Optimization based input preview filtering for dynamical systems

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***Abstract*— This paper is about filtering uncertain forecast information to update a preview model of inputs to a linear dynamical system, as may be useful in predictive control schemes. A moving horizon optimization approach is proposed, with a view to smoothing abrupt changes in order based forecast information and to manage error, given observations of the dynamics. Numerical examples are used to illustrate a potential application of this approach within the context of processing demand profile requests in a water distribution system.**

I. INTRODUCTION

For many systems information is available about future disturbances or reference signals. This is commonly referred to as a forecast or preview information.[1] It has long been established that preview can be of benefit for control in particular for improving transient performance [1]. Indeed, it is often included in control problems, especially moving horizon schemes such as model predictive control; e.g., in-vehicle suspension [2], HVAC systems [3], wind-turbines [4], irrigation networks [5]. The forecast, however, is typically uncertain to some extent.

As an example, consider an irrigation channel where farmers place orders for desired flow at some time in the future. The demand forecast at a given time can be defined by the orders in place. However, a farmer could cancel or add an order and hence the received forecast could change abruptly. It can be desirable to filter or smooth the raw forecast before it is used for decision making; e.g., in model predictive control. Further, the farmer may take a different flow-rate to the order amount, or with modified timing, which will impact the observable dynamics of the channel. Here, it is desirable to utilizes observations of the dynamics to correct the preview model.

In practice, it is common to use the most recent forecast, which may be uncertain or change abruptly. Alternatively, uncertainty in the forecast is dealt with as part of the control design; e.g., using $\mathcal{H}_\infty$ techniques [6], robust-MPC [7] or minimizing variance through $\mathcal{H}_2$ [8] or using Stochastic-MPC [9] in the case of a stochastic forecast. By contrast, the focus of this paper is on filtering to update a preview model of the system inputs, in addition to observations of the system dynamics and the forecast information over a sliding horizon. How this may relate to the subsequent use of the input preview updates in decision making is the topic of future work.

Funded in part by the Australian Research Council (LP160100666).

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[1]Preview and forecast are used interchangeably throughout this paper.

Below, a discrete-time linear time-invariant model is used for the system dynamics. This is augmented with a simple delay-line model that represents the preview of inputs. Such a model is often used within the context of so-called preview control; e.g., see [10], [8]. The model incorporates additive terms that represent forecast uncertainty, motivated by ideas from the theory of unknown input observers; e.g., see [11], [12]. A receding horizon optimization approach is used as the filtering method for updating the preview model. It is shown that under certain conditions the additive disturbance model of the uncertainty can be completely decoupled from the system, allowing a modified moving horizon estimation (MHE) problem to be formed. This method is compared against standard MHE for a couple of realistic examples related to the operation of large-scale irrigation networks. The advantages and disadvantages of the decoupling method are discussed.

The paper develops as follows. In Section II, a delay-line model of input preview and a correspondingly augmented system model are introduced, along with a formulation of the robust preview filtering problem. This is followed by an introduction to moving horizon estimation in Section III, before showing how the uncertainty can be decoupled to form an equivalent system in section III-A. This equivalent system is used to form the decoupled MHE approach in section III-B, and estimator stability is established. In section IV a numerical example is presented before concluding with an outline of future work in Section V.

A. Notation

The sets of natural and real numbers are denoted by $\mathbb{N}$ and $\mathbb{R}$, respectively. For any two matrices $A \in \mathbb{R}^{n\times m}$ and $B \in \mathbb{R}^{p\times q}$, $A \otimes B \in \mathbb{R}^{np\times mq}$ is the Kronecker product. An n-dimensional vector of ones is denoted by $\mathbb{1}_n$. The identity matrix in $\mathbb{R}^{n\times n}$ is denoted by I_n and the matrix of zeros in $\mathbb{R}^{n\times m}$ is denoted $0_{n\times m}$. When the dimension is clear from the context, the simplified notations $\mathbb{1}$, I and 0 are adopted. For any semi-positive definite matrix $Q \in \mathbb{R}^{n\times n}$ and vector $x \in \mathbb{R}^n$ the quadratic term $x^\top Qx$ is denoted $\|x\|_Q^2$.

II. PROBLEM FORMULATION

Consider the discrete-time system with input u, output y, and a disturbance input d, modeled by

$$x(t+1) = Ax(t) + Bu(t) + Ed(t) \tag{1a}$$

$$y(t) = Cx(t) + Du(t) + Fd(t) \tag{1b}$$

where $x(0) = x_0$, and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, and $E \in \mathbb{R}^{n\times q}$, $C \in \mathbb{R}^{p\times n}$, $D \in \mathbb{R}^{p\times m}$, $F \in \mathbb{R}^{p\times q}$. Suppose that at each time $t \geq 0$, an uncertain measurement of $u(t+T)$

is available in $y_{T+}(t) \in \mathbb{R}^m$, where $T \in \mathbb{N}$ is the preview horizon. Further, forecast information with linear dependence on the inputs $u(t), \dots, u(t+T-1)$ is available in vector $y_p(t) \in \mathbb{R}^s$. Respectively, these measurements are given by

$$y_{T^+}(t) = u(t+T) + F_{T^+} d_p(t) \tag{2a}$$

$$y_p(t) = C_p \begin{bmatrix} u(t) \\ u(t+1) \\ \vdots \\ u(t+T-1) \end{bmatrix} + D_p u(t+T) + F_p d_p(t), \tag{2b}$$

