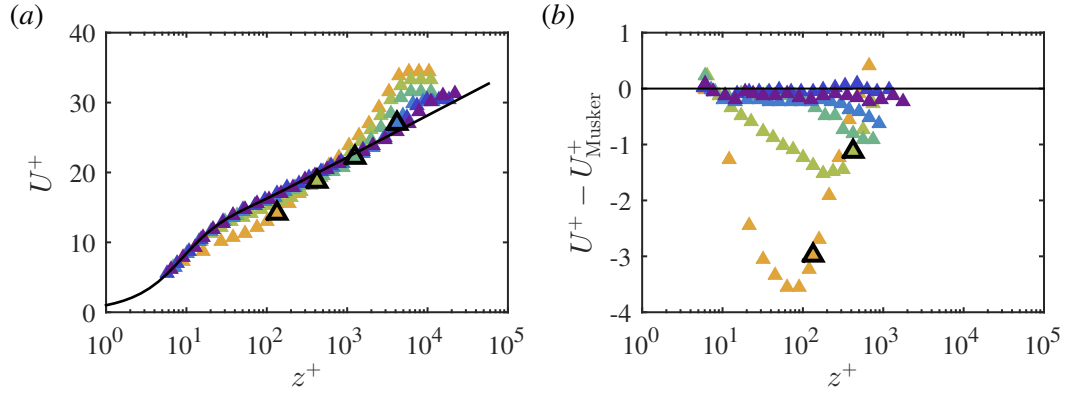


FIGURE 6.10: (a) Inner-normalised mean velocity profile for case Re07ks16. The solid line is a Musker profile (Chauhan et al., 2009). (b) The difference between the measured mean streamwise velocity U^+ and the Musker profile U_{Musker}^+ for the same case below the wake region ($z/\delta_{99} < 0.15$). In both figures, shading of the symbols changes from yellow to green and then to violet as the fetch increases. The colour scheme is summarised in detail in figure 6.1. The profiles shown in the figure are obtained at $\hat{x}/\delta_0 = 0.2, 0.8, 3.4, 15.7, 39.4$ and 112. The symbols with thick black outlines show the location of the IBL (detailed in §6.5) in each profile. Note that the mean velocity profiles are shown only at every second streamwise location measured for clarity.

The mean velocity deficit profiles normalised by the friction velocity U_{τ_2} obtained locally at the wall are shown in figure 6.11(a). A smooth-wall reference (Marusic et al., 2015) obtained at a Reynolds number matched to the last streamwise location is shown by the dashed magenta line. A collapse on the smooth-wall reference is observed in the outer layer only for the most downstream profiles (at $\hat{x}/\delta_0 = 39.4$ and 112, as shown by the dark blue and violet symbols) where the flow has completely recovered to an equilibrium state with the smooth-wall condition. The deficit profiles closer to the step change, however, are above the reference profile, and they gradually relax back to the reference as the fetch increases. The lack of collapse in the inner-normalised velocity deficit suggests an alternative velocity scale in the outer layer above the IBL. We assume the outer-layer velocity scale $U_{\tau, \text{out}} = U_{\tau_0}$ as a constant, and present the velocity deficit profiles normalised by this velocity scale in figure 6.11(b). Under this scaling, the velocity deficit profiles above the IBL collapse reasonably well on the rough-wall reference, confirming that the flow beyond the IBL is in equilibrium with the upstream characteristics. For the two most downstream locations (dark blue and violet symbols), δ_i has reached δ_{99} , therefore no collapse with the rough-wall reference is found under outer scaling.

The IBL thicknesses (shown by the symbols with thick black outlines), although determined from the streamwise turbulence intensity profiles, coincide with the location where the mean velocity deficit profile deviates from the rough-wall reference. This is also true for other cases in the current study and the figures are not shown here for brevity. This implies that for the range of Re_{τ_0} and k_{s0}^+ investigated, the IBL thickness determined from the mean streamwise velocity and the streamwise turbulence intensity profiles is similar. A similar observation has also been reported by Pendergrass and Arya (1984) in an experimental study of an internal layer formed over a rough-to-rougher transition, as well as in an open-channel DNS study with a rough-to-smooth transition (Rouhi et al., 2019b) where the intersection

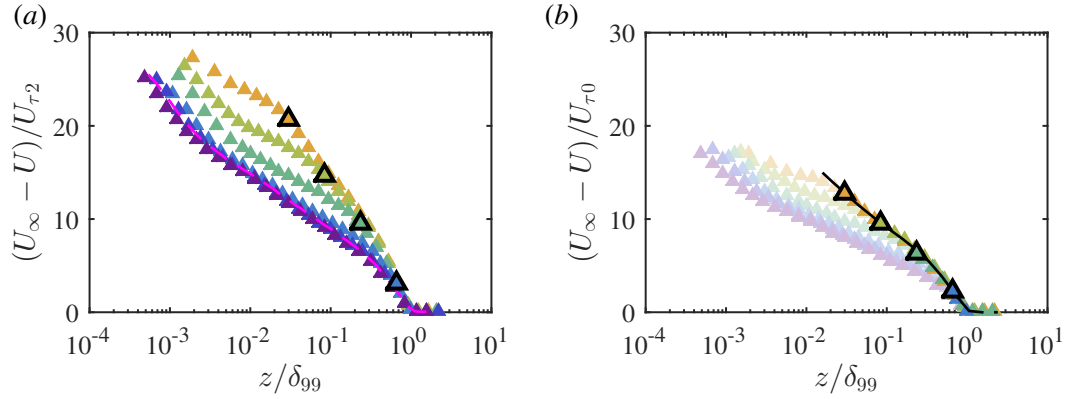


FIGURE 6.11: Mean velocity defect $U_{\infty} - U$ for case Re07ks16 normalised by (a) $U_{\tau 2}$, the local friction velocity obtained on the smooth wall and (b) $U_{\tau 0}$, the friction velocity just upstream on the roughness transition over the rough surface. The abscissa is the outer-scaled wall-normal location z/δ_{99} . The dashed magenta line in (a) is a reference smooth-wall hotwire profile measured at $Re_{\tau} = 1.0 \times 10^4$, a Reynolds number matched with the most downstream profile (Marusic et al., 2015), and the solid black line in (b) is the reference rough-wall hotwire profile obtained in the present study, with the datum adjusted to the smooth surface. The shading of the symbols indicates the downstream fetch, with the specific streamwise locations detailed in the caption of figure 6.10. Data points within the IBL are shown with a lighter tint in (b) to highlight the outer-layer behaviour. Note that in the two most downstream locations (dark blue and violet symbols), δ_i has already reached δ_{99} , therefore the entire profile is shown in the lighter tint and no IBL location is marked.

of two logarithmic laws in the mean velocity profile coincides with the wall-normal location where the turbulence intensity profile deviates from its upstream counterpart.

6.7 Streamwise turbulence intensity

Viscous-scaled streamwise turbulence intensity profiles are shown in figure 6.12(a) for case Re07ks16. Close to the step change, the peak magnitude of $\overline{u^2}^+$ in the outer layer exceeds the inner peak at $z^+ = 15$ as a result of the highly turbulent rough-wall flow beyond the IBL. The location of the ‘outer peak’ coincides with the edge of the IBL, and the magnitude of this outer peak decreases with $\hat{x}$. A more gradual decreasing trend with $\hat{x}$ can also be observed at the inner peak, which eventually reaches the same value as a smooth-wall profile in equilibrium (dashed magenta line). In §4 we suggested that the collapse in the small-scale energy could be used as a surrogate method of extracting the smooth-wall velocity scale $U_{\tau 2}$ in situations where direct measurements in the viscous sublayer were not possible. Figure 6.12(b) shows the small-scale contribution $\overline{u_s^2}^+$ to the broadband streamwise turbulence intensity. A threshold is selected as $T^+ = 300$ (to be detailed in §6.9), roughly equivalent to $\lambda_x^+ = 3000$ at the inner-peak location. Profiles at all streamwise locations collapse well at $z^+ = 15$, while the excess energy is clearly present at $z \gtrsim \delta_i$ (the IBL location is shown by the symbols with the bold outline). As noted previously in §4, the small-scale contribution recovers almost immediately downstream of a roughness step change, and the decreasing trend of the broadband turbulence intensity observed in figure 6.12(a) at the inner peak is due

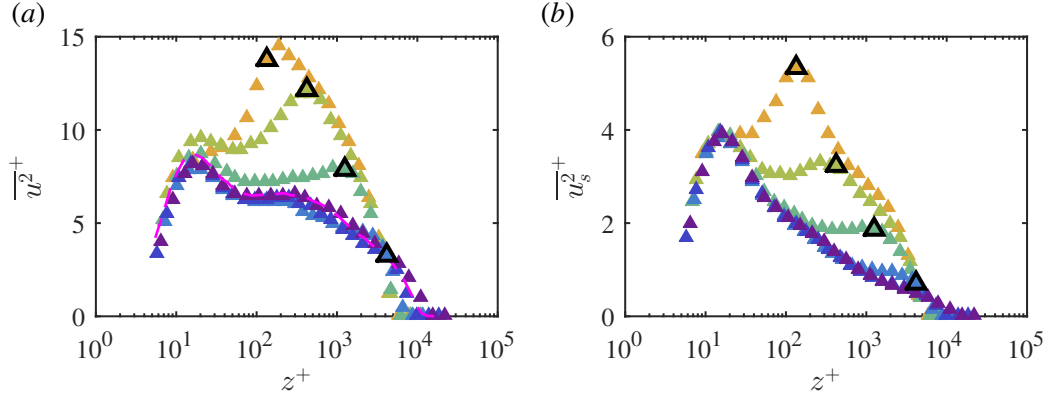


FIGURE 6.12: (a) Viscous-scaled streamwise turbulence intensity $\overline{u^2}^+$ profiles and (b) small-scale contribution to the broadband $\overline{u^2}^+$ for case Re07ks16. The symbols with thick black outlines show the location of the IBL (detailed in §6.5) in each profile. The shading of the symbols indicates the downstream fetch, with the specific streamwise locations detailed in the caption of figure 6.10. The dashed magenta line in (a) is a reference smooth-wall hotwire profile measured at $Re_\tau = 1.0 \times 10^4$, a Reynolds number matched with the most downstream profile (Marusic et al., 2015).

to the footprint of turbulent motions at larger scales. This will be discussed more in depth in §6.9.

The outer (z/δ_{99}) scaling of streamwise turbulence intensity profiles with both the local velocity scale $U_{\tau 2}$ and the upstream rough-wall velocity scale $U_{\tau 0}$ is examined in figure 6.13(a) and (b), respectively. These plots can give some indication of the degree of the outer-layer similarity observed. When normalised by $U_{\tau 2}$, the friction velocity obtained locally over the smooth wall, $\overline{u^2}^+$ is much higher than the equilibrium smooth-wall reference (shown by the dashed magenta line) even beyond the IBL, as shown in 6.13(a). Only for the two most downstream locations ($\hat{x}/\delta_0 = 39.4$ and 112, shown by the dark blue and violet symbols) does the outer part of the variance profile scaled in this manner show signs of collapsing to the equilibrium smooth-wall reference. This is presumably because at this downstream location, δ_i has grown close to the edge of the boundary layer, and the entire boundary layer is now in equilibrium with the smooth wall boundary condition. Note that the dashed magenta line in (a) and solid black line in (b) are expected to collapse under Townsend's assumption of outer-layer similarity, which has been shown to hold in fully-rough turbulent boundary layers developed on a similar roughness (Squire et al., 2017). In figure 6.13(b), the outer-layer similarity between the measurements downstream of the step change and the rough-wall reference is observed beyond the IBL, when the upstream friction velocity is chosen as the velocity scale. This is similar to observations made in figure 6.11 for the mean velocity defect.

6.8 Evolution of the outer layer

Traditionally, studies on the flow downstream of a roughness transition have been focusing on the developing IBL and the local friction velocity. The length and velocity scales in the

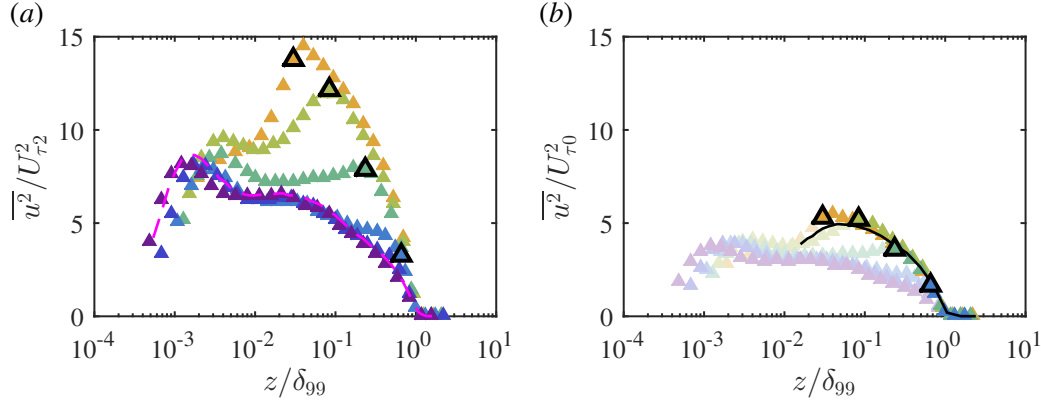


FIGURE 6.13: Streamwise turbulence intensity $\overline{u^2}$ for case Re07ks16 normalised by (a) $U_{\tau 2}$, the local friction velocity obtained on the smooth wall and (b) $U_{\tau 0}$, the friction velocity just upstream on the roughness transition over the rough surface. The abscissa is the outer-scaled wall-normal location z/δ_{99} . The dashed magenta line in (a) is a reference smooth-wall hotwire profile measured at $Re_{\tau} = 1.0 \times 10^4$, a Reynolds number matched with the most downstream profile (Marusic et al., 2015), and the solid black line in (b) is the reference rough-wall hotwire profile, with $z = 0$ shifted downward by $|\Delta H|$ to the smooth surface. The shading of the symbols indicates the downstream fetch, with the specific streamwise locations detailed in the caption of figure 6.10. Data points within the IBL are shown with a lighter tint in (b) to highlight the outer-layer behaviour. Note that in the two most downstream locations (dark blue and violet symbols), δ_i has already reached δ_{99} , therefore the entire profile is shown in the lighter tint and no IBL location is marked.

outer layer are usually assumed to remain constant and the evolution of the outer layer in the streamwise direction is often neglected. In addition, the mean velocity profile in the outer layer is often characterised by a logarithmic law only and the boundary layer thickness δ_{99} does not enter the picture. These assumptions may be adequate in a short fetch downstream of the transition, where the recovery of the flow is predominant. However, even in a turbulent boundary layer developing on a homogeneous surface, the streamwise evolution of the flow leads to an increase in the boundary layer thickness and a decrease in the friction velocity with an increasing x . In this section we examine the outer-layer velocity scale and the boundary layer thickness of the current dataset to study the evolution of the outer layer, and determine whether it is affected by the rough-to-smooth change at the wall.

6.8.1 Velocity scale

Elliott's model of the developing flow downstream of a rough-to-smooth transition assumes that above the IBL, $U_{\tau 0}$ (the friction velocity over the upstream rough surface) is the correct velocity scale. This assumes that the over-energised outer turbulence continues to reflect the upstream roughness velocity scale, even far downstream of the transition, and does not decay or relax downstream. This assumption has been questioned by previous experimental results, and here we aim to investigate it further.

By performing a least-squares regression, Hanson and Ganapathisubramani (2016) calculated the friction velocity $U_{\tau, \text{out}}$ required to force the collapse of the mean velocity deficit above the internal layer. They reported that all $C_{f, \text{out}} (\equiv 2U_{\tau, \text{out}}^2/U_{\infty}^2)$ values are less than C_{f0} , the skin-friction coefficient on the upstream rough surface. $C_{f, \text{out}}$ was observed to decay

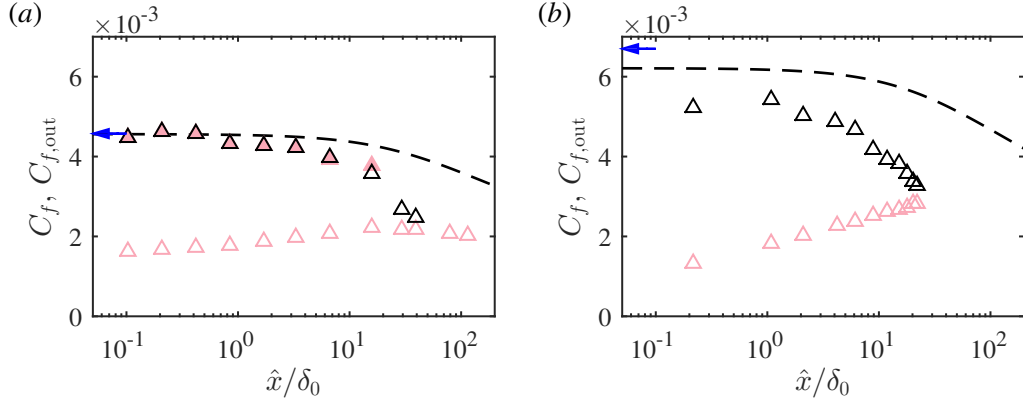


FIGURE 6.14: C_f obtained within the IBL and above the IBL via curve fitting for (a) case Re07ks16 and (b) case grit adapted from Hanson and Ganapathisubramani (2016). Empty pink triangles in both figures are C_f measured in the near-wall region, while empty black triangles are $C_{f,out}$ computed assuming an outer-layer similarity in the mean velocity deficit with δ_i estimated following the method of Antonia and Luxton (1971b). Filled pink triangles in (a) are $C_{f,out}$ computed using the same approach but with δ_i estimated with the thresholding approach as detailed in §6.5. The black dashed line is C_f of a homogeneous rough-wall boundary layer without the change of surface condition at x_0 computed by integrating the von Kármán momentum integral equation (Monty et al., 2016), and the blue arrow shows the C_f at the reference upstream rough-wall measurement.

downstream and approaches (though has not reached) the local smooth wall C_f at the last measuring location at $\hat{x}/\delta_0 \approx 20$. Data from their original figure 19(a) are reproduced here in figure 6.14(b) for comparison.

$C_{f,out}$ for case Re07ks16 computed following the same procedure as in Hanson and Ganapathisubramani (2016) is shown in figure 6.14(a). Note that similar behaviours are observed in all other cases, and are not shown here for brevity. Two definitions of δ_i are tested: the U versus $z^{1/2}$ definition originally proposed by Antonia and Luxton (1971b) and used in Hanson and Ganapathisubramani (2016), and a new definition based on the turbulence intensity profile as detailed in §6.5. In the present study, δ_i is defined as the upper limit of the fluctuating rough/smooth interface, while δ_i used by Hanson and Ganapathisubramani (2016) appears to be closer to the averaged location of the fluctuating interface. As δ_i grows and approaches the local δ_{99} , the fitting range diminishes and eventually there are not enough data points to perform the regression, making the $C_{f,out}$ trend in the limit of large $\hat{x}/\delta_0$ unattainable. There is little difference between $C_{f,out}$ with the two definitions of δ_i except for the last few downstream locations, where $C_{f,out}$ with the current definition of δ_i is absent due to the vanishing of the fitting region as δ_i approaches δ_{99} . With Antonia & Luxton's definition of δ_i , a larger portion of the velocity profile which has in fact been affected by the new smooth-wall condition is included in the fit, therefore, a distinct decrease presents in $C_{f,out}$, qualitatively similar to the near collapse in $C_{f,out}$ and C_f at their most downstream station as reported by Hanson and Ganapathisubramani (2016).

$C_{fR}(\hat{x})$, the skin-friction coefficient for a homogeneously rough surface (with the same roughness as the upstream surface) as a function of $\hat{x}$ is also included for comparison in figure 6.14 (black dashed line), which is computed by integrating the von Kármán momentum

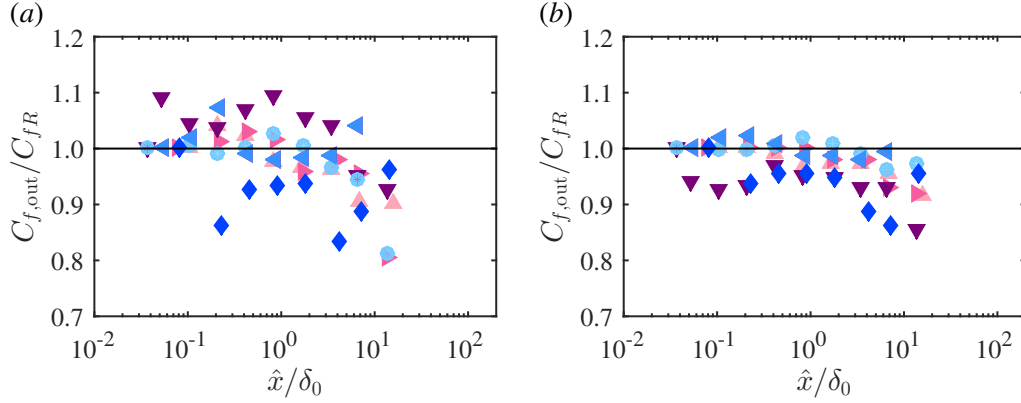


FIGURE 6.15: $C_{f,\text{out}}$ normalised by C_{fR} , the skin-friction coefficient for a homogeneously rough surface. $C_{f,\text{out}}$ is calculated by assuming an outer-layer similarity in (a) mean velocity deficit and (b) turbulence intensity profiles. The solid black line is $C_{f,\text{out}}/C_{fR} = 1$.

integral equation (Monty et al., 2016) in the streamwise direction¹. $C_{fR}(0)$ obtained from the measured upstream rough-wall profile is shown by the blue arrow on the logarithmic abscissa. Figure 6.14 shows that $C_{f,\text{out}}$ follows the predicted value for a homogeneously rough surface closely in the present dataset, while an immediate decrease in $C_{f,\text{out}}$ compared to its value on the rough wall upstream is reported by Hanson and Ganapathisubramani (2016), which can readily be observed in (b). In figure 6.14(b), $C_{f,\text{out}}$ at the first downstream location is lower than the rough-wall reference $C_{fR}(0)$ by 22%. Such a sudden change in the velocity scale in the outer layer challenges the traditional view that the effect of the new surface condition gradually modifies the interior of the flow through the growth of the IBL. Future work is required to further verify the existence of this behaviour and to understand the mechanisms behind it.

We further examine the change of $C_{f,\text{out}}$ with $\hat{x}$ for all cases. Figure 6.15 shows $C_{f,\text{out}}(\hat{x})$ normalised by $C_{fR}(\hat{x})$ for all cases. $C_{f,\text{out}}$ is computed by assuming an outer-layer similarity in the mean velocity deficit and turbulence intensity profiles in (a) and (b), respectively. If the velocity scale (i.e. rough-wall friction velocity) in the outer layer continues to develop as before without being affected by the rough-to-smooth transition, then $C_{f,\text{out}}/C_{fR}$ should be equal to 1, as shown by the solid black line. Note that the focus here is on the trend of $C_{f,\text{out}}/C_{fR}$ in the streamwise direction rather than to assess how well the predicted $C_{fR}(0)$ matches the measured C_f on the rough wall, we divide all the downstream points by $C_{f,\text{out}}/C_{fR}$ at the first downstream measuring station. This reduces the scatter of the data and facilitates the observation of a general trend. The correction is usually less than 15% in (a) and 3% in (b), and it does not show any dependence on $Re_{\tau 0}$ or k_s^+ .

Due to the uncertainty of the measurement and the curve fit, $C_{f,\text{out}}$ computed from the mean velocity deficit (figure 6.15a) exhibits $\pm 10\%$ scatter, while a less scattered trend is observed when the streamwise turbulence intensity profile is used (figure 6.15b). Overall, $C_{f,\text{out}}/C_{fR}$ shows a slight decreasing trend, and for all cases, it is below 1 (shown by the

¹To calculate the rough-wall $C_{fR}(\hat{x})$ for Hanson and Ganapathisubramani's dataset, we use $\Pi_j = 0.71$ as reported for their homogeneous smooth-wall boundary layer. k_s is calculated from the reported viscous-scaled roughness length z_0^+ of the rough surface. $\kappa = 0.41$ and $B = 5.0$ are used in the evolution to be consistent with their choice of the constants.

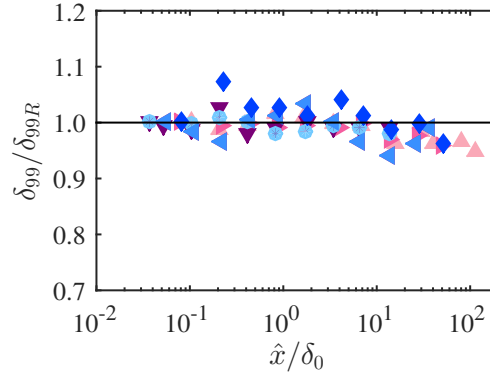


FIGURE 6.16: δ_{99} determined from the measured mean velocity profiles downstream of the rough-to-smooth normalised by δ_{99R} , the boundary layer thickness for a homogeneously rough surface. The solid black line is at $\delta_{99}/\delta_{99R} = 1$.

solid black line) at $\hat{x}/\delta_0 \approx 15$, the last streamwise location where $C_{f,\text{out}}$ is attainable. For the homogeneous rough surface, C_{fR} decreases in the streamwise direction as the boundary layer grows and the Reynolds number increases. The decreasing trend of the ratio $C_{f,\text{out}}/C_{fR}$ suggests that the velocity scale in the outer layer decreases marginally more aggressively than the Reynolds number trend of C_{fR} , i.e. $C_{f,\text{out}}/C_{fR}$ decreases slightly faster after a rough-to-smooth change than it would without the roughness transition. The physical interpretation is, the production of turbulent energy at the wall decreases after a rough-to-smooth change, and with a reduced supply of turbulent energy, the outer layer cannot sustain a rough-wall friction velocity that is no longer compatible with the new wall condition. Therefore, the velocity scale in the outer layer decreases more aggressively than the Reynolds number trend.

Despite the small difference between $C_{f,\text{out}}$ and C_{fR} , this again confirms that the velocity scale in the flow above the IBL is very close to the friction velocity of the rough wall upstream, and may in fact prove to be a reasonable modelling assumption in Elliott's approach.

6.8.2 Boundary layer thickness

The growth of the boundary layer thickness δ_{99} downstream of the rough-to-smooth transition is compared with δ_{99R} , the thickness that a boundary layer developing on a homogeneously rough surface would have. Similar to C_{fR} , δ_{99R} is also computed by integrating the von Kármán momentum integral equation (Monty et al., 2016) in the streamwise direction. The ratio $\delta_{99}(\hat{x})/\delta_{99R}(\hat{x})$ for all cases is presented in figure 6.16. δ_{99}/δ_{99R} is very close to 1 (shown by the solid black line) in the vicinity of the roughness transition, suggesting that the growth rate of a rough-wall boundary layer thickness is initially sustained after the transition. Farther downstream, a consistent deviation from unity is observed, especially in case Re07ks16 which has the longest fetch on the downstream smooth wall. A δ_{99}/δ_{99R} ratio smaller than 1 indicates that the boundary layer thickness after a rough-to-smooth change is smaller than its rough-wall counterpart at the same fetch. Eventually, after a sufficiently long fetch, the boundary layer is expected to asymptote to an equilibrium state with the new wall condition (i.e. smooth wall) and the growth rate of the boundary layer thickness should also follow

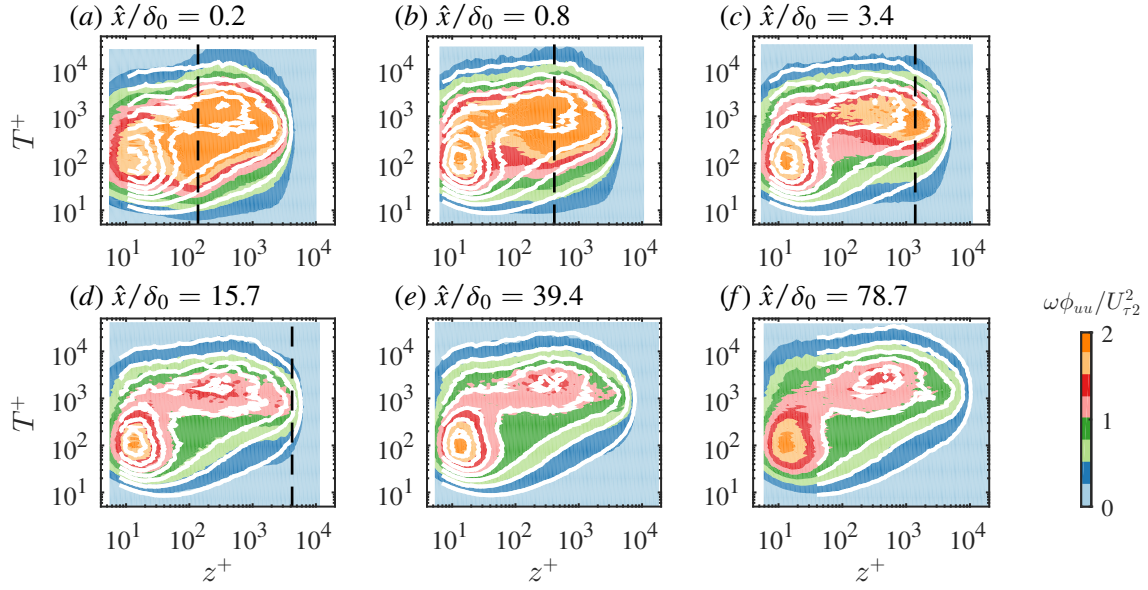


FIGURE 6.17: Viscous-scaled premultiplied energy spectrum $\omega\phi_{uu}/U_{\tau_2}^2$. The colour contours correspond to the rough-to-smooth case Re07ks16, and the white contour lines are interpolated from a reference smooth-wall experimental dataset to matched Re_τ . Contour levels are chosen at $\omega\phi_{uu}/U_{\tau_2}^2 = 0$ to 2 with an increment of 0.25. The vertical black dashed line represents the location of δ_i .

that of a smooth-wall boundary layer, which is smaller than the growth rate of a rough-wall boundary layer. Therefore, such a decrease in δ_{99}/δ_{99R} is fully expected.

