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Optimal Power Flow for Active Distribution Networks: Advanced Formulations, Practical Considerations and Laboratory Demonstration

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Optimal Power Flow for Active Distribution Networks

**Advanced Formulations, Practical Considerations
and Laboratory Demonstration**

by

Michael Z. Liu

ORCID ID: 0000-0002-9609-4544

A thesis submitted to The University of Melbourne for the degree of
Doctor of Philosophy

October 2020

Department of Electrical and Electronic Engineering

Melbourne School of Engineering

This thesis is submitted for the partial fulfilment of this degree.

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ABSTRACT

The rapid growth of renewable distributed generation (DG) has introduced unconventional challenges for distribution companies (e.g., dealing with voltage rise). To enable future DG growth, a promising alternative (to the otherwise capital-intensive and time-consuming network reinforcements) is the real-time orchestration of DG and existing network assets using advanced schemes. In this context, the operational usage of Optimal Power Flow (OPF)—an optimisation-based technique traditionally found in transmission network applications, albeit using simplified formulations—as a decision-making engine has gained tremendous interest in recent literature. Nonetheless, before such schemes can be readily integrated in the control room of distribution networks, there are several practical challenges that must be addressed.

Firstly, the operational usage of OPF requires a fast and scalable formulation that can handle the size (thousands of nodes) and complexity (phase unbalances, discrete devices) of typical distribution networks. Furthermore, since the differences in device-specific characteristics in the sub-minute scale (delays, ramp rates and deadbands) may lead to coordination issues when multiple devices are being controlled simultaneously, additional adaptations are necessary to ensure OPF-based setpoints can be implemented in real-world applications. Finally, while active power curtailment is inevitable at times, since such actions has a direct impact on the return on investment for DG owners, the implications from different fairness objectives (e.g., removing disparity in renewable energy harvesting or financial benefits) as well as the trade-offs between fairness (reducing disparity) and efficiency (aggregated performance) need to be first understood.

In this PhD project, the following research is carried out to address the aforementioned challenges:

- A linearised, three-phase AC OPF is developed to cater for multi-voltage level distribution feeders and integer variables. Its performance is demonstrated using a realistic MV-LV residential feeder (from the primary substation down to individual connection points of 4,626 single-phase consumers) with over 4,900 nodes.

-
- The necessary adaptations in existing device controllers and the OPF formulation are proposed, allowing network participants and assets to be successfully controlled using OPF-based schemes in an operational setting with minute-scale control actions. Particularly, the importance of the proposed adaptations in preventing short-term voltage spikes are demonstrated using a rural distribution feeder with multiple actively managed on-load tap changers and wind farms.
 - The implications and trade-offs from different fairness considerations are investigated using several OPF-based schemes, each considering a unique and contrasting fairness objective. The findings highlight the multi-facet nature of curtailment fairness and the importance of identifying the most appropriate objective for a given application. Furthermore, it can help operators/policymakers to make informed decisions when a portfolio of DG is to be managed.
 - A hardware-in-the-loop demonstration platform is built using commercially available software and hardware at the Smart Grid Lab of The University of Melbourne. This implementation extends beyond static plots and tables by introducing a rich and interactive user interface, and thus enabling a more realistic and engaging way of showcasing advanced schemes to industry.

DECLARATION

This is to certify that

1. the thesis comprises only of my original work towards the degree of Doctor of Philosophy except where indicated in the preface,
2. due acknowledgement has been made in the text to all other material used, and
3. the thesis is less than 100,000 words in length, exclusive of tables, maps bibliographies and appendices.

Michael Z. Liu, October 2020

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PREFACE

Parts of this thesis have been carried out in collaboration with Dr Andreas T. Procopiou (The University of Melbourne), Dr Kyriacos Petrou (The University of Melbourne), Dr Luis Gutierrez-Lagos (The University of Manchester), Prof Steven H. Low (California Institute of Technology). The list below identifies the publications that correspond to each chapter, as well as the contributions of each collaborator.

Chapter 2

1. L. Gutierrez-Lagos, **M. Z. Liu**, and L. F. Ochoa, "Implementable Three-Phase OPF Formulations for MV-LV Distribution Networks: MILP and MIQCP," in *Proc. 2019 IEEE PES Innovative Smart Grid Technologies Conference - Latin America (ISGT Latin America)*, pp. 1-6.

It should be noted that publication (1) is a comparison paper between two OPF formulations that are developed independently. The first formulation is developed by me which is also presented in Chapter 2 of this thesis. The second formulation is developed by Dr Luis Gutierrez-Lagos. Dr Luis Gutierrez-Lagos also provided the necessary network data to perform the benchmarks.

Chapter 3

2. **M. Z. Liu**, L. F. Ochoa, S. H. Low, "On the Implementation of OPF-Based Set-Points for Active Distribution Networks", *IEEE Transactions on Smart Grids*, in Review.

Prof Steven H. Low provided support in reviewing publication (2).

Chapter 4

3. **M. Z. Liu**, A. T. Procopiou, K. Petrou, L. F. Ochoa, T. Langstaff, J. Harding and J. Theunissen, "On the Fairness of PV Curtailment Schemes in Residential Distribution Networks", *IEEE Transactions on Smart Grids*, Accepted March 2020.

Dr Andreas T. Procopiou and Dr Kyriacos Petrou provided support in developing the household-centric metrics, which is a part of the overall methodology developed by me to analyse and assess the fairness implications of different PV curtailment schemes (e.g., identifying the different perspectives of fairness as well as the trade-offs between fairness and efficiency). Particularly, these metrics are used to quantify the impact of PV curtailment on *individual* households. The analysis in terms of

fairness, which is the core contribution of publication (3) and Chapter 4 of this thesis, is built on top of these metrics and focuses on the differences among *all* households.

Dr Andreas T. Procopiou and Dr Kyriacos Petrou also provided support in developing the network model used in the case study.

As per regulation of The University of Melbourne, it is hereby declared that I, Michael Z. Liu, have contributed to the majority (at least 51%) of the content found in publication (3) and Chapter 4 of this thesis.

“Genius is one percent inspiration and ninety-nine percent perspiration.”

– **Thomas Edison**

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DEDICATION

*To Xingfu and Liqun,
my loving parents.*

Thank you for raising me into the person I am today.

敬爱的父亲，母亲：谢谢你们一直以来
无私的付出和牺牲，无条件的鼓励与支持。

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First and foremost, I would like to express the sincerest gratitude to my supervisor Prof. Luis F. Ochoa—a respected mentor, an amazing boss, and a caring friend. Thank you for the support and guidance over the past years. You have been a constant source of inspiration to me, not only as researcher, but also as an engineer and as a leader.

Special thanks to my fellow colleagues at Team Nando: Kyriacos, William, Dillon, Andreas, Luis, Yolanda and Arthur. It has been a pleasure working with everyone, sharing crazy-yet-brilliant ideas over often-extendedly-long team meetings and still-educated conversations after ~~one~~ way-too-many beers at team BBQs. I would also like to thank my friends and colleagues, at The University of Melbourne and beyond, your friendship has made this journey even more worthwhile. Looking forward to sharing a beer and some laughter with you again, sometime, somewhere.

Finally, I would like to thank my beloved partner Yan. Thank you for your accompany, support and understanding during this important chapter of my life. Just like the saying “the little things in life matter more than anything else”, coming home every day to a gentle smile and a warm hug has always helped me to get through the toughest and most stressful moments.

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TABLE OF CONTENTS

1	INTRODUCTION	1
1.1	The Emergence of Active Distribution Networks	1
1.1.1	Passive Distribution Networks	1
1.1.2	The Rise of Distributed Generation.....	4
1.1.3	Impacts of Renewable DG.....	4
1.1.4	Towards Active Distribution Networks.....	6
1.1.5	Advanced Schemes for Active Distribution Networks	8
1.2	Challenges.....	8
1.2.1	Scalability and Speed of OPF Formulations	8
1.2.2	Real-World Implementation of OPF-Based Setpoints	9
1.2.3	Ensuring Fairness in Curtailment Schemes	9
1.2.4	Realistic Demonstration of OPF-Based Schemes	10
1.3	Research Questions.....	10
1.4	Objectives	11
1.4.1	A Scalable and Fast OPF Formulation	11
1.4.2	Necessary Adaptations	11
1.4.3	Trade-Offs among Different Fairness Objectives	12
1.4.4	Realistic Demonstration Platform	12
1.5	Main Contributions	12
1.5.1	Advanced AC OPF for Operational Usage in Distribution Networks ...	12
1.5.2	Necessary Adaptations to Implement OPF-Based Setpoints.....	13
1.5.3	Different Fairness Objectives of Curtailment Schemes	14
1.5.4	A Hardware-in-the-Loop Demonstration Platform	14
1.6	Publications.....	15
1.6.1	Journal Papers.....	15
1.6.2	Conference Papers	15

1.6.3	Technical Reports	15
1.6.4	Magazine Articles	16
1.7	Thesis Outline	16
2	A FAST AND SCALABLE THREE-PHASE OPF FOR DISTRIBUTION NETWORKS .	19
2.1	Introduction	19
2.2	Literature Review	20
2.2.1	Categories of Mathematical Optimisation Problems	20
2.2.2	Variations of AC Power Flow Equations	22
2.2.3	Three-Phase OPF Formulations.....	26
2.2.4	Summary of Research Gaps.....	28
2.3	Linearised Three-Phase OPF.....	28
2.3.1	Households.....	29
2.3.2	Distribution Lines	30
2.3.3	Delta-Wye Transformers	35
2.3.4	Nodal Constraints	41
2.3.5	Power Balance	41
2.3.6	Objective Function.....	42
2.3.7	Summary of the OPF Formulation.....	42
2.3.8	Solution Process.....	43
2.4	Case Study.....	44
2.4.1	Co-Simulation Platform.....	44
2.4.2	Assessment Methodology and Metrics	44
2.4.3	PV-Rich Distribution Network	45
2.4.4	Investigated Cases.....	46
2.4.5	Simulation Results	47
2.4.6	OPF Solution Time	52
2.5	Error in Linearised OPF Formulation	52

2.5.1	Error in Linearised Three-Phase Line Model	52
2.5.2	Error in Linearised Delta-Wye Transformer Model.....	58
2.6	Summary.....	61
3	IMPLEMENTING OPF-BASED SETPOINTS: CHALLENGES AND SOLUTIONS.....	63
3.1	Introduction.....	63
3.2	Literature Review	63
3.3	Necessary Adaptations.....	66
3.3.1	Decision-Making Process with Control Cycles.....	66
3.3.2	Understanding Operating Characteristics.....	67
3.3.3	Challenges of Implementing OPF-Based Setpoints	69
3.3.4	Adaptations in Device Controllers	71
3.3.5	Adaptations in OPF Formulations	73
3.4	Case Study	75
3.4.1	EHV1 Network	75
3.4.2	Modelling Considerations	77
3.4.3	Cases Investigated	79
3.4.4	Results	80
3.4.5	Sensitivity Analysis of Scaling Factors	91
3.4.6	Remarks on Solution Speed.....	93
3.5	Summary.....	94
4	FAIRNESS OF ACTIVE POWER CURTAILMENT SCHEMES: A RESIDENTIAL CONTEXT.....	95
4.1	Introduction.....	95
4.2	Literature Review	96
4.2.1	Fairness in PV Curtailment Schemes	96
4.2.2	Different Perspectives of Fairness.....	97
4.2.3	Fairness vs. Efficiency.....	100
4.2.4	Fairness Metrics.....	104

4.3	OPF-Based PV Curtailment Schemes	106
4.3.1	OPF-Generation	106
4.3.2	OPF-Export	107
4.3.3	OPF-Financial	107
4.3.4	OPF-Alpha-Fairness	108
4.4	Case Study: Different Perspectives of Fairness	108
4.4.1	PV-Rich Distribution Network	108
4.4.2	Simulation Setup	109
4.4.3	Results	110
4.4.4	Remarks on Solution Speed	120
4.5	Case Study: Fairness-Efficiency Trade-Offs	121
4.5.1	Scenarios Investigated	121
4.5.2	Results	122
4.6	Summary	126
5	A HARDWARE-IN-THE-LOOP DEMONSTRATION PLATFORM	127
5.1	Introduction	127
5.2	Literature Review	128
5.3	Architecture	129
5.3.1	Distribution Network	130
5.3.2	Communication Infrastructure	130
5.3.3	Control Room	130
5.4	Implementation of a HIL Demonstration Platform	130
5.4.1	Real-Time Simulator	131
5.4.2	Communication Infrastructure	132
5.4.3	Advanced Network Management System	132
5.4.4	Data Visualisation	133
5.5	Live Demonstration	135
5.6	Summary	137

6	CONCLUSION	139
6.1	Research Gaps and Contributions.....	139
6.1.1	Advanced OPF Formulation.....	140
6.1.2	Practical Considerations: Real-World Implementation of OPF-Based Setpoints	141
6.1.3	Practical Considerations: Fairness of Curtailment Schemes	142
6.1.4	Laboratory Demonstration.....	143
6.2	Key Findings.....	144
6.2.1	Linearised Three-Phase OPF Formulation	144
6.2.2	OPF-Based Schemes in an Operational Environment.....	144
6.2.3	Fairness of Curtailment Schemes in Residential Feeders.....	145
6.3	Future Works	146
6.3.1	Distributed Algorithms for OPF	146
6.3.2	More Accurate Forecast vs. Shorter Control Cycles	146
6.3.3	Multi-Period Optimisation.....	147
6.3.4	More Scalable Formulation of OPF-Alpha-Fairness.....	147
6.3.5	Fairness Implications when Additional Sources of Flexibility are Considered.....	147
	REFERENCES.....	149

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LIST OF TABLES

Table 2-1. AC power flow equations used in existing OPF literature.....	28
Table 2-2. Summary of the proposed linearised OPF.	43
Table 2-3. Summary simulation setup for C1-C8.....	47
Table 2-4. Summary of results for C1-C8.	48
Table 2-5. Reference values for the unknown variables.	53
Table 3-1. Objective function.	78
Table 3-2. Summary of cases.	79
Table 3-3. Conventional AVR settings.	79
Table 3-4. Selected network-level metrics.	81
Table 3-5. Sensitivity matrix (energy harvested).	92
Table 3-6. Sensitivity matrix (tap changes).....	92
Table 3-7. Sensitivity matrix (reactive power).....	92
Table 4-1. Aggregated energy statistics.	112
Table 4-2. Household data.....	113
Table 4-3. Jain's Fairness Index for household-centric metrics.....	117
Table 4-4. Modified Gini Coefficient for household-centric metrics.....	117
Table 4-5. Summary of cases.	121
Table 4-6. Aggregated energy statistics for different α values.	123
Table 4-7. Jain's Fairness Index and modified Gini Coefficient for different α values.	124
Table 4-8. Marginal change as α increases.	125

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LIST OF FIGURES

Figure 1-1. Unidirectional power flows in a passive distribution network.	2
Figure 1-2. Technical issues from excessive reverse power flows.....	5
Figure 1-3. Managed bidirectional power flows in an active distribution network. ...	7
Figure 2-1. Illustration of convexity.....	21
Figure 2-2. A single-phase distribution line.	23
Figure 2-3. Visualisation of the piecewise linearisation technique.	35
Figure 2-4. A delta-wye transformer.	36
Figure 2-5. Voltage phasors for balanced and unbalanced scenarios.....	39
Figure 2-6. Data flow of the co-simulation platform.	44
Figure 2-7. Topology of the 22kV feeder.....	45
Figure 2-8. Boxplot of nodal voltages and asset utilisations for C1-C8.....	49
Figure 2-9. Boxplot of APE (in %) for nodal voltages and asset utilisations for C1-C8.	51
Figure 2-10. Test LV feeder.	53
Figure 2-11. Impact of inaccuracies in estimated power terms on the linearised line model.	55
Figure 2-12. Impact of inaccuracies in estimated voltage terms on the linearised line model.....	56
Figure 2-13. Approximation error of the linearised thermal model.	57
Figure 2-14. Test LV feeder with a delta-wye transformer.....	58
Figure 2-15. Impact of load unbalances on the linearised delta-wye transformer model.	60
Figure 3-1. Decision-making process using control cycles.....	66
Figure 3-2. OLTC-fitted transformer with AVR.....	68
Figure 3-3. Principles of AVR operation.	68
Figure 3-4. Timing diagrams of control events: a) uncoordinated scenario, b) coordinated scenario.	70

Figure 3-5. Different ramping behaviours.	73
Figure 3-6. EHV1 network.	76
Figure 3-7. Demand and generation profiles.	77
Figure 3-8. Time-series voltage profiles.....	81
Figure 3-9. Time-series utilisation profiles for lines and transformers.	82
Figure 3-10. Uncoordinated execution of set-points (Case 2).	83
Figure 3-11. Coordinated execution of setpoints (Case 3).	85
Figure 3-12. Uncoordinated execution of setpoints (Case 1).	85
Figure 3-13. Impacts of not including operational constraints in the OPF formulation (Case 2).	87
Figure 3-14. Adequate consideration of operational constraints in the OPF formulation (Case 3).	88
Figure 3-15. Number of tap changes per OLTC.....	89
Figure 3-16. Wind power and demand at node 1115.....	90
Figure 3-17. Inadequate behaviour due to deadbands and voltage drifts in OLTCs (Case 2).	90
Figure 3-18. Improvements in OLTC behaviour from the proposed direct control method (Case 3).	90
Figure 4-1. Household with positive net demand.	98
Figure 4-2. A simple LV feeder.....	100
Figure 4-3. Illustration of the solutions to (a) maximum throughput, (b) proportional fairness and (c) max-min fairness.	101
Figure 4-4. Fairness-efficiency spectrum.	103
Figure 4-5. Demand and generation profiles.	109
Figure 4-6. Modified Volt-Watt curve.....	110
Figure 4-7. Time-series voltage profiles for all 4,626 households.	111
Figure 4-8. Time-series plots for two households: towards the head of a feeder (A) and towards the end of a feeder (B).	115

Figure 4-9. Time-series plots for two representative households: with high demand (C) and with low demand (D).....	115
Figure 4-10. Time-series scattered plot of household-centric metrics.	117
Figure 4-11. Illustration of quartiles.....	119
Figure 4-12. Boxplots of average PHI, EEI and NBI for all households shown in four quartiles.	119
Figure 4-13. Illustration of the differences in average EEIs due to non-defined instances.	120
Figure 4-14. Time-series voltage profiles for all 4,626 households.....	123
Figure 4-15. Average network voltages for different for different α values.....	123
Figure 4-16. Boxplots of average PHI for different α values.	124
Figure 5-1. Architecture of the HIL demonstration platform.....	129
Figure 5-2. Implementation of a HIL demonstration platform.....	131
Figure 5-3. Data flows in the proposed setup.....	133
Figure 5-4. Screenshot of dashboard one.	134
Figure 5-5. Main design elements of dashboard one.....	134
Figure 5-6. Screenshot of dashboard two.	135
Figure 5-7. A live demonstration of an OPF-based scheme for active distribution networks.	136

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ABBREVIATIONS

AC	Alternating Current
AVR	Automatic Voltage Regulator
DC	Direct Current
DG	Distributed Generation
DMS	Distribution Management System
EEI	Energy Export Index
GC	Genie Coefficient
HIL	Hardware-in-the-Loop
JFI	Jain's Fairness Index
LP	Linear Program
LV	Low Voltage
MILP	Mixed-Integer Linear Program
MV	Medium Voltage
NBI	Net Benefit Index
NLP	Non-linear Program
OLTC	On-Load Tap Changer
OPF	Optimal Power Flow
PHI	PV Harvesting Index
POC	Point of Connection
PV	Photovoltaic
QCP	Quadratically-Constrained Program
RTS	Real-Time Simulator
SCADA	Supervisory Control and Data Acquisition
SDP	Semi-Definite Program
SOCP	Second-Order Cone Program

NOMENCLATURE

Sets

H	Households
G	Households with PV systems
N	Buses/nodes
Φ	Phases (electrical)
L	Lines
T	Transformers
S	Sources

Parameters

P^D, Q^D	Household active and reactive power demand
P^{PV}	Maximum available PV power
$\bar{P}^G$	PV inverter capacity
V^-, V^+	Upper and lower voltage limits of a bus/node
R, X	Physical impedance of a line
R', X'	Equivalent impedance of a line
$\vec{V}$	Approximated complex voltage phasor
I^+	Ampacity of a line
τ	Tap ratio of an on-load tap changer
$\rho, \bar{\Delta}, \Lambda$	Approximation constants to model the thermal capacity

Variables

P^G, Q^G	PV inverter output, active and reactive
$p^{Curtail}$	PV curtailed active power
P^{ND}, Q^{ND}	Household net demand, active and reactive
v	Squared line-to-neutral voltage of a bus/node
P, Q	Active and reactive power flow into an element
$\kappa_1 \dots \kappa_5$	Auxiliary variables
P^+, P^-, Q^+, Q^-	Approximation variables to model the thermal capacity
$\Delta^P, \Delta^Q, \lambda$	
k	Status of a tap position (binary) in OLTC-fitted transformer

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1 INTRODUCTION

Tackling climate change has become a global focus in recent decades. Particularly, in the electricity sector, there has been tremendous support for renewable energy as an alternative to fossil fuel. In fact, as of 2018, at least 162 countries have adopted a national target for power generation from renewables [1]. With notable names such as Germany and Denmark leading the charge—80% and 100% by 2050, respectively—this collective effort has resulted in a more than tripling of the total renewable generation capacity in less than two decades (from 754 GW in 2000 to over 2356 GW in 2018) [2].

Undoubtedly, renewable generation has a crucial role in decarbonising the electricity sector. However, integrating large volumes of renewable generation into the existing power system is a rather challenging task. This is especially true considering that renewable generation (e.g., from rooftop PV systems) are increasingly being connected to the distribution network—the part of a power system that is typically associated with the last-mile delivery of power to consumers, not necessarily designed for bidirectional power flows as a result of local generation. Consequently, this rapid transformation of the electricity generation mix necessitates a paradigm shift in the way distribution networks are managed: moving from *passive distribution networks* (with conservative design principles to cope with the worst-case scenarios) towards *active distribution networks* (where network issues are tackled at the operational stage through real-time monitoring and control).

1.1 The Emergence of Active Distribution Networks

1.1.1 Passive Distribution Networks

In traditional power systems, distribution networks are predominantly operated as passive circuits with the assumption of unidirectional power flows. The typical structure is shown in Figure 1-1 where power is received in bulk from the transmission network and distributed to individual consumers (e.g., houses, businesses, factories, etc.) through a radially connected system of poles and wires (commonly referred to as feeders).

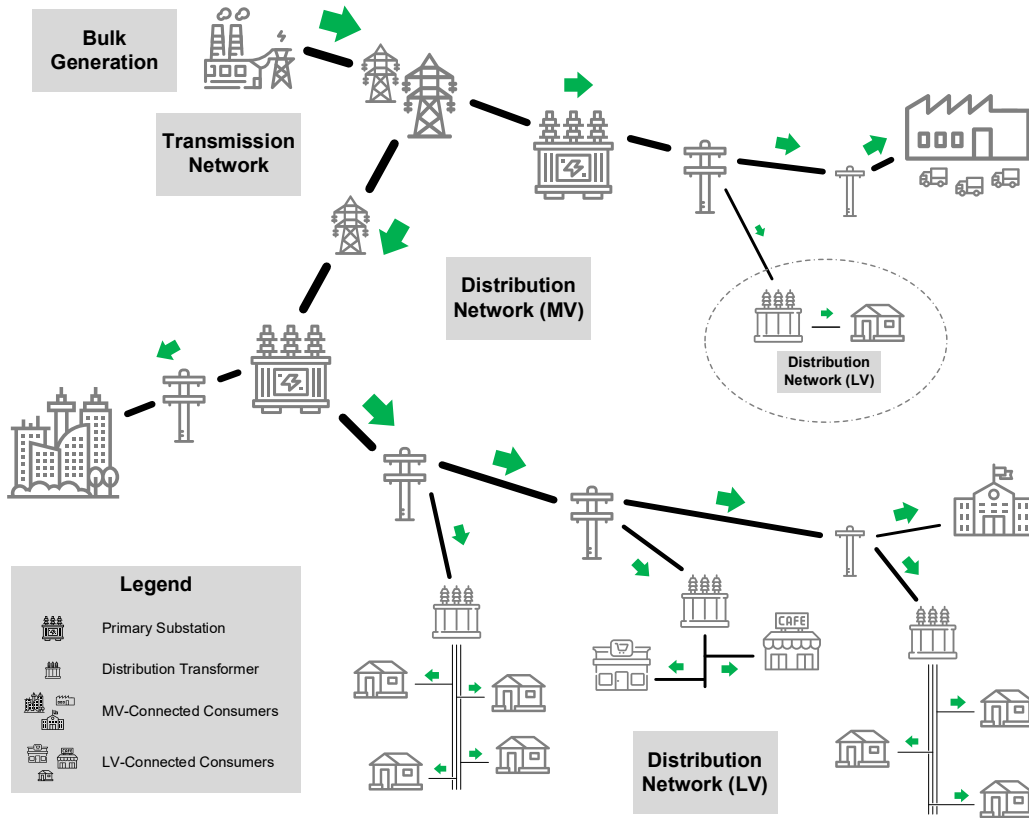


Figure 1-1. Unidirectional power flows in a passive distribution network.

Being a passively operated infrastructure, there is often limited observability and controllability in distribution networks. Consequently, they are built to cope with the worst-case scenarios without any external interventions. Furthermore, as new issues develop over time, the predominant solution is network augmentation (where the existing infrastructure is upgraded based on the newly determined worst-case scenarios). This largely conservative approach is often referred to as *fit-and-forget*. Distribution networks that are operated based on this design philosophy are henceforth referred to as *passive distribution networks*.

Among the technical constraints considered in the day-to-day operation of distribution networks, two dominant ones are voltage and thermal limits (and thus are the focus of this thesis). Particularly, this means ensuring voltage limits at critical nodes (e.g., consumer connection points) and thermal limits of network assets (e.g., distribution lines and transformers) are not violated during critical periods. When distribution networks only need to cater for demand, these critical periods (or worst-case scenarios) naturally correspond to periods with maximum demand, as it results in the largest power flows and voltage drops in distribution feeders. To this end, there are

several well-established ways of managing voltage and thermal issues in passive distribution networks.

From a thermal perspective, the existing assets (e.g., lines and transformers) are designed according to the expected peak demand, and thus prevents being thermally overloaded. In terms of voltage management, apart from sizing the conductor appropriately to minimise ohmic losses (and thus voltage drops), there are a few other mechanisms in place. For instance, primary substations are equipped with on-load tap changers (OLTCs), autonomous devices that adjusts the transformation ratio of the corresponding transformer (and thus the voltage) according to the variations in demand throughout a day. Furthermore, distribution transformers are often configured to provide a voltage boost, albeit being fixed, to compensate for voltage drops in LV feeders. Lastly, capacitor banks and voltage regulators can also be installed along a feeder for voltage regulation purposes.

Finally, it is worth noting that there are two distinct voltage levels in a distribution network: medium voltage (MV) and low voltage (LV) [3], also shown in Figure 1-1. Although the specific voltages used may vary around the world, the design principles are largely the same.

MV feeders start from primary substations and extend throughout the serviced area. Since each primary substation can service a large geographical area (e.g., across multiple suburbs), transporting electricity at the MV level has obvious technical benefits (i.e., lower losses). Typical voltages used in MV networks can range from several kV to tens of kV; for instance, the most common voltages are 6.6, 11 and 22 kV in Australia. Large consumers such as high-rise buildings and factories are also connected directly to MV feeders for increased power capacity (e.g., peak demands over hundreds of kVA).

In contrast, LV feeders are created for groups of small consumers (e.g., residential properties along a street) and interfaced to the MV feeders through distribution transformers. Around the world, the (line-to-line) voltages used in LV networks largely fall within two categories: around 400 V (e.g., China and across Europe) and 200 V (e.g., Japan, the US and across South America). In Australia, the nominal voltage is 400 V (line-to-line) for three-phase connections and 230 V (line-to-neutral) for single-phase connections. Depending on the population density, each LV feeder

can host from a few to dozens of consumers and each MV feeder can contain hundreds of these LV feeders.¹

1.1.2 The Rise of Distributed Generation

To decarbonise the electricity sector, more and more renewable generation will be connected to the distribution network. Consequently, this proliferation of renewable technologies is drastically changing where electricity is generated in modern power systems: from large-scale, centralised power stations that are located away from urban areas to small-scale, decentralised power generating units that are connected to distribution networks [4]. Collectively, this power generation within the distribution networks are referred to as distributed generation (DG).

In LV distribution networks, the growth of renewable DG is predominantly associated with rooftop solar PV systems [5]. Since rooftops are often underutilised areas and has minimal obstruction from sunlight, they naturally become prime locations for solar PV systems. Depending on the available area, the installed capacity can range from multiple kW (e.g., on top of residential properties) to hundreds of kW (e.g., on top of large factories).

On the other hand, in MV distribution networks, renewable DG often comes in the form of megawatt-scale wind farms and solar farms [5]. This is particularly common in rural areas as real estate is more readily available (than urban areas) and utilising the existing distribution network eliminates the need for additional transmission infrastructure.

1.1.3 Impacts of Renewable DG

The non-dispatchable nature of renewable generation creates an inherent mismatch between generation (from DG units) and demand within the distribution networks. Consequently, due to the reverse power flows during periods of surplus generation, technical issues such as voltage rise and asset congestion are becoming increasingly common in renewable-rich distribution networks [6, 7], as illustrated in Figure 1-2. In general, voltage and thermal limits are the main aspects considered to determine the

¹ It is worth highlighting that a European-style distribution networks (as adopted in Australia), is depicted here. In contrast, American-style distribution networks typically feature much smaller LV feeders; the distribution transformers are located much closer to the consumers and each distribution transformer only supplies power to a few consumers. The rest of the thesis will focus on European-style distribution networks only.

technical viability of DG installations during normal day-to-day operation. Nonetheless, there are other technical issues, such as increased fault currents and poor power factors. However, these are beyond the scope of this thesis.

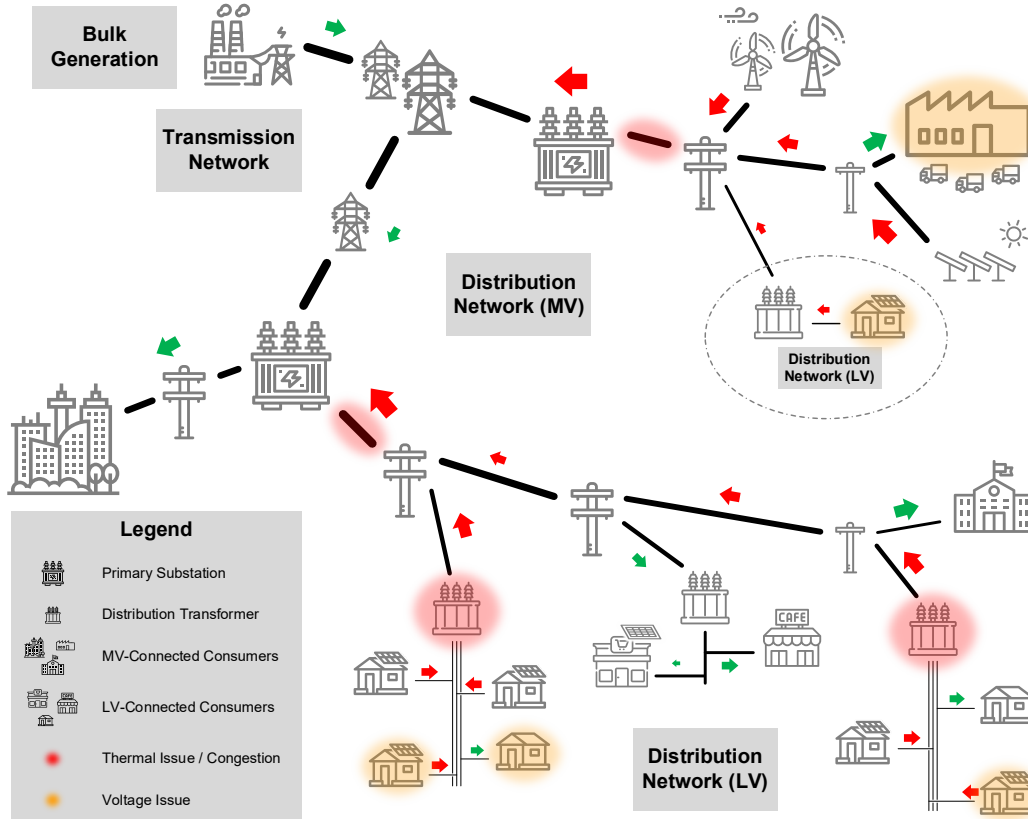


Figure 1-2. Technical issues from excessive reverse power flows.

For instance, in residential LV feeders, peak PV generation often coincide with low residential demand (e.g., due to low occupancy around noon on weekdays), and thus resulting in reverse power flow towards the distribution transformers. Since the existing infrastructure is designed to accommodate voltage drop—the exact opposite of voltage rise caused by reverse power flows—the likelihood of voltage violations is largely increased during these periods excessive reverse power flows. Particularly, to fully exploit the available voltage headroom (e.g., $\pm$ a few % from the nominal), the voltage at the head of a feeder is often boosted towards the upper limit. Consequently, while this strategy ensures the voltage at the end of a feeder remains within the lower limit during maximum demand, it can be problematic when dealing with reverse power flows as it results in little headroom from the upper voltage limit. Furthermore, unlike the inherent diversification of demand from many residential consumers, peak

generation from rooftop PV systems is largely correlated in any given region (e.g., a residential suburb or across an area of a few squared kilometres). As a result, there is an increased risk of asset congestion in not only the LV feeders, but also the upstream MV feeder.

The underlying causes of technical issues in MV distribution networks largely follow the same principles as in LV networks: a mismatch between peak generation and peak demand. Furthermore, due to longer distances of rural MV feeders (which is where most renewable generation is being connected to at the MV level), the corresponding infrastructure is substantially weaker than their urban counterparts (e.g., more susceptible to voltage issues due to the higher impedances, and thus higher magnitude of voltage rise for the same volume of reverse power flows). Consequently, this significant amplifies the risk of technical issues from excessive reverse power flows.

1.1.4 Towards Active Distribution Networks

The rapid uptake of DG in recent decades has changed the role of modern distribution networks at a fundamental level; it is no longer just the last mile delivery of power to consumers. As a result, the passive way of tackling voltage and thermal issues at the planning stage through network augmentation may no longer be effective; due to the bidirectional power flows in a modern distribution network, issues from two extreme scenarios—*maximum generation and minimum demand* as well as *maximum demand and minimum generation*—must be dealt with simultaneously. To this end, the concept of *active distribution networks*, where network issues are solved at the operational stage through the real-time monitoring and control of network assets and participants, is becoming an increasingly attractive option. Particularly, by leveraging the various sources of flexibility in real-time, network operators can fully exploit the capability of existing infrastructure, and thus defer (or eliminate) the need of costly network augmentation.

An example of an active distribution network is shown in Figure 1-3 where a centralised control scheme is adopted to manage network assets and participants in real-time, and thus mitigates potential technical issues from the bidirectional power flows. As depicted here, the coordination of OLTC-fitted transformers and the reactive power capability of inverter-based DG units can help to tackle voltage issues [8].

Although not explicitly shown (and also beyond the scope of this thesis), emerging technologies such as battery storage systems [9] also offer additional flexibility to optimise power flows in the distribution network. Finally, as a last resort, active power curtailment of DG units (i.e., limiting their maximum output) [8] can be used during critical periods to prevent constraint violations.

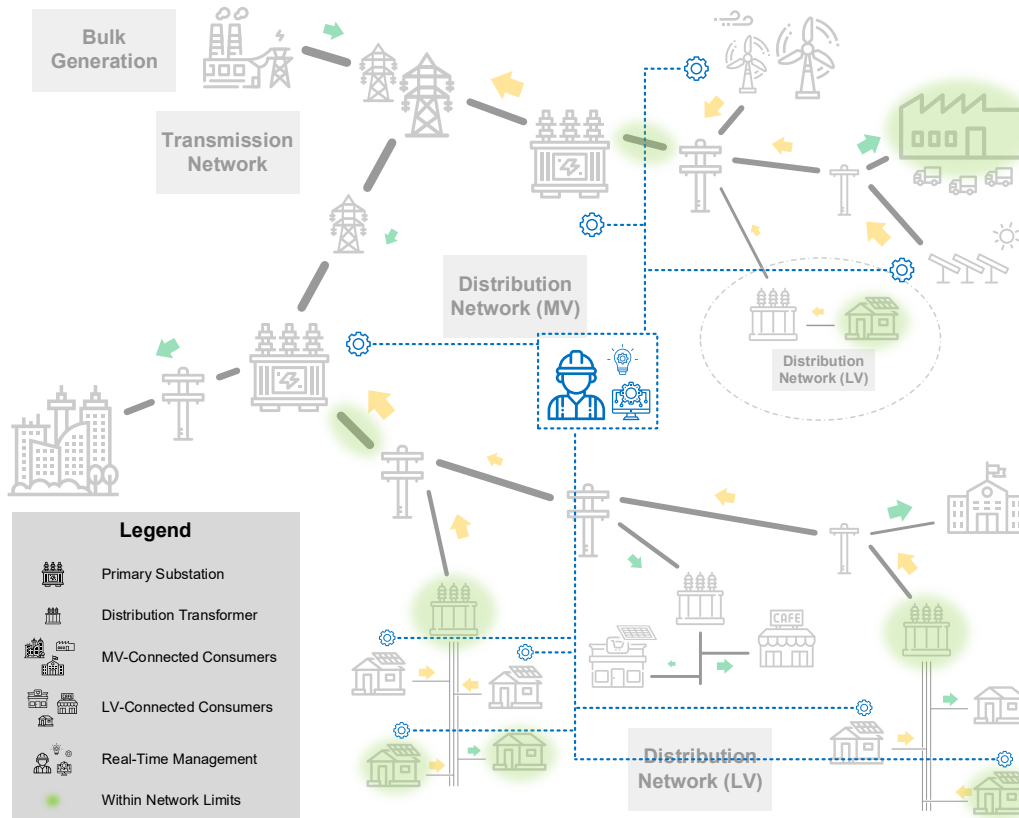


Figure 1-3. Managed bidirectional power flows in an active distribution network.

It is clear that the transition to active distribution networks will unlock new avenues to facilitate the bidirectional power flows in an active way (as opposed to only relying on reinforcing existing assets). In fact, early trials [10, 11] have already demonstrated that by actively controlling the output of DG units, albeit using relatively simple rule-based schemes, the existing distribution network is able to accommodate substantially higher penetrations of renewable generation. Furthermore, thanks to the progressive deployment of smart grid technologies (e.g., communication infrastructure, smart meters, etc.) [12], the observability and controllability (a key technical barrier) of existing distribution networks have improved significantly, making their real-time monitoring and control increasingly plausible nowadays.

1.1.5 Advanced Schemes for Active Distribution Networks

Given the size (e.g., large number of assets and participants across multiple voltage levels) and complexity (e.g., phase unbalances, discrete devices) of distribution networks, developing adequate control schemes using rule-based approaches (as commonly adopted by industry) can be prohibitively cumbersome. Considering that more and more sources of flexibility are becoming available, this is especially true as many variables (e.g., the settings of controllable devices) and constraints (e.g., network limits) must be considered simultaneously by the network operator. Therefore, to fully utilise these various sources of flexibility, advanced approaches that can cater for these challenges associated with realistic distribution networks is necessary.

To this end, AC Optimal Power Flow (OPF), a technique that combines optimisation with an electrical model of the distribution network, has emerged as a promising alternative for more advanced schemes [13]. Particularly, by formulating the decision-making process as a mathematical optimisation problem, an OPF-based approach enables a more systematic way of determining the most appropriate setpoints for controllable devices in an active distribution network while also respecting the corresponding constraints. Nonetheless, there are several technical challenges that must be overcome before OPF-based schemes are readily adopted in the control rooms of future distribution networks. These challenges are discussed in the following section.

1.2 Challenges

1.2.1 Scalability and Speed of OPF Formulations

The classical AC OPF formulation is non-convex [14] due to the non-linearity of AC power flow equations. In general, this type of optimisation problems is known to suffer scalability and speed issues (i.e., solvable fast enough to be practically relevant) as a) there are generally no effective way of finding the global optimum [15] and b) a local optimum can be very sub-optimal (i.e., substantially worse in performance compared with the global optimum) [16]. Consequently, this becomes a major challenge for the operational usage of OPF-based approaches due to the associated constraints on solution time (i.e., within minutes).

Furthermore, this issue is exacerbated by the need of modelling discrete devices found in typical distribution networks (e.g., OLTCs and capacitor banks) where integer

programming is required. Particularly, due to the combinatorial nature of solving a mixed-integer program, it often requires an exhaustive search (i.e., going through all possible combinations) to find the optimal solution [17]. Consequently, the non-convex nature of classical OPF formulations is expected to make this an extremely time-consuming process.

1.2.2 Real-World Implementation of OPF-Based Setpoints

Since there are different types of controllable devices in a distribution network, such as legacy devices (e.g., OLTCs and capacitor banks) and inverter-interfaced devices (e.g., modern wind farms and PV systems), their operating characteristics (e.g., delays, ramp rates and deadbands) can vary significantly. Consequently, this introduces several practical challenges when OPF-based setpoints are to be implemented in real-world applications. For instance, due to the design for autonomous operation, there is usually a relatively long delay (e.g., around a minute) [18] associated with conventional OLTC controllers. As a result, this introduces coordination issues with inverter-interfaced devices that can typically respond to new setpoints within seconds [19]. Therefore, since the execution of setpoints do not happen instantaneously, this and similar sub-minute scale interactions among multiple controllable devices (due to the differences in their operating characteristics) may lead to undesirable behaviours not envisaged by the OPF-based scheme, such as temporary constraint violations and unnecessary control actions.

1.2.3 Ensuring Fairness in Curtailment Schemes

Although the definition of fairness—ensuring equality or removing disparity—is relatively simple to understand, assessing fairness in the context of distribution networks is a more challenging task. Firstly, it requires defining an appropriate metric to quantify the benefits for DG owners, which is not necessarily straightforward given that there are many perspectives that can be used to quantify the impact due to active power curtailment (e.g., PV harvesting and monetary benefits) [20]. Furthermore, it also requires an adequate understanding (and assessment) of the trade-offs between fairness (i.e., equally for everyone) and efficiency (i.e., aggregated performance) [21], a topic that is rarely investigated in distribution network studies.

1.2.4 Realistic Demonstration of OPF-Based Schemes

Traditionally, distribution network studies are done offline using PC-based software. While this is indispensable from a research and development perspective, it often lacks the ability to from a demonstration perspective as it is difficult to fully capture the physical interactions among the various components in an active distribution network (e.g., network-connected devices, communication infrastructure, SCADA system, advanced control schemes, etc.). Therefore, as more and more advanced concepts are being developed for future distribution networks, it is also increasingly important to demonstrate these concepts to industry through more realistic means, such as using hardware-in-the-loop (HIL) techniques [22]. Nonetheless, developing such a HIL demonstration platform is not a trivial task, as it involves specialised software and hardware from different domains that extend beyond conventional tools used in typical power system studies.

1.3 Research Questions

The decarbonisation of the electricity sector will inevitably introduce a large volume of renewable DG. Consequently, there is a need for more advanced schemes to manage the bidirectional power flows in renewable-rich distribution networks. Therefore, acknowledging the potential benefits of the OPF technique and the associated challenges of its real-world application, this thesis aims to demonstrate the technical feasibility of integrating OPF-based schemes in the control rooms of future distribution networks. Particularly, this thesis aims to answer the following key research questions:

- What modifications (or adaptations) are needed to conventional AC OPF formulations so as to handle the size and complexity of realistic distribution networks in an operational environment?
- What considerations are required to ensure OPF-based setpoints are adequately implemented in practice (i.e., the real world)?
- What considerations are required to ensure adequate fairness is incorporated in OPF-based curtailment schemes?
- How to demonstrate advanced schemes for future distribution networks to the industry in a realistic and engaging way?

