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STABILIZING DEAD-BEAT CONTROLLERS FOR TWO CLASSES OF WIENER-HAMMERSTEIN SYSTEMS

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Abstract. Two classes of block oriented models of Wiener-Hammerstein type are considered. We prove that a generic condition is sufficient for a null controllable system of this form to have a stabilizing minimum-time dead-beat controller. In case the condition is violated, we show how to design a non-minimum time stabilizing (dynamic) dead-beat controller. The result can be used to obtain stabilizability conditions for these systems, which are the same as the linear conditions: all uncontrollable modes should be stable.

Résumé. Pour deux classes de systèmes du type Wiener-Hammerstein, on démontre qu'une condition structurelle générique est suffisante pour construire un régulateur à réponse pile en temps minimum. Ce résultat permet de formuler des conditions de stabilisabilité identiques aux conditions linéaires : tous les modes non commandables doivent être stables.

Keywords :controllability, discrete-time, Wiener-Hammerstein systems, generalized Hammerstein systems, dead-beat, nonlinear, stabilizing.

1. INTRODUCTION

It is well known that minimum-time dead-beat controller always stabilizes a null controllable linear system, since it places all the poles of the closed loop systems inside the unit disc, that is at the origin. However, in the nonlinear context, these controllers render the origin of the closed loop system globally attractive [9] whereas they may not be stabilizing, violating in this way the most basic requirement for their implementation.

Stability analysis of minimum-time dead-beat controllers for general nonlinear systems leads to computationally intractable problems even for "mild" nonlinearities and low order systems. Therefore, it appears to be necessary to consider simpler classes of nonlinear systems in order to carry out stability analysis successfully.

Such an important class of models, which are very often used in black-box identification of nonlinear systems, are of Wiener-Hammerstein type [2]. The basic building blocks for these systems are parallel (Figure 1) and series (Figure 2) connections of linear dynamical blocks and monomial static nonlinearities of the form $N(\cdot) = (\cdot)^q, q \in \mathbb{N}$.

Null (state dead-beat) controllability conditions for these systems were presented in the recent articles [3, 4, 6]. We show below that if a generic condition is satisfied for a null controllable system of this form, there exists a minimum-time dead-beat controller which is globally stabilizing. Moreover, the condition for stabilizing properties of minimum-time dead-beat controllers is not necessary (we present a counter-example to this statement). If the condition is violated, we show how it is possible to design a stabilizing dynamic non-minimum-time dead-beat controller.

The result is to best of our knowledge first of this kind for a class of nonlinear systems. Its importance is reflected in the fact that we use it in the second part of the paper to prove necessary and sufficient conditions for stabilizability of these classes of Wiener-Hammerstein systems. The stabilizability conditions are the same as in the linear case: all uncontrollable modes should be stable.

The paper is organized as follows. After presenting preliminaries, notation and definitions in Section 2, we present some results from literature in Section 3. The main results are presented in in Section 4. Examples are presented in Section 5 and summary is given in the last section.

2. PRELIMINARIES

Sets of real, natural and complex numbers are respectively denoted as $\mathbb{R}$, $\mathbb{N}$ and $\mathbb{C}$. We consider SISO generalized Hammerstein systems of the form (Figure 1):

$$\begin{aligned} \Sigma_1 : x_1(k+1) &= Ax_1(k) + bu(k) \\ \Sigma_2 : x_2(k+1) &= Fx_2(k) + g(u(k))^q \\ y(k) &= cx_1(k) + hx_2(k) \end{aligned} \quad (1)$$

or SISO simple Wiener-Hammerstein discrete-time systems (Figure 2):

$$\begin{aligned} \Sigma_1 : x_1(k+1) &= Ax_1(k) + bu(k) \\ \Sigma_2 : x_2(k+1) &= Fx_2(k) + g(cx_1(k))^q \\ y(k) &= hx_2(k) \end{aligned} \quad (2)$$

where $x_i \in \mathbb{R}^{n_i}$, $i = 1,2$, $n_1 + n_2 = n$, $u \in \mathbb{R}$, $q \in \mathbb{N}, q > 1$ and matrices A, F, b, g are of appropriate dimensions. It is assumed that $c \neq (0 \dots 0)$. In the sequel we often refer to the following decomposition of the simple Wiener-Hammerstein system. We say that the linear dynamical block $W_1(z)$ is the subsystem S_1 and that the series connection of the static nonlinearity $(\cdot)^q$ and the linear block $W_2(z)$ is the system S_2 (see Figures 1 and 2).

