Service Level Agreement with Dynamic Inventory Policy: The Effect of the Performance Review Period and the Incentive Structure

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Abstract

Performance measures are often outlined in the section of the service-level agreement (SLA) of the contract between a supplier and a retailer. They are monitored periodically, and penalty and/or bonus payments are imposed in each performance review period, according to the SLA clauses. Previous studies have mostly considered a static inventory policy in analysing SLAs. However, in practice, the supplier may have an opportunity to adjust the stock level in each inventory review period, according to the observed performance. This study analyses the dynamic stocking decision for a supplier facing an SLA where the supplier sells a single product to the retailer. The ready rate is used to measure the performance in an SLA. To this end, models for both lump-sum and linear penalty/bonus structures are developed, and the optimal stocking decisions for a strategic supplier are calculated using the stochastic dynamic programming approach. The results are then compared with the optimal static inventory policy, and new insights are derived to efficiently design an inventory system for the suppliers that are subject to service-level incentives. In addition, we investigate the impact of SLA parameters – such as the length of the performance review period and incentive structures – on a supplier's performance, with the probability of meeting or exceeding the target service levels and the supplier's cost. We also consider the impact of demand distribution and inventory holding costs. Results show that under lump-sum incentives, a longer performance review period benefits both the supplier and the buyer, given that the average ready rate increases with less variability as the length of the performance review period increases, leading to decrements in the supplier's total costs. In this scenario, there is a higher chance of gaining bonuses/avoiding penalties for a strategic supplier that adopts a dynamic inventory policy. On the other hand, under linear incentives, the impact of the performance review period on the supplier's cost and the performance measure (i.e., ready rate) is complicated and depends on the magnitude of the holding cost and the bonus and/or penalty structure of the contract. Under this scheme, the performance of a static inventory policy is highly dependent on the holding cost because a high holding cost may lead to failure to meet the contract requirements in terms of the service level.

Keywords: Service-level agreement, Dynamic base-stock policy, Inventory management, Ready rate, Supply chain performance

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INTRODUCTION

Performance- or outcome-based contracts are commonly used in a decentralised supply chain to facilitate supply chain collaboration by focusing on designing contracts based on outcomes rather than prescribed products or services (Selviaridis & Wynstra, 2015; Kim, Cohen, & Netessine, 2007; Nowicki, Kumar, Steudel, & Verma, 2008). A study by McKinsey (2013) estimates that implementing outcome-based payments in healthcare spending in the USA would reduce costs by US$1 trillion over the next decade, and, at the same time, would improve patient well-being. The service-level agreement (SLA) is a type of performance-based contract in which the desired performance is enforced by a predefined (target) service level, along with financial penalty structures to incentivise required performance over a specific review period (Hefley & Loesche, 2010; Abbasi, Hosseinifard, Alamri, Thomas, & Minase, 2018). Therefore, in an SLA contract, the retailer does not need to micromanage the supplier or dictate how the work is to be performed. Instead, the supplier is responsible for reaching the target service level at the time intervals when performance is measured and reviewed (i.e., performance review points). In inventory applications of SLA contracts, for example, the supplier's inventory service levels are measured over the performance review periods, and incentives or penalties are introduced to induce the supplier to invest in inventory. A supplier may incur a penalty for not meeting this target, but may also receive a bonus for meeting or exceeding another (higher) target (Sieke, Seifert, & Thonemann, 2012). One challenge in designing a service-level agreement involves deciding on the contract parameters. To design an SLA, contractors should outline the performance measures, the performance review period, performance targets, and the penalty and/or bonus structures. In this paper, we study the impact of SLA terms and the supplier's stocking decision on the supplier's performance and cost. The two SLA terms investigated in this research are the length of the performance review period and the incentive structures. The impacts of these parameters on the supplier's performance are explored through a distribution of the performance measure and the probability of meeting or exceeding the target service levels. The analysis provides critical insights for both contract parties in SLA design and negotiation.

SLAs are widely applied in different areas of operations management such as inventory systems, production management, and quality control. A survey conducted across a broad range of industries and companies in 2007 by Oblicore Inc. (Oblicore Inc, 2007) found that 91% of organisations use SLAs to manage their suppliers. Thonemann *et al.* (2003) found that SLAs are used by 70% of retailers in Germany to monitor the performance of suppliers. There are several measures used to quantify the supplier's performance (Hefley & Loesche, 2010; Sieke, Seifert, & Thonemann, 2012). A study by Chen and Thomas (2018) examined 70 SLAs collected from US-based retailers from several sectors, including general merchandise, grocery, and automotive aftermarket part providers. They found that in 73% of contracts, retailers measure the supplier's performance by defining 100% complete delivery on the scheduled due date. The ready rate is a frequently used performance measure that calculates the long-run fraction of periods for which demands are met in full (Schneider, 1981; Choi, Dai, & Song, 2004). Axsäter (2006) defined the ready rate as "the fraction of time that the net inventory level is positive". The ready rate is the α-type service level that measures the inventory/service availability and is equal to the probability of no stock-out over all order periods. The ready rate is used by service providers such as contact/call centres as a key performance indicator (KPI). These service providers usually measure the quality of service by defining the percentage of contacts served within a set threshold. For example, the 80/20 rule in call centres states that the 80% of calls should be answered within 20 seconds, since customers waiting more than 20 seconds are more likely to abandon calls (Jackson, 2002; Milner & Olsen, 2008). Contact service centres use SLA contracts across different services such as phone calls, email, live chat, instant messaging, and social media channels. The fill rate is another measure used in inventory systems. It is calculated based on the long-run fraction of demands that are immediately satisfied by the supplier (Schneider, 1981; Johnson & Scudder, 1999; Zhang & Zhang, 2007; Abbasi, Hosseinifard, Alamri, Thomas, & Minase, 2018).

To guarantee the desired performance, incentives are usually imposed in SLA design. Underperformance can result in penalties, and the supplier may receive bonuses for good performance. However, some SLA contracts use only a penalty structure. The retailer can employ either lump-sum or linear-based incentives to manage the supplier's performance. In a linear incentive structure, the amount of penalty or bonus depends on the deviation of the supplier's performance from that of the target(s). For example, Advance Auto Parts Inc., an automotive aftermarket parts provider, imposes the following penalty where suppliers fail to comply with contract requirements in inventory management: *"10% of the cost of the unit gap needed to meet vendor specific fill goal"* (Advance Auto Parts, 2019). Pep Boys (2018), on the other hand, imposes 5% of the cost on items not shipped where the supplier's service level is below the target fill rate. Under a lump-sum penalty scheme, in contrast, suppliers receive a fixed penalty or provide credit for future services to the buyer if they fail to meet the target service level. For example, CVS Pharmacy – the largest pharmacy retail chain in the USA – applies a lump-sum penalty to suppliers that fail to supply 95% of orders in a month (CVS, 2017). Similarly, dm-drogerie-markt – a retailer of cosmetics, healthcare products, and health food in Europe – applies a lump-sum penalty to suppliers that do not achieve the target service level (Mostberger, 2006). Cloud providers such as Amazon, Microsoft, and Google provide their cloud-based products/services to customers through SLAs, with service commitments measured based on the monthly uptime percentage. Amazon, for example, suggests a 100% service credit (penalty) for a monthly uptime of less than 95.0% in a monthly billing cycle. Amazon also suggests a 30% penalty for a monthly uptime between 95%, and 99%, and a 10% penalty for performance between 99% - 99.99% (Amazon, 2018). Bonuses are particularly attractive to suppliers and are included in SLAs:

> *"Some SLAs will even include a service level bonus for performance above a stated level. Where service level credits essentially decrease the price, service level bonuses increase the price triggered by higher performance levels."* (Network World, 2005)

> *"Bonuses or incentive schemes are appropriate and helpful ... where the customer could receive significant benefit from the supplier meeting a performance deadline or other target that will be difficult to*

achieve and which cannot be guaranteed by the supplier ... In business transformation outsourcing where there are not detailed service descriptions and service levels, bonuses can play an important role as part of the overall risk-reward incentivisation mechanisms." (Latham & Watkins, 2018)

Using both penalty and bonus mechanisms is also common practice. Selviaridis and van der Valk (2019) investigated the payment mechanism and the performance incentives of 38 suppliers for packaging solution and automobile manufacturing companies, as well as international food retailers. From semi-structured interviews with the suppliers, the penalty-and-bonus scheme was found to be used by suppliers to support buyers, and to achieve cost efficiency: *"we needed a tool, so, that was the SLA with the penalty and bonus scheme and we would have something to gain as well"*. Selviaridis and van der Valk (2019) suggested that the penalty-and-bonus structure can lead to productive supplier responses among the three investigated incentive schemes of bonus-only, penalty-only, and penalty-and-bonus.

Suppliers in inventory systems make decisions on stocking levels in order to achieve target service levels. They replenish their inventory at specified time intervals, called replenishment or inventory review periods, which are usually on a daily, weekly, or monthly basis. Suppliers can also monitor their performance during the performance review period and adjust their inventory levels strategically, which is commonly perceived as an undesirable behaviour by retailers/buyers. However, with this stocking policy, the impact of SLA terms, such as the length of the performance review period (PPR) and the incentive structure on the supplier's performance (i.e., realised service level), is not trivial. This knowledge can assist suppliers in SLA negotiation, as well as in supporting buyers when designing an SLA contract. As an application of SLAs in inventory management, we have observed that a supplier's policy in dynamic stock level adjustment, based on the observed performance history, can affect the supplier's performance and total cost. Here we focus on the supply management of SLA contracts in a multi-period performance review setting. The supplier can invest in inventory to supply a single product for the retailer (buyer) in order to meet the target service levels. This study aims to understand the effect of incentive schemes and the length of the performance review period on the supplier's cost and realised service level. In this study, the ready rate is used as the service-level measure. The ready rate is the preferred performance measure in many industries for which there is no value for buyers unless the demand is fully satisfied. An example is the availability of operating rooms for certain hours of operation requested by hospital emergency departments. We consider cases of lump-sum and linear incentives when both the penalty and bonus are included in the contract, as well as penalty-only structures, and provide a mathematical formulation of and solution to the supplier's optimal stock level in each inventory period. The results provide managerial insights for suppliers and retailers in negotiating SLA terms.

In summary, the research questions in this study are:

- How can the supplier facing an SLA decide on the stock level required at each inventory review point?
- What is the impact of applying a dynamic inventory policy rather than a static policy on the supplier's total

cost, the observed service level, and the probability of meeting or exceeding targets?

- How does the supplier's performance, cost, and observed service level change with different lengths of performance review periods, incentive values, and incentive structures, as well as different holding costs and demand distributions under static and dynamic inventory policies?

The key findings of this study are as follows:

- Implementing a dynamic stocking policy, not surprisingly, reduces the supplier's costs, compared to a static policy under all different SLA settings of performance review periods and incentive structures. More importantly, we find that this cost saving is insignificant when the holding cost is negligible, the length of the performance review is large, and a linear incentive scheme is applied.
- Under a penalty-and-bonus regime, the supplier's cost saving using a dynamic inventory policy is significantly higher with longer performance review periods if a lump-sum structure is applied. If a linear incentive structure is applied, the cost saving depends on the holding costs, irrespective of the penalty-to-bonus ratio.
- Under the supplier's optimal stocking policy, a longer performance review period results in an increased average ready rate with less variability if a lump-sum incentive structure is imposed. However, under this incentive scheme, adopting a static policy may lead to a supplier's underperformance in terms of meeting the penalty threshold when the inventory holding cost is high.
- With a linear incentive structure, the ready rate distribution depends on the magnitude of the holding cost, bonus, and penalty. However, for high holding costs, using a static stocking policy may significantly impact the supplier's performance regarding the average ready rate and the probability of meeting the penalty threshold.

The rest of the paper is organised as follows: The next section is a review of related literature. In the Model Setup Section, we describe the structure of the system and the notations used throughout this study, and we present the ready rate calculations. The optimal stock-level calculations for dynamic and static stocking policies under SLAs with a lump-sum penalty-and-bonus structure are provided before providing optimal inventory models under a linear incentive structure. Then, we summarise the numerical experiments and results. Next, analytical calculations are provided for the probability of exceeding the target service level using a Markov chain process. Finally, we provide some discussion and conclusions.

LITERATURE REVIEW

In a review of service supply chain management, Wang *et al.* (2015) highlights the importance of research on supply chain contracting and performance-based contracts. A critical element of performance-based contracts and service-level agreements (SLAs) is measuring the performance (i.e., service level) of a supplier (Kim, Cohen,

& Netessine, 2007; Hefley & Loesche, 2010; Sieke, Seifert, & Thonemann, 2012). Several studies have discussed inventory service-level measures for the supplier in supply chains. Schneider (1981) provided a comprehensive classification on various SLA measures, including the ready rate and the fill rate. The inventory service-level measures can be assessed over an infinite or a finite horizon. Most research in this area focuses on infinite horizon models (Larsen, 2011; Tempelmeier, 2007; Larsen & Thorstenson, 2014). However, in an agreement between a supplier and a retailer, the inventory service level (e.g., ready rate) is often measured over a finite horizon, and is then compared with the targets to monitor the supplier's performance. The infinite horizon assumption does not provide a basis to identify underperformance of the supplier. Therefore, we focus on previous studies that have considered a finite horizon in measuring service levels.

Under a finite multi-period horizon, the supplier's inventory performance is reviewed every T periods (e.g., T days), called the performance review period. The length of T can impact both parties in terms of cost and performance. Chen and Thomas (2018)'s study of 70 SLA contracts showed that the majority of SLA contracts outline a shorter performance review period (79%) rather than longer review horizons (such as biweekly or longer). A longer T provides the supplier with an opportunity to make up for earlier underperformance, while a shorter T will increase the administrative cost by micromanaging the supplier. Thomas (2005) carried out a simulation study to examine the distribution of a finite-horizon fill rate for a stationary base-stock inventory system. He concluded that the average fill rate decreases when the performance review period length is increased; however, volatile fill rates are expected in very short horizons. Banerjee and Paul (2005) proved that the average fill rate decreases when the performance review period is increased in a base-stock inventory system. Katok *et al.* (2008) used a controlled laboratory setting to study both the performance review period length and the magnitude of the bonus for meeting or overreaching the target service level. They used the fill rate to measure the supplier's performance and concluded that longer performance review periods may be more effective than shorter ones due to enhanced feedback and the opportunity to improve service. They recommended further analysis using more complex policies such as a dynamic inventory system in future research. Abbasi *et al.* (2018) designed a simulation study to investigate the fill-rate behaviour for a supplier with multiple customers when the supplier adopts inventory pooling models to fulfil demands in SLA contracts. They used a static base-stock policy in their settings. Recently, Abbasi *et al.* (2020) extended this work by proposing effective resource allocation policies where heterogeneous service level requirements, non-identical performance review periods and various demand distributions are expected. Kloos and Pibernik (2020) investigated the allocation planning problem when the fill rate is used to measure the performance in an SLA with a linear penalty. They determined the optimal penalty-minimising allocation decisions for customers using the stochastic dynamic programming technique. Alamri *et al.* (2018) investigated the ready rate behaviour for a supplier with two retailers using a static base-stock policy. In this paper, we investigate the impact of the length of a performance review period when the ready rate is used as the measure of performance in an SLA.

