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# ESSAYS IN INDUSTRIAL ORGANISATION

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SVETLANA DANILKINA

(ORCID: 0000-0002-4893-1432)

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# Abstract

This thesis contains three essays in Industrial Organisation. The first two are united by a common theme: the role of bounded rationality in shaping market behaviour.

In Chapter 2, we provide a novel explanation of the relationship between product quality and variety. We develop a theoretical model with nearly rational consumers who randomize among all alternatives that yield the same utility for them but can mistakenly choose a slightly worse alternative. We show that a firm price discriminates more effectively if it offers more varieties of its higher-quality, more-profitable, products. When the market is thin at the very top quality level, the number of varieties first increases and then decreases as quality increases.

In Chapter 3, we consider a model of Bertrand oligopoly when consumers are boundedly rational and make their purchase decisions probabilistically, according to the Luce model. We consider three different cases: first, we characterize equilibrium when firms face boundedly rational consumers with the fixed irrationality parameter  $\lambda$ ; second, we discuss the case of obfuscating oligopoly, when firms can invest in order to confuse consumers, i.e. to increase their  $\lambda$ ; and third, we consider educating oligopoly, when firms can choose to invest to decrease  $\lambda$ . We show that while it is worthwhile for the firms to confuse the consumers, it is only optimal to educate them if they are sufficiently rational at default. We also analyze how the social welfare, consumer surplus and the firms' profits depend on the number of firms.

The final essay, Chapter 4, examines the optimal procurement mechanisms when the procurer tests potential contractors for reliability either before or after the auction. Testing suppliers before the procurement auction is the procedure typically followed by US supply chains, while public procurement in the European Union typically take place by running an auction first and then testing the reliability of the low bidders afterwards. We compare both approaches and show that when testing can be performed at no cost and the procurer and contractors are risk neutral, the timing of the test (before vs after the auction) has no effect on the procurer's expected surplus, the contractors' expected payoffs, or the total surplus. In case of costly testing a risk neutral procurer would prefer testing after the auction.



# Declarations

This is to certify that:

1. the thesis comprises only my original work towards the Ph.D. except where indicated in the Preface;
2. due acknowledgment has been made in the text to all other material used; and
3. the thesis is fewer than 100,000 words in length, exclusive of tables, figures, bibliographies, and appendices.

Svetlana Danilkina



# Preface

This thesis contains original research in Chapters 2 through 4.

Chapter 2 is based on joint work with S. Basov. The theoretical model we developed was combined with empirical work done by D. Prentice, which resulted in the following published paper:

Basov, S., S. Danilkina, D. Prentice (2020). When Does Variety Increase with Quality? *Review of Industrial Organization*, 56(3), pp. 463–487.

Chapter 3 is based on the following published paper:

Basov, S., S. Danilkina (2015). Bertrand Oligopoly with Boundedly Rational Consumers. *The B.E. Journal of Theoretical Economics*, 15(1), pp. 107-123.

Chapter 4 is based on joint work with C. Mezzetti.

All co-authorship has taken place in accordance with the Graduate Research Training Policy of the University of Melbourne.



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My research benefited from discussions and many useful comments provided by many colleagues over the course of my PhD, those who participated in seminars, workshops and conferences or gave me their comments privately.

Special thank you goes to Lawrence J. White, the editor of *Review of Industrial Organization*, for his insightful and thorough comments that helped to improve our paper significantly.

And last but not least, I would like to thank my wonderful coauthors: Suren Basov, Claudio Mezzetti and David Prentice.



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# Chapter 1

## Introduction

This thesis contains three essays in Industrial Organisation. The first two are united by a common theme: the role of bounded rationality in shaping market behavior. The third essay explores the optimal procurement auction when the potential ability of contractors to perform the task is not known beforehand and must be tested. The crucial issue is whether such testing should occur before or after the auction is conducted.

The concept of bounded rationality was first proposed by Simon (1957). From that time a rich literature has developed describing systematic errors made by experimental subjects (see, for example, Arkes and Hammond, 1986; Hogarth, 1991; Kahneman et al., 1982; Nisbett and Ross, 1980) and incorporating bounded rationality into decision making. The latter models can be divided into two groups: static and dynamic.

In static models choice is assumed to be probabilistic. It is typical in this type of models to impose some intuitive restrictions on the choice probabilities and study the probability distributions that satisfy these restrictions. They were introduced into economics by Luce (1959) and have already found their application. See, for example, McKelvey and Palfrey (1995, 1998), Chen et al. (1997), Sonnemans et al. (1998), and Anderson et al. (1998, 2001).

In dynamic models individuals are assumed to adjust their choices over time in directions that appear to be beneficial. The dynamic approach originated in the work of Bush and Mosteller (1955), was introduced in economics by Arrow and Hurwicz (1960), and is represented, for example, by papers of Foster and Young (1990), Fudenberg and Harris

(1992), Kandori et al. (1993), Young (1993), Friedman and Yellin (1997), Anderson et al. (2004), Friedman (2001), and Basov (2003).

In the steady state the dynamic models also result in a probabilistic choice between the alternatives. Therefore, in these essays I model boundedly rational behavior as a probabilistic choice. However, see Basov (2005) for some caveats to this approach.

It is often argued that the deviations from rational behaviour by individual agents, though real, are sufficiently small to affect aggregate economic outcomes in any significant way. If these deviations are also independent across the agents, then they tend to cancel each other. In the first two essays, I will argue for the opposite point of view, by providing specific examples in which consumers exhibit boundedly rational behaviour, and indeed, are nearly rational, yet the consequences of their behaviour are quite significant in shaping the market interactions.

The first two essays are based on the probabilistic choice model, first introduced by Luce (1959), where probability of choosing alternative  $j$  from the set of available options  $M$  is given by:

$$prob(j) = \frac{V_j}{\sum_{h \in M} V_h}, \quad (1.1)$$

where the function  $V : M \rightarrow \mathbb{R}$  is called a scaling function. Let us consider the function  $U = \ln V$  and interpret it as the utility function. Then the above probability distribution, as a function of  $U$ , is the same as the logistic one. A mathematically identical model for choice probabilities is popular in empirical Industrial Organization, where it is known as the logit model (see, for example, Anderson et al., 1992), where it is interpreted as a model of unobserved heterogeneity in tastes. Despite their mathematical similarity, the Luce model and the logit model are conceptually different. In the Luce model each agent's choice is probabilistic, while in the logit model the choice of individual agent is deterministic utility maximizing choice, which only looks to be probabilistic to an econometrician who does not observe the horizontal component of preferences. The latter observation has important welfare consequences. For example, adding an extra alternative to the menu will always weakly improve utility of the agents under the logit interpretation, but in the Luce model, it may under some circumstances decrease the expected utility of

the agent. I will return to this point in my essays.

The thesis is organized in the following way.

The first essay, Chapter 2, applies a model of boundedly rational probabilistic choice to provide a novel explanation of the relationship between product quality and variety and is based on the joint work with S. Basov. The theoretical model we developed was combined with empirical work done by D. Prentice, which resulted in the publication by Basov et al. (2020). Chapter 2 develops a simple theoretical model with nearly rational consumers, who randomize among all alternatives that yield the same utility for them but could make a mistake of choosing a slightly worse alternative. It is shown that it is optimal for a firm to offer more varieties of its higher quality, more profitable, products, in order to price discriminate more effectively. When the market is thin at the very top quality level, the number of varieties first increases and then decreases, resulting in the overall hump-shaped relationship between the number of varieties and quality. These predictions found empirical support in the Australian car market, see Basov et al. (2020). The model also makes an important theoretical observation: under bounded rationality the Revelation Principle no longer holds. Though that by itself is not surprising, an interesting observation is that a small deviation from rational choice on the part of the consumers could lead to a significant change in the monopolist's optimal product line, adding an extra dimension to it, multiple varieties at each quality level.

The second essay, Chapter 3, considers a model of Bertrand oligopoly and is based on Basov and Danilkina (2015). The behaviour of the consumers is modelled in the same way as in Chapter 2, i.e. they are boundedly rational and make their purchase decisions probabilistically, according to Luce's model. Three different cases are considered: first, we characterize equilibrium when firms face boundedly rational consumers with the fixed irrationality parameter  $\lambda$ ; second, we discuss the case of obfuscating oligopoly, when firms can invest in order to confuse consumers, i.e. to increase their  $\lambda$ ; and third, we consider educating oligopoly, when firms can choose to invest to decrease  $\lambda$ . We show that while it is worthwhile for the firms to confuse the consumers, it is only optimal to educate them if they are sufficiently rational at default. We also analyze how the

social welfare, consumer surplus and the firms' profits depend on the number of firms. In this framework competition has two beneficial effects from the social point of view: it increases the chance that a consumer will buy the product and decreases the amount of obfuscation. The second effect arises because obfuscation is a public good from the firms' point of view and, therefore, the firms try to free ride on each other. From the consumers' point of view, there is another beneficial effect of competition - it leads to decrease in price.

The third essay, Chapter 4, examines the optimal procurement auctions, when the ability of contractors to complete the project is not known beforehand and must be tested. The crucial issue is whether such testing should occur before or after the auction is conducted. We compare both approaches and show that when testing can be performed at no cost and the procurer and contractors are risk neutral, the timing of the test (before vs after the auction) has no effect on the procurer's expected surplus, the contractors' expected payoffs, or the total surplus. In case of costly testing a risk neutral procurer would prefer testing after the auction.

Chapter 5, concludes.

## Chapter 2

# When does variety increase with quality?

## 2.1 Introduction

A casual look at the shelves of a supermarket or at producers' web sites reveals that higher quality products often come in more varieties than do lower-quality products within the same category. For example, branded products almost always have more varieties than the stores' generic brand, and premium products (e.g., Pickwick tea) often have more varieties than their entry-level peers (e.g., Lipton tea).

The profitability of product lines has been analyzed empirically (Thomadsen, 2012) and theoretically. The existing theoretical literature—such as Neven and Thisse (1990)—has studied how the relationship between variety and quality might arise from consumer preferences. This paper instead concentrates on how the relationship between quality and variety might emerge through the actions of a firm that implements second-degree price discrimination to consumers who value quality but not variety.

We study a standard probabilistic choice model of consumer behavior, Luce's (1959) model, and introduce nearly rational consumers who predominantly randomize among products that yield the same utility. We demonstrate that the monopolist finds it optimal to offer more varieties of higher-quality, more-profitable products to increase the likelihood that those products are chosen by consumers. If the market is thin at higher-quality levels, then the number of varieties is shown to be hump-shaped rather than always increasing with quality.

This chapter makes two main contributions to the literature: First, the theoretical model provides a new explanation for a systematic relationship between quality and variety: driven by price discrimination rather than solely by the distribution of preferences or costs. The model predicts either a positive or a hump-shaped relationship between quality and variety. Second, in contrast to most of the existing literature, such as Lancaster (1990), we show that a monopolist can produce a greater-than-optimal number of varieties: Variety proliferation is wasteful.

## 2.2 Consumer Behavior

The model that is developed in this paper combines two strands of literature: the probabilistic choice literature, and the monopolistic screening model. In this section, we adapt a standard probabilistic choice model—the Luce (1959) model—that introduces nearly rational consumers who randomize among products that yield them the same utility.

Anderson et al. (1992) summarized four types of reasons that lead to probabilistic choice.

Mattsson and Weibull (2002) showed that choice probabilities,  $\hat{p}_i$ , under plausible assumptions, can be written as:

$$\hat{p}_i = \frac{q_i \exp(u_i/\lambda)}{\sum_{j=1}^n q_j \exp(u_j/\lambda)}, \quad (2.1)$$

where:  $q_i$  is the default probability of choosing alternative  $i$ ;  $u_i$  is the corresponding utility; and  $\lambda$  is a positive scalar.<sup>1</sup>

If by default all alternatives are equally likely to be chosen, one arrives at the Luce model:

$$\hat{p}_i = \frac{\exp(u_i/\lambda)}{\sum_{j=1}^n \exp(u_j/\lambda)}. \quad (2.2)$$

---

<sup>1</sup>Mattsson and Weibull interpret  $\lambda$  as capturing deliberation costs and representing the relative weight that the decision-maker attaches to the disutility of choosing probability assignment across outcomes other than  $q$ .

We interpret  $\lambda$  as the parameter of irrationality, with higher values corresponding to less rational behavior. When  $\lambda$  approaches zero, the consumer is almost rational: She assigns equal positive probabilities to all utility-maximizing options and zero probabilities to the other alternatives. When  $\lambda$  approaches infinity, the consumer selects all alternatives with equal probabilities, without any regard for the utility that they yield to her.

The Luce model predicts that the introduction of different alternatives of the same product will significantly increase the overall demand for that product. This is usually considered to be a problematic feature but we argue that it is, in fact, a useful property for our analysis. This implication of the Luce model is usually criticized by using the blue-bus-red-bus paradox: Adding an alternative—which differs in an irrelevant aspect, such as the bus’s color—reduces the probability of other alternatives (such as going by car) being chosen: see, for example, Ben-Akiva and Lerman (1985). In principle, adding a perfect substitute to an option would most likely result in no change to the probabilities of choosing other options.

However, there is experimental evidence that shows that offering greater variety is associated with greater market share (Huber and Puto, 1983; Berger et al., 2007). Indeed, Huber and Puto (1983) argue that it is possible for the attraction effect of additional varieties to offset the substitution effect of adding a close substitute which results in an overall increase in the firm’s market share as suggested by the simple Luce model.

The attraction that is associated with additional varieties seems potentially more applicable to our case. Consider a buyer who walks into a car dealership and sees two cars on display: a Mercedes-Benz and a BMW. Suppose, given the prices, that the buyer is indifferent between the alternatives. Hence, she will choose each with 50% probability. Now assume another variety of Mercedes-Benz is on display, such that the two differ in only a minor way. It seems plausible that having both models on display may well increase the probability that a Mercedes-Benz is chosen. In the Mattsson and Weibull (2002) formulation, this corresponds to the default probabilities of choosing any of the three cars as being similar.

Hence, we adopt and build upon the Luce model because it captures, in the simplest

possible way, the property that the products with more varieties are chosen more often, everything else being equal. This feature is present in almost all of Mattsson and Weibull's classes of models.<sup>2</sup>

### 2.2.1 The Nearly Rational Consumer

Assume that the probability of different choices by the consumer is given by a continuously differentiable function:

$$\hat{p}(\cdot) : \mathbb{R}^n \times \mathbb{R}_+ \rightarrow \Delta^n, \quad (2.3)$$

which assigns probability of choosing each option for every utility profile given the consumer's irrationality parameter  $\lambda$ , such that  $u_i = u_j$  implies  $\hat{p}_i(u, \lambda) = \hat{p}_j(u, \lambda)$ : Alternatives with the same utilities are chosen with the same probabilities; and  $\hat{p}(u, 0)$  is a uniform distribution over the utility-maximizing alternatives.<sup>3</sup>

Let  $M$  be the set of the utility maximizers:

$$M = \{u_i : u_i = \max\{u_1, \dots, u_n\}\}. \quad (2.4)$$

**Definition.** A consumer whose choice probabilities are given by (2.3) is said to be *nearly rational* for a given set of alternatives if

$$\lambda \ll \delta = \min_{u_j \in M, u_k \notin M} (u_j - u_k). \quad (2.5)$$

In words, a consumer is nearly rational if the parameter of irrationality  $\lambda$  is *much smaller* (sign  $\ll$  reads "much smaller") than the difference between the utilities of the best and the second-best alternatives. The exact meaning of "much smaller" depends on the extent of randomness to be permitted in decision-making.

Thus, a nearly rational consumer chooses all utility-maximizing alternatives with equal

---

<sup>2</sup>Note that we are not considering the effects of evaluation costs, as is highlighted in Kuksov and Villas-Boas (2010) who assume that greater variety may prevent consumers from participating in the market. In our model, unlike Villas-Boas (2009), adding a horizontal dimension (varieties) adds depth to the product line without affecting its length. The reason for this, again, is the absence of evaluation costs.

<sup>3</sup>Luce probabilities, as in equation (2.2), satisfy these properties.

probability and chooses all other, suboptimal, options with negligibly small probabilities.<sup>4</sup> Note that this definition applies to the general class of probabilistic models, not only to the Luce model.

## 2.3 The Monopolistic Screening Model

Now let us briefly review the basic screening model, in the tradition of Mussa and Rosen (1978), with rational consumers, which is the second building block of our theoretical model. We analyze whether the monopolist should offer the second-best contracts (i.e. quality/price bundles) that are derived from it to the consumers who are only nearly, but not fully, rational.

Consider a risk-neutral monopolist that produces a unit of a good of quality  $x$  at a cost  $C(x)$ , where  $C(\cdot)$  is a strictly convex, twice differentiable function. A consumer of type  $\theta$  has a unit demand and her utility,  $v(\theta, x, t)$ , from buying it at price  $t$  is given by:

$$v(\theta, x, t) = \theta x - t.$$

If the consumer does not purchase the good, she receives zero utility. The consumer's type  $\theta$  is the private information of the consumer, where  $\theta \in \{\theta_1, \dots, \theta_N\}$  with  $0 < \theta_1 < \theta_2 < \dots < \theta_N$  and  $\Pr(\theta = \theta_i) = p_i > 0$ .

