A hybrid model of networked control systems implemented on WirelessHART networks under source routing configuration

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***Abstract*— A Network control system (NCS) is a control system in which communication between subsystems takes place over a digital network. Numerous results exist in the literature on modelling, analysis and design of NCSs in the presence of specific communication constraints such as packet dropouts, delays, data rates, quantization, etc. However, when analysing NCSs implemented on real physical networks, the existing results are based on restrictive assumptions. We consider NCSs over WirelessHART, the first international standard for industrial process control. With the goal of closing the gap between theory and practice, we propose for the first time a hybrid control-oriented model of WirelessHART NCSs under source routing configuration. Moreover, asymptotic and exponential stability results are presented under reasonable conditions.**

I. Introduction

A control system where plant and controller communicate over non-transparent communication links is called a networked control system (NCS). There have been numerous works in the literature dealing with constraints that typically affect NCSs such as scheduling, dropouts, quantization, and power and data-rates constraints [1]. Although useful to have better insight in how these constraints affect the stability and performance of the NCS, these results cannot be directly applied to specific communication networks given their complexity (e.g. WirelessHART, CAN, FlexRay, etc). In this paper, we consider a recent network standard used in industrial control called WirelessHART [2]. It was developed by the HART Communication Foundation in 2007 specifically for advanced process control and monitoring.

For the purpose of this work, existing results concerning WirelessHART can be summarised as either simulation based studies of network control systems, see for example [3]–[5], theoretical investigations of optimal link scheduling and channel assignment [7], [9], or controller-communications co-design [6], [8], [10]. The aforementioned control-oriented results model network as Markov chains that take into account packet losses between nodes. However, linear and discrete time plant/controller models are considered, together with equidistant transmission instants. Such assumptions are hard to implement in real WirelessHART networks, where extra features need to be taken into consideration (e.g., TDMA communications, time-varying transmission times, nonlinear plant/controller). Therefore, as the functionalities supported by the WirelessHART standard exceed that considered by existing models, the development of a richer class of NCS models is crucial. With a view to bridging the extant gap between existent control-oriented results and actual WirelessHART NCSs implemented in practice, in this paper we adopt the modelling formalism of Hybrid Control Systems (HCS) presented in [11]. It allows us to obtain higher fidelity models that cover many network features (like time varying transmission intervals, dropouts, protocol models, quantization, nonlinear plants and controllers, etc) making them convenient for analysis and design. Recent work [12], [13] uses this formalism to model a specific communication network used in automotive control called FlexRay. This model is then used to derive results on emulation and observer design.

As our main contribution, we propose a novel hybrid model of NCSs over WirelessHART by carefully studying each component and characteristic of the network and making the corresponding assumptions accordingly. In particular, we are able to cover inter-sampling behaviour, time varying transmission instants, and nonlinear plant and controller, which have not yet been considered in the literature. Interestingly, the resulting model fits a general class of NCSs previously studied in [14], therefore, as a second contribution, we state off-the-shelf stability results for such models.

The remainder of this paper is organized as follows: Notation and preliminaries are given in Section II. Section III describes in detail the WirelessHART standard. We present the hybrid model in Section IV and stability results are stated in Section V. A numerical example to illustrate our results is given in Section VI, whilst Section VII draws conclusions.

II. Preliminaries

Denote by $\mathbb{R}$ the set of real numbers. Let $\mathbb{R}_{\geq 0} \doteq [0, \infty)$, $\mathbb{Z}_{\geq 0} \doteq \{0, 1, 2, \dots\}$, and $\mathbb{Z}_{>0} \doteq \{1, 2, 3, \dots\}$. For vector arguments, $|\cdot|$ denotes the Euclidean norm. The same notation is used to denote the induced 2-norm of a matrix. To shorten notation, we often use $(x, y) \doteq [x^T \ \ y^T]^T$. A function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to be of class $\mathcal{K}$ if it is continuous, zero at zero and strictly increasing. It is said to be of class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$ and it is unbounded. A function $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to be of class $\mathcal{KL}$ if $\beta(\cdot, t)$ is of class $\mathcal{K}$ for each $t \geq 0$, and $\beta(s, \cdot)$ is nonincreasing and satisfies $\lim_{t\to\infty} \beta(s, t) = 0$ for each $s \geq 0$. A function $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to be of class $\mathcal{KLL}$ if, for each $r \geq 0$, $\beta(\cdot, r, \cdot)$ and $\beta(\cdot, \cdot, r)$ belong to class $\mathcal{KL}$. Given $t \in \mathbb{R}$ and a piecewise continuous function $f : \mathbb{R} \to \mathbb{R}^n$, we use the notation $f(t^+) \doteq \lim_{s \to t, s > t} f(s)$.

