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Revisiting fits to $B^0 \rightarrow D^{*-} \ell^+ \nu_\ell$ to measure $|V_{cb}|$ with novel methods and preliminary LQCD data at nonzero recoil

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We present a study of fits to exclusive $B^0 \rightarrow D^{*-} \ell^+ \nu_\ell$ measurements for the determination of the Cabibbo-Kobayashi-Maskawa matrix element magnitude $|V_{cb}|$, based on the most recent Belle untagged measurement. Results are obtained with the Caprini-Lellouch-Neubert (CLN) and Boyd-Grinstein-Lebed (BGL) form factor parametrizations, with and without the inclusion of preliminary lattice QCD (LQCD) measurements of form factors at nonzero hadronic recoil from the JLQCD Collaboration. The CLN and BGL fits are also studied in different scenarios with reduced theoretical assumptions and at higher-order expansions, respectively. To avoid bias from high systematic error correlations, we employ a novel technique in the field of B -physics phenomenology with a toy Monte Carlo method using a Cholesky decomposition of the covariance matrix. Using additional input from lattice QCD calculations of form factors at nonzero recoil, in collaboration with JLQCD, allows for well-defined fit results with reduced model dependence in CLN and BGL. The results obtained are consistent between different configurations, ultimately providing a method for a more model-independent exclusive measurement of $|V_{cb}|$. Using preliminary LQCD inputs and assuming a value of $\mathcal{F}(1) = 0.904 \pm 0.012$, we find $|V_{cb}| = (38.48 \pm 0.32 \pm 0.97 \pm 0.46) \times 10^{-3}$ in BGL(2,2,2) and $(38.42 \pm 0.35 \pm 1.05 \pm 0.46) \times 10^{-3}$ in CLNnoHQS, where the errors are statistical, systematic, and related to $\mathcal{F}(1)$, respectively.

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I. INTRODUCTION

The Cabibbo-Kobayashi-Maskawa matrix element $|V_{cb}|$ is a fundamental parameter of the Standard Model, describing the weak decay of b quarks, and must be measured. A long-standing discrepancy between inclusive and exclusive decay mode determinations limits our present understanding of this parameter. The world-average value from combined inclusive results is based on measurements of semileptonic B meson decays of the type $B \rightarrow X_c \ell \nu$, where X_c denotes all possible hadronic states in the $b \rightarrow c \ell \nu$ transition. The inclusive value is reported to be [1]

$$|V_{cb}| = (42.2 \pm 0.8) \times 10^{-3}. \quad (1)$$

The exclusive measurements are determined from both $B^0 \rightarrow D^{*-} \ell^+ \nu_\ell$ and $B^0 \rightarrow D^- \ell^+ \nu_\ell$ decays. The results from $B^0 \rightarrow D^{*-} \ell^+ \nu_\ell$ decays are

$$|V_{cb}| = (38.4 \pm 0.7 \pm 0.5 \pm 1.0) \times 10^{-3}, \quad (2)$$

using $\mathcal{F}(1) = 0.904 \pm 0.012$, where the first uncertainty is experimental, the second is from lattice QCD (LQCD), and the third is an additional uncertainty to compensate for the truncation of power series in a fit. The results from $B \rightarrow D \ell^+ \nu_\ell$ decays are

$$|V_{cb}| = (40.1 \pm 1.0) \times 10^{-3}, \quad (3)$$

using $\mathcal{G}(1) = 1.054 \pm 0.004 \pm 0.008$. Both exclusive results are significantly lower than that from the inclusive approach [1]. These modes must be studied in further detail to understand where the discrepancies could be originating.

Measurements of $|V_{cb}|$ from $B^0 \rightarrow D^{*-} \ell^+ \nu_\ell$ decays have typically used two different parametrizations for models in order to calculate these extrapolations; one by Caprini, Lellouch, and Neubert (CLN) [2] and another by Boyd, Grinstein, and Lebed (BGL) [3]. Both the CLN and BGL parametrizations are built from the same foundations of operator product expansions, where analytic properties of form factors can also introduce some constraints. The CLN parametrization makes use of heavy quark effective theory relations and its constraints on the $B^0 \rightarrow D^{*-} \ell^+ \nu_\ell$

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form factors to reduce the number of independent free parameters with respect to BGL.