Sellers typically react to consumer heterogeneity by devising product lines where each product is targeted to a specific type of consumer. Let us make an assumption that guarantees that all types of consumers are served,  $x_i > 0$ : The following inequality holds for all  $i$  :

$$\theta_i - \frac{(1 - P_i)}{p_i}(\theta_{i+1} - \theta_i) > 0, \quad (2.6)$$

where  $P_i = \Pr(\theta \leq \theta_i) = \sum_{j=1}^i p_j$ .

Then the equilibrium qualities  $x_i$  satisfy the following condition:<sup>5</sup>

<sup>4</sup>Exponentially small in a Luce model.

<sup>5</sup>Note that  $P_i < 1$  for  $i < N$  and  $P_N = 1$ ; therefore, type  $\theta_N$  is served efficiently, and the quality consumed by all the other types is downward biased. This observation was first made by Mussa and Rosen (1978).

$$C'(x_i) = \theta_i - \frac{(1 - P_i)}{p_i}(\theta_{i+1} - \theta_i) \quad (2.7)$$

and in the equilibrium the higher type buys a good of a higher quality:  $x_{i+1} > x_i$ .

To find the corresponding optimal prices  $t_i$ , recall that the constraint reduction theorem (Stole, 1999) implies that the consumer of type  $\theta_i$  with  $i > 1$  is indifferent between her own quality/price contract  $(x_i, t_i)$  and that designed for type  $\theta_{i-1}$ , but strictly prefers it to all other contracts; while the consumer of type  $\theta_1$  is indifferent between her own contract and not participating, but strictly prefers it to any other contract. Therefore, the optimal prices  $t_i$  are determined from binding incentive-compatibility constraints:

$$\theta_i x_i - t_i = \theta_i x_{i-1} - t_{i-1} \text{ and } \theta_1 x_1 = t_1. \quad (2.8)$$

Define  $\pi_i$  as the equilibrium profit that is earned on a contract that is designed for consumer of type  $\theta_i$ , by

$$\pi_i = t_i - C(x_i). \quad (2.9)$$

It is straightforward to show that the monopolist earns a higher profit on a contract that is designed for a higher type consumer: For all  $i$

$$\Delta\pi_i = \pi_i - \pi_{i-1} > 0. \quad (2.10)$$

The qualities that are specified by (2.7) and the corresponding set of prices define the *menu of the second-best contracts* for rational consumers that implements second-degree price discrimination.

To determine whether it is optimal for the monopolist to offer these contracts to nearly rational consumers, let us first define near-rationality in this context: The difference between the utility that consumer of type  $i$  obtains from selecting the contract designed for her and that from the contract that is designed for type  $j$  is given by

$$v(\theta_i, x_i, t_i) - v(\theta_i, x_j, t_j) = \theta_i(x_i - x_j) - (t_i - t_j). \quad (2.11)$$

Let

$$\delta = \min_{i,j \notin \{i-1,i\}} (\theta_i(x_i - x_j) - (t_i - t_j))^6 \quad (2.12)$$

Then, according to (2.5), the consumer is nearly rational, given the menu of the second-best contracts, if

$$\lambda \ll \delta. \quad (2.13)$$

For the remainder of this paper, we assume that condition (2.13) holds: The consumers are nearly rational.

When the monopolist offers this second-best menu to rational consumers, each consumer of each type  $\theta_i$  selects the contract  $(x_i, t_i)$  that is designed for her, even though she is indifferent between it and the contract designed for type  $\theta_{i-1}$ .<sup>7</sup> However, a nearly rational consumer of type  $\theta_i$  will randomize between these two utility-maximizing contracts and will also choose, with negligible probability, the suboptimal contracts that are designed for other types.

Therefore, if the monopolist offers this menu to nearly rational consumers, approximately half of them will choose the contract that is designed for them, another half will choose a contract that is designed for the type just below them, and a negligibly small fraction of the consumers will choose one of the strictly dominated alternatives. Compared with the equilibrium with rational consumers, just over half of consumers choose contracts that return lower profits per unit sold.

The monopolist, as a result, will experience a drop in its profits that is quite large:  $O(\Delta\pi)$ , where  $\Delta\pi$  is the average of the  $\Delta\pi_i$ 's, which is of a higher order of magnitude than  $\lambda$ .

If the consumers are fully rational, an infinitely small price change is enough to force them to choose their own contract, but it is not sufficient for nearly rational consumers. Because small price changes lead to a small change in probabilities, almost half of the consumers will still choose the wrong contract. To alter consumer behavior substantially, the monopolist will have to reduce prices in such a way as to violate the binding constraints

<sup>6</sup>For  $i = 1$  option  $i - 1$  is interpreted as the outside option: not buying.

<sup>7</sup>This outcome can be justified as a Bayes-Nash equilibrium.

by some finite amount.<sup>8</sup>

The better alternative to a significant price cut is for the monopolist to create multiple varieties for each quality level, which will ensure that consumers choose their own contract more often. Next, we investigate the variety proliferation model to explore this possibility.

## 2.4 The Variety Proliferation Model

In this section we analyze how variety proliferation makes price discrimination more profitable by overcoming the problem of consumers' randomizing away from the most-profitable product for their type. We demonstrate that the relationship between the number of varieties and quality is either increasing or hump-shaped if the market is very thin at high quality levels.

If the monopolist offers  $n_i$  different varieties of the good of quality  $x_i$  then the proportion  $q_i$  of consumers will buy it:

$$q_i = p_i \frac{n_i}{n_{i-1} + n_i} + p_{i+1} \frac{n_i}{n_i + n_{i+1}}, \quad (2.14)$$

where:  $p_i$  is the proportion of consumers of type  $\theta_i$  in the population;  $n_i/(n_{i-1} + n_i)$  is the proportion of type  $\theta_i$  consumers who choose their own contract;  $n_i/(n_i + n_{i+1})$  is the proportion of consumers of type  $\theta_{i+1}$ , who also choose the contract that is designed for type  $\theta_i$ ; while the proportion of consumers who choose a strictly dominated alternative is negligibly small. We assume that the outside option, of not buying, is unique:  $n_0 = 1$ . It is important to note that varieties at the same quality level yield the same utility to each consumer.

Assume that the marginal cost of the process of adding a new variety,  $c$ , is positive and does not depend on the quality of the good.<sup>9</sup> We will use the qualities and prices that are optimal for rational consumers as an approximation for those that are optimal for

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<sup>8</sup>Basov (2009) shows that the required change in prices in an optimal contract is of the order of  $\lambda \ln \lambda$ , which corresponds to the proportion of consumers who make a mistake as being of order  $\lambda$ .

<sup>9</sup>This is the incremental set-up costs of getting a new variety into production. The costs of the production process itself are accounted for in equation (2.9).

nearly rational consumers.<sup>10</sup> The monopolist, therefore, chooses the number of varieties of each quality to maximize profit:

$$\max_{\{n_i\}_{i=1}^N} \sum_{i=1}^N (q_i \pi_i - c n_i),$$

where probabilities  $q_i$  are given by (4.2), and  $\pi_i$ —the profit that is earned on the consumer who chooses quality  $x_i$ —is given by (2.9). After some algebra one arrives at:

$$\max_{\{n_i\}_{i=1}^N} \sum_{i=1}^N \left( p_i \pi_i \frac{n_i}{n_{i-1} + n_i} + p_i \pi_{i-1} \frac{n_{i-1}}{n_{i-1} + n_i} - c n_i \right). \quad (2.15)$$

Ignoring the constraint that  $n_i$  should be an integer, one can write the first-order conditions:

$$\frac{p_i \Delta \pi_i n_{i-1}}{(n_{i-1} + n_i)^2} - \frac{p_{i+1} \Delta \pi_{i+1} n_{i+1}}{(n_i + n_{i+1})^2} = c, \quad i = 1, \dots, N, \quad (2.16)$$

where we defined  $p_{N+1} = 0$ ,  $n_{N+1} = 0$  and  $n_0 = 1$ .

Let us introduce the following dimensionless parameters:

$$k_i = \frac{p_{i+1} \Delta \pi_{i+1}}{p_i \Delta \pi_i}, \quad \varepsilon_i = \frac{c}{p_i \Delta \pi_i}, \quad i = 1, \dots, N. \quad (2.17)$$

where  $k_N = 0$  and  $\varepsilon_i = \varepsilon_N k_{N-1} \dots k_i$ .

The amount  $p_{i+1} \Delta \pi_{i+1}$  represents the potential loss of profit if all consumers of type  $\theta_{i+1}$  switch from the product that is designed for them to the product that is designed for the type just below them:  $\theta_i$ . To make this more concrete, recall that in section 4, when the monopolist offered one variety of each quality level, approximately half of the consumers of type  $\theta_{i+1}$  choose the product that is designed for  $\theta_i$ , leading to the loss of  $\frac{1}{2} p_{i+1} \Delta \pi_{i+1}$ .

The parameter  $k_i$  characterizes the relative importance of the potential gains to be achieved from the two adjacent types— $\theta_{i+1}$  and  $\theta_i$ —if they are to buy the product that is

<sup>10</sup>It is not optimal to adjust qualities for nearly rational consumers since that adjustment will disrupt the trade-off between information-rent extraction and efficient provision of low types. Therefore, one has to choose between adding new varieties and adjusting prices. Results in Basov (2009) and condition (23) imply that the former is more profitable.

designed for them. When  $k_i$  is close to zero, the profit to be gained on the higher-type consumer is insignificant compared to the profit from the lower type: due either to the lower profit margin  $\Delta\pi_{i+1}$  or to the much smaller number of customers  $p_{i+1}$ . If  $k_i$  is close to one, the monopolist cares equally about ensuring that each type buys her own product.

The parameter  $\varepsilon_i$  measures the cost of introducing a variety relative to the overall potential gain in profit. The first order conditions can now be rewritten as:

$$\frac{n_{i-1}}{(n_{i-1} + n_i)^2} - k_i \frac{n_{i+1}}{(n_i + n_{i+1})^2} = \varepsilon_i, \quad i = 1, \dots, N. \quad (2.18)$$

As we can see from the equation, the way the optimal number of varieties changes with quality depends crucially on the parameter  $k_i$  and the marginal cost  $c$ .

Next, we establish the qualitative predictions of our theoretical model: When the marginal cost of adding an extra variety is very small, the number of varieties dramatically increases with quality, unless the market becomes thin at very high-quality levels. In the latter case, the number of varieties decreases at those higher qualities, resulting in the overall hump-shaped relationship between the number of varieties and quality.

To establish these results, we consider two limit cases: first, the case with  $k_i \sim 1$  (i.e.,  $k$  is of order of magnitude 1) for all  $i$ ; second, the case of a thin market at the top: when there exists index  $j$  such that  $k_i \ll 1$  for all  $i > j$ , while  $k_i \sim 1$  for all  $i \leq j$ , as in the first case. In both cases we will determine how the number of varieties depends on quality when the marginal cost  $c$  of the process of introducing a new product is sufficiently small.

**Proposition 1.**

*If  $k_i \sim 1$  for all  $i$ , then, as marginal cost  $c \rightarrow +0$ , the number of varieties,  $n_i$ , goes to infinity, but in such a way that there are many more varieties at higher qualities:*

$$\lim_{c \rightarrow +0} \frac{n_i}{n_{i+1}} = 0. \quad (2.19)$$

*Indeed, the number of varieties is given by*

$$n_i = a_i \varepsilon_N^{-\frac{i}{N+1}} + O(\varepsilon_N^{\frac{1}{N+1}}). \quad (2.20)$$

**Proof.** Using (2.18), it is straightforward to verify that as  $\varepsilon_i \rightarrow +0$  for all  $i$  (as marginal cost  $c \rightarrow +0$ ), the number of varieties is given by

$$n_i = a_i \varepsilon_N^{-\frac{i}{N+1}} + O(\varepsilon_N^{\frac{1}{N+1}}), \quad (2.21)$$

where coefficients  $a_i$  satisfy the second-order difference equation

$$\frac{a_{i+1}a_{i-1}}{a_i^2} = k_i, \quad (2.22)$$

subject to the following initial and terminal conditions:

$$a_0 = 1, \quad \frac{a_{N-1}}{a_N^2} = 1. \quad (2.23)$$

Therefore, the ratio of numbers of varieties in the limit goes to zero:

$$\lim_{c \rightarrow +0} \frac{n_i}{n_{i+1}} = 0. \quad (2.24)$$

■

The corresponding cost of variety proliferation at quality level  $x_i$  is of order  $F_i$ , given by:

$$F_i = ca_i \varepsilon_N^{-\frac{i}{N+1}} = a_i (p_N \Delta \pi_N) \varepsilon_N^{\frac{N+1-i}{N+1}}. \quad (2.25)$$

Variety proliferation is approximately optimal as long as:

$$\max_i F_i \ll \lambda \ln \lambda \ll \delta, \quad (2.26)$$

where the first inequality ensures that the monopolist will prefer variety proliferation to introducing slack into the binding constraints as explained above, and the second one is sufficient to guarantee that the consumers are nearly rational.

The intuition for this result is that the monopolist is comparing the cost of adding varieties to the expected gains. When gains on all types are similar ( $k_i \sim 1$ ), the mo-

nopolist wants to ensure that as many consumers as possible choose the product that is designed for them instead of the product for the type just below. As the consumers randomize between the two, the only way to achieve this is to add increasing numbers of varieties for higher-type consumers. This is worth doing when the cost of introducing varieties is relatively small compared to the profit to be gained.

Now, let us consider the second limit case, when the market becomes very thin at the top because consumers with strong preferences for quality (snobs) are rare:  $p_{i+1}/p_i \ll 1$ . This observation is supported, for example, by sales numbers from Table 1. Thus, there exists index  $j$  such that  $p_{i+1}/p_i \ll 1$  for all  $i > j$ , which leads to  $k_i \ll 1$ .

Note that the scenario when  $k_i \ll 1$  may fail to hold, even if  $p_{i+1}/p_i \ll 1$ , due to  $\Delta\pi_{i+1} \gg \Delta\pi_i$ , is improbable since to earn a high profit margin on type  $\theta_{i+1}$ , the monopolist has to introduce a significant downward distortion on lower types, which is unprofitable when higher types are rare and, thus, the information rent paid to them is not significant.

**Proposition 2.**

*If the market becomes very thin at the top—if there exists index  $j$  such that  $k_i \sim 1$  for all  $i \leq j$ , but  $k_i \ll 1$  for all  $i > j$ —then for small <sup>11</sup> values of the marginal cost  $c$  of introducing a new variety, the number of varieties first increases and then decreases.*

**Proof.** For  $i > j$ , for small marginal cost and small  $p_i$ , we have  $\varepsilon_i \sim 1$ . (Note that if marginal cost is so small such that  $\varepsilon_i$  is much smaller than  $k_i$ , then Proposition 1 applies.) Given that  $k_i \ll 1$  and  $\varepsilon_i \sim 1$ , we can drop the second term on the left-hand side of equation (2.18) and easily verify that  $n_{i+1} < n_i$  as long as  $4n_i\varepsilon_{i+1} > 1$ . Since  $n_i \geq 1$ , a sufficient condition is  $\varepsilon_{i+1} > 1/4$ : If the cost of proliferation is not too small and there are very few high-type consumers, the number of varieties will eventually fall.

For  $i \leq j$ , the same type of argument as in Proposition 1 applies; thus, the number of varieties is increasing in quality.

Therefore, this case results in an overall hump-shaped set of varieties. ■

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<sup>11</sup>But not too small. See the proof for more details.

The intuition for the difference between this case and that of Proposition 1 is that, because there are many fewer consumers who prefer high-quality products, the gains to be made on them by introducing more varieties are not significant and are smaller than the costs involved. Thus, we get fewer varieties at higher-quality levels when the number of potential customers decreases sufficiently.

Note that our model shows that a small deviation from rational choice on the part of the consumers could lead to a significant change in the monopolist's optimal product line: The change could add an extra dimension to it: multiple varieties at each quality level.<sup>12</sup>

The excessive variety that is provided by the monopolist is socially inefficient. As consumers have no direct preference for varieties, introducing more varieties comes at a cost for the firm but does not provide consumers with greater utility. Thus, from a social perspective, variety proliferation is always wasteful. If one assumes instead that varieties are directly valued, the wastefulness is only partially mitigated.<sup>13</sup>

This prediction is in contrast to most of the existing literature that tends to find a smaller-than-optimal number of varieties, for reasons that are similar to those of a monopolist that produces a lower-than-optimal quantity (Lancaster, 1990). It should also be distinguished from the larger-than-efficient product lines, a possibility predicted by Mussa and Rosen (1978)<sup>14</sup> in a model with continuous types. These models do not have any horizontal variety, while the vertical size of the product line in these models is the same as in our model.