From the inventory management perspective, most studies that model finite horizon service levels focus on the static stocking policy. Dynamic inventory is a policy that obtains the optimal base-stock level at the beginning of each inventory review period. The dynamic base-stock policy is used in the case of non-stationary demand in the supply chain (Topkis, 1968). Here we review some of the papers that have applied the dynamic base-stock policy in inventory systems. Neale and Willems (2009) applied a dynamic base-stock concept for non-stationary demand to support monthly safety stock planning at Microsoft and to design the inventory policy at Case New Holland. Another work to design an inventory system for a non-stationary demand process was proposed by Graves (1999). Benjaafar *et al.* (2011) applied a dynamic base-stock policy – a so-called state-dependent base-stock policy – in an assembly system. Li and Ryan (2013) developed a dynamic base-stock policy based on the updated demand information at each stage using a Bayesian model. Selçuk (2013) proposed an adaptive base-stock policy for repairable item inventory control. He used stock-out situations and priority shipments to update the base-stock level. de Kok *et al.* (2005) applied a dynamic base-stock policy to the case where the demand distribution changes over time. They updated the demand distribution at each inventory review point and used it to obtain the base-stock level for each period. Liang and Atkins (2013) investigated supply chain coordination in SLA contract design. They analysed the effects of imposing penalty or bonus terms on the supplier's costs under linear and lump-sum schemes and recommended a linear-penalty SLA over a lump-sum structure to mitigate the dynamic investment behaviour of a strategic supplier.

This study builds on previous research in SLA design and negotiations by considering the dynamic stocking policy for a supplier of a single product under an SLA with penalty and bonus (or penalty-only) factors. We investigate the impact of SLA parameters such as the length of the performance review period, incentive values, and structure on the supplier's cost and performance, with consideration of the holding cost and demand distribution effects. A paper by Liang and Atkins (2013) is closely related to our work; however, they did not explore the distribution of the ready rate under different SLA parameter settings such as the performance review period and incentive values. They also studied penalty and bonus terms separately in their work. In a penalty-only (or bonus-only) setting, Liang and Atkins (2013) recommended that buyers enforce a linear structure rather than a lump-sum in designing an SLA to avoid the dynamic behaviour of suppliers in inventory adjustment. However, the result of this study shows that even with a lump-sum incentive structure, both parties can benefit from the longer performance review periods. Under a linear incentive SLA, on the other hand, the longer performance review period has a negative impact on the average realised ready rate under both inventory policies. In this paper, we formulate the supplier's optimal stocking decision based on the observed ready rate service level and cost during the performance review period. We provide analytical results for a strategic supplier that uses a dynamic base-stock policy, and is committed to an SLA with penalty and/or bonus terms. We also address the inventory system design, assuming that the supplier employs a static inventory policy. The results are then compared under each incentive scheme (linear and lump-sum).

MODEL SETUP

We consider an inventory supply chain system in which a supplier manages the supply of a single product for the retailer. The demand at each period (d_t) is stochastic, independent, and identically distributed (*i.i.d.*) with mean λ and standard deviation σ. We consider the case where the SLA outlines:

- The length of the performance review period (T)
- The amount and type (lump-sum or linear) of penalty and bonus
- The lower target service level (i.e., penalty threshold) – when the supplier performs below this target in a performance review period, a financial penalty (lump-sum or linear) is applied to the supplier
- The upper target service level (i.e., bonus threshold) – when the supplier performs above this target in a performance review period, a financial bonus is paid to the supplier

The supplier replenishes the inventory at the beginning of the inventory review period t to S_t to satisfy the demand of period t. Then the retailer's demand is observed, and the unmet demand is backordered. The supplier does not incur the backorder (shortage) cost; thus, this cost is not directly included in our model. However, the retailer incorporates this cost into the contract by setting the target service level and penalty structures. We assume that excess inventory at the end of the period is discarded (Rosenfield, 1989; Chen & Sapra, 2013; Çetinkaya & Parlar, 2010; Chiang, 2007; Qin, Wang, Vakharia, Chen, & Seref, 2011). Since the end of the period is discarded, the unit holding cost in our model is composed of the unit storing cost plus the unit purchase price and the unit disposal cost minus the unit salvage value. There is a constant unit inventory holding cost h per item held at the end of each inventory review period. The supplier uses a periodic-review base-stock policy with replenishment. The supplier makes a decision on the base-stock at each inventory review period based on the demand distribution and the current service-level performance. Therefore, at the beginning of period t, the supplier determines the stocking level and places an order. Here we consider an inventory setting of a supply chain with zero lead-time, similarly to Thomas (2005) and Abbasi *et al.* (2018). We consider both static and dynamic inventory policies and study a supplier's problem under an optimal contract. Figure 1 illustrates the problem setting.

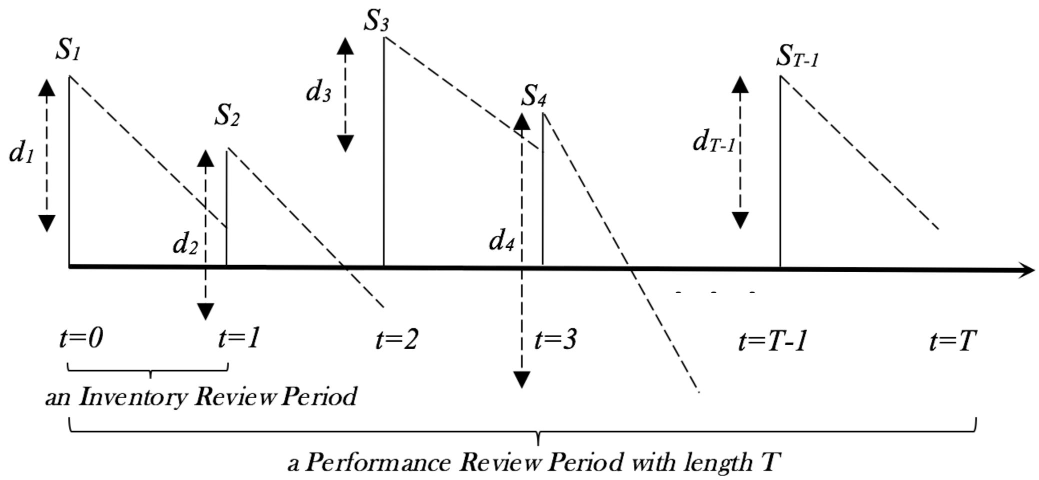


Figure 1: The setting of the problem. S_i's are the decision variables; stocking decisions are made at the beginning of each inventory period. The fill rate has been satisfied in periods 1, 3, and T-1 but it has not been met in periods 2 and 4.

The following notations are used:

T	The length of the performance review period
S_t	Base-stock in the inventory review period t $(t = 1, 2, ..., T)$
d_t	Demand in the inventory review period t
α^{T_L}	Penalty threshold: a lower target ready rate such that if the supplier's ready rate in a period falls below that, a penalty is imposed on the supplier
α^{T_U}	Bonus threshold: an upper target ready rate such that if the supplier's ready rate in a period meets or exceeds that threshold, a bonus is paid to the supplier
α^L	Long-run ready rate
α_T^A	Average ready rate given T
α	The ready rate in a performance review period; α is a random variable
h	Holding cost per item per unit of time
$\mathcal{L}^-$	The lump-sum penalty incurred if the target ready rate is not achieved (note that $\mathcal{L}^-$ is a linear function of T, i.e., $\mathcal{L}^- = \theta T$)
$\mathcal{L}^+$	The lump-sum bonus incurred if the target ready rate is passed (note that $\mathcal{L}^+$ is a linear function of T, i.e., $\mathcal{L}^+ = \theta' T$)
$\mathcal{L}^{-l}$	The linear penalty incurred if the target ready rate is not achieved
$\mathcal{L}^{+l}$	The linear bonus incurred if the target ready rate is passed

Ready Rate Analysis

In this section, we develop a model in which the ready rate is used as the service-level measure. The supplier's service level is a random variable under a finite performance review period. As discussed previously, the ready rate is defined as the fraction of periods for which the demand is immediately met in full. We let X_t be the inventory performance indicator for period t, where $X_t = \mathbf{1}(d_t \leq S_t)$ is a Bernoulli random variable, and $X_t = 1$ means no demand is delayed. The mathematical expression for the ready rate (α) measured in a performance review period with a length of T is defined as:

$$\alpha := \frac{\sum_{t=1}^{T} \mathbf{1}(d_t \leq S_t)}{T}$$

where $\sum_{t=1}^{T} \mathbf{1}(d_t \leq S_t)$ is the cumulative inventory performance during a performance review period (T). The average performance of the supplier is obtained as $\alpha_T^A = E(\alpha)$. Liang and Atkins (2013) showed that under a static base-stock inventor policy, the average ready rate has a normal distribution as T reaches a large value. If $T \to \infty$, the obtained ready rate is not a random variable anymore and is referred to as the long-run ready rate, defined as $\alpha^L := \underset{T\to\infty}{lim} (\sum_{t=1}^{T} \mathbf{1}(d \leq S_t)/T)$.

The probability that the lower target ready rate is not satisfied in a performance review period is denoted by $Pr(\alpha < \alpha^{T_L}) = Pr(\sum_{t=1}^{T} \mathbf{1}(d_t \geq S_t) < T\alpha^{T_L})$, where $\mathbf{1}(d_t \geq S_t) \sim Bernoulli(F_{d_t}(S_t))$ and $F_{d_t}(.)$ is the cumulative distribution function (CDF) of the demand in period t. We denote $Pr(\alpha \geq \alpha^{T_U})$ as the probability that in a performance review period, the upper target ready rate is satisfied. Thus, $Pr(\alpha \geq \alpha^{T_L}) = Pr(\sum_{t=1}^{T} \mathbf{1}(d_t \geq S_t) \geq T\alpha^{T_U})$. Under a dynamic inventory policy, $\sum_{t=1}^{T} \mathbf{1}(d_t \leq S_t)$ is the sum of T Bernoulli random variables with different probabilities of success. Therefore, it has a Poisson-binomial distribution. Thus, for a dynamic inventory policy, we have:

$$\Pr\big(\alpha < \alpha^{T_L}\big) = \sum_{l=1}^{T\alpha^{T_L}-1} \sum_{A \in W_l} \prod_{i \in A} F_{d_i}(S_i) \prod_{j \in A^c} (1 - F_{d_j}(S_j))$$

$$\Pr\big(\alpha \geq \alpha^{T_U}\big) = \sum_{l=T\alpha^{T_U}}^{T} \sum_{A \in W_l} \prod_{i \in A} F_{d_i}(S_i) \prod_{j \in A^c} (1 - F_{d_j}(S_j)$$

where W_l is the set of all subsets of l integers that can be selected from $\{1, 2, 3, ..., T\}$. The above equations compute the probabilities for all combinations of periods for which the demand is satisfied. If $T\alpha^{T_L}$ and $T\alpha^{T_U}$ are not integer values, they are rounded down to the closest integer value for $T\alpha^{T_L}$ and rounded up to the closest integer number for $T\alpha^{T_U}$. Note that under a static base-stock policy, $\sum_{t=1}^{T} \mathbf{1}(d_t \leq S_t)$, and the cumulative inventory performance during a performance review period (T) has a binomial distribution with a probability of success equal to $F_d(S)$, where S is the base-stock level for the static policy.

SLA MODEL WITH THE READY RATE AND LUMP-SUM PENALTY AND BONUS

Under an SLA with a lump-sum penalty and bonus structure, if the supplier's ready rate in a performance review period with length T does not exceed the penalty threshold α^{T_L}, the supplier compensates the retailer via a lump-sum penalty $\mathcal{L}^-$. However, when the supplier's ready rate exceeds the bonus threshold α^{T_U} during a performance review period (T), the supplier will receive a lump-sum bonus $\mathcal{L}^+$ from the retailer.

The supplier's cost function has two components: holding and penalty costs. Therefore, we can write the supplier's objective function per period with a lump-sum penalty/bonus structure and backorder assumptions as:

$$\underset{S_1,\ldots,S_T}{Min}\ C_s = h\sum_{t=1}^{T} E([S_t - d_t]^+) + \mathcal{L}^- Pr(\alpha < \alpha^{T_L}) - \mathcal{L}^+ Pr(\alpha \geq \alpha^{T_U}) \tag{1}$$

As discussed in Ready Rate Analysis Section, regarding the probability of exceeding the upper target ready rate and not satisfying the lower target, we can rewrite Equation 1 as:

$$\underset{S_1,\ldots,S_T}{Min}\ C_s = h\sum_{t=1}^{T} E([S_t - d_t]^+) + \mathcal{L}^- E(\mathbf{1}(\sum_{t=1}^{T}\mathbf{1}(d_t \leq S_t)) < T\alpha^{T_L}) - \mathcal{L}^+ E(\mathbf{1}(\sum_{t=1}^{T}\mathbf{1}(d_t \leq S_t)) \geq T\alpha^{T_U})$$

Solution Structure

For a strategic supplier who chooses a dynamic base-stock policy, the decisions on $S_1, \ldots, S_T$ do not need to be made simultaneously. Thus, we can formulate it as a stochastic dynamic programming model with T stages, where each stage is a replenishment cycle:

$$\begin{aligned}\underset{S_i}{Min} C_s(\tilde{v}_i = \sum_{t=1}^{i}\mathbf{1}(d_t \leq S_t)) =& h(\sum_{t=1}^{i-1}[S_t - d_t]^+ + E([S_i - d_i]^+) + E(\mathbf{1}(d_i \leq S_i))V_{i+1}(\tilde{v}_{i+1} = \tilde{v}_i + 1) \\ &+ E(\mathbf{1}(d_i > S_i))V_{i+1}(\tilde{v}_{i+1} = \tilde{v}_i)\end{aligned}$$

where i is the index for the i^{th} inventory review period (e.g., the i^{th} day); therefore, the cost of the previous $i-1$ inventory review period is observed as $h\sum_{t=1}^{i-1}[S_t - d_t]^+$, and the cost associated with the decision variable S_i is $hE([S_i - d_i]^+) + E(\mathbf{1}(d_t \leq S_t))V_{i+1}(\tilde{v}_{i+1} = \tilde{v}_i + 1) + E(\mathbf{1}(d_i > S_i))V_{i+1}(\tilde{v}_{i+1} = \tilde{v}_i)$. $V_i(\tilde{v}_i)$ is the cost to go function (or value function) in period i when the state of the system is $\tilde{v}_i$. The state of the system is defined as the number of periods in which the supplier has fully satisfied the demand so far, i.e., $\tilde{v}_i = \sum_{t=1}^{i}\mathbf{1}(d_t \leq S_t)$. As $E(\mathbf{1}(d_t \leq S_t)) = F_{d_t}(S_t)$, we can rewrite this as:

$$\begin{aligned}\underset{S_i}{Min} C_s(\tilde{v}_i = \sum_{t=1}^{i}\mathbf{1}(d_t \leq S_t)) =& h(\sum_{t=1}^{i-1}[S_t - d_t]^+ + E([S_i - d_i]^+) + F_{d_i}(S_i)V_{i+1}(\tilde{v}_{i+1} = \tilde{v}_i + 1) \\ &+ (1 - F_{d_i}(S_i))V_{i+1}(\tilde{v}_{i+1} = \tilde{v}_i)\end{aligned}$$

Therefore, the cost-to-go function in the last period ($i = T$), i.e., $V_i^*(\tilde{v}_i)$ and S_T^* is obtained according to Table 1. For $i(i = 1, \ldots, T-1)$, we have:

$$V_i(\tilde{v}_i) = hE([S_i - d_i]^+) + F_{d_i}(S_i)V_{i+1}^*(\tilde{v}_{i+1} = \tilde{v}_i + 1) + (1 - F_{d_i}(S_i))V_{i+1}^*(\tilde{v}_{i+1} = \tilde{v}_i) \tag{2}$$

Table 1: The optimal S^* and the value function in the last period: lump-sum penalty and bonus

