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Event-triggered transmission for linear control over communication channels

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Abstract

We consider an exponentially stable closed loop interconnection between a continuous-time linear plant and a continuous-time linear controller, and we study the problem of interconnecting the plant output to the controller input through a digital channel. We propose an event-triggered transmission policy whose goal is to transmit the measured plant output information as little as possible while preserving closed-loop stability. Global asymptotic stability is guaranteed when the plant state is available or when an estimate of the state is available (provided by a classical continuous-time linear observer). Under further assumptions, the transmission policy guarantees global exponential stability of the origin.

Key words: event-triggered control, hybrid system, asymptotic stability

1 Introduction

The study of event-triggered and self-triggered systems [2,3,4,11,12,18,20,21,14] led to a significant amount of research results where the core problem under consideration is that of two nodes (the sensing node and the actuating one) communicating through a (low capacity) digital channel where the transmission policy is determined based on suitable Lyapunov-like conditions involving some (more or less coarse) measurement of the plant state. A natural way to represent and suitably write the dynamics of this specific two-nodes configu-

ration is to use the hybrid systems notation, namely a state-space description wherein the state flows according to some continuous-time rules and, at some specific times, called jump times, it jumps following some discrete-time jump rule. A framework for the representation of hybrid systems that has been recently proposed and is surveyed in [7,8] allows for a quite natural description of these phenomena with useful Lyapunov like results that have been proven to apply to large classes of systems described using this framework. This framework was used in connection with event-triggered control in [5], which is a preliminary version of the results of this paper. Then, in [14], Lyapunov tools are used to model ISS properties of networked control systems and the MATI (maximum allowable transfer interval), to preserve asymptotic stability. Later, in [16] a similar approach to the one of this paper (and [5]) has been developed for the nonlinear case.

Here, we consider a closed-loop system that consists of a linear controller driving a linear plant to guarantee closed-loop asymptotic stability, as shown in Figure 1, and we break the continuity of the transmission of the measured plant output y to the controller input u by introducing transmission devices/sensors which measure the output y and decide whether or not sending this

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measurement to the controller input u through a transmission channel, based on non-periodic Lyapunov-based policies. The ideal scenario where this approach is relevant corresponds to cases where due to some technological constraint, there is a transmission line between a location where all the sensors are installed and a second location where the actuators are placed with a transmission channel inbetween (see Figure 1). An example of an industrial control problem where this type of constraints appear is described in [10] and corresponds to oil well drilling operations where the stick-slip self excited oscillations happening at the bit need to be measured locally and then suppressed by the action of an actuator which is located at the top of the drill string.

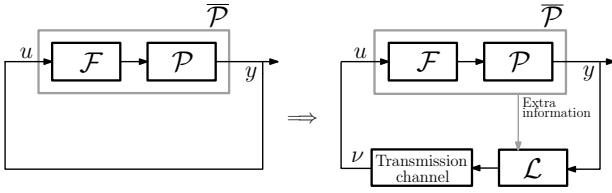


Figure 1. Nominal closed-loop $\mathcal{S}$ and event-triggered closed-loop system $\mathcal{S}_L$.

Our paper is a constructive solution along the general lines of [3,12,14], where Lyapunov tools and the hybrid framework are used in similar contexts. Our approach pairs with many interesting results on event-triggered control ([11,18,20,14] and references therein). Additional work sharing the scenario of Figure 1 is that of [17,13] and references therein, where the feedback signal is affected by an undesired quantization effect, rather than the presence of the communication channel. We show in the paper how restricting the attention to linear systems (whereas [18,20] considers nonlinear systems) allows us to design transmission policies which lead to improved results, both in terms of architectures and of achievable performance, as compared to those in [18,20,14] where, since a much more general nonlinear scenario is considered, the results obtained are more conservative. Finally, this work extends the results proposed in [5] by enforcing a dwell-time between transmissions and by introducing exponential bounds on the asymptotic stability guaranteed by the transmission policies. Then, the practical stability results of the output feedback case in [5] are here strengthened to asymptotic (and exponential) stability results.

The paper is organized as follows. We introduce nominal and event-triggered closed-loop systems in Section 2. The state-feedback transmission policy is illustrated in Section 3. The output-feedback case and robustness of the policy are considered in Sections 4 and 5. Simulation examples are given in Section 6.

Notation: Given a vector v , v^T denotes the transpose vector of v . Given two vectors w and v , $\langle v, w \rangle = w^T v$. Given a set

$a = \{a_1, \dots, a_n\}$ where $a_i \in \mathbb{R}$ for each $i = 1 \dots, n$, $\text{diag}(a)$ denotes a diagonal matrix having the entries of a on the main diagonal. Both the Euclidean norm of a vector and the corresponding induced matrix norm are denoted by $|\cdot|$. For a vector $v \in \mathbb{R}^n$ and a set $\mathcal{A} \subset \mathbb{R}^n$ $|v|_{\mathcal{A}} := \inf_{y \in \mathcal{A}} |y - v|$. Given a set $\mathcal{A} \in \mathbb{R}^n$, the set $\mathcal{A} + \varepsilon \mathbb{B}$, $\varepsilon \geq 0$, is the set of vectors v such that $|v|_{\mathcal{A}} \leq \varepsilon$. A continuous function $\alpha : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$; it belongs to class $\mathcal{K}_{\infty}$ if, moreover, $\lim_{r \rightarrow +\infty} \alpha(r) = +\infty$. For any $s \in \mathbb{R}$, consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(s) = 0$ if $|s| \leq 1$, and $f(s) = \text{sgn}(s)(|s| - 1)$ if $|s| \geq 1$. Then, for any $s = [s_1 \dots s_n]^T \in \mathbb{R}^n$, the deadzone function $\text{dz} : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is given by $\text{dz}(s) = \text{diag}(f(s_1), \dots, f(s_n))$.

2 The event-triggered closed-loop system

2.1 The hybrid system framework

The modeling tools adopted in the paper are based on the hybrid system framework [9,15] well summarized in [7,8]. In a nutshell, a hybrid system $\mathcal{H}$ is a tuple $(\mathcal{C}, \mathcal{D}, F, G)$, where $\mathcal{C} \subseteq \mathbb{R}^n$ and $\mathcal{D} \subseteq \mathbb{R}^n$ are, respectively, the *flow set* and the *jump set*, while $F : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ and $G : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ are set-valued mappings, called *flow map* and *jump map*. A hybrid system is usually represented by

$$\mathcal{H} : \begin{cases} \dot{x} \in F(x) & x \in \mathcal{C} \\ x^+ \in G(x) & x \in \mathcal{D} \end{cases}, \quad (1)$$

and its behavior can be roughly summarized as follows: the state *flows* (continuous-time evolution) through $\mathcal{C}$, by following the dynamics given by F , or it *jumps* (discrete-time evolution) from $\mathcal{D}$, according to G . The reader is referred to [7,8] for a detailed characterization of several fundamental concepts on hybrid systems, like the notions of hybrid time and solution, the crucial role of the so-called basic conditions, and the standard definitions of stability and related concepts.

2.2 The nominal closed-loop system

Consider a *nominal closed-loop system*, $\mathcal{S}$, defined by the plant-controller cascade $\mathcal{P}$, given by

$$\mathcal{P} : \begin{cases} \dot{x} = Ax + Bu \\ y = Cx, \end{cases} \quad (2)$$

(A, B, C matrices, $x \in \mathbb{R}^n$, and $u, y \in \mathbb{R}^q$), and by the interconnection relation $u = y$ between the controller input u and the plant measured output y . The nominal closed-loop system reads

$$\mathcal{S} : \begin{cases} \dot{x} = (A + BC)x \\ y = Cx. \end{cases} \quad (3)$$

We consider the following standing assumption.