We must point out that the determination of both $C_{f,out}$ and δ_{99} suffer from a degree of uncertainty, however, we still observe a weak trend in both of them that they fall below the corresponding rough-wall value in the far field. The general picture is that when a turbulent boundary layer encounters a rough-to-smooth change, the change is barely felt in the outer layer at first and the flow in the outer layer continues to evolve in streamwise as if the roughness transition was not there. Farther downstream, both $C_{f,out}$ and δ_{99} begin to slowly adapt to the smooth-wall state. A noticeable deviation from the rough-wall trend only appears after approximately $10\delta_0$ downstream of the transition.

6.9 Premultiplied energy spectrum

Most previous laboratory measurements only cover a downstream fetch of approximately $20\delta_0$ or less, where usually, the flow still has not reached a full equilibrium with the new wall condition. In the present study, we are able to measure up to $120\delta_0$ for case Re07ks16, which enables us to study the recovery of the flow in the far field.

Accordingly, the premultiplied energy spectrum $\omega\phi_{uu}/U_{\tau_2}^2$ of case Re07ks16 is shown in figure 6.17, where $\omega = 2\pi/T$ is the angular frequency, T is the time period (corresponding to the wavelength in spatial domain), ϕ_{uu} is the energy spectrum of the streamwise velocity fluctuation ($\int_0^\infty \phi_{uu} d\omega = \overline{u^2}$), and U_{τ_2} is the friction velocity measured from the OFI experiments. The spectrograms are computed from hotwire time series as the flow is heterogeneous in x , we refrain from converting the spectrum from the temporal to the spatial

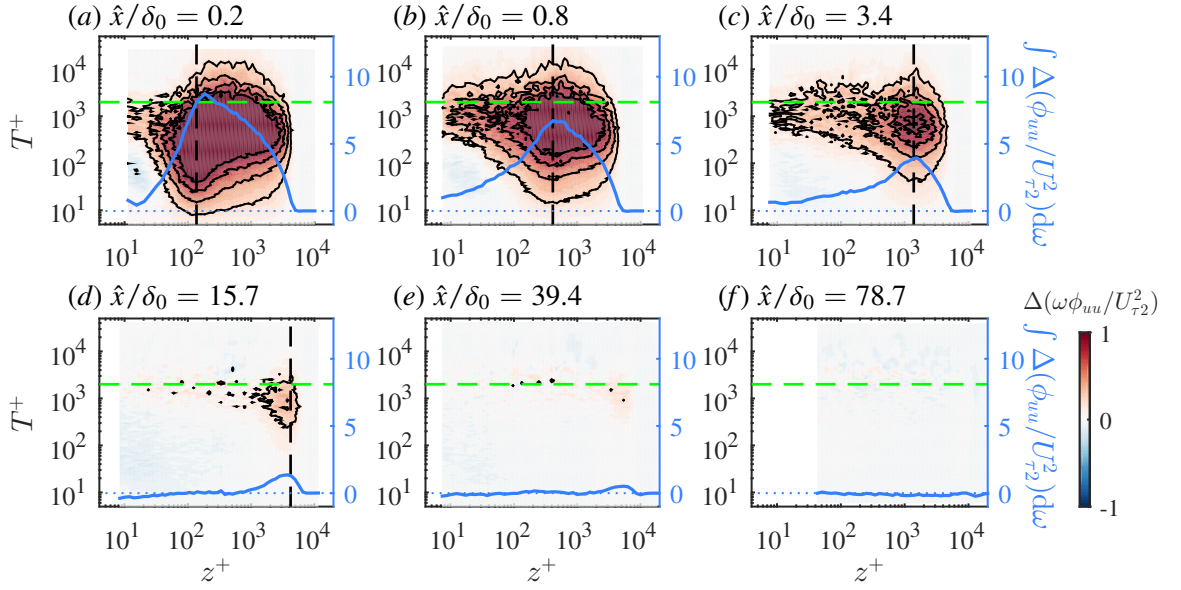


FIGURE 6.18: The difference between the viscous-scaled premultiplied spectrum of Re07ks16 and the smooth-wall reference (matched Re_τ) at streamwise locations corresponding to figure 6.17. The four black contour lines indicate $\Delta(\omega\phi_{uu}/U_{\tau_2}^2) = 0.25, 0.5, 0.75$ and 1. The vertical dashed black line represents the location of δ_i and the horizontal green dashed line is at $T^+ = 2 \times 10^3$, representing the location of the large-scale footprint. The blue line is the difference in $\phi_{uu}/U_{\tau_2}^2$ integrated across all wavelengths.

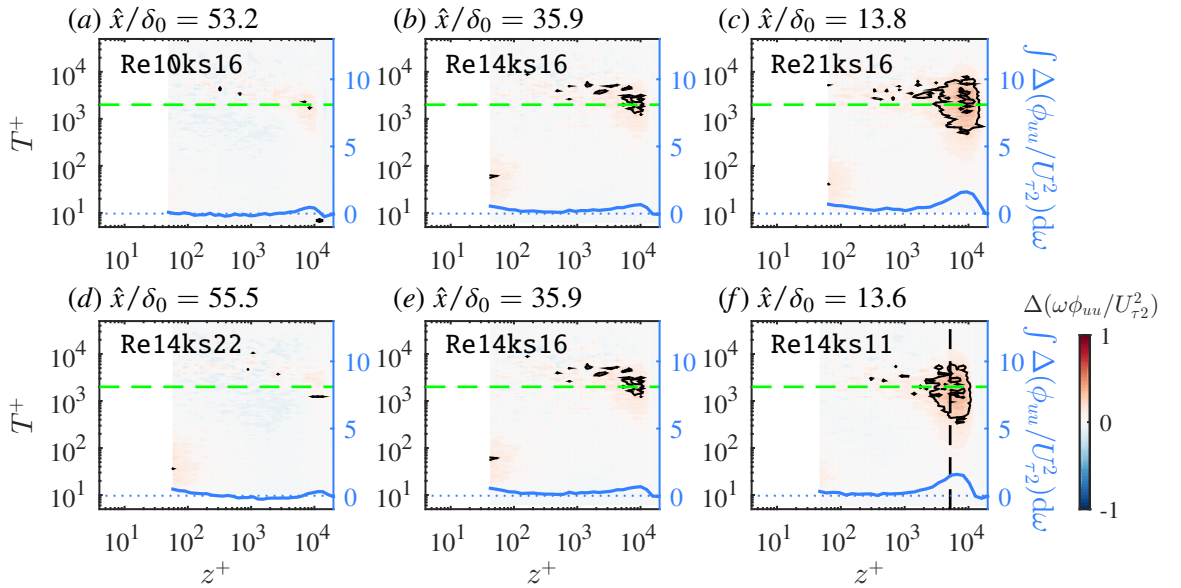


FIGURE 6.19: The difference between the viscous-scaled premultiplied spectrum and the smooth-wall reference (matched Re_τ) at the most downstream measurement station. The four black contour lines indicate $\Delta(\omega\phi_{uu}/U_{\tau_2}^2) = 0.25, 0.5, 0.75$ and 1. The vertical dashed black line represents the location of δ_i and the horizontal green dashed line is at $T^+ = 2 \times 10^3$, representing the location of the large-scale footprint. The blue line is the difference in $\phi_{uu}/U_{\tau_2}^2$ integrated across all wavelengths. The top row shows cases in Group I (excluding Re07ks16 which has been shown previously) with an increasing $Re_{\tau 0}$ from left to right, and the bottom row shows cases in Group II with a decreasing k_{s0}^+ from left to right. Note that the ‘crossing-point’ case Re14ks16 is repeated in both groups.

domain, which has been shown to have limited accuracy in rough-wall flows (Squire et al., 2017). The coloured contours are the current rough-to-smooth data, and the white contour

lines are interpolated from a reference smooth-wall experimental dataset (Marusic et al., 2015; Squire et al., 2016b) at matched Re_τ , which ensures that the energy diminishes at the same wall-normal height in viscous units in both the rough-to-smooth case and the smooth-wall reference. Figure 6.18 shows the difference between the rough-to-smooth spectrum and the reference smooth-walled spectrum, defined as,

$$\Delta(\omega\phi_{uu}/U_{\tau_2}^2) \equiv (\omega\phi_{uu}/U_{\tau_2}^2)_{R \rightarrow S} - (\omega\phi_{uu}/U_\tau^2)_S. \quad (6.5)$$

A complete recovery of the energy spectrum is achieved at $\hat{x}/\delta_0 = 78.7$, as shown in figure 6.17(f) and figure 6.18(f). Note that the complete recovery of the energy spectrum can be expected somewhere in the range from $39.4\delta_0$ to $78.7\delta_0$ downstream of the roughness transition. We lack an intermediate measurement between these locations due to a logarithmic spacing of streamwise measurement stations. Regardless, the noteworthy feature from these spectra is that it takes a longer fetch downstream for the energy spectrum to relax completely to the smooth-wall state than for the IBL to outgrow the original boundary layer ($\approx 26.5\delta_0$ as predicted by extrapolating the power-law fit (6.4) in §6.5). The difference between the rough-to-smooth spectrum and the reference smooth-wall spectrum at the most downstream measurement station for the other three *Group I* cases are shown in figure 6.19. The limitation imposed by the length of the tunnel working section means that no measurement is available beyond $\hat{x}/\delta_0 = 40$ and $\hat{x}/\delta_0 = 15$ for cases Re14ks16 and Re21ks16, respectively. Nevertheless, the difference in the premultiplied energy spectrum at the most downstream location in each case is similar to case Re07ks16 at matched $\hat{x}/\delta_0$ locations, suggesting that the downstream fetch required for a full recovery in energy spectrum may have little Reynolds number dependence when scaled by δ_0 . This agrees with the observations made previously in §6.4 and §6.5 that the streamwise evolution of both C_f and δ_i has little Reynolds number dependence when the fetch is normalised by δ_0 within the range of parameters investigated in this study.

Comparing figure 6.6 and figure 6.18, it appears that, within experimental uncertainty, C_f achieves the complete recovery from the roughness transition in a shorter fetch ($20\delta_0$) compared to the energy spectrum ($40\delta_0 - 80\delta_0$). Similar observations have been reported by Rouhi et al. (2019b), Ismail et al. (2018b) and Sridhar (2018) in their numerical studies.

The streamwise location where C_f has reached the complete recovery seems to coincide with the location where the energetic large-scale footprint in the near-wall region vanishes. Although at $\hat{x}/\delta_0 = 15.7$ (figure 6.18d), this footprint is already becoming very weak for $z^+ < 100$ (see the horizontal dashed lines in figure 6.18).

To further examine the recovery behaviour of the large-scale footprints in the near-wall region, 1-D energy spectra are extracted at $z^+ = 30$ for the rough-to-smooth cases and the smooth-wall reference and shown in figure 6.20. The excess energy related to the rough wall upstream resides mostly in the intermediate scales of $T^+ \approx 2 \times 10^3$, while a good collapse with the smooth-wall reference at matched Re_τ is observed in the small scales ($T^+ < 100$) at all streamwise locations. This confirms the findings detailed in §4 at higher Reynolds numbers, and thus the applicability of the surrogate method of estimating the local friction

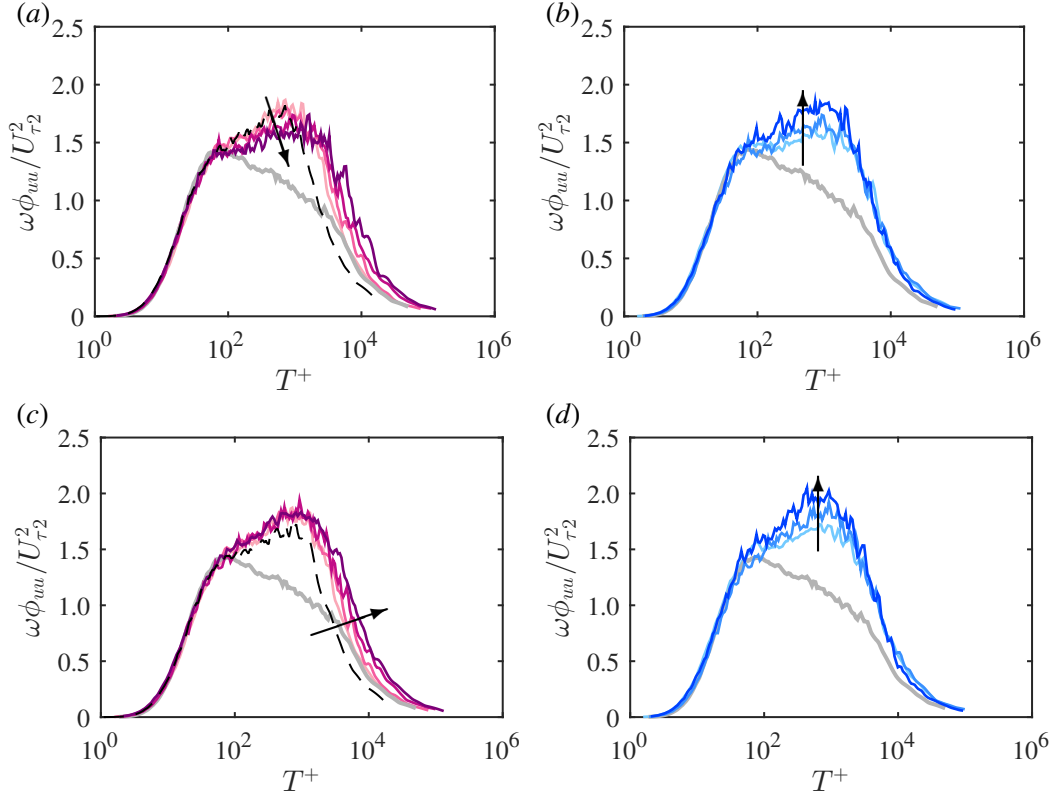


FIGURE 6.20: Premultiplied energy spectrum at a wall-normal location of $z^+ = 30$ and at streamwise locations (a,b) $\hat{x}/\delta_0 = 0.8$ and (c,d) $\hat{x}U_{\tau 0}/\nu = 6000$. The left column (a,c) is *Group I* and the right column (b,d) is *Group II*. The black dashed line in (a,c) is case Re04ks13, and the pink and blue colours of the lines indicate $Re_{\tau 0}$ and k_{s0}^+ , respectively, following the colour scheme as listed in table 3.5. The black arrow in the left and right column indicates the direction of increasing $Re_{\tau 0}$ and k_{s0}^+ , respectively. The thick grey line is the reference smooth-wall data interpolated at an Re_{τ} matched with Re14ks16 and the same wall-normal location of $z^+ = 30$.

velocity from the energy spectrum. For *Group II* cases, as all of them share a nominal friction Reynolds number $Re_{\tau 0} = 14000$, $\hat{x}U_{\tau 0}/\nu = 6000$ (figure 6.20d) is essentially $\hat{x}/\delta_0 = 0.4$. The excess energy around $T^+ \approx 2 \times 10^3$ increases with k_{s0}^+ , implying that a stronger large-scale footprint presents with a rougher (higher k_{s0}^+) surface upstream. In this study, a higher k_{s0}^+ leads to a higher $U_{\tau 0}$ to local $U_{\tau 2}$ ratio, which means that the $U_{\tau 0}$ -scaled rough-wall structures in the outer layer are more energetic compared to the $U_{\tau 2}$ -scaled small-scale energy near the wall. Hence, the trend shown in figure 6.20(b) and (d) is entirely expected.

For *Group I* cases, at matched $\hat{x}/\delta_0 = 0.8$, as $Re_{\tau 0}$ increases, the location of the peak in the energy spectrum appears to shift to a greater T^+ and the its magnitude decreases, as highlighted by the arrow in figure 6.20(a). A better agreement between cases is observed at matched viscous-scaled fetch $\hat{x}U_{\tau 0}/\nu = 6000$. Note that $\hat{x}U_{\tau 0}/\nu = 6000$ is equivalent to $\hat{x}/\delta_0 = 1.5, 0.8, 0.6, 0.4$ and 0.3 for cases Re04ks13, Re07ks16, Re10ks16, Re14ks16 and Re21ks16, respectively. As shown in figure 6.20(c), all cases collapse for $T^+ < 1 \times 10^3$, and similar to canonical smooth-wall boundary layers (Hutchins et al., 2009), the large-scale energy increases with $Re_{\tau 0}$. Here we speculate that strength of the footprints is dependent on $\delta_i U_{\tau 0}/\nu$. At a matched $\hat{x}U_{\tau 0}/\nu = 6000$, $\delta_i U_{\tau 0}/\nu$ is approximately the same for all *Group I*

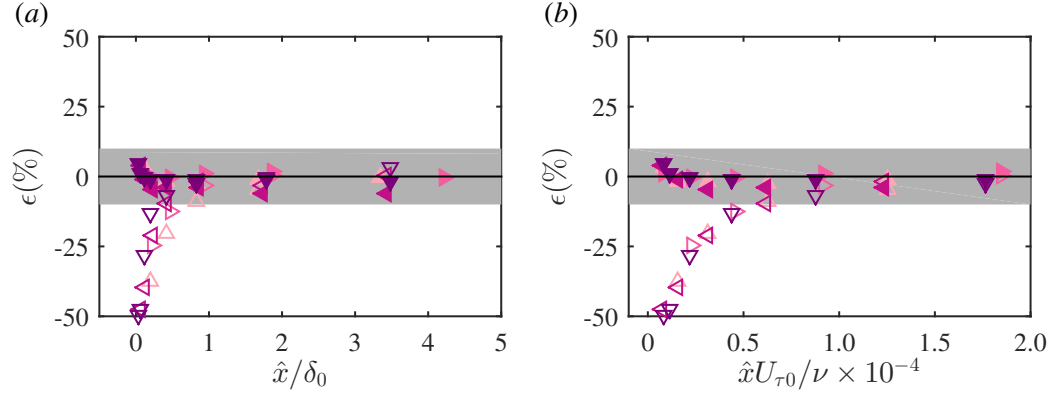


FIGURE 6.21: ϵ , the error in the estimated C_f relative to the OFI results, as defined in (6.6). Open symbols represent buffer region fit and solid symbols are for the spectrum fit. The shaded band covers -10% to 10% on the vertical axis. The horizontal axis is the fetch normalised by the outer length scale δ_0 in (a) and the viscous scale in (b).

cases as δ_i is almost linearly proportional to $\hat{x}$ according to equation (6.4), and $\delta_i U_{\tau 0} / \nu$ only varies from 600 to 800 when $Re_{\tau 0}$ increases from 7100 to 21000. The physical interpretation of figure 6.20(c) is that for a given upstream roughness condition (fixed k_{s0}^+), when the rough-wall structures are at approximately the same distance above the wall in viscous units, the footprints they leave in the near-wall region have the same strength. However, figure 6.20(d) show evidence that the magnitude of the excess outer energy is a function of k_{s0}^+ .

6.9.1 Estimating $U_{\tau 2}$ from the premultiplied energy spectrum

In §4.5, we detailed the recovery behaviour of the premultiplied energy spectrum of a lower Reynolds number case, and based on the observation that the energy distribution at the high-frequency end in the near-wall region recovers to the smooth-wall equilibrium state faster than the lower frequencies, we developed an alternative method to extract $U_{\tau 2}$ from the premultiplied energy spectrum when a direct wall-shear stress measurement such as OFI is not available.

Here we provide a brief summary of the method described previously. A rectangular region in the spectrum plot bounded by the limits $10 < z^+ < 30$ and $5 < T^+ < 90$ is chosen, and the difference between the viscous-scaled energy spectra for the rough-to-smooth and the smooth case $\Delta(\omega \phi_{uu} / U_{\tau 2}^2)$ is then minimised across this region by varying $U_{\tau 2}$ for the rough-to-smooth case. We have compared the error in the estimation of the spectrum fit method with that from a buffer-region fit of the mean velocity profile (it represents conventional methods that assume a canonical or equilibrium mean velocity profile, see §4 for details), and the former exhibited much better performance, especially immediately downstream of the roughness transition. More specifically, we defined ϵ as the relative error in the skin-friction coefficient with the OFI results as the standard reference:

$$\epsilon \equiv \frac{C_{f|M} - C_{f|OFI}}{C_{f|OFI}} \times 100\%, \quad (6.6)$$

and presented the comparison in figure 4.9. The subscript M stands for the method, which can be either the spectrum fit (solid symbols) or the buffer-region fit (open symbols). The error of the spectrum fit falls between $\pm 10\%$ at all streamwise locations, while an underestimation over 25% is observed in the buffer-region fit result close to the roughness transition.

To continue testing the efficacy of this method over a wider range of Re_{τ} , we present results from *Group I* cases in a similar fashion here in figure 6.21(a). As shown in the figure, the spectrum fit method continues to have a good agreement with the reference OFI data for all cases, and the buffer-region fit produces an underestimation close to the roughness transition. We have previously reported a minimum fetch of $5\delta_0$ after the roughness transition before the buffer-region method can be safely used. An even longer fetch ($\hat{x} \gtrsim 8h$) is required for the open-channel DNS dataset (which has a much lower $Re_{\tau 0}$). For the cases shown in figure 6.21(a), the error in the buffer-region fit diminishes in a fetch that is typically smaller than $2\delta_0$, and the fetch decreases even further as $Re_{\tau 0}$ increases. We speculate that this minimum fetch scales on the viscous unit rather than δ_0 or h , and we present ϵ versus $\hat{x}U_{\tau 1}/\nu$ in figure 6.21(b). A nominally zero ϵ of the buffer-region fit method is observed at $\hat{x}U_{\tau 0}/\nu \gtrsim 1.0 \times 10^4$.

We may conclude that, for a certain k_{s0}^+ (and possibly a certain step height), the minimum fetch required before a buffer-region fit can give good wall shear stress estimates is approximately constant when scaled by the upstream viscous units. For the buffer-region fit method to be valid, it essentially requires a full recovery of the mean velocity profile within the buffer region, i.e. the edge of the equilibrium layer should exceed the upper limit of the fitting region. As the upper limit of the buffer region is usually fixed in viscous units, it is expected that the minimum fetch for the equilibrium layer to reach such a height will also be a constant when scaled by viscous units.

6.10 Summary and conclusion

Results of the measurement campaign in the HRNBLWT are presented in this chapter, focusing on the effect of $Re_{\tau 0}$ and k_{s0}^+ on the flow recovery. The main findings are detailed below:

1. The initial flow condition prior to the roughness transition has been verified to be the same as planned: a nominally constant $k_{s0}^+ \approx 160$ is achieved for all *Group I* cases with $Re_{\tau 0}$ ranging from 7100 to 21000, while a nominally constant $Re_{\tau 0} = 14000$ is achieved for all *Group II* cases with k_{s0}^+ ranging from 111 to 228. Townsend's outer-layer similarity hypothesis appears to be satisfied, and these profiles in the upstream of the roughness transition behave similarly as a turbulent boundary layer developed on a homogeneous rough surface in a quasi-equilibrium state.
2. The recovery of C_f follows a single trend with $\hat{x}/\delta_0$ when normalised by C_{fe} (the skin-friction coefficient expected at the same Reynolds number using the smooth-wall relationship), showing little $Re_{\tau 0}$ or k_{s0}^+ dependence. The recovering C_f can be approximated by a log-linear relation with the fetch.

3. A method to extract δ_i , the IBL height, from the turbulence intensity profile is described. δ_i values obtained using this method are compared with the results following the definition of Antonia and Luxton (1971b) (which is a commonly used method in the literature), and we show that the present definition leads to a slightly higher δ_i as it tends to pick up the upper limit of the IBL. δ_i computed using both definitions follows a power-law growth trend in streamwise with an exponent close to 0.8. Little $Re_{\tau 0}$ or k_{s0}^+ dependence is observed in δ_i .
4. The streamwise evolution of the mean velocity and turbulent intensity profiles are examined. We confirm that the flow in the IBL is not in complete equilibrium with the new surface condition: the mean velocity profile can be scaled by the local $U_{\tau 2}$ only very close to the wall or at a long fetch downstream of the roughness transition. $U_{\tau 0}$, the friction velocity on the upstream rough wall appears to be a good approximation of the velocity scale in the outer layer.
5. By assuming an outer-layer similarity above the IBL, the velocity scale in the outer layer is found to be very close to the friction velocity on the upstream rough wall, and it follows a marginally more aggressive Reynolds number trend of a turbulent boundary layer developing on the same upstream roughness without transitioning to a smooth surface. The boundary layer thickness δ_{99} is found to be slightly below that of a rough-wall boundary layer in the far field.
6. A complete recovery of the energy spectrum occurs within a fetch between $40\text{--}80\delta_0$ for case Re07ks16, longer than the fetch required for δ_i to outgrow δ_{99} as predicted by the power-law relation. The recovery length has shown little dependence on $Re_{\tau 0}$ or k_{s0}^+ when normalised by δ_0 . The over-energised inner peak observed in the streamwise turbulence intensity profiles is found to be contributed by the large-scale footprints in the near-wall region, which reside at a time period of $T^+ \approx 2000$. The energy distribution at smaller time periods remains unmodified from the smooth-wall reference. Furthermore, at a given z^+ in the near-wall region, the strength of the footprints increases with an increasing k_{s0}^+ , while for a fixed k_{s0}^+ , the footprint strength of difference cases collapses at the same viscous-scaled fetch, which may suggest a dependence on the distance between the rough-wall structures and the wall in viscous units.
7. The spectrum fit method to extract local friction velocity as detailed in §4.5 continues to have a good performance over a wide range of $Re_{\tau 0}$. The error in the estimation from a buffer-region fit of the mean velocity profile is shown to diminish after $\hat{x}U_{\tau 0}/\nu \gtrsim 1.0 \times 10^4$.

Chapter 7

Evaluating Elliott's model with the experimental dataset

We examine the performance of Elliott's model in predicting the flow recovery from a rough-to-smooth change in the surface condition using the experimental dataset presented in the previous chapter. The predictions of the IBL thickness δ_i and the local friction velocity $U_{\tau 2}$ are compared against the experimental measurements at matched flow conditions. We find that the assumption of a log-law mean velocity profile in Elliott's model is responsible for its incapability to have a good prediction of δ_i and $U_{\tau 2}$ simultaneously. To reconcile this issue, we define a new IBL thickness $\delta_{i,log}$ based on the log-law profile in Elliott's model scaled by the correct $U_{\tau 2}$ and we further investigate its relation with the IBL thickness defined from the measured profile.

7.1 Chapter outline

In the previous chapter, we have reported an experimental dataset obtained in the HRNBLWT at controlled flow conditions. A thorough examination of the data is presented, including the flow recovery trend of a single case and also the effect of different $Re_{\tau 0}$ and k_{s0}^+ values. Moving forward, in order to enhance our ability to eventually predict the flow response to a roughness transition, here we first evaluate the performance of a classical model with the experimental dataset to identify any opportunities for improvement. We will start with a brief overview of Elliott's (1958) model, and then we compare the measured and predicted friction velocity $U_{\tau 2}$ and IBL thickness δ_i downstream of the roughness transition. Finally, we provide a further discussion on the result. The knowledge gained through the comparison here will be further utilised in the following chapter to develop a more realistic model of the mean velocity profile.

7.2 A brief revision of Elliott's model

In the seminal study of Elliott (1958), a theoretical model predicting the flow recovery from a step change in the surface roughness is established. The flow condition for a rough-to-smooth transition is illustrated in figure 7.1(a) as an example. The incoming flow can be fully described by the roughness length z_{01} and friction velocity $U_{\tau 1}$ of the upstream surface, while the only information about the downstream surface is the roughness length z_{02} . After the roughness transition at $x = x_0$ (i.e. $\hat{x} = 0$), an IBL containing the modified flow develops on the downstream surface and a local friction velocity $U_{\tau 2}$ evolves with the growth of the IBL. In this system, the roughness lengths z_{01} and z_{02} are constant. $U_{\tau 1}$ is also assumed to be constant, therefore, we have $U_{\tau 1}(\hat{x}) = U_{\tau 1}(0) \equiv U_{\tau 0}$, to be consistent with previous notations. This leaves two unknowns, $\delta_i(\hat{x})$ and $U_{\tau 2}(\hat{x})$, as functions of $\hat{x}$, the fetch on the downstream surface. Therefore, two governing equations are required to fully close the system and provide solutions of δ_i and $U_{\tau 2}$.

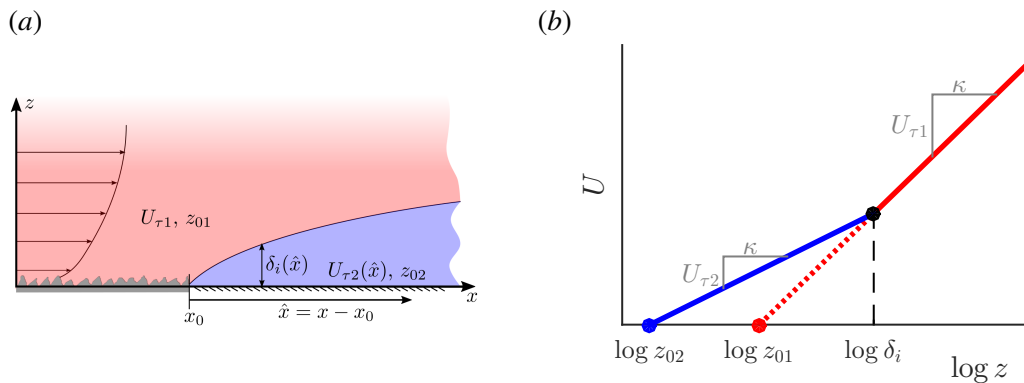


FIGURE 7.1: (a) Sketch of the flow over a rough-to-smooth change in the streamwise direction. The IBL is shown by blue, and the outer layer is shown by red. (b) Sketch of the mean velocity profile in Elliott's model corresponding to the flow condition in (a). The local smooth-wall log-law profile is shown by blue, and the outer rough-wall log-law profile is shown by red.