1.4 Objectives

To answer the aforementioned questions, the following four main objectives have been identified (and briefly discussed in the remainder of this section):

- Developing a scalable and fast OPF formulation.
- Identifying the necessary adaptations in existing device controllers and OPF formulation.
- Understanding different aspects of fairness and quantify the trade-offs from different fairness objectives.
- Developing a realistic and engaging demonstration platform to showcase advanced schemes to industry.

1.4.1 A Scalable and Fast OPF Formulation

A crucial step to enable the operational usage of OPF is to develop an adequate formulation that can be implemented by industry. Particularly, the formulation needs to cater for the size and complexity of a realistic distribution network, such as being able to handle as several thousand nodes across multiple voltage levels and multiple discrete devices. Furthermore, the solution time must be sufficiently small for operational usage where control decisions are needed within minutes.

To achieve this, a linearisation approach is adopted to convexify the classical OPF formulation, and thus significantly improves its scalability and speed when dealing with large and complex distribution network models. Furthermore, thanks to the operational usage of the proposed formulation, real-time measurements of the network can be exploited as linearisation points of the proposed formulation, and thus used as warm-start conditions to improve the accuracy of the linearised formulation.

1.4.2 Necessary Adaptations

Apart from being able to calculate setpoints using the OPF technique, how these setpoints are being implemented by controllable devices is equally important. Particularly, in addition to the calculated setpoints, satisfying the power flow constraints, adequate considerations related to the device control are necessary to ensure the expected outcome is achieved in practice; particularly when multiple devices (with vastly different operating characteristics) are being controlled at the same time.

To achieve this, a high granularity modelling is adopted to realistically capture the interactions among controllable devices in real-time (i.e., at a sub-minute scale). Then, through a better understanding of the complications arising from the real-time operating characteristics of controllable devices, the necessary adaptations can be determined.

1.4.3 Trade-Offs among Different Fairness Objectives

Active power curtailment is often necessary to mitigate technical issues during critical periods. Therefore, a fair allocation of the required curtailment among many DG owners could be of great importance as it directly affects the benefits these owners receive from their investments.

To achieve this (in a residential context), household-centric metrics are adopted to realistically quantify the impacts due to active power curtailment, each focusing on a different (and potentially contrasting) perspective. Then, several OPF-based schemes, each with a different fairness objective, are compared to understand the associated trade-offs among different perspectives of fairness as well as between fairness and efficiency.

1.4.4 Realistic Demonstration Platform

Before the real-world deployment of any advanced scheme, it is beneficial to validate and demonstration its performance in a more realistic way that goes beyond conventional offline analysis.

To achieve this, commercially available software and hardware packages are used in a HIL setup to replicate the key components of an active distribution networks under a laboratory environment.

1.5 Main Contributions

1.5.1 Advanced AC OPF for Operational Usage in Distribution Networks

A linearised, three-phase AC OPF is proposed in this thesis for operational usage in future distribution networks. The three-phase formulation caters for the inherent unbalances in distribution networks and the linearisations (of the non-linear OPF formulation) provide significant improvements in terms of the scalability and speed

for large networks. Furthermore, being a linear program (LP), the proposed formulation can leverage recent advances in solving mixed-integer linear programs (MILP) [23] to cater for discrete devices (which can be modelled as integer variables). Lastly, a linearised delta-wye transformer model—the most common configuration for step-down transformers in distribution networks—is developed to accurately model multi-voltage level networks (e.g., integrated MV-LV networks).

The performance of the proposed OPF formulation (for operational usage) is successfully demonstrated for a realistic MV-LV distribution feeder that starts from the primary substation and ends at the connection points of individual residential consumers. In total, there are over 4,500 single-phase connected consumers and 4,000 three-phase buses.

1.5.2 Necessary Adaptations to Implement OPF-Based Setpoints

Through the adopted high granularity modelling in a co-simulation environment (which is necessary to capture the sub-minute implications from different delay, ramp rate and deadband settings), potential issues when OPF-based setpoints are being implemented in practice are demonstrated for the first time. For instance, when OLTCs and DG units are being used for voltage management, short term constraint violations (e.g., around a minute) are observed due to a mismatch in their delays and ramp rates, despite the setpoints being requested at the same time. Furthermore, due to the design of conventional OLTC controllers being tailored for autonomous operation, the associated deadbands render a centralised, OPF-based schemes less effective as the calculated voltage target cannot be precisely realised, leading to excessive tap changes.

To ensure the expected outcome is achieved when OPF-based setpoints are implemented in practice, the required adaptations in the device controllers (of OLTCs and DG units) and the OPF formulation are proposed in this thesis. For instance, this involves the alignment of their delays and ramp rates as well as eliminating large deadbands associated with conventional OLTC controllers. Furthermore, the operational limits of controllable devices and the minimisation of control actions are incorporated in the OPF formulation to ensure calculated setpoints are technically feasible for actual implementation.

Overall, the results show that, a high granularity modelling is essential to realistically assess the performance of OPF-based schemes. Furthermore, the proposed

adaptations are extremely crucial to ensure adequate coordination when multiple controllable devices are being centrally managed using OPF-based (or similar) schemes. Conversely, inadequate considerations can result in undesirable outcomes, such as network constraints violations and accelerated device wear-and-tear.

1.5.3 Different Fairness Objectives of Curtailment Schemes

An assessment methodology using several household-centric metrics are proposed to realistically capture the impact of curtailment schemes (in the context of residential distribution networks). Then, the existence of multiple and, potentially, contrasting fairness objectives are demonstrated.

Firstly, there are different perspectives of fairness (e.g., PV harvesting, energy export and financial benefits), each with associated trade-offs in terms of the other perspectives. For instance, a scheme that is designed to be fair in terms of PV harvesting for all households will result in noticeable disparity in terms of their energy exports. Secondly, the trade-offs between fairness (removing disparity) and efficiency (aggregated performance) are quantified, using the concept of α -fairness [24] (which is typically found in the literature of internet traffic control). Particularly, it has been shown that improving fairness for individuals can result in compromises in the aggregated performance.

Overall, the results highlight that, although the definition of fairness is relatively straightforward, i.e., ensuring equality/removing disparity, developing a fair curtailment scheme is a rather challenging task. Therefore, the proposed methodology and findings can provide valuable insights for decision-makers on how to adequately cater for fairness when active power curtailment is required.

1.5.4 A Hardware-in-the-Loop Demonstration Platform

The architecture of a realistic demonstration platform (for advanced control schemes in distribution networks) is developed. The proposed design closely mimics key elements expected in future active distribution networks, including the SCADA system, the distribution management system (DMS) and the advanced (e.g., OPF-based) control scheme. Furthermore, the design also focuses on the visualisation of real-time simulation data in a control-room-like environment. Based on this, the implementation details of a hardware-in-the-loop (HIL) demonstration platform using commercially available software and hardware are presented. Apart from the

validation aspect (which is typically associated with HIL simulations), the proposed platform enables a more realistic and engaging experience when showcasing advanced schemes to industry.

1.6 Publications

The following publications have been produced as part of the research carried out for (or related) to this thesis.

1.6.1 Journal Papers

- **M. Z. Liu**, A. T. Procopiou, K. Petrou, L. F. Ochoa, T. Langstaff, J. Harding and J. Theunissen, “On the Fairness of PV Curtailment Schemes in Residential Distribution Networks”, *IEEE Transactions on Smart Grids*, vol. 11, no. 5, pp. 4502-4512, 2020.

DOI link: <https://www.doi.org/10.1109/TSG.2020.2983771>

- **M. Z. Liu**, L. F. Ochoa, S. H. Low, “On the Implementation of OPF-Based Set-Points for Active Distribution Networks”, *IEEE Transactions on Smart Grids*, in Review (R2).

1.6.2 Conference Papers

- **M. Z. Liu** and L. F. Ochoa, "Hardware-In-the-Loop Demonstration of Advanced Control Schemes for Active Distribution Networks," in *Proc. 2019 IEEE PES Innovative Smart Grid Technologies Conference - Latin America (ISGT Latin America)*, pp. 1-6.

DOI link: <https://www.doi.org/10.1109/ISGT-LA.2019.8895493>

- L. Gutierrez-Lagos, **M. Z. Liu**, and L. F. Ochoa, "Implementable Three-Phase OPF Formulations for MV-LV Distribution Networks: MILP and MIQCP," in *Proc. 2019 IEEE PES Innovative Smart Grid Technologies Conference - Latin America (ISGT Latin America)*, pp. 1-6.

DOI link: <https://www.doi.org/10.1109/ISGT-LA.2019.8895008>

1.6.3 Technical Reports

The following technical report was produced for the consultancy project “Smart Grid Test and Evaluation” funded by QinetiQ Australia.

- **M. Z. Liu**, L. F. Ochoa, Deliverable 1 "Testing Facilities Report", prepared for QinetiQ Australia, 2018

1.6.4 Magazine Articles

- Dubey, A. Bose, **M. Liu**, and L. N. Ochoa, "Paving the Way for Advanced Distribution Management Systems Applications: Making the Most of Models and Data," *IEEE Power and Energy Magazine*, vol. 18, no. 1, pp. 63-75, 2020.

DOI link: <https://www.doi.org/10.1109/MPE.2019.2949442>

1.7 Thesis Outline

A summary for each of the remaining chapters is presented below.

Chapter 2 - A Fast and Scalable Three-Phase OPF for Distribution Networks

presents the linearised, three-phase AC OPF formulation that is used as the basis for all subsequent chapters. The proposed formulation is based on the branch flow model [25] and utilises squared voltages and power injections terms as state variables to model the power flow equations. It is adapted from the single-phase formulation in [26] and the process is documented in this chapter, including the extension to a three-phase models, the introduction of a linearised model for delta-wye transformers (to cater for the modelling of multi-voltage level distribution feeders) and the usage of integer variables (to cater for the discrete devices in distribution networks).

Chapter 3 - Implementing OPF-Based Setpoints: Challenges and Solutions

investigates the necessary adaptations in device controllers and the OPF formulation when OPF-based setpoints are to be implemented in practice. This chapter focuses on the active management of OLTCs and DG units (e.g., wind farms) in rural distribution network. Due to the diverse operating characteristics of different types of controllable devices (e.g., OLTCs and inverter-interfaced DG units), an alignment process of their delays and ramp rates as well as the modifications to remove large deadbands are identified. Then, the benefits of the proposed changes are demonstrated using a rural, UK-style distribution network considering the active management of wind farms and OLTCs. Particularly, it has been shown that, inadequate considerations from a device control perspective may lead to temporary voltage violations and excessive control actions. On the other hand, the proposed adaptations allow OPF-based setpoints to be successfully implemented (i.e., achieving the outcome as intended by the OPF) in an operational environment.

Chapter 4 - Fairness of Active Power Curtailment Schemes: A Residential Context investigates how to adequately cater for fairness when active power

curtailment is required in a residential context. This first requires defining several household-centric metrics that are related to how performance/impact can be quantified for residential prosumers (consumers with PV systems). To this end, three metrics in the perspective of energy harvesting (of PV systems), energy export (of households) and financial benefit (from the PV investment) are considered. Furthermore, the concept of α -fairness is adopted to model the relative priority between fairness and efficiency. Based on these, several OPF-based schemes that incorporate different fairness objectives are proposed. Finally, the implications from different fairness objectives are demonstrated using a PV-rich MV-LV residential feeder with over 4,500 households. The results highlight that, fairness can be achieved from multiple perspectives, each with their associated trade-offs. Furthermore, the marginal improvement in terms of fairness can result in substantial compromises in terms of the aggregated performance.

Chapter 5 - A Hardware-in-the-Loop Demonstration Platform first introduces the proposed architecture of a realistic demonstration platform for advanced schemes in future distribution networks. Then, a specific implementation at the Smart Grid Lab of The University of Melbourne using a real-time simulator as well as commercially available software and hardware packages is presented. Lastly, this platform is used to create a live demonstration of the advanced control scheme (to actively manage a renewable-rich distribution network) presented in Chapter 3.

Chapter 6 - Conclusion summarises the key findings of this thesis and highlights the potential future works.

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2 A FAST AND SCALABLE THREE-PHASE OPF FOR DISTRIBUTION NETWORKS

2.1 Introduction

Optimal power flow (OPF) is a mathematical optimisation problem that seeks to find the set of optimal control actions (e.g., output of generators) according to a given objective (e.g., minimise costs) while satisfying the corresponding constraints (e.g., the physics of power flows, operational limits, etc.). Since its first introduction by Carpentier in 1962 [27], OPF has played a significant role as an operation tool for modern transmission networks.

The classical formulation of an AC OPF can be very difficult to solve due to the non-convex (and non-linear) nature of AC power flow equations—the most accurate form to capture the underlying physics. Therefore, its operational usage in transmission networks is largely based on a simplified version of a full AC OPF, i.e., a DC OPF [28, 29], which only captures the active power and excludes the losses. Furthermore, a single-phase DC OPF is usually sufficient due to the balanced nature of power flows at the transmission level. One such example is in Australia, where a DC OPF-based engine is used by the Australian Energy Market Operator (AEMO) to dispatch scheduled generators in real-time, once every five minutes [28]. Similar examples can also be found around the world, such as the US [30, 31] and across Europe [32].

In recent decades, driven by the rapid growth of distributed generation (DG), as well as the emergence of other distributed energy resources, there has been a significant interest in the application of OPF in distribution networks. Particularly, by combining mathematical optimisation with the electrical model of a distribution network, an OPF can become a powerful decision-making engine to manage (i.e., determine the most appropriate setpoints) the various sources of flexibility in an active distribution network (e.g., OLTCs, output of DG units, etc.).

However, due to the size and complexity of distribution networks, the operational usage of OPF is significantly more challenging than transmission networks. Firstly, the number of elements (e.g., buses, lines, etc.) in a distribution network can be several

orders of magnitude larger than a transmission network. Furthermore, due to the higher resistance/reactive (i.e., R/X) ratio in distribution networks, both the active and reactive power must be modelled, and thus requiring the full AC OPF formulation. This is exacerbated by the inherent topological and load unbalances in distribution networks, resulting in the need of modelling all three phases and their mutual effects on each other. Lastly, to accurately represent discrete devices¹ in distribution networks (e.g., OLTCs and capacitor banks), integer variables are needed, and thus introducing extra difficulty in solving the AC OPF problem within limited time frames.

Given the various challenges discussed above, a crucial step to enable the operational usage of OPF is to develop a scalable (i.e., tractable) and fast (i.e., sufficiently fast solution time) formulation that can cater for the size and complexity of distribution networks. To this end, a linearised, three-phase OPF is proposed and analysed in this chapter.

2.2 Literature Review

This section first presents an overview of the different categories of mathematical optimisation problems. This is followed by a summary of the variations of power flow equations used in existing OPF formulations. Lastly, existing work on three-phase OPF and the research gaps are presented.

2.2.1 Categories of Mathematical Optimisation Problems

All mathematical optimisation problems can be expressed in the generic form shown in (2.1). It contains an objective function $f_0(x)$, the set of variables $x = \{x_1, \dots, x_n\}$ and the set of constraints $\{f_1, \dots, f_m\}$.

$$\begin{aligned} & \text{minimise} && f_0(x) \\ & \text{subject to} && f_i(x) \leq 0, \quad i = 1, 2, \dots, m \end{aligned} \tag{2.1}$$

Depending on the types of functions used (e.g., linear, quadratic, cubic, trigonometric, logarithmic, etc.), the relative difficulty² of optimisation problems can

¹ The setpoints for these devices only take discrete values, e.g., $\{1, 2, 3\}$, as opposed to any value within a range, e.g., $[0, 3]$. Hence, they are referred to as discrete devices.

² In this thesis, the term ‘difficulty’ refers to whether the solution time of an OPF problem is adequate for operational usage (e.g., when decisions are needed within minutes).

vary significantly. Based on this, they are classified into three representative categories: linear programs (LPs), quadratically-constrained programs (QCPs) and non-linear programs (NLPs), in order of increasing difficulty.¹

An LP only contains linear functions; a QCP is a special case of an NLP, where all non-linear constraints are quadratic functions; if other non-linear functions (apart from quadratic ones) are present, the resulting formulation is an NLP.

A main contributing factor to this difference in difficulty is the convexity of an optimisation problem [14]. To illustrate this, a simple convex and a non-convex (minimisation) problem are shown in Figure 2-1. In the convex case, there is only one feasible region (the blue line) and one optimal solution. Therefore, it is relatively straightforward to find the global optimum (the red dot) of a convex problem. On the other hand, in a non-convex problem, there are often multiple feasible regions (the disjoint blue lines) and each feasible region may contain more than one optimal solution. Additionally, which optimal solution is obtained by an algorithm can be heavily affected by the initial starting point. Therefore, finding the global optimum (the red dot) as well as differentiating it from the local optima (the orange dots) is much more challenging in non-convex problems.

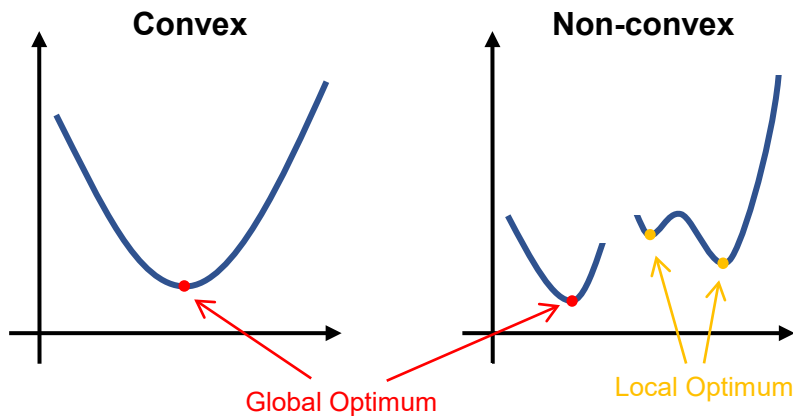


Figure 2-1. Illustration of convexity.

Based on the reasoning above, an LP is intrinsically easier to solve as it is always convex [14]. On the other hand, since both QCP and NLP are often non-convex, they

¹ The classifications presented here only focus on the constraints. Nonetheless, different sub-categories also exist depending on the type of function used in the objective function. For instance, linear constraints and quadratic objective function is classified as a quadratic program (QP).

are generally considered as harder problems than an LP due to various numerical difficulties of finding an optimal solution (global or local) [33]. While quadratic constraints are technically non-linear, the special category of QCP exists due to the various established techniques of convexifying a QCP, such as through semi-definite program (SDP) relaxation and second-order cone program (SOCP) relaxation [14]. In contrast, there are generally no effective methods for solving NLP, and thus NLPs are often regarded as the most difficult category of optimisation problems.

It is also worth noting that, the above discussions only represent a general rule of thumb for comparable problems. For instance, an LP with over thousands of constraints is likely to be more difficult than an NLP with only one constraint.

Lastly, LPs also have an added benefit when dealing with integer variables (e.g., when modelling discrete devices). Thanks to the simplex algorithm—an algorithm that is only applicable to LPs—the non-integral solutions found at the parent node during the branching stage of solving a mixed-integer linear program (MILP) can be utilised as warm start values (also known as providing an advanced basis) in the subsequent child nodes [34], resulting in significant speed boosts. Furthermore, combined with the recent advances in solving MILP by commercial solvers [23], this makes linearised formulations of OPF even more preferable when dealing with discrete variables.

2.2.2 Variations of AC Power Flow Equations

A core component of any AC OPF formulation (henceforth referred to as OPF) is the power flow equations that model the underlying physics. In general, there are four main variations used by existing OPF literature, ranging from non-linear functions to quadratic functions. Consequently, the difficulty of the corresponding OPF formulation can vary as discussed earlier in Section 2.2.1 (i.e., LP $\rightarrow$ QCP $\rightarrow$ NLP, in increasing order of difficulty). These four variations are introduced in this sub-section. For simplicity, a single-phase line is used here to illustrate the key differences among these variations.

For a line l as shown in Figure 2-2, the purpose of power flow equations is to model the relationships between the complex voltage phasors (e.g., $\vec{V}_{l_x}$ and $\vec{V}_{l_y}$) and the

complex nodal power injections (e.g., $\vec{S}_{l_x} = P_{l_x} + jQ_{l_x}$ and $\vec{S}_{l_y} = P_{l_y} + jQ_{l_y}$).¹ Here, l_x and l_y indicates the start node and end node of line, respectively. The electrical properties of the line can be expressed in terms of either the impedance $\mathbf{Z}_l$ (i.e., $\mathbf{R}_l + j\mathbf{X}_l$) or the admittance $\mathbf{Y}_l$ (i.e., $\mathbf{G}_l + j\mathbf{B}_l$), where $\mathbf{Y}_l = \mathbf{Z}_l^{-1}$. The current phasor $\vec{I}_l$ can also be used as an alternative to the nodal power injections based on the relationships $\vec{S} = \vec{V} \cdot \vec{I}^*$, where $*$ denotes the complex conjugate.

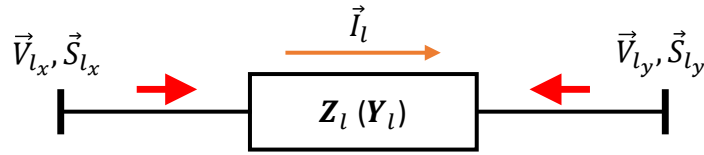


Figure 2-2. A single-phase distribution line.

Variation I

The first variation expresses the voltage phasors using polar coordinates, e.g., $\vec{V}_{l_x} \rightarrow V_{l_x} \angle \theta_{l_x}$ where V_{l_x} is the voltage magnitude and θ_{l_x} is the voltage angle, as shown in (2.2)-(2.5). This has been adopted by works such as [35, 36].

$$P_{l_x} = \mathbf{G}_l V_{l_x}^2 - V_{l_x} V_{l_y} \left[\mathbf{G}_l \cos(\theta_{l_x} - \theta_{l_y}) + \mathbf{B}_l \sin(\theta_{l_x} - \theta_{l_y}) \right] \quad (2.2)$$

$$Q_{l_x} = -\mathbf{B}_l V_{l_x}^2 - V_{l_x} V_{l_y} \left[\mathbf{G}_l \sin(\theta_{l_x} - \theta_{l_y}) - \mathbf{B}_l \cos(\theta_{l_x} - \theta_{l_y}) \right] \quad (2.3)$$

$$P_{l_y} = \mathbf{G}_l V_{l_y}^2 - V_{l_x} V_{l_y} \left[\mathbf{G}_l \cos(\theta_{l_y} - \theta_{l_x}) + \mathbf{B}_l \sin(\theta_{l_y} - \theta_{l_x}) \right] \quad (2.4)$$

$$Q_{l_y} = -\mathbf{B}_l V_{l_y}^2 - V_{l_x} V_{l_y} \left[\mathbf{G}_l \sin(\theta_{l_y} - \theta_{l_x}) - \mathbf{B}_l \cos(\theta_{l_y} - \theta_{l_x}) \right] \quad (2.5)$$

Since it focuses on the nodal power injections, it is commonly referred to as the bus injection model [25]. Furthermore, due to the usage of trigonometry functions (i.e., sines and cosines), the resulting constraints are non-linear.

¹ Note, the convention adopted in this thesis is that a negative value for S corresponds to an injection into the node. In contrast, a positive value for S corresponds to an injection into the line, as depicted.

Variation II

The second variation is also based on the bus injection model, as shown in (2.6)-(2.9). In contrast to Variation I, it adopts rectangular coordinates for voltage phasors, e.g., $\vec{V}_{l_x} \rightarrow V_{l_x}^{\Re} + jV_{l_x}^{\Im}$ where the real and imaginary components are denoted by $\Re$ and $\Im$, respectively. It has been adopted by works such as [25, 37].

$$P_{l_x} = \mathbf{G}_l \left[(V_{l_x}^{\Re})^2 + (V_{l_x}^{\Im})^2 - V_{l_x}^{\Re} V_{l_y}^{\Re} - V_{l_x}^{\Im} V_{l_y}^{\Im} \right] + \mathbf{B}_l \left(V_{l_x}^{\Re} V_{l_y}^{\Im} - V_{l_x}^{\Im} V_{l_y}^{\Re} \right) \quad (2.6)$$

$$Q_{l_x} = \mathbf{G}_l \left(V_{l_x}^{\Re} V_{l_y}^{\Im} - V_{l_x}^{\Im} V_{l_y}^{\Re} \right) - \mathbf{B}_l \left[(V_{l_x}^{\Re})^2 + (V_{l_x}^{\Im})^2 - V_{l_x}^{\Re} V_{l_y}^{\Re} - V_{l_x}^{\Im} V_{l_y}^{\Im} \right] \quad (2.7)$$

$$P_{l_y} = \mathbf{G}_l \left[(V_{l_y}^{\Re})^2 + (V_{l_y}^{\Im})^2 - V_{l_x}^{\Re} V_{l_y}^{\Re} - V_{l_x}^{\Im} V_{l_y}^{\Im} \right] - \mathbf{B}_l \left(V_{l_x}^{\Re} V_{l_y}^{\Im} - V_{l_x}^{\Im} V_{l_y}^{\Re} \right) \quad (2.8)$$

$$Q_{l_y} = -\mathbf{G}_l \left(V_{l_x}^{\Re} V_{l_y}^{\Im} - V_{l_x}^{\Im} V_{l_y}^{\Re} \right) - \mathbf{B}_l \left[(V_{l_y}^{\Re})^2 + (V_{l_y}^{\Im})^2 - V_{l_x}^{\Re} V_{l_y}^{\Re} - V_{l_x}^{\Im} V_{l_y}^{\Im} \right] \quad (2.9)$$

The use of rectangular coordinates (in Variation II) simplifies the non-linear constraints (in Variation I) to equivalent forms using quadratic constraints. At the same time, a potential drawback of this variation is the need of additional equations to obtain the voltage magnitudes—a more useful quantity than the complex values—as shown in (2.10) and (2.11).

$$V_{l_x}^2 = (V_{l_x}^{\Re})^2 + (V_{l_x}^{\Im})^2 \quad (2.10)$$

$$V_{l_y}^2 = (V_{l_y}^{\Re})^2 + (V_{l_y}^{\Im})^2 \quad (2.11)$$

Variation III

The third variation also adopts rectangular coordinates, as shown in (2.12) and (2.13). In contrast to Variation II, it uses the complex current (i.e., $\vec{I}_l \rightarrow I_l^{\Re} + jI_l^{\Im}$) as state variables instead of power injections. This has been adopted by works such as [38, 39].

$$V_{l_x}^{\Re} - V_{l_y}^{\Re} = \mathbf{R}_l I_l^{\Re} - \mathbf{X}_l I_l^{\Im} \quad (2.12)$$

$$V_{l_x}^{\Im} - V_{l_y}^{\Im} = \mathbf{R}_l I_l^{\Im} + \mathbf{X}_l I_l^{\Re} \quad (2.13)$$

The voltage magnitude can be recovered using the previous equations (2.10) and (2.11). The relationships to the power injections (only shown for the start node) are described by (2.14) and (2.15); equations for the end node can be derived accordingly.

$$V_{l_x}^2 \cdot I_l^{\Re} = P_{l_x} V_{l_x}^{\Re} + Q_{l_x} V_{l_x}^{\Im} \quad (2.14)$$

$$V_{l_x}^2 \cdot I_l^{\Im} = P_{l_x} V_{l_x}^{\Im} - Q_{l_x} V_{l_x}^{\Re} \quad (2.15)$$

When considering the squared voltage magnitudes as an optimisation variable (i.e., V^2), the resulting constraints are also quadratic. Nonetheless, there is the same drawback as Variation II where additional equations are needed to obtain the magnitude of voltages and currents.

Variation IV

The fourth, and final, variation is obtained by eliminating the voltage angles in voltage phasors, as shown in (2.16) and (2.17). The term I_l represents the magnitude of the current phasor $\vec{I}_l$. This variation is often referred to as the branch flow model [25], as it focuses on the variables associated with a branch instead of the nodal injections. It has been used in works such as [25, 26, 40].

$$V_{l_x}^2 - V_{l_y}^2 = 2(\mathbf{R}_l P_{l_x} + \mathbf{X}_l Q_{l_x}) - I_l^2(\mathbf{R}_l^2 + \mathbf{X}_l^2) \quad (2.16)$$

$$V_{l_x}^2 \cdot I_l^2 = (P_{l_x}^2 + Q_{l_x}^2) \quad (2.17)$$

The power injections at the end node can be derived by subtracting the losses in a line, as shown in (2.18) and (2.19).

$$P_{l_y} = -(P_{l_x} - I_l^2 \mathbf{R}_l) \quad (2.18)$$

$$Q_{l_y} = -(Q_{l_x} - I_l^2 \mathbf{X}_l) \quad (2.19)$$

Variation IV also results in quadratic constraints when the squared voltage magnitudes and square current magnitudes are considered as optimisation variables (i.e., V^2 and I^2). In contrast to variations II and III, the benefit of variation IV is that voltage magnitudes are already embedded in the formulation; no additional equations

are needed. However, since information on voltage angles is eliminated from the formulation, a potential drawback is that it cannot cater for meshed sections. Nonetheless, this is unlikely to be a significant issue since distribution feeders are mostly operated radially.

2.2.3 Three-Phase OPF Formulations

Many early works on OPF formulations have focused on single-phase models (e.g., [25, 26, 35-40]). However, since distribution networks are inherently unbalanced, the applicability of these single-phase formulations is very limited as the interactions among phases due to any topological and load unbalances cannot be captured. Particularly, this involves not only modelling each phase individually but also considering the mutual impedances among phases. Therefore, the rest of this chapter will focus on three-phase OPF formulations only.

In the context of distribution networks, the main challenges of using OPF in an operational environment is its scalability (tractability) and speed. For instance, when dealing with a model that contains thousands of nodes—a rather common size for realistic distribution feeders—a non-linear formulation is unlikely to be solved in a short amount of time (e.g., within minutes), if it is even technically feasible. Therefore, several approaches have been proposed in the existing literature to address these challenges.

The first approach is developing advanced algorithms to solve the non-convex OPF formulation. For instance, a convex iteration algorithm is proposed in [41], where the non-convex OPF problem is approximated as a sequence of convex problems; the solution to the non-convex problem can be recovered once the iterative algorithm converges. The use of advanced primal-dual interior point methods, a well-known iterative algorithm that guides the search for an optimal solution within a feasible region, have been investigated by [42-44]. Sequential quadratic programming with multiple starting point is investigated in [45] where the non-linear problem is solved iteratively by creating and solving a quadratic subproblem in each iteration. However, there is a common drawback associated with these algorithms: it is often difficult to guarantee their convergence. Furthermore, this is expected to worsen as the size and complexity of the OPF problem grows. Lastly, how to cater for discrete devices (e.g., OLTCs) is largely un-investigated by these and similar works.

The second approach is through relaxation techniques, after which, convex OPF formulations are obtained. For instance, the use of SDP relaxation and SOCP relaxation for the non-convex quadratic constraints have been proposed in [46, 47] and [48], respectively. However, since the feasible region is extended by the relaxation process, exactness (i.e., when the optimal solution of a relaxed problem is also the optimal solution to the original non-relaxed problem) is only guaranteed under special conditions, and thus significantly limiting its applicability in more general applications. Particularly, existing derivations on exactness conditions are largely based on an objective function that aims to minimise the total network losses (or generation cost). Therefore, from the perspective of maximising renewable energy harvesting, a highly relevant objective function in the context of renewable-rich distribution networks [8, 36], the sufficient conditions are largely invalidated. This is due to the fact that, the process of maximising renewable energy harvesting will inherently increase the network losses (which is the exact opposite as considered in aforementioned works) due to higher volumes of reverse power flows.

The third, and final, approach involves approximations to simplify the non-convex constraints in an OPF formulation, typically to achieve a fully linear formulation. To this end, linear approximations of the non-linear power flow equations have been proposed in [49, 50] (for Variation I), [51, 52] (for Variation III) and [53, 54] (for Variation IV). The linearisation point is typically based on the last known state of the network (e.g., recent SCADA measurements). In an operational environment, this is expected to be sufficiently accurate as the state of the network will not change dramatically from consecutive OPF runs. Nonetheless, existing literature on three-phase OPF has largely focused on linearised models for distribution lines; modelling of OLTC-fitted delta-wye transformers—the most common type in distribution networks—has not been adequately investigated (apart from [49] where the delta-wye transformers are considered without modelling the OLTC).

For completeness, the classification of the aforementioned literature based on the four variations presented in Section 2.2.2 is summarised in Table 2-1.

Table 2-1. AC power flow equations used in existing OPF literature.

Variation	Publications
I	[35, 36, 49, 50]
II	[25, 37, 41, 44]
III	[38, 39, 42, 43, 51, 52]
IV	[25, 26, 40, 45-48, 53, 54]

2.2.4 Summary of Research Gaps

OPF can become a powerful decision-making engine in an operational environment for future distribution networks. However, due to the non-linear and non-convex nature of AC power flow equations, and thus the resulting OPF formulation, there are significant challenges in terms of its scalability and speed. To address these challenges, three main approaches have been identified in the existing literature:

- developing advanced algorithms to solve non-convex OPF formulations,
- adopting relaxation techniques to convexify OPF formulations, and
- applying linear approximations to simplify the OPF formulation.

There are clear advantages to the linearisation approach due to the various benefits of formulating the OPF problem as a LP, such as its convexity and the techniques to handle discrete variables (as discussed in Section 2.2.1). However, in the current literature, the modelling of OLTC-fitted delta-wye transformers have not been adequately investigated. This is particularly crucial to cater for the modelling of multiple voltage levels, such as an integrated MV-LV model [55], where the step-down transformers are often in the delta-wye configuration. This will be addressed in the remainder of this chapter.

2.3 Linearised Three-Phase OPF

The proposed linearised, three-phase OPF formulation is presented in this section. It is adapted from the single-phase, branch flow model-based formulation in [26], which corresponds to Variation IV discussed earlier. This is chosen for two main reasons. Firstly, the voltage magnitudes are directly available as variables, which is

beneficial for voltage management schemes since no extra equations (such as those found in variations I and III) are needed. Furthermore, the quadratic terms in Variation IV correspond to the line losses, which typically small compared with the corresponding power flows. Consequently, it offers accuracy advantages when applying linear approximations (as discussed later).

For benchmark purposes, an objective function is required to create an OPF problem. Unlike typical OPF formulations for transmission networks that carry an economic emphasis (e.g., minimisation of generation cost), the application of OPF in distribution networks is often more technical [35, 36, 56]. Therefore, the maximisation of energy harvesting from rooftop PV systems (i.e., maximising PV generation) in a PV-rich residential distribution network is considered. Furthermore, to better demonstrate the scalability and speed of the proposed formulation, an integrated MV-LV feeder that fully models the interconnections from the primary station (in the MV network) down to the connection point of individual residential consumers (in the LV network) is considered (also discussed later).

Parameters (i.e., constants) are highlighted as bolded symbols (e.g., $\mathbf{C}$). Per unit (p.u.) values are used where appropriate. A nomenclature for the key terms is provided at the start of this thesis (Page XXVI).

2.3.1 Households

For a given household $h \in H$, its location (i.e., the node) and the phase connection are denoted by $\beta_h \in N$ and $\varphi_h \in \Phi = \{A, B, C\}$, respectively. All households are assumed to have a single-phase connection, which is the most common type in Australia (and most countries around the world); nonetheless, other connections types can also be modelled if necessary.

The demand of each household is represented by $\mathbf{P}_h^D$ (for active power) and $\mathbf{Q}_h^D$ (for reactive power). These are assumed to be uncontrollable (i.e., they are constants in the OPF).

Power generation for households with PV systems (i.e., $h \in G \subset H$) is represented by P_h^G . This is assumed to be controllable (i.e., it is a variable in the OPF). Furthermore, the total generation is limited by the maximum power available from the PV panels $\mathbf{P}_h^{PV}$ (which is determined by the irradiance) and the inverter capacity $\bar{\mathbf{P}}_h^G$, as well as

the any curtailment being imposed $P_h^{curtail}$, as shown in (2.20). On the other hand, for households without PV systems, P_h^G is zero as shown in (2.21).

$$P_h^G = \min(\mathbf{P}_h^{PV}, \bar{\mathbf{P}}_h^G) - P_h^{curtail}, \quad \forall h \in G \quad (2.20)$$

$$P_h^G = 0, \quad \forall h \notin G \quad (2.21)$$

For simplicity, a unity power factor is assumed for all PV systems, and thus the reactive power is always zero, as shown in (2.22). If necessary, the reactive power can also be incorporated such as through the power factor of PV inverters.

$$Q_h^G = 0, \quad \forall h \in H \quad (2.22)$$

Finally, the net demand of each household at the point of connection to the distribution network is given by (2.23) and (2.24).

$$P_h^{ND} = \mathbf{P}_h^D - P_h^G, \quad \forall h \in H \quad (2.23)$$

$$Q_h^{ND} = \mathbf{Q}_h^D - Q_h^G, \quad \forall h \in H \quad (2.24)$$

2.3.2 Distribution Lines

Power Flow Model

The linearised power flow model for a three-phase distribution line l is derived from the complex three-phase voltage drop equation shown in (2.25). Here, $\vec{V}$, $\vec{I}$ and $\vec{Z}$, correspond to the complex voltage, current and impedance, respectively. The phases are denoted by ϕ and ψ (both in the set Φ). The start node and the end node of a line are denoted by l_x and l_y (both in the set N), respectively.

$$\vec{V}_{l_x, \phi} - \vec{V}_{l_y, \phi} = \sum_{\psi \in \Phi} \vec{Z}_{l, \phi, \psi} \vec{I}_{l, \psi}^*, \quad \forall l \in L, \forall \phi \in \Phi \quad (2.25)$$

The electrical parameters are modelled based on the self-impedance (when $\phi = \psi$) and the mutual-impedance (when $\phi \neq \psi$), as shown in (2.26). The current in a given phase is related to the complex power injection $\vec{S} = P + jQ$, as shown in (2.27).

$$\vec{\mathbf{Z}}_{l,\phi,\psi} = \mathbf{R}_{l,\phi,\psi} + j\mathbf{X}_{l,\phi,\psi}, \quad \forall l \in L, \forall \phi \in \Phi, \forall \psi \in \Phi \quad (2.26)$$

$$\vec{I}_{l,\phi} = \left(\frac{P_{l_x} + jQ_{l_x}}{\vec{V}_{l_x,\phi}} \right)^*, \quad \forall l \in L, \forall \phi \in \Phi \quad (2.27)$$

By taking the magnitude on both sides of (2.25), this complex voltage drop equation can be rewritten as a single scalar equation with only real terms, as shown in (2.28) with the corresponding substitutions in (2.29) and (2.30). Here, the squared voltage magnitude is denoted by v (i.e., $\sqrt{v} = |\vec{V}|$) and the voltage angle is denoted by θ . Furthermore, auxiliary terms κ_1 , κ_2 and κ_3 are introduced for compactness.

$$v_{l_x,\phi} - v_{l_y,\phi} = 2\kappa_1 - \frac{(\kappa_1)^2 + (\kappa_2)^2}{v_{l_x,\phi}}, \quad \forall l \in L, \forall \phi \in \Phi \quad (2.28)$$

$$\kappa_1 = \sum_{\psi \in \Phi} (R'_{l,\phi,\psi} P_{l_x,\psi} + X'_{l,\phi,\psi} Q_{l_x,\psi})$$

$$\kappa_2 = \sum_{\psi \in \Phi} (X'_{l,\phi,\psi} P_{l_x,\psi} - R'_{l,\phi,\psi} Q_{l_x,\psi})$$

$$R'_{l,\phi,\psi} = \frac{V_{l_x,\phi}}{V_{l_x,\psi}} [\mathbf{R}_{l,\phi,\psi} \cos(\kappa_3) - \mathbf{X}_{l,\phi,\psi} \sin(\kappa_3)], \quad \forall l \in L, \forall \phi \in \Phi \quad (2.29)$$

$$X'_{l,\phi,\psi} = \frac{V_{l_x,\phi}}{V_{l_x,\psi}} [\mathbf{R}_{l,\phi,\psi} \sin(\kappa_3) + \mathbf{X}_{l,\phi,\psi} \cos(\kappa_3)], \quad \forall l \in L, \forall \phi \in \Phi \quad (2.30)$$

$$\kappa_3 = \theta_{l_x,\psi} - \theta_{l_x,\phi}$$

In (2.29) and (2.30), the primed terms R' and X' can be considered as the ‘equivalent’ resistance and reactance, respectively, which are scaled versions of the physical impedance based on the voltage phasors.

The power injections at the end node of a line can be calculated by subtracting the ohmic losses, as shown in (2.31). Here, the term $\vec{I}_{l,\psi}^* \vec{I}_{l,\phi} \vec{\mathbf{Z}}_{l,\phi,\psi}$ is equivalent to the well-known $I^2 R$ expression for pure resistive losses.

$$\vec{S}_{l_y,\phi} = - \left(\vec{S}_{l_x,\phi} - \sum_{\psi \in \Phi} \vec{I}_{l,\psi}^* \vec{I}_{l,\phi} \vec{\mathbf{Z}}_{l,\phi,\psi} \right), \quad \forall l \in L, \forall \phi \in \Phi \quad (2.31)$$

By taking the magnitude of both sides of (2.31) as well as using the substitutions in (2.26) and (2.27), the power injections at the end node can also be obtained as scalar equations. These are shown in (2.32) and (2.33) along with new auxiliary terms κ_4 and κ_5 for compactness.

$$P_{l_y, \phi} = -P_{l_x, \phi} + \frac{P_{l_x, \phi} \kappa_4 - Q_{l_x, \phi} \kappa_5}{v_{l_x, \phi}}, \quad l \in L, \forall \phi \in \Phi \quad (2.32)$$

$$Q_{l_y, \phi} = -Q_{l_x, \phi} + \frac{P_{l_x, \phi} \kappa_5 + Q_{l_x, \phi} \kappa_4}{v_{l_x, \phi}}, \quad l \in L, \forall \phi \in \Phi \quad (2.33)$$

$$\kappa_4 = \sum_{\psi \in \Phi} (R'_{l, \phi, \psi} P_{l_x, \psi} + X'_{l, \phi, \psi} Q_{l_x, \psi})$$

$$\kappa_5 = \sum_{\psi \in \Phi} (X'_{l, \phi, \psi} P_{l_x, \psi} - R'_{l, \phi, \psi} Q_{l_x, \psi})$$

To linearise the power flow model in (2.28)-(2.30), (2.32) and (2.33), estimated terms (ones with the $\sim$ accent) for voltages (i.e., $\tilde{v}$ and $\tilde{\theta}$) and powers (i.e., $\tilde{P}$ and $\tilde{Q}$) are introduced to reduce all non-linear expressions to linear expressions. The final forms are shown in (2.34)-(2.38)¹. These estimated terms can come from recent SCADA measurements or, in the absence of recent measurements, the flat start values² used to run power flows.

A key advantage of this linearisation approach is its relatively low impact on accuracy. For (2.34), (2.37) and (2.38), the approximated terms only appear in expressions related to the losses in lines. Consequently, since the magnitude of losses is much smaller than the actual power flows, the overall impact on accuracy is expected to be negligible.

For (2.35) and (2.36), the approximation only affects the mutual impedances (cases when $\phi \neq \psi$). In contrast, when $\phi = \psi$, (2.35) and (2.36) are reduced to the constants

¹ Note, in these equations, the terms in bold represents an estimated version of a corresponding variable. Overall, this allows the original non-linear equation to be approximated with a linear version. For instance, the non-linear term $1/v_{l_x, \phi}$ is converted to a constant by using an estimated value of $v_{l_x, \phi}$, i.e., $\tilde{v}_{l_x, \phi}$.

² Typically, flat start values assume voltage magnitudes are at nominal and power injections are zero. Alternatively, historical SCADA values can also be used; nonetheless, this would depend on how relevant/accurate these historical values are, which then needs to be assessed per application.

