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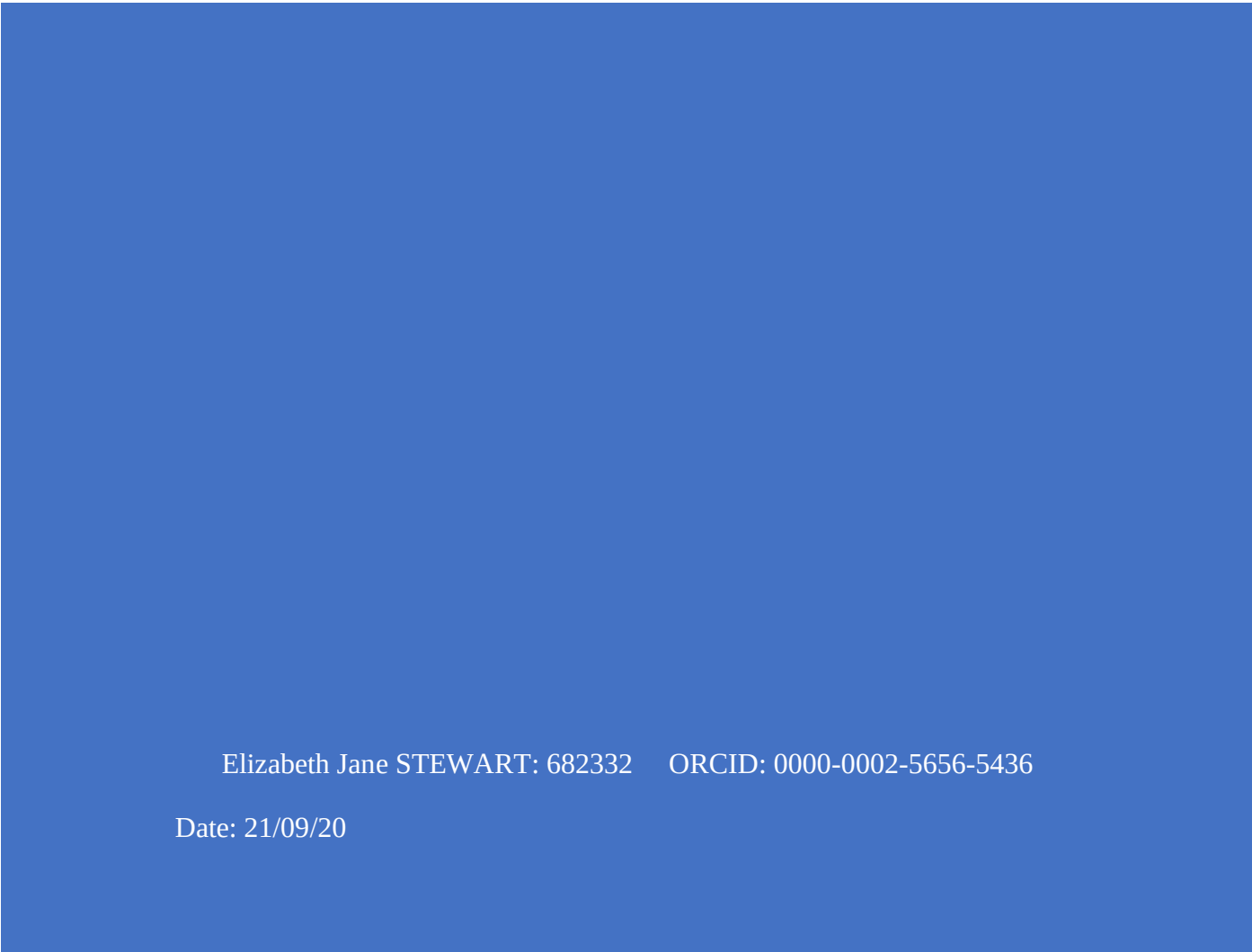
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An Investigation of the Implementation of a Problem-Solving Intervention in Two Primary Classrooms

Submitted in partial fulfilment of the degree of Master of Education
(960BA)

Melbourne Graduate School of Education
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Abstract

Problem-solving in mathematics is an important component of curricula around the world and it has been identified as essential that students develop this capacity in order to achieve success in mathematics. Studies have found that more teachers need to teach their students strategies to problem-solve in mathematics. The aim of this case study was to investigate the implementation of a problem-solving intervention by two primary school teachers over two lessons each. It focussed on their perceptions of the effectiveness of the intervention and how it might improve their teaching of problem-solving in mathematics in the future. It also focussed on how they implemented the intervention and how their students responded to the intervention. The problem-solving intervention was designed based on features identified in problem-solving literature and in discussion with the two teachers. Particular features that were incorporated into the intervention included: enabling and extending prompts; the provision of periods of time in which students were left to ‘struggle’ with trying to solve the problems themselves; and the provision of periods in which students shared problem-solving strategies with peers. The teachers were interviewed separately before and after teaching the lessons. The researcher observed all four lessons and collected student work samples from each lesson. Data was analysed using a content analysis strategy.

The results suggest that the two teachers perceived that the intervention had both positive and negative impacts on their students’ problem-solving abilities. They found that the enabling prompts supported and extended their students’ thinking in the lessons and commented that their students enjoyed being challenged in the lessons. The two teachers perceived that it was often not beneficial for some of their students to struggle with problems in the lessons due to perceived resilience and confidence issues. Both teachers deviated from the intervention in the lessons in order to reduce the amount of struggle their students experienced. However, where students were given time to struggle in the lessons, they were able to formulate and record a greater range of problem-solving strategies. There appeared to be a tension for the teachers between providing time for their students to struggle with problems and preserving some of their students’ confidence. One of the teachers facilitated student share time in the middle of one of her lessons which allowed students to experience both struggle and success. This approach could serve as a compromise between these two tensions. The two teachers perceived that the intervention had a positive impact on their teaching practice. One

teacher commented that she intended to implement problem-solving lessons based on the intervention in the future and the other suggested that she would incorporate more manipulatives in her problem-solving lessons.

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Declaration of Originality

I, Elizabeth Jane Stewart, declare that this thesis comprises only my work and has not been submitted in any form for another degree at any university or other institute of tertiary education. Information derived from the published work of others has been acknowledged in the text and a list of references has been provided. The thesis is less than 22 000 words in length, exclusive of tables, figures, bibliographies and appendices.

Elizabeth Jane Stewart

Table of Contents

Table of Contents.....	6
1. Introduction and Background	10
1.1 Importance of Problem-solving	10
1.2 The Australian Curriculum.....	10
1.3 Victorian Curriculum	11
1.4 Background to study: School Problem-Solving Approach.....	12
1.5 The Case Study	12
1.6 Research Questions	13
1.7 Thesis Structure.....	13
2. Literature Review	14
2.1 Problem-solving Processes and Strategies	14
2.2 Problem-solving in School Mathematics	17
2.3 Problem-solving Interventions	19
3. Methodology	24
3.1 Research Questions	24
3.2 Case Study.....	24
3.3 Research Design	25
3.4 Participant Selection.....	26
3.5 Problem-Solving Intervention Design	27
3.6 Problem-solving Lesson Plans	32
3.6.1 Lesson Plan A: Coin Problem	34
3.6.2 Lesson Plans B (Cookie Problem), C (Counter Problem) and D (Over and Over Problem).....	35
3.6.3 Teacher Choices	35
3.7 Research Instruments	36
3.7.1 Research Instrument 1: Interviews.....	36
3.7.2 Research Instrument 2: Lesson Observations	39
3.7.3 Research Instrument 3: Student Work Samples	39
3.8 Data Analysis Strategy	40
3.8.1 Deductive Approach.....	40
3.8.2 Inductive Approach	42
3.9 Ethical Considerations	43
4 Results: Teacher Reflections	44
4.1 Intervention Design Discussion and Lesson Plan Choices	44
4.2 Interviews	47
4.2.1 Theme One: Struggle	47
4.2.2 Theme Two: Challenging problem features and assumptions	54
4.2.3 Theme Three: Prompts	59
4.2.4 Theme Four: Student share time	63

4.2.5	Theme Five: Teachers' Problem-solving practice	65
5	<i>Results: Student Work Samples</i>	69
5.1	Correct Responses	71
5.2	Recording of Problem-solving Strategies.....	77
5.3	Misunderstandings	84
6.	<i>Results: Lesson Observations</i>	88
6.1	Problem-solving Strategies	88
6.2	General and problem-specific prompts	90
6.3	Lesson Deviations.....	95
7.	<i>Discussion</i>	102
7.1	Struggle.....	102
7.2	Teachers anticipate student responses.....	105
7.3	Prompts	106
7.4	Student Share Time	108
7.5	Problem-solving Practice.....	109
7.6	Strengths and Limitations of the Study	110
8.	<i>Conclusion.....</i>	112
	<i>Appendices.....</i>	119
	<i>References.....</i>	189

List of Figures

<i>Figure 1. Research design</i>	26
<i>Figure 2. Research design: Intervention design</i>	28
<i>Figure 3. Problem-solving lesson structure-WCP (whole class prompts), Page 1 and Page 2</i>	31
<i>Figure 4. Research design: Intervention discussion</i>	32
<i>Figure 5. Teachers' options for lesson choices.</i>	32
<i>Figure 6. Utterance recorded in an interview transcript (Rachel)</i>	38
<i>Figure 7. Research design: Lessons</i>	39
<i>Figure 8. Research design: Lessons</i>	39
<i>Figure 9. Code relating to prompts: Beth (BL2)</i>	41
<i>Figure 10. Combined codes relating to struggle: Rachel (Interviews)</i>	42
<i>Figure 11. Teachers' lesson choices</i>	45
<i>Figure 12. Statements from Rachel's interview transcripts relating to the concept of struggle</i>	48
<i>Figure 13. Statements from Beth's interview transcripts relating to the concept of struggle</i>	52
<i>Figure 14. Share time reflections (Rachel)</i>	63
<i>Figure 15. Share time reflections (Beth)</i>	64
<i>Figure 16. Annotated work sample: Student 6 (BL1)</i>	70
<i>Figure 17. Nine correct responses: Student 1 (RL1)</i>	72
<i>Figure 18. One correct response: Student 4 (RL2)</i>	73
<i>Figure 19. Seven correct responses for original total and four correct responses for new total: Student 1 (BL1)</i>	74
<i>Figure 20. One correct response: Student 7 (BL2)</i>	75
<i>Figure 21. Six correct responses for original total and three correct responses for new total: Student 6 (BL1)</i>	76
<i>Figure 22. Draw a picture and number sentences strategies: Student 6 (BL2)</i>	79
<i>Figure 23. Number sentences and look for a pattern strategies: Student 3 (BL1)</i>	80
<i>Figure 24. Number sentences, write a story and highlighting key words strategies: Student 5 (RL2)</i>	81
<i>Figure 25. Working out an answer in more than one way strategy: Student 3 (RL2) ...</i>	81
<i>Figure 26. Look for a pattern strategies: Student 7 (BL1)</i>	83
<i>Figure 27. Look for a pattern strategies: Student 1 (RL2)</i>	83
<i>Figure 28. Misunderstanding – included coins that do not exist: Student 3 (RL1)</i>	85
<i>Figure 29. Misunderstanding - added range of numbers to reach total: Student 2 (BL2)</i>	86
<i>Figure 30. Students encouraged to apply peers' strategies after middle of the lesson share time (RL2)</i>	98
<i>Figure 31. Beth's number sentence visual for introduction to Coin Problem (BL1)...</i>	100
<i>Figure 32. Students encouraged to adopt number sentence strategy in introduction to Coin Problem (BL1)</i>	100

List of Tables

Table 1. <i>Data sources links to research questions</i>	26
Table 2. <i>Problem-solving intervention features and rationale for their inclusion</i>	29
Table 3. <i>Challenging Problems used in the four lesson plans</i>	33
Table 4. <i>Assumptions behind the challenging problems used in the lessons</i>	34
Table 5. <i>Whole class prompts added to Lesson Plan A: Coin Problem</i>	35
Table 6. <i>Interview 1 questions and targeted research questions</i>	37
Table 7. <i>Interview 2 questions and targeted research questions</i>	38
Table 8. <i>Krippendorff's components of content analysis including researcher's procedure</i>	42
Table 9. <i>Planning to teach the intervention: Teacher choices</i>	46
Table 10. <i>Challenging problem features teachers did and did not discuss in interviews</i>	56
Table 11. <i>Assumptions behind the problems used in the lessons that teachers did and did not discuss during interviews</i>	58
Table 12. <i>Prompts teachers indicated using: all lessons</i>	60
Table 13. <i>Analysis of Student 6's work sample from BL1</i>	70
Table 14. <i>Number of correct responses for Coin Problem recorded by students in RL1</i>	72
Table 15. <i>Number of correct responses for Cookie Problem recorded by students in RL2</i>	73
Table 16. <i>Number correct responses for Coin Problem recorded by students in BL1</i> ...	74
Table 17. <i>Number of correct responses for Coin Problem recorded by students in BL2</i>	75
Table 18. <i>Frequency of problem-solving strategies identified in student work samples from all lessons</i>	78
Table 19. <i>Frequency of types of patterns identified in student work samples</i>	82
Table 20. <i>Problem misunderstandings identified in student work samples for all lessons</i>	84
Table 21. <i>Problem-solving strategies used by students, as observed in RL1 (Coin Problem)</i>	89
Table 22. <i>Problem-solving strategies used by students, as observed in RL2 (Cookie Problem)</i>	89
Table 23. <i>Problem-solving strategies used by students, as observed in BL1 (Coin Problem)</i>	90
Table 24. <i>Problem-solving strategies used by students, as observed in BL2 (Over and Over Problem)</i>	90
Table 25: <i>General and problem-specific prompts identified in RL1 (Coin Problem) lesson observation data</i>	92
Table 26. <i>General and problem-specific prompts identified in RL2 (Cookie Problem) lesson observation data</i>	93
Table 27. <i>General and problem-specific prompts identified in BL1 (Coin Problem) lesson observation data</i>	94
Table 28. <i>General and problem-specific prompts identified in BL2 (Over and Over Problem) lesson observation data</i>	95
Table 29. <i>Summary of the deviations made by the two teachers in all lessons</i>	96
Table 30. <i>Deviations to intervention when introducing the problem in all four lessons</i>	99

1. Introduction and Background

This study investigates two teachers' perceptions of the impact of a problem-solving intervention on their students' problem-solving skills and on their own teaching practice. The study is situated in Victoria, Australia.

Problem-solving in mathematics is an important component of curricula around the world (Liledahl, Santos-Trigo, Malaspida, & Bruder, 2016), and it has been identified as important that students develop this capacity in order to achieve success in mathematics. Buchanan (1987) defines mathematical problems as "non-routine problems that required more than ready-to-hand procedures or algorithms in the solution process" (p. 402). Stacey, Vincent, Stephens and Holton (2015), in a national review of mathematics school education in Australia, found that more teachers need to teach their students strategies to problem-solve in mathematics. McCormick (2016) and Cheeseman (2018) provided reasons why teachers might be reluctant to teach their students to undertake problem-solving. These reasons related to teachers' perceptions about their students' abilities to problem solve and about their own teaching practice. They will be discussed in section 2.2.

1.1 Importance of Problem-solving

A major focus for the teachers of today is preparing students for the rapidly changing world that we are living in. In this context, one of the 21st century learning skills that teachers need to encourage their students to develop, is the ability to undertake problem-solving (English, & Sriraman, 2009). Furthermore, problem-solving has been identified as an important focus of mathematics education (e.g., Butterfield, 2012; Cheeseman, 2018; Liljedahl et al., 2016; Shiakalli, & Zacharos, 2014). The process of teaching students to develop problem-solving skills starts in primary school.

1.2 The Australian Curriculum

In Australia, The Australian Curriculum, Assessment and Reporting Authority (ACARA) is responsible for providing relevant curriculum requirements for Australian teachers and students (ACARA, n.d.). The ACARA website provides relevant

information about the teaching of mathematics and more specifically about problem-solving. The ACARA website also provides expectations for each year level in problem-solving. For example, the curriculum states that Year Four students are expected to: “formulate model and record authentic situations involving operations, comparing large numbers with each other, comparing time durations and using properties of numbers to continue patterns” (ACARA, n.d.).

The ACARA website also provides problem-solving specific annotated samples. For example, in a Year One sample of Problem-Solving in the Number and Algebra strand, the following question is provided with sample annotated student work:

“Yesterday Mr Lee had a full basket of skipping ropes. Today he only has five left. How many ropes could have been in the basket yesterday and how many went missing?” (ACARA, n.d.).

The annotated example displays a student’s work sample containing responses recorded in a variety of ways. It includes pictorial representations, number lines and number sentences. The information provided to teachers highlights types of problems that may be used to teach problem-solving and illustrate that a number of problem-solving strategies can be applied to problems.

1.3 Victorian Curriculum

The Victorian Curriculum and Assessment Authority (VCAA) is responsible for the state-based curriculum in Victoria which is derived from the Australian Curriculum (VCAA, n.d.). The VCAA provide four proficiencies in mathematics including problem-solving, understanding, fluency and reasoning (VCAA, n.d.). Teachers are expected to help students apply these across all of the three strands of mathematics: number and algebra; measurement and geometry; and statistics and probability. The VCAA defines problem solving as, “the ability of students to make choices, interpret, formulate, model and investigate problem situations, select and use technological functions and communicate solutions effectively” (VCAA, n.d., para. 4). The VCAA states that “capable problem-solvers solve unfamiliar and meaningful problems, create their own investigations, use prior knowledge and strategies to aide in their solving of problems and check the viability of their solutions” (VCAA, n.d., para 4).

The VCAA website provides sample programs for teachers of each year of primary schooling (VCAA, n.d.). Included in the sample programs are brief descriptions of particular problems that could be solved when teaching different areas of the Number and Algebra strand. The resource provides teachers with ideas about how to link problem-solving tasks with the content descriptors but does not provide guidance about teaching strategies for implementing problem-solving lessons in mathematics.

1.4 Background to study: School Problem-Solving Approach

At the time that the study was conducted, the school in the study required teachers and students to use Peter Maher's approach to problem-solving (Maher, 2007). In Maher's text book for teachers: *All you need to teach problem-solving*, he provides teachers and students with a list of problem-solving strategies to apply to problems including, *look for the important words in the question, look for a pattern, have a go, use a table or chart, use a drawing, work backwards, try an easier problem and make a model*. Maher also provides teachers and students with problems that correlate with particular problem-solving strategies. Maher suggests that while some students are capable of choosing and applying their own strategy from the list, other students need to be directed towards appropriate strategies and some students require teacher modelling of an appropriate strategy. Teachers and students at the participating school were encouraged to implement Peter Maher's approach fortnightly with their students as they see appropriate.

1.5 The Case Study

Problem-solving literature was examined to help identify what kinds of interventions might improve the effectiveness of the teaching of problem-solving (see section 2.3). An intervention was created by the researcher which included a number of features from the literature examined (see section 3.5).

The aim of the study was to investigate the implementation of a problem-solving intervention by two primary school teachers, focusing on their perceptions of the effectiveness of the intervention and how it might improve their teaching of problem-solving in the future.

1.6 Research Questions

Based on the aim of the research, the follow four key questions were investigated:

RQ1: What do teachers perceive to be the impact of the intervention on students' problem-solving abilities?

RQ2: What do teachers perceive to be the impact of the intervention on their teaching practice?

RQ3: How did the teachers implement the intervention in the lessons?

RQ4: How did students respond to the intervention in the lessons?

1.7 Thesis Structure

Chapter Two provides a literature review on problem-solving in mathematics, including the importance of problem-solving, impediments to the teaching of problem-solving and problem-solving interventions. Chapter Three provides the research methodology for the development of the intervention, interviews with the teachers and obtaining their input to the design of the intervention, observation of the lessons and the student work samples. Chapters four, five and six present the results. In Chapter Four the teachers' perspectives are presented, including the collaborative element of the intervention design and the interview data. The lesson observation data is provided in Chapter Five and the student work samples data is presented in Chapter Six. These results are then combined and analysed as the key findings of the research project in Chapter Seven. Finally, Chapter Eight answers the research questions and provides recommendations for future research.

2. Literature Review

Problem-solving involves solving problems that are unfamiliar to the student (Kilpatrick, Swafford, & Findell, 2001), and that are cognitively demanding (McCormick, 2016). Schoenfeld (1992) argues that in order for a learner to engage in meaningful and purposeful problem-solving, a problem cannot be solved using a known method. Rather, it needs to be a complex process whereby students are required to use a range of higher-order thinking skills.

The first section of the review considers the importance of effective teaching of problem-solving in mathematics. The second section focuses on current problem-solving practice in Victorian primary schools. The third section considers research that is relevant to the design of effective problem-solving interventions.

2.1 Problem-solving Processes and Strategies

In this study, *problem-solving processes* refer to the stages or phases problem-solvers may move through as they solve a problem. An example is where a problem-solver chooses a particular problem-solving strategy to apply to a problem. *Problem-solving strategies* describe approaches that students use when they are trying to solve a problem.

The terms *problem-solving strategies* and *heuristics* are sometimes used interchangeably in the literature because heuristics are also used to describe problem-solving approaches. For example, Swanson (2016) uses “heuristics” and Millard, Oaks and Sanders (2002) use “problem-solving strategies” to describe approaches problem-solvers apply to a given problem.

Polya (1945) provides a four-phase problem-solving process: *understand the problem*, *devise a plan*, *carry out the plan* and *look back*. Polya includes problem-solving strategies, questions, and explanations to assist problem-solvers (i.e. students) in each phase. In the *understand the problem* phase, problem-solvers identify the important parts of the problem using strategies such as, drawing a figure or writing down

important words that may assist in solving the problem. In the *devise a plan phase*, problem-solvers use their knowledge of problems solved previously and make connections with the current problem. They choose an appropriate problem-solving strategy or strategies they will use to attempt to solve the problem. If the problem-solver cannot solve the problem, Polya suggests they try and solve an accessible related problem or solve part of the problem. Next, in the *carry out the plan phase*, the problem solver applies their chosen problem-solving strategy or strategies to the problem and checks that the steps taken have enabled them to arrive at a correct answer. Finally, in the *look back phase*, the problem solver checks their result, makes connections with problems they have previously solved and makes generalisations about other potential problems. Polya does not provide guidance for instances in which an incorrect solution is reached. He claims that these phases and their relevant strategies, questions and explanations are appropriate for all students engaging in the problem-solving process.

Mason, Stacey and Burton (1985) provide a three-phase problem-solving process: *entry*, *attack* and *review*, which aligns with Polya's (1945) problem-solving process. In the *entry* phase the problem-solver gains an understanding of the problem and decides on appropriate problem-solving strategies. In the *attack* phase the problem-solver attempts to solve the problem using their chosen problem-solving strategies. In the *review* phase, problem-solvers check for accuracy, reflect on their thinking and make connections with other problems. The *entry* phase provided by Mason et al. (1985) is similar to Polya's *understand the problem* and *devise a plan* phase. In addition, the *attack* phase is equivalent to Polya's *carry out the plan* phase and the *review* phase is equated to his *look back* phase.

Problem-solving strategies are included within the *entry*, *attack* and *review* phases of the problem-solving process (Mason et al., 1985). In the *entry* phase, they suggest that problem-solvers *specialise* through trialling a possible *case* of what the question is asking. For example, when searching for two numbers that make twenty, a problem-solver may try selecting two numbers at random and combine them, to help decide whether their problem-solving approach is appropriate. This *specialising* strategy also helps the problem-solver to gain an understanding of the question and makes the given problem seem less intimidating. Specialising in this phase aids the problem-solver's

comprehension of the question and helps them consider potential problem-solving strategies to use in the *attack* phase.

Specialising continues in the *attack* phase when the problem-solver is “stuck” and *generalising* is introduced (Mason et al. 1985). When *specialising*, the problem-solver engages with a second *case* of the problem to help them consider another problem-solving approach. In the *attack* phase a problem-solver may use more systematic *specialising*. For example, they may trial *cases* of the problem, where they vary numbers systematically, to search for a pattern within the results. When *generalising*, the problem-solver observes a pattern within the results and reflects upon its implications.

The third stage is the *review* phase. In this phase, Mason et al. suggest that *specialising* is used to check that a pattern is correct. They also suggest that *generalising* occurs where problem-solvers apply their new understandings to other problems or scenarios, for example, a similar problem where the numbers are different. Mason et al. suggest that students bring their own individual ideas to problem-solving and recommend the provision of broader problem-solving strategies that allow for individual differences such as, *specialising* and *generalising*.

Throughout the three stages of the problem-solving process, Mason et al. (1985) acknowledge that a problem-solver may get “stuck” because their problem-solving strategies are not effective. There are two different states students may experience when they are stuck which Mason et al. described as *distilling* and *mulling*. In the *distilling* state the student articulates the difficulties they are having with the problem to a peer or records it in a journal. In the *mulling* state the student abandons the problem for a period of time. This *mulling* state allows the problem-solver to process their thoughts. They may formulate another *specialising* idea while they are not solely focussing their attention on the problem. Perkins (2000) uses breakthrough thinking to describe a similar process in which a problem-solver suddenly arrives at a solution without solely focussing on the problem.

Schoenfeld (1992) stresses that different individuals bring their own problem-solving strategies to the problem-solving process. Schoenfeld believes that an individual’s prior

knowledge of problems, metacognition, decision making abilities and dispositions towards problem-solving have a large effect on how they approach a problem. Polya's problem-solving strategies were identified as too abstract and unclear for problem-solvers to understand and use (Schoenfeld, 1987).

Liljedahl (2004) argues that not all problems can be solved through one-size-fits-all problem-solving processes and strategies. He believes that problem-solving in mathematics is more of a creative process. Liljedahl claims that problem-solvers may suddenly formulate an idea or gain some insight that assists in solving the problem, and that this may change their approach or strategy.

In conclusion, a number of different problem-solving processes and strategies for problem-solving have been presented in the literature. An example of a generalise approach is Polya's (1945) four step problem solving process. A number of the processes and strategies that have built on Polya's ideas are designed to further encourage students to bring individual creativity to the problem-solving process (Liljedahl, 2004; Schoenfeld, 1992). In line with this notion, Mason et al. (1985) provide problem-solving strategies that are less prescriptive and allow for individual differences. These include specialising and generalising strategies.

2.2 Problem-solving in School Mathematics

In this section, literature about the teaching of problem-solving in school mathematics will be explored. The scope of the literature will be limited to the primary years of schooling.

In the review of mathematics pedagogies and learning resources in Australian education (Years Foundation to Year 12) Stacey et al. (2015) identified gaps in several areas. In relation to problem solving, Stacey et al. claim that despite curriculum and research having stressed the importance of teaching problem-solving in schools, students are spending the majority of their time engaging with rote tasks that do not require higher order thinking. They argued that mathematics in schools needs to broaden to focus on the mathematic proficiencies of problem-solving, fluency, reasoning and understanding in the curriculum. This implies that more teachers need to teach the mathematical

content strands in ways that encourage students to develop the proficiencies. Stacey et al. argue that teachers perceive that there are not enough problem-solving teaching resources available and that they are too time poor to teach problem-solving. Stacey et al. found that although there are sufficient problem-solving resources available, many of them are poor quality and difficult to find. As a consequence, it is time consuming for teachers to create curriculum-aligned planning documents.

McCormick (2016) surveyed 108 upper primary teachers from VIC, NSW and TAS about their implementation of complex problems and also found that problem-solving is being overlooked in primary classrooms. Teachers in the study preferred to explicitly teach their students mathematics content areas before requiring them to undertake problem-solving. Students were rarely given the opportunity to undertake problem-solving in mathematics, instead engaging with routine tasks that do not encourage the development of problem-solving strategies.

In addition, in a study focussing on enhancing early years' practice, Cheeseman (2018) found that teachers are reluctant to engage their students in problem-solving. In the study, teachers in three Victorian schools received professional development about problem-solving and were provided with twenty-two problems in the Year One, Number Strand. Three months into the project, the teachers were surveyed about their perspectives on the difficulties of incorporating problem-solving in their classrooms. Some teachers reported that problem-solving was too novel for their students. Other teachers were concerned that the problems were too open for their students. Similar to McCormick (2016), Cheeseman (2018) found that some teachers believe that their students needed explicit teaching of mathematics content areas before they are capable of problem-solving.

Cheeseman (2018) also found that some teachers are unsure of their ability to implement problem-solving lessons. For example, some teachers find it difficult to choose appropriate problems for their students. Others have difficulties making decisions about when to intervene when their students are struggling with problems. Some teachers do not see the value of allowing their students to struggle with mathematical problems. Similar to Stacey et al. (2015), Cheeseman (2018) found that

teachers attribute time restraints and accessibility to appropriate resources as barriers to teaching problem-solving.

Pineiro, Castro-Rodriguez and Castro (2018) also found that many teachers believed that students needed mathematical concepts in order to undertake problem solving. They conducted a survey of 51 fourth-year pre-service teacher trainees and found that 98% believed this to be the case. In addition, Pineiro et al., found that 98% of these teachers regarded routine and applied problems as appropriate for problem-solving.

Encouragingly, Savardi, Cavalcante and Polotskaia (2018), reported a study of 18 Years One to Three teachers who were able to reflect on their teaching and adopt problem-solving practices. Savardi et al. developed a teaching approach intervention that the teachers trialled and also provided a professional development program (15 sessions) over a three-year period. The study aimed to shift teachers' problem-solving paradigms from operational (routine problems) to relational (non-routine problems). As a result of this study, they found that teachers were able to change paradigms and perceive engagement with non-routine problems as more effective in developing their students' mathematical concepts.

In conclusion, there appears to be a gap between mathematics curriculum requirements for the teaching of problem-solving and the frequency in which it is occurring. As a result, it is important for teachers to reflect on their practices and consider approaches to teaching mathematics that encourage their students to develop problem-solving skills.

2.3 Problem-solving Interventions

A number of problem-solving interventions have been designed, which aim to enable teachers to successfully implement problem-solving lessons in their classrooms. Studies have analysed the implementation of these problem-solving interventions (e.g. Roche, Clarke, Sullivan, & Cheeseman, 2013; Cheeseman, Downton, & Livy, 2017).

As part of the project 'Encouraging Persistence Maintaining Challenge' (EPMC), Roche et al. (2013) found that students' problems-solving skills were developed if they were given time to struggle with problems. By struggle, they mean that students spend some

time “grappling” with a problem and are required to use higher order thinking skills (e.g. trying multiple ways to solve the problem in order to arrive at a solution). They found that problems need to be engaging and challenging and that students need clear expectations about their working environment (e.g. they needed to understand whether they were required to record their thinking, work independently or as part of a group, and how much time they would be given to work on a problem). They also found that teachers need to listen to their students intently to understand their thinking throughout the problem-solving process.

In a survey of 758 primary students, Sullivan, Clarke, Cheeseman, Mornane, Roche, Sawatzki and Walker (2014) found that students prefer to struggle with problems before being given solutions. The students’ teachers were supported to trial some tasks before the 758 primary aged students were surveyed. They found that 88% of the students favoured working on problems that were as hard or even harder than the challenging problems they engaged with in the trial.

Ingram, Holmes, Linsell, Livy and Sullivan (2016) conducted a study as part of the EPMC project where teachers trialled a range of strategies designed to encourage students to persist with challenging problems. Ten teachers participated in the study along with their students aged nine to eleven. After the lessons were trialled, teachers were surveyed and asked to identify specific teaching strategies that contributed to student success with persistence. Teachers highlighted a number of strategies. These included: encouraging students to collaborate and exchange knowledge; providing students with access to relevant materials; and encouraging students to reflect on their ability to persist with the challenging problems. Teachers found value in posing probing questions that were carefully formulated throughout the lessons to direct student thinking. They also found it helpful to implement extending and enabling prompts. Enabling prompts are prompts that support students to understand and attempt problems. Extending prompts encourage students to find more responses to problems and look for patterns and rules in the results (further discussed in 3.5). Cheeseman, Downton and Livy (2017) also reported that teachers found extending and enabling prompts helpful in supporting their students to understand and access problems. They surveyed 37 teachers of Years Three, Four, Five or Six and found that teachers believed that the prompts assisted them to differentiate problems by supporting and extending

their students' thinking. In addition, it encouraged them to better analyse their students' thought processes.

In another investigation which was also part of the EPMC study, Sullivan, Borcek, Walker and Rennie (2016) found that the particular way in which an intervention was structured was an essential part of the problem-solving lessons that were successful. In this study, 34 Year Three and Four teachers trialled ten lessons based around challenging problems that are not easily solved. Twenty-nine of the teachers indicated that they would plan future lessons using the intervention structure. The lesson structure includes four different phases. The first phase is *posing the problem* where a common language about mathematical concepts relevant to the problem is constructed. In this phase, possible approaches to the given problem are avoided. The second phase is *differentiating the problem*, where students work on the problem independently for some time before teachers prompt students to increase or decrease the difficulty of the task. For example, to make a problem simpler, numbers may be lowered, steps may be reduced, or visual representations may be provided. To make a problem more difficult, numbers may be increased or students may be asked to identify patterns in the results. In the third phase, *reviewing student activity on the problem*, students share their problem-solving strategies with their peers. The final phase is *consolidating the learning* where follow-up problems are provided that vary slightly from the original problem. These enable students to make connections by applying what they have learned from the original problem to new problems.

Before implementing the ten lessons based on the intervention, teachers were required to solve the problems themselves. Sullivan et al. included this requirement in order to aid teachers to better anticipate their students' responses to the given problems. Teachers were also provided with a range of enabling prompts (e.g. Could pictures help?) to help with the *differentiating the problem* phase. In a post-lesson survey, Sullivan et al. found that 26 of the teachers believed the intervention's structure was successful in terms of student learning and rich conversations. In particular, teachers found that the *differentiating the problem* part of the structure encouraged their students to develop the ability to persist with problems and engage in deep thinking. They also found the *reviewing student activity on the problem* phase was beneficial as students explained their thinking and identified the most efficient problem-solving strategies.

Sullivan et al. concluded that the workshops where teachers solved the problems themselves partially ensured that they knew when to intervene appropriately in the lessons. However, they acknowledged that teachers' abilities to provide suitable prompts to individual students was imperative.

In contrasting results, Russo and Hopkins (2017) trialled two different problem-solving lesson structures and found that the structures themselves did not have a significant impact on the teaching and learning of problem-solving. Russo and Hopkins undertook a study with 75 Year One and Two students and three teachers from Melbourne, Victoria. Russo and Hopkins trialled the problem-solving interventions in the area of Number and Algebra in the three teachers' respective classrooms. The *problem-first* intervention structure consisted of a challenging problem and then a discussion. This was followed by consolidation worksheets and then a teacher-led summary of the lesson. The *teach-first* intervention structure consisted of a discussion, consolidating worksheets, a challenging problem and then a teacher-lead summary of the lesson. The two lesson structures were compared in terms of students' problem-solving results and teacher and student preference of each intervention. Russo and Hopkins found that the lesson structure did not impact the problem-solving outcomes of the students in Year One and Year Two. They also found that the three teachers perceived that the *problem-first* intervention encouraged creativity, student discovery, persistence, and the conversations were richer. However, teachers perceived that the *teach-first* intervention was more focussed, efficient and supportive for their students. Russo and Hopkins found that some students liked the *teach-first* intervention as they could prepare for the problem by completing the consolidating worksheets. In contrast, other students liked the *problem-first* intervention because it required them to think deeply about the problem in order to arrive at possible solutions. Although Russo and Hopkins' results indicated that lesson structure had little impact on the teaching of problem-solving in the lessons, there was no account of the type of teacher and student interactions that took place in these lessons. Furthermore, it is not clear whether the type of support provided by teachers was altered to suit each lesson structure in order for an accurate comparison to be made between the two structures.

In conclusion, the literature indicates that there are effective interventions that encourage and enable teachers to successfully implement problem-solving lessons. For

example, it appears that structuring lessons so that the central focus of the lesson is about students solving challenging problems, improves students' problem-solving abilities (Sullivan et al., 2016). Some of the other factors that contribute to successful problem-solving lessons are the ability of teachers to establish common language surrounding the wording of the challenging problems and anticipate their students' responses (Sullivan et al.). An important feature of successful interventions is that they should require teachers to allocate time for their students to struggle with problems before providing enabling or extending prompts if required (Cheeseman et al., 2017, Sullivan et al.). In addition, students benefit from collaborating and sharing their knowledge with their peers (Ingram et al., 2016). These key components of successful problem-solving interventions were used by the researcher to create a problem-solving intervention.

3. Methodology

This chapter describes how the study was conducted and explains the data analysis strategy. Section 3.1 provides the research questions, with section 3.2 providing a rationale for the case study approach used in this study. Section 3.3 describes the research design and explains the link between the data sources and the research questions. This is followed by the methodology for the selection of participants in section 3.4. Section 3.5 describes how the researcher designed the intervention. The lessons that were planned based on the intervention are described in section 3.6. Section 3.7 explains the three research instruments used in the study and their purpose. Section 3.8 describes the data analysis strategy. Section 3.9 focusses on the ethical considerations.

This research project investigated the implementation of a problem-solving intervention by two primary school teachers. The focus was on their perceptions of the impact of the intervention on their students' problem-solving abilities and their own teaching of problem-solving.

3.1 Research Questions

As mentioned in section 1.6 the research questions were:

RQ1: What do teachers perceive to be the impact of the intervention on students' problem-solving abilities?

RQ2: What do teachers perceive to be the impact of the intervention on their teaching practice?

RQ3: How did the teachers implement the intervention in the lessons?

RQ4: How did students respond to the intervention in the lessons?