Note that variety proliferation in our model is not even constrained efficient, since a social planner who is subject to the same information constraints would have chosen a single variety and introduced slacks into binding incentive-compatibility constraints by varying prices to match the probabilities of sales that are achieved by the monopolist. Being pure transfers, prices do not affect total welfare; therefore, the social planner

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<sup>12</sup>The technical reason for this is the failure of lower hemicontinuity of the choice correspondence in the parameter that captures the degree of irrationality  $\lambda$  (see Basov (2009)). This situation is quite general and is not restricted to the Luce model.

<sup>13</sup>Even in this case proliferation will be wasteful as long as  $\lambda \ll F_i \ll \lambda \ln \lambda$  for all  $i$ .

<sup>14</sup>In the Mussa and Rosen (1978) model quality is downward biased. For the alternative assumptions that lead to upward bias see Donnenfeld and White (1990), Srinagesh and Bradburd (1989).

would have achieved the same levels of consumption and production and saved on the proliferation costs.

The model can be generalized from a monopoly to an oligopoly along the lines of Champsaur and Rochet (1989). It can be shown that under some reasonable conditions there exists an equilibrium where each producer specializes in a particular range of qualities. Qualitatively, the outcome is similar to the monopoly outcome. Proliferation of varieties in the oligopolistic case will be due to two effects: competition among the producers, and price discrimination among consumers of different types who buy from the same producer.

## 2.5 Conclusions

In this chapter we have analyzed the links between quality and variety. The theoretical model has two components: First, consumers are modeled as caring about quality but indifferent among varieties of the same quality. In addition, we adapt a standard probabilistic choice model to introduce a very small amount of randomness into consumer behavior. The second component is a monopolist who engages in second-degree price discrimination. Faced with consumers who randomize, the profitability of price discrimination is increased when the monopolist offers more varieties at higher qualities, since the likelihood that consumers choose high-price products increases. However, if the markets become sufficiently thin at high-quality levels, the profit-maximizing number of varieties falls, which yields a hump-shaped relationship between variety and quality.

The theoretical predictions of this chapter were tested and found empirical support on the Australian car market, see Basov et al. (2020).

Another important contribution of the paper is that the number of varieties that are offered by the market can be higher than the social optimum even in the monopoly case, which is opposite to the conclusions that are drawn from the previous literature (see Lancaster, 1990).

## Chapter 3

# Bertrand oligopoly with boundedly rational consumers

### 3.1 Introduction

Recent years saw proliferation of economic models that incorporate boundedly rational behavior on the part of economic agents. This explosion followed the seminal paper by Conlisk (1996) and includes Sonnemans et al. (1998) in the area of provision of public goods, and Anderson et al. (1998) in Tullock contests. For the review of the literature in the mechanism design theory, see chapter by Basov et al. (2011).

More recently, there emerged considerable interest in studying the consequences of boundedly rational behavior on the oligopolistic pricing and other activities of the firms - see Ellison (2006) for a detailed review of this literature. Here we will mention only a few papers related to the current topic.

Grossman (1981) and Milgrom (1981) demonstrated that if information disclosure is costless, the firms will reveal all the relevant information, i.e. they will educate rather than obfuscate the consumers. However, Ellison and Ellison (2009) argue that this prediction is not born out by many real world observations. Ellison and Wolitzky (2012) developed a model where obfuscation is theoretically justified in a search-theoretic framework. In their model obfuscation takes a form of an unobservable action that makes it more difficult to inspect the product and learn its price or quality. One way to think about it is to assume that the consumers have to spend time to make a product selection. They go through the

list of product's characteristics to determine its value. If the list is too long they may end up making a mistake in their assessment. By devising concise and relevant description of the product, a firm may ease the task of the consumer, thereby decreasing the probability of a mistake. On the contrary, by putting a lot of irrelevant characteristics on the list or by engaging in an uninformative advertising, firms can make it more difficult for the consumers to go through the product description, increasing the probability of an error. One just needs to remember the plethora of different mobile plans to see the point. The complexity of different rates and features leaves most consumers being unable to compare mobile plans, let alone to choose the best one.

Ellison and Wolitzky (2012) describe several scenarios for achieving obfuscation. For example, they suggest that in online shopping application the firm can choose the number of screens the consumer has to go through to find all the relevant information and they may instruct the salespeople how long to talk to consumers during face-to-face shopping. Ellison and Wolitzky assume that consumers go sequentially through description of products supplied by the firms. The consumers' search costs depend on the sum of obfuscation levels chosen by the firms that they are visiting. Consumers have rational expectations about the obfuscation levels and can rationally choose to stop the search.

Our paper is similar in spirit to Ellison and Wolitzky (2012). However, instead of assuming that consumers experience direct costs from reading the products' description and learning relevant information about their quality and price, we assume that this process affects quality of consumers' decision making; the longer and more confusing the product description, the lower the quality of decision making. The crucial difference of this assumption from the one used by Ellison and Wolitzky is that while in the latter model the consumers are fully aware of the search costs and will rationally stop search at some point, in our model consumers unaware of the effect of obfuscation on their quality of the decision making. For example, a consumer who already has spent a couple of hours at a computer screen reading description of different house insurance policies, will still continue going through the exhaustive list, since she does not experience direct costs from search and is not aware that the very act of continuing the search will decrease

the quality of her decision making.<sup>1</sup> For another example, think about choosing among different mobile plans and trying to understand how much they will cost per month.

A related but different model will be one where the value of the products has a potentially heterogeneous element that differs across firms. In such a model, advertising can be seen as a way for firms to teach consumers to differentiate between the good and the bad versions of the product. In such a model firms will act as experts conveying information to the consumers and the model will have potential links with cheap talk literature (see, for example, Hagenbach and Koessler, 2010) where the dominant insight is that if there are multiple experts around, there always exists equilibrium with perfect communication. Morgan and Krishna (2001) reached a similar conclusion<sup>2</sup> in a model of expertise, provided that experts have opposite biases. In our framework, however, the products are homogeneous and quality is not an easily communicable piece of information, but can instead be learned only by reading a lengthy description of the product.

To formalize the idea of quality of choice we use the Luce model of probabilistic choice (Luce, 1959). In the Luce model the degree of bounded rationality is captured by a single parameter,  $\lambda$ , with  $\lambda = 0$  corresponding to rational behavior and higher  $\lambda$  corresponding to more irrational behavior. Note that consumers in our model are boundedly rational in two ways: they may fail to choose optimal product and they fail to take into account the effect of going through lengthy product description on  $\lambda$ .

We consider three different cases: first, we characterize equilibrium when firms face boundedly rational consumers with the fixed irrationality parameter  $\lambda$ ; second, we discuss the case of obfuscating oligopoly, when firms can invest in confusing consumers, i.e. in increasing their  $\lambda$ ; and third, we consider educating oligopoly, when firms can choose to invest in decreasing  $\lambda$ .

In this paper we arrive at the following results. First, firms will in general engage in obfuscating activities, but will only engage in educating ones if consumers are sufficiently rational at default. It is in contrast to the conclusion of full information revelation reached

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<sup>1</sup> Since the consumer will always go through the exhaustive set of the alternatives, which will affect the overall quality of choice, a decision of one firm to increase level of obfuscation will affect the relative probabilities of all choices.

<sup>2</sup> The conclusion does not hold, however, if one of the senders can veto messages. See Newton (2014).

by Grossman (1981) and Milgrom (1981). Though definition of education and obfuscation is formally different in our framework and that of Grossman (1981) and Milgrom (1981), comparison of the conclusion is still illuminating. Second, in this framework competition has two beneficial effects from the social point of view: it increases the chance that a consumer will buy the product and decreases the amount of obfuscation. From the consumers' point of view, there is another beneficial effect of competition - it leads to decrease in price.

The chapter is organized in the following way. In Section 3.2 we briefly remind the reader the Luce model of boundedly rational behavior. In Section 3.3 we consider price competition when firms face boundedly rational consumers with given  $\lambda$ .<sup>3</sup> In Section 3.4 we allow firms to engage in obfuscating activities. In Section 3.5 we assume that the firms cannot engage in obfuscating activities, but may choose to educate consumers (assume, for example, that the new law requires that all advertising and product description must be informative and simple). Section 3.6 concludes.

## 3.2 The Luce model

In this paper we assume that the consumers are boundedly rational. To capture it we assume that their choice is probabilistic, i.e. the utilities associated with different choices determine the probabilities with which these choices are made. The first probabilistic choice model in economics was proposed by Luce (1959). He showed that if one requires the choice probabilities to be independent of a sequence in which choices are made, then they must be represented by:<sup>4</sup>

$$q_i = \frac{\exp(u_i/\lambda)}{\sum_{j=1}^n \exp(u_j/\lambda)}, \quad (3.1)$$

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<sup>3</sup> We restrict our attention to pure strategy Nash equilibria.

<sup>4</sup> These probabilities are also known in empirical IO as a logit probabilities, but motivation there is different.

where  $n$  is the number of alternatives,  $q_i$  is the probability that alternative  $i$  is chosen, and  $u_i$  is the utility associated with alternative  $i$ . Note that according to this model any two alternatives that have the same utility are selected with the same probabilities. Parameter  $\lambda$ , which can take values from zero to infinity, can be usefully thought of as representing the degree of the consumer's irrationality. If  $\lambda = 0$  then

$$q_i = \begin{cases} 1/k, & \text{if } u_i = \max\{u_1, \dots, u_n\} \\ 0, & \text{otherwise} \end{cases}, \quad (3.2)$$

where integer  $k$  is the cardinality of the set of the utility maximizers, which is consistent with rational behavior. At the other extreme, as  $\lambda \rightarrow \infty$  the choice probabilities converge to  $1/n$ , i.e. the choice becomes totally random, independent of the utility level - the case of fully irrational consumer.

### 3.3 The Bertrand model with boundedly rational consumers: the case of fixed $\lambda$

In this Section we assume that each of  $n$  firms sets a price and consumers respond by choosing whether to buy a good and from which firm. We assume that the consumers never buy more than one unit of a good, i.e. each consumer has  $(n + 1)$  different choices. The probability a consumer, who values the good at  $v$ , will decide to buy the good from firm  $i$  is given by:

$$q_i = \frac{\exp((v - p_i)/\lambda)}{1 + \sum_{j=1}^n \exp((v - p_j)/\lambda)}, \quad (3.3)$$

while the probability that she will decide not to buy the good at all is:

$$q_0 = \frac{1}{1 + \sum_{j=1}^n \exp((v - p_j)/\lambda)}. \quad (3.4)$$

Here  $p_i$  is the price charged by oligopolist  $i$ .

It is instructive to compare the demand shares implied by equation (3.3) with the ones implied by the Hotelling model. Though parameter  $\lambda$  plays a role somewhat similar to the transportation costs, there are important differences. For example, in our model consumers are not partitioned into groups that purchase from a particular firm. Also, in the latter model, the demand shares will depend only on price differences as long as the consumers' value is sufficiently high, so that all consumers purchase the good in equilibrium.<sup>5</sup> In our model the result will only obtain asymptotically, as  $v \rightarrow +\infty$ . Also, as we will see below, unlike the case of the Hotelling model, for finite  $n$  the equilibrium price depends on  $v$ .

Assuming the marginal cost is constant and equal to  $c$  for all firms, the profits of firm  $i$  are given by:

$$\Pi_i(p_i, p_{-i}) = (p_i - c)q_i(p_i, p_{-i}), \quad (3.5)$$

here  $p_{-i}$  denotes the vector of prices chosen by its rivals. The first order conditions are:

$$\frac{1}{p_i - c} = -\frac{\partial \ln q_i}{\partial p_i}, \text{ for } i = 1, \dots, n. \quad (3.6)$$

In the unique<sup>6</sup> symmetric equilibrium  $p_i = p^{(n)}$ ,<sup>7</sup> where  $p^{(n)}$  solves:

$$p^{(n)} = c + \lambda \frac{1 + n \exp((v - p^{(n)})/\lambda)}{1 + (n - 1) \exp((v - p^{(n)})/\lambda)}. \quad (3.7)$$

One can write equation (3.7) as

$$p^{(n)} = F(p^{(n)}), \quad (3.8)$$

where  $F(\cdot)$  is defined as:

$$F(p^{(n)}) = c + \lambda \frac{1 + n \exp((v - p^{(n)})/\lambda)}{1 + (n - 1) \exp((v - p^{(n)})/\lambda)}. \quad (3.9)$$

---

<sup>5</sup> In our model for any profile of prices and any finite  $v$  some consumers will choose not to purchase the good.

<sup>6</sup> We restrict our attention to the pure strategy equilibria. For a discussion of a mixed strategy equilibrium that allows firms to earn positive profits in the standard Bertrand game, see Klemperer (2004).

<sup>7</sup> From here on, a subscript  $i$  will refer to the price charged by firm  $i$ , while a superscript  $(n)$  will refer to the price charged by any firm in a symmetric equilibrium with  $n$  firms.

It is straightforward to establish that

$$F'(p^{(n)}) = -\frac{\exp((v - p^{(n)})/\lambda)}{[1 + (n - 1) \exp((v - p^{(n)})/\lambda)]^2} < 0. \quad (3.10)$$

Therefore, the right hand side of (3.7) is monotonically decreasing in  $p^{(n)}$ , while the left hand side monotonically increases from zero to infinity, which implies that the solution exists and is unique. Moreover, since an increase in the number of firms will decrease the right hand side of equation (3.7) for any  $p^{(n)}$ , the equilibrium price will uniformly decrease from the monopoly price  $p^{(1)}$  to  $c + \lambda$ , when the number of firms approaches infinity. Therefore, we arrive at the following Lemma:

**Lemma 1** *For any  $n$ , the equilibrium price is above the marginal cost and is decreasing in the number of firms.*

Let us concentrate on an important special case when the consumers' valuation of the good is large compared with both the marginal cost and the irrationality parameter. Then for  $n \geq 2$  equation (3.7) implies:

$$p^{(n)} = c + \frac{n\lambda}{n-1} + O\left(\exp\left(-\frac{v-c}{\lambda}\right)\right). \quad (3.11)$$

Note that the equilibrium price is always much smaller than the valuation, therefore one can neglect the possibility that the consumers do not buy the good (the probability of it is proportional to  $\exp(-(v - p_n)/\lambda)$ , which is exponentially small).

### 3.3.1 The monopoly pricing

In this Subsection we analyze the behavior of a monopoly, derive a general formula for  $p^{(1)}$  and provide simple expressions for two opposite cases, when the consumers are nearly rational or strongly irrational.

The monopoly price can be obtained from equation (3.7) by setting  $n = 1$  and it is the unique solution of the equation:

$$p^{(1)} = c + \lambda (1 + \exp(v - p^{(1)})/\lambda). \quad (3.12)$$

One can rewrite equation (3.12) as:

$$\frac{v - c}{\lambda} = \Phi\left(\frac{v - p}{\lambda}\right), \quad (3.13)$$

where

$$\Phi(x) = 1 + x + \exp(x). \quad (3.14)$$

Since function  $\Phi(\cdot)$  is strictly increasing, its inverse function,  $\Phi^{-1}(\cdot)$ , is well defined.

Therefore, we arrive at the following Lemma:

**Lemma 2** *The equilibrium monopoly price is given by:*

$$p^{(1)} = v - \lambda \Phi^{-1}\left(\frac{v - c}{\lambda}\right). \quad (3.15)$$

It is interesting to note that  $p^{(1)} > v$  provided that  $(v - c)/\lambda < 2$ , i.e. if either total social surplus,  $v - c$ , is small or the consumers are sufficiently irrational, i.e.  $\lambda$  is large, the monopolist will charge the price above the valuation. To get some intuition for that result let us consider the case when value is close to the marginal cost. Though charging price below value increases the probability of a purchase, the monopolist will make more money by charging price above value and capitalizing on irrationality of consumers.<sup>8</sup>

Now let us look at the special case when the consumers are strongly irrational. Then the equilibrium price satisfies

$$p^{(1)} = v - \lambda \left[ x_0 + O\left(\frac{v - c}{\lambda}\right) \right], \quad (3.16)$$

where  $x_0$  solves equation

$$\Phi(x_0) = 0. \quad (3.17)$$

Solving it numerically one obtains  $x_0 = -1.28$ , which implies in the main order of approximation

$$p^{(1)} = v + 1.28\lambda. \quad (3.18)$$

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<sup>8</sup> Making a mistake that leads to pay above one's value is not qualitatively different from buying from a wrong firm.

In the opposite case, when  $(v - c)/\lambda$  is large, i.e. either the consumers are nearly rational or the total social surplus,  $v - c$ , is large, the optimal monopoly price is approximately equal to:

$$p^{(1)} = v - \lambda \ln \frac{v - c}{\lambda}, \quad (3.19)$$

where we used that

$$\Phi^{-1}(y) = \ln y + O\left(\frac{\ln y}{y}\right). \quad (3.20)$$

### 3.3.2 Number of firms and social welfare

Next, we would like to study how social welfare changes depending on the number of firms. Recall, that in the case of fully rational buyers the social welfare under Bertrand competition is independent of the number of firms. However, this conclusion does not hold under bounded rationality.

