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Reflection Paper

Reimagining Mathematics Education for the 21st Century in the 21st Century

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Abstract

The 21st Century is characterised by technological advances which is the Fourth Industrial Revolution, climate change, and the COVID19 pandemic, for examples. The role of mathematics in each of these phenomena has been central and crucial. As such, it is an opportune time now to take stock of events that are (re-)shaping the world, so that we can better facilitate mathematics education in schools. Three themes are identified and discussed in this article, namely the convergence of mathematics pedagogical approaches, mathematics proficiencies, and students' mathematical wellbeing.

Keywords: mathematical proficiencies, mathematical wellbeing, pedagogical convergence, values

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I. INTRODUCTION

The importance of developing mathematical competency amongst students for mathematics education into the future has often been emphasised, and the need for change in the ways of supporting students to develop their mathematical competency has increased. However, this need for change has radically become more urgent recently, as much has happened during the first two decades of the 21st Century which had altered the geopolitical, commercial and natural landscapes of the world. For instance, artificial intelligence technologies have improved in leaps and bounds in the new millennium, when it finds itself being embedded in day-to-day applications such as mobile phones and virtual assistants (e.g. Amazon Alexa, Google Home, Siri, Cortana, and Bixby). Also, the world has been crippled by the COVID-19 pandemic the scale of which has created unprecedented levels of crises across medical, economic and political arenas. Given the socio-cultural nature of mathematics education and given also the desire of many education systems to educate their young citizens to engage meaningfully with the future using their mathematics knowledge, skills and dispositions, it is perhaps important for us to pause at this juncture in time in the 21st Century, reflect on what we had set out to achieve with mathematics education at the turn of the century, evaluate the implications of unforeseen but significant changes in the world, and reposition mathematics education in our respective economies so that it achieves the aims and objectives associated with it.

It is also an opportune time, given the various public release and dissemination of studies and policies which inform the planning, execution and/or assessing of school mathematics lessons. Examples of such knowledge sources include cycles of international comparative studies such as Programme for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS), study reports commissioned by the private sector such as those published by McKinsey Global Institute (Manyika et al, 2017), as well as reports put together by professional associations such as USA National Research Council (2012).

A context of this rethinking exercise is that generally across the world, students' mathematics performance has not seen any statistically significant improvement in the 'new' century. For example, "there has … been no real overall improvement in the learning outcomes of students in OECD countries, even though expenditure on schooling rose by more than 15% over the past decade alone" (Schleicher, 2019, p. 5). According to data published by OECD (2019), less than half of 69 economies (i.e., 32) had been demonstrating positive overall performance trajectories. In the period 2012~2018, only 12 of 65 economies were seeing a statistically significant percentage-point decrease in the number of low achieving students (i.e., below OECD's Level 2) while 49 economies were registering no change in the number of students performing below Level 2 during this period. Specifically, the performance in PISA mathematics assessments of students in USA and Australia had been stagnant in the period 2000~2018 and 2000~2015 respectively; in PISA2018, Australian students' performance took a dip. In Korea, even though the 15-yearold students' mathematics performance in PISA2018 have continued to be amongst the very best in the world, the proportion of low achieving students (i.e., below OECD's Level 2) has increased and that of high achieving students (i.e., OECD's Level 6) has decreased when compared to PISA2003 and PISA2012 (KICE, 2019). These happened, despite billions of dollars being spent to try to improve students' performance in mathematics in these economies.

In this paper, we will identify and discuss three general themes which we believe are important factors in the evolution of mathematics education as the 21st Century continues to unfold. These themes are, namely, a convergence of mathematics pedagogical approaches, students' mathematical wellbeing, and the mathematical proficiencies. By unpacking how the themes might affect mathematics pedagogy, the intention of this paper is for readers to be mindful of how the themes might underlie the reimaginations and rethinking of school mathematics education, thus drawing their attention to three of the foundational values which shape the development of mathematics education during the 21st Century, which are *convergence, wellbeing, and proficiencies*.

To illustrate how the themes can affect the development of mathematics education, references will be made with the articles in the 2021 Special Issue of the 'Journal of Educational Research in Mathematics', which has the theme, 'Reimagining Mathematics Education for the 21st Century in the 21st Century'.