3.2 Case Study

The study was a qualitative Case Study of two teachers' implementation of a problem-solving intervention. Yin (2015) describes a Case Study as a methodology that enables researchers to gain in-depth understandings of the complexity of a phenomenon in its

real-life context. Corbett-Whittier and Hamilton (2013) support the use of Case Studies in educational settings, arguing that Case Studies allow for context-specific and meaningful data to be collected. They make comparisons between Case Studies in educational research and quantitative approaches that attempt to make generalisations about groups across educational settings. They criticise quantitative approaches and argue that educational contexts are far too complex for widespread generalisations to be made due to the diversity of settings, individuals and pedagogies.

This study attempted to gain rich data about the two teachers' implementation of the problem-solving intervention through a Case Study approach. The teachers' implementation of the problem-solving intervention was the phenomenon of interest with a focus on their perspectives and reflections. A range of research instruments were adopted in order to gain multiple perspectives on the implementation of the intervention (refer to 3.7.1, 3.7.2 and 3.7.3).

3.3 Research Design

Based on the research questions and the Case Study methodology, the research design for the project comprised the following key elements (refer to Figure 1):

- Intervention Design: The creation of the problem-solving intervention (consisting of two different lesson structures that were used to plan four separate lessons), drawing on the relevant literature
- Interview 1: A semi-structured interview with each teacher before they taught the two lessons
- Intervention Discussion: The intervention was adapted in discussion with each of the two teachers separately
- Interview 2: A semi-structured interview with each teacher after they taught the two lessons
- The observation of all four lessons (two for each teacher), with the recording of lesson observation notes
- Following each lesson, the collection of student work samples; these were copied and then returned to the students

- The analysis of all data using content analysis and determination of relevant themes

Intervention Design Researcher	Interview 1 and Intervention Discussion Teacher 1 or 2	Lesson 1 Teacher 1 or 2	Lesson 2 Teacher 1 or 2	Interview 2 Teacher 1 or 2
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Figure 1. Research design

Table 1 displays how the data sources link to each of the research questions.

Table 1. *Data sources links to research questions*

Data Source	Research Question 1	Research Question 2	Research Question 3	Research Question 4
Interviews	Yes	Yes	No	No
Lesson Observation Notes	No	No	Yes	Yes
Student Work Samples	No	No	No	Yes

3.4 Participant Selection

The participants were selected through convenience sampling as participants were selected from the primary school where the researcher works.

The Principal of the school gave permission for the study to be undertaken at his school. Two teachers volunteered to be part of the study. Each teacher was teaching a composite Grade One/Two class at the time that the research was undertaken. Rachel (pseudonym) was in her third year of teaching and her third year of teaching Grade One/Two at the participating school. Beth (pseudonym) was in her fourth year of teaching and had taught a Grade One class at her previous school. They were invited individually and were not unduly influenced to participate in the study. The teachers' respective classes of students were aged between six and eight. They were invited to participate in the study. Seven students from Beth's class and eight students from Rachel's class gave permission or their work samples to be used in the study. All of the

participants understood that they could withdraw from the project at any point. The small sample size was due to the in-depth Case Study approach.

The government primary school where the study was undertaken is in Victoria and had approximately seven hundred and fifty students enrolled when the research was conducted. It is important to note that the researcher was employed at the school at the time that the research was undertaken.

Mutually appropriate times were scheduled for the tasks to be completed. All tasks were completed at the participants' school. Due to the researcher's working relationship with the school and due to the small number of participants, the school, teachers and students may be identifiable in the results. However, teachers were given pseudonyms and student work was de-identified to reduce this probability. It is also acknowledged that there may have been some bias as the researcher acted as an observer in all four of the lessons trialled.

Appropriate protocols were established between the researcher and the teachers due to the potential conflict of interest (see Appendix X).

3.5 Problem-Solving Intervention Design

The problem-solving intervention was created by the researcher and was based on features that were identified in the literature (see section 2.3) as being part of quality problem-solving interventions. The features are listed in Table 2. The intervention comprised two slightly different lesson structures:

- Lesson structure-WCP (see Appendix A) which includes a section for Whole Class Prompts as well as a section for individual prompts and a requirement that teachers anticipate students' responses to a given problem (it includes all features listed in Table 2).
- Lesson structure-IP (see Appendix A) which includes a section for Individual Prompts, but does not include a section for whole class prompts (it includes all features listed in Table 2 except for a whole class prompt section and does not require teachers to anticipate their students' responses to the given problem).

The reason why two features from the problem-solving literature were included in Lesson structure-WCP, but not in Lesson structure-IP, was to enable conclusions to be drawn about the impact of these particular features. A section for whole class prompts was included in only one of the lesson structures so that insights could be gained into whether the two teachers preferred using both whole class prompts as well as individual prompts, or solely individual prompts. Furthermore, the teachers were only asked to anticipate their students' responses to the problem in Lesson structure-WCP in order to determine whether this better prepared the teachers to prompt their students effectively in the lessons.

Intervention Design Researcher	Interview 1 and Intervention Discussion Teacher 1 or 2	Lesson 1 Teacher 1 or 2	Lesson 2 Teacher 1 or 2	Interview 2 Teacher 1 or 2
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Figure 2. Research design: Intervention design

Table 2 lists each feature included in the intervention and provides a rationale for the inclusion of each feature for both teachers and students.

Table 2. *Problem-solving intervention features and rationale for their inclusion*

Intervention Feature	Description of Intervention Feature	Rationale for inclusion of intervention feature for students	Rationale for inclusion of intervention feature for teachers
Challenging problem	A challenging problem is the focus of the problem-solving lesson.	Focus on the process of trialling problem-solving strategies in order to find a solution (Sullivan et. al., 2016).	Clear expectations about the focus of the lesson.
Introduce challenging problem	Time is allocated for a class discussion about the meaning of the problem, but teachers avoid providing approaches to problems.	Understand the problem and devise own strategy to solve the problem (Sullivan, Borcek, Walker & Rennie., 2016).	Clear expectations about how to introduce the problem.
Struggle time	Time is allocated for students to struggle with problems.	Learn to persist. (Ingram, et.al, 2016).	Time to observe their students and prepare relevant prompts.
Problem-solving strategy recording	Time is allocated for students to record problem-solving strategies.	Express and organise their thinking	Understand their students thought processes and use of problem-solving strategies (Sullivan et. al., 2016).
Problem-solving strategy sharing	Time is allocated for students to share problem-solving strategies.	Verbalise thinking to peers and determine most efficient problem-solving strategies (Sullivan et. al., 2016).	Understand their students thought processes and use of problem-solving strategies.
Teachers anticipate student responses (only included in Lesson structure-WCP)	Teachers are asked to anticipate student responses prior to the lesson implementation.	Receive prompts that better support them to engage with the problem.	Become more aware of the mathematical features and assumptions that are specific to the problem and be prepared to prompt students more appropriately (Sullivan et. al., 2016).
Enabling and extending prompts	Time is allocated for teachers to prompt their students to increase or decrease the difficulty of the problem.	Receive support to engage with the problem or to extend their thinking (Sullivan et al., 2016).	Support students to engage with the problem and extend students' thinking (Cheeseman et al., 2017).
General prompts	Prompts are to be added to a template for teachers to use to support and extend the problem for their students.	Receive support to engage with the problem or to extend their thinking (Sullivan et al., 2016).	Support students to engage with the problem and extend students' thinking Can also be used in other problem-solving lessons.
Problem-specific prompts	Prompts are to be added to a template for teachers to use to support and extend the problem for their students.	Receive support to engage with the problem or to extend their thinking (Sullivan et al., 2016).	Support students to engage with the problem and extend students' thinking. Prompts are specific to the particular problem so can target specific misconceptions or provide specific guidance.
Whole class prompts (only included in Lesson structure-WCP)	Two allocated times slots for teachers to prompt their students as a class.	Receive support to engage with the problem or to extend their thinking (Sullivan et al., 2016).	Support and/or extend the entire class at the same time.

Figure 3 provides an outline of Lesson structure-WCP from the problem-solving intervention. Each feature of the lesson structure as explained in Table 2 is annotated onto Figure 3. Figure 3 includes two pages:

- Page 1 makes provision for the periods of time that are allocated to each element of the lesson structure. A description of what the teacher and students are doing at each stage of the structure are given, to help the teacher to implement the problem-solving lesson. Page 1 also provides time for whole class prompts to be given.
- Page 2 provides an outline of the types of prompts that can be added when planning a lesson (see Appendix A for both Lesson structure-WCP and Lesson structure-IP).

The researcher also decided to include an opportunity for the teachers to have input into the design of the intervention. This was decided in order to allow the teachers the opportunity to suggest appropriate modifications to the intervention. This was intended to ensure that the intervention was aligned with their teaching style and to increase their ownership (see section 3.51 for more detail).

The intervention outlines the general structure and key features that are to be included in each lesson that is developed as part of the intervention. But it does not specify the detailed elements that are to be included in any particular lesson. These need to be added when planning a lesson within the intervention. As such, the following elements need to be added by a researcher or teacher in order to plan a specific lesson:

- A challenging problem (a problem that is not easily solved).
- Individual prompts (the actual prompts are not included in the intervention) (see Table 2 for prompts and their rationale):
 - General enabling prompts (e.g. Could a picture help?)
 - General extending prompts (e.g. Is there a pattern?)
 - Problem-specific enabling prompts (e.g. Could skip counting help?)
 - Problem-specific extending prompts (e.g. What if the total was increased to one dollar?).
- Whole class prompts that are specific to the challenging problem (these are included only in Lesson structure-WCP)

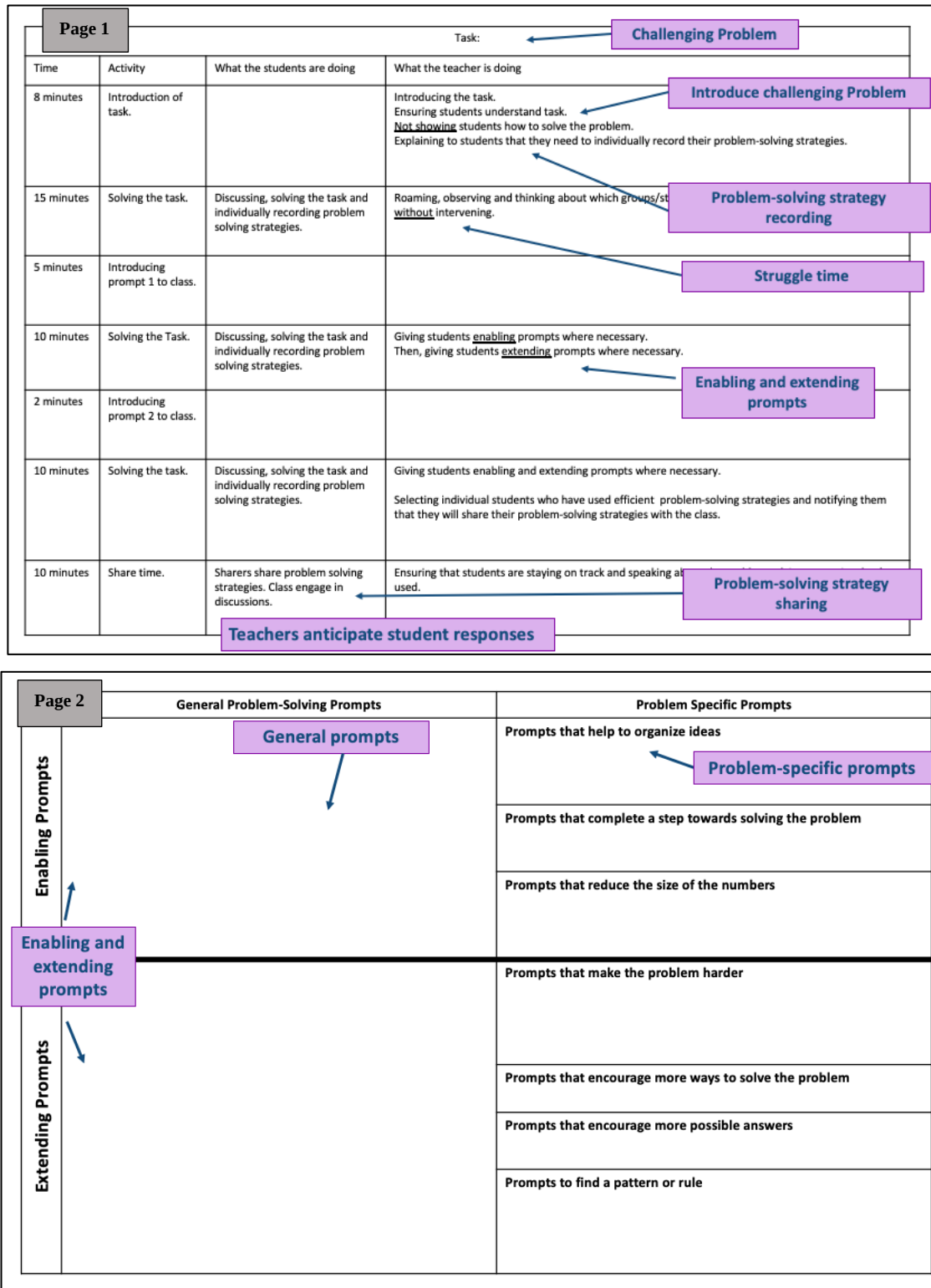


Figure 3. Problem-solving lesson structure-WCP (whole class prompts), Page 1 and Page 2

3.5.1 Teacher Intervention Design Discussion

The two teachers each participated individually in a discussion with the researcher to refine the intervention. Teachers were asked if they would like to alter the intervention. Following the discussion and the making of any amendments to the intervention, teachers were provided with the lesson plans based on the intervention (refer to Appendix B to E).

Intervention Design	Interview 1 and Intervention Discussion	Lesson 1	Lesson 2	Interview 2
Researcher	Teacher 1 or 2	Teacher 1 or 2	Teacher 1 or 2	Teacher 1 or 2

Figure 4. Research design: Intervention discussion

3.6 Problem-solving Lesson Plans

Four lesson plans based on four different problems were planned by the researcher using the intervention. These lesson plans focus on the content area of Number and Algebra in Level 1 and Level 2 of the Victorian Curriculum with a particular focus on the proficiency of problem-solving (VCAA, 2017).

- Lesson Plan A was planned using Lesson structure-WCP (see Appendix B)
- Lesson Plans B, C and D were planned using Lesson structure-IP (see Appendix C-E).

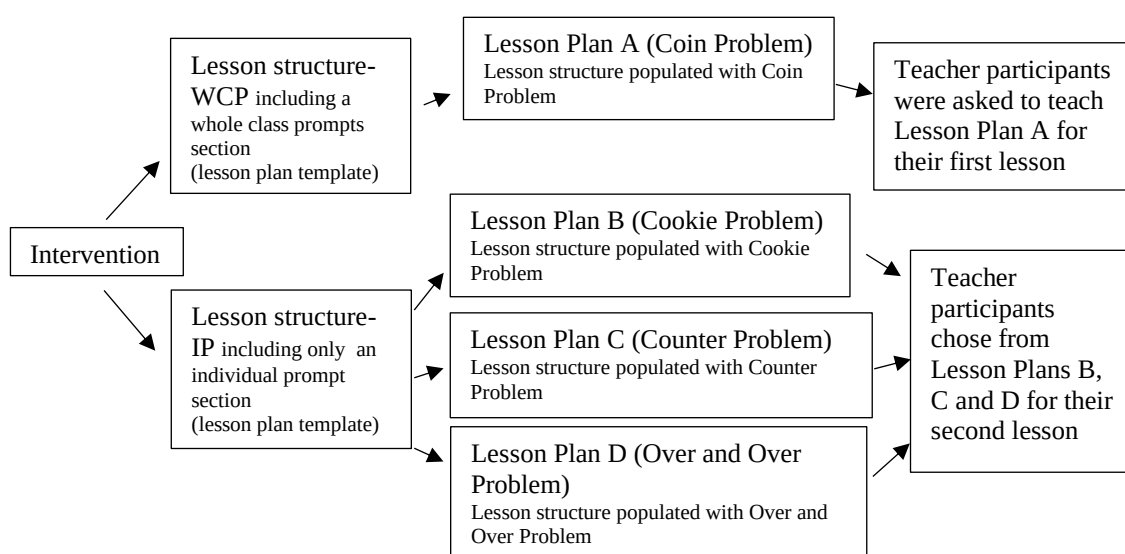


Figure 5. Teachers' options for lesson choices.

Table 3. *Challenging Problems used in the four lesson plans*

Problem
<i>Coin Problem:</i> How many ways can you make 50c using coins?
<i>Cookie Problem:</i> Everyone got the same amount of cookies. How many cookies may there have been?
<i>Counter Problem:</i> Yesterday I put 12 counters into equal groups. I can't remember the number of groups there were. How many may there have been?
<i>Over and Over Problem:</i> I added the same number over and over until I reached 16. Which number may I have started with?

Each of the problems were chosen because of their particular problem features. Stein, Grover and Henningsen (1996) discuss problem features as features that teachers see as important in order for student thinking, reasoning and sense making to occur. For example, this includes problems that allow for multiple solutions and multiple representations.

The four problems had the following mathematical features:

- There are multiple ways to solve the problem
- The problem caters for a range of abilities as students can just make a start, or alternatively can complete the problem and can explain patterns or rules in the results
- There are multiple answers

Students needed to be aware of particular assumptions if they were to properly understand each problem (refer to Table 4). These assumptions were not made explicit to the teachers.

Table 4. *Assumptions behind the challenging problems used in the lessons*

Problem in lesson plan	Problem assumptions
Coin Problem: How many ways can you make 50c using coins?	1. Australian coins as currently used 2. The coins (50 cent coin, 20 cent coin, 10 cent coin, 5 cent coin) are known 3. The same value coin can be used once or several times
Cookie Problem: Everyone got the same number of cookies. How many cookies may there have been?	1. Students know the meaning of the term, “same amount” 2. Each person gets a whole number of cookies 3. Students choose the amount of people and the number of cookies each person received
Counter Problem: Yesterday I put 12 counters into equal groups. I can’t remember the number of groups there were. How many may there have been?	1. Students understand the meaning of the term “equal groups” 2. Students know that they are calculating the number of groups not counters in each group
Over and Over Problem: I added the same number over and over until I reached 16. Which number may I have started with?	1. Students know the meaning of the terms, “same” and “over” 2. Students know their answers will be single numbers rather than a number sentence

3.6.1 Lesson Plan A: Coin Problem

Lesson Plan A (Coin Problem) was planned using Lesson structure-WCP (see Appendix B for the lesson plan including the individual prompts that were added by the researcher).

As described in section 3.5, teachers were asked to anticipate their students’ responses to the problem and to add particular whole class prompts to the Lesson structure. See the whole class prompts and their rationale in Table 5.

Table 5. *Whole class prompts added to Lesson Plan A: Coin Problem*

Whole class prompt number	Whole class prompt	Supporting resource for prompt	Purpose of prompt
Prompt One	John said there were three solutions to the problem, is he correct? Why? Why not?	Visual that only included solutions that had repeated sets of like coins (see Appendix F)	Visual is displayed to the entire class. Students understand that they can use combinations of coins to find solutions.
Prompt Two	Kath said there were 13 ways you could make 50 cents. Is she correct? Why? Why not?		Extend students by prompting them to search for more coin combinations

3.6.2 Lesson Plans B (Cookie Problem), C (Counter Problem) and D (Over and Over Problem)

Lesson Plans B, C and D were planned using Lesson structure-IP.

As mentioned in section 3.5, teachers were not asked to anticipate their students' responses to the problems used in these lessons and whole class prompts were not included. However, a range of suggested prompts designed to be used with individuals were included in these lessons by the researcher (see Appendix B to E).

3.6.3 Teacher Choices

The two teachers each taught two of the lesson plans that the researcher developed in accordance with the intervention (see section 4.1):

- Both teachers were asked to teach Lesson Plan A: Coin Problem (Lesson structure-WCP).
- Both teachers were required to choose between the following for their respective second lessons (Lesson structure-IP):
 - Lesson Plan B: Cookie Problem
 - Lesson Plan C: Counter Problem
 - Lesson Plan D: Over and Over

The teachers were asked if they would like to change anything about the lesson plans such as:

- Would you like to alter anything in these lesson plans?

- Would you like to change the wording of the problems?
- Would you like to add any prompts to the list?

The teachers also had to make choices around the following:

- They could use student recording sheets with or without visuals for the problem in Lesson Plan A: Coin Problem (see Appendix G for examples)
- They could prompt students verbally or by giving students paper strips with prompts written on them

The researcher prepared any materials that the teachers required for their lessons. See Figure 5 for an illustration of the lesson plans the teachers were required to make choices between for their two respective lessons.

3.7 Research Instruments

In this section, the three different research instruments used to collect data are discussed. These include the interviews, lesson observations and student work samples.

3.7.1 Research Instrument 1: Interviews

Each teacher participated in two individual interviews conducted by the researcher in a private office in the school. Interview 1 was undertaken prior to the lessons and Interview 2 was undertaken after the teachers had taught the lessons. The interviews were semi-structured. Birmingham and Wilkinson (2003) explain that semi-structured interviews focus on a predetermined set of questions however they provide participants with an opportunity to guide the direction of the interview. The same questions were planned for both teachers. All interview data was audio recorded and transcribed by the researcher. The teachers were provided with the transcripts to check for accuracy and to suggest any amendments.

3.7.1.1 Interview 1

The questions asked in Interview 1 and their targeted research questions are provided in Table 6.

There were three foci:

- To determine teachers' views on problem-solving (Q3)
- To determine teachers' previous experiences with problem-solving (Q4, Q5 and Q6)
- To introduce the Coin Problem included in Lesson Plan A. This was so that the teachers could anticipate their students' responses to the problem as featured in Lesson structure-WCP and consider adding prompts that may support and extend their students (Q1, Q2)

Table 6. *Interview 1 questions and targeted research questions*

Question	Interview 1 Question	Research Question Targeted
1	If you were to give this kind of problem to your current class, what kind of correct solutions would you expect to see?	RQ1
2	How do you think your students would find their solutions?	RQ2
3	What does problem solving in mathematics mean to you?	RQ1
4	What specific strategies do you use in your problem-solving lessons	RQ1
5	If you were teaching a problem-solving lesson with your class what would it look like?	RQ1, RQ2
6	How would you use a problem like this with your class?	RQ1, RQ2

3.7.1.2 Interview 2

See Table 7 for the questions included in Interview 2.

Interview 2 had three foci:

- To find out teachers' views on their implementation of the lessons (Q1, Q2, Q3, Q4, Q5, Q8)
- To determine teachers' views on how their students responded to the lessons (Q3, Q5, Q6, Q7, Q8, Q9)
- To determine whether the teachers perceive they have benefited from trialling the lessons (Q4, Q10)

Student participants' work samples were referred to throughout Interview 2. This was so that the teachers could make observations about how their students responded to the lessons.

Table 7. *Interview 2 questions and targeted research questions*

Question	Interview 2 Question	Research Question Targeted
1	Can you please tell me how you think the lessons went?	RQ1, RQ2
2	Was there any aspect of the lesson plans that you deviated from while you were teaching?	RQ1, RQ2
3	Were you happy with the period where the students were working on the problem without your support...?	RQ1, RQ2
4	What would you change if you were to teach these lessons again and what would be the impact of any changes?	RQ1, RQ2
5	Which prompts did you find most useful? Why?	RQ1, RQ2
6	Can you please tell me how you think your students responded to the lessons?	RQ1
7	Were there any student work samples that you thought were interesting? Why?	RQ1
8	How did you feel about the sharing at the end of the lesson?	RQ1, RQ2
9	Did you notice any specific ways in which the lessons impacted on your students' problem-solving abilities?	RQ1
10	Is there anything in particular that you gained from trialling the lessons? How is what you have gained likely to impact on your future practice?	RQ2

Each quote from an interview transcript is referenced with a label such as R2002 (see Figure 6). R2002 references the following: R (Rachel), 2 (Interview 2) and 002 (second utterance). Sections of the interview transcripts may be referenced with a range of utterances such as, (R2002 – R2004) rather than (R2002). This is due to the researcher making an utterance in response to a participant answering a question. See Figure 6.

R2002: Um, so I think the second lesson surprisingly went better, um, even though the question was a lot more open ended... so the tricky one. Um I found that they both were really successful. I love how open ended they are. I think that it's achievable for all of the kids. I just think that some of the kids didn't have that resilience.

I2003: /Yep.

R2004: And they didn't have that because, it's because it is so open. They didn't know how to approach it and a lot of them just sort of shut down. Um and the kids that surprised me the most were the ones that normally would be probably, lower academically. They're the ones that got in there and actually did a lot better than maybe the kids that always know the answer to typical questions.

Figure 6. Utterance recorded in an interview transcript (Rachel)

3.7.2 Research Instrument 2: Lesson Observations

Observations of all four of the lessons were used and notes were taken to capture how the teachers and students responded to the intervention in their respective two lessons.

Figure 7 illustrates that this section focuses on the lessons section of the research design. This is where the lesson observation data is collected.

Intervention Design Researcher	Interview 1 and Intervention Discussion Teacher 1 or 2	Lesson 1 Teacher 1 or 2	Lesson 2 Teacher 1 or 2	Interview 2 Teacher 1 or 2
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Figure 7. Research design: Lessons

The two teachers taught the two lessons to their classes in their regular classroom environment. Observational notes were taken by the researcher, who acted as a silent observer. Lesson observations were chosen as an instrument as they provide researchers with the opportunity to see complex social situations directly. Birmingham and Wilkinson (2003) support this view and further argue that through observation, researchers can understand the complexity of social settings more deeply than through stand-alone interviews with participants.

3.7.3 Research Instrument 3: Student Work Samples

Figure 8 illustrates that this section focuses on the lessons section of the research design. This is where the student work samples data is collected.

Intervention Design Researcher	Interview 1 and Intervention Discussion Teacher 1 or 2	Lesson 1 Teacher 1 or 2	Lesson 2 Teacher 1 or 2	Interview 2 Teacher 1 or 2
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Figure 8. Research design: Lessons

Student work samples were collected and photographed at the end of each of the lessons. This data was collected in order to analyse how students recorded their responses to problems. The work samples were available in Interview 2 and teachers were invited to refer to any student work samples of interest when discussing how students responded to the intervention.

3.8 Data Analysis Strategy

The three data sets (i.e., interview transcripts, student work samples, lesson observation notes) were examined using a content analysis strategy. Weber (1990) suggests that a content analysis is undertaken by systematically and objectively making valid inferences by examining texts that represent social communication. To implement a content analysis, the following method of coding was used. The data sets were read three times each to familiarise the researcher with their content. Then a combination of deductive (refer to 3.8.1) and inductive coding (refer to 3.8.2) was undertaken. The features of the intervention that were identified in the problem-solving literature guided the deductive analysis. This was to ensure that key features of the intervention were the focus of the analysis. Inductive coding was also used in order to reduce bias and ensure that new insights that arose from the data were identified. The data sets were revisited multiple times throughout this process in order to check that the findings correctly reflected the data.

3.8.1 Deductive Approach

In order to guide the deductive analysis, the researcher used the features from the intervention that were identified in the problem-solving literature.

To reiterate, these features were (3.5 for more detail):

1. Challenging Problem (Sullivan et. al., 2016)
2. Introduce challenging problem (Sullivan et al., 2016)
3. Struggle time (Ingram et al., 2016)
4. Problem-solving strategy recording (Sullivan et. al., 2016)
5. Problem-solving strategy sharing (Sullivan et. al., 2016)
6. Teachers anticipate student responses (Sullivan et. al., 2016) (only in Lesson structure-WCP)
7. Extending and enabling problem-solving prompts. (Cheeseman et al., 2017)
8. General problem-solving prompts
9. Problem-specific prompts
10. Whole class prompts (only in Lesson structure-WCP)

The data sets were coded based on words, phrases and portions of the data that related to one or more of the features listed above. Figure 9 provides an example of a code highlighted in Beth's transcript from Interview 2 related to prompts. "What if the number was six?" was highlighted. Here, Beth reflected on her use of the prompt "What if the number was six?" and indicated that she used it to reduce the complexity of the Over and Over Problem for a few of her students in BL2.

B2081: And I think I'd sort of done that but somehow I've missed that strip, which I wished I'd had of done. And then I saw the other which was like six. "What if the number was six?" So there was a few that got the easier option which I think was good.

Figure 9. Code relating to prompts: Beth (BL2)

Codes were combined and findings were drawn from these combined codes (see Figure 10 for an example). The literature was revisited multiple times throughout this process in order to ensure that the comparison between the findings and the research were accurate.

Figure 10 provides an example of some codes that were combined from Rachel's interview transcripts relating to struggle (see section 4.2.1 for all of the combined codes relating to struggle). A number of codes were combined that related to the heading, "Teaching strategies Rachel uses to reduce the amount of struggle for students". For example, "modelling an example" was coded because Rachel indicated that she uses the teaching strategy of modelling problem-solving approaches for her students in order to reduce the amount of struggle they experience. This code was combined with another code where Rachel reported using the teaching strategy of prompting her students when the intervention required her students to work on the problem without support.

"So, I was meant to roam without intervening in the first fifteen minutes of their exploration but there were a few kids who I could see were just weren't gonna have a go so I jumped in before I should have," (R2022)

Teaching strategies Rachel uses to reduce the amount of struggle for students

- "Even just discuss with the kids, you know, how many people they could have..." (R2040)
- "Um, so tryna [trying to] create those links between what we already know so they're not quite stumped and stuck on what they think they don't understand and then practice." (R1116)
- "Model an example" (R2040)
- "So, I was meant to roam without intervening in the first fifteen minutes of their exploration but there were a few kids who I could see were just weren't gonna [going to] have a go so I jumped in before I should have." (R2022)
- "This question matches up quite well with, ah the fact that we're doing multiplication... they're using strategies that we're using in class at the moment and it really suits the question" (R2010)

Figure 10. Combined codes relating to struggle: Rachel (Interviews)

3.8.2 Inductive Approach

This part of the content analysis was based on Krippendorff's components of content analysis (Krippendorff, 2004). Krippendorff provides six components that are designed to be used systematically when analysing content. He argues that the components do not have to be used linearly, as certain components may need to be repeated or revisited. Table 8 describes these components and explains how the researcher worked through each component.

Table 8. *Krippendorff's components of content analysis including researcher's procedure*

Krippendorff's content analysis component	Description of Krippendorff's content analysis component	What the researcher did
Unitising	Determining sections of the data sets that are of interest to the analysis	Familiarised self with the content of each of the data sets and documented interesting points (e.g. It appears that both teachers used a range of general and problem-specific prompts in their respective lessons) which related to the research questions
Sampling	Focussing on examples of sections of the data that are of interest and represent a range of other sections	Continued to read sections of the data sets to familiarise self with content and documented interesting points which related to the research questions
Recording coding	Coding relevant words and phrases from the data sets	Found words, phrases and portions of the results that linked to the interesting points that related to the research questions
Reducing data to manageable representations	Summarising data	Linked codes with similar meaning or that related to the same concept
Abductively inferring contextual phenomena	Analysing	Drew inferences to create findings
Narrating the answer to the research question	Recording findings	Recorded the findings in the results

3.9 Ethical Considerations

The methodology of the Case Study was approved by the University of Melbourne's Human Ethics Advisory Group. It met their relevant requirements in relation to ethical considerations for a minimal risk project. It was also approved by the Department of Education and Training.

The principal and the teacher participants were given Plain Language Statements and Consent Forms which were signed and held by the researcher. Teacher participants distributed Plain Language Statements and Consent Forms to parents of potential student participants. These were provided, collected and held by the researcher.

One issue that arose in the process was that a limited number of parents of student participants gave consent to participate in the research. This meant that work samples were only able to be collected from seven students in RL1, RL2, BL1 and eight students in BL2. However, observation of the lessons suggested that the work samples provided by those who consented reflected the range of responses observed in the lessons. Nevertheless, it is acknowledged that the sample size of participants was small which makes generalising difficult. However, as mentioned earlier the Case Study methodology was chosen to gain in-depth rich understandings of the two teachers' implementation of the intervention.

The results from the research are explored in the next chapter.

4 Results: Teacher Reflections

In this chapter, the results of the researcher and teachers' intervention design discussion are presented. Choices the teachers made about the problems, the prompts and the student recording sheet to be used in the lessons are also presented. The results from the interview transcripts (including from the interviews conducted after the teachers implemented the lessons) are then discussed.

4.1 Intervention Design Discussion and Lesson Plan Choices

The lesson plans based on the intervention were provided to teachers (refer to section 3.5.1 and Appendix B to E). Although the two teacher participants were individually invited to make alterations to the lesson structures of the draft intervention prior to the implementation of the lessons, neither of them altered the intervention. As a result, each teacher had Lesson structure WCP and IP, as shown in Appendix A, for the given lessons they taught.

Lesson plans teachers planned to use for their lessons and their corresponding problems

The lesson plans that Rachel and Beth chose for their respective two lessons are displayed in Figure 11. Rachel and Beth both used Lesson Plan A for their first lessons (i.e., for RL1 and BL1 respectively). They chose different lesson plans for their second lessons. Rachel chose Lesson Plan D (i.e., for RL2) and Beth chose Lesson Plan B (i.e., for BL2). For RL2, Rachel slightly changed the wording of the Cookie Problem in Lesson Plan D to include “in our class” (Refer to Table 9). However, it is clear through all of the data that she interpreted “in our class” to be a class comprising of any number of people which is consistent with the original Cookie Problem.

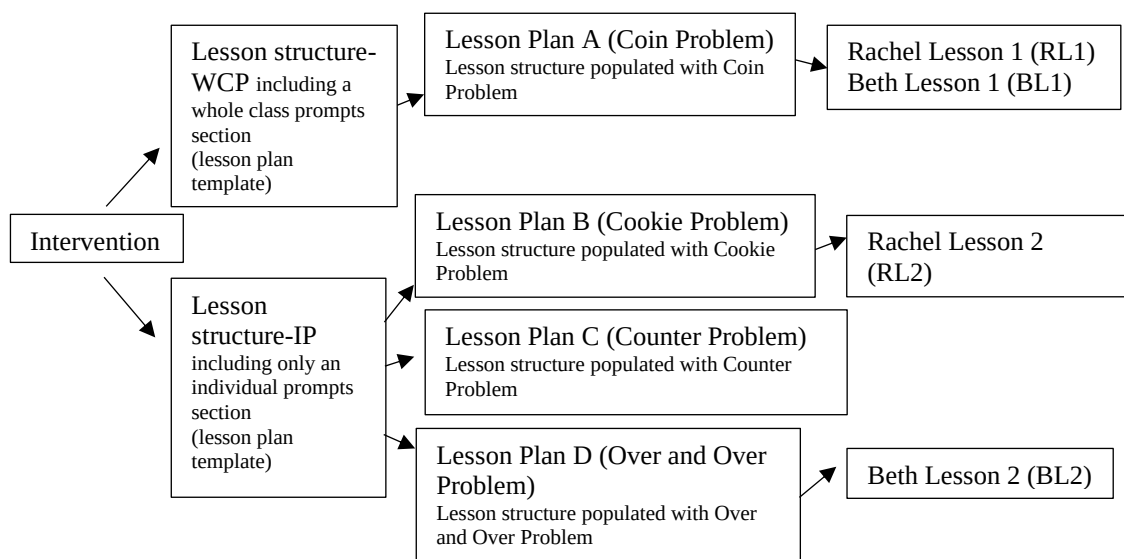


Figure 11. Teachers' lesson choices

Planned approaches for prompts

Rachel planned to prompt her students verbally in both of her lessons (Table 9). She added one additional prompt to Lesson Plan D (RL2) which was, “Could you use pictures of cookies?” Rachel planned to use the student recording sheets containing visuals which was offered as a choice in Lesson Plan A (see Appendix G).

Beth chose to prompt her students with written prompts. She added an additional prompt to Lesson Plan A (BL1) which was, “Could you skip count?”. Beth also planned to use the student recording sheets containing visuals which was offered as a choice in Lesson Plan A (see Appendix G).

Table 9. *Planning to teach the intervention: Teacher choices*

Teacher and Lesson number	Lesson Plan	Problem	Adaptations to problem wording	Planned approach for use of problem-solving prompts	Additional problem-solving prompts added to the lesson	Recording sheet with visuals planned to be provided to students
Rachel Lesson 1	A	Coin Problem: How many ways can you make fifty cents using coins?	No	Verbal prompt to be provided by teacher	No	Yes
Rachel Lesson 2	B	Cookie Problem: Everyone was given the same number of cookies. How many cookies may there have been?	Everyone in <i>our class</i> was given the same number of cookies. How many cookies may there have been?	Verbal prompt to be provided by teacher	Yes “Could you use pictures of cookies?”	N/A
Beth Lesson 1	C	Coin Problem: How many ways can you make fifty cents using coins?	No	Students provided with prompts written on paper strips as needed	Yes “Could you skip count?”	Yes
Beth Lesson 2	D	Over and Over Problem: I added a number over and over until I reached sixteen. Which number may I have started with?	No	Students provided with prompts written on paper strips as needed	No	N/A

4.2 Interviews

From analysis of the interview transcripts (See Appendices D to M), five themes were identified: struggle, challenging problem features, prompts, student share time and problem-solving practice. These themes are discussed in 4.2.1 to 4.2.5 below.

4.2.1 Theme One: Struggle

Theme One focuses on the two teachers' perceptions concerning their students struggling with problems (Appendices H and I). The results were grouped under the following headings, in relation to struggle (see section 3.8.1):

- Negative impact of struggle on students
- Positive impact of struggle on students
- Impact of struggle on lower achieving students
- Teaching strategies teachers use to reduce the amount of struggle for their students
- Impact of the teaching strategies teachers use to reduce the amount of struggle for their students

4.2.1.1 Rachel

Rachel's comments grouped under the main aspects related to struggle are illustrated in Figure 12.