First, note that since  $v > c$ , the social efficiency requires that all consumers buy a unit of the good. Therefore, the welfare loss,  $WL$ , is proportional to the fraction of the consumers who do not buy the good.

More precisely, welfare loss is given by

$$WL(n) = q_0(v - c) = \frac{v - c}{1 + n \exp((v - p^{(n)})/\lambda)}. \quad (3.21)$$

Let us formulate the following Proposition.

**Proposition 1.** *If the consumers are boundedly rational both the total welfare and the consumer surplus increase when there are more firms in a Bertrand competitive industry. A firm's profits decrease as the number of firms increases, but are always bounded away from zero.*

**Proof.** It is easy to see that the welfare loss declines in the number of firms. Indeed

$$\frac{\partial WL(n)}{\partial n} = - \frac{(v - c) \exp((v - p^{(n)})/\lambda)}{[1 + n \exp((v - p^{(n)})/\lambda)]^2} \left(1 - \frac{n}{\lambda} \frac{dp^{(n)}}{dn}\right) < 0 \quad (3.22)$$

where we used that according to Lemma 1

$$\frac{dp^{(n)}}{dn} < 0. \quad (3.23)$$

Since increase in  $n$  leads to both an increase of consumer participation and a decrease in price, the consumer surplus increases.

A firm's profits are given by:

$$\pi = \frac{1 - q_0}{n} (p^{(n)} - c) = \frac{p^{(n)} - c}{n + \exp(-(v - p^{(n)})/\lambda)}. \quad (3.24)$$

Therefore

$$\frac{d\pi}{dn} = \frac{\partial\pi}{\partial n} + \frac{\partial\pi}{\partial p^{(n)}} \frac{dp^{(n)}}{dn}. \quad (3.25)$$

It is straightforward to show that

$$\frac{\partial\pi}{\partial n} < 0. \quad (3.26)$$

On the other hand

$$\frac{\partial\pi}{\partial p^{(n)}} = \frac{n\lambda - (p^{(n)} - c - \lambda) \exp(-(v - p^{(n)})/\lambda)}{\lambda [n + \exp(-(v - p^{(n)})/\lambda)]^2}. \quad (3.27)$$

Define

$$H(z) = n\lambda - (z - c - \lambda) \exp(-(v - z)/\lambda). \quad (3.28)$$

Straightforward differentiation shows that

$$H'(z) = -\frac{(z - c) \exp(-(v - z)/\lambda)}{\lambda}. \quad (3.29)$$

The numerator of (3.27) is equal to  $H(p^{(n)})$  and according to (3.29) is decreasing in  $p^{(n)}$  as long as  $p^{(n)} > c$ , which is always true according to Lemma 1. Therefore, since  $p^{(n)}$  is decreasing in  $n$ ,

$$H(p^{(n)}) > H(p^{(1)}) = (n - 1)\lambda \geq 0, \quad (3.30)$$

where we made use of equation (3.12). Thus,

$$\frac{\partial \pi}{\partial p^{(n)}} > 0 \quad (3.31)$$

together with (4.1) this implies that

$$\frac{d\pi}{dn} < 0. \quad (3.32)$$

Since, according to Lemma 1, the price remains above marginal cost a firm's profits are bounded away from zero. ■

Note that the derivative in equation (3.22) can be represented as a sum of two terms. The first term captures the reduction of the social loss due to the increase in the number of efficient options, while the second captures the decrease of probability of staying out of the market due to the reduction of price. The main result of this section is that if the consumers are boundedly rational both the total social surplus and the consumer surplus increase when there are more firms in a Bertrand competitive industry. It is also worth mentioning that

$$\lim_{n \rightarrow +\infty} q_0(n) = 0, \quad (3.33)$$

therefore taking into account equation (3.11) a firm's profits behave as

$$\pi = \frac{\lambda}{n} + o\left(\frac{1}{n}\right) \quad (3.34)$$

and the total profits of the industry converge to  $\lambda$ .

### 3.4 An obfuscating oligopoly

In this Section we assume that the consumers are rational at default ( $\lambda = 0$ ); however, the firms can spend resources to confuse the consumers. This will lead to an increase in the value of  $\lambda$ . In the Introduction, we discussed why resources spent by one firm affect probabilities of choice between any alternatives. For simplicity, we also make, admittedly strong, assumption that the single parameter  $\lambda$  captures all these effects. One way to

justify it, is to assume that a consumer goes through the list of the alternatives in a random order.

We will assume the following timing. At date 1 all firms simultaneously decide on the amount of resources they spend confusing the consumers. If firm  $i$  devotes resources  $z_i$  to the obfuscating activities, then

$$\lambda = \varphi \left( \sum_{i=1}^n z_i \right), \quad (3.35)$$

where  $\varphi(\cdot)$  is a twice differentiable, strictly increasing, concave function, such that  $\varphi(0) = 0$ , and satisfying the Inada conditions. We will refer to it as an “obfuscating technology.” At the beginning of date 2 the firms observe  $\lambda$  and play the simultaneous Bertrand game.

Introduce

$$\Pi_i(\lambda, n, p^{(n)}) = \frac{(p^{(n)} - c) \exp((v - p^{(n)})/\lambda)}{1 + n \exp((v - p^{(n)})/\lambda)}, \quad (3.36)$$

and define  $G(\lambda, n)$  to be the profits firm  $i$  earns at date 2, assuming the firms at that date play the symmetric Nash equilibrium obtained in the previous section. Then

$$G(\lambda, n) = \Pi_i(\lambda, n, p^{(n)}(\lambda)), \quad (3.37)$$

where  $p^{(n)}(\lambda)$  is determined by equation (3.7). At date 1 firm  $i$  chooses the amount it invests in obfuscation to solve:

$$\max_{z_i} \left( G \left( \varphi \left( \sum_{j=1}^n z_j \right), n \right) - z_i \right). \quad (3.38)$$

The first order conditions are:

$$G_\lambda(\lambda, n) \varphi' \left( \sum_{j=1}^n z_j \right) = 1. \quad (3.39)$$

In a symmetric Nash equilibrium with  $n$  firms  $z_i = z^{(n)}$  and

$$G_\lambda(\varphi(nz^{(n)}), n) \varphi'(nz^{(n)}) = 1. \quad (3.40)$$

Equation (3.40) allows us to find equilibrium amount of obfuscation for any number of firms. Inada conditions guarantee that if at default consumers are rational, the equilibrium amount of obfuscation is positive.

Let us determine the equilibrium amount of obfuscation when the number of competitors becomes very large. In that situation, we can write

$$p^{(n)} = c + \lambda + O\left(\frac{1}{n}\right) \quad (3.41)$$

and

$$G(\lambda, n) = \frac{\lambda}{n + e * \exp\left(\frac{c-v}{\lambda}\right)} + O\left(\frac{1}{n}\right). \quad (3.42)$$

Let  $Z^{(n)} = nz^{(n)}$  be the total amount of obfuscation in the equilibrium. Next, we show that the amount of resources the industry devotes to obfuscating activities converges to zero as the number of firms increases to infinity.

**Lemma 3.** *As the number of firms tends to infinity,  $Z^{(n)}$  converges to zero, i.e.*

$$\lim_{n \rightarrow +\infty} Z^{(n)} = 0. \quad (3.43)$$

**Proof.** Equation (3.42) implies  $G_\lambda \rightarrow 0$ , therefore equation (3.40) implies that  $\varphi'(nz^{(n)}) \rightarrow \infty$  and the Inada conditions imply that  $Z^{(n)} = nz^{(n)} \rightarrow 0$ . ■

If, instead of assuming that consumers are rational at default, one assumes that they are characterized by some positive  $\lambda_0$ , there will exist  $n_0 \in \mathbb{N}$  such that whenever number of firms is at least  $n_0$ , no resources are spent on obfuscation in equilibrium. If  $\lambda_0$  is sufficiently large, then value of  $n_0$  can be one, thus, resources will never be spent on obfuscation: even monopoly would find it unprofitable.

The reason for the result stated in the Lemma 3 is that the obfuscation is, from the point of view of the firms, a public good and the firms, therefore, tend to free ride at each other's expenses.

It is hard to further analyze equation (3.40) in general and we consider a special case

when the surplus,  $v - c$ , is large compared to the equilibrium level of  $\lambda = \lambda^*$ .

We now show that in this special case the incentive to obfuscate is decreasing in competition: the total amount of resources spent on obfuscation monotonically decreases in number of firms. Indeed, the price is approximately given by equation (3.11) for  $n \geq 2$  and (3.12) in the case of monopoly and with the same precision

$$G(\lambda) = \frac{\lambda^*}{n-1} + O\left(\frac{\lambda^*}{v-c}\right) \quad (3.44)$$

in the proper oligopoly case,  $n \geq 2$ . In that case, neglecting  $O$ -terms, (3.40) can be rewritten as:

$$\varphi'(Z^{(n)}) = \varphi'(nz^{(n)}) = n-1, \quad (3.45)$$

where  $z^{(n)}$  is the equilibrium value of  $z$ , when the number of firms is  $n$ . Since  $\varphi(\cdot)$  is assumed to be strictly concave, equation (3.45) implies that the total amount of resources devoted to obfuscation decreases in the number of firms, i.e.  $Z^{(2)} > Z^{(3)} > Z^{(4)} \dots$

Let us look at a specific example. If

$$\varphi(z) = \sqrt{z} \quad \text{and} \quad v - c = 100, \quad (3.46)$$

then for the oligopoly case ( $n \geq 2$ ) we have:

$$z^{(n)} = \frac{1}{4n(n-1)^2}, \quad Z^{(n)} = \frac{1}{4(n-1)^2}, \quad \lambda_n = \varphi(Z^{(n)}) = \frac{1}{2(n-1)}. \quad (3.47)$$

Note that equilibrium value of  $\lambda$  is 0.5 for a duopoly, and monotonically decreases to zero as  $n$  goes to infinity, always remaining much smaller than  $v - c$ .

For a monopoly, the special case when  $v - c$  is large compared to the equilibrium level of  $\lambda$  never realizes, since the monopoly will always choose the level of obfuscation high enough to bring  $\lambda$  sufficiently close to  $v - c$ .

We will refer to a situation when the total level of obfuscation is decreasing in the number of firms as a regular case. It always realizes when the total surplus is sufficiently greater than the irrationality parameter.

Results of this Section show that from the social point of view increased competition is beneficial for two different reasons. First, as emphasized in the previous section, it decreases the probability that a consumer will choose not to buy the good. Second, it decreases the total amount of resources spent on obfuscation of the consumers.

### 3.5 An educating oligopoly

In this Section we show that it is only optimal for the firms to spend resources to educate boundedly rational consumers who are sufficiently rational at default.

Assume that the consumers have  $\lambda = \lambda_0 > 0$ , i.e. consumers are boundedly rational at default. The firms can spend resources to educate the consumers, for example, they can engage in informative advertising that will lead to a decrease in the value of  $\lambda$ .

The timing is the same as in the previous section. If firm  $i$  devotes resources  $\zeta_i$  to the educating activities, then

$$\lambda = \phi \left( \sum_{i=1}^n \zeta_i \right), \quad (3.48)$$

where  $\phi(\cdot)$  is a twice differentiable, strictly decreasing, convex function, satisfying the Inada conditions, such that

$$\phi(0) = \lambda_0, \lim_{\zeta \rightarrow \infty} \phi(\zeta) = 0, \quad (3.49)$$

i.e. spending infinite resources on educating activities will make the consumers fully rational. We will refer to  $\phi(\cdot)$  as an “educating technology.”

Note that educating activities of one firm affect the probability of choices between any pair of products. Intuition for this assumption is that products produced by the firms are homogenous; therefore, better understanding of characteristics of one product helps to understand characteristics of all the products. Again, we make an assumption, admittedly strong, that the effect is symmetric.

At the beginning of date 2 the firms observe each other’s choice of  $\zeta_i$  and play the simultaneous Bertrand game, assuming that the market is populated by the consumers whose rationality parameter is given by (3.48). Denote by  $\Pi_i(\lambda, n)$  the profits firm  $i$  earns at date 2, assuming the firms at that date play the symmetric Nash equilibrium (obtained

in the previous section). Then

$$\Pi_i(\lambda, n) = \frac{(p^{(n)} - c) \exp((v - p^{(n)})/\lambda)}{n \exp((v - p^{(n)})/\lambda) + 1}, \quad (3.50)$$

where  $p^{(n)}$  is determined by equation (3.7).

We will argue that firms spend resources on educating the consumers only if consumers are sufficiently rational at default and spend no resources if the consumers are strongly irrational. This happens for two reasons. First, if there are at least two firms on the market, any potential increase in profits due to the higher degree of the consumers' participation will be eroded by price competition. The investment in educating consumers in the case of at least two firms in the market will be decreased further because such an investment represents a public good from the point of view of the firms. Second, even in the case of the monopoly, the firm will find it more profitable to cut the price than to spend resources on educating the consumers. The above discussion suggests that if there going to be any investment in educating consumers at all, it should happen in the case of a monopoly.<sup>9</sup> So, we concentrate on this case and argue that even a monopoly will choose not to educate the consumers.

**Proposition 2.** *If the consumers are sufficiently irrational, i.e.*

$$\lambda_0 > \min \left( \frac{v - c}{2}, \frac{v - c}{\zeta^*} \right), \quad (3.51)$$

where  $\zeta^*$  solves

$$\frac{\zeta^* \exp(\zeta^*)}{1 + \exp(\zeta^*)} = -\frac{1}{\phi'(0)}, \quad (3.52)$$

the monopolist will spend no resources educating consumers.

**Proof.** The monopolist solves:

$$\max_{p, \lambda, \zeta} \left( \frac{(p - c) \exp((v - p)/\lambda)}{1 + \exp((v - p)/\lambda)} - \zeta \right) \quad (3.53)$$

---

<sup>9</sup> At the end of the previous Section we saw that, at least in the case when  $v - c$  is large compared to the equilibrium level of  $\lambda$ , the total resources spent on obfuscation decrease in the number of firms and referred to it as the regular case. A similar result can be obtained for the case of educating oligopoly. We will assume that the regular case is realized.

$$s.t. \lambda = \phi(\zeta), \quad (3.54)$$

$$\zeta \geq 0. \quad (3.55)$$

Here we dropped superscript <sup>(1)</sup> from the equilibrium price for the notational simplicity. The non-negativity constraint was never binding in the obfuscating oligopoly case due to the Inada conditions. The Kuhn-Tucker first order conditions are:

$$\left\{ \begin{array}{l} p = c + \lambda(1 + \exp(\frac{v-p}{\lambda})) \\ \mu = -\frac{(v-p)(p-c)\exp((v-p)/\lambda)}{\lambda^2(1 + \exp((v-p)/\lambda))^2} \\ \gamma = 1 - \mu\phi'(\zeta) \\ \lambda = \phi(\zeta) \\ \gamma\zeta = 0, \gamma \geq 0, \zeta \geq 0 \end{array} \right. , \quad (3.56)$$

where  $\mu$  and  $\gamma$  are Lagrange multipliers on constraints (3.54) and (3.55) respectively. Equations (3.48) and (3.49) together with the properties of function  $\phi(\cdot)$  imply that  $\lambda > 0$ . The first equation in system (3.56) now implies that  $p > c$ . Using the first equation in system (3.56), the second equation can be re-written as

$$\mu = -\frac{(v-p)\exp((v-p)/\lambda)}{\lambda(1 + \exp((v-p)/\lambda))}. \quad (3.57)$$

Note that if

$$\frac{v-c}{\lambda} < 2 \quad (3.58)$$

then  $p > v$  and  $\mu > 0$ , in which case, the monopolist's problem has the following solution: price  $p$  is given by (3.15) and satisfies  $p > v$ ,  $\zeta = 0$ ,  $\lambda = \lambda_0$ ,  $\mu > 0$  is given by (3.57), and  $\gamma = 1 + \mu\phi'(0) > 0$ . On the other hand, if

$$\frac{v-c}{\zeta^*} < \lambda_0 < \frac{v-c}{2}, \quad (3.59)$$

then  $\mu < 0$  and

$$\gamma = 1 - \mu\phi'(\zeta) > 1 - \mu\phi'(0) = 1 + \frac{(v-p)\exp((v-p)/\lambda)}{\lambda(1+\exp((v-p)/\lambda))}\phi'(0). \quad (3.60)$$

Since  $p > c$  and  $\phi'(0) < 0$  one obtains

$$\gamma > 1 + \frac{(v-c)\exp((v-c)/\lambda)}{\lambda(1+\exp((v-c)/\lambda))}\phi'(0). \quad (3.61)$$

Define  $\zeta^*$  to be the solution of (3.52). Then for any  $\lambda > (v-c)/\zeta^*$  one obtains  $\gamma > 0$ , which implies  $\zeta = 0$ ,  $\lambda = \lambda_0$ . ■

Note that though the Proposition holds when

$$\lambda_0 > \min\left(\frac{v-c}{2}, \frac{v-c}{\zeta^*}\right), \quad (3.62)$$

the case  $\lambda_0 > (v-c)/2$  is not very interesting, since when the irrationality parameter is not sufficiently smaller than the total surplus the case may not be regular. However, if  $\phi(\cdot)$  is sufficiently flat at the origin, the condition  $\lambda_0 > (v-c)/\zeta^*$  can be consistent with the regular case. Numerical simulations tell us that, for example, when  $\phi'(0) = -0.2$  then  $\zeta^* = 5.03$ , when  $\phi'(0) = -0.1$  then  $\zeta^* = 10$ . In general, for  $\phi'(0) > -0.1$  one can approximate  $\zeta^* = -1/\phi'(0)$ . Therefore, in the regular case, the analysis is reduced to that of Section 3 with  $\lambda = \lambda_0$ .