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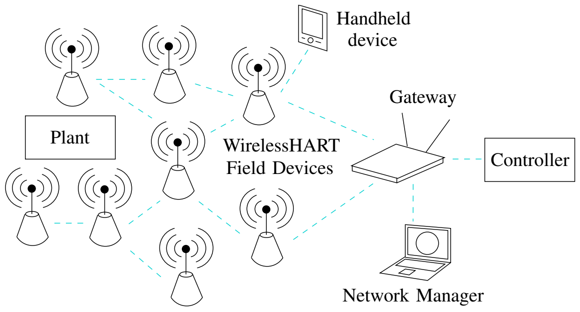

Fig. 1. WirelessHART Architecture

We will show that NCSs implemented over WirelessHART networks can be represented by a hybrid system of the form:

$$\begin{array}{rcll} \dot{\xi} & = & \mathcal{F}(\xi), & \xi \in C, \\ \xi^+ & = & \mathcal{G}(\xi), & \xi \in D, \end{array} \tag{1}$$

where $\xi \in \mathbb{R}^n$ denotes the state, $C, D \subset \mathbb{R}^n$ are the flow and jump sets respectively, and $\mathcal{F}, \mathcal{G}$ are the flow and jump maps respectively [11]. We now recall some important definitions used in the formulation of hybrid models [11].

Definition 1 (Hybrid time domains): A subset $\mathcal{D} \subset \mathbb{R}_{\geq 0} \times \mathbb{Z}_{\geq 0}$ is called a *compact hybrid time domain* if

$$\mathcal{D} = ([t_0, t_1], 0) \cup ([t_1, t_2], 1) \cup \cdots \cup ([t_{J-1}, t_J], J-1),$$

for some finite sequence of times $0 = t_0 \leq t_1 \leq \cdots \leq t_J$ and $J \in \mathbb{Z}_{>0}$. It is a *hybrid time domain* if for all $(T, J) \in \mathcal{D}$, the set $\mathcal{D} \cap ([0, T] \times \{0, 1, \ldots, J\})$ is a compact hybrid domain. ■

Definition 2 (Hybrid arc): A function $\xi : \mathcal{D} \to \mathbb{R}^n$ is a *hybrid arc* if $\mathcal{D}$ is a hybrid time domain and $\xi(\cdot, j)$ is locally absolutely continuous for each $j \in \mathbb{Z}_{\geq 0}$. ■

Definition 3 (Solution to (1)*):* A hybrid arc ξ : dom $\xi \to \mathbb{R}^n$ is a *solution to the hybrid system* (1) if $\xi(0, 0) \in C \cup D$, and

1) for all $j \in \mathbb{Z}_{\geq 0}$ and almost all t such that $(t, j) \in$ dom ξ, $\xi(t, j) \in C$ and $\dot{\xi}(t, j) = \mathcal{F}(\xi(t, j))$;
2) for all $(t, j) \in$ dom ξ such that $(t, j + 1) \in$ dom ξ, $\xi(t, j) \in D$ and $\xi(t, j + 1) = \mathcal{G}(\xi(t, j))$. ■

III. WirelessHART Network

In this section, we provide a detailed description of the WirelessHART standard and state the assumptions underlying the model development to follow in Section IV.

A. General structure

The general architecture of a WirelessHART network is shown in Fig. 1. It consists in a mesh configuration with basics components including *field devices* that communicate with the plant process (e.g. sensor/actuators), *handheld devices* which are portable computers used to run diagnostics, *gateways* that enable communications between host applications and field devices (usually allocated with the *controller*), and a *network manager* responsible of the scheduling between devices and network configuration.

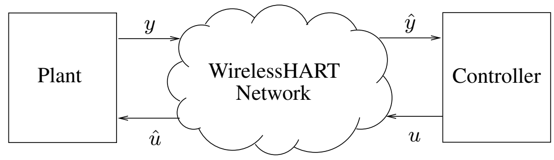

Fig. 2. Block diagram of a NCS over WirelessHART.

B. Key communication features

In its physical layer, WirelessHART is based mostly on the IEEE 802.15.4-2006 physical layer and operates in the 2.4 GHz ISM radio band with a maximum data rate of 250 kbits/s and 16 channels. In the data link layer, WirelessHART defines a slotted TDMA technology to provide deterministic communications without collisions. WirelessHART also enables channel hopping, that is, multiple transmission links can be active on the same time slot by using different frequency channels. Where interference is an ongoing issue for specific channels, WirelessHART quarantines their use by way of a channel blacklist. WirelessHART networks also support multiple access timeslots, where multiple devices can share a specific channel, with transmissions permitted by any such device whenever the channel is determined to be free by that device. In this case, WirelessHART uses CSMA/CA mechanisms to avoid collisions. Consequently, we start by stating the following assumption.