where $d_p(t) \in \mathbb{R}^r$ is an unknown disturbance to model forecast uncertainty, $C_p \in \mathbb{R}^{s \times mT}$, $D_p \in \mathbb{R}^{s \times m}$, $F_p \in \mathbb{R}^{s \times r}$ and $F_{T^+} \in \mathbb{R}^{m \times r}$. The preview of inputs can be modeled with a T-step delay-line, leading to the following augmented system model:

$$\tilde{x}(t+1) = \tilde{A}\tilde{x}(t) + \tilde{B} y_{T^+}(t) + \tilde{E}\tilde{d}(t) \tag{3a}$$

$$\tilde{y}(t) = \tilde{C}\tilde{x}(t) + \tilde{D} y_{T^+}(t) + \tilde{F}\tilde{d}(t), \tag{3b}$$

where

$$\tilde{x}(t) = \begin{bmatrix} x(t) \\ u(t) \\ u(t+1) \\ \vdots \\ u(t+T-1) \end{bmatrix}, \tilde{d}(t) = \begin{bmatrix} d(t) \\ d_p(t) \end{bmatrix}, \tilde{y}(t) = \begin{bmatrix} y(t) \\ y_p(t) \end{bmatrix},$$

$$\tilde{A} = \begin{bmatrix} A & B & 0 & \dots & 0 \\ 0 & 0 & I_m & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & & 0 & I_m \\ 0 & \dots & & 0 & 0 \end{bmatrix}, \tilde{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ I_m \end{bmatrix},$$

$$\tilde{E} = \begin{bmatrix} E & 0 \\ 0 & E_p \end{bmatrix} + \begin{bmatrix} 0_{n+(T-1)m \times q} & 0_{n+(T-1)m \times r} \\ 0_{m \times q} & -F_{T^+} \end{bmatrix},$$

$$\tilde{C} = \begin{bmatrix} C & \begin{bmatrix} D & 0 \end{bmatrix} \\ 0 & C_p \end{bmatrix} \tilde{D} = \begin{bmatrix} 0 \\ D_p \end{bmatrix}, \tilde{F} = \begin{bmatrix} F & 0 \\ 0 & F_p - D_p F_{T^+} \end{bmatrix}.$$

Note that (3) is formulated such that there is a known input $y_{T^+}(t)$ and an unknown input $\tilde{d}(t)$. This leads to the following robust preview filtering problem. Also $y_p(t)$ may contain information about other parts of the forecast mechanism through C_p; e.g., it may be possible to observe parts but not all of the delay-line, as in the case of LIDAR sensors on wind turbines [8].

PROBLEM 1 (ROBUST PREVIEW FILTER PROBLEM): Given, at time $t \in \mathbb{N}$, measurements $y(k)$, $y_p(k)$ and $y_{T^+}(k)$ for $k = t-H, \dots, t$, with $H \in \mathbb{N}$, determine an estimate of the state $\tilde{x}(t)$ (i.e. state $x(t)$ and the input preview) in (3) that is insensitive (or robust) to the bounded but unknown disturbances d and d_p.

EXAMPLE 1: Consider a situation in which the input, $u(\cdot)$, consists of two parts, one part is measured exactly for the entire preview horizon and the other part is unknown; i.e., $u(k) = u_1(k) + u_2(k)$, $y_p(k) = \begin{bmatrix} u_1(k)^\top & \dots & u_1(k+T-1)^\top \end{bmatrix}^\top$, $y_{T^+}(k) = u_1(k+T)$. In this case, Problem 1 is to determine an estimate for the overall input $u(k)$ for times $k = t, \dots, t+T$ given the measurements of only $y_p(k), y_{T^+}(k)$ for $k = t-H, \dots, t$. This can be modeled using the aforementioned framework by allowing the unknown disturbance $d_p(t) = u_2(t+T)$, $C_p = I_{Tm}$, $D_p = 0_{Tm \times m}$, $E_p = \begin{bmatrix} 0_{Tm \times m} \end{bmatrix}$, $F_p = -\mathbb{1}_T \otimes I_m$ and $F_{T^+} = -I_m$.

REMARK 2.1: Although the augmented model (3) has increased in dimension from n to $n + Tm$, the system is highly structured. In fact, the augmented system can be modeled as a cascade of T smaller systems. It is expected that recent moving horizon estimation approaches that exploit such structure could be used for this type of problem [13].

As describe below, a moving horizon estimation approach is pursued in this paper, instead of alternative approaches, because it aligns with the way the preview model might be used within the context of receding horizon model predictive control (i.e. MPC), which is the topic of ongoing work.

III. MOVING HORIZON ESTIMATION

In moving horizon estimation (MHE) the aim is to determine the state of a system given observations of inputs and outputs over a finite horizon. For (3), the standard MHE formulation involves an optimization problem at each time t that is parametrized by the past $H \in \mathbb{N}$ inputs $u(t+T-H), \dots, u(t+T)$ and outputs $\tilde{y}(t-H), \dots, \tilde{y}(t)$, which includes forecast information y_p and the system output y. However, the input $u(t+T)$ to the augmented system (3) is not available directly into the past. Only the past H uncertain measurements y_{T^+} are known. Let $x_p(t)$ denote the estimate of the state $\tilde{x}(t-H)$ at H steps in the past, as determined in the previous time step. The component $\xi^*(t)$ of the solution of the following optimization problem is taken to be the estimate for $\tilde{x}(t)$:

$$\min_{\xi,\omega} \; J_t(\xi, \omega; x_p, \tilde{y}, u) \tag{4a}$$

$$\text{s.t} \quad \xi(k+1) = \tilde{A}\xi(k) + \tilde{B} y_{T^+}(k) + \omega(k), \quad k = t-H, \dots, t-1, \tag{4b}$$