$\mathbf{R}_{l,\phi,\psi}$ and $\mathbf{X}_{l,\phi,\psi}$, respectively, which do not depend on the approximated terms. Consequently, since the contribution to voltage drops for a given phase is typically dominated by the power flows in that phase, the overall impact on accuracy is also expected to be negligible.

$$v_{l_x,\phi} - v_{l_y,\phi} = 2\kappa_1 - \frac{\kappa_1 \tilde{\kappa}_1 + \kappa_2 \tilde{\kappa}_2}{\tilde{v}_{l_x,\phi}}, \quad \forall l \in L, \forall \phi \in \Phi \quad (2.34)$$

$$\kappa_1 = \sum_{\psi \in \Phi} (\tilde{\mathbf{R}}'_{l,\phi,\psi} P_{l_x,\psi} + \tilde{\mathbf{X}}'_{l,\phi,\psi} Q_{l_x,\psi})$$

$$\kappa_2 = \sum_{\psi \in \Phi} (\tilde{\mathbf{X}}'_{l,\phi,\psi} P_{l_x,\psi} - \tilde{\mathbf{R}}'_{l,\phi,\psi} Q_{l_x,\psi})$$

$$\tilde{\kappa}_1 = \sum_{\psi \in \Phi} (\tilde{\mathbf{R}}'_{l,\phi,\psi} P_{l_x,\psi} + \tilde{\mathbf{X}}'_{l,\phi,\psi} Q_{l_x,\psi})$$

$$\tilde{\kappa}_2 = \sum_{\psi \in \Phi} (\tilde{\mathbf{X}}'_{l,\phi,\psi} P_{l_x,\psi} - \tilde{\mathbf{R}}'_{l,\phi,\psi} Q_{l_x,\psi})$$

$$\tilde{\mathbf{R}}'_{l,\phi,\psi} = \frac{\tilde{V}_{l_x,\phi}}{\tilde{V}_{l_x,\psi}} [\mathbf{R}_{l,\phi,\psi} \cos(\tilde{\kappa}_3) - \mathbf{X}_{l,\phi,\psi} \sin(\tilde{\kappa}_3)], \quad \forall l \in L, \forall \phi \in \Phi \quad (2.35)$$

$$\tilde{\mathbf{X}}'_{l,\phi,\psi} = \frac{\tilde{V}_{l_x,\phi}}{\tilde{V}_{l_x,\psi}} [\mathbf{R}_{l,\phi,\psi} \sin(\tilde{\kappa}_3) + \mathbf{X}_{l,\phi,\psi} \cos(\tilde{\kappa}_3)], \quad \forall l \in L, \forall \phi \in \Phi \quad (2.36)$$

$$\tilde{\kappa}_3 = \tilde{\theta}_{l_x,\psi} - \tilde{\theta}_{l_x,\phi}$$

$$P_{l_y,\phi} = -P_{l_x,\phi} + \frac{P_{l_x,\phi} \tilde{\kappa}_4 - \tilde{Q}_{l_x,\phi} \tilde{\kappa}_5}{\tilde{v}_{l_x,\phi}}, \quad l \in L, \forall \phi \in \Phi \quad (2.37)$$

$$Q_{l_y,\phi} = -Q_{l_x,\phi} + \frac{\tilde{P}_{l_x,\phi} \tilde{\kappa}_5 + Q_{l_x,\phi} \tilde{\kappa}_4}{\tilde{v}_{l_x,\phi}}, \quad l \in L, \forall \phi \in \Phi \quad (2.38)$$

$$\tilde{\kappa}_4 = \sum_{\psi \in \Phi} (\tilde{\mathbf{R}}'_{l,\phi,\psi} \tilde{\mathbf{P}}_{l_x,\psi} + \tilde{\mathbf{X}}'_{l,\phi,\psi} \tilde{\mathbf{Q}}_{l_x,\psi})$$

$$\tilde{\kappa}_5 = \sum_{\psi \in \Phi} (\tilde{\mathbf{X}}'_{l,\phi,\psi} \tilde{\mathbf{P}}_{l_x,\psi} - \tilde{\mathbf{R}}'_{l,\phi,\psi} \tilde{\mathbf{Q}}_{l_x,\psi})$$

Thermal Constraint

The thermal limits of a distribution line is modelled through its ampacity I_l^+ as shown in (2.39). This constraint can be approximated using the piecewise linearisation technique adopted in [26].

$$P_{l_x,\phi}^2 + Q_{l_x,\phi}^2 \leq v_{l_x,\phi} (I_l^+)^2 \quad (2.39)$$

This linear approximation of (2.39) is shown in (2.40)-(2.49). The underlying mechanism of this approach is briefly explained below; detailed explanations can be found in [26].

$$\sum_{\lambda} \rho_{l,\phi,\lambda} (\Delta_{l_x,\phi,\lambda}^P + \Delta_{l_x,\phi,\lambda}^Q) \leq v_{l_x,\phi} (I_l^+)^2, \quad \forall l \in L, \forall \phi \in \Phi \quad (2.40)$$

$$P_{l_x,\phi} = P_{l_x,\phi}^+ - P_{l_x,\phi}^-, \quad \forall l \in L, \forall \phi \in \Phi \quad (2.41)$$

$$Q_{l_x,\phi} = Q_{l_x,\phi}^+ - Q_{l_x,\phi}^-, \quad \forall l \in L, \forall \phi \in \Phi \quad (2.42)$$

$$P_{l_x,\phi}^+ + P_{l_x,\phi}^- = \sum_{\lambda} \Delta_{l_x,\phi,\lambda}^P, \quad \forall l \in L, \forall \phi \in \Phi \quad (2.43)$$

$$Q_{l_x,\phi}^+ + Q_{l_x,\phi}^- = \sum_{\lambda} \Delta_{l_x,\phi,\lambda}^Q, \quad \forall l \in L, \forall \phi \in \Phi \quad (2.44)$$

$$0 \leq \Delta_{l_x,\phi,\lambda}^P \leq \bar{\Delta}_{l,\phi}, \quad \forall l \in L, \forall \phi \in \Phi, \forall \lambda \in \{1, 2, \dots, \Lambda\} \quad (2.45)$$

$$0 \leq \Delta_{l_x,\phi,\lambda}^Q \leq \bar{\Delta}_{l,\phi}, \quad \forall l \in L, \forall \phi \in \Phi, \forall \lambda \in \{1, 2, \dots, \Lambda\} \quad (2.46)$$

$$P_{l_x,\phi}^+, P_{l_x,\phi}^-, Q_{l_x,\phi}^+, Q_{l_x,\phi}^- \geq 0, \quad \forall l \in L, \forall \phi \in \Phi \quad (2.47)$$

$$\rho_{l,\phi,\lambda} = (2\lambda - 1)\bar{\Delta}_{l,\phi}, \quad \forall l \in L, \forall \phi \in \Phi, \forall \lambda \in \{1, 2, \dots, \Lambda\} \quad (2.48)$$

$$\bar{\Delta}_{l,\phi} = V_{n,\phi}^+ I_l^+ / \Lambda, \quad \forall l \in L, \forall \phi \in \Phi \quad (2.49)$$

The intuitive interpretation of constraints (2.40)-(2.49) is a more conservative way of enforcing the thermal limit in (2.39). This is achieved by creating a ‘linearised upper bounds’ of $P_{l_x,\phi}^2 + Q_{l_x,\phi}^2$; i.e., $P_{l_x,\phi}^2 + Q_{l_x,\phi}^2 \leq \text{linearised upper bound} \leq v_{l_x,\phi} (I_l^+)^2$.

To illustrate this, consider a simplified case of (2.39) where the reactive power is zero, i.e., $P_{l_x,\phi}^2 \leq v_{l_x,\phi} (I_l^+)^2$. As shown in Figure 2-3, the piecewise linearisation technique will produce several line segments (red dotted lines) that jointly form a

linearised upper bound for $P_{l_x,\phi}^2$ (black line). Furthermore, the number of discretisation steps is determined by the tuning parameter Λ ; in Figure 2-3, $\Lambda = 2$ is depicted.

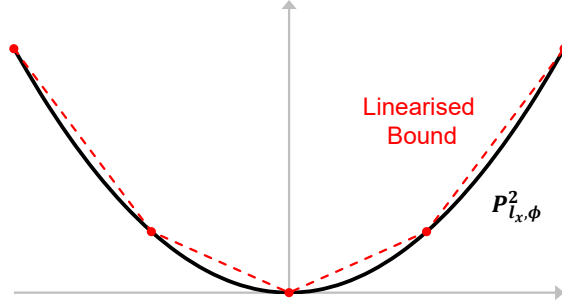


Figure 2-3. Visualisation of the piecewise linearisation technique.

In a more generic case where both the active and reactive powers are none-zero, the resulting linearised upper bound will consist of multiple planes on a three-dimensional plot (instead of the line segments shown in Figure 2-3) above the paraboloid formed by $P_{l_x,\phi}^2 + Q_{l_x,\phi}^2$.

Finally, it is worth noting that, unlike a conventional piecewise linearisation approach, an adjacency order is not required for the segments (i.e., $\Delta_{l_x,\phi,\lambda}^P$ and $\Delta_{l_x,\phi,\lambda}^Q$). This is a special scenario when being used to model the thermal constraint. Particularly, this constraint only becomes a binding constraint when the thermal limit is reached. In this situation, the constraint in (2.40) and objective function of maximising generation will inherently force the adjacency order for the Δ terms due to the monotonically increasing scaling factor $\rho_{l,\phi,\lambda}$. On the other hand, when the thermal limit of a line is not a constraining factor, the terms $\Delta_{l_x,\phi,\lambda}^P$ and $\Delta_{l_x,\phi,\lambda}^Q$ essentially become free variables.

2.3.3 Delta-Wye Transformers

The linearised mathematical model for a delta-wye transformer—the most commonly used configuration for step-down transformers in distribution networks [57]—is presented below. The delta connection corresponds to the primary side (higher voltage level) and the wye connection corresponds to the secondary side (lower voltage level).

Power Flow Model

The modelling of a physical three-phase delta-wye transformer t is separated into two virtual parts: a lossless transformer (to model the transformations of powers and voltages) and three single-phase line segments (to model the losses and thermal limits in each winding), as shown in Figure 2-4.¹ Here, the squared voltage magnitudes at the start node (i.e., the primary side) and the end node (i.e., the secondary side) are represented by $v_{t_x,\phi}$ and $v_{t_y,\phi}$, respectively; similarly, the power flow terms are represented by $P_{t_x,\phi}$, $P_{t_y,\phi}$, $Q_{t_x,\phi}$ and $Q_{t_y,\phi}$.

The derivation of the linearised power flow model for the lossless transformer is presented below. The linearised power flow model for the single-phase line segments can be extracted from the three-phase model in (2.34)-(2.49) by considering only one of three phases; this is omitted for compactness. The impedance values for these line segments are selected to match the transformer's losses characteristics (e.g., the copper losses and leakage reactance) [58].

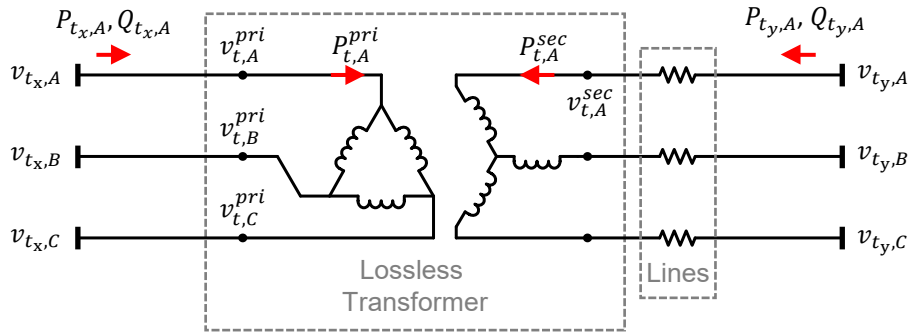


Figure 2-4. A delta-wye transformer.

For better clarity, additional terms (ones with superscripts *pri* and *sec* that denote the primary side and secondary side, respectively) are introduced to distinguish the intermediate terms for a *virtual* lossless transformer (which is introduced for modelling purposes) from the *physical* delta-wye transformer; selected terms are indicated in Figure 2-4.

¹ Note, the same sign convention as lines are used here for the power injection terms.

The linearised power equations for the lossless transformer is derived based on the phasor model proposed in [59]; an example is illustrated through (2.50)-(2.54).

Due to the phase-to-phase connection on the primary side of a delta-wye transformer, a (phase-to-neutral) power on phase A at the secondary terminal (i.e., $\vec{S}_{t,A}^{sec} = P_{t,A}^{sec} + jQ_{t,A}^{sec}$) will result in a (phase-to-phase) current between phases A and C on the primary side, as shown in (2.50).

$$(\vec{I}_{t,A \rightarrow C}^{pri})^* = \frac{-\vec{S}_{t,A}^{sec}}{\vec{V}_{t,A}^{pri} - \vec{V}_{t,C}^{pri}}, \quad \forall t \in T \quad (2.50)$$

Using (2.50) and the complex power formula $\vec{S} = \vec{V} \cdot \vec{I}^*$, the corresponding power injections at phase A of the primary side can be calculated by (2.51).

$$\vec{S}_{t,A}^{pri} = \frac{\vec{V}_{t,A}^{pri}}{\vec{V}_{t,A}^{pri} - \vec{V}_{t,C}^{pri}} \vec{S}_{t,A}^{sec}, \quad \forall t \in T \quad (2.51)$$

To linearise (2.51), the following conditions for relatively balanced three-phase voltages are assumed: $|\vec{V}_{t,A}^{pri}| \approx |\vec{V}_{t,C}^{pri}|$ and $\angle \vec{V}_{t,C}^{pri} - \angle \vec{V}_{t,A}^{pri} \approx 120^\circ$. For MV networks, where these transformers are usually being connected to, this assumption is generally valid due to the diversity of power flows at an aggregated level.

Based on the above, the approximated form of (2.51) is shown in (2.52). Also, by separating the real and imaginary parts of (2.52), the linear equations in (2.53) and (2.54) are obtained. It is worth noting that, pure active power on the secondary side of a delta-wye transformer actually results in both active and reactive powers on the primary side.

$$\vec{S}_{t,A}^{pri} = \left(\frac{1}{2} - j \frac{1}{2\sqrt{3}} \right) \vec{S}_{t,A}^{sec}, \quad \forall t \in T \quad (2.52)$$

$$P_{t,A}^{pri} = \frac{1}{2} P_{t,A}^{sec} + \frac{1}{2\sqrt{3}} Q_{t,A}^{sec}, \quad \forall t \in T \quad (2.53)$$

$$Q_{t,A}^{pri} = \frac{1}{2} Q_{t,A}^{sec} + \frac{1}{2\sqrt{3}} P_{t,A}^{sec}, \quad \forall t \in T \quad (2.54)$$

Finally, following the same process presented above, the linearised power relationships for the whole lossless transformer can be derived; the final result is shown in (2.55) and (2.56).

$$\begin{bmatrix} P_{t,A}^{pri} \\ P_{t,B}^{pri} \\ P_{t,C}^{pri} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_{t,A}^{sec} \\ P_{t,B}^{sec} \\ P_{t,C}^{sec} \end{bmatrix} + \frac{1}{2\sqrt{3}} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} Q_{t,A}^{sec} \\ Q_{t,B}^{sec} \\ Q_{t,C}^{sec} \end{bmatrix}, \quad \forall t \in T \quad (2.55)$$

$$\begin{bmatrix} Q_{t,A}^{pri} \\ Q_{t,B}^{pri} \\ Q_{t,C}^{pri} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} Q_{t,A}^{sec} \\ Q_{t,B}^{sec} \\ Q_{t,C}^{sec} \end{bmatrix} - \frac{1}{2\sqrt{3}} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_{t,A}^{sec} \\ P_{t,B}^{sec} \\ P_{t,C}^{sec} \end{bmatrix}, \quad \forall t \in T \quad (2.56)$$

The voltage equations for a lossless transformer is derived from the standard phasor formula in [57]; an example is illustrated through (2.57)-(2.62).

The phasor voltage relationships from the primary side to the secondary side of a delta-wye connection is shown in (2.57). If a tap changer (either on-load or off-load) is present, the current tap position can be modelled by its tap ratio τ_t (assuming it is installed on the primary side, which is typically true for step-down transformers); otherwise, $\tau_t = 1$.

$$\sqrt{3}\tau_t \vec{V}_{t,A}^{sec} = \vec{V}_{t,A}^{pri} - \vec{V}_{t,C}^{pri}, \quad \forall t \in T \quad (2.57)$$

By taking the magnitude on both sides of (2.57), the complex equation can be reduced to the scalar form in (2.58).

$$3\tau_t^2 v_{t,A}^{sec} = v_{t,A}^{pri} + v_{t,C}^{pri} - |\vec{V}_{t,A}^{pri}| |\vec{V}_{t,C}^{pri}| \cos(\angle \vec{V}_{t,A}^{pri} - \angle \vec{V}_{t,C}^{pri}), \quad \forall t \in T \quad (2.58)$$

In a perfectly balanced network, the three voltage phasors form an equilateral triangle (i.e., $\vec{V}_{t,A}^{pri} + \vec{V}_{t,B}^{pri} + \vec{V}_{t,C}^{pri} = 0$), as shown on the left of Figure 2-5. Consequently, (2.59) can be obtained by applying the Law of Cosines, which then can be used to simplify (2.58) into the linear form in (2.60)

$$|\vec{V}_{t,B}^{pri}|^2 = |\vec{V}_{t,A}^{pri}|^2 + |\vec{V}_{t,C}^{pri}|^2 - 2|\vec{V}_{t,A}^{pri}||\vec{V}_{t,C}^{pri}|\cos(\angle\vec{V}_{t,A}^{pri} - \angle\vec{V}_{t,C}^{pri}), \quad \forall t \in T \quad (2.59)$$

$$3\tau_t^2 v_{t,A}^{sec} = 2v_{t,A}^{pri} + 2v_{t,C}^{pri} - v_{t,B}^{pri}, \quad \forall t \in T \quad (2.60)$$

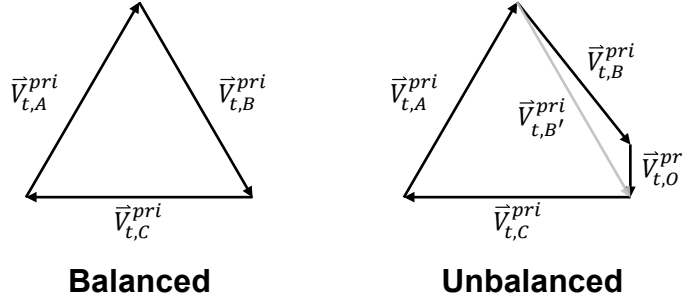


Figure 2-5. Voltage phasors for balanced and unbalanced scenarios.

While (2.59) is no longer true in an unbalanced network, a similar condition still holds when a small offset phasor $\vec{V}_{t,O}^{pri}$ is introduced to obtain $v_{t,B'}^{pri}$, as shown on the right of Figure 2-5; the offset vector is exaggerated on the diagram for better clarity. Based on this, (2.60) can be adapted to (2.61) to account for the voltage unbalances.

$$3\tau_t^2 v_{t,A}^{sec} = 2v_{t,A}^{pri} + 2v_{t,C}^{pri} - v_{t,B'}^{pri}, \quad \forall t \in T \quad (2.61)$$

Combining (2.61) with the relationship $\vec{V}_{t,B'}^{pri} = \vec{V}_{t,A}^{pri} + \vec{V}_{t,B}^{pri} = \vec{V}_{t,B}^{pri} + \vec{V}_{t,O}^{pri}$, an linear approximation of (2.61) with the physical squared voltage magnitudes (i.e., $v_{t,A}^{pri}$, $v_{t,B}^{pri}$ and $v_{t,C}^{pri}$) can be obtained, as shown in (2.62). Here, the offset expression $|\vec{V}_{t,A}^{pri} + \vec{V}_{t,C}^{pri}|^2 - |\vec{V}_{t,B}^{pri}|^2$ can be calculated based on the available data for the corresponding transformer to account for the unbalances among phases.

$$3\tau_t^2 v_{t,A}^{sec} = 2v_{t,A}^{pri} + 2v_{t,C}^{pri} - v_{t,B}^{pri} - \left(|\vec{V}_{t,A}^{pri} + \vec{V}_{t,C}^{pri}|^2 - |\vec{V}_{t,B}^{pri}|^2 \right), \quad \forall t \in T \quad (2.62)$$

Finally, following the same process presented above, the linearised voltage relationships for the whole lossless transformer can be derived; the final result is shown in (2.63).

$$\boldsymbol{\tau}_t^2 \begin{bmatrix} v_{t,A}^{sec} \\ v_{t,B}^{sec} \\ v_{t,C}^{sec} \end{bmatrix} = \begin{bmatrix} 2 & -1 & 2 \\ 2 & 2 & -1 \\ -1 & 2 & 2 \end{bmatrix} \begin{bmatrix} v_{t,A}^{pri} \\ v_{t,B}^{pri} \\ v_{t,C}^{pri} \end{bmatrix} - \begin{bmatrix} |\tilde{\mathbf{v}}_{t,A}^{pri} + \tilde{\mathbf{v}}_{t,C}^{pri}|^2 - |\tilde{\mathbf{v}}_{t,B}^{pri}|^2 \\ |\tilde{\mathbf{v}}_{t,A}^{pri} + \tilde{\mathbf{v}}_{t,B}^{pri}|^2 - |\tilde{\mathbf{v}}_{t,C}^{pri}|^2 \\ |\tilde{\mathbf{v}}_{t,B}^{pri} + \tilde{\mathbf{v}}_{t,C}^{pri}|^2 - |\tilde{\mathbf{v}}_{t,A}^{pri}|^2 \end{bmatrix}, \quad \forall t \in T \quad (2.63)$$

Thermal Constraint

As mentioned earlier, the thermal constraint is modelled through the line segments (shown in Figure 2-4). Since it reuses the formulation presented in Section 2.3.1, further discussions are omitted.

Voltage Regulation Capability

Integer variables can be introduced to (2.63) when an OLTC is present and its tap position is considered to be controllable. In this case, the extra modelling required to replace (2.63) is presented below.

For an OLTC-fitted transformer, the physical tap positions are represented by the set Π_t (indexed by π) and the corresponding tap ratio (of each physical tap position) is denoted by $\boldsymbol{\tau}_{t,\pi}$. Furthermore, a binary variable $k_{t,\pi}$ is introduced for each tap π to represent whether this tap is active (or not). To ensure that only one tap position can be active at all times, constraint (2.64) is introduced.

$$\sum_{\pi} k_{t,\pi} = 1 \quad (2.64)$$

The regulation capability of the OLTC is modelled by (2.65) and (2.66). This is adapted from (2.63) and the auxiliary term H_t that represents the right-hand side of (2.63) is introduced for compactness.

$$H_t \geq 3\boldsymbol{\tau}_{t,\pi}^2 \begin{bmatrix} v_{t,A}^{sec} \\ v_{t,B}^{sec} \\ v_{t,C}^{sec} \end{bmatrix} - (1 - k_{t,\pi})\mathbf{M}, \quad \forall t \in T, \forall \pi \in \Pi_t \quad (2.65)$$

$$H_t \leq 3\boldsymbol{\tau}_{t,\pi}^2 \begin{bmatrix} v_{t,A}^{sec} \\ v_{t,B}^{sec} \\ v_{t,C}^{sec} \end{bmatrix} + (1 - k_{t,\pi})\mathbf{M}, \quad \forall t \in T, \forall \pi \in \Pi_t \quad (2.66)$$

$$H_t = \begin{bmatrix} 2 & -1 & 2 \\ 2 & 2 & -1 \\ -1 & 2 & 2 \end{bmatrix} \begin{bmatrix} v_{t,A}^{pri} \\ v_{t,B}^{pri} \\ v_{t,C}^{pri} \end{bmatrix} - \begin{bmatrix} |\tilde{\mathbf{V}}_{t,A}^{pri} + \tilde{\mathbf{V}}_{t,C}^{pri}|^2 - |\tilde{\mathbf{V}}_{t,B}^{pri}|^2 \\ |\tilde{\mathbf{V}}_{t,A}^{pri} + \tilde{\mathbf{V}}_{t,B}^{pri}|^2 - |\tilde{\mathbf{V}}_{t,C}^{pri}|^2 \\ |\tilde{\mathbf{V}}_{t,B}^{pri} + \tilde{\mathbf{V}}_{t,C}^{pri}|^2 - |\tilde{\mathbf{V}}_{t,A}^{pri}|^2 \end{bmatrix}$$

The Big M method [33] is used in (2.65) and (2.66) where $\mathbf{M}$ is a sufficiently large constant; if p.u. values are used for voltages, $\mathbf{M} \geq 5$ is sufficient. When a given tap position π is active, (2.65) and (2.66) become an equality constraint that represents the voltage transformation from the corresponding tap ratio $\tau_{t,\pi}$. On the other hand, when the tap position π is inactive, (2.65) and (2.66) become inactive constraints due to the earlier choice of $\mathbf{M}$.

2.3.4 Nodal Constraints

Voltage limits at each node is enforced through (2.67) for the squared, line-to-neutral voltage magnitude $v_{n,\phi}$ (which are also used as state variables in distribution lines and delta-wye transformers). Here, $\mathbf{V}_{n,\phi}^+$ and $\mathbf{V}_{n,\phi}^-$ are the adopted upper and lower limits, respectively.

$$(\mathbf{V}_{n,\phi}^-)^2 \leq v_{n,\phi} \leq (\mathbf{V}_{n,\phi}^+)^2, \quad \forall n \in N, \forall \phi \in \Phi \quad (2.67)$$

The interconnections with the upstream network (indexed by $s \in S \subset N$) are assumed to be able to import/export both active power ($P_{s,\phi}^S$) and reactive power ($Q_{s,\phi}^S$) as required by the distribution network. This is shown in (2.68) and (2.69). Furthermore, one of these nodes is also selected as the slack bus for the modelled network where the voltage magnitude is fixed (e.g., at 1.0 p.u.).

$$P_{s,\phi}^S \in \mathbb{R}, \quad \forall s \in S, \forall \phi \in \Phi \quad (2.68)$$

$$Q_{s,\phi}^S \in \mathbb{R}, \quad \forall s \in S, \forall \phi \in \Phi \quad (2.69)$$

2.3.5 Power Balance

The active and reactive power balance (for each node n and each phase ϕ) is described by Kirchhoff's Current Law, as shown in (2.70) and (2.71), respectively.

Here, each of the P and Q terms refer to the active and reactive power flow into a network element (i.e., a household, line, transformer or source), which are already described in previous subsections (i.e., Section 2.3.1 - 2.3.3).

$$\begin{aligned} \sum_{l \in L \mid l_x = n} P_{l_x, \phi} + \sum_{l \in L \mid l_y = n} P_{l_y, \phi} + \sum_{t \in T \mid t_x = n} P_{t_x, \phi} + \sum_{t \in T \mid t_y = n} P_{t_y, \phi} \\ + \sum_{h \in H \mid \beta_h = n, \varphi_h = \phi} P_h^{ND} + \sum_{s \in S \mid s = n} P_{s, \phi}^S = 0, \quad \forall n \in N, \forall \phi \in \Phi \end{aligned} \quad (2.70)$$

$$\begin{aligned} \sum_{l \in L \mid l_x = n} Q_{l_x, \phi} + \sum_{l \in L \mid l_y = n} Q_{l_y, \phi} + \sum_{t \in T \mid t_x = n} Q_{t_x, \phi} + \sum_{t \in T \mid t_y = n} Q_{t_y, \phi} \\ + \sum_{h \in H \mid \beta_h = n, \varphi_h = \phi} Q_h^{ND} + \sum_{s \in S \mid s = n} Q_{s, \phi}^S = 0, \quad \forall n \in N, \forall \phi \in \Phi \end{aligned} \quad (2.71)$$

2.3.6 Objective Function

Finally, the objective function J , i.e., the total power injections from rooftop PV systems, is given by (2.72). The investigated OPF problem will aim to maximise J (in order to maximise the renewable energy harvesting from residential PV systems, as discussed at the start of this section).

$$J = \sum_{h \in G} P_h^G \quad (2.72)$$

2.3.7 Summary of the OPF Formulation

A summary of the linearised AC OPF formulation is shown in Table 2-2.

Table 2-2. Summary of the proposed linearised OPF.

Modelled Element	Formulation
Households	(2.20)-(2.24)
Three-Phase Distribution Lines	(2.34)-(2.38), (2.40)-(2.49)
OLTC-Fitted Delta-Wye Transformers	(2.55), (2.56), (2.63)-(2.66) + single-phase line for losses
Single-Phase Lines	Extracted from a three-phase line model by taking a single phase only
Nodal Constraints and Power Balance	(2.67)-(2.71)
Objective Function	(2.72)

2.3.8 Solution Process

Since the proposed formulation relies on estimated values, i.e., constants with the ‘ $\sim$ ’ accent in (2.34)-(2.38) and (2.63), the accuracy of these estimated terms is expected to affect the overall accuracy of the formulation. To this end, there are two ways of improving the overall accuracy.

The first option is using the latest SCADA measurements as warm start values. Since, in an operational environment, the state of the network is unlikely to change dramatically from minute to minute, warm start values based on recent measurements is likely to be much closer to the actual system state (i.e., voltage magnitudes and power flows) than common flat start values for power flow calculations (i.e., voltage at 1.0 p.u. and power flows at zero). Therefore, the use of SCADA measurements is expected to bring noticeable improvements in terms of the overall accuracy of the OPF formulation.

The second option is solving the OPF problem twice [26], where the calculated results from the first iteration is used as the estimated terms in the second iteration. For instance, the calculated values of the squared voltages (i.e., $v_{l_x,\phi}$) in the first iteration can be used as the estimated voltage (i.e., $\tilde{V}_{l_x,\phi}$) in the second iteration. This

iteration process will also improve the quality of the estimated terms, and thus the overall accuracy of the OPF formulation.

2.4 Case Study

The speed and accuracy of the proposed OPF formulation is demonstrated using a realistic PV-rich MV-LV distribution network in Victoria, Australia. The simulation setup and results are presented in this section.

2.4.1 Co-Simulation Platform

The validation is carried out in a co-simulation environment that consists of OpenDSS [60] (the network simulator), AIMMS [61] with the solver CPLEX v12.8 [62] (the optimisation engine) and a Python [63] script (acting as a proxy of the SCADA system to facilitate data transfers between the simulated network and optimisation engine). This co-simulation setup allows the calculated setpoints (from the OPF) to be validated in a network simulator (as a proxy of a real network). The data flow of the co-simulation platform is shown in Figure 2-7.

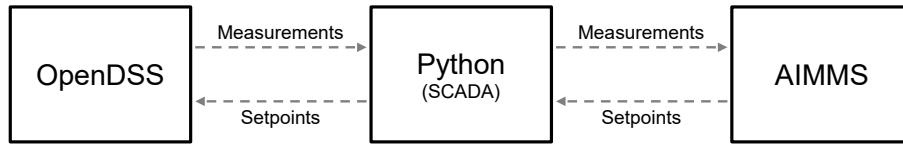


Figure 2-6. Data flow of the co-simulation platform.

2.4.2 Assessment Methodology and Metrics

The accuracy of the OPF formulation is assessed by implementing the calculated setpoints in the network simulator. Particularly, the performance is quantified by calculating the absolute percentage error (APE), as shown in (2.73), between the results from the network simulator (NS), i.e., the actual values observed, and the results from the optimisation engine, i.e., the expected values by the OPF.

$$APE = \left| \frac{NS - OPF}{NS} \right| \quad (2.73)$$

The speed of the OPF is assessed based on the solution time reported by the solver. This is done assuming that the proposed formulation is used in an operational environment where the decision-making interval falls in the scale of several minutes, i.e., the solution time must be less than one (or two) minutes.

2.4.3 PV-Rich Distribution Network

This case study considers a real 22 kV residential feeder from Victoria, Australia [64]. The topology of the MV feeder is shown in Figure 2-7.

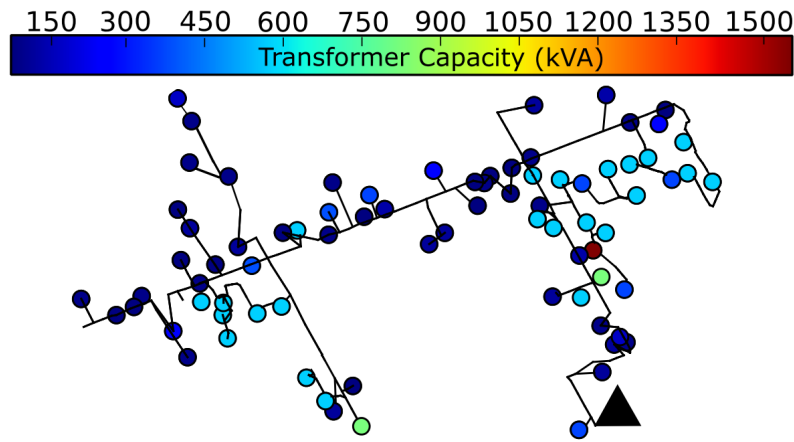


Figure 2-7. Topology of the 22kV feeder.

The MV feeder is connected to the upstream transmission network through a 66/22 kV OLTC-fitted delta-wye transformer at the primary substation (triangle in Figure 2-7). The primary side of this transformer is assumed to be the slack bus with a reference voltage of 66 kV (1.0 p.u.). The OLTC has a total of 18 tap positions and the corresponding tap ratios range from 1.0715 to 0.8284 in steps of 0.0143. The nominal tap position has a tap ratio of 1.0.

In total, there are 79 22/0.433 kV distribution transformers (circles in Figure 2-7) in this MV feeder. They are designed to provide a natural voltage boost of 8.25% from the nominal voltage (0.4 kV) to accommodate voltage drops during periods of peak demand. All distribution transformers also have a delta-wye configuration.

There are 4,626 single-phase households across all LV feeders connected to the 79 distribution transformers. A 50% PV penetration is considered in this case study, i.e., 50% of all households have rooftop PV systems installed. For simplicity, the installed

capacity of each household is assumed be 5 kW. A large pool of anonymized smart meter data (at 30-min time resolution) is used as the demand profile for each household. Nonetheless, only two snapshots are considered in this analysis, as discussed in the following sub-section.

2.4.4 Investigated Cases

In addition to quantifying the accuracy and speed of the proposed formulation (as discussed in Section 2.4.2), the potential improvements from warm start values and second OPF iteration (as discussed in Section 2.3.8) are also investigated.

To this end, two different snapshots (i.e., time instances) on a weekday are considered for this case study. The first instance is *noon*, where there is sufficient sunlight such that all rooftop PV systems can generate at their maximum rated capacity. Due to the coincidence of high PV generation and low residential demand, curtailment is expected during this period. The second instance is *late afternoon* where, due to the lower irradiance, PV systems can only generate at 20% of their rated capacity. In contrast to noon, much lower curtailment (if any) is expected as a result of the simultaneous reduction in PV generation and increase in demand (e.g., from people returning home after work). For both snapshots, three potential tap positions are considered: nominal (0), one up (+1) to boost voltage and one down (-1) to lower voltage.

Based on the above, there are a total of eight cases considered in this analysis; a summary of the setup in each case is shown in Table 2-3.

Table 2-3. Summary simulation setup for C1-C8.

Case	Time	PV Availability (% of Installed Capacity)	Level of Demand	Initial Values for Estimated Terms	OPF Iterations
1	Noon	100%	Relatively Low	Flat Start	1
2				Flat Start	2
3				Warm Start	1
4				Warm Start	2
5	Late Afternoon	20%	Relatively High	Flat Start	1
6				Flat Start	2
7				Warm Start	1
8				Warm Start	2

2.4.5 Simulation Results

Distribution Network

For each case, the total demand (fixed as per demand profile) as well as the total generation from PV systems and the tap ratio of the OLTC (setpoints calculated by the OPF) are shown in Table 2-4. For simplicity, the aggregated PV generation setting is shown. Furthermore, the total potential generation (if no curtailment is applied) are 11,290 kW and 2,258 kW for noon and late afternoon, respectively.

A boxplot of the nodal voltages (at the point of connection for each household) and asset utilisation (for each line and transformer) for each case is shown in Figure 2-8. Furthermore, for each boxplot, the adopted upper limits (253 V for nodal voltages, as per Electricity Distribution Code for Victoria, Australia [65], and 100% for asset utilisations) are also highlighted by the red dotted lines. These results are retrieved from the network simulator after the corresponding setpoints have been applied.

As shown in Table 2-4, at noon (i.e., C1-C4), there is a large volume of reverse power flows as the total generation is around 4x as much as the total demand. A lower tap position is also commanded at the substation to lower the reference voltage at the head of the MV feeder, and thus increase the upper voltage headroom. Nonetheless,

Table 2-4. Summary of results for C1-C8.

Time	Case	Total Demand (kW)	Total Generation (kW)	Tap Position
Noon	1	2362	9,454	-1
	2		9,603 (+1.6%)	-1
	3		9,526	-1
	4		9,604 (+0.8%)	-1
Late Afternoon	5	3,074	2,258	Nominal
	6		2,258	Nominal
	7		2,258	Nominal
	8		2,258	Nominal

despite this effort, the upper voltage limit is still a binding constraint for this feeder, as shown by the boxplots in Figure 2-8; the maximum nodal voltages for C1-C4 are at the upper limit of 253 V while the maximum asset utilisation is hovering around 80%. Consequently, active power curtailment (around 1,700 kW from the maximum potential generation of 11,290 kW) is needed at noon to ensure the upper voltage limits are not breached.

On the other hand, for late afternoon (i.e., C5-C8), the exact opposite is observed where the demand is now higher than generation, as shown by Table 2-4. Consequently, this results in significantly lower nodal voltages and asset utilisations, as shown by Figure 2-8. Since voltage headroom is no longer a binding constraint, no tap changes nor active power curtailment is required.

Also shown in Table 2-4, for the noon snapshot, there are several noticeable differences in the total PV generation among cases 1 to 4. For instance, when only one OPF iteration is adopted, using flat start values resulted in a lower PV generation than when warm start values are used, i.e., $C1 < C3$. Furthermore, when a second iteration is run, there is an improvement for both of the previous cases, i.e., $C2 > C1$ and $C4 > C2$. Lastly, when two iterations are run, the relative contribution from the initial values (i.e., flat start or warm start) in the first iteration is largely reduced, i.e., $C2 \approx C4$.

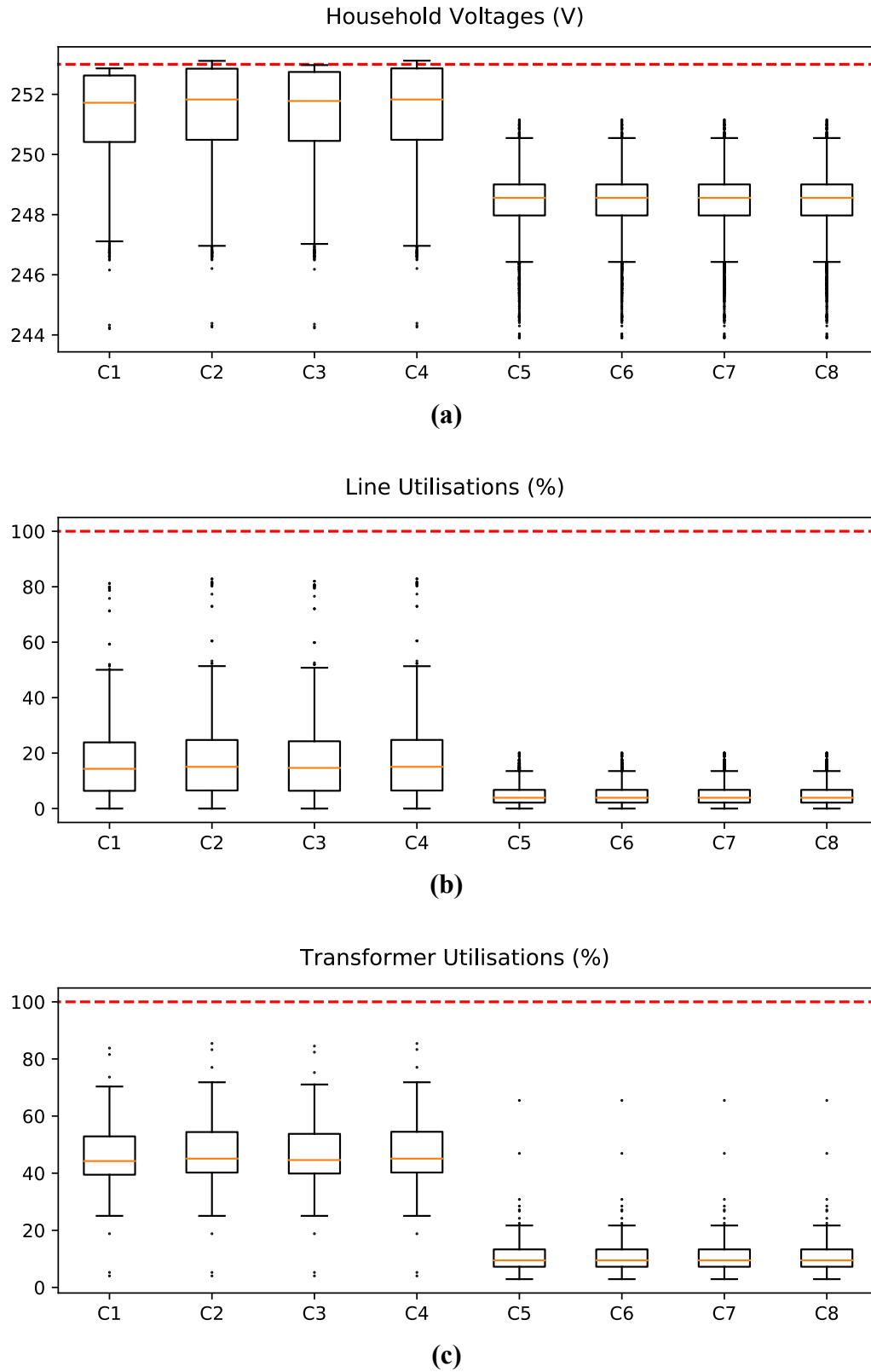


Figure 2-8. Boxplot of nodal voltages and asset utilisations for C1-C8.

The above observation is mainly due to the ‘quality’ of the estimated terms used in each OPF run. In the first iteration, there is a larger error associated with the estimated terms. Furthermore, this error is expected to be more significant from flat start values (i.e., in C1) than warm start values (i.e., in C3). Nonetheless, for both of these cases, this gap is significantly reduced by running a second iteration.

In contrast, for the late afternoon snapshot, all cases showed little variation in terms of the total PV generation. Since no curtailment is required, all cases actually resulted in exactly the same setpoints for the PV systems. This also highlights that the impact from the estimated terms is proportional to the magnitude of net power flows in the network, i.e., $|\text{total demand} - \text{total generation}|$. Particularly, during periods with relatively lower net power flows, the benefit from having a warm start and a second OPF iteration is less apparent. Nonetheless, even for the worst-case scenario (i.e., C1, where flat start values and only one iteration is adopted), the actual difference is still quite low; at a total of 1.6% for over 2,000 households that have PV systems.

Lastly, it is worth noting that, in both C2 and C4, the nodal voltages of a few households are slightly beyond the statutory limit of 253V. This is largely due to the inherent numerical inaccuracy associated with linear approximations of the non-linear AC power flow equations. Nonetheless, this magnitude of this error is extremely small. For instance, the highest voltage in C4 is only 253.1 V, only 0.1 V (or 0.04%) over the considered limit of 253 V.

Accuracy of the OPF

The relatively accuracy of the proposed formulation, as well as the benefits of using warm start values and a second OPF iteration are further illustrated in Figure 2-9.

Firstly, and as expected, it is apparent that a second iteration helps to significantly reduce the error between the expected values (from the OPF) and the actual values (from the network simulator), i.e., in terms of error, $C2 < C1$, $C4 < C3$, etc. Furthermore, when the same number of iterations is adopted, using warm start values also helps to improve the accuracy, i.e., in terms of error, $C3 < C1$, $C4 < C2$, etc.

It is also apparent that, the relative error is positively correlated with the magnitude of net power flow within the network. Since the noon snapshot (i.e., C1-C4) results in higher net power flows in the network, the error is also consistently higher than the late afternoon snapshot (i.e., C5-C8).