Assumption 1 The nominal closed-loop system $\mathcal{S}$ in (3) is exponentially stable.

2.3 The event-triggered closed-loop system

The event-triggered policy is realized through the introduction of transmission sensors $\mathcal{L}$ within the feedback interconnection, as shown in Figure 1. These intelligent sensors monitor the measured output y and decide autonomously when transmitting a new sample of the output, denoted by ν , to the control input u , with the twofold goal of preserving the stability of the closed-loop system while breaking the continuity on the transmission of the measured plant output. Looking at Figure 1, the arising *event-triggered closed-loop system* $\mathcal{S}_{\mathcal{L}}$, namely the closed-loop system of (2) through the interconnection $u = \nu$, is a hybrid system which combines together the continuous dynamics of the plant-controller cascade $\mathcal{P}$ and the discrete dynamics of the transmission sensors $\mathcal{L}$. Its continuous dynamics can be modeled by

$$\begin{cases} \dot{x} = Ax + B\nu \\ \dot{\nu} = 0, \end{cases} \quad (4)$$

where x takes into account the plant-controller cascade dynamics while $\nu \in \mathbb{R}^q$ denotes the *state* of the transmission sensors, (each element ν_i of ν is related to the i th-input/output channel), which replaces the interconnection $u = y$ of $\mathcal{S}$ with $u = \nu$. ν is kept constant during flows by $\dot{\nu} = 0$. The discrete dynamics is given by

$$\begin{cases} x^+ = x \\ \nu^+ = y, \end{cases} \quad (5)$$

where x does not change during jumps while ν is updated to the current value of the output y .

Finally, we equip the transmission sensors with a bounded timer to guarantee a non-zero dwell-time between updates.

$$\begin{cases} \dot{\tau} = 1 - \text{dz}(\frac{\tau}{\rho}) & \tau \geq 0 \\ \tau^+ = 0 & \tau \geq \Delta, \end{cases} \quad (6)$$

where $0 < \Delta < \rho$ are design parameters. Combined with the use of the deadzone function in the flow map of (6), they guarantee that τ is bounded by 2ρ , it has rate 1 for $\tau \leq \rho$, and it may be reset to zero only if $\tau \geq \Delta$.

The hybrid dynamics of the event-triggered closed-loop system can be summarized as follows:

$$\mathcal{S}_{\mathcal{L}} : \begin{cases} \dot{x} = Ax + B\nu \\ \dot{\nu} = 0 \\ \dot{\tau} = 1 - \text{dz}(\frac{\tau}{\rho}) \\ x^+ = x \\ \nu^+ = y = Cx \\ \tau^+ = 0 \end{cases} \quad \begin{matrix} (x, \nu, \tau) \in \mathcal{C}_{\Delta} \\ (x, \nu, \tau) \in \mathcal{D}_{\Delta} \end{matrix} \quad (7)$$

Within the model proposed in (7), the event-triggered transmission policy is defined by the sets $\mathcal{C}_{\Delta}$ and $\mathcal{D}_{\Delta}$ and by the parameters Δ and ρ . These quantities characterize the *event* that triggers the transmission.

Remark 1 The proposed sample transmission model is an abstraction disregarding transmission delays, noise corruption of the samples, packet drop, and many other features of digital transmission. While these phenomena concur to the evaluation of the closed-loop performance, within a certain bounded magnitude they will not affect the main stability results established below. In particular, relying on the robustness to small perturbations guaranteed by the hybrid framework [8], the stability of our closed loop still holds in the presence of (small) transmission noise and delays, as shown in Section 5. $\lrcorner$

3 State-feedback transmission policy

3.1 The error dynamics

The exposition of the transmission policy is simplified by the adoption of error coordinates given by $(x, e) := (x, \nu - y)$. In those coordinates (7) reads

$$\begin{cases} \dot{x} = F_{11}x + F_{12}e \\ \dot{e} = F_{21}x + F_{22}e \\ \dot{\tau} = 1 - \text{dz}(\frac{\tau}{\rho}) \\ x^+ = x \\ e^+ = 0 \\ \tau^+ = 0 \end{cases} \quad \begin{matrix} (x, e) \in \overline{\mathcal{C}} \text{ or } 0 \leq \tau \leq \Delta \\ (x, e) \in \overline{\mathcal{D}} \text{ and } \tau \geq \Delta \end{matrix} \quad (8)$$

$$y = Cx$$

where $F_{11} := (A + BC)$ is Hurwitz by Assumption 1, $F_{12} := B$, $F_{21} := -C(A + BC)$, $F_{22} := -CB$, and the relation between flow sets and jump sets before and after the coordinate transformation is given by $\mathcal{C}_{\Delta} :=$

$$\{(x, \nu, \tau) \in \mathbb{R}^n \times \mathbb{R}^q \times \mathbb{R} \mid (x, \nu - y) \in \overline{\mathcal{C}} \text{ or } 0 \leq \tau \leq \Delta\}, \quad (9)$$

and by $\mathcal{D}_{\Delta} :=$

$$\{(x, \nu, \tau) \in \mathbb{R}^n \times \mathbb{R}^q \times \mathbb{R} \mid (x, \nu - y) \in \overline{\mathcal{D}} \text{ and } \tau \geq \Delta\}. \quad (10)$$

In what follows we use $F := \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$.

By coordinate transformation, the closed-loop state is now directly related to the error e between the current value of the output y and the latest sample ν transmitted to the controller. Moreover, from (8), the behavior of the event-triggered closed loop is to transmit a new sample only if the pair (x, e) satisfies the criterion $(x, e) \in \overline{\mathcal{D}}$ and if at least Δ units of time have elapsed from the last transmission.

3.2 The transmission policy

The transmission policy is characterized by the definition of the quantities $(\bar{\mathcal{C}}, \bar{\mathcal{D}}, \Delta, \rho)$ and is based on two main parameters $\gamma_e > 0$ and $\gamma_x > 0$, and on a suitably defined (Lyapunov) function V .

- $\gamma_e > 0$ guarantees a proportionality between the norm of the error e and the current state x , to avoid an asymptotic growth to infinity of the error e while x remains bounded. The conditions on $\gamma_e > 0$ for the stability of the closed loop are very mild: every $\gamma_e > 0$ guarantees stability;
- $\gamma_x > 0$ specifies a bound on the decay rate of a given Lyapunov function V . This bound is directly connected to the frequency of the sample transmissions, since a transmission is required when the desired decrease on the Lyapunov function is not guaranteed anymore.
- The Lyapunov function V is provided in condition (S1) below, and is used in condition (S2) below to characterize $\bar{\mathcal{C}}$ and $\bar{\mathcal{D}}$.

(S1) Take $\gamma_x > 0$, $Q = Q^T > 0$, and $V : \mathbb{R}^n \times \mathbb{R}^q \rightarrow \mathbb{R}_{\geq 0}$,

$$V(x, e) := \frac{1}{2} \begin{bmatrix} x \\ e \end{bmatrix}^T \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix}, \quad (11)$$

such that

- $P := \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$ is symmetric and positive definite,
- $F_{11}^T P_1 + P_1 F_{11} \leq -Q$,
- $\gamma_x I < Q$.