Assumption 1: Attention is restricted to WirelessHART networks that are configured only for TDMA communications, i.e., collision-free and deterministic scheduling. ■

Remark 1: Assumption 1 is a fair standard assumption. In fact, WirelessHART typically adopts TDMA communications [2] because in process control industry guaranteed process data delivery is essential and TDMA meets the need. It allows for CSMA usage if required by some specific applications and is not commonly used. CSMA was studied in the context of NCSs in [15], and we believe it can be adapted to WirelessHART. ■

C. TDMA superframe structure

All communications in the WirelessHART network are defined with respect to a *superframe*. A superframe is an a priori fixed period of time $T > 0$, contiguous in real time with other superframes, that is divided into a sequence of timeslots as depicted in Fig. 3. Each time slot is strictly $\tau_{ts} = 10\,ms$ in duration. Within this period, a complete single data packet and its corresponding acknowledgement are transmitted between two field devices. The transmission delay required for the delivery of a packet from a transmitting device to a receiving device, in each i-th timeslot, is denoted by $t_i^r - t_i^s$ and it depends on the packet size. At the end of every i-th timeslot, the time it takes to acknowledge such a packet is denoted by τ_i^{ACK}. Given Assumption 1, the network manager assigns to each field device a specific timeslot within the superframe to transmit, and therefore protocols such as Round Robin (RR) can readily be implemented [16]. In order to have effective TDMA communications, all devices need to be synchronized, so there exists a short

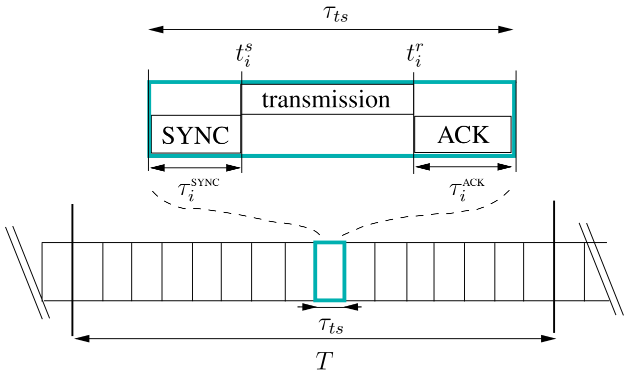

Fig. 3. WirelessHART superframe and its i-th timeslot.

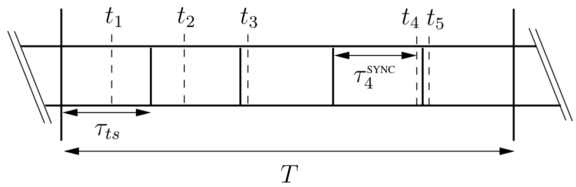

Fig. 4. Feasible superframe in WirelessHART communications after Assumption 2.

delay τ_i^{SYNC} in the beginning of each time slot for this specific operation. For simplicity, the following additional assumption is adopted.

Assumption 2:

(a) Data are transmitted instantaneously in each timeslot, i.e., $t_i \doteq t_i^s = t_i^r$ for all $i \in \mathbb{Z}_{>0}$. We refer to t_i as transmission instant.

(b) Acknowledgement time is negligible in each timeslot, i.e., $\tau_i^{\text{ACK}} = 0$ for all $i \in \mathbb{Z}_{>0}$.

(c) One successful transmission between devices occurs within each timeslot. ■

Items (a) and (b) are reasonable when packets are short (relative to the maximum packet length). If this is not the case, transmission delays $t_i^r - t_i^s$, and acknowledgement times τ_i^{ACK} can be modelled as time delays. We believe modelling and stability results presented later in this paper can be extended to cover this case by using [17]. We adopt item (c) so as to restrict attention to the effects of the transmission instants t_i in the modelling. Packet drops will be considered in future work. Given Assumption 2, a feasible superframe that may arise in WirelessHART is shown in Fig. 4.

Remark 2: Note that transmission instants t_i, depending on synchronization time τ_i^{SYNC}, may not be equally spaced between each other (see Fig. 4). This models the fact that real-life networks in general have time-varying transmission instants and not necessarily equidistant as often modelled. τ_i^{SYNC} may also represent delays in the network generated by changes in the environment (common phenomena in wireless networks), connectivity issues, etc. ■

In this paper we parametrize our model with the so-called *maximal allowable transmission interval* (MATI) [18], which we denote as $\tau_{\text{MATI}} > 0$. It is a measurement of how fast the network needs to transmit in order to preserve stability of the NCS. MATI is directly related to the superframe by $\tau_{\text{MATI}} = 2\tau_{ts}$ (see Fig. 4). In other words, a packet must be transmitted at most in τ_{MATI} seconds after the previous packet transmission. The following lemma follows directly from Assumption 2 and the latter discussion.

Lemma 1: If Assumption 2 holds, then the transmission instants $\{t_i\}_{i\in\mathbb{Z}_{>0}}$ in the WirelessHART network satisfy $t_{i+1} - t_i \in [\varepsilon, \tau_{\text{MATI}}]$, for all $i \in \mathbb{Z}_{>0}$ and $\varepsilon > 0$. ■

In Lemma 1, ε corresponds to the minimum synchronization time, i.e., $\min_i \tau_i^{\text{SYNC}}$.