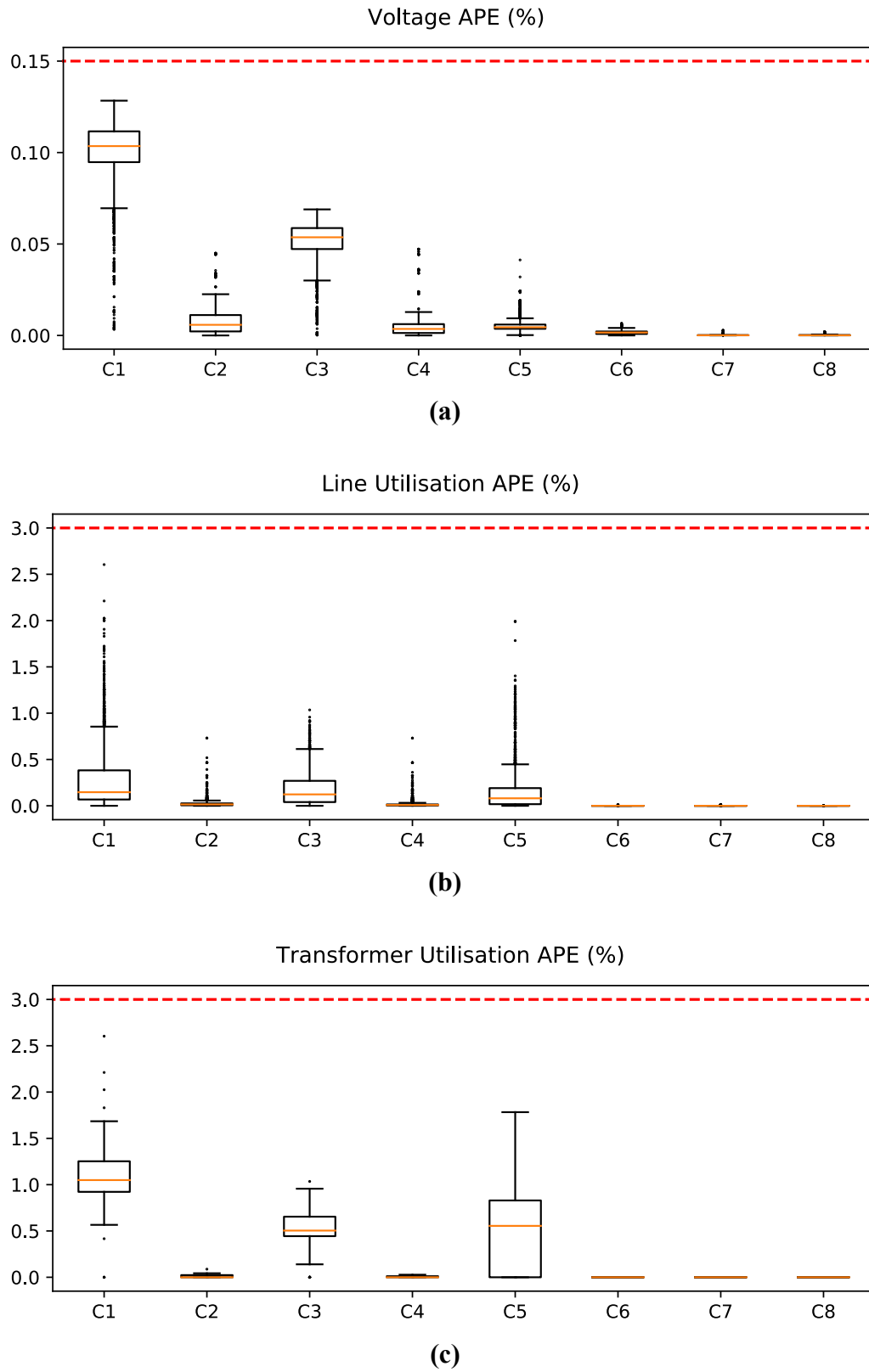


Figure 2-9. Boxplot of APE (in %) for nodal voltages and asset utilisations for C1-C8.

Finally, despite noticeable accuracy improvements from using warm start values and a second OPF iteration, the accuracy of the worst-case (i.e., C1 where only flat start values and one OPF iteration is adopted) is still fairly small: below 0.15% for nodal voltages and 3% for asset utilisation. Therefore, although two iterations are desirable from an accuracy perspective, it will incur extra computation time as the same OPF problem must be solved twice.

2.4.6 OPF Solution Time

The simulation PC used has an i7 processor with 32GB of RAM. The computation time of the first iteration is around 25 seconds. For cases 2, 4, 6 and 8, the second iteration around 15 seconds, which is consistently faster than the first iteration.

2.5 Error in Linearised OPF Formulation

For completeness, and also to complement the findings in Section 2.4.5, an additional analysis of the error associated with the linearised three-phase line model (presented in Section 2.3.2) and delta-wye transformer model (presented in Section 2.3.3) are investigated in this subsection.

2.5.1 Error in Linearised Three-Phase Line Model

Power Flow Model

As discussed in Section 2.3.2, the accuracy of the linearised power flow model for three-phase lines depend on the accuracy of estimated voltage and power terms, i.e., the terms with the $\sim$ accent in (2.28)-(2.30). Therefore, the following analysis aims to explore the impact of different inaccuracies in these estimated terms on the overall power flow model. Furthermore, the potential improvement from a second OPF iteration (as discussed in Section 2.3.8) is also investigated.

The test network (an LV feeder) for this analysis is shown in Figure 2-10. It is modelled based the MV-LV network presented in Section 2.4.3. The head of this LV feeder is assumed to be the slack bus and its voltage is set to 433 V line-to-line (i.e., 250 V line-to-neutral). The total impedance is shown in Figure 2-10, which is based on an 800m long LV feeder (a typical length for the investigated network). Furthermore, to exaggerate the voltage drop in this LV feeder, all single-phase loads are lumped at the end of the feeder, also shown in Figure 2-10. It is worth noting that,

normally, these loads would be spread out along the feeder, and thus reduces the maximum voltage drop along the feeder.

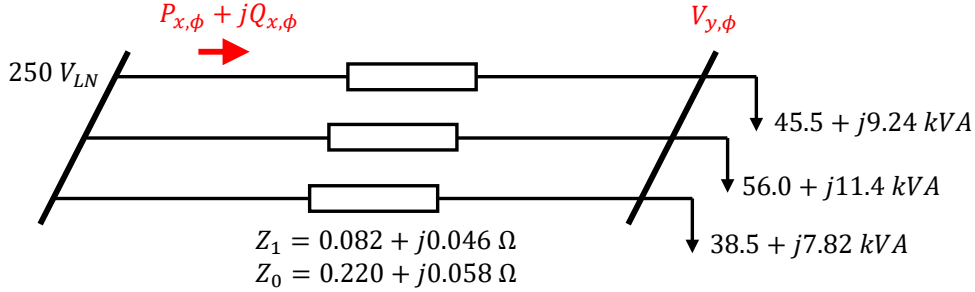


Figure 2-10. Test LV feeder.

The LV feeder in Figure 2-10 represents a typical situation where a power flow model (e.g., the linearised model proposed in Section 2.3.2) can be used to calculate the unknown variables shown in red: the voltage magnitude (line-to-neutral) at the end of the feeder $V_{y,\phi}$ and the power flow into the start of the feeder $P_{x,\phi} + jQ_{x,\phi}$. Furthermore, the true values (i.e., reference values) for each phase are shown in Table 2-5, which are obtained by running a power flow in the network simulator OpenDSS.

Table 2-5. Reference values for the unknown variables.

	Phase A	Phase B	Phase C
$V_{y,\phi}$ (V)	233.3	223.7	237.5
$P_{x,\phi}$ (kW)	48.7	62.1	40.3
$Q_{x,\phi}$ (kvar)	10.4	14.8	9.4

The impact of an inaccuracy¹ in the estimated power terms (i.e., $\tilde{P}_{x,\phi}$ and $\tilde{Q}_{x,\phi}$) on the linearised power flow model is shown in Figure 2-11. Here, the true values in Table 2-5 are first used as a ‘starting point’ for $\tilde{P}_{x,\phi}$ and $\tilde{Q}_{x,\phi}$ when solving the linearised

¹ For better clarify, the term ‘inaccuracy’ will be used when referring to the values used for the estimated power terms. These are the inputs to the linearised power flow model. On the other hand, the term ‘error’ will be used when referring to the results from the linearised power flow model. These are the outputs when using the linearised power flow model to calculate the unknowns in Figure 2-10.

power flow model. It is worth noting that this ‘starting point’ also corresponds to the case when there are no inaccuracies associated with the estimated power terms. Then, different inaccuracies are introduced to both $\tilde{P}_{x,\phi}$ and $\tilde{Q}_{x,\phi}$ to produce the resulted shown in Figure 2-11. It is also worth noting that, for simplicity, the inaccuracies in $\tilde{P}_{x,\phi}$ and $\tilde{Q}_{x,\phi}$ are increased by the same value. Furthermore, no inaccuracy is assumed for the estimated voltage term $\tilde{V}_{x,\phi}$ in this scenario (i.e., the true value as shown in Figure 2-10 are used).

From Figure 2-11, three observations are immediately apparent. Firstly, the results from the linearised power flow model matches exactly with the true values (obtained using the network simulator) if there are no inaccuracies associated with the estimated terms. This situation is represented by the ‘0’ point on the x-axis in Figure 2-11. Secondly, there is a positive correlation between any inaccuracies in the estimated power terms and the resulting error of the linearised power flow model. Finally, it is also evident that a second OPF run (represented by the dotted line) can drastically reduce the error associated the linearised power flow model. As discussed previously in Section 2.3.8, this can be particularly useful when there is no adequate data (e.g., recent SCADA measurements) available and flat start values have to be used as these estimated terms.

A similar analysis is also carried out to investigate the impact of any inaccuracy in the estimated voltage terms (i.e., $\tilde{V}_{x,\phi}$) on the linearised power flow model. The results are shown in shown in Figure 2-12. In contrast to the analysis above, the true values of estimated powers (as per Table 2-5) are used and inaccuracies are gradually introduced to the estimated voltage $\tilde{V}_{x,\phi}$. Since the observations are very similar to the previous scenario, further discussions are omitted.

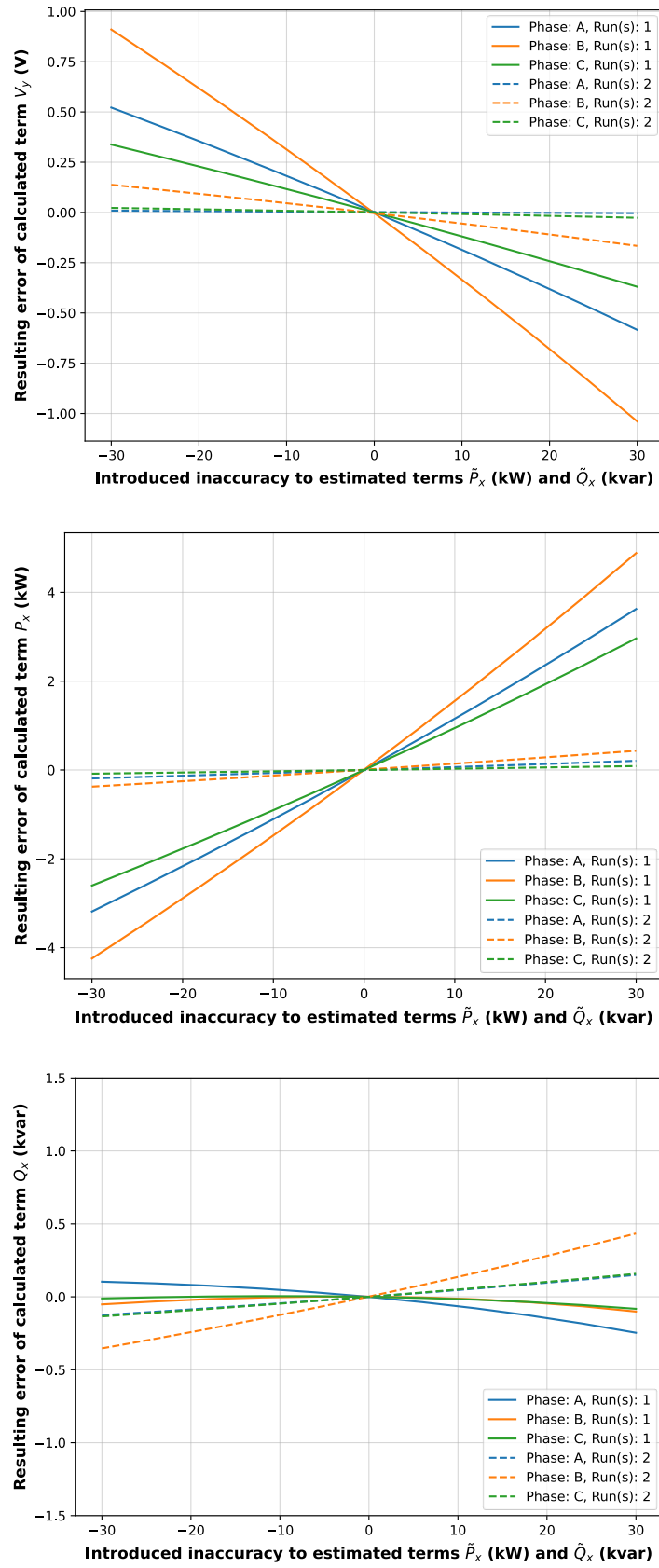


Figure 2-11. Impact of inaccuracies in estimated power terms on the linearised line model.

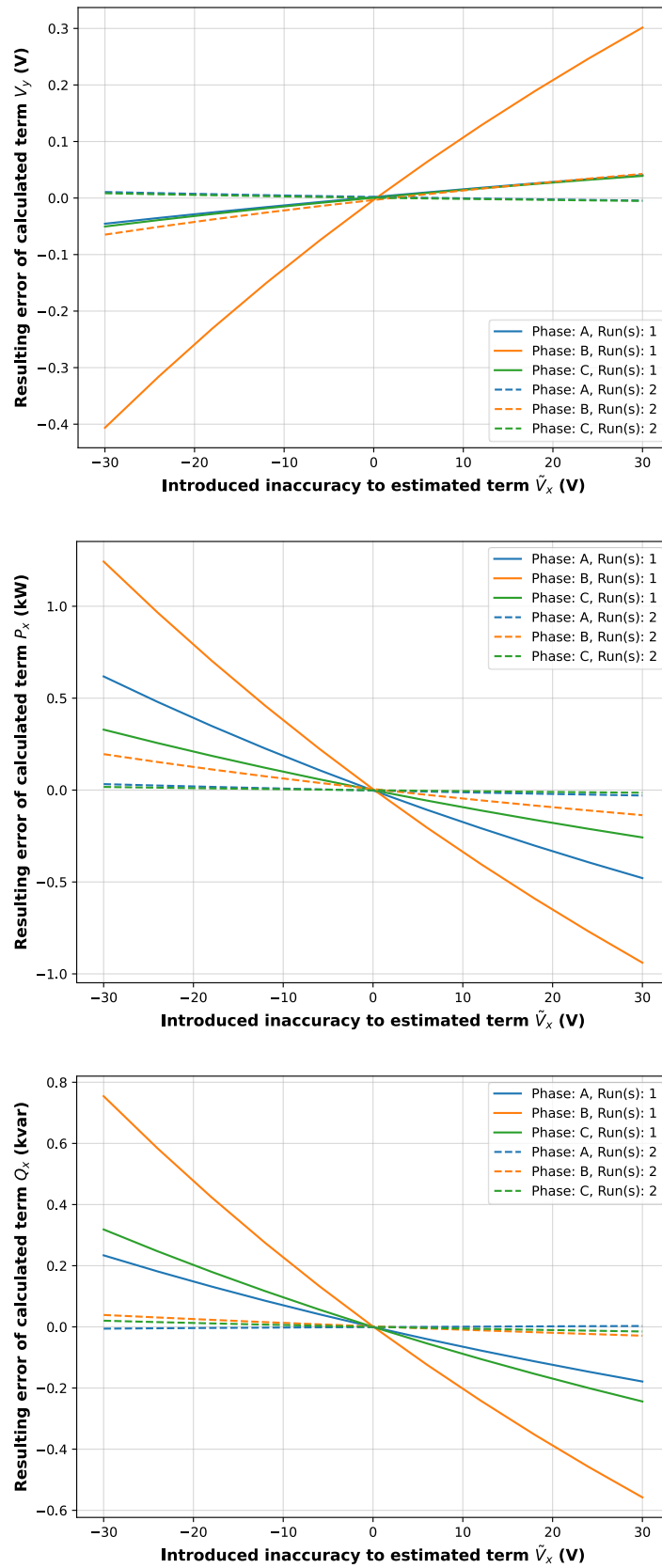


Figure 2-12. Impact of inaccuracies in estimated voltage terms on the linearised line model

Piecewise Linear Approximation of Thermal Constraints

As discussed in Section 2.3.2, the thermal constraint is modelled using a piecewise linearisation technique defined by (2.40)-(2.49). Since the linearised model is equivalent to an upper bound for the actual line utilisation level, a potential side-effect is that the maximum allowed line utilisation may never reach the rated capacity. The impact of this side-effect can be reduced by using more linear segments, which are defined by the parameter Λ . At the same time, since more linear segments will result in more constraints in the OPF problem, there is an inherent trade-off between accuracy and model size.

The differences in performance for various values of Λ are illustrated in Figure 2-13. This analysis is also carried out for different power factors to study the effect of different ratios between the active power and reactive power¹. As shown in Figure 2-13, the linearised model may significantly over estimate the actual line utilisation for low values of Λ (e.g., 1), and thus not allowing the utilisation level to reach the full capacity. Furthermore, this phenomenon is most severity when the ratio of active power and reactive power approaches one. On the other hand, for Λ values beyond 6, the marginal improvement (of using more linear segments) decreases significantly.

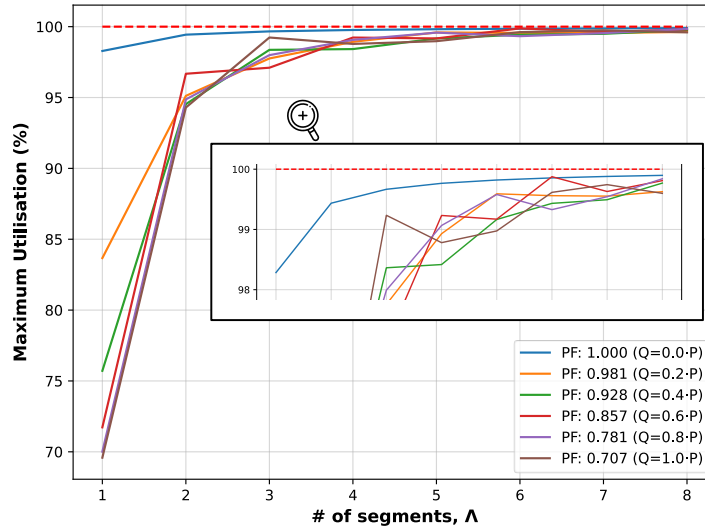


Figure 2-13. Approximation error of the linearised thermal model.

¹ As discussed in Section 2.3.2, the linearised upper bound consist of planes on a 3D plot that covers the paraboloid created by the active power and reactive power. Therefore, for a given apparent power, the distance from this plane to the paraboloid can vary depending on the ratio between the active power and reactive power.

2.5.2 Error in Linearised Delta-Wye Transformer Model

The error associated with the lossless delta-wye transformer model (or transformer model for short) is analysed here. Since the losses part of a transformer is modelled using the a linearised line model, the associated error follows the results above (i.e., in Section 2.5.1).

A key assumption for the linearised transformer model is that the aggregated power flow is relatively balanced on the primary side. In fact, if the entire network is perfectly balanced, there will be no error associated with the linearised transformer model. Therefore, the following analysis aims to investigate the associated error due to unbalances in the network.

The test feeder for this analysis is shown in Figure 2-14. It is loosely modelled after one of the farthest transformer from the primary substation in the network presented in Section 2.4.3. The length of the MV feeder is doubled compared with the one in the network to exaggerate the magnitude of voltage differences among the phases in unbalanced situations. The corresponding impedances are shown in Figure 2-14. In terms of the investigated transformer, the nominal voltages are 22 kV and 0.433 kV for the primary and secondary terminal, respectively. The power rating of the transformer is 500 kVA.

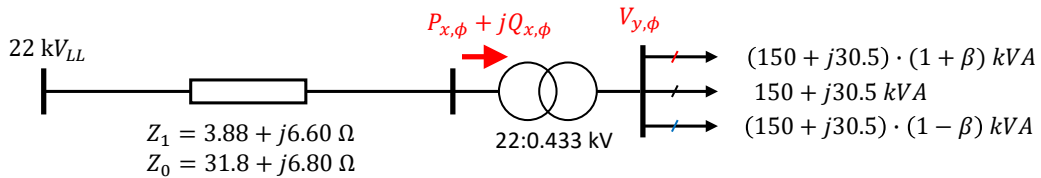


Figure 2-14. Test LV feeder with a delta-wye transformer.

The error analysis is carried out considering a total loading on the transformer of approximately 92% (i.e., around 460 kVA), as shown in Figure 2-14. Furthermore, the error due to load unbalances on the linearised model is assessed by varying the unbalance factor β (also shown in Figure 2-14). For instance, when $\beta = 0.5$, the power on Phase A will be 50% higher than the average loading across all three phases.

The impact of load unbalances on the linearised model is shown in Figure 2-15. The error is obtained by comparing the differences in results from the network

simulator OpenDSS and the linearised transformer model. As shown in Figure 2-15, it is evident that the error increases as the load unbalances increases. Nonetheless, even at an unbalanced factor of 0.6, the error introduced is negligible compared with the actual power flows in the network. For instance, the maximum error in $P_{x,\phi}$ is only around 2.5 kW, which is just over 1.5% compared with the average power flow of 150 kW.

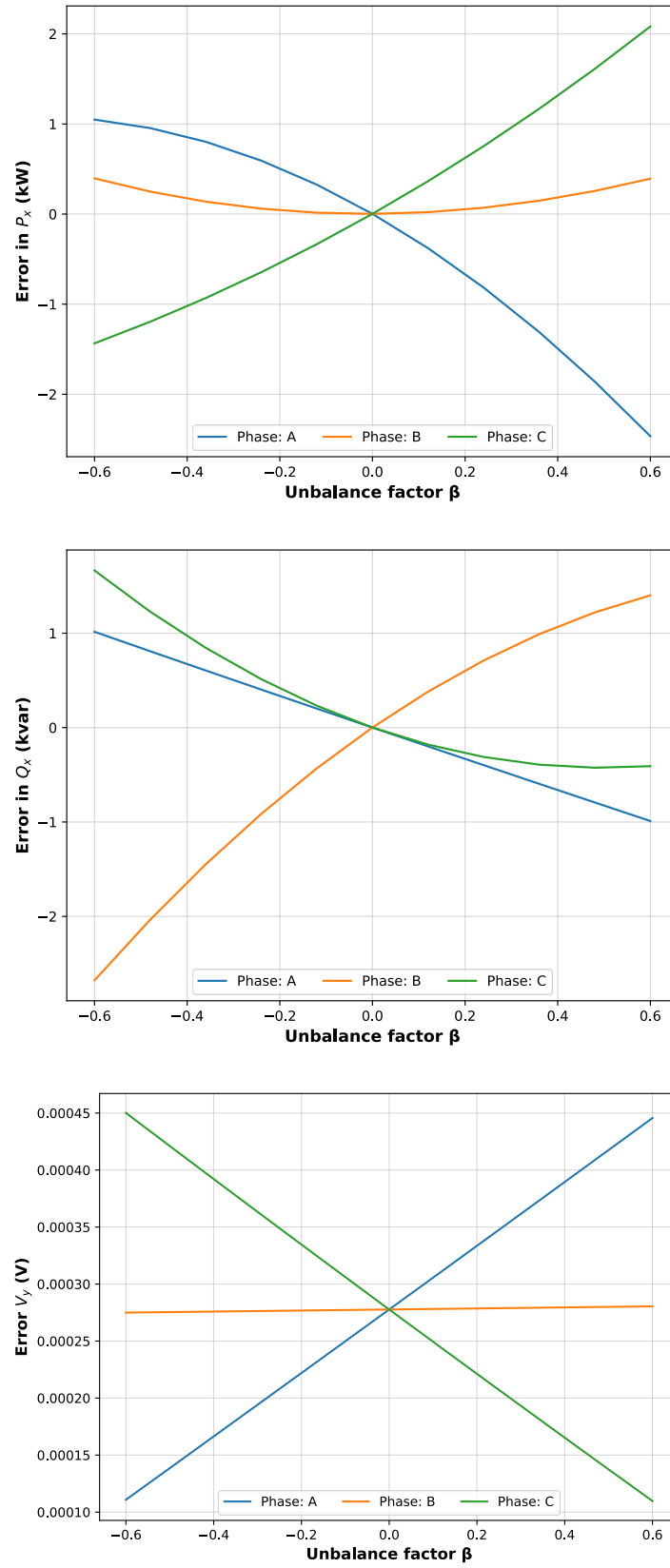


Figure 2-15. Impact of load unbalances on the linearised delta-wye transformer model.

2.6 Summary

Driven by the rapid growth of DG in recent decades, there has been significant interests in the real-world application of OPF-based control schemes in distribution networks. However, due to the non-linear and non-convex nature of classical AC OPF formulations, there are significant scalability (tractability) and speed challenges of using OPF in an operational environment.

To this end, a linearised, three-phase AC OPF is proposed in this chapter. The linearised formulation significantly improves its scalability when dealing with large-scale distribution networks. Furthermore, thanks to the maturity of mixed-integer linear programming techniques, it offers an effective way of handling discrete devices in distribution networks, such as OLTCs. Lastly, a linearised model for delta-wye transformers is proposed, allowing the realistic modelling of distribution networks across multiple voltage levels (where the interface is commonly a delta-wye transformer), such as integrated MV-LV feeders.

The accuracy and speed of the proposed OPF formulation is validated using a realistic residential MV-LV feeder in Australia that consists of over 4,600 households. Overall, the proposed linearised, three-phase AC OPF formulation is found to be sufficiently accurate for determining the optimal setpoints for controllable devices such as OLTCs and PV systems. Particularly, it achieved an error of less than 1% for both voltage magnitudes and power flows when validated in a network simulator. Furthermore, the computation time is found to be within a minute, making its operational usage technically feasible for similar sized networks.

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3 IMPLEMENTING OPF-BASED SETPOINTS: CHALLENGES AND SOLUTIONS

3.1 Introduction

The rapid uptake of DG is driving a paradigm shift in future distribution networks where network assets and participants are managed in real-time so as to facilitate the bidirectional power flows and ensure network integrity. In this context, and as demonstrated in Chapter 2, OPF allows network operators to determine the most appropriate setpoints according to their objective (e.g., maximisation of renewable energy harvesting) while catering for the distribution network constraints (e.g., steady-state voltage and thermal limits).

At the same time, apart from being able to calculate these setpoints, the physical implementation of these setpoints is another crucial aspect that requires careful considerations in a real-world application of OPF-based schemes. For instance, since the execution of these setpoints do not happen instantaneously, sub-minute scale interactions among multiple devices with different delays and ramp rates may lead to undesirable outcomes that are typically not (or difficult to be) captured by the OPF problem. Therefore, when multiple devices are being controlled at the same time, potential differences in their operating characteristics (e.g., delays, ramp rates, deadbands, etc.) must be catered for so as to ensure OPF-based setpoints can be adequately implemented in reality.

In this context, the necessary adaptations within existing device controllers as well as the OPF formulation will be investigated in this chapter, focusing on the adequate management of MW-scale DG in distribution networks.

3.2 Literature Review

Recognising that OPF can play a crucial role in the day-to-day operation of future distribution network, there has been significant efforts in the existing literature that investigates how OPF can be used to orchestrate the various sources of flexibility from network-connected assets and participants (collectively referred to as controllable devices), and thus facilitate high penetrations of renewable generation. For instance,

an OPF-based congestion management scheme is proposed in [66] to minimise the active power curtailment (of DG units) while ensuring thermal limits are not breached. The extension to cater for voltage issues are proposed in [8, 67, 68] through the coordinated control of both the reactive power (of DG units) and existing OLTCs. Other avenues such as battery storage systems (e.g., [69, 70]), flexible demand (e.g., [71]) and network reconfiguration (e.g., [72]) are also proposed to further maximise the harvesting of renewable energy.

The integration of OPF with other techniques has also been a key focus so as to cater for the inherent variability of demand and renewable generation. For instance, chance-constrained OPF (e.g., [35]) is proposed to cater for the uncertainty in wind power. Particularly, it ensures that the probability of satisfying the OPF constraints is above a certain threshold for multiple wind generation scenarios. In contrast to this probabilistic approach, the application of robust optimisation (e.g., [73]) is proposed where the corresponding constraints for multiple scenarios are satisfied simultaneously, and thus being robust to these scenarios. Lastly, similar to the non-probabilistic approach of robust optimisation, the application of information gap decision theory (e.g., [74]) is also studied to cater for severe uncertainty in demand.

Among these and similar studies, time-series simulations are predominantly used in order to capture the continuous changes in demand and generation. However, this is usually done using relatively large time-steps that range from multiple minutes to hours. Although this is sufficient from an energy perspective (to demonstrate the underlying concept), it is not adequate to assess the performance of OPF-based schemes in an operational environment. Particularly, large time-steps cannot capture the real-time behaviours of controllable devices and the interactions among them, which typically happen in the sub-minute scale [75, 76]. Therefore, in addition to capturing the power flows, the real-time and inter-temporal characteristics of devices (e.g., delays, ramp rates and deadbands) as well as their effects from a control perspective (e.g., volume of control actions and impacts on the network) must also be accounted for.

Understanding this, the authors in [36] adopted a 5s time-step to realistically capture the sequences of control events in an operational environment, from the calculation of setpoints by using an OPF to the final realisation of these setpoints by the controllable devices in a network simulator. Then, using this setup, the

performance of an OPF-based scheme that produces new setpoints for DG units and OLTCs every five minutes is evaluated. It has been shown that, although the OPF solution may satisfy the considered constraints, voltage violations may still occur in real-time due to the deadbands (an operating characteristic that is typically not modelled in the OPF) of conventional OLTC controllers. Furthermore, it has been highlighted that, given the available headroom in a distribution network, there are often multiple feasible control actions. Consequently, an inadequately designed OPF may produce discontinuous setpoints from consecutive OPF runs, and thus results in unnecessary control actions and accelerated wear-and-tear. The proposed solution to mitigate these issues is by operating away from the statutory limit. While this approach helps to reduce the impact associated with the deadbands, it does not fully address the drawbacks associated with conventional OLTC controllers. Furthermore, potential coordination issues when OLTCs and DG units are being controlled at the same time were not considered.

This coordination between OLTCs and DG units have been discussed in [77], where an OPF-based voltage management scheme is proposed. Particularly, a sequential process is proposed for coordination purposes: reactive power setpoints for DG units are executed first, then followed by the setpoints for OLTCs. However, scenarios where the setpoints of these devices are executed at the same time were not considered. Consequently, potential issues due to the simultaneous actions of OLTCs and DG units still need to be investigated and tackled.

To this end, the main challenges associated with ensuring OPF-based setpoints can be adequately implemented in a real-world application are outlined below and further discussed/addressed in the remainder of this chapter:

- Temporal coordination issues among multiple devices with different delays and ramp rates
- Deadbands in device controllers (e.g., OLTCs) preventing the precise enactment of the requested setpoints
- Multiple sets of feasible control actions resulting from the flexibility of several controllable devices

3.3 Necessary Adaptations

3.3.1 Decision-Making Process with Control Cycles

An OPF-based voltage and congestion management scheme is proposed in this chapter with the aim of maximising renewable energy harvesting from DG units. It adopts minute-scale control cycles to routinely calculate and dispatch setpoints for the controllable devices (i.e., DG units and OLTCs) in an active distribution network, as shown in Figure 3-1. Due to limited variations of minute-to-minute outputs from typical renewable DG units, such as wind farms [78], the use of short control cycles lasting a few minutes has been proven to be an effective way to mitigate constraint violations [35, 36]; significantly reducing the uncertainties faced by longer control cycles or other schemes that make decision in a longer time frame.

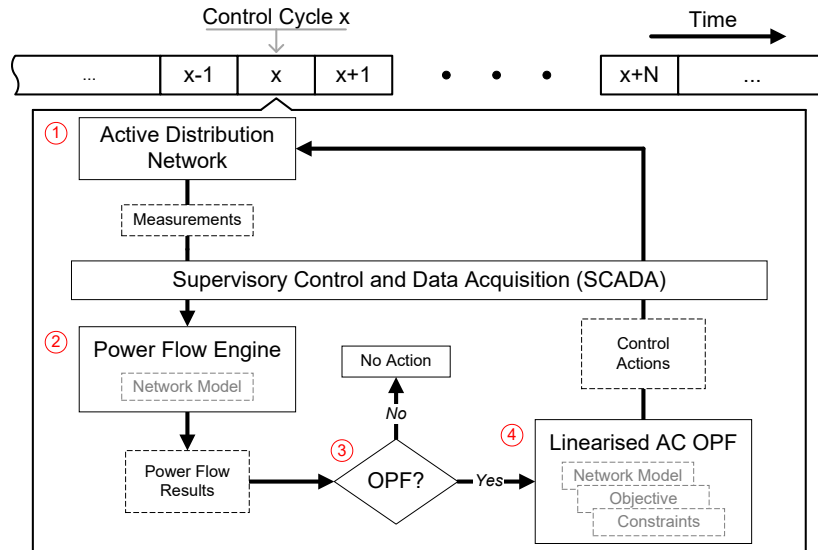


Figure 3-1. Decision-making process using control cycles.

As depicted in Figure 3-1, within each control cycle, the required network measurements to run an OPF (e.g., expected demand and generation as well as current setpoints of controllable devices) are first collected from the network. Then, this data is used in combination with the network model to run a normal three-phase power flow. This allows the network operator to check for potential issues (e.g., a voltage violation), and thus determine whether an OPF is required to dispatch new setpoints. If so, a three-phase OPF is solved to obtain the optimal setpoints according to the considered objective, in this case, the maximisation of renewable energy harvesting. Furthermore,

OPF is also triggered if active power curtailment has been applied to DG units to re-evaluate the curtailment settings. Once new setpoints are obtained for a given control cycle, they are dispatched to all controllable devices (at the same time) and maintained throughout the corresponding control cycle.

After receiving these setpoints, the actual implementation is handled individually by each of the controllable devices, subject to their own delays, ramp rate restrictions, etc. Consequently, several adaptations to the device controllers and the OPF formulation are required to ensure adequate coordination is achieved among multiple controllable devices with different operating characteristics. Nonetheless, it is first important to understand these underlying differences in terms of their delays, ramp rates and deadbands, as discussed in the following sub-section.

Lastly, it is worth mentioning that using control cycles can be referred to as a preventative approach where the optimal setpoints are routinely evaluated. An alternative (to control cycles) is through problem persistency (e.g., as investigated in in [56]) where control actions are needed only if a problem *persists* longer than a pre-defined threshold (e.g., a few minutes). In contrast to control cycles, problem persistency can be seen as a reactive approach. Nonetheless, regardless which control architecture (to trigger the calculation of new setpoints) is adopted, the same considerations to ensure setpoints are adequately executed still applies.

3.3.2 Understanding Operating Characteristics

OLTC-Fitted Transformer

A typical OLTC-fitted transformer and the commonly used tap controller, also known as an automatic voltage regulator (AVR), are shown in Figure 3-2. The OLTC is a mechanical device that allows minor adjustments to the winding ratio of a transformer, and thus achieving voltage regulation. Physically, these predefined ratios can be selected by the AVR through one of the taps, as shown in Figure 3-2.

Conventionally, OLTCs are designed as autonomous devices where the movement of taps is government by the AVR. A simplified case of regulating the voltage at the secondary terminal of the transformer is depicted here; nonetheless, it can also be used to regulate voltages at a remote node using line drop compensation (i.e., by accounting for the voltage drops based on power flows). Furthermore, the pre-set reference voltage (i.e., V_{ref} as shown in Figure 3-2) is typically fixed throughout the day/season.

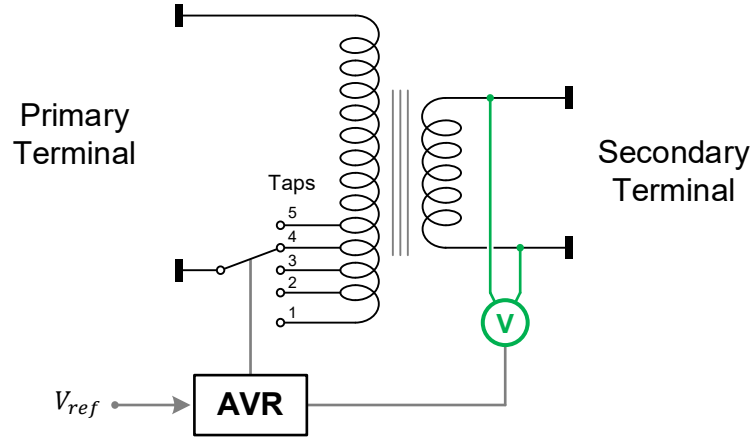


Figure 3-2. OLTC-fitted transformer with AVR.

Since each tap change results in some degradation of the electrical contacts (as it needs to interrupt power flows at very high voltages), certain design considerations have been incorporated into the tap changing algorithms of existing AVR to prevent unnecessary tap changes. As a result, these designs have largely contributed to the specific operating characteristics of OLTCs, as illustrated in Figure 3-3.

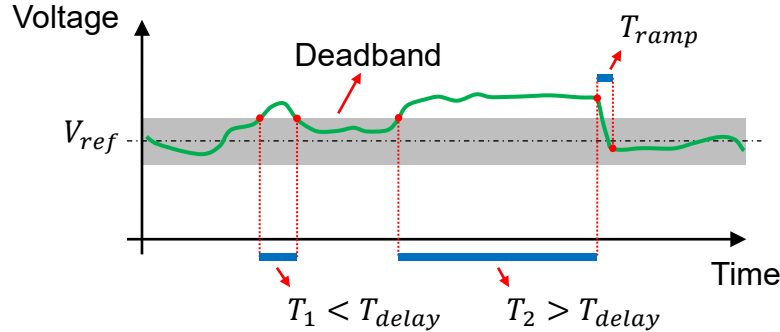


Figure 3-3. Principles of AVR operation.

Firstly, due to the discrete nature of an OLTC's taps, a deadband (shaded area in Figure 3-3) is needed around the reference voltage V_{ref} . Consequently, any voltage variations within this deadband will not trigger any tap changes. In typical distribution networks, the range of this deadband is around 1~2% of the nominal voltage. To prevent unnecessary tap changes due to temporary voltage fluctuations, a tap change is only initiated if a voltage excursion beyond the deadband persists for a certain duration (i.e., T_{delay}); this delay is typically around 1-2 minutes for OLTCs in distribution networks. For instance, in Figure 3-3, no tap changes are triggered when

the regulated voltage (green line) exceeded the deadband for the first time as T_1 is less than T_{delay} . In contrast to this relatively long delay, once a tap change is deemed necessary, the physical movement from one tap position to another is significantly faster, only a few seconds per tap. Consequently, the ramp rate of OLTCs, albeit discrete, can be considered very high.

Inverter-Interfaced DG Units

The operating characteristics of modern inverter-interfaced DG units often differ substantially from OLTCs. Firstly, being continuous devices, similar delays and deadbands (like those found in OLTCs) are typically not found in DG units. Furthermore, while DG units are technically capable of ramping up/down very rapidly, the actual ramp rate is often restricted through a connection agreement with the network operator.

This restriction on ramp rate is to prevent extreme fluctuations in their output, and thus avoid negative impacts on the network (e.g., a sudden voltage spike). For instance, a ramp down from 0% output to 100% output can take from mere seconds (for PV systems) to a few minutes (for modern wind systems). One such example is in Germany—one of the global leaders in wind generation—where the restrictions on ramp rates is 10% (of the installed capacity) per minute. Though less common, similar restrictions on ramp rates can also be applied to the reactive power/power factor.

3.3.3 Challenges of Implementing OPF-Based Setpoints

Based on the above, it can be concluded that, conventionally, OLTCs exhibit long delays and large deadbands while having a relatively fast ramp rate. In contrast, DG units often have negligible delay and deadbands while the ramp rate is restricted due to local connection agreements (despite its technical capabilities). Consequently, when both OLTCs and DG units are being controlled at the same time, practical issues (discussed in detail below) are expected due to this mismatch in their operating characteristics.

Temporal Coordination

The first issue is related to temporal aspects due to their delays and ramp rates. To illustrate this, timing diagrams (as shown in Figure 3-4) are used to highlight the

various events during a control cycle for two key scenarios: uncoordinated and coordinated.

In Figure 3-4, the start and end of a control cycle are denoted by 0 and T_{CC} , respectively. The time required for network operators to calculate and dispatch new setpoints (i.e., collect network measurements, check constraint violations, solve the OPF and dispatch to controllable devices, as depicted in Figure 3-1) is collectively represented by ΔT_{OPF} . The remaining Δ terms are used to represent the *execution times*, i.e., the time required for a setpoint to be fully reached after accounting for the associated delays and ramp rates. Here, the execution times of a DG unit's active power setpoint (ΔT_{DG}^P) and power factor setpoint (ΔT_{DG}^{PF} , as a proxy of the reactive power) as well as the OLTC's tap setpoint (ΔT_{OLTC}) are shown. Within each of the execution times, the components associated with the delay (shaded region) and ramp rate (solid region) are also highlighted.

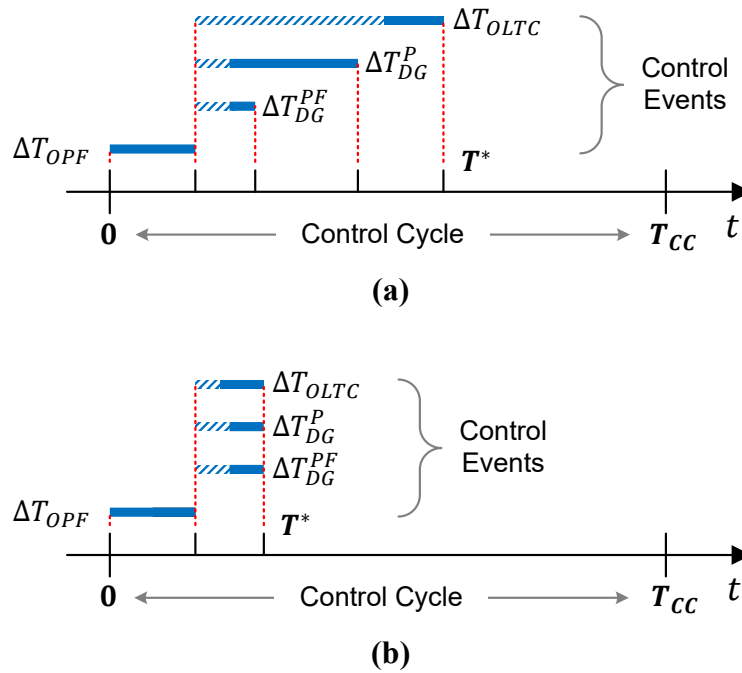


Figure 3-4. Timing diagrams of control events: a) uncoordinated scenario, b) coordinated scenario.

As illustrated by Figure 3-4-a, the uncoordinated scenario occurs when there is a significant misalignment of their execution times. Despite all setpoints are being dispatched at the same time (i.e., after ΔT_{OPF}), the full execution of each of these can

happen at different times as shown. Since the solution determined by the OPF is only valid after T^* , unexpected behaviours in the network can occur during the transition period leading up to T^* . On the other hand, the more desirable coordinated scenario is shown in Figure 3-4-b. Here, the delays and ramp rates are aligned to ensure the adverse impacts during the transition period is minimised.

Overrun

In addition to the alignment of execution times, a short execution time is also desirable. For instance, if the delay is comparable (or even longer) than the duration of the control cycle, the requested setpoint may never happen. At the same time, this also places a technical limit on how fast a control cycle can be due to the various delays associated with physical systems.