Conditions	$V_T(\tilde{v}_T)$	S_T^*	**Explanation**
$\tilde{v}_T \geq T\alpha^{T_L}$ and $\tilde{v}_T + 1 < T\alpha^{T_U}$	0	0	The supplier receives no penalty or bonus
$\tilde{v}_T + 1 < T\alpha^{T_L}$	$\mathcal{L}^-$	0	The supplier receives a lump-sum penalty
$\tilde{v}_T \geq T\alpha^{T_U}$	$-\mathcal{L}^+$	0	The supplier receives a lump-sum bonus
$\tilde{v}_T < T\alpha^{T_L}$ *and* $\tilde{v}_T + 1 \geq T\alpha^{T_L}$ *and* $\tilde{v}_T + 1 < T\alpha^{T_U}$	$hE([S_T - d_T]^+) + \mathcal{L}^-(1 - F_{d_T}(S_T))$	$arg.Min(hE([S_T - d_T]^+) + \mathcal{L}^-(1 - F_{d_T}(S_T))$	There is a chance to avoid a penalty
$\tilde{v}_T \geq T\alpha^{T_L}$ *and* $\tilde{v}_T + 1 \geq T\alpha^{T_U}$	$hE([S_T - d_T]^+) - \mathcal{L}^+ F_{d_T}(S_T)$	$arg.Min(hE([S_T - d_T]^+) - \mathcal{L}^+ F_{d_T}(S_T))$	There is a chance to obtain a bonus
$\tilde{v}_T < T\alpha^{T_L}$ *and* $\tilde{v}_T + 1 \geq T\alpha^{T_U}$	$hE([S_T - d_T]^+) + \mathcal{L}^-(1 - F_{d_T}(S_T)) - \mathcal{L}^+ F_{d_T}(S_T)$	$arg.Min(hE([S_T - d_T]^+) + \mathcal{L}^-(1 - F_{d_T}(S_T)) - \mathcal{L}^+ F_{d_T}(S_T))$	A bonus or a penalty is applied. This situation occurs when T is small

A non-strategic supplier, on the other hand, uses a static base-stock policy. Under this stocking policy, the supplier uses the same stock level in every inventory period. In Equation 1, the objective function for the static inventory policy under the lump-sum penalty and bonus terms with the assumption of *i.i.d.* demands can be rewritten as:

$$\begin{aligned} \underset{S}{Min} C_s =& T^* h^* E([S-d]^+) + \mathcal{L}^- \sum_{i=0}^{\lfloor T\alpha^{T_L} - \epsilon \rfloor} \binom{T}{i} [F_d(S)]^i [1 - F_d(S)]^{T-i} \\ &- \mathcal{L}^+ \sum_{i=\lceil T\alpha^{T_U} \rceil}^{T} \binom{T}{i} [F_d(S)]^i [1 - F_d(S)]^{T-i} \end{aligned} \tag{3}$$

where $\lfloor . \rfloor$ and $\lceil . \rceil$ are the floor and roof functions, respectively. The supplier's average cost per period under the static and dynamic inventory policies are $\frac{C_S^*}{T}$ and $\frac{C_S^*(V_1(0))}{T}$, respectively. Given that the analytical results are not available, we will conduct an extensive numerical analysis.

SLA MODEL WITH THE READY RATE AND LINEAR PENALTY AND BONUS

Under a dynamic inventory policy, the total cost function for the model with the ready rate and linear penalty/bonus components is expressed as:

$$\begin{aligned} \underset{S_1,\ldots,S_T}{Min} C_s =& h \sum_{t=1}^{T} E([S-d]^+) + \mathcal{L}^{-l} E([T\alpha^{T_L} - \sum_{t=1}^{T} \mathbf{1}(d_t \leq S_t)]^+) \\ &- \mathcal{L}^{+l} E([\sum_{t=1}^{T} \mathbf{1}(d_t \leq S_t) - T\alpha^{T_U}]^+) \end{aligned}$$

Solution Structure

In each period, decisions are made only on the stocking level of that period. The model can be expressed as the following stochastic dynamic programming formulation:

$$\underset{S_i}{Min} C_s(\tilde{v}_i = \sum_{t=1}^{i} \mathbf{1}(d_t \leq S_t)) = h(\sum_{t=1}^{i-1}[S_t - d_t]^+ + E([S_i - d_i]^+) + F_{d_i}(S_i)V_{i+1}(\tilde{v}_{i+1} = \tilde{v}_i + 1)$$
$$+(1 - F_{d_i}(S_i))V_{i+1}(\tilde{v}_{i+1} = \tilde{v}_i)$$

where $V_i(\tilde{v}_i)$ is the cost-to-go function in period i when the state of the system is $\tilde{v}_i$. The state of the system is defined as the number of periods where the supplier could fully satisfy the demand, i.e., $\tilde{v}_i = \sum_{t=1}^{i} \mathbf{1}(d_t \leq S_t)$. The cost-to-go function in the last period, i.e., $V_T(\tilde{v}_T)$ and optimal S_T^*, are obtained in Table 2.

Table 2: The optimal S^* and value function in the last period: linear penalty and bonus

Conditions	$V_T(\tilde{v}_T)$	S_T^*	**Explanation**
$\tilde{v}_T \geq T\alpha^{T_L}$ *and* $\tilde{v}_T + 1 < T\alpha^{T_U}$	0	0	The supplier receives no penalty or bonus
$\tilde{v}_T + 1 < T\alpha^{T_L}$	$hE([S_T - d_T]^+)$ $+\mathcal{L}^{-l}(T\alpha^{T_L} - \tilde{v}_T - F_{d_T}(S_T))/T$	$arg.Min(hE([S_T - d_T]^+)$ $+\mathcal{L}^{-l}(T\alpha^{T_L} - \tilde{v}_T - F_{d_T}(S_T))/T$	There is a chance to avoid a penalty
$\tilde{v}_T \geq T\alpha^{T_U}$	$hE([S_T - d_T]^+)$ $-\mathcal{L}^{+l}(-T\alpha^{T_U} + \tilde{v}_T + F_{d_T}(S_T))/T$[1]	$arg.Min(hE([S_T - d_T]^+)$ $-\mathcal{L}^{+l}(T\alpha^{T_L} - \tilde{v}_T - F_{d_T}(S_T))/T$	There is a chance to receive a bonus
$\tilde{v}_T < T\alpha^{T_L}$ *and* $\tilde{v}_T + 1 \geq T\alpha^{T_L}$ *and* $\tilde{v}_T + 1 < T\alpha^{T_U}$	$hE([S_T - d_T]^+)$ $+\mathcal{L}^{-l}(1 - F_{d_T}(S_T))(T\alpha^{T_L} - \tilde{v}_T)/T$	$arg.Min(hE([S_T - d_T]^+)$ $+\mathcal{L}^{-l}(1 - F_{d_T}(S_T))(T\alpha^{T_L} - \tilde{v}_T)/T$	There is a chance to avoid a penalty
$\tilde{v}_T \geq T\alpha^{T_L}$ *and* $\tilde{v}_T + 1 \geq T\alpha^{T_U}$	$hE([S_T - d_T]^+)$ $-\mathcal{L}^{+l}F_{d_T}(S_T)(-T\alpha^{T_U} + \tilde{v}_T + 1)/T$	$arg.Min(hE([S_T - d_T]^+)$ $-\mathcal{L}^{+l}F_{d_T}(S_T)(-T\alpha^{T_U} + \tilde{v}_T + 1)/T$	There is a chance to receive a bonus
$\tilde{v}_T < T\alpha^{T_L}$ *and* $\tilde{v}_T + 1 \geq T\alpha^{T_U}$	$hE([S_T - d_T]^+)$ $+\mathcal{L}^{-l}(1 - F_{d_T}(S_T))(T\alpha^{T_L} - \tilde{v}_T)/T$ $-\mathcal{L}^{+l}F_{d_T}(S_t)(-T\alpha^{T_U} + \tilde{v}_T + 1)/T$	$arg.Min(hE([S_T - d_T]^+)$ $+\mathcal{L}^{-l}(1 - F_{d_T}(S_T))(T\alpha^{T_L} - \tilde{v}_T)/T$ $-\mathcal{L}^{+l}F_{d_T}(S_t)(-T\alpha^{T_U} + \tilde{v}_T + 1)/T$	There is a chance to receive a bonus or a penalty

Using a backward procedure, we can solve the stochastic dynamic programming model as: $V_{T-1}(\tilde{v}_{T-1}) = hE([S_{T-1} - d_{T-1}]^+) + F_{d_{T-1}}(S_{T-1})V_T^*(\tilde{v}_T = \tilde{v}_{T-1} + 1) + (1 - F_{d_{T-1}}(S_{T-1}))V_T^*(\tilde{v}_T = \tilde{v}_{T-1})$. Therefore, for $i = 1, ..., T-1$, we have:

$$V_i(\tilde{v}_i) = hE([S_i - d_i]^+) + F_{d_i}(S_i)\, V_{i+1}^*(\tilde{v}_{i+1} = \tilde{v}_i + 1) + (1 - F_{d_i}(S_i)\, V_{i+1}^*(\tilde{v}_{i+1} = \tilde{v}_i) \qquad (4)$$

The objective function for the static policy with a linear penalty and bonus, assuming *i.i.d.* demand, can be

[1]The formula is obtained from: $hE([S_T - d_T]^+) - F_{d_t}(S_T)\mathcal{L}^{+l}(\tilde{v}_T + 1 - T\alpha^{T_U})/T - (1 - F_{d_t}(S_T))\mathcal{L}^{+l}(\tilde{v}_T - T\alpha^{T_U})/T$

expressed as:

$$\begin{aligned} \underset{S}{Min} C_s =& T^* h^* E([S-d]^+) + \mathcal{L}^- \sum_{i=0}^{\lfloor T\alpha^{T_L}-\epsilon \rfloor} \frac{(T\alpha_{T_L}-i)}{T} \binom{T}{i} [F_d(S)]^i [1-F_d(S)]^{T-i} \\ & - \mathcal{L}^+ \sum_{i=\lceil T\alpha^{T_U} \rceil}^{T} \frac{(i-T\alpha_{T_U})}{T} \binom{T}{i} [F_d(S)]^i [1-F_d(S)]^{T-i} \end{aligned} \tag{5}$$

The supplier's average costs per period under the static and dynamic inventory policies are $\frac{C_S^*}{T}$ and $\frac{C_S^*(V_1(0))}{T}$, respectively.

COMPUTATIONAL RESULTS

In this section, we perform experimental studies in order to investigate the impact of SLA parameters such as the length of the performance review period (T), incentive structure, and incentive magnitude on the supplier's cost function under a dynamic inventory replenishment policy, compared to a static policy. We also investigate the impact of these parameters on the supplier's performance and the probability of meeting or exceeding the target service levels. The computational study is based on a simulation of an inventory system using the analytical calculations provided in the previous sections. The comparison is undertaken using both lump-sum and linear incentive schemes where both penalty and bonus structures are embedded in the model. An analysis is also performed for a penalty-only structure, and results are summarised in Appendix B. The numerical results demonstrate that, as might be expected, the supplier's cost under a dynamic base-stock policy is lower than under a static policy. The amount of cost saving is impacted by the key parameters, i.e., the inventory holding cost, incentive values, performance review period, and demand distribution. In addition, an optimal static inventory policy performs better in terms of the average ready rate for small-to-medium holding costs. However, when the inventory holding cost is high, there is an increased chance that a static policy may fail to meet the penalty targets (or contract requirements) and will underperform in terms of the average ready rate.

On the assumption that the demand distribution can have an impact on inventory replenishment decision models (Käki, Salo, & Talluri, 2013), in this study we consider Poisson and normal distributions to model the stochastic demand. Management insights are derived based on these two distributions. However, the formulations for the optimal stock levels presented in in ready rate analysis section are general and not limited to any distribution. With a normal demand (μ, σ^2) assumption, we discretise and truncate the distribution such that: $f(x) = \phi((x-\mu+1)/\sigma) - \phi((x-\mu)/\sigma)$ and $F(x) = \phi((\lfloor x \rfloor + 1 - \mu)/\sigma)$; therefore, only non-negative demands are included in the analysis. The results for the normal distribution (15,5) are provided in the Supplementary Materials. We have also presented the analytical solutions of these models for a special case with Uniform demand and a performance review period of length 2 in Appendix A.

In this section, we use the Poisson distribution with $\lambda = 5$ for the demand data and analyse the results for various lengths of performance review periods of $T = 10, 20, 30, 40, 50, 60, 70, 80$, and 90. For the shorter horizon of $T = 1$, the dynamic inventory policy is the same as the static policy; therefore, it is not included in the simulation study. The target service level for the bonus (i.e., the bonus threshold) is assumed to be $\alpha^{T_U} = 0.9$. The penalty threshold (i.e., the lower target service level) is set to $\alpha^{T_L} = 0.8$. The reason for choosing the above values for the upper and lower incentive thresholds is to have the $T\alpha^{T_U}$ and $T\alpha^{T_L}$ as integer values to enable a comparison in the simulation analysis. Otherwise, the non-integer values will affect the result, as they will correspond to higher incentive thresholds. The non-integer values impose unintended changes in the implied targets; for example, when $T = 15$ and $\alpha^{T_L} = 0.9$, the demands of at least 14 periods should be met to avoid a penalty, which is equivalent to a higher penalty target of 0.93. The inventory holding cost per replenishment period per unit is assumed to be $h = 0.5, 1, 1.5$ and 2 to investigate the impact of wide-range holding costs on the models.

The optimal stock levels, S^*, under the static and dynamic inventory policies, are obtained using the solutions provided in the modelling sections for lump-sum and linear incentives, and are calculated by solving the models numerically. For example, under linear incentives with a static stocking policy, Equation 5 was solved by the enumeration method in the range of 0 and a high S. In this study, a high value of S equal to 40 is used for all models. We then performed an experimental study using a simulation with 18,445 iterations to investigate the impact of SLA parameters such as the incentive structure, magnitude, and performance review period on the supplier's cost and performance under static and dynamic stocking policies. The number of simulation iterations was calculated using the Dvoretzky-Kiefer-Wolfowitz (DKF) inequality (Kosorok, 2008), with an error in the estimation of 0.01 and a 95% confidence level.

Lump-sum Incentives

In this section, we consider an SLA subject to lump-sum incentives (penalty and bonus) while the target penalty and bonus threshold are predefined. The penalty is assumed to be equal to 5 per period ($\mathcal{L}^- = 5T$). i.e., $\mathcal{L}^- = 50, 100, 150, 200, 250, 300, 350, 400$, and 450. A penalty is imposed on the supplier if the calculated ready rate at each performance review point is below 0.8. A bonus of 3 per period is considered (i.e., $\mathcal{L}^+ = 30, 60, 90, 120, 150, 180, 210, 240$, and 270) and is applied at each performance review point if the calculated ready rate is above or equal to 0.9.

Supplier's Stocking Decision

Figure 2 presents the supplier's optimal average costs per inventory period using a static base-stock S^*, compared to the dynamic policy with different lengths of performance review periods. In this figure, the analysis is presented for the inventory holding cost per unit of 0.5, 1, 1.5, and 2; in addition, the variation in cost in the simulated inventory system is included as one standard deviation from the mean (the shaded areas in the plots). The penalty-to-bonus ratio in this example is 1.67. The impact of the incentive magnitude or ratio on the supplier's costs is shown in this section. The analysis for a lump-sum penalty SLA is presented in Appendix B. It can be seen that the supplier may significantly benefit from a dynamic policy through cost savings under different performance review period settings. The reason is that a dynamic inventory model provides suppliers with an opportunity to dynamically adjust their stock levels throughout the review period so that the total costs are minimised. In terms of variation in the average cost, the upper confidence limits are consistently lower in a dynamic policy, compared to the static model across all performance review periods with different holding costs. The optimal stock S^* under the static inventory policy in this study is equal to 8 for all performance review periods and for all h values from 0.5 to 2, except for the unit holding cost of 2 ($h = 2$) when $T = 10$ and 20. In this case, the optimal stock under the static inventory policy is equal to 7 ($S^* = 7$). Therefore, this has caused spikes within a one standard deviation interval of the mean for $h = 2$ and $T = 10, 20$, as seen in Figure 2. However, we have observed that the optimal cost values in these two periods are insignificantly different from the optimal $S^* = 8$ obtained for the other performance review periods. The optimal dynamic stock levels are presented in Figure 3, and are compared to the static model.