D. Routing configurations

The WirelessHART standard allows for two types of routing configurations:

- **Source routing:** In order to send a packet from the controller to the plant (and vice-versa) the network manager determines a specific route of field devices to complete the task. That is, it writes an ordered list of devices for the packet to go through in the network header. In other words, the network adopts a line topology communication.
- **Graph routing:** A routing graph is a collection of paths that connects network nodes between plant and controller. The possible transmission paths are created by the network manager and downloaded to each field device. This specifies the list of neighbours of each device to which the packet may be forwarded depending on their signal strength. That is, the transmitting field device checks the received output power of its neighbours and sends its packet to the one with higher signal strength. In this case we have a mesh topology in which each field device have different paths to route the packet.

Due to the clear difference between both configurations, we believe it is important to develop hybrid models for each type of configuration. In this paper, we focus on source routing models only.

IV. Hybrid Model of NCSs over WirelessHART

We model NCSs implemented over WirelessHART networks in source routing configuration. Consider the NCS shown in Fig. 2, where the implemented controller has been designed to stabilize the plant while ignoring the WirelessHART network[1]. We model this controller as

$$\dot{x}_c = f_c(x_c, \hat{y}), \quad u = g_c(x_c), \tag{2}$$

where $x_c \in \mathbb{R}^{n_c}$ is the state of the controller, $\hat{y} \in \mathbb{R}^{n_y}$ is the plant output after being transmitted through the network, $u \in \mathbb{R}^{n_u}$ is the control signal, $f_c(\cdot,\cdot)$ and $g_c(\cdot)$ are nonlinear functions, and $n_c, n_y, n_u \in \mathbb{Z}_{>0}$. We consider

$$\dot{x}_p = f_p(x_p, \hat{u}), \quad y = g_p(x_p) \tag{3}$$

as the plant model, where $x_p \in \mathbb{R}^{n_p}$ is the state, $\hat{u} \in \mathbb{R}^{n_u}$ is the control signal after being transmitted through the network, $y \in \mathbb{R}^{n_y}$ is the plant output, $f_p(\cdot,\cdot)$ and $g_p(\cdot)$ are nonlinear functions, and $n_p \in \mathbb{Z}_{>0}$.

[1]This fact becomes important in Section V where the so-called emulation approach is used to obtain stability results for this NCS [16], [18].

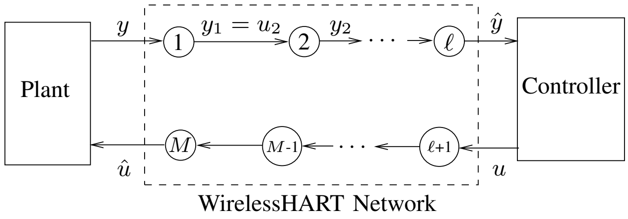

Fig. 5. Source routing block diagram.

Consider the NCS in Fig. 2 where plant and controller are as in (2)-(3), and where the WirelessHART network has been configured to transmit by using source routing. As discussed in Section III-D, this situation can be modelled as a line topology as shown in Fig. 5. Consider there are ℓ field devices in the plant-to-controller path and $M-\ell$ in the controller-to-plant path. We label them as shown in Fig. 5. We use k as the index for field devices, i.e., $k = 1, \ldots, M$. suppose each field device k has an input u_k and output y_k. According to WirelessHART specifications, each field device must act as a router for data from/to neighbouring field devices. Therefore, we model field devices as buffers, where their inputs and outputs satisfy the following equations:

$$\dot{y}_k(t) = 0, \quad t \in [t_i, t_{i+1}], \tag{4a}$$
$$y_k(t_i^+) = u_k(t_i) + h_k(i, e(t_i)), \tag{4b}$$

for all $i \in \mathbb{Z}_{>0}$ and $k = 1, \ldots, M$. In other words, field devices forward their inputs to the next field device when they are scheduled to transmit at time instant t_i. In (4b), $h_k(i, e(t_i))$ is a function that determines which field device gets to transmit at time instant t_i, as will be explained in detail in Remark 3, and e is the network induced error

$$e \doteq (e_1, \ldots, e_M) \in \mathbb{R}^{n_e}, \tag{5}$$

where $n_e \doteq \ell n_y + (M-\ell)n_u$, and $e_k \doteq y_k - u_k$, for $k = 1, \ldots, M$, are the errors introduced by each field device, i.e., the difference between their inputs and outputs. In addition, because of the line topology we can state the following connectivity conditions (see Fig. 5):

$$u_1 = y, \ u_k = y_{k-1}, \ \hat{y} = y_\ell, \ u_{\ell+1} = u, \ \hat{u} = y_M, \tag{6}$$

for all $k \in \{2, \ldots, \ell, \ell+2, \ldots, M\}$.