Elliott approximated the recovering mean velocity profile with a piecewise logarithmic function (an illustration is shown in figure 7.1b):

$$U(z) = \begin{cases} \frac{U_{\tau 1}}{\kappa} \ln(z/z_{01}), & z > \delta_i \\ \frac{U_{\tau 2}}{\kappa} \ln(z/z_{02}), & z < \delta_i, \end{cases} \quad (7.1)$$

which implicitly assumes that the mean velocity profile within the IBL is in full equilibrium with the local wall condition. From this expression of the mean velocity profile, the first equation to close the system rises naturally as the matching condition, i.e. the two log laws intersecting at $z = \delta_i$:

$$\frac{U_{\tau 1}}{\kappa} \ln(\delta_i/z_{01}) = \frac{U_{\tau 2}}{\kappa} \ln(\delta_i/z_{02}). \quad (7.2)$$

A second equation is obtained from a momentum equation with the flow within the IBL as the control volume:

$$\frac{d}{dx} \int_{z_{02}}^{\delta_i} U^2 dz - U(\delta_i) \frac{d}{dx} \int_{z_{02}}^{\delta_i} U dz = U_{\tau 2}^2 - U_{\tau 1}^2. \quad (7.3)$$

With these two equations, the system will be closed and the streamwise evolution of $U_{\tau 2}$ and δ_i can be predicted.

7.3 Friction velocity

In §6.4, we presented the results of C_f , and discussed the possibilities of a universal recovery trend under appropriate normalisation. Here we would like to revisit the problem based on dimensional analysis in the context of Elliott's model in a deep surface layer where neither δ_0 nor U_∞ (therefore C_f) is well-defined. Note that this model is also applicable to an engineering-scale flat plate boundary layer where the internal layer thickness δ_i is only a small fraction of the entire boundary layer. Based on the dimensional argument, the flow should be independent of δ_0 in the limit of a small δ_i/δ_0 . In the case of a rough-to-smooth transition, the downstream surface condition is already prescribed, and the recovering flow is completely dependent on the upstream conditions k_s (or equivalently z_{01}) and $U_{\tau 0}$. As the downstream surface is a smooth wall, the kinematic viscosity of air ν (effectively the Reynolds number) is introduced to the problem despite the fully-rough conditions upstream. Quantities such as $U_{\tau 2}$ and δ_i are properties of the recovering flow, which are functions of $\hat{x}$ and dependent on the upstream conditions. When choosing $U_{\tau 0}$ as the velocity scale and $\nu/U_{\tau 0}$ as the length scale, the incoming flow condition can be fully described by a nondimensional number k_{s0}^+ , provided that δ_0 is large so it does not enter the problem. This implies that the friction velocity data in *Group I* where k_{s0}^+ is held constant should collapse to a single trend despite the changes in $Re_{\tau 0}$. This behaviour is approximately confirmed over a limited range of $0 < \hat{x}U_{\tau 0}/\nu < 1 \times 10^5$ in figure 7.2(a), and the predictions from Elliott's original model are shown by the lines for comparison. *Group II* data are presented in figure 7.2(b), where a greater drop in the friction velocity downstream is observed with a higher k_{s0}^+ . The

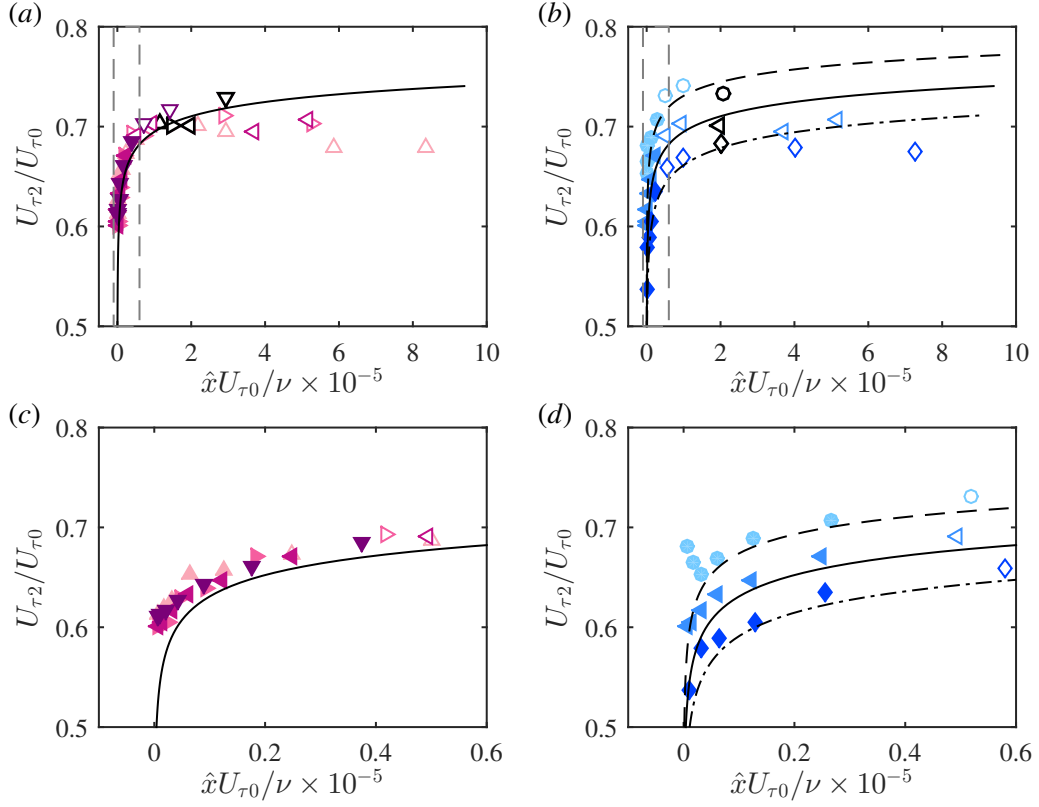


FIGURE 7.2: Friction velocities on the smooth surface normalised by U_{τ_0} , the friction velocity on the rough surface versus the viscous-scaled fetch $\hat{x}U_{\tau_0}/\nu$ for (a) *Group I* and (b) *Group II*. (c) and (d) are the corresponding magnified view in the near field. The solid lines in all figures are predictions using Elliott's model with $k_{s0}^+ = 160$, while the dashed and dash-dotted lines in (b) and (d) are with $k_{s0}^+ = 110$ and $k_{s0}^+ = 230$, respectively. Data points with $\delta_i/\delta_{99} < 0.15$ are shown by solid symbols, while the rest are shown by open symbols. Symbols with a thick black outline are at $\delta_i/\delta_{99} \approx 0.6$.

good agreement between the datapoints and the model prediction indicates that such a dependence on k_{s0}^+ is well captured by Elliott's model. As the mean velocity within and above the IBL are modelled by two equilibrium logarithmic laws, the flow recovery in the near vicinity of the step change will not be captured correctly by Elliott's model due to the non-equilibrium effect as detailed in §4. This is reflected by the under-estimation in the near-field zoomed views as shown in figures 7.2(c) and (d). In addition, the adequacy of the model is also expected to drop when δ_i exceeds the upper limit of the logarithmic layer (chosen as $z/\delta_{99} = 0.15$ in this study), as Elliott's model fails to capture the wake. In figure 7.2, the data points with $\delta_i/\delta_{99} < 0.15$ are shown by solid symbols, while the rest are shown by open symbols. For all solid symbols, the model prediction follows the measured results closely (excluding the near field). This good agreement still exists even after δ_i exceeds $0.15\delta_{99}$, until it reaches approximately $0.6\delta_{99}$ (estimated by equation (6.4) and shown by the symbols with a thick black outline). It is still not clear why the model is valid for a wider range of δ_i/δ_{99} than would be expected. One possible explanation is that the effects from those factors which are not accounted for in Elliott's original model, such as the inclusion of a wake function or the growth of the outer layer downstream, tend to cancel out. This will be discussed in more detail in §9. It is also noticed that cases with a higher Re_{τ_0} in *Group I* follow the

model prediction for a longer $\hat{x}U_{\tau 0}/\nu$, as a longer $\hat{x}U_{\tau 0}/\nu$ is required before δ_i/δ_{99} reaches 0.6.

After δ_i exceeds $0.6\delta_{99}$, $U_{\tau 2}/U_{\tau 0}$ starts to decrease with $\hat{x}U_{\tau 0}/\nu$ in both groups. For cases in *Group I*, $U_{\tau 2}/U_{\tau 0}$ is observed to fan out and increase with $Re_{\tau 0}$, which is dictated by the asymptotic behaviour of the quasi-equilibrium boundary layer. Below is a theoretical analysis of the aforementioned far-field behaviour which is not captured by Elliott's model. We continue using $U_{\tau 2}$ and $U_{\tau 0}$ to denote the friction velocities of the smooth-wall and rough-wall boundary layers, respectively, although in the following derivation we assume these two boundary layers are in equilibrium with the local surface condition (rather than in the non-equilibrium recovering state as discussed above). The boundary layer thicknesses on the smooth wall and rough wall are denoted by δ and δ_0 , respectively. Note that δ is larger than δ_0 due to the growth of the boundary layer downstream of the step change. The two boundary layers are assumed to share the same wake function $\mathcal{W}(z/\delta)$ (or $\mathcal{W}(z/\delta_0)$ on the rough wall). U_{∞}^+ for a canonical smooth-wall turbulent boundary layer can be expressed as

$$\frac{U_{\infty}}{U_{\tau 2}} = \frac{1}{\kappa} \ln\left(\frac{\delta U_{\tau 2}}{\nu}\right) + B + \frac{2\Pi}{\kappa} \mathcal{W}(1), \quad (7.4)$$

while its rough-wall counterpart is

$$\frac{U_{\infty}}{U_{\tau 0}} = \frac{1}{\kappa} \ln\left(\frac{\delta_0 U_{\tau 0}}{\nu}\right) + A'_{FR} - \frac{1}{\kappa} \ln(k_{s0}^+) + \frac{2\Pi}{\kappa} \mathcal{W}(1). \quad (7.5)$$

Therefore, the friction velocity ratio is

$$\begin{aligned} \frac{U_{\tau 2}}{U_{\tau 0}} &= \frac{\frac{1}{\kappa} \ln\left(\frac{\delta_0 U_{\tau 0}}{\nu}\right) + A'_{FR} - \frac{1}{\kappa} \ln(k_{s0}^+) + \frac{2\Pi}{\kappa} \mathcal{W}(1)}{\frac{1}{\kappa} \ln\left(\frac{\delta U_{\tau 2}}{\nu}\right) + B + \frac{2\Pi}{\kappa} \mathcal{W}(1)} \\ &= \frac{\frac{1}{\kappa} \ln\left(\frac{\delta_0 U_{\tau 0}}{\nu}\right) + A'_{FR} - \frac{1}{\kappa} \ln(k_{s0}^+) + \frac{2\Pi}{\kappa} \mathcal{W}(1)}{\frac{1}{\kappa} \ln\left(\frac{\delta_0 U_{\tau 0}}{\nu}\right) + \frac{1}{\kappa} \ln\left(\frac{U_{\tau 2}}{U_{\tau 0}}\right) + \frac{1}{\kappa} \ln\left(\frac{\delta}{\delta_0}\right) + B + \frac{2\Pi}{\kappa} \mathcal{W}(1)} \end{aligned} \quad (7.6)$$

For *Group I* cases, k_{s0}^+ is held constant. Therefore, we can define two constants

$$C_0 = A'_{FR} - \frac{1}{\kappa} \ln(k_{s0}^+) + \frac{2\Pi}{\kappa} \mathcal{W}(1)$$

$$C = B + \frac{2\Pi}{\kappa} \mathcal{W}(1)$$

and two nondimensional quantities

$$U_{\tau 2}^* = \frac{U_{\tau 2}}{U_{\tau 0}}$$

$$\delta^* = \frac{\delta}{\delta_0}.$$

Also note that $Re_{\tau 0} \equiv U_{\tau 0} \delta_0 / \nu$, thus equation (7.6) can be rearranged to

$$F(U_{\tau 2}^*, Re_{\tau 0}, \delta^*) = U_{\tau 2}^* \left(\frac{1}{\kappa} \ln Re_{\tau 0} + \frac{1}{\kappa} \ln U_{\tau 2}^* + \frac{1}{\kappa} \ln \delta^* + C \right) - \frac{1}{\kappa} \ln Re_{\tau 0} - C_0 = 0 \quad (7.7)$$

For a prescribed δ^* (which is approximately equivalent to a given fetch), the dependence of $U_{\tau 2}^*$ on $Re_{\tau 0}$ can be studied using the chain rule. Equation (7.7) is differentiated with regard to $Re_{\tau 0}$ while δ^* is held constant:

$$0 = \frac{\partial F}{\partial Re_{\tau 0}} + \frac{\partial F}{\partial U_{\tau 2}^*} \frac{dU_{\tau 2}^*}{dRe_{\tau 0}}$$

The derivative is solved as

$$\frac{dU_{\tau 2}^*}{dRe_{\tau 0}} = \frac{\frac{1}{\kappa Re_{\tau 0}} (1 - U_{\tau 2}^*)}{\frac{1}{\kappa} \ln Re_{\tau 0} + \frac{1}{\kappa} \ln U_{\tau 2}^* + \frac{1}{\kappa} \ln \delta^* + C}, \quad (7.8)$$

which is positive as $U_{\tau 2}^*$ is smaller than 1. This indicates that $U_{\tau 2}^*$ increases with $Re_{\tau 0}$ in the equilibrium asymptote in the far field, in good agreement with the measurements shown in figure 7.2(a).

The dependence of $U_{\tau 2}^*$ on δ^* for a constant $Re_{\tau 0}$ can be obtained via a similar approach. The derivative is found as

$$\frac{dU_{\tau 2}^*}{d\delta^*} = \frac{-\frac{U_{\tau 2}^*}{\kappa \delta^*}}{\frac{1}{\kappa} \ln Re_{\tau 0} + \frac{1}{\kappa} \ln U_{\tau 2}^* + \frac{1}{\kappa} \ln \delta^* + C} \quad (7.9)$$

indicating a decreasing trend in $U_{\tau 2}^*$ as δ^* increases. This agrees with the observation that in the far field, $U_{\tau 2}^*$ decreases with $\hat{x} U_{\tau 0} / \nu$ in each case in figure 7.2(a).

7.4 IBL height

The experimentally determined δ_i values (see §6.5) are also compared with the predictions using Elliott's model at the corresponding k_{s0}^+ in figure 7.3. Overall, in the limit of a large fetch, the measured δ_i appears to fall below the model prediction (see figures 7.3a and b). Elliott's model predicts that the viscous-scaled IBL thickness $\delta_i U_{\tau 0} / \nu$ will grow slightly faster with a higher upstream k_{s0}^+ , as shown in figure 7.3(b). The comparison with Elliott's model suggests that for the range of k_{s0}^+ investigated here, we would struggle to see any difference in the development of δ_i in experimental data. In future, if k_{s0}^+ effects on the growth rate of δ_i are to be investigated, a much larger variation of k_{s0}^+ will be required. In terms of experimentally determining δ_i , the internal layer becomes more dispersed as it grows, and the difference between neighbouring turbulence intensity profiles diminishes farther downstream. Therefore, the experimentally determined δ_i values are subject to higher uncertainties at a large fetch and

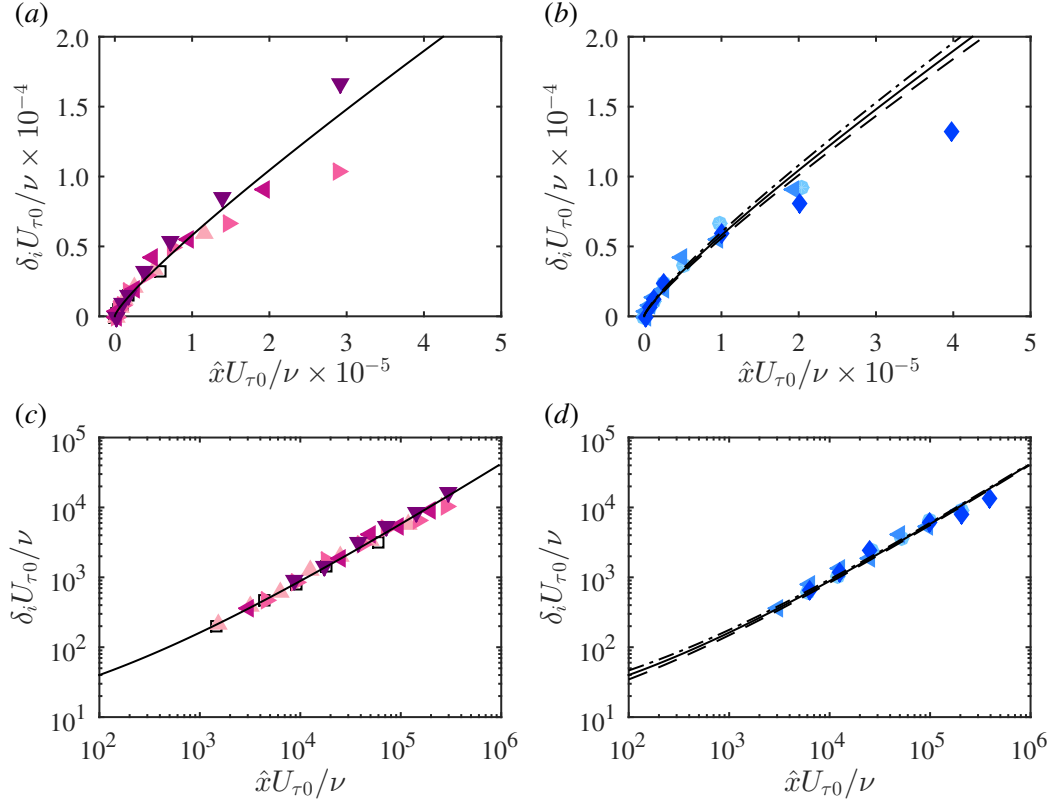


FIGURE 7.3: IBL thickness δ_i versus the fetch $\hat{x}$ over the downstream smooth surface, both normalised by the viscous scale $\nu/U_{\tau 0}$, where $U_{\tau 0}$ is the friction velocity on the rough surface in each case (symbols). *Group I* cases are shown in (a,c) and *Group II* cases are shown in (b,d). (c) and (d) are plots (a) and (b) shown in log scale, highlighting the near-field growth of δ_i . The solid lines in both columns are predictions using Elliott's model with $k_{s0}^+ = 160$, while the dashed and dash-dotted lines in (b,d) are Elliott's prediction with $k_{s0}^+ = 110$ and $k_{s0}^+ = 230$, respectively.

any interpretation on the trend should be treated with caution. Despite the underprediction of $U_{\tau}/U_{\tau 1}$ in the vicinity of the step change (see figure 7.2), the model prediction follows the measured $\delta_i U_{\tau 1}/\nu$ closely in the same region, as shown in the logarithmic axis in panels (c) and (d). In order to understand why Elliott's model has satisfactory prediction in only one of the two coupled outputs, here we take a closer look at the model assumption.

As mentioned in §7.2, $U_{\tau 2}$ and δ_i are coupled through the assumed mean velocity profile in Elliott's model. The mean velocity profile is modelled by a piecewise logarithmic function, with a slope proportional to $U_{\tau 2}$ below δ_i , and $U_{\tau 0}$ above δ_i . Here we examine the validity of this assumption by reconstructing the inner logarithmic profile as $U_{log}^+ = \frac{1}{\kappa} \ln(z^+) + B$, where $^+$ denotes an inner normalisation with the local $U_{\tau 2}$ measured by OFI. The comparison between U^+ and U_{log}^+ is shown in figure 7.4. 'Elliott's δ_i ' is defined as the wall-normal location where $U^+ - U_{log}^+ = 0$, as represented by the empty pink symbols, and we will denote it by $\delta_{i,log}$ in the remaining part of the thesis. This definition is essentially dependent on the local $U_{\tau 2}$ value rather than the shape of the measured profiles. If the mean velocity profile is the same as assumed by Elliott, then $\delta_{i,log}$ should coincide with the profile-based δ_i results. As can be seen in the figure, however, at small $\hat{x}/\delta_0$, the two profile-based δ_i values (determined from the variance profile following the approach in §6.5 as shown by the

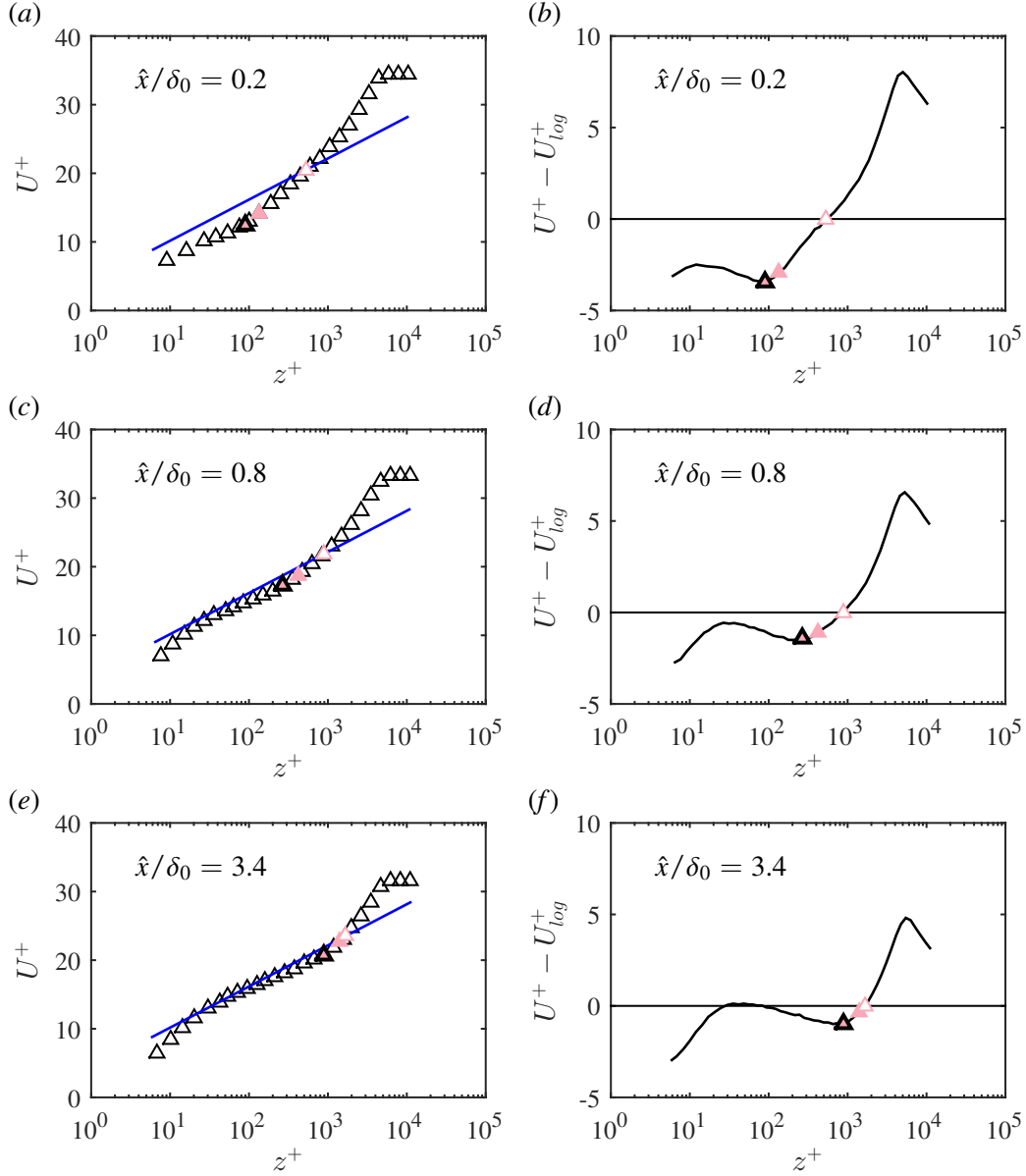


FIGURE 7.4: (a,c,e) Comparison between U^+ (empty triangles), the viscous-scaled mean velocity profile downstream of the step change and a logarithmic profile $U_{log}^+ = \frac{1}{\kappa} \ln(z^+) + B$ (solid blue line) for case Re07ks16. The right column (panels b, d and f) shows the difference between U^+ and U_{log}^+ at streamwise locations corresponding to the left column. Symbols with a pink outline represents the wall-normal position where $U^+ - U_{log}^+ = 0$, solid pink symbols represent δ_i computed using the variance-profile-based approach detailed in §6.5, and the symbols with a thick black outline represent δ_i defined as the inflection point in the U versus $z^{1/2}$ profile (Antonia and Luxton, 1971b).

solid pink symbol, and from U versus $z^{1/2}$ profile following Antonia and Luxton (1971b) as shown by the solid pink symbol with a thick black outline) are in good agreement with each other despite a small systematic difference, while both of them are much lower than Elliott's $\delta_{i,log}$ which is shown by the open pink symbol. As presented on a logarithmic abscissa, this difference between the δ_i determined by these three methods decreases further downstream of the transition. A comparison of the δ_i values within the logarithmic region is shown

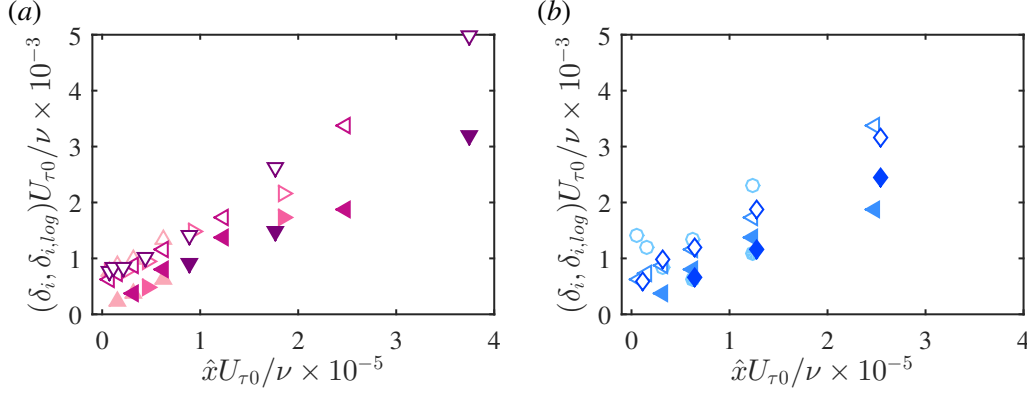


FIGURE 7.5: Comparison of δ_i determined using the current variance-profile-based approach (solid symbols) and $\delta_{i,log}$, the wall-normal position where $U^+ - U_{log}^+ = 0$ (empty symbols) for (a) *Group I* and (b) *Group II* cases.

in figure 7.5. Note that the method to compute Elliott’s $\delta_{i,log}$ is only expected to have a good performance within the logarithmic region: when $\delta_{i,log}$ enters the wake region, the intersecting location will remain as the upper limit of the logarithmic region regardless of $\delta_{i,log}$. Therefore, only $\delta_{i,log} < 0.15\delta_{99}$ are shown in figure 7.5. In both *Group I* and *Group II*, Elliott’s $\delta_{i,log}$ is systematically higher than the δ_i calculated following our current approach. This observation reiterates our previous conclusion that the flow within the IBL is not fully in equilibrium with the new wall condition, and it also provides some useful clues in refining Elliott’s original model to accommodate the non-equilibrium effect. In fact, $\delta_{i,log}$ can be approximated by the following empirical equation at least in the near field:

$$\frac{\delta_{i,log} U_{\tau 0}}{\nu} = \frac{\delta_i U_{\tau 0}}{\nu} + \Delta \delta_i^+ \quad (7.10)$$

The additive constant $\Delta \delta_i^+ = 500$ is obtained based on empirical observation, and it appears to be universal within the range of $Re_{\tau 0}$ and k_{s0}^+ available in this study, which may be interpreted as a constant level of deviation from the equilibrium state. It is not clear whether this additive constant will change in the case of extreme $Re_{\tau 0}$ or k_{s0}^+ values. Future work is required to fully address this question.

7.5 Summary and conclusions

In this chapter, we evaluate the performance of Elliott’s model in predicting the flow recovery downstream of a rough-to-smooth change using the dataset presented in §6. The main findings are summarised as follows:

1. Elliott’s model is able to capture the general recovery trend of the local friction velocity ratio $U_{\tau 2}/U_{\tau 0}$, but an under-prediction is observed in the very near field and an over-prediction is seen in the far field. The under-prediction in the near field is rooted in the non-equilibrium behaviour of the mean velocity profile not captured by the log law used in Elliott’s model. The over-prediction in the far field is dictated by the governing equation (7.3) that $U_{\tau 2}$ will continue to increase and asymptote to $U_{\tau 1}$, which fails to

capture the decrease of $U_{\tau 2}$ with an increasing Reynolds number once a full recovery is reached. It might also be related to the absence of the wake function in the outer layer.

2. The prediction of $U_{\tau 2}$ and δ_i is coupled through the assumed form of the mean velocity profile in the IBL. The logarithmic velocity profile assumed in Elliott's model does not account for the non-equilibrium behaviour in the IBL, therefore, it cannot predict the correct relation between $U_{\tau 2}$ and δ_i . To reconcile this issue, we distinguish the IBL thickness derived from the correct $U_{\tau 2}$ and Elliott's assumed log-law profile ($\delta_{i,log}$) from the one defined based on the measured profile (δ_i). An empirical relation (7.10) between δ_i and $\delta_{i,log}$ is extracted from the data.

Chapter 8

Blending model of the mean velocity profile

In this chapter, based on the non-equilibrium behaviour observed in §4 and §6, we propose a semi-empirical approach to model the mean velocity profile within the IBL, which takes these non-equilibrium effects into consideration. The key assumption of the model is that the flow in the IBL gradually transitions from resembling a smooth-wall boundary layer at the wall to resembling a rough-wall boundary layer at the upper limit of the IBL, which can be parametrised as a ‘blending’ process between the corresponding equilibrium smooth- and rough-wall profiles. A functional form and coefficients are proposed using the two groups of HRNBLWT hotwire datasets. The blending function exhibits no dependence on $Re_{\tau 0}$ or k_{s0}^+ within the range of data under examination. The effect of the step height on the blending model has also been studied utilising the lower Reynolds number dataset, and it has been found that with a slight modification, the blending model can be applied to cases with various step heights where the downstream surface is lower than the upstream one. In its current form, the blending model unfortunately cannot accurately describe the flow where the downstream surface is higher than the upstream one. A new functional form will be required for such scenarios.