The systems (1) and (2) consist respectively of a parallel and series connection of two linear dynamical blocks

$$\begin{aligned} W_1(z) &= \frac{y_1(z)}{u_1(z)} = c(zI - A)^{-1}b = \frac{b_1(z)}{a_1(z)}, \\ W_2(z) &= \frac{y_2(z)}{u_2(z)} = h(zI - F)^{-1}g = \frac{b_2(z)}{a_2(z)} \end{aligned} \quad (3)$$

interconnected via the static nonlinearity. Roots of polynomials $a_i(z)$ and $b_i(z)$ are respectively poles and zeros of $W_i(z)$. To simplify the exposition of the results, we assume without loss of generality that there are no feed-through terms for transfer functions $W_i(z)$. We emphasize, however, that our main results hold in this case.

We denote a sequence of controls $\{u(0), u(1), \dots\}$ as U where $u(i) \in \mathbb{R}$ and its truncation of length N, that is $\{u(0), \dots, u(N-1)\}$, as U_N. The state of the system (1) or (2) at time step N, which is obtained when a sequence U_N is applied to the system and which emanates from the initial state $x(0)$, is denoted as $x(N, x(0), U_N)$. We give below the definitions that are used in the sequel:

Definition 1 *The system is null controllable if there exists an integer N such that for any initial state $x(0) \in \mathbb{R}^n$ there exists a control sequence U_N such that $x(N, x(0), U_N) = 0$.*

Definition 2 *The system is completely controllable if there exists an integer N, such that $\forall x(0), x^* \in \mathbb{R}^n$ there exists $U_k, k \leq N$ such that $x(k, x(0), U_k) = x^*$.*

The characteristic polynomial of a matrix F is denoted as $P_F(\lambda) = \det(\lambda I - F)$. Given a polynomial:

$$P(\lambda) = \lambda^t + a_{t-1}\lambda^{t-1} + \dots + a_1\lambda + a_0$$

we introduce a new polynomial $P^q(\lambda)$ which is obtained from $P(\lambda)$ when all the coefficients a_i are taken with a power $q > 0, q \in \mathbb{N}$, that is we write:

$$P^q(\lambda) = \lambda^t + a_{t-1}^q\lambda^{t-1} + \dots + a_1^q\lambda + a_0^q$$

Also, if we are given a polynomial $H = \lambda^s + h_{s-1}\lambda^{s-1} + \dots + h_1\lambda + h_0$, we use the notation:

$$\begin{aligned} (P \cdot H)^q(\lambda) &= \lambda^{t+s} + (h_{s-1} + a_{t-1})^q\lambda^{t+s-1} \\ &+ \dots + (h_1a_0 + h_0a_1)^q\lambda + (a_0h_0)^q \end{aligned}$$

If $q \in \mathbb{N}$, we can use repeatedly the binomial formula $(a+b)^q \doteq \sum_{i=0}^{q} \binom{q}{j} a^{q-j}b^j$, where $\binom{q}{j} = \frac{q!}{j!(q-j)!}$, to find the coefficients of $(P \cdot H)^q(\lambda)$. Hence, the polynomial $(P \cdot H)^q(\lambda)$ is obtained when we first multiply the polynomials P and H and then take qth powers of all the coefficients of the product polynomial. Notice that $(P \cdot H)^q(\lambda) = (H \cdot P)^q(\lambda)$.

Definition 3 *[1] A greatest common divisor of polynomials P_1, P_2 is a polynomial G such that: G divides P_1 and P_2; if P is another polynomial which divides P_1 and P_2, then P divides G. When G has these properties we write $G = GCD(P_1, P_2)$.*

If we have that $GCD(P_1, P_2) = 1$, we say that P_1 and P_2 are coprime.