## 3.6 Conclusions

In this chapter we developed a model of Bertrand oligopolistic competition with boundedly rational consumers. We used the Luce model of probabilistic choice (Luce, 1959) to capture the degree of the consumers' rationality that could be influenced by firms engaging either in educating or obfuscating activities. We have shown that contrary to the results of Grossman (1981) and Milgrom (1981), firms will in general engage in obfuscating activities, but will engage in educating ones only if consumers are sufficiently rational at default.

In this framework competition has two beneficial effects from the social point of view:

it increases the chance that a consumer will buy the product and decreases the amount of obfuscation. The second effect arises because obfuscation is a public good from the firms' point of view and, therefore, the firms try to free ride on each other. From the consumers' point of view, there is another beneficial effect of competition - it leads to decrease in price.



## Chapter 4

# Before or after? Qualifying contractors in procurement auctions

### 4.1 Introduction

The enviable skill of every good procurer is the ability to allocate a project to a contractor (or choose a supplier) who can provide reliable quality at a reasonable cost. In order to identify the potential contractor who can complete a project at the lowest cost, the procurer can, for example, allocate the project by a competitive bidding procedure conducted through an auction. Regrettably, low cost contractors are also more likely to fail to meet the specific requirements (standards) of the procurer. This feature makes the risk of nonperformance an important issue which must be dealt with by the procurer.

The failure of the procurement department to properly screen contractors for reliability is very costly. Typically, the supply chain management of U.S. firms solves the problem of potential nonperformance by conducting costly and time consuming screening checks in order to qualify all potential suppliers before allowing them to participate in a procurement auction.

Another approach to addressing the contractors' potential nonperformance is to rule out excessively low bids outright. This is commonly done in many European countries, where abnormally low tenders for public works are automatically excluded from the valid bids (see Engel et al., 2006). Due to its anticompetitive flavour, though, this procedure has been opposed by the EU Commission who instead mandates that low bid contractors

should be allowed to win an auction but, before the project is awarded, their reliability must be assessed in an after-the-auction audit carried out by the procurer. Thus, this approach of conducting the auction followed by testing is the opposite of the one adopted by the commercial supply chains in the U.S., where all contractors are tested before the auction.

In this paper, we employ a mechanism design approach to address the question of when (before or after the auction) the procurer should test contractors for reliability. We characterize the optimal procurement mechanism in the case of both testing before and after the auction and compare the corresponding surplus of the procurer, the payoffs of the contractors, and total social surplus.

We consider a procurer who wishes to allocate a particular project to one contractor out of a pool of  $N$  potential candidates, who possess two characteristics: the ability to complete the project satisfactorily (a binary variable) and the production cost (a continuous variable). Each contractor privately knows his own cost of completing the project, but faces the possibility that he might fail to complete the project to the standard required by the procurer. If a contractor is able to complete the project, we will call him a "good match" for the procurer; otherwise, he is a "bad match".

We assume that a contractor's ability to complete the project – i.e., of being either good or bad match – is unobservable by either the procurer or the contractor himself until a test is carried out, as the procurer might not have enough information about the contractor and the contractor might be unsure about the details of the project and the specific requirements of the procurer. The test is fully revealing – only contractors who are good matches are able to pass it.

We consider the case when a contractor's probability of being a good match for the procurer is a monotone – increasing or decreasing – function of his production cost.

Assuming that this conditional probability is increasing (a contractor with a higher production cost is more likely to be a good match) is equivalent to assuming that the two variables/characteristics, the quality of the match and the production cost, are **affiliated**, as defined by Milgrom and Weber (1982). The case when this probability is

decreasing (i.e., a contractor with a higher production cost is less likely to be a good match) corresponds then to the case of **negative affiliation** between the two, extending Milgrom and Weber's definition for it to mean that quality of the match and the negative of the cost are affiliated.

This assumption of monotone conditional probability implies that the contractor's ability to complete the project is correlated with his production cost; the case when the two are independently distributed is a special case of our model.

Particularly interesting is the case of affiliation (thus, positive correlation) between a contractor's production cost and his ability to complete the project, which may occur in many realistic situations and, to our knowledge, has not been considered in the literature.

Let us discuss a few examples of both public works procurement and supply chain management. Positive correlation (affiliation) between cost and completion may occur, for example, because firms that do not exert as much effort on cost cutting are more likely to have higher quality standards; or because small local firms who tend to be low cost are also more vulnerable to financial stress and more likely to default; or because the suppliers with higher costs may have excess capacity available in case the procurer needs it and, therefore, are more likely to satisfy the procurer's requirements.

We identify the optimal procurement mechanisms for testing before and after the auction when procurer and contractors are risk neutral.

Similar to the standard case in the literature, we characterize the set of implementable (i.e., incentive compatible and individually rational) mechanisms the procurer could choose from in order to maximise her surplus. Similar to the standard case, we are able to obtain the expression for the procurer's expected surplus in terms of the contractors' virtual costs. However, due to the affiliation between the production cost and the ability to complete the project, the virtual cost includes conditional probabilities instead of the unconditional ones in the standard expression.

We show that the optimal procedure with testing before the auction is a standard auction (e.g., FPA or SPA) with a reserve price, in which all contractors who passed the test participate. Note that though the number of participants in the procurement

auction may be different (it is a random variable whose realization is determined after the test is conducted), the reserve price is the same and does not depend on the number of contractors who passed the test; this is analogous to the standard result without non-completion risk.

The optimal mechanism with testing after the auction could also be implemented by a standard auction with a reserve price, where after the auction all contractors are ranked based on their bids from the lowest to the highest and are tested in that order until a contractor with the lowest bid who is a good match (i.e., can pass the test) is found and awarded the project. The reserve price is the same whether testing is done before or after the auction.

An important question arises for the procurer - is it more beneficial for her to conduct testing **before or after** the auction?

On one hand, if testing is conducted before the auction, then the contractors know that they all have been qualified and so “every bid counts”, pushing them to bid more aggressively during the auction, while in a mechanism with bidding first and testing after, some bids may be discarded due to the failure of a bidder to pass the after-the-auction test and so bidders might feel less pressure to bid aggressively.

On the other hand, because the number of the participants in the mechanism is the same, whether the test is performed before or after the auction, all of them will bid in the auction if it is carried out before the test, while a smaller, random number of contractors will participate in the auction if testing is done first, before the auction. Therefore, from the point of view of the number of the participants, auctions with testing done after exert more competitive pressure on the contractors than auctions where testing is done before, forcing contractors to bid more aggressively in after-the-auction testing mechanisms.

The two effects work in the opposite direction and, as we show, cancel each other exactly when the procurer and contractors are risk neutral. The “every bid counts” effect present in testing before the auction is completely offset by the effect due to the presence of a smaller number of bidders.

Thus, we prove that, in the case of risk neutrality, the “surplus/payoff equivalence”

result holds – the timing of the test (before or after the auction) does not matter. In both cases, the project is awarded to the same good match contractor (if there is one) who has the lowest cost; the expected surplus of the procurer, the expected payoffs of contractors and the total expected social surplus are exactly the same.

Our result implies that even if testing can be performed at no cost, there are no benefits from testing before the auction – a risk neutral procurer facing risk neutral contractors can achieve the same result by testing after the auction.

When testing is costly, even if the cost is infinitesimally small, it is clearly advantageous for the procurer to test after the auction, because it involves a smaller number of tests, as contractors are sequentially tested in the order of their bids, until a good match is found. Testing before the auction, on the other hand, requires to test all potential contractors before they are allowed into the auction, leading to a higher testing cost.

Both the economics and management literatures have studied remedies for the risk of non-performance. The former literature considers standard auctions and concentrates on arrangements such as penalties, performance bonds, and third party guarantees like letters of credit and surety bonds (Waehrer, 1995; Calveras et al., 2004; Board, 2007). The basic assumption is that the probability of nonperformance depends on the bidding strategy, with default being the best choice when the realized costs of production are high. Our approach differs from this literature as we want to study the situation when the non-performance is due to external circumstances; in order to capture this idea we assume that the probability of nonperformance is exogenously given. The management literature on nonperformance in supply chain relationships studies such arrangements as deferred payments, payments contingent on performance, and dual sourcing (Yang et al., 2009; Babich and Tang, 2012; Yang et al., 2012).

The paper most closely related to ours is Wan and Beil (2009), who also focus on the trade-off between testing suppliers before and after the auction. Unlike Wan and Beil, who assume that a contractor's ability to complete the project (to qualify) and his production cost are independent, we consider the case where they are affiliated - for example, with positive correlation the contractors with higher costs are more likely to

complete the project; the case of independence is, therefore, a special case of our model. Our result that there are no benefits for a risk neutral procurer to pre-qualify suppliers even when it could be done at zero cost differs from the results in Wan and Beil (2009). They argue that there is a trade off between testing before and after the auction and that a risk neutral procurer would find it beneficial to implement (partial) pre-testing if it is not too costly. Their results rely heavily on the assumption that there are an infinite number of suppliers available and that the procurer selects  $n$  of them to be admitted to the bidding process. This implies that testing before the auction allows the procurer to select a pool of bidders with a better than average probability of completing the project, while when testing after the auction the procurer must select the  $n$  bidders randomly from the infinite pool. Thus, their conclusion that testing before might be beneficial is driven not by the relative merits of testing before vs testing after (e.g., how it affects the bidding behaviour of the participants), but by the ability to select a fixed number of bidders from an infinite pool; but if there is an infinite pool of bidders, why not let them all bid and test them afterwards?

The paper is organized as follows. Section 4.2 introduces the model. Section 4.3 studies the benchmark case in which testing is not available. Section 4.4 extends the analysis to the case in which a risk neutral procurer can test the risk neutral contractors' ability to complete the project, both before and after the auction, and compares the two alternatives. Section 4.5 concludes.

## 4.2 The Model

A procurer is seeking to allocate a project to one of  $N$  contractors. If the project is completed, the procurer obtains benefit  $v$  and makes a payment to the contractor who completed it; otherwise, she obtains zero benefit and makes no payment. Each contractor incurs a production cost whenever he completes the project.

Contractor  $i$ 's production cost  $c_i$  is randomly drawn from a distribution function  $G(c_i)$  with positive density function  $g(c_i)$  over the interval  $[\underline{c}, \bar{c}]$ , all of which is common

knowledge. Each contractor  $i$ , but not the procurer or the other contractors, knows his own production cost  $c_i$ .

If contractor  $i$  is allocated the project, he may be unable to complete it up to the procurer's standards. To simplify the exposition, we will call a contractor who is able to complete the project (and deliver the benefit  $v$  to the procurer) a “**good match**” and a contractor who is unable to complete the project a “**bad match**”. Whether any particular contractor is good or bad is unknown to either the contractor himself or the procurer.

These two characteristics of a contractor, his production cost and the quality of the match, need not be independent; in many interesting applications they are either positively or negatively correlated. We will assume that contractor  $i$ , conditional on his production cost being  $c_i$ , is a good match with probability  $\Phi(c_i)$  and a bad match with the remaining probability, where the function  $\Phi : [\underline{c}, \bar{c}] \rightarrow [0, 1]$  is differentiable. Note that the probability to be a good match – i.e., to successfully complete the project – conditional on the contractor's production cost, depends only on his cost, and not on the bid he places during the auction. If a contractor fails to deliver the project, it is due to exogenous reasons and not because of his low bid. To avoid non-performance by a contractor, the procurer can carry out a test (at no cost) to determine whether the contractor is a good or a bad match. The test is fully revealing: contractor  $i$  passes the test if and only if he is a good match.

We will focus attention on the case when the conditional probability of being a good match,  $\Phi(c_i)$ , is a monotone function. When the conditional probability is increasing, the contractor's match quality (i.e. his ability to complete the project) is affiliated<sup>1</sup> and, therefore, positively correlated with his production cost. A reason for this could be that good, safe firms do not gamble on cost reduction (for example, they employ a more qualified workforce and use better quality materials) and hence, on average, have higher production costs. Another possibility is that smaller firms have lower costs on

<sup>1</sup>To see that  $\Phi(c)$  increasing implies affiliation, let  $c_1 < c_2$ . We need to show that:  $\Pr(\text{good}, c_2) \cdot \Pr(\text{bad}, c_1) = \Phi(c_2)g(c_2) \cdot [1 - \Phi(c_1)]g(c_1) \geq \Phi(c_1)g(c_1) \cdot [1 - \Phi(c_2)]g(c_2) = \Pr(\text{good}, c_1) \cdot \Pr(\text{bad}, c_2)$ , which clearly holds since  $\Phi(c_2)[1 - \Phi(c_1)] \geq \Phi(c_1)[1 - \Phi(c_2)]$ .

average, but also lower financial strength, resulting in a higher probability of default. In the opposite case, when  $\Phi(c_i)$  is decreasing, there is negative affiliation (and so negative correlation) between the quality of the match and the production cost - as when some contractors have developed an expertise in particular projects and thus have a superior ability to deliver high quality projects at a low cost. When the probability  $\Phi(c_i)$  is constant, the quality of the match and the production cost are independent; this is a special case of our model. The standard case in the literature corresponds to  $\Phi(c_i) = 1$  - contractors are always “good”; that is, always capable of delivering the project.

We use a mechanism design approach to identify the optimal procurement mechanisms when some hidden information - the quality of the match between the procurer and a contractor - could be discovered by the procurer through testing; particularly, we are interested in determining when the procurer should discover this information. Is it better to carry out the testing before or after the auction?

By the revelation principle, we restrict attention, without loss of generality, to direct incentive-compatible mechanisms, for each case: testing before the auction and testing after the auction.<sup>2</sup> In the case of testing before the auction: first, all contractors are tested to identify the good matches; second, the good contractors (those who survived the test) report their true production costs; third, based on those reports, the procurer allocates the project and makes payments. In the case of testing after the auction: after the contractors have reported their costs, they are ranked according to a priority order and the procurer tests the highest ranked contractor and allocates the project to him if he is a good match. If the highest ranked contractor fails the test, the procurer tests the second ranked contractor, and so on till a good match is found and the project allocated, or if there are no good matches the project is not allocated. After the allocation decision, payments to the contractors are made.

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<sup>2</sup>We will show that allocations corresponding to the optimal mechanisms can be implemented by auctions; therefore, we will say *testing before or after the auction*, instead of saying *testing at the beginning or at the end of the optimal mechanism*.

### 4.3 A benchmark: optimal procurement without testing

We start by considering the case when testing is not available so whenever the procurer happens to assign the project to a contractor who is a bad match for her, the project is not delivered. It is a useful benchmark against which we can compare testing either before or after the auction and it allows us to develop the main techniques we will employ in the rest of the paper. Alternatively, the case of no-testing can be thought of as a situation when the chosen contractor does undergo testing before a project could be assigned to him, but if this contractor fails the test and turns out to be a bad match, then the project cannot be reassigned to a different contractor and is, therefore, not completed.

In this section we identify incentive compatible (IC) mechanisms (Lemma 1), derive the expression for the procurer's surplus in terms of the contractors' virtual costs, characterize the optimal mechanisms (Proposition 1) and compare the results to the standard case in the literature (when a contractor is always able to complete a project if assigned).

Denote by  $C_{-i}$  the set of the costs of contractor  $i$ 's opponents with generic element  $c_{-i}$ , the associated probability density function  $g_{-i}(c_{-i}) = \prod_{j \neq i} g(c_j)$  and the cdf  $G_{-i}(c_{-i})$ . We will restrict our attention, without loss of generality, to the mechanisms where only the contractor who completes the project is paid. Let  $\pi_i(c_i, c_{-i})$  be the probability that the project is assigned to contractor  $i$ . If he turns out to be a good match for the procurer (which happens with probability  $\Phi(c_i)$ ), then he completes the project, incurs production cost  $c_i$  and receives the payment  $p_i(c_i, c_{-i})$ ; otherwise, his payoff is zero.

Contractor  $i$ 's expected payoff from participating in a procurement mechanism when he reports  $c$ , given that his true production cost is  $c_i$ , while all the other contractors report their costs truthfully, is

$$u_i(c, c_i) = \Phi(c_i) \int_{C_{-i}} [p_i(c, c_{-i}) - c_i] \pi_i(c, c_{-i}) dG_{-i}.$$

First, we will identify the mechanisms that are incentive compatible, i.e. under what conditions all contractors report their production costs truthfully.