Deadbands

The final issue is due to the presence of deadbands in certain devices; in this case, ones adopted by conventional AVR.

As highlighted by [36] and in Figure 3-3, the precise voltage target calculated by an OPF cannot be strictly followed by OLTCs. As a result, the actual voltage observed in the network may drift within the deadband. While a small drift in one OLTC is unlikely to cause major issues, the compounding effect due to multiple installed OLTCs across multiple voltage levels in the same feeder can exacerbate the problem. For instance, the actual voltage measured at downstream sections of the network may be significantly different from the OPF solution, and thus creating unintended behaviours.

Furthermore, the constantly varying nature of renewable resources may cause large fluctuations in voltages that exceeds this deadband, and thus result in excessive tap changes.

3.3.4 Adaptations in Device Controllers

The following modifications to the local controllers of OLTCs and DG units are proposed. Specifically, the aim of these modifications is to align and reduce their execution times (as shown in Figure 3-4-b) as well as to eliminate the undesirable deadbands of OLTCs.

OLTCs

The existing design of AVR is largely contributed by the autonomous design for passive operations. As a result, the associated delays and deadbands are, fundamentally, less compatible with OPF-based schemes where network devices are actively managed. Therefore, this work proposes the direct control of each OLTC's tap position, bypassing the mechanism that takes a voltage target as an input.

Through this approach, the undesirable initial delay is completely eliminated, allowing the OLTC to respond swiftly after receiving the required tap position. Furthermore, requesting a precise tap position helps to eliminate the inherent voltage drifts associated with a deadband.

At the same time, this means the precise tap position must also be considered as a variable in the OPF formulation; this can be catered for using integer variables as discussed in Chapter 2.

DG Units

For DG units, the main issue is associated with slow ramp rates as a result of local restrictions. Nonetheless, since the ramp rate is often specified in terms of the maximum change per minute, e.g., 1 MW per minute, how this is achieved in the sub-minute scale can be exploited to ensure better coordination with OLTCs.

Two different ramping behaviours are illustrated in Figure 3-5. The most commonly adopted behaviour in literature is a linear ramp (solid line in Figure 3-5) between the original setpoint P^0 and the new setpoint P^* . However, a potential drawback of this approach is the relatively long execution time (as indicated by $\Delta P_{DG}^{P,2}$). Instead, the propose ramping behaviour has a faster initial ramp (dashed line in Figure 3-5) to achieve the desired execution time (as indicated by $\Delta P_{DG}^{P,1}$).

At the same time, this requires the OPF to consider both the connection agreement and the full ramping capability of the DG unit so as to ensure the calculated setpoint P^* is technically feasible.

Lastly, no ramp rate restrictions for the reactive power of DG units is assumed. Nonetheless, the aforementioned approach can also be applied if needed.

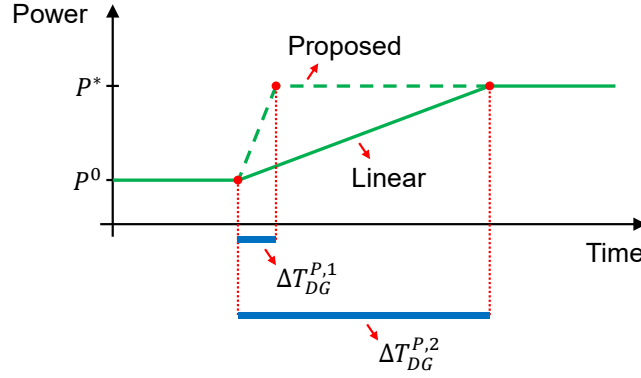


Figure 3-5. Different ramping behaviours.

3.3.5 Adaptations in OPF Formulations

Additional modelling in the OPF is also required to ensure the calculated setpoints are technically feasible when being implemented. Particularly, this requires the inclusion of operational constraints for the OLTC and DG units. The corresponding modifications to the OPF formulation presented in Chapter 2 are outlined below.

Modelling of OLTCs

In addition to the modelling of physical tap positions, a further constraint on the maximum allowed number of tap changes per control cycle (K) is introduced, as shown in (3.1). Firstly, this restriction guarantees the maximum execution time to achieve better coordination with DG units. Secondly, this also prevents a large step change in the voltages due to many tap changes being requested at once.

$$|\pi_t^* - \pi_t^0| \leq K, \quad \forall t \in T \quad (3.1)$$

Modelling of DG Units

For a DG unit w (in the set W), its active and reactive power are represented by P_w and Q_w . Its location (i.e., which node it is connected to) in the network is denoted by $\beta_w \in N$. For simplicity, its output is assumed to be balanced across all three phases; only three-phase connected DG units are considered.

The active power is limited by the installed capacity P_w^+ and the estimated resource availability P_w^{avail} , as shown in (3.2). Furthermore, the restriction on the maximum allowed change in its active power setpoint (ΔP_w) for each control cycle is given by

(3.3) where $\mathbf{P}_w^0$ is the setpoint in the previous control cycle. The correspond value for $\Delta \mathbf{P}_w$ must satisfying the ramping requirement as discussed in Section 3.3.4.

$$P_w \leq \mathbf{P}_w^{avail} \leq \mathbf{P}_w^+, \quad \forall w \in W \quad (3.2)$$

$$|P_w - \mathbf{P}_w^0| \leq \Delta \mathbf{P}_w, \quad \forall w \in W \quad (3.3)$$

The control of reactive power is assumed to be based on the power factor. While the power factor (i.e., PF_w) is not explicitly modelled as a variable, it can be extracted from P_w and Q_w using (3.4). Furthermore, the physical capability is enforced through (3.5) where $\mathbf{PF}_w^+$ is the maximum feasible power factor.

$$PF_w = \tan^{-1} \left(\frac{Q_w}{P_w} \right), \quad \forall w \in W \quad (3.4)$$

$$|Q_w| \leq P_w \tan(\cos^{-1} \mathbf{PF}_w^+), \quad \forall w \in W \quad (3.5)$$

Nodal Power Balance

Due to the addition of DG units, the nodal power balance in (2.70) and (2.71) are updated accordingly, as shown in (3.8) and (3.9), respectively.

$$\begin{aligned} \sum_{l \in L | l_x = n} P_{l_x, \phi} + \sum_{l \in L | l_y = n} P_{l_y, \phi} + \sum_{t \in T | t_x = n} P_{t_x, \phi} + \sum_{t \in T | t_y = n} P_{t_y, \phi} \\ + \sum_{w \in W | \beta_w = n} \frac{1}{3} P_w + \sum_{h \in H | \beta_h = n, \varphi_h = \phi} P_h^{ND} + \sum_{s \in S | s = n} P_{s, \phi}^S = 0, \end{aligned} \quad (3.6)$$

$\forall n \in N, \forall \phi \in \Phi$

$$\begin{aligned} \sum_{l \in L | l_x = n} Q_{l_x, \phi} + \sum_{l \in L | l_y = n} Q_{l_y, \phi} + \sum_{t \in T | t_x = n} Q_{t_x, \phi} + \sum_{t \in T | t_y = n} Q_{t_y, \phi} \\ + \sum_{w \in W | \beta_w = n} \frac{1}{3} Q_w + \sum_{h \in H | \beta_h = n, \varphi_h = \phi} Q_h^{ND} + \sum_{s \in S | s = n} Q_{s, \phi}^S = 0 \end{aligned} \quad (3.7)$$

$\forall n \in N, \forall \phi \in \Phi$

Optimisation Objective

Due to the various sources of flexibility as well as the available headroom in distribution networks, there are often multiple control actions that can achieve the

same amount of total generation. Therefore, in addition to the maximisation of renewable energy harvesting, a secondary objective of minimising the control effort (i.e, the volume of control actions) is also considered. The control effort for OLTCs and DG units are given by u_t in (3.8) and u_g in (3.9), respectively.

$$u_t = |\pi_t^* - \pi_t^0|, \quad \forall t \in T \quad (3.8)$$

$$u_w = |Q_w|, \quad \forall w \in W \quad (3.9)$$

The combined objective is given by (3.10). The scaling factors c_1 , c_2 and c_3 are selected such that the secondary objective (i.e., $c_2 \sum u_t + c_3 \sum u_w$) will have minimal impact on the primary objective (i.e., $c_1 \sum P_w$) while still within the tolerances of the solver. This can be done by first normalising each objective (based on their upper bound) and then scaling them to different orders of magnitudes (based on their priorities).

$$\text{maximise} \left(c_1 \sum_{w \in W} P_w - c_2 \sum_{t \in T} u_t - c_3 \sum_{w \in W} u_g \right) \quad (3.10)$$

3.4 Case Study

The proposed adaptations are assessed in a generic UK distribution network with high penetrations of wind generation. The analysis is carried out for a period of 24-hours using the same co-simulation platform as Chapter 2. A 1 second time-step is also adopted to accurately capture the real-time behaviours from a control perspective; for instance, the execution of setpoints subject to different operating characteristics and the interactions among multiple controllable devices.

3.4.1 EHV1 Network

The EHV1 network [79], a UK-style rural distribution network is used in this case study. A simplified one-line diagram is shown in Figure 3-6.

Bulk power is received at the Grid Supply Point (GSP) with a nominal voltage of 132 kV. This is also treated as the slack bus for the simulation. The supply voltage is stepped down to 33 kV at the substation transformer (SS) and distributed to the 33/11 kV primary substations. The aggregated demand across 18 all primary

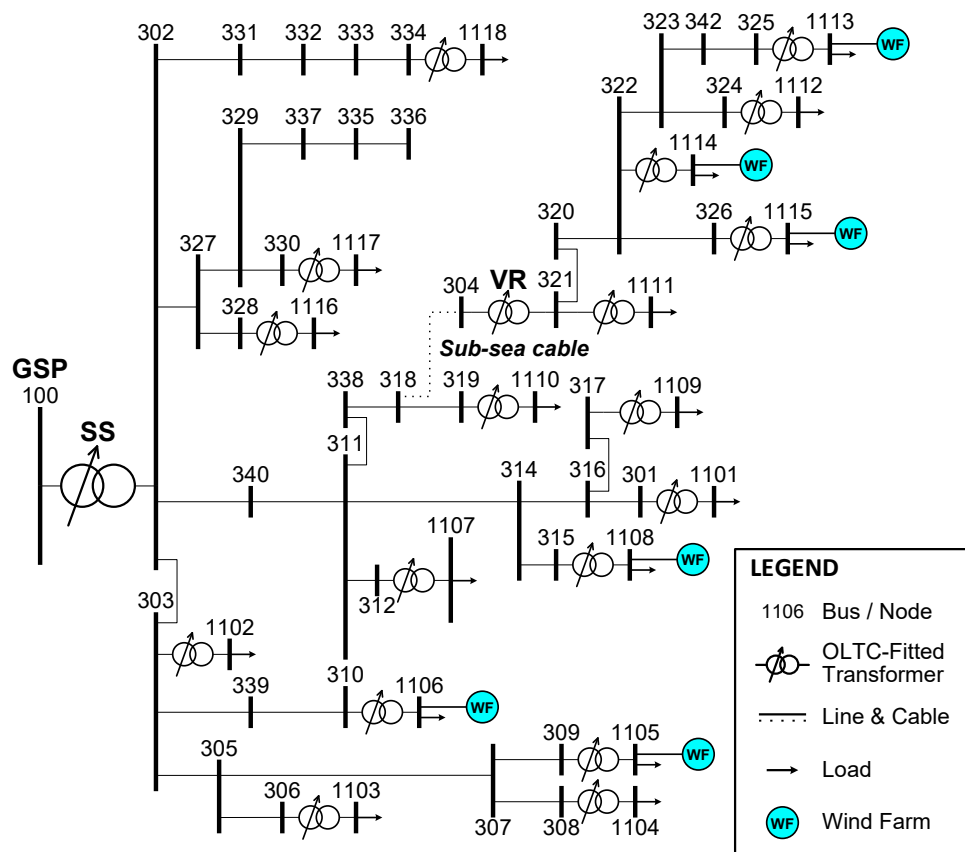


Figure 3-6. EHV1 network.

substations is approximately 38 MW. There is also an inline 33/33 kV voltage regulator (VR) after the sub-sea cable. All transformers are fitted with OLTCs. For simplicity, the same configuration is assumed: there are 21 taps in total with tap ratios ranging from 0.85 to 1.05 in steps of 0.01; the time required for each physical tap movement is 3 seconds.

As shown in Figure 3-6, it is assumed that six wind farms have been connected to buses 1105, 1106, 1108, 1113, 1114 and 1115 due to abundant resource availability. Their installed capacities are based on a previous planning study in [8]; these are 8, 12, 4.5, 1.5, 7 and 12 MW, respectively. For simplicity, it is also assumed that each wind farm is able to operate at a power factor of up to 0.95 (inductive/capacity).

While not explicit shown in Figure 3-6, geographically, the latter three wind farms (i.e., at nodes 1113, 1114 and 1115) are located on an island that has much a weaker infrastructure (e.g., longer distances from the GSP, and thus higher line impedances). Consequently, voltage issues are expected to be more severe on the island than the mainland.

Real normalised demand (at one-hour resolution, available from [79]) and generation (at one-minute resolution, from historical data of a site in England) are used, as shown in Figure 3-7; these are linearly interpolated to 1 second resolution to match the adopted simulation time-step. The demand profile is based on the aggregated residential demand and it is used for all 18 primary substations. Generation profile Wind 1 is used for the wind farms at buses 1105, 1106 and 1108 whereas generation profile Wind 2 is used for remaining three wind farms. Furthermore, to account for geographical separation, each of the available profile is shifted by 30 minutes and 60 minutes to create two other profiles (hence, three in total).

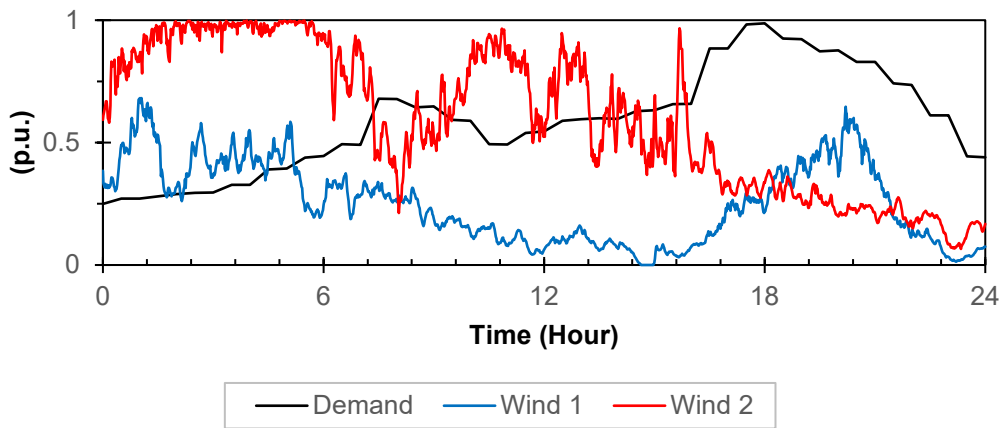


Figure 3-7. Demand and generation profiles.

3.4.2 Modelling Considerations

OPF Formulation

Based on UK standards, the statutory steady-state voltage limits for the 11 kV and 33 kV buses are $\pm 6\%$ from the nominal value. However, instead of the full voltage range, more conservative values of $\pm 5.5\%$ are used in the OPF to account for any numerical inaccuracies. Furthermore, to account for potential voltage drops in the corresponding 11 kV feeders (which are modelled as lumped loads), it is assumed that the voltages at these nodes (1101 to 1118 in Figure 3-6) should be kept above nominal; i.e., the adopted lower voltage limit in the OPF formulation becomes nominal (e.g., 1.0 p.u., rather than -5.5%) for these nodes.

The scaling factors used in the objective function (3.10) and the corresponding range for each of the three objectives are shown in Table 3-1. These values are selected

to prioritise renewable energy harvesting (over control actions). Furthermore, using reactive power is more preferable than triggering tap changes.

Table 3-1. Objective function.

Objective Function	Scaling Factor (c_i)	Range
$c_1 \sum_{w \in W} P_w$	$c_1 = \left(\sum_{w \in W} P_w^{avail} \right)^{-1}$	[0, 1]
$c_2 \sum_{t \in T} u_t$	$c_2 = \left(10 \sum_{t \in T} 3 \right)^{-1}$	[0, 0.1]
$c_3 \sum_{w \in W} u_g$	$c_3 = \left[100 \sum_{w \in W} P_w^{avail} \tan(\cos^{-1} P_w^+) \right]^{-1}$	[0, 0.01]

Control Cycles

A two-minute control cycle is considered for the case study. Furthermore, the concept of persistent forecast is adopted in each control cycle. Therefore, when new setpoints are calculated, the estimated demand and wind resource availability will be based on the corresponding averages over the previous minute.

Practical Considerations

During each control cycle, the time required to compute new setpoints (i.e., ΔT_{OPF} in Figure 3-4) is assumed to be 10 seconds. The desired execution time is also assumed to be within 10 seconds; this will be used as a basis to align the execution times of OLTCs and wind farms, as discussed below.

For all wind farms, the proposed ramping behaviour in Figure 3-5 is adopted for both the active power and power factor. Therefore, after receiving a new setpoint, the active power/power factor is linearly ramped to the setpoint in 10 seconds. This is also in line with the capability of typical wind turbine controllers [19]. The ramp rate restriction per local connection agreements is assumed to be 10% of the installed capacity/minute. Therefore, the adopted value for ΔP_w in (3.3) is set to $0.1 P_w^+$.

For OLTCs under the proposed direct control method, a limit of 3 tap changes per control cycle is imposed. This helps to align the execution time with DG units (i.e., within 10 seconds) and prevent any large step changes in voltages.

3.4.3 Cases Investigated

Three cases are considered to demonstrate the proposed adaptations in this chapter. The differences are summarised in Table 3-2.

Table 3-2. Summary of cases.

Case	Objective Function	OLTC Direct Control	Wind Farm Ramping Behaviour	Operational Constraints in OPF
1	Generation	No	Linear	No
2	Generation + Control Actions	No	Linear	No
3	Generation + Control Actions	Yes	Proposed	Yes

In Case 1, the objective function only considers the maximisation of renewable generation. For OLTCs, the conventional AVR (as shown in Figure 3-2) is used and the adopted delays and deadbands are shown in Table 3-3; for the wind farms, the linear ramping of their active power (as shown in Figure 3-5) is adopted; for the OPF formulation, operational constraints (3.1) and (3.3) are excluded.

Table 3-3. Conventional AVR settings.

Transformer	Delay	Deadband
132kV/33kV (SS)	60s	2.00%
33kV/33kV (VR)	75s	1.75%
33kV/11kV	90s	1.50%

In Case 2, the minimisation of control actions is also included in the objective function; all other settings are still the same as Case 1. This is expected to significantly reduce the volume of control actions.

Lastly, in Case 3, all proposed adaptations (i.e., direct control of OLTCs, fast initial DG ramp and operational constraints in the OPF) are adopted. This is expected to achieve the best performance.

At the start of the simulation for all cases, all wind farms are assumed to be fully curtailed and all OLTCs are assumed to be at the nominal tap position (i.e., a tap ratio of 1.00).

3.4.4 Results

An overview of the network-level performances in terms of voltage management, congestion management, renewable energy harvesting and volume of control actions is first presented. This is followed by more in-depth analysis on the necessity of the proposed adaptations to ensure OPF-based setpoints are adequately implemented in a real-world application.

For easy identification using the network diagram in Figure 3-6, a naming convention based on the node that a network device is being connected to are adopted; e.g., the 33/11 kV OLTC-fitted transformer between nodes 326 and 1115 is referred to as ‘Tx. 326-1115’ and the wind farm at node 1115 is referred to as ‘WF 1115’.

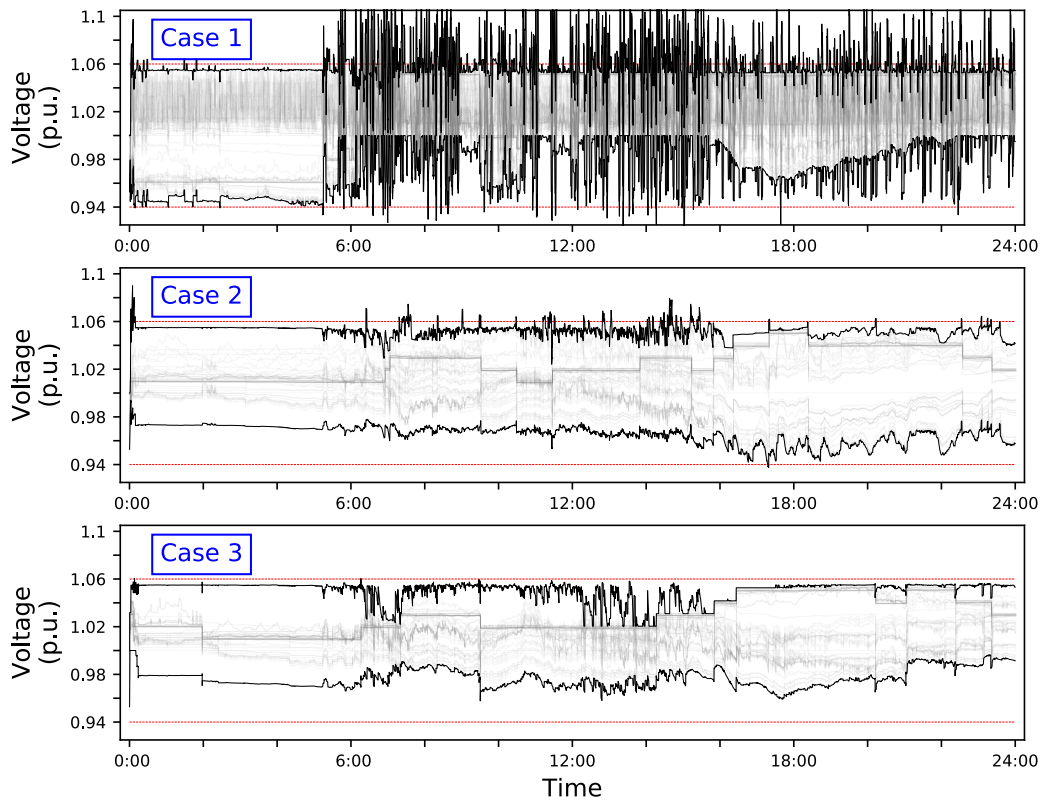
Network-Level Performance

The resulting voltage profiles (at 1 second resolution) are shown in Figure 3-8. Here, the nodal voltages are shown in grey and the envelope that encompasses all voltage profiles are shown in black. Furthermore, the statutory limits are shown in red.

In terms of voltage management, Case 1 immediately stood out with frequent voltage spikes well exceeding the statutory limit. While Case 2 also exhibits voltage spikes, the magnitude and frequency are much lower than Case 1. In contrast to the previous cases, the voltage levels in Case 3 are always kept within the statutory limits. A summary of the maximum voltage recorded for each case is also shown in Table 3-4. While the results obtained for Case 1 may look erroneous, the underlying cause is actually similar to the voltage spikes seen in Case 2, which is contributed by the inadequate considerations in temporal coordination (as explained later).

Table 3-4. Selected network-level metrics.

	Case 1	Case 2	Case 3
Maximum Voltage (p.u.)	1.16	1.09	1.06
Energy Harvested (MWh)	411	409	409
Number of Tap Changes	27,374	138	71
Reactive Power (Mvarh)	57.6	40.6	36.2

**Figure 3-8. Time-series voltage profiles.**

In terms of congestion management, the results are very similar across all three cases. The utilisation profiles of all lines and transformers are shown in Figure 3-9 with similar conventions as voltage profiles. Unlike the issues seen in terms of voltages, all three cases managed to keep the asset utilisation within their thermal rating. Furthermore, it is worth noting that, from a thermal perspective, there is little differences across the three cases. This is largely expected due to the fact that the delays associated with voltage management devices (e.g., OLTCs) do not significantly affect the steady-state power flow characteristics (i.e., P and Q) in the network.

In terms of renewable energy harvesting, there are also little variations across the three cases (within 1% from each other), as shown in Table 3-4. On the other hand, the volume of control actions (measured by the number of tap changes and reactive power absorbed by the wind farms) showed significant discrepancies among the three cases.

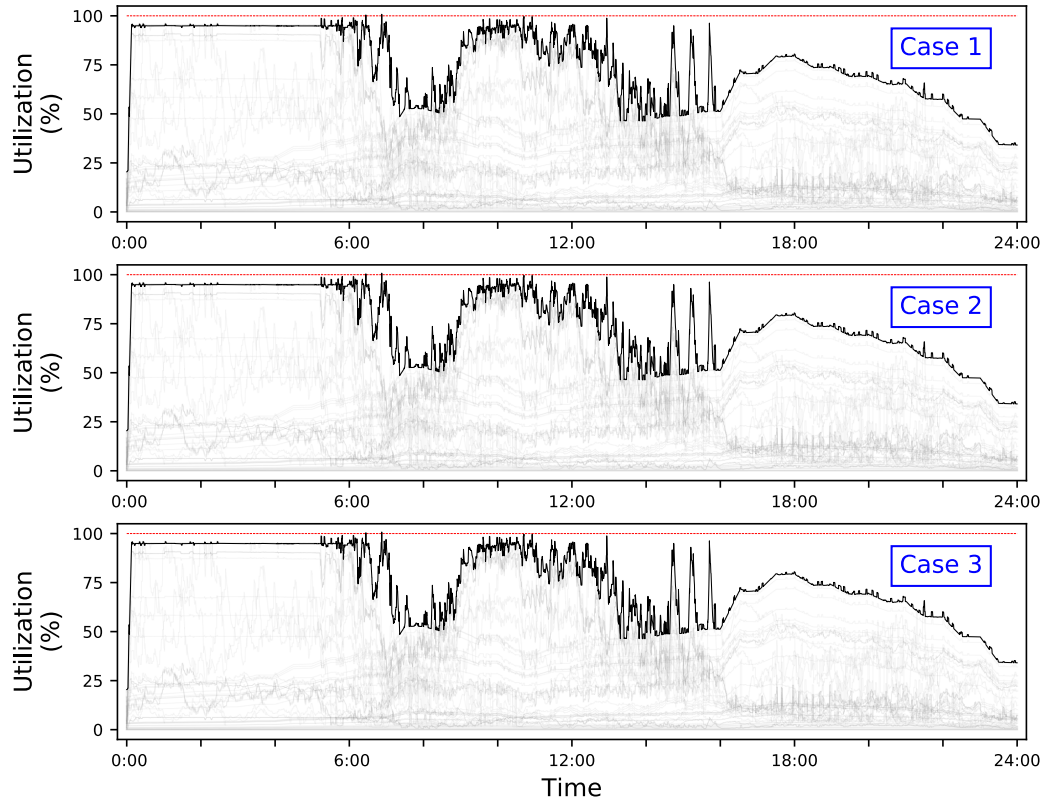


Figure 3-9. Time-series utilisation profiles for lines and transformers.

Particularly, the results for Case 1 is very unrealistic (over 27,000 tap changes) due to the poor design of the OPF formulation where excessive control actions are not being penalised. At the same time, despite incorporating the minimisation of control actions in Case 2, it is not as effective as Case 3; there are almost twice as many tap changes and 12% more reactive power absorbed. Overall, these findings further reinforce the need to assess the performance of a control scheme both in terms of the power flows and the control aspect. Detailed discussions in a control perspective are presented next.

Temporal Coordination

As discussed earlier, the root cause of the behaviour in Case 1 is very similar to Case 2. Therefore, a comparison between Case 2 and Case 3 are first presented below.

The voltage spikes in Case 2 are caused by inadequate alignment of execution times between the OLTCs and DG units. This is illustrated for the two-minute control cycle starting at 15:12:00 in Figure 3-10, using node 1115, Tx. 100-302, Tx. 326-1115 and WF 1115 as an example. Here, the top plot shows the voltage profiles at the secondary side of Tx. 100-302 (i.e., node 302, shown by the blue line) and Tx. 326-1115 (i.e., node 1115, shown by the black line). Additionally, the deadbands of the corresponding AVRs (as a proxy of the calculated voltage targets by the OPF) are also shown as shaded regions. The tap positions of the corresponding OLTCs are shown in the middle plot. Lastly, the lower plot shows the actual power factor (solid line) and the power factor setpoint (dotted line) of WF 1115. For the setpoints, a step change is used to illustrate the instance when these setpoints are received by a controllable device (i.e., after ΔT_{OPF} as shown on the timing diagram in Figure 3-4).

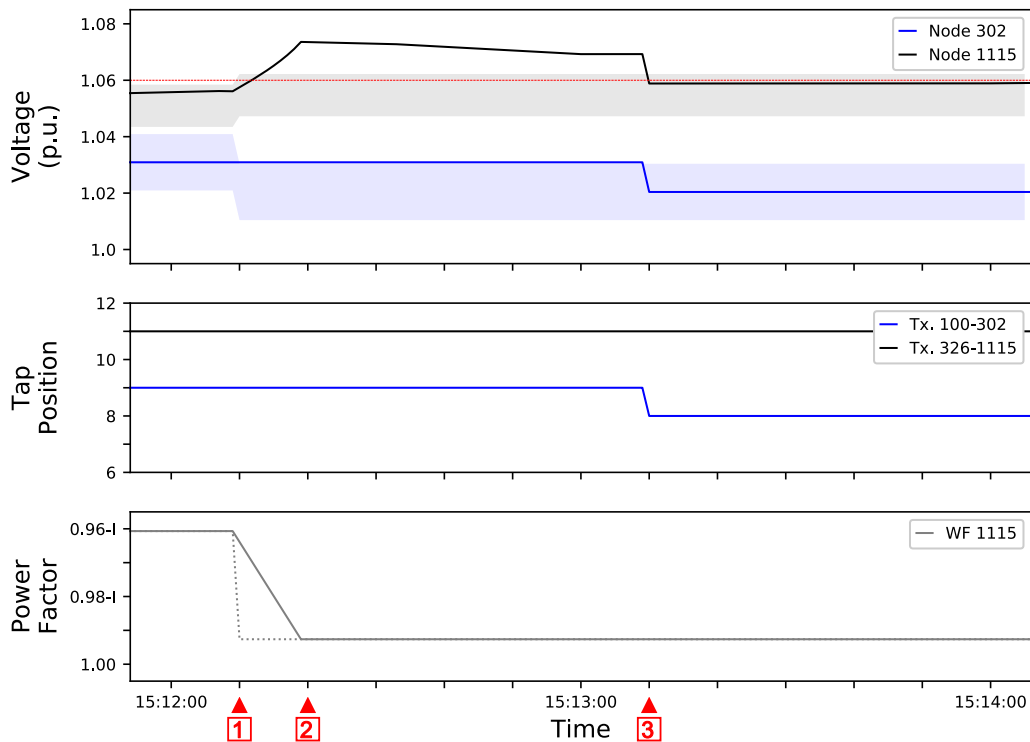


Figure 3-10. Uncoordinated execution of set-points (Case 2).

In this example (for Case 2), three new setpoints have been defined by the OPF: new voltage targets for Tx. 100-302 and Tx. 326-1115 as well as new power factor for WF 1115. While the change to a unit power factor (from an inductive power factor) in WF 1115 is expected to raise voltage at node 1115, the lower voltage target for the upstream transformer Tx. 100-302 should compensate for this increase. However, the

uncoordinated execution of setpoints between Tx. 100-302 and WF 1115 resulted in a temporary voltage spike beyond the statutory limit.

As illustrated by the real-time plot in Figure 3-10, all new setpoints are received at 15:12:10 (10 seconds after the start of the control cycle), as indicated by the time marker [1]. Since there is a negligible delay for WF 1115, the transition to the new power factor is initiated immediately, and thus resulting a gradual increase in the voltage at node 1115 over the next 10 seconds until 15:12:20 (indicated by [2]). Although a lower voltage target is also requested from the upstream OLTC Tx. 100-302 to compensate for this increase, due to adopted delays, the response did not occur until a minute later (around 15:13:10 as indicated by [3]). Consequently, this voltage spike persisted until the response from Tx. 100-302 is observed.

In contrast, Case 3 results in much more desirable behaviour when the responses of Tx. 100-302 and WF 1115 are better synchronised. This is shown in Figure 3-11 for the control cycle starting at 9:30:00¹. Since the OLTC is being directly controlled, the tap position setpoint (dotted line) is being plotted instead of the AVR deadband. By adopting the proposed direct control of OLTC's taps, the problematic delay associated with AVR is fully eliminated. Consequently, the calculated setpoints for Tx. 100-302 and WF 1115 are now being executed at the same time, starting from 9:30:10 (as indicated by [1]) and fully completed by 9:30:20 (as indicated by [2]). Thanks to this coordination, the intended behaviour from the OPF-based setpoints is achieved and the potential undesirable voltage spikes normally seen in Case 2 are avoided.

Based on the above analysis, the erratic behaviour in Case 1 can now be explained. This is shown in Figure 3-12 for the 30-minute period between 13:30 and 13:30 where Tx. 326-1115 and WF 1115 are used to as examples. Note that each x-axis tick here represents the start of a new two-minute control cycle.

The first issue is similar to Case 2, where the uncoordinated execution of setpoints by the Tx. 326-1115 and the WF 1115 is causing voltage spike beyond the statutory limit. The second issue is the frequency of these voltage spikes. This is caused by the oscillatory behaviour of the dispatched setpoints. For instance, as shown in Figure 3-12,

¹ Due to the nature of simulations, where the state of the network depends on the previous control actions, it is difficult to find instances where all three schemes produced the same setpoints. Therefore, the comparison between Case 2 and Case 3 uses different time instances that exhibits similar characteristics (in terms of the state of the network and the control actions determined by the OPF).

both the voltage target (for Tx. 326-1115) and power factor (for WF 1115) setpoints are constantly changing between consecutive control cycles.

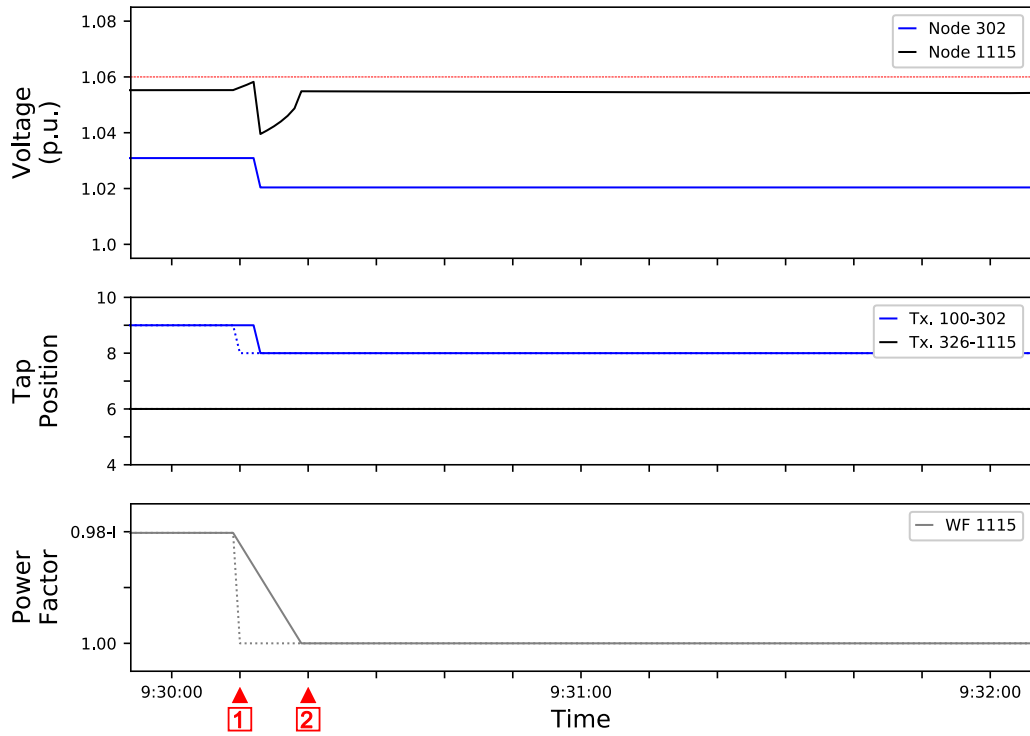


Figure 3-11. Coordinated execution of setpoints (Case 3).

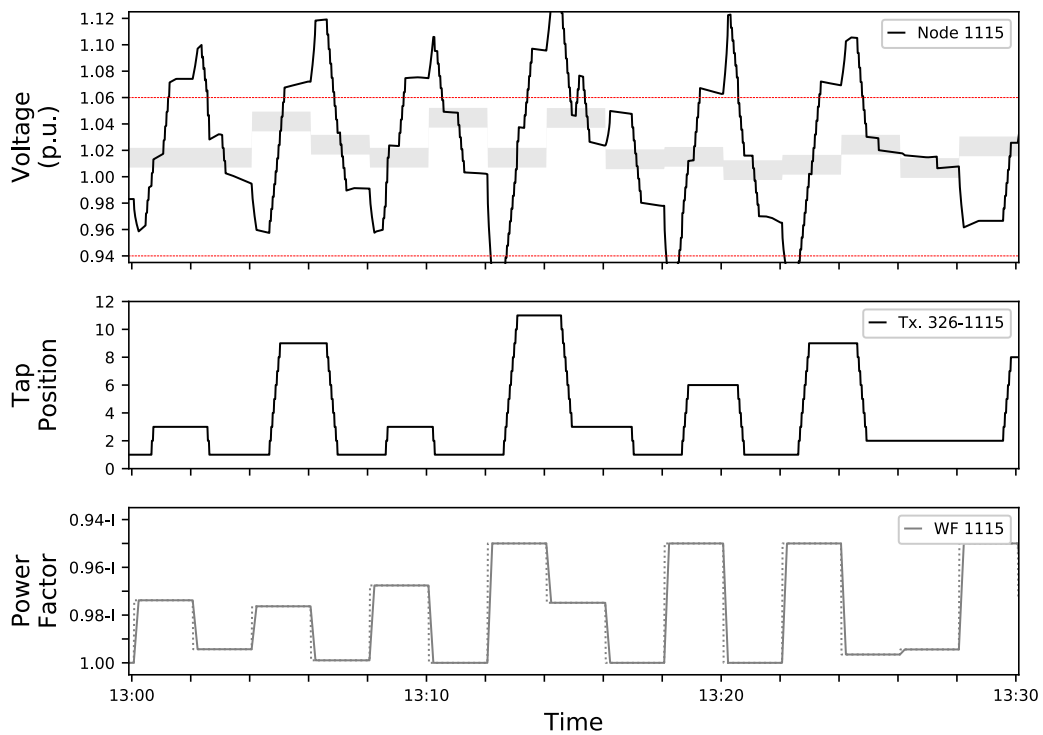


Figure 3-12. Uncoordinated execution of setpoints (Case 1).

Operational Constraints

The consequences of not considering the relevant operational constraints (e.g., ramp rate constraint for DG units) are demonstrated for Case 2, as shown in Figure 3-13. Here, in addition to the voltage profiles, tap position and power factors, the active power generation of WF 1115 is also plotted. Since the wind farm is fully curtailed at the start of the simulation (i.e., at 0:00:00), an increase in its active power output is expected. In this time frame, the maximum feasible amount around 9 MW before the upper voltage limits are reached.

Given that the ramp rate constraint (10% of installed capacity/minute) as per local connection agreement is not being considered by the OPF in Case 2, the calculated active power may not be achievable within a given control cycle. This exact behaviour is seen for the first three control cycles between 0:00:00 and 0:06:00 in Figure 3-13, where the active power is gradually ramping up towards the calculated setpoint of 9 MW yet never fully reaches this setpoint. Consequently, this resulted in premature responses from Tx. 326-1115 in an attempt to regulate the voltage target for node 1115 that has been calculated based a 9 MW setpoint for WF 1115, as indicated by marker [1]. After this premature attempt to regulate the voltage at node 1115, the voltages at node 1115 continued to climb as the active power of WF 1115 gradually increase towards the 9 MW setpoint. As a result, voltage violations and unnecessary tap changes (to undo the premature actions) are observed in two separate occasions, as indicated by [2] and [3], respectively. Eventually, the tap position for Tx. 326-1115 returned to where it started after 0:08:00.

In contrast, Case 3 does not exhibit the same behaviour as the ramp rate constraint of WF 1115 is being considered by the OPF formulation. Consequently, as shown in Figure 3-14, no unnecessary tap changes are observed at Tx. 326-1115.

Lastly, it is worth noting that the effective ramp rate of Case 3 is lower than Case 2. This is due to the values adopted in (3.3) where the ramp rate of 10%/minute considered per two-minute control cycle, effectively halving the maximum observed ramp rate to 5%/minute.

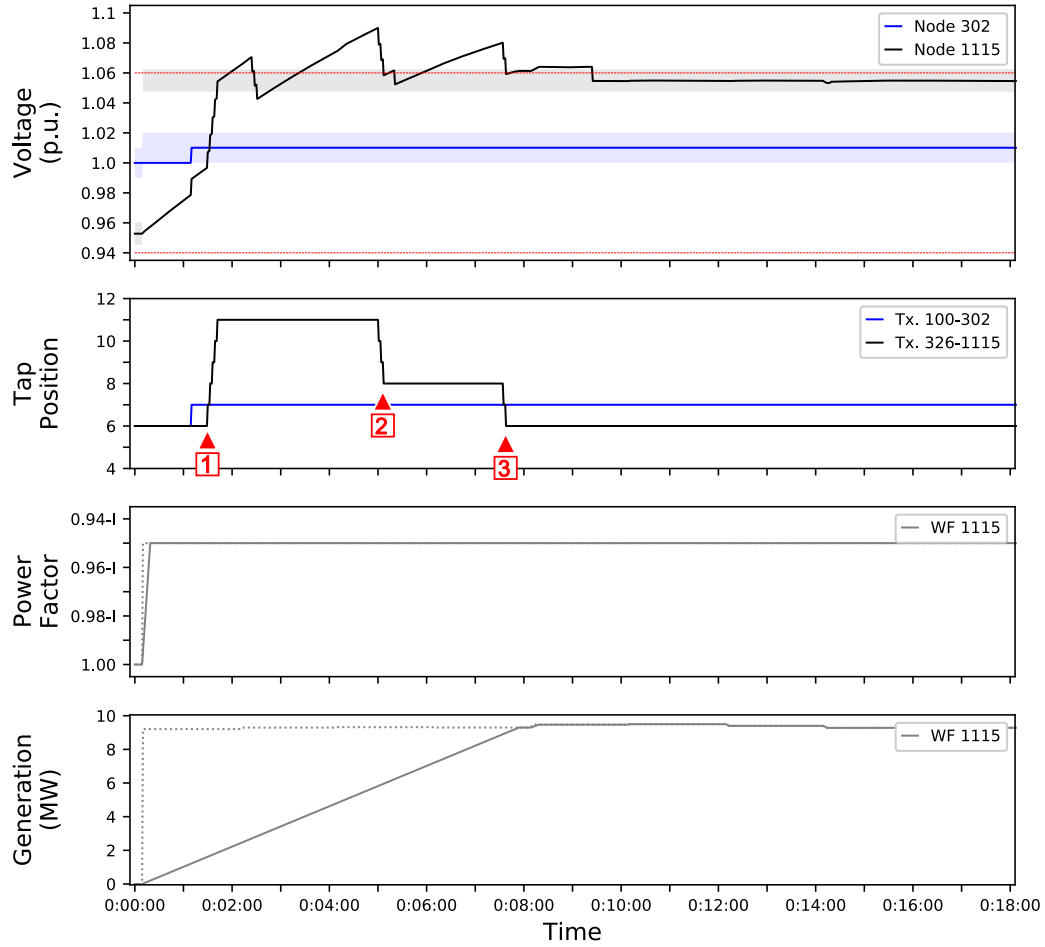


Figure 3-13. Impacts of not including operational constraints in the OPF formulation (Case 2).