Positive impact of struggle on students

- “All except for the few, the two that struggled, they were so excited. They said, “Can we do it again?” (R2054)
- “I think the fact that..., they had to just push through. Even if they got maybe one, one way of expressing their thoughts out, the fact that it’s so open to interpretation worked quite well.” (R2014)
- “Okay, well I’ll just try [student]” (R2014)

Impact of struggle on lower-achieving students

- “I’m probably gonna [going to] get it wrong or it doesn’t matter if I do” (R2008)

Rachel: Struggle

Negative impact of struggle on students

- “Sort of explaining the question without explaining how to approach it was really hard for me. Especially for those kids who kept going, “I don’t get it, I don’t get it I don’t get”. Which was quite a few of them.” (R2016)
- “I didn’t want to say how many people were in the class and I didn’t want to say that you didn’t have to do the number of people in our class. But some of the kids were looking at me like they had no idea and no clue” (R2018)
- “The only kids that really got any help which wasn’t many of them were the ones that really looked like they were probably going to cry.” (R2014)
- “across both of the lessons some of the kids were consistent of not knowing what to do and couldn’t, just Couldn’t even start” (R2008)
- “because it’s so open, they don’t know how to approach it and a lot of them just sort of shut down” (R2004)
- “I love how open ended they are. I think that it’s achievable for all of the kids. I just think that some of the kids didn’t have that resilience” (R2042)
- “Student three. Ah, her sample... On the first one she completely gave up and refused almost to participate because she didn’t understand, obviously it all. So was a bit tricky because we don’t do maths (money) til next term so it was something... it did it did come from the side and they were, they weren’t really sure what was going on. Some of them. Um, but she is an extension student who I thought would easily be able to do this activity and she scribbled (R2064).
- “I don’t understand it so I’m not gonna do it attitude” (R2042)
- “some of the kids were looking at me like they had no idea and no clue and so tryna push some of the other kids who I could see understood the question to share rather than having me but without picking the kids I knew would have the answer straight away. Tryna [trying to] get some of those kids who were less confident.” (R2018)
- “so, I was meant to roam without intervening in the first fifteen minutes of their exploration but there were a few kids who I could see were just weren’t gonna [going to] have a go so I jumped in before I should have. So, I... because I know two of those kids would not have given it a go. They would’ve just sat there, probably cried and that would’ve just set the tone for the rest of the lesson. They wouldn’t have approached the activity” (R2002).
- “all except for the few, the two that struggled, they were so excited” (R2054).
- “I could see frustration from a lot of kids especially in the first ten minutes” (R2042)

Teaching strategies Rachel uses to reduce the amount of struggle for students

- “Even just discuss with the kids, you know, how many people they could have...” (R2040)
- “Um, so tryna [trying to] create those links between what we already know so they’re not quite stumped and stuck on what they think they don’t understand and then practice.” (R1116)
- “Model an example” (R2040)
- “So, I was meant to roam without intervening in the first fifteen minutes of their exploration but there were a few kids who I could see were just weren’t gonna [going to] have a go so I jumped in before I should have.” (R2022)
- “This question matches up quite well with, ah the fact that we’re doing multiplication... they’re using strategies that we’re using in class at the moment and it really suits the question” (R2010)

Impact of the teaching strategies teachers use to reduce the amount of struggle for their students

- “I think it would be more approachable” (R2042)
- “Help them be a little more positive about it” (R2042)

Figure 12. Statements from Rachel’s interview transcripts relating to the concept of struggle

The following sections discuss each of the aspects in Figure 12 in more detail.

Positive impact of struggle on students

Rachel indicated that most students responded positively to the demands of the problem-solving lessons. She commented on her students' desire to engage with similar problems in the future (e.g. "They said, can we do it again? We wanna [want to] do another one. They were really, really, really engaged. They loved these activities." R2054). In relation to giving students the opportunity to struggle, Rachel commented on both positive and negative impacts that struggle had on some of her students. In relation to positive impacts, Rachel said: "All except for a few, the two that struggled, they were so excited" (R2054). However, as outlined in the next section, Rachel elaborated in some detail about the negative impacts that struggle had on some students.

Negative impact of struggle on students

A number of times throughout Interview 2, Rachel indicated that some of her students responded negatively to the lessons ("I could see frustration from a lot of kids especially in the first ten minutes", R2042 and "because it's so open, they don't know how to approach it and a lot of them just sort of shut down", R2004). She indicated that the students in her extension group found it difficult to persist when struggling with the task ("the kids that are in my extension group, they couldn't figure out how to approach this," R2004). It appears that Rachel believes resilience was a factor that impacted on her students' capacity to struggle with the problems. For example, Rachel noted that Student 3, "completely gave up" (R2064), when the Coin Problem required struggle, even though she thought that this student "would easily be able to do this activity" (R2064) Rachel noted that the topic of money had not been studied and that this may have contributed to why Student Three was unable to persist.

"Um. Student three. Ah, her sample... On the first one she completely gave up and refused almost to participate because she didn't understand, obviously it all. So was a bit tricky because we don't do maths (money) til next term" (R2064-R2066).

However, as discussed in the section, Rachel perceived that the negative impacts of struggle were less for students who find mathematics difficult.

Impact of struggle on lower-achieving students

Rachel perceived that the students who usually find mathematics difficult were not perturbed by having to struggle with the problems in the lessons because they often struggle with mathematics (e.g. “as horrible as it sounds those kids are used to maybe failing or not doing as well,” R2002). Rachel noted that these students think that they are “probably gonna [going to] get it wrong or it doesn’t matter if I [they] do” (R2008), so they tend to persist with problems because they are not expecting to be correct.

Teaching strategies Rachel uses to reduce the amount of struggle for her students and their impact

Rachel identified four teaching strategies that she uses in her teaching practice to reduce the struggle her students experience. These include:

- creating links between what students do and do not know
- providing students with problems that correlate with the current topic being taught
- modelling examples or discussing approaches to problems
- prompting students as soon as they struggle with a problem.

Each strategy is discussed in more detail below.

Rachel reported that she creates links for her students between what they do and do not know to reduce the amount of struggle they experience. In response to the question asking how she would implement the Coin Problem in Lesson Plan One, Rachel indicated that she would “create those links between what we already know” (R1116). She provided examples such as, “remind them that this is skip counting, it’s addition... multiplication” (R1116) which support students to understand the mathematical skills that need to be applied to the problem. She explained that this strategy would ensure that her students are “not quite stumped and stuck on what they think they don’t understand”, (R1116). It appears Rachel believes that if she supports her students to make connections to their prior mathematical knowledge and identify mathematics which will be helpful for solving the problem, their struggle will be reduced.

Rachel deviated from the structure of the lesson in RL2 to prompt her students as soon as they struggled with the Cookie Problem. Rachel intervened in the fifteen-minute

period where the intervention indicated for students to work on the problem without assistance from the teacher. Rachel perceived that some of her students were not going to persist with the problem, so she intervened.

“I was meant to roam without intervening in the first fifteen minutes of their exploration but there were a few kids who I could see were just weren’t gonna [going to] have a go so I jumped in before I should have” (R2022).

In this instance, Rachel chose not to provide time for her students to struggle with the problem.

To reduce the amount of struggle her students experience, Rachel reflected upon a change she would make to the introduction of the Cookie Problem in the future. She would use the strategy of “model(ling) an example” (R2040) or having a “discuss(ion) with the kids” (R2040) during the introduction to the problem. Rachel’s reasoning for changing the lesson introduction was to support her students to enthusiastically get started with the problem.

“[the problem] would be more approachable for some of the kids. I think that was really open ended and I loved it [i.e. the Cookie Problem] but I could see frustration from a lot of kids especially in the first ten minutes. I think that might just help them be a little bit more positive about it. (R2042)”

Rachel indicated awareness of struggle as being an element of problem solving (“they’re meant to be in a state of frustration and confusion,” R2042). However, she preferred to reduce the amount of struggle her students experience as she believes it “would be a good way to engage them in the activity rather than just having that, “I don’t understand it so I’m not gonna [going to] do it attitude” (R2042). Rachel indicated that it is important to foster her students’ resilience “some of the kids didn’t have the resilience” (R2042). One interpretation of these comments is that Rachel perceives that if her students do not experience success immediately then they will not persist.

4.2.1.2 Beth

In this section, Beth’s perceptions associated with her students struggling with problem-solving is discussed. See Figure 13 below for direct quotes from Beth’s transcripts relating to struggle.

Positive impact of struggle on students

- “So, your repeated addition worked in well with the multiplication, but the currency was kind of, a little bit more out there... and I think the kids coped really well with it which was good for me to see.” (B2060 - B2062)
- “They’ve got a real growth mindset but I wanno [want to] maintain that so as much as I love to challenge... And I do challenge them a lot (B1070).
- “I think they really enjoyed it. The kids love a challenge and they’re always asking me, “Can you test me?” So, um, yeah, I think they really... It’s strange, but they seem to really thrive in those conditions. They like, they love a challenge (B2175).
- “And I don’t think anyone sort of did the whole, “I can’t do this.” (B2073)

Impact of struggle on lower-achieving students

- “So, the real challenge, I’ll maintain for the higher group... Rather than frustrating... I think that’s the problem in maths, that a lot of teachers just move onto the next and the next. And this kid, you can see, you can literally see them shrink before your eyes. Losing it. Losing their confidence. Losing their enjoyment. Their will to live (B1072 – B1074).

Beth: Struggle

Negative impact of struggle on students

- “you know something that they will be confident to answer. Just to help bring them back into it so that they haven’t, you haven’t lost them (B1131).
- I think that’s the problem in maths, that a lot of teachers just move onto the next and the next. And this kid, you can see, you can literally see them shrink before your eyes. Losing it. Losing their confidence. Losing their enjoyment. Their will to live (B1074).
- “If I had have let it go to three digit, I just felt like I would’ve shattered the confidence of three quarters of the room to be honest. Not even half. Three quarters of the room woulda just gone, “It’s too hard. I can’t do this.” So, doing the two digit allowed most, if not all to experience success (B1078-B1080).

Strategies Rachel uses to reduce the amount of struggle for students

- “And I think sometimes when you do see that they’re not participating, you sort of set it up so that there’s an easier option or a, you know something that they will be confident to answer” (B1131)
- I didn’t want to give it to them, but I wanted to say it’s the same number and then we’re going to... and I did model on the board something plus something equals sixteen. Something plus something plus something. And I did say that would be the same number plus the same number plus the same number, trying to get them to see that it had to be the same number... Yeah and you wanno [want to] scaffold it but you don’t really wanno [want to] give them the answer or... (B20109-B2111)

Impact of the teaching strategies teachers use to reduce the amount of struggle for their students

- So, doing the two digit allowed most, if not all to experience success Allowed most, if not all to experience success.” (B1080)

Figure 13. Statements from Beth’s interview transcripts relating to the concept of struggle

The following sections address Figure 13 in more detail.

Positive impact of struggle on students

Beth perceived that many of her students excelled when attempting the problems in the lessons, noting that her students “seem to really thrive in those conditions. They like, they love a challenge” (B2175). Beth also noted that her students managed well with the Coin Problem even though it was not a familiar topic.

“the repeated addition worked in well with the multiplication but the currency was kind of, a little bit more out there... and I think the kids coped really well with it which was good for me to see.” (B2060 - B2062).

It appears that Beth was pleased that her students were able to persist with a problem based on a less known content area and perceived that many of her students responded positively to struggle during the lesson.

Negative impact of struggle on students

Beth made three comments related to the negative impact of struggle on her students in Interview 1 (B1078 - B1080, B1131, B1074). These comments related to her perception that if students struggle with a problem and do not know the answer, then their confidence is negatively impacted. Beth did not make any comments in relation to the negative impact of struggle in Interview 2 so it appears that she did not focus on the negative impact of struggle in relation to the impact of the implementation of the lessons.

Impact on lower-achieving students

Although Beth reported that struggle had a positive impact on some students in the lessons, she indicated in Interview 1 that the impact of struggle on lower achieving students was an issue in general. She noted that:

“So, the real challenge, I’ll maintain for the higher group... Rather than frustrating... I think that’s the problem in maths, that a lot of teachers just move onto the next and the next. And this kid, you can see, you can literally see them shrink before your eyes. Losing it. Losing their confidence. Losing their enjoyment. Their will to live (B1072 - B1074).”

It appears that Beth believes that unless her students are in her higher group, she needs to ensure that they do not struggle to the point that they experience frustration and lose confidence. This implies that Beth believes that most of her students should not be given periods of time to struggle that can cause frustration and undermine confidence.

Teaching strategies Beth uses to reduce the amount of struggle for her students and their impact

Beth identified two teaching strategies that she uses to reduce the struggle her students experience. These include:

- providing students with an easier question that they can confidently answer
- modelling an example or providing a scaffold to a problem.

Beth reported that that if her students are not engaging with a task (i.e., “they’re not participating,” B1131), then she maintains their confidence by giving them a question “that they will be confident to answer,” (B1131). It appears that Beth perceives that it is more important for her students to achieve success in finding a correct answer than having to struggle and persist with particular problems in mathematics.

Beth reported that she models approaches to problems to reduce the amount of struggle her students experience. For example, in Interview 2, Beth reflected on her introduction to the Over and Over Problem in BL2 noting that she did not want to provide answers for her students but she wanted to help them approach the problem.

“I didn’t want to give it to them but I wanted to say it’s the same number and then we’re going to... and I did model on the board something plus something equals sixteen. Something plus something plus something” (B2109-B2111).

Beth modelled an approach to the problem to guide her students when she introduced the problem. However, she indicated that “you wanno [i.e. want to] scaffold it but you don’t really wanno [i.e. want to] to give them the answer” (B2011). This comment seems to suggest that she wanted to provide help with strategies and scaffold their problem-solving rather than provide her students with the opportunity to struggle to find their own approach to the problem.

4.2.2 Theme Two: Challenging problem features and assumptions

Theme Two focuses on the two teachers’ perceptions concerning the specific features and assumptions of the challenging problems used in the lessons (Appendices J and K). Each problem had certain mathematical features and assumptions (see section 3.6). Where teachers refer to a mathematical feature in their interview it is assumed that they perceive its relevance to problem-solving (see 4.2.2.1). In addition, if the teachers refer to an assumption behind a problem being relevant, it is assumed that they perceive it as necessary for understanding the problem (see 4.2.2.2). The two teachers’ reflections related to the features and assumptions behind the challenging problems are then discussed.

4.2.2.1 Challenging problem features: Rachel and Beth

The relevant mathematical features of the problems in the lessons that Rachel and Beth did and did not discuss in their interviews are illustrated in Table 10. Both teachers reported on all three of the mathematical features of the Coin Problem used in RL1 and BL1 (see Table 10). As a result, it appears that the two teachers perceived that these features were relevant to the development of their students' problem-solving skills. For RL2, Rachel indicated that there are multiple ways to solve the Cookie Problem in the lesson, however, she did not mention that the problem caters for a range of abilities or that there are an infinite number of solutions depending on the number of cookies. Beth did not report on any of the three challenging features of the Cookie Problem in BL2.

It is important to note that the two teachers were prompted to anticipate their students' responses to the Coin Problem (Lesson structure-WCP) in Interview 1, before they taught the lesson. Therefore, they were prompted to think and discuss the features of the problem for RL1 and BL1 in Interview 1. Teachers were not prompted to anticipate their students' responses to the Cookie Problem and the Over and Over Problem (Lesson structure-IP) used in RL2 and BL2 respectively. This may be an indication of why less features were discussed in relation to the problems used in these lessons. This may have also impacted on the results about assumptions as discussed below.

Table 10. *Challenging problem features teachers did and did not discuss in interviews*

Teacher and Lesson Number	Code	Problem used in lesson	Mathematical features of problem discussed in interviews	Mathematical features of problem NOT discussed in interviews
Rachel Lesson 1	RL1	Coin Problem: How many ways can you make fifty cents using coins?	There are multiple ways to solve the problem: hands on materials, patterns (repetition and breaking down of coins), tables, the four operations The problem caters for a range of abilities as students can make a start or complete the problem and explain its patterns or rules There are a number of ways (13) 50 cents can be made (R1020-R1026) and (R1028)	-
Rachel Lesson 2	RL2	Cookie Problem: Everyone in our class was given the same number of cookies. How many cookies may there have been?	There are multiple ways to solve the problem: hands on materials, diagrams, drawings, pattern (increasing or decreasing the number of cookies each person receives), tables, the four operations. (R2066) and (R2070)	The problem caters for a range of abilities as students can make a start or complete the problem and explain its patterns or rules There are an infinite number of solutions depending on the number of cookies.
Beth Lesson 1	BL1	Coin Problem: How many ways can you make fifty cents using coins?	There are multiple ways to solve the problem: hands on materials, patterns (repetition and breaking down of coins), tables, the four operations The problem caters for a range of abilities as students can make a start or complete the problem and explain its patterns or rules There are a number of	-

			ways (13) 50 cents can be made (B1032-B1036), (B1024), (B1048), (B1050), (B1056), (B2227), (B2235), (B2241) and (B2255)
Beth Lesson 2	BL2	Over and Over Problem: I added a number over and over until I reached sixteen. Which number may I have started with?	There are multiple ways to solve the problem: hands on materials, drawings, pattern factors of sixteen, repetition, tables, the four operations The problem caters for a range of abilities as students can make a start or complete the problem and explain its patterns or rules There are four different numbers that could be repeated to make 16

4.2.2.2 Problem assumptions: Rachel and Beth

Beth and Rachel reported on a limited number of assumptions behind the problems that were used in the lessons. For the Coin Problem, both teachers discussed the importance of their students knowing the coins (see Assumption Two, Table 11). However, neither of the teachers discussed the importance of Assumption One or Three for the Coin Problem. Rachel did not discuss any of the problem assumptions for the Cookie Problem and Beth only discussed the importance of understanding the key words in the Over and Over Problem for her second lesson (see Assumption One, Table 11). It appears that the two teachers were not aware or did not see the significance of the assumptions behind the problems used in the lessons. This can be problematic as assumptions need to be made explicit in order for students to understand what a problem entails.

Table 11. *Assumptions behind the problems used in the lessons that teachers did and did not discuss during interviews*

Teacher and Lesson Number	Code	Problem used in lesson	Assumptions behind problem discussed in interviews	Assumptions behind problem NOT discussed in interviews
Rachel Lesson 1	RL1	Coin Problem: How many ways can you make fifty cents using coins?	2.Students know the coins (50 cent coin, 20 cent coin, 10 cent coin, 5 cent coin) (R1114)	1. Australian coins as currently used 3. The same value coin can be used once or several times
Rachel Lesson 2	RL2	Cookie Problem: Everyone in our class was given the same number of cookies. How many cookies may there have been?		1.Students understand the meaning of the word “same” 2.The Cookies stay whole 3.Students share cookies with the amount of people in their class
Beth Lesson 1	BL1	Coin Problem: How many ways can you make fifty cents using coins?	2.Students know the coins (50 cent coin, 20 cent coin, 10 cent coin, 5 cent coin) (B2026)	1. Australian coins as currently used 3. The same value coin can be used once or several times
Beth Lesson 2	BL2	Over and Over Problem: I added a number over and over until I reached sixteen. Which number may I have started with?	1.Students understand the meaning of the words “same” and “over” (B2099)	2. Answers identified will be single numbers

Beth indicated that she focussed on the words “same” and “over” to reach “16” in her introduction to the Over and Over Problem in BL2 (Assumption One). However, she perceived that some of her students just focussed on the numbers 16 and added a range of number to equal 16.

“We talked about the key words being the “number”, “sixteen”. We talked about repeated... Um, the “same number” so we had that, um, really, really reinforced at the start of the lesson and some students have gone off and not really... They’ve been focussed on one element of that... like the “sixteen” being the important thing but the “repeated” being not so important” (B2099).

It appears that Beth perceived that although she made Assumption One explicit to her students, they misunderstood what the Over and Over Problem entailed and did not repeatedly add the same number to reach 16.

4.2.3 Theme Three: Prompts

Theme Three focuses on the two teachers' perceptions concerning the prompts used in the lessons (refer to 3.8.1). The prompts the teachers reported using in the lessons and teachers' reflections about the prompts used are discussed (Appendices L and M).

4.2.3.1 Prompts teachers reported using in the lessons

The prompts the teachers reported using in the lessons were organised into the four categories that were included in the intervention (Table 12).

Table 12. *Prompts teachers indicated using: all lessons*

Teacher participant	General enabling prompts *prompts not provided on lesson plan	General extending prompts *prompts not provided on lesson plan	Problem-specific enabling prompts *prompts not provided on lesson plan	Problem-specific extending prompts *prompts not provided on lesson plan
Rachel	Could you use manipulatives? (R2044) Could you use pictures? (R2044)	How many different ways could you do it? (R2044) *What could you do to change it? (R2050) How would you do it differently? (R2050) *How could you make this more challenging for yourself? (R2050) *How could you use the same strategy but do it to a different problem? (R2050)	Reducing the size of the numbers: So, what if there were three people? (R2044) How could you change the number of people? (R2044) Could you change the number of cookies that each person gets? (R2044)	Increasing the size of the numbers (R2044)
Beth	Could you use manipulatives? (B2159) Could you draw a picture? (B2143) *Look at the important words in the question “What does it say? (B2067)	“Is there another way?” (B2042)	Reducing the size of the numbers (B2141) Could you halve it? (B2141)	Increasing the size of the numbers (B2075)

The two teachers reported using a range of general and problem-specific enabling and extending prompts in their respective two lessons (see Table 12).

Rachel reported 11 different extending and enabling prompts that she used in her lessons. Seven of these prompts were general extending prompts (R2044; R2048; R2050) and four were problem-specific prompts (R2044). Rachel used both the general and problem-specific prompts. However, she used three more general prompts than problem-specific prompts.

Beth reported on seven of the extending and enabling prompts she used in the lessons (see Table 12). Four of these prompts were general prompts (B2163; B2143) and three were problem-specific prompts (B2142; B2075). Beth used both types of prompts.

Both teachers reported using the following prompts:

- Could you use manipulatives? (B2163, R2044)
- Could you draw a picture? (R2044, B2143)
- Is there another way? (R2044, B2042)
- Reducing the size of the numbers (R2044, B2141)
- Increasing the size of the numbers (R2044, B2075)

Beth reported using one extra prompt that was not listed on the lesson plan (“look at the important words in the question,” B2067).

Rachel reported using three prompts that were not listed on the lesson plan (“What could you do to change it?” R2050; “How could you make this more challenging for yourself?” R2050 and “How could you use the same strategy but do it to a different problem?” R2050).

Neither teacher reported using the whole class prompts in the lessons based on Lesson structure-WCP.

4.2.3.2 Teachers’ reflections on their use of prompts

Rachel

Rachel indicated that the prompts enabled her to support or extend her students’ thinking effectively in the lessons.

“I thought they were fantastic because there were some kids that I didn’t want to lead directly but I didn’t know exactly how to tell them where to go without giving them an answer” (R2012).

It appears that Rachel perceived that the prompts helped her guide her students towards formulating their own approaches without providing them with problem-solving strategies or solutions. Rachel also reported that the general prompts were optimal because they encouraged her students to endure most of the thinking. “I tried to keep it

really general. I tried to keep a lot of my conversation in questions, so it was pushing them to do the thinking” (R2050). It appears that Rachel preferred to use the general prompts more than the problem-specific prompts because they provide a minimal amount of scaffolding to support her students’ thinking.

Rachel reported three different prompts that she found effective in supporting her students and explained why the prompts were effective:

- “Could you change the number of cookies each person gets?” for the Cookie Problem (R2044)
 - Rachel explained that this prompt was effective because it encouraged her students to consider providing people with more than one cookie each (“most people did one cookie per person,” R2044).
- “What if there were three people?” for the Cookie Problem (R2044)
 - Rachel explained that this prompt helped her students understand the problem as they could visualise what the problem entailed. (“I think because it’s for such a small number. It’s easy for them to even picture it mentally” R2044).
- Prompts that increased the total for her students for the Coin Problem (R2046)
 - Rachel commented that these prompts encouraged her students to use a larger range of Australian currency (“a combination of notes and coins would be great...two different forms of money,” R2046).

Rachel prompted her students earlier than the intervention suggested due to perceived resilience issues as mentioned in 4.2.1.1.

Beth

Beth noticed that a student who was prompted to reduce her problem’s total from 16 to six for the Over and Over Problem in BL2, completed the simpler problem and then was able to revisit the original problem. Beth noted that by prompting this student to reduce the total to six it “helped her to realise that she could do more for the sixteen” (B2083). Beth perceived that as a result of prompting her student to find an approach to the problem with a reduced total, the student could then apply the approach to the larger number in the original problem. Upon reflection on her use of the prompts, Beth

indicated that she would give more of her students this total of six as it would “help build the confidence and it would help the students to realise...if they can do the six, then the 16 would seem achievable “(B2133-B2135).

Beth also perceived she could have prompted her students earlier in the lessons when they were struggling with the problems (B2097).

4.2.4 Theme Four: Student share time

Theme Four focuses on the two teachers’ perceptions concerning their students sharing their problem-solving strategies in the lessons (Appendices N and O). Rachel and Beth’s reflections about the share time at the end of the lessons is discussed, followed by a discussion of Rachel’s reflections on the share time she facilitated in the middle of her lessons.

4.2.4.1 Student share time at the end of the lesson

In share time, students share their most effective problem-solving strategies with their peers. Teachers choose students selectively based on the problem-solving strategies they have used in the lesson. Share time is featured in both of the lesson structures included in the intervention as a ten-minute section at the end of the problem solving lesson (see Figure 14 and 15).

Rachel: Student Share Time			
How many cookies?			
Time	Activity	What the students are doing	What the teacher is doing
10 minutes	Introduction of task: Everyone in our class got the same amount of cookies. How many cookies may there have been?		Introducing the task. Ensuring students understand task. <u>Not showing</u> students how to solve the problem. Explaining to students that they need to individually record their <u>problem solving</u> strategies.
15 minutes	Solving the task.	Engaging in discussions, attempting to solve the task and individually recording problem solving strategies.	Roaming, observing and thinking about which groups/students need to be given particular prompts <u>without</u> intervening.
25 minutes			Giving students <u>enabling</u> prompts where necessary. Then, giving students <u>extending</u> prompts where necessary.
			Selecting individual students from groups who have used efficient <u>problem solving</u> strategies and notifying them that they will share their problem solving strategies with the class.
10 minutes	Share time.	Sharers share problem solving strategies. Class engages in discussions.	Ensuring that students are staying on track and speaking about the <u>problem solving</u> strategies they've used.

Reflection

“Um, so the middle of the lesson sharing... I think worked really well in both sessions. Um, so those kids that were doing really well and showing their understanding in a range of ways got to sort of have that success with the class and they got to share it and enjoy it. And then the kids that maybe didn't have as many strategies or many ways to approach it were able to sort of steal ideas from people that were sharing...” (R2080)

Reflections

- “I think the end of, end of the lesson sharing was a bit too short but that's something that happens in my grade. So, um, I think that next time I'd probably give myself more time to go and wander and pick people a bit more selectively,” (R2082).
- “Yeah and I didn't have enough share time. I mean, It's always tricky when... you never have enough share time.” (R2030)

Figure 14. Share time reflections (Rachel)

Beth: Student Share Time			
How many cookies?			
Time	Activity	What the students are doing	What the teacher is doing
10 minutes	Introduction of task: Everyone in our class got the same amount of cookies. How many cookies may there have been?		Introducing the task. Ensuring students understand task. <u>Not showing</u> students how to solve the problem. Explaining to students that they need to individually record their problem-solving strategies.
15 minutes	Solving the task.	Engaging in discussions, attempting to solve the task and individually recording problem solving strategies.	Roaming, observing and thinking about which groups/students need to be given <u>particular prompts without</u> intervening.
25 minutes			Giving students <u>enabling</u> prompts where necessary. Then, giving students <u>extending</u> prompts where necessary. Selecting individual students from groups who have used efficient problem-solving strategies and notifying them that they will share their problem-solving strategies with the class.
10 minutes	Share time.	Sharers share problem solving strategies. Class engages in discussions.	Ensuring that students are staying on track and speaking about the problem-solving strategies they've used.

Reflections

- “Um, I think we discussed it briefly... but not as in-depth as we could of [have]. I guess because these aren’t specifically topics we’re doing at the moment.” (B2054-B2056)
- “Yeah, I wish we’d done it a little bit more. And I think now I might actually share some of this with the students...” (B2319)
- “Discussing. Oh, we didn’t discuss at the end of the first lesson. I do remember that. And I don’t remember why, um, and even in the second lesson coz [because] I still had some students at the table.” (B2044)

Figure 15. Share time reflections (Beth)

Rachel

Rachel perceived that her “end of lesson sharing was a bit too short” (R2082), in both lessons (Figure 14). She suggested that “next time I’d probably give myself more time” (R2082) for this part of the lessons. She also proposed that she could “wander and pick people a bit more selectively” (R2082) to share their problem-solving strategies with their peers in the share time. It appears that Rachel perceived that she could have spent more time facilitating share time in her lessons and chosen students who used effective problem-solving strategies to share.

Beth

Beth perceived that her end of lesson share time was too short in both lessons (“I wish we’d done it a little bit more”, B2054). However, she attributed the short share time to the fact that she was not focussing on the mathematical topics related to the Coin Problem or the Over and Over Problem at the time of the study (Figure 15). “Um, I think we discussed it briefly... but not as in-depth as we could of [have]. I guess

because these aren't specifically topics we're doing at the moment." (B2054-B2056). This implies that Beth prefers her students to engage with and discuss problem-solving strategies that relate to problems based on familiar and current content areas.

4.2.4.2 Student share time in the middle of the lesson

Rachel

Rachel indicated that her students observed their peers' problem-solving strategies that were shared in the middle of the lesson and applied them to the problem in the second half of the lessons (RL1 and RL2). Rachel perceived that it "worked really well in both sessions" (R2080). She suggested that "the kids that maybe didn't have as many strategies or many ways to approach it could sort of steal ideas from people who were sharing" (R2080). It appears that Rachel perceived that the benefit of facilitating share time in the middle of the lesson was that her students who were having difficulty identifying strategies could learn from their peers.

4.2.5 Theme Five: Teachers' Problem-solving practice

Theme Five focuses on the two teachers' perceptions concerning their problem-solving practice (Appendix P and Q). The findings from Rachel's interviews that relate to her problem-solving practice are discussed, before the findings from Beth's interviews.

4.2.5.1 Rachel

Before trialling the intervention, Rachel described problem-solving as "open ended tasks" (R1044), which involve students working on tasks that do not have immediately obvious solutions, before trialling the intervention. Furthermore, she suggested that students are given "an opportunity to explore a range of processes to getting to an answer" (R1044). Rachel indicated the problem-solving skills that she encouraged her students to develop. These included encouraging her students "to use pictures" (R1046), "us(ing) hands on materials" (R1048), encouraging them to "work(ing) in groups" (R1048) and to be "flexible in their thinking" (R1074). In addition, she indicated that she encourages students to understand that mathematical content areas overlap by "apply(ing) knowledge that they have from other areas" (R1074).

After trialling the lessons, Rachel indicated that she would continue planning and implementing lessons based on the intervention, after trialling the lessons. She reported that she had “planned a lesson based on this sort of format” (R2054) and that she’s “gonna [i.e., going to] use this sort of approach probably weekly or fortnightly” (R2084). This implies that Rachel perceives the intervention had a positive impact on her teaching of problem-solving. Rachel provided a reason for her positive response to the intervention.

“I think for a lot of those kids it’s created that enjoyment of maths... whereas that traditional maths of doing sums and equations and, and things where they either know the answer or they don’t...I think this is a great way for my, my lower students to engage and enjoy maths again...”(R2056)

It appears that Rachel perceives that the openness of the problems in the lessons was engaging as there is less focus on finding a correct answer.

Following her involvement in the study, Rachel reported that she had planned a lesson based on the intervention in the content area of multiplication because she was teaching her students about “multiplication at the moment” (R2058). The lesson was based on the problem, “How many ways can you make an array with one total number?” (R2058). One interpretation of this is that Rachel prefers to align problems with content areas she is currently teaching in mathematics to reduce the amount of struggle her students experience. For this lesson she altered the structure of the intervention by prompting her students to alter the problem’s total at the start of the independent work time. This is instead of prompting them only after they have “struggled” with the problem for fifteen minutes as indicated in the intervention. Furthermore, she assigned different totals to different students, giving her “low kids” (R2058) the total of “ten” (R2058), the “middle group” (R2058) the total “thirty” (R2058) and her “top group” (R2060) the total of “fifty or one hundred” (R2060). This was to ensure the students were able to successfully manage the numbers they were working with in the problem (“So, they’re all using the same question. It’s differentiated based on their needs. They can do lots of different strategies or use lots of different materials,” R2062). It appears that Rachel perceived that the intervention had a positive impact on students’ problem-solving abilities, however, she intends to alter it to reduce the amount of struggle students experience.

4.2.5.2 Beth

Prior to trialling the intervention, it appears that Beth perceived that undertaking problem-solving was the same as answering a routine mathematical question, prior to trialling the intervention, noting that “A problem is just a question that we’re needing to answer” (B1062) and that “a problem is not a problem. It just means an algorithm” (B1062). However, Beth discussed a number of problem-solving skills she encourages her students to develop including, encouraging her students to “not give up and just try a different way,” (B1127) and “the importance of reading the key words,” in problems (B1173). Beth also discussed some enabling prompts that she would use to implement the Coin Problem used in BL1. These included, “Oh, have you thought of all the ways?” (B1197), “I wonder how many ways people have?” (B1197) and “what if you mixed up some coins?” (B1201).

In Interview 2 following the lessons, Beth indicated that she had gained a lot from trialling the intervention in the lessons, however, she provided limited examples of how it improved her teaching of problem-solving. She noted that she would use manipulatives and visual representations in her future lessons in response to being asked how trialling the intervention impacted on her problem-solving practice.

“When you’ve got the manipulatives or when you’ve got the students drawing it, they can see it. It’s something concrete it’s not just... a blur of numbers,” (B2341).

It appears that Beth’s involvement in the study has helped her recognise the importance of involving manipulatives and visual representations in problem-solving lessons to support visual learners.

4.2.5.3 Conclusions

Five themes related to the two teachers’ perceptions about the intervention’s impact on their students problem-solving abilities and their problem-solving practice were discussed in this chapter. The results indicate the following:

- Struggle
 - The two teachers often do not perceive that providing time for their students to struggle with problems can positively impact on their students’ problem-solving skills

- The two teachers perceive that they limit the amount of struggle their students experience
- Task features
 - The two teachers perceived that the mathematical features behind the Coin Problem used in RL1 and BL1 were relevant to problem-solving.
 - When the two teachers were prompted to discuss the mathematical features and assumptions behind the problems, more were identified.
- Prompts
 - Both teachers indicated that they used general and problem-specific enabling and extending prompts in their lessons to support and extend their students' thinking
 - Beth perceived that she could have prompted her students earlier on in the lessons she taught
 - Rachel preferred the general prompts over the problem-specific prompts because she could provide a minimal amount of scaffolding to support her students' thinking.
- Student share time
 - The two teachers perceive that their end of lesson share time was too short
 - Rachel perceived that she had success facilitating share time in the middle of both of her lessons. Beth did not facilitate share time in the middle of either of her lessons
- Problem-solving practice
 - The two teachers perceive that their involvement in the study will have a positive impact on their future problem-solving teaching practice
 - Rachel indicated that she will use a modified version of the intervention in the future and Beth indicated that she will incorporate visuals and manipulatives into her problem-solving lessons

In the next section, the results from the student work samples are discussed

5 Results: Student Work Samples

The results from the student work samples are provided in this section. Each student work sample was analysed in relation to four categories:

- Number of correct responses (discussed in section 5.1)
- Recording of problem-solving strategies (discussed in section 5.2)
- Number of strategies used (discussed in section 5.2)
- Other observations that emerged from the data (discussed in section 5.3)

A work sample from Student 6 is provided as an example of how work samples were analysed. Student 6's annotated work sample from BL1 (Coin Problem) is provided in Figure 16 (see Table 13 for the analysis). Student 6 recorded two different problem-solving strategies: *Number sentences* and *Look for a pattern*. Student 6 recorded six correct responses (numbers 1-6) for the original total of 50 cents out of 13 possible responses. Student 6 also recorded three correct responses (numbers 1-3) for the new total of \$1 as a result of teacher prompting. Student 6 recorded a coin worth zero cents and a coin worth 30 cents which are incorrect and imply that an important aspect of the problem was lost.