Remark 3: Recall that network manager is the decision maker behind routing and scheduling within the WirelessHART network. As we adopt TDMA communications, the network manager needs to schedule field devices to transmit into corresponding timeslots within the superframe. To model this situation we have incorporated the functions $h_k(\cdot, \cdot)$. For instance, if network manager determines field device 2 has to transmit at timeslot 5, this is equivalent to $h_2(5, e(t_5))$ being equal to zero (see (4b)). ∎

In order to have a better notation for analysis, define

$$x \doteq (x_p, x_c), \quad h(i, e) \doteq (h_1(i, e), \ldots, h_M(i, e)). \tag{7}$$

Combining (2)–(7) allow us to write a model, which we call an *impulsive model*, for the block diagram in Fig. 5 between and at transmission instants. In particular,

$$\dot{x}(t) = f(x(t), e(t)), \quad t \in [t_i, t_{i+1}], \tag{8a}$$
$$\dot{e}(t) = g(x(t), e(t)), \quad t \in [t_i, t_{i+1}], \tag{8b}$$
$$x(t_i^+) = x(t_i), \tag{8c}$$
$$e(t_i^+) = h(i, e(t_i)), \tag{8d}$$

where $f(\cdot, \cdot)$ and $g(\cdot, \cdot)$ are obtained from straightforward calculations from (2)–(4). We refer to (8d) as the protocol equation, as it contains the function $h(\cdot, \cdot)$ which is responsible for the scheduling of all field devices in the superframe. Depending on the scheduling protocol implemented by the network manager, $h(\cdot, \cdot)$ will have different definitions. We provide an example in the following remark.

Remark 4: An important protocol that can be implemented in source routing communications is the so-called Round Robin (RR), presented in [16]. This protocol assigns a field device to a timeslot in a predetermined and cyclic manner. The definition for the protocol equation (8d) in this particular case is $h(i, e) \doteq (I - \Delta(i))e$, where

$$\Delta(i) \doteq \text{diag}\left\{\delta_1(i) I_{n_y}, \ldots, \delta_\ell(i) I_{n_y}, \delta_{\ell+1}(i) I_{n_u}, \ldots, \delta_M(i) I_{n_u}\right\},$$

and

$$\delta_k(i) \doteq \begin{cases} 1, & \text{if } i = k + \sigma M, \ \sigma \in \mathbb{Z}_{>0}, \\ 0, & \text{otherwise.} \end{cases}$$

In other words, it resets to zero error e_1 (field device 1 transmits) at time t_1 and so forth until device M transmits at time t_M ending the superframe, which then repeats. Note the superframe period in this case is $T = M\tau_{ts}$. ∎

We now embed the impulsive model (8) into a hybrid system of the form (1) and specify the mappings $\mathcal{F}, \mathcal{G}$ and the sets C, D. To that end, we introduce a clock variable $\tau \in \mathbb{R}_{\geq 0}$, which represents the time elapsed since the last transmission. We also introduce $\kappa \in \mathbb{Z}_{\geq 0}$, which counts the number of transmissions (or "jumps"). Let $\xi \doteq (x, e, \tau, \kappa)$. In that way, by using (8) and Lemma 1, we can present the following hybrid model for NCSs implemented over WirelessHART networks using source routing configuration:

$$\begin{aligned} \dot{\xi} &= (\dot{x}, \dot{e}, \dot{\tau}, \dot{\kappa}) \\ &= (f(x,e), g(x,e), 1, 0) \doteq \mathcal{F}(\xi), \quad \xi \in C, \end{aligned} \tag{9a}$$
$$\begin{aligned} \xi^+ &= \left(x^+, e^+, \tau^+, \kappa^+\right) \\ &= (x, h(\kappa, e), 0, \kappa+1) \doteq \mathcal{G}(\xi), \quad \xi \in D, \end{aligned} \tag{9b}$$

where the flow and jump sets are given by $C \doteq \mathbb{R}^{n_x} \times \mathbb{R}^{n_e} \times [0, \tau_{\text{MATI}}] \times \mathbb{Z}_{\geq 0}$ and $D \doteq \mathbb{R}^{n_x} \times \mathbb{R}^{n_e} \times [\varepsilon, \tau_{\text{MATI}}] \times \mathbb{Z}_{\geq 0}$.

Given that we embedded (8) into the hybrid system (9), is important to note that (9) may generate more solutions than (8). Consequently, guaranteeing well behaved solutions in (9), ensures that solutions of the "smaller" system (8) will behave properly as well.

V. STABILITY RESULTS

In order to obtain closed loop stability for WirelessHART NCSs in source routing configuration, in this section we pursue the *emulation approach* proposed in [18] and further developed in [14], [16]. That is, we first design a controller for the plant by ignoring the network, then we implement the controller via the WirelessHART network and demonstrate that under reasonable assumptions, the stability of the resulting closed loop is preserved under sufficiently high rate of transmissions (i.e. small enough MATI). As the obtained hybrid model (9) fits the NCS framework in [14], these stability results follow by application of tools from [14]. These results are summarised in the remainder of this section.