8.1 Introduction

In the seminal work of Elliott (1958), a theoretical model of a turbulent boundary layer adapting to a step change in the surface condition was developed by evolving an assumed mean velocity profile in the streamwise direction using the momentum equation. The flow both within and above the IBL is modelled by a logarithmic law with different values of friction velocity, representing the stress jump that is assumed to occur at the edge of the IBL. Within the IBL, the logarithmic region scales with the local friction velocity, while beyond the IBL, the logarithmic region scales with the upstream reference friction velocity (prior to the transition in surface condition). The assumption of a logarithmic profile in the IBL implies that the mean velocity profile there is the same as that of a canonical equilibrium turbulent boundary layer. However, many recent studies of a rough-to-smooth transition have suggested a persistence of non-equilibrium behaviour within the IBL (Antonia and Luxton, 1971b; Ismail et al., 2018b; Rouhi et al., 2019b; Li et al., 2019b). Similar observations have also been made in the current dataset in §4 and §6. Therefore, in this chapter we would like to propose a semi-empirical approach to model the mean velocity profile within the IBL, which takes the non-equilibrium behaviour of the flow into consideration. This velocity profile model will later be used to construct the full prediction of the flow evolution in §9.

8.2 Description of the model

A schematic of the model is illustrated in figure 8.1(a). Downstream of a rough-to-smooth change, the flow very close to the wall is expected to have fully adapted to the new wall condition, so it can be approximated by an equilibrium smooth-wall profile. Above the IBL, the flow is expected to follow the upstream rough-wall scaling and is approximated by an equilibrium rough-wall profile. We use blue and red colours, respectively, to represent the fully-smooth and fully-rough state of the flow. With an increasing z , we expect the mean flow to gradually transition from the smooth-wall asymptote and be increasingly similar to the rough-wall one, as illustrated by the transition from blue to red in figure 8.1(a).

A mathematical description of the qualitative model outlined above is detailed as follows. In the near-wall region where the flow is in full equilibrium with the local smooth wall condition (shown by the blue coloured region in figure 8.1a), the mean velocity distribution satisfies

$$\frac{U}{U_{\tau 2}} = f(z^+) = \frac{U_S}{U_{\tau S}}, \quad (8.1)$$

where $U_{\tau 2}$ is the local friction velocity measured by OFI on the smooth wall, $z^+ = zU_{\tau 2}/\nu$ for the rough-to-smooth profile and $z^+ = z_S U_{\tau S}/\nu$ for the smooth-wall reference. The subscript $(\cdot)_S$ denotes quantities of the smooth-wall reference. Similarly, above the IBL, the outer-layer similarity is satisfied when normalised by the correct velocity scale, of which $U_{\tau R}$ has been found to be a good approximation when the fetch is not too large (see §6.7). The region where the upstream rough-wall scaling is supposed to hold is shown by the red patch

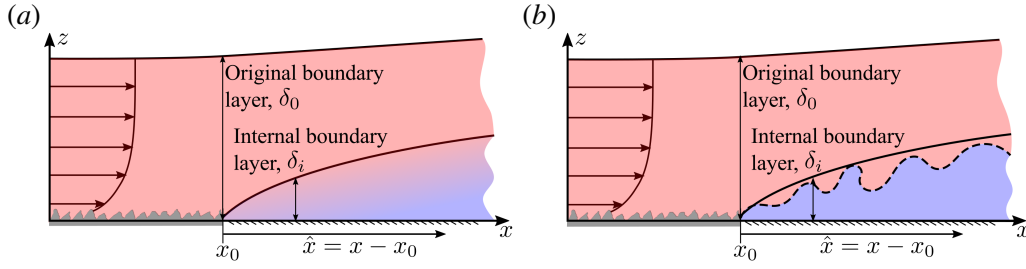


FIGURE 8.1: Sketch of (a) time-averaged and (b) instantaneous field of a turbulent boundary layer over a rough-to-smooth change. The flow is from left to right. The red colour represents regions with rough-wall flow features, and the blue colour represents regions with smooth-wall features. The solid curve in both figures indicates the upper limit of the IBL, and the dashed curve in (b) is an (assumed) instantaneous interface of the IBL.

in figure 8.1(a). The velocity deficit in the outer layer can be expressed as

$$\frac{U_\infty - U}{U_{\tau R}} = g(\eta) = \frac{U_\infty - U_R}{U_{\tau R}}, \quad (8.2)$$

where η is the wall-normal distance normalised by the boundary layer thickness of the corresponding velocity profile, i.e. $\eta = z/\delta_{99}$ for the rough-to-smooth profile and $\eta = z_R/\delta_R$ for the rough-wall reference. The subscript $(\cdot)_R$ denotes quantities of the rough-wall reference. Note that here the velocity scale is selected as $U_{\tau R}$, the rough-wall friction velocity, for both upstream and downstream mean velocity profiles. (8.2) essentially reduces to $U(\eta) = U_R(\eta)$, therefore a rough-wall velocity profile can be recovered above the IBL (which will include a log and a wake region).

In order to combine the two limiting cases described in (8.1) and (8.2) and demonstrate how the recovering profile transitions from the smooth-wall to the rough-wall asymptote as the wall-normal distance increases, we define

$$U_R^{*+} = \frac{U_R}{U_{\tau 2}}, \quad (8.3)$$

and plot it against

$$z_R^{*+} = \eta \frac{\delta_{99} U_{\tau 2}}{\nu} = \frac{\delta_{99}}{\delta_R} \frac{z_R U_{\tau 2}}{\nu} \quad (8.4)$$

on the inner-normalised mean velocity profiles as shown in figure 8.2. Note that by multiplying the viscous-scaled z_R by the ratio of δ_{99}/δ_R , we account for the growth of the boundary layer as it evolves downstream. The U_R^{*+} versus z_R^{*+} profile is designed to match the viscous-scaled U^+ versus z^+ profile beyond the IBL through an assumed outer-layer similarity (8.2), representing the outer layer which is scaled on with the upstream rough-wall velocity scale. The asterisk in the notation is to distinguish it from the conventional inner normalisation where the corresponding friction velocity $U_{\tau R}$ is used. The smooth-wall reference is chosen as a composite velocity profile (Chauhan et al., 2009) with the same Re_τ as the rough-to-smooth profile, and the rough-wall reference is obtained from the hotwire survey just upstream of the roughness transition at matched flow conditions.

To model such a transition in the wall-normal direction, we consider a conceptual model

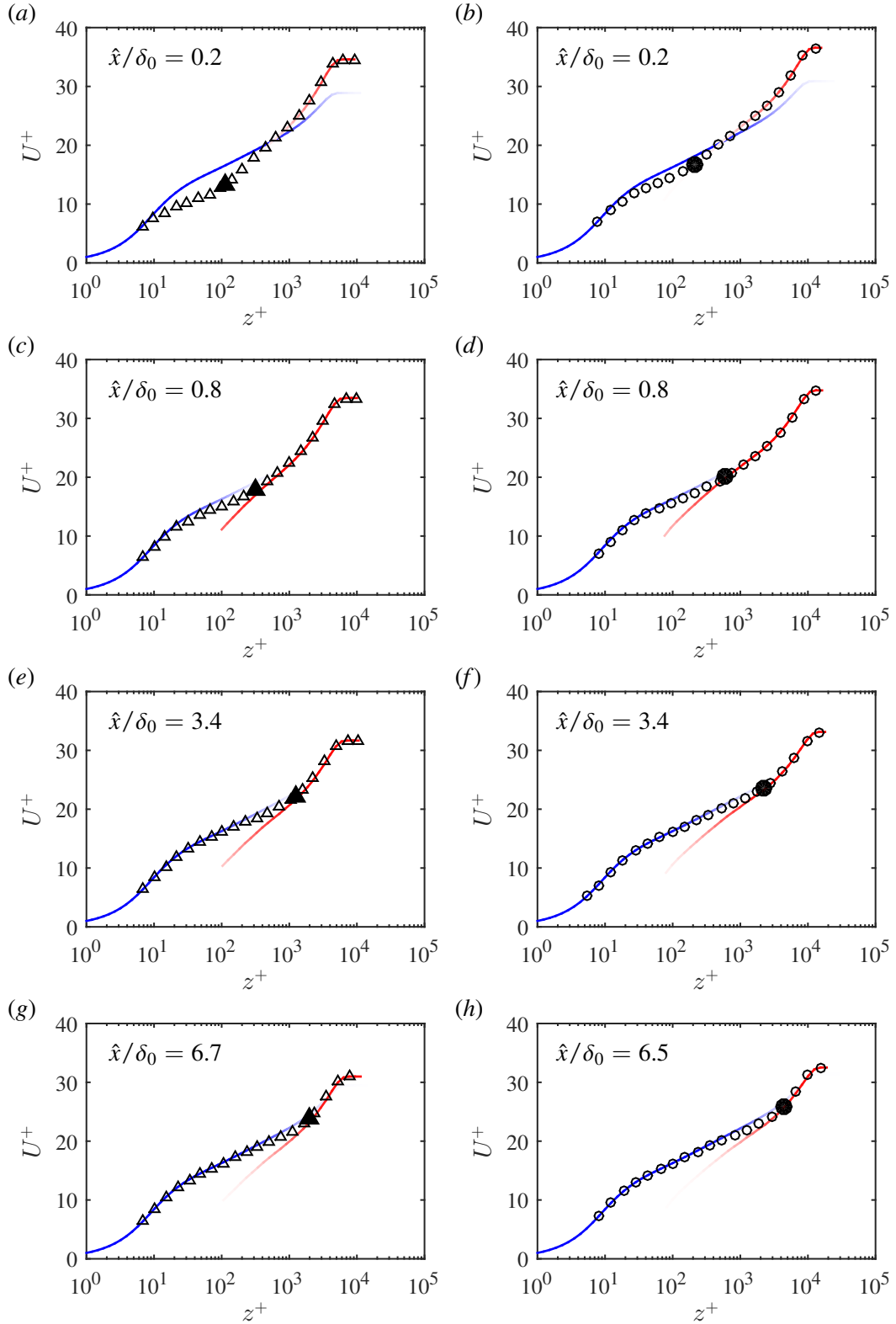


FIGURE 8.2: Viscous-scaled mean velocity profiles downstream of a rough-to-smooth change (empty symbols) compared to the equilibrium rough-wall and smooth-wall profiles. The blue line is U_S^+ and the red line is U_R^+ . The solid black symbol represents the location of δ_i extracted from the turbulence intensity profile as detailed in §6.5. (a,c,e,g) are from case Re07ks16 and (b,d,f,h) are from case Re14ks11.

of an instantaneous interface between the structures which still preserve the upstream rough-wall scales and those that have adapted to the new downstream wall conditions, as illustrated in figure 8.1(b). This interface bears some resemblance with the turbulent/non-turbulent interface (TNTI) at the edge of a boundary layer (Chauhan et al., 2014; Krug et al., 2017), but instead of separating the turbulent regions from the non-turbulent free stream as is the case for the TNTI, the interface here separates two ‘flavours’ of turbulence: one with rough-wall (red) and the other with smooth-wall (blue) characteristics. Similar to a TNTI, this interface also fluctuates between a lower bound close to the wall and an upper bound of δ_i^+ (as dictated by its definition) in the current study. There is some evidence of the existence of such a fluctuating interface as captured by identifying the uniform momentum zones in the near field of an aggressive smooth-to-rough change (Hearst and Ganapathisubramani, 2019). However, a sharp interface probably does not exist further downstream or in the case of a less aggressive roughness transition and we would like to emphasise that by showing figure 8.1(b) we do not claim the existence of such a distinct separation between regions with fully-equilibrium rough-wall and smooth-wall structures.

Without losing generality, we express the recovering mean velocity profile as

$$U^+ = U_S^+ E + U_R^{*+} (1 - E), \quad (8.5)$$

following Krug et al. (2017). Here we do not prescribe a functional form of the blending function $E(z^+)$. At this stage, the only requirement of $E(z^+)$ is that it decreases monotonically from 1 to 0 when z^+ increases from 0 to δ_i^+ , corresponding to the limiting cases as the flow fully follows the smooth-wall profile in the vicinity of the wall, and it entirely preserves the upstream rough-wall scaling above the IBL.

The blending function can be derived by restating equation (8.5) as

$$E = \frac{U^+ - U_R^{*+}}{U_S^+ - U_R^{*+}}. \quad (8.6)$$

Here U_S^+ is the equilibrium smooth-wall composite profile, U_R^+ comes from the measured rough-wall profile upstream of the transition, and we have measured U^+ at multiple locations downstream of the transition. The experimentally obtained blending function $E(z^+)$ is shown by the symbols in figure 8.3 for cases Re07ks16 (a) and Re14ks11 (b) as an example. The experimentally measured behaviour of E appears to be well captured by a log-normal distribution of the interface location:

$$E(z^+; \mu, \sigma) = \frac{1}{2} \left[1 - \operatorname{erf} \left(\frac{\ln(z^+) - \mu}{\sqrt{2}\sigma} \right) \right], \quad (8.7)$$

as shown by the solid lines in figure 8.3. The mean μ and standard deviation σ obtained by fitting (8.7) for all cases are summarised in figure 8.4. As expected, the averaged location of the flapping interface (captured by μ) increases as δ_i^+ increases with an increasing downstream fetch (figure 8.4a), while the standard deviation of the flapping interface (σ) appears to remain approximately constant for the entire range. Little $Re_{\tau 0}$ or k_{s0}^+ dependence

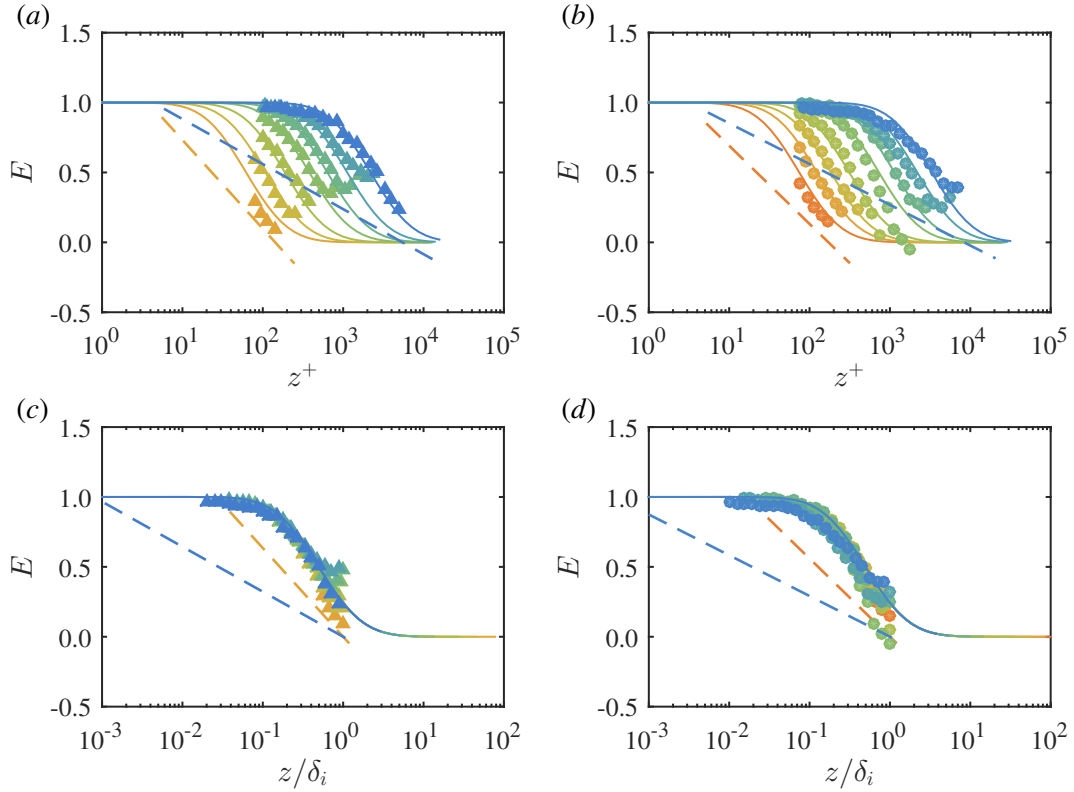


FIGURE 8.3: E for (a, c) Re07ks16 and (b, d) Re14ks11. Shading indicates the downstream fetch: $\hat{x}$ increases as the colour changes from yellow to blue in each case. The symbols are computed from each hotwire profile from the wall up to the edge of the IBL following (8.6), and the solid lines with corresponding colours represent E reconstructed using (8.7) with the parameters given by (8.8). The dashed lines are equation (8.10) at corresponding streamwise locations. (c) and (d) are the original and reconstructed E plotted against z/δ_i .

is observed from the data, and a general trend for all cases is found to be

$$\mu = \ln(\delta_i^+) - 0.7, \quad \sigma = 1.0. \quad (8.8)$$

The solid coloured lines in figure 8.3 show the profiles of E obtained from equations (8.7) and (8.8), which compare very well with the experimentally determined E (shown by the symbols). Subsequently, the recovering mean velocity profile can be obtained by substituting the reconstructed E into (8.5). The results are shown by the thick lines with a colour transitioning from blue to red in figure 8.5, where they are compared to the experimental data (symbols) and the assumed equilibrium profiles (in red and blue). The reconstructed mean velocity profiles are in a better agreement with the experimental data compared to the equilibrium U_s^+ profiles. The improvement is especially notable close to the roughness transition.

After substituting the fitted expressions (8.8) into (8.7), it is found that the blending function reduces to a self-similar form:

$$E(z/\delta_i) = \frac{1}{2} \left[1 - \operatorname{erf} \left(\frac{\ln(z/\delta_i) + 0.7}{\sqrt{2}} \right) \right]. \quad (8.9)$$

Here, δ_i is the characteristic length scale and E is a function of one single variable, z/δ_i . The

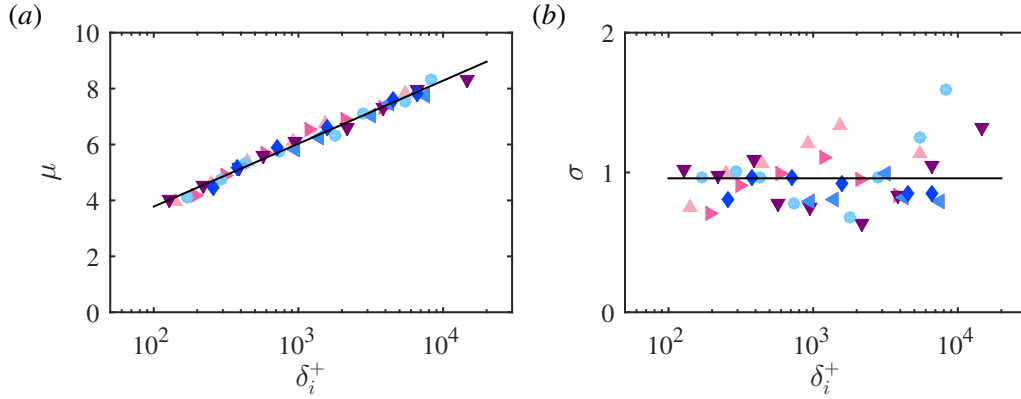


FIGURE 8.4: (a) The mean μ and (b) standard deviation σ of all cases obtained by fitting (8.7) to $E(z^+)$ determined experimentally from (8.6). Expressions for the solid lines in the figures are given by (8.8).

existence of the self-similar behaviour of E is further confirmed by plotting the measured E against z/δ_i . As shown in figure 8.3(c,d), data from all streamwise locations in both cases collapse on to a single trend that follows (8.9) closely (shown by the solid line). Note that although the level of scatter in E appears to increase slightly in the vicinity of $z/\delta_i = 1$, the agreement between the measured and reconstructed mean velocity profile is barely affected since the denominator $U_S^+ - U_R^{*+}$ in (8.6) is small in this region.

Chamorro and Porté-Agel (2009) proposed a similar method to model the non-equilibrium velocity profile with a blending function rearranged to be consistent with notations in this study as

$$E_c(z^+) = 1 - \ln \left(\frac{z^+}{k_{s0}^+ \exp(-\kappa A'_{fr})} \frac{U_{\tau R}}{U_{\tau 2}} \right) \bigg/ \ln \left(\frac{\delta_i^+}{k_{s0}^+ \exp(-\kappa A'_{fr})} \frac{U_{\tau R}}{U_{\tau 2}} \right). \quad (8.10)$$

This blending function E_c at the first and the last streamwise locations is shown in figure 8.3 by the dashed lines with corresponding colours. Note that the equilibrium smooth-wall and rough-wall profiles were originally modelled by the logarithmic law only, therefore a fair comparison can only be drawn in the expected logarithmic region. A better agreement with the symbols is observed closer to the step change, while the predicted E_c falls below the measured data points downstream, resulting in an over-estimation of the contribution from the rough-wall profile during blending. Regardless, equation (8.7) captures the trend of the data points better than the log-linear relationship in (8.10), therefore we will continue with our formulae of the blending function for the remainder of this study.

8.3 Influence of the step height

So far, a mean velocity blending model has been developed with a dataset where the step heights for all cases are the same. Here, we would like to make use of the dataset reported in §5 (cases H0-H4) and examine the applicability of the model to various step-height cases.

We start with determining the blending function and the coefficients following the same procedure as described in §8.2. Using (8.6), the blending function E is evaluated for all four cases, and the coefficients μ and σ are obtained subsequently via fitting the data to an

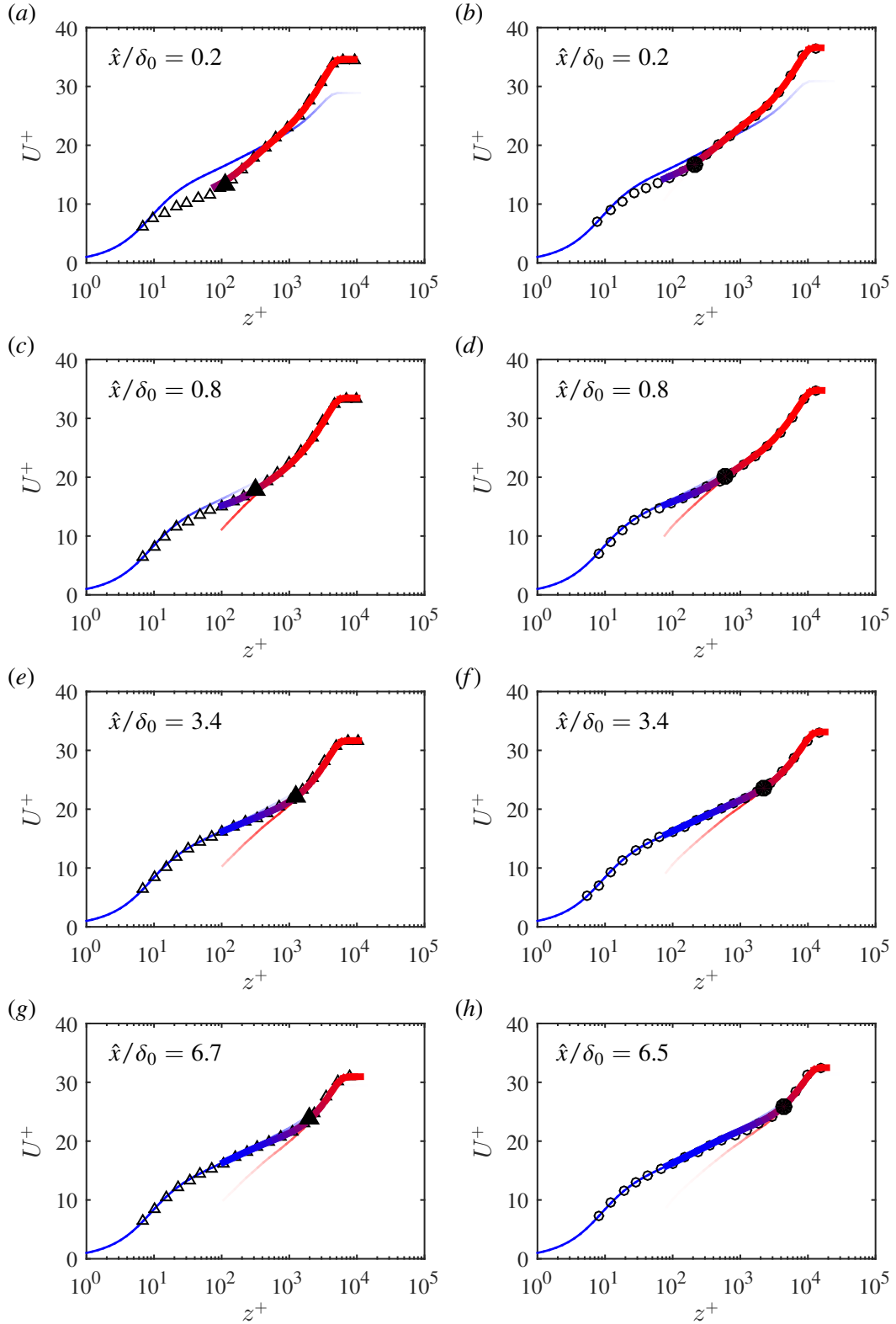


FIGURE 8.5: Comparison of the measured (empty symbols) and reconstructed (thick lines with colour transitioning from blue to red) mean velocity profiles downstream of a rough-to-smooth change. The reconstructed velocity profile is obtained using (8.6) and (8.7) with parameters given by (8.8). The blue line is the equilibrium smooth-wall profile U_S^+ and the red line is U_R^+ . The solid black symbol represents the location of δ_i extracted from the turbulence intensity profile as detailed in §6.5. (a,c,e,g) are from case Re07ks16 and (b,d,f,h) are from case Re14ks11.

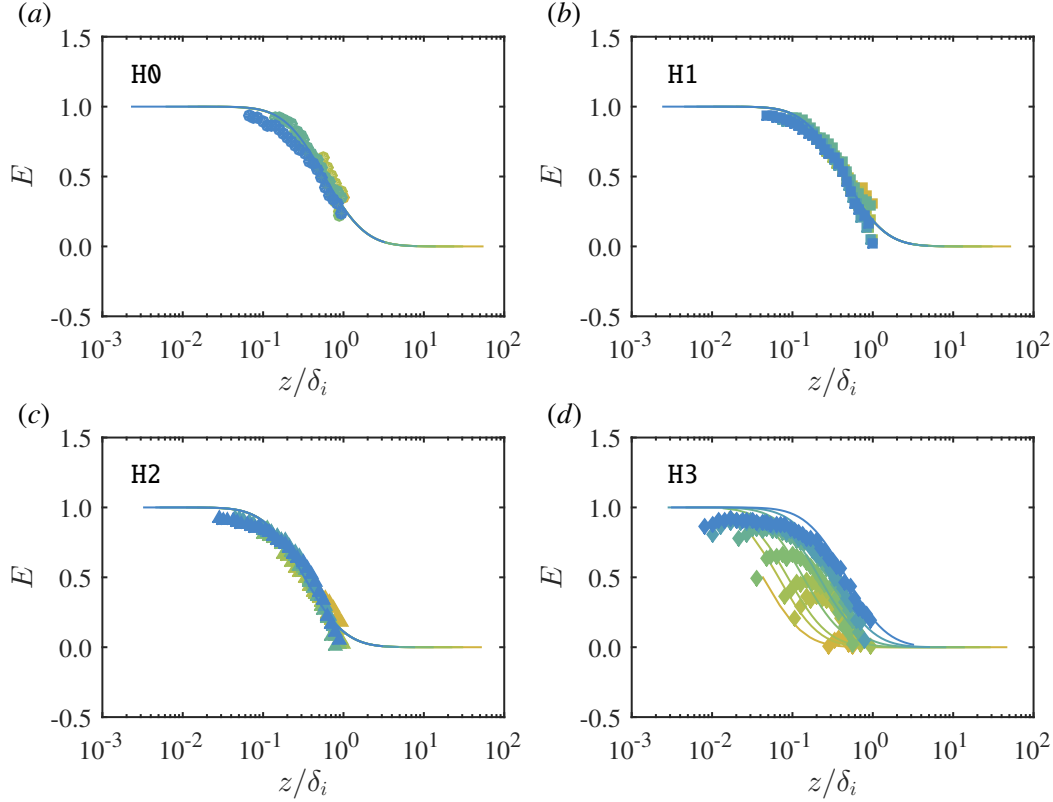


FIGURE 8.6: E for cases H0-H3. Shading indicates the downstream fetch: $\hat{x}$ increases as the colour changes from yellow to blue in each case. The symbols are computed from each hotwire profile from the wall up to δ_i^+ following (8.6), and the solid lines with corresponding colours represent E reconstructed using (8.7) with μ and σ obtained from the fit as shown in figure 8.7.

error function (8.7). The results are shown in figures 8.6 and 8.7, respectively. The blending function E follows the trend of an error function closely in cases H0-H2, while in case H3, the shape of E is different and no longer exhibits self-similarity when plotted against z/δ_i . For case H3, E is no longer monotonically decreasing (with an increasing z/δ_i) in the near-wall region. This cannot be captured by the error function model, suggesting that an alternative functional form of E is required for ‘step-up’ cases where $\Delta H > 0$. The coefficients μ and σ also show a different trend in H3 compared to the highly similar behaviour in cases H0-H2. Figure 8.7 shows that the first three cases (H0-H2) have very similar trends, exhibiting a weak dependence on the step height with μ slightly decreasing from H0 to H2 (indicating that when normalised by δ_i , the blending occurs at a location closer to the smooth wall in H2 compared to H0). The standard deviation of the flapping interface σ is observed to fluctuate about 1 with no distinguishable trend with the step height. Comparing figure 8.7 with figure 8.4, we may conclude that the coefficients μ and σ of cases H0-H2 are very similar to those obtained from the HRNBLWT dataset. Cases H0-H2 have an $Re_{\tau 0} \approx 4100$, which is well below the Reynolds number range of 7100 to 21000 in the HRNBLWT dataset, suggesting that the blending model (and the related coefficients) exhibits universality across a wide range of Reynolds numbers.