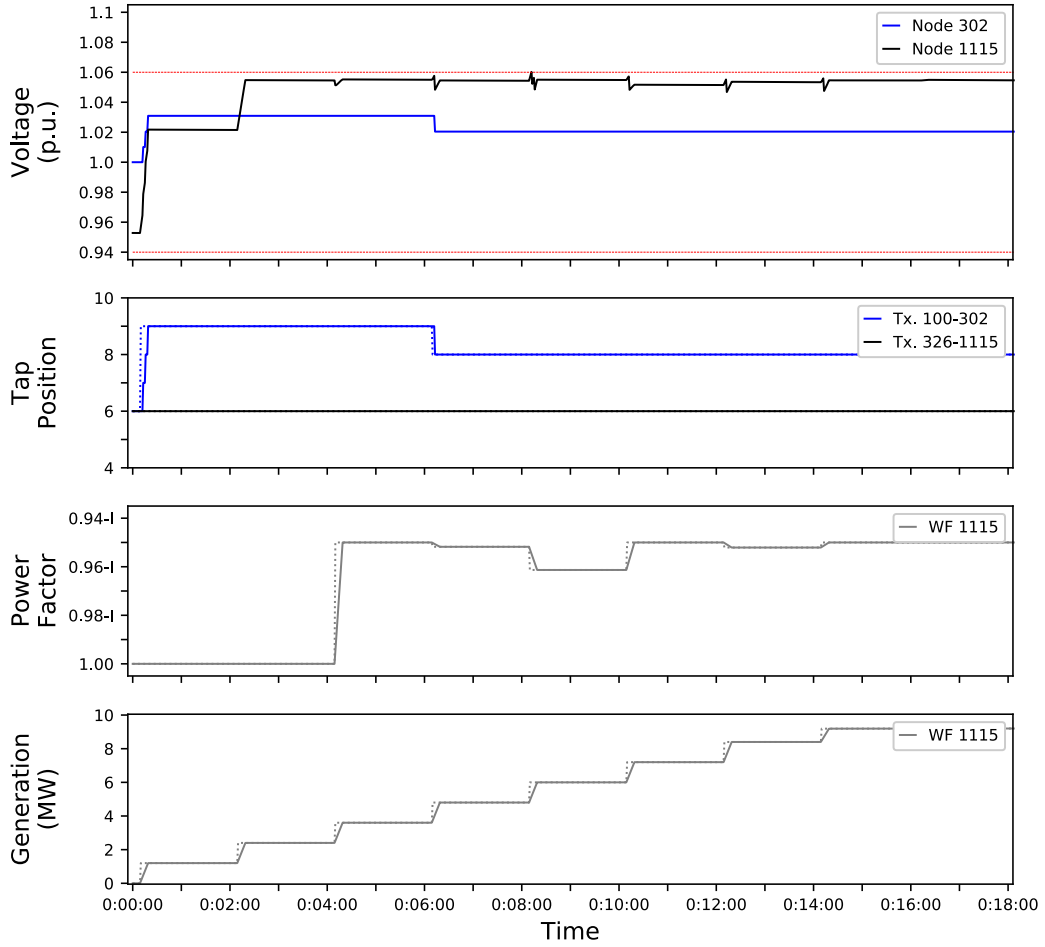


Figure 3-14. Adequate consideration of operational constraints in the OPF formulation (Case 3).

Inadequacy of OLTC Deadbands

In addition to causing temporary voltage spikes due to inadequate coordination between OLTCs and DG units, the deadband of conventional AVR also poses issues due to the constant fluctuations in wind power.

There are two main contribution factors to the inadequacy of AVR in this regard: the relatively large fluctuations in voltage (due to the constantly varying wind generation) compared to the range of deadbands and the compounding effect of voltage drifts within the deadbands from multiple OLTCs along a feeder. This is demonstrated by Case 2 between 6:40 and 7:20, as shown in Figure 3-16 and Figure 3-17.

As shown in Figure 3-16, during this period, the available wind power varies substantially for the wind farm at node 1115. Consequently, this is expected to cause appreciable voltage fluctuations at node 1115 and excessive tap changes at

Tx. 326-1115. Since excessive tap changes is undesirable, strategies to minimise the wear-and-tear of OLTCs are desirable.

For OLTCs controlled by conventional AVR, this can, theoretically, be achieved by setting a voltage target that tracks the busbar voltage at the secondary side of Tx. 326-1115 (i.e., node 1115). However, when this strategy is being implemented in practice, voltage drifts in upstream OLTCs (e.g., Tx. 304-321) renders it less effective for downstream OLTCs (e.g., Tx. 326-1115). As shown in Figure 3-17, the actual voltages at node 321 and 1115 (solid line) are consistently lower than the expected value by the OPF (dotted lines), albeit still within the deadband (shaded region) most of the time. This is exacerbated by the fact that the magnitude of voltage fluctuations is often larger than the adopted deadband for Tx. 326-1115. Consequently, despite the continuous efforts to adjust the reference voltage for Tx. 326-1115, a total of five tap changes is still recorded during this period, as indicated in Figure 3-17.

On the other hand, in Case 3, using the proposed direct control of OLTCs eliminates the aforementioned issues with deadbands. As shown in Figure 3-18, this approach is much more effective in minimising the number tap changes; during the same period, no tap changes are observed for Tx. 326-1115.

Finally, a breakdown of the total tap changes per OLTC is shown in Figure 3-15. It can be verified that the extra tap changes in Case 2 is indeed concentrated around the wind farms on the island (i.e., near nodes 1112 to 1115).

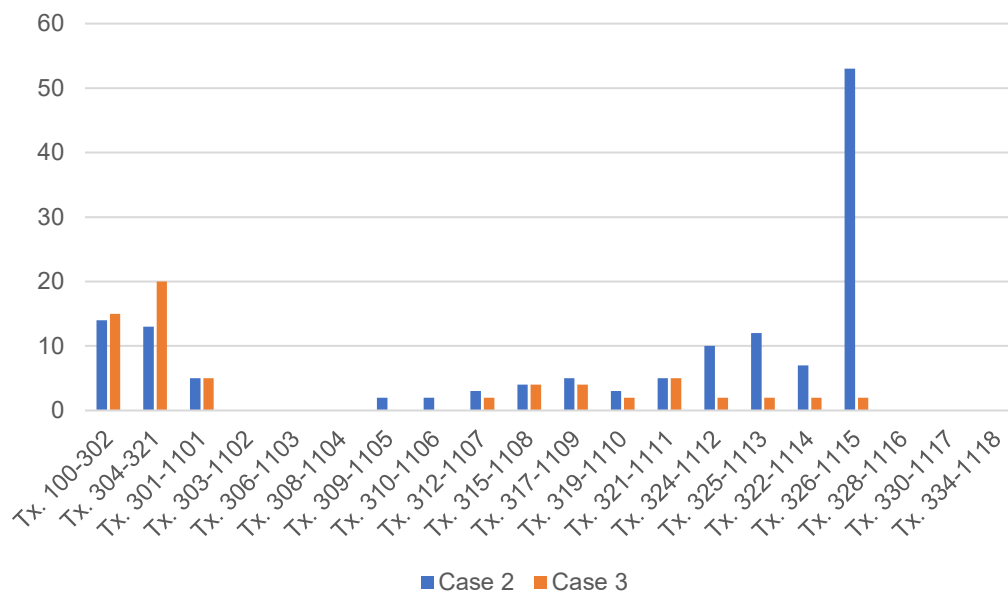


Figure 3-15. Number of tap changes per OLTC.

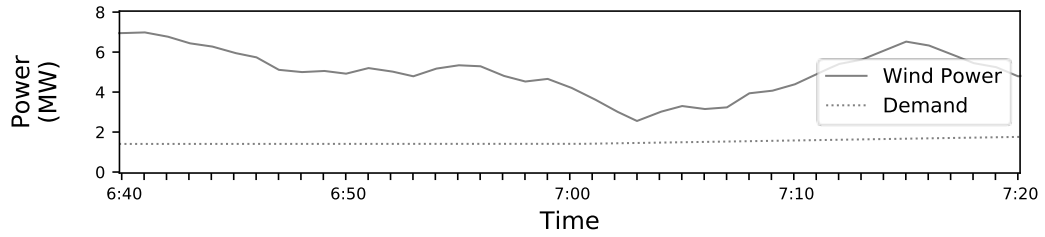


Figure 3-16. Wind power and demand at node 1115.

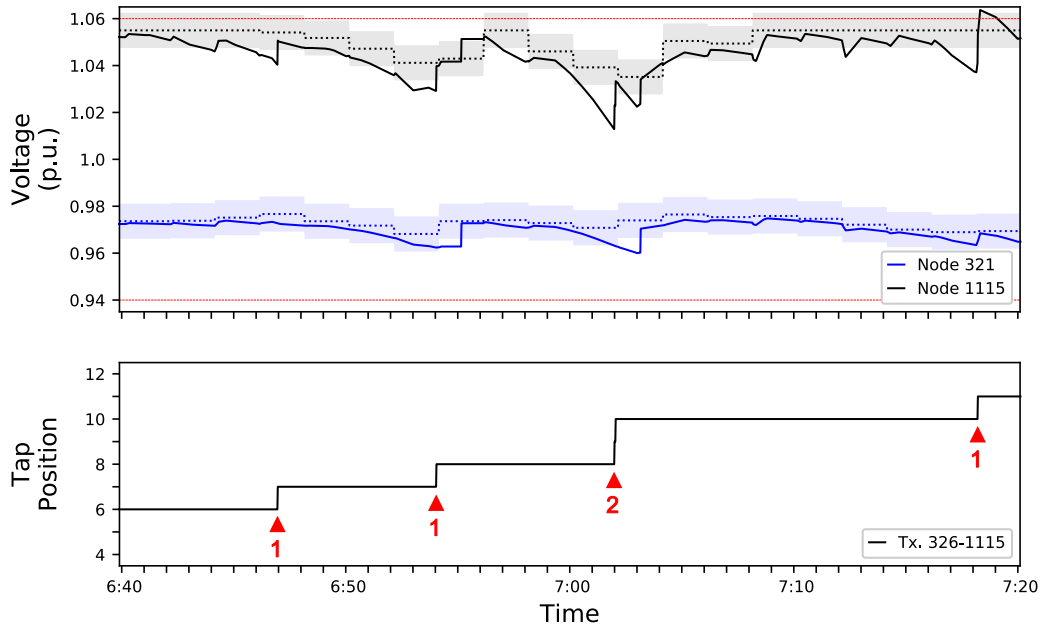


Figure 3-17. Inadequate behaviour due to deadbands and voltage drifts in OLTCs (Case 2).

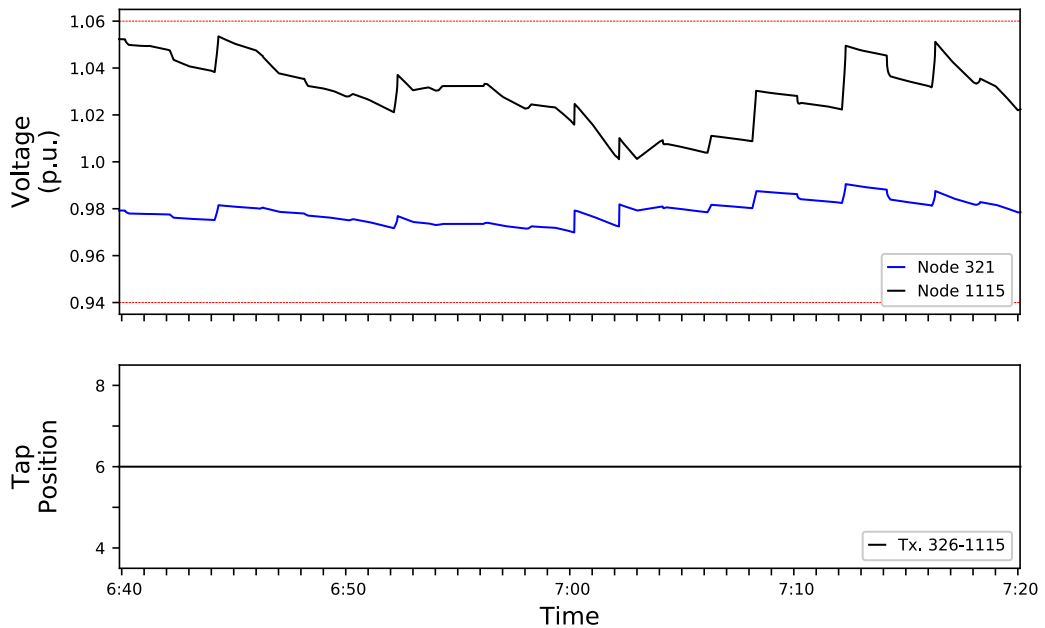


Figure 3-18. Improvements in OLTC behaviour from the proposed direct control method (Case 3).

3.4.5 Sensitivity Analysis of Scaling Factors

The sensitivity of the scaling factors c_2 (for OLTC tap changes) and c_3 (for DG reactive power) on renewable energy harvesting and the volume of control actions is investigated. The scheme in Case 3 is used as a basis for this analysis and the new scaling factors are chosen based on the values shown in Table 3-1, denoted by c_2^* and c_3^* here. Particularly, new scaling factors are created by either moving up or down an order of magnitude (i.e., multiply or divide by 10) from the original values used in Case 3 (i.e., as per second column of Table 3-1).

The resulting sensitivity matrices are shown in Table 3-5 to Table 3-7. Note that an increase in the scaling factor means the corresponding control action is penalised more by the OPF. As expected, the penalty factor is negatively correlated with the total energy harvested, i.e., an increase in penalty factor for either/both control actions will result in less energy harvested. Furthermore, an increase in the penalty factor for one control action (e.g., tap changes) leads to a decrease in volume for that control action (i.e., tap changes) and an increase in volume for the other control action (i.e., reactive power). Lastly, apart from these intuitive findings, there are also a few worth noting observations from these sensitivity matrices.

Table 3-5. Sensitivity matrix (energy harvested).

Effect on Energy Harvested (MWh)		Tap Change Penalty		
		$\frac{1}{10}c_2^*$	c_2^*	$10c_2^*$
Reactive Power Penalty	$\frac{1}{10}c_3^*$	410	408	390
	c_3^*	410	408	389
	$10c_3^*$	410	408	387

Table 3-6. Sensitivity matrix (tap changes).

Effect on Number of Tap Changes		Tap Change Penalty		
		$\frac{1}{10}c_2^*$	c_2^*	$10c_2^*$
Reactive Power Penalty	$\frac{1}{10}c_3^*$	312	61	46
	c_3^*	245	71	46
	$10c_3^*$	290	112	59

Table 3-7. Sensitivity matrix (reactive power).

Effect on Reactive Power (Mvarh)		Tap Change Penalty		
		$\frac{1}{10}c_2^*$	c_2^*	$10c_2^*$
Reactive Power Penalty	$\frac{1}{10}c_3^*$	28.6	39.5	53.4
	c_3^*	17.4	36.1	53.7
	$10c_3^*$	15.4	30.4	39.5

As shown in Table 3-5, while a decrease in the penalty factor only produced minor improvements of the total energy harvested (up to +0.5%), an increase in the penalty factor for tap changes (i.e., column $10c_2^*$) has a much more significant impact on the total energy harvest (around -4.5%).

As shown in Table 3-6, a very low penalty factor for tap changes is potentially undesirable as it leads to a substantial spike in the number of tap changes. Furthermore, since tap changes is often less preferred than reactive power due to the associated wear-and-tear, it is also desirable to ensure the penalty factor for tap changes is higher than that for the reactive power.

3.4.6 Remarks on Solution Speed

Overall, the resulting OPF formulation has approximately 3,750 constraints and 5,900 variables (including 140 binary variables.).

The simulation PC has an i7 processor and 32GB of RAM. The OPF problem in Case 3 (which is formulated as a MILP) requires an average computation time of 0.46 second by CPLEX v12.8. Furthermore, the maximum recorded time is 1.2 second.

3.5 Summary

The adequate implementation of OPF-based setpoints requires practical considerations on the real-time operating characteristics and behaviours (e.g., delays, ramp rates and deadbands) of the devices being controlled. Particularly, due to the diversity in their temporal characteristics (i.e., delays and ramp rates), coordination challenges may arise when multiple devices are being controlled at the same time. Furthermore, existing controller designs (e.g., the AVR for an OLTC) may be geared towards autonomous and passive operation, which creates compatibility issues with an OPF-based scheme where the network is being centrally managed in real-time. As a result, in addition to the modelling of steady-state power flows (e.g., voltages, currents, etc.), these control aspects must also be captured to realistically assess the performance of an OPF-based control scheme.

In this chapter, necessary adaptations in the device controllers and the OPF formulation are proposed to ensure the calculated setpoints from the OPF problem are properly realised in an operational environment. From a device perspective, the direct control of OLTCs' taps and a new active power ramping method for DG units are proposed to align their delays and ramp rates. Furthermore, the proposed OLTC control eliminates the undesirable deadbands associated with conventional AVR. In terms of the OPF formulation, operational constraints (to ensure the calculated setpoints are technically feasible) and a secondary objective (to minimise control actions) are proposed.

The necessity of the proposed adaptations is demonstrated in a realistic co-simulation environment with high granularity (1 second time-step) to capture the control aspects that happens in the sub-minute scale. The results show that, when OPF-based setpoints are being implemented in practice, the proposed adaptations are extremely crucial in ensuring the expected behaviour as well as preventing temporary constraint violations and unnecessary control actions.

4 FAIRNESS OF ACTIVE POWER CURTAILMENT SCHEMES: A RESIDENTIAL CONTEXT

4.1 Introduction

Over the recent decade, residential PV systems have seen an unprecedented growth [80]. At the same time, excessive reverse power flows are also creating significant operational challenges for network operators to cope with issues such as voltage rise and congestion. This is particularly evident in residential feeders due to the high coincidence of peak PV generation and low residential demand (e.g., around noon on weekdays when occupancy declines). As a result, active power curtailment is often necessary to maintain network integrity. To date, two curtailment schemes are commonly adopted by network operators around the world to reduce the volume of reverse power flows in PV-rich residential networks: through the Volt-Watt functionality found in modern PV inverters [81, 82] or through a fixed limit on the households' power exports (either in kW [83] or as a percentage of the installed PV capacity [84, 85]).

While PV curtailment has been shown as an effective solution to mitigate network issues [86] (since it eliminates the cause of these issues), ensuring adequate fairness¹ among the affected households can be a rather challenging task. For instance, since typical distribution networks are operated radially, households farther away from substations are expected to experience higher voltages during periods of high reverse power flows [64]. Consequently, a Volt-Watt scheme inherently introduces locational penalisation that mostly affects households at remote locations (i.e., these households will typically see higher volumes of curtailment). Furthermore, since the impacts on households (from PV curtailment) can be quantified in different, and potentially conflicting, perspectives (e.g., PV harvesting, electricity bills, etc.), whether a scheme can be considered as fair or not is not necessarily a straightforward process. Therefore,

¹ In this context, fairness is defined as the quality of a PV curtailment scheme to treat households equally; in other words, unfairness arises if one household receives more curtailment than others.

the most appropriate perspective needs to be determined first before even attempting to improve the fairness aspect of curtailment schemes.

As the volume of curtailment directly affects households' return on investment for their PV systems, appropriate measurements are desirable to ensure curtailment is applied effectively and fairly. Therefore, the implications from different perspectives on fairness are first investigated in this chapter so as to better understand how to adequately cater for fairness when active power curtailment is necessary. Following this, the trade-offs between fairness (i.e., eliminating disparity) and efficiency (i.e., improving aggregated performance) is also assessed.

4.2 Literature Review

4.2.1 Fairness in PV Curtailment Schemes

As mentioned earlier, for a Volt-Watt-based approach, the voltage at each house's point of connection (POC) is used to determine the volume of curtailment (for that household); e.g., higher voltage results in more curtailment. While this is a readily implementable solution to mitigate technical issues (e.g., voltage rise), the underlying mechanism is inherently unfair. Particularly, due to the radial nature of most distribution feeders, houses located farther away from the substation/head of feeder are expected to experience higher voltages during periods of high reverse power flows. As a result, a Volt-Watt scheme naturally penalises these households, and thus can be considered as unfair.

On the other hand, while the second strategy of adopting a fixed limit for all households can be argued as a fairer approach (as the same limit is given to everyone), it is often overly prohibitive. For instance, such strategies are often implemented as a region-wide policy based on the worst-case scenarios (i.e., maximum generation and minimum demand) and relatively high PV penetrations. Consequently, households in areas with low PV penetration and/or strong network infrastructure can still be unnecessarily penalised.

Therefore, it is clear that the existing schemes adopted by industry has significant drawbacks that may hinder the future adoption of residential PV systems. To address this, several different approaches have been proposed in the existing literature to apply curtailment in a more effective and fairer way.

For instance, a model-free approach is proposed in [87] to tackle voltage issues by coordinating PV curtailment through distress signals. Particularly, when over-voltage occurs, a distress signal is broadcasted to other PV systems to initiate the active power curtailment process. Then, a time-based algorithm where, after receiving the signal, each household will curtail at the same rate until the distress signal disappears (i.e., after sufficient curtailment is achieved to restore the voltage levels). Since each household is curtailing at the same rate, curtailment is naturally shared among them in a fair way. Another model-free approach is proposed in [88] where the network state between two time instances (one before voltage issues are detected and one after) are used to linearly interpolate the required volume of curtailment that eliminates voltage issues. Once the total volume of curtailment is computed, fairness is achieved by equally distributing curtailment among all PV systems.

A hybrid scheme that consists of both central and local control of PV systems is proposed in [89] to improve the unfairness of Volt-Watt schemes. Particularly, a centralised optimisation problem is solved every 15 minutes to determine individual Volt-Watt settings for all PV systems such that that similar volume of PV curtailment is achieved while mitigating network issues.

Lastly, fully centralised approaches using OPF are considered in [90, 91] to determine (and dispatch) fair curtailment settings throughout the day. In [90], this is achieved by adding a constraint to the OPF formulation that ensures the amount of curtailment is the same for each PV system. On the other hand, in [91], a penalty factor is introduced to the objective function to minimise the variances of curtailment among multiple PV systems.

Despite these various efforts in the existing literature to tackle fairness in PV curtailment schemes, two key aspects related to fairness are still yet to be adequately investigated: the different perspectives of fairness (in a residential context) and how fairness is interpreted when also considering the trade-offs with efficiency. These two aspects are further explained in the following sub-sections.

4.2.2 Different Perspectives of Fairness

While the mechanisms of applying PV curtailment largely differ in the aforementioned works, they all share the same perspective when treating fairness: fairness is considered and assessed only based on the impacts at the PV system (i.e.,

the total PV generation). However, in a residential context, there are other perspectives that should also be considered, such as the net energy imports and exports at the POC (i.e., at the electricity meter) as well as the overall monetary benefit received by the household from the PV investment.

For instance, as shown in Figure 4-1, if the local demand (red arrow) of a house is higher than the PV generation (blue arrow), the net demand (green arrow) at the meter is still positive (i.e., the household is still importing power), and thus not technically contributing to any reverse power flows. Consequently, if the local demand is not being factored in (as commonly done in the literature), it is arguable that these households have been unnecessarily penalised. Furthermore, it is necessary to acknowledge that monetary benefits (e.g., a reduction in electricity bills) is one of the main reasons behind the investment in residential PV systems. Therefore, the assessment of the net monetary benefits is also extremely valuable, which also requires the consideration of both energy imports and exports. This is even more so given that the cost of electricity (when importing power from the grid) can be very different from the feed-in tariff (when exporting power back to the grid) in many regions around the world. This is because a kWh of surplus generation exported does not necessarily mean the same as a kWh reduction in energy import (due to the PV generation).

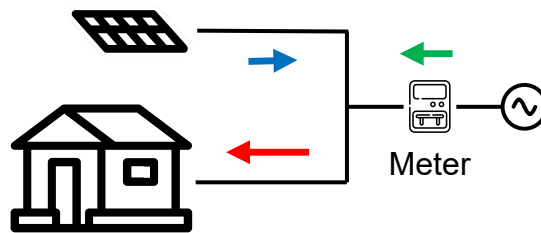


Figure 4-1. Household with positive net demand.

Therefore, to realistically assess the impacts from PV curtailment and the associated implications on fairness, the performance must be quantified considering all these different perspectives. To this end, the household-centric metrics proposed in [20] (albeit not used in the context of fairness) are adopted (and adapted) in this chapter. The definitions of these metrics are introduced below. The same terminology for household variables/constants as presented in Section 2.3.1 are used and each metric is defined for a given snapshot $i \in I$ in a time-series simulation.

PV Harvesting Index (PHI)

The PHI is used to quantify the overall utilisation of the PV system. It is calculated as a ratio between the actual PV generation (i.e., P_h^G) and the maximum available power at the PV panels (i.e., P_h^{PV}), as shown in (4.1). It has a range of $[0,1]$ where 1 means the PV system is fully utilised (i.e., no PV curtailment).

$$PHI_h(i) = \frac{P_h^G(i)}{P_h^{PV}(i)}, \quad \forall h \in H, \forall i \in I \quad (4.1)$$

Energy Export Index (EEI)

The EEI is used to quantify how much surplus generation from the PV system can be exported back to the grid. Therefore, it is only defined when the power available at the PV panels (P_h^{PV}) is greater than the local demand (P_h^D). It is calculated as a ratio between the actual energy export (i.e., $P_h^G - P_h^D$) and the potential energy export if no PV curtailment is imposed (i.e., $P_h^{PV} - P_h^D$), as shown in (4.2). It has a range of $[0,1]$ where 1 means all potential energy export is allowed. (i.e., no PV curtailment).

$$EEI_h(i) = \begin{cases} \frac{(P_h^G(i) - P_h^D(i))}{(P_h^{PV}(i) - P_h^D(i))}, & \forall h \in H, \forall i \in \{I | P_h^{PV}(i) > P_h^D(i)\} \\ N/A, & \text{otherwise} \end{cases} \quad (4.2)$$

Net Benefit Index (NBI)

The NBI is used to quantify the derived financial benefit (e.g., in a dollar value) for a household from the PV investment. Since the cost of electricity (μ^E) and the feed-in tariff (μ^{FIT}) is usually different in most regions, the financial benefit from import reduction, i.e., $\mu^E \cdot \min(P_h^D, P_h^G)$, must be quantified separately from energy exports, i.e., $\mu^{FIT} \cdot \max(P_h^G - P_h^D, 0)$. For instance, if the actual PV generation (P_h^G) is more than the local demand (P_h^D), the equivalent financial benefit would consist of both the import reduction portion $\mu^E P_h^D$ and the energy export portion $\mu^{FIT}(P_h^G - P_h^D)$. The formula for NBI is shown in (4.3). It has a range of $[0,1]$ where 1 means all potential financial benefit is derived. (i.e., no PV curtailment).

$$NBI_h(i) = \frac{\mu^E \cdot \min(P_h^D(i), P_h^G(i)) + \mu^{FIT} \cdot \max(P_h^G(i) - P_h^D(i), 0)}{\mu^E \cdot \min(P_h^D(i), P_h^{PV}(i)) + \mu^{FIT} \cdot \max(P_h^{PV}(i) - P_h^D(i), 0)}, \quad (4.3)$$

$$\forall h \in H, \forall i \in I$$

4.2.3 Fairness vs. Efficiency

In the existing literature, fairness is often considered as an even allocation of curtailment among all households (either as a kW value or as a percentage of the PV installed capacity) to remove any disparity. Nonetheless, there are alternative ways of interpreting fairness when the overall efficiency is being considered at the same time. While relatively uncommon in the field of power systems, such concepts have been well established in the field of internet traffic control and is introduced here. Particularly, there are three representative scenarios (or methods) commonly studied in the existing literature: maximum throughput, proportional fairness [92] and max-min fairness [93]. These concepts are explained using the simple LV feeder shown in Figure 4-2.

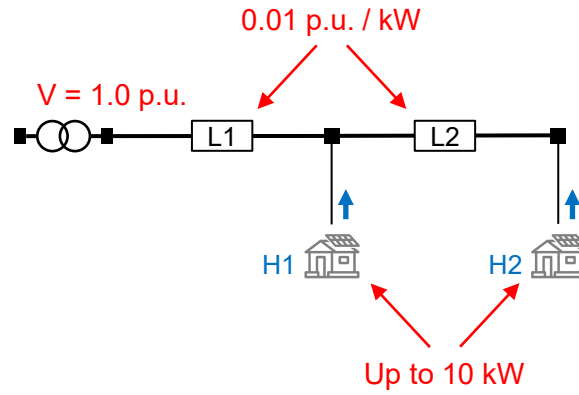


Figure 4-2. A simple LV feeder.

In Figure 4-2, two houses ($H1$ and $H2$) connected to the same phase are shown, each wanting to inject up to 10 kW into the grid from their PV systems; for simplicity, no demand is considered. The impedances of lines $L1$ and $L2$ have the values such that the voltage rise per kW of reverse power flow is 0.01 p.u. The reference voltage at the head of the feeder is assumed to be fixed at 1.00 p.u. Lastly, the statutory voltage limit is assumed to be 1.10 p.u. (i.e., 10% above the nominal).

Based on this setup, it can be immediately verified that, if both houses are to inject 10 kW into the grid, the voltage at H1 and H2 would rise to 1.20 p.u. and 1.30 p.u., well above the statutory limit of 1.10 p.u. Consequently, without other means of mitigating voltage rise, active power curtailment is inevitable. At the same time, depending on how fairness is considered, the curtailment seen in each house can vary significantly. To this end, the three unique solutions for maximum throughput, proportional fairness and max-min fairness are shown in Figure 4-3.

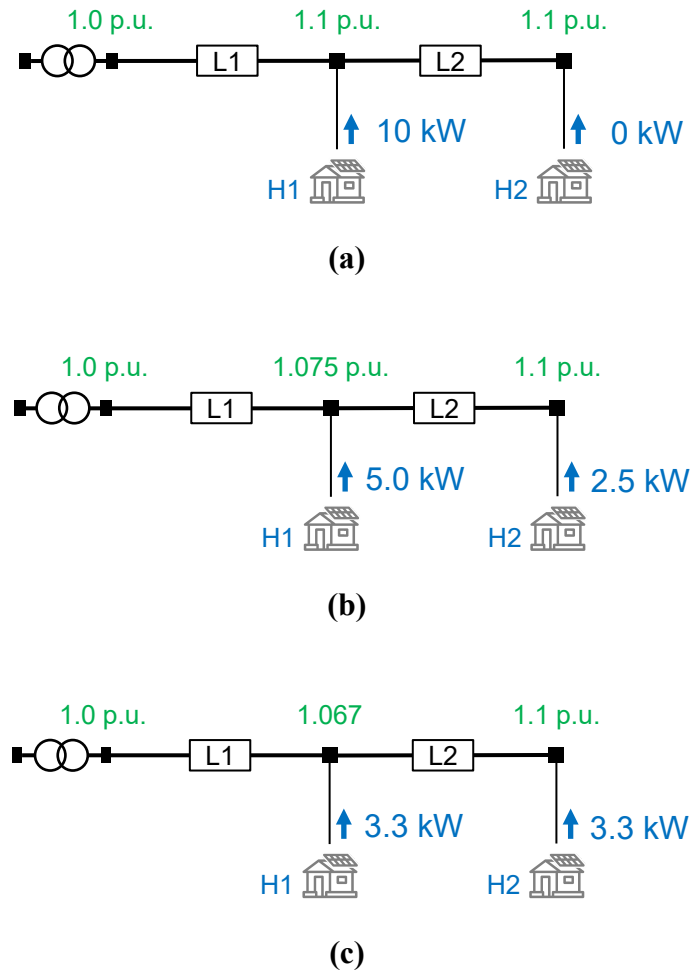


Figure 4-3. Illustration of the solutions to (a) maximum throughput, (b) proportional fairness and (c) max-min fairness.

As the name suggests, maximum throughput refers to the scenario when the aggregated generation is maximised. In this example, this is achieved by allowing H1 to inject at 10 kW while H2 is fully curtailed, as shown in Figure 4-3-a. Since H2 is farther away from the head of the feeder (i.e., has higher impedance), it is more

susceptible to voltage rise than H1. Particularly, if only H2 is to inject power, only 5 kW can be accommodated before the upper voltage limit is reached at H2. Therefore, in terms of being efficient, H1 is preferred. At the same time, the unfairness of this allocation method is extremely obvious as there is a huge disparity between the volume of curtailment H1 (i.e., 0%) and H2 (i.e., 100%) received.

The solution to the max-min fairness method (i.e., Figure 4-3-c) is the same as how fairness is interpreted in most of the aforementioned literature on PV curtailment schemes, where the volume of curtailment is equally shared between H1 and H2. It is also worth highlighting that this is a special case of the max-min fairness when dealing with voltage issues. This is because every household with a PV system in a feeder will contribute to voltage rise to a certain degree. In a more generic sense, max-min fairness attempts to minimise the curtailment for the least fortunate household (H2 in this case).

Lastly, the solution for the proportional fairness method is shown in Figure 4-3-b. While, at a first glance, this solution does not resemble a fair allocation of curtailment, as there is still a disparity in how much curtailment is applied for H1 (5 kW, or 50% curtailment) and H2 (7.5 kW, or 75% curtailment). Nonetheless, it is fair if the relative impact on each other is also factored in. To illustrate this, consider a small deviation from this solution where the curtailment of H2 is reduced by 1% (i.e., H2 will inject at 2.6 kW instead of 2.5 kW). In this new scenario, to still satisfy the voltage constraints, a further 2% curtailment from H1 (i.e., H1 will inject at 4.8 kW instead of 5.0 kW) is needed. Therefore, based on the relative impact on each other, this method can be argued as fair. In a more generic sense, the proportional fairness method will find a solution such that any marginal gain in one household will result in more marginal losses in all other households combined.

From an optimisation perspective, the aforementioned three scenarios (i.e., maximum throughput, proportional fairness and max-min fairness) can be encompassed by a more generic concept known as α -fairness [24]. Mathematically, it can be formulated through the objective function in (4.4) where P_h^{inj} is the individual household injections used in the previous examples. Then, through different α values, the solution to the maximisation problem in (4.5) with the corresponding network constraints (not shown) can result in different allocation methods (including the three considered in this sub-section).

$$J(P_h^{inj}) = \begin{cases} \frac{(P_h^{inj})^{1-\alpha}}{1-\alpha}, & \alpha \geq 0, \alpha \neq 1 \\ \log(P_h^{inj}), & \alpha = 1 \end{cases} \quad (4.4)$$

$$\text{maximise } \sum_h J(P_h^{inj}) \quad (4.5)$$

Particularly, when $\alpha = 0$, the maximum throughput method is obtained; the optimisation objective simply becomes maximise $\sum P_h^{inj}$ which aims to maximise the aggregated injections (i.e., maximum efficiency). When $\alpha = 1$, the proportional fairness method is obtained; the optimisation objective becomes maximise $\log(P_h^{inj})$. Normally, the max-min fairness method is obtained when $\alpha \rightarrow \infty$; however, as demonstrated in Figure 4-3-c, this is equivalent to having the same limit for everyone when dealing with voltage issues. The theoretical derivations can be found in [24], and are beyond the scope of this work.

Based on the above, the intuitive interpretation of α -fairness is illustrated on the fairness-efficiency spectrum shown in Figure 4-4. In general, it is typically considered as fairer (i.e., in terms of eliminating disparity) when the value of α increases. Consequently, max-min fairness is typically considered as the fairest based on this concept. On the other hand, decreasing α general results in more efficiency (i.e., higher aggregated PV generation). For instance, these can be verified based on the results in Figure 4-3: the maximum throughput method resulted in the highest aggregated injection of 10 kW and the largest disparity whereas the max-min method resulted in the least total injections of 6.6 kW (or 33%) but with no disparity between H1 and H2.

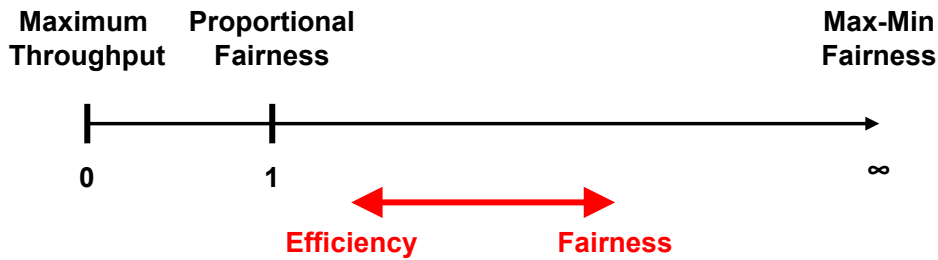


Figure 4-4. Fairness-efficiency spectrum.

4.2.4 Fairness Metrics

Qualitatively, fairness is relatively easy to grasp as it is often immediately apparent that a scheme is fair (when all households are injecting the same amount) or not fair (when there is a disparity among households). However, this binary way of classifying schemes does not allow a quantitative comparison of how much one scheme is unfair compared to another as any scheme that resulting in an uneven allocation is classified as unfair. Therefore, several fairness metrics have been proposed in the existing literature to allow more meaningful comparisons. In general, these metrics are largely based on the relative spread (i.e., disparity) of the benefits (e.g., power injections to the grid) among multiple households. For illustration purposes, the benefit each household h receives will be denoted by b_h .

The simplest form of a fairness metric is based on the maximum benefit (i.e., $\max_h\{b_h\}$) and minimum benefit (i.e., $\min_h\{b_h\}$). This can be done either as a difference, as shown in (4.6), or as a ratio, as shown in (4.7). While this gives a general sense of fairness based on the extremes (households with the maximum benefit and minimum benefit), it does not factor in all other households in between. Therefore, it is often inadequate for a large number of households.

$$\text{Max-Min Difference} = \max_h\{b_h\} - \min_h\{b_h\} \quad (4.6)$$

$$\text{Max-Min Ratio} = \frac{\min_h\{b_h\}}{\max_h\{b_h\}} \quad (4.7)$$

The variance can also be used to quantify fairness, as it considers the relative spread of all benefits from the mean value; the formula is shown in (4.8) where $|H|$ is the cardinality of the set H and $\bar{b}$ is the mean of all b_h values. A scaled version of the variance based on the mean can also be used, which is commonly referred to as the coefficient of variation (COV), as shown in (4.9). However, a potential drawback of both the variance and COV is its unboundedness, i.e., they are in range of $[0, \infty)$, and thus becomes difficult for comparison purposes.

$$\text{Variance} = \sum_{h \in H} \frac{b_h - \bar{b}}{|H|} \quad (4.8)$$

$$\text{COV} = \frac{\text{Variance}}{\text{Mean}} \quad (4.9)$$

Based on the drawbacks of the previous examples, the authors in [94] have proposed the following desirable properties of a fairness metric that allows more intuitive and meaningful comparisons in the context of fairness:

1. Population size independence: the metric can be applied to any number of households greater than one.
2. Scale and metric independence: regardless of the unit used to quantify the benefits (e.g., kW or MW), the metric should yield the same result.
3. Boundedness: the metric should be in the range of $[0,1]$, where 0 can be interpreted as 0% fair and 1 is 100% fair (or vice versa). This allows intuitive and more straightforward comparisons. For instance, a metric value of 0.6 is 10% more fair than 0.5.
4. Continuity: there is no discrete jumps in the metric such that any small change in the allocation of benefits will result in a change in the metric.

Acknowledging these requirements, two commonly used metrics to quantify fairness that both satisfy the aforementioned properties are considered in this thesis: the Jain's Fairness Index (JFI) [94] and the modified Gini Coefficient (mGC) [95]. Their formulas are shown in (4.10) and (4.11), respectively.

$$\text{JFI} = \frac{(\sum_{h \in H} b_h)^2}{|H| \cdot \sum_h b_h^2} \quad (4.10)$$

$$\text{mGC} = 1 - \frac{\sum_{i \in H} \sum_{j \in H} |b_i - b_j|}{2|H|^2 \bar{b}} \quad (4.11)$$

For both the JFI and the mGC, a value of 1 is obtained when all households receive the same benefit (i.e., the fairest). The modification to the GC is done such that a value of 1 in mGC corresponds to the fairest scenario; conventionally, the fairest scenario is when $\text{GC} = 1$. It is worth noting that the JFI is related to the COV by $\text{JFI} = (1 + \text{COV}^2)^{-1}$ and the mGC is related to the original GC through $\text{mGC} = 1 - \text{GC}$.

4.3 OPF-Based PV Curtailment Schemes

As discussed in earlier in this chapter, tackling curtailment fairness in residential distribution networks is not a straightforward process of simply allocating the same level of curtailment for all households. Furthermore, the trade-offs between fairness and efficiency is another important aspect that should not be overlooked. Therefore, in this section, several OPF-based PV curtailment schemes are proposed to demonstrate the how different fairness considerations will affect the households. The necessary modifications based on the OPF formulation in Chapter 2 are introduced.¹ Furthermore, for easy identification, the OPF-based scheme in Chapter 2 is referred to as *OPF-Total*, which is also equivalent to the maximum throughput method discussed in Section 0.

To investigate the different perspectives of fairness, three schemes based on the max-min fairness method are proposed: *OPF-Generation*, *OPF-Export* and *OPF-Financial*. Particularly, these schemes are designed to be fair by ensuring each household (with PV systems) will receive the same benefit in terms of PV harvesting, energy export and financial benefit. These are used in the first case study presented in this chapter.

To investigate the associated efficiency-fairness trade-offs, *OPF-Total* and *OPF-Generation* are used as a basis to formulate different objectives using the α -fairness method. This is referred to as *OPF-Alpha-Fairness*. These are used in the second case study presented in this chapter.

4.3.1 OPF-Generation

This scheme is designed according to the PHI defined in (4.1) where all PV systems are curtailed based on the maximum available power at their PV panels (as a proxy of their installed capacity). The formulation to define the necessary PV curtailment $P_h^{Curtail}$ is shown in (4.12) where $\gamma^G \in [0,1]$ has the same interpretation as the PHI. Finally, the proxy optimisation objective of minimising PV curtailment is shown in (4.13).

$$P_h^{Curtail} = (1 - \gamma^G) P_h^{PV}, \quad \forall h \in G \quad (4.12)$$

¹ Note: only modifications to the modelling of households in Section 2.3.1 and the objective function in Section 2.3.4 are required.

$$\text{maximize } \gamma^G \quad (4.13)$$

As shown in (4.12), PV curtailment is only affected by the power available at the PV systems (which is based on the solar irradiance). Therefore, the local demand does not have any influence on the overall PV curtailment. Furthermore, it is worth highlight that this perspective of fairness is similar to what is commonly considered in the existing literature of PV curtailment schemes (e.g., [87-90]).

4.3.2 OPF-Export

This scheme is designed according to the EEI defined in (4.2) where all PV systems are curtailed based on the potential household exports at the meter. The formulation to define the necessary PV curtailment is shown in (4.14) where γ^E has the same interpretation as the EEI. As shown here, no curtailment is enforced if the household can fully utilise the available PV generation (i.e., when $P_h^{PV} \leq P_h^D$). Finally, the proxy optimisation objective of minimising PV curtailment is shown in (4.15).

$$P_h^{Curtail} = \begin{cases} (1 - \gamma^E)(P_h^{PV} - P_h^D) & P_h^{PV} > P_h^D \\ 0 & P_h^{PV} \leq P_h^D \end{cases}, \quad \forall h \in G \quad (4.14)$$

$$\text{maximize } \gamma^E \quad (4.15)$$

4.3.3 OPF-Financial

This scheme is designed according to the NBI defined in (4.3) where all PV systems are curtailed based on the potential financial benefits for each household. Since there are two ways of deriving benefits (i.e., reduction in energy imports and energy exports), PV generation is separated into two parts: one for import reductions (i.e., $P_h^{G,IR}$) and one for energy exports (i.e., $P_h^{G,EE}$), as shown in (4.16). Furthermore, since energy export is only possible after the local demand is fully covered, a binary variable e_h is necessary to model whether a household is exporting power, as shown in (4.17) and (4.18). Based on this, the formulation to define the necessary PV curtailment is shown in (4.19) and (4.20). Finally, the proxy optimisation objective of minimising PV curtailment is shown in (4.21).

$$P_h^G = P_h^{G,IR} + P_h^{G,EE}, \quad \forall h \in G \quad (4.16)$$

$$e_h \mathbf{P}_h^D \leq P_h^{G,IR} \leq \mathbf{P}_h^D, \quad \forall h \in G \quad (4.17)$$

$$P_h^{G,EE} \leq e_h \max(\mathbf{P}_h^{PV} - \mathbf{P}_h^D, \mathbf{0}), \quad \forall h \in G \quad (4.18)$$

$$P_h^{Curtail} = \mathbf{P}_h^{PV} - (P_h^{G,IR} + P_h^{G,EE}), \quad \forall h \in G \quad (4.19)$$

$$\gamma^F = \frac{P_h^{G,IR} \boldsymbol{\mu}^E + P_h^{G,EE} \boldsymbol{\mu}^{FIT}}{\min(\mathbf{P}_h^{PV}, \mathbf{P}_h^D) \boldsymbol{\mu}^E + \max(\mathbf{P}_h^{PV} - \mathbf{P}_h^D, \mathbf{0}) \boldsymbol{\mu}^{FIT}}, \quad \forall h \in G \quad (4.20)$$

$$\text{maximize } \gamma^F \quad (4.21)$$

4.3.4 OPF-Alpha-Fairness

Instead of the curtailment variable ($P_h^{Curtail}$), the PV system output variable (P_h^G) is used directly in this formulation. The formulation incorporating α -fairness is given by (4.22) and (4.23).

$$J(P_h^G) = \begin{cases} \frac{(P_h^G)^{1-\alpha}}{1-\alpha}, & \alpha \geq 0, \alpha \neq 1 \\ \log(P_h^G), & \alpha = 1 \end{cases}, \quad \forall h \in H \quad (4.22)$$

$$\text{maximise } \sum_h J(P_h^G) \quad (4.23)$$

It is worth highlighting that, the proposed formulation for OPF-Alpha-Fairness only considers the impact on total PV generation. Nonetheless, energy exports or financial benefits can also be adopted in a similar fashion.