where

$$J_t(\xi, \omega; x_p, \tilde{y}, u) = \|(\xi(t-H) - x_p(t))\|_P^2 + \sum_{k=t-H}^{t} \|\tilde{C}\xi(k) + \tilde{D} y_{T^+}(k) - \tilde{y}(k))\|_R^2 + \sum_{k=t-H}^{t-1} \|\omega(k)\|_Q^2,$$

and $P \in \mathbb{R}^{(n+Tm)\times(n+Tm)}$, $R \in \mathbb{R}^{(s+p)\times(s+p)}$ and $Q \in \mathbb{R}^{(n+Tm)\times(n+Tm)}$ are all positive-semi definite matrices. Assuming that R is positive definite, $(\tilde{A}, \tilde{C})$ is detectable and Q is semi-positive definite, with $(\tilde{A}, Q)$ stabilizable, and P is chosen such that its maximum singular value is sufficiently small, the estimation error is stable and converges to a ball of the origin [14], [15].

The second term in $J_t(\xi, \omega; x_p, \tilde{y}, u)$ encodes two objectives. One corresponds to minimization of the estimation error associated with the linear system dynamics (1). The other corresponds to minimization of the error between the received forecast (component y_p in $\tilde{y}$) and the estimated forecast as modelled by the delay-line dynamics for the preview inputs.

The decision variables ω provides scope to compensate for the absence of the disturbances d and d_p in the model (16b), as these are not observed. A large penalty Q on ω reflects high confidence in the observations matching the model (16b), i.e. a small $\tilde{d}$.

The approach just described does not fully exploit the structure of the model (3) in that $\tilde{E}, \tilde{F}$ and $\tilde{d}$ do not appear in (4). Consideration of this aspect of the model is explored in the next section.

A. Decoupled Preview System

This section presents a reformulated system for (3) to remove the dependence on $\tilde{d}$. Towards this end, ideas from unknown input observer theory are applied. In particular, the measured $\tilde{y}$, given by (3b), is used to modify the model input as presented in the following lemma, where

$$\bar{y}(t) = (I_{s+p} - \tilde{F}\tilde{F}^{\dagger})\tilde{y}(t) \quad \forall\ t, \tag{5a}$$

$$\bar{C} = (I_{s+p} - \tilde{F}\tilde{F}^{\dagger})\tilde{C}, \quad \bar{D} = (I_{s+p} - \tilde{F}\tilde{F}^{\dagger})\tilde{D}, \tag{5b}$$

$$\bar{B} = \tilde{E}(I_{q+r} - \tilde{F}^{\dagger}\tilde{F}) \left[\bar{C}\tilde{E}(I_{q+r} - \tilde{F}^{\dagger}\tilde{F})\right]^{\dagger}, \tag{5c}$$

$$\bar{A} = (I_{n+Tm} - \bar{B}\bar{C})(\tilde{A} - \tilde{E}\tilde{F}^{\dagger}\tilde{C}), \tag{5d}$$

$$\begin{aligned}\bar{G} = (I_{n+Tm} - \bar{B}\bar{C})&(\tilde{B} - \tilde{E}\tilde{F}^{\dagger}\tilde{D}) \\ &- \bar{A}\bar{B}(I_{s+p} - \tilde{F}\tilde{F}^{\dagger})\tilde{D},\end{aligned} \tag{5e}$$

$$\bar{B}_y = (I_{n+Tm} - \bar{B}\bar{C})\tilde{E}\tilde{F}^{\dagger} + \bar{A}\bar{B}(I_{s+p} - \tilde{F}\tilde{F}^{\dagger}). \tag{5f}$$

LEMMA 3.1: If $(I_{n+Tm} - \bar{B}\bar{C})\tilde{E}(I_{q+r} - \tilde{F}^{\dagger}\tilde{F}) = 0$, then the model

$$\zeta(t+1) = \bar{A}\zeta(t) + \bar{B}_y\tilde{y}(t) + \bar{G}y_{T^+}(t) \tag{6a}$$

$$z(t) = \bar{C}\zeta(t) + \bar{D}y_{T^+}(t), \tag{6b}$$

is equivalent to (3) in that

$$\zeta(t) = \tilde{x}(t) + \bar{B}\bar{y}(t) - \bar{B}\bar{D}y_{T^+}(t) \quad \forall t. \tag{7}$$

Proof: This proof follows similar steps to [12] except here the terms associated with the input $y_{T^+}(t)$ are explicitly included. From (3b) the generalized solution, see [16], for $\tilde{d}(t)$ is given by

$$\tilde{d}(t) = \tilde{F}^{\dagger}(\tilde{y}(t) - \tilde{C}\tilde{x}(t) - \tilde{D}y_{T^+}(t)) + (I_{q+r} - \tilde{F}^{\dagger}\tilde{F})\bar{d}(t), \tag{8}$$

where $\tilde{F}^{\dagger}$ is the generalized inverse such that $\tilde{F}\tilde{F}^{\dagger}\tilde{F} = \tilde{F}$ and $\bar{d}(t) \in \mathbb{R}^{r+q}$ can be considered as a new disturbance. Substituting (8) into (3a) gives

$$\begin{aligned}\tilde{x}(t+1) &= \tilde{A}\tilde{x}(t) + \tilde{B}y_{T^+}(t) + \tilde{E}(\tilde{F}^{\dagger}(\tilde{y}(t) - \tilde{C}\tilde{x}(t) \\ &\quad - \tilde{D}y_{T^+}(t)) + (I_{q+r} - \tilde{F}^{\dagger}\tilde{F})\bar{d}(t)) \\ &= (\tilde{A} - \tilde{E}\tilde{F}^{\dagger}\tilde{C})\tilde{x}(t) + (\tilde{B} - \tilde{E}\tilde{F}^{\dagger}\tilde{D})y_{T^+}(t) \\ &\quad + \tilde{E}\tilde{F}^{\dagger}\tilde{y}(t) + \tilde{E}(I_{q+r} - \tilde{F}^{\dagger}\tilde{F})\bar{d}(t)\end{aligned} \tag{9}$$