To further quantify the influence of the step height on the blending model, we continue

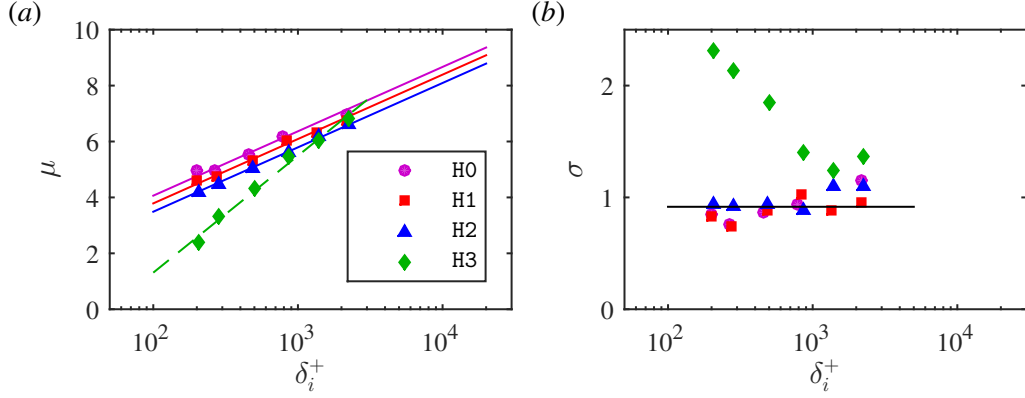


FIGURE 8.7: (a) The mean μ and (b) standard deviation σ of all step height cases obtained by fitting (8.7) to $E(z^+)$ determined experimentally from (8.6). The solid lines in (a) are (8.11) with $\mu_0 = 0.5, 0.8$ and 1.1 , respectively, and the dashed line is the best fit to the green diamonds yielding $\mu = 1.8 \ln(\delta_i^+) - 7.1$. The solid black line in (b) shows the averaged σ from cases H0-H2, which is approximately 1.

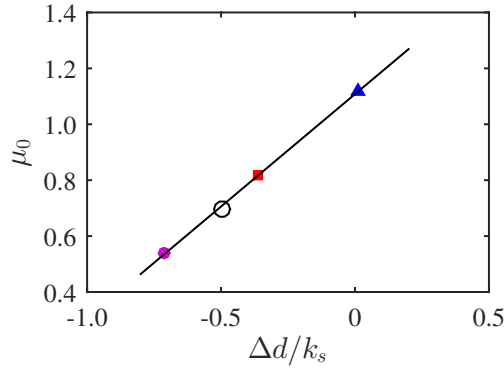


FIGURE 8.8: The additive constant μ_0 in (8.11) plotted against the normalised step height $\Delta d/k_s$, with coloured symbols corresponding to cases H0-H2 and the empty black circle the HRNBLWT dataset. The solid black line is (8.12).

with the assumption of self-similar E profiles and fit a linear trend to μ , with a modified version of equation (8.8) that incorporates a step dependent additive constant μ_0 ,

$$\mu = \ln(\delta_i^+) - \mu_0. \quad (8.11)$$

This additive constant μ_0 , as obtained from the fit above for cases H0-H2 is plotted against the normalised step height, $\Delta d/k_s$, in figure 8.8. The result from the HRNBLWT dataset is also included in the figure for comparison (as the open symbol). The data points follow a linear trend with $\Delta d/k_s$ and a curve fit results in an expression of

$$\mu_0 = 0.8(\Delta d/k_s) + 1.1. \quad (8.12)$$

In summary, combining equations (8.7), (8.11) and (8.12), the blending function E can be expressed as

$$E(z/\delta_i) = \frac{1}{2} \left[1 - \operatorname{erf} \left(\frac{\ln(z/\delta_i) + 0.8(\Delta d/k_s) + 1.1}{\sqrt{2}} \right) \right]. \quad (8.13)$$

The blending model above is developed for a rough-to-smooth change in the surface condition accompanied by a small displacement in the virtual origin. Both the roughness height and the step height should be small compared to the boundary layer thickness, and the model has only been tested in a certain range of step heights where the flow resembles a backward-facing step. The model may not work in scenarios where the change is more aggressive. We have also shown that modification to the current form of E is required to accommodate ‘step-up’ cases, where the E obtained from the measured velocity profiles have a very different shape and are not self-similar. To further validate the trend in figure 8.8, more data covering a wider range of roughness size, geometry and step height are necessary in future works.

8.4 Summary and conclusions

In this chapter, we utilise the extensive experimental dataset presented in §5 and §6 to develop a blending model of the mean velocity profile. The modelled mean velocity profile approaches an equilibrium smooth-wall velocity profile in the near-wall region, and it asymptotes to the upstream rough-wall profile above the IBL. The non-equilibrium behaviour of the mean velocity profile within the IBL is captured in the model as a gradual change from the smooth-wall to the rough-wall state with z increasing from 0 to δ_i , which is characterised by a blending function E decreasing monotonically from 1 to 0. The main findings are summarised below:

1. The blending function E can be well approximated by an error function (8.7). The coefficients μ and σ obtained by fitting the data to (8.7) appear to be universal across the range of $Re_{\tau 0}$ and k_{s0}^+ examined in this study. We also show that after substituting the fitted μ and σ values, E reduces to a self-similar form, which is a function of one single variable z/δ_i .
2. The blending model of the mean velocity profile can be related to a conceptual model of a flapping interface separating the flow regions instantaneously occupied by rough-wall and smooth-wall structures.
3. The blending model can also be applied to cases with various step heights with a slight modification of the coefficient μ , whereas σ is barely affected by the change of step height. Note that the aforementioned self-similar form of E is only applicable to cases with a negative ΔH (i.e. the downstream smooth wall is located below the upstream roughness crest).

Chapter 9

Evolution model

In this chapter, we develop an analytical model to predict the mean flow downstream of a step-change in the surface roughness where upstream flow conditions are given. We first revisit the classical model of Elliott (1958), who modelled the velocity distribution within and above the internal layer with a piecewise logarithmic profile, and evolved the velocity profile with the streamwise momentum equation. A numerical solution of the coupled system is developed following a similar approach as in Monty et al. (2016), enabling the study of velocity profiles with more complicated forms than a simple logarithmic law and applying more constraints to the system. With the aid of the numerical implementation, we examine the effect to the final predictions of various refinements, including adding a wake function to the velocity profile, using a more realistic shear stress profile in the momentum equation and considering the growth of the entire boundary layer in the streamwise direction. In particular, we implement the blending model (see §8) to account for the non-equilibrium behaviour of the flow within the internal layer, leading to an improvement in the agreement between the prediction and the measurement especially in the vicinity of the rough-to-smooth change.

9.1 Numerical implementation of Elliott's model

In the seminal work by Elliott (1958), a theoretical model of an atmospheric boundary layer over a step change of roughness was proposed. The original model is applicable to step changes between two arbitrary roughness levels, including the rough-to-smooth case which is the focus of the present study. The roughness length z_0 is used to represent the surface roughness, which can be related to the equivalent sand grain roughness k_s of a rough surface by

$$z_0 = k_s \exp(-\kappa A'_{fr}) \quad (9.1)$$

and the wall unit of a smooth surface by

$$z_0 = \frac{\nu}{U_{\tau 2}(\hat{x})} \exp(-\kappa B). \quad (9.2)$$

Here, the constant values are chosen as $\kappa = 0.384$, $B = 4.17$ (Chauhan et al., 2009) and $A'_{fr} = 8.5$ (Nikuradse, 1950).

A schematic of the flow configuration is shown in figure 9.1(a). A flow fully in equilibrium with the upstream surface goes through a change of the surface roughness at $\hat{x} = 0$, and an IBL with thickness of $\delta_i(\hat{x})$ forms downstream. While flow beyond the IBL is still in equilibrium with the upstream surface, the flow inside the IBL is considered to be in equilibrium with the new wall condition with a local friction velocity $U_{\tau 2}(\hat{x})$ that evolves downstream. The mean velocity profile is modelled as a piecewise logarithmic function:

$$U(z) = \begin{cases} \frac{U_{\tau 1}}{\kappa} \ln\left(\frac{z}{z_{01}}\right), & \text{for } z \geq \delta_i, \\ \frac{U_{\tau 2}}{\kappa} \ln\left(\frac{z}{z_{02}}\right), & \text{for } z < \delta_i. \end{cases} \quad (9.3)$$

One constraint that rises naturally from the assumed velocity profile is the continuity of the piecewise function at $z = \delta_i$:

$$U(\delta_i) = \frac{U_{\tau 1}}{\kappa} \ln\left(\frac{\delta_i}{z_{01}}\right) = \frac{U_{\tau 2}}{\kappa} \ln\left(\frac{\delta_i}{z_{02}}\right). \quad (9.4)$$

The velocity profile is then evolved downstream with an integrated momentum equation of the flow within the IBL:

$$\frac{d}{dx} \int_{z_{02}}^{\delta_i} U^2(\hat{x}, z) dz - U_i(\hat{x}) \frac{d}{dx} \int_{z_{02}}^{\delta_i} U(\hat{x}, z) dz = \frac{\Delta\tau(\hat{x})}{\rho}, \quad (9.5)$$

where $\Delta\tau(\hat{x}) = \tau(\hat{x}, 0) - \tau(\hat{x}, \delta_i(\hat{x}))$ is the net shear stress applied on the control volume. The shear stress at the wall is $\tau(\hat{x}, 0) = \rho U_{\tau 2}^2(\hat{x})$ by definition, while Elliott assumed that the upper boundary of the control volume resides within the constant-stress layer which is in equilibrium with the upstream surface, so the shear stress can be expressed as $\tau(\hat{x}, \delta_i) = \rho U_{\tau 1}^2$. Therefore, $\Delta\tau(\hat{x}) = \rho U_{\tau 2}^2(\hat{x}) - \rho U_{\tau 1}^2$. The lower bound of the integral, z_{02} , is chosen as the lowest wall-normal position before U becomes negative according to the mean velocity profile in (9.3). To simplify the notation, we denote the mean velocity at $z = \delta_i$ as U_i , i.e.

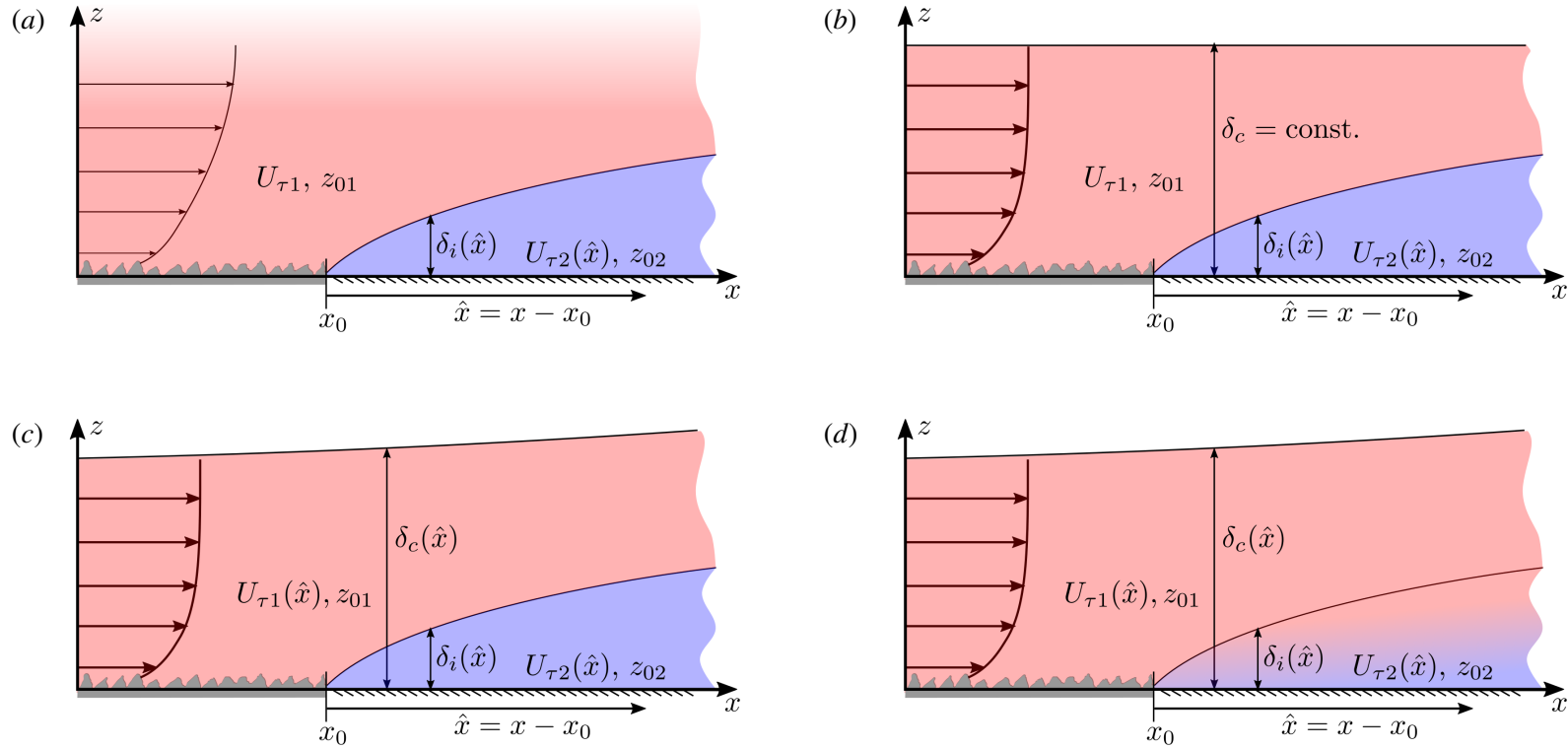


FIGURE 9.1: Schematics of the IBL with various modelling assumptions. The flow direction is from left to right. A step change from rough to smooth is currently depicted in the figure, but Elliott's model is also applicable to other scenarios (e.g. smooth-to-rough or rough-to-rougher). (a) Original Elliott's model, where the outer layer thickness is assumed to be infinite, as summarised in §9.1. (b) A finite layer thickness δ_c and a wake function is taken into consideration (§9.3). (c) Streamwise evolution of the outer layer is taken into consideration (i.e. both δ_c and $U_{\tau 1}$ are now functions of $\hat{x}$, see §9.5). (d) The mean velocity profile in the IBL is modelled by the blending model (§9.6).

$$U_i(\hat{x}) \equiv U(\hat{x}, \delta_i).$$

To summarise, so far $\hat{x}$ has been used as the independent variable. The model has two unknowns, $U_{\tau 2}(\hat{x})$ and $\delta_i(\hat{x})$ that are dependent on $\hat{x}$. Two equations (9.4 and 9.5) are chosen to close the system. To obtain the functional form of $U_{\tau 2}(\hat{x})$ and $\delta_i(\hat{x})$, Elliott (1958) solved the ordinary differential equation (9.5) analytically. This approach is highly dependent on the analytical form of the mean velocity profile and can be extremely tedious or even impossible to find an algebraic solution when the functional form of the mean velocity profile changes. Alternatively, we may also consider the integral form of (9.5):

$$\hat{x}(\delta_i) = \int_{G_2(\delta_i(0))}^{G_2(\delta_i)} \frac{\rho}{\Delta\tau(\delta_i)} dG_2(\delta_i) - \int_{G_1(\delta_i(0))}^{G_1(\delta_i)} \frac{\rho U_i(\delta_i)}{\Delta\tau(\delta_i)} dG_1(\delta_i) \quad (9.6)$$

$$= \int_{\delta_i(0)}^{\delta_i} \frac{\rho}{\Delta\tau(\delta_i)} G_2'(\delta_i) d\delta_i - \int_{\delta_i(0)}^{\delta_i} \frac{\rho U_i(\delta_i)}{\Delta\tau(\delta_i)} G_1'(\delta_i) d\delta_i \quad (9.7)$$

where

$$G_1(\delta_i) = \int_{z_{02}}^{\delta_i} U(\delta_i, z) dz, \quad G_2(\delta_i) = \int_{z_{02}}^{\delta_i} U^2(\delta_i, z) dz \quad (9.8)$$

and the prime (') denotes differentiation. Here, δ_i rather than $\hat{x}$ is considered as the independent variable for computational convenience. Thus, U_i will be a function of δ_i and the two-dimensional mean velocity field $U(\hat{x}, z)$ will be a function of δ_i and z instead. The initial condition of the IBL evolution $\delta_i(0)$, i.e. $\delta_i = \delta_i(0)$ at $\hat{x} = 0$. The choice of $\delta_i(0)$ will be discussed in more detail in §9.2. In practice, an array of δ_i locations is generated, which subsequently determines $U_{\tau 2}(\delta_i)$ and the mean velocity profile $U(\delta_i, z)$ using (9.4), the relation from the continuity of the mean velocity profile at δ_i . The fetch $\hat{x}$ corresponding to each δ_i will then be computed from (9.6) numerically. Compared to the original solution of Elliott (1958), this approach can easily accommodate velocity profiles with more complicated functional forms, which will largely serve our intention on refining the model assumptions. We also note that when presenting the model predictions graphically, we revert to the previous

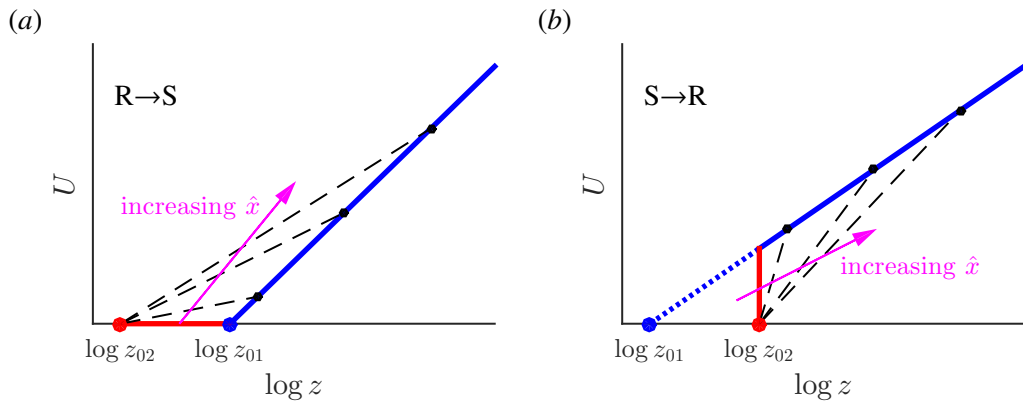


FIGURE 9.2: An illustration of the mean velocity profile (9.3) for (a) $z_{01} > z_{02}$ (rough-to-smooth) and (b) $z_{01} < z_{02}$ (smooth-to-rough). The blue lines in both panels are the upstream velocity profile, and the red line is the velocity profile within the IBL at $\hat{x} = 0$. The black dots are a few representative locations of δ_i farther downstream and the black dashed lines are the velocity profile within the IBL corresponding to these δ_i values.

convention of using $\hat{x}$ as the independent variable (i.e. plotting physical quantities against $\hat{x}$) to be consistent with the literature.

9.2 Initial condition

The initial condition is chosen as $\delta_i(0) = \max(z_{01}, z_{02})$, as illustrated by the red lines in figure 9.2. This is the smallest value δ_i can take without resulting in a negative U or $U_{\tau 2}$. The initial value of $U_{\tau 2}$ is determined subsequently via (9.4). It is possible to start the evolution with a higher δ_i at $\hat{x} = 0$ and a few examples are shown in figure 9.3(a) and (b). This figure shows the evolution of $\delta_i U_{\tau 1} / \nu$ as a function of $\hat{x} U_{\tau 1} / \nu$, computed using the numerical implementation of Elliott's model as described in equations (9.6)-(9.8).

Rough-to-smooth and smooth-to-rough cases are shown in figure 9.3(a) and (b), respectively. The line colours, red, orange, green and blue correspond to increasing values of the IBL thickness $\delta_i(0)$. A higher $\delta_i(0)$ will lead to a higher δ_i farther downstream. If we choose the red line with $\delta_i(0) = \max(z_{01}, z_{02})$ from figure 9.3(a) and (b) as the baseline case, and shift the other three lines downstream until the first point in each line falls on top of the red line, we can see an almost perfect collapse for the remaining part of the curves. This scenario

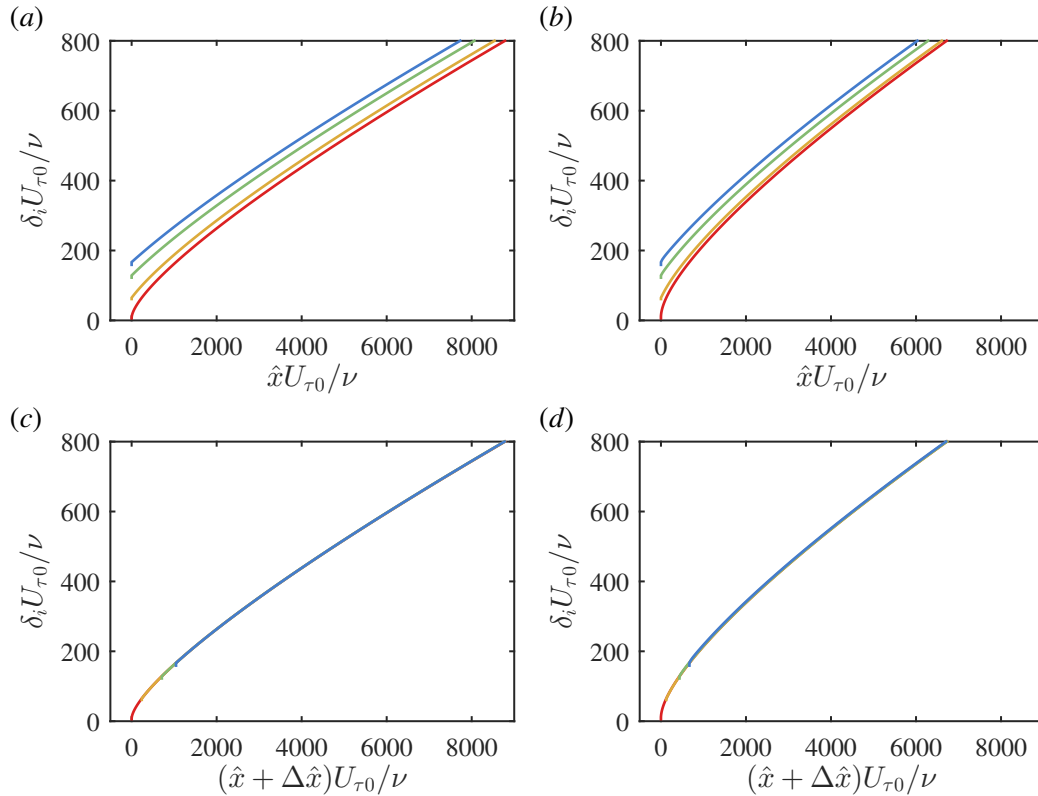


FIGURE 9.3: Predicted IBL thicknesses δ_i using equations (9.3) and (9.6) for (a) rough-to-smooth and (b) smooth-to-rough cases. (c) and (d) show the same data as in (a) and (b) but with the abscissa shifted by $\Delta \hat{x}$ (see equation (9.10) for the analytical expression) for each curve. Line colours red, orange, green and blue correspond to $\delta_i(0) / \max(z_{01}, z_{02}) = 1, 10, 20$ and 26 , respectively. Note that $\delta_i(0) / \max(z_{01}, z_{02}) = 26$ is approximately $\delta_i(0) / \max(k_{s1}, k_{s2}) = 1$. For both rough-to-smooth and smooth-to-rough cases, $\max(k_{s1}, k_{s2}) U_{\tau 1} / \nu = 160$.

is present in figure 9.3(c) and (d). In other words, using a different starting point of $\delta_i(0)$ does not change the shape of the predicted δ_i versus $\hat{x}$ curve: it is essentially equivalent to displacing the curve in x by a certain amount. To demonstrate this analytically, we consider two different initial values $\delta_{i1}(0)$ and $\delta_{i2}(0)$ and compute the corresponding fetch $\hat{x}_1$ and $\hat{x}_2$ following (9.7):

$$\begin{aligned}\hat{x}_2(\delta_i) &= \int_{\delta_{i2}(0)}^{\delta_i} \frac{\rho}{\Delta\tau(\delta_i)} G'_2(\delta_i) d\delta_i - \int_{\delta_{i2}(0)}^{\delta_i} \frac{\rho U_i(\delta_i)}{\Delta\tau(\delta_i)} G'_1(\delta_i) d\delta_i \\ &= \left(\int_{\delta_{i2}(0)}^{\delta_{i1}(0)} + \int_{\delta_{i1}(0)}^{\delta_i} \right) \frac{\rho}{\Delta\tau(\delta_i)} G'_2(\delta_i) d\delta_i - \left(\int_{\delta_{i2}(0)}^{\delta_{i1}(0)} + \int_{\delta_{i1}(0)}^{\delta_i} \right) \frac{\rho U_i(\delta_i)}{\Delta\tau(\delta_i)} G'_1(\delta_i) d\delta_i \\ &= \int_{\delta_{i2}(0)}^{\delta_{i1}(0)} \frac{\rho}{\Delta\tau(\delta_i)} [G'_2(\delta_i) - U_i(\delta_i) G'_1(\delta_i)] d\delta_i + \hat{x}_1(\delta_i).\end{aligned}\quad (9.9)$$

Therefore, $\Delta\hat{x}$, the shift in $\hat{x}$ required for the two predictions with different initial conditions to match can be expressed as

$$\Delta\hat{x} = \hat{x}_2(\delta_i) - \hat{x}_1(\delta_i) = \int_{\delta_{i2}(0)}^{\delta_{i1}(0)} \frac{\rho}{\Delta\tau(\delta_i)} [G'_2(\delta_i) - U_i(\delta_i) G'_1(\delta_i)] d\delta_i. \quad (9.10)$$

With $\delta_{i1}(0)$ and $\delta_{i2}(0)$ as the upper and lower limits of the integral being constants, it is apparent that $\Delta\hat{x}$ is also a constant.

9.3 Wake function

Elliott (1958) originally considered a thick surface layer where the boundary layer thickness is irrelevant in the problem and the mean velocity profile can be modelled by a logarithmic law as high as needed to predict the quantities of interest. In most engineering applications, however, the boundary layers have a finite thickness and the growth of the IBL may be affected when it exceeds the logarithmic layer and enters the wake region (figure 9.1b). In this section, we attempt to include a wake function in the mean velocity profile and examine its impact on the flow recovery.

Here we adopt the expression of the wake function given by Jones et al. (2001):

$$\mathcal{W}(\eta) = -\frac{1}{3\kappa}\eta^3 + \frac{\Pi}{\kappa}[2\eta^2(3-2\eta)], \quad (9.11)$$

where $\eta = z/\delta_c$ and δ_c is Jones' boundary layer thickness. The mean velocity profile has zero gradient at $z = \delta_c$, and typically $\delta_c \approx 1.2\delta_{99}$ for a smooth-wall turbulent boundary layer profile at a moderate Reynolds number. Here, $\Pi = 0.56$ (corresponding to a Coles wake strength of $\Pi_c = 0.42$, as reported by Marusic et al. (2015) from data obtained in the HRNBLWT). Further, the growth of the outer layer in the streamwise direction is neglected, i.e. both δ_c and $U_{\tau 1}$ are assumed to be constant. Therefore, throughout the evolution $U_{\tau 1}(\hat{x}) = U_{\tau 1}(0)$, which is equivalent to $U_{\tau 0}$ to be consistent with our previous notation. To summarise, the streamwise momentum equation remains the same as (9.6) while the mean

velocity profile is modified as:

$$U(z) = \begin{cases} \frac{U_{\tau 1}}{\kappa} \ln\left(\frac{z}{z_{01}}\right) + U_{\tau 1} \mathcal{W}\left(\frac{z}{\delta_c}\right), & \text{for } z \geq \delta_i, \\ \frac{U_{\tau 2}}{\kappa} \ln\left(\frac{z}{z_{02}}\right) + U_{\tau 2} \mathcal{W}\left(\frac{z}{\delta_c}\right), & \text{for } z < \delta_i. \end{cases} \quad (9.12)$$

The continuity equation of U at $z = \delta_i$ will consequently be

$$U(\delta_i) = \frac{U_{\tau 1}}{\kappa} \ln\left(\frac{\delta_i}{z_{01}}\right) + U_{\tau 1} \mathcal{W}\left(\frac{\delta_i}{\delta_c}\right) = \frac{U_{\tau 2}}{\kappa} \ln\left(\frac{\delta_i}{z_{02}}\right) + U_{\tau 2} \mathcal{W}\left(\frac{\delta_i}{\delta_c}\right). \quad (9.13)$$

By adding a wake into the assumed profile in this manner, an additional Reynolds number $Re_{\tau c} \equiv \delta_c U_{\tau 1} / \nu$ is introduced into the problem. The predicted $\delta_i(\hat{x})$ and $U_{\tau 2}(\hat{x})$ for both rough-to-smooth and smooth-to-rough cases are shown in figures 9.4 and 9.5, in comparison with a reference case with the original pure logarithmic mean velocity profile. The different coloured curves indicate different Reynolds numbers, with red, orange, green and blue

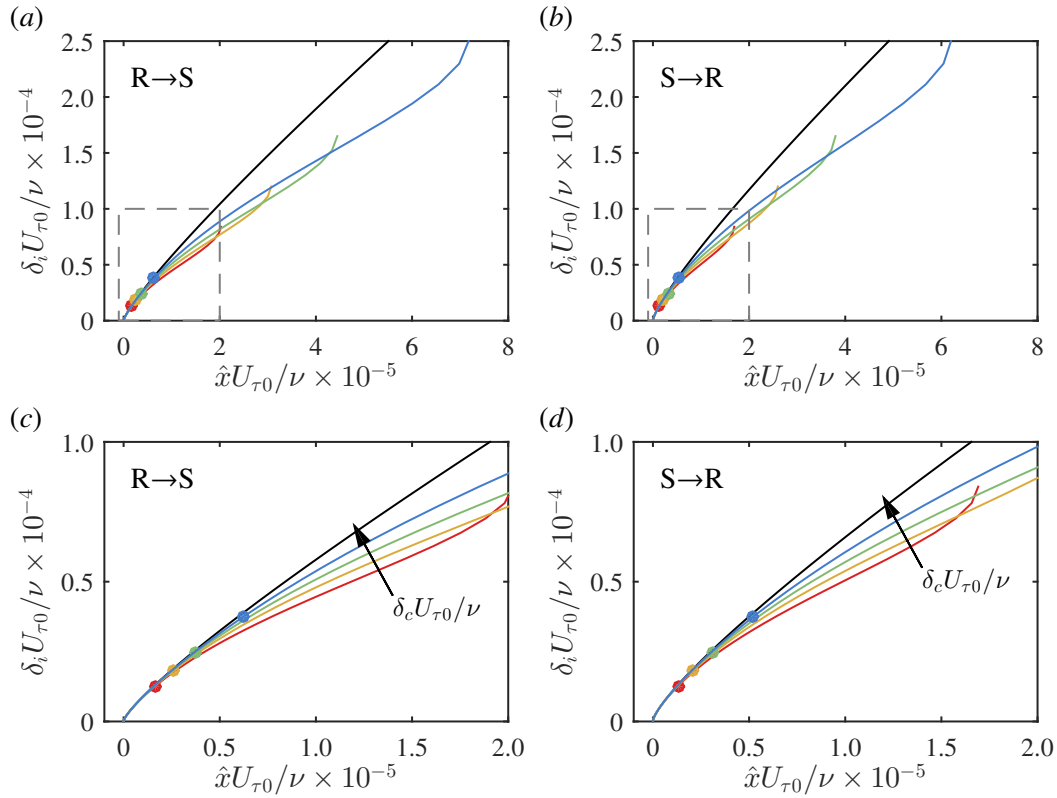


FIGURE 9.4: Comparison of the model prediction of δ_i with (coloured) and without (black) the inclusion of a wake function in the mean velocity profile. The left column is for the rough-to-smooth case and the right column is for the smooth-to-rough case. The top row is the IBL thickness and the bottom row is the zoomed view of the corresponding figures close to the step change (shown by the dashed box). For both rough-to-smooth and smooth-to-rough cases, $\max(k_{s1}, k_{s2}) U_{\tau 1} / \nu = 160$ (matched to the experimental data in *Group I*). The colours indicate the friction Reynolds number: red, orange, green and blue correspond to $\delta_c U_{\tau 1} / \nu = 8400, 12000, 16500$ and 25000 . The coloured dot on each curve marks the location where $\delta_i = 0.15 \delta_{c0}$, representing the onset of the wake region.

corresponding to $Re_{\tau_c} = 8400, 12000, 16500$ and 25000 . The coloured dot marks the location of $\delta_i = 0.15\delta_{c0}$ on each curve, representing the onset of the wake region. In both figures, plots (a) and (b) show the rough-to-smooth and smooth-to-rough cases, respectively, and plots (c) and (d) show zoomed views of the behaviour close to the surface transition ($0 < \hat{x}U_{\tau 1}/\nu < 1 \times 10^5$). As expected, the inclusion of a wake function makes a negligible difference in the prediction within the logarithmic region (to the left of the dots in the figures). This is particularly noticeable in the zoomed views given in plots (c) and (d). Farther downstream where δ_i has grown beyond the logarithmic region, the δ_i values predicted with the inclusion of the wake profile are lower than those from Elliott's original model for both rough-to-smooth and smooth-to-rough cases. A friction Reynolds number trend is also present in the data: as $\delta_c U_{\tau 1}/\nu$ increases, the predicted δ_i increases and asymptotes to the result from Elliott's original model, which is essentially a wake-included model in the limit of $\delta_c U_{\tau 1}/\nu \rightarrow \infty$. Similar behaviours are also observed in the $U_{\tau 2}$ predictions, although including the wake function leads to a higher prediction for the rough-to-smooth case and a lower one for the smooth-to-rough case. When $\delta_c U_{\tau 1}/\nu$ increases, the ratio $U_{\tau 2}/U_{\tau 1}$ at the most downstream location where a full recovery is achieved also increases for the rough-to-smooth case and decreases for the smooth-to-rough case (see the arrows in figure 9.5a and b). This far-field behaviour agrees with the trend predicted analytically in equation (7.8).