II. CONVERGENCE OF MATHEMATICS PEDAGOGICAL APPROACHES

Due to several global trends such as convenience of air travel and ease of information sharing/exchange, never before have we witnessed so much similarities in mathematics pedagogical approaches around the world. On the one hand, we see some mathematics lessons in the West taking on features of pedagogical approaches that were traditionally associated with Asian mathematics pedagogy. For example, the Japanese Lesson Study approach to teacher professional development is becoming increasingly visible in mathematics classrooms in Western countries, such as Australia, Canada, Ireland and USA. Western mathematics education systems such as those in the USA have also learnt to develop more coherent curricula at the national level. Some of these have gone further to strike a better balance between conceptual understanding and procedural fluency. On the other hand, and to a much greater extent, mathematics lessons in China and in other Asian education systems have also been taking on more Western teaching practices. These would include examples such as group discussions, learning through inquiry, and reorienting mathematics education outcomes from high-stakes examinations (alone) to the development of mathematical practices.

Thus, we see a convergence of mathematics pedagogical practices around the world. In part, this has been brought about by globalization which has resulted in stronger inter-relationships and interactions amongst individuals, institutions and societies from different cultures. The ease with which people move around the globe due to ever interconnected networks of air travel, for example, has allowed mathematics education researchers and academics to meet more often at conferences and other academic events. Although adopting pedagogical practices or interventions that have been successful in one education system in another is never easy (Kim, 2019; Li, 2019), these events are fertile grounds for effective pedagogical initiatives to be introduced, clarified and discussed by delegates from different mathematics educational systems. In turn, they would bring back to their respective mathematics educational systems for local adaptation.

The current information age, characterised by free movement of information, also contributes to the convergence of mathematics pedagogical practices. The availability of research journal articles in electronic formats and through academic library access is a far cry from the days of hardcopy journals which are only accessible when nobody else in the library membership has borrowed them. More recently, in the 21st Century, this availability is expanded further as increasing numbers of journal articles are freely accessible anytime anywhere in the form of Open Access literature.

The extent to which the amount of cross-cultural flow of innovative pedagogical knowledge is similar amongst major education traditions is not known, however. Even though there might have been more Asian scholars studying, researching and working in the West than there have been Western peers engaging in such academic activities in the East, say, this phenomenon does not necessarily indicate that Asian scholars bring back more Western pedagogical ideas than they do influencing teaching practices in the West when they were there. That more Asian researchers are able to understand English at academic conferences held in Western countries than Western researchers being able to understand Asian academic languages if they attend conferences held in Asia is also a factor for differential access to – and engagement with – the latest pedagogical knowledge.

However, it is the various international comparative assessments, such as TIMSS and PISA which probably exerted one of the strongest – if not the strongest – influences leading to the convergence of mathematics pedagogical approaches. 5 of the 7 rounds of TIMSS assessments, and all 7 rounds of PISA, took place in the 21st Century. Even though the ranking of students' performance by countries and economies might not have been the main intention of these comparative assessments, it was convenient for governments and attractive for the media to refer to performance scores and rankings as success indicators. These 'league tables' led to education systems emulating the pedagogical practices of different 'successful' countries (Baker & LeTendre, 2005) such as China, Finland, Japan and Singapore at different times over the last two decades, with the Chinese and Singapore systems being the centres of attention in the last decade or so, and Finland and Japan much earlier at the beginning of the 21st Century. Indeed, according to Baker and LeTendre (2005), it was the poor performance of USA students in the first TIMSS in 1995 which led to policy advocates using the results to rationalize the need for the No Child Left Behind educational reform. Angel Gurría, OECD Secretary-General, alluded to this trend when she wrote that "more and more countries are looking beyond their own borders for evidence of the most successful and efficient education policies and practices" (Schleicher, 2019, p. 2).

Some 'Western' mathematics pedagogical ideas are also being valued and implemented globally in the 21st Century. Teacher noticing is an example which is enjoying much uptake in Asian mathematics pedagogical systems, and Pang and Sunwoo (2021)'s study shows how it is a focus of skill development in a Korean university offering pre-service elementary mathematics teaching courses. Teacher noticing is also a theme in the 2021 volume of the ZDM journal (Dindyal, Choy, Schack, & Sherin, 2021) in which the 4 guest editors were made up of 2 researchers from USA, led by 2 others from Singapore.

The production and use of mathematics videos might use to have been a luxury reserved for first world, financially rich education systems, but it has become more affordable in increasingly more countries around the world. One no longer needs the most advanced mobile phone to document mathematics content in video format and uploads it, and anyone can view it via platforms such as YouTube as long as they can receive mobile phone signals. Domingues and Borba's (2021) paper, for example, discusses the relationship between mathematics videos and the 21st Century mathematics classroom in the Brazilian context. Mathematics videos, together with other asynchronous modes of communication, have especially become more accepted during the COVID-19 pandemic. This is another example of the convergence of mathematics pedagogical approaches that characterises the current millennium.