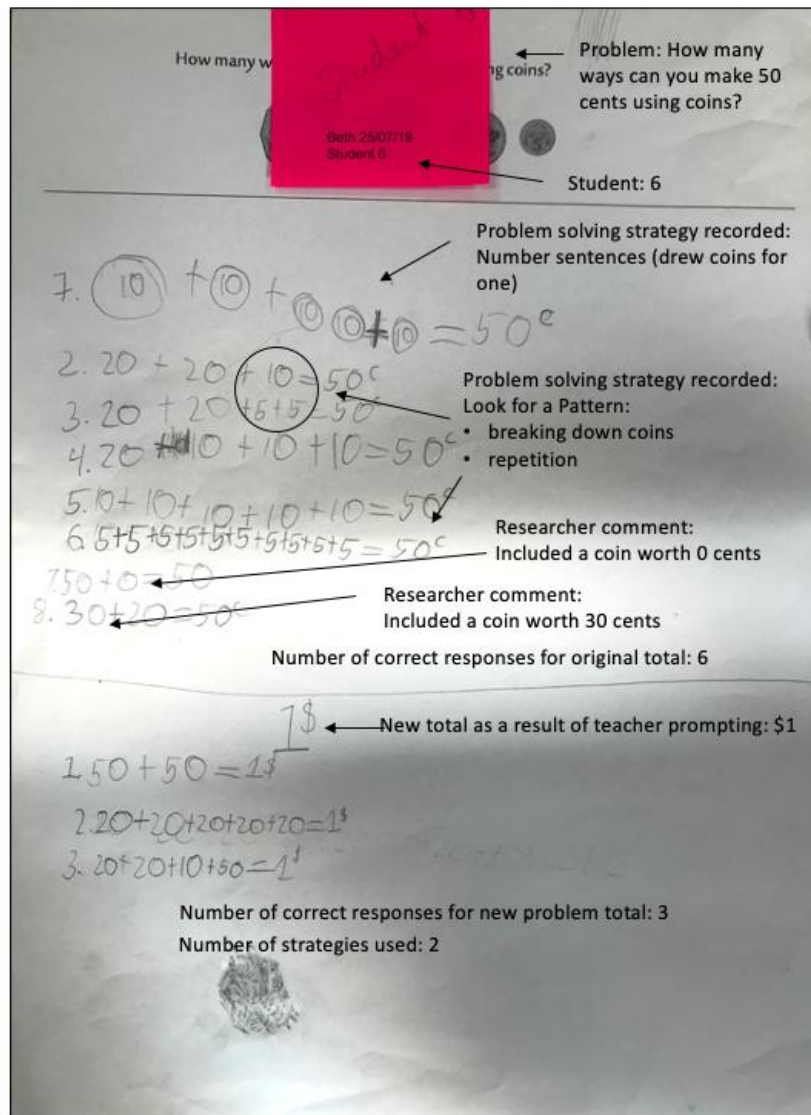


Figure 16. Annotated work sample: Student 6 (BL1)

Table 13. Analysis of Student 6's work sample from BL1

Problem	Student	Problem-solving strategies recorded	Number of strategies used	Number of correct responses for original total (out of 13 possible responses)	New total as a result of teacher prompting	Number of correct responses for new problem total (out of 50 possible responses)	Researcher comments
How many ways can you make 50 cents using coins?	6	Number sentences (drew coins for one) Look for a pattern: breaking down coins And repetition	2	6	\$1	3	Included a coin worth 0. Included a 30 cent coin.

See Appendix R for an example of an annotated work sample and its analyses from each of the other three lessons (RL1, RL2 and BL2). See Appendices S to V for analyses of all of the work samples from each of the four lessons respectively.

5.1 Correct Responses

Student work samples were analysed through identifying the correct responses that emerged in the data. Table 14 to 17 display the number of correct responses recorded by students for all four lessons. The tables include new problem totals students were given as a teacher prompt (if applicable), and the number of correct responses students recorded for these new totals. A student work sample referencing the number of correct response(s) recorded for each respective lesson are displayed in Figure 19 to 21.

In all four lessons, there was only one instance in which a student was unable to record a correct response (see Table 14 to 17). This was Student 4, for the Coin Problem in RL1 (see Table 14). This suggests that most of the students responded positively to the problem's mathematical feature of catering for a range of abilities (see section 3.6). This is because some students made a start to the problem while others were able to identify many correct responses.

In RL2 there was a large range in the number of responses students recorded. In RL2, Students Four, Five and Eight recorded one correct response to the Cookie Problem, whereas Student One recorded 14 responses and Student Three recorded 23 responses (see Table 15). This was the largest range of correct responses recorded by students in a lesson. This also suggests that most of the students responded positively to the problem's mathematical feature of catering for a range of abilities (see section 3.6) because it appears that some students made a start to the problem whereas others identified up to 23 correct responses.

Table 14. *Number of correct responses for Coin Problem recorded by students in RL1*

Student	Number of correct responses for original total of 50c (out of 13 possible responses)	New total for Coin Problem given as a teacher prompt	Number of correct responses for new problem total (out of 50 possible responses)
1	9	N/A	N/A
2	3	N/A	N/A
3	2	N/A	N/A
4	0	N/A	N/A
5	8	N/A	N/A
6	8	N/A	N/A
7	3	N/A	N/A
8	N/A	N/A	N/A

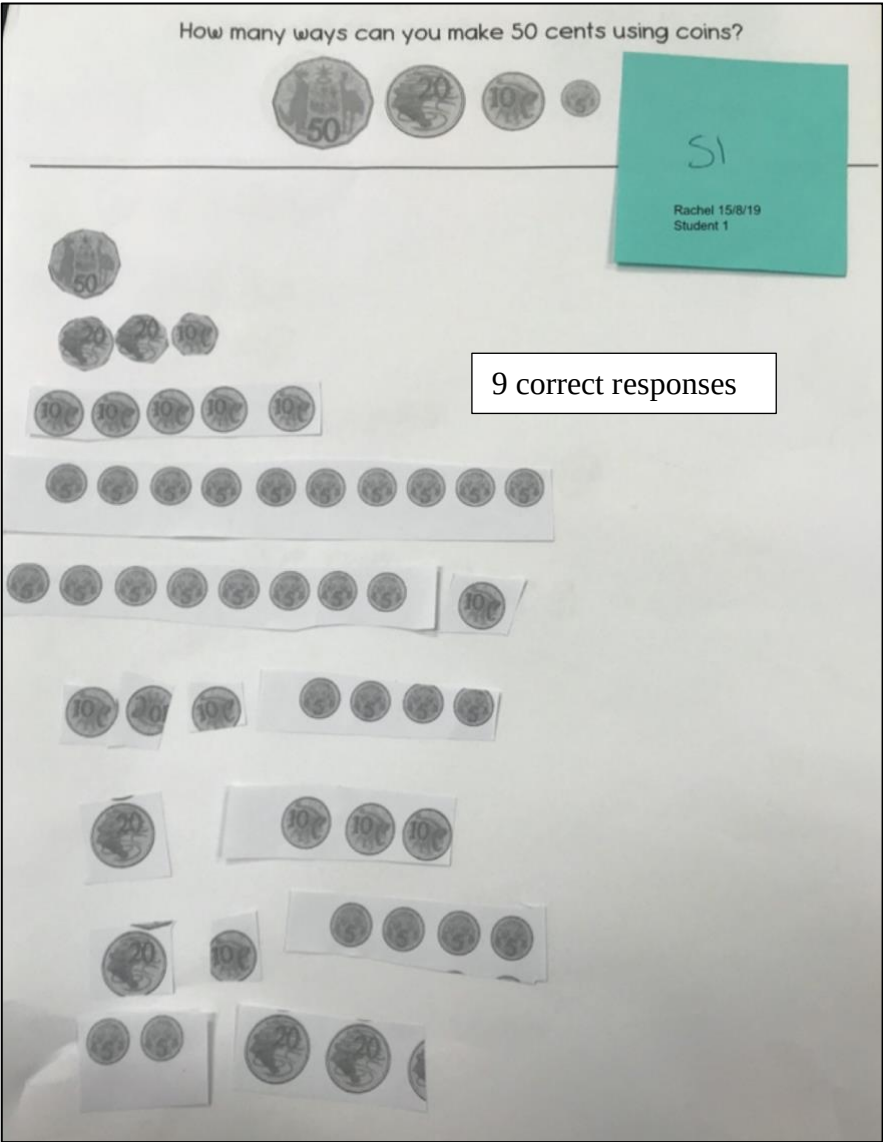


Figure 17. Nine correct responses: Student 1 (RL1)

Table 15. Number of correct responses for Cookie Problem recorded by students in RL2

Student	Number of correct responses for original total (out of infinite possible responses)	New total for Cookie Problem as a result of teacher prompting	Number of correct responses for new problem total
1	14	N/A	N/A
2	2	N/A	N/A
3	23	N/A	N/A
4	1	N/A	N/A
5	1	N/A	N/A
6	4	N/A	N/A
7	2	N/A	N/A
8	1	N/A	N/A

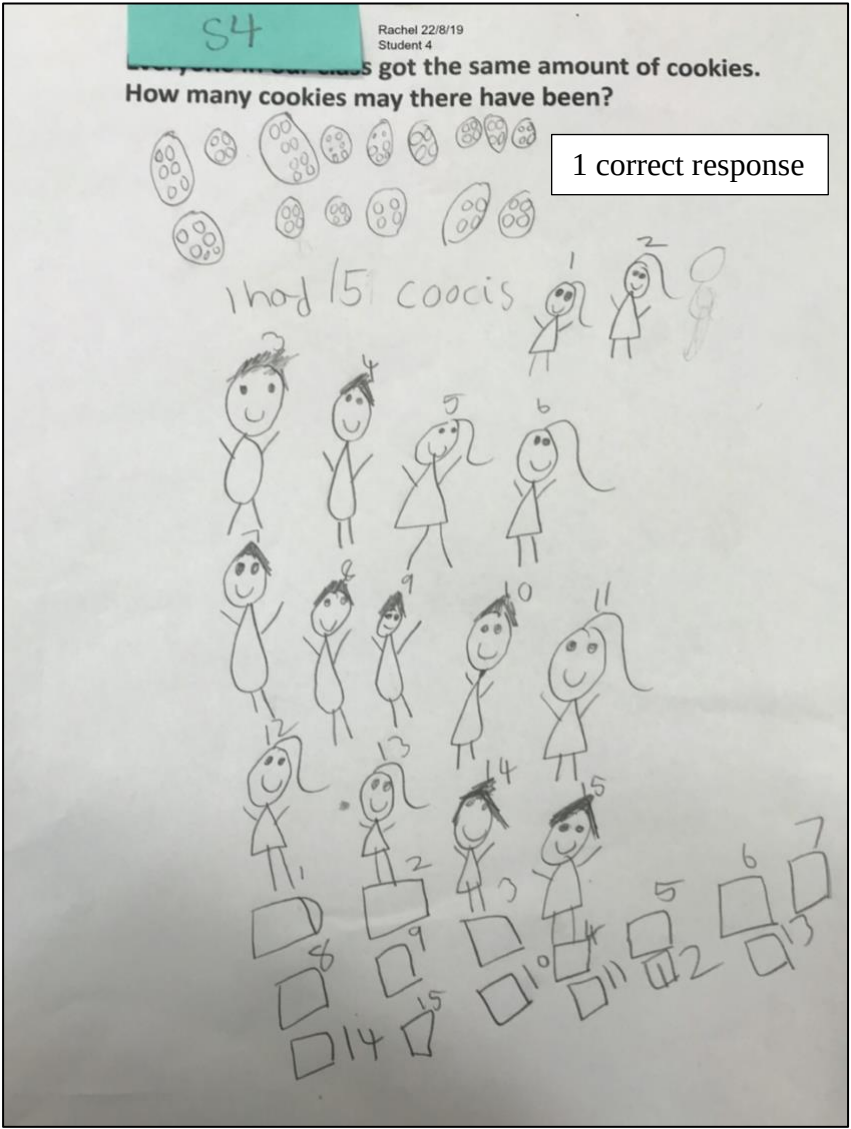


Figure 18. One correct response: Student 4 (RL2)

Table 16. Number correct responses for Coin Problem recorded by students in BL1

Student	Number of correct responses for original total (out of 13 possible responses)	New total as a result of teacher prompting	Number of correct responses for new problem total (out of 50 possible responses)
1	7	\$1	4
2	1	N/A	N/A
3	6	N/A	N/A
4	3	N/A	N/A
5	3	N/A	N/A
6	6	\$1	3
7	3	N/A	N/A

How many ways can you make 50 cents using coins?

Student 1

Beth 25/7/19
Student 1

1. $5^c + 5^c + 5^c + 5^c + 5^c + 5^c + 5^c + 5^c = 50^c$

2. $50^c + 0^c = 50^c$

Incorrect response

3. $10^c + 10^c + 10^c + 10^c + 10^c = 50^c$

4. $20^c + 20^c + 10^c = 50^c$

5. $20^c + 20^c + 5^c + 5^c = 50^c$

6. $10^c + 20^c + 5^c + 5^c + 10^c = 50^c$

7. $10^c + 5^c + 10^c + 5^c + 10^c + 5^c + 5^c = 50^c$

8. $20^c + 10^c + 10^c + 10^c = 50^c$

1. $50^c + 50^c = //$

2. $20^c + 20^c + 20^c + 20^c + 20^c = //$

3. $10^c + 10^c + 10^c + 10^c + 10^c + 10^c + 10^c + 10^c + 10^c + 10^c = //$

4. $5^c + 5^c + 5^c + 5^c + 5^c + 5^c + 5^c + 5^c + 5^c + 5^c + 5^c + 5^c + 5^c + 5^c + 5^c + 5^c + 5^c + 5^c + 5^c + 5^c = //$

Incorrect response

7 correct responses for original total

4 correct responses for new total as a result of teacher prompting

Figure 19. Seven correct responses for original total and four correct responses for new total: Student 1 (BL1)

Table 17. Number of correct responses for Over and Over Problem recorded by students in BL2

Student	Number of correct responses for original total (out of 4 possible responses)	New total for Over and Over Problem as a result of teacher prompting	Number of correct responses for new total
1	1	89	0 (out of 0 possible)
2	1	6	1 (out of 3 possible)
3	4	6	3 (out of 3 possible)
4	1	6	1 (out of 3 possible)
5	2	N/A	N/A
6	1	6	1 (out of 3 possible)
7	1	N/A	N/A

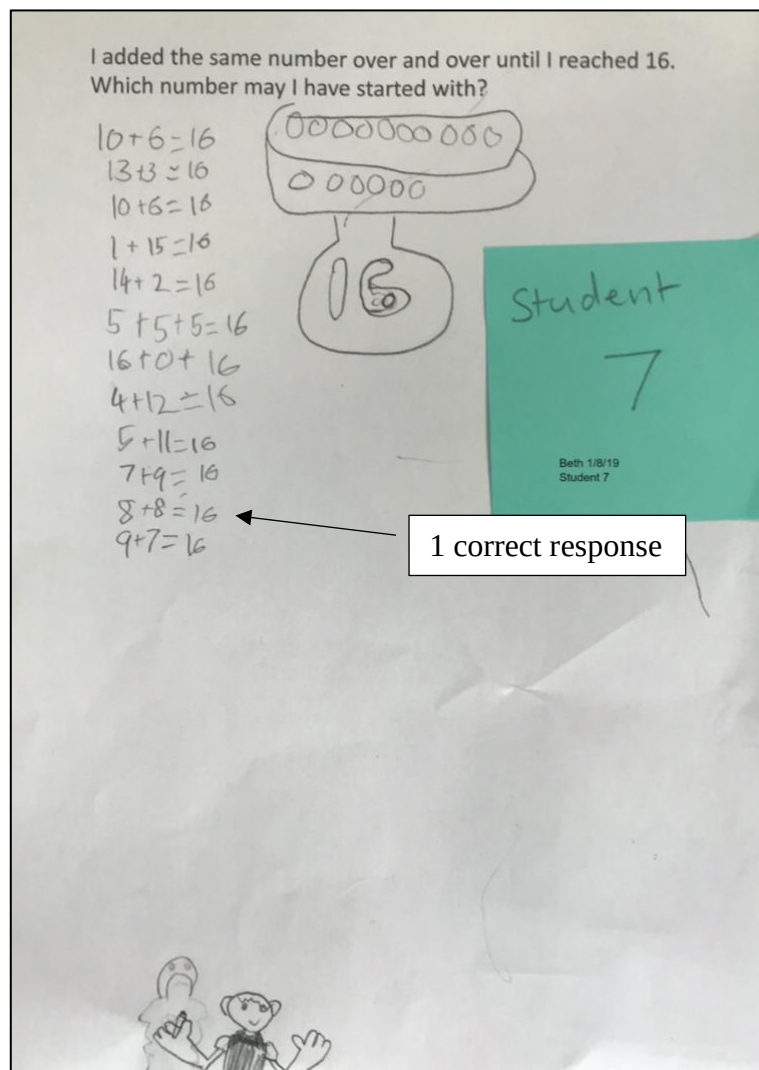


Figure 20. One correct response: Student 7 (BL2)

There were three instances in which students (Student 1, BL1; Student 6, BL1; Student 1, BL2) did not record all of the correct responses for the original problem, but did

record responses for an increased total (example provided in Figure 21). It is inferred that these students were prompted to work with a higher total before they had found all of the responses for the original total. In BL1, Student 1 identified seven and Student 6 (Figure 21) identified six out of the thirteen possible coin combinations for the Coin Problem. However, they recorded responses for the increased total of one dollar (see Table 16). In addition, In BL2, Student 1 identified only one correct response out of four for the initial total in the Over and Over Problem, yet recorded responses for the increased total of 89 (see Table 17). It is not known whether these students could have found more of the responses for the initial problem totals or identified a pattern or rule in the results if they were given the encouragement to do so.

How many ways can you make 50¢ using coins?

Beth 25/07/19
Student 6

7. $\textcircled{10} + \textcircled{10} + \textcircled{10} + \textcircled{10} + \textcircled{10} = 50^c$
 2. $20 + 20 + 10 = 50^c$
 3. $20 + 20 + 5 + 5 = 50^c$
 4. $20 + 10 + 10 + 10 = 50^c$
 5. $10 + 10 + 10 + 10 + 10 = 50^c$
 6. $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 50^c$
 7. $50 + 0 = 50$
 8. $30 + 20 = 50^c$

6 correct responses out of 13 for the original total.

Incorrect responses

1\$

1. $50 + 50 = 1^s$
 2. $20 + 20 + 20 + 20 + 20 = 1^s$
 3. $20 + 20 + 10 + 50 = 1^s$

3 correct responses for new total as a result of teacher prompting

Figure 21. Six correct responses for original total and three correct responses for new total: Student 6 (BL1)

5.2 Recording of Problem-solving Strategies

Student work samples were analysed through identifying the different problem-solving strategies that emerged in the data and the results are summarised in Table 18. Six different problem-solving strategies were identified across the work samples from the four lessons: *draw a picture*, *number sentences*, *look for a pattern*, *write a story*, *highlighting key words* and *working out an answer in more than one way*. This suggests that the students responded positively to one of the mathematical features of the problems which was that they could be solved in a number of ways (see section 3.6 for more information). In RL2, all six of the problem-solving strategies were identified in the work samples. This was the largest range of strategies identified for a problem out of the four lessons (see Table 18). More than one of these strategies were identified in Student 1, 2, 3, 5 and 6's work samples. The most common problem-solving strategy identified in the work samples from all four of the lessons was *number sentences* (19 out of 29). *Number sentences* were identified in all of the work samples from BL1 and BL2 (see Table 18). See Figure 22 to 25 for students work samples displaying each of the problem-solving strategies identified in the work samples from all lessons.

Table 18. *Frequency of problem-solving strategies identified in student work samples from all lessons*

Problem-solving strategies	Frequency of problem-solving strategies identified in RL1 (out of 7 students)	Frequency of problem-solving strategies identified in RL2 (out of 8 students)	Frequency of problem-solving strategies identified in BL1 (out of 7 students)	Frequency of problem-solving strategies identified in BL2 (out of 7 students)	Frequency of problem-solving strategies identified in all 4 lessons (out of 29 work samples)
1. Draw a picture	4 (Student #1, #2, #4, #5)	6 (#2, #3, #4, #6, #7, #8)	-	4 (#1, #2, #6, #7)	14
2. Number sentences	4 (#3, #5, #6, #7)	1 (#5)	7 (All students)	7 (All students)	19
3. Look for a Pattern	6 (#1, #2, #3, #5, #6, #7)	2 (#1, #3)	7 (All students)	-	15
4. Write a story	-	5 (#1, #2, #3, #5, #6)	-	-	5
5. Highlighting key words	-	1 (#5)	-	-	1
6. Working out an answer in more than one way	-	2 (#2, #5)	-	1 (#6)	3

Student 6
Beth 1/8/19
Student 6

I added the same reached 16.
Which number may I have started with? Are there more answers? How do you break it into groups.

I know there is more ways because you draw 16 break it into groups.

array 8 columns
2 arrays

$8+8=16$

$2+14=16$

$7+9=16$

$3+13=16$

$000+0000000000000000=16$

1. Draw a picture
2. Number sentences

Figure 22. Draw a picture and number sentences strategies: Student 6 (BL2)

Student 3

Thursday 25th of July 2014

Make 50 cents using coins?

Both 25/7/14
Student 3

20c 10c 5c

1. $10 + 10 + 10 + 10 + 10 = 50$
2. $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 50$
3. $20 + 20 + 10 = 50$
4. $20 + 10 + 10 + 10 = 50$
5. $50 + 0 = 50$
6. $5 + 5 + 10 + 10 + 20 = 50$
7. $5 + 5 + 5 + 5 + 5 + 20 = 50$
8. $10 + 10 + 10 + 10 + 5 + 5 = 50$
9. $5 + 5 + 5 + 10 + 10 + 10 = 50$
10. $20 + 20 = 50$
- 11.

2. Number sentences

3. Look for a pattern

Figure 23. Number sentences and look for a pattern strategies: Student 3 (BL1)

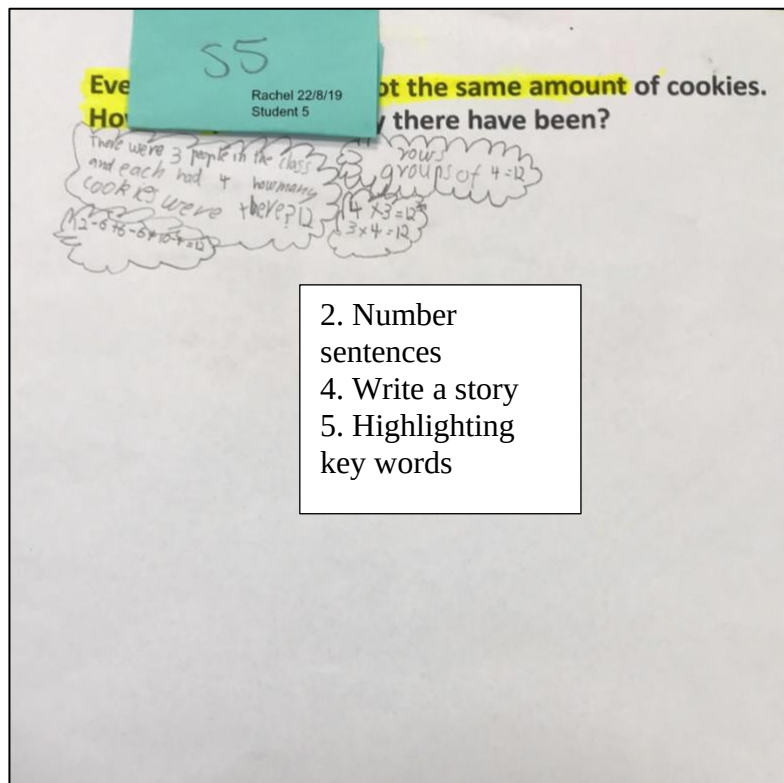


Figure 24. Number sentences, write a story and highlighting key words strategies: Student 5 (RL2)

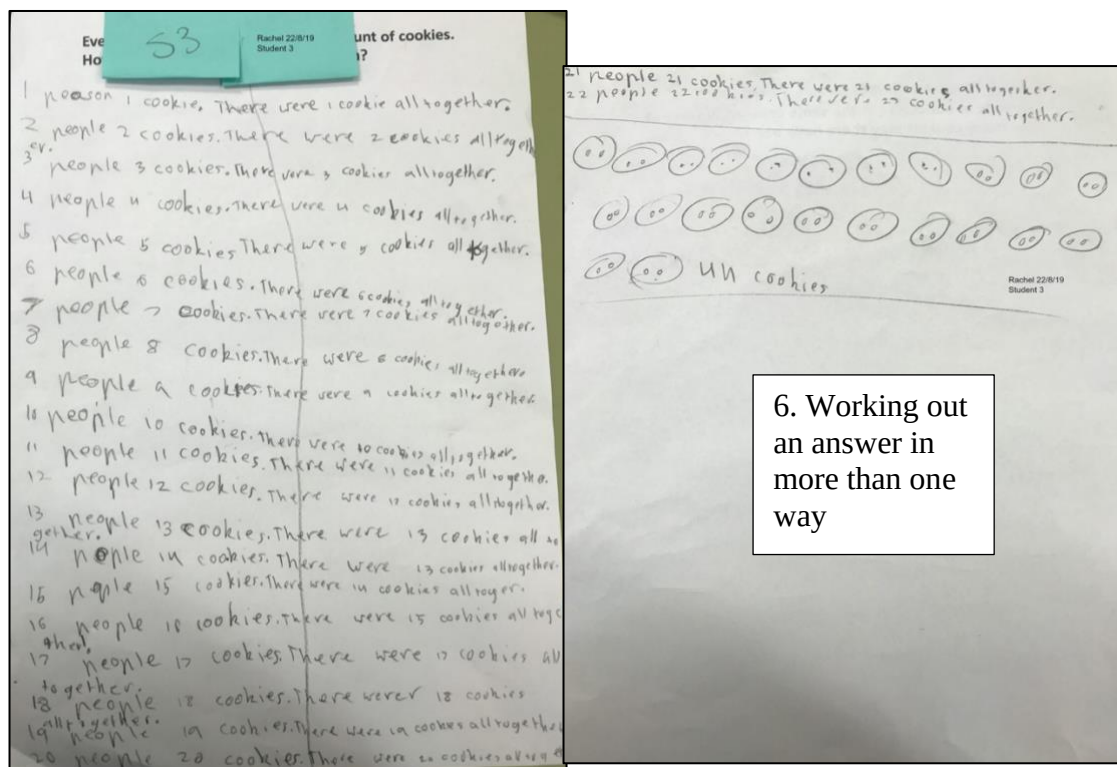


Figure 25. Working out an answer in more than one way strategy: Student 3 (RL2)

The second most common problem-solving strategy identified in the work samples was *Look for a Pattern* (15 out of 29). There were two main categorisations of patterns that were identified in the work samples: *repetition* and *breaking down coins/numbers*. These are displayed in Table 19.

The pattern of *repetition* to record combinations of like coins in the Coin Problem was identified in six out of seven work samples from RL1 and all seven work samples from BL1 (See Figure 26 for an example). The pattern of *repetition* was not identified in any of the work samples from BL2 and RL2.

The pattern of *breaking down coins* in the Coin Problem (see Figure 26 for an example) was identified in Student 1, 3 and 6's work samples (BL1) and Student 1, 3, 5 and 6's work samples (RL1). The recordings on the work samples show that coins were substituted from previous solutions. However, no student work samples indicated that these strategies had been used to identify all of the possible solutions. In addition, the pattern of *increasing or decreasing the number of cookies and people* in the Cookie Problem (see Figure 27 for an example) was identified in Student 1 and Student 3's work samples (RL2). This strategy was not identified in any of the work samples from the Over and Over Problem (BL2).

Table 19. *Frequency of types of patterns identified in student work samples*

Type of Pattern	Frequency of problem-solving strategies identified in RL1 (out of 7 students)	Frequency of problem-solving strategies identified in RL2 (out of 8 students)	Frequency of problem-solving strategies identified in BL1 (out of 7 students)	Frequency of problem-solving strategies identified in BL2 (out of 7 students)	Frequency of problem-solving strategies identified all 4 lessons (out of 29 work samples)
Repetition	6 (#1, #2, #3, #5, #6, #7)	-	7 (All students)	-	13
Breaking down coins/numbers	3 (#1, #3, #6)	2 (#1, #3)	4 (#1, #3, #5, #6)	-	9

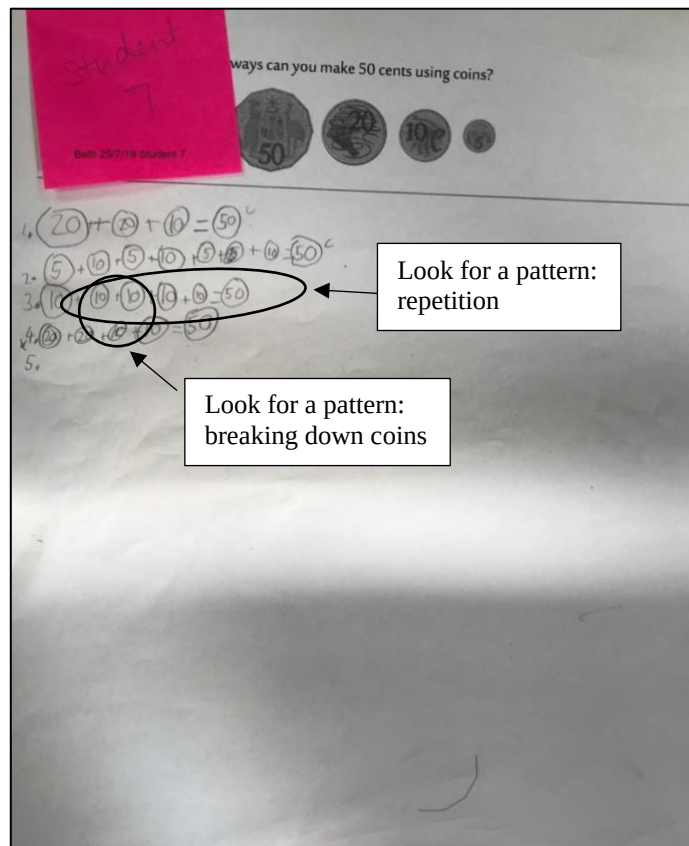


Figure 26. Look for a pattern strategies: Student 7 (BL1)

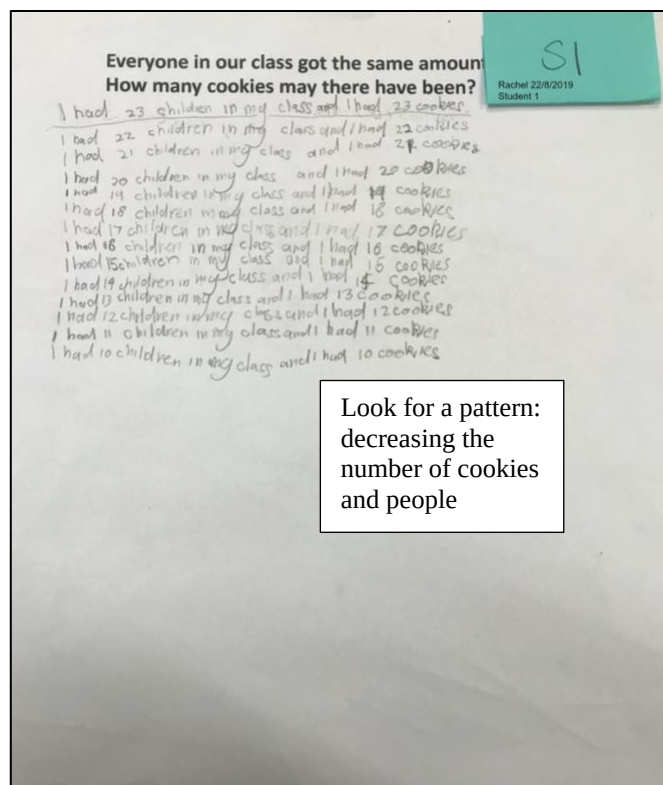


Figure 27. Look for a pattern strategies: Student 1 (RL2)

5.3 Misunderstandings

Seven “Misunderstandings” emerged from the data. The misunderstandings were identified through analysing the student work samples (see Appendix S to V). Table 20 sets out the misunderstandings identified in work samples for each lesson. There was evidence of misunderstandings identified in 15 out of 21 work samples and there were 18 misunderstandings identified in total (see Table 20). The results are discussed in detail below the table.

Table 20. *Problem misunderstandings identified in student work samples for all lessons*

Teacher and Lesson Number	Code	Problem	Students’ misunderstanding	Number of students work samples with each particular misunderstanding	Number of student work samples that included one or more of the misunderstandings
Rachel Lesson 1	RL1	Coin Problem: How many ways can you make 50 cents using coins?	Included coin worth zero cents Included coins that do not exist Included two cent coins Drew coins for no reason	2 (out of 7) (Student #4, #6) 2 (out of 7) (#3, #7) 1 (out of 7) (37) 1 (out of 7) (#4)	4 (out of 7) (Student #3, #4, #6 and #7)
Beth Lesson 1	BL1	Coin Problem: How many ways can you make 50 cents using coins?	Included coin worth zero cents Included coins that do not exist	5 (out of 7) (#1, #3, #4, #5, #6) 1 (out of 7) (#6)	5 (out of 7) (#1, #3, #4, #5, #6)
Beth Lesson 2	BL2	Over and Over Problem: I added the same number over and over until I reached 16. What number may I have started with (BL2)?	Added a range of numbers to equal sixteen	6 (out of 7) (#1, #2, #4, #5, #6, #7)	6 (out of 7) (#1, #2, #4, #5, #6, #7)
In all four lessons		N/A	N/A	18	15 (out of 21)

In RL1, some students recorded coins in their responses that do not exist or that cannot be used to reach a total of fifty cents for the Coin Problem. For example, Student 3

included a three cent coin in one of her responses (see Figure 28). This implies that these students were unaware of current Australian coins (see assumptions for the Coin Problem in 3.6).

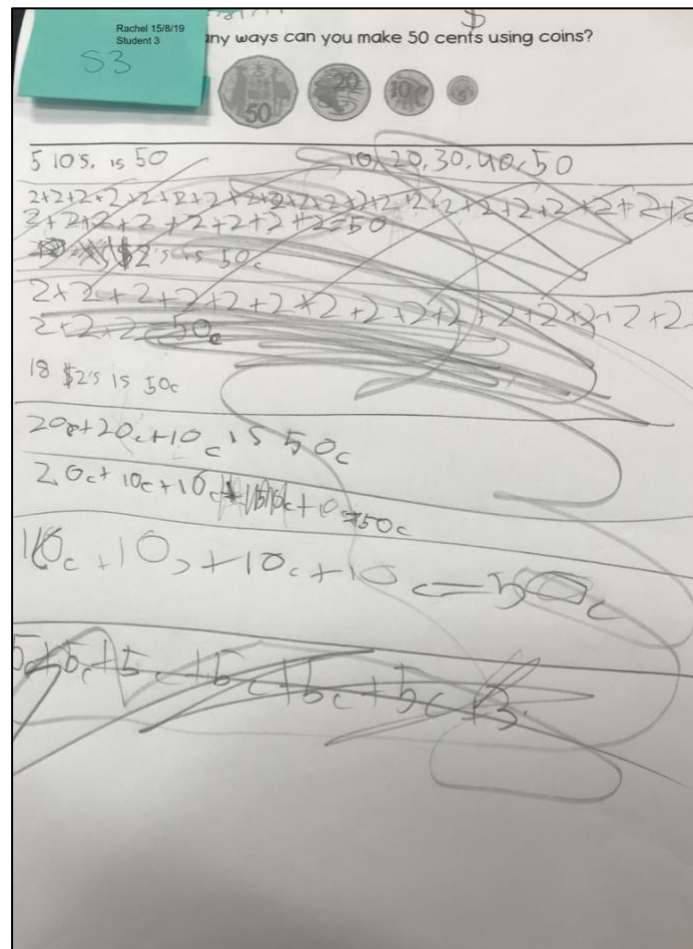


Figure 28. Misunderstanding – included coins that do not exist: Student 3 (RL1)

In BL2, five out of seven students showed that they misunderstood that the Over and Over Problem required them to repeat the same number over and over to reach the total of 16. Rather, the students recorded a range of numbers to equal 16 (Figure 29). This implies that these students did not know that they had to add the same number over and over (see assumptions in 3.6). This is also a major misunderstanding and the goal of the problem was lost.

- Draw a picture
- Number sentences
- Look for a patten
- Write a story
- Highlighting key words
- Working out an answer in more than one way
- The largest range of problem-solving strategies were evident in the student work samples from the Cookie Problem in RL2 (six)
- In BL1 and BL2, number sentences were evident in all of the student work samples
- Misunderstandings
 - Five out of seven students misunderstood the assumption that they were to add the same number over and over to reach 16 in the Over and Over Problem in BL2
 - Nine out of fourteen students included coins that do not exist or that cannot be used to reach 50 cents for the Coin Problem in RL1 and BL1
 - No misunderstandings were identified in the work samples from RL2 (Cookie Problem)

In the next section, the results from the lesson observation data is discussed

6. Results: Lesson Observations

In this section, the findings from the lesson observations are discussed. These findings are related to problem-solving strategies, prompts and deviations from the intervention. See Appendix W for the lesson observation notes.

6.1 Problem-solving Strategies

Problem-solving strategies that the researcher observed being applied and being discussed by students during the lessons were identified in the lesson observation notes. Table 21 to 24 summarise the resultant data of the problem-solving strategies from all lessons.

Six different problem-solving strategies were identified across the observations from the four lessons: *draw a picture*, *number sentences*, *look for a pattern*, *write a story*, *make a list*, *concrete materials* (see Tables 21 to 24). Students were observed using three different problem-solving strategies in RL1, six in RL2, three in BL1 and two in BL2 (see Tables 21 to 24). These results suggest that students responded positively to one of the key mathematical features of the problems used in the lessons, namely, that the problems can be solved in a variety of ways (see section 3.6 for more detail about the mathematical features of the problems used in the lessons).

Six problem-solving strategies were observed in RL2 (Cookie Problem). This was the largest number out of all of the lessons (see Table 22).

In both of Beth's lessons, students were observed recording *number sentences* and *looking for patterns* (see Tables 23 and 24). However, students were also observed using *concrete materials* in BL1.