(S2) For any $\gamma_e > 0$, define

$$\begin{aligned} \bar{\mathcal{C}} &:= \{(x, e) \mid \langle \nabla V(x, e), F \begin{bmatrix} x \\ e \end{bmatrix} \rangle \leq -\gamma_x |x|^2 \text{ and } |e| \leq \gamma_e |x|\} \\ \bar{\mathcal{D}} &:= \{(x, e) \mid \langle \nabla V(x, e), F \begin{bmatrix} x \\ e \end{bmatrix} \rangle \geq -\gamma_x |x|^2 \text{ or } |e| \geq \gamma_e |x|\}. \end{aligned} \quad (12)$$

We can now state the main result of the section, whose proof is provided in Section 3.4.

Theorem 1 *Under Assumption 1, consider a transmission policy $(\bar{\mathcal{C}}, \bar{\mathcal{D}}, \Delta, \rho)$ which satisfies (S1) and (S2). Then, there exists $\Delta > 0$ and $\rho > \Delta$ (sufficiently small) such that the compact set*

$$\mathcal{A} := \{0\} \times \{0\} \times [0, 2\rho] \subset \mathbb{R}^n \times \mathbb{R}^q \times \mathbb{R} \quad (13)$$

is globally asymptotically stable for the event-triggered closed-loop system (8). Moreover, if B in (2) is full column rank, then $\mathcal{A}$ is globally exponentially stable.

The reader will notice that the asymptotic stability of the set $\mathcal{A}$ in (13) guarantees asymptotic convergence of

x to zero (by construction $\bar{\mathcal{C}} \cup \bar{\mathcal{D}} = \mathbb{R}^n \times \mathbb{R}^q \times \mathbb{R}_{\geq 0}$, therefore every maximal solution to (8) is complete). If B is full column rank the convergence is exponential. In this sense, the exponential stability of the nominal closed-loop system is preserved by the transmission policy.

Under Assumption 1, it is straightforward to see that the inequalities in (S1) are always feasible under Assumption 1. In fact, for any given γ_x , there exists a matrix $Q = Q^T > 0$ such that $\gamma_x I < Q$. Then, by Assumption 1, there exists a matrix $P_1 = P_1^T > 0$ which satisfies condition $F_{11}^T P_1 + P_1 F_{11} \leq -Q$. Note that inequality $\gamma_x I < Q$ guarantees that $\bar{\mathcal{C}} \neq \emptyset$. To see this, take $e = 0$, then $\langle \nabla V(x, e), F \begin{bmatrix} x \\ e \end{bmatrix} \rangle \leq -x^T Q x < -\gamma_x |x|^2$.

Since the average sample transmission frequency is connected to the decay rate of the function V , this frequency can be partially regulated by suitably choosing P_1 , P_2 , γ_x , and γ_e . In the typical scenario, the transmission of one sample resets the error e , which typically increases during the flow interval after the sample transmission, weighted by P_2 . Then, possibly, the boundary of the set $\bar{\mathcal{C}}$ is reached and a new sample is transmitted. In particular, from the definition of $\bar{\mathcal{C}}$, for an initial condition $e = 0$ and $x \neq 0$, smaller values for P_2 guarantee longer flow intervals. In fact, for smaller values of P_2 , (x, e) satisfies $\langle \nabla V(x, e), F \begin{bmatrix} x \\ e \end{bmatrix} \rangle > -\gamma_x |x|^2$ for larger values of e . In the limit, that is, for $P_2 = 0$ and $\gamma_e = \infty$ (i.e. no bound on e), e may grow unbounded and $\mathcal{A}$ is not necessarily stable. But this is forbidden by condition (S1).

3.3 Comparison with selected literature

The proposed transmission policy is closely related to the event-triggered control approaches [1,2,16,18,20,14] (which consider a more general class of nonlinear control systems) and [4,11], which propose transmission policies based on inequalities on the measured output and on the state of the plant. In particular, the transmission policy in [18, Section IV] is similar to our policy, as shown by [18, Equation (13)] which also highlights some differences between our approach and the one in [18]. Specifically, for any choice of the event-triggering conditions used in [18] (which are expressed simply as inequalities involving the norms $|x|$ and $|e|$), there always exist an equivalent choice of the sets $\bar{\mathcal{C}}$ and $\bar{\mathcal{D}}$ in (12) and of Δ in (6) (with $\Delta < \tau$ and τ given by [18, Corollary IV.1]) yielding exactly the same events; the converse is not true, in general, due to the restricted dependence from $|x|$ and $|e|$ (instead of x, e) in [18] (the same comment applies to [14] which once again treat the nonlinear case using input-to-state stability from $|e|$). See also the simulation results of Section 6 where we compare these two approaches. The work in [16] proposes two event-triggering strategies and generalizes the assumptions in [1,2,18,20]. In particular, the proposed strategies are based on the relaxation of the input-to-state stability assumptions on the underlying (not triggered) system to global asymp-

otic stability assumptions. Unfortunately, these generalizations do not provide improvements on the transmission rate for LTI systems since on LTI systems the two properties are equivalent. Both [4] and [11] focus on more transmission-related implementation issues, thus being less related to the basic stabilization problem considered here. In [4] the optimization of a stochastic performance index measuring the state variance is considered for a network of simple dynamical systems, whereas in [11] the issue of balancing energy consumption in different nodes is tackled. Finally, alternative approaches are simpler to implement but require more bandwidth, like [3], which guarantees transmissions to happen before the expiration of the maximum allowable delay compatible with stability preservation.

In comparison, the transmission policy proposed in the paper provides some advantages and novelties. First, in general, global exponential stability is guaranteed with less conservative bounds with respect to the current literature (see e.g. the above discussion about [18]). Then, the formulation within the hybrid systems framework of [7,8] automatically provides some levels of robustness which are guaranteed by the framework itself, as well as several analysis tools that make it easier to establish some relevant properties and additional results (see e.g. the output feedback results in Section 4). Finally, an additional novelty is the introduction of a timer (the state τ) within the sensors; while simple to implement, such modification enforces a minimum interval between consecutive transmissions of samples, meanwhile preserving stability (more recently, the same idea was used in [16]). Finally, our transmission policy forces a synchronous update on each channel. This specific feature can be relaxed, as shown in [5]. See also [6]

3.4 Proof of Theorem 1

The proof technique is inspired by [7, Example 27]. From the assumptions of the theorem, we provide a Lyapunov function W in (14), we show that W is non-increasing at each sample transmission (at jumps) in (15), and we show that W is non-increasing during flows, by decomposing the analysis in two parts: (i) for $\tau \in [0, \Delta]$, and (ii) for $\tau \geq \Delta$ and $(x, e) \in \bar{\mathcal{C}}$. Combining these results with the invariance principle for hybrid systems in [7, Theorem 23] and [15], we prove global asymptotic stability. Finally, based on a recent result in [19, Theorem 2], we strengthen the asymptotic convergence to an exponential one, by using the hypothesis that B is full column rank.

Lyapunov function: using X for the aggregate state $[x^T e^T \tau]^T$, consider the following Lyapunov function

$$W(X) = x^T P_1 x + \exp((2\rho - \tau)\lambda) e^T P_2 e \quad (14)$$

where $\lambda > 0$ is selected later. Using the definitions $\underline{\alpha} := \lambda_{\min}(P)$ and $\bar{\alpha} := \max\{\lambda_{\max}(P_1), \exp(2\rho\lambda)\lambda_{\max}(P_2)\}$

we get $\underline{\alpha}|X|_{\mathcal{A}}^2 \leq W(X) \leq \bar{\alpha}|X|_{\mathcal{A}}^2$ (radially unbounded).

Lyapunov function at jumps: we have that

$$W(X^+) - W(X) = -\exp((2\rho - \tau)\lambda) e^T P_2 e \leq -e^T P_2 e \quad (15)$$

for each X such that $(x, e) \in \bar{\mathcal{D}}$ and $\tau \geq \Delta$.