Another example is collaborative problem-solving, "a critical and necessary skill used in education and in the workforce … [in which] individuals pool their understanding and effort and work together to solve … problems" (OECD, 2017, p. 3). Zhang, Chan, Clarke, and Cao's (2021) report of how mainland Chinese students participated in collaborative problem-solving, highlights academic attention on how student participants engaged in this form of pedagogy in the East. A secondary analysis of PISA2015 data by Hwang and Ham (2021) not only highlighted Korean interest in collaborative problem-solving in the new millennium, but also emphasized the cultural sensitivity needed when applying pedagogical practices across cultures (in this case, Korea and USA).

Indeed, can (the same) mathematics be taught effectively using the same approach in different countries? Perhaps, there is something we can learn from the professional experiences of migrant teachers of mathematics. Seah's (2005) research, for example, revealed that immigrant mathematics teachers in Australian schools had to confront culturally based differences in how mathematics was taught in classrooms across different countries, even if the mathematics topics were similar to what was learnt in their respective home countries. Over time, different cultures develop their own ways to teach the various mathematical concepts. Two-digit numbers can be multiplied using Vedic mathematics, Russian Peasant method, partial products method, Lattice method, for examples. Whole-class chorus response may be frowned upon in some Western mathematics education systems, but it is an effective and psychologically safe teaching strategy to promote learning for all students in many parts in China (Dong, Seah, Cao & Clarke, 2019); and it's similarly shown in Korea. Clearly, there is a need for mathematics education systems in different cultures to evaluate if and how what works elsewhere can be introduced into the local classrooms in culturally relevant ways, despite pressures to adopt pedagogical approaches wholesale.

When England introduced the Shanghai-style mathematics mastery reform in 2014 to "address England's stagnant performance in mathematics in international league tables" (Blausten, Gyngell, Aichmayr, & Spengler, 2020, p. 29), it was interesting that the changes made went beyond how teachers were expected to teach, to also include re-designing of the schools' daily routines. This acknowledged that teaching is culturally referenced, that it would be futile to change teacher practice if supporting structures such as school timetables are not modified as well. But, the learning context extends beyond the boundaries of school campuses, and even the mastery approach itself began to take on different forms when it was applied across England: "some schools are adopting mastery schemes created by a variety of publishers, some are using government-approved textbooks, some are using schemes created by White Rose Maths Hub and others are implementing their own take on the mastery approach" (Blausten, Gyngell, Aichmayr, & Spengler, 2020, p. 45). Also, Drury (2019) asserted that it was professional development and support – not just mastery teaching alone – which led to Shanghai's success in PISA. Furthermore, she emphasised the need for "a proper implementation process grounded in the reality of school life in the UK" (n.p.).

III. MATHEMATICAL PROFICIENCIES

The beginning of the 21st Century coincides roughly with the start of the Fourth Industrial Revolution, when the biological, physical and digital worlds are brought together to define how we better work, live and relate to one another. One of the implications for mathematics education is that new technologies (including artificial intelligence) are getting increasingly better at using and applying mathematics skills, and at making predictions and decisions based on mathematical modelling. If the new technologies associated with the Fourth Industrial Revolution are creating the inequalities around the world now and in the near future (Mirza et al., 2019), then mathematics education needs to evolve so that students are not only introduced to mathematics knowledge and skills, not only develop positive dispositions, but are also motivated to embrace relevant competencies and soft skills so that these convictions provide them with a life compass to help them navigate through the uncertainties and challenges in life. This emerging focus on proficiencies and competencies was given a boost during the COVID-19 pandemic, when schools all over the world were forced to close at short notice in 2020 and lessons had to be delivered remotely.

Proficiencies and competences need not be regarded only as soft skills (as opposed to hard skills). The USA National Research Council (2012), for instance, conceptualised competence as falling into three domains, namely, cognitive, intrapersonal, and interpersonal.

In the context of mathematics education, we may want to consider if the five mathematical proficiencies identified by a group of American scholars (National Research Council, 2001) – strategic competence, productive disposition, procedural fluency, adaptive reasoning, conceptual understanding –are still relevant, and if it needs expansion, leading to the formulation of what Seah (2020) called '21st Century Competencies 2.0'. At the same time, the Australian Curriculum for mathematics (ACARA, 2013) emphasises fluency, problem-solving, reasoning, and understanding. The current UK mathematics curriculum likewise emphasises fluency, problem-solving, and reasoning (DfE, 2013). The most recent Chinese, Singapore, and Korean mathematics curriculum reform also resulted in a focus on facilitating students' development of competencies.