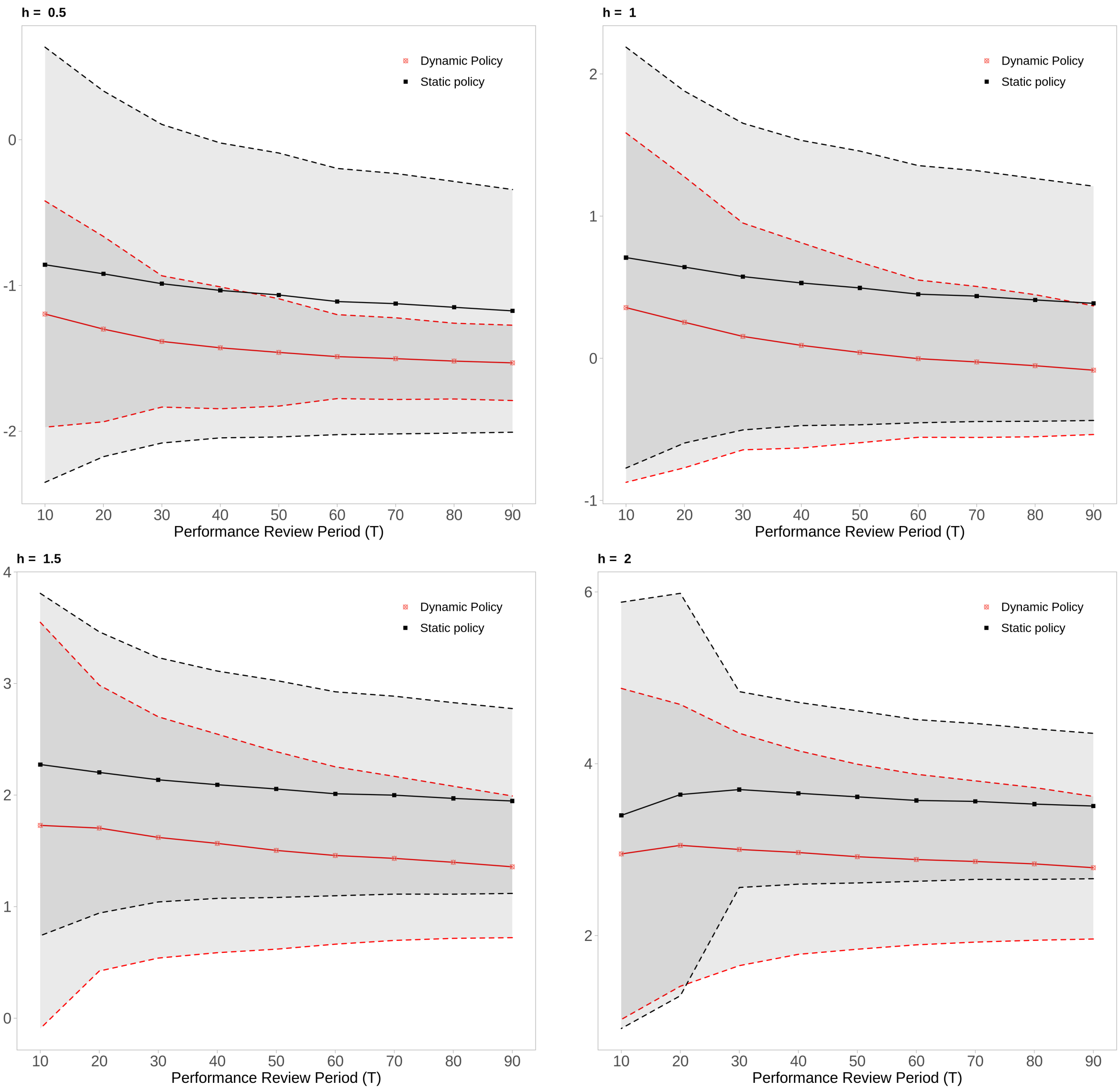
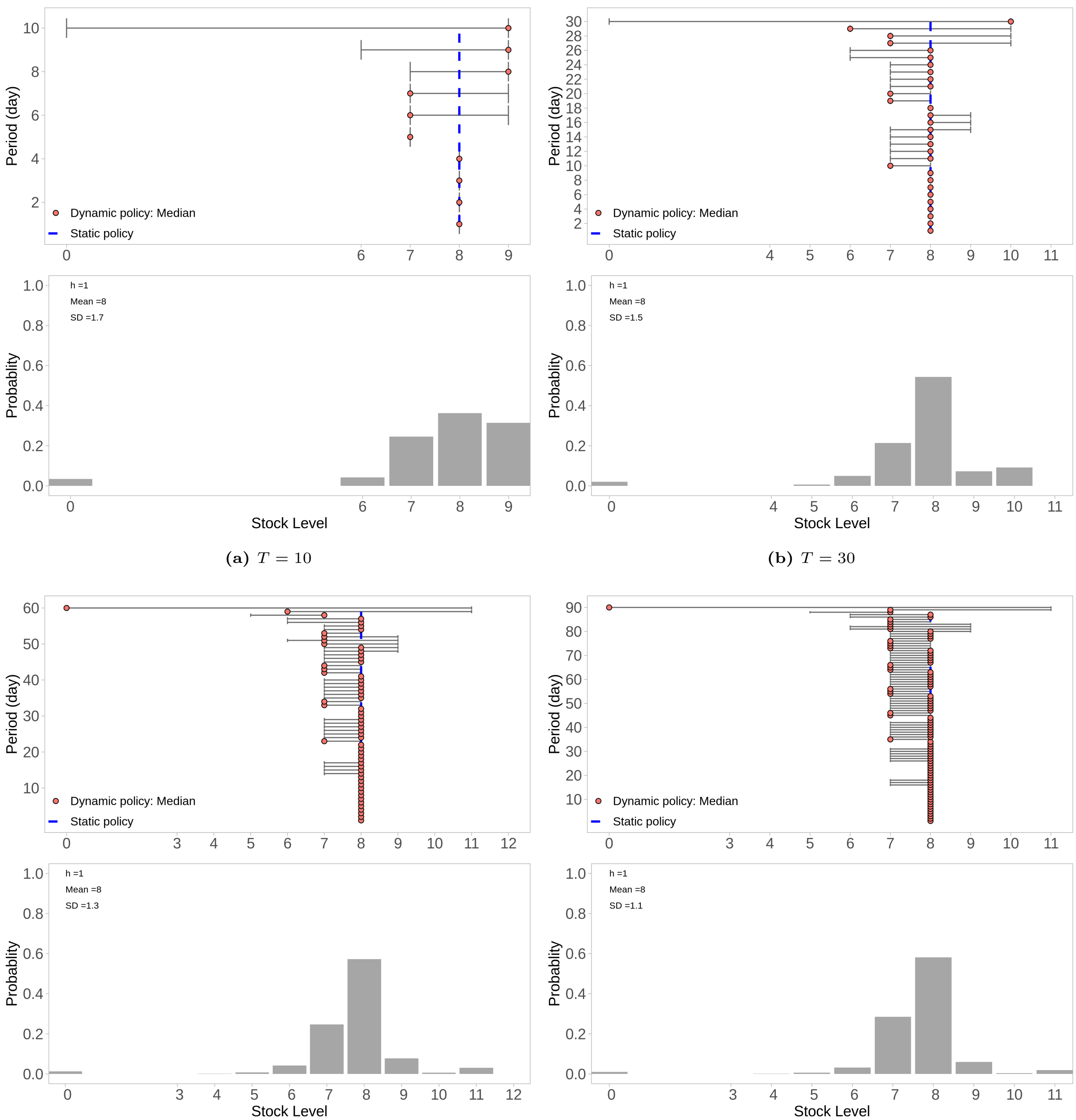


Figure 2: Lump-sum incentives SLA: Supplier's average cost within one standard deviation of the means per period for different performance review periods and holding costs – comparison for the dynamic and static stocking policies

Remark 1. ***Impact of PRP and holding costs on the supplier's cost:*** *Under the lump-sum incentive (or penalty-only) scheme, the supplier's average cost is significantly lower using a dynamic stocking policy for different performance review periods and inventory holding costs, as might be expected. The supplier's average cost per period is not monotonic for high holding costs with respect to the performance review period under either inventory system. However, the supplier's average cost decreases for the longer performance review periods,* T.



(a) $T = 10$ (b) $T = 30$ (c) $T = 60$ (d) $T = 90$

Figure 3: Distribution of the optimal stock level under a dynamic inventory policy for a lump-sum incentive SLA (with unit holding cost of 1).

In Figure 3, the distributions of optimal stocking levels in this simulation study are presented for the cases of $T = 10, 30, 60$, and 90, and $h = 1$ for a dynamic base-stock inventory policy under an SLA with lump-sum incentives. Further, in Figure 3, the distributions of the optimal dynamic stock decisions in each period (e.g., on a daily basis) are provided using line charts over $T = 10, 30, 60$, and 90. The upper and lower limits in these lines correspond to the first and third quartiles. The median stock values are also presented in this figure. For instance, when $T = 10$, the median optimal dynamic stock level is 8, with an interquartile of zero for days 1-4; however, from days 5-10, the median stock level fluctuates between 7 and 9, depending on the realised ready rate calculated so far. Note that in this example, the dynamic stocking decision in the last period ($T = 10$) is between 0 and 9, with a median of 9. Not surprisingly, this is due to the strategic supplier's opportunity to adjust stock decisions in the last period based on the realised service level to avoid a penalty or obtain a bonus.

Experienced Service Level

Given the optimal base-stocks under the parameters presented in Lump-sum Incentives subsection, we calculated the ready rate for the static and dynamic stocking policies in a simulated inventory system. Over a finite horizon, the ready rate is a random variable. The expected ready rate for the static stock can be obtained from $\alpha_T^A = \sum_{i=0}^{T} \frac{i}{T} \binom{T}{i} [F_d(S)]^i [1 - F_d(S)]^{T-i}$. In the following, the results of the service-level distributions in the simulation study are presented for the lump-sum incentive scheme in Figure 4 and summarised in Table 3.

Figure 4 shows the distribution of the ready rate for dynamic and static inventory policies for $h = 0.5, 1, 1.5,$ and 2 and $T = 10, 30, 60,$ and 90 when the supplier is operating under an SLA with lump-sum incentives. The horizontal dashed lines in these plots denote the upper and lower target ready rates. Outliers are not shown in this figure; however, their impacts are reported in Table 3 with the probability of receiving a penalty or bonus and the standard deviation in each case. The invariability seen in Figure 4 for the dynamic stocking policy demonstrates the behaviour of a strategic supplier reluctant to exceed the upper target service level. For example, for $T = 10$ and $h = 0.5$, there is a 96.1% chance that the realised ready rate will be equal to the target (see Table 3), while the chance of exceeding the upper target of $\alpha^{T_U} = 0.9$ is zero. However, when the inventory holding cost per unit is high ($h = 2$), and the value of T is small ($T = 10$), the supplier's performance under a dynamic policy is highly volatile. Figure 4 also shows that the distribution of the ready rate in a static policy is highly dispersed, compared to the distribution under a dynamic stocking policy; yet, this variation decreases as T increases.

The probabilities of meeting or exceeding the upper target service level and failing to meet the lower target service level over the performance review periods are presented in Table 3. These probabilities can also be obtained analytically from the chance of absorption in each absorbing state of the corresponding Markov chain (see the Analytical Calculation for the Probability of Exceeding the Target Service Level Section.)

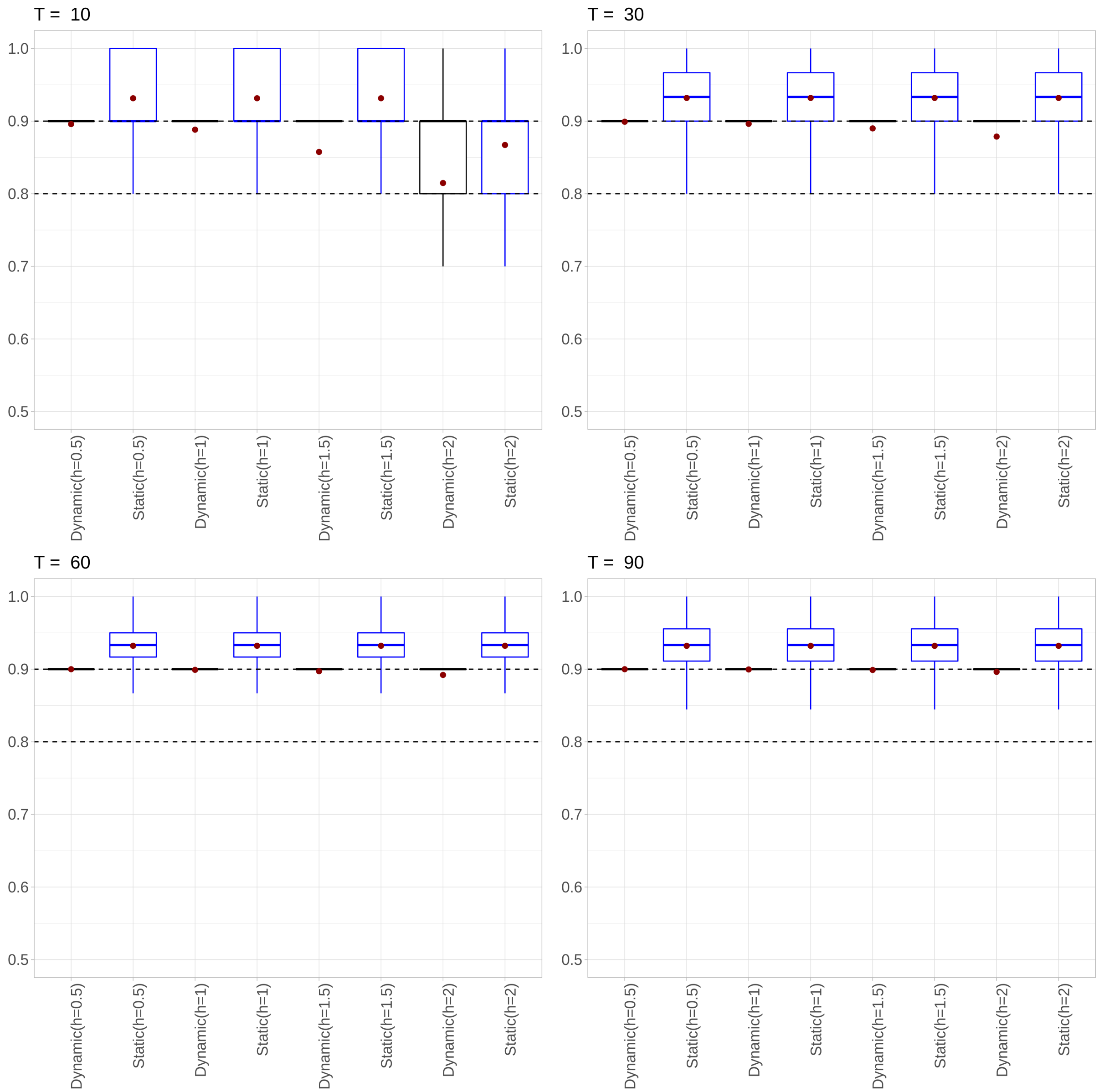


Figure 4: Distribution of the ready rate under a lump-sum incentive SLA – comparison of dynamic and static stocking policies. The red dot point presents the average ready rates

Here we observe that, under both stocking policies with a lump-sum incentive scheme, the average ready rate falls above the lower target service levels. However, under a static stocking policy, there is a greater chance that the supplier performance levels will fall below the lower limit. In practice, very low realised ready rates are more likely under a static policy. For example, with $T = 10$ and $h = 0.5$, there is a 2.6% chance that the ready rate will fall below 80% using a static inventory policy, versus approximately 0.1% under a dynamic stocking policy. The probability that the realised ready rate is above 90% (the upper service level target) is near zero for $h = 0.5$ under a dynamic stocking policy. With the dynamic inventory policy, a higher ready rate is expected to be observed in practice as the length of the performance review period increases (Table 3). On average, the ready rate is higher with more variability under a static stocking policy, compared to a dynamic policy. When h, the holding cost, is small, suppliers tend to keep more inventory in stock, which means that the chance of receiving bonuses and avoiding penalties is higher. Interestingly, under the lump-sum penalty-only structure and for high holding costs, the static policy can lead to a high chance of frequent failure to meet the contract requirements in terms of the average ready rate. Given that in this case the optimal static policy suggests less investment in inventory, the chance of receiving a penalty is significant. The results are consistent under symmetric and skewed distributions.