Definition 4: For the hybrid system (9), the set $\{(x, e, \tau, \kappa) : x = 0, e = 0\}$ is *uniformly globally asymptotically stable (UGAS)* if there exists $\beta \in \mathcal{KLL}$ such that, for each initial condition $x(0,0) \in \mathbb{R}^{n_x}$, $e(0,0) \in \mathbb{R}^{n_e}$, $\tau(0,0) \in \mathbb{R}_{\geq 0}$, $\kappa(0,0) \in \mathbb{Z}_{\geq 0}$, and each corresponding solution we have

$$|(x(t,j), e(t,j))| \leq \beta\left(|(x(0,0), e(0,0))|, t, \epsilon j\right), \tag{10}$$

for all (t, j) in the solution's domain and $\epsilon > 0$. The set is *uniformly globally exponentially stable (UGES)* if β can be taken to have the form $\beta(s, t, k) = Ks\exp(-\lambda(t + k))$ for some $K > 0$ and $\lambda > 0$. ■

Assumption 3: There exist a function $W : \mathbb{Z}_{\geq 0} \times \mathbb{R}^{n_e} \to \mathbb{R}_{\geq 0}$ that is locally Lipschitz in its second argument, a continuous function $H : \mathbb{R}^{n_x} \to \mathbb{R}_{\geq 0}$, real numbers $\lambda \in (0, 1)$, $L \geq 0$, $\gamma > 0$, and $\underline{\alpha}_W$, $\bar{\alpha}_W \in \mathcal{K}_\infty$, such that, $\forall \kappa \in \mathbb{Z}_{\geq 0}$ and $e \in \mathbb{R}^{n_e}$,

$$\underline{\alpha}_W(|e|) \leq W(\kappa, e) \leq \bar{\alpha}_W(|e|), \tag{11a}$$

$$W(\kappa + 1, h(\kappa, e)) \leq \lambda W(\kappa, e), \tag{11b}$$

and for all $\kappa \in \mathbb{Z}_{\geq 0}$, $x \in \mathbb{R}^{n_x}$ and almost all $e \in \mathbb{R}^{n_e}$,

$$\left\langle \frac{\partial W(\kappa, e)}{\partial e}, g(x, e) \right\rangle \leq LW(\kappa, e) + H(x). \tag{12}$$

Moreover, there exists a locally Lipschitz, positive definite, radially unbounded function $V : \mathbb{R}^{n_x} \to \mathbb{R}_{\geq 0}$, and a continuous, positive definite function ϱ, such that, for all $e \in \mathbb{R}^{n_e}$, all $\kappa \in \mathbb{Z}_{\geq 0}$, and almost all $x \in \mathbb{R}^{n_x}$,

$$\begin{aligned}\langle \nabla V(x), f(x, e)\rangle \leq& -\varrho(|x|) - \varrho(W(\kappa, e)) \\ &- H(x)^2 + \gamma^2 W(\kappa, e)^2.\end{aligned} \tag{13}$$

■

Remark 5: Assumption 3 is not restrictive, in fact, all its conditions are checkable. Condition (11) requires that the scheduling rule $h(\cdot, \cdot)$ satisfies the property of being UGAS. As already mentioned, a possible scheduling rule is to use RR protocols (see Remark 4) which indeed satisfies this condition. Condition (12) is also checkable and it requires that W is globally Lipschitz in e, which is the case for RR protocols. In particular, a possible Lyapunov function satisfying these conditions for RR protocols is given by $W(i, e) = \sqrt{\sum_{j=i}^{\infty} |\phi(j, i, e)|^2}$, where $\phi(j, i, e)$ denotes the solution of $e(i + 1) = h(i, e(i))$ at time j starting at time i and initial condition e (see [16] for more details). The last condition (13) may be ensured when designing the controller before taking into account the actual network. ■

Theorem 1: Under Assumption 3, if τ_{MATI} satisfies

$$\tau_{\text{MATI}} \leq \begin{cases} \frac{1}{Lr}\arctan\left(\frac{r(1-\lambda)}{2\frac{\lambda}{1+\lambda}(\frac{\gamma}{L}-1)+1+\lambda}\right), & \gamma > L, \\ \frac{1}{L}\frac{1-\lambda}{1+\lambda}, & \gamma = L, \\ \frac{1}{Lr}\text{arctanh}\left(\frac{r(1-\lambda)}{2\frac{\lambda}{1+\lambda}(\frac{\gamma}{L}-1)+1+\lambda}\right), & \gamma < L, \end{cases}$$

where $r \doteq \sqrt{|(\frac{\gamma}{L})^2 - 1|}$, then, for the hybrid NCS (9), the set $\{(x, e, \tau, \kappa) : x = 0, e = 0\}$ is UGAS. If, in addition, in (11) you have $\underline{\alpha}_W(|e|) = \underline{\alpha}_W|e|$ and $\bar{\alpha}_W(|e|) = \bar{\alpha}_W|e|$, for $\underline{\alpha}_W, \bar{\alpha}_W \in \mathbb{R}$, and there exist $a_1, a_2, a_3 \in \mathbb{R}_{>0}$ such that $a_1|x|^2 \leq V(x) \leq a_2|x|^2$ and $\varrho(s) \geq a_3 s^2$, then this set is UGES. ■

Theorem 1 tells us how fast the WirelessHART network needs to transmit in order to preserve stability of the NCS.