A timely finetuning of mathematical proficiencies meant for the 21st Century will optimise one's opportunity to engage in meaningful work that remains or becomes relevant during and after the Fourth Industrial Revolution.

There is little doubt, however, that proficiencies are now part of curriculum design considerations. For example, the conceptualisation and trial of the Constructive Learning Design in Singapore (Lee, Lee, & Wong, 2021) might appear to be a move from the Direct Instruction approach to a 'problem-solving first, instruction later' mindset, but it is clear in the paper that intrapersonal and interpersonal competencies (National Research Council, 2012) had been given consideration in addition to the cognitive ones.

If this approach above reminds readers of the Japanese problem-solving approach to mathematics pedagogy (see, for example, Hiraoka & Yoshida-Miyauchi, 2007), it is worth noting that several Japanese researchers have been working on progressing this teaching approach further to incorporate more explicit proficiencies. Hattori, Fukuda and Baba's (2021) paper was an update on their research in school classrooms incorporating the nurturing of social judgement skills in the context of an ethical framework. In this novel teaching approach, social values such as *fairness and economic efficiency* played a central role across the different stages of the teaching cycle.

Indeed, discussions about proficiencies or competencies often make more explicit the attributes of mathematics and of learning that are valued by the (local) culture. More so than cognitive-related decisions, there are often many more choices available amongst proficiencies to be given attention to, and each of these involves selections and preferences amongst values.

The crucial role of values and valuing in (mathematics) education will thus be made more prominent as we navigate the 21st Century. Taking the COVID-19 pandemic as an example, Gooya and Gholamazad (2021) argued how a new set of values needed to be developed amongst students so that the remote education that was inevitable in many countries could be implemented more effectively. Amongst the values which the authors listed were *responsibility, perseverance, collaboration*, and *autonomy*.

IV. STUDENTS' MATHEMATICAL WELLBEING

The COVID-19 pandemic which has led to global lockdown of communities, resulting in remote teaching/learning for students, has highlighted the significance of emotional wellbeing in people's lives and students' learning. For example, a survey of 4,666 students in New Zealand revealed that they "were four times as likely as principals and teachers to report that they never feel happy" (New Zealand Education Review Office, 2021, p.5) during the pandemic. In particular, older students were more likely to be experiencing wellbeing issues than their younger peers. In addition, amongst the oldest students who were in Grades 11-13, more of them (29%) were not feeling positive about the rest of the year, compared to those (22%) who were. A schoolteacher's remark in the Australian Education Survey (Ziebell, Acquaro, Pearn, & Seah, 2020) probably echoed the opinions of the profession when she commented that "good teaching will soon fill any gaps created by online teaching …. It is the social-emotional wellbeing of our young people, particularly those at risk in their homes, that is my biggest concern" (p. 7). The New Zealand survey mentioned above also revealed something more troubling: "students were struggling more [in terms of wellbeing] after lockdown than they were in lockdown" (New Zealand Education Review Office, 2021, p. 4).

For us, we adopt Tiberius' (2018) definition of wellbeing, in which "well-being is served by the successful pursuit of a relatively stable set of values that are emotionally, motivationally, and cognitively suited to the person" (p. 13). This means that if we are concerned about fostering the wellbeing of students in relation to their mathematics learning, what are the subjective and personal values that need to be fulfilled?

We call this subject-specific wellbeing: mathematical wellbeing [MWB], which refers to a sense of "feeling good and functioning well" (Huppert & So, 2013, p. 839) in relation to learning and using mathematics. Although wellbeing is a (positive) psychology concept which is no stranger to school education policies and research, it is rarely considered in association with specific subjects. For mathematics, MWB was conceptualized by Clarkson, Bishop and Seah (2010) well in the 21st Century. It describes a state that is associated with a range of affective variables such as anxiety, helplessness, stress, satisfaction and happiness. Positive MWB enhances self-confidence, positive attitudes, and engagement with mathematics, leading to gains in mathematics understanding and an appreciation of the importance of mathematics in daily life and work. On the other hand, negative MWB would affect the growth of these cognitive and affective aspects of mathematics learning.