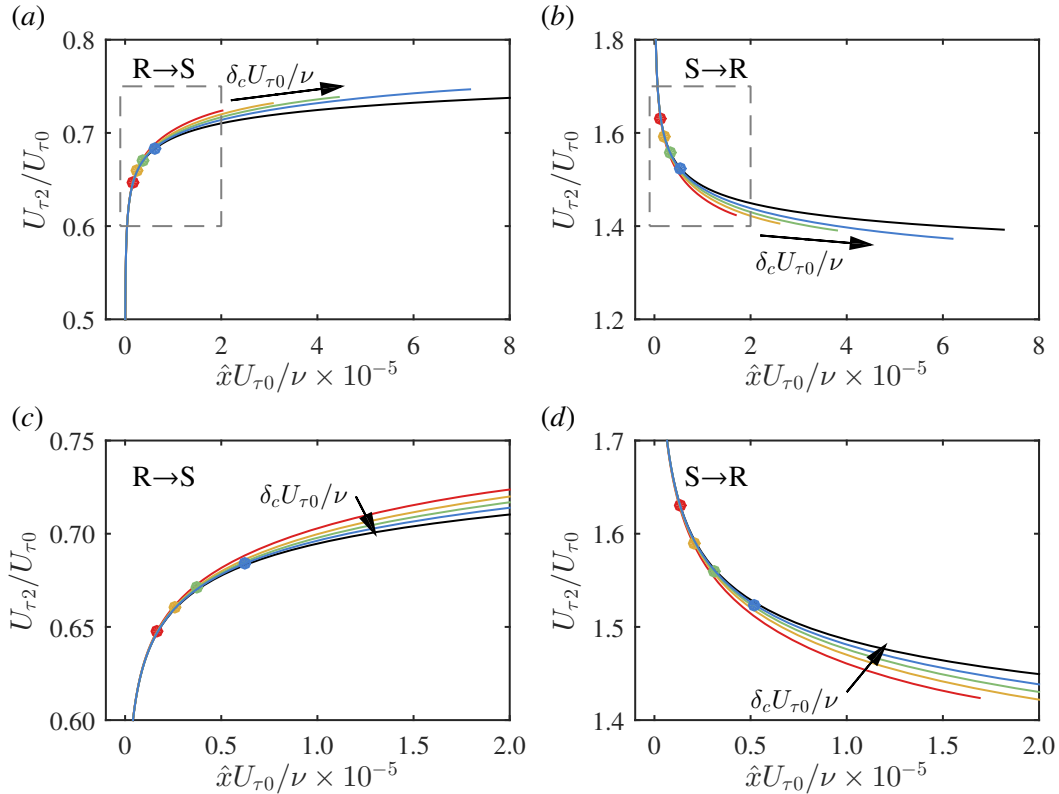


FIGURE 9.5: Comparison of the model prediction of $U_{\tau 2}$ with (coloured) and without (black) the inclusion of a wake function in the mean velocity profile. The legends are the same as in figure 9.4.

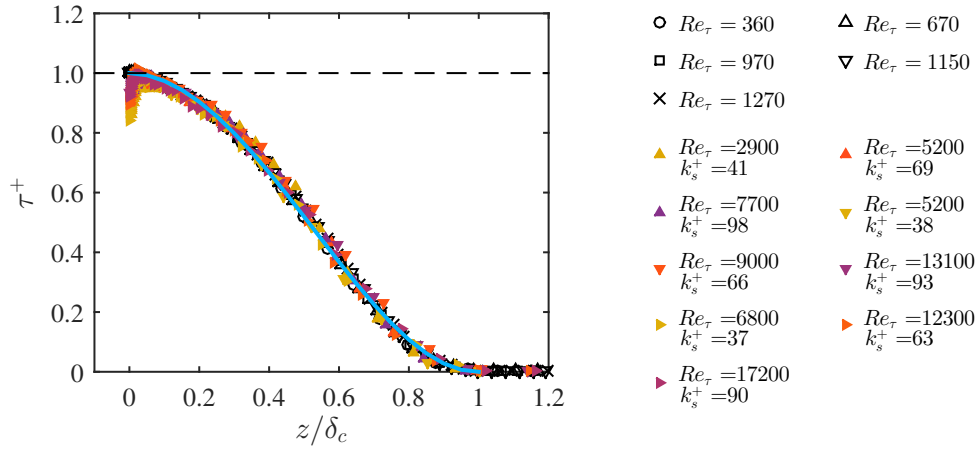


FIGURE 9.6: Shear stress normalised by the friction velocity at the wall and plotted against the wall position normalised by Jones' boundary layer thickness. The coloured symbols are multi-wire hot-wire data obtained from turbulent boundary layers developed on a sand-paper roughness (Morrill-Winter et al., 2017). The black empty symbols are DNS data of a smooth-wall boundary layer (Schlatter and Örlü, 2010). The thick blue line is equation (9.15) and the horizontal dashed line is at $\tau^+ = 1$.

9.4 Shear-stress correction

In a turbulent boundary layer, the total shear stress τ is contributed by the wall-normal gradient in the mean flow and also the Reynolds shear stress:

$$\tau^+ = \frac{dU^+}{dz^+} - \overline{uw}^+. \quad (9.14)$$

The first term only dominates in the viscous sublayer, and the contribution is mainly from the Reynolds shear stress (the second term) beyond the buffer region. In the limit of infinite Reynolds numbers, a constant-stress layer forms in the logarithmic region with τ^+ remaining at unity as determined by the equations of motion (Tennekes and Lumley, 1972, p. 54). This assumption is adequate in Elliott's original model because δ_i remains in the logarithmic layer throughout the evolution, but its validity diminishes with the inclusion of a wake function. In the wake region, the Reynolds shear stress is observed to decrease significantly to $0.5\tau_w$ at $z \approx 0.5\delta_c$ (Antonia and Luxton, 1972; Schlatter and Örlü, 2010; Morrill-Winter et al., 2017). Shear stress data from both experiments and DNS for canonical boundary layers are summarised in figure 9.6. Both rough-wall and smooth-wall boundary layer data with a wide range of Re_τ and k_s^+ are included. τ^+ decreases steadily from 1 in the very near-wall region to 0 in the freestream. The approximation of constant stress, $\tau^+ = 1$, is reasonable for $z/\delta_c < 0.15$, while the deviation from this approximation is significant above the logarithmic region. A good collapse of the data at various Reynolds numbers is observed under the current scaling. An empirical fit to the DNS data in the range of $0 < z/\delta_c < 1$ results in

$$\tau^+ = \left[1 - \left(\frac{z}{\delta_c} \right)^{1.9} \right]^{2.1}. \quad (9.15)$$

Here we propose a refinement of the shear stress difference term $\Delta\tau$ by replacing the stress at $z = \delta_i$ with the aforementioned empirical fit which captures the decay of the shear stress due to a finite Reynolds number effect beyond the logarithmic region. Therefore, the shear-stress difference $\Delta\tau$ becomes

$$\Delta\tau = \rho U_{\tau 1}^2 \left[1 - \left(\frac{\delta_i}{\delta_c} \right)^{1.9} \right]^{2.1} - \rho U_{\tau 2}^2. \quad (9.16)$$

The predicted results with the shear-stress correction are summarised in figure 9.7. For the rough-to-smooth case shown in figure 9.7(a) and (c), the shear-stress correction (red dot-dashed lines) leads to a lower prediction of $\delta_i U_{\tau 1} / \nu$ and $U_{\tau 2} / U_{\tau 1}$ compared to the results with a wake refinement only (dashed blue lines), which is expected because in (9.6) the integrand is inversely proportional to $\Delta\tau$. For a given δ_i , considering the decay of the shear stress in the outer layer results in a smaller $\Delta\tau$, thus, a larger $\hat{x}$, which essentially indicates

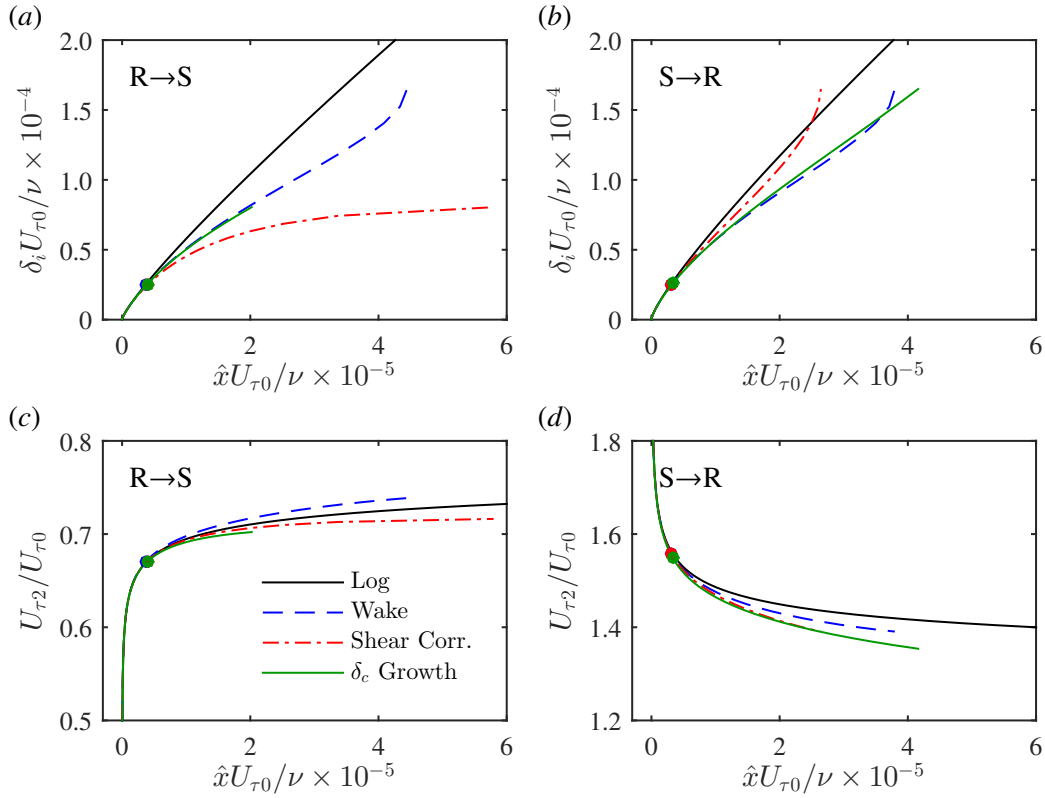


FIGURE 9.7: Comparison of the predictions from Elliott's original model ('Log' as in the legend), the refined model with the a wake function ('Wake'), with both wake and shear-stress correction ('Shear Corr.') and finally with a δ_c that evolves in the streamwise direction on top of all the refinements included in Shear Corr. (' δ_c Growth'). The left column is for the rough-to-smooth case and the right column is for the smooth-to-rough case. The top row is the IBL thickness and the bottom row is the ratio between the downstream and upstream friction velocity. Note that only the data with $\Delta\tau > 0$ are shown for the rough-to-smooth case. The coloured dot on each curve marks the location where $\delta_i = 0.15\delta_{c0}$, representing the onset of the wake region. For both rough-to-smooth and smooth-to-rough cases, $\max(k_{s1}, k_{s2})U_{\tau 1} / \nu = 160$ and $\delta_c U_{\tau 1} / \nu = 16500$. The flow conditions of the rough-to-smooth case are matched to case Re14ks16.

a slower recovery. It is noted that the shear-stress correction model features both the wake and the shear-stress refinement. For the smooth-to-rough case shown in figure 9.7(b) and (d), as $U_{\tau 1}$ is consistently lower than $U_{\tau 2}$, the magnitude of $\Delta\tau$ is expected to increase after applying the shear-stress correction, leading to a faster recovery as indicated by the higher $\delta_i U_{\tau 1} / \nu$ and lower $U_{\tau 2} / U_{\tau 1}$.

In the rough-to-smooth case, as δ_i / δ_c increases from 0 to 1, the sign of $\Delta\tau$ changes from positive to negative with a zero-crossing point in between, which will cause a division-by-zero issue in equation (9.6). A physical interpretation of the singularity is that when there is a change of sign in $\Delta\tau$ (the net force on the control volume), the monotonic growth of δ_i in the streamwise direction cannot be sustained, meaning that δ_i will asymptote to a value lower than δ_c , and a complete recovery where $\delta_i = \delta_c$ can never be achieved. For the smooth-to-rough case, on the other hand, applying the shear-stress correction will only make $\Delta\tau$ more negative as $U_{\tau 1}$ is consistently lower than $U_{\tau 2}$.

9.5 Streamwise evolution of the outer layer

In this section, δ_c , the thickness of the entire boundary layer (Jones et al., 2001) is allowed to grow with $\hat{x}$ and $U_{\tau 1}$ is no longer treated as a constant. Therefore, two more unknowns $\delta_c(\hat{x})$ and $U_{\tau 1}(\hat{x})$ are added to the problem, and two more equations are required to close the system. A refined schematic of the problem is shown in figure 9.1(c).

We first consider the mean velocity profile in the outer layer at $z = \delta_c$:

$$\frac{U_\infty}{U_{\tau 1}} = \frac{1}{\kappa} \ln\left(\frac{\delta_c U_{\tau 1}}{\nu}\right) + B - \Delta U_1^+ + \frac{2\Pi}{\kappa} \mathcal{W}(1), \quad (9.17)$$

Note that now both $U_{\tau 1}$ and δ_c are functions of $\hat{x}$ and their values will change in the streamwise direction as the outer layer evolves. We can also introduce the streamwise momentum equation of the entire boundary layer:

$$\hat{x} = \int \frac{U_\infty^2}{U_{\tau 2}^2} d\theta, \quad (9.18)$$

where $\theta = \int_{z_{02}}^{\delta_c} \frac{U}{U_\infty} (1 - \frac{U}{U_\infty}) dz$ is the momentum thickness of the entire boundary layer. These two equations, together with (9.4) and (9.6) which govern the flow within the IBL, can fully describe the system.

In practice, we solve the coupled system numerically following the procedure described in figure 9.8. Similar to the numerical implementation of the original Elliott's model, δ_i is chosen as the independent variable. For every given δ_i , an initial guess of δ_c is made. As (9.17) involves only two unknowns: $U_{\tau 1}$ and δ_c , for any given δ_c , the local velocity scale of the outer layer $U_{\tau 1}(\hat{x})$ can be easily determined. Next, $U_{\tau 1}$ and δ_c can be fed into (9.4) to solve for $U_{\tau 2}$. Finally, $\hat{x}$ can be computed explicitly from both (9.6) and (9.18). We denote the $\hat{x}$ computed from these two equations as $\hat{x}_1$ and $\hat{x}_2$, respectively. If $\hat{x}_1 = \hat{x}_2$, then it means the initial guess of δ_c is adequate, otherwise the guess will have to be improved until the difference between $\hat{x}_1$ and $\hat{x}_2$ is smaller than a predefined threshold. In this way, we

have restated the problem of solving a coupled ODE system to an optimisation problem: we need to find the optimal δ_c so that the objective function $|\hat{x}_1 - \hat{x}_2|$ is minimised. In practice, the threshold of $|\hat{x}_1 - \hat{x}_2|$ is chosen as 1×10^{-10} , with a maximum iteration of 1000 and the tolerance in δ_c as 1×10^{-10} .

$U_{\tau 2}$ and δ_i predicted by the model are shown by the solid green lines in figure 9.7. Note that these green curves now have the wake and the shear-stress refinements, and also permitting an evolving layer thickness $\delta_c(\hat{x})$. For the rough-to-smooth case, comparing the solid green line (' δ_c Growth') against the red dot-dashed line ('Shear Corr.'), it is apparent that considering the growth of δ_c in the streamwise direction leads to a higher δ_i at a given $\hat{x}$, while its effect on the $U_{\tau 2}/U_{\tau 0}$ is very limited. For the smooth-to-rough case, on the other hand, considering the growth of δ_c results in a reduction of δ_i and a slight decrease of $U_{\tau 2}/U_{\tau 0}$.

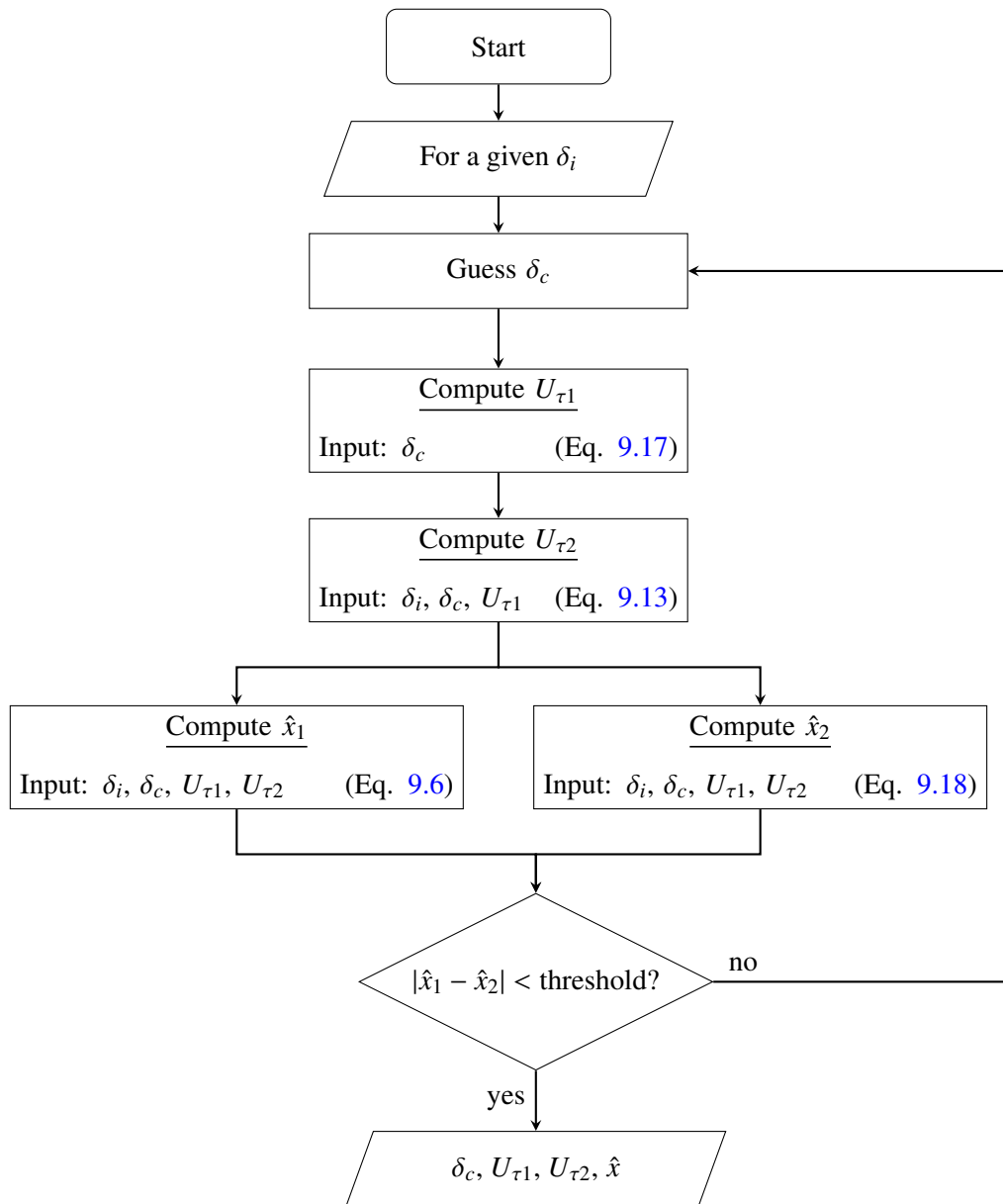


FIGURE 9.8: Flowchart of the procedure to compute flow parameters δ_c , $U_{\tau 1}$, $U_{\tau 2}$ and $\hat{x}$ for every given δ_i .

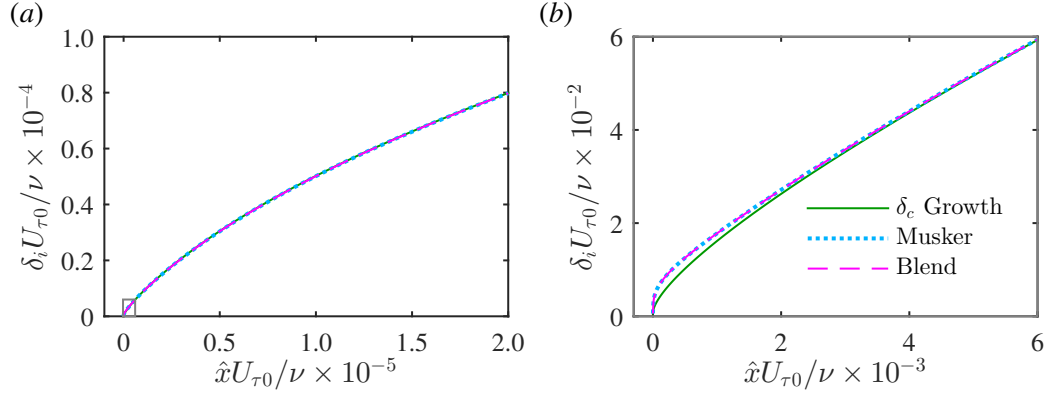


FIGURE 9.9: A comparison of the predicted δ_i after including the growth of δ_c (' δ_c Growth' as in the legend), a Musker profile on the smooth wall ('Musker') and the blending model ('Blend') with $\Delta\delta_i^+ = 0$, $\mu_0 = 0$ and $\sigma = 0$. (b) is a zoomed view of the area marked by the rectangular box in (a). Note the change of axis labels and limits in (b).

9.6 Non-equilibrium flow in the IBL

So far, the mean velocity profile within the IBL has been modelled by a logarithmic law with a slope determined by the local friction velocity $U_{\tau 2}(\hat{x})$. In this section, we would like to further refine the model by incorporating the well-recognised non-equilibrium features of the flow in the IBL with the blending model formulated in §8, focusing on rough-to-smooth transitions.

For a better application of the blending model, we first improve the mean velocity representation in the near-wall region with a Musker profile (Chauhan et al., 2009). The full velocity profile can be expressed as

$$U(z) = \begin{cases} \frac{U_{\tau 1}}{\kappa} \ln\left(\frac{z}{z_{01}}\right) + U_{\tau 1} \mathcal{W}\left(\frac{z}{\delta_c}\right), & \text{for } z \geq \delta_i, \\ U_{\tau 2} U_{\text{Musker}}^+ \left(\frac{z U_{\tau 2}}{\nu}\right) + U_{\tau 2} \mathcal{W}\left(\frac{z}{\delta_c}\right), & \text{for } z < \delta_i, \end{cases} \quad (9.19)$$

where U_{Musker}^+ stands for the Musker profile (composite velocity profile excluding the wake function) as used by Chauhan et al. (2009) (the expression of the Musker profile is detailed in §9.A). The continuity equation will then be

$$U(\delta_i) = \frac{U_{\tau 1}}{\kappa} \ln\left(\frac{\delta_i}{z_{01}}\right) + U_{\tau 1} \mathcal{W}\left(\frac{\delta_i}{\delta_c}\right) = U_{\tau 2} U_{\text{Musker}}^+ \left(\frac{\delta_i U_{\tau 2}}{\nu}\right) + U_{\tau 2} \mathcal{W}\left(\frac{\delta_i}{\delta_c}\right). \quad (9.20)$$

If we consider the lower bound of the logarithmic region as 100 wall units, then for a flow with $Re_\tau \sim O(10^5)$, the viscous sublayer and buffer layer together will only take up 0.1% of the entire boundary layer thickness. Therefore, any refinement of the mean velocity representation in this region is expected to make a negligible difference in the final prediction. Figure 9.9 compares the predictions using the logarithmic profile (green) and Musker profile (dotted blue) in the near-wall region. Note that all refinements detailed to this point are included in these predictions (wake, shear-stress and δ_c evolution). Using a Musker velocity profile in place of a logarithmic law results in almost identical predictions, except for a slight over-prediction in δ_i extremely close to the step change. Figure 9.9(b) shows a zoomed view

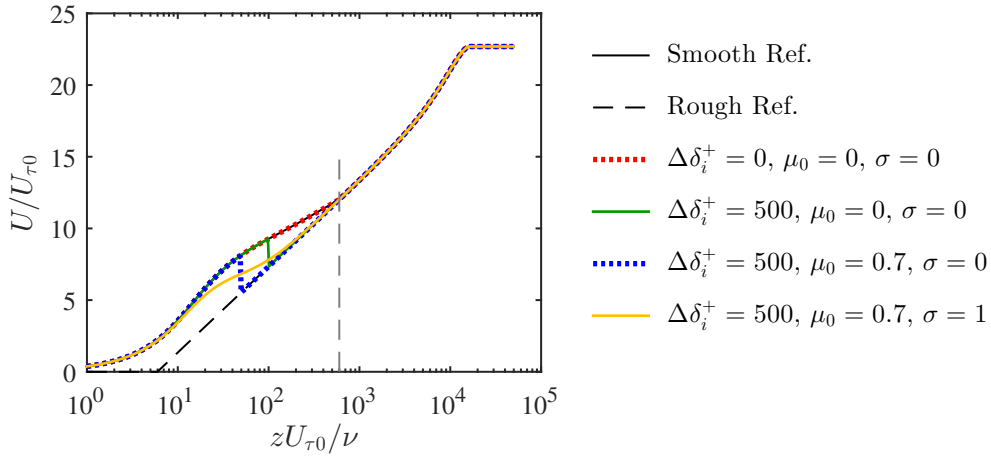


FIGURE 9.10: Mean velocity profiles normalised by $U_{\tau 0}$ with various blending parameters. The red, green and blue coloured lines corresponds to the baseline cases in figures 9.11–9.13 (shown by the black dashed lines). For all profiles shown here, the upstream conditions are $\delta_c U_{\tau 0}/\nu = 16500$ and $k_s U_{\tau 0}/\nu = 160$, with the IBL thickness chosen as $\delta_{i,log} U_{\tau 0}/\nu = 600$ (shown by the vertical dashed line in the figure). $U_{\tau 1} \equiv U_{\tau 0}$ is assumed for all cases.

in the region immediately downstream of the surface transition. The upper limit in δ_i of figure 9.9(b) is approximately 4% of δ_c , demonstrating that using the Musker profile effects only the very initial growth of the IBL. Despite the negligible effect on the predicted results, the inclusion of the Musker profile is useful to facilitate the direct comparison of the mean velocity profiles between the model prediction and experiments. Using the Musker profile also permits further modifications to the velocity profile within the IBL to model the non-equilibrium effect to be detailed below.