Table 3: Summary of the ready rate distribution under an SLA with lump-sum incentives

		Dynamic stocking policy				Static stocking policy			
h	T	Average (%)	SD (%)	$Pr(\alpha < \alpha^{T_L})$ (%)	$Pr(\alpha \geq \alpha^{T_U})$ (%)	Average (%)	SD (%)	$Pr(\alpha < \alpha^{T_L})$ (%)	$Pr(\alpha \geq \alpha^{T_U})$ (%)
	10	89.59	2.55	0.13	96.10	93.15	8.01	2.67	85.23
	20	89.70	1.78	0.01	96.59	93.14	5.67	1.07	84.49
	30	89.91	0.98	0.00	98.83	93.19	4.60	0.36	85.61
0.5	40	89.93	0.87	0.00	98.93	93.23	3.96	0.11	86.76
	50	89.94	0.80	0.00	99.17	93.19	3.57	0.07	87.61
	60	89.99	0.43	0.00	99.70	93.22	3.24	0.01	89.06
	70	89.99	0.43	0.00	99.66	93.17	3.00	0.01	89.58
	80	90.00	0.29	0.00	99.72	93.16	2.82	0.00	90.32
	90	89.99	0.35	0.00	99.67	93.21	2.69	0.01	91.18
	10	88.82	4.92	0.73	90.45	93.15	8.01	2.67	85.23
	20	89.11	3.15	0.17	90.90	93.14	5.67	1.07	84.49
	30	89.63	1.87	0.01	95.59	93.19	4.60	0.36	85.61
1	40	89.73	1.61	0.00	96.58	93.23	3.96	0.11	86.76
	50	89.82	1.33	0.01	97.71	93.19	3.57	0.07	87.61
	60	89.91	0.95	0.00	98.58	93.22	3.24	0.01	89.06
	70	89.92	0.85	0.00	98.66	93.17	3.00	0.01	89.58
	80	89.94	0.71	0.00	98.88	93.16	2.82	0.00	90.32
	90	89.96	0.61	0.00	99.19	93.21	2.69	0.01	91.18
	10	85.75	11.82	4.05	75.21	93.15	8.01	2.67	85.23
	20	88.59	4.29	0.44	86.17	93.14	5.67	1.07	84.49
	30	88.99	3.11	0.10	89.43	93.19	4.60	0.36	85.61
1.5	40	89.25	2.60	0.02	91.57	93.23	3.96	0.11	86.76
	50	89.57	1.98	0.02	94.57	93.19	3.57	0.07	87.61
	60	89.71	1.58	0.00	96.14	93.22	3.24	0.01	89.06
	70	89.77	1.45	0.00	97.10	93.17	3.00	0.01	89.58
	80	89.85	1.16	0.00	97.89	93.16	2.82	0.00	90.32
	90	89.89	1.03	0.00	98.39	93.21	2.69	0.01	91.18
	10	81.47	20.88	10.38	71.08	86.71	10.77	13.88	60.79
	20	85.71	11.63	3.15	71.29	86.65	7.65	12.02	49.08
	30	87.87	5.83	0.59	79.65	93.19	4.60	0.36	85.61
2	40	88.59	3.64	0.12	85.01	93.23	3.96	0.11	86.76
	50	88.95	3.03	0.07	88.46	93.19	3.57	0.07	87.61
	60	89.20	2.64	0.01	90.65	93.22	3.24	0.01	89.06
	70	89.44	2.22	0.02	92.87	93.17	3.00	0.01	89.58
	80	89.55	1.98	0.02	94.28	93.16	2.82	0.00	90.32
	90	89.63	1.86	0.02	95.47	93.21	2.69	0.01	91.18

* We observe that 96.1% of the realised ready rates are actually equal to $\alpha^{T_U} = 0.9$, while this is near zero for values greater than the upper target level. The same behaviour is observed for the remaining of values in this column ($h = 0.5, 1, 1.5$).

Remark 2. ***Dynamic vs. static inventory policy:*** *Under a lump-sum incentive SLA, the average ready rate is slightly higher when a static policy is adopted; however, its distribution is more volatile. On the other hand, the* *probability of meeting or exceeding the bonus threshold is higher when a dynamic policy is adopted, compared to the static one. Under a lump-sum penalty-only structure and high holding costs, a static policy performs less well than a dynamic one with a high chance of a penalty and a low average service level.*

Remark 3. ***Impact of PRP for a strategic supplier:*** *Using a dynamic stocking policy under a lump-sum incentive SLA contract, the average ready rate increases with less variability when the length of the performance review period is increased. However, with a static stocking policy, this positive impact of the performance review period length on the average ready rate is mainly observed for higher holding costs, given that an optimal static policy recommends holding inventory stocks.*

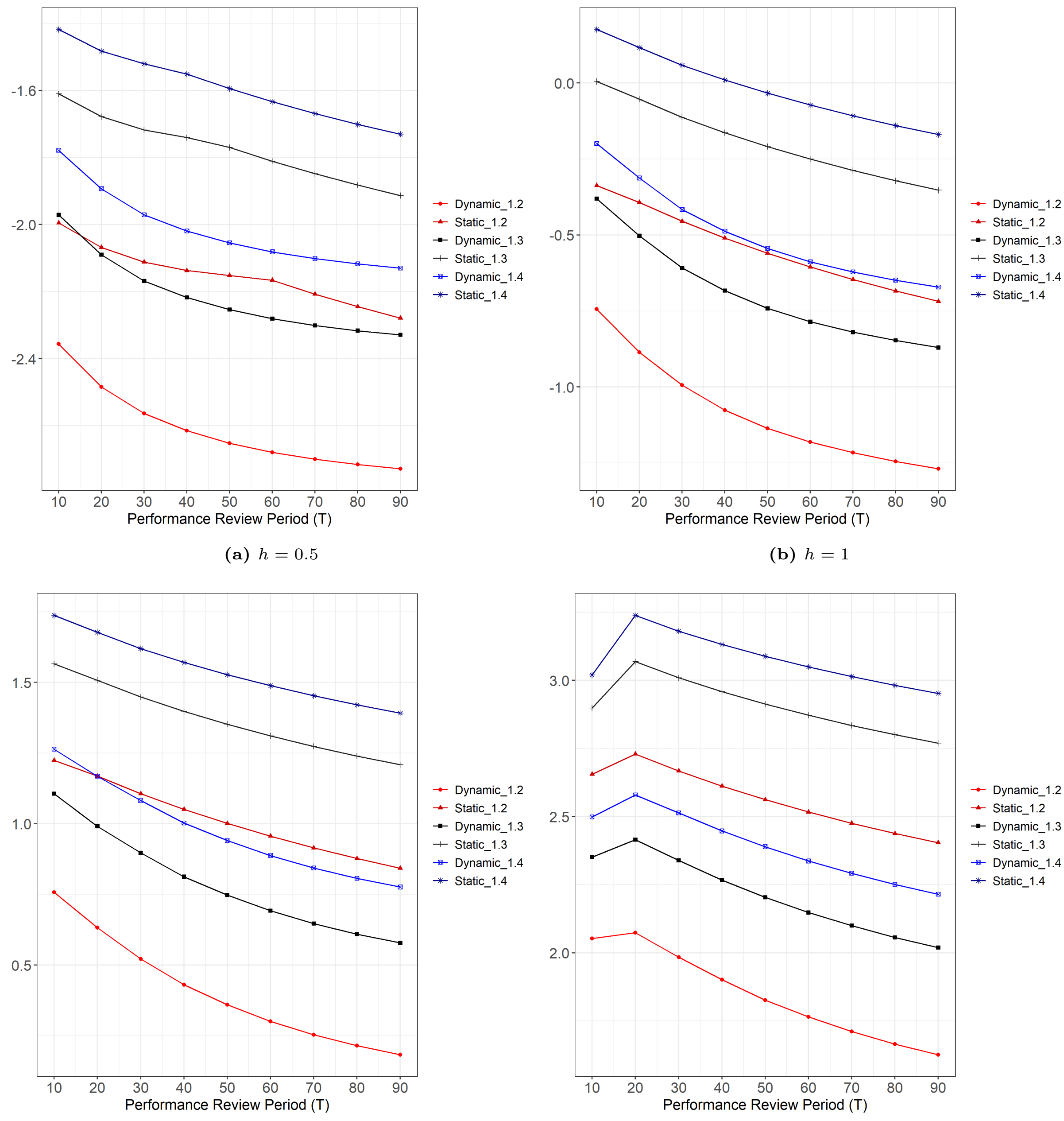
Remark 4. ***Impact of PRP for SLA negotiation:*** *Under a lump-sum incentive scheme, as the length of the performance review period increases, there is a higher chance of meeting or exceeding the bonus threshold, while the chance of receiving a penalty decreases. This result is consistent under both inventory policies, while the dynamic inventory policy slightly outperforms the static policy. This result is not impacted by the holding cost.*

From this simulation study, we see that increasing the holding costs mainly affects the static and dynamic inventory policies in terms of the ready rate distribution for the shorter performance review periods.

Impact of Incentives on the Supplier's Expected Cost and Cost Saving

In this section, we investigate the impact of incentive values on the supplier's cost and cost saving. To account for the impact of a bonus and penalty together, we looked at the penalty-to-bonus ratio. The supplier's average costs are calculated analytically with the expected values of the total cost, as explained in the modelling sections for lump-sum and linear incentives, which are, $\frac{C_S^*}{T}$ and $\frac{C_S^*(V_1(0))}{T}$ under the static and dynamic inventory policies, respectively. The penalty-to-bonus ratio is considered to be $1.2, 1.3$, and 1.4 when the lump-sum penalty is equal to 5. The parameters used in the numerical analysis are based on the numerical study explained in Computational Results section.

Figure 5 shows the supplier's optimal expected costs per inventory period using a dynamic base-stock policy, compared to a static policy. The results are presented with different lengths of performance review periods for $h = 0.5, 1, 1.5$, and 2. It can be seen that the higher ratio corresponds to the higher total expected cost for the supplier. In other words, suppliers may meet the bonus threshold more often when the difference between the penalty and bonus is small. The negative values in this figure represent the supplier's gained profit under this model. The results are consistent under both static and dynamic inventory policies. As discussed previously (see Remark 1), the supplier's cost is reduced for the longer performance review periods, T. We have summarised the results in terms of the supplier's cost savings in a dynamic policy in Table 4 and Figure 6.



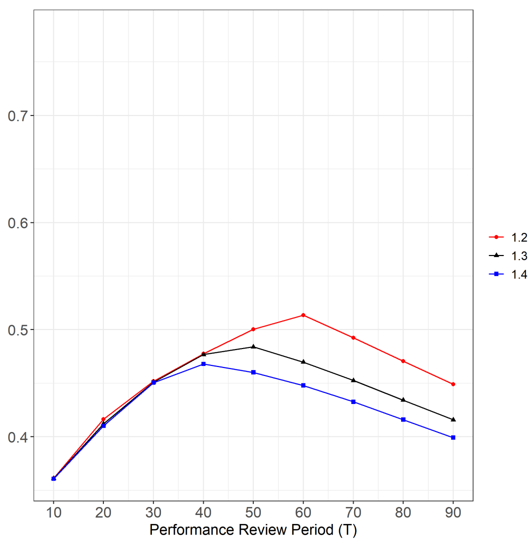
(a) $h = 0.5$ **(b)** $h = 1$ **(c)** $h = 1.5$ **(d)** $h = 2$

Figure 5: Lump-sum incentives SLA: Supplier's cost per period for different performance review periods and holding costs (across the penalty-to-bonus ratio) – comparison for dynamic and static stocking policies

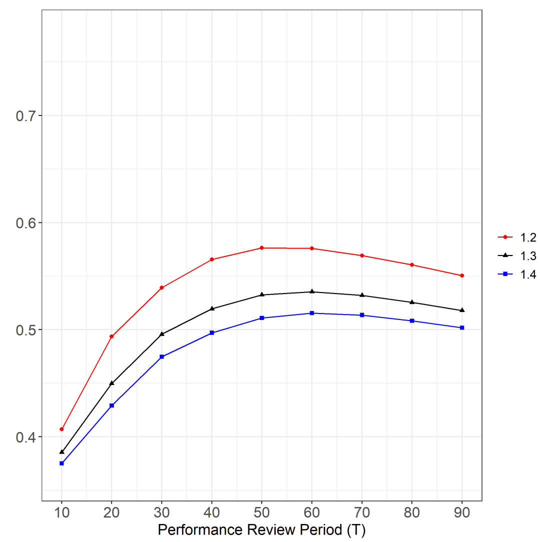
Table 4 provides a summary of the supplier's cost saving under a dynamic policy with lump-sum penalty and bonus terms for different lengths of the performance review period and inventory holding costs. The results are also illustrated in Figure 6. The same scale on the y-axis is used to show the impact of the inventory holding cost, h. The supplier's cost saving is calculated based on the saved cost of using a dynamic policy relative to the cost of a static policy. Figure 6 and Table 4 indicate that the smaller incentive ratio (i.e., penalty-to-bonus ratio) will result in a greater cost saving for a strategic supplier. Further, as the holding cost increases, this cost saving will increase for the longer performance review periods.