Let $\bar{y}(t) = (I_{s+p} - \tilde{F}\tilde{F}^{\dagger})\tilde{y}(t) = \bar{C}\tilde{x}(t) + \bar{D}y_{T^+}(t)$. Then

$$\begin{aligned}\bar{y}(t+1) &= \bar{C}\tilde{x}(t+1) + \bar{D}y_{T^+}(t+1) \\ &= \bar{C}(\tilde{A} - \tilde{E}\tilde{F}^{\dagger}\tilde{C})\tilde{x}(t) + \bar{C}\tilde{E}\tilde{F}^{\dagger}\tilde{y}(t) \\ &\quad + \bar{C}(\tilde{B} - \tilde{E}\tilde{F}^{\dagger}\tilde{D})y_{T^+}(t) + \bar{D}y_{T^+}(t+1) \\ &\quad + \bar{C}\tilde{E}(I_{q+r} - \tilde{F}^{\dagger}\tilde{F})\bar{d}(t).\end{aligned} \tag{10}$$

From (10) the general solution for $\bar{d}(t)$ is given by

$$\begin{aligned}\bar{d}(t) = &\left[\bar{C}\tilde{E}(I_{q+r} - \tilde{F}^{\dagger}\tilde{F})\right]^{\dagger} \left(\bar{y}(t+1) - \bar{D}y_{T^+}(t+1)\right. \\ &-\bar{C}(\tilde{A} - \tilde{E}\tilde{F}^{\dagger}\tilde{C})\tilde{x}(t) - \bar{C}\tilde{E}\tilde{F}^{\dagger}\tilde{y}(t) \\ &\left.-\bar{C}(\tilde{B} - \tilde{E}\tilde{F}^{\dagger}\tilde{D})y_{T^+}(t)\right) + (I_{s+p}- \\ &\left.\left[\bar{C}\tilde{E}(I_{q+r} - \tilde{F}^{\dagger}\tilde{F})\right]^{\dagger} \bar{C}\tilde{E}(I_{q+r} - \tilde{F}^{\dagger}\tilde{F})\right) \bar{d}_2(t)\end{aligned} \tag{11}$$

Substituting (11) into (9) gives

$$\begin{aligned}\tilde{x}(t+1) &= \bar{A}\tilde{x}(t) + \bar{B}\bar{y}(t+1) + (I_{n+Tm} - \bar{B}\bar{C})\tilde{E}\tilde{F}^{\dagger}\tilde{y}(t) \\ &+ (I_{n+Tm} - \bar{B}\bar{C})(\tilde{B} - \tilde{E}\tilde{F}^{\dagger}\tilde{D})y_{T^+}(t) - \bar{B}\bar{D}y_{T^+}(t+1) \\ &+ (I_{n+Tm} - \bar{B}\bar{C})\tilde{E}(I_{q+r} - \tilde{F}^{\dagger}\tilde{F})\bar{d}_2(t)\end{aligned} \tag{12}$$

Introducing new variables $\zeta(t) \in \mathbb{R}^{n+Tm}$ and $z(t) \in \mathbb{R}^{s+p}$ defined as

$$\zeta(t) = \tilde{x}(t) + \bar{B}\bar{y}(t) - \bar{B}\bar{D}y_{T^+}(t) \tag{13}$$

$$z(t) = \bar{C}\zeta(t) + \bar{D}y_{T^+}(t), \tag{14}$$

it follows from (12) and by the hypothesis $(I_{n+Tm} - \bar{B}\bar{C})\tilde{E}(I_{q+r} - \tilde{F}^{\dagger}\tilde{F}) = 0$, that

$$\zeta(t+1) = \bar{A}\zeta(t) + \bar{B}_y\tilde{y}(t) + \bar{G}y_{T^+}(t) \tag{15}$$

as claimed. ■

B. MHE via decoupled Unknown Input

By using the decoupled system (6) the MHE system (4) can be reformulated as follows

$$\min_{\zeta,\gamma}\ \bar{J}_t(\zeta, \gamma; x_p, \tilde{y}, y_{T^+}) \tag{16a}$$

$$\begin{aligned}\text{s.t} \quad \zeta(k+1) = \bar{A}\zeta(k) + \bar{B}_y\tilde{y}(k) + \bar{G}y_{T^+}(k) + \gamma(k), \\ k = t-H, \dots, t-1,\end{aligned} \tag{16b}$$

where

$$\begin{aligned}\bar{J}_t(\zeta, \gamma; x_p, \tilde{y}, y_{T^+}) &= \|(\zeta(t-H) - \tilde{x}_p(t))\|_P^2 \\ &+ \sum_{k=t-H}^{t}\|z(k) - z_m(k)\|_R^2 + \sum_{k=t-H}^{t-1}\|\gamma(k)\|_Q^2,\end{aligned} \tag{17}$$

$z_m(k) = (I_{s+p} + \bar{C}\bar{B})\bar{y}(k) - \bar{C}\bar{B}\bar{D}y_{T^+}(k)$, $z(k), \bar{y}$ are defined in (6b) and (5a) respectively and $\tilde{x}_p(t) = x_p(t) - \bar{B}\bar{y}(t-H) + \bar{B}\bar{D}y_{T^+}(t-H)$. Let $\zeta^* = \left[\zeta^*(t-H)^\top \cdots \zeta^*(t)^\top\right]^\top$ denote the optimal solution of (16). The estimate for $\tilde{x}(t)$ is then given by

$$\hat{x}(t) = \zeta^*(t) + \bar{B}\bar{y}(t) - \bar{B}\bar{D}y_{T^+}(t). \tag{18}$$

THEOREM 3.2: Provided the pair $(\bar{A}, \bar{C})$ is H step observable and $(I_{n+Tm} - \bar{B}\bar{C})\tilde{E}(I_{q+r} - \tilde{F}^{\dagger}\tilde{F}) = 0$, with $P = 0$

and R, Q both positive definite, the estimate $\hat{x}(t)$ given by (18) converges in the sense that $\hat{x}(t) - \tilde{x}(t) \to 0$.