We attempt to account for the non-equilibrium behaviour of the mean flow within the IBL by replacing the mean velocity profile (9.12) with a blending model as formulated empirically in §8 and illustrated in figure 9.1(d):

$$U^+ = U_S^+ E + U_R^{*+} (1 - E) \quad (9.21)$$

where E is given by an error function (originally equation 8.7)

$$E(z^+; \mu, \sigma) = \frac{1}{2} \left[1 - \operatorname{erf} \left(\frac{\ln(z^+) - \mu}{\sqrt{2}\sigma} \right) \right] \quad (9.22)$$

with the parameters modelled as

$$\mu = \ln(\delta_i^+) - \mu_0, \quad \mu_0 = 0.7 \quad \text{and} \quad \sigma = 1.0. \quad (9.23)$$

Here, δ_i^+ is the viscous-scaled IBL thickness as determined from the ‘kink’ in the mean velocity or variance profile. In §6.5, we have previously shown that as the mean flow within the IBL is not in equilibrium with the new wall condition, δ_i determined directly from the

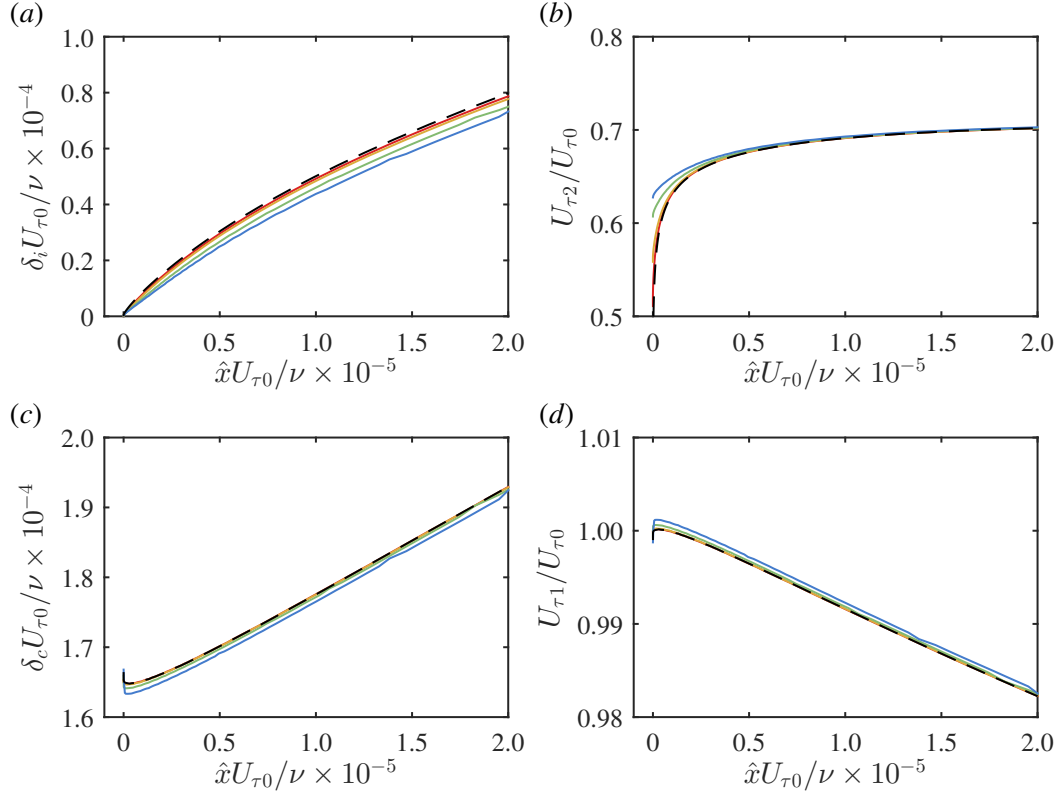


FIGURE 9.11: A comparison of the predicted (a) δ_i , IBL thickness; (b) $U_{\tau 2}$, friction velocity at the wall; (c) δ_c , thickness of the entire boundary layer and (d) $U_{\tau 1}$, friction velocity scale of the outer layer with different $\Delta\delta_i^+$ values. The black dashed line has $\Delta\delta_i^+ = 0$ and line colours red, yellow, green and blue correspond to $\Delta\delta_i^+ = 100, 200, 500$ and 800 . For all lines, $\mu_0 = 0$ and $\sigma = 0$. The flow conditions are matched to case Re14ks16 with $k_{s1}U_{\tau 1}/\nu = 160$ and $\delta_c U_{\tau 1}/\nu = 16500$.

‘kink’ in the measured profile is consistently lower than $\delta_{i,log}$, which is defined as the wall-normal location where the assumed equilibrium smooth-wall mean velocity profile with the local $U_{\tau 2}$ intersects the outer-layer mean velocity profile. If the mean velocity in the IBL is in equilibrium with the local wall condition, then $\delta_i = \delta_{i,log}$, as in the case with all the model predictions we have made so far. From this point on, the non-equilibrium behaviour in the IBL is modelled with the aid of a blending velocity profile, and we must distinguish the difference between δ_i and $\delta_{i,log}$, which is assumed to be a constant shift when scaled by $U_{\tau 0}$:

$$\frac{\delta_{i,log}U_{\tau 0}}{\nu} = \frac{\delta_i U_{\tau 0}}{\nu} + \Delta\delta_i^+, \quad (9.24)$$

The constant shift is chosen as $\Delta\delta_i^+ = 500$, as suggested by the data in figure 7.5. Equation (9.4) is still employed to relate $U_{\tau 2}$ to $\delta_{i,log}$. With a non-zero $\Delta\delta_i^+$, the initial condition is modified as $\delta_{i,log}(0) = z_{01} + \Delta\delta_i^+ \nu / U_{\tau 0}$ to ensure a positive δ_i^+ at the starting point.

In summary, in the blending velocity model described above, there are 3 empirically determined parameters, namely $\Delta\delta_i^+$ (see §6.5), μ_0 and σ (see §8). For a combination of $\Delta\delta_i^+ = 0$, $\mu_0 = 0$ and $\sigma = 0$, (9.21) is equivalent to the piecewise velocity profile (9.19) where the non-equilibrium effect is not modelled, and it is shown by the dashed magenta line (labelled ‘Blend’) in figure 9.9 as a validation of the algorithm. The prediction with the

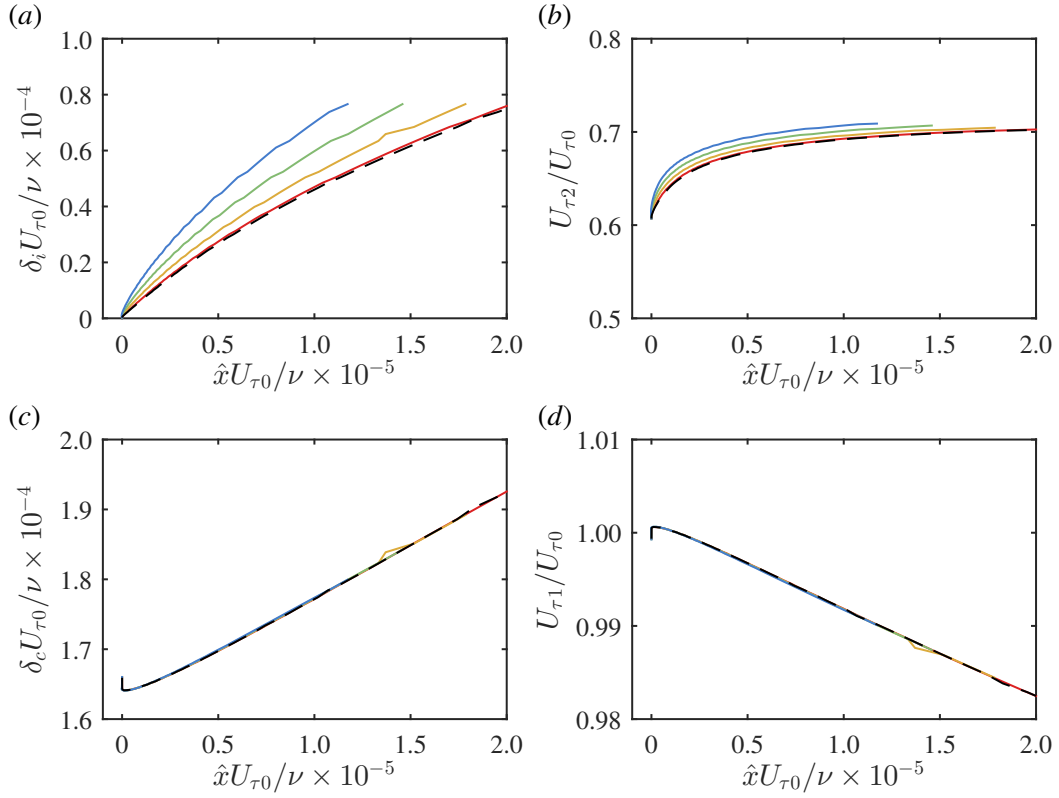


FIGURE 9.12: A comparison of the predicted (a) δ_i , IBL thickness; (b) $U_{\tau 2}$, friction velocity at the wall; (c) δ_c , thickness of the entire boundary layer and (d) $U_{\tau 1}$, friction velocity scale of the outer layer with different μ_0 values. The black dashed line has $\mu = 0$, which is the same as the green line in figure 9.11, and line colours red, yellow, green and blue correspond to $\mu_0 = 0.1, 0.4, 0.7$ and 1 . For all lines, $\Delta\delta_i^+ = 500$ and $\sigma = 0$. The flow conditions are matched to case Re14ks16 with $k_{s1}U_{\tau 1}/\nu = 160$ and $\delta_c U_{\tau 1}/\nu = 16500$.

blending refinement using these parameters are identical with the results from the previous step (‘Musker’), confirming the consistency with the refined model developed up to this stage.

The effect of $\Delta\delta_i^+$, μ_0 and σ are examined systematically to reveal their contribution to the final prediction. We continue to present the IBL thickness in δ_i , which is more directly comparable with the IBL thickness determined from the ‘kink’ in the measured profile. The wake refinement, shear-stress correction and the growth of δ_c are considered in all the predictions. Results with $\Delta\delta_i^+$ increasing from 0 to 800 are summarised in figure 9.11, where the baseline case (dashed black line) is the same as case ‘Blend’ in figure 9.9 with $\Delta\delta_i^+ = 0$, $\mu_0 = 0$ and $\sigma = 0$. The latter two parameters, μ_0 and σ remain 0 while $\Delta\delta_i^+$ varies. The effect of $\Delta\delta_i^+$ on the mean velocity profile is quantified in equation (9.24) and also shown graphically in figure 9.10 by comparing the green solid line and the red dotted line. Including a non-zero $\Delta\delta_i^+$ essentially stitches the smooth-wall and rough-wall reference profiles at $\delta_i U_{\tau 0}/\nu$. Because $U_{\tau 2}$ is determined so that the two profiles will intersect at $\delta_{i, \log} U_{\tau 0}/\nu$, this will lead to a discontinuity in the mean velocity profile. Figure 9.11(a) shows the predicted IBL thickness δ_i . A close agreement between cases is observed very close to the roughness transition, but the difference in $\Delta\delta_i^+$ manifests downstream and $\delta_i U_{\tau 0}/\nu$ is slightly lower in cases with a higher $\Delta\delta_i^+$. The predicted $U_{\tau 2}$ increases with an increasing $\Delta\delta_i^+$, and the effect

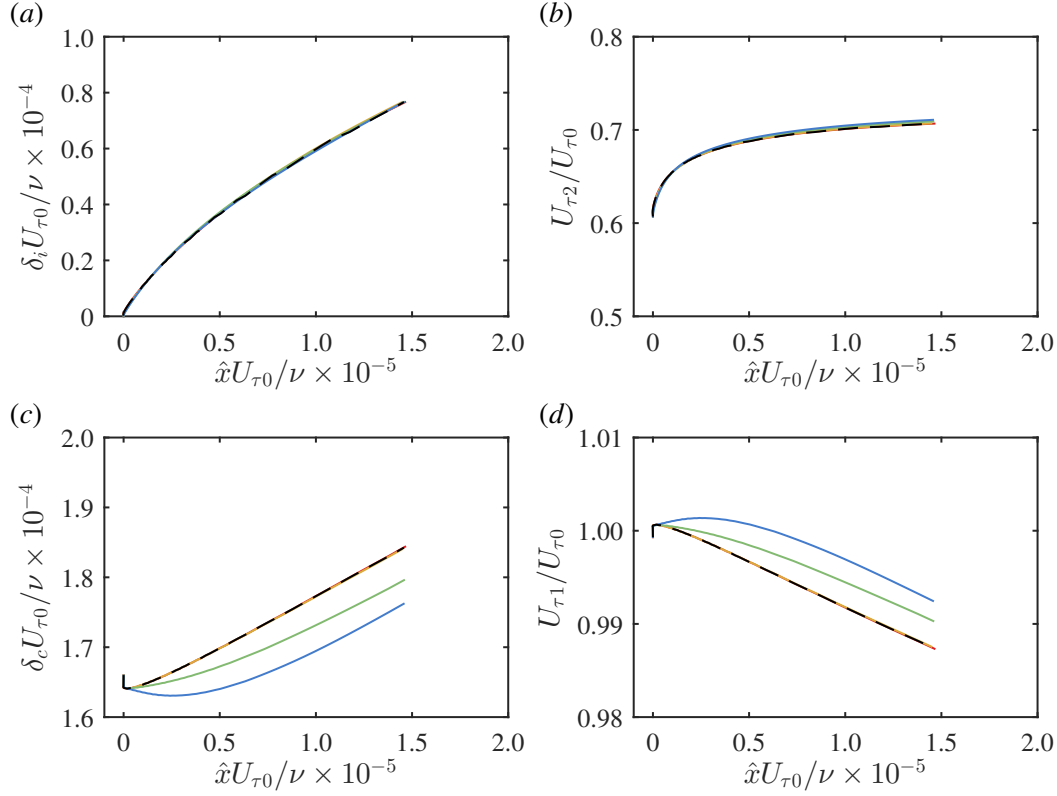


FIGURE 9.13: A comparison of the predicted (a) δ_i , IBL thickness; (b) $U_{\tau 2}$, friction velocity at the wall; (c) δ_c , thickness of the entire boundary layer and (d) $U_{\tau 1}$, friction velocity scale of the outer layer with different σ values. The black dashed line is the same as in figure 9.11, and line colours red, yellow, green and blue correspond to $\sigma = 0.1, 0.5, 1$ and 1.2 . For all coloured solid lines, $\Delta\delta_i^+ = 500$ and $\mu_0 = 0.7$. The flow conditions are matched to case Re14ks16 with $k_{s1}U_{\tau 1}/\nu = 160$ and $\delta_c U_{\tau 1}/\nu = 16500$.

is mainly limited to the vicinity of the step change, as shown in figure 9.11(b). There is little difference in the outer layer quantities δ_c and $U_{\tau 1}$ within the range of $\Delta\delta_i^+$ examined.

We further investigate the effect of the mean location of the blending function μ_0 by fixing $\Delta\delta_i^+ = 500$ and $\sigma = 0$, and increasing μ_0 from 0 (baseline case, shown by the black dashed line) to 1. The predicted results are presented in figure 9.12. The baseline case here is $\Delta\delta_i^+ = 500$, $\sigma = 0$ and $\mu_0 = 0$, the same as the case shown by the green curves in figure 9.11 (and it is also represented by the green line in figure 9.10). By comparing the solid green line and dotted blue line in figure 9.10, it is revealed that including a non-zero μ_0 has a similar effect as $\Delta\delta_i^+$: a larger discontinuity in the mean velocity profile is created by stitching the smooth-wall and rough-wall reference profiles at $\delta_{i,\log}U_{\tau 0}/(\nu \exp(\mu_0)) - \Delta\delta_i^+$, an even lower wall-normal position. Unlike $\Delta\delta_i^+$ which modifies the ‘stitching location’ through a simple subtraction of the original δ_i , μ_0 acts through a multiplication, the effect of which does not diminish farther downstream where $\delta_i U_{\tau 0}/\nu$ has become much greater than $\Delta\delta_i^+$. δ_i and $U_{\tau 2}$ are found to increase with μ_0 at all streamwise locations, while the streamwise evolution of the outer-layer properties appears to follow the unaltered trajectories.

Finally, comparing the dotted blue line and solid orange line in figure 9.10, the effect of the last parameter σ is found to smoothen the discontinuity in the mean velocity profile. In

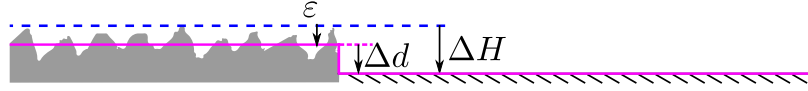


FIGURE 9.14: Schematic of a rough-to-smooth transition accompanied by a change in the virtual origin. The flow goes from left to right of the figure. The blue dashed line represents the location of the roughness crest, and the magenta solid line shows the virtual origin.

figure 9.13, σ is varied from 0 to 1.2 with $\Delta\delta_i^+ = 500$ and $\mu_0 = 0.7$ unchanged. On the contrary to $\Delta\delta_i^+$ and μ_0 , σ is found to mainly affect the outer layer, especially at large σ values where a decrease in δ_c and increase in $U_{\tau 1}$ is observed. The inner parameters δ_i and $U_{\tau 2}$ are little affected by σ .

To conclude, including a positive $\Delta\delta_i^+$ increases the prediction of $U_{\tau 2}$ close to the step change, while μ_0 provides a more significant increase farther downstream. Both parameters make a negligible difference in the outer-layer properties. The smoothing parameter, σ , contributes little to δ_i and $U_{\tau 2}$, but does slow down the streamwise development of the outer-layer quantities slightly.

Refinements made to the original Elliott's model in §9.3–9.6 are summarised in table 9.1. Their contribution to the predicted δ_i and $U_{\tau 2}$ are visualised in figure 9.15. Many refinements have cancelling effects: for example, in the case of a rough-to-smooth change, considering the wake refinement, the shear stress correction and a non-zero $\Delta\delta_i^+$ in the blending function will lead to a reduction in the predicted δ_i , while a non-zero μ_0 in the blending function and considering the growth of the outer layer will lead to an increase in the predicted δ_i . The cancellation of the increasing or decreasing trend in the prediction can partially explain why the original Elliott's model can still give an adequate prediction even without modelling the process realistically.

9.7 Change of the virtual origin

The possibility of modelling the effect of a step change in the virtual origin is discussed in this section. For a rough surface, the virtual origin usually falls between the roughness crest and trough, while it is located on the surface without any ambiguity for a smooth wall. We first consider that in a rough-to-smooth transition, the virtual origin of the rough wall is above the smooth wall downstream by an amount of Δd (see figure 9.14). Note that Δd is not equivalent to the step height ΔH defined in §3.2.1, but these two quantities can be related by

$$\Delta H = \Delta d + \varepsilon, \quad (9.25)$$

where the absolute value of ε is the distance between the roughness crest and the negative sign of ε indicates that the virtual origin is located below the roughness crest. Both ΔH and Δd are negative for a 'step-down' case.

Refinement	Case	δ_i	$U_{\tau 2}$	δ_c	$U_{\tau 1}$
Wake	R→S	↓ in the far field; ↑ with $\delta_c U_{\tau 0} / \nu$	↑ in the far field; ↓ with $\delta_c U_{\tau 0} / \nu$	const.	const.
	S→R	↓ in the far field; ↑ with $\delta_c U_{\tau 0} / \nu$	↓ in the far field; ↑ with $\delta_c U_{\tau 0} / \nu$	const.	const.
Shear Corr.	R→S	↓ in the far field	↓ in the far field	const.	const.
	S→R	↑ in the far field	↓ in the far field	const.	const.
δ_c Growth	R→S	↑ in the far field	slight ↓ in the far field	↑	slight ↓
	S→R	↓ in the far field	slight ↑ in the far field	↑	slight ↓
Musker	R→S	slight ↑ in the near field	slight ↑ in the near field	slight ↑	slight ↓
Blend - $\Delta\delta_i^+$	R→S	↓ in the far field	↑ in the near field	slight ↓	slight ↑
Blend - μ_0	R→S	↑	↑	-	-
Blend - σ	R→S	-	-	↓	↑

TABLE 9.1: Effect of various refinements to the original Elliott’s model on the predicted flow parameters.

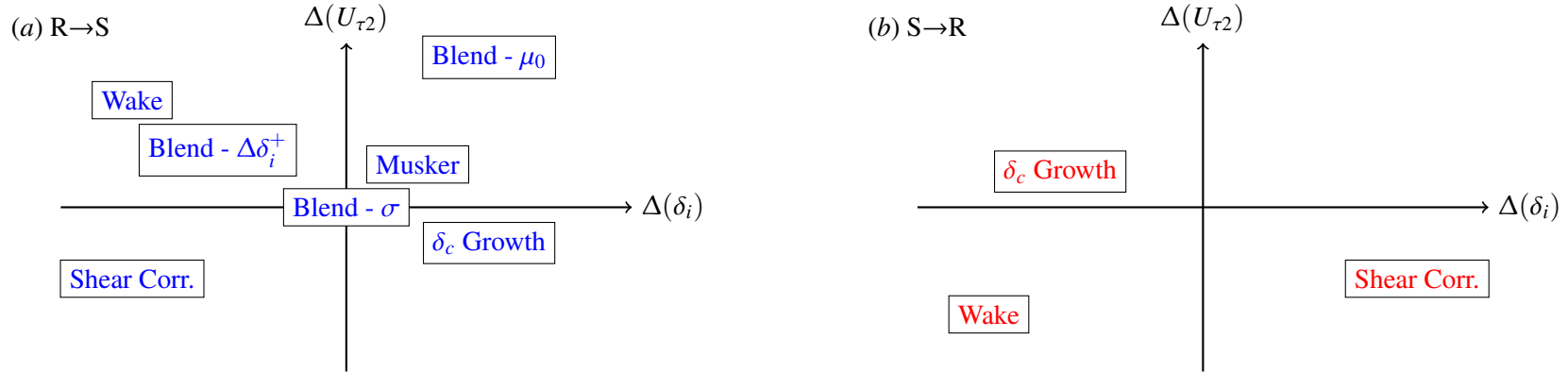


FIGURE 9.15: Schematic showing the effect of various model refinements on δ_i and $U_{\tau 2}$. A positive sign of a coordinate indicates an increase in the corresponding prediction, and a negative sign indicates a decrease: e.g. a refinement that falls in the first quadrant increases the prediction of both δ_i and $U_{\tau 2}$.

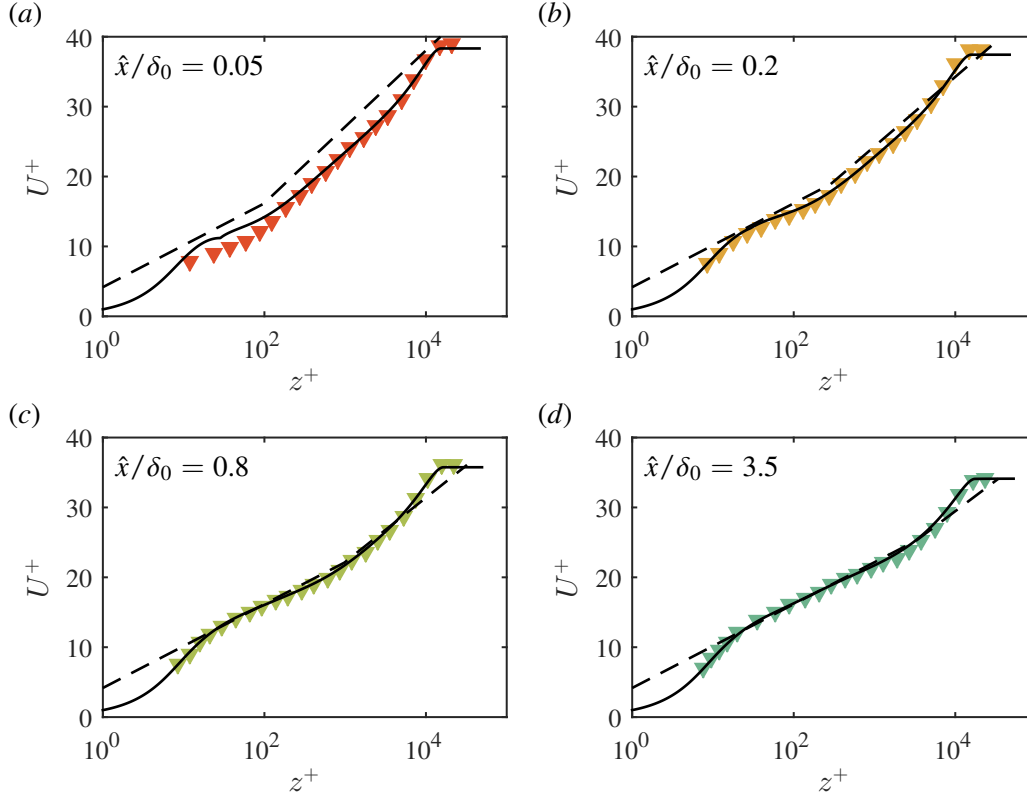


FIGURE 9.16: Mean velocity profiles normalised by the local $U_{\tau 2}$ for case Re21ks16. The experimental data are shown by the symbols (colour corresponds to $\hat{x}/\delta_0$ following figure 6.1), while the solid and dashed black lines represent the refined and original Elliott's model. The four panels are at $\hat{x}/\delta_0 = 0.05, 0.2, 0.8$ and 3.5 , corresponding to $\hat{x}U_{\tau 0}/\nu = 1000, 4400, 18000$ and 75000 , respectively.

We attempt to model the step change of the virtual origin by a wall-normal displacement in the upstream velocity profile, which ensures the continuity of the mean velocity along the streamwise direction. Following Jackson (1976), we state the initial condition of δ_i as $\delta_i(0) = \max(z_{01}, z_{02}) + \max(-\Delta d, 0)$. For a step-down case ($\Delta d < 0$), $\delta_i(0) = \max(z_{01}, z_{02}) - \Delta d$, i.e. the IBL starts forming at the edge of the backward-facing step in the upstream. For a step-up case ($\Delta d > 0$), $\delta_i(0) = \max(z_{01}, z_{02})$ remains unchanged, as the IBL is assumed to start forming at the top of the forward facing step.

9.8 Comparison with the experimental data

Finally, the performance of the theoretical model is tested by comparing the model prediction against the experimentally obtained data in this section. The mean velocity is modelled by (9.21) with $\Delta\delta_i^+ = 500$, $\mu_0 = 0.7$ and $\sigma = 1$. The wake function refinement, the shear-stress correction and the growth of the outer layer are all taken into consideration. The change in the virtual origin is $\Delta d = 0.6$ mm as determined from the surface scan (see figure 3.2c). To draw a direct comparison with the experimental results, a prediction is made for each case using the same $Re_{\tau 0}$ and k_{s0}^+ as the experimental results.

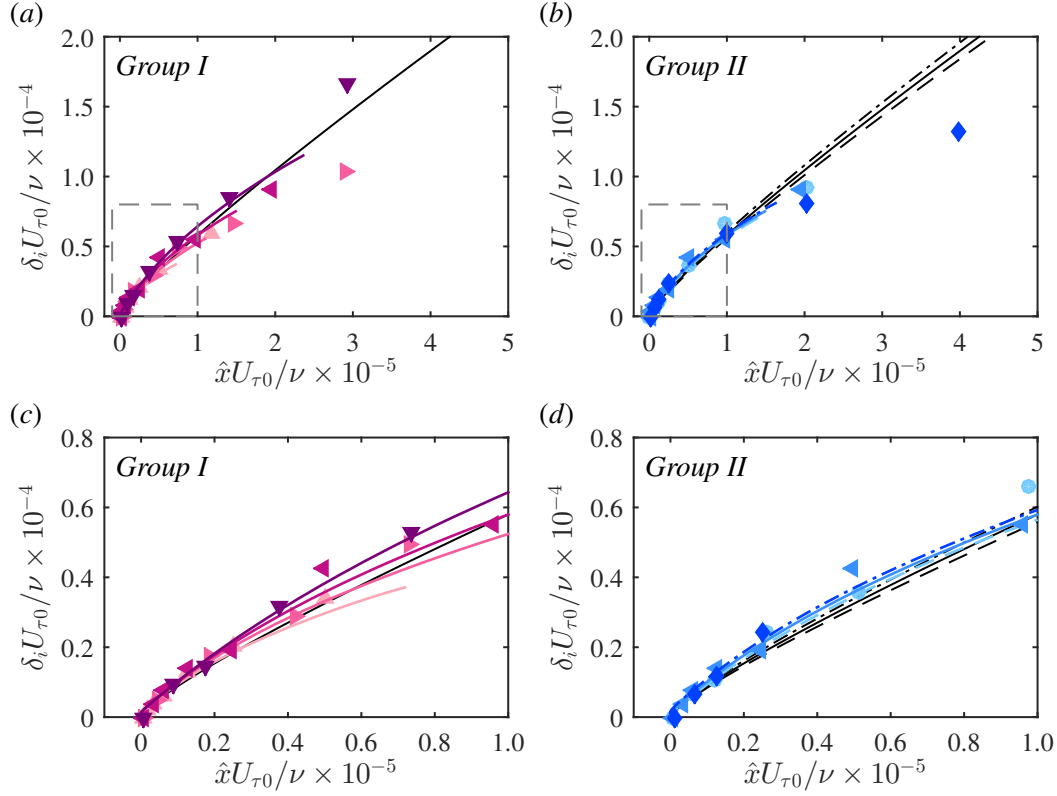


FIGURE 9.17: Profile-based IBL thickness δ_i obtained from experimental measurements (symbols) and blending model predictions (coloured lines). Predictions using the original Elliott's model are shown by the black lines; dashed, solid and dot-dashed lines correspond to $k_{s0}^+ = 111, 160$ and 230 , respectively. Note that the relationship between δ_i and $\delta_{i,\log}$ for the blending model is given by equation (9.24). *Group I* and *Group II* cases are shown in (a) and (b), respectively, while (c) and (d) are the zoomed views of (a) and (b) in the vicinity of the step change.