Table 4: Supplier's cost saving under an SLA using a dynamic policy with lump-sum incentives – comparison of incentive ratio, holding cost, and PRP

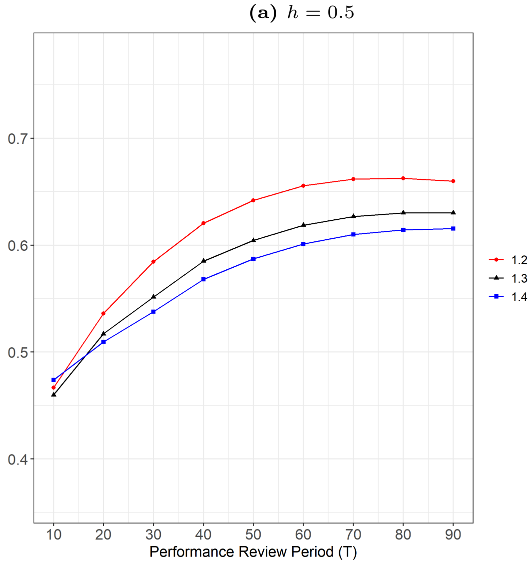
		Performance review period (T)								
Ratio	h	10	20	30	40	50	60	70	80	90
	0.50	0.36	0.42	0.45	0.48	0.50	0.51	0.49	0.47	0.45
1.2	1.00	0.41	0.49	0.54	0.57	0.58	0.58	0.57	0.56	0.55
	1.50	0.47	0.54	0.58	0.62	0.64	0.66	0.66	0.66	0.66
	2.00	0.60	0.66	0.68	0.71	0.74	0.75	0.76	0.77	0.78
	0.50	0.36	0.41	0.45	0.48	0.48	0.47	0.45	0.43	0.42
1.3	1.00	0.39	0.45	0.50	0.52	0.53	0.54	0.53	0.53	0.52
	1.50	0.46	0.52	0.55	0.59	0.60	0.62	0.63	0.63	0.63
	2.00	0.55	0.65	0.67	0.69	0.71	0.72	0.74	0.74	0.75
	0.50	0.36	0.41	0.45	0.47	0.46	0.45	0.43	0.42	0.40
1.4	1.00	0.38	0.43	0.47	0.50	0.51	0.52	0.51	0.51	0.50
	1.50	0.47	0.51	0.54	0.57	0.59	0.60	0.61	0.61	0.62
	2.00	0.52	0.66	0.67	0.68	0.70	0.71	0.72	0.73	0.74



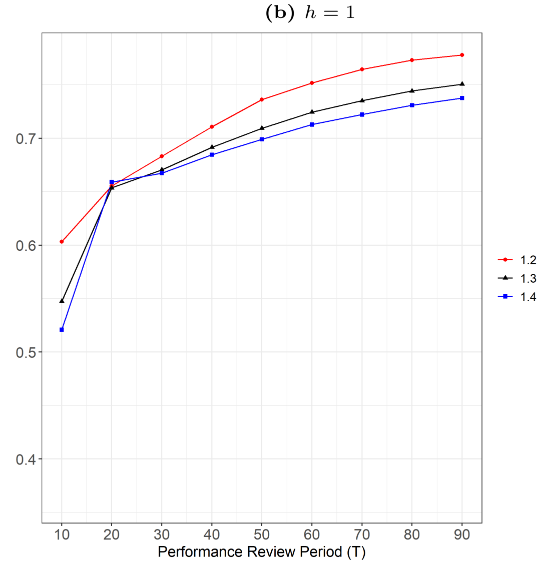
(a) $h = 0.5$



(b) $h = 1$



(c) $h = 1.5$



(d) $h = 2$

Figure 6: Lump-sum incentives SLA: Supplier's cost saving under an SLA using a dynamic policy with lump-sum incentives – comparison of the incentive ratio, holding cost, and PRP

Remark 5. ***Impact of incentives on supplier's cost saving:*** *The smaller difference between the penalty and bonus values results in higher cost saving for the supplier using a dynamic stocking policy under a lump-sum incentive SLA contract. The holding cost does not impact this trend. However, with a higher holding cost and longer performance review period, the supplier will benefit more by adopting a dynamic stocking policy.*

Linear Incentives

Under a linear incentive SLA, the amount of the penalty/bonus applied to the supplier is a proportion of the deviation of the realised service level from the target service levels (i.e., α^{T_L} and α^{T_U}). Hence, in each performance review period with length T, the supplier's ready rate is compared to the upper and lower target levels. For example, if the supplier's ready rate does not exceed the penalty threshold α^{T_L}, the supplier pays a penalty of $\mathcal{L}^{-l}(\alpha^{T_L} - \alpha_r)$ to the retailer, where α_r is the supplier's realised ready rate. To design an experiment for the impact of linear incentives with different performance review periods (T), we considered a fixed ratio incentive per replenishment period (e.g., per day in T periods). That means, for each performance review period, the linear incentives are multiplied by T. In this study, the penalty is assumed to be equal to 30 per period (i.e., $\mathcal{L}^{-l} = 25T$), and a bonus of 15 per period is considered to apply to performances above α^{T_U} (i.e., $\mathcal{L}^{+l} = 15T$). Then, unit incentives are multiplied by the deviation from the targets; for example, when $T = 10$, $\alpha^{T_L} = 0.8$, the realised ready rate is equal to 0.7, then the penalty is calculated as $25 \times 10 \times (0.8 - 0.7) = 25$. Similarly for $T = 20$, we have $25 \times 20 \times (0.8 - 0.7) = 50$, where the unit penalty per replenishment period is constant and is equal to 2.5 in both cases.

Note that, for numerical studies, the linear and lump-sum incentives cannot be set to be equivalent. The performance of each scheme (i.e., lump-sum and linear) depends on the distribution of the ready rate, which itself is a function of the incentive (i.e., penalty and bonus) units. In our numerical study, the lump-sum incentive values ($\mathcal{L}^{-} = 5T$ and $\mathcal{L}^{+} = 3T$) are set to be equivalent to the linear incentive values with a magnitude of a 0.2 deviation from the target ready rate. The remaining parameters used in the numerical analysis are explained in Computational Results Section.

Supplier's Stocking Decision

Figure 7 shows the supplier's cost function per inventory period with different lengths of performance review periods under dynamic and static base-stock policies with linear incentives. In this figure, the comparison is provided for holding costs of 0.5, 1, 1.5, and 2. The shaded areas in these plots show one standard deviation from the mean costs. In this figure, the spike in the one-standard-deviation interval from the means occurs with $T = 70$ when the optimal base-stock S^* changes from 7 to 6 for the remaining PRPs ($T = 70, 80, 90$). We observe that the cost function under this condition has non-significantly improved. Similar to a lump-sum incentive structure, the dynamic inventory model outperforms a static model in terms of cost saving under a linear incentive SLA with different performance review periods. Although this result is not improbable, the values of the incentives can slightly impact this pattern. The impact of the incentive magnitude or ratio on the supplier's costs is shown in this section in Figure 10. Further, the upper confidence limits are consistently lower in the dynamic policy than in the static model across all performance review periods with different holding costs. The analysis for a linear penalty SLA is presented in Appendix B.

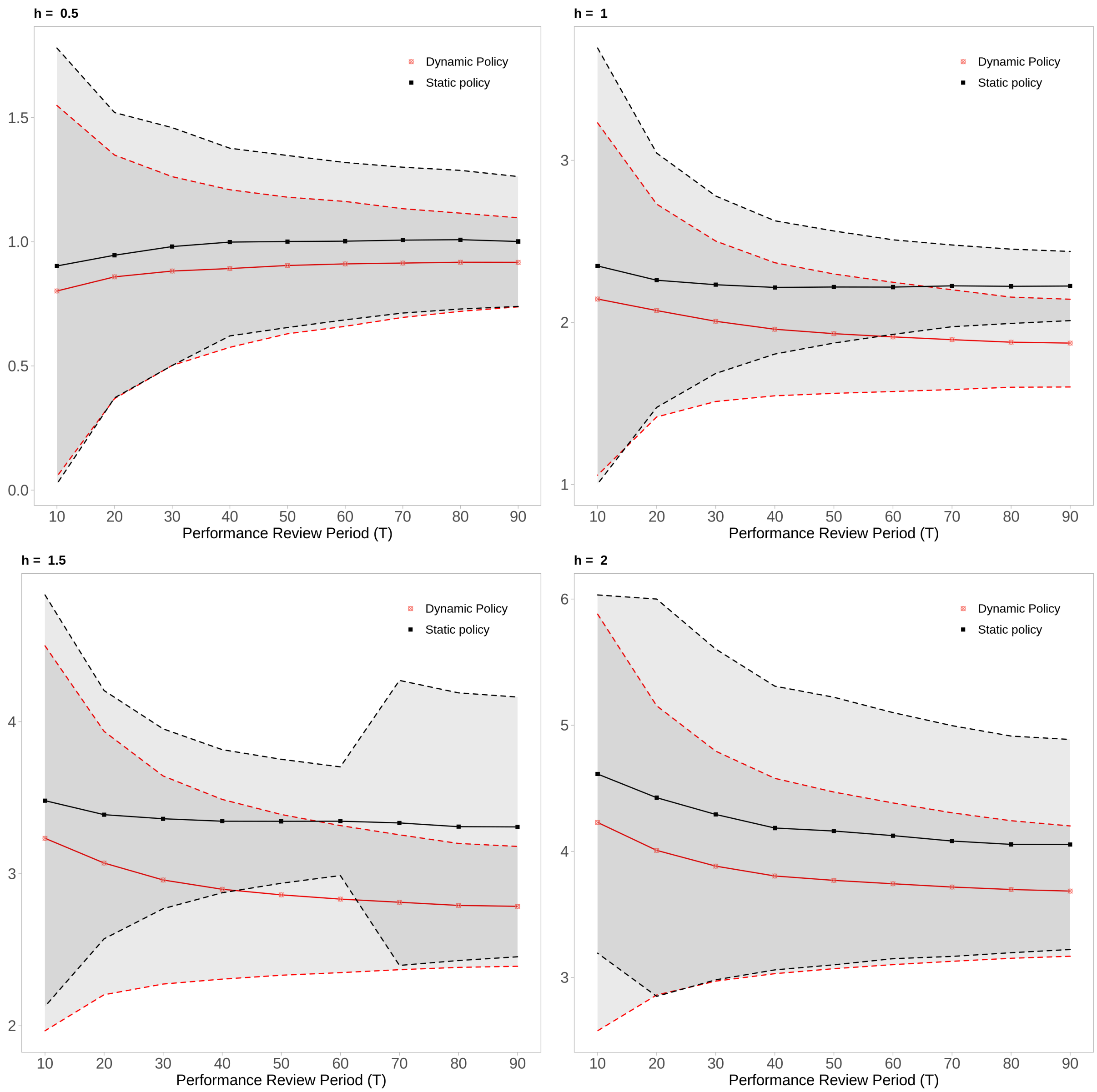
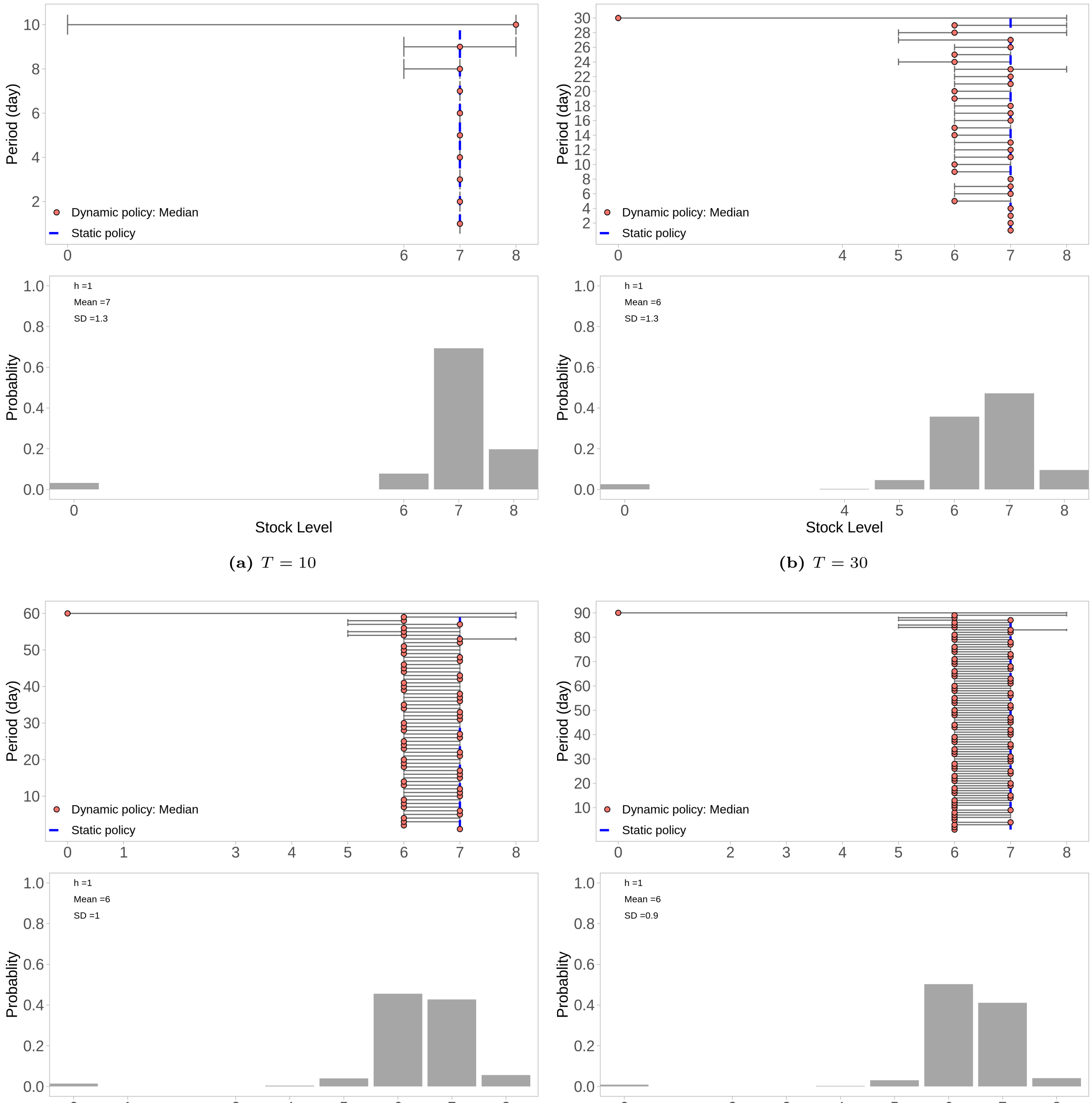


Figure 7: Linear incentives SLA: Supplier's cost per period for different performance review periods and holding costs with CI – comparison for the dynamic and static stocking policies

Remark 6. *Under the linear incentives SLA, the supplier's cost per inventory period is significantly lower using the optimal dynamic inventory policy, as might be expected. The supplier's cost function under a static or a dynamic base-stock policy is not unimodal with respect to the performance review length.*



(a) $T = 10$ **(b)** $T = 30$

(c) $T = 60$ **(d)** $T = 90$

Figure 8: Distribution of the optimal stock level under a dynamic inventory policy for a linear incentive SLA (with a holding cost equal to 1)

Figure 8 shows the stocking levels under linear incentives for a dynamic base-stock inventory policy with performance review periods of $T = 10, 30, 60$, and 90. Only the case of an inventory holding cost of $h = 1$ is shown. The figure also shows the distributions of the optimal dynamic stock decisions in each period (e.g., on a daily basis) using line charts over T = 10, 30, 60, and 90. The upper and lower limits in these lines correspond to the first and third quartiles, while the median values are presented by dot points. The plots show that under a dynamic inventory model, the interquartile range mainly lies between 6 and 7, although it may fluctuate to zero in the last period as a consequence of the supplier's attempt to minimise costs while meeting the contract requirements. The optimal static base-stock, S^*, in this case for $h = 1$, is equal to 7.

Experienced Service Level

Figure 9 presents the distribution of the ready rate under linear incentives for different performance review periods. In this figure, the dynamic stocking policy is compared to the static policy when the holding costs per unit are equal to 1 and 2. The horizontal dashed lines represent the upper and lower target rates for each case.

Under the linear incentive scheme, the impact of the SLA parameters on the supplier's performance is not straightforward and depends on the magnitude of the holding cost under both inventory policies. As seen in Figure 9, the realised ready rate for a strategic supplier that adopts the dynamic stocking policy remains close to the lower performance target, which is the reason for the in-variations observed in the boxplots. Note that outliers are not shown in this figure, but their impacts are reported in Table 5 with the probability of receiving a penalty or bonus and the standard deviations. Although the supplier can benefit from a static policy with a low chance of failing the penalty targets under low holding costs and long performance review periods, this result is not consistent with the linear penalty-only structure.

Table 5 summarises the expected ready rate, standard deviation, probability of exceeding the upper target service levels, and falling under the lower target service level over the performance review periods, and inventory holding costs.