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**Lemma 1. IC for the case of no-testing**

A direct mechanism  $\pi_i(c_i, c_{-i}), p_i(c_i, c_{-i}), i = 1, \dots, N$  is incentive compatible if and only if for every  $i$

a) contractor  $i$ 's expected probability of being assigned the project

$$\Pi_i(c_i) = E_{c_{-i}} [\pi_i(c_i, c_{-i})] = \int_{C_{-i}} \pi_i(c_i, c_{-i}) dG_{-i}$$

is weakly decreasing in his production cost  $c_i$ , and

b) the expected payment to contractor  $i$  with cost  $c_i$ , conditional on his being a good match for the procurer, is given by

$$P_i(c_i) = \int_{C_{-i}} p_i(c_i, c_{-i}) \pi_i(c_i, c_{-i}) dG_{-i} = \frac{U_i(\bar{c})}{\Phi(\bar{c})} + \Pi_i(c_i)c_i + \int_{c_i}^{\bar{c}} \Pi_i(z) dz, \quad (4.1)$$

i.e. it is uniquely determined by  $\Pi_i(\cdot)$ , his expected probability of being assigned the project, and by  $U_i(\bar{c})$ , the payoff that the contractor with the highest cost obtains in the mechanism.

**Proof.** See the appendix. ■

Note that  $U_i(c_i)$ , contractor  $i$ 's expected payoff in the mechanism, is therefore also pinned down by  $U_i(\bar{c})$  and  $\Pi_i(\cdot)$ :

$$U_i(c_i) = u_i(c_i, c_i) = U_i(\bar{c}) \frac{\Phi(c_i)}{\Phi(\bar{c})} + \Phi(c_i) \int_{c_i}^{\bar{c}} \Pi_i(z) dz. \quad (4.2)$$

This lemma mirrors the corresponding incentive compatibility result in the standard literature (particularly, that the assignment probability is decreasing), which is somewhat surprising. In any incentive compatible mechanism the procurer must assign the project with higher probability to a contractor with a lower production cost, even though she knows that he is more likely to be a bad match and, thus, to fail to complete the project.

When assigned the project by the procurer, contractor  $i$  completes it with probability  $\Phi(c_i)$ , delivering surplus  $v - p_i(c_i, c_{-i})$  to the procurer; otherwise, with probability  $1 -$

$\Phi(c_i)$ , he defaults on the project, leaving the procurer with zero surplus. Therefore, without testing, the procurer's expected surplus from the project when contractors report truthfully is given by

$$S^{NoT} = \sum_{i=1}^N \int_{C_i} \int_{C_{-i}} \Phi(c_i) [v - p_i(c_i, c_{-i})] \pi_i(c_i, c_{-i}) dG_{-i}(c_{-i}) dG(c_i).$$

In order to maximise her expected surplus, the procurer considers all implementable mechanisms and selects the optimal one. To be implementable, a mechanism must be both incentive compatible (with all contractors reporting their costs truthfully) and individually rational (with all contractors obtaining non-negative expected payoffs:  $U_i(c_i) \geq 0$  for all types  $c_i$ ).

To obtain the expression for the procurer's surplus in any incentive compatible and individually rational mechanism, we substitute (4.1) into the surplus formula:

$$\begin{aligned} S^{NoT} &= \sum_{i=1}^N \int_{C_i} \int_{C_{-i}} \left\{ \Phi(c_i) (v - c_i) \pi_i(c_i, c_{-i}) - \Phi(c_i) \int_{c_i}^{\bar{c}} \Pi_i(z) dz \right\} dG_{-i}(c_{-i}) dG(c_i) \\ &\quad - \sum_{i=1}^N U_i(\bar{c}) \frac{1}{\Phi(\bar{c})} \int_{C_i} \Phi(c_i) dG(c_i). \end{aligned}$$

This expression is similar to the one for the standard mechanism design problem but now includes both the probability that a contractor is assigned the project and the probability that he is able to complete it.

We normalized the contractor's payoff when he does not participate in the bidding to be zero; thus, his individual rationality constraint is satisfied as long as  $U_i(\bar{c}) \geq 0$ , due to (4.2). Therefore, in the optimal mechanism the procurer sets the utility of the contractor with the highest production cost to be zero:  $U_i(\bar{c}) = 0$ , as that minimizes the payment to contractor  $i$  with any value of cost. Therefore, the last term in the above formula for the procurer's surplus is zero.

Next, we will derive the expression for the procurer's surplus in terms of the contractors' virtual costs. The procurer obtains surplus only if the project is completed – i.e., whenever a contractor is a good match; therefore, it is the conditional densities of the

production cost that matter. Let  $h(c_i)$  be the density of a contractor's production cost  $c_i$ , conditional on his being a good match for the procurer; we denote the corresponding cdf by  $H(c_i)$ . Applying Bayes' Rule, we obtain

$$h(c_i) = \frac{\Pr(\text{good} | c_i)g(c_i)}{\Pr(\text{good})} = \frac{\Phi(c_i)g(c_i)}{\Pr(\text{good})},$$

where the probability that a contractor is a good match,  $\Pr(\text{good})$ , is determined by the total probability theorem:  $\Pr(\text{good}) = \int_{\underline{c}}^{\bar{c}} \Phi(z)g(z)dz$ .

We can now obtain the procurer's surplus in terms of conditional densities as:<sup>3</sup>

$$S^{NoT} = \int_C \left\{ \sum_{i=1}^N \pi_i(c_i, c_{-i}) \Phi(c_i) \left[ v - c_i - \frac{H(c_i)}{h(c_i)} \right] \right\} dG. \quad (4.3)$$

Therefore, the procurer's surplus depends on the contractors' virtual costs  $c + \frac{H(c)}{h(c)}$ , as in the standard literature, but we now need to use the conditional pdf and cdf in the formula for the virtual cost instead of the unconditional ones.

How does the virtual cost in our case compare with the standard virtual cost  $c + \frac{G(c)}{g(c)}$ ?

The virtual cost is weakly smaller than the standard virtual cost when the production cost and the quality of match are affiliated (and thus positively correlated), i.e., when  $\Phi(c)$  is increasing. On the contrary, in the case of negative affiliation (negative correlation) the virtual cost is weakly larger than the standard virtual cost, with equality at the lowest possible value of production cost.<sup>4</sup> It is useful to think of the difference between virtual cost and cost as the information rent that a contractor must obtain to reveal his

<sup>3</sup>Interchanging the order of integration in the double integral gives us

$$\begin{aligned} \int_{C_i} \left( \int_{c_i}^{\bar{c}} \Pi_i(z) dz \right) \Phi(c_i) dG(c_i) &= \Pr(\text{good}) \int_{\underline{c}}^{\bar{c}} \left( \int_{c_i}^{\bar{c}} \Pi_i(z) dz \right) dH(c_i) = \\ \Pr(\text{good}) \int_{\underline{c}}^{\bar{c}} \left[ \int_{\underline{c}}^z dH(c_i) \right] \Pi_i(z) dz &= \Pr(\text{good}) \int_{\underline{c}}^{\bar{c}} H(z) \Pi_i(z) dz = \\ \int_{\underline{c}}^{\bar{c}} \frac{H(c_i)}{h(c_i)} \Phi(c_i) \Pi_i(c_i) dG(c_i) &= \int_{C_i} \int_{C_{-i}} \frac{H(c_i)}{h(c_i)} \Phi(c_i) \pi_i(c_i, c_{-i}) dG_{-i}(c_{-i}) dG(c_i) \end{aligned}$$

<sup>4</sup>When  $\Phi(c)$  is increasing (decreasing), we have

$$\frac{H(c)}{h(c)} = \frac{\int_{\underline{c}}^c \Phi(z)g(z)dz}{\Phi(c)g(c)} = \frac{1}{g(c)} \int_{\underline{c}}^c \frac{\Phi(z)}{\Phi(c)} g(z)dz \stackrel{\leq}{\geq} \frac{1}{g(c)} \int_{\underline{c}}^c g(z)dz = \frac{G(c)}{g(c)}$$

cost. The contractor must be discouraged to try and increase his expected payment from the procurer by reporting that his cost is higher than his true cost. When the probability to complete the project increases with the production cost, there are countervailing incentives. A lower cost is associated with a lower probability of completion and lower powered incentives (lower information rents) are sufficient to induce truthful reporting. On the contrary, with negative affiliation/correlation there are no countervailing incentives; higher cost contractors are also less likely to complete the project, pushing the virtual cost up – that is, making higher powered incentives optimal.

Clearly, beside a difference in virtual costs and hence information rents, the procurer’s surplus when a contractor’s quality of match (the ability to complete the project) is correlated with his production cost is also different from that in the standard case in the literature because the corresponding revenue is decreased by a factor  $\Phi(c_i)$ , due to the probability that the selected contractor fails to complete the assigned project and thus brings no surplus to the procurer.

Define

$$B(c_i) = \left[ v - c_i - \frac{H(c_i)}{h(c_i)} \right] \quad \text{and} \quad \Gamma(c_i) = \Phi(c_i)B(c_i);$$

$B(c_i)$  is the procurer’s surplus “associated” with a contractor with cost  $c_i$  (it takes into account the IC constraint and includes the information rent that has to be given to contractors), conditional on completing the project, and  $\Gamma(c_i)$  is the corresponding unconditional surplus. We can then write the procurer’s expected surplus as follows

$$S^{NoT} = \int_C \left\{ \sum_{i=1}^N \pi_i(c_i, c_{-i}) \Phi(c_i) B(c_i) \right\} dG_{-i}(c_{-i}) dG(c_i) \quad (4.4)$$

$$= \int_C \left\{ \sum_{i=1}^N \pi_i(c_i, c_{-i}) \Gamma(c_i) \right\} dG_{-i}(c_{-i}) dG(c_i). \quad (4.5)$$

Now we are in a position to characterize the optimal contracts for the procurer when testing is not available. We will concentrate on the **regular case** - when the function  $\Gamma(c) = \Phi(c)B(c)$  is decreasing; we will call this condition regularity condition 2 (R2). For comparison, the assumption made in the standard literature is that  $B(c)$  is decreasing –

i.e., virtual cost  $c + \frac{H(c)}{h(c)}$  is increasing; we will call this latter condition regularity condition 1 (R1).

When the probability  $\Phi(c)$  is increasing, our R2 assumption is stronger than assumption R1, as we need  $\Phi(c)B(c) = \Phi(c) \left[ v - c - \frac{H(c)}{h(c)} \right]$  to be decreasing; therefore, this puts some restrictions on how quickly the probability  $\Phi(c)$  of being a good match can increase with cost. When  $\Phi(c)$  is decreasing, R2 is a weaker assumption than R1 – in that case, an increasing virtual cost is sufficient for the  $\Gamma(c)$  to be decreasing.

**Proposition 1. Optimal mechanisms for the case of no-testing.**

*When the function  $\Gamma(c) = \Phi(c)B(c)$  is decreasing (Regular case 2), the project is either assigned to the contractor with the lowest cost  $c_i = \min_j c_j$  conditional on  $B(c_i) \geq 0$ ; or the project is not assigned at all if  $B(c_i) < 0$  for  $c_i = \min_j c_j$ . Let  $c^*$  be the maximum value of  $c_i$  such that  $B(c_i) \geq 0$  (i.e.,  $c^*$  either satisfies  $B(c^*) = 0$ , or, if  $B(c) > 0$  for all values, then  $c^* = \bar{c}$ ). An optimal payment to contractor  $i$  is:*

$$p_i(c_i, c_{-i}) = c_i + \int_{c_i}^{\bar{c}} \frac{\Pi_i(z)}{\Pi_i(c_i)} dz = c_i + \int_{c_i}^{c^*} \left( \frac{1 - G(z)}{1 - G(c_i)} \right)^{N-1} dz, \quad (4.6)$$

*if he is assigned and completes the project and zero otherwise.*<sup>5</sup>

**Proof.** Let us disregard for the moment the fact that in any incentive compatible mechanism the probability  $\Pi_i(c_i)$  must be weakly decreasing (as proved in Lemma 1). Then, according to expression (4.5), to maximise surplus the procurer should assign the project to the contractor with the highest value of the function  $\Gamma(c)$  as long as it is weakly positive; otherwise the project should not be assigned at all. Therefore, in Regular case 2 when  $\Gamma(c)$  is decreasing, the project should be assigned to the lowest cost bidder (as long as the corresponding surplus is weakly positive). As a result, the expected probability that the project is assigned to contractor  $i$  is  $\Pi_i(c) = (1 - G(c))^{N-1}$  for  $c \leq c^*$  and 0 otherwise, which is weakly decreasing, as required by incentive compatibility.

<sup>5</sup>It is only one of possible payment functions - the one that does not depend on  $c_i$ ; there are many other possibilities as in the optimal mechanism only the expected payment  $P(c_i)$  is fixed.

To find an optimal payment, consider the equality

$$\int_{c_i}^{\bar{c}} \Pi_i(c) dc = \int_{C_{-i}} (p_i(c_i, c_{-i}) - c_i) \pi_i(c_i, c_{-i}) dG_{-i}(c_{-i}).$$

It is thus possible to choose a payment function  $p_i$  that is independent of  $c_{-i}$ , by setting

$$\int_{c_i}^{\bar{c}} \Pi_i(c) dc = [p_i(c_i, \cdot) - c_i] \Pi_i(c_i), \quad \text{or}$$

$$p_i(c_i, \cdot) = c_i + \int_{c_i}^{\bar{c}} \frac{\Pi_i(z)}{\Pi_i(c_i)} dz.$$

■

For comparison, consider the non-regular case when the function  $\Gamma(c)$  is increasing. In such a case the project should be assigned to the bidder with the highest cost. However, this would violate incentive compatibility as  $\Pi_i(c_i)$  would be weakly increasing, and therefore is not feasible. The optimal incentive compatible mechanism in this case is a fair lottery which allocates the project to each contractor with the same probability,  $1/N$ , if  $E_c[\Gamma(c)] \geq 0$  and does not assign it otherwise. The payment to contractor  $i$  is  $p_i(c_i, c_{-i}) = \bar{c}$  if he is assigned and completes the project, and is zero otherwise.

Proposition 1 is the analog of a result in the standard literature, but note that the requirement of increasing virtual cost is no longer sufficient. We need the regularity assumption that the expected surplus  $\Gamma(c) = \Phi(c)B(c)$  associated with a contractor is decreasing to guarantee that the project is assigned to the lowest cost contractor, as long as his cost is below the threshold  $c^*$ . In this case, any incentive compatible mechanism that implements this project allocation is optimal for the procurer. In particular, a first or second-price auction will implement the optimal procurement procedure, provided the reserve price is set equal to  $c^*$ .

We conclude this section by comparing the social surplus achieved in an optimal mechanism without testing to that of the standard case, when contractors are always a good match (i.e., when  $\Phi(c) = 1$ ). As discussed above, the virtual cost is smaller than the standard one when the probability  $\Phi(c)$  is increasing and larger when it is decreasing.

When the virtual cost is smaller, the corresponding reserve price  $c^*$  is higher, so the project is assigned more often than in the standard case – the mechanism is more efficient in the sense that a higher proportion of the possible social surplus is implemented. The reason is that, because the procurer is unsure whether any contractor whom she assigns the project to will be able to complete it, and contractors with higher costs are more likely to be a good match for her, she finds it optimal to assign the project to a contractor with a higher cost than she otherwise would have, were she confident that a contractor would always be able to complete the project. In the opposite case, when  $\Phi(c)$  is decreasing, the contractors with lower production costs have a higher probability to complete the project and, as a result, the virtual cost is higher; in this case the procurer finds it optimal to assign the project less often - to contractors with a lower cost on average.

Note that comparing the values of social surpluses themselves, and not what proportion of the first best surplus is obtained, is not very informative, as the social surplus is always higher in the standard case because contractors never fail to complete the project.

We summarise this discussion in the following corollary.

**Corollary. No-testing versus the standard case: welfare implications.**

*When a contractor's match quality (the ability to complete the project) and his production cost are positively (negatively) affiliated (and thus, correlated), a higher (lower) proportion of the first best social surplus is achieved in the optimal mechanism, compared to the standard case when contractors can always complete the project.*

## 4.4 Optimal procurement with testing

In this section we characterize the optimal procurement mechanisms for a procurer who tests the reliability of the contractors either before or after the auction. The test is fully revealing: a contractor with cost  $c_i$  passes the test if and only if he is able to complete the project, i.e., with probability  $\Phi(c_i)$ . We then compare the procurer's surplus, contractors' profits and the social surplus for the two different timings of the test.

### 4.4.1 Testing before the auction

With **before-the-auction testing** all  $N$  contractors are tested first. Let  $\{i_k, k = 1, \dots, K\}$  be the set of contractors who passed the test and, therefore, are good matches for the procurer. These  $K$  survivors participate in the procurement mechanism. The number of survivors  $\mathbb{K}$  is a discrete random variable which takes integer values between 0 and  $N$ .