Definition 4 *Given a polynomial P_1 with roots $\{\sigma_1, \dots, \sigma_{n_1}\}$ and polynomial P_2 with roots $\{\zeta_1, \dots, \zeta_{n_2}\}$, their resultant is denoted as $Res(P_1, P_2)$ and is defined as*

$$Res(P_1, P_2) = \prod_{i,j}(\sigma_i - \zeta_j)$$

Two polynomials have a common root if and only if their resultant is equal to zero. If $Res(P_1, P_2) \neq 0$, the polynomials P_1 and P_2 are coprime. Resultant of two polynomials can be obtained as a function of coefficient of the polynomials by using the Sylvester matrix [1].

3. NULL CONTROLLABILITY AND MINIMUM-TIME DEAD-BEAT CONTROLLERS

Null controllability conditions for systems (1) and (2) were presented in [3, 4, 6]. We repeat them here for convenience:

Theorem 1 *The generalized Hammerstein system (1) (simple Wiener Hammerstein system (2)) is null controllable if and only if:*

1. *q is odd:*
   (a) *$rank[\lambda I - A : b] = n_1, \forall\lambda \in \mathbb{C} - \{0\}$.*
   (b) *$rank[\lambda I - F : g] = n_1, \forall\lambda \in \mathbb{C} - \{0\}$.*
2. *q is even:*
   (a) *$rank[\lambda I - A : b] = n_1, \forall\lambda \in \mathbb{C} - \{0\}$.*
   (b) *$rank[\lambda I - F : g] = n_1, \forall\lambda \in \mathbb{C} - \{0\}$.*
   (c) *F has no real positive eigenvalues $\lambda_i(F) \in \mathbb{R}, \lambda_i(F) > 0$.*

Complete controllability conditions are somewhat different [3, 4, 6], but are not cited here for space reasons.

Theorem 1 can be interpreted as follows: the system (1) or system (2) is null controllable if and only both subsystems S_i are null controllable. The result does not hold for the case when $q = 1$, that is the purely linear case. For instance, in order to have null controllability of system (1) with $q = 1$, the subsystems should not have common non-zero poles.

Minimum-time dead-beat controllers for polynomial systems can be designed by using a procedure based on the QEPCAD symbolic computation package [7, 9]. QEPCAD is used to compute the sets:

$$\begin{aligned} S_0 &= \{x : \exists u \in \mathbb{R},\ f(x,u) = 0\} \\ S_k &= \{x : \exists u \in \mathbb{R},\ f(x,u) \in S_{k-1}\} \end{aligned}$$

The set S_k is a set of states $x \in \mathbb{R}^n$ for which minimum-time necessary to transfer x to the origin is at most $k+1$. The following sets are also important:

$$\hat{S}_0 = S_0, \hat{S}_k = S_k - S_{k-1}, \forall k = 1, 2, \ldots, N-1$$

since they represent sets of states for which the minimum time to transfer them to the origin is equal to k. After the sets $\bar{S}_k$ have been computed using QEPCAD, the design of a minimum-time dead-beat controller follows easily.

Expressions which define sets S_k are denoted as $S_k(x)$. We illustrate the operation of the algorithm and our notation by the following example:

Example 1 *[9] Consider the generalized Hammerstein system:*

$$\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & -2 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u(k) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u^2(k) \qquad (4)$$

By using the procedure described in [7, 9] we obtain:

$$\begin{aligned} S_0 &= \{x : x_2 - x_1^2 = 0 \ \wedge \ x_3 = 0\} \\ S_1 &= \{x : 2x_3 + x_2 \geq 0 \ \wedge \ 2x_2x_3 \\ &\quad +x_3^2 - 6x_1^2x_3 + x_2^2 - 2x_2x_1^2 + x_1^4 = 0\} \qquad (5) \\ S_2 &= \mathbb{R}^3 \end{aligned}$$

Using our notation, we have that $S_0(x) \equiv (x_2 - x_1^2 = 0) \ \wedge \ (x_3 = 0)$, $S_1(x) \equiv (2x_3 + x_2 \geq 0) \ \wedge \ (2x_2x_3 + x_3^2 - 6x_1^2x_3 + x_2^2 - 2x_2x_1^2 + x_1^4 = 0)$ and $S_2(x) \equiv (0 = 0)$.