Lyapunov function on flows: the analysis is developed by considering two cases: (i) $\tau \in [0, \Delta]$, and (ii) $\tau \geq \Delta$.

For (i), using $\varphi(\rho, \lambda, \tau) := \exp((2\rho - \tau)\lambda)$, considering that $1 - \text{dz}(\frac{\tau}{\rho}) = 1$, and defining $\lambda_0 := \lambda_{\min}(Q)$, $\gamma_1 := 2|P_1 F_{12}|$, $\gamma_2 := 2|P_2 F_{21}|$, $\gamma_3 := 2|P_2 F_{22}|$, and $\gamma_4 := \lambda_{\min}(P_2)$, we get

$$\begin{aligned} \dot{W} &\leq -x^T Q x + 2x^T P_1 F_{12} e \\ &\quad + 2\varphi(\rho, \lambda, \tau) e^T P_2 (F_{21} x + F_{22} e) \\ &\quad - \lambda \varphi(\rho, \lambda, \tau) e^T P_2 e \\ &\leq -\lambda_0 |x|^2 + \gamma_1 |x| |e| + \gamma_2 \varphi(\rho, \lambda, \tau) |e| |x| \\ &\quad + \gamma_3 \varphi(\rho, \lambda, \tau) |e|^2 - \lambda \gamma_4 \varphi(\rho, \lambda, \tau) |e|^2. \end{aligned} \quad (16)$$

Exploiting the inequality $ab \leq \frac{1}{\varepsilon} a^2 + \varepsilon b^2$, where $\varepsilon > 0$, and $a, b \in \mathbb{R}$, from (16) we get

$$\begin{aligned} \dot{W} &\leq -\lambda_0 |x|^2 + \gamma_1 \frac{\varepsilon}{\varphi(\rho, \lambda, \tau)} |x|^2 + \gamma_1 \frac{\varphi(\rho, \lambda, \tau)}{\varepsilon} |e|^2 \\ &\quad + \gamma_2 \varepsilon |x|^2 + \gamma_2 \frac{\varphi(\rho, \lambda, \tau)^2}{\varepsilon} |e|^2 \\ &\quad + \gamma_3 \varphi(\rho, \lambda, \tau) |e|^2 - \lambda \gamma_4 \varphi(\rho, \lambda, \tau) |e|^2 \\ &\leq (-\lambda_0 + \gamma_1 \varepsilon + \gamma_2 \varepsilon) |x|^2 \\ &\quad + \varphi(\rho, \lambda, \tau) \underbrace{\left(\frac{\gamma_1}{\varepsilon} + \frac{\gamma_2 \varphi(\rho, \lambda, 0)}{\varepsilon} + \gamma_3 - \lambda \gamma_4 \right)}_{=: \lambda_1} |e|^2 \\ &\leq -\frac{1}{2} \lambda_0 |x|^2 - \varphi(\rho, \lambda, \tau) \lambda_1 |e|^2 \quad \forall \tau \in [0, \Delta], \end{aligned} \quad (17)$$

where the first term of the last inequality follows from the selection of $\varepsilon := \frac{\lambda_0}{2(\gamma_1 + \gamma_2)}$, while $\lambda_1 > 0$ is achieved by picking λ, ρ such that $\lambda > \frac{\gamma_3}{\gamma_4} + \frac{\gamma_1}{\gamma_4 \varepsilon} + \frac{\gamma_2 \varphi(\rho, \lambda, 0)}{\gamma_4 \varepsilon}$, which can always be satisfied by picking $\lambda > 0$ sufficiently large and $\rho > \Delta > 0$ sufficiently small (i.e. for the design parameter Δ sufficiently small). For instance, define $c_1 := \frac{\gamma_3}{\gamma_4} + \frac{\gamma_1}{\gamma_4 \varepsilon}$ and $c_2 := \frac{\gamma_2}{\gamma_4 \varepsilon}$, the inequality above reads $\lambda > c_1 + c_2 \varphi(\rho, \lambda, 0)$, which holds for $\lambda := c_1 + 2c_2$ and ρ sufficiently small, since $\varphi(\rho, \lambda, 0) \rightarrow 1$ as $\rho \rightarrow 0$.

For (ii), $\tau > \Delta$ implies $(x, e) \in \bar{\mathcal{C}}$. Thus, considering $1 - \text{dz}(\frac{\tau}{\rho}) \geq 0$, and using $\varphi(\rho, \lambda, \tau) := \exp((2\rho - \tau)\lambda)$, we get

$$\begin{aligned} \dot{W} &\leq -x^T (F_{11}^T P_1 + P_1 F_{11}) x + 2x^T P_1 F_{12} e \\ &\quad + 2\varphi(\rho, \lambda, \tau) e^T P_2 (F_{21} x + F_{22} e) \\ &= \langle \nabla V(x, e), F \begin{bmatrix} x \\ e \end{bmatrix} \rangle \end{aligned} \quad (18)$$

$$\begin{aligned}
 & + 2(\varphi(\rho, \lambda, \tau) - 1)e^T P_2(F_{21}x + F_{22}e) \\
 & \leq -\gamma_x|x|^2 + 2(\varphi(\rho, \lambda, \tau) - 1)e^T P_2(F_{21}x + F_{22}e) \\
 & \leq -\gamma_x|x|^2 + 2(\varphi(\rho, \lambda, \tau) - 1)(\gamma_2|e||x| + \gamma_3|e|^2) \\
 & \leq -\gamma_x|x|^2 + 2(\varphi(\rho, \lambda, \Delta) - 1)(\gamma_2\gamma_e|x|^2 + \gamma_3\gamma_e^2|x|^2) \\
 & \leq -\frac{\gamma_x}{2}|x|^2,
 \end{aligned}$$

where the last inequality holds for $\rho > \Delta$ sufficiently small. For example, take $\rho = 2\Delta$, then $\varphi(\rho, \lambda, \Delta) = \varphi(2\Delta, \lambda, \Delta) = \exp(3\Delta\lambda) \rightarrow 1$, as $\Delta \rightarrow 0$. As in (17), the decreasing of W in (18) is achieved by selecting the design parameter Δ sufficiently small.

GAS of the set $\mathcal{A}$ by invariance principle: since the event-triggered closed-loop system in (8) satisfies the basic conditions of [7], from the inequalities above, following [7, Theorem 23] or [15], $\mathcal{A}$ is stable. Moreover, for any given $\mu > 0$, consider the level curve given by $\ell(\mu) = \{X \mid W(X) = \mu\}$. Suppose now that $X(0, 0) \in \ell(\mu) \cap \overline{\mathcal{D}}$ with $e(0, 0) \neq 0$. Then, from (15) $W(X)$ decreases. Thus, suppose $X(0, 0) \in \ell(\mu) \cap \overline{\mathcal{D}}$ with $e(0, 0) = 0$. From the definition of $\overline{\mathcal{D}}$ in (12), necessarily $x = 0$, thus $W(X) = 0$ (in fact, for $e = 0$ and $x \neq 0$, $X \notin \overline{\mathcal{D}}$). During flows each solution X such that $X(0, 0) \in \ell(\mu) \cap \overline{\mathcal{C}}$ and $x(0, 0) \neq 0$ guarantees that $W(X)$ decreases (by (17) and (18)). For $X \in \ell(\mu) \cap \overline{\mathcal{C}}$ and $x(0, 0) = 0$, considering $\tau \leq \Delta$, we have that W decreases (by (17)), while considering $\tau > \Delta$ we have $(x, e) \in \overline{\mathcal{C}}$ which implies $|e| \leq \gamma_e|x| \leq 0$, thus $W(X) = 0$. Thus, using the fact that $W(X)$ is radially unbounded and no complete solutions remain within $\ell(\mu)$, by [7, Theorem 23], the set $\mathcal{A}$ is globally asymptotically stable.