4.4 Case Study: Different Perspectives of Fairness

In this case study, a detailed comparison of OPF-Total, OPF-Generation, OPF-Export, OPF-Financial and a Volt-Watt scheme is presented to demonstrate the different perspectives of fairness.

4.4.1 PV-Rich Distribution Network

The same PV-rich MV-LV distribution network (as discussed in Section 2.4.2) is used in this case study. For simplicity, the OLTC at the primary substation is assumed to be fixed at the nominal tap position (i.e., $\tau = 1$) so as to focus on the impact of PV

curtailment on fairness. Therefore, the only controllable variable is the volume of active power curtailment.

This case study considers a 50% PV penetration and the installed capacity of PV systems range from either 3, 4 or 5 kW. Furthermore, a sunny weekday (i.e., clear sky irradiance and weekday demand) are considered to represent the worst-case scenario. Based on this, the normalised demand (based on the aggregated peak demand of 4.15MW for all 4,626 households) solar irradiance profiles are shown in Figure 4-6.

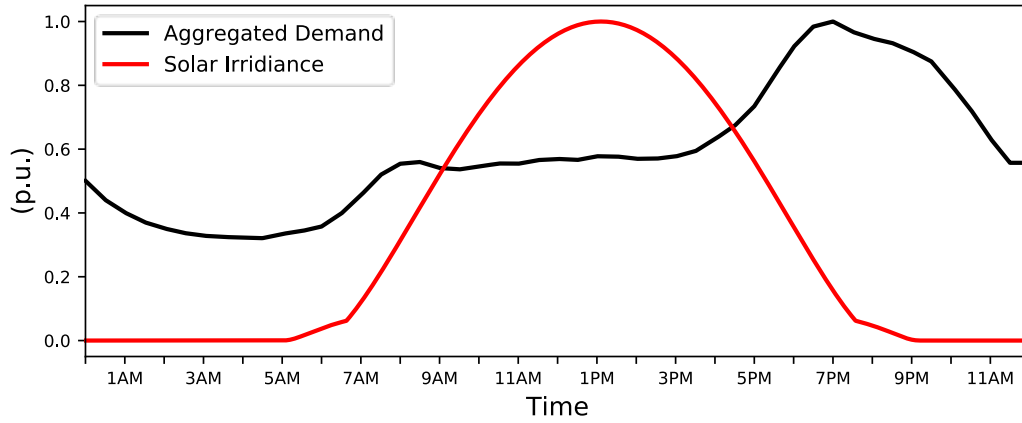


Figure 4-5. Demand and generation profiles.

Voltage compliance is assessed using the Electricity Distribution Code [65] which specifies that the voltage at the POC must be within +10% and -6% from the nominal voltage of 230 V (line-to-neutral). The electricity price μ^E and feed-in tariff μ^{FIT} are set to 0.28 and 0.099 \$/kWh, respectively, based on current Victorian prices.

4.4.2 Simulation Setup

The same co-simulation platform (as discussed in Section 2.4.1) is used. The assessment period corresponds to 7AM and 7PM on a weekday, as it potentially requires PV curtailment. Furthermore, while control cycles (similar to the case study in Chapter 3) are used in this case study, the simulation time-step and control cycle durations are both set to 1 minute; sub-minute scale interactions are beyond the scope of this analysis.

For the Volt-Watt scheme, a modified Volt-Watt curve based on the Australian standard [81] is adopted, as shown in Figure 4-6. Since the original standard imposes full curtailment when the voltage at the POC exceeds 1.15 p.u., it is not able to fully

address voltage issues. Therefore, this modification is required to ensure that the voltage at the POC will not exceed the statutory limit of 1.10 p.u., and thus allowing a meaningful comparison with the OPF-based schemes (which will not allow the voltage to exceed 1.10 p.u.).

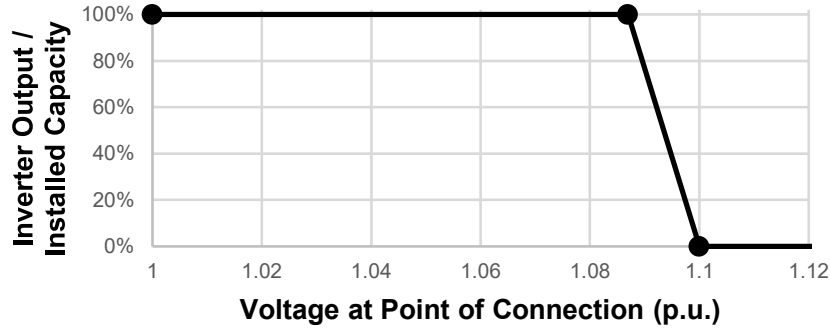


Figure 4-6. Modified Volt-Watt curve.

4.4.3 Results

Distribution Network

The voltage profiles of all households (at the POC and as time-series plots) are shown in Figure 4-7. They are plotted as an intensity map to emphasize the coincidence of the same voltage from multiple households at each time-step. For instance, a darker region means higher coincidence of the same voltage. The statutory upper voltage limit of 1.10 p.u. is also shown as a red line.

From Figure 4-7, it can be seen that all five schemes are able to keep the voltages within the statutory limit. Furthermore, an interesting observation can be made between the schemes with fairness considerations (i.e., OPF-Generation, OPF-Export and OPF-Financial) and the ones without (i.e., OPF-Total and Volt-Watt): in the fair schemes, voltages are concentrated slightly below the statutory limit (the darker region is slightly below the red dotted line); in contrast, for the other two schemes, voltages are concentrated around the upper limit (the darker region is pushed up against the red dotted line).

Since fairness is not being considered in OPF-Total, it can assign vastly different curtailment settings as long as they maximise the aggregated generation. Consequently, this inherently pushes the voltages as close to the statutory limit as possible to maximise the available voltage headroom. At the same time, this means households

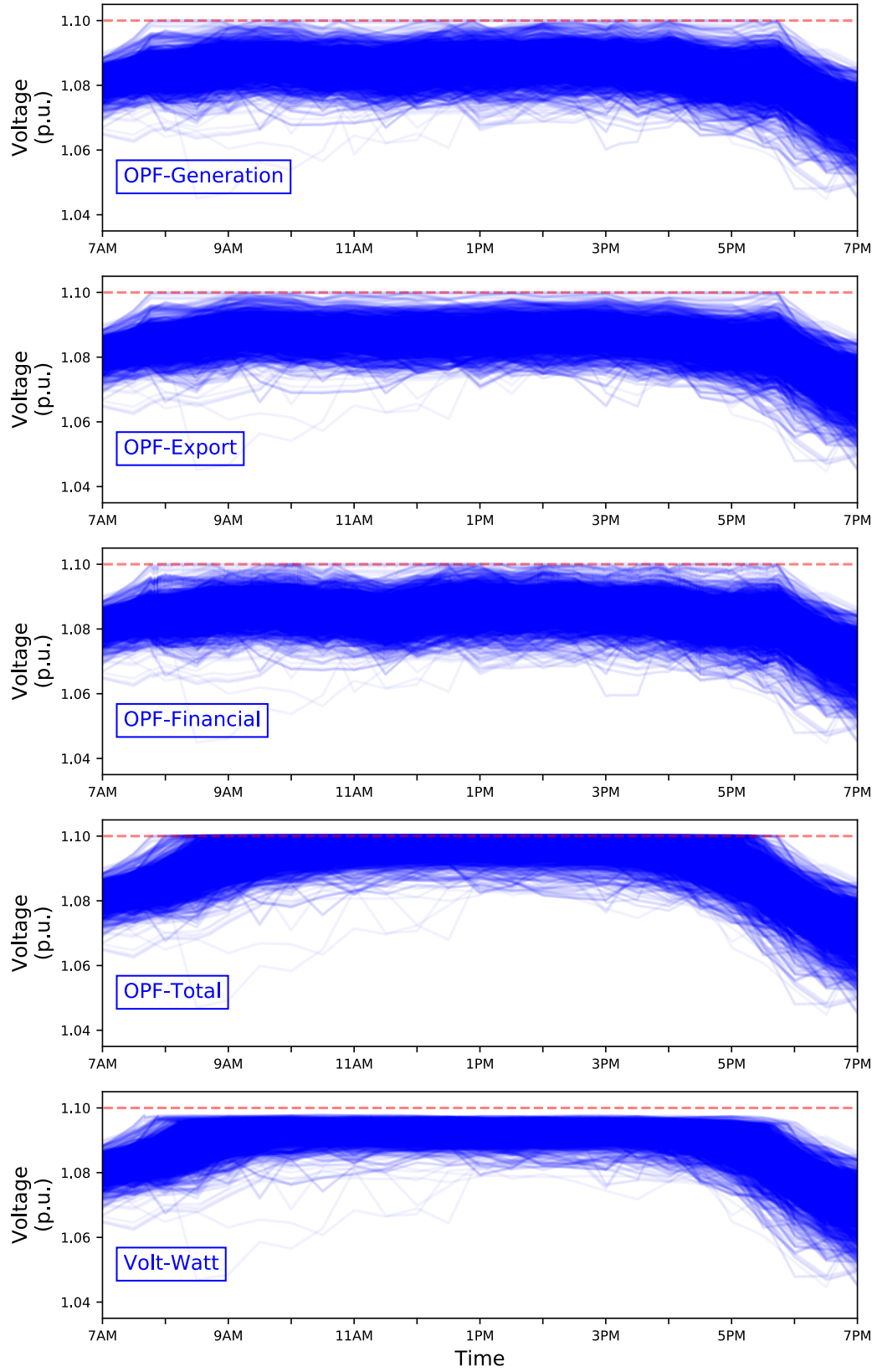


Figure 4-7. Time-series voltage profiles for all 4,626 households.

that are less sensitive to voltage rise issues (e.g., closer to a substation or the head of a feeder) are preferred. Particularly, this is very similar to the phenomenon demonstrated earlier in Figure 4-3-a where H1 is being favoured by the maximum throughput method. This is also evident based on the aggregated energy statistics shown in Table 4-1 where OPF-Total resulted in much higher aggregated energy harvesting (total across all PV systems) and energy exports (at the substation). Additionally, since a Volt-Watt scheme also inherently favours households who are less sensitive to voltage rise, a similar behaviour to OPF-Total is also observed. Nonetheless, being a passive and non-optimisation-based scheme, it is not as effective as OPF-Total to fully exploit the available voltage headroom. Finally, across the three schemes that incorporated fairness considerations, OPF-Export particularly stood out as to achieve the highest energy harvested and energy exported. This intuitive observation is also aligned with the objective function of OPF-Export, i.e., to maximise the energy exported.

Table 4-1. Aggregated energy statistics.

	OPF- Generation	OPF- Export	OPF- Financial	OPF- Total	Volt- Watt
Energy Harvested (MWh)	33.4	35.0	32.3	60.5	47.2
Energy Exported (MWh)	4.3	5.8	3.3	31.0	18.0

Individual Households

Four representative households (based on their demands and locations in the feeder, as shown in Table 4-2) are selected to illustrate the relative impacts due to different fairness considerations. Time-series plots of their demand and PV generation profiles are shown in Figure 4-8 (for households A and B) and Figure 4-9 (for households C and D). Here, the power generated by the PV systems are shown in red, orange, cyan, blue and green for OPF-Generation, OPF-Export, OPF-Financial, OPF-Total, and Volt-Watt, respectively. Furthermore, each household's demand and the maximum

available power from the PV panels (based on the adopted solar irradiance) are shown in black and grey, respectively.

Table 4-2. Household data.

Household	Feeder Location	Relative Demand	Installed PV Capacity
A	Start	Low	4 kW
B	End	Low	4 kW
C	Middle	High	3 kW
D	Middle	Low	3 kW

From Figure 4-8, penalisation of households in remote locations by OPF-Total and Volt-Watt is immediately obvious. Although both households have very little demand during the day, their PV generation profiles are significantly different. Particularly, Household B is curtailed 100% (i.e., no PV generation) most of the time between 10AM to 4PM by OPF-Total. While it is less severe for the Volt-Watt scheme, a huge disparity still exists. In contrast, for the remaining fair schemes (i.e., OPF-Generation, OPF-Export and OPF-Financial), the resulting generation profiles are largely comparable between households A and B (i.e., curtailment has been allocated evenly regardless of their location or sensitivity to voltage issues).

To better illustrate the differences among OPF-Generation, OPF-Export and OPF-Financial, two additional households (i.e., C and D) with very different demand profiles throughout the day are compared, as shown in Figure 4-9. The PV generation profiles for OPF-Total and Volt-Watt are omitted for clarity.

For OPF-Generation, it can be seen that the level of demand does have any effect on PV curtailment; particularly, the PV systems for both households received the exact same setpoints (red lines in Figure 4-9) throughout the assessment period. Although, from the perspective of PV energy harvesting, OPF generation can be considered as fair to both households, this is no longer valid when their respective demands are also being factored in (i.e., when considering the net demand at the electricity meter).

For instance, as shown in Figure 4-9, Household C has to import energy from the grid as the demand (black line) is higher than the output of the PV system (red line)

between 9AM and 4PM, even though this demand can be fully covered by the PV system (grey line). In contrast, with OPF-Export, Household C is able to better utilise its PV system to cover the local demand between 9AM and 4PM; not only achieving zero energy import, but also some energy export (when the orange line is above the black line in Figure 4-9). Although not immediately apparent, it can be verified that, during periods when energy export is possible (i.e., grey line is above the black line), both households are allowed to export the same proportion (i.e., distance between the orange line and black line) with respect to their total potential energy export (i.e., distance between grey line and black line). Therefore, from an energy export perspective, OPF-Export can be considered as fair.

While the PV generation profiles for OPF-Generation and OPF-Export showed distinct characteristics for households C and D, the results for OPF-Financial is quite the opposite. This is mainly due to the fact that curtailment is applied not only based on their demand and generation, but also the demand and generation of other households as well as the adopted cost of electricity and feed-in tariff. Nonetheless, the results can still be analysed using the metric to quantify financial benefits (i.e., the NBI). Particularly, households C and D both received the same average NBI¹ of 0.547 for the assessment period, and thus OPF-Financial can be considered as fair from the perspective of financial benefits.

¹ Firstly, calculated for each time-step. Then, averaged throughout the assessment period.

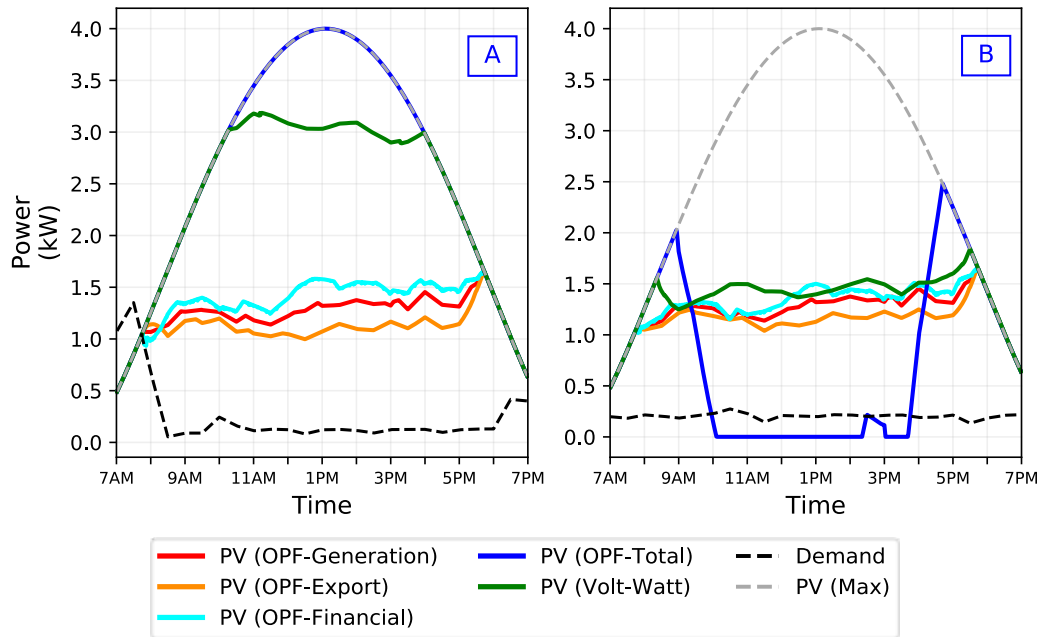


Figure 4-8. Time-series plots for two households: towards the head of a feeder (A) and towards the end of a feeder (B).

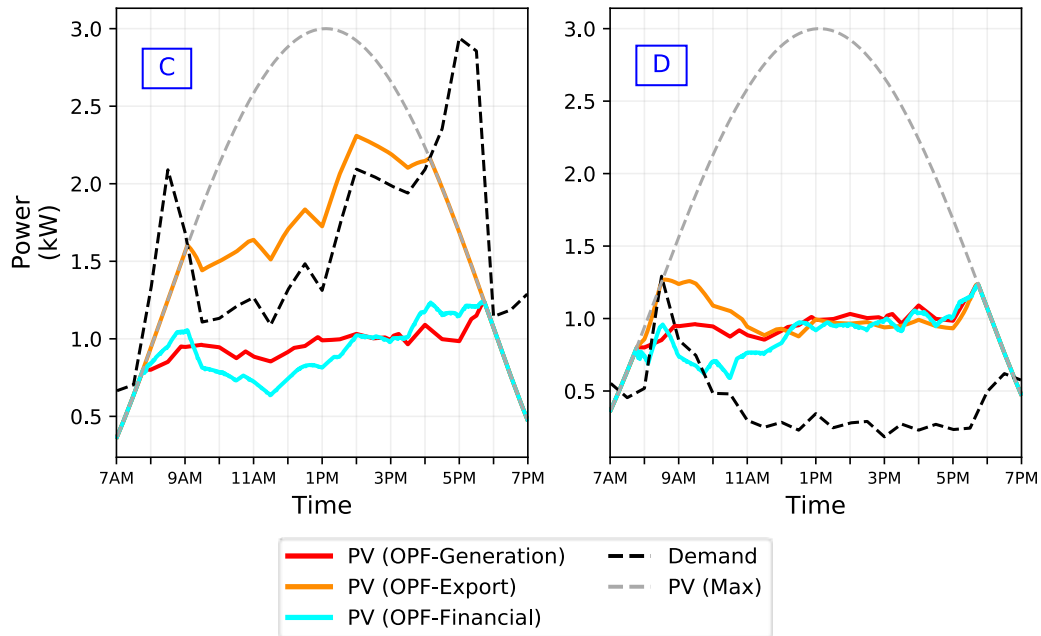


Figure 4-9. Time-series plots for two representative households: with high demand (C) and with low demand (D).

Fairness Assessment

To extend the previous analysis (on the different perspectives of fairness) for all households in the network, the proposed household-centric metrics (i.e., PHI, EEI and NBI, as discussed in Section 4.2.4) are evaluated for each household and at each simulation time-step; these are shown as scatted plots in Figure 4-10. The key feature of this figure is the spread at each timestep, which is an indicator for the disparity among households. For instance, if all the points overlap, it means no disparity (i.e., absolute equality for all households). It is also worth noting that, for the EEI plots, a ‘N/A’ is used to indicate the instances when EEI is not applicable to a household, i.e., time-steps with no surplus PV generation.

From these household-centric metrics, the JFI and mGC are also evaluated per time-step to quantitatively compare the fairness of the investigated schemes. The average values of JFI and mGC throughout the assessment period are shown in Table 4-3 and Table 4-4, respectively. As discussed previously, a value of 1 means all households received the same benefit (i.e., absolute equity). Furthermore, the highest values (i.e., the fairest scheme) in terms of the four perspectives (i.e., PV harvesting, import reduction, energy export and financial benefit) are highlighted in **bold**.

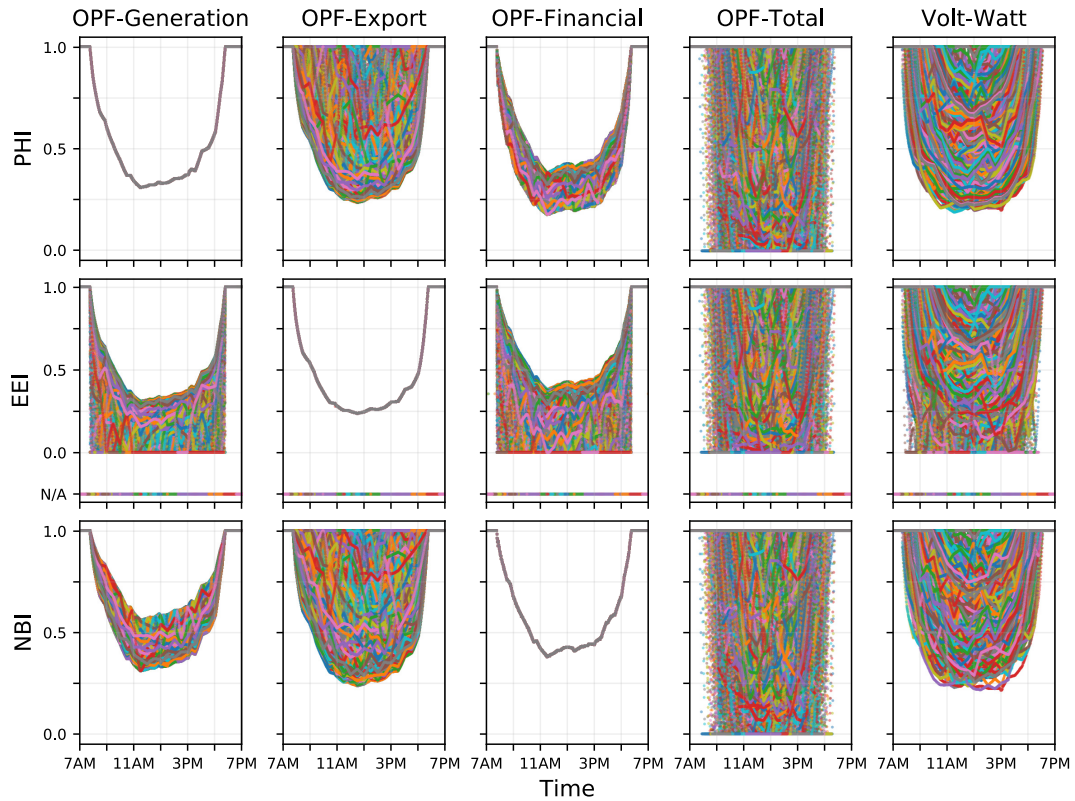
Based on the results, it is apparent that, from the perspective of PV harvesting, OPF-Generation is the fairest scheme. As shown in Figure 4-10, the setpoints calculated by OPF-Generation ensures that the PHI for all households is exactly the same in every time-step. Consequently, both the JFI and mGC evaluated for the PHI are also 1, indicating that there is no disparity among all households in terms of PV harvesting. In contrast, the PHI for each time-step can vary significantly for the other four schemes (as shown in Figure 4-10), and thus resulting in lower JFI and mGC (as shown in Table 4-3 and Table 4-4, respectively). Therefore, it can be argued that, from the perspective of PV harvesting, the remaining four schemes are less effective in removing the disparity among households, and thus not as fair as OPF-Generation. Similar observations (to OPF-Generation) can also be made for OPF-Export and OPF-Financial: they are the most effective in removing disparity in terms of energy export and financial benefit, respectively, while also having the associated trade-offs in terms of the other metrics. Lastly, it is worth noting that the correlation between JFI and mGC are very high. At the same time, the mGC tends to place a heavier emphasis on disparity as the mGC is consistently lower than JFI for the same scenario.

Table 4-3. Jain's Fairness Index for household-centric metrics.

	OPF- Generation	OPF- Export	OPF- Financial	OPF- Total	Volt- Watt
JFI_{PHI}	1.000	0.943	0.979	0.907	0.958
JFI_{EEI}	0.891	1.000	0.796	0.893	0.919
JFI_{NBI}	0.988	0.943	1.000	0.918	0.970

Table 4-4. Modified Gini Coefficient for household-centric metrics.

	OPF- Generation	OPF- Export	OPF- Financial	OPF- Total	Volt- Watt
mGC_{PHI}	1.000	0.901	0.928	0.883	0.900
mGC_{EEI}	0.843	1.000	0.742	0.872	0.859
mGC_{NBI}	0.947	0.885	1.000	0.894	0.913

**Figure 4-10. Time-series scattered plot of household-centric metrics.**

The unfair nature of OPF-Total and Volt-Watt can also be seen from Figure 4-10, as the disparity in PHI, EEI and NBI among households can become very high. While both of these schemes can be considered as unfair, quantitatively, Volt-Watt is still fairer than OPF-Generation based on the JFI and mGC. This is largely expected as Volt-Watt is still inherently distributing curtailment among every household; since the volume of curtailment is based on the voltage at the POC, naturally, some households will curtail more than the others. In contrast, being an optimisation problem, OPF-Total can ‘intentionally’ assign 100% curtailment to some households (e.g., those at the end of a feeder) as long as it improves the objective function (i.e., reducing the aggregated curtailment). Therefore, by design, and as demonstrated by the results here, OPF-Total tends to be less fair than Volt-Watt.

Lastly, there is another interesting observation between OPF-Generation and OPF-Financial: their results are very similar across the board. This is largely driven by the typical demand pattern in residential networks. Since, on a weekday, most households have relatively low demand, improving their financial benefit (which is the objective of OPF-Financial) is highly correlated with increasing their PV generation (which is the objective of OPF-Total). Conversely, and also as demonstrated for Household C in Figure 4-9, these two schemes can be very unfair to households with relatively high demand on weekdays. Therefore, this further highlight that a scheme that is designed to eliminate disparity in one perspective (e.g., PV harvesting) may not be so when a different perspective is considered (e.g., energy export).

Locational Penalisation

A boxplot of the average PHI, EEI and NBI for each household is shown in Figure 4-12 to better visualise the locational penalisation of certain schemes (e.g., OPF-Total and Volt-Watt). Here, the time-series metrics for each household in Figure 4-10 are averaged and separated into four different groups (i.e., quartiles) based on the distance to the head of their respective LV feeder (as illustrated in Figure 4-11). For instance, the group Q1 contains the first 25% of the houses by distance from the header of the feeder.

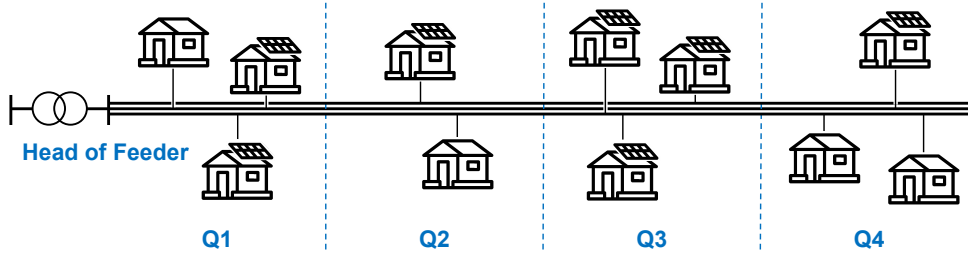


Figure 4-11. Illustration of quartiles.

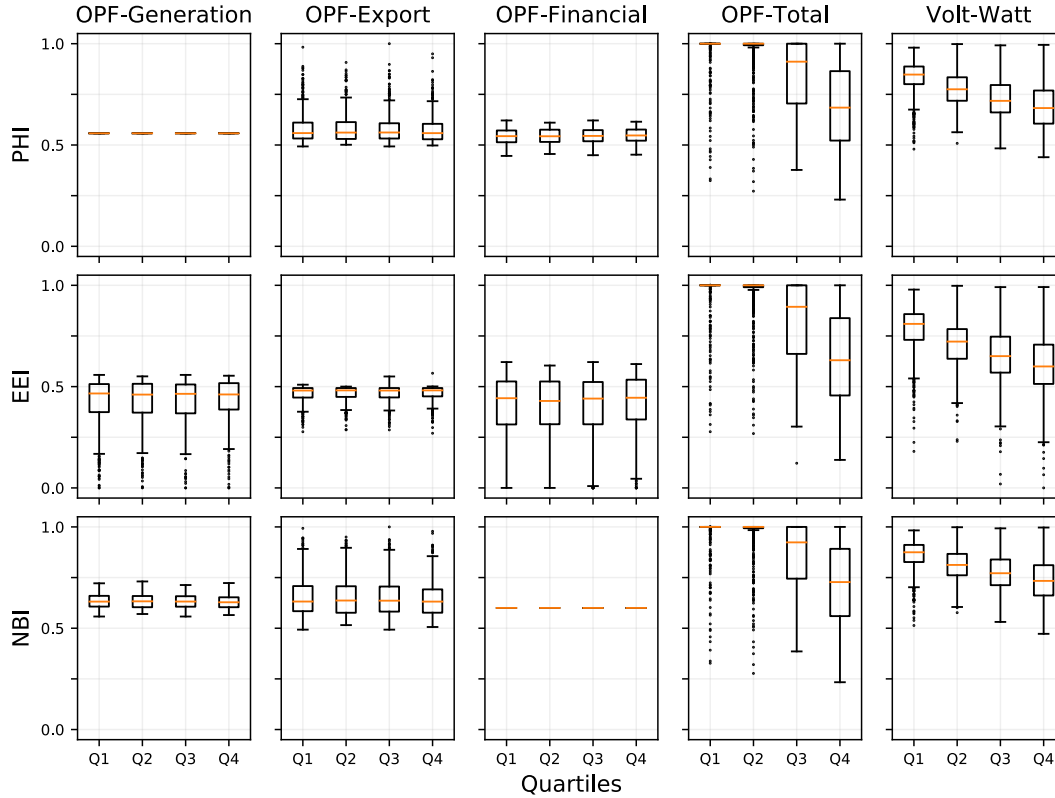


Figure 4-12. Boxplots of average PHI, EEI and NBI for all households shown in four quartiles.

As expected, for OPF-Generation, the average PHI is the same across all four quartiles, which is also in line with the results presented in Figure 4-10. Furthermore, while there is an appreciable spread for the EEI and NBI, which is due to the variations in demands, there is still a consistency across all the quartiles; this means, households are not penalised based on their location. The results for OPF-Financial is very similar to OPF-Generation, and thus further discussion are omitted.

On the other hand, for OPF-Export, the same ‘flat line’ is not observed for the EEI as with the previous two cases. This is because the EEI is not always defined for a household (i.e., when there is no surplus generation from the PV system, as indicated

by ‘N/A’ in Figure 4-10) and the occurrence of such instances also varies from households to households. Consequently, when these time-series values of EEI in Figure 4-10 are averaged for the assessment period, a small spread is expected; this is illustrated using coloured blocks in Figure 4-13 where a missing block indicates the instances where EEI is not defined. Nonetheless, the behaviour is still consistent across the four quartiles, i.e., OPF-Export does not penalise households that are located towards the end of a feeder.

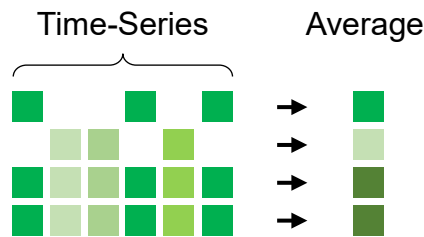


Figure 4-13. Illustration of the differences in average EEIs due to non-defined instances.

Finally, for the remaining two cases (OPF-Total and Volt-Watt), locational penalisation is immediate apparent, as shown by the noticeable drop in medians from Q1 to Q4 as well as the much larger spread in all metrics. At the same time, it can also be seen that, despite being unfair to certain households, the medians of each quartile are still higher than the previous schemes. Therefore, this result highlights that there is an inherent trade-off between fairness and efficiency. To better understand this aspect, further analyses are presented in the next case study.

4.4.4 Remarks on Solution Speed

For OPF-Generation and OPF-Export, the resulting OPF formulation has approximately 213,000 constraints and 463,900 variables. For OPF-Export, the resulting formulation has approximately 231,100 constraints and 472,930 variables (including 2258 binary variables).

The simulation PC has an i7 processor and 32GB of RAM. The two linear OPF formulations (i.e., OPF-Generation and OPF-Export) is solved to optimality with an average computation time of 15 seconds. The mixed-integer, linear OPF formulation (i.e., OPF-Financial) requires an average solution time of 40 seconds.

4.5 Case Study: Fairness-Efficiency Trade-Offs

This case study aims to analyse the trade-offs between fairness and efficiency, taking PV harvesting as an example. To this end, the formulation of OPF-Alpha-Fairness is used along with different values of α to create the cases that represent different points on the fairness-efficiency spectrum (as shown in Figure 4-4).

The same PV-rich distribution network and simulation setup (as presented in the previous case study) are used.

4.5.1 Scenarios Investigated

The investigated cases are summarised in Table 4-5. In total, there are five cases, each corresponding to a different value for α .

Table 4-5. Summary of cases.

α	Objective Function	Remarks
0	$\sum P_h^G$	Same as OPF-Total
1	$\sum \log(P_h^G)$	Formulation is an NLP
4	$\sum -\frac{1}{3(P_h^G)^3}$	Formulation is an NLP
7	$\sum -\frac{1}{6(P_h^G)^6}$	Formulation is an NLP
∞	Implemented through same P_h^G for all households	Same as OPF-Generation

It is worth noting that the cases when $\alpha = 0$ (i.e., the maximum throughput scenario) and $\alpha \rightarrow \infty$ (i.e., the max-min fairness scenario) are also present in the previous case study (Section 4.4): when $\alpha = 0$, the formulation is identical to OPF-Total; when $\alpha \rightarrow \infty$, the formulation is identical to OPF-Generation. The newly introduced scenarios include $\alpha = 1$ (i.e., the proportional fairness scenario) as well as $\alpha = 4$ and $\alpha = 7$ (which represents intermediate stages between proportional fairness and max-min fairness) on the fairness-efficiency spectrum (Figure 4-4).

Since the OPF formulations for α values of 1, 4 and 7 (i.e., cases B, C and D, respectively) are non-linear, the OPF problem is solved using the non-linear solver CONOPT v4.0 [96]. Furthermore, although the investigations of higher values of α is desirable, since α appears as a power term in the objective function of OPF-Alpha-Fairness, the resulting OPF problem becomes numerically unstable for the adopted solver. Therefore, higher values of α are not considered in this thesis (as tackling this numerical issue is beyond the scope). Furthermore, since the analysis in this case study is to highlight the trade-offs between fairness and efficiency, not to demonstrate the scalability of the proposed OPF formulation when dealing with non-linear objective functions. Therefore, the computation times for new scenarios investigated in this case study (which are expected to be much longer than a linear formulation) are not assessed.

Finally, it is worth noting that, in the non-linear formulations (when α values of 1, 4 and 7), the objective functions are all quasi-convex. Consequently, a bisection method using a sequence of convex linear programs [14] can be utilised to obtain global optimality and avoid the use of a non-linear solver. However, this is also beyond the scope of this work.

4.5.2 Results

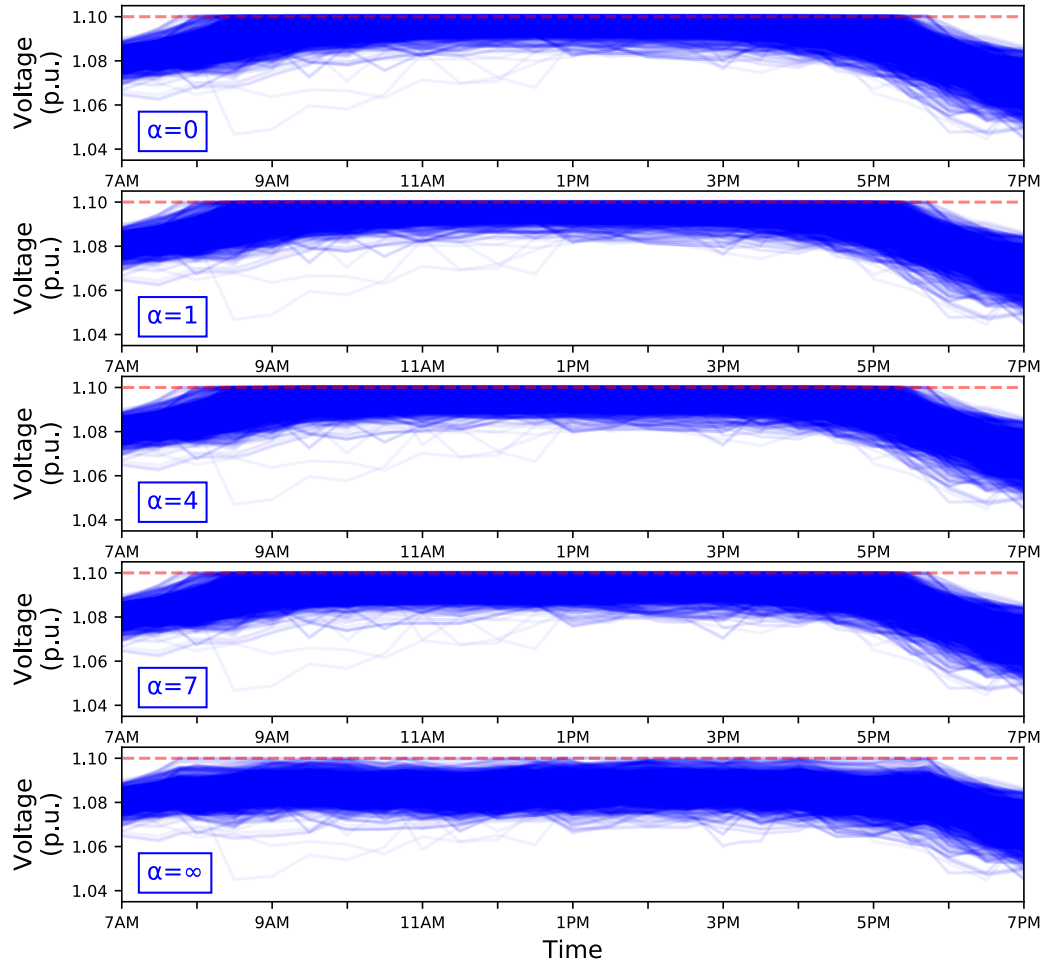
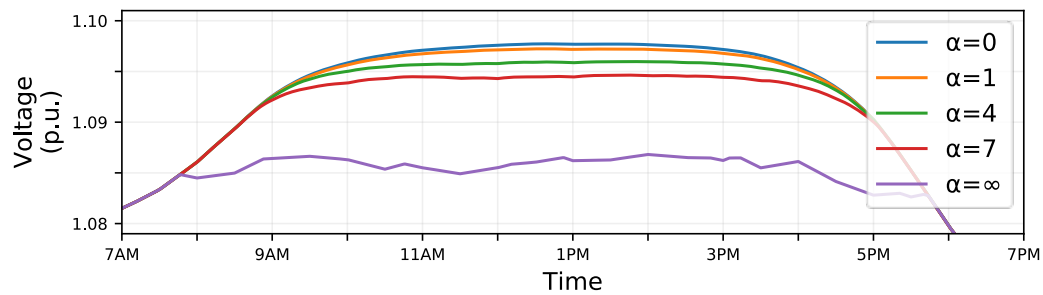
Distribution Network

As shown in Figure 4-14, and also as expected, the network voltages are always maintained within the statutory limit. Furthermore, apart from the max-min scenario (i.e., when $\alpha \rightarrow \infty$), a similar pattern to maximum throughput scenario (i.e., when $\alpha = 0$) is seen for the other α values (i.e., 1, 4 and 7) where the voltages are much more concentrated towards the upper limit.

While the differences in voltage plots for α values of 0, 1, 4 and 7 is not obvious from Figure 4-14, a plot of the average voltage across all houses, as shown in Figure 4-15, reveals a subtle decrease as the optimisation objective travels from favouring efficiency to fairness (i.e., as the value of α increases). This is also accompanied by a steady decrease in the total energy harvested, as shown in Table 4-6.

Table 4-6. Aggregated energy statistics for different α values.

α	0	1	2	3	∞
Energy Harvested (MWh)	60.5	59.3	56.3	53.3	33.4

**Figure 4-14. Time-series voltage profiles for all 4,626 households.****Figure 4-15. Average network voltages for different for different α values.**

Fairness Assessment

A boxplot of the average PHI for all households (adopting the same four quartiles as before) is shown in Figure 4-16. Here, there are two distinct features as the value of α increases (i.e., becoming increasingly fair based on the fairness-efficiency spectrum). Firstly, as indicated by the red shaded region, the relative spread between the medians steadily decreases, which means, the scheme is becoming increasingly fair by reducing the disparity in curtailment. Secondly, as indicated by the red line, the minimum PHI (which corresponds to the most heavily penalised household) steadily increases, which means, the least fortunate household is increasingly being protected as fairness improves. This gradual increase in fairness is also reflected through the two adopted fairness metrics (i.e., JFI and mGC), as shown in Table 4-7, where the JFI and mGC continuous to improve as α increases.

Table 4-7. Jain's Fairness Index and modified Gini Coefficient for different α values.

α	0	1	4	7	∞
JFI	0.907	0.951	0.960	0.962	1.000
mGC	0.883	0.904	0.909	0.910	1.000

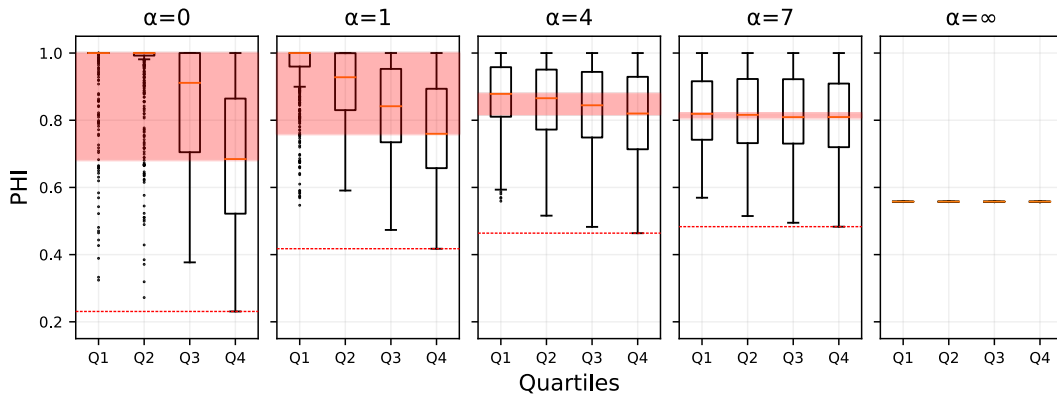


Figure 4-16. Boxplots of average PHI for different α values.