Figure 9.16 shows the comparison between the mean velocity profiles measured experimentally (symbols), predicted by the original Elliott's model (dashed lines) and the refined Elliott's model detailed in this chapter (solid lines). The comparison for case Re21ks16 is shown here as an example, and qualitatively similar results are observed in all other cases. Despite an obvious absence of the wake in the outer layer as well as an inner profile in the buffer region and below, Elliott's model also fails to capture the velocity profile in the logarithmic region close to the roughness transition (see figure 9.16a and b). The refined model has an improved agreement with the measured profile compared to the original one. We would like to note that the predictions of the original and refined models are tested against the experimental dataset acquired in this study, which has also been used to obtain the coefficients in the blending model of the mean velocity profile. For future works, a wider range of experimental data from various sources and flow conditions is necessary to further evaluate the performance of the refined model.

The performance of the original and refined Elliott's model can be further assessed by comparing the model parameters. The IBL thickness δ_i is shown in figures 9.17. δ_i extracted from the measured profiles are shown by symbols, while the predictions using the original and refined Elliott's model are shown by black and coloured curves. The same legend will

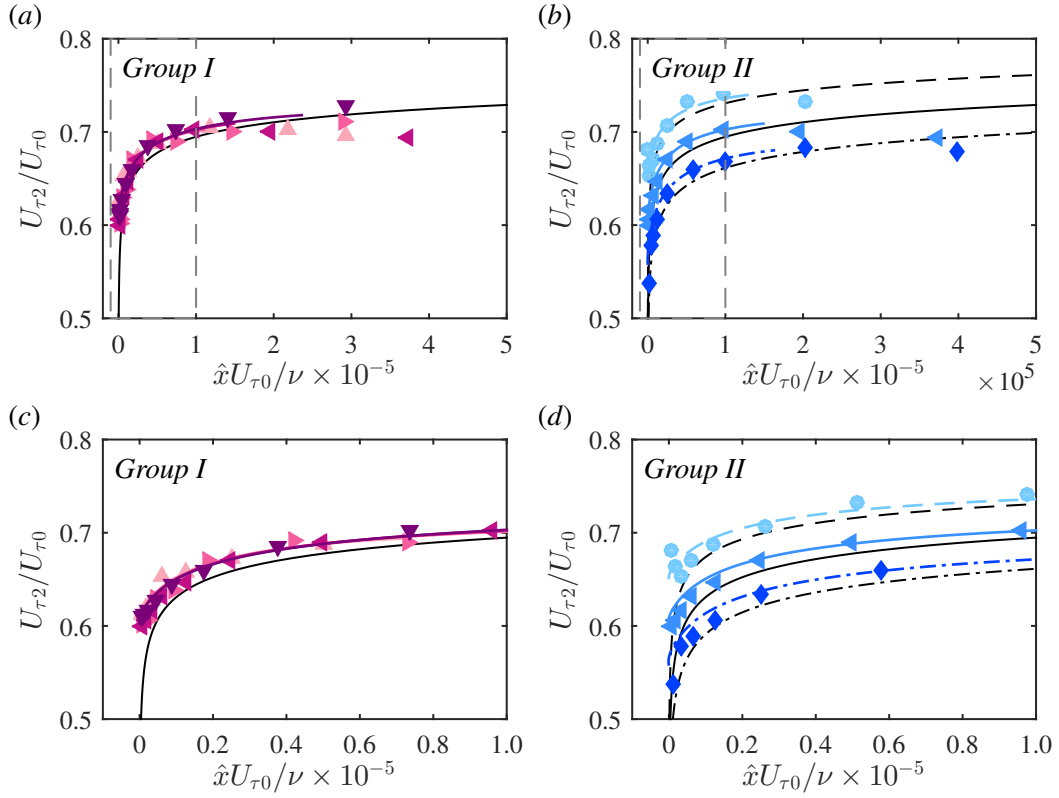


FIGURE 9.18: $U_{\tau 2}$, the friction velocity on the downstream smooth wall obtained from experimental measurements (symbols) and blending model predictions (coloured lines). Legends are the same as in figure 9.17. *Group I* and *Group II* cases are shown in (a) and (b), respectively, while (c) and (d) are the zoomed views of (a) and (b) in the vicinity of the step change.

be adhered to for the following figures. The prediction using the refined model is close to the results from the original one but shows dependence on the friction Reynolds number: for a higher $Re_{\tau 0}$, a higher $\delta_i U_{\tau 0}/\nu$ is observed in the far field. The $Re_{\tau 0}$ dependence primarily results from the inclusion of the wake function, and it is absent in the original Elliott's model. Both of the refined and the original model predictions appear to have a good agreement with the experimental data close to the step change. The comparison is inconclusive in the very far field ($\hat{x}/\delta_c(0) > 20$) as the model terminates at an earlier stage than the experiments. The early termination is a result of the shear-stress correction, which introduces a singularity point in the ODE (9.5) and stops the evolution to a larger δ_i . Similar to the prediction from the original Elliott's model, the slight increase of δ_i with k_{s0}^+ is also reflected in the prediction from the refined model as shown in figure 9.17(d). We lack the range of k_{s0}^+ to see the trend of δ_i with k_{s0}^+ in the current experiments.

The local friction velocity $U_{\tau 2}$ on the downstream smooth wall is shown in figure 9.18. Similar to the previous figure, here we use coloured symbols to represent $U_{\tau 2}$ measured experimentally, black and coloured curves for predictions by the original and refined Elliott's model, respectively. Close to the roughness transition, $U_{\tau 2}$ predicted using the refined model is higher than that predicted by the original model, leading to an improved agreement between the data and prediction in the near field for both *Group I* and *Group II*. This is particularly

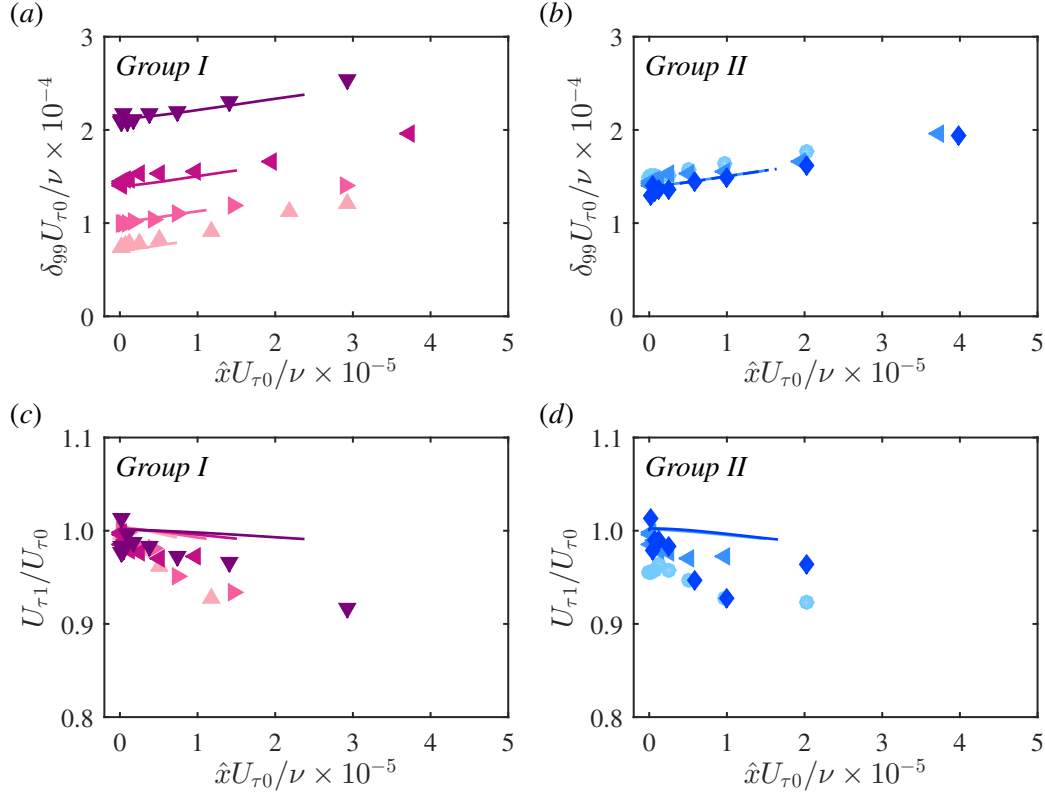


FIGURE 9.19: Comparison of δ_{99} and $U_{\tau 1}$ obtained from experimental data and predicted by the refined model. The experimental data are shown by the corresponding symbols while the predictions are shown by coloured lines.

noticeable in the zoomed views given in plots (c) and (d). This improvement is mainly contributed by the refinement of $\Delta\delta_i^+$ and Δd very close to the step change (where the fetch is in the order of a few hundred viscous units), while the non-equilibrium effect described by μ_0 is the predominant factor in the intermediate range (where the fetch is $\sim O(10^4)$ viscous units). The difference between the two predictions diminishes in the far field before the prediction terminates. Contrary to the IBL thickness results, the $Re_{\tau 0}$ dependence of $U_{\tau 2}$ is not significant within the range investigated in this study.

The outer quantities, δ_{99} and $U_{\tau 1}$ are shown in figure 9.19. Note that although we use δ_c (the boundary layer thickness following Jones et al.'s (2001) definition) in the refined model, here we present the results of δ_{99} interpolated from each predicted profile to allow an easier comparison with the experiments. In the original Elliott's model, δ_{99} (or any form of the boundary layer thickness) is absent and $U_{\tau 1}$ is assumed to be equal to $U_{\tau 0}$ at all streamwise locations, so here we only show the coloured lines (predictions from the refined model) in the figure to compare with the experimental results. The growth of δ_{99} becomes less aggressive after the roughness transition compared to a rough-wall turbulence boundary layer (see §6.8). As shown in figure 9.19(a) and (b), the growth of δ_{99} is well captured by the refined model in both *Group I* and *Group II*. The outer-layer friction velocity scale $U_{\tau 1}$ is shown in figure 9.19(c) and (d). The symbols represent $U_{\tau 1}$ computed from the measured turbulence intensity profiles with an assumed outer-layer similarity, following the method detailed in §6.8. Despite the good agreement between the predicted and measured δ_{99} , the

decrease of experimental $U_{\tau 1}$ with an increasing $\hat{x}$ is much slower than the predicted trend. For instance, at $\hat{x}U_{\tau 0}/\nu = 1.5 \times 10^5$, the measured $U_{\tau 1}$ for case Re21ks16 has decreased by 3% compared to its upstream value while the prediction shows less than 1% drop. For a given δ_c , the measured $U_{\tau 1}$ appears to be lower than the results computed using the outer-layer relationship (9.17). In future work, a new relationship other than (9.17) may be required to account for the decay of $U_{\tau 1}$ in the streamwise direction.

9.9 Summary and conclusion

In this chapter, we numerically restate Elliott's (1958) classical model of the mean velocity profile evolution in the streamwise direction after a roughness transition, and provide further refinements of the model by considering more physically realistic assumptions. The main findings are summarised below:

1. The initial condition of the evolution (the value of δ_i at $\hat{x} = 0$) does not affect the shape of the δ_i versus $\hat{x}$ trajectory as predicted by Elliott's model. It only introduces a streamwise shift $\Delta\hat{x}$, which is a function of $\delta_i(0)$.
2. On the basis of Elliott's original model, we consider several refinements, including adding a wake function to the logarithmic velocity profile, modelling the decay of the shear stress when δ_i exceeds the constant-stress layer, modelling the streamwise evolution of the outer layer with two additional governing equations, and replacing the inner log-law profile with the blending model to account for the non-equilibrium behaviour in the IBL. After implementing these refinements, an improved agreement between the model prediction and the experimental data is observed. So far, the refined model has only been tested against the present experimental dataset. Comparisons with a wider range of datasets from various sources are required in future works.
3. Compared to the original Elliott (1958) model which is intended for a deep surface layer, the refinements developed here make the model suitable for a spatially developing turbulent boundary layer with a finite thickness. Some refinements, including the consideration of a wake profile, the shear stress profile and the streamwise growth of the entire boundary layer thickness can also be applied to rougher-to-smoother or smooth-to-rough transitions. The blending mean velocity profile has only been tested in the rough-to-smooth case. In the future, the validity of the concept should be explored in other roughness heterogeneity scenarios such as smooth-to-rough, rough-to-less rough and a sequence of roughness transitions (as explored by Bou-Zeid et al. (2004) for example).
4. The refinements we introduce to the model have cancelling effects on the final prediction. For instance, for a rough-to-smooth change, both wake and shear-stress refinements lead to a reduction in the predicted δ_i , which is balanced by an increase resulting from the streamwise evolution of the outer layer and the blending mean velocity model. Therefore, the prediction of the original Elliott's model is in reasonable agreement with the experimental results even without including any of these refinements.

9.A Appendix: Modified Musker mean velocity profile

We adopt the expression of the inner mean velocity profile of a smooth-wall turbulent boundary layer proposed by Chauhan et al. (2009). The Musker inner profile is given by

$$U_{\text{inner}}^+ = \frac{1}{\kappa} \ln \left(\frac{z^+ - a}{-a} \right) + \frac{R^2}{a(4\alpha - a)} \left[-\frac{a}{R} \frac{\sqrt{(z^+ - \alpha)^2 + \beta^2}}{z^+ - a} \right. \quad (9.26)$$

$$\left. + \frac{\alpha}{\beta} (4\alpha + 5a) \left(\arctan \left(\frac{z^+ - \alpha}{\beta} \right) + \arctan \left(\frac{\alpha}{\beta} \right) \right) \right] \quad (9.27)$$

with $\alpha = (-1/\kappa - a)/2$, $\beta = \sqrt{-2a\alpha - \alpha^2}$ and $R = \sqrt{\alpha^2 + \beta^2}$. For the consistency with the logarithmic law constants $\kappa = 0.384$ and $B = 4.17$, a is chosen as -10.3061 . To account for the undershoot of the indicator function $z^+ (dU^+/dz^+)$ at around $z^+ \approx 50$, an exponential term is added to the original expression (Monkewitz et al., 2007):

$$U_{\text{inner}}^+(\text{modified}) = U_{\text{inner}}^+ + \frac{\exp[-\ln^2(z^+/30)]}{2.85}. \quad (9.28)$$

Chapter 10

Conclusions and future work

10.1 Summary of principal results

In this study, we present two unique experimental datasets to systematically investigate the flow recovery after a rough-to-smooth transition. Multiple experimental techniques are employed, including a direct measurement of the wall shear stress by OFI. The measurements are unique in their scope, with a wide parameter range of roughness Reynolds number ($111 < k_{s0}^+ < 228$) and friction Reynolds number ($4100 < Re_{\tau 0} < 21000$) with velocity profiles obtained up to a fetch of 120 boundary layer thicknesses (δ_0) downstream of the roughness transition. The main findings are summarised below.

1. Conventional ‘indirect’ techniques that only have access to velocity information above the viscous sublayer consistently underestimate the magnitude of τ_w immediately downstream of a rough-to-smooth transition. This might be responsible for part of the scatter in the C_f recovery trends reported in the literature. The mean flow within the buffer region (defined as $10 < z^+ < 30$) only recovers to an equilibrium state after approximately 10,000 viscous length scales downstream of the transition ($\hat{x}U_{\tau 0}/\nu \gtrsim 1.0 \times 10^4$). An alternative approach to estimate the wall-shear stress from the premultiplied energy spectra is proposed when no direct measurement of the wall-shear stress is available, based on the observation that the small scales attain a universal form over relatively short recovery distances after a rough-to-smooth transition. This method offers improved performance relative to more conventional techniques, which are based purely on the mean velocity profile in the buffer region, for experiments conducted at a wide range of $Re_{\tau 0}$.
2. When normalised by the smooth-wall skin-friction coefficient C_{fe} , the recovery of C_f follows a single trend with $\hat{x}/\delta_0$, showing little $Re_{\tau 0}$ or k_{s0}^+ dependence. A full recovery in C_f is observed in a fetch of $\hat{x}/\delta_0 \approx 20$. We also compare the internal boundary layer (IBL) height δ_i extracted from the turbulence intensity profile and the mean velocity profile following the definition of Antonia and Luxton (1971b). The former is slightly higher than the latter, as it tends to pick up the upper limit of the IBL. δ_i computed using both definitions follows a power-law growth trend in the streamwise direction with an exponent close to 0.8. Little $Re_{\tau 0}$ or k_{s0}^+ dependence is observed in the IBL growth rates.

3. The step height between the upstream rough and downstream smooth surface influences a number of flow quantities, including C_f , δ_i , mean velocity profiles and various orders of the fluctuating streamwise velocity moments. The effect is the strongest in the vicinity of the roughness transition before gradually diminishing over a fetch of $\hat{x}/\delta_0 = 2 - 4$. A comparison of the experimental results in this study with an accompanying DNS dataset shows similar flow recovery trends, suggesting that Δd , the step height in the virtual origin may be a better length scale than the boundary layer thickness prior to the surface transition (δ_0) for normalising the fetch for comparison between different flow conditions.
4. A blending model of the mean velocity profile that captures the non-equilibrium behaviour in the IBL is developed. The modelled mean velocity profile approaches an equilibrium smooth-wall velocity profile in the near-wall region, and it approaches the upstream rough-wall profile above the IBL. The blending function E can be well approximated by an error function and the coefficients appear to be universal across the range of $Re_{\tau 0}$ and k_{s0}^+ examined in this study. The blending model is then incorporated into the original Elliott's model by replacing the simple logarithmic velocity profile, together with other refinements, such as adding a wake function, considering the decrease of shear stress in the wall-normal direction and also the growth of the outer layer in the streamwise direction. The refined Elliott's model leads to an improved agreement with the experimental data compared to the original one.

10.2 A final note on the common practice in the shipping industry

In order to highlight the significance of the present study, we would like to briefly discuss the application of the evolution model on a practical problem in the shipping industry. It has been well-recognised that biofouling can significantly increase the drag penalty, thus the fuel consumption, of a ship. For example, at the cruising speed, the power penalty of a heavy calcareous fouling is estimated to be 86% (Schultz, 2007). Therefore, it is advisable to regularly monitor the roughness level of the underwater ship hull and design a cleaning schedule (dry-docking or underwater cleaning) accordingly.

In order to calculate the drag penalty introduced by the surface roughness, a common practice is to first measure the peak-to-trough surface roughness height with a hand-held profilometer at a number of points on the ship hull. The simple average of these readings, commonly referred to as the Averaged Hull Roughness (AHR), is then substituted into an empirical equation to determine the drag penalty (International Marine Coatings, 2004). Despite the caveats such as the lack of Reynolds number dependence in the empirical equation, as well as the incapability of a single topographical parameter of the roughness to represent its effect on the flow, here we would like to investigate whether the inherent averaging process in the AHR can successfully predict the drag penalty in the case of heterogeneous surface roughness.

Heterogeneity of the biofouling roughness can be created in several ways: some marine creatures, for example barnacles, tend to settle close to each other and form colonies

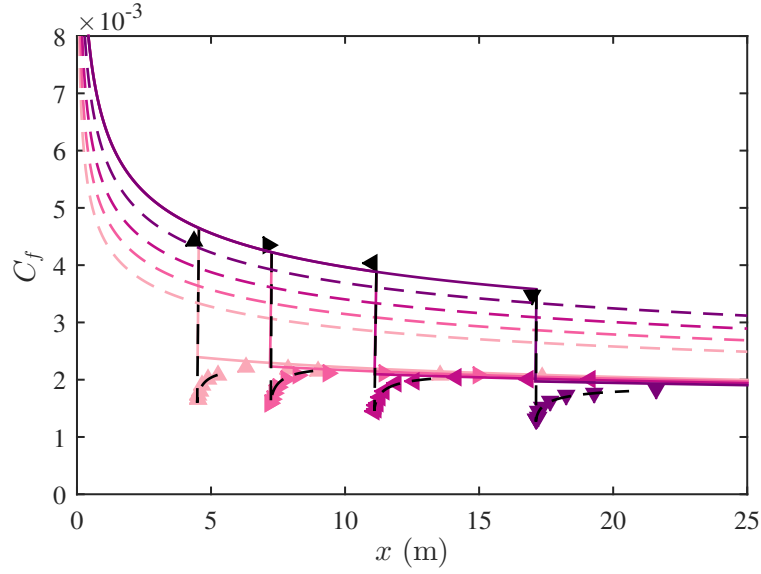


FIGURE 10.1: Skin-friction coefficient C_f of *Group I* cases. The black dashed line is predicted by the refined Elliott's model as detailed in §9, the coloured solid line is calculated by assuming the equilibrium relation on each surface patch, and the coloured dashed line is calculated by assuming a homogeneous AHR covering the entire plate. The black symbols are obtained from a log fit of the measured rough-wall mean velocity profiles, while the coloured ones are obtained by OFI.

(Knight-Jones and Crisp, 1953); besides, underwater cleaning or dry docking process may not be able to access the entire ship hull so that patches of roughness remain after cleaning (under docking blocks for example). Furthermore, at various parts of the ship hull, the local flow condition can vary greatly, so difference in the surface fouling level is expected. To summarise, heterogeneity in biofouling roughness is inevitable. To demonstrate the accuracy of estimating the overall drag penalty with the AHR, we consider the experiments conducted in the HRNBLWT (where a direct measurement of the recovering skin-friction coefficient C_f is available) as a test case. The flat plate has a length of $L = 25$ m in stream-wise, the upstream portion of which is covered by a uniform roughness and the downstream portion remains a smooth surface. Figure 10.1 shows C_f of *Group I* cases predicted by the refined Elliott's model (as shown by the black dashed line) and by assuming a homogeneous AHR for the entire length of the plate (coloured dashed line). This figure also shows the drag curves resulting from an assumption of full equilibrium on each surface patch (coloured solid line). The drag coefficient of the entire flat plate $\overline{C_F}$ is expressed as the average of the local skin-friction coefficient:

$$\overline{C_F} = \frac{1}{L} \int_0^L C_f(x) dx, \quad (10.1)$$

and the results corresponding to various modelling approaches are shown in figure 10.2. A 10% difference is observed in the $\overline{C_F}$ computed using the refined Elliott's model and the AHR approach. Clearly, simply taking the arithmetic mean of the observed heterogeneous patches is insufficient, with the possibly of introducing large errors. The model developed here offers a possible tool for bypassing the reliance on the AHR, and using more sophisticated and validated methods to account for heterogeneity. For the current case, resolving

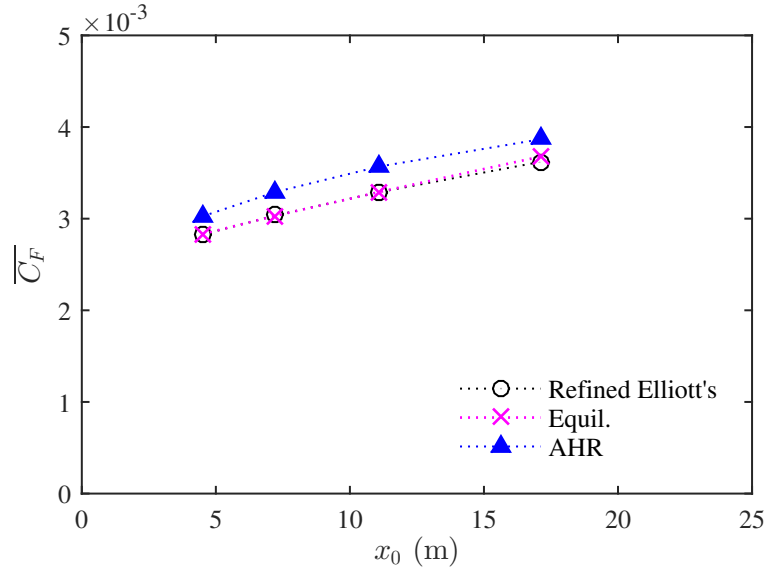


FIGURE 10.2: Drag coefficient $\overline{C_F}$ of *Group I* cases versus the sandpaper length x_0 . The black circles are computed from the refined Elliott's model results (shown by black dashed lines in figure 10.1), the magenta crosses are computed by assuming the equilibrium relation on each surface patch (coloured solid lines in figure 10.1), and the blue triangles are computed from the AHR results (coloured dashed lines in figure 10.1).

the recovering flow downstream of the roughness transition using the refined Elliott's model gives very similar results as employing the assumption of full equilibrium on each surface patch (denoted by 'Equil.'). This is because there is only one roughness transition on the entire plate and the flow is in equilibrium with the local surface condition over the majority of the length. For more complicated roughness heterogeneity with multiple smaller patches, the flow might not be in full equilibrium with the local wall condition before encountering a further roughness transition downstream, and the assumption of full equilibrium on each surface patch can lead to larger errors in this case. Future work is required to understand the flow response to such surface conditions and to further develop the predictive tool detailed here to accommodate such scenarios.

10.3 Suggestions for future work

1. **Experiments on a wider range of surface roughness.** So far, we have only tested a rough-to-smooth surface heterogeneity with two types of sandpaper. There are several possibilities for future research. First, we may examine flows over smooth-to-rough transitions with the focus on the non-equilibrium behaviour within the IBL, and also explore the possibility of a mean velocity profile blending model similar to the one described in §8. Second, a wider range of k_{s0}^+ should be considered. In §9, we have shown a weak increasing trend in the predicted $\delta_i U_{\tau 0} / \nu$ with an increasing k_{s0}^+ . However, the k_{s0}^+ range of our current data is not sufficient to confirm or disprove the trend predicted. Third, the selection of roughness should be extended to sandpaper with various grits and other surface textures. All the experiments conducted in the HRNBLWT

share the same $k_s = 2.43$ mm. Although using the same sandpaper (therefore the same physical k_s) reduces the uncertainty when comparing different cases, a range of k_s is required in order to assess the performance of k_s as a length scale. For a certain k_s in the air flow, the only way to increase k_{s0}^+ is to increase the friction velocity $U_{\tau 0}$, which is roughly proportional to the freestream velocity U_∞ . Case Re14ks22, which has the highest k_{s0}^+ in the present study, is obtained at a freestream velocity of $U_\infty = 31 \text{ ms}^{-1}$. Considering that the maximum freestream velocity achievable in the HRNBLWT is approximately 40 ms^{-1} , pushing the tunnel speed to its limit can only expand the range of k_{s0}^+ slightly. Referring back to the previous point, a much rougher surface is necessary to further increase k_{s0}^+ .

2. **Experiments with multiple patches in the streamwise direction.** This proposed study ties in closely with the biofouling induced drag penalty prediction as detailed in §10.2. In reality, the naturally formed biofouling patches might be smaller in size. The fetch on an approximately homogeneous patch might not be long enough for the boundary layer to fully adjust to the new wall condition. However, neither the experiments nor the model developed in this study accounts for such scenarios. With these additional experiments, we can closely examine the response of the recovering flow to another roughness transition downstream, leading towards developing an analytical model to capture the mechanism.
3. **Two-point measurements to investigate the inner-outer interaction after a rough-to-smooth transition.** A model introduced by Marusic et al. (2010) consolidates the current understanding of the interaction between large- and small-scale motions in a turbulent boundary layer, and demonstrates the possibility to predict the flow statistics in the near-wall region by accounting for the superposition and modulation effects on a universal small-scale signal. To date, systematic studies to calibrate the model parameters have been conducted on both homogeneous smooth-wall (Mathis et al., 2011a; Baars et al., 2016a) and rough-wall (Squire et al., 2016a) boundary layers. In fact, flows over a rough-to-smooth change are an ideal case to examine the response of the small-scale motions to over-energised large-scale structures in the outer layer. We have shown it in §6.9 with a single-point measurement that when compared to an equilibrium smooth-wall reference, it exhibits a higher level of energy in the near-wall large-scale motions due to the ‘footprint’ of the over-energised rough-wall structures in the outer layer. More direct evidence, such as two-point correlations, is required to confirm this point.
4. **Developing an analytical model for flows over a roughness transition in internal geometries.** The applicability of the present model (and Elliott’s original model) to internal geometries such as channel or pipe flows require careful considerations. Compared to turbulent boundary layers, channel or pipe flows have an additional constraint: the mass conservation of the flow across the cross-section, while the streamwise pressure gradient varies in x as a consequence. In fact, a strong adverse pressure gradient is observed immediately downstream of the rough-to-smooth change in a channel (Ismail

et al., 2018b), which corresponds to the deceleration of the mean streamwise velocity above the IBL. To conclude, these differences in the driving mechanism must be fully accounted for to successfully model the flow evolution after a step change of roughness in a confined geometry and therefore need to be examined in future works.

5. **Tailoring LES wall models for flows after a rough-to-smooth change.** The wall model usually provides U_τ and a boundary condition (such as a slip velocity or a velocity gradient) at the first grid point above the wall for the outer LES. Equilibrium assumptions (e.g. the law of the wall) are usually made when computing U_τ and the boundary condition, which may be violated immediately downstream of the rough-to-smooth change. To improve the LES model performance for such scenarios, we propose to use the refined Elliott's model to calculate U_τ and the slip velocity at the first grid point downstream of the roughness transition. The fetch where the refined Elliott's model operates is dependent on the grid size, but in principle the refined Elliott's model should be used until the resulting U_τ and the slip velocity are very similar to that of a conventional equilibrium wall model. To improve the estimation of near-wall small-scale fluctuations, the IOIM can be incorporated following Sidebottom et al. (2014). The IOIM parameters (e.g. the universality of small-scale energy spectrum and the transfer kernel) will first need to be characterised in the flow after a rough-to-smooth change, as discussed above in point 3.
6. **Investigation of the passive scalar transport in flows over a heterogeneous roughness.** The importance of this study is two-folds. First, from a practical point of view, the proposed study represents common scenarios such as the dispersion of air pollutants in the outskirts of a city. Second, from a theoretical point of view, the diffusion-analogy type models (see §2.3.2) rely on the assumption that the IBL propagates in the same manner as a passive contaminant, and the growth rate of δ_i is proportional to the vertical diffusion intensity w_{rms} . Experimental evidence from the proposed study can provide valuable insight to assess the above assumption. In addition, considering the great difficulty in capturing the instantaneous IBL interface from the velocity field after a rough-to-smooth transition, introducing a passive scalar might increase the feasibility. These results will benefit further developments of the blending model.
7. **Introducing active scalars to flows over a heterogeneous roughness.** On the basis of the previous proposed study, we can also introduce an active scalar such as temperature or humidity to the problem and examine the effect of the active scalar on the flow perturbed by the roughness heterogeneity. The insight gained from laboratory studies under controlled conditions is meaningful in advancing our understanding of the microclimate in coastal regions and improving the predictive weather models.

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