Notice that once we have computed the sets S_k, the minimum-time dead-beat controller is obtained easily. Indeed, we know that $\forall x \in S_k, \exists u \in \mathbb{R}$ such that $S_{k-1}(f(x,u))$ is satisfied. The time-optimal control action is obtained as a real solution u to the polynomial expression $S_{k-1}(f(x,u))$ if $x \in S_k$.

For instance, a minimum time state DB (feedback) controller for Example 1 is given below:

$$u(x) = \text{any real root } u \text{ to}$$
$$\begin{cases} (x_1 + u = 0) \wedge (-x_2 - 2x_3 + u^2 = 0), \text{ if } x \in \hat{S}_0 \\ (x_3 - (x_1+u)^2) = 0) \wedge (-x_2 - 2x_3 + u^2 = 0), \\ \qquad \text{if } x \in \hat{S}_1 \\ (2(-x_2 - 2x_3 + u^2) + x_3 \geq 0) \wedge \\ (2x_3(-x_2 - 2x_3 + u^2) + (-x_2 - 2x_3 + u^2)^2 \\ -6(x_1+u)^2(-x_2 - 2x_3 + u^2) + x_3^2 - \\ 2x_3(x_1+u)^2 + (x_1+u)^4 = 0), \text{ if } x \in \hat{S}_2 \end{cases} \qquad (6)$$

Obviously, the control u is obtained as a real solution to a set of polynomial equations for $x \in \hat{S}_0$ or $\hat{S}_1$. On the other hand, a polynomial equation and an inequality should be solved for $x \in \hat{S}_2$. We can first solve the equation and then check which solutions satisfy the inequality. Since we may have non unique solutions, the above given minimum time controller actually represents a family of minimum-time dead-beat control laws.

If the system is null controllable, it was shown in [6] that a minimum-time dead-beat controller is of the form:

$$u(k) = \text{any real } u \text{ root to}$$
$$\begin{cases} f(x,u) = 0, & \text{if } x \in \hat{S}_0, \\ S_0(f(x,u)), & \text{if } x \in \hat{S}_1, \\ S_1(f(x,u)), & \text{if } x \in \hat{S}_2, \\ \cdots & \cdots \\ S_{N-2}(f(x,u)), & \text{if } x \in \hat{S}_{N-1} \end{cases} \qquad (7)$$

By definition, if $x \in \hat{S}_k$ then there exists a real solution to the expression $S_{k-1}(f(x,u))$. If we denote any such solution as $u_k(x)$ for $x \in S_k$, we can write the minimum-time dead-beat controller in the following form:

$$u(x(k)) = u_i(x(k)), \quad \text{if } x(k) \in \hat{S}_i, i = 0, 1, \ldots, N-1 \qquad (8)$$

4. STABILIZING PROPERTIES OF DEAD-BEAT CONTROLLERS

In this section we present the main results of the paper. In Theorems 2 and 3 we give conditions for minimum-time dead-beat controllers to be stabilizing. The condition is generic for null controllable systems (1) and (2). Then in Theorem 4 we show that if the condition is violated, but the system (1) or (2) is null controllable, we can still design a dynamic state feedback dead-beat controller which is stabilizing. With this result we prove necessary and sufficient conditions for stabilizability of systems (1) and (2) (Theorem 5).

The theorems are stated hereafter without proofs. Detailed proofs can be found in an extended version of the paper [5].

Theorem 2 *There exists a stabilizing minimum-time dead-beat controller for the generalized Hammerstein system (1) if the system is null controllable and polynomials $a_1^q(z)$ and $a_2(z)$ are coprime.*

Theorem 3 *There exists a stabilizing minimum-time dead-beat controller for the simple Wiener-Hammerstein system (2) if the system is null controllable and polynomials $b_1^q(z)$ and $a_2(z)$ are coprime.*

The proof of these theorems are based on the following propositions.