VI. NUMERICAL EXAMPLE

To illustrate how the number of field devices (i.e., hops) affect stability of a WirelessHART NCS, we consider an unstable batch reactor which transmits its measurements via a WirelessHART network in source routing configuration. The linearized model of an unstable batch reactor is a two-input two-output system that can be written as

$$\dot{x}_p = A_p x_p + B_p u, \quad y = C_p x_p,$$

where

$$A_p = \begin{bmatrix} 1.38 & -0.2077 & 6.715 & -5.676 \\ -0.5814 & -4.29 & 0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix},$$

$$B_p = \begin{bmatrix} 0 & 0 \\ 5.679 & 0 \\ 1.136 & -3.146 \\ 1.136 & 0 \end{bmatrix}, \quad C_p = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

For simplicity, assume that only plant measurements are sent through the WirelessHART network, to a PI controller whose state-space realization is

$$\dot{x}_c = A_c x_c + B_c \hat{y}, \quad u = C_c x_c + D_c \hat{y},$$

where

$$A_c = 0_{2\times 2}, B_c = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, C_c = \begin{bmatrix} -2 & 0 \\ 0 & 8 \end{bmatrix}, D_c = \begin{bmatrix} 0 & -2 \\ 5 & 0 \end{bmatrix}.$$

By defining the error vector as in (5), the equations that we need to consider take the form:

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} \Longleftrightarrow \dot{z} = Az,$$

where $z \doteq [x^T \; e^T]^T$, $A \doteq \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, and

$$A_{11} = \begin{bmatrix} A_p + B_p D_c C_p & B_p C_c \\ B_c C_p & A_c \end{bmatrix}, A_{12} = \begin{bmatrix} B_p D_c [I \; \cdots \; I] \\ B_c [I \; \cdots \; I] \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} -[C_p \;\; 0] A_{11} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} -[C_p \;\; 0] A_{12} \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

First we compute the stability bound on τ_{MATI} by Theorem 1 for different numbers of hops in the wireless network. Assume the TDMA communication within the network is using a Round-Robin protocol. It is possible to show that for this protocol: (11) and (12) are satisfied with $\lambda = \sqrt{(\ell-1)/\ell}$ and $L = \sqrt{\ell}|A_{22}|$, respectively (recall ℓ stands for the number of hops in the plant-to-controller path). Moreover, by using the μ-synthesis toolbox in Matlab it is possible to find the gain γ in (13), which also depends on the number of hops in the network. Then, by using Theorem 1 we obtain the bounds on τ_{MATI} that ensure stability of the NCS for different number of field devices (see Table I).

TABLE I

SUMMARY OF BOUNDS ON MATI.

	Number of hops				
	1	2	3	4	5
Theoretical bound via Theorem 1 (in [ms])	63.1	6.3	2.7	1.5	0.97
Real bound on MATI (in [ms])	131.5	65.7	43.8	32.8	26.3
$\tau_{\text{REAL}}/\tau_{\text{THEO}}$	≈ 2	≈ 10	≈ 16	≈ 22	≈ 27

Assume now that transmission instants of the network are equidistant. For this scenario, it is possible to show that the impulsive model (8) can be reduced to the following periodically time-varying linear system (see [16]):

$$x(t_{i+1}^+) = F_{11}x(t_i^+) + F_{12}e(t_i^+),$$
$$e(t_{i+1}^+) = F_{21}(i)x(t_i^+) + F_{22}(i)e(t_i^+),$$

where the matrices $F_{11}, F_{12}, F_{21}(i)$ and $F_{22}(i)$ depend on the exponential matrix of $A\tau_{\text{MATI}}$, and also $F_{21}(i), F_{22}(i)$ depend on the protocol $h(i,e)$ of Remark 4, which gives periodicity to the system. This scenario allows us to compute bounds for τ_{MATI} analytically by an eigenvalue computation. We call this bound *the real bound*. Recall that the bound in Theorem 1 is a sufficient condition, therefore the real bound of the NCS could be potentially bigger (less conservative) and still preserve stability. Table I enumerates these bounds for different numbers of hops.

As intuition suggests, increasing the number of hops in the network yields a smaller MATI. In other words, the longer the path from plant to controller (implying large delays in arrival packets) the faster the WirelessHART network needs to transmit its packets in order to preserve stability. By comparing the theoretical bounds and the real simulated bound it can be appreciated that the real bound is indeed larger than the conservative theoretical bound. For a small number of hops, the bound provided in Theorem 1 is slightly more conservative than the real bound. However, this conservatism increases with the number of hops. In addition, we can empirically observe that, when transmissions are equidistant, MATI satisfies $\tau_{\text{REAL},k} = \tau_{\text{REAL},1}/k$.