Proof: At time t let decision variables $\zeta(k)$ for $k = t-H, \ldots, t$ be chosen such that $\zeta(k) = \tilde{x}(k)$ where $\tilde{x}(k)$ is given by (3a). Using the result of Lemma 3.1 and that $P = 0$ results in a cost

$$\bar{J}_t(\zeta, \gamma; x_p, \tilde{y}, y_{T^+}) = 0. \tag{19}$$

Denote the optimal cost at time t by J_t^*. Observability guarantees that a solution to (16) exists since the second term can be represented by a strongly convex term of the initial state estimate $\xi(t-H)$. By definition

$$J_t^* \leq \bar{J}_t(\zeta, \gamma; x_p, \tilde{y}, y_{T^+}) = 0 \quad \forall t, \tag{20}$$

and since $P = 0$ and R and Q are positive definite then $J_t^* \geq 0$ hence $J_t^* = 0$ $\forall t$. By positive definiteness of Q and R this implies $\gamma(t) = 0$ and $\bar{C}\zeta^*(t) + \bar{D}y_{T^+}(t) - \bar{C}\bar{B}\bar{y}(t) - \bar{C}\bar{B}\bar{D}y_{T^+}(t) = \bar{y}(t)$ i.e., $z^*(t) = z(t)$ where $z(t)$ is the actual decoupled measurement. Consider the output of a system (6) the associated measurement vector is given by

$$\mathbf{z}_t = \begin{bmatrix} z(t) \\ \vdots \\ z(t+H) \end{bmatrix} = \mathcal{O}\zeta(t) + \mathcal{B}\bar{\mathbf{y}} + \mathcal{G}\mathbf{y}_{T^+}, \tag{21}$$

where $\bar{\mathbf{y}} = \begin{bmatrix} \bar{y}(t)^\top & \cdots & \bar{y}(t+H-1) \end{bmatrix}^\top$, $\mathbf{y}_{T^+} = \begin{bmatrix} \bar{y}_{T^+}(t)^\top & \cdots & \bar{y}_{T^+}(t+H)^\top \end{bmatrix}^\top$,

$$\mathcal{O} = \begin{bmatrix} \bar{C}^\top & (\bar{C}\bar{A})^\top & \ldots & (\bar{C}\bar{A}^{H-1})^\top \end{bmatrix}^\top,$$

$$\mathcal{B} = \begin{bmatrix} 0 & \cdots & \cdots & 0 \\ \bar{C}\bar{B}_y & 0 & \cdots & 0 \\ \bar{C}\bar{A}\bar{B}_y & \bar{C}\bar{B}_y & 0 & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ \bar{C}\bar{A}^{H-1}\bar{B}_y & \bar{C}\bar{A}^{H-2}\bar{B}_y & \ldots & \bar{C}\bar{B}_y \end{bmatrix}$$

$$\mathcal{G} = \begin{bmatrix} \bar{D} & \cdots & \cdots & \cdots & 0 \\ \bar{C}\bar{G} & \bar{D} & \cdots & \cdots & 0 \\ \bar{C}\bar{A}\bar{G} & \bar{C}\bar{G} & \bar{D} & 0 & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \bar{C}\bar{A}^{H-1}\bar{G} & \bar{C}\bar{A}^{H-2}\bar{G} & \ldots & \bar{C}\bar{G} & \bar{D} \end{bmatrix},$$

Since $(\bar{C}, \bar{A})$ is observable $\mathcal{O}$ is full column rank and hence $\mathcal{M} := (\mathcal{O}^\top\mathcal{O})^{-1}$ exists and is unique. Hence

$$\zeta(t) = \mathcal{M}\mathcal{O}^\top\mathbf{z}_t - \mathcal{M}\mathcal{O}^\top\mathcal{B}\bar{\mathbf{y}} - \mathcal{M}\mathcal{O}^\top\mathcal{G}\mathbf{y}_{T^+} \tag{22}$$

The difference between the actual state $\zeta(t)$ and the solution $\zeta^*(t)$ of (16) is therefore given by

$$\zeta(t) - \zeta^*(t) = \mathcal{M}\mathcal{O}^\top(\mathbf{z}_t - \mathbf{z}_t^*) \tag{23}$$

$$\Rightarrow \exists\kappa > 0 \text{ s.t } |\zeta(t) - \zeta^*(t)| \leq \kappa\|\mathbf{z} - \mathbf{z}^*\|. \tag{24}$$

Since $z^*(t) = z(t)$ then $|\zeta^*(t) - \zeta(t)| = 0$ which from (7) implies $\hat{x}(t) = \tilde{x}(t)$. ■

REMARK 3.1: The proof of Theorem 3.2 requires $P = 0$. It should be possible to be relax this in order to allow for positive definite P that satisfies appropriate other conditions, e.g., sufficiently small maximum singular value. The example in the subsequent numerical results section demonstrates this by use of a non-zero, but small, P. In practice P allows the designer to link the previous estimate with the new estimate, potentially allowing one to achieve smaller time horizons H for similar performance.