Proposition 1 *Consider equation:*

$$u^q + t_1 u^{q-1} + \ldots + t_{q-1}u + t_q = 0 \tag{9}$$

where $t_i = t_i(x, v_0, v_1, \ldots, v_k), i = 1, \ldots, q$, with $t_i(0, 0, \ldots, 0) = 0$, are functions in $x \in \mathbb{R}^n$ and $v_j = v_j(x)$. Suppose that the following holds: $\forall E > 0$, $\exists \delta_u > 0$ such that if $||x|| < \delta_u$ then $|t_i| < E$, $\forall i = 0, 1, \ldots, q-1$. Denote the set of roots u_i to the equation (9) as Σ. Then it holds that $\forall \epsilon_u > 0$, $\exists \delta_u > 0$ such that if $||x|| < \delta_u$ then $|u_i| < \epsilon_u, \forall u_i \in \Sigma$.

This proposition can be interpreted in the following way: if we can make coefficients t_i of the equation (9) arbitrarily small, then all the roots $u_i \in \Sigma$ to (9) can be made arbitrarily small. Proposition 1 is instrumental in proving the main result. We introduce the "small control property", see for instance [10]:

Definition 5 *A control law $u = u(x)$ is said to have small control property (SCP) if $\forall \epsilon_u > 0$, $\exists \delta_u > 0$ such that if $||x|| < \delta_u$ then $|u(x)| < \epsilon_u$.*

The following proposition shows that if the conditions of Theorems 2 or 3 are satisfied, then there exists a minimum-time dead-beat controller respectively for system (1) or (2), $u = k(x)$, which has SCP.

Proposition 2 *Consider system (1). If $a_1^q(z)$ and $a_2(z)$ are coprime, then there exists a minimum-time dead-beat controller $u = u(x)$, which has SCP.*

Consider system (2). If $b_1^q(z)$ and $a_2(z)$ are coprime, then there exists a minimum-time dead-beat controller $u = u(x)$, which has SCP.

Comment 1 *Notice that in Example 1 $a_1^2(z)$ and $a_2(z)$ are not coprime but the minimum-time dead-beat controller has SCP and it is therefore stabilizing. This shows that conditions of Theorem 2 are not necessary.*

Theorem 4 *There exists a non-minimum-time stabilizing dead-beat controller for a system (1) (system (2)) if and only if the the system is null controllable.*

Theorem 4 shows that we can make a trade-off between the performance (stability of the closed loop) and the dead-beat time in cases when minimum-time dead-beat controller is not stabilizing.

The stabilizing properties of dead-beat controllers can be used in a constructive way to prove the stabilizability of systems (1) and (2) as stated in the next theorem. The statement is the same as in the linear case: all uncontrollable modes should be stable. In other words, the considered block oriented models are stabilizable if and only if they are *asymptotically controllable.*

Theorem 5 *The system (1) or (2) is stabilizable if and only if it is asymptotically controllable.*

5. EXAMPLES

By examples we illustrate the statements of the main results.

Example 2 Consider the simple Wiener Hammerstein system with:

$$W_1(z) = \frac{z+b}{z^2+1}; \quad W_2(z) = \frac{1}{z^2-1}; \quad q = 3$$

where the parameter $b \in \mathbb{R}$ is free and we analyze its effect on the stabilizing properties of minimum-time dead-beat controllers. The first subsystem is controllable for any value of $b \in \mathbb{R}$ and the second subsystem is also controllable. Hence, we can write state equations of both subsystems in the controllability canonical form:

$$\begin{aligned} x_1(k+1) &= x_2(k) \\ x_2(k+1) &= -x_1(k) + u(k) \\ x_3(k+1) &= x_4(k) \\ x_4(k+1) &= x_3(k) + (bx_1(k) + x_2(k))^3 \end{aligned} \tag{10}$$

Theorem 3 states that a minimum-time dead-beat controller is stabilizing if the polynomials $b_1^3(z) =$

$z + b^3$ and $a_2(z) = z^2 - 1$ are coprime. Since poles of $W_2(z)$ are equal to ± 1, we have that the value $b^3 = \pm 1$ is the critical one which needs to be avoided in order to have stability.