Finally, it is worth acknowledging that, for the scenario considered here, the marginal improvement in fairness may result in appreciable impact on the overall efficiency (i.e., total energy harvest), as shown in Table 4-8. This is especially so after the proportional fairness case (i.e., for $\alpha > 1$). For instance, while an increase of α

from 4 to 7 resulted in a marginal loss of 5% in the total energy harvested, the corresponding marginal gain in fairness is less than 1% for both the JFI and mGC.

Table 4-8. Marginal change as α increases.

α	0	1	4	7	...	∞
ΔEnergy Harvested	-	-2%	-5%	-5%	...	-37%
ΔJFI	-	5%	1%	0.2%		4%
ΔmGC	-	2%	0.6%	0.04%		10%

4.6 Summary

While active power curtailment is a simple yet effective solution to mitigate the issues associated with excessive reverse power flows, applying it in a fair and effective way is not necessarily a straightforward process in a residential context, as demonstrated in this chapter.

The first challenge is due to the many, and potentially conflicting, perspectives that can be used to quantify the impact on households, such as PV harvesting, energy exports and financial benefits. As demonstrated in this chapter, a curtailment scheme that is designed to be fair in one of these perspectives may not be so in others.

The second challenge is due to the trade-offs between achieving absolute fairness (where there is no disparity among households for a given perspective) and its impact on overall efficiency (e.g., aggregated PV harvesting). Since there is an inherent trade-off between fairness and efficiency, this may justify the need for alternative avenues to address the resulting disparities, such as through compensation schemes.

Overall, the results highlight that, since fairness is a multi-facet problem, an ultimate fair scheme may not exist. Consequently, there is always the need for decision-makers to determine the most appropriate objectives and compromises that best aligns with their respective policies. To this end, the methodology presented in this chapter can help to compare different schemes so as to allow more informed decisions.

5 A HARDWARE-IN-THE-LOOP DEMONSTRATION PLATFORM

5.1 Introduction

Distribution networks studies are typically carried out offline, using PC-based power flow analysis packages; a few notable names include OpenDSS [60], MATPOWER [97] and PSS®SINCAL [98]. Furthermore, to cater for advanced schemes, co-simulation setups are often adopted, such as the one introduced in Section 2.4.1 that uses a Python script and AIMMS, as these power flow analysis packages are typically not built with such functionality.

Although these offline approaches are indispensable to the research and development of new concepts, there are limitations from a validation and demonstration perspective. Firstly, they are based on software models, and thus it is difficult to realistically capture all the intricacies of physical systems. For instance, in the context of active distribution networks, using scripts/code is not possible to realistically model the physical interactions between the control room (and the associated infrastructure such as SCADA, distribution management system, advanced schemes, etc.) and the devices in a distribution network (e.g., DG units, OLTCs, etc.). Furthermore, it often lacks the flexibility (or capability) for live demonstration purposes, which require the real-time visualisation of simulation data rather than creating tables and plots after the simulation has finished. Therefore, in addition to the development of adequate schemes for future distribution networks, new approaches to facilitate the real-time validation and demonstration of these schemes are also desirable. Particularly, this helps to boost industry's confidence in adopting these advanced approaches in real networks.

To this end, this can be accomplished through an online approach using a real-time simulator (RTS). An RTS is specialised hardware that not only can carry out network simulations in real-time¹ but also features extensive interfaces to facilitate the

¹ Real-time simulation is achieved when, given an adopted simulation time-step (e.g., around microseconds for electromagnetic transient simulations in power systems), the computation time is guaranteed to be less than the adopted time-step.

interactions of physical systems/hardware with the simulated network; notable examples of an RTS include the RTDS Simulator [99] and the Opal-RT Simulator [100]. Therefore, leveraging these capabilities of an RTS, the development of a realistic hardware-in-the-loop (HIL) demonstration platform for advanced schemes in future distribution networks is presented in this chapter.

5.2 Literature Review

In general, HIL-based studies can be classified in two categories: controller HIL and power HIL. Their main differences lie in the physical hardware being tested and the interfaces required to the simulated network (in the RTS). For a controller HIL simulation, it only involves low voltage data communications (e.g., through an Ethernet connection or analogue signals) to the RTS; all power system components (e.g., lines, transformers and DG units) are modelled in the RTS. One such example is [101] that investigates the usage of demand-response signals to manage voltages in active distribution networks. In the proposed setup, the proposed network controller that is responsible for calculating the required demand-response signals is implemented on a PC and interacts with the simulated network (in an RTS) via both Ethernet and low-voltage analogue signals. Similar setups can also be found in [102] (for a Volt-VAR optimisation), [103] (for state estimation in low-voltage networks) and [104] (for adaptive protection schemes). In contrast, a power HIL simulation also involves real power devices to be interfaced with the RTS; this can range from a few kW (e.g., a rooftop PV system in [105] and a residential battery system in [106]) to hundreds of kW (e.g., a wind generator in [107]). To facilitate this, power amplifiers (also known as grid simulators) are required to replicate the responses of the simulated network such that the real power devices can be operated and tested as if they are physically connected to the simulated networks.

There are also novel approaches to extend the capability of a conventional HIL setup. For instance, a key limitation of an RTS is the size of the network that can be modelled; due to the finite computing power and the rigid restriction on computation time to guarantee real-time simulation, there is an inherent limit on the size of the network that can be modelled by an RTS. To tackle this, a hybrid setup that consists of both offline software and an RTS is developed in [108] to model larger distribution networks that are technically not feasible for an RTS.

Although HIL studies are becoming increasingly common in the literature, the main focus has largely been on the validation aspect. Consequently, there are limited implementations that are also designed for live demonstration purposes; this is not a trivial task as it also involves specialised software and hardware from other domains. Therefore, to address this, the architecture of such a HIL demonstration platform and an example implementation is presented in the remainder of this chapter.

5.3 Architecture

The architecture of the proposed HIL demonstration platform is shown in Figure 5-1. The design is based on the key elements found in existing distribution networks and the functionality of each element is explained below.

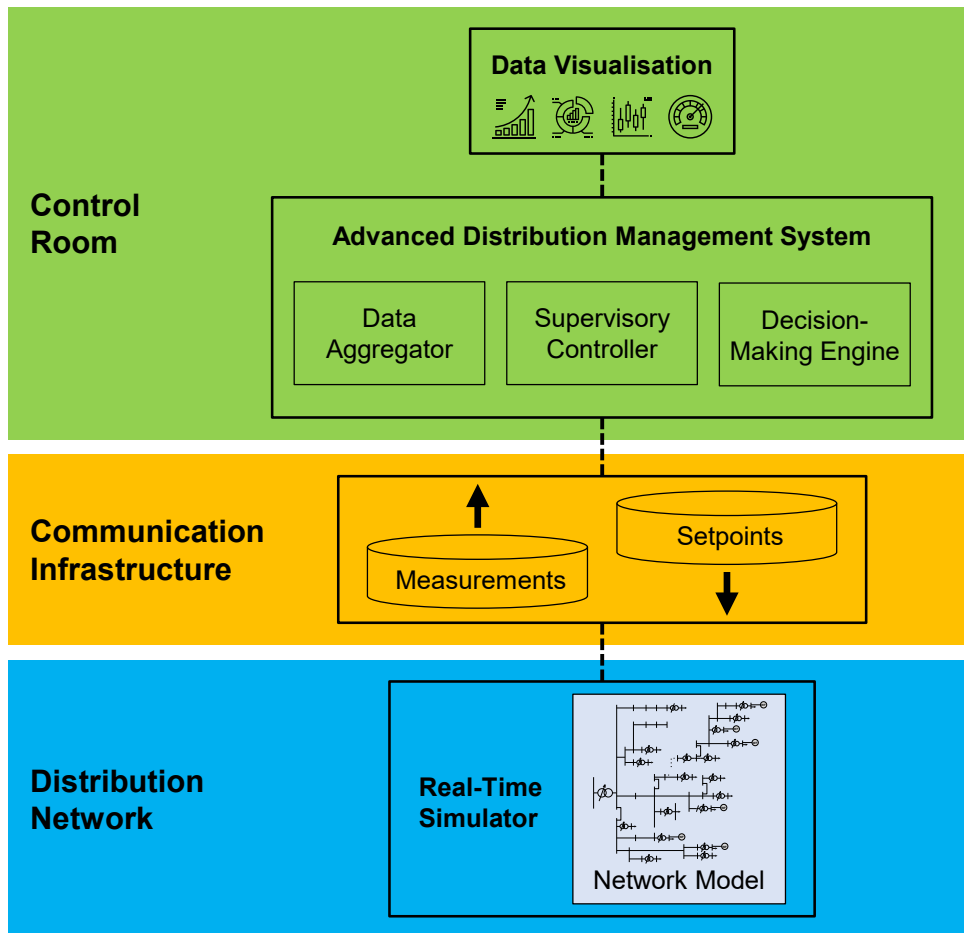


Figure 5-1. Architecture of the HIL demonstration platform.

5.3.1 Distribution Network

A controller HIL setup is considered where the distribution network is modelled in the RTS. This includes the power system elements (e.g., lines, transformers, loads and DG units as well as the associated device controllers) that utilise the available model of an RTS and the communication elements that utilise the hardware interfaces of the RTS (e.g., DNP3 [109] over Ethernet).

5.3.2 Communication Infrastructure

Physical communication infrastructure (based on the hardware capability of the RTS) is used to facilitate data transfers between the RTS and the control room. This includes the necessary network measurements (RTS $\rightarrow$ control room) for the advanced scheme and the setpoints (control room $\rightarrow$ RTS) determined by the advance scheme.

5.3.3 Control Room

Inside the control room, an advanced distribution management system (ADMS) that contains a data aggregator, a supervisory controller and an advanced decision-making engine is created. The data aggregator is responsible for handling the communication with the distribution network; this is necessary for compatibility reasons as several different protocols can be used in typical distribution networks. The supervisory controller is responsible for overseeing the real-time operation of the distribution network, such as identifying constraint violations. The supervisory controller is also responsible for accessing the decision-making engine (if required) to determine new setpoints for the controllable devices in the distribution network; this can be done in regular intervals or when constraint violation is detected.

In addition to the ADMS, a rich data visualisation interface is also created inside the control room. This is typically achieved through an interactive user interface displayed on multiple large monitors.

5.4 Implementation of a HIL Demonstration Platform

Based on the proposed architecture, the implementation details of a HIL demonstration platform (shown in Figure 5-2) at the Smart Grid Lab of The University of Melbourne [110] is presented in this section. This is combined with the network

setup in Chapter 3 to create a live demonstration of an active distribution network where an OPF-based scheme is used to actively manage the wind farms and OLTCs.

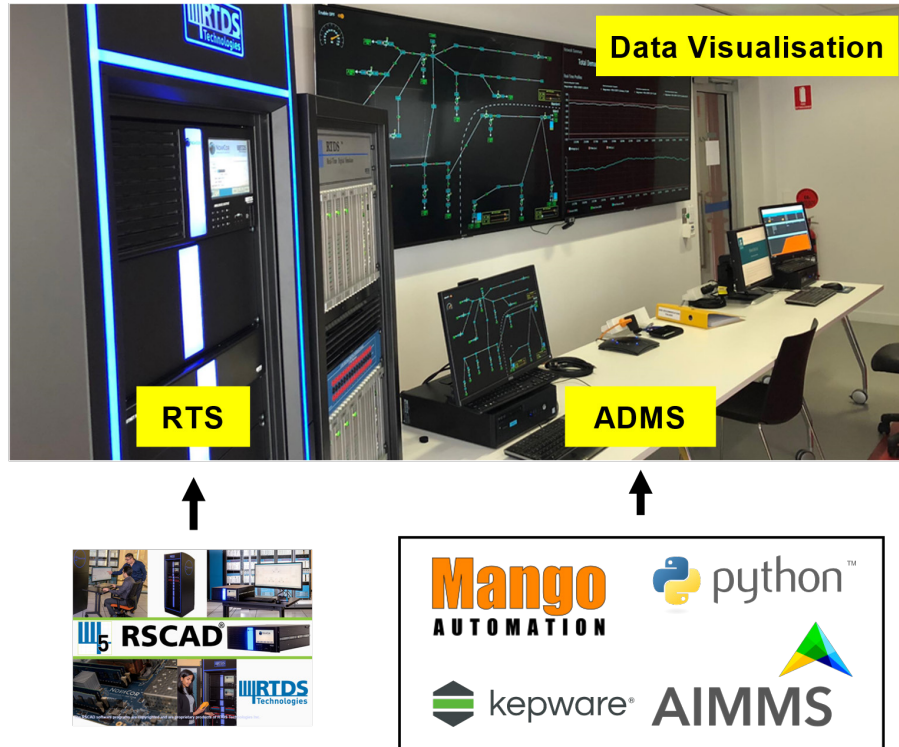


Figure 5-2. Implementation of a HIL demonstration platform.

5.4.1 Real-Time Simulator

The RTDS Simulator is used as the RTS. It consists of a hardware component known as the NovaCor (depicted on the top left of Figure 5-2) and a software component known as RSCAD. RSCAD is responsible for creating the network model (using the Draft module) and managing the simulation hardware (using the Runtime module). On the other hand, the actual power flow calculations and external interfaces to the control room are handled by the NovaCor.

The RTDS Simulator offers two distinctly different simulation modes (normal and distribution), each with associated trade-offs between the maximum size of the network and the minimum simulation time-step. As discussed previously, this inherent trade-off exists due to a finite computing power (e.g., number of processors). In the normal mode, a much smaller time-step of $50\mu\text{s}$ is possible while the maximum number of nodes that can be modelled is only around two hundred. This mode is primarily targeted at dynamic studies that requires granular data within an AC cycle.

On the other hand, while the distribution mode uses a much larger time-step of 150 μ s, it is capable of handling over two thousand nodes. Since typical distribution studies only require steady-state power flow data, this mode is much more suitable due to the increased modelling capability to handle larger networks. Therefore, the distribution mode is adopted in this setup.

Lastly, while the Runtime module also can be used for visualisation purposes, the available resources is limited to create a rich user interface. Therefore, an external software is used (as discussed later); the built-in features are used for debugging purposes only.

5.4.2 Communication Infrastructure

The RTDS Simulator supports an extensive range of protocols found in power systems. For this implementation, DNP3 over TCP/IP, a commonly used protocol in distribution networks, is used. Physically (not shown in Figure 5-2), a direct Ethernet connection is adopted between the NovaCor and the computers inside the control room.

While aspects related to an imperfect communication channel (e.g., data loss and latency) are not investigated, more advanced communication network simulators (e.g., WANem [111]) can be incorporated in the proposed setup to achieve this purpose.

5.4.3 Advanced Network Management System

The ADMS in the control room is implemented on two computers (PC-1 and PC-2) using several commercially available software. The data aggregator is implemented on PC-1 using the Kepware KEPServerEX [112]. It is responsible for data translation between DNP3 (protocol used by the RTDS) and OPC DA [113] (protocol used by the supervisory controller). The remaining elements of the ADMS are implemented on PC-2. Particularly, the supervisory controller is implemented using Mango Automation [114], an integrated platform for SCADA applications in industrial plants. The decision-making engine consists of a Python script and AIMMS, which is similar to the co-simulation setup presented in Section 2.4.1.

The supervisory role (i.e., checking for constraint violations and triggering the decision-making engine) is handled by the Mango Automation, if new setpoints are required, Mango Automation will invoke the Python script which, then, launches

AIMMS to calculate the new setpoints. For this implementation, the same control cycle-based approach (discussed in Section 3.3.1) is adopted.

An overview of data flows in the proposed setup is highlighted in Figure 5-3.

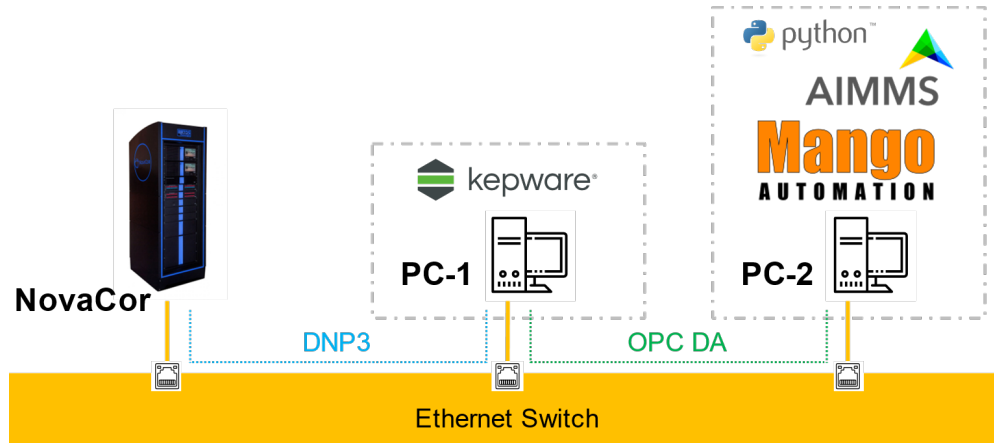


Figure 5-3. Data flows in the proposed setup.

5.4.4 Data Visualisation

The built-in dashboard designer of Mango Automation is used to create custom, HTML-based dashboards for data visualisation. Particularly, two dashboards are created, one for each of the wall-mounted TVs shown in Figure 5-2.

Dashboard one, shown in Figure 5-4, features the network diagram in a map-like format to highlight the instantaneous power flows within the network. The main features include nodal voltages, asset utilisation, demand, OLTC tap positions and wind farm controls, as shown in Figure 5-5. The base design of the dashboard is drawn as a static image in the scalable vector graphics (SVG) format. Then, live operational data are used to animate the base design using CSS functions (commonly known as CSS Animations), e.g., changing the nodal voltages and animating flashing signals.

Dashboard two, shown in Figure 5-6, offers the ability to plot (selectable) network measurements over time, such as the nodal voltages and output of wind farms. This is achieved using the built-in graphing features of Mango Automation.

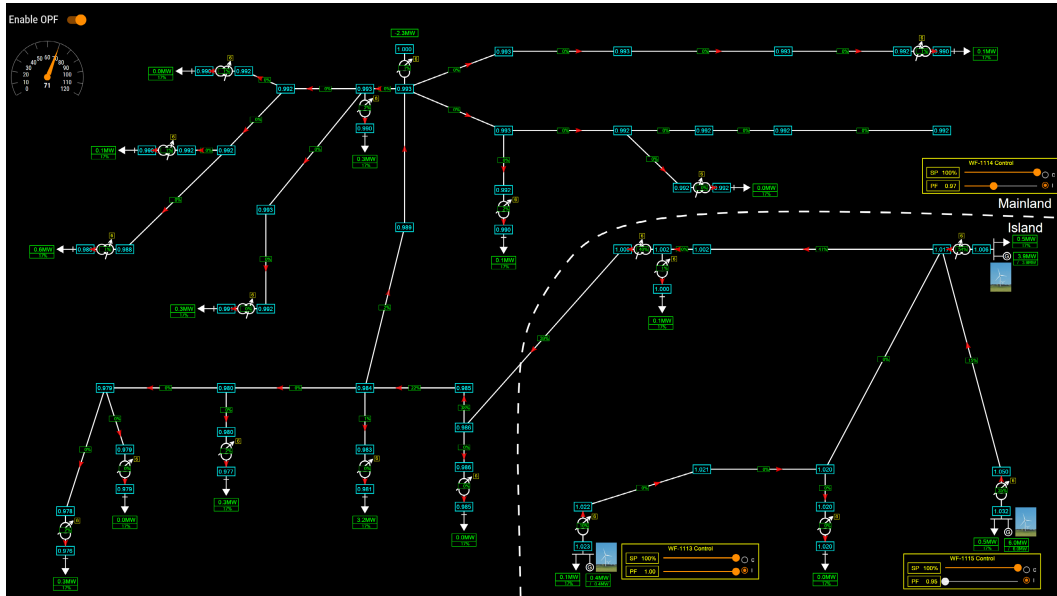


Figure 5-4. Screenshot of dashboard one.

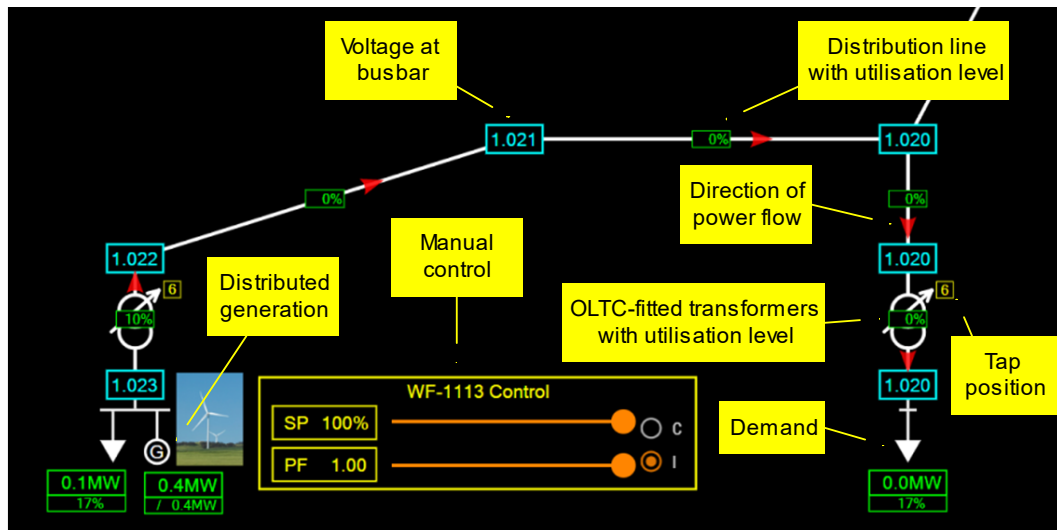


Figure 5-5. Main design elements of dashboard one.

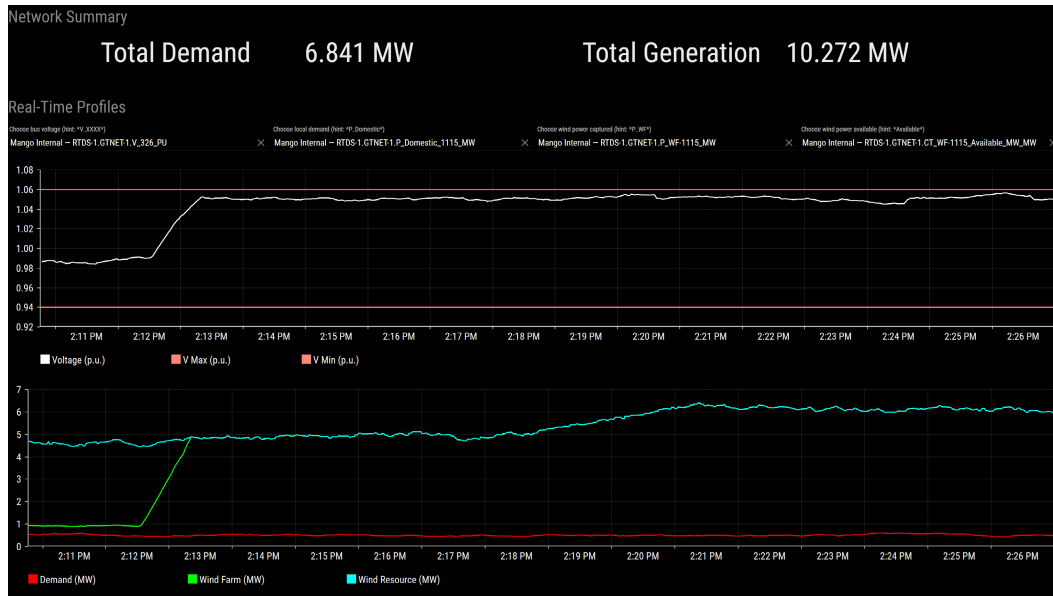


Figure 5-6. Screenshot of dashboard two.

5.5 Live Demonstration

A live demonstration of the case study in Chapter 3 is shown in Figure 5-7, where an OPF-based scheme is used to control several wind farms and the OLTCs in an active distribution network. A short video can also be found on youtu.be/1TxaNIqTno4 [115].

In Figure 5-7, the voltage at node 326 (white line) and the statutory limits (pink lines) are shown on the dashboard 2 (as indicated by the orange arrows). Furthermore, the demand at node 1115 (red line), active power output of wind farm 1115 (WF. 1115) and the available wind power (blue line) are also shown.

Here, it can be seen that, prior to 2:12PM, a heavy curtailment is replaced on WF 1115, indicated by the gap between the available wind power and actual output. Immediately after 2:12PM, the OPF-based scheme is enabled to routinely evaluate the optimal setpoints for all wind farms and OLTCs. As a result, throughout the rest of the period depicted in Figure 5-7, the wind farms are available to fully harvest the available wind resource and the voltages are also maintained within the statutory limits (despite the continuous changes in wind generation).

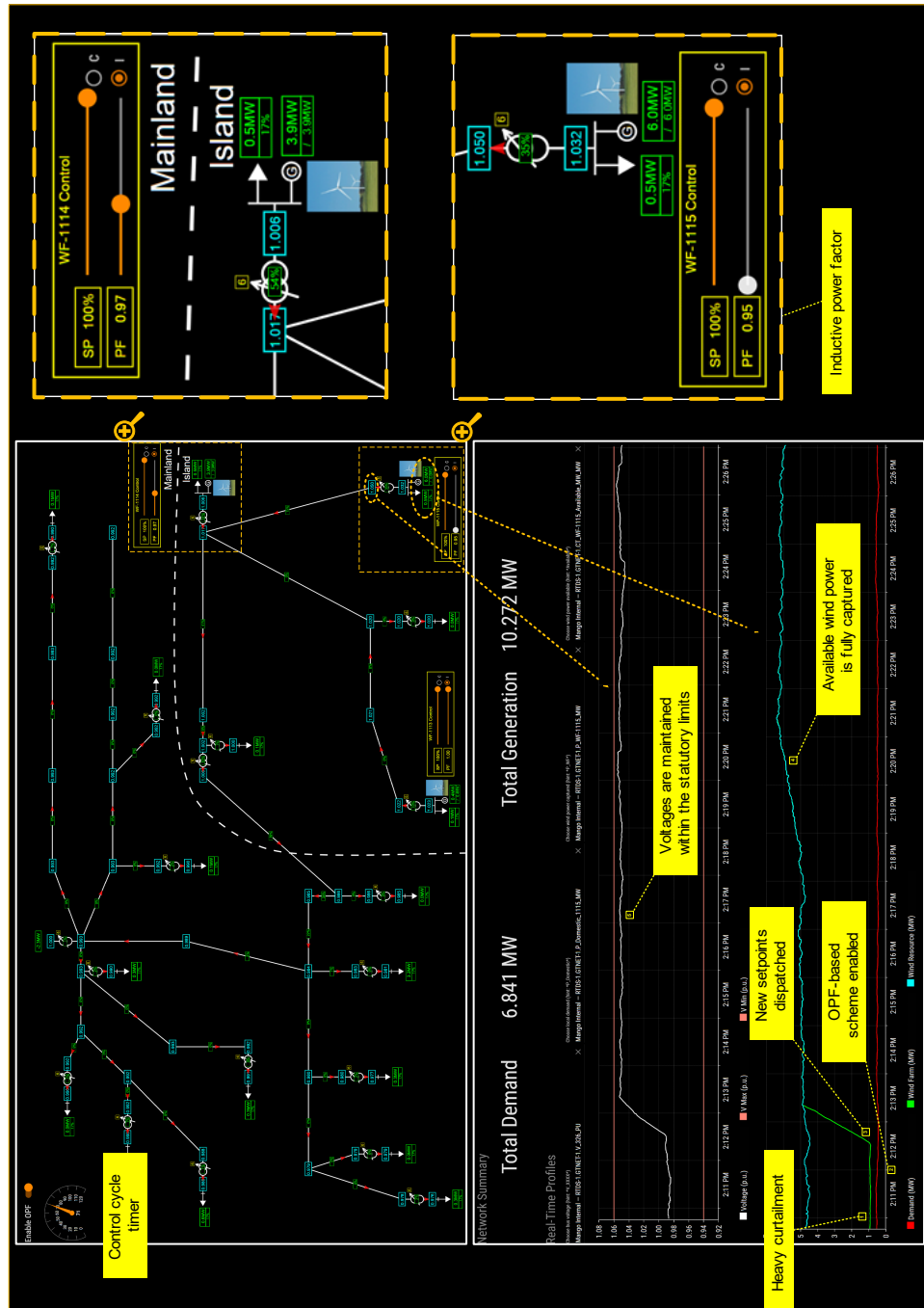


Figure 5-7. A live demonstration of an OPF-based scheme for active distribution networks.

5.6 Summary

Traditionally, distribution networks studies are carried out offline, using PC-based software. Although indispensable, there are several limitations from a validation and demonstration perspective as it is difficult to capture all the intricacies of physical systems using software models. Furthermore, while the validation aspect has been examined in the existing literature through the usage of HIL simulation, the demonstration aspect is less frequently discussed.

To this end, the architecture of a HIL platform (with a particular focus on the demonstration and data visualisation) is proposed. Based on this, a specific implementation using the RTDS Simulator and several commercially available software at the Smart Grid Lab of The University of Melbourne is presented. The implemented HIL platform is then used to demonstrate the active control of wind farms and OLTCs using an OPF-based scheme.

A highlight of such HIL setups is the ability to showcase (in a control room-like environment) that advanced concepts for future distribution networks, and thus helping to prove that these advanced concepts are getting closer and closer to real-world implementation.

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6 CONCLUSION

In this chapter, a summary of the research gaps and contributions is first presented. Then, the key findings of this thesis are outlined. Finally, potential improvements and extensions based on the concepts explored in this thesis are introduced.

6.1 Research Gaps and Contributions

Driven by the increasing environmental awareness and financial incentives, renewable DG, from MW-scale wind farms in rural areas to kW-scale PV systems on city's rooftops, has seen a tremendous growth over the last decade. At the same time, this sudden influx of power generation within the distribution network—the part of a power system that is typically associated with the last-mile delivery of power to consumers, not necessarily facilitating the bidirectional power flows as a result of local generation—has introduced significant operational challenges for distribution companies. Particularly, technical issues such as voltage rise and asset congestion are becoming increasingly common in renewable-rich distribution networks.

In this context, the concept of active distribution networks, where network issues are solved at the operational stage through the real-time monitoring and control of network assets and participants, is becoming an increasingly attractive alternative to the otherwise capital-intensive and time-consuming network augmentation. For instance, existing OLTCs and the reactive power capability of DG units can be simultaneously exploited to alleviate voltage issues. As a last resort, active power curtailment can also provide temporary relief from voltage and thermal violations by reducing the volume of reverse power flows during constrained periods.

However, as more and more sources of flexibility are becoming available to network operators, existing rule-based approaches (commonly adopted by industry) are also becoming prohibitively cumbersome due to the fact that many variables (e.g., setpoints for each of the controllable devices) and constraints (e.g., voltage limits at multiple nodes) must be to be considered at the same time. To this end, AC OPF—a mathematical optimisation problem that seeks the optimal setpoints while satisfying the network constraints—has shown great potential (albeit mostly in academia) as the

decision-making engine for more advanced schemes, and thus enabling a more systematic approach to the active management of these various sources of flexibility.

Nonetheless, before OPF-based schemes can be readily integrated in the control rooms of future network operators, there are several challenges that must be addressed. These challenges as well as the corresponding contributions and findings of this thesis are highlighted in the following sub-sections.

6.1.1 Advanced OPF Formulation

Due to the non-linearity and non-convexity of AC power flow equations, classical OPF formulations are known to have scalability and speed issues for operational usage where there is limited computation time. This is exacerbated by the added complexity of modelling distribution networks, such as the need for a three-phase model and integer variables. While alternative formulations have been proposed in the existing literature, ranging from convex relaxations to linear approximations, there are two areas that have not been adequately addressed: a linearised model for delta-wye transformers and the modelling of discrete devices in distribution networks.

Firstly, a linear model of delta-wye transformers has not been considered by existing three-phase OPF formulations, which is necessary to accurately model multi-voltage level distribution networks (e.g., from the primary substation in the MV feeder down to residential consumers in LV feeders). Consequently, the extension to also cater for the discrete nature of OLTCs' taps and any other integer variables have not been investigated in the context of multi-voltage level distribution networks.

To address these gaps, a linearised, three-phase OPF formulation is proposed for operational usage in active distribution networks. The main contributions and advantages of the proposed formulation are summarised below:

- A linearised delta-wye transformer model is proposed to cater for the modelling of multi-voltage level networks. Furthermore, an approximation method using an offset factor (calculated from the latest network measurement) is proposed to improve the accuracy of the linearised model.
- The proposed formulation is fully linear (i.e., a LP, the simplest form of mathematical optimisation problems), and thus significantly improves its scalability and speed over the classical non-linear formulation. Furthermore,

established techniques of solving MILP (e.g., branch and cut, advanced basis, etc.) can be exploited to effectively handle the modelling of discrete devices as well as any integer variables in general.

- The proposed OPF formulation is readily implementable using commercially available solver; no bespoke algorithms required.

6.1.2 Practical Considerations: Real-World Implementation of OPF-Based Setpoints

While OPF-based schemes for active distribution networks have received significant attention in the existing literature, the main focus has been on the determination of setpoints. In contrast, how these setpoints are realised in real-time (i.e., at a sub-minute scale) by each of the controllable devices is rarely investigated. This is particularly important when multiple devices are being managed simultaneously by an OPF-based scheme. For instance, temporal coordination issues may arise due to a mismatch in their delays and ramp rates, leading to short term constraint violations when new setpoints are requested at the same time. Furthermore, the presence of a deadband in certain device controllers introduces compatibility issues with OPF-based schemes as the precise setpoint may not be achieved.

In this thesis, the necessary practical considerations from a device control perspective are investigated, focusing on the active management of OLTCs and DG units. Particularly, a high granular time-step of 1 second is adopted to realistically capture the sub-minute interactions among controllable devices and the necessary adaptations are proposed to ensure the expected outcome from the OPF-based setpoints are achieved in a real-world application. The main contributions are summarised below:

- The device-specific operating characteristics that affect the real-time implementation of OPF-based setpoints are identified; these are the delays, ramp rates and deadbands of the local device controllers.
- The necessary adaptations in existing device controllers are proposed, which involves the direct control of OLTCs' taps and a new active power ramping behaviour for DG units. The proposed changes ensure an adequate alignment

of delays and ramp rates (between OLTCs and DG units) as well as eliminate the compatibility issues associated with OLTCs' deadbands.

- The need of incorporating operational constraints (e.g., ramp rate of controllable devices) and the minimisation of unnecessary control actions (e.g., through a penalty factor in the objective function) in the OPF formulation are demonstrated. This ensures the setpoints produced by an OPF-based scheme is technically feasible and prevents accelerated equipment wear-and-tear.
- Overall, the results highlight that, to ensure OPF-based schemes can be adequately implemented in an operational environment, it is necessary to extend the analysis beyond conventional power flow studies that mostly focus on power and energy. Particularly, it is crucial to also consider the implications arising from device control, such as the differences in controllable devices' operating characteristics at a sub-minute scale.

6.1.3 Practical Considerations: Fairness of Curtailment Schemes

While active power curtailment is a simple yet effective way of mitigating network issues, it also reduces the return on investment for DG owners. Therefore, adequate fairness considerations are crucial for curtailment schemes. However, existing literature on curtailment fairness is mostly limited to a single perspective that focuses on the impact at the PV system (e.g., the total PV generation), which is not necessarily the most appropriate measure for every application. Particularly, other (and potentially conflicting) perspectives also exist (e.g., the impact on electricity bills) which can result in vastly different impacts on the DG owners. Furthermore, the trade-offs between fairness (removing disparity) and efficiency (aggregated performance) is another crucial aspect that has not been investigated.

Therefore, to explore the implications of different fairness considerations in a residential context, several OPF-based curtailment schemes are proposed, each with a unique and contrasting fairness objective. Then, the performance of the proposed schemes is analysed using a PV-rich distribution feeder. The main contributions are summarised below:

- An assessment methodology using several household-centric metrics is proposed to realistically capture the impact of curtailment schemes from multiple perspectives.

- The different perspectives of fairness (e.g., PV harvesting, energy export and financial benefits) are identified and the associated trade-offs for different fairness objectives are quantified. For instance, a scheme that is designed to be fair in terms of PV harvesting for all households will result in noticeable disparity in terms of their energy exports.
- For the first time in the context of distribution networks, the concept of alpha-fairness (commonly used in internet traffic management) is applied as a way to quantify the relative trade-offs between fairness and efficiency.
- Overall, the results highlight the need of adequate fairness considerations (by network operators, policymakers, etc.) that best align with the corresponding objective when active power curtailment is used.

6.1.4 Laboratory Demonstration

While PC-based software is typically used in distribution network studies, it is often difficult to capture the physical interactions among different systems in an actual distribution network using software models. To this end, HIL simulations has been widely used in the existing literature for validation purposes of advanced schemes as it allows physical hardware to tested. However, there are limited implementations in the existing literature of HIL setups that also focuses on the demonstration aspect, which can be extremely useful as an initial step to showcase advanced schemes to industry (before their eventual trials). Instead, the results are often analysed post simulation using static plots and tables, which is similar to how it is traditionally done using PC-based software.

To address this, the architecture of a HIL demonstration platform is proposed, with a particularly focus on the visualisation of live data. Based on this, a specific implementation is created at the Smart Grid Lab of The University of Melbourne to demonstrate OPF-based schemes for active distribution networks. The main contributions and advantages of the proposed setups are summarised below:

- The process of building a realistic HIL demonstration platform (for advanced schemes in active distribution networks) from various commercially available software and hardware packages is documented.

- The proposed setup extends beyond static plots and tables by introducing a rich and interactive user interface, and thus enabling a more realistic experience for demonstration purposes.

6.2 Key Findings

6.2.1 Linearised Three-Phase OPF Formulation

The scalability and speed of the proposed OPF formulation is evaluated using a realistic MV-LV feeder in Victoria, Australia, which starts from the primary substation and ends at the connection points of individual residential consumers. In total, the modelled feeder consists of 4,947 three-phase nodes and 4,626 single-phase connected residential consumers. The main findings are:

- The proposed linearised OPF is highly accurate in terms of the calculated voltages and power flows (when compared with the actual values from a network simulator after the calculated setpoints are implemented) even if flat start values are used for the estimated terms in the linearised formulation.
- The accuracy of the linearised formulation can be improved through warm start values (e.g., using recent network measurements) as well as running a second iteration of the OPF (where the results from the first OPF run is used in the second iteration). Furthermore, while the latter is found to give the largest improvement, it also requires the OPF problem to be solved twice (i.e., it compromises on the solution speed).
- The solution time is sufficiently fast (within a minute for one iteration) even when integer variables are introduced (e.g., 3 binary variables to model the OLTC's taps, as investigated in Chapter 2, and over 2,000 binary variables to model the export status of households, as investigated in Chapter 4).

6.2.2 OPF-Based Schemes in an Operational Environment

The operational usage of OPF-based schemes is demonstrated using a UK-style rural distribution feeder with 21 OLTCs and 6 wind farms. The main findings are:

- Due to the mismatch between the delays and ramp rates of existing device controllers (of OLTCs and wind farms), temporary issues can arise when OPF-based setpoints are implemented in practice. In contrast, once their delays and

ramp rates are aligned through the propose adaptations, the desirable behaviour can be achieved.

- The presence of deadbands (e.g., within existing controllers for OLTCs) introduces compatibility issues with OPF-based schemes as the requested setpoint cannot be precisely realised, leading to excessive number of control actions. In contrast, the direct control of OLTCs' taps (as proposed) is found to be much more effective as the tap position calculated by the OPF can be precisely implemented in practice.
- Due to the various sources of flexibility within distribution networks, inadequate design of a control scheme (i.e., not penalising unnecessary control actions) may result in an oscillatory behaviour among multiple sets of feasible control actions, and thus accelerated equipment wear-and-tear. Therefore, the operational usage of OPF-based schemes must ensure the unnecessary control actions are penalised (e.g., through a penalty factor in the objective function).
- High granularity modelling is essential to realistically capture the aforementioned complications that arise from the different operating characteristics of controllable devices in the sub-minute scale.

6.2.3 Fairness of Curtailment Schemes in Residential Feeders

The implications and trade-offs from different fairness considerations are investigated using several OPF-based schemes, each considering a unique and contrasting fairness objective. The main findings are:

- Since the impact from curtailment can be quantified in multiple ways, e.g., based on PV harvesting, energy export or financial benefits, there are also multiple perspectives of fairness. Furthermore, improving fairness in one particular perspective may lead to a decrease in fairness for the other perspectives.
- There is an inherent trade-off between fairness (i.e., equality) and efficiency (i.e., aggregated performance). Particularly, when voltage is the limiting factor in radial distribution feeders, the marginal gain in fairness may require substantial compromises in terms of the overall efficiency.

- While the definition of fairness is relatively simple, i.e., ensuring equality, incorporating fairness in curtailment schemes is not necessarily a straightforward process due to the multi-facet nature of fairness in the context of distribution networks. Therefore, before attempting to improve fairness, it is necessary to first define the most appropriate fairness objective for a given application.

6.3 Future Works

6.3.1 Distributed Algorithms for OPF

In this thesis, the OPF formulation is solved as single problem. While the solution time has been adequate for the investigated networks, alternative approaches that can further improve the solution time is also desirable to cater for larger/more complex network models. One such approach is through a distributed algorithm that breaks down the single OPF problem into multiple smaller sub-problems that can be solved in parallel. Particularly, an interesting aspect of a distributed approach is the improvement in solution time when many integer variables are considered (e.g., when many OLTCs are being modelled, the formulation for OPF-Export in Chapter 4 or OPF formulations that involves batteries where binary variables are often needed to model the charging/discharging state [20]). Due to the combinatory nature of mixed-integer programming where an exhaust search is often required to find the optimal solution, the process of breaking down the main problem into multiple sub-problems is expected to bring significant benefits. For instance, a single OPF with $2x$ binary variables will result in 2^{2x} different combinations; in contrast, two sub-problems each with x binary variables (still $2x$ in total) only consists of 2^x combinations (for each sub-problem), which is an exponential reduction from 2^{2x} . Therefore, the significant reduction in complexity from a mixed-integer programming perspective can outweigh the extra complexity associated with distributed algorithms.

6.3.2 More Accurate Forecast vs. Shorter Control Cycles

In Chapter 3, persistent forecast is adopted for simplicity where the resource availability (for wind farms) and the demand is expected to remain the same when new setpoints are calculated. Therefore, the extent of improvements from more accurate forecast is worth exploring. Furthermore, the potential benefits of using short control

cycles (as opposed to more accurate forecast, but with longer control cycles) should be explored. Particularly, given that, the level of uncertainty in forecasts is expected to reduce as the duration of each control cycle shortens, the benefits from more accurate forecast algorithm is also likely to reduce.

6.3.3 Multi-Period Optimisation

The proposed scheme only considered a single snapshot (i.e., the upcoming control cycle) when decisions are made. An extension of this work is to quantify the potential benefits from performing a multi-period optimisation, such as using the receding-horizon technique, where multiple snapshots in the future are modelled. This can be particularly beneficial in reducing the number of unnecessary control cycles. Nonetheless, it also has the drawbacks of longer computation time.

6.3.4 More Scalable Formulation of OPF-Alpha-Fairness

While the concept of alpha-fairness is successfully used (in an offline analysis) to quantify the trade-offs between fairness and efficiency, due to the nonlinear nature of the corresponding objective functions, the resulting formulation (i.e., OPF-Alpha-Fairness in Chapter 4) is not necessarily practical in an operational environment. For instance, a single snapshot can require as long as ten minutes to solve. Therefore, alternative formulations are desirable to improve the solution speed, and thus enabling the operational usage of OPF-Alpha-Fairness.

6.3.5 Fairness Implications when Additional Sources of Flexibility are Considered

In Chapter 4, only active power curtailment is considered. Therefore, it is worth extending the analysis to incorporate other sources of flexibility, such as the reactive power of PV inverters and the OLTC commonly found at primary substations. Naturally, the OLTC is expected to bring benefits for all households in a PV-rich distribution network where voltage rise is the issue. However, due to the apparent power limits of PV inverters, using the reactive power is expected to bring additional complications depending on whether active power or reactive power is prioritised.

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