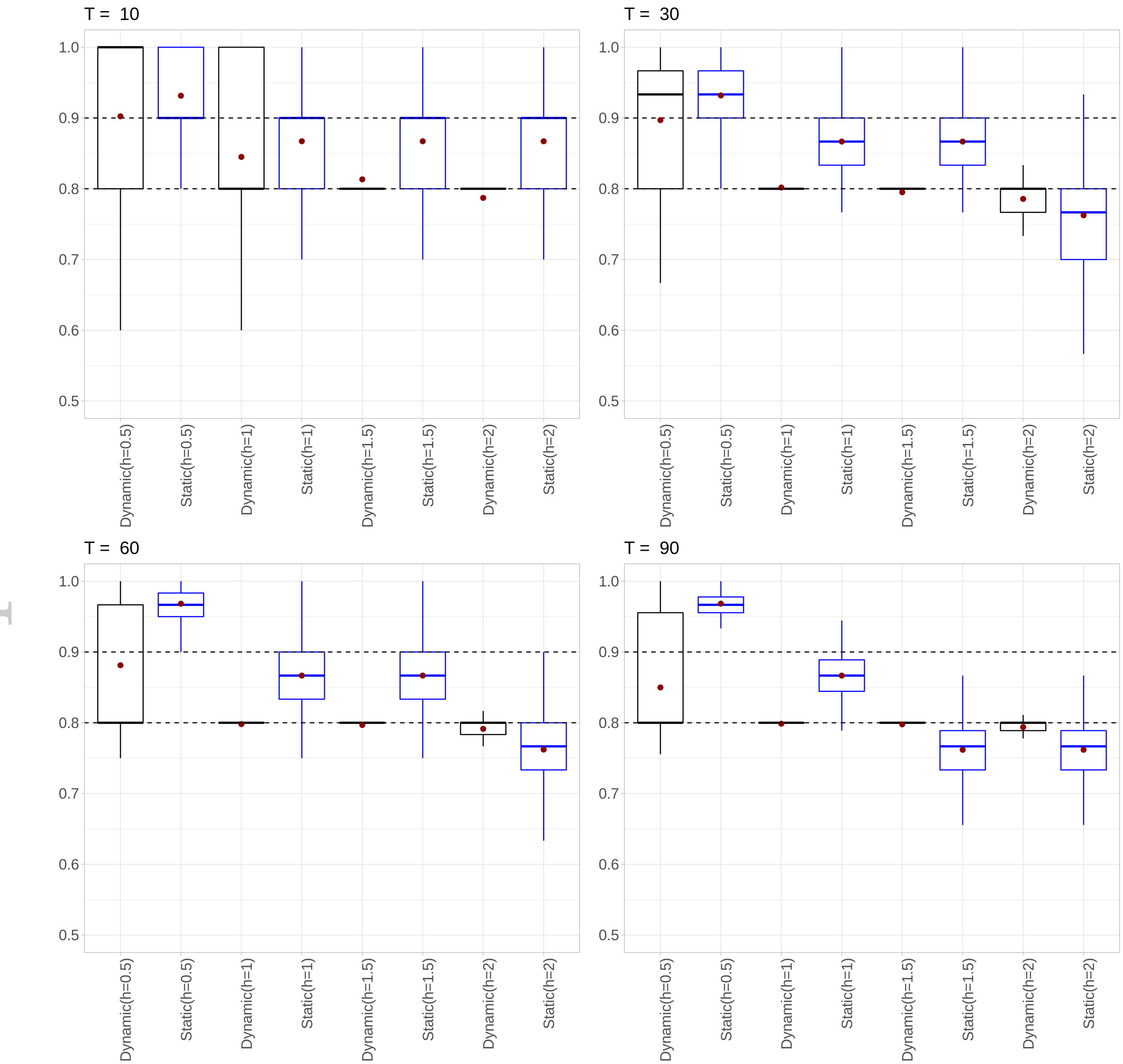


Figure 9: Distribution of the ready rate for a linear incentive SLA – comparison of the dynamic and static policies. The red dot point presents average ready rates

With linear incentive (penalty and bonus) terms, we observe that for a high holding cost ($h = 2$), the probability of receiving a penalty is significant under a static policy. The probability of not meeting the penalty target under a dynamic stocking policy increases as the length of the performance review period T increases (analytical calculations using an absorbing Markov chain are presented in the next section). The impact of the performance review period on the probability of exceeding a penalty target level is analysed in Appendix C. We also observe that for medium-to-high horizon lengths ($T \geq 30$), the expected ready rate is higher with less variability under the dynamic stocking policy, compared to the static model.

Table 5: Summary of the ready rate distribution under an SLA with linear incentives

h	T	Dynamic stocking policy Average (%)	SD (%)	$Pr(\alpha < \alpha^{T_L})$ (%)	$Pr(\alpha \geq \alpha^{T_U})$ (%)	Static stocking policy Average (%)	SD (%)	$Pr(\alpha < \alpha^{T_L})$ (%)	$Pr(\alpha \geq \alpha^{T_U})$ (%)
0.5	10	90.24	10.19	1.78	53.05	93.15	8.01	2.67	85.23
	20	89.97	9.15	1.89	55.88	93.14	5.67	1.07	84.49
	30	89.70	8.83	1.58	55.86	93.19	4.60	0.36	85.61
	40	89.17	8.74	1.32	53.35	96.85	2.75	0.00	99.20
	50	88.47	8.71	1.43	49.44	96.82	2.50	0.00	99.55
	60	88.13	8.68	1.86	47.63	96.82	2.27	0.00	99.71
	70	86.74	8.44	1.84	39.62	96.80	2.10	0.00	99.79
	80	85.89	8.18	1.95	34.76	96.78	1.98	0.00	99.90
	90	84.99	7.81	2.21	29.50	96.83	1.87	0.00	99.89
1	10	84.50	9.86	8.18	28.09	86.71	10.77	13.88	60.79
	20	82.46	6.94	8.33	16.69	86.65	7.65	12.02	49.08
	30	80.18	3.18	9.36	3.20	86.67	6.22	9.44	41.80
	40	79.87	1.73	9.29	0.82	86.75	5.31	7.14	37.30
	50	79.80	1.01	9.99	0.14	86.65	4.82	6.25	32.72
	60	79.81	0.78	10.10	0.05	86.67	4.40	5.05	29.81
	70	79.83	0.59	10.33	0.00	86.66	4.03	4.14	26.36
	80	79.86	0.52	10.02	0.01	86.62	3.80	3.46	24.32
	90	79.86	0.48	10.27	0.00	86.66	3.63	2.80	22.66
1.5	10	81.33	7.72	10.17	13.35	86.71	10.77	13.88	60.79
	20	79.79	3.77	12.95	3.21	86.65	7.65	12.02	49.08
	30	79.51	1.84	13.61	0.35	86.67	6.22	9.44	41.80
	40	79.58	1.22	14.13	0.02	86.75	5.31	7.14	37.30
	50	79.63	1.01	15.10	0.01	86.65	4.82	6.25	32.72
	60	79.68	0.87	15.02	0.00	86.67	4.40	5.05	29.81
	70	79.73	0.74	15.09	0.00	76.27	5.06	71.70	0.23
	80	79.76	0.65	15.37	0.00	76.24	4.70	74.23	0.12
	90	79.78	0.61	15.30	0.00	76.18	4.49	76.13	0.10
2	10	78.71	8.37	22.85	8.45	86.71	10.77	13.88	60.79
	20	78.20	4.12	28.08	0.99	76.18	9.48	53.55	11.13
	30	78.57	2.76	28.79	0.04	76.25	7.79	58.78	5.13
	40	78.88	2.11	29.41	0.00	76.39	6.67	62.73	2.44
	50	79.02	1.83	30.67	0.00	76.22	6.09	66.36	1.01
	60	79.14	1.59	31.41	0.00	76.21	5.51	69.61	0.54
	70	79.26	1.39	31.32	0.00	76.27	5.06	71.70	0.23
	80	79.32	1.25	32.21	0.00	76.24	4.70	74.23	0.12
	90	79.39	1.14	32.07	0.00	76.18	4.49	76.13	0.10

Remark 7. ***Dynamic vs. static inventory policy:*** *Under a linear incentive SLA, a static stocking policy outperforms the dynamic one in terms of the average service level (ready rate). In particular, with small holding costs and long performance review periods, it leads to almost no chance of failing the contract requirements. However, for a long performance review period and high holding costs, the dynamic inventory policy significantly performs better than the static policy, given that the average ready rate is higher with less variability, and there is a lower chance of receiving a penalty. The chance of receiving a penalty is significant if the holding cost is high and the static inventory policy is adopted.*

Remark 8. ***Impact of PRP for a strategic supplier:*** *With a linear penalty-only SLA and dynamic stocking* *policy, the average ready rate remains unchanged with less variability as the length of the performance review period increases (refer to Appendix B), while the chance of receiving a penalty increases. However, this behaviour may change as the bonus term is augmented in the contract (linear penalty and bonus regimes). A negative relationship between the average ready rate and the performance review period is observed.*

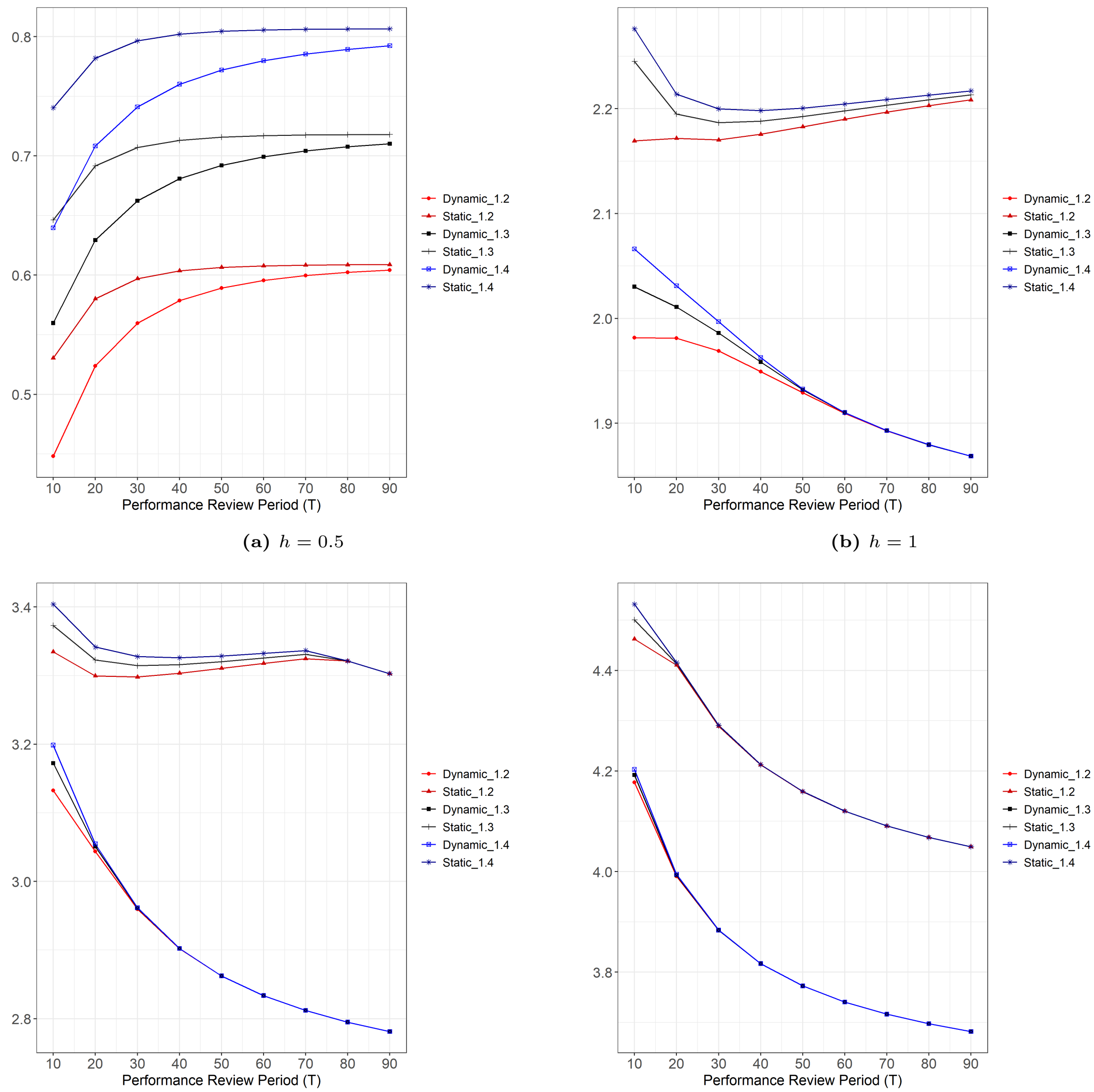
Remark 9. ***Impact of PRP for SLA negotiation:*** *With a linear incentive SLA contract, as the length of the performance review period increases, the average ready rate decreases, while there is a higher chance of receiving a penalty and a small probability of a gaining a bonus. The result is consistent under both inventory policies.*

Impact of Linear Incentives on Supplier's Expected Cost and Cost saving

In this section, we investigate the impact of incentive values on the supplier's expected cost and cost saving by using the penalty-to-bonus ratio. The supplier's average costs are calculated analytically as the expected values of the cost functions, as explained in modelling sections for lump-sum and linear incentives under the static ($\frac{C_S^*}{T}$) and dynamic ($\frac{C_S^*(V_1(0))}{T}$) inventory policies, respectively.

In Figure 10, we used a penalty-to-bonus ratio of $1.2, 1.3$, and 1.4 when the linear penalty is equal to 25 per period in order to show the impact of the (linear) incentive values on the supplier's average cost using a dynamic stocking policy, compared to a static policy. It can be seen that for small holding costs (Figure 10 (a)), the supplier's average cost under both stocking policies (static and dynamic) is reduced as the penalty-to-bonus ratio decreases, particularly for smaller performance review periods. In other words, with closer penalty and bonus values, suppliers may easily reach the target threshold under shorter performance review periods; therefore, their average total costs are reduced under both stocking policies. This figure also shows that the supplier's average costs in static and dynamic policies converge for small holding costs as the length of the performance review period increases, provided that the penalty and bonus values are close (with a ratio of 1.2). Otherwise, the supplier's cost is much lower in a dynamic stocking policy.

A summary of the supplier's cost saving under linear incentives is provided in Table 6.