Exponential stability of the set $\mathcal{A}$: it follows from the application of [19, Theorem 2]. For instance, decompose the state of the event-triggered closed-loop system in $\xi_1 = (x, e)$ and $\xi_2 = \tau$. Then, conditions 1)-3) of [19, Assumption 1] are satisfied. Moreover, B full column rank implies the observability of the pair $([I_n \ 0], F)$. In fact, using the linear transformation $T := \begin{bmatrix} I & 0 \\ -C & I \end{bmatrix}$, we have $T^{-1} = \begin{bmatrix} I & 0 \\ C & I \end{bmatrix}$ from which $\begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} = T^{-1}FT$ and $[I_n \ 0] = [I_n \ 0]T$. Thus, the observability of the pair $([I_n \ 0], F)$ can be established via observability PBH test on the pair $([I_n \ 0], \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix})$, that is,

$$\text{rank} \begin{bmatrix} A - sI & B \\ 0 & -sI \\ I_n & 0 \end{bmatrix} = n + q \quad \forall s \in \mathbb{C}, \quad (19)$$

which holds when B is full column rank. Therefore, combining the observability of $([I_n \ 0], F)$ with (15)-(18) and with the bound on W given after (14), condition 4) of [19, Assumption 1] is satisfied. Finally, the jumps of the event-triggered closed-loop system satisfy a dwell-time constraint, since for each solution X we have that

$(t, j) \in \text{dom } X$ implies $j \leq \frac{t}{\Delta}$, which satisfies condition 5) of [19, Assumption 1]. Thus, from [19, Theorem 2], $\mathcal{A}$ is globally exponentially stable. $\square$

4 Output-feedback transmission policy

4.1 The transmission policy

The state information used by the transmission policy can be replaced by an estimation provided by a classical linear continuous-time observer, which can always be designed because Assumption 1 implies detectability of the pair (C, A) .

From the even-triggered closed-loop system in (7), the introduction of an observer dynamics $\hat{x}$ leads to the following formulation.

$$\begin{cases} \dot{x} = Ax + B\nu \\ \dot{\hat{x}} = A\hat{x} + B\nu + L(y - C\hat{x}) \\ \dot{\nu} = 0 \\ \dot{\tau} = 1 - \text{dz}(\frac{\tau}{\rho}) \end{cases} \quad (\hat{x}, \nu, \tau) \in \mathcal{C}_\Delta \quad (20a)$$

$$\begin{cases} x^+ = x \\ \hat{x}^+ = \hat{x} \\ \nu^+ = C\hat{x} \\ \tau^+ = 0 \end{cases} \quad (\hat{x}, \nu, \tau) \in \mathcal{D}_\Delta \quad (20b)$$

$$y = Cx. \quad (20c)$$

where the flow dynamics is enriched by the observer dynamics with gain $L \in \mathbb{R}^{n \times q}$, and where $\hat{y} := C\hat{x}$ replaces $y = Cx$ in the transmission updates. Thus, looking at the definition of flow and jump sets in (20), transmissions depend now on the state ν of the sensors and the estimate $\hat{x}$ of the controller-plant cascade state.

The following stability results extend the result of Theorem 1 to the output feedback case under the assumption that $A + LC$ is Hurwitz (it is always possible to find an L satisfying this, under Assumption 1).

Theorem 2 *Suppose that the hypotheses of Theorem 1 are satisfied and that $A + LC$ is Hurwitz. Then, there exists $0 < \Delta < \rho$ (sufficiently small) such that the set*

$$\mathcal{A} := \{0\} \times \{0\} \times \{0\} \times [0, 2\rho] \subset \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^q \times \mathbb{R} \quad (21)$$

is globally asymptotically stable for the closed-loop system (20). Moreover, if B in (2) is full column rank, then $\mathcal{A}$ in (21) is globally exponentially stable.

The key point of Theorem 2 is in showing that the transmission policy does not need any modification when implemented in feedback from the estimated state. Since

now $\hat{y} := C\hat{x}$ replaces $y = Cx$, the transmission event is now based on $\hat{x}$, through a comparison between the quantities $\nu - C\hat{x}$ and $\hat{x}$. Thus, for example, the jump dynamics of the event-triggered closed loop is now given by $\nu^+ = C\hat{x}$ which allows for a mismatch dynamics at jumps given by $\nu^+ - C\hat{x}^+ = C\hat{x} - C\hat{x} = 0$, paralleling the jumps dynamics of the state-feedback case in which $\nu^+ = y = Cx$ guarantees that $\nu^+ - Cx^+ = y - Cx = 0$. Following this approach, the transmission policy operates on the subsystem $(\hat{x}, \nu)$, whose state is available, and the stability of the whole closed-loop system follows from the convergence of $\hat{x}$ to x , which is guaranteed by the property of $A + LC$ and by the dwell-time between jumps enforced by the timers dynamics.

4.2 Proof of Theorem 2

Error dynamics and Lyapunov function: using the coordinate transformation $(\hat{x}, e, \eta) = (\hat{x}, \nu - C\hat{x}, x - \hat{x})$ we can rewrite (20) as follows

$$\begin{cases} \dot{\hat{x}} = F_{11}\hat{x} + F_{12}e + LC\eta \\ \dot{e} = F_{21}\hat{x} + F_{22}e - CLC\eta \\ \dot{\eta} = (A - LC)\eta \\ \dot{\tau} = \bar{1} - \text{dz}(\frac{\tau}{\rho}) \\ \hat{x}^+ = \hat{x}, e^+ = 0 \\ \dot{\eta}^+ = \eta, \tau^+ = 0 \end{cases} \quad \begin{aligned} &(\hat{x}, e) \in \bar{\mathcal{C}} \text{ or } 0 \leq \tau \leq \Delta \\ &(\hat{x}, e) \in \bar{\mathcal{D}} \text{ and } \tau \geq \Delta \end{aligned} \quad (22)$$

Using the aggregate state $X := [\hat{x}^T, e^T, \tau]^T$ and $Y := [\hat{x}^T, e^T, \tau, \eta]^T$, consider $W(X)$ defined in (14), and define $P_o = P_o^T > 0$ such that $(A + LC)^T P_o + P_o(A + LC) \leq -I$, from which we can define the Lyapunov function $V : \mathbb{R}^n \times \mathbb{R}^q \times \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$ given by

$$V(Y) := W(X) + \gamma \eta^T P_o \eta, \quad (23)$$

where $\gamma > 0$. From the definition of V , using the bounds on W , there exists $\underline{\alpha}, \bar{\alpha} > 0$ such that $\underline{\alpha}|Y|_{\mathcal{A}}^2 \leq V(Y) \leq \bar{\alpha}|Y|_{\mathcal{A}}^2$.

Lyapunov function at jumps: using (16), we have

$$V(Y^+) - V(Y) \leq -e^T P_2 e \quad (24)$$

for each Y such that $(\hat{x}, e) \in \bar{\mathcal{D}}$ and $\tau \geq \Delta$.