Whilst the aforementioned method achieves robustness with respect to the unknown disturbance, it relies on the strong decoupling condition, which may not be satisfied in general. The approach also relies on an accurate model for this decoupling to hold exactly. It may be difficult to determine $\tilde{F}$ and $\tilde{E}$ to be consistent with the situation of interest; such issues are explored in [17, Chapter 5]. In addition, the decoupled system (6) appears to have lost the cascade-structure mentioned in Remark 2.1, possibly making it more difficult to apply distributed methods. How to tune the sensitivity to the disturbance via a penalty function or by using some other information about the disturbance, e.g., an error bound, or a probability distribution, is the subject of future work.

IV. NUMERICAL EXAMPLE

In this section the aforementioned preview estimation method is demonstrated on an example from an automated irrigation channel. In this simulation an irrigation channel consists of a string of pools linked by actuated flow regulation structures. When the pools are operated under a decentralized distant-downstream control regime, each pool can be modeled using an integrator-delay model, where the delay is approximated using padé approximation, in closed-loop with a PI-type feedback controller; e.g., see [18]. This can be represented via a state-space representation which consists of 4 states per pool. Within each pool there is an off-take(s) that extracts flow from the pool to deliver to the farm. Farmers are required to place an order for their planned extraction ahead of time, which is used to make up the forecast. However, this order may not match what is actually taken. In the numerical example here a 3 pool system is used with an off-take for each pool, the parameters are the same as the first three pools used in [19]. A measurement of the water level and flow at each regulator is available. The load for each pool is modeled by a pulse with a given start-time s_i, duration l_i and magnitude m_i. There is a full, i.e., $C_p = I_{(T)m}$, forecast of horizon $T = 29$ available for each off-take. Two scenarios are simulated

1) The actual off-takes are parameterized by $(s_i)_{i=1}^3 = (120, 60, 60)$, $(l_i)_{i=1}^3 = (180, 360, 360)$ and $(m_i)_{i=1}^3 = (0, 0.05, 0.1)$. The forecast has no information about the order in pool 2 i.e., it is zero for entire horizon. The initial forecast for this scenario can be parameterized by $(s_i)_{i=1}^3 = (120, 60, 60)$, $(l_i)_{i=1}^3 = (180, 360, 360)$ and $(m_i)_{i=1}^3 = (0.105, 0.0, 0.105)$. However, at time $t = 105$ (15 minutes before start time of order 1) the order in first pool is canceled and this is reflected in the received forecast going to zero for $t > 105$.
2) The actual off-takes are parameterized by $(s_i)_{i=1}^3 = (120, 60, 60)$, $(l_i)_{i=1}^3 = (180, 360, 360)$ and $(m_i)_{i=1}^3 =$

$(0.1, 0.0, 0.1)$. In this scenario, two forecast processes are simulated. The first process gives the forecast for $k = t + 15, \ldots t + 29$ and has a 20% magnitude error for pool 1 and an incorrect start time of 10 for pool 3, whilst the second has a completely accurate forecast for has a 5% magnitude error for pool 1 and a completely accurate forecast for pool 3 and gives the forecast for $k = t, \ldots, t + 14$.

The penalty matrices are chosen as $P = 10^{-3} I_{n+(T)m}$, $Q = \begin{bmatrix} 10000 I_n & \\ 0 & 1000 I_{(T)m} \end{bmatrix}$, and $R = \begin{bmatrix} 1000 I_p & 0 \\ 0 & I_m \otimes r \end{bmatrix}$ where r =diag$([10, 10 - 0.35, 10 - 2 \times 0.35, \ldots, 0.1]^\top)$. Both the standard MHE (4) and decoupled MHE (16) are simulated. The penalty matrix R is structured to more heavily penalize the forecast estimation error the closer it is to hitting the dynamics. This reflects diminishing confidence in the forecast the further into the future it predicts. For the decoupled MHE for scenario 1 $E_p = [0_{(T)m \times m}]$, $F_p = -\mathbb{1}_{29} \otimes I_m$ and $F_{T+} = -I_m$ and for scenario 2 $E_p = [0_{Tm \times 2m}]$, $F_p = [-\mathbb{1}_{29} \otimes I_m, \begin{bmatrix} 0_{(14)m \times m} \\ \mathbb{1}_{15} \otimes I_m \end{bmatrix}]$ and $F_{T+} = [-I_m, \ I_m]$. The horizon length for the MHE is chosen to be $H = 15$. The following notation is adopted for each pool:

$$
\begin{aligned}
y_p(t) &= \begin{bmatrix} y_{p1}(t)^\top & y_{p2}(t)^\top & \cdots & y_{p29}(t)^\top \end{bmatrix}^\top \\
&= \begin{bmatrix} u(t)^\top & u(t+1)^\top & \cdots & u(t+28)^\top \end{bmatrix}^\top + F_p d_p(t)
\end{aligned} \tag{25}
$$

$$
\hat{x}(t) = \begin{bmatrix} \hat{x}_d(t) \\ \hat{x}_p(t) \end{bmatrix}, \tag{26}
$$

where $\hat{x}_p(t) = \begin{bmatrix} \hat{x}_{p1}(t) & \hat{x}_{p2}(t) & \cdots & \hat{x}_{p29}(t) \end{bmatrix}^\top$ denotes the estimated forecast at time t.