Let  $\pi_{i_k}^B(c_{i_k}, c_{-i_k})$  be the probability that the project is assigned to contractor  $i_k$ , where  $k = 1, \dots, K$ . When he completes the project, he receives the payment  $p_{i_k}^B(c_{i_k}, c_{-i_k})$ . Both probability and payment will depend on the number  $K$  of contractors participating in the auction. To simplify the notation, we are omitting reference to it.

We can summarise the properties of the incentive compatible mechanisms when testing is carried out before the auction (see Lemma 2 in the Appendix) in the same way as in Lemma 1 of the previous section. The incentive compatible mechanism is the same as for the standard procurement auction with  $K$  bidders (i.e., when all bidders are good matches for the procurer), with the only difference being that we need to use the conditional pdf of the production cost for each contractor  $i_k$  – it is conditional on his being a good match.

Before the auction, but after having tested the reliability of all contractors, the procurer updates her beliefs about the distribution of the production cost of each surviving contractor who is about to participate in the procurement auction. Due to affiliation/correlation between the quality of the match and the production cost, the contractors who pass the test on average have higher costs if the correlation is positive and lower costs if the correlation is negative. As this is common knowledge, the contractors take it into account when bidding at the auction; and so does the procurer when she is searching for the optimal mechanism.

When testing is done before the auction, the pdfs of the production cost of contractor  $i_k$  and all other contractors that passed the test are given by

$$g^B(c_{i_k}) = \frac{\Phi(c_{i_k})g(c_{i_k})}{\Pr(\text{good})} = \frac{\Phi(c_{i_k})g(c_{i_k})}{\int_{\underline{c}}^{\bar{c}} \Phi(z)g(z)dz} = h(c_{i_k});$$

$$g_{-i_k}^B(c_{-i_k}) = \frac{\prod_{j_k \neq i_k} \Phi(c_{j_k})g(c_{j_k})}{[\Pr(\text{good})]^K} = \prod_{j_k \neq i_k} h(c_{j_k}) = h_{-i_k}(c_{-i_k});$$

their distribution functions are denoted by  $G^B$  and  $G_{-i_k}^B$  respectively.

Now that we know which mechanisms are implementable when testing is done before the auction and exactly  $K$  contractors are identified as good matches who will be participating in it, we can look for the optimal auction. Let's consider all incentive compatible mechanisms and take into account that in the optimal mechanism the IR constraint of the highest cost contractor binds (i.e.,  $U_{i_k}(\bar{c}) = 0$ ).

As shown in more detail in the appendix, the expected surplus of the procurer with testing before the auction, conditional on the set of good match contractors being  $\{i_k, k = 1, \dots, K, K \geq 1\}$ , is

$$\begin{aligned} S_K^B &= \sum_{k=1}^K \int_{C_{i_k}} \int_{C_{-i_k}} [v - p_{i_k}(c_{i_k}, c_{-i_k})] \pi_{i_k}(c_{i_k}, c_{-i_k}) dG_{-i_k}^B(c_{-i_k}) dG^B(c_{i_k}) \\ &= \int_C \left\{ \sum_{k=1}^K \pi_{i_k}(c_{i_k}, c_{-i_k}) B(c_{i_k}) \right\} \frac{\Phi(c_{i_1}) \cdot \dots \cdot \Phi(c_{i_K})}{[\Pr(\text{good})]^K} dG_{-i_k}(c_{-i_k}) dG(c_{i_k}). \end{aligned}$$

The overall expected surplus in the mechanism is the sum of the expected surpluses generated by  $K$  good contractors bidding in an auction, weighted by the probability that exactly  $K$  contractors are a good match for the procurer.

$$S^B = \sum_{K=1}^N S_K^B \binom{N}{K} [\Pr(\text{good})]^K [1 - \Pr(\text{good})]^{N-K}.$$

It is important to point out that even though all contractors who bid in an auction are good matches and can complete the project for sure, the probability of completion  $\Phi(c_{i_k})$  still appears in the formula for the surplus – it comes through the updated beliefs of the procurer and contractors about the distribution of the production cost conditional on passing the test. As a result, perhaps surprisingly, the virtual cost is again different from the standard  $c + \frac{G(c)}{g(c)}$ ; instead, it is the same as with no-testing:  $c + \frac{H(c)}{h(c)}$ .

Under R1, the standard regularity assumption in the literature,  $B(c_{i_k})$  is strictly decreasing (i.e., the virtual cost  $c_{i_k} + \frac{H(c_{i_k})}{h(c_{i_k})}$  is strictly increasing) and the optimal mechanism assigns the project to the good match contractor with the lowest cost, as long as  $B(c_{i_k}) \geq 0$ . Note that different regularity assumptions are required for different cases:

in the case of testing before the auction, we require regularity condition R1, but in case of no-testing (and, as we show later, in case of testing after the auction), we require regularity condition R2, that  $\Gamma(c_{i_k}) = \Phi(c_{i_k})B(c_{i_k})$  is strictly decreasing. We summarise the main result for testing before the auction in the following proposition:

**Proposition 2. Optimal mechanisms for testing before the auction.**

*In regular case R1, the project is either assigned to the contractor with the lowest cost  $c_{i_k} = \min_j c_j$  conditional on  $B(c_{i_k}) \geq 0$ , or if  $B(c_{i_k}) < 0$  for  $c_{i_k} = \min_j c_j$  the project is not assigned at all. An optimal payment to contractor  $i_k$  is  $p_{i_k}(c_{i_k}, c_{-i_k}) = \min \{c^*, 2^{nd} \text{ lowest cost}\}$  if he is assigned the project and zero otherwise.<sup>6</sup> If there is only one good contractor, then he is paid  $c^*$ , the minimum value of  $c$  such that  $B(c) \geq 0$ .*

**Proof.** Similar to the proof of Proposition 1 and omitted.

The optimal mechanism could again be implemented by a first or a second price auction with reserve price  $c^*$ . The payment to the procurer depends on the number  $K$  of good match contractors who bid in the auction. Auctions with a smaller number of participants will exert less competitive pressure on contractors; their resulting bids for delivery of the project will be higher.

We now discuss the implications for efficiency of the optimal mechanism. In the optimal mechanism the project is assigned to the lowest cost contractor if and only if  $c \leq c^*$ . The threshold cost (reserve cost)  $c^*$  is the maximum value of  $c$  such that  $B(c) \geq 0$ . For high values  $v$  of the project to the procurer,  $v > \bar{c} + \frac{1}{h(\bar{c})}$ , the virtual value  $B(c)$  is positive for all costs (and so  $c^* = \bar{c}$ ) and then the project is always assigned, which is efficient. For low values of the project,  $v < \underline{c}$ ,  $B(c)$  is negative for all costs,  $c^*$  is not defined and the project is never assigned, which is also efficient. For intermediate values of the project to the procurer,  $v$  in the range  $[\underline{c}, \bar{c} + \frac{1}{h(\bar{c})}]$ , the reserve cost  $c^*$  satisfies  $B(c^*) = 0$ ; in this case  $c^*$  divides the contractors into those that could be assigned the project if they happen to have the lowest production cost ( $c \leq c^*$ ) and those that are never assigned the project ( $c > c^*$ ). Assigning the project when the lowest cost satisfies  $c < c^*$  is efficient, while not assigning it when  $c > c^*$  is efficient if  $c^* < v < c$  and

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<sup>6</sup>As each contractor is a good match, he will complete the project if assigned it.

inefficient whenever  $c^* < c < v$ .

Note that the reserve price with testing before the auction is the same as the reserve price without testing (and does not depend on  $K$ , the number of good match contractors), but it is different from the reserve price of a standard problem, when all contractors are a good match. It is higher than the latter when the match quality and the production cost are affiliated, and lower when they are negatively affiliated (See footnote 4). Therefore, whenever high cost contractors are more likely to be good match for the procurer, uncertainty about whether any particular contractor is good or bad leads the procurer to run a mechanism which is more efficient, in the sense that a higher proportion of the total possible social surplus is realized; the reserve cost is higher, more contractors have an opportunity to deliver the project and hence the project is delivered more often. In the case of negative affiliation, the optimal mechanism leads to more inefficiency, as the reserve cost is lower, and the project gets implemented less often.

Now, let's compare welfare in the cases of testing-before-the-auction and no-testing. Though the reserve prices when testing before the auction and in the no-testing regimes are the same, the procurer's surplus, total surplus and contractors' payments are not – they are (weakly) higher in the case of testing before the auction. The intuition for the result is very simple – even though the project is always assigned to the lowest cost contractor as long as his cost  $c_{i_k}$  is below the reserve price  $c^*$ , in the case of no-testing this contractor fails to complete the project with probability  $1 - \Phi(c_{i_k})$ , resulting in zero surplus and payments, while with pre-testing the project is always assigned to the contractor who can complete the project (which may be the same contractor as in the case of no-testing or a contractor with higher cost if the lowest cost contractor turns out to be a bad match), leading to positive total surplus, procurer's surplus and contractors' payments. To be precise, whenever the project is assigned to the same contractor with testing before the auction and no-testing (or not assigned at all), the corresponding welfare measures/payoffs are the same; when the contractors who are assigned the project are different, there is zero associated welfare in case of no-testing and positive welfare in case of testing. It is also possible for the project to be assigned in the case of no-testing

and not to be assigned at all in the case of testing before the auction – that happens when the lowest cost contractor has a cost lower than the reserve price but fails the test, while the lowest cost contractor who passes the test has a cost above the reserve price; the associated welfare measures in that case are the same in the two scenarios – they are all zero. In conclusion, testing before the auction Pareto dominates no-testing.

#### 4.4.2 Testing after the auction

Now let's analyze the situation when the procurer chooses to test after the auction. In a direct incentive compatible mechanism all  $N$  contractors report their production costs at the start, then the procurer runs the testing and the assignment procedure, which is known before the auction starts.

First, the procurer assigns the project to contractor  $i$  with probability  $\pi_i^{(1)}(c_i, c_{-i})$ . She carries out the test; with probability  $\Phi(c_i)$  the contractor turns out to be a good match and passes the test, completes the project and surplus is realized. He is paid  $p_i(c_i, c_{-i})$ . Note that the procurer may choose to assign the project with probability less than 1, or even not at all.

Second, if the first chosen contractor fails the test, then the procurer reassigns the project to a different contractor  $j$  with probability  $\pi_j^{(2)}(c_j, c_{-j})$ . She then conducts the test; with probability  $\Phi(c_j)$  the contractor happens to be a good match, completes the project and surplus is realized. He is paid  $p_j(c_j, c_{-j})$ . If a contractor fails the test at some stage, he is never again assigned the project in the future. This procedure continues until a good match contractor is found or there are no more contractors to assign the project to.

Let  $\pi_i^A(c_i, c_{-i})$  be the total probability that the project is assigned to contractor  $i$  (i.e., at some stage). If he completes the project, he receives the payment  $p_i^A(c_i, c_{-i})$ ; otherwise, his payoff is zero. To better explain the intuition behind our analysis, we will start by deriving a surplus formula for the special case of **two contractors**. The total probability that contractor 1 is assigned the project,  $\pi_1^A(c_1, c_2)$ , is equal to the sum of his probability to be assigned the project first and his probability to be assigned the project

second, which happens when the other contractor is assigned the project first, but turns out to be a bad match for the procurer. It is given by

$$\pi_1^A(c_1, c_2) = \pi_1^{(1)}(c_1, c_2) + \pi_2^{(1)}(c_1, c_2) [1 - \Phi(c_2)] \pi_1^{(2)}(c_1, c_2).$$

Similarly, the total probability for contractor 2 is given by

$$\pi_2^A(c_1, c_2) = \pi_2^{(1)}(c_1, c_2) + \pi_1^{(1)}(c_1, c_2) [1 - \Phi(c_1)] \pi_2^{(2)}(c_1, c_2).$$

The procurer's surplus is therefore

$$\begin{aligned} S^A = & \int_C \left[ \left\{ \Gamma(c_1) + \Gamma(c_2) [1 - \Phi(c_1)] \pi_2^{(2)}(c_1, c_2) \right\} \pi_1^{(1)}(c_1, c_2) + \right. \\ & \left. + \left\{ \Gamma(c_2) + \Gamma(c_1) [1 - \Phi(c_2)] \pi_1^{(2)}(c_1, c_2) \right\} \pi_2^{(1)}(c_1, c_2) \right] dG(c_2) dG(c_1), \end{aligned}$$

where the first term is the surplus obtained when contractor 1 is assigned the project first, and the second term is the surplus when it is contractor 2 who is assigned the project first. We can see from this expression that the project will be reassigned to contractor  $j$  after the failure of contractor  $i$  only if the corresponding surplus  $\Gamma(c_j)$  is weakly positive; otherwise, the project will not be reassigned and will be left uncompleted. The same is true for the first round – the appropriate virtual value must be weakly positive for the project to be assigned in the first place. We can also see that it is the contractor  $i$  with the highest virtual value  $\Gamma(c_i)$  who gets assigned the project first; the contractor with the lowest surplus will be assigned the project second, only if the first contractor fails.

Simple algebra manipulation allows us to rewrite the surplus in terms of the total probability:

$$S^A = \int_C \left\{ \sum_{i=1}^{N=2} \pi_i^A(c_i, c_{-i}) \Gamma(c_i) \right\} dG(c_2) dG(c_1). \quad (4.7)$$

Note that the total probabilities of assignment don't have to sum up to be less than or equal to 1, as there is reassignment. If the project was first assigned with probability one to contractor 1, then the total probability for contractor 1 is one; but if he fails the test (which happens with prob  $1 - \Phi(c_1)$ ), then the project could be reassigned to contractor

2 with probability one, so his total probability would be  $1 - \Phi(c_1)$ .

Now, let's return to the general case of  $N$  contractors. We describe incentive compatible mechanisms for testing after the auction in Lemma 3.

**Lemma 3. IC for testing after the auction.**

A direct mechanism  $\pi_i^{(k)}(c_i, c_{-i}), p_i^A(c_i, c_{-i}), i, k = 1, \dots, N$  is incentive compatible if and only if for every  $i$ :

a) contractor  $i$ 's expected total probability of being assigned the project (derived from the probabilities  $\pi_i^{(k)}(c_i, c_{-i})$ ) is

$$\Pi_i^A(c_i) = E_{c_{-i}} [\pi_i^T(c_i, c_{-i})] = \int_{C_{-i}} \pi_i^T(c_i, c_{-i}) dG_{-i}(c_{-i})$$

is weakly decreasing in  $c_i$  and

b) the expected payment to contractor  $i$  with cost  $c_i$  conditional on being a good match is given by

$$P_i^A(c_i) = \int_{C_{-i}} p_i(c_i, c_{-i}) \pi_i^A(c_i, c_{-i}) dG_{-i}(c_{-i}) = \frac{U_i(\bar{c})}{\Phi(\bar{c})} + \Pi_i^A(c_i)c_i + \int_{c_i}^{\bar{c}} \Pi_i^A(z) dz.$$

**Proof.** The proof is the same as for Lemma 1 and is omitted. ■

The expected payoff of contractor  $i$  in the mechanism,  $U_i(c_i)$ , is, therefore,

$$U_i(c_i) = u_i(c_i, c_i) = U_i(\bar{c}) \frac{\Phi(c_i)}{\Phi(\bar{c})} + \Phi(c_i) \int_{c_i}^{\bar{c}} \Pi_i^A(c) dc.$$

Lemma 3 establishes that the incentive compatible mechanisms for the case of testing after the auction look exactly the same as in our benchmark case of no-testing, if we use the **total probability** of being assigned the project (i.e., at all possible stages). Recall that a contractor's total probability may be derived, for example, from a zero probability of being assigned the project first and second and probability one of being assigned the project third if contractors  $i$  and  $j$  with costs  $c_i$  and  $c_j$ , who were assigned the project first and second, fail the test; in this example the contractor's total probability is  $[1 - \Phi(c_i)][1 - \Phi(c_j)]$ .

With testing after the auction, the procurer's expected surplus in any IC mechanism, taking into account that in the optimal mechanism the high cost IR constraint binds, is given by

$$\begin{aligned}
S^A &= \sum_{i=1}^N \int_C [v - p_i(c_i, c_{-i})] \Phi(c_i) \pi_i^A(c_i, c_{-i}) dG_i(c_{-i}) dG(c_i) \\
&= \sum_{i=1}^N \int_C \Phi(c_i) \left[ v - c_i - \frac{H(c_i)}{h(c_i)} \right] \pi_i^A(c_i, c_{-i}) dG_i(c_{-i}) dG(c_i) \\
&= \int_C \left\{ \sum_{i=1}^N \pi_i^A(c_i, c_{-i}) \Phi(c_i) B(c_i) \right\} dG_i(c_{-i}) dG(c_i) \\
&= \int_C \left\{ \sum_{i=1}^N \pi_i^A(c_i, c_{-i}) \Gamma(c_i) \right\} dG_i(c_{-i}) dG(c_i). \tag{4.8}
\end{aligned}$$

**Proposition 3. Optimal mechanisms when testing after the auction.**

*Under regularity condition R2, the optimal mechanism discards contractors with costs  $c_i > c^*$  (i.e., with  $\Gamma(c_i) < 0$ ) and ranks all other contractors from the lowest to the highest cost. Starting with the lowest cost contractor, the procurer tests the contractor with the highest rank (lowest cost) among those who have not yet failed the test and assigns the project to and pays the first contractor that passes the test.*

In the optimal mechanism the project ends up being assigned and completed by the lowest cost contractor who is a good match if and only if his production cost  $c \leq c^*$ . This will generate a weakly decreasing expected probability  $\Pi_i^A(c)$  as required.