(a) $h = 0.5$

(b) $h = 1$

(c) $h = 1.5$

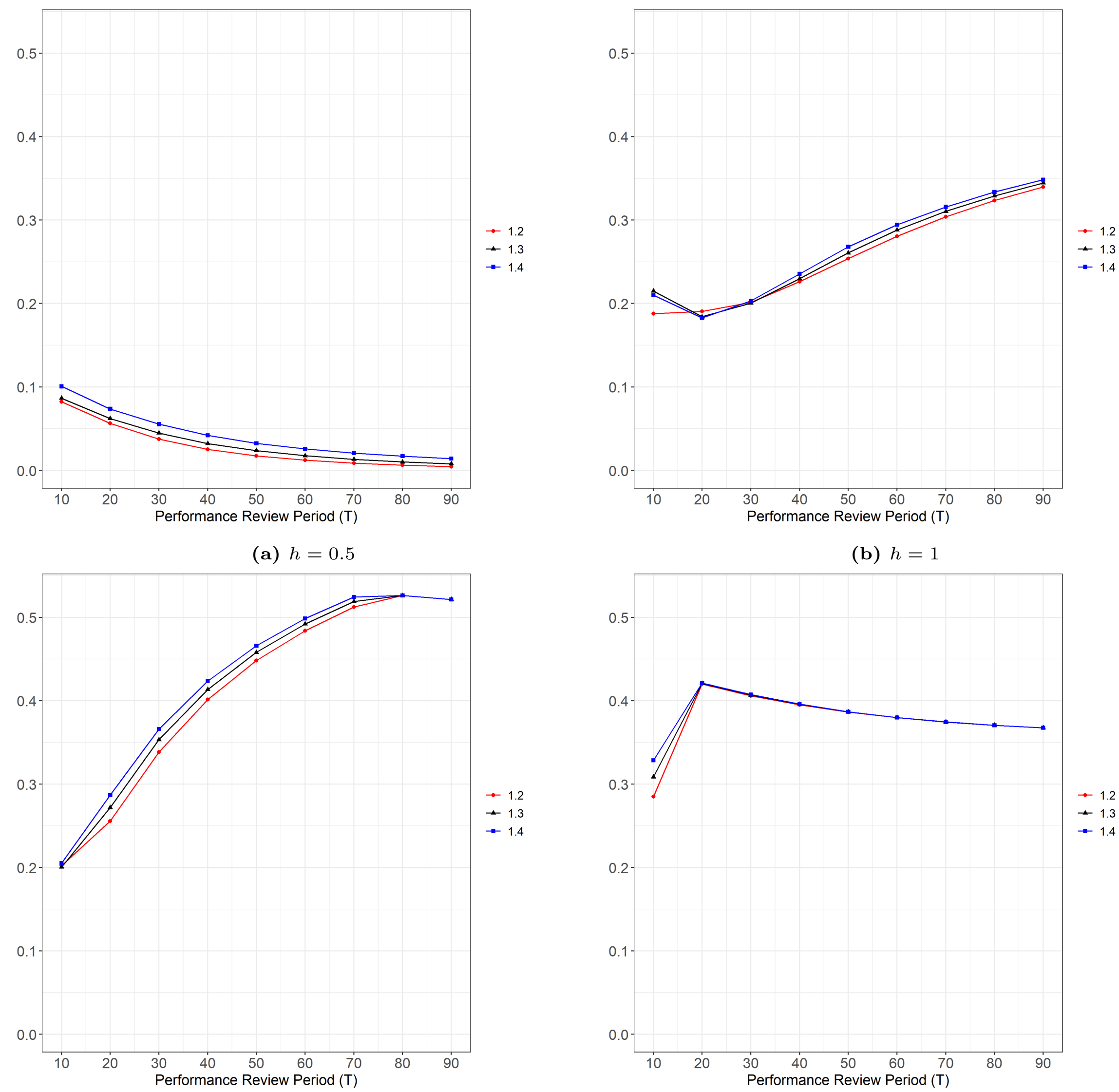
(d) $h = 2$

Figure 10: Linear incentives SLA: Supplier's cost per period for different performance review periods and holding costs (across the penalty-to-bonus ratio) – comparison of the dynamic and static stocking policies

Table 6 and Figure 11 show the supplier's cost saving using a dynamic policy under a linear penalty and bonus scheme for different lengths of performance review periods and inventory holding costs. We also examined the impact of the penalty-to-bonus ratio. The supplier's cost saving is calculated based on the proportion of the saved cost of using a dynamic policy to the cost of the static policy. The parameters used in the numerical analysis are based on the numerical study explained in Computational Results Section. We have used the same scale on the y-axis in Figure 11 to showcase the impact of the inventory holding cost, h. It can be seen that for a small holding cost (Figure 11-(a)), the supplier's cost saving is small for longer performance review periods.

Table 6: Supplier's cost saving under an SLA using a dynamic policy with linear incentives – comparison of the incentive ratio, holding cost, and PRP

		Performance review period (T)								
Ratio	h	10	20	30	40	50	60	70	80	90
	0.50	0.08	0.06	0.04	0.03	0.02	0.01	0.01	0.01	0.00
1.2	1.00	0.19	0.19	0.20	0.23	0.25	0.28	0.30	0.32	0.34
	1.50	0.20	0.26	0.34	0.40	0.45	0.48	0.51	0.53	0.52
	2.00	0.29	0.42	0.41	0.40	0.39	0.38	0.37	0.37	0.37
	0.50	0.09	0.06	0.04	0.03	0.02	0.02	0.01	0.01	0.01
1.3	1.00	0.21	0.18	0.20	0.23	0.26	0.29	0.31	0.33	0.34
	1.50	0.20	0.27	0.35	0.41	0.46	0.49	0.52	0.53	0.52
	2.00	0.31	0.42	0.41	0.40	0.39	0.38	0.37	0.37	0.37
	0.50	0.10	0.07	0.06	0.04	0.03	0.03	0.02	0.02	0.01
1.4	1.00	0.21	0.18	0.20	0.24	0.27	0.29	0.32	0.33	0.35
	1.50	0.21	0.29	0.37	0.42	0.47	0.50	0.52	0.53	0.52
	2.00	0.33	0.42	0.41	0.40	0.39	0.38	0.37	0.37	0.37



(a) $h = 0.5$

(b) $h = 1$

(c) $h = 1.5$

(d) $h = 2$

Figure 11: Linear incentives SLA: Supplier's cost saving under an SLA using a dynamic policy with linear incentives – comparison of the incentive ratio, holding cost, and PRP

Remark 10. ***Impact of incentives on the supplier's cost saving:*** *Under a linear incentive SLA, the incentive values (penalty-to-bonus ratio) does not significantly impact the supplier's cost under either inventory replenishment policy. Further, for very small holding costs and long performance review periods, there is not much difference between the two inventory policies in terms of cost.*

ANALYTICAL CALCULATION FOR THE PROBABILITY OF EXCEEDING THE TARGET SERVICE LEVEL

The supplier's dynamic inventory policy is a Markov chain process, which we demonstrate in Figure 12 for a performance review period with length T. In this plot, each circle is presented by $i, \tilde{v}_i$: where i represents the stage i ($i = 1, ..T$), in which the stocking decisions are made (S_i) at the beginning of the performance review period i; and state $\tilde{v}_i$ presents the possible conditions of full demand fulfilment until this stage. As shown in this figure, stage $T' = T + 1$ is absorbing.

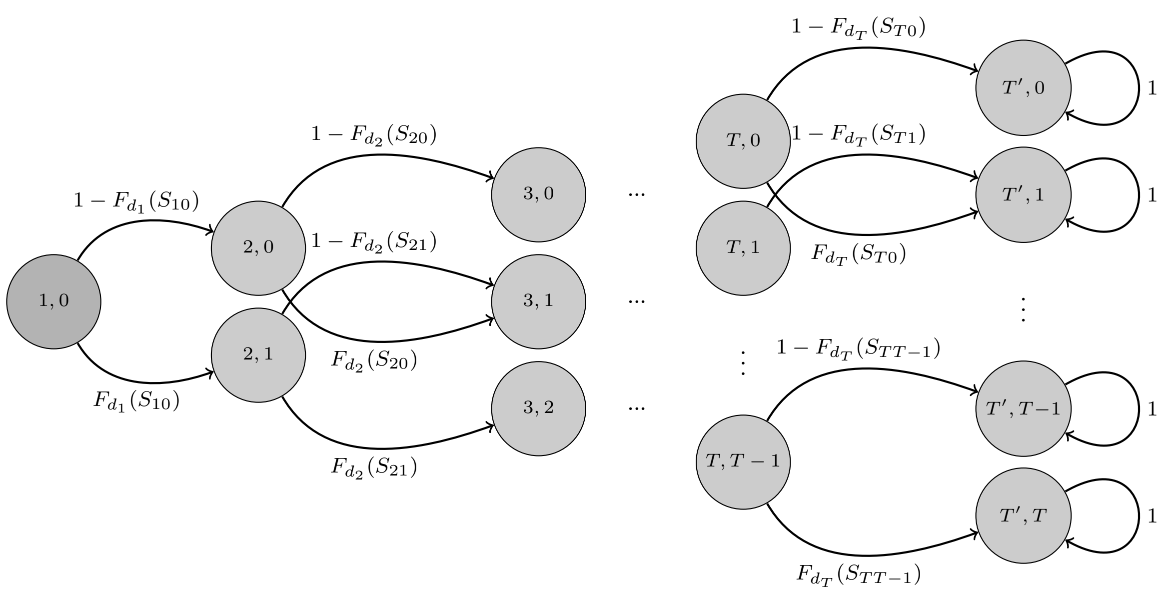


Figure 12: Graphical presentation of the dynamic inventory policy with the performance review period T

$T' = T + 1$ is the last period of the performance review period after a stocking decision is made. There are T absorbing states and $t = \sum_{i=1}^{T} i = T(T + 1)/2$ transient states in this model. The transition matrix ($\mathbf{S}$) has the following canonical form:

$$\begin{array}{cc}
\begin{array}{ccccccccc} \mathbf{1,0} & \mathbf{2,0} & \mathbf{2,1} & \dots & \mathbf{T',0} & \mathbf{T',1} & \dots & \mathbf{T',T-1} & \mathbf{T',T} \end{array} & \\
\left[\begin{array}{cccc|ccccc}
0 & 1-F_{d_t}(S_{1,0}) & F_{d_t}(S_{1,0}) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \dots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1-F_{d_t}(S_{1,0}) & F_{d_t}(S_{1,0}) & \dots & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1-F_{d_t}(S_{1,0}) & F_{d_t}(S_{1,0}) & \dots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \dots & \dots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \dots & 1-F_{d_t}(S_{1,0}) & F_{d_t}(S_{1,0}) \\
\hline
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right] &
\begin{array}{l} \mathbf{1,0} \\ \mathbf{2,0} \\ \mathbf{2,1} \\ \vdots \\ \mathbf{T,0} \\ \mathbf{T,1} \\ \vdots \\ \mathbf{T,T-1} \\ \mathbf{T',0} \\ \mathbf{T',1} \\ \vdots \\ \mathbf{T',T} \end{array}
\end{array}$$

If we denote Q as an t-by-t matrix and R as a nonzero t-by-T' matrix, we have:

$$\mathbf{S} = \left[\begin{array}{c|c} \mathbf{Q} & \mathbf{R} \\ \hline \mathbf{0} & \mathbf{I} \end{array}\right]$$

where I is an T'-by-T' identity matrix, and 0 is a zero matrix with T' rows and t columns. Let B be the matrix with b_{ij} entries, with the probability that an absorbing chain will be absorbed in the absorbing state s_j if it starts in the transient state s_i. Then, to calculate the probability that the process ends up in a given absorbing state, we have $B = NR$, where $N = (I - Q)^{-1}$ is the fundamental matrix. N gives the expected number of times that the process is in the transient state s_j if it is started in the transient state s_i. B is an t-by-T' matrix. The comparison of the analytical calculations with the computational results is provided in Appendix D.

DISCUSSION AND CONCLUSION

This paper has shed light on the impacts of the length of the performance review period, incentive structures, incentive values, demand distribution, and inventory stocking policies on suppliers' performance and cost for managers facing SLA contracts. We examined the supplier's optimal stocking decision in a supply chain, including a single buyer (retailer). Suppliers can observe their performances and dynamically adjust their inventory level for each inventory review period. Subsequently, the strategic supplier decides on the base-stock at the beginning of each replenishment (inventory review) period based on the realised measured service level and cost. We also addressed the impact of SLA parameters, assuming that the supplier employs a static inventory policy and compares the results with those obtained under a dynamic stocking policy.The numerical results demonstrated that under an SLA with either linear or lump-sum incentives, the supplier's costs are lower when using a dynamic stocking policy, compared to a static policy. This is also the case even when a penalty-only regime is employed. The dynamic inventory policy makes an optimal stocking decision to minimise the cost functions during a performance review period using the observed performance in that period. To implement a dynamic inventory policy, the supplier needs to determine the order quantity based on the performance history in the current performance review period.

Under lump-sum penalty and/or bonus schemes with a static stocking policy, there is a higher chance that the supplier's performance level will fall below the lower target limit with less of a chance to receive a bonus, compared to a dynamic stocking policy. This is accompanied by a higher cost for the supplier under a static base-stock policy. For a strategic supplier that adopts a dynamic inventory policy with a lump-sum incentive regime, as the length of the performance review period increases, the supplier benefits from a higher expected performance level (i.e., ready rate) with less variability. These results are valid for small changes in the holding costs. However, when linear penalty and/or bonus schemes are employed, a more complex functionality in terms of the supplier's optimal cost and performance level average is observed. Interestingly, with a penalty-only linear term, we expect that increasing the length of the performance review will result in a higher performance level, with less variability and less optimal cost under a dynamic stocking policy. In contract negotiations, it is usually understood that the value of the bonus and penalty has a direct impact on the supplier's optimal cost and expected performance measures. However, this study identified the length of the performance review period as a cornerstone in SLA ready rate-based contract negotiations. The key findings are summarised below:

- The supplier's cost under a dynamic stocking policy is, unsurprisingly, significantly lower, compared to a static policy. However, this cost saving is insignificant when the holding cost is negligible and the performance review period is long under a linear incentive scheme.
- The performance of a static inventory policy in terms of the ready rate distribution is volatile with respect to the magnitude of the holding cost. In practice, there is more of a chance that the supplier's performance level will fall below the lower target limit (penalty threshold) as the holding cost increases under a static stocking policy. In other words, for high holding costs, there is a high chance that a static policy will fail to meet the contract requirements and will underperform, compared to a dynamic policy.
- Under lump-sum penalty and/or bonus SLAs:
  - The supplier's cost decreases as the length of the performance review period increases. This result is consistent under both inventory policies and with small changes in the holding cost.

- The supplier's average ready rate is slightly higher if a static replenishment policy is employed and the holding cost is not high. However, the probability of meeting or exceeding the bonus threshold is higher when a dynamic policy is applied, compared to the static one.
- As the holding cost increases, the static inventory policy tends to hold less stock for the entire period. In practice, this might result in a low ready rate, on average, for long performance review periods. Therefore, adopting a static inventory policy under high inventory holding costs may not fulfil the requirements in a penalty SLA contract in terms of meeting the penalty threshold.
- Under a dynamic inventory policy, the average ready rate increases with less variability as the length of the performance review period is increased.

- Under linear incentives (penalty and/or bonus), the impact of the length of the performance review period is complicated and depends on the magnitude of the holding cost and the incentive structure offered.
- Under a linear penalty SLA:
  - The supplier's cost decreases as the length of the performance review period increases. This result is consistent under both inventory policies.
  - The supplier's average ready rate is slightly higher if a static policy is applied. However, the chance of receiving a penalty is significant with a static inventory policy for a high per-unit holding cost.
  - Under a dynamic inventory policy, the average ready rate increases with less variability as the length of the performance review period increases.
  - When a bonus term is added to the contract (i.e., a linear penalty and bonus SLA), these behaviours may change, depending on the magnitude of the incentive.
- Changing the demand distribution from Poisson to normal does not impact the results of the SLA model analysis. The findings are valid when the demand follows a normal distribution.

A dynamic inventory policy might give suppliers an opportunity to achieve the required service level with a lower cost. However, it adds a computational burden for implementation in practice, compared to a static policy, as optimal stock levels must be calculated for every inventory review period. In this paper, we assumed that the excess inventory is discarded at the end of the period. Therefore, future research can investigate the impact of the performance review period and holding cost on the ready rate while relaxing this assumption. In such cases, the remaining inventory will carry over to the next period and the dynamic inventory model should calculate the optimal base-stock where the period-ending inventory is not more than the next period's base-stock level. To handle this situation, the period-ending inventory should be included in the state-space of the dynamic programming model, which increases the computational time for solving the problem.

An extension of this work might consider an investigation under multiple products with aggregated target service levels. Future research could also study the impact of other performance review measures, such as the fill rate, and extend our analysis to other inventory policies. Furthermore, our analysis can be extended to stocking decision policies for suppliers facing SLA contacts, with ready rates and heterogeneous service-level requirements for multiple customers.

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Supplemental Materials

Additional supporting materials for this paper can be found in the Online Supplementary Materials.