Table 21. *Problem-solving strategies used by students, as observed in RL1 (Coin Problem)*

Problem	Problem-solving strategy	Observations of students using problem-solving strategies in RL1
Coin Problem: How many ways can you make 50 cents using coins?	Draw a Picture	Pictures of coins to represent different possibilities Drawings of coins to represent different possibilities Tracings of coins to represent different possibilities
	Look for a pattern	Skip counted to find possibilities
	Number sentences	Written number sentences of coin values added to reach the total

Table 22. *Problem-solving strategies used by students, as observed in RL2 (Cookie Problem)*

Problem	Problem-solving strategy	Observations of students using problem-solving strategies in RL2
Cookie Problem: Everyone in our class got the same amount of cookies. How many cookies may there have been?	Draw a picture	Drawings of people and cookies to represent different possibilities Drawings of arrays or groups to represent people and cookies for different possibilities Drawings of diagrams of the classroom with people, cookies and “keys” to represent different possibilities
	Number sentences	Written number sentences to display an answer
	Make a list	Lists of responses where the amount of people and cookies are increasing by one each time
	Write a story	Stories about cookies being shared between a number of people
	Look for a pattern	The amount of people and cookies are increasing by one each time
	Concrete materials	Counters used to represent different possibilities

Table 23. *Problem-solving strategies used by students, as observed in BL1 (Coin Problem)*

Problem	Problem-solving strategy	Observations of students using problem-solving strategies in BL1
Coin Problem: How many ways can you make 50 cents using coins?	Numbers sentences	Written number sentences of coin values added to reach the total
	Concrete materials	Coins used to represent different possibilities
	Look for a pattern	Skip counted to find possibilities

Table 24. *Problem-solving strategies used by students, as observed in BL2 (Over and Over Problem)*

Problem	Problem-solving strategy	Observations of students using problem-solving strategies in BL2
Over and Over Problem: I added the same number over and over until I reached 16. What number may I have started with?	Numbers sentences	Written number sentences to represent possibilities
	Look for a pattern	Repeated the same number over and over to reach the total

6.2 General and problem-specific prompts

The lessons based on the intervention included both general and problem-specific problem-solving prompts included (see section 3.6). Teachers could choose from a range of suggested prompts and use any other prompts throughout their lessons. Rachel planned to prompt her students verbally and Beth planned to provide her students with prompt strips, prior to trialling the lessons. The lesson that was planned by populating Lesson structure-WCP (Coin Problem) included whole class prompts for the teachers to use in the lesson. In the lesson observation notes, the researcher recorded utterances or descriptions of prompting in the lessons. The prompts were then organised into the four categories of problem-solving prompt:

- General enabling prompts
- General extending prompts

- Problem-specific enabling prompts
- Problem-specific extending prompts

The results were identified in the lesson observation data through classifying the different references to prompts. Table 25 to 28 summarise the resultant data for prompts used in all lessons.

In all four lessons the two teachers were observed using prompts from all four categories (See Tables 25 to 28). The most frequently used prompts used in all of the lessons were the general extending prompts. Rachel's use of the prompt "try to solve the problem another way" was the most frequently used prompt out of all of her lessons. It was used nine times in RL1 and eight times in RL2 (see Table 25 and 26). The most frequently used prompt by Beth was the general prompt, "Are there any more ways/answers?" (see Table 27 and 28). It was used six times in BL1 and four times in BL2. However, Beth also used the general enabling prompt, "could you draw a picture" six times in BL2 (see Table 28).

In RL2 (Cookie Problem), Rachel, was observed prompting with general prompts (26 times) more frequently than problem-specific prompts (9 times). The prompt Rachel used in this lesson most frequently was used eight times (Is there another way?). Therefore, it appears that Rachel preferred using general prompts more than the problem-specific prompts in RL2.

In BL2 (Over and Over Problem), Beth, was observed prompting her students with 19 enabling prompts and only seven extending prompts (see Table 28). She did, however, use the same enabling prompt five times (Could you draw an array?). This was the most frequently used problem-specific prompt out of all of the lessons.

The two teachers used many of the prompts that were included in the lesson plans. Rachel used prompts from the lesson plans 15 times in RL1 and 21 times in RL2. Beth used prompts from the lesson plans 13 times in BL1 and 22 times in BL2.

The two teachers also introduced extra prompts to the lessons as they were teaching. Rachel used extra prompts nine times in RL1 and 14 times in RL2. Beth used extra prompts six times in BL1 and four times in BL2.

Table 25: *General and problem-specific prompts identified in RL1 (Coin Problem) lesson observation data*

General enabling prompts *prompts that were not included in the lesson plan	General extending prompts *prompts that were not included in the lesson plan	Problem-specific enabling prompts *prompts that were not included in the lesson plan	Problem-specific extending prompts *prompts that were not included in the lesson plan
*Reminding students of the question (1 time)	“Could you try it another way?” (10 times)	*“Can you skip count?” (1 time)	“Try mixing coins together” (1 time)
*Reminded about reading the key words (2 times)		*“Could you use a 100s chart?” (1 time)	Prompt on board: If you make 50 cents as many ways as possible, try \$5. (1 time)
		*“Could you count by another number?” (1 time)	
		*“Can you add those for me?” (1 time)	
		*“Do we have 90 cent coins?” (1 time)	
		“Use the same strategy with a different coin.” (1 time)	
		“That’s \$50. Try and use your strategy to make 50 cents.” (1 time)	
		*“Let’s count on from 10.” (1 time)	
		“Could you use 10s?” (1 time)	
		“Use the coins.” (1 time)	
3 times in total	10 times in total	10 times in total	2 times in total
	13 times in total		12 times in total

Table 26. *General and problem-specific prompts identified in RL2 (Cookie Problem) lesson observation data*

General enabling prompts *prompts that were not included in the lesson plan	General extending prompts *prompts that were not included in the lesson plan	Problem-specific enabling prompts *prompts that were not included in the lesson plan	Problem-specific extending prompts *prompts that were not included in the lesson plan
*“Read the question again. What is it asking you?” (2 times)	“Can you show one of your answers in another way?” (1 time)	Reminded about using picture cut outs. (1 time)	Teacher asked if anyone had given people a different number of cookies? (1 time)
*“What should you do first?” (1 time)	*“What could you change?” (2 times)	*“How many people in the class?” (1 time)	“Could you change the number of people?” (1 time)
“Could you make it simpler?” (1 time)	“Could you show your thinking another way?” (2 times)	“What if there were 3 people?” (1 time)	*Teacher told students to go back to their tables after share time and find another way. “Think about the strategies people showed and use one that might work for you.” (1 time)
*“You’ve done the first part of the question. What next?” (1 time)	“How could you explain your thinking differently?” (1 time)	*“How many people are there?” (1 time)	
*“Let’s look at the important words” (1 time)	“Is there another way?” (8 times)	*“How many cookies are there?” (1 time)	
*Directed to strategy poster (3 times)	“How could you find another answer?” (1 time)	*“How many do they each get?” (1 time)	
“How could you show your thinking?” (2 times)			
11 times in total	15 times in total	6 times in total	3 times in total
	26 times in total		9 times in total

Table 27. *General and problem-specific prompts identified in BL1 (Coin Problem) lesson observation data*

General enabling prompts *prompts that were not included in the lesson plan	General extending prompts *prompts that were not included in the lesson plan	Problem-specific enabling prompts *prompts that were not included in the lesson plan	Problem-specific extending prompts *prompts that were not included in the lesson plan
*Teacher underlined key words in question. (1 time)	“Are there any more ways?” (6 times)	*“Don’t get gold coins.” (1 time)	“How many ways can you make \$1?” (3 times)
*“Mmm think a bit more.” (1 time)		Teacher handed out coins. (1 time)	
*“Did you finish that one?” (1 time)		*“Do you need 6 10s?” (1 time)	
		*Asked a girl who was making 20c to make 50c (1 time)	
		“What could you count by?” (1 time)	
		“Do you know how to skip count?” (2 times)	
3 times in total	6 times in total	7 times in total	3 times in total
	9 times in total		10 times in total

Table 28. *General and problem-specific prompts identified in BL2 (Over and Over Problem) lesson observation data*

General enabling prompts *prompts that were not included in the lesson plan	General extending prompts *prompts that were not included in the lesson plan	Problem-specific enabling prompts *prompts that were not included in the lesson plan	Problem-specific extending prompts *prompts that were not included in the lesson plan
*“Mmm think a bit more.” (1 time)	“Are there more answers?” (4 times)	“Could you draw an array?” (5 times)	“What if the number was 89?” (3 times)
*“Did you finish that one?” (1 time)		“How could halving help?” (1 time)	
“Could you draw a picture?” (6 times)		“What if the number you reached was 6?” (3 times)	
*Reminded about reading the key words (2 times)			
10 times in total	4 times in total	9 times in total	3 times in total
	14 times in total		12 times in total

Neither teacher was observed using the whole class prompts in the Coin Problem lesson based on Lesson structure-WCP which includes whole class prompts (BL1 or RL1).

6.3 Lesson Deviations

Deviations that the teachers made were points in the lessons where the teachers did something different to what the intervention indicated. Deviations were identified in the lesson observation data by comparing the intervention with what each teacher did in their respective lessons. There were also instances where deviations made by teachers were not referenced in the lesson observation notes. This occurred where the deviations involved a teacher omitted doing something that was indicated in the lesson plan. For example, when the whole class prompts were not implemented in the Coin Lesson (Lesson structure-WCP). Appendix W provides a comparison between the intervention and what the teachers did in each of the lessons. Table 29 provides relevant intervention instructions that were provided for the teacher in the lessons, the teacher’s deviation from the intervention and the potential impact of the deviation.

Table 29. *Summary of the deviations made by the two teachers in all lessons*

Lesson	Intervention instructions for teacher	Deviation from intervention	Impact of deviation
RL1	Ensuring students understand task. <u>Not showing</u> students how to solve the problem	Did not ensure that the assumptions behind the problem were explicit	Students may have developed misunderstandings about the goal of the problem
	Roaming, observing and thinking about which groups/students need to be given prompts <u>without</u> intervening	Prompted students	Students were not provided with time to struggle with the problem
	Introducing whole class prompt 1 and prompt 2 to class	Did not introduce the whole class prompts	The whole class missed two opportunities to be prompted
	Selecting students who have using efficient problem-solving strategies and notifying them that they will share their problem-solving strategies with the class	Did not select specific students for share time	Students were chosen to share at random and not based on whether they had used efficient problem-solving strategies
RL2	Ensuring students understand task. <u>Not showing</u> students how to solve the problem	Added the words “in our class” to the problem, however, asked students to make up the class size which was consistent with the original problem	Students may have been initially confused about the goal of the problem
	Roaming, observing and thinking about which groups/students need to be given prompts <u>without</u> intervening	Prompted students	Students were not provided with time to struggle with the problem
	Giving students enabling and extending prompts where necessary	Facilitated share time in the middle of the lesson	Students were given time to apply their peers’ strategies to the problem in the second half of the lesson
	Selecting students who have using efficient problem-solving strategies and notifying them that they will share their problem-solving strategies with the class	Did not select specific students for share time	Students were chosen to share at random and not based on whether they had used efficient problem-solving strategies

BL1	Ensuring students understand task. <u>Not showing</u> students how to solve the problem	Did not ensure that the assumptions behind the problem were explicit. Modelled an approach to the problem	Students may have developed misunderstandings about the goal of the problem Students may have copied the teacher's modelled approach
	Roaming, observing and thinking about which groups/students need to be given prompts <u>without</u> intervening	Prompted students	Students were not provided with time to struggle with the problem
	Introducing whole class prompt 1 and prompt 2 to class	Did not introduce the whole class prompts	The whole class missed two opportunities to be prompted
	Selecting students who have using efficient problem-solving strategies and notifying them that they will share their problem-solving strategies with the class	Did not select specific students for share time	Students were chosen to share at random and not based on whether they had used efficient problem-solving strategies
BL2	Ensuring students understand task. <u>Not showing</u> students how to solve the problem	Did not ensure that the assumptions behind the problem were explicit. Modelled an approach to the problem	Students may have developed misunderstandings about the goal of the problem Students may have copied the teacher's modelled approach
	Roaming, observing and thinking about which groups/students need to be given prompts <u>without</u> intervening	Prompted students	Students were not provided with time to struggle with the problem
	Selecting students who have using efficient problem-solving strategies and notifying them that they will share their problem-solving strategies with the class	Did not select specific students for share time	Students were chosen to share at random and not based on whether they had used efficient problem-solving strategies
	Ensuring that students are staying on track and speaking about the problem-solving strategies they have used	Encouraged some students to work through share time	Some students did not discuss and determine problem-solving strategies that were efficient

Each teacher deviated from the intervention in both of their lessons (Table 29).

In all four lessons, the two teachers deviated in the introductions to the problems (see section 6.3.1 for more information) and by prompting their students during the time where the intervention indicated not to intervene (see Table 29). They also did not choose specific students to share their strategies in the share time of their lessons. As mentioned in section 6.2 the whole-class prompts were not used in RL1 and BL1 (Lesson structure-WCP).

Rachel deviated from the intervention in RL2 (Cookie Problem) by facilitating share time in the middle of the lesson. At the end of the middle of the lesson share time, Rachel encouraged students to apply peers' strategies to try to solve the problem (See

Figure

Rachel: "If you're stuck please stay on the floor. If not go back to your table and find another way. Think about the strategies people shared and use one that might work for you."
"What if there were 3 people? How could you show your thinking?"
Student: "Draw"
Rachel: "How else could you show it? Another?"
Researcher: "Lots of people at tables tried different ways of working it out".

Figure 30. Students encouraged to apply peers' strategies after middle of the lesson share time (RL2).

It appeared that students followed this instruction and applied different problem-solving strategies to the problem.

Beth deviated in BL2 (Over and Over Problem) by allowing her students to continue working through the share time period of the lesson. So, her students did not have the opportunity to learn from their peers.

6.3.1 Introducing the problem

Both teachers deviated in the section of the intervention where they introduced the problems. In this section of the intervention, teachers were required to ensure that their students understood the problem but refrain from modelling possible approaches to the problems. To ensure that students understood the problem, there were a range of assumptions behind each of the problems that the teacher needed to make explicit

(section 3.6). A deviation from the structure of the introduction to the lessons is considered as a point in which a teacher either did not make one of the assumptions explicit to the students or where a teacher modelled an approach to a problem. Table 30 provides relevant intervention instructions that were provided for the teacher in the lessons, the teacher's deviation from the intervention and the potential impact of the deviation.

Table 30. *Deviations to intervention when introducing the problem in all four lessons*

Lesson	Intervention instructions for teacher	Problem used in lesson	Deviation from intervention	Impact
RL1	Ensuring students understand task. <u>Not showing</u> students how to solve the problem	Coin Problem	Students were not told that the same value coin could be used once or several times (Assumption 3)	Limits number of responses as students may think they can only use each coin once
RL2		Cookie Problem	Students were not told that their responses would include whole cookies (Assumption 2) Students were told to choose their own class size which contradicted with the wording in the problem	Responses may include parts of cookies Students may be confused about the meaning of "class size" in the problem
BL1		Coin Problem	Modelled or shared an approach to the problem Students were not reminded of current Australian coins (Assumption 1) Students were not told that the same value coin could be used once or several times (Assumption 3)	Students may adopt the modelled problem-solving strategy and not devise their own strategy Students may use coins that do not exist Limits number of responses as students may think they can only use each coin once
BL2		Over and Over Problem	Modelled or shared an approach to the problem Students were not told that their responses would be single numbers (Assumption 2)	Students may adopt the modelled problem-solving strategy and not devise their own strategy. Students may represent the total as their answer, rather than the repeated number

Both teachers deviated by either not making assumptions clear and/or modelling approaches to the problems in all of the introductions to their lessons (Table 30).

In BL1, Beth did not make Assumption One (i.e. use Australian coins as currently used) behind the Coin Problem explicit. As a result, her students may have misunderstood which coins they could and could not use in their responses. Neither teacher made it explicit to their students that coin values could be used more than once in a coin combination in the Coin Problem (Assumption Three). This could limit the amount of responses for the problem.

Beth deviated in BL1 and BL2 by modelling approaches to the problems when the intervention indicated not to show students an example or give them an approach. Figure 31 shows the approach to the Coin Problem that Beth drew on her whiteboard in BL1. Beth referred to the list of number sentences when explaining the Coin Problem. Notably, one of the number sentences Beth modelled ($___ + ___ = 50c$) was not able to be completed as no two coins add to make fifty cents.

1.	$______ + ______ = 50c$
2.	$______ + ______ + ______ = 50c$

Figure 31. Beth's number sentence visual for introduction to Coin Problem (BL1)

Furthermore, Beth encouraged her students to use the approach she modelled for the Coin Problem ("This is the easiest way"), which could limit the range of problem-solving strategies her students used (Figure 32).

Beth: "Any more questions?"
Student: "Can you do it another way or do we do it like it is on the board?"
Beth: "Don't write words. This is the easiest way. You could draw coins" (she drew an example).
Student: "Could we draw pictures and do addition?"
Beth: "Yes you can."

Figure 32. Students encouraged to adopt number sentence strategy in introduction to Coin Problem (BL1)

Conclusion

The results from the lesson observation notes indicate the following:

- Problem-solving strategies

- Six different problem-solving strategies were identified across the observations from the four lessons: *draw a picture, number sentences, look for a pattern, write a story, make a list, concrete materials*
- Students were observed using three different problem-solving strategies in RL1, six in RL2, three in BL1 and two in BL2
- The largest range of problem-solving strategies were evident in RL2 (Cookie Problem)
- Prompts
 - In all four of the lessons, the respective teacher was observed using prompts from all of the four categories
 - The two teachers used a range of problem-specific extending prompts infrequently in RL1 (ten prompts once each) and BL1 (five prompts once each and one prompt twice). However, they used a small range of general extending prompts frequently in RL1 (one prompt ten times) and BL1 (one prompt six times).
 - The two whole class prompts were not implemented in RL1 and BL1 (Coin Problem)
- Deviations
 - There were six areas in which the two teachers deviated from the intervention structure in the lessons.
 - Beth and Rachel both deviated in the section of the intervention where they were to ensure their students understood the task without modelling approaches to the problems.
 - After Rachel deviated by facilitating share time in the middle of RL2, students were observed applying new strategies to the Cookie Problem.

The next section discusses the implications of all of the findings outlined in the last three Chapters for the aim and goals of the Case Study.

7. Discussion

The aim of the study was to investigate the implementation of a problem-solving intervention by two primary school teachers. Each teacher implemented a problem-solving lesson populated on Lesson structure-WCP which included whole class prompts and a lesson populated on Lesson structure-IP which included only individual prompts. Interviews were used to gain insight into their perceptions of the effectiveness of the intervention and how it might improve their teaching of problem-solving in the future. The researcher collected work samples and recorded lesson observation notes for all four lessons to gain insight into how the teachers taught the lessons and how the students responded to the lessons. Five main findings were identified through combining and analysing all of the results. These findings are related to struggle (see section 7.1), teachers anticipating student responses (see section 7.2), prompts (see section 7.3), share time (see section 7.4) and problem-solving practice (see section 7.5).

7.1 Struggle

As mentioned in section 2.3 the opportunity for students to struggle with problems aids them in developing problem-solving skills (Roche et al., 2013). As a result, the intervention was designed to give students time to struggle in the lessons by suggesting that the teacher refrain from modelling approaches to problems in the introductions and from intervening during the first 15 minutes of the independent work time. However, based on the comments the two teachers made in their interviews, it appears that they did not perceive that some of their students' problem-solving skills would necessarily improve when given the opportunity to struggle with the problems in the lessons (see section 4.2.1).

Although Rachel commented that most of her students enjoyed the lessons, her main concern in relation to struggle was that some of her students did not have the resilience to struggle with the problems. She referred to a particular student (Student 3) who had scribbled out her work when attempting the Coin Problem (see section 4.2.1.1), concluding that the problem was too difficult for this student. However, this student recorded three correct responses on her work sample (see Figure 32), so even though

Rachel believed the problem was too hard for Student 3, there was evidence that this student achieved success. There was only one instance in which a student was unable to record a correct response across the four lessons (see 5.1). This suggests that the students were able to respond positively to the mathematical features of the problems to find one or more correct responses. Beth did not directly comment on any negative impact of struggle in Interview 2, however in Interview 1 she discussed teaching strategies she applies to reduce the amount of struggle for her students. These teaching strategies entailed providing students with an easier problem that they can confidently answer, and modelling approaches to problems (4.2.1.2). It appears that Beth avoids giving some of her students time to struggle with problems, and therefore discounts the potential positive impact struggle can have on their problem-solving abilities.

Both teachers valued their students being confident and achieving success with problem-solving. They perceived that if some of their students struggled with the problems then their confidence would be affected negatively (see section 4.2.1). As a result, the two teachers deviated from the intervention in the lessons in order to reduce the amount of struggle their students experienced (see section 6.3). For example, both teachers deviated by prompting their students when the intervention indicated that the students should work on a problem without support. This resulted in the students spending very little time working on the problems independently before being supported by a teacher.

Beth also modelled approaches to the problems during the introductions and each of her students copied her method (see 6.3.1). Beth encouraged her students to adopt her problem-solving strategy which tended to inhibit them from using their own problem solving skills to choose and apply strategies. Beth's modelling of the problem also caused unnecessary struggle for her students in the 'over and over' problem in BL2 (see section 6.3). Beth modelled number sentences as an approach to the problem in her introduction. However, misunderstandings about the goal of the problem were identified in her students' work samples (see section 5.3). It appears that her students focussed on creating number sentences. This was instead of attending to the key words in the problem. Beth reflected upon this misunderstanding in interview 2 (see section 4.2.2.1). It appears that although Beth was aiming to reduce the amount of struggle for her

students, in this case, Beth's modelling narrowed her students focus to the procedure of recording number sentences. The modelling appeared to divert her students from properly understanding what the problem was asking.

Rachel perceived that her students who find mathematics difficult were used to struggling and therefore, were more inclined to persist with the problems in the lessons than her other students (see section 4.2.1.1). She explained that this was because her students that find mathematics difficult did not expect to easily find correct responses to the problems, so were not distressed when the answer was not immediately obvious. In turn, she commented that her extension students did not respond positively to having to struggle with the tasks. It appears that Rachel's extension students were perturbed by not being able to find the correct answer straight away. This suggests that students might benefit from opportunities to struggle because it may improve their willingness to struggle.

The largest range of problem-solving strategies were recorded by students when attempting the Cookie Problem in RL2 (see section 5.2). This could have been due to the openness of the problem in so far that it lent itself to a range of problem-solving strategies. A factor that may have contributed to this was that Rachel did not model an example of an approach or strategy to the problem in this particular lesson (see section 6.3). Beth modelled recording *number sentences* to both of the problems in her lesson and all of her students also recorded *number sentences*. There were three other strategies recorded over her two lessons too. However, upon reflection Rachel said that, due to resilience issues, she would support her students more if she were to teach the lesson again by providing an example or having more in-depth discussions about the Cookie Problem (see section 4.2.1.1). It appears that Rachel was focussed on her students' resilience and overlooked the positive impact that not providing a strategy had on her students. In RL2, her students were given time to struggle and were able to choose and apply their own problem-solving strategies to the problem.

Rachel attributed some of this lesson's success to another interesting factor relating to struggle. She perceived that it was optimal for students to learn relevant mathematical content skills before being required to problem-solve. Furthermore, RL2 was based around a content area she was teaching at the time of the study. She perceived that her

students were applying the mathematical content they were learning in class to the problem-solving lesson. Consequently, Rachel chose to plan a lesson based on the intervention that was also within the content area she was teaching at the time. Beth also commented about the importance of teaching mathematical content before requiring problem-solving in her interviews (see section 4.2.1.2). This has been reported as common in current problem-solving literature (Cheeseman, 2018, McCormick, 2016) and it appears that the two teachers perceive that by posing problems related to the mathematical content their students are studying, the amount of struggle they experience is minimised.

In conclusion, the two teachers often do not perceive that it is beneficial for all of their students to struggle with problems in order to improve their problem-solving skills due to perceived resilience and confidence issues. As a result, they deviated from the intervention in the lessons in ways that aimed to reduce the amount of struggle their students experienced. Rachel perceived that her students who are used to struggling with mathematics coped better than her other students when struggling with the problems in the lessons. This suggests that because her students had previously been given opportunities to struggle, they were less perturbed by having to struggle in the lessons.

The results from this study further support the finding in the problem-solving literature that many teachers often do not see the benefit of their students struggling with problems (Cheeseman, 2018). Furthermore, in one of her lessons, Rachel deviated from the intervention by facilitating share time in the middle of one of her lessons. This may provide a productive compromise between these different approaches. This is discussed in detail in section 7.4.

7.2 Teachers anticipate student responses

As mentioned in section 3.5, the two teachers were asked to anticipate their students' responses to the Coin Problem before implementing RL1 And BL1 (Lesson structure-WCP). This was in order to determine whether this anticipation would effectively prepare the two teachers to support their students in the lessons. This feature was not

included in Lesson structure-IP so that the impact of this feature could be evaluated by comparing the two Lesson structures.

A relatively small number of misunderstandings were identified in student work samples from the coin lessons (see section 5.3) which suggests that the two teachers supported their students' thinking appropriately in the lessons. In addition, the two teachers referred to a number of the problem features in their interviews which suggests that they were aware of what the problems entailed (see section 4.2.2). In contrast, for the Over and Over Problem in BL1 (Lesson structure-IP), where the teacher was not required to anticipate his/her students' responses for the problems, there were some serious misunderstandings about the goal of the problem (see results from the work samples in section 5.3). This suggests that Beth found it difficult to support her students to comprehend the problem. In addition, the two teachers did not discuss as many of the problem features for RL2 and BL2 in their interviews (see section 4.2.2). This implies that the teachers had a deeper understanding of what the Coin Problem entailed as a result of anticipating their students' responses to the problem prior to teaching the lessons. This supports a similar finding that teachers are better prepared to support their students if they anticipate their students' possible responses before a problem is implemented, as mentioned in section 2.3 (Sullivan et al., 2016).

7.3 Prompts

One of the features of the intervention was that it provided time for teachers to implement enabling and extending prompts. Both lesson structures included a template for prompts to be added. These included general and problem-specific enabling and extending prompts. Lesson structure-WCP included whole class prompts, and also provided time for whole class prompts to be implemented.

Both teachers used a range of general and problem-specific enabling and extending prompts in their lessons (see section 6.2). Some of the prompts that the teachers used were included in the lesson plans. However, they introduced some extra prompts as they were teaching. It appears that the teachers were helped by the provision in the lesson plans of a number of prompts that could be used in the lessons. However, it appears it

was beneficial for the teachers to use further prompts in order to directly respond to their students' thinking during the lessons.

The two teachers displayed their understanding of using the prompts in the lesson. For example, Beth gave an instance in which a student used an enabling prompt to successfully revisit the initial problem (see section 4.2.3.2). For the Over and Over Problem (I added the same number over and over until I reached 16. What number may I have started with?) this student was prompted to reduce the size of the numbers (What if the total was six?). After finding all of the possible answers to the smaller total, the student revisited the original total and had success.

Rachel indicated that she found the prompts useful as she could support her students' thinking without specifically telling them how to approach the problems (see section 4.2.3.2). Rachel also reported that the general prompts were optimal because they encouraged her students to persevere with their thinking. She was able to articulate how a number of the prompts that she used in the lessons impacted on her students' thinking in the lessons. For example, for the Cookie Problem (Everyone in our class got the same amount of cookies. How many cookies may there have been?), Rachel reported noticing that many of her students were providing people with one cookie each. She indicated that she used the prompt "Could you change the number of cookies each person gets?" to encourage her students to extend their thinking and consider providing people with more than one cookie each. This suggests that Rachel was able to identify the support her students needed and implement a prompt that specifically targeted her students' thinking.

At times, both teachers had difficulties around the timing of prompts. Both teachers deviated from the intervention by prompting their students in the time allocated for students to struggle with the problems (see section 6.3). Rachel commented that she prompted her students during this time due to perceived resilience issues (4.2.1.1). Beth did not provide any reasoning behind this deviation. They both also deviated by not implementing the whole class prompts in the RL1 and BL1 (Lesson structure-WCP which included whole class prompts). As a result, a comparison between the inclusion and exclusion of this feature could not be drawn. Neither teacher discussed their reasoning for disregarding these particular prompts in their interviews (see section

4.2.3.2). It is assumed that they overlooked the importance of highlighting the mathematical features that these prompts emphasised.

There were instances in which students misunderstood what a problem was asking but were given prompts that did not support them to focus on the meaning of the problem. For example, when working on the ‘over and over’ problem, Beth prompted her students to draw pictures when they were using number sentences to add a range of different numbers to equal 16, rather than the same number over and over (see section 4.2.3.2 and section 6.3). This prompt did not appear to support the students’ thinking in order to understand the goal of the problem. This is because this prompt just encourages students to use drawings instead of number sentences. It does not support students to understand that they are required to add the same number over and over to reach 16 instead of a range of numbers. A more appropriate prompt to target this misunderstanding would be, “Look at the important words in the question. Have you added the same number over and over to reach 16?” This highlights that teachers need to carefully select prompts based on the thought processes of their students.

Analysis of student work samples also suggests that students were given prompts that increased the problem totals when they were yet to find all of the responses for the original problem (see section 5.1). Before these students were prompted to increase the total, they could have used their thinking skills to determine whether they had found all of the possibilities or to identify patterns or rules in the results.

In conclusion, the two teachers used a range of problem-specific and general enabling and extending prompts in their two respective lessons. They understood that the prompts were included in the intervention in order to support and extend their students’ thinking. However, at times the two teachers had difficulty with the timing of the prompts used in the lessons.

7.4 Student Share Time

A share time section was included in the final ten minutes of the intervention so that students could share and determine the most effective problem-solving strategies for the challenging problem used in the lessons (see section 3.5). Both Beth and Rachel

reported that the share time they facilitated in each of their respective lessons was too short and that they should have facilitated it for longer (see section 4.2.4.1).

Rachel deviated by facilitating share time in the middle of RL2 (see section 6.3). In the first half of the problem-solving lesson, students were provided with an opportunity to struggle with the problem. Then after the middle-of-the-lesson share time, her students were encouraged to apply the strategies they observed from their peers to the given problem. Many students were observed applying new strategies to the Cookie Problem after the share time finished (see section 6.3). This suggests that the students paid close attention to the strategies they were being shown by their peers because they were invested in the potential solution(s) and wanted to achieve success with the problem. Rachel also reflected on her success of encouraging her students to apply the strategies to the problem in the second half (see section 4.2.4.2). This suggests that she believes that this deviation was beneficial. It appears that the middle-of-the-lesson share time had a positive impact on her students' problem-solving abilities and encouraged her students to achieve success with the problem.

The notion of facilitating share time in the middle of the lesson differs from the successful problem-solving intervention discussed in section 2.3 (Roche et al., 2013). This is because Roche et al. structure share time between the completion of the original problem and the beginning of work on a consolidating problem. With Roche et al.'s structure, students may fail the original problems but are permitted to apply shared strategies only to a new consolidating problem. Essentially, in Rachel's Cookie Lesson a structure that positioned share time in the middle of a lesson enabled her students to experience success with the original problem.

7.5 Problem-solving Practice

Both Rachel and Beth stated that their involvement in the project had positively impacted on their problem-solving practice. However, neither teacher spent much time commenting on their students' problem-solving skills or giving specific examples of how the intervention impacted on their practice in relation to their students' problem-solving skills. Rachel stated that she intends to plan future problem-solving lessons based on the structure of the intervention (4.2.5.1). She indicated that she liked the fact

that the intervention was based on challenging, open ended problems that gave her students the opportunity to use materials, think and apply lots of different problem-solving strategies. She also reported that she really liked the extending and enabling prompts and said that she could guide her students' thinking by prompting them (4.2.5.2). However, she also identified that she will alter the intervention and adopt teaching strategies that will reduce the amount of struggle for her students. She indicated that she would prompt students to reduce or increase the size of the totals before the start of the independent work time and align the topic to the one she is currently teaching. She did not explain how these changes would improve her teaching of problem-solving. The only specific gain that Beth mentioned was that she would encourage her students to use more manipulatives and pictures in her problem-solving lessons in the future (see section 4.2.5.2). She explained that this would make the mathematics within the problems more concrete for her students.

Although the teachers viewed the impact of the intervention on their problem-solving practice as positive, they did not detail how it had impacted their practice. Rachel explained that she would adopt the lesson structure in an altered form and Beth explained that she would incorporate manipulatives into her lessons.

In summary, the two teachers often do not perceive that it is beneficial for all of their students to struggle with problems in order to improve their problem solving skills due to perceived resilience and confidence issues. The two teachers used a range of prompts and understood the purpose of prompts, however they experienced difficulties around timing of prompts. Facilitating student share time in the middle of a lesson allowed students to experience both struggle and success. The two teachers perceived that their implementation of the intervention positively impacted on their teaching practice. However, they did not directly provide many specific ways in which it impacted their practice.

7.6 Strengths and Limitations of the Study

A strength of the research was that multiple methods of data collection were used to gather data about the implementation of the intervention. This was facilitated by the

small sample size. However, the small sample size was also a limitation as the results from the study are not easily generalised.

Another limitation was that only around a third of the students in each class gave consent for their work samples to be photographed. However, the work samples that were collected provided examples of problem-solving strategies for all of the problems in the lessons. This meant that it was possible to get a good indication of the range of problem-solving strategies the students applied to the problems in the lessons.

Another limitation was that in some cases, it was difficult to identify from an analysis of their work samples if students had been prompted. An example in which it was easy to draw an inference that a useful prompt had been given was when a student was prompted to reduce or increase the total of a problem. This would result in a change in the size of the numbers recorded. However, it was difficult to identify whether a student had changed his/her approach to a problem in response to other prompts. This reduced the ability to analyse the effectiveness of the prompts used in the lessons. However, some insight into the implementation of the prompts was provided by the teachers in their interviews.

8. Conclusion

The study aimed to investigate the implementation of problem-solving lessons by two primary school teachers. It focussed on their perceptions of the effectiveness of the lessons, how their students responded to the intervention and how it might improve their teaching of problem-solving in the future. The research study investigated four research questions, each of which is addressed in turn below. The implications and recommendations for future studies are then considered.

RQ1: What do teachers perceive to be the impact of the intervention on students' problem-solving abilities?

The two teachers appeared to perceive that the intervention had both positive and negative impacts on their students' problem-solving abilities.

The teachers made a number of comments that suggest they perceived that the intervention had positive impacts on their students' problem-solving skills. For example, in Interview 2, Rachel commented that the prompts she used in the lessons supported and extended her students' thinking. In Interview 2, Beth commented that her students enjoyed being challenged in the lessons. However, she did not elaborate on the specific ways in which her students were challenged. The two teachers also made positive comments about the overall impact of the intervention, including about how the students generally enjoyed the lessons. Although these comments made by the two teachers support the inference that they perceive that the intervention had positive impacts on their students' problem-solving skills, neither of the teachers spoke at length about the positive impacts.

The teachers' comments suggest that they thought a negative impact of the intervention was that it affected some of their students' confidence and inhibited these students from fully exploring problem-solving strategies. Rachel reported that the intervention had a negative impact insofar as it put some of her students in a position where they tended to have to struggle with solving the intervention's problems. Rachel reflected upon the two lessons she taught and indicated that if she were to teach the lessons again, she would

alter her practice in order to reduce the amount of struggle for her students due to its impact on some of her students' confidence. Beth did not directly state that the intervention itself negatively impacted on her students' problem-solving abilities. However, it was confirmed by the lesson observation notes that Beth deviated from the intervention presumably to reduce the amount of struggle her students experienced. She also reported strategies that she had used in the past to reduce the amount of struggle some of her students experienced. Therefore, it can be inferred that Beth perceived the feature of struggle as negative for some of her students in the intervention.

Although the central focus of the intervention was on problem-solving, the two teachers spent limited time discussing their students' problem-solving skills in the interviews. However, as the two teachers were able to identify positive and negative impacts that the intervention had on their students' problem-solving abilities, it appears that they carefully considered the intervention.

RQ2: What do teachers perceive to be the impact of the intervention on their teaching practice?

In Interview 2, after trialling the intervention, the two teachers were directly asked to describe the impacts that trialling the intervention had had on their problem-solving teaching practice. Both teachers reported that the impacts were positive.

Rachel reported that she planned to create lessons based on the intervention for her future problem-solving lessons. It can be inferred that she believes the features of the intervention had a positive impact on her teaching practice. She also reported strategies that she will use in the future when she uses the intervention that will reduce the amount of struggle for her students. These include prompting students at the start of the independent work time and aligning topics with content she is currently teaching. It can be inferred that Rachel will not incorporate into her problem-solving teaching practice the features from the intervention that provide time for students to struggle with problems. Rachel also specifically commented that she found the prompts very useful for supporting and extending her students' thinking. This suggests that Rachel perceives that the feature of using prompts is an effective teaching strategy.

Although Beth reported that she had gained a lot from trialling the intervention, her comments related to the impacts on her practice were limited. She indicated that she would encourage her students to use more manipulatives and visuals in her problem-solving lessons in the future to make mathematics more concrete for her students. It appears that Beth perceived that the use of manipulatives was an effective problem-solving strategy for her students to adopt.

It would have been optimal to gain more specific insights into how Rachel and Beth believe the intervention impacted on their problem-solving practice. However, Rachel intends to use the intervention in the future which implies that the intervention has had a positive impact on her teaching practice. Beth proposed to incorporate manipulatives in her future problem-solving lessons as a result of her trialling the lessons which implies that her practice had also been positively impacted.

RQ3: How did the teachers implement the intervention in the lessons?

The teachers implemented most parts of the intervention as intended. They posed the challenging problems, implemented the enabling and extending prompts and asked their students to record and share their problem-solving strategies in the lessons. However, they appeared to deviate from the intervention in a number of ways. In particular, they both deviated in order to reduce the amount of struggle for their students. Both teachers also deviated in the following ways:

- Some assumptions behind the problems were not made clear in the introductions
- Students were prompted in the part of the lesson where they were expected to work on the problems without support
- Whole class prompts were not implemented in Lesson structure-WCP
- Specific sharers were not chosen to share in the share time sections

Rachel also deviated by facilitating share time in the middle of one of her lessons.

Because both of the teachers deviated from some of the features of the intervention, it is difficult to evaluate certain features of the intervention. Based on the two teachers' comments, it can be assumed that a number of the deviations were made in order to reduce the amount of struggle for some of their students. It is also possible that the

teachers deviated in certain ways because they either did not understand the reasoning behind some of the features in the intervention or did not perceive that some of the features had a positive impact on students' problem-solving skills.