Unfortunately, the argument by Hill, Kern, Seah and van Driel (2020) that there is "a poor sense of student well-being in many Australian mathematics classrooms" (p. 2) would apply to most if not all countries around the world. Students are generally not thriving in their mathematics learning experience, and not feeling good nor happy with it as well. For instance, TIMSS2015 reveals that on average across the world, 19% and 38% of Grades 4 and 8 students respectively did not like learning mathematics (Mullis et al., 2016). Here, Korean students registered the highest (35%) and second highest (58%) percentages of students who self-reported their dislike for mathematics learning for Grades 4 and 8 respectively. Their equally high-performing peers in Japan and Taiwan also felt the same. This is not a desirable outcome for any mathematics education system, not just because of the young lives that are lost, but also the emotional health of the others. There are certainly implications for student disengagement with mathematics, and for living and working in the 21st Century which requires us to develop and exercise our mathematical way of thinking and competencies that we learn and hone through mathematics education.

As alluded to above, Tiberius' (2018) value fulfilment theory of wellbeing suggests that positive MWB might be fostered by facilitating students' pursuit of relevant values. However, what might these values be, and how varied might these be from student to student, culture to culture?

A recent Australian study conducted by Hill, Kern, Seah and van Driel (2020) had aimed to explore if there might be a set of values that is relatively stable at the national level. Preliminary findings from 488 Grade 8 students in Melbourne suggested that the following seven 'themes' were associated with positive wellbeing: relationships, engagement, cognitive, accomplishment, positive emotions, perseverance, and meaning. These seven themes are currently being validated with elementary school students in Chengdu, China. With the 'wellbeing as values fulfillment' context, the seven themes have been re-named as seven values, namely, *relationships, engagement, learning, accomplishment, bliss, perseverance*, and *meaningfulness*. The implication of these work is that insofar that MWB is concerned, positive experiences of it in the context of mathematics learning might be achieved through facilitating students' successful fulfillment of a finite set of values, possibly the seven listed above. We certainly hope that more research will be conducted in this area to deepen our collective knowledge and skills, so that students not only feel and function well physically, mentally, socially, and cognitively with mathematics learning, but also contribute to the overall sense of wellbeing amongst students in the 21st Century.

V. NEXT STEPS

The three themes we discussed briefly above—namely the convergence of mathematics pedagogical approaches, mathematics proficiencies, and students' mathematical wellbeing—provide clues about how we can/should rethink mathematics education for the (rest of the) 21st Century, while we are living in the 21st Century itself. When reimagining mathematics education for the 21st century, globalization of education will lead to mathematics pedagogical approaches becoming increasingly convergent, students' need for MWB will be more urgent, and the changing educational needs arising from the Fourth Industrial Revolution will call for a change in the nature of mathematics and the state of mathematics education. Over the past half century, student-centered curriculum and mathematics-centered curriculum have been at odds with each other for a long time. From a historical point of view, it is self-evident that finding a balance between the two curricula has not been easy. In the 21st century, we are facing a similar dilemma in relation to an increasingly technological society and a rapidly changing globalized landscape. On the one hand, we are faced with the important mission of how to foster students' MWB through student-centered curriculum, and on the other hand, we are also faced with how to develop the crucial task of fulfilling the needs of the Fourth Industrial Revolution through mathematics-centered curriculum. In other words, we are faced with the question of "How to balance the unbalanceable between two curricula?" (Sfard, 2001). Responding to this question will involve addressing some unanswered questions such as "What is the nature of mathematics in future-oriented mathematics, such as coding mathematics, interdisciplinary mathematics, and AI and Mathematics for the 21st century?", "What are mathematical proficiencies in the context of mathematics for the 21st century?", "How to foster our students' MWB through future-oriented mathematics?", "What is the role of educational globalization in defining the nature of future-oriented mathematics and in fostering students' MWB?", and so on.

They are by no means the only themes that will surface when we reimagine mathematics education for the 21st Century, of course (see, for example, Bakker, Cai & Zenger, 2021). The threat to globalization as we know it, and the flow-on effects of the ongoing trade war between the world's two biggest economies, might also shape mathematics education in ways which we cannot yet predict. Nevertheless, the three themes discussed here should provide a glimpse into the range of research and thinking we can possibly engage in to inform this reimagination process, so that our students and their children can thrive for the rest of this Century and beyond with the mathematical knowledge, skills, dispositions and motivations they acquire from school and apply in life.

CONFLICTS OF INTEREST

No potential conflict of interest relevant to this article was reported.

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