VII. CONCLUSIONS AND FUTURE WORK

We have presented a hybrid model for networked control systems implemented over WirelessHART networks in source routing configuration, that incorporates its practical communication features and functionalities. In contrast to existing models in the literature, we cover nonlinear systems, time-varying transmission instants and the assumptions we used are reasonable within the standard specifications. We were able to use a previously presented stability framework to state UGAS and UGES properties to this model. Future work will focus on getting hybrid models for WirelessHART NCSs under graph routing configuration. Such configuration would lead to genuinely new interesting models for which we plan to obtain results on emulation and observer design, while considering other communication constraints such as time-delays or packet dropouts.

REFERENCES

[1] J. Hespanha, P. Naghshtabrizi, and Y. Xu, “A survey of recent results in networked control systems,” *Proceedings of the IEEE*, vol. 95, no. 1, pp. 138–162, January 2007.

[2] “HART Communication Protocol,” http://en.hartcomm.org/.

[3] M. D. Biasi, C. Snickars, K. Landernas, and A. Isaksson, “Simulation of process control with WirelessHART networks subject to packet losses,” in *Proceedings of the IEEE International Conference on Automation Science and Engineering*, 2008.

[4] S. Han, X. Zhu, K. M. Aloysius, M. Nixon, T. Blevins, and D. Chen, “Control over WirelessHART network,” in *Proceedings of the 36th Annual Conference on IEEE Industrial Electronics Society*, 2010.

[5] P. Ferrari, A. Flammini, M. Rizzi, and E. Sisinni, “Improving simulation of wireless networked control systems based on WirelessHART,” *Computer Standards & Interfaces*, vol. 35, no. 6, pp. 605–615, 2013.

[6] B. Demirel, Z. Zou, P. Soldati, and M. Johansson, “Modular co-design of controllers and transmission schedules in WirelessHART,” in *Proceedings of the 50th IEEE Conference on Decision and Control and European Control Conference (CDC-ECC)*, 2011.

[7] H. Zhang, P. Soldati, and M. Johansson, “Time-and channel-efficient link scheduling for convergecast in WirelessHART networks,” in *Proceedings of the 13th IEEE International Conference on Communication Technology (ICCT)*, 2011.

[8] Z. Zou, B. Demirel, and M. Johansson, “Minimum-energy packet forwarding policies for guaranteed LQG performance in wireless control systems,” in *Proceedings of the 51st Conference on Decision and Control*, 2012.

[9] H. Zhang, P. Soldati, and M. Johansson, “Performance bounds and latency-optimal scheduling for convergecast in WirelessHART networks,” *IEEE Transactions on Wireless Communications*, vol. 12, no. 6, pp. 2688–2696, 2013.

[10] B. Demirel, Z. Zou, P. Soldati, and M. Johansson, “Modular design of jointly optimal controllers and forwarding policies for wireless control,” *IEEE Transactions on Automatic Control*, vol. 59, no. 12, pp. 3252–3265, 2014.

[11] R. Goebel, R. Sanfelice, and A. Teel, *Hybrid Dynamical Systems: modeling, stability, and robustness*. Princeton University Press, 2012.

[12] W. Wang, D. Nešić, and R. Postoyan, “Emulation-based stabilization of networked control systems implemented on FlexRay,” *Automatica*, vol. 59, pp. 73–83, 2015.

[13] W. Wang, D. Nešić, and R. Postoyan, “Design of observers implemented over flexray networks,” in *Proceedings of the Australian Control Conference (AUCC)*, 2014.

[14] D. Carnevale, A. Teel, and D. Nešić, “A lyapunov proof of an improved maximum allowable transfer interval for networked control systems,” *IEEE Transactions on Automatic Control*, vol. 52, no. 5, p. 892, 2007.

[15] M. Tabbara, D. Nešić, and A. Teel, “Stability of wireless and wireline networked control systems,” *IEEE Transactions on Automatic Control*, vol. 52, no. 9, pp. 1615–1630, 2007.

[16] D. Nešić and A. Teel, “Input-output stability properties of networked control systems,” *IEEE Transactions on Automatic Control*, vol. 49, no. 10, pp. 1650–1667, 2004.

[17] W. Heemels, A. Teel, N. van de Wouw, and D. Nešić, “Networked control systems with communication constraints: Tradeoffs between transmission intervals, delays and performance,” *IEEE Transactions on Automatic Control*, vol. 55, no. 8, pp. 1781–1796, 2010.

[18] G. Walsh, O. Beldiman, and L. Bushnell, “Asymptotic behavior of nonlinear networked control systems,” *IEEE Transactions on Automatic Control*, vol. 46, no. 7, pp. 1093–1097, jul 2001.