Prior to the implementation of the lessons, the teachers were given the opportunity to engage in a discussion with the researcher about the features included in the intervention and the reasons for their inclusion. However, it appears that these discussion did not result in the teachers valuing all the features of the intervention, including the provision of opportunities for students to struggle. In these circumstances, the provision of professional development for the teachers that highlighted the importance of the features from the problem-solving literature that were applied to the intervention, may have further ensured that the teachers were making informed decisions about whether to implement certain features of the intervention as intended.

The fact that the teachers did not always implement the intervention as intended in the lessons, could be viewed in a positive light. It implies that the two teachers were using their professional judgement about effective problem-solving practices during the lessons. Rachel had success when she deviated in one of her lessons by facilitating share time in the middle. Her students were observed adopting and applying their peers' problem-solving strategies after the share time period. This deviation from the intervention appeared to have a positive impact on Rachel's students' problem-solving.

RQ4: How did students respond to the intervention in the lessons?

The students appeared to respond positively to the intervention. This was evidenced by the range of problem-solving strategies students recorded in the lessons, such as: *draw a picture, number sentences, look for a pattern, write a story, highlighting key words and working out the answer in more than one way.*

Furthermore, when the features of the intervention were implemented as intended, it appeared that students struggled in ways that caused them to formulate their own problem-solving approaches to the problems. For example, when a teacher refrained from modelling approaches to the problems in the introductions to the lessons, students used a greater variety of problem-solving strategies. In contrast, when a teacher

modelled approaches to the introduction, students tended to adopt and apply the teacher's problem-solving strategy. It appears that the intervention's feature of posing the problem without providing a solution, positively impacted on the students' problem-solving abilities.

There were limited problem-solving strategies discussed by students during the share-time session that was scheduled in the intervention for the end of the lesson. This may have been due to the small amount of time the teachers provided for students to reflect upon their problem-solving strategies. This may also be due to the two teachers deviating from the intervention by not selectively choosing students to share effective problem-solving strategies in the share time section lessons. The two teachers commented that they should have spent more time facilitating share time at the end of their lessons. In addition, Rachel reported that she should have been more selective about the students who she chose to share their problem-solving strategies. It appeared that limiting the share time provided at the end of the lessons did not have a positive impact on students' problem-solving abilities.

The students were able to record a range of problem-solving strategies in the lessons. When the intervention was implemented as intended, students were able to formulate and apply their own problem-solving strategies. However, their discussion of problem-solving strategies in the share time section of the intervention was limited, but this may have been due to the short time the teachers provided.

This study shows that teaching problem-solving in mathematics is particularly difficult. Many teachers are used to nurturing their students and naturally help them when they are struggling. Problem-solving differs from other pedagogies as teachers are required to relinquish some of their support. It also requires planning and thinking prior to implementing lessons regarding when to prompt individual students effectively, without providing approaches.

8.2 Implications and recommendations for future studies

The two teachers in the study focussed on struggle as being detrimental to some of their students' abilities to problem-solve. This raises some key questions: What is success in problem-solving? Is it about confidence and producing correct responses? Or is it more about the problem-solving process? There appears to be tension between teachers' perceptions of students developing problem-solving skills through struggling with problems, and maintaining some of their students' confidence. However, students appeared to respond positively to the opportunities to struggle.

Key implications of the case study for teaching practice include providing time for students to struggle with problems before intervening, refraining from providing problem-solving approaches and preparing for problem-solving lessons by anticipating students' responses and gaining awareness of the assumptions behind particular problems.

As the study was based on recommendations from problem-solving literature it has implications for the field. Notably, the study confirmed and reinforced some problem-solving research that suggested that teachers are reluctant to provide time for their students to struggle with problems. It also raised a possible approach that could help resolve this tension. Sharing in the middle of problem-solving lessons may provide students with the opportunity to struggle and achieve success.

Key implications for future studies include exploring in greater detail the impact of the provision of opportunities to struggle on the development of problem-solving skills. Investigating the relationship between teachers' and students' perceptions of the provision of opportunities to struggle on the development of students' problem-solving skills.

Future studies could investigate the timing of enabling and extending prompts and the impact that different timings have on students' problem solving. They could also investigate how teachers' prompting of students could be targeted more effectively in order to support students to use effective problem-solving strategies. Furthermore, these studies could focus on investigating the impacts of prompting students after they have had appropriate "struggle" time and have attempted to find all of the possible answers to

the immediate problem. This recommendation has been made because the two teachers experienced difficulties with the timing of the prompts they used in the lessons.

Future studies could also investigate the effectiveness of teachers facilitating share time in the middle of problem-solving lessons. They could investigate in greater depth the impact of this promising teaching strategy on students' problem-solving abilities. This recommendation has been made because there was success with share time being facilitated in the middle of a problem solving lesson as students were able to adopt and apply problem-solving strategies that their peers discussed.

Appendices

Appendix A

Lesson structure-WCP which includes whole class prompts

Problem:			
Time	Activity	What the students are doing	What the teacher is doing
8 minutes	Introduction of task.		Introducing the task. Ensuring students understand task. <u>Not showing</u> students how to solve the problem. Explaining to students that they need to individually record their problem-solving strategies.
15 minutes	Solving the task.	Discussing, solving the task and individually recording problem solving strategies.	Roaming, observing and thinking about which groups/students need to be given <u>particular prompts without</u> intervening.
5 minutes	Introducing prompt 1 to class.		
10 minutes	Solving the Task.	Discussing, solving the task and individually recording problem solving strategies.	Giving students <u>enabling</u> prompts where necessary. Then, giving students <u>extending</u> prompts where necessary.
2 minutes	Introducing prompt 2 to class.		
10 minutes	Solving the task.	Discussing, solving the task and individually recording problem solving strategies.	Giving students enabling and extending prompts where necessary. Selecting individual students who have used efficient problem-solving strategies and notifying them that they will share their problem-solving strategies with the class.
10 minutes	Share time.	Sharers share problem solving strategies. Class engage in discussions.	Ensuring that students are staying on track and speaking about the problem-solving strategies they've used.

Enabling Prompts	General Problem-Solving Prompts	Problem-Specific Prompts
		Prompts that help to organize ideas (C)
		Prompts that complete a step towards solving the problem (D)
		Prompts that reduce the size of the numbers (E)
Extending Prompts		Prompts that make the problem harder (F)
		Prompts that encourage more ways to solve the problem (G)
		Prompts that encourage more possible answers (H)
		Prompts to find a pattern or rule (I)

Lesson structure-IP which includes only individual prompts

Problem:			
Time	Activity	What the students are doing	What the teacher is doing
10 minutes	Introduction of task:		Introducing the task. Ensuring students understand task. <u>Not showing</u> students how to solve the problem. Explaining to students that they need to individually record their problem-solving strategies.
15 minutes	Solving the task.	Engaging in discussions, attempting to solve the task and individually recording problem solving strategies.	Roaming, observing and thinking about which groups/students need to be given <u>particular prompts without</u> intervening.
25 minutes			Giving students <u>enabling</u> prompts where necessary. Then, giving students <u>extending</u> prompts where necessary. Selecting individual students from groups who have used efficient problem-solving strategies and notifying them that they will share their problem-solving strategies with the class.
10 minutes	Share time.	Sharers share problem solving strategies. Class engages in discussions.	Ensuring that students are staying on track and speaking about the problem-solving strategies they've used.

	General Problem-Solving Prompts	Problem-Specific Prompts
		Prompts that help to organize ideas (C) Prompts that complete a step towards solving the problem (D) Prompts that reduce the size of the numbers (E)
Enabling Prompts		
Extending Prompts		Prompts that make the problem harder (F)
		Prompts that encourage more ways to solve the problem (G)
		Prompts that encourage more possible answers (H)
		Prompts to find a pattern or rule (I)

Appendix B

Lesson Plan A: Coin Problem

How many ways can you make 50 cents using coins?			
Time	Activity	What the students are doing	What the teacher is doing
8 minutes	Introduction of task.		Introducing the task. Ensuring students understand task. <u>Not showing</u> students how to solve the problem. Explaining to students that they need to individually record their problem solving strategies.
15 minutes	Solving the task.	Discussing, solving the task and individually recording problem solving strategies.	Roaming, observing and thinking about which groups/students need to be given particular prompts <u>without</u> intervening.
5 minutes	Introducing prompt 1 to class.		Displaying and discussing John's solution to the problem: "John said there were 3 ways you can make 50 cents. Is he correct? Why? Why not?"
10 minutes	Solving the Task.	Discussing, solving the task and individually recording problem solving strategies.	Giving students <u>enabling</u> prompts where necessary. Then, giving students <u>extending</u> prompts where necessary.
2 minutes	Introducing prompt 2 to class.		Discussing Kath's solution: "Kath said there were 13 ways to make 50 cents. Is she correct? Why? Why not?"
10 minutes	Solving the task.	Discussing, solving the task and individually recording problem solving strategies.	Giving students enabling and extending prompts where necessary. Selecting individual students who have used efficient problem solving strategies and notifying them that they will share their problem solving strategies with the class.
10 minutes	Share time.	Sharers share problem solving strategies. Class engage in discussions.	Ensuring that students are staying on track and speaking about the problem solving strategies they've used.

	General Problem Solving Prompts	Problem Specific Prompts
Enabling Prompts	Could you use a table? (A1) Could pictures help? (A2) Can you think of an easier question like this? (A3) Can you think of a problem like this? (A4) Could you have a guess? (A5) Could you make a list? (A6) Could you act it out? (A7) Could you start with the answer and work backwards? (A8) Could you draw a picture? (A9) Could materials help? (A9)	Prompts that help to organize ideas (C) Could "cut and paste" pictures of coins help? (C1) Coin fake coins help? (C2)
		Prompts that complete a step towards solving the problem (D) Make 50 cents using only 10 cent coins. (D1) <u>Could you skip count? (added by Beth)</u>
		Prompts that reduce the size of the numbers (E) Could you use a strategy to work out how many ways can you make 10c or 20c? (E1)
Extending Prompts	Are there more answers? How do you know? (B1) How could someone find all of the answers? (B2) Is there another way that you could solve this problem? (B3) How many different ways could you solve this problem? (B4) What pattern can you see? (B5) How would you teach someone how to solve this problem? (B6)	Prompts that make the problem harder (F) What strategies could you use to make \$1, \$2, \$10, 70 cents or \$1.50 in as many ways as possible? (F1) What if you could no longer include 20 cent coins in your ways to make 50 cents? (F2)
		Prompts that encourage more ways to solve the problem (G)
		Prompts that encourage more possible answers (H) Can you use more than one coin in your combination to make 50 cents? (H1)
		Prompts to find a pattern or rule (I) How can you explain to John that he is wrong? (I1) If you were finding out how many ways you can make \$1 would you just double your answer? Why? Why not? (I2)

Appendix C

Lesson Plan B: Cookie Problem

How many cookies?			
Time	Activity	What the students are doing	What the teacher is doing
10 minutes	Introduction of task: Everyone got the same amount of cookies. How many cookies may there have been?		Introducing the task. Ensuring students understand task. <u>Not showing</u> students how to solve the problem. Explaining to students that they need to individually record their problem solving strategies.
15 minutes	Solving the task.	Engaging in discussions, attempting to solve the task and individually recording problem solving strategies.	Roaming, observing and thinking about which groups/students need to be given particular prompts <u>without</u> intervening.
25 minutes			Giving students <u>enabling</u> prompts where necessary. Then, giving students <u>extending</u> prompts where necessary. Selecting individual students from groups who have used efficient problem solving strategies and notifying them that they will share their problem solving strategies with the class.
10 minutes	Share time.	Sharers share problem solving strategies. Class engages in discussions.	Ensuring that students are staying on track and speaking about the problem solving strategies they've used.

	General Problem Solving Prompts	Problem Specific Prompts
Enabling Prompts	Could you use a table? (A1) Could pictures help? (A2) Can you think of an easier question like this? (A3) Can you think of a problem like this? (A4) Could you have a guess? (A5) Could you make a list? (A6) Could you act it out? (A7) Could you start with the answer and work backwards? (A8) Could you draw a picture? (A9) Could materials help? (A10)	Prompts that help to organize ideas (C) Could counters help? (C1) Could drawing an array help? (C2) <u>Could you use pictures of cookies? (added by Rachel)</u>
		Prompts that complete a step towards solving the problem (D) What if there were 3 people? (D1) What if each person got 2 cookies? (D2) Could 12 be an answer? (D3) Does checking this number help you find others?
		Prompts that reduce the size of the numbers (E)
Extending Prompts	Are there more answers? How do you know? (B1) How could someone find all of the answers? (B2) Is there another way that you could solve this problem? (B3) How many different ways could you solve this problem? (B4) What pattern can you see? (B5) How would you teach someone how to solve this problem? (B6)	Prompts that make the problem harder (F) What if one person didn't get the same amount of cookies as everyone else? (F1)
		Prompts that encourage more ways to solve the problem (G)
		Prompts that encourage more possible answers (H) Could you change the number of people? (H1) Could you change the number of cookies that each person gets? (H2)
		Prompts to find a pattern or rule (I) How does changing the amount of people or the amount of cookies they receive change your answers? (I1)

Appendix D

Lesson Plan C: Counter Problem

12 Counters. How many groups?			
Time	Activity	What the students are doing	What the teacher is doing
10 minutes	Introduction of task: Yesterday I put 12 counters into equal groups. I can't remember the amount of groups there were. How many may there have been?		Introducing the task. Ensuring students understand task. <u>Not showing</u> students how to solve the problem. Explaining to students that they need to individually record their problem solving strategies.
15 minutes	Solving the task.	Engaging in discussions, attempting to solve the task and individually recording problem solving strategies.	Roaming, observing and thinking about which groups/students need to be given particular prompts <u>without</u> intervening.
25 minutes			Giving students <u>enabling</u> prompts where necessary. Then, giving students <u>extending</u> prompts where necessary. Selecting individual students from groups who have used efficient problem solving strategies and notifying them that they will share their problem solving strategies with the class.
10 minutes	Share time.	Sharers share problem solving strategies. Class engages in discussions.	Ensuring that students are staying on track and speaking about the problem solving strategies they've used.

	General Problem Solving Prompts	Problem Specific Prompts
Enabling Prompts	Could you use a table? (A1) Could pictures help? (A2) Can you think of an easier question like this? (A3) Can you think of a problem like this? (A4) Could you have a guess? (A5) Could you make a list? (A6) Could you act it out? (A7) Could you start with the answer and work backwards? (A8) Could you draw a picture? (A9) Could materials help? (A10)	Prompts that help to organize ideas (C) Could counters help? (C1) Could you draw circles around each group? (C2) Could you draw an array to help? (C3)
		Prompts that complete a step towards solving the problem (D) What if there were 6 counters in each group? (D1)
		Prompts that reduce the size of the numbers (E) What if I started with 4, 6 or 8 counters? (E1)
Extending Prompts	Are there more answers? How do you know? (B1) How could someone find all of the answers? (B2) Is there another way that you could solve this problem? (B3) How many different ways could you solve this problem? (B4) What pattern can you see? (B5) How would you teach someone how to solve this problem? (B6)	Prompts that make the problem harder (F) What if I started with 32, 84 or 364 counters (F1) What if there were 17 counters? (F2)
		Prompts that encourage more ways to solve the problem (G)
		Prompts that encourage more possible answers (H) Could you change the size of the groups? (H1) Could you change the amount of groups? (H2)
		Prompts to find a pattern or rule (I) How does changing the amount of groups effect your answer? (I1) How does changing the size of the groups effect your answer? (I2)

Appendix E

Lesson Plan D: Over and Over Problem

Adding Over and Over			
Time	Activity	What the students are doing	What the teacher is doing
10 minutes	Introduction of task: I added the same number over and over until I reached 16. Which number may I have started with?		Introducing the task. Ensuring students understand task. <u>Not showing</u> students how to solve the problem. Explaining to students that they need to individually record their problem solving strategies.
15 minutes	Solving the task.	Engaging in discussions, attempting to solve the task and individually recording problem solving strategies.	Roaming, observing and thinking about which groups/students need to be given particular prompts <u>without</u> intervening.
25 minutes			Giving students <u>enabling</u> prompts where necessary. Then, giving students <u>extending</u> prompts where necessary. Selecting individual students from groups who have used efficient problem solving strategies and notifying them that they will share their problem solving strategies with the class.
10 minutes	Share time.	Sharers share problem solving strategies. Class engages in discussions.	Ensuring that students are staying on track and speaking about the problem solving strategies they've used.

	General Problem Solving Prompts	Problem Specific Prompts
Enabling Prompts	Could you use a table? (A1) Could pictures help? (A2) Can you think of an easier question like this? (A3) Can you think of a problem like this? (A4) Could you have a guess? (A5) Could you make a list? (A6) Could you act it out? (A7) Could you start with the answer and work backwards? (A8) Could you draw a picture? (A9) Could materials help? (A10)	Prompts that help to organize ideas (C) Could counters help? (C1) Could a 100s chart help? (C2) Could you draw an array? (C3)
		Prompts that complete a step towards solving the problem (D) Could 4 be an answer? (D1) Does checking this number help you find others?
		Prompts that reduce the size of the numbers (E) What if the number you reached was 4, 6 or 10? (E1)
Extending Prompts	Are there more answers? How do you know? (B1) How could someone find all of the answers? (B2) Is there another way that you could solve this problem? (B3) How many different ways could you solve this problem? (B4) What pattern can you see? (B5) How would you teach someone how to solve this problem? (B6)	Prompts that make the problem harder (F) What if the number you reached was 18, 89 or 364? (F1) If the number I was trying to reach was 32, would I just double the amount of answers? Why? Why not? (F2)
		Prompts that encourage more ways to solve the problem (G)
		Prompts that encourage more possible answers (H) Could you change the number that you're adding? (H1)
		Prompts to find a pattern or rule (I) Explain how halving numbers can help you solve the problem?

Appendix F

Lesson Plan A (Coin Problem): Supporting visual resource for whole class prompt One

John's Solution

How many ways can you make 50 cents using coins?





Appendix G

Lesson Plan A (Coin Problem): Teachers choose between student recording sheets with and without visuals

How many ways can you make 50 cents using coins? 	
How many ways can you make 50 cents using coins?	

Appendix H

Rachel: Struggle (Theme One)

Code	Rachel	Analysis
I1111 R1112 I1113 R1114 I1115 R1116 I1117 R1118	Interview 1 Ah. Well I guess just if this is the problem you picked up, how would you think it would work best in your class? Um. I would have a lot of hands on materials for this one. Mmm. Um. Do a... Make sure the kids understand the difference between the coins obviously. Remind them that you know, from smallest to biggest is the simplest way to remember. Mmm. And remind them that this is skip counting. It's addition. For those top kids this would be multi, multiplication coz that's what we're touching on now. Um, so tryna create those links between what we already know so they're not quite stumped and stuck on what they think they don't understand and then practice. So, I would leave the fifty cents off completely and maybe just have a look... have a hundreds chart out and do the tens and then skip count by tens. Mmm. See how many we need and then dr, write it and draw it on the board.	Modelling an example of problem solving tasks reduce the amount of struggle students experience
R2002 I2003 R2004 I2005 R2006 I2007 R2008	Final Interview Um, so I think the second lesson surprisingly went better, um, even though the question was a lot more open ended... so the tricky one. Um I found that they both were really successful. I love how open ended they are. I think that it's achievable for all of the kids. I just think that some of the kids didn't have that resilience. Yep. And they didn't have that because, it's because it is so open. They didn't know how to approach it and a lot of them just sort of shut down. Um and the kids that surprised me the most were the ones that normally would be probably, lower academically. They're the ones that got in there and actually did a lot better than maybe the kids that always know the answer to typical questions. And why do you think that is? Um, I think because they get to explore and because in our class, we're very big on if you make a mistake it doesn't matter. Even if you rip the page out, I don't care, as long as I can see that you're trying, and I guess, er, as horrible as it sounds those kids are used to maybe failing or not doing as well... Mmm. so, they just go in with that attitude of, well I'm probably gonna get it wrong or it doesn't matter if I do. And they're used to it, whereas the kids that are in my extension group, they couldn't figure out how to approach this. And, um, across both of the lessons some of the kids were consistent of not knowing what to do and couldn't, just couldn't even	The problems were very open ended and some of the students did not have the resilience to work through the state of struggle The students who were lower academically were able to deal with struggle better because they were used to finding tasks difficult The students who were higher academically found it very difficult to work out how to approach the tasks.

I2009 R2010	start, so I had to actually use some of the enabling questions for kids that I expected to do a lot better than everyone else. Mmm. And so why do you think the second one, the one about the cookies went better than the... um, money one? Because overall I think there was a lot more success in the grade. I think they... the first one was more of a test run and this question, um, matches up quite well with, ah, the fact that we're doing multiplication. And even though some of the kids didn't see it at first, when they saw another kid doing a strategy, they, they sort of, they had that light bulb where they went "Oh, they're doing groups of so I might draw groups or I'll do an array." Or they'll write, um, you know this many rows of something is this and so they're using strategies that we're using in class at the moment and it really suits the question.	
R2014 I2015 R2016 I2017 R2018 I2019	Um. I think the fact that., they had to just push through. Even if they got maybe one, one way of expressing their thoughts out, the fact that it's so open to interpretation worked quite well. They didn't have much direction. The only kids that really got any help which wasn't many of them were the ones that really looked like they were probably going to cry. And just sat there in a ball but the rest of them just said, "Okay, well I'll just try." And, and having the hands on materials but also watching the kids use strategies that we use all the time. So, you know a lot of them turned around and said, "Can I draw and write?" And they're strategies which we're really trying to push with the kids so they can show their thinking in a range of ways. And is there anything that you think didn't work very well? Um. I found it really tricky not to lead the kids. So especially with the introduction. Sort of explaining the question without explaining how to approach it was really hard for me. Especially for those kids who kept going, "I don't get it, I don't get it I don't get". Which was quite a few of them. And so tryna figure out how to break down the question in a way that's accessible without giving them an answer or a strategy was really hard. Okay, so can you tell me more about, um, how that didn't work well? Like, what was the, um, your feeling about it? Um. Well in the second question, talking about how everyone in our class got the same amount of cookies. I didn't want to say how many people were in the class and I didn't want to say that you didn't have to do the number of people in our class. But some of the kids were looking at me like they had no idea and no clue and so tryna push some of the other kids who I could see understood the question to share rather than having me but without picking the kids I knew would have the answer straight away. Tryna to get some of those kids who were less confident. Um., yeah. Yeah. That makes sense. Um and which aspects or were there any aspects of the lesson that you deviated from while you were teaching?	Students were helped with the tasks if they were crying due to the level of struggle they were experiencing. It is difficult not to give students examples of how to solve tasks or strategies to help them solve tasks as students' have to struggle with tasks. One lesson was deviated from and students were assisted in the part of the lesson where they were encouraged to struggle with the task individually because they did not have the resilience.

R2054	Um. At the end of that, all except for the few, the two that struggled, they were so excited. They said, “Can we do it again?” “We wanna do another one.” They were really, really, really engaged. They loved these activities, um, and because of the engagement I’ve actually planned a lesson based on this sort of format for next week and they’re really excited. So, I think for a lot of those kids it’s created that enjoyment of maths...	do more lessons like the ones trialled
R2064	Um. Student three. Ah, her sample... On the first one she completely gave up and refused almost to participate because she didn’t understand, obviously it all. So was a bit tricky because we don’t do maths (money) til next term so it was something... it did it did come from the side and they were, they weren’t really sure what was going on. Some of them. Um, but she is an extension student who I thought would easily be able to do this activity and she scribbled... you can see her thought process and her emotions... [laughter]	Student Three struggled to the point of not being able to participate in BL1. Rachel perceived that this was because she had not taught a unit about money to her students yet
I2065 R2066	Yeah. [laughter] towards it and then in the second session she’s worked through the entire page and she’s even worked on to the back of it because she’s got so many strategies and... so many different combinations and she’s modelled her thinking in three or four different ways so I thought that was fantastic	

Appendix I

Beth: Struggle (Theme One)

Code	Beth	Analysis
B1070	Initial Interview and I would've lost them. So, they've got a really healthy mindset. They've got a real growth mindset but I wanna maintain that so as much as I love to challenge... And I do challenge them a lot.	Students' enjoy being challenged. Opportunities and time for struggle are avoided unless in the most capable group. Success is more important than opportunity to struggle.
I1071	Mmm.	
B1072	And I always joke and say I'm going to try and trick you. You're too hard to trick. I'm gonna find a different way or a different problem. I still make it so that it's not... So, the real challenge, I'll maintain for the higher group.	
I1073	Mmm. Mmm.	
B1074	Rather than frustrating... I think that's the problem in maths, that a lot of teachers just move onto the next and the next. And this kid, you can see, you can literally see them shrink before your eyes. Losing it. Losing their confidence. Losing their enjoyment. Their will to live.	
I1075	Yeah, so how do you keep that..?	
B1076	Well, like I said, in that lesson I just had then, even though a couple were ready to move onto three digit, there was no way. We were doing a whole class problem solving.	
I1077	Mmm.	
B1078	If I had have let it go to three digit, I just felt like I would've shattered the confidence of three quarters of the room to be honest.	
I1079	Yep.	
B1080	Not even half. Three quarters of the room woulda just gone, "It's too hard. I can't do this." So, doing the two digit allowed most, if not all to experience success.	
B1131	And I think sometimes when you do see that they're not participating, you sort of set it up so that there's an easier option or a, you know something that they will be confident to answer. Just to help bring them back into it so that they haven't, you haven't lost them.	If a student is struggling it is important to maintain their confidence by giving them a question they can answer.
B2058	Final Interview Even though it was good to do a little snapshot and something different. Um, because it's not the topic we're doing currently and we sort of... as a whole unit we have to teach subtraction and then we have to teach multiplication.	Students coped with problem solving in a content area that was less familiar and that they were not focussing on at the time of the study The fact that they coped and are adaptable assumes that there was more of an opportunity to struggle with a less familiar task.
I2059	Mmm. Yep.	
B2060	So, your repeated addition worked in well with the multiplication but the currency was kind of, a little bit more out there. So...	
I2061	And yeah, how did you feel about that?	
B2062	Fine and I think the kids coped really well with it which was good for me to see because I do think...	
I2063	Mmm.	

B2063 I2064 B2065	they are adaptable. I think these students do go with the flow and don't question me and go, "But we're doing subtraction. Why are we doing this?" [laughter] Although, with the repeated addition you may've heard one or two students said, "Can I do a subtraction?"	
I2072 B2073 I2074 B2075	Um. And so, were you happy with the period where the students were working on the problem without support? Yeah, I think so. I really do think so. And I don't think anyone sort of did the whole, "I can't do this." Or was really heavily reliant on me and trying to monopolise my time. Mmm. And stop me from getting other, to other students.	Students were able to struggle through a problem solving task without support.
B2109 I2110 B2111 I2112 B2113	... I didn't want to give it to them but I wanted to say it's the same number and then we're going to... and I did model on the board something plus something equals sixteen. Something plus something plus something. And I did say that would be the same number plus the same number plus the same number, trying to get them to see that it had to be the same number. Mmm. Mmm. But, I guess for a turn around and for a child in grade one or two, a turn around is the same number. You're just reversing the order, so I could see the logic with that. Um. Yeah and you wanno scaffold it but you don't really wanno give them the answer or... Mmm. you want to see what they can do..., well that's what I assumed the task was. Maybe I read the task wrong. [laughter]	Modelling examples but not wanting to tell students how to answer the problem. Students' need a certain level of support and opportunity to struggle
I2174 B2175 I2176 B2177	Okay. Um, and yeah, so how do you think your students responded to the lessons? I think they really enjoyed it. The kids love a challenge and they're always asking me, "Can you test me?" So, um, yeah, I think they really... It's strange, but they seem to really thrive in those conditions. They like, they love a challenge. Mmm. In our classroom we have a philosophy that maths is fun. It's actually on my timetable. Maths is fun. And The children have sort of adopted that, that real growth mindset. Legitimately. It's fully embedded and, um, yeah they, they approach it, thinking it's fun. They're a detective. They're solving a problem. And a problem's not a bad thing. Um, yeah, so they, they have a really good attitude to maths and they really enjoy it. So, to do some different is always fun.	Students thrive when given tasks that give them the opportunity to struggle

Appendix J

Rachel: Challenging Problem Features (Theme Two)

Code	Rachel	Analysis
R1018 I1019 R1020 I1021 R1022 I1023 R1024 I1025 R1026	Initial Interview Oh okay. Um, so for I guess for my 1, lower kids. I don't like saying that, but fifty cents hopefully is one. [laughter] For my lower kids I expect fifty cents and possibly twenty, twenty and ten. Yep. And then for my mid group, I'd expect both of those and then possibly five, ten cent pieces. Ten, ten, ten, ten, ten and I do talk while I'm doing that. [laughter] And then for my extension, I'd expect all of that and then the five cent pieces. Umm., ten of those. Sorry, my brain. Yep. Ah. Ten of those and then I would expect a combination of, um, a replacement. So, if you have two twenties and a ten, then maybe swapping one of the twenty cent pieces. So, you'd have a twenty cent piece and three tens or having a combination using five cent pieces or, um, having you know, four ten cent pieces and two fives. So, just using a complete range and seeing how many they can come up with and then tryna exhaust every single option	Students using strategies such as repetition and breaking down of coins. Discusses Low ceiling and high floor and range of possible responses
I1027 R1028 I1029 R1030	Yeah, okay, no, that makes sense. And how do you think they'd find these solutions? Like, what would they do, do you think? Um, hopefully if they've got concrete materials they'd be able to do that. Um so that would be exposing them to all the different pieces and explaining the value of it. For higher, for my grade twos, most of my grade twos, I wouldn't necessarily think that they'd need the materials. Mmm. But I think in the junior school it's always handy to have them on the off chance that they want that physical making the group. And, um, I tend to use coloured bits of paper so they can actually put them on almost like lily pads or so they can group them off	Concrete materials would help students work on the problem.
I1111 R1112 I1113 R1114	Ah. Well I guess just if this is the problem you picked up, how would you think it would work best in your class? Um. I would have a lot of hands on materials for this one. Mmm. Um. Do a... Make sure the kids understand the difference between the coins obviously. Remind them that you know, from smallest to biggest is the simplest way to remember.	Discuss the importance of students knowing the values of the coins.
R2018	Um. Well in the second question, talking about how everyone in our class got the same amount of cookies. I didn't want to say how many people were in the class and I didn't want to say that you didn't have to do the number of people in our class. But some of the kids were looking at me like they had no idea and no clue and so tryna push	Discusses not encouraging students to use the number of children in the class as the question states.

	some of the other kids who I could see understood the question to share rather than having me but without picking the kids I knew would have the answer straight away. Tryna to get some of those kids who were less confident. Um., yeah	
R2066:	Towards it and then in the second session she's worked through the entire page and she's even worked on to the back of it because she's got so many strategies and... so many different combinations and she's modelled her thinking in three or four different ways so I thought that was fantastic.	Refers to the modelling of thinking in a variety of ways.
R2070:	No fives, no tens. So that was really tricky for her, um, but doing multiplication and doing the second lesson... She didn't even need any support. Um, that was a person I thought I would have to work with the most. Um, she just took off. She modelled her thinking in, you know, three different ways and was able to articulate it to me so I just thought that was fantastic.	Refers to the modelling of thinking in a variety of ways.

Appendix K

Beth: Challenging Problem Features (Theme Two)

Code	Beth	Analysis
B1032 I1033 B1034 I1035 B1036 I1037 B1038	Interview 1 Most of them would do the fifty cent. That would be the obvious one. Mmm. Most of them would get the two twentys and a ten. Mmm. Some would get the five tens and others would get the ten fives and then probably my top three or four would do mixed denominations. Yep But probably only the top. When I say mixed denominations, a lot would get the two twentys and a ten. But I don't believe a lot would do combinations of twentys and fives or tens and fives	Students using strategies such as repetition and mixed denominations. Discusses low ceiling and high floor Discusses range of possible answers
B1044 I1045 B1046 I1047 B1048 I1049 B1050 I1051 B1052 I1053 B1054 I1055 B1056 I1057 B1058	Um, they're pretty good at skip counting. Mmm. So, I think skip counting would definitely come into play. They're also very visual. I am too. So That's why I like this sheet better... Mmm. coz it's got the visuals. Um. So yeah, some of them would probably want the coins, the maths resource to use to work it out. As well as being able to skip count they'd want... they're very tactile... Yep. so, they'd want something, manipulatives to work with as well. Mmm. Um. Was that, was that the question? Basically, how would they work it out? Yeah, like how would they work it out? Yeah, I think. Are there any other things they might do? Actually, they work... they're quite logical so they do sort of work mentally. They don't always need the resources but I think coz they are young I like to try and provide the resources... Mmm. just for that safety net and just to scaffold it. And, um, give them that bit of confidence and more chance of success. Yeah	Use of manipulatives, skips counting, pictures, work mentally
B2026 I2027	Final Interview So, they sort of... "Oh got coins. I want coins". And I think for some children it may've helped but the coins were clearly visible at the top of the page. And I don't think I said it. I could be wrong but I don't think I said that... Oh, maybe I... I probably did, knowing me... That they had their names on them. Mmm.	Ensured students knew values of coins

B2028 I2029 B2030	So, if you were going to try and work out how to skip count or what the coins were worth their names are on them. And it is pretty clear. There... So technically they probably didn't need the resource coz the resource is pictured on the page. Whereas the other one's just a bit of text. It's not as, um, visual for your visual learners. Mmm. This is quite visual but that's the one that had the resources for... With the coins. Yeah	
B2097 I2098 B2099	And I'd go straight to the default setting of six, which I should've done earlier with some of the students who've just gone... Mmm. Um, started to do random addition and subtraction sums to equal sixteen. Even though I clearly scaffolded it at the start of the lesson and made it very explicit that the number was sixteen. We talked about the key words being the "number", "sixteen". We talked about repeated... Um, the "same number" so we had that, um, really, really reinforced at the start of the lesson and some students have gone off and not really... They've been focussed on one element of that... like the "sixteen" being the important thing but the "repeated" being not so important.	Attempted to have students understand the significance of the word repeated.
B2117 I2118 B2119 I2120 B2121 I2122 B2123	Like I said, the repeated addition would be really handy for me with multiplication which we're just starting. But also, with the coins and the currency. It was just good to see which kids knew which coins and I think someone did have an eight cent or a twelve cent coin which always appears when I teach, um, Australian currency in early years. Mmm. There's always that one coin that I just don't recognise how we've managed to come up with it. [laughter] It's not in any of your work samples. Thank goodness. [laughter] But I'm sure someone had a twelve or an eight cent coin and was like, "Wow!" But it always happens. Always. I don't think I've ever taught a year where it hasn't appeared	Discusses experience with students using coins other than the ones that exist.
B2227	Yeah, with the coins I think they probably experienced more success. So, student six for example, has got the repeated five lots of tens cents. They've got the two lots of twenty and a ten. And then they've broken the ten down further to get the two lots of twenty and the two fives. The twenty plus three tens. They've really come up with lots of different options.	Students using strategies such as repetition and breaking down of coins.
B2241	Um, and he's actually got, he's got that correct. So, he's done the two twentys and the ten but then he's broken it down further to the three tens, four fives option. And then the five tens... Um, he's not quite calculated this correctly. So, he's got the two twentys and two tens which is sixty cents, not fifty. But, he's trying	Students using strategies such as repetition and breaking down of coins.