Fig. 1 shows the resulting total error between estimated forecast and actual shifted input signal for the two scenarios. The decoupled MHE method is able to perform outright better in this metric in the three cases where there was a magnitude error in the raw forecast. Figures 2 and 3 show how the forecast signal corrects itself from the initial incorrect magnitude when using the decoupled MHE, in comparison to the standard MHE which maintains incorrect values. For the cases with incorrect start-time in the forecast there are periods when the raw (measured) forecast is inconsistent with the delay-line dynamics and in some instances abrupt changes between consecutive runs. In these cases both MHE methods attempt to "smooth" the result to remain close to the dynamics and avoid sudden changes in the forecast between consecutive time steps, which is highlighted in Fig. 4. Future work is to explore the effect or potential benefit of this smoothing when the forecast is used for optimization based decision making, e.g., MPC.

V. CONCLUSION AND FUTURE WORK

A preliminary exploration of an optimization based preview filtering problem is presented in this work. A simple linear model is used to model the uncertainty in forecast information. By using unknown input observer theory the

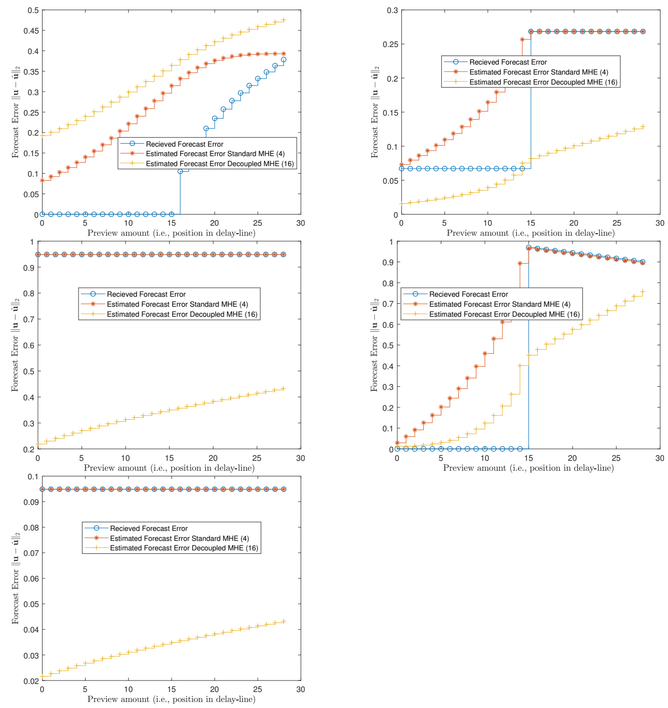

Fig. 1: The forecast error for scenario 1 (left) for pools 1-3 (top-bottom) and scenario 2 (right) for pools 1 and 3 (top and middle).

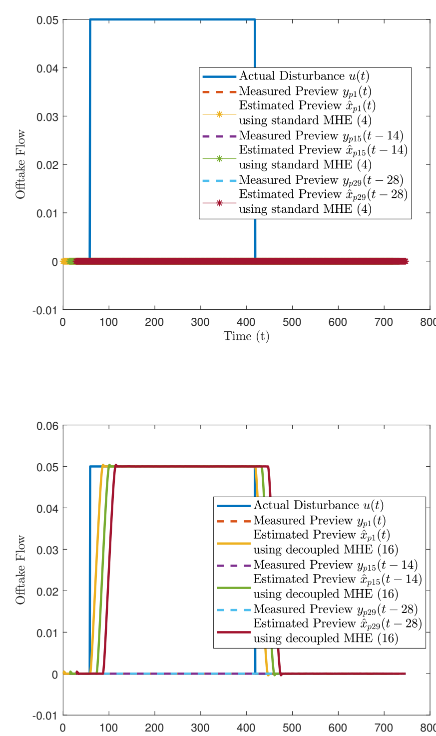

Fig. 2: The measured y_{pk} and estimated forecast $\hat{x}_{pk}$ signal for scenario 1, pool 2, for different locations k in the forecast vector, for standard MHE (top) and decoupled MHE (bottom).

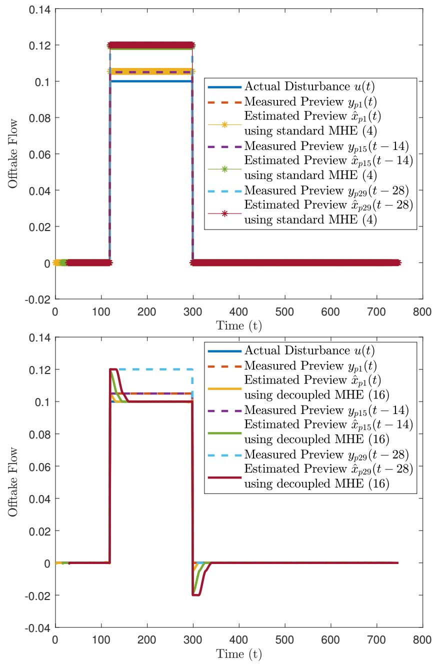

Fig. 3: The measured y_{pk} and estimated forecast $\hat{x}_{pk}$ signal for scenario 2 pool 1 for different locations k in the forecast vector, for standard MHE (top) and decoupled MHE (bottom).