First, to simplify analysis we introduce a non-singular state-feedback transformation

$$u(k) = x_1(k) + v(k) \tag{11}$$

With the new control input, the state equations become:

$$\begin{aligned} x_1(k+1) &= x_2(k) \\ x_2(k+1) &= v(k) \\ x_3(k+1) &= x_4(k) \\ x_4(k+1) &= x_3(k) + (bx_1(k) + x_2(k))^3 \end{aligned} \tag{12}$$

Since the system is null controllable, there exists a minimum-time dead-beat controller which transfers any initial state to the origin in finite time, say less than N steps [6]. Hence, sets $\hat{S}_i, i = 0, 1, 2, \ldots, N-1$ can be computed. Minimum-time dead-beat controllers have the form (8), and it renders the origin of the closed loop globally attractive. We do not need the exact description of these sets in order to prove stabilizing properties of (8).

Let us compute the controls which transfer the states from $\hat{S}_k$ to $\hat{S}_{k-1}$. Notice that this can be done by considering the state of the system (12) at time step k and computing the control $v(0)$. If we do that for steps $k = 1, 2, \ldots, N$, we obtain values of controls that are applied on sets $\hat{S}_k$ in (8).

In order to shorten notation we write below $x_i(0) = x_i$. Notice, that on $\hat{S}_0$ we know that there exists a feedback control strategy $v_0(x)$ which is such that $\forall x \in \hat{S}_0$ we have:

$$\begin{aligned} 0 &= x_2 \\ 0 &= w_0 \\ 0 &= x_4 \\ 0 &= x_3 + (bx_1 + x_2)^3 \end{aligned} \tag{13}$$

and it is easily seen from the second equation that $w_0 = 0$, which implies $W_0(x) = \{0\}$. It follows that $v_0(x) \in W_0(x), \forall x \in \hat{S}_0$, that is $v_0(x) = 0$. If the state is in $\hat{S}_1$, we consider:

$$\begin{aligned} 0 &= w_1 \\ 0 &= w_0 \\ 0 &= x_3 + (bx_1 + x_2)^3 && (14) \\ 0 &= x_4 + (bx_2 + w_1)^3 && (15) \end{aligned}$$

Notice that we exploit open loop arguments to analyze a closed-loop control law.From the second equation we have again that $w_0 = 0$, that is $W_1(x) = \{0\}$. Hence, closed loop minimum-time dead-beat controller (8) applies $u(x) = -x_1$ on the sets $\hat{S}_0$ and $\hat{S}_1$. Obviously SCP holds on these sets.

Consider now the dead-beat control action on the set $\hat{S}_2$.

$$\begin{aligned} 0 &= w_1 \\ 0 &= w_0 \\ 0 &= x_4 + (bx_2 + w_2)^3 \\ 0 &= x_3 + (bx_1 + x_2)^3 + (bw_2 + w_1)^3 \end{aligned} \tag{16}$$

Solutions w_2 on the set $\hat{S}_2$ satisfies third equation:

$$x_4 + b^3x_2^3 + 3b^2x_2^2w_2 + 3bx_2w_2^2 + w_2^3 = 0$$

SCP still holds on $\hat{S}_2$. Notice that on the set $\bar{S}_2$, the minimum-time dead-beat control law for the original system is of the form $u(x) = x_1 + w_2(x_2, x_4)$, and $w_2(x_2, x_4)$ must satisfy the above given equation.

Finally, consider the dead-beat control action on the set $\hat{S}_3$, where $x(4) = 0$. From the first two equations we have that $w_0 = w_1 = 0$. We can write the last two equations, as follows:

$$\begin{pmatrix} b^3 & 1 \\ 1 & b^3 \end{pmatrix} \begin{pmatrix} w_3^3 \\ w_2^3 \end{pmatrix} + \begin{pmatrix} x_3 + (bx_1 + x_2)^3 + 3b^2w_2^2w_3 + 3bw_2w_3^2 \\ x_4 + b^3x_2^3 + 3b^2x_2^2w_3 + 3bx_2w_3^2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{17}$$

If the matrix

$$\begin{pmatrix} b^3 & 1 \\ 1 & b^3 \end{pmatrix} \tag{18}$$

is singular, that is $b^3 = \pm 1$, the polynomials $b_1^3(z)$ and $a_2(z)$ have a common root, which is a critical situation according to Theorem 3.To see this, we multiply the second equation by -1 and add in to the first one, without affecting the solutions of (17). We obtain that w_3 must satisfy:

$$\begin{aligned} x_3 - x_4 + (bx_1 + x_2)^3 + 3b^2w_2^2w_3 + 3bw_2w_3^2 \\ -(b^3x_2^3 + 3b^2x_2^2w_3 + 3bx_2w_3^2) \end{aligned} \tag{19}$$

It is not clear anymore whether SCP for w_2 would imply the same property for w_3.