B2255	Student five. No surprises. He's done quite well. Um, he's come up with a number of options that are different so he hasn't, um, repeated any, so that was good. He's got the fives repeated. He's got the tens repeated. He's got the fifty plus zero. Two twentys and a ten. Three tens and a twenty. He could've done more and I think, he probably needs more scaffolding to think outside the square and think of more options like a twenty and fives...	As above
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Appendix L

Rachel: Prompts (Theme Three)

Code	Rachel	Analysis
	Final Interview Did not mention not implementing class prompts	Did not use whole class prompts
R2044	Um., I really liked., um the prompts talking about how to organise. So, using the materials ... so the pictures or the actual fake coins in the first one and I liked the, um, extension ones, talking about instead of making fifty cents, could you, what strategies would you use making ten dollars, making a dollar fifty? How many different ways would you do it? And.., um these prompts here. How could you change the number of people? Could you change the number of cookies that each person gets? Because everyone, most people did one cookie per person, um, and then this one in particular I used quite a few times. So, what if there were 3 people? Which seemed to get a lot of kids started very quickly.	Prompts used: • Using materials • Pictures • Increasing the size of the numbers • How many different ways could you do it? • How could you change the number of people? • Could you change the number of cookies that each person gets? • So, what if there were 3 people? • How would you do it differently? • How could you make this more challenging for yourself? • What could you do to change it? • How could you use the same strategy but do it to a different problem? Rachel liked “Could you change the number of cookies that each person gets? Because everyone it prompted students to give people more than one cookie. Rachel liked keeping the prompts general because she perceived
I2045	Oh. Okay. And then so why did you like those ones? You’ve touched on that but...	
R2046	Yep. So that one I think because it’s for such a small number. It’s easy for them to even picture it mentally. They can use those mental strategies. Um, and for this, I knew that a lot of my extension kids would... fifty cents would be really easy for them and they would come up with you know more than six ways quite quickly. So, having that where they could use a combination of notes and coins would be great. For one thing they’re really comfortable with money anyway but it gets them to use two different forma, two different forms of money.	
I2047	Okay. Cool. And are there... did you use any other prompts outside of these ones throughout the lesson?	
R2048	Um. I said, I think I said a few times, quite a few times, how did, would you do it differently?	
I2049	Yep.	
R2050	So, I tried to keep it really general. I tried to keep a lot of my conversation in questions, so it was pushing them to do the thinking. Um, how could you do it differently? How could you make this more challenging for yourself? Just those really general... What could you do to change it? So, you’re using... how could you use the same strategy but do it to a different problem? So really, really general but trying to get them to do the thinking.	

		that her students would do the thinking.
I2011 R2012	Okay. And so what else, like what do you think worked well in general with the lessons? Um. The, I love the questions on the lesson plans. The enabling and the, what is it? The extending prompts... I thought they were fantastic because there were some kids that I didn't want to lead directly but I didn't know exactly how to tell them where to go without giving them an answer. So, using those questions was a really good way of saying it without saying it directly.	Rachel really like the extending and enabling prompts

Appendix M

Beth: Prompts (Theme Three)

Code	Beth	Analysis
B1101 Interview 1 Coz it says, “How many ways?” and I’d refer back to the importance of reading the question. “And It’s not asking me to name a way I can make fifty cents. It’s asking me how many ways can I make fifty cents.” So, I’d be looking to maximise their thinking. “Is that as many ways as possible? Is there any other way possible? “ I1102 Yep. B1103 “Any other way you can think of?” I’d probably even point out, there’s room on the back of that sheet. “If you run out of room on the front, you can always turn it over and do some on the back. “ I1104 /Yep. B1105 Just to set the bar high and make them think. “Oh, there’s enough that I could do two pages.”		Use of prompting to encourage students to find more ways and to look at the important words in the question.
B1115 I’d monitor but then I’d probably be roving because if they would be doing this independently, coz this they’d be doing independently... So, any time we are doing something like that I’m definitely roving. And as I rove around the room, I’ll be looking... and I’ll try and put out any spot fires or identify anyone who’s, um, slow to get started... I1116 /Yep. B1117 slow to think of alternate options, um, and I’d just sort of suggest, you know, “Oh, have you thought of all the ways?” or “How, I wonder how many ways people have.” So, it sort of phrases things like that so, “I wonder how many ways, I wonder how many different ways or different options people are coming up with.” I1118 /Mmm. B1119 “I wonder if people have got like five options or I wonder if anyone’s got six options. Six different ways they could make the coins.” And I’d sort of plant that seed but walk around looking for whose going strong and doesn’t need the support but who’s actually needing a bit of support. I1120 Yep. Okay. No. That makes sense. B1121 And I’d probably direct and I’d go, “Oh, what if you mixed up some coins? Could you make a mixture of coins? Like some of those and some of those?” And then I just sort of plant the seed and still walk away. I don’t want to give them the answer.		Discussing using enabling prompts.
	Interview 2 Didn’t mention not doing whole class prompts	Did not discuss that the whole class prompts were not implemented.
B2018	Um, even this one. It’s interesting to see when you’ve said, “Can you draw a picture?” which is obviously the strip I’ve given the child... so they’ve drawn a picture of cats... [laughter]	Discussing using enabling prompts.

B2042	...like it was test conditions. And I think they had the strips for this as well. So, a lot of them did do what the strip suggested, "Could you draw a picture?" or "Is there another way?" They really did think about it.	
B2067	And I had to re, reaffirm what the question has stated and I said, "What does it say? Repeated..," And they all answered "addition." They knew. But sometimes because they know about turn arounds they wanted to...	
B2075	And stop me from getting other, to other students. One thing I think I did wrong was the strip for eighty-nine as the alternate to sixteen, which was the original question. So, extension kids. I went straight to the eighty-nine coz I didn't see the thirty. Was it thirty-six or thirty-two?	Reflected on giving extending prompt of 89 (which has no solutions). Would have rather had given the 32 strip.
I2076	Yeah, I think it was...	
B2077	There was another strip and I should've given that strip first. Obviously normally I would've, and I think I'd sort of sorted the strips on the sideline into two groups...	
I2078	/Yeah. Yeah.	
B2079	...like the real modification, simplification and the other side was more the extension, um, higher order thinking.	
I2080	Yep.	
B2081	And I think I'd sort of done that but somehow, I've missed that strip, which I wished I'd had of done. And then I saw the other which was like six. "What if the number was six?" So, there was a few that got the easier option which I think was good.	
I2082	/Mmm.	
B2083	And that was good for scaffolding and building the knowledge. And there was one student who once she'd done the easier one went and flipped her page and thought, "Oh wait, I can do more for the sixteen." Coz the six helped her realise she could do more for the sixteen. So, I wish I'd seen the thirty whatever number, what thirty-two I think it was.	
B2095	Definitely that with the repeated addition. I would have it laid out and mapped out.	Reflected positively on giving students the option to reduce the total to six.
I2096	/Yep.	
B2097	And I'd go straight to the default setting of six, which I should've done earlier with some of the students who've just gone...	
I2098	/Mmm.	
B2099	Um, started to do random addition and subtraction sums to equal sixteen.	
I2126	Um, and what would be the impact of the changes you'd make? So, I think you mentioned giving the thirty-two strip.	One student completed the easier version of the problem (to six) and then came back to the original problem.
B2127	/Yeah.	
I2128	You kind of... You did explain.	
B2129	/Yeah.	
I2130	That and then also giving the six out earlier.	
B2131	/Yeah.	
I2132	Like you said, what, how would that change..?	
B2133	Oh. I just think it would help build the confidence and it would help the students to realise, "Oh wait if I can do the	

I2134 B2135	six and I've worked this out because it's more achievable..." So, if they can do the six... /Yep. then the sixteen would seem achievable or more achievable as well. Once you get into two digits some of the, um, yeah, some of the grade ones sort of will go, "Oh. two digits. It's over ten." Um, whereas a lot of them really enjoy the challenge but I think to scaffold it by starting with the single digit. Then building to double, would be the more obvious.	
I2136 B2137	/Mmm. Mmm. And then to go from the sixteen to the thirty-two and then if they can then go to the eighty-nine. But, yeah, I definitely would scaffold it more so they can achieve success and get the stepping stones for how to problem solve before you give them the more challenging problem.	
I2138 B2139 I2140 B2141	Yep. Okay. That answers that question definitely. Um, which prompts did you find the most useful out of the prompt strips? Oh. That was funny. [laughter] Because I remember in the second one, I think I even said to you at one stage... I flashed the strip and said, "This is popular." Because there were some... I actually had to recycle. I had... You'd given me plenty. You'd given me plenty of strips but there was a really popular one. I think it might've been "the six". Um. I used "halve it" a little bit.	Discussing that the prompt to reduce the number to six should have been given out earlier to students who were adding random number to make sixteen.
I2142 B2143	/Yep. But "the six" was definitely very popular and I think it was "draw it".	Liked
I2144 B2145	/Mmm. "Array" was interesting. I gave "array" to one student and she looked at me and said, "What's an array?" And I thought, "It's what we did yesterday."	Reduce the total to six
I2146 B2147	[laughter] But, you know, we've only done arrays twice so then when I gave her the strip "What's an array?" I thought, okay swap that to "Could you draw a picture?" Instead of "Could you draw an array?" But "Could you draw a picture?" Was the most popular prompt and "Making six" was popular as well.	Halve it
I2148 B2149	/Mmm. Coz I think I did realise probably a little later than I should of that sixteen... We were going off on random tangents that weren't that helpful. I if had have brought it back to six I'd probably get more useful data...	Draw a picture
I2150 B2151 I2152	/Mmm. and we could actually see what the students could do. Okay. And what about the... Why was the picture one possible? The "draw a picture."	Array
B2153	Oh. Definitely "picture" because I think, um, for your visual learners... I think they need to see it. It's just not numbers on a page. There's a visual...	
I2154 B2155	/Mmm. Mmm.	

	that can help them with basically counting, um, and keeping track of the count. So, yep definitely “picture” helped for that.	
I2156 B2157 I2158 B2159 I2160 B2161 I2162 B2163 I2164 B2165 I2166 B2167	Yep. Okay. Great. And did you use any other prompts throughout the lesson that weren’t... That you didn’t have strips for, for example? I don’t think so. We had the coins for the coin lesson but for the repeated addition there wasn’t too much I could give them other than the strips. Um, there were counters at the front of the room, but I don’t even think anyone really used the counters. /Mmm. Which was ironic because they were there. I didn’t really highlight it too much, but I do think I said it at one stage, that there were counters. And they were clearly visible. And the students know that there’s usually always counters at the front of the room when we’re doing a maths lesson. So that was interesting too, that they didn’t. Actually, in hindsight too, I guess, if I had’ve given counters, that possibly would’ve made more students ... /Ah. be able to see that if they split into half or if they tried a few... /Mmm. with manipulatives, they may’ve been able to work out the sixteen more successfully. I think I still woulda had a lot of... Well I wouldn’t have had this. I wouldn’t have had nineteen minus three. /Yeah. Because they wouldn’t have had nineteen counters so. [laughter] I don’t know. Hindsight’s a wonderful gift. [laughter]	Reflected upon the fact that manipulatives would have been useful in BL2.
B2197	sometimes that to his detriment. So, sometimes he just goes, “Yeah I got this.” Does it and then doesn’t challenge himself or think further. So, I’m trying to get him to think more critically and to, um, employ more strategies or look for more options. And I know I gave him the strip to say, “Are there more options?” Um, and yeah he just hasn’t really got the task. But I found this in a few assessments where, um, he’s reversed numbers in a subtraction problem. Where, they were baking a cake with twenty-four apples and they used six. How many are left? So, he’s done the subtraction problem and he said, “Six are left coz they subtracted eighteen.” And I’m thinking “but they didn’t cook with eighteen apples. You’ve got the right numbers. Wrong order.” Coz He’s not thinking it through because he’s overly confident.	Student doesn’t challenge himself. Beth prompted student and “are there more options?”
B2203	Now this student is a student who struggles with math. He’s been in Kumon and withdrawn from Kumon for maths but stays in Kumon for everything else. Um, Mum’s asked me... should she get him a math tutor. He does sort of struggle a bit with maths. He’s grade two. But, um, he’s drawn the picture to try and help him but it clearly hasn’t	The halve it prompt can narrow students’ options.

I2204 B2205 I2206 B2207 I2208 B2209 I2210	helped. So, it just... I guess it makes me see... I just need to bring it back to basics and do a lot more manipulatives and drawing to really get him to see the number and how he can break that number apart. Mmm. Is there a prompt that can, would've, would help him? The "half one" and probably starting with "the six". /Mmm. Okay. "Could you halve it?" And then, but then, I think maybe that would narrow down the options with six. Once you've halved it and you've got three and three, I don't think they would see two plus two plus two. Mmm. So Sometimes the prompts can help but sometimes they can narrow... /Yep.	
B2223	option all the way to sixteen. And for some reason I think I may've done... He might be one of the ones that I've accidentally given the eighty-nine not realising the thirty-two was there. Because I'd want to challenge him more. I wouldn't have given him the six, but in hindsight, being a grade one, six would've been a great option...	Could have given a student the prompt of reducing the number to six because he was in Grade One.
B2243 I2244 B2245 I2246 B2247 I2248 B2249 I2250 B2251	And I think had I given him the dollar... it just would've been good to see more of what he could do. So sometimes if you've got the one option it's more limiting to capture how much knowledge the student really has. /Mmm. Okay. You give them the second option, you get more insight into their problem solving skills. Can you tell me more about that... What, how, would, how would..? I think if this student seven had have been given the dollar or even the easier option of twenty cents... /Mmm. he would've come up with... Um, I think it just would've helped him build his problem solving skills and strategies. /Okay. So, if you gave him higher numbers he would have higher order thinking because he'd have more practise with the lower numbers.	If students are given a higher option after a lower option then you see more of their thinking
B2255 I2256 B2257	Student five. No surprises. He's done quite well. Um, he's come up with a number of options that are different so he hasn't, um, repeated any, so that was good. He's got the fives repeated. He's got the tens repeated. He's got the fifty plus zero. Two twentys and a ten. Three tens and a twenty. He could've done more and I think, he probably needs more scaffolding to think outside the square and think of more options like a twenty and fives... /Mmm. or something like that but given more time and, um, more prompting I think he would have arrived at that conclusion himself.	Student 5 needed prompting

B2293	And then, he hasn't, he hasn't... even though I've given him the prompt, he hasn't really tried other options. I think sometimes he looks for the extension task rather than focussing on the task at hand.	This student is always looking for the extension task.
I2294	/Yep.	
B2295	So, he, I think he did actually call out at one stage, "Do I need that one?" coz he saw someone else get the extension task...	
I2296	/Yep.	
B2297	so, he was so focussed on that he gave me one answer, um, for the repeated, for sixteen. He should've done a lot more. Here he's done a lot more and he's even mixed it up with the ten plus five plus ten plus five plus ten plus five. So, he has the ability to do this but he just needs a lot of scaffolding and a lot of reminders to stay on task and really think this through. And not, not just think, "Yeah, I've got this." and not try.	
I2298	/Mmm.	
B2299	Um, then when I've given him the dollar option for the extension, which would've been what he was waiting for... So, he's done, he's done eight options for the fifty cents which is quite good. Um, when he's got to the dollar: one, two, three, four, five. He's got five options. He's numbered four of them but he's got five options: thirty, fifty, sixty, eighty. He's only got to eighty on that one. Maybe he hadn't quite finished that one when I packed the task up. I'm not sure. Um. It's interesting that he's numbered the numbers underneath too., Oh Coz he would've known that five twentys is a hundred.	

Appendix N

Rachel: Share time (Theme Four)

Code	Rachel	Analysis
R1086 I1087 R1088	Initial Interview And then we'd come back together for a reflection or a quick share. /Yep. At the end. If there's time.	Usually have a quick share time at the end of a lesson if there is time.
R1124 R1126 R1128 R1130 R1132 R1134	And the expectation would be that their, you know, their minimum would be five. But it... You know, they already know they can find more than that. Um, my middle group would probably be aiming for about five different ways and then talking, working in pairs to discuss which ones they've done and having a look at each other's to see if they... and then looking and seeing "Oh well that persons done.., two different coins in the same combination. I didn't think to do that." And then sort of going off, re-evaluating and then having another go. So, I'd probably stop groups halfway through, get them to talk about what they're doing and then get back... so, they're going back with fresh eyes Get them to come back to the floor. Um, I'd probably designate people a few minutes before to share. Ah, ask people if they would like to share. I don't tend to just pick people. And get them to share the strategies that they used. What sort of process they worked through. Um, ask the kids, the extension kids what was their most challenging one was and why. So, using as many different coins in one combination I would assume would be that. Talking to the kids in the middle. Asking them what they learnt when they discussed, talked to each other and what combinations they didn't notice but then they started seeing working with other people. And then getting the kids on the floor to explain probably back what we did, you know skip counting and then double checking our counts.	Usually have share time in the middle of a lesson Students share strategies and then continue to work on the task
I2029 R2030 I2031 R2032	Final Interview That, They're the things that you feel like, were the points in which you deviated? Yeah and I didn't have enough share time. I mean, it's always tricky when... you never have enough share time. Um. I think because I stopped, oh I did stop in the middle of the lesson which is the way that I teach. I always stop in the middle and pick out kids who I think are, are doing a fantastic strategy so it gives other kids that idea to go and then have a go. Um, so my share time tends to be in the middle of the session which is what I did in both sessions now that I think about it. Um... /Mmm. and then the sharing at the end was really quick. You know they did this really well. There's four different strategies.	Rachel's share time in the lessons was too short She never has enough share time Rachel does her share time in the middle of lessons

	But I tend to share in the middle so then the kids can have a go afterwards.	
I2079	And so how did you feel about the sharing at the end of the lesson and can you tell me a bit more about the middle of the lesson sharing too?	Share time in the middle of the lessons worked well.
R2080	Um, so the middle of the lesson sharing... I think worked really well in both sessions. Um, so those kids that were doing really well and showing their understanding in a range of ways got to sort of have that success with the class and they got to share it and enjoy it. And then the kids that maybe didn't have as many strategies or many ways to approach it were able to sort of steal ideas from people that were sharing...	Students had success with the class.
I2081	Mmm.	Students who did not have many strategies were able to use other students' strategies.
R2082	which was great because it doesn't matter obviously how they get there as long as they're having a go. Um, so I think that worked quite well. I think the end of, end of the lesson sharing was a bit too short but that's something that happens in my grade. So, um, I think that next time I'd probably give myself more time to go and wander and pick people a bit more selectively.	End of the lesson sharing was too short

Appendix O

Beth: Share time (Theme Four)

Code	Beth	Analysis
B2044	Discussing Oh, we didn't discuss at the end of the first lesson. I do remember that. And I don't remember why, um, and even in the second lesson coz I still had some students at the table.	There was not any share time at the end of Beth's first lesson
B2046	And I think I said, "Oh, you know your brains are imploding. Let's come to the floor." And I sort of gave them that out if they wanted it.	This was because some students were still working on the task
B2048	But, I still had those children, including one girl, who I knew was exhausted and had exhausted her options... "Oh no. No. I wanna try. I really like to challenge myself."	This was also because the topic the task was based on was not of focus at the time
B2050	So, I let them go and then, you know, two or three minutes later I'll check in. "Do you reckon you're exhausted now?"	
B2052	Do you wanna..?" So, yeah, probably not the share time at the end...	
B2054	like, I could have done. And I hope that wasn't detrimental	
B2056	to um your observations.	
B2058	Um, I think we discussed it briefly...	
B2060	but not as in-depth as we could of. I guess because these aren't specifically topics we're doing at the moment. Even though it was good to do a little snapshot and something different. Um, because it's not the topic we're doing currently and we sort of... as a whole unit we have to teach subtraction and then we have to teach multiplication. So, your repeated addition worked in well with the multiplication but the currency was kind of, a little bit more out there. So...	
I2318	Yeah, wonderful. Um, and oh, you've sort, you've touched on this before, but how did you feel about the sharing at the end? You mentioned...	The sharing could have been done more
B2319	Yeah, I wish we'd done it a little bit more. And I think now I might actually share some of this with the students...	The student work samples may be used as models for future lessons
I2320	/Mmm.	
B2321	to show them because there's nothing like the examples they've done to use... Rather than me drawing it on the board and writing it on the board.	This is so that students can experience success
I2322	/Mmm.	
B2323	These are your examples. Look what you were able to do. Look at the options you've thought of. And then to show all these different options... and if I use this students work to model, well then the students might come up with one or two more options. But I also think it allows her to experience success and go, "Wow! Yeah. I did come up with a lot of options." Um, and you could start the lesson with, by going, "Oh I could've just done twenty plus twenty plus ten. That's fifty. Jobs done."	It is also so students may come up with other options
I2324	/Mmm. Mmm.	It is also so students can see how many different strategies could be used
B2325	But you could show... look at how many other options there are. And I think it probably would encourage the students to think that there's... coz we always talk about	

I2326 B2327 I2328 B2329	how there's more than one way of getting the answer in maths. /Mmm. So, this really shows it. So that's a great example. Same with the repeated addition. Both are excellent, um, examples of how there are so many different ways... /Mmm. of achieving the result. So long as the answers right, there's so many different ways of getting there. We all think differently. We all problem solve differently. There's no right or wrong answer as long as we arrive at fifty cents. As long as we arrive at sixteen.	
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Appendix P

Rachel: Problem-solving practice (Theme Five)

Code	Rachel	Analysis
I1043	Initial Interview Yeah. Yep. Okay. Cool. Okay, then, so when someone says to you, you know, what does, what is problem solving in maths? What does that mean to you?	Problem-solving involves engaging students with open ended tasks and giving them the opportunity to find multiple ways of finding solutions, using different strategies, using hands on materials, working in groups and ensuring that mistakes are seen as part of the process of arriving at a correct answers.
R1044	To me it's more open ended tasks or it's giving chil, kids an opportunity to explore a range of processes to getting to an answer. So, it's not necessarily that there's... there might be a correct answer to a problem but there's a whole range of ways to get there.	
I1045	/Mmm.	
R1046	And giving them different strategies. So being able to use pictures.	
I1047	/Mmm.	
R1048	And using hands on materials and working in groups and making sure that failure is part of the process.	
I1049	/Mmm.	
B1050	So, it's not as simple, it's gonna take me five minutes to do this. Or thirty seconds. And it's not just, alright I understand the formula now I can do it. It's trying a few different ways to get to the answer first.	
I1051	Mmm. Cool. Okay. Um, so then in... if you do a um, problem solving lesson, what does that look like at the moment? If you were teaching...	Rachel teaches problem-solving using Peter Maher problem-solving cards and by teaching Peter Maher's nine problem-solving strategies
R1052	At the moment we're doing... so we're doing the Peter Maher problem solving cards.	
I1053	/Mmm.	Rachel chooses a Peter Maher card that requires the use of a similar problem-solving strategy to the card students will use for their independent work time
R1054	Um, and so at the beginning of the year I explicitly taught the nine different strategies that we've got.	
I1055	/Mmm.	
R1056	And I can't remember all of them off the top of my head. Um, but there was make a table, look at the important words in a question, draw a picture... Um, I can't remember the rest but explicitly went through those and then, um, every question we did, we looked back at the problem solving strategies, picked two that we thought were appropriate and then went through one of the questions and modelled one on the board.	
I1057	/Okay.	
R1058	As a whole class focus. And I tended to pick, tend to pick one where I found... in the box you can get quite a few that are similar and so I used one, where I knew that they'd be ones that they could go and then explore themselves that were using the same strategies. So, we would do one together then talk about if it was effective or not and I tend to make mistakes on purpose. So, they'd corrected mine and then I would work with one group. Um. Ah, generally extension or my lower kids and then everyone else would be off trying to work independently or in pairs.	

I1059	Okay. Um, so, so you'd initially be out the front and then you'd work with a group while everyone else was working independently or in groups?	
I1069 R1070 I1071 R1072 I1073 R1074 I1075 R1076	Oh cool. Okay and um what kind of skills would you be ex, like expect your class developing? Like, you touched on it before but what kind of skills? What do you mean? So, I suppose, what kind of problem solving skills would you? Umm., I would hope that they were... I don't know. I don't know how to explain it sorry. [laughter] Um. I'd want them to be able to be really flexible in their thinking. I want them to look at a question and not just... to see beyond it I guess. And try and apply knowledge that they have from other areas and understand that mathematics in particular... It overlaps. Any of the strategies that you use in addition, you could use in subtraction and multiplication but you could also apply some of that to measurement and... /Mmm. Just them understanding that we're not just done with a unit. I've taken those skills rather than just I can do a formula or algorithm, and using that and applying it to different situations.	Students need to be flexible with their thinking, be able to analyse focus questions and apply their mathematical skills
I1077 R1078 I1079 R1080 I1081 R1082 I1083 R1084 I1085 R1086 I1087 R1088	Mmm. Okay. Yep. That makes sense and in the lesson that you'd be teaching, um., what, so your students would be working in groups or at tables or on the floor. Would that be for the... Would the group that you'd be with... How long would you be with them? Um, the entire lesson, part of the lesson? What... Um, so if you give whole class focus, ten to fifteen minutes. /Mmm. Probably, maybe twenty minutes. Um, and for this sort of activity it would be... I'd be working with my, ah, my low kids. Mmm. Mmm. Because I think they would need more of that explicit understanding of the difference between the coins and, um, just tryna remind them of any strategies and any real world situations they've used it. Yep. And so, for the, for, for the entire lesson that would be, or part of it? /The main... most of it. /Yep And then we'd come back together for a reflection or a quick share. /Yep. At the end. If there's time. But yeah, so it would be most of the lesson.	Rachel would work with her students that find maths most challenging and at the end of the lesson students would share their problem-solving strategies with their peers
I1091	[laughter] Okay. Cool. Um and then you did mention this, but in your problem solving lessons, um, what specific	Rachel teaches her students to use the problem-solving

R1092 The strategies I would use? I1093 Yeah. R1094 Um. At the moment we're working particularly on, ah, using pictures and looking for key words. I1095 /Mmm. R1096 We're doing a lot of worded problems because we've got a lot of hands on activities. But then when it gets to the recording and writing of a lot of tasks, that's where the knowledge seems to drop off. Um, so we're doing a lot of worded problems and I'm getting them to explicitly underline or highlight what their... I1097 /Mmm. R1098 what they think is important and then we come back and actually analyse what words they've picked and why and tend to lead kids in the direction if they're picking words that aren't, aren't the important ones. I1099 [laughter] R1100 Yep.	strategies do you use when you're, um, in your group, like with the groups or doing the whole class focus? strategies: use pictures and look for key words
I1107 Um and so this particular problem here... R1108 /Yeah. I1109 like, if you were going to use that with your class, um, what would, yeah, what would you do with this one? R1110 Um, I would talk about, ah, so do you just mean as a whole class or as a... with the group or..? I1111 Ah. Well I guess just if this is the problem you picked up, how would you think it would work best in your class? R1112 Um. I would have a lot of hands on materials for this one. I1113 Mmm. R1114 Um. Do a... Make sure the kids understand the difference between the coins obviously. Remind them that you know, from smallest to biggest is the simplest way to remember. I1115 /Mmm. R1116 And remind them that this is skip counting. It's addition. For those top kids this would be multi, multiplication coz that's what we're touching on now. Um, so tryna create those links between what we already know so they're not quite stumped and stuck on what they think they don't understand and then practice. So, I would leave the fifty cents off completely and maybe just have a look... have a hundreds chart out and do the tens and then skip count by tens. I1117 Mmm. R1118 See how many we need and then dr, write it and draw it on the board. I1119 Mmm. And so, and so that's what you'd do as your focus... whole class focus. That's what you mean R1120 /Yep. I1121 And then what would happen next? R1122 Um. So, I would send... I'd probably send off my, my extension group. They, I would give them some coins but I doubt they'd need it and just say that they have to come up with every possible way. I1123 /Mmm. Mmm.	If Rachel were to use the focus problem: How many ways can you make 50 cents using coins? When introducing the task, she would create links for her students between their mathematical knowledge and the problem. She would also model an example answer In the middle of the lesson, Rachel would ask her students to share their problem solving strategies with each other before further attempting the task At the end of the lesson her students would share their problem solving strategies again for a short time

R1124	And the expectation would be that their, you know, their minimum would be five. But it... You know, they already know they can find more than that. Um, my middle group would probably be aiming for about five different ways and then talking, working in pairs to discuss which ones they've done and having a look at each other's to see if they... and then looking and seeing "Oh well that persons done.., two different coins in the same combination. I didn't think to do that." And then sort of going off, re-evaluating and then having another go. So, I'd probably stop groups halfway through, get them to talk about what they're doing and then get back...	
I1125	/Mmm.	
R1126	: so, they're going back with fresh eyes and then my group on the floor, we would be physically skip counting and putting the coins in, in rows or in lines and then trying to figure out how many we sort of need. And I wouldn't expect them to know that if you've got that many fives then you need half the amount of tens and...	
I1127	Mmm. Yep and so you'd be with them on the floor and then, um, at the end of the lesson, like what would happen then?	
R1128	Get them to come back to the floor. Um, I'd probably designate people a few minutes before to share. Ah, ask people if they would like to share. I don't tend to just pick people.	
I1129	[laughter]	
R1130	And get them to share the strategies that they used. What sort of process they worked through. Um, ask the kids, the extension kids what was their most challenging one was and why. So, using as many different coins in one combination I would assume would be that.	
I1131	/Mmm.	
R1132	Talking to the kids in the middle. Asking them what they learnt when they discussed, talked to each other and what combinations they didn't notice but then they started seeing working with other people.	
I1133	/Mmm.	
R1134	And then getting the kids on the floor to explain probably back what we did, you know skip counting and then double checking our counts.	
I1135	/Mmm.	
R1136	Making sure that it's the actual value that we're aiming for.	
R2054	Final Interview Um. At the end of that, all except for the few, the two that struggled, they were so excited. They said, "Can we do it again?" "We wanna do another one." They were really, really, really engaged. They loved these activities, um, and because of the engagement I've actually planned a lesson based on this sort of format for next week and they're really excited. So, I think for a lot of those kids it's created that enjoyment of maths...	After trialling the lessons, Rachel has planned a lesson based on the intervention
R2056	whereas that traditional maths of doing sums and equations and, and things where they either know the answer or they don't. Rather than being able to explore. Um, those kids,	She will use the question: How many ways can you make an array with one total number?

R2058	you know, they tend to miss out a lot so I think this is a great way for my, my lower students to engage and enjoy maths again. So, we're doing multiplication at the moment and we're doing arrays as a strategy. So, I've got a question I think my que, I can't remember exactly. Um. How many ways can you make an array with one total number? So, for my low kids it would be, um, ten. How many different ways can they make it using counters or drawing? So, they have to change it to do different arrays so two rows of five, one row of ten, five rows of two. And then for my middle group it would be, ah twenty, I think it was thirty... and then for my top group it would be fifty or one hundred.	She will differentiate the task by giving her ability groupings different totals to work with Rachel create activities based on the intervention once a week or once a fortnight
R2060	So, they're all using the same question. It's differentiated based on their needs they can do lots of different strategies or use lots of different materials.	Rachel indicated that the intervention is beneficial because it is simple, open ended, all that is needed is some materials and students' who do not usually achieve success can achieve success
R2062	Yeah. I'm gonna use this sort of approach probably weekly or fortnightly and try and create activities in this format.	
R2084	Really, really open ended. Really problem solving based but so simple that they don't need, they don't need any worksheets. They just need some materials and some time to have a think and try whatever they can. So yeah, I'm gonna change some of my lessons probably reg, regularly to, to work this way and sort of see what I get out of students I don't expect...	
R2096	Well, I wanno... I've always done... ah, not always. In the last year I've done problem solving from the program that we use, ah, Back to Front Maths which is quite wordy. So, you've got to spend a lot of time scaffolding that language because obviously it's not about the reading skill, it's about the maths, um, which I've always found to be a drawback. And I try and do them as little as possible now because I think it just takes too much time and when they've actually understood the question... by that time they've sort of lost the interest in the maths aspect. So, I think this approach... really simple language... really the instruction is clear but you can approach it however you want. This way is a lot easier. Especially for one twos.	When comparing to the Back the Front Maths program that Rachel has used she said that it contains much more complex wording than the intervention which results in students' losing interest in the tasks

Appendix Q

Beth: Problem-solving practice (Theme Five)

Code	Beth	Analysis
I1059 B1060 I1061 B1062	Initial Interview When someone says to you problem solving in maths, what do you... what does that mean to you? Oh! we talk about that all of the time ... [laughter] /Yeah. in the classroom. And we always say a problem is not a problem. It just means an algorithm. A problem is just a question that we're needing to answer. And we always talk about how maths is fun and that we're detectives and the answer is right there. It's right in front of your face and you've just got to work it out.	Beth indicates that a problem in problem-solving is just a question that needs to be answered
I1100 B1101 I1102 B1103 I1104 B1105 I1106 B1107 I1108 B1109 I1110 B1111	Cool. Um, so then, yeah. So, what specific strategies would you use in your problem solving lessons? Oh, okay. So, we use the Peter Maher problem solving cards. /Yep We try to do visuals, like on the board, so I can show it. And then I'll ask the students to name the strategy they used... /Yep. or the method that they used and then I also point out that, yeah look we did it three different ways but no way is the right way or the wrong way. /Mmm. As long as we're getting the correct answer and we're able to work it out. We all do things differently. /Mmm. but and I think that's kind of reassuring for kids' coz, just coz they can't skip count or just because they're good at skip counting, but they don't know that other strategy, they know they can always default to skip counting, coz that's their strength. /Mmm. Mmm. So, I always try and show multiple ways of doing it and I try to just consolidate everything and approach it from lots of different angles. Lots of different materials. Lots of different lessons. The same thing over and over again. Multi context.	Beth uses Peter Maher cards Beth displays visual images for her students Beth asks her students to share their problem-solving strategies with their peers Beth stresses to her students that there are many ways to arrive at an answer and she models many different ways
I1124 B1125 I1126 B1127 I1128 B1129	Yep. Yep. No. That makes sense. And then so how do the, how do the, the... you've kind of answered this already but I guess, how does the strategy that you would use, help develop their problem solving skills? Hang on, so you mean the strategies I use... In your teaching... Oh. Okay. Um, I think because where there's a big focus on maths being fun and not giving up and just trying a different way... /Mmm. and just persevering and trying something that doesn't work try something else. So, I think that they look for	Beth encourages her students to persist with difficult tasks while focussing on them having fun

I1130 B1131 I1132 B1133 I1134 B1135	alternative solutions and they, um... I think coz we do a lot of things as a group too, so there's that support of the group. I did have one student say today, "Oh but I didn't put my hand up." And I said, "Oh yeah but I still choose you." [laughter] And I think sometimes when you do see that they're not participating, you sort of set it up so that there's an easier option or a, you know something that they will be confident to answer. Just to help bring them back into it so that they haven't, you haven't lost them. /Yep. Coz I think it's just easy, like in that situation today. I could've moved onto three digit but it woulda been the worst possible thing to do because I just woulda lost so many of them. So, I think the strategies I use mainly is just to try multiple different ways of doing it. Yeah. Yep. And yeah, I suppose in the teaching, I often have hands on stuff. I often have like, um, small group where they're working with me. I'll have, um, different worksheet options or different, um, self-directed.	
I1140 B1141 I1142 B1143 I1144 B1145 I1146 B1147 I1148 B1149 I1150 B1151	Yeah. Yeah. Yep. Okay. Cool. Um, so then what would a, so I guess what would a problem solving lesson that you teach, um, look like? Oh. Okay, so. I often set it up... So, like we do a warm and we'll do the mini lesson. So, the warmup to get the brain switched on and thinking mathematically. Then I'll do the mini lesson where we'll do, like, a whole group, sort of, focus. And we might go through one of the Peter Maher problem solving cards together. /Mmm. Then I might put another one up and, or I might send them off in their three... they're mainly in three groups. Sometimes I send them of in pairs or little trios to work together coz I like the collaborative approach to the problem solving. /Mmm. So, if I've already modelled one on the board, they can go off and do one as a trio or in their maths group. Or it might be that there's a grade one and a grade two. /Yep. Off you go. Here is the problem on the board. We've discussed it... /Mmm. but now I need you work out the answer. But I'll fully scaffold it before I'd sent them of to answer it because I realise that some of them, even reading the question might be the challenge. And I don't want a child to not be able to answer it coz their literacy level isn't high enough. /Mmm. Yep. Okay. Generally speaking, their literacy level is high enough but just for the sake of it being that it could be a literacy thing, I don't want that to stop the child from being able to problem solve.	Beth scaffolds tasks for her students by modelling a Peter Maher problem-solving card before they work independently Her students work in three ability groups and sometimes work collaboratively

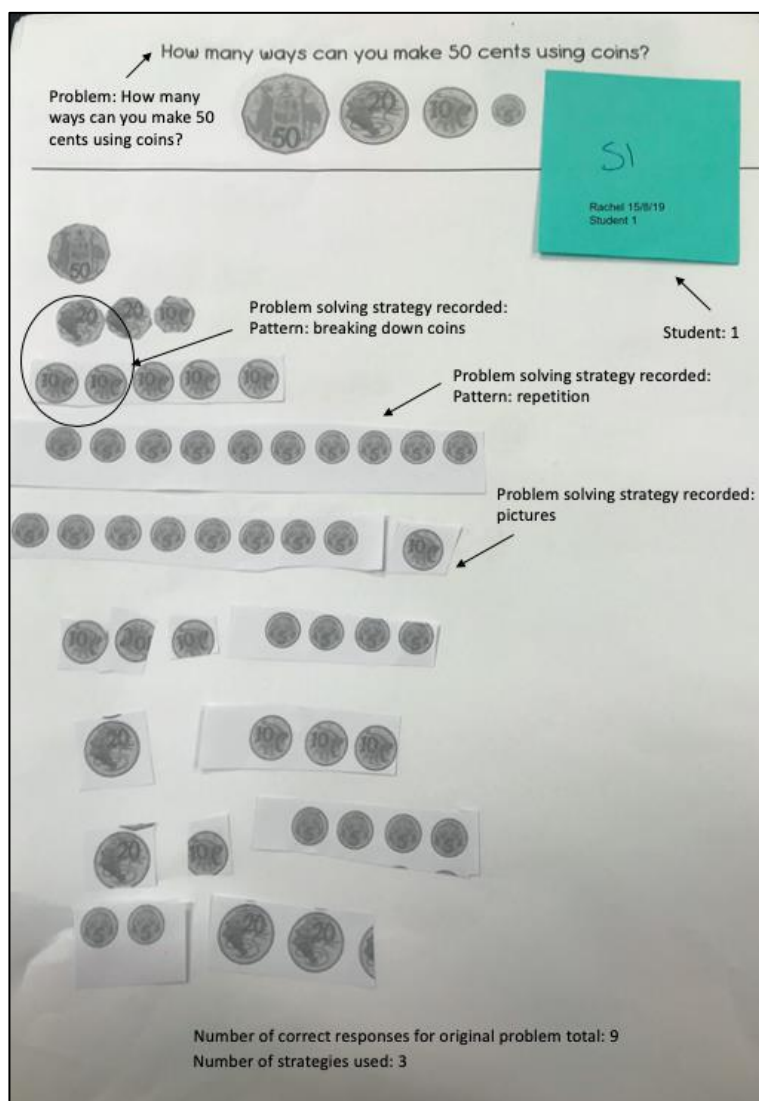
B1163	I normally always have a group with me. Um, so yeah, it would be rare that I'm roving in maths. And if I don't have a group, I'd be roving but that's kind of rare. I'm usually always with a group or supporting students who need the support.	Beth usually works with one of her three groups while the other students work independently
I1164	Yep.	
B1165	And just answering the million questions that I get.	In one of Beth's
I1166	Yeah, okay. And what kind of questions might they ask?	problem-solving
B1167	Ah. Sometimes it'll be... Like even though we've just clarified... I... The lesson we just had then. We had to put four insects in order. It was a problem solving lesson actually. And, um, there were four insects to put in order. And it was just interesting that even though I gave a visual where I picked a really tiny little student, and then my grade six helper and then me and I said, "Look. I've lined us up. Shortest to tallest..."	lessons her students had to put four insects in order
I1168	/Mmm.	A strategy Beth's students focus on is reading the key words in questions
B1169	"You can see Jill, Ewan and Ms B. We're in order, Shortest to tallest." So, some kids just wrote the insects or drew them. Didn't put them in order and some did the reverse order, so they did the longest insect to the shortest.	
I1170	/Ah.	
B1171	Which doesn't really, really matter but when you think about it with worded problems and the importance of getting the answer correct, if the question is which is the shortest insect and you've started off at the longest. Incorrect answer.	
I1172	/Mmm.	
B1173	So, we do a lot focus on that too I guess, on the importance of reading the key words. Yeah.	
I1174	Yep, of the question. Cool. So then how would you use a problem like this particular one here? Um, about the, "How many ways can you make fifty cents using coins?" How would you use that problem in your lesson?	If Rachel were to use the focus problem: How many ways can you make 50 cents using coins? Beth would encourage her students to make sure they have found all of the possible options. She would prompt her students by referring to the important words in the question and prompting her students by asking: Is that as many ways as possible?
B1175	So I would have kids who would go off and do two ways and go, "Done it".	
I1176	/Yep.	
B1177	And sometimes in my classroom it's a race to be first. And I always point out that it's not about being first, it's about checking you're work and making sure you've got all the options.	
I1178	/Yep.	
B1179	Like with the problem solving cards, it always has the extension at the bottom and I encourage the kids to always look for that, which they do seem to notice. Again, that sort of helps with the worded problems. But with this, if I had a kid who rushed through and came up with two options, I'd go, "Oh, is that all of the ways?"	
I1180	/Mmm.	Beth would model an example of a similar task
B1181	Coz it says, "How many ways?" and I'd refer back to the importance of reading the question. "And It's not asking me to name a way I can make fifty cents. It's asking me how many ways can I make fifty cents." So, I'd be looking to maximise their thinking. "Is that as many ways as possible? Is there any other way possible? "	Beth would then rove and prompt students who need support. She would not give