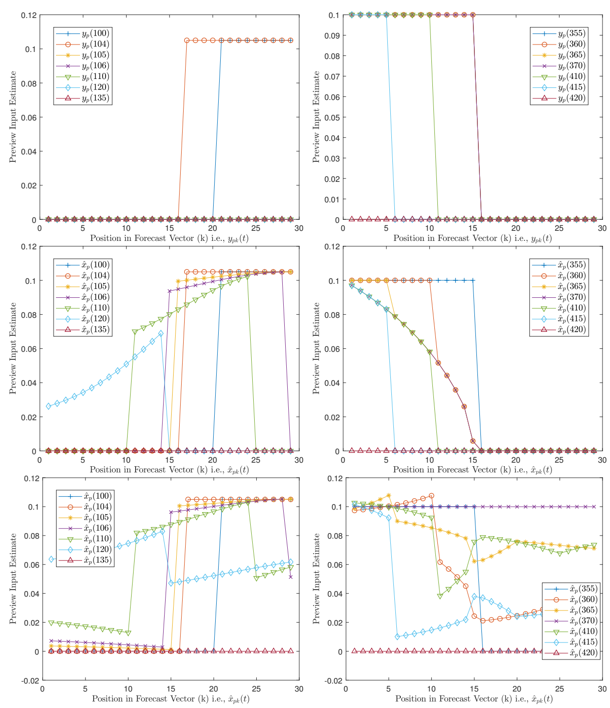

Fig. 4: The measured forecast $y_p(t)$ (top), estimated forecast $\hat{x}_p(t)$ using standard MHE (middle) and estimated forecast $\hat{x}_p(t)$ using decoupled MHE (bottom), for different times t. The left figures are for pool 1, scenario 1, for times around when the order is canceled. The right figures are for pool 3, scenario 2, for times around when the order is scheduled to end.

conditions to decouple this uncertainty from the system, via use of the measured dynamics and forecast, are given. This leads to a decoupled moving horizon estimation method, which is demonstrated in a numerical example from automated irrigation channel to offer advantages for order-based forecasts with certain types of uncertainties. Future work includes consideration of alternative methods for input preview estimation from uncertain forecast information and how to use such estimates to improve optimization based decision making for control.

References

[1] M. Tomizuka and D. E. Whitney, "Optimal Discrete Finite Preview Problems (Why and How Is Future Information Important?)," *Journal of Dynamic Systems, Measurement, and Control*, vol. 97, pp. 319–325, 12 1975.

[2] M. Tomizuka, "Optimum Linear Preview Control With Application to Vehicle Suspension—Revisited," *Journal of Dynamic Systems, Measurement, and Control*, vol. 98, pp. 309–315, 09 1976.

[3] Y. F. Du, L. Jiang, C. Duan, Y. Z. Li, and J. S. Smith, "Energy consumption scheduling of hvac considering weather forecast error through the distributionally robust approach," *IEEE Transactions on Industrial Informatics*, vol. 14, pp. 846–857, March 2018.

[4] J. H. Laks, L. Y. Pao, and A. D. Wright, "Control of wind turbines: Past, present, and future," in *2009 American Control Conference*, pp. 2096–2103, June 2009.

[5] M. Kearney and M. Cantoni, "MPC-based reference management for automated irrigation channels," in *2012 2nd Australian Control Conference (AUCC)*, pp. 349–354, Nov 2012.

[6] A. Kojima, "H∞ preview control with uncertain information," in *2015 54th IEEE Conference on Decision and Control (CDC)*, pp. 6643–6649, Dec 2015.

[7] M. B. Saltık, L. Özkan, J. H. Ludlage, S. Weiland, and P. M. V. den Hof, "An outlook on robust model predictive control algorithms: Reflections on performance and computational aspects," *Journal of Process Control*, vol. 61, pp. 77 – 102, 2018.

[8] R. H. Moroto, R. R. Bitmead, and B. Sleegers, "The information structure of feedforward/preview control using forecast data," *IFAC Proceedings Volumes*, vol. 47, no. 3, pp. 176 – 181, 2014. 19th IFAC World Congress.

[9] M. A. Sehr and R. R. Bitmead, "Stochastic output-feedback model predictive control," *Automatica*, vol. 94, pp. 315 – 323, 2018.

[10] A. J. Hazell and D. J. N. Limebeer, "A Framework for Discrete-Time H2 Preview Control," *Journal of Dynamic Systems, Measurement, and Control*, vol. 132, 04 2010.

[11] M. Hou and P. C. Muller, "Disturbance decoupled observer design: a unified viewpoint," *IEEE Transactions on Automatic Control*, vol. 39, pp. 1338–1341, June 1994.

[12] M. Hou and R. J. Patton, "Optimal filtering for systems with unknown inputs," *IEEE Transactions on Automatic Control*, vol. 43, pp. 445–449, March 1998.

[13] M. Guo, A. Lang, and M. Cantoni, "Structured moving horizon estimation for linear system chains*," in *2019 18th European Control Conference (ECC)*, pp. 1830–1835, June 2019.

[14] A. Alessandri, M. Baglietto, and G. Battistelli, "Receding-horizon estimation for discrete-time linear systems," *IEEE Transactions on Automatic Control*, vol. 48, pp. 473–478, March 2003.

[15] J. B. Rawlings and D. Q. Mayne, *Model predictive control: Theory and design*. Nob Hill Pub. Madison, Wisconsin, 2009.

[16] C. Rao and S. K. Mitra, *Generalized Inverse of Matrices and Its Applications*. Probability and Statistics Series, Wiley, 1971.

[17] J. Chen and R. Patton, *Robust model-based fault diagnosis for dynamic systems*. Kluwer international series on Asian studies in computer and information science: 3, Boston : Kluwer Academic, c1999., 1999.

[18] M. Cantoni, E. Weyer, Y. Li, S. K. Ooi, I. Mareels, and M. Ryan, "Control of large-scale irrigation networks," *Proceedings of the IEEE*, vol. 95, pp. 75–91, Jan 2007.

[19] A. R. Neshastehriz, M. Cantoni, and I. Shames, "Water-level reference planning for automated irrigation channels via robust mpc," in *2014 European Control Conference (ECC)*, pp. 1331–1336, June 2014.