The optimal mechanism outlined in Proposition 3 can be implemented by a first or a second price auction with reserve price  $c^*$ , subject to the provision that only a good match contractor can be declared the winner.

### 4.4.3 Testing before vs. after the auction: a comparison

First, let's compare the **total social surplus** generated while testing before or after the auction. As shown in the previous sections, in both cases the project ends up being completed either by exactly the same contractor, or not at all. With testing before the

auction, the project is allocated to the good contractor who has the lowest production cost, as long as it is below the reserve price  $c^*$ . With testing after the auction, the procurer assigns the project to a contractor with the lowest cost if it is below the same reserve price  $c^*$ , and keeps testing the contractors until a good match is found. Therefore, for any realization of the quality of a match and the production costs of the contractors, the same contractor completes the project (if at all), and total social surplus is exactly the same, whether testing is done before or after the auction.

While total surplus is the same, its distribution between the procurer and contractors does not have to be the same. Particularly (as discussed in an introduction), two different effects play a role in how surplus is divided. When testing is carried out before the procurement auction, the contractors have an incentive to bid more aggressively, as “every bid counts” - all participants are good matches and can complete the project, so overbidding on cost will lead to a loss of a project, there are no second chances. On the other hand, with testing first and bidding second, only good match contractors get to bid in an auction; therefore, the total number of bidders in a procurement auction is smaller, providing less competitive pressure on them to bid aggressively. These two effects work in the opposite direction; the following proposition establishes the main result of this section, that these two effects cancel each other exactly; there is no difference for the procurer whether to test before or after the auction; she will have to share her surplus with the contractors in exactly the same way.

**Proposition 4. Surplus/payoff equivalence in a risk neutral case.**

*The total social surplus, the expected surplus of the procurer and the payoffs of the contractors (conditional on the cost  $c_i$ ) are exactly the same, whether testing is done before or after the auction. There is no surplus/payoff equivalence between testing and no-testing: testing (weakly) dominates no-testing.*

**Proof.** The proof that the total surplus is the same has been provided above, before the statement of the proposition.

Let’s show that the expected payment to a good match contractor with production cost  $c_i$  is the same, whether the testing is done before or after the auction, i.e.  $P_i^A(c_i) =$

$P_i^B(c_i)$ . That would mean that the payoff of any contractor is the same across the two regimes, and so is the surplus of the procurer herself.

We start by comparing the probability that contractor  $i$ , who is a good match and has production cost  $c_i$ , is assigned the project. In any realization of uncertainty, each contractor has a production cost and a quality of match. Consider all realizations where contractor  $i$  is good and has cost  $c_i$ , but all other contractors could be either good or bad and, particularly, could have different costs, higher or lower than that of contractor  $i$ . In both testing before and after the auction, this contractor could be assigned the project only if his cost is below the reserve price  $c^*$ , otherwise he is never assigned the project and his payment and payoff is zero. Therefore, we need to concentrate on the case when his cost is below the reserve price.

The expected probability  $\Pi_i^A(c_i)$  that contractor  $i$  is assigned the project in the mechanism where testing is done **after** the auction is the probability that he has the lowest cost out of everyone who is a good match for the procurer. As we assumed that contractor  $i$  is a good match, this probability is equal to the probability that he has the lowest cost plus the probability that he has the second lowest cost but the lowest cost contractor turns out to be a bad match, plus the probability that he has the third lowest cost but both the first and second lowest cost contractors turn out to be bad matches, and so on...

The expected probability  $\Pi_i^{B,K}(c_i)$  that contractor  $i$  is assigned the project in the mechanism with testing done **before** the auction and exactly  $K$  good match contractors are participating in bidding is equal to the probability that he has the lowest cost out of those  $K$  contractors. To obtain the overall probability, we need to average over all possible situations with different values of the number of good match contractors  $K = 1, 2, \dots, N$ , therefore, it is given by

$$\Pi_i^B(c_i) = \sum_{K=1}^N \Pi_i^{B,K}(c_i) \binom{N}{K} [\Pr(\text{good})]^K [1 - \Pr(\text{good})]^{N-K}.$$

Thus, the probability to be assigned the project when testing before vs. after the auction is, in both cases, equal to the probability that contractor  $i$  has the lowest cost

out of all good match contractors. This probability is obtained in two different ways. In testing before the auction the procurer first finds all the good matches and then finds the one with the lowest cost. In testing after the auction, the procurer first ranks the contractors in order of their costs, and then looks for the first good match. Thus, we conclude that  $\Pi_i^A(c_i) = \Pi_i^B(c_i)$ .

Recalling that the expected payments are uniquely determined by the expected probabilities (as derived previously), with:

$$\begin{aligned} P_i^A(c_i) &= \Pi_i^A(c_i)c_i + \int_{c_i}^{c^*} \Pi_i^A(z)dz, \\ P_i^B(c_i) &= \sum_{k=1}^N P_i^{B,k}(c_i) \binom{N}{K} [\Pr(\text{good})]^K [1 - \Pr(\text{good})]^{N-K}, \\ P_i^{B,k}(c_i) &= \Pi_i^{B,k}(c_i)c_i + \int_{c_i}^{c^*} \Pi_i^{B,k}(z)dz, \end{aligned}$$

we conclude that the expected payments to contractors do not depend on when testing is done. When testing is carried out before the auction, a contractor is paid more if he competes with a smaller number of other good match contractors and less if there is more competition; but, averaging over all possible events and numbers of competitors, his payment is exactly the same as when testing is done after and all contractors, good and bad, bid at the auction (but bids from bad matches are discarded). Thus, the payoffs of contractors and the surplus of the procurer do not depend on the timing of the test.

The proof that testing before the auction Pareto dominates no-testing was provided above. Total social surplus, the surplus of the procurer and the payoffs of the contractors are higher with testing than without it. ■

We have thus established a surplus/payoff equivalence between the two regimes: testing before and after the auction. It does not matter for the procurer when to test the contractors for reliability in order to identify which ones are good matches for her.

## 4.5 Conclusions

In this chapter we have studied the procurement mechanisms when the procurer is unsure whether potential contractors are able to complete the project and has to test them for reliability. We identified and compared optimal mechanisms when testing is done before or after the auction and when no testing is performed. We showed that when the procurer and the contractors are risk neutral and the testing could be carried out costlessly, it does not matter whether the testing is done before or after the procurement auction: in both cases the project is awarded to the same contractor (or not at all), the procurer's expected surplus and the contractors' payoffs are the same. In future research, first, it would be interesting to explore whether a mixed regime, in which some contractors are tested before and some after is beneficial to the procurer. Second, as our conclusions depend on the risk neutrality of procurer and contractors, it would be interesting to study what happens once one introduces risk aversion. In several public project applications, for example, it would seem appropriate to model the procurer and perhaps the bidders as risk averse.

## Chapter 5

# Conclusion

In this thesis I present three essays in Industrial Organisation.

The second and third chapters contribute to our understanding of how bounded rationality on the part of the consumers affects pricing, product variety and competition in monopolistic and oligopolistic markets.

The second chapter provides a novel explanation of the relationship between product quality and variety. It introduces the concept of a nearly rational consumer and makes an important theoretical observation: even a small deviation from rational choice on the part of the consumers could lead to a significant change in the monopolist's optimal product line, adding an extra dimension to it, namely, multiple varieties at each quality level. It also identifies the technical reason for this: failure of lower hemicontinuity of the choice correspondence in the parameter that captures the degree of irrationality. The concept of a nearly rational agent provides a useful framework for future research: it could be used to analyse many Mechanism Design problems with binding incentive compatibility constraints.

The third chapter introduces a pioneering model where the degree of irrationality exhibited by agents is endogenous. It examines oligopolistic market with boundedly rational consumers, where Bertrand oligopolists can choose to manipulate the degree of irrationality of their consumers by either obfuscating or educating them. It shows that while it is worthwhile for the firms to confuse the consumers, it is only optimal to educate them if they are sufficiently rational at default. It also argues that competition has two

beneficial effects from the social point of view: it increases the chance that a consumer will buy the product and decreases the amount of obfuscation. This chapter builds the tools for analysing other markets or contractual arrangements, where degree of irrationality of the agents can be manipulated.

The fourth chapter contributes to the literature on the optimal procurement auctions. It considers the case when the reliability of contractors is not known beforehand and must be tested for. The crucial issue is whether such testing should occur before or after the auction is conducted. When the procurer and bidders are risk neutral, the timing of the test is irrelevant. Further work is needed to investigate the case when the procurer and bidders exhibit risk aversion of a general form.

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# Appendices



## Appendix A

# Appendices for Chapter 4

### A.1 Proof of Lemma 1

First, let's show that in any IC mechanism a) and b) must be satisfied (only if part).

a) Contractor  $i$ 's expected payoff from participating in a procurement mechanism when he reports  $c$ , given that his true cost is  $c_i$ , while all the other contractors report their types truthfully is

$$u_i(c, c_i) = \Phi(c_i) \int_{C_{-i}} [p_i(c, c_{-i}) - c_i] \pi_i(c, c_{-i}) dG_{-i} = \Phi(c_i) [P_i(c) - \Pi_i(c) c_i],$$

where  $\Pi_i(c) = \int_{C_{-i}} \pi_i(c, c_{-i}) dG_{-i}$  is contractor  $i$ 's expected probability to be assigned a project and  $P_i(c) = \int_{C_{-i}} p_i(c, c_{-i}) \pi_i(c, c_{-i}) dG_{-i}$  is his expected payment conditional on completing it.

Due to the incentive compatibility of the mechanism, contractor  $i$  weakly prefers to report his true type  $c_i$  to reporting any other type  $c$ :  $u_i(c, c_i) \leq u_i(c_i, c_i)$ , or

$$\Phi(c_i) [P_i(c) - \Pi_i(c) c_i] \leq \Phi(c_i) [P_i(c_i) - \Pi_i(c_i) c_i]$$

which gives us

$$P_i(c) - \Pi_i(c) c_i \leq P_i(c_i) - \Pi_i(c_i) c + \Pi_i(c_i) c - \Pi_i(c_i) c_i = [P_i(c_i) - \Pi_i(c_i) c] - \Pi_i(c_i) [c_i - c]$$

and multiplying by  $\Phi(c)$  we have

$$\Phi(c) [P_i(c) - \Pi_i(c) c_i] \leq \Phi(c) [P_i(c_i) - \Pi_i(c_i) c] - \Phi(c) \Pi_i(c_i) [c_i - c].$$

Using incentive compatibility again:  $u_i(c_i, c) \leq u_i(c, c)$ , or

$$\Phi(c) [P_i(c_i) - \Pi_i(c_i) c] \leq \Phi(c) [P_i(c) - \Pi_i(c) c],$$

and substituting into the above inequality, we obtain

$$[P_i(c) - \Pi_i(c) c_i] \leq [P_i(c) - \Pi_i(c) c] - \Pi_i(c_i) [c_i - c],$$

or

$$[\Pi_i(c_i) - \Pi_i(c)] [c_i - c] \leq 0.$$

Thus, in any incentive compatible mechanism, the expected probability  $\Pi_i(c_i)$  of being assigned the project must be weakly decreasing in  $c_i$ .

Note that the proof does not rely on  $\Phi(c_i)$  being increasing at all.

b) Let  $U_i(c_i) = u_i(c_i, c_i)$  be the expected payoff of the agent in the mechanism when everyone reports truthfully. Then by using the envelope theorem we obtain

$$\frac{d}{dc_i} \left( \frac{U_i(c_i)}{\Phi(c_i)} \right) = \frac{\partial}{\partial c_i} \left( \frac{u_i(c, c_i)}{\Phi(c_i)} \right) \Big|_{c=c_i} = -\Pi_i(c) \Big|_{c=c_i} = -\Pi_i(c_i),$$

Integrating, we obtain the formula for the expected payoff of contractor  $i$  :

$$U_i(c_i) = U_i(\bar{c}) \frac{\Phi(c_i)}{\Phi(\bar{c})} + \Phi(c_i) \int_{c_i}^{\bar{c}} \Pi_i(z) dz.$$

The corresponding expected payment to contractor  $i$  with cost  $c_i$  conditional on his being a good match is then given by

$$\begin{aligned} P_i(c_i) &= U_i(c_i)/\Phi(c_i) + \Pi_i(c_i)c_i = \\ &= \int_{C_{-i}} p_i(c_i, c_{-i}) \pi_i(c_i, c_{-i}) dG_{-i} = \frac{U_i(\bar{c})}{\Phi(\bar{c})} + \Pi_i(c_i)c_i + \int_{c_i}^{\bar{c}} \Pi_i(z) dz. \end{aligned}$$

(If part) - a) and b) imply IC - the proof is standard and is omitted. ■

## A.2 Lemma 2

### Lemma 2. IC for testing before the auction

A direct mechanism  $\pi_{i_k}^B(c_{i_k}, c_{-i_k}), p_{i_k}^B(c_{i_k}, c_{-i_k}), i = 1, \dots, K$  is incentive compatible if and only if for every  $i_k$

a) contractor  $i_k$ 's expected probability of being awarded the project

$$\Pi_{i_k}^B(c_{i_k}) = E_{c_{-i_k}}^B \left[ \pi_{i_k}(c_{i_k}, c_{-i_k}) \right] = \int_{C_{-i_k}} \pi_{i_k}(c_{i_k}, c_{-i_k}) dH_{-i_k}$$

is weakly decreasing in  $c_{i_k}$ .

b) the expected payment to contractor  $i_k$  with cost  $c_{i_k}$  conditional on being a good match is given by

$$\begin{aligned} P_{i_k}^B(c_{i_k}) &= U_{i_k}(c_{i_k}) + \Pi_{i_k}^B(c_{i_k})c_{i_k} = \\ &= \int_{C_{-i_k}} p_{i_k}^B(c_{i_k}, c_{-i_k}) \pi_{i_k}^B(c_{i_k}, c_{-i_k}) dH_{-i_k} = U_{i_k}(\bar{c}) + \Pi_{i_k}^B(c_{i_k})c_{i_k} + \int_{c_{i_k}}^{\bar{c}} \Pi_{i_k}^B(z) dz. \end{aligned}$$

Note that  $U_{i_k}(c_{i_k})$ , contractor  $i_k$ 's expected payoff in the mechanism conditional on passing the test, is pinned down by the payoff of the contractor with the highest cost  $U_{i_k}(\bar{c})$  and his own expected probability of being assigned the project  $\Pi_{i_k}(\cdot)$ :

$$U_{i_k}(c_{i_k}) = u_{i_k}(c_{i_k}, c_{i_k}) = U_{i_k}(\bar{c}) + \int_{c_{i_k}}^{\bar{c}} \Pi_{i_k}(z) dz. \quad (\text{A.1})$$

**Proof.** The proof is standard and is omitted. ■

The expected surplus of the procurer with testing before the auction, conditional on

the set of good matches being  $\{i_k, k = 1, \dots, K, K \geq 1\}$ , is

$$\begin{aligned}
S_K^B &= \sum_{k=1}^K \int_{C_{i_k}} \int_{C_{-i_k}} \left[ v - p_{i_k}(c_{i_k}, c_{-i_k}) \right] \pi_{i_k}(c_{i_k}, c_{-i_k}) dG_{-i_k}^B(c_{-i_k}) dG^B(c_{i_k}) \\
&= \sum_{k=1}^K \int_C \left[ v - c_{i_k} - \frac{H(c_{i_k})}{h(c_{i_k})} \right] \pi_{i_k}(c_{i_k}, c_{-i_k}) dG_{-i_k}^B(c_{-i_k}) dG^B(c_{i_k}) \\
&= \sum_{k=1}^K \int_C \left[ \pi_{i_k}(c_{i_k}, c_{-i_k}) \left( v - c_{i_k} - \frac{H(c_{i_k})}{h(c_{i_k})} \right) \right] \frac{\Phi(c_{i_1}) \cdot \dots \cdot \Phi(c_{i_K})}{[\Pr(\text{good})]^K} dG_{-i_k}(c_{-i_k}) dG(c_{i_k}) \\
&= \int_C \left\{ \sum_{k=1}^K \pi_{i_k}(c_{i_k}, c_{-i_k}) B(c_{i_k}) \right\} \frac{\Phi(c_{i_1}) \cdot \dots \cdot \Phi(c_{i_K})}{[\Pr(\text{good})]^K} dG_{-i_k}(c_{-i_k}) dG(c_{i_k}).
\end{aligned}$$