I1182	/Yep.	her students the answers but would prompt them with: Have you thought of all of the ways; What if you mixed up some coins?
B1183	“Any other way you can think of?” I’d probably even point out, there’s room on the back of that sheet. “If you run out of room on the front, you can always turn it over and do some on the back.”	
I1184	/Yep.	
B1185	Just to set the bar high and make them think. “Oh, there’s enough that I could do two pages.”	
I1186	Yeah. Yeah. Um, and then, so what would you do? Like how would you introduce it and all of that kind of thing?	
B1187	Oh. I’d probably model it but I wouldn’t use that specific example.	
I1188	/Mmm.	
B1189	Sometimes I would. It depends on the context. But if I wanted to know... To get them thinking, I might do it with twenty cent first. And just go, “What are the ways could I make the twenty cent?, How many ways can I make the twenty cents using these coins?” The twenty, the tens and the fives.	
I1190	/Mmm.	
B1191	And then hopefully doing that modelling someone would pick up on the ten and two fives. So, they’d see the multiple denomination option. Um, but yeah, I’d preferably model it but not with the fifty cent. I don’t think... I think I’d probably use a different option to get the warmup and the thinking and then send them off to do this task if I want to know what they know, not just remember what I’ve done five minutes ago.	
I1192	/Yep. Okay.	
B1193	Which often proves to be not much, even though I’ve just done it five minutes ago.	
I1194	[laughter] And then what would you be doing when... So, you said you’d send them off to work in groups.	
B1195	I’d monitor but then I’d probably be roving because if they would be doing this independently, coz this they’d be doing independently... So, any time we are doing something like that I’m definitely roving. And as I rove around the room I’ll be looking... and I’ll try and put out any spot fires or identify anyone who’s, um, slow to get started...	
I1196	/Yep.	
B1197	slow to think of alternate options, um, and I’d just sort of suggest, you know, “Oh, have you thought of all the ways?” or “How, I wonder how many ways people have.” So, it sort of phrases things like that so, “I wonder how many ways, I wonder how many different ways or different options people are coming up with.”	
I1198	/Mmm.	
B1199	“I wonder if people have got like five options or I wonder if anyone’s got six options. Six different ways they could make the coins.” And I’d sort of plant that seed but walk around looking for whose going strong and doesn’t need the support but who’s actually needing a bit of support.	
I1200	Yep. Okay. No. That makes sense.	
B1201		

I1202 B1203	And I'd probably direct and I'd go, "Oh, what if you mixed up some coins? Could you make a mixture of coins? Like some of those and some of those?" And then I just sort of plant the seed and still walk away. I don't want to give them the answer. /Mmm. Same as when I hear students telling each other the answer. I always say, "Oh, don't tell him the answer. Tell him how to work out the answer or give him assistance but don't give him the answer coz that's not actually helping him."	
I2330 B2331 B2337 B2339	Final Interview Mmm. That makes sense. Um, is there anything in particular that you've gained from trialling the lessons? Oh, heaps. Absolutely heaps and I really mean that. I think, I will not just say it, I will actually use these as examples to teach a mini lesson. I think I will. I always try to get that one on one time to sit down with the kids. It's just never there but... using their language, they'll be able to explain it in a way that they'll understand. But yeah, I would definitely use this. I've used it for data. Um, and I think I will use it to inform some lessons and maybe teach some mini lessons as well.	After trialling the lessons, Beth indicated that she will use the student work samples as the focus for class introductions and she has used it as data to inform her teaching of concepts
I2340 B2341	Okay and is there... and is it... will it impact on your future practice of teaching problem solving? Yeah. I think with the resources and the visuals. Like, you always think about it but I guess... and you know you've got visual learners but I think with numbers, maths sometimes can become a blur. But when you've got the manipulatives or when you've got the students drawing it they can see it. It's something concrete it's not just...	When prompted to explain how her participation in the study has impacted on her teaching of problem-solving, Beth indicated that concrete materials and visuals are really important for students to use

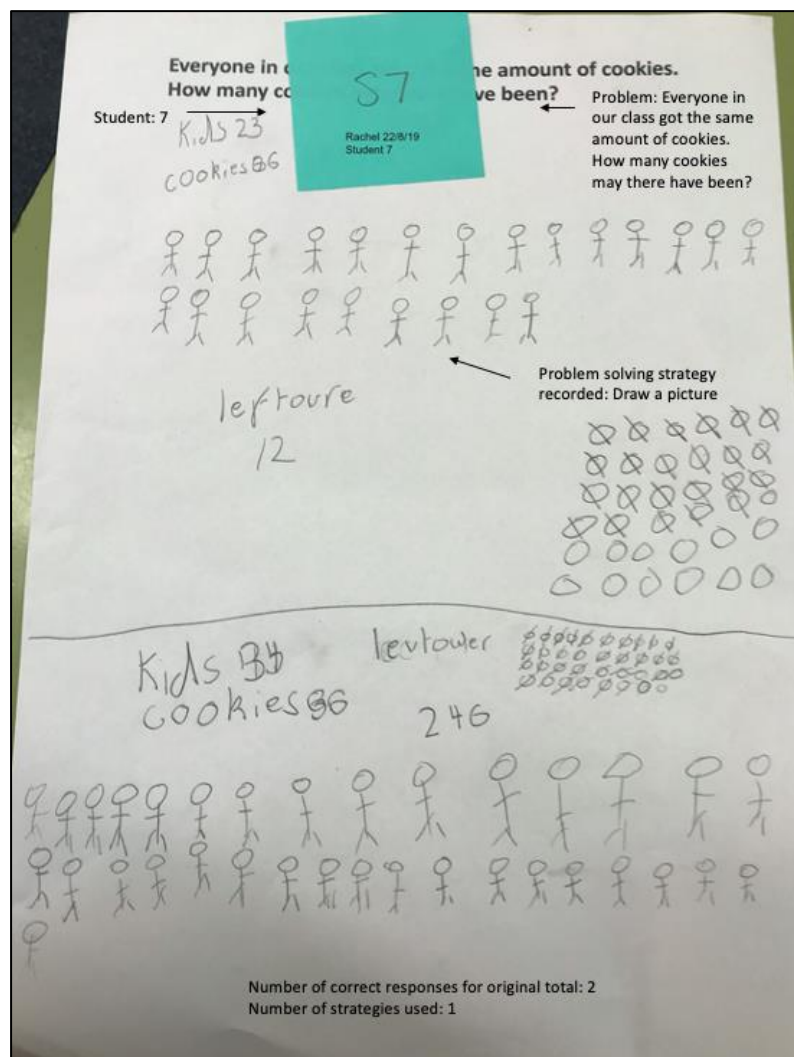
Appendix R

Annotated work sample from RL1



Problem	Student	Problem-solving strategies recorded	Number of strategies used	Number of correct responses for original total (out of 13 possible responses)	New total as a result of teacher prompting	Number of correct responses for new problem total (out of 50 possible responses)	Researcher comments
How many ways can you make 50 cents using coins?	1	Draw a Picture Look for a Pattern (repetition and breaking down coins)	2	9	N/A	N/A	N/A

Annotated work sample from RL2.



Problem	Student	Problem-solving strategies recorded	Number of strategies used	Number of correct responses for original total	New total as a result of teacher prompting	Number of correct responses for new problem total	Researcher comments
Everyone in our class got the same amount of cookies. How many cookies may there have been?	7	Draw a picture	1	6	N/A	N/A	N/A

Annotated work sample from BL2

I added the same number over and over until I reached 16.
Which number may I have started with?

10+6=16
13+3=16
10+6=16
1+15=16
14+2=16
5+5+5=16
16+0=16
4+12=16
5+11=16
7+9=16
8+8=16
9+7=16

Problem: I added the same number over and over until I reached 16. Which number may I have started with?

Student: 7

Problem solving strategy recorded: Draw a picture

Problem solving strategy recorded: Number sentences

Researcher comment: Misunderstood the question as numbers were not repeated

Number of correct responses for original total: 1
Number of strategies used: 2

Problem	Student	Problem-solving strategies recorded	Number of strategies used	Number of correct responses for original total (out of 4 possible responses)	New total as a result of teacher prompting	Number of correct responses for new problem total (out of 50 possible responses)	Researcher comments
I added the same number over and over until I reached 16. Which number may I have started with?	7	Number sentences Draw a picture	2	1	N/A	N/A	Misunderstood the question as numbers weren't repeated.

Appendix S

RL1: Student Work Sample Analysis

Task: How many ways can you make 50 cents using coins?

Student	Problem-solving Strategies Recorded	Number of strategies used.	Number of correct responses for original total (out of 13 possible responses)	New total as a result of teacher prompting	Number of correct responses for new problem total (out of 50 possible responses)	Researcher Comments
1	Draw a picture Look for a Pattern: breaking down coins Look for a Pattern: repetition of like coins	2	9	N/A	N/A	N/A
2	Draw a picture: Look for a Pattern: repetition of like coins	2	2	N/A	N/A	Didn't use combinations of coins. Included 2 and 1 cent coins.
3	Number sentences Look for a Pattern: breaking down coins Look for a Pattern: repetition of like coins	2	3	N/A	N/A	Resilience (scribbled out work). Used \$2 coins to add to 50 cents Accuracy
4	Draw a picture	1	0	N/A	N/A	Appeared to not understand the task. Drew a 5c, 10c and 0c.
5	Number sentences Draw a picture: Look for a Pattern: repetition	3	8	N/A	N/A	Added and subtracted correctly

	of like coins					
6	Number sentences Look for a Pattern: breaking down coins Look for a Pattern: repetition of like coins	2	8	N/A	N/A	Included a coin worth 0. Added and subtracted correctly
7	Number sentences Look for a Pattern: repetition of like coins	2	3	N/A	N/A	Added and subtracted correctly Used a \$2 coin and \$99 coin

Appendix T

RL2: Student Work Sample Analysis

Task: Everyone in our class got the same amount of cookies. How many cookies may there have been?

Student	Problem-solving Strategies Recorded	Number of strategies used.	Number of correct responses for original total	New total as a result of teacher prompting	Number of correct responses for new problem total	Researcher Comments
1	Write a story Look for a pattern: people and cookies increasing	2	14	N/A	N/A	Did not alter the number of cookies people were given.
2	Draw a picture Represented an answer in more than one way	3	2	N/A	N/A	
3	Write a story Look for a pattern: people and cookies increasing Draw a picture	3	23	N/A	N/A	
4	Draw a picture	1	1	N/A	N/A	Did not alter the number of cookies people were given.
5	Write a story Number sentences Highlighted key words Represented an answer in more than one way	4	1	N/A	N/A	
6	Draw a picture Write a story	2	4	N/A	N/A	Halved a cookie.
7	Draw a picture	1	2	N/A	N/A	Used leftovers Did not alter the number of

8	Draw a picture	1	1	N/A	N/A	cookies people were given.
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Appendix U

BL1: Student Work Sample Analysis

Task: How many ways can you make 50 cents using coins?

Student	Problem-solving Strategies Recorded	Number of strategies used.	Number of correct responses for original total (out of 13 possible responses)	New total as a result of teacher prompting	Number of correct responses for new problem total (out of 50 possible responses)	Researcher Comments
1	Number sentences Look for a Pattern: breaking down coins Look for a Pattern: repetition of like coins	2	7	\$1	4	Included a coin worth 0.
2	Number sentences Look for a Pattern: repetition of like coins	2	1	N/A	N/A	Accuracy Tried doing part part whole.
3	Number sentences Look for a Pattern: breaking down coins Look for a Pattern: repetition of like coins	2	6	N/A	N/A	Included a coin worth 0. Accuracy.
4	Number sentences Look for a Pattern: repetition of like coins	2	3	N/A	N/A	Included a coin worth 0. Accuracy. Tried doing part part whole.
5	Number sentences Look for a Pattern: repetition of like coins Look for a Pattern: breaking down coins	2	3	N/A	N/A	Included a coin worth 0.

6	Number sentences) Look for a Pattern: breaking down coins Look for a Pattern: repetition of like coins	2	6	\$1	3	Included a coin worth 0. Included a 30 cent coin.
7	Number sentences Look for a Pattern: repetition of like coins	2	3	N/A	N/A	Didn't appear to break down coins.

Appendix V

BL2: Student Work Sample Analysis

Task: I added the same number over and over until I reached 16. Which number may I have started with?

Student	Problem-solving Strategies Recorded	Number of strategies used	Number of correct responses for original total (out of 4 possible responses)	New total as a result of teacher prompting	Number of correct responses for new problem total	Researcher comments
1	Number sentence Draw a picture	2	1	89	0 (out of 0 possible)	With extension task didn't repeat same number One answer before given extending prompt.
2	Number sentences Draw a picture	2	1	6	1 (out of 3 possible)	Misunderstood Question: didn't repeat same number in both number sentences and pictures
3	Number sentences	1	4	6	3 (out of 3 possible)	After finding all solutions for the original question was given lower total.
4	Number sentences	1	1	6	1 (out of 3 possible)	Misunderstood Question: didn't repeat same number Wrote prompt "Can you draw a picture?" on page and drew some cats.

5	Number sentences	1	2	N/A	N/A	Misunderstood Question: didn't repeat same number
6	Number sentences Draw a picture Represented an answer in more than one way	3	1	6	1 (out of 3 possible)	Misunderstood Question: didn't repeat same number Wrote prompt "Are there more answers?" on page and wrote a response explaining that you break the number into groups.
7	Number sentences Draw a picture	2	1	N/A	N/A	Misunderstood Question: didn't repeat same number in both number sentences and pictures.

Appendix W

RL1: Lesson Observations

Deviations: Yellow
Green

Problem-solving Strategies: Pink

Prompts:

11:33 Teacher 2 15/8/19

How Many Ways Can Make 50c Using Coins?

What are we doing? Who can tell me? * Money.

Turn and talk about what you know about money.

I'll pick 3 people to share.

 ← On wall Someone saw it. Teacher moved it.

What do you know? * There can be notes and ~~coins~~ ^{numbers}.

Can anyone add? * There are coins * You can get change

Does anyone know any coins? * \$2 (5c) (10c) (20c) (50c) (\$1)

* What about (1c)? Put it on board. * It's not Australian money * It used to exist.

Is there anything else we should put up? * Notes.

* \$100 Draw a dollar sign. Where does it go? Start or end?

* start. * \$50 \$20 \$10 \$5 \$2 \$1 Is there 13? Hands up for yes. Hands up for no. * No. \$10.

Put them in order from smallest to largest. (talk)

What's the first one * 5. 5 what? (5c) (1c) (20c) (\$1) ^{some people shaking heads}

* What can we do (50c) (\$1) (\$2) (\$5). Scaffolded until they were ordered correctly.

Held up question sheet and said... I'm not going to help you. Here I have pics of coins or fake coins. You can draw. You can write. You can use our problem solving strategies.

Read question. Said to record all thinking on the page.

If you are in cannots work on the floor.

Feas and Beetoots work at your tables.

11:50. Some kids cutting coins. Some writing addition equations. Some skip counting. Some drawing coins.

Group on the floor with lots of coins in the middle. All coins.

* Do we draw coins? Yeah sure.

Teacher asked students on the floor what she did.

* Got 10s. Counted by 10s. * Can you use coins? * Yes.

Teacher 2 15/6/19
Teacher not really supporting groups on floor. They're using the coins.

Teacher reminding students of question if needed.

Students talking about what they're doing.

On board if you make 50c as many ways as possible, try 15c.

Someone did $2+2+...+50$ Teacher said that would make 50c.

Try and use your strategy to make 50c.

Teacher on floor - says on floor. Student 13 Teacher: let's count on from 10. 15.

Could we try it another way? Aww. Aww. Aww. Aww.

Could you use 10s? Yep. Could you think of another way? Could you use counting by another number?

Stopped class. Remember the question asks you how many ways you can do it. Push yourself.

Some people tracing coins. Some putting 3 signs in front of coins. Some subtracting from \$1 etc.

Teacher hasn't looked at what people around the room have done. Teacher is on the floor.

Teacher helping child on the floor skip count by 5s. Get out 100 chart.

Teacher: Which coins can we use?

Lots of students using 32 and \$1 and subtracting.

Can you add those for me? Student came up to teacher.

Teacher helping this child skip count on 100 chart.

12:13pm.

4 students out the front waiting for the teacher.

12:16pm. Can you make it another way?

12:17pm. stopped class. 4 more minutes.

People on floor. I'm going to go and check how the people at their tables are going. Some students came out to teacher.

Do we have 40c coins?

Is there another way? Use the same strategy with a different coin. Record it however you please. No subtraction - find another way. Use the coins. Try mixing coins together. Quite a few times.

Teacher still out the front.

12:22pm. Everyone stand up 3 people need to share 1 way they solved the problem. Teacher picked 3 people with hands up.

Hand up if it was tricky? Easier? Teacher 2 15/8/19.

■■■■ tell us one way you made it?

$50 + 50 - 50 = 50$

⁵⁰
Hand up if you did 50c by itself? Did anyone think you couldn't do it.

■■■■ share one way you made it and how.

* 10c coins. I was counting by 10s.

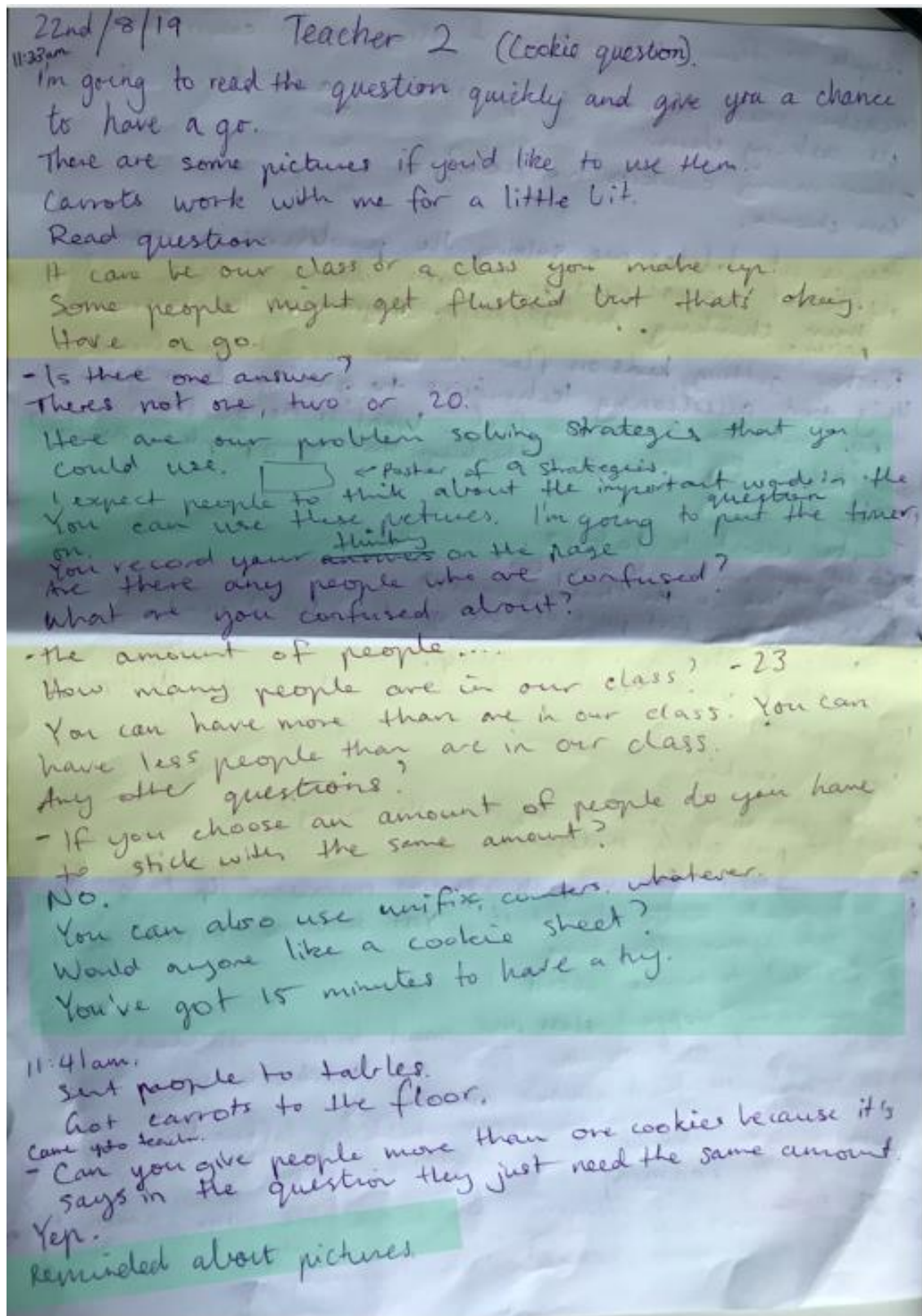
Did anyone else skip counting?

Was it easy or hard and why? * When we count by 10s we land on 50.

Last sharer * 20c 10c 10c 5c 5c.

RL1: Lesson Deviations: Red Text

How many ways can you make 50 cents using coins?			
Time	Activity	What the students are doing	What the teacher is doing
8 minutes	Introduction of task.		Introducing the task. Ensuring students understand task. Not showing students how to solve the problem. coin can be used once or several times. Explaining to students that they need to individually record their problem solving strategies.
15 minutes	Solving the task.	Discussing, solving the task and individually recording problem solving strategies.	Roaming, observing and thinking about which groups/students need to be given particular prompts without intervening. Rachel prompted her students during this period of the lesson.
5 minutes	Introducing prompt 1 to class.		Displaying and discussing John's solution to the problem: "John said there were 3 ways you can make 50 cents. Is he correct? Why? Why not?" Rachel did not introduce prompt 1.
10 minutes	Solving the Task.	Discussing, solving the task and individually recording problem solving strategies.	Giving students enabling prompts where necessary. Then, giving students extending prompts where necessary.
2 minutes	Introducing prompt 2 to class.		Discussing Kath's solution: "Kath said there were 13 ways to make 50 cents. Is she correct? Why? Why not?" Rachel did not introduce prompt 2.
10 minutes	Solving the task.	Discussing, solving the task and individually recording problem solving strategies.	Giving students enabling and extending prompts where necessary. Selecting individual students who have used efficient problem solving strategies and notifying them that they will share their problem solving strategies with the class. Rachel did not select specific students to share their problem solving strategies.
10 minutes	Share time.	Sharers share problem solving strategies. Class engage in discussions.	Ensuring that students are staying on track and speaking about the problem solving strategies they've used.



Teacher 2 22/10/19
People on floor were asking if they could count the cookies.

Teacher said to read the question again. Asked what it's asking them.

- How many cookies are there?

You choose.

People at tables are solving the problem in many different ways: pics, equations, drawing, lists, writing their thinking. Arrays, keys, diagram of the classroom.

Teacher getting kids on floor to ~~read~~ reread question using good questioning techniques to guide floor kids eg. what should you do first? What is the question asking? Explain your thinking. How many people in the class?

11:49 am

Teacher went to tables.

- * Could you make it simpler?
- * Go at look at the strategy cards?
- * You've done the first part of the question. What next?

Someone said she's getting mixed up.

Teacher asked what student was doing. she said "groups of" 10 groups of 7

↑
people cookies

- * Could you show your thinking in another way?
- * Look at prob solving strategies

Someone asked "Can we pick random numbers" / up to you. Do I have to write about it? / You tell me. I just need to see your thinking. He decided to write about it.

11:50 am Teacher stopped class and said 5 more minutes until they'd meet on the floor for a check in.

- * How could you find another answer?
- * How could you explain your thinking differently?
- * Could you change the number of people?

Teacher still roaming.

Student said "Ahh everyone won't get the same amount?"

- * What could you change? "Ah the amount, people"

Teacher 22/8/19.
★ How could you find another way?
- Change the numbers.

12:00 - 15 mins before we meet on the floor.
Teacher looked at prompts.

★ Is there another way you could show me?

★ Is there another way? Again - Again - again.

★ Let's look at important words Again.

★ Went through strategy poster with a student.

12:05 stayed class. - Names on sheets.

- 1 more minute before stopping.

12:06 Please quickly come to the floor. Bring your sheet.
Some people were annoyed about coming to the floor.

Hands up if you're finding it tricky - (about 8 hands)

Hands up if you've found a solution (lots of hands).

Who can share their strategies?

Listed - numbers getting bigger each time.

Picture - cookies above people's heads and a story.

Picture - ↑ cookies in hands - 1 per person.

★ Did anyone give people a different amount of cookies?
24 and 36
not cookies - explained that she worked out it
wasn't right.

★ What tells us you can't do that in the question? (same amount).

4 cookies per person and drew an array.

Picture - drew people with numbers above their heads.

I used counters.

12:14 pm.

If you're stuck please stay on the floor. If not go
back to your table and find another way. Think
about the strategies people shared and use one that
might work for you.

★ What if there were 3 people?

How could you show your thinking? Draw.

★ How else could we show it? Another?

Lots of people at tables tried different ways of
working it out.

★ What are you going to change? 15 ppl again?

★ Find another way.

12:19 pm 5 minutes and then we're packing up.

- Teacher 2 22/8/19.
- * How many people are there?
 - * How many cookies are there?
 - * How many do they each get?
 - * Can you show one of your answers in another way?
- Student said "I like this activity"

12:26pm. Pack up

Sabine (said he didn't want to share).

Jama and Floss... can you share?

- I wrote a story. Teacher asked if anyone else did stories? 3 hands.
- Read out stories. Teacher asked how she changed for the second one... She said she changed the no.
- I did an array, groups, story, sum, addition, vertical (repeated).
- I did examples. I did one and 1 person miscel a cookies but I tried again and made the person happy they were sad.

RL2: Lesson Deviations: Red Text

How many cookies?			
Time	Activity	What the students are doing	What the teacher is doing
10 minutes	Introduction of task: Everyone in our class got the same amount of cookies. How many cookies may there have been?		Introducing the task. Ensuring students understand task. <u>Not showing</u> students how to solve the problem. Explaining to students that they need to individually record their problem solving strategies. Rachel prompted her students to chose any amount of children for their class. She did not remind students that they need to use whole cookies.
15 minutes	Solving the task.	Engaging in discussions, attempting to solve the task and individually recording problem solving strategies.	Roaming, observing and thinking about which groups/students need to be given particular prompts without intervening. Rachel prompted her students during this section of the lesson.
25 minutes			Giving students <u>enabling</u> prompts where necessary. Then, giving students <u>extending</u> prompts where necessary. Rachel stopped her students in the middle of the lessons and facilitated "share time" . Her students then continued to work on the task. Rachel did not select students to share in this section. Selecting individual students from groups who have used efficient problem solving strategies and notifying them that they will share their problem solving strategies with the class.
10 minutes	Share time.	Sharers share problem solving strategies. Class engages in discussions.	Ensuring that students are staying on track and speaking about the problem solving strategies they've used.

Teacher 1 25/7/19.
 Lost lesson plan. Offered another one...
 Recap'd what they've been learning and they all knew exactly what they've been learning.

Money - what do we know
 cents, money, \$, coins, notes
 not not

Coins have names on them. They tell you how much they are.

There is some fake money here. If you need coins you can use them. Try without them though.

Read question - said "don't tell me"
 How many ways do you think you could do it?
 10, 7, 15, 13 - oo you're thinking

Draw or write e.g. 1. $\text{---} + \text{---} = 50c$
 2. $\text{---} + \text{---} + \text{---} = 50c$

Could I make 75c? No

What are the key words in the question?
 How many ways? Make 50c - went on about 50c

Any questions? 3. $\text{---} + \text{---} + \text{---} + \text{---} + \text{---}$

How many do we do? For
talked
about
addition
and
plus.

Where does the cents go (after).

Any more questions?

Can you ~~draw~~ do it another way or do we do it like it is on the board? Don't write words. This is the easiest way. You could draw coins. $0+0+0=50c$

Could we draw pictures and do addition. Yes you can.

Tables at 11:45am.

Teacher 1 25/7/19
Kids went to get coins. Teacher said "don't get gold ones."

Someone spoke. Teacher asked them to stop talking. Reminded students to look at the coins on their page and to remember which number they were trying to make.

Kids discussing ideas with each other.

Teacher reminded kids to work on their own. Teacher walked around handing out coins. She said "I need to give them enough of particular coins because I may be limiting their answers. The majority of students chose to use coins. Reminded kids to do their own work."

"Mummi just asked me for 6 tens to make 50? Do you need 6?"

Doing your own work.

Child started coin rubbing. - Teacher underlined key words.

Child said they were finished. Teacher said

"I'm in this a bit more."

"Is there another way you could do it?"

11:58

asked a girl who was making 20c to make 50cents.

Told students to stop talking.

2:20 - asked to put x in.

• 8 What could you count by?

Child asked if there were anymore. Teacher said another kid says 8.

• Extension strip - How many ways can you make \$1?

• Do you know how to skip count?

• Again to someone else.

There are a few kids who have all incorrect solutions.

• Circled 8s in someone's work and said are there 8c coins.

He was still confused... Could you skip count?

Again and said "How many ways can you make \$1?"
How many cents in \$1?

- Another \$1...
- Did you finish that one?
- Are there anymore ways?
- again
- Is there a 30cent coin?
- This is 40. lets skip count.
- again
- again.

Teacher 1 25/7/19

Teacher
Checking
Work

Teach

- more ways.

Teacher said that kids were getting to the stage where they were being silly.
12:15pm brought kids to the floor.

Who can tell me a way you can make 50c?

$20 + 20 + 10$

$50 + 0$ (isn't really a zero coin)

What were some strategies that you use?
We used addition didn't we.

- 10s facts - to add the fives.

- count on.

- 100s chant. - were you counting on?

Maybe you use some without even knowing?
Can you all breathe?

- skip counting - what by - 5s 10s.

BL1: Lesson Deviations: Red Text

How many ways can you make 50 cents using coins?			
Time	Activity	What the students are doing	What the teacher is doing
8 minutes	Introduction of task.		Introducing the task. Ensuring students understand task. Not showing students how to solve the problem.be used once or several times Explaining to students that they need to individually record their problem solving strategies. Beth modelled an example of how they could approach the problem. Beth did not mention that the use of Australian coins as currently used Did not mention that the same value coin can
15 minutes	Solving the task.	Discussing, solving the task and individually recording problem solving strategies.	Roaming, observing and thinking about which groups/students need to be given particular prompts without intervening. Beth prompted her students during this period of the lesson.
5 minutes	Introducing prompt 1 to class.		Displaying and discussing John's solution to the problem: "John said there were 3 ways you can make 50 cents. Is he correct? Why? Why not?" Beth did not introduce prompt 1.
10 minutes	Solving the Task.	Discussing, solving the task and individually recording problem solving strategies.	Giving students enabling prompts where necessary. Then, giving students extending prompts where necessary.
2 minutes	Introducing prompt 2 to class.		Discussing Kath's solution: "Kath said there were 13 ways to make 50 cents. Is she correct? Why? Why not?" Beth did not introduce prompt 2.
10 minutes	Solving the task.	Discussing, solving the task and individually recording problem solving strategies.	Giving students enabling and extending prompts where necessary. Selecting individual students who have used efficient problem solving strategies and notifying them that they will share their problem solving strategies with the class. Beth did not select specific students to share their problem solving strategies.
10 minutes	Share time.	Sharers share problem solving strategies. Class engage in discussions.	Ensuring that students are staying on track and speaking about the problem solving strategies they've used.

Deviations: Yellow
Green

BL2: Lesson Observations
Problem-solving Strategies: Pink

Prompts:

11:32am. Teacher 1: Adding a number to make 16 1/8/19
On board as students walked in

$__ + __ = \square$ $__ + __ + __ = \square$ $__ + __ + __ + __ = \square$

Warm-up - skip counting vid to 100. 2s 5s 10s 20s 25s 50s
What did we just do? What did we skip count by? 2s 5s 10s

- If we've got a problem, what is it? - We learn a lot from mistakes - A problem in maths is an algorithm.
- What is a problem? - It's a question.
- I could put problems into these and find an answer. What could I do? - 100+100
- What about the second one? - You do repeated addition or a few plus.
- What about the bottom? - You're adding more numbers.
- So I could add a number again and again and again. When do I stop? - At the number it equals. = means stop.
- You might have to work backwards. You might be given an answer and you have to come up with the algorithm.
- Read question. Asked about key words. (16)
- Why's that important? - It's the answer.
- Same
- over and over.
- What's another word for over and over - repeated addition.
- added.
- So we're not doing subtraction are we.
- Do you do it as many ways as you can? • You can.
- Can you draw? • Yes.
- Make sure you reread the question when you get back to your table so that you know what to do. Off you go.

11:45am.
Some kids get it completely. Others are adding different numbers together to make 16.

11:45
Could you draw an array slip. x 5.
What if the number was 39? x 3
Could you draw a picture? x 4.
Are there more answers? 3

Teacher | 1/8/19
11:50am Could you draw a picture was given to students who were adding different numbers together.

- Can I do minus?

• Reread the question and gave "Can you draw a picture?"

For some of the (89) ^{to answer} How could halving help?

* What if the number you reached was 6? $\times 3$.

For some of the (Are there any more answers?) ^{could you draw a picture.}

11:57am Reminded about reaching the key words.

Teacher mentioned to me that she'd like ^{$\times 2$} more that said to draw a picture... She recycled and gave some to others.

She said that it helps her see what kids don't know.

12:03 She said she'd need another number between 6 and 89.

(The one about doubling amount of answers for 32 wasn't used.

People could finish if they had had enough.

^{Some} People were using a range of numbers, not the same number to add to make the number.

12:08.

A few people wanting to continue at tables.

• What did we do? ² - A response using different numbers.

• What were the key words? - same and 16.

- I used a trick X. ^{responses} ^{2 people}

• Who drew a picture? Who did an array?

12:11

Teacher asked class to stop. (people who were still going.

One person did the enable and then ^{eva.} went back
to the original problem. Teacher! 1/8/19

BL2: Lesson Deviations: Red Text

Adding Over and Over			
Time	Activity	What the students are doing	What the teacher is doing
10 minutes	Introduction of task: I added the same number over and over until I reached 16. Which number may I have started with?		Introducing the task. Ensuring students understand task. Not showing students how to solve the problem. Explaining to students that they need to individually record their problem solving strategies. Beth modelled an example of how her students could approach the problem and she did not remind her students that their answers will be single numbers.
15 minutes	Solving the task.	Engaging in discussions, attempting to solve the task and individually recording problem solving strategies.	Roaming, observing and thinking about which groups/students need to be given particular prompts <u>without</u> intervening. Beth prompted students in this section of the lesson.
25 minutes			Giving students <u>enabling</u> prompts where necessary. Then, giving students <u>extending</u> prompts where necessary. Beth did not select students to share in this section. Selecting individual students from groups who have used efficient problem solving strategies and notifying them that they will share their problem solving strategies with the class.
10 minutes	Share time.	Sharers share problem solving strategies. Class engages in discussions.	Ensuring that students are staying on track and speaking about the problem solving strategies they've used. Some students were able to continue working during this section of the lesson.

Appendix X

Researcher and Participant Protocols: Roles and Responsibilities

Researcher's Roles and Responsibilities:

- Act in a professional and respectful manner.
- Act in accordance with the participants' school's rules and requirements.
- Protect the rights, safety, and welfare of participants' involved in the research.
- Facilitate the discussions and interviews and ensure they finish on time.
- Keep information about the study confidential.
- Confine discussions about the project to our agreed meeting times.
- Provide any materials the participants' request for the lessons.

Teacher Participants' Roles and Responsibilities:

- Act in a professional and respectful manner.
- Act in accordance with the school's rules and requirements.
- Protect the rights, safety, and welfare of student participants involved in the research.
- Keep information about the study confidential.
- Confine discussions about the project to our agreed meeting times.
- Show up at scheduled appointments on time and inform the researcher within a reasonable time if an appointment needs to be rescheduled.
- Provide truthful answers to questions asked during interviews.

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