Example 3 Consider the generalized Hammerstein system:

$$\begin{aligned} x_1(k+1) &= -x_1(k) + u(k) \\ x_2(k+1) &= bx_2(k) + u^2(k) \end{aligned} \tag{20}$$

From Theorem 3, it follows that the minimum time dead-beat controller is stabilizing if $b \neq -1$.

Suppose first that $b = -1$. Then we can easily compute the minimum-time dead-beat controller:

$$u(x) = \begin{cases} x_1 & , x \in \hat{S}_0 \\ \frac{x_2 + x_1^2}{2x_1} & , x \in \hat{S}_1 \\ \Delta(x) & , x \in \hat{S}_2 \end{cases}$$

where the sets are computed to be:

$$\begin{aligned}\hat{S}_0 &= \{x : -x_2 + x_1^2 = 0\} \\ \hat{S}_1 &= \{x : x_1 \neq 0\} - \hat{S}_0 \\ \hat{S}_2 &= \mathbb{R}^2 - \hat{S}_0 - \hat{S}_1\end{aligned} \quad (21)$$

The function $\Delta(x)$ should satisfy $\Delta(x) \neq 0, x \neq 0$. Hence, we can choose the function so that it satisfies SCP. However, on the set $\hat{S}_1$ we have for any $x_2 \neq 0$ and $x_1 \to 0$ that $|u(x)| \to \infty$.

We prove instability of the closed loop system. Fix any $\epsilon^* > 0$. Consider any $\delta > 0$. By choosing $x_2(0) = \delta/2$ and letting $x_1(0) \to 0$, we have that $||x(1, x(0))|| > \epsilon^*$ and the closed loop system is unstable.

Consider now the situation $b = -2$. Minimum time dead-beat controller is:

$$u(x) = \begin{cases} x_1 & , x \in \hat{S}_0 \\ \frac{-2x_1 \pm \sqrt{8x_1^2 + 16x_2}}{-2} & , x \in \hat{S}_1 \\ \Delta(x) & , x \in \hat{S}_2 \end{cases}$$

where

$$\begin{aligned}\hat{S}_0 &= \{x : -2x_2 + x_1^2 = 0\} \\ \hat{S}_1 &= \mathbb{R}^2 - \hat{S}_0 - \hat{S}_1\end{aligned} \quad (22)$$

and $\Delta(x)$ is (any) solution to the inequality:

$$x_1^2 - 4x_2 - 2x_1\Delta + 3\Delta^2 \geq 0$$

There is lots of freedom in choosing $\Delta(x)$. However, it is obvious that we can chose it so that it has SCP. One such choice is $\Delta(x) = 0$. SCP holds also on sets $\hat{S}_0$ and $\hat{S}_1$ and we conclude that there exists a minimum-time dead-beat controller which stabilizes the system.

6. CONCLUSION

Two basic models arising in black-box identification of nonlinear systems (simple Wiener-Hammerstein and generalized Hammerstein systems) are considered. We present conditions which guarantee that minimum-time dead-beat controllers for these systems are stabilizing. If the condition is violated, we show how it is possible to design a dynamic state feedback dead-beat controller, which is stabilizing but not time-optimal. The maximal order of the dynamic feedback is determined and, moreover, it is shown how to construct it. The results are then used to state necessary and sufficient conditions for stabilizability of these models.

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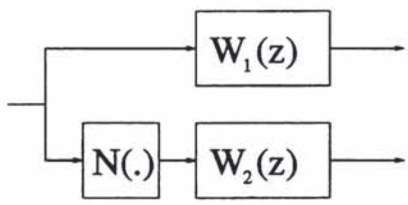

Figure 1: Generalised Hammerstein model

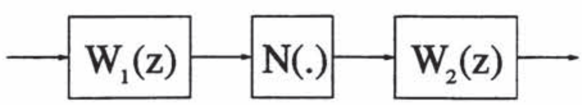

Figure 2: Simple Wiener-Hammerstein model