Lyapunov function on flows: (i) for $0 \leq \tau \leq \Delta$, using (17) and λ_0, λ_1 given in the last inequality of (17), pick λ and ρ as in the proof of Theorem 1 and define $\gamma_0 := \frac{\lambda_0}{2}$, $\gamma_1 := \lambda_1 \exp((2\rho - \Delta)\lambda)$, $\gamma_2 := 2|P_1 LC|$, $\gamma_3 := 2 \exp(2\rho\lambda)|P_2 CLC|$. Then, we get

$$\begin{aligned} \dot{V}(Y) &\leq -\gamma_0|\hat{x}|^2 - \gamma_1|e|^2 + 2\hat{x}^T P_1 LC\eta \\ &\quad - 2 \exp((2\rho - \tau)\lambda) e^T P_2 CLC\eta - \gamma|\eta|^2 \\ &\leq -\gamma_0|\hat{x}|^2 - \gamma_1|e|^2 + \gamma_2|\hat{x}||\eta| + \gamma_3|e||\eta| - \gamma|\eta|^2 \\ &\leq -\frac{\gamma_0}{2}|\hat{x}|^2 - \frac{\gamma_1}{2}|e|^2 - \frac{\gamma}{2}|\eta|^2 \end{aligned} \quad (25)$$

where the last inequality is established by using $\varepsilon a^2 + \frac{1}{\varepsilon} b^2 \geq ab$, $a, b \in \mathbb{R}_{\geq 0}$, $\varepsilon > 0$, for $\gamma > 0$ sufficiently large.

(ii) For $\tau \geq \Delta$, $(\hat{x}, e) \in \bar{\mathcal{C}}$, using (18) with ρ sufficiently small, we get

$$\begin{aligned} \dot{V}(Y) &\leq -\frac{\gamma_x}{2}|\hat{x}|^2 + \gamma_2|\hat{x}||\eta| + \gamma_3|e||\eta| - \gamma|\eta|^2 \\ &\leq -\frac{\gamma_x}{2}|\hat{x}|^2 + (\gamma_2 + \gamma_3\gamma_e)|\hat{x}||\eta| - \gamma|\eta|^2 \\ &\leq -\frac{\gamma_x}{4}|\hat{x}|^2 - \frac{\gamma}{2}|\eta|^2 \end{aligned} \quad (26)$$

where, as before, the last inequality holds for $\gamma > 0$ sufficiently large.

GAS of $\mathcal{A}$ by invariance principle: from the inequalities above we can establish global asymptotic stability of the set $\mathcal{A}$ following the argument of the proof of Theorem 1. For instance, define $\ell(\mu) := \{Z \mid W(Z) = \mu\}$. For each solution $Y := (\hat{x}, \eta, e, \tau)$ such that $Y(0, 0) \in \ell_\mu$, we have that (i) on flows, when $\hat{x}, \eta \neq 0$, V decreases; (ii) on flows, when $\hat{x} = 0$ and $\eta = 0$, either V decreases or $(\hat{x}, e) \in \bar{\mathcal{C}}$, from which $e = 0$, that is, $V(Y) = 0$; (iii) on jumps, V does not increase but after each jump the system must flow for a Δ interval of time thus, necessarily, $\ell(\mu)$ is not an invariant set. global asymptotic stability follows from [7, Theorem 23].

Exponential stability of the set $\mathcal{A}$: exponential stability of the set $\mathcal{A}$ in (21) can be established by using [19, Theorem 2], as in the proof of Theorem 1. In fact, the pair

$$\left(\begin{bmatrix} I_n & 0 & 0 & 0 \\ 0 & 0 & I_n & 0 \end{bmatrix}, \begin{bmatrix} F_{11} & F_{12} & LC \\ F_{21} & F_{22} & CLC \\ 0 & 0 & A - LC \end{bmatrix} \right) \quad (27)$$

is observable when B is full column rank (by linear transformation and PBH-test). Thus, decomposing the state in $\xi_1 = (\hat{x}, e, \eta)$ and $\xi_2 = \tau$, using (24)-(26), and observing that $(t, j) \in \text{dom } X$ implies $j \leq \frac{t}{\Delta}$, every condition of [19, Assumption 1] is satisfied. Therefore, $\mathcal{A}$ is globally exponentially stable from [19, Theorem 2].

5 Robustness of the transmission policies

A fundamental feature of the proposed hybrid model (7) or (20) is that asymptotic stability is robust. These two models satisfy the so-called *basic conditions* [7], which guarantee several regularity properties of the space of solutions to the hybrid system. This regularity is exploited to establish several robustness results for hybrid systems stability. Based on these results, we show in this section that the stability proven in Theorems 1 and 2 is indeed robust. This is one of the main advantages of modeling the transmission policies within the hybrid systems framework proposed of [9],[7].

We will not enter into the details of robustness theory for hybrid systems. The interested reader is referred to [8].

Instead, we will show how to characterize parameter perturbations, measurements noise, and transmission delays as a perturbed hybrid model $\mathcal{H}_\delta$ where δ represents a perturbation radius with respect to the original system $\mathcal{H}$. Then, for example, we may invoke [7, Theorem 17] to establish that the asymptotic stability guaranteed by the transmission policy on the unperturbed system turns to practical stability when parameters perturbations, measurement noise, and delays are sufficiently small. We do not propose any formal statement here. We will keep the exposition at the level of a discussion.

Robustness to *measurement noise*, *parameter uncertainties* and *transmission delays* can be characterized as shown in [7, p.57]. Using the error model of the state-feedback policy for simplicity, let us rewrite (8) in the following simplified form

$$\begin{cases} \dot{x} = f_1(x, e) \\ \dot{e} = f_2(x, e) \\ \dot{\tau} = f_3(\tau) \end{cases} \quad \begin{array}{l} (x, e) \in \overline{\mathcal{C}} \\ \text{or } 0 \leq \tau \leq \Delta \end{array} \quad (28)$$

$$\begin{cases} x^+ = x \\ e^+ = 0 \\ \tau^+ = 0 \end{cases} \quad \begin{array}{l} (x, e) \in \overline{\mathcal{D}} \\ \text{and } \tau \geq \Delta \end{array}$$

where the definitions of f_1 , f_2 and f_3 follow from the comparison to (8). Different kinds of perturbations can be addressed by looking at the following perturbed hybrid model.

$$\begin{cases} \dot{x} \in f_1(x, e) + \delta_1 \mathbb{B} \\ \dot{e} \in f_2(x, e) + \delta_2 \mathbb{B} \\ \dot{\tau} \in f_3(\tau + d_1) + \delta_3 \mathbb{B} \end{cases} \quad \begin{array}{l} (x + d_2, e + d_3) \in \overline{\mathcal{C}} + \delta_4 \mathbb{B} \\ \text{or } 0 \leq \tau \leq \Delta \end{array}$$

$$\begin{cases} x^+ = x \\ e^+ = 0 + d_4 \\ \tau^+ = 0 \end{cases} \quad \begin{array}{l} (x + d_5, e + d_6) \in \overline{\mathcal{D}}. \\ \text{and } \tau \geq \Delta \end{array} \quad (29)$$

where d_i , $i \in \{1, \dots, 6\}$ are disturbance signals, and $\delta_i > 0$ are constants. Note that $f_i(x, e) + \delta_i \mathbb{B}$ is now a set-valued mapping for each $i \in \{1, 2, 3\}$.

We can identify the following three cases of interest.

- Consider $\delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$ and suppose that The signals $d_1, \dots, d_6$ are typically small, thus bounded within a ball of radius $\delta_0 \mathbb{B}$, $\delta_0 > 0$. They characterize possible *measurement noise* that perturbs the system. Respectively, d_1 enforces a drift on the internal timer of the transmission sensors, d_2, d_3, d_5, d_6 introduce a perturbation on the transmission decision, $e = 0 + d_4$, that is, $v = y + d_4$ corrupts the transmitted sample.
- Suppose $d_i = 0$ for each $i \in \{1, \dots, 6\}$, $\delta_4 = 0$, and $\delta_i > 0$ for each $i \in \{1, 2, 3\}$. Then $f_i(x, e) + \delta_i \mathbb{B}$ is

a (outer semicontinuous, convex and not-empty) set valued mapping of radius δ_i , centered at $f_i(x, e)$. The use of set-valued mappings is a possible approach to the characterization of *parameters uncertainty* on the model.

- Suppose $d_i = 0$ for each $i \in \{1, \dots, 6\}$, $\delta_i = 0$ for each $i \in \{1, 2, 3\}$, and $\delta_4 > 0$. Since the flow set is now larger, also the space of solutions to the perturbed system is larger than the space of solutions to the original hybrid model (8). The new solutions have an “excess” of flow which exactly characterizes *transmission delays*. In fact, the excess of flow represents a scenario in which a sample transmission should be performed but the system continues to flow for a given (small) amount of time.

Using the fact that the original model (8) satisfies the basic conditions (see e.g. [7,8]), the continuity of the flow map in (8), and the fact that each signal d_i is bounded by $\delta_0 \mathbb{B}$, we invoke [7, Theorem 17] (as done in [7, p.57]) to establish that for any given compact set of initial conditions $\mathcal{K}$ and any $\varepsilon > 0$, there exist sufficiently small values of $\delta_i > 0$, $i \in \{0, 1, 2, 3, 4\}$, such that all the solutions to (28) asymptotically (and uniformly) converge to the set $\mathcal{A} + \varepsilon \mathbb{B}$. It follows that the transmission policy is robust to (small) measurement noise, parameters uncertainty and transmission delays. This property corresponds to semiglobal practical stability, where *practical* refers to the fact that solutions converge to a neighborhood of $\mathcal{A}$ given by $\mathcal{A} + \varepsilon \mathbb{B}$, while *semiglobal* refers to the fact that the initial compact set $\mathcal{K}$ can be taken arbitrarily large.

Remark 2 While this section illustrates the robustness of our scheme for small disturbances and/or perturbations, it is also of interest to characterize some level of robustness in the presence of large seldom events, such as packet corruption or loss within the transmission network. This phenomenon can be captured by adding an extra state to (4), (5), which acts like an input matched disturbance to the right hand side of the first equation in (4) and remains constant between jumps. Robustness to this (non-small) disturbance can then be addressed by making assumptions on its persistency (for example one may use dwell-time or average dwell-time assumptions between packet losses, that is nonzero occurrences of this disturbance) and exploiting the strict decrease given by our Lyapunov functions to dominate the effect of this large but not persistent disturbance. Developing this analysis is beyond the scope of this paper. $\square$

6 Simulation examples

Consider an unstable plant given by the transfer function $\frac{s+2}{(s+1)(s-3)}$, which can be stabilized by negative static output feedback, for example by using the gain $k = 9$. The controller-plant cascade is represented by the fol-

lowing state space equations

$$\begin{cases} \dot{x} = \begin{bmatrix} 2 & 1.5 \\ 2 & 0 \end{bmatrix} x + \begin{bmatrix} -18 \\ 0 \end{bmatrix} u_c \\ y_p = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} x, \end{cases} \quad (30)$$

and the nominal closed-loop system is given by (30) through the interconnection $u_c = y_p$.

For $P_1 = \begin{bmatrix} 0.091 & 0.067 \\ 0.067 & 0.573 \end{bmatrix}$, $0 < \gamma_x \ll 1$, $\gamma_e \gg 1$, and $P_2 \in \{0.1, 10\}$, conditions (S1) of Section 3 are satisfied and the effect of the transmission sensors on the trajectories of the system in the state-feedback case is summarized in Figures 2 and 3. In all the simulations the state ν is initialized at zero for simplicity, $\Delta := 0.01$ and $\rho := 0.02$. The transmission instants are identified by the vertical lines within the input signal traces in the rightmost graphs.

The top row of Figure 2 represents the time evolution of the event-triggered system (x and ν) with state information available and without corruption of the transmitted samples for $P_2 = 0.1$ (black curve) and $P_2 = 10$ (gray curve). Note that the solid curve, corresponding to a smaller value of P_2 results in a significantly reduced data rate, as compared to the other selection (gray) whose trace is almost coincident with the nominal closed-loop (thin red curve), namely the closed loop with no transmission channel. The bottom row of Figure 2 shows the same simulations with the addition of a uniform random number between -0.1 and 0.1 added to each transmitted sample, representing transmission noise affecting the communication channel. Comparing the solid traces in the upper and bottom rows it appears that the channel noise negatively affects the transmission rate but, as indicated in Section 5, closed-loop (practical) stability is preserved.

Figure 3 proposes a comparison between the case $P_2 = 0.1$ (solid curves in all traces of Figures 2 and 3) and the state-feedback policy in [18]. To apply the results of [18] to this example, we consider $V = x^T P_1 x$ with the same P_1 used by our algorithm (defined above). From (8) we have that $\dot{V} \leq -\alpha|x|^2 + \gamma|x||e|$ where $\alpha = 1$ and $\gamma = 4.046$. Thus, following the notation in [18, Equation (8)], we enforce a transmission of a sample when $\gamma|e| \geq \sigma\alpha|x|$, for $\sigma = 0.9 < 1$ (which preserves the decrease of V along the solutions). The plots clearly illustrate the reduced transmission rate of our approach as compared to the policy based on [18, Equation (8)]. In particular, when the initial condition is balanced $x(0,0) = [1 \ 1]^T$ in the top row of Figure 3), a reduced transmission rate is already visible. However, the greatest advantage is experienced with the unbalanced initial condition $x(0,0) = [10 \ 1]^T$ of the bottom row. It should be also recalled that the reduced transmission rate that we achieve is due to the fact that we restrict our attention to linear control systems, while the work in [18] ad-

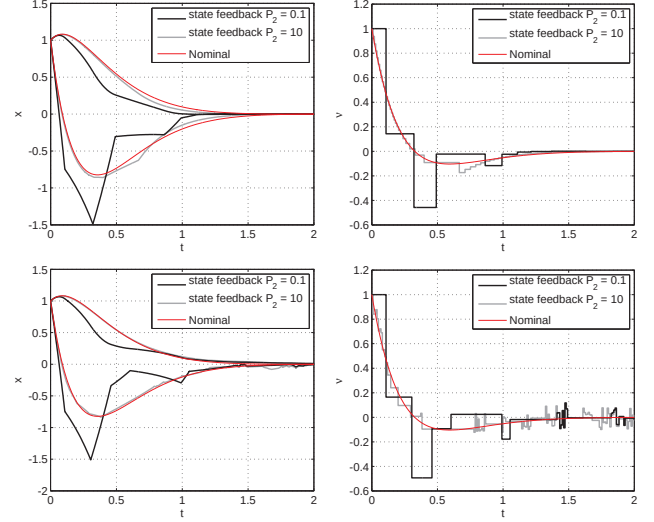


Figure 2. State-feedback transmission policy: time evolution of state x and input u_c for $P_2 \in \{0.1, 10\}$. TOP - no sample corruption. BOTTOM - sample corruption by uniform distributed noise between -0.1 and 0.1 .

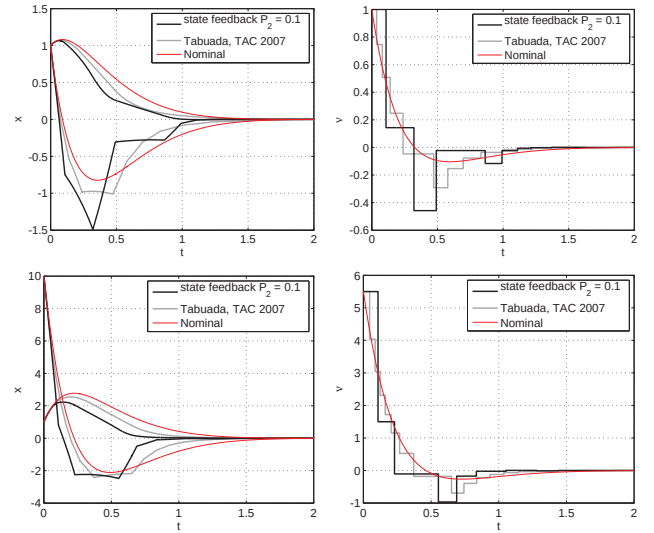


Figure 3. Comparison between the state-feedback transmission policy for $P_2 = 0.1$ and the policy of [18]. TOP - comparison from the (balanced) initial condition $x(0,0) = [1 \ 1]^T$. BOTTOM - comparison from the (unbalanced) initial condition $x(0,0) = [10 \ 1]^T$.

resses nonlinear systems for which more conservative bounds need to be used in general.

Finally Figure 4 represents the time evolution of the transmission policy from the estimated state $\hat{x}$ and without any noise. The observer gain is $L = \begin{bmatrix} -14.77 \\ -6.68 \end{bmatrix}$. For a fixed value of $P_2 = 0.1$, the convergence to zero depends on the initial mismatch between $\hat{x}$ and x .

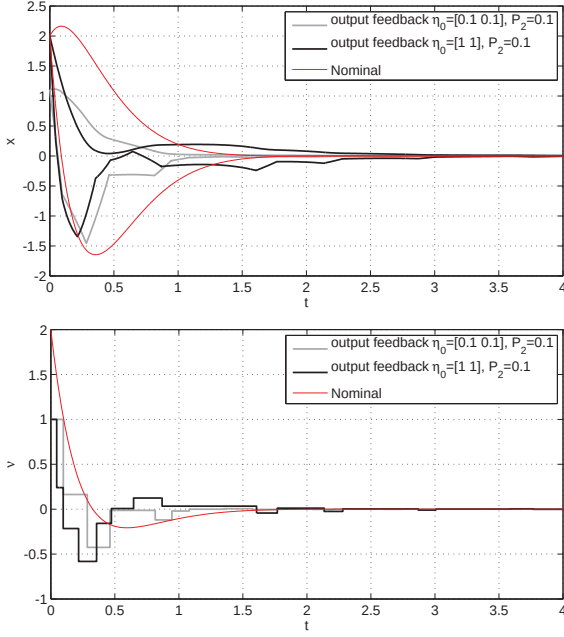


Figure 4. Output-feedback transmission policy: time evolution of state x and input u_c . $P_2 = 0.1$. Initial conditions: $x(0,0) = [1 \ 1]^T$, and $\eta(0,0) = [0.1 \ 0.1]^T$ - small estimation error (gray line), or $\eta(0,0) = [1 \ 1]^T$ - large estimation error (black line).

7 Conclusions

We proposed an event-triggered transmission policy that preserves the asymptotic stability of the original closed-loop system. Global exponential stability is guaranteed if the input matrix of the plant-controller cascade is full column rank. The transmission policy is based on the knowledge of the state, but the information on the state can be replaced by the state-estimation provided by a linear continuous-time observer (the exponential stability bounds are preserved). Robustness to small perturbations is guaranteed by classical results on hybrid systems. Simulations illustrate the effectiveness of the transmission policy and improved performance with respect to earlier event-triggered transmission policies.

References

- [1] A. Anta and P. Tabuada. Self-triggered stabilization of homogeneous control systems. In *American Control Conference*, pages 4129–4134, 2008.
- [2] A. Anta and P. Tabuada. To sample or not to sample: Self-triggered control for nonlinear systems. *IEEE Transactions on Automatic Control*, 55(9):2030–2042, 2010.
- [3] D. Carnevale, A.R. Teel, and D. Nešić. Further results on stability of networked control systems: A Lyapunov approach. In *American Control Conference, 2007. ACC '07*, pages 1741–1746, July 2007.
- [4] A. Cervin and T. Henningsson. Scheduling of event-triggered controllers on a shared network. In *47th IEEE Conference on*

- Decision and Control, 2008. CDC 2008.*, pages 3601–3606, December 2008.
- [5] F. Forni, S. Galeani, D. Nešić, and L. Zaccarian. Lazy sensors for the scheduling of measurement samples transmission in linear closed loops over networks. In *49th IEEE Conference on Decision and Control*, pages 6469–6474, 2010.
- [6] F. Forni, S. Galeani, D. Nešić, and L. Zaccarian. Event-triggered transmission for linear control over communication channels. arXiv:1310.0932, 2013.
- [7] R. Goebel, R. Sanfelice, and A.R. Teel. Hybrid dynamical systems. *IEEE Control System Magazine*, 29(2):28–93, April 2009.
- [8] R. Goebel, R. Sanfelice, and A.R. Teel. *Hybrid Dynamical Systems: modeling, stability, and robustness*. Princeton University Press, 2012.
- [9] R. Goebel and A.R. Teel. Solutions to hybrid inclusions via set and graphical convergence with stability theory applications. *Automatica*, 42(4):573 – 587, 2006.
- [10] R.B. Jijón, S. Canudas de Wit, C. and Niculescu, and J. Dumon. Adaptive observer design under low data rate transmission with applications to oil well drill-string. In *American Control Conference*, pages 1973–1978. IEEE, 2010.
- [11] M. Mazo and P. Tabuada. On event-triggered and self-triggered control over sensor/actuator networks. In *47th IEEE Conference on Decision and Control, 2008. CDC 2008.*, pages 435–440, December 2008.
- [12] D. Nešić and A.R. Teel. Input output stability properties of networked control systems. *IEEE Trans. Automat. Contr.*, 49:1650–1667, 2004.
- [13] D. Nešić and D. Liberzon. A unified framework for design and analysis of networked and quantized control systems. *IEEE Transactions on Automatic Control*, 54(4):732–747, 2009.
- [14] R. Postoyan, A. Anta, D. Nešić, and P. Tabuada. A unifying lyapunov-based framework for the event-triggered control of nonlinear systems. In *50th IEEE Conference on Decision and Control and European Control Conference*, pages 2559–2564, 2011.
- [15] R.G. Sanfelice, R. Goebel, and A.R. Teel. Invariance principles for hybrid systems with connections to detectability and asymptotic stability. *IEEE Transactions on Automatic Control*, 52(12):2282–2297, 2007.
- [16] Alexandre Seuret, Christophe Prieur, Nicolas Marchand, et al. Stability of nonlinear systems by means of event-triggered sampling algorithms. *IMA Journal of Mathematical Control and Information*, 2013.
- [17] Y. Sharon and D. Liberzon. Input to state stabilizing controller for systems with coarse quantization. *IEEE Transactions on Automatic Control*, 57(4):830–844, 2012.
- [18] P. Tabuada. Event-triggered real-time scheduling of stabilizing control tasks. *IEEE Transactions on Automatic Control*, 52(9):1680–1685, September 2007.
- [19] A.R. Teel, F. Forni, and L. Zaccarian. Lyapunov-based sufficient conditions for exponential stability in hybrid systems. *IEEE Transactions on Automatic Control*, 58(6):1591–1596, 2013.
- [20] X. Wang and M.D. Lemmon. Event design in event-triggered feedback control systems. In *47th IEEE Conference on Decision and Control, 2008. CDC 2008.*, pages 2105–2110, December 2008.
- [21] X. Wang and M.D. Lemmon. Event-triggered broadcasting across distributed networked control systems. In *American Control Conference, 2008*, pages 3139–3144, June 2008.