The 2019 untagged Belle measurement of $|V_{cb}|$ [4] measured binned yields of $B^0 \rightarrow D^{*-}\ell^+\nu_\ell$ and fits to the CLN and BGL parametrizations, measuring $\mathcal{F}(1)\eta_{EW}|V_{cb}| = (35.06 \pm 0.15 \pm 0.56) \times 10^{-3}$ and $(34.93 \pm 0.23 \pm 0.59) \times 10^{-3}$, respectively, where the latter is determined at a fixed order of power series truncation, defined later. These results showed consistency between the two parametrizations at a given order in the BGL expansion, in contrast to the results based on a preliminary measurement with tagged Belle data [5], as covered by Refs. [6,7] and others. Both fit studies acknowledge that the observed discrepancy may have been a feature of the tagged dataset. The Belle results are consistent with a four-dimensional analysis from *BABAR* [8], with $\mathcal{F}(1)\eta_{EW}|V_{cb}| = (35.02 \pm 0.77) \times 10^{-3}$ in CLN and $(34.98 \pm 0.82) \times 10^{-3}$ in BGL, where the BGL parametrization also required truncation. However, various studies [9,10] have shown that the choice of configuration in CLN and BGL and the implementation of systematic uncertainties has an effect on the measured value of $|V_{cb}|$ and the relevant form factors in exclusive measurements. In this study, we use both the CLN parametrization and the BGL parametrization in fits to further explore the Belle results, checking for consistency in measurements between parametrizations and the effects of systematic correlations. We apply a technique in which the Cholesky decomposition of a covariance matrix is used to generate a toy Monte Carlo sample in a novel approach for fits to B -meson decays. We then include additional data from preliminary LQCD calculations to allow for fits to higher order in the BGL parametrization and to explore fits with different theoretical assumptions for CLN that are less model dependent.

This paper is organized as follows. Section II is a description on the techniques used to extract results from the data provided by the 2019 Belle measurement, followed

by a brief summary of the conventions used in this analysis. In Sec. IV, we discuss a method in which systematic uncertainties can be taken into account when correlations between observed measurements are very high (predominantly from scale uncertainties). Sections V and VI explore fits to the CLN and BGL parametrizations with more degrees of freedom, followed by the incorporation of additional LQCD data in Sec. VII. The results are then summarized and discussed in Sec. VIII.

II. ANALYSIS OVERVIEW

The exclusive value of $|V_{cb}|$ is typically extracted from fits to yields of $B^0 \rightarrow D^{*-}\ell^+\nu_\ell$ decays as a function of the kinematic observables: the hadronic recoil w and three angular variables $\cos\theta_\ell$, $\cos\theta_v$, and χ . Hadronic recoil is defined as

$$w = \frac{m_{B^0}^2 + m_{D^{*\pm}}^2 - q^2}{2m_{B^0}m_{D^{*\pm}}}, \quad (4)$$

where q^2 is the invariant mass squared of the lepton-neutrino system and m_{B^0} and $m_{D^{*\pm}}$ are the B^0 and $D^{*\pm}$ meson masses, respectively. The observable θ_ℓ is the angle between the direction of the lepton and the direction opposite to the B meson in the W boson rest frame, and θ_v is the angle between the direction of the D^0 meson and the direction opposite to the B meson in the D^* meson rest frame. The observable χ is the angle between the planes formed by the decays of the W and D^* mesons in the rest frame of the B meson. The analysis described here follows the approach taken in the 2019 Belle analysis [4], where a theoretical differential decay rate is calculated in ten bins for each of the observables and then forward folded with the detector response. A partial integration of the full four-dimensional differential decay rate,

$$\begin{aligned} \frac{d\Gamma(B^0 \rightarrow D^{*-}\ell^+\nu_\ell)}{dw d\cos\theta_\ell d\cos\theta_v d\chi} &= \frac{\eta_{EW}^2 3m_{B^0}m_{D^{*\pm}}^2}{4(4\pi)^4} G_F^2 |V_{cb}|^2 \sqrt{w^2 - 1} (1 - 2wr + r^2) \\ &\times \{ (1 - \cos\theta_\ell)^2 \sin^2\theta_v H_+^2 + (1 + \cos\theta_\ell)^2 \sin^2\theta_v H_-^2 + 4\sin^2\theta_\ell \cos^2\theta_v H_0^2 - 2\sin^2\theta_\ell \sin^2\theta_v \cos 2\chi H_+ H_- \\ &- 4\sin\theta_\ell (1 - \cos\theta_\ell) \sin\theta_v \cos\theta_v \cos\chi H_+ H_0 + 4\sin\theta_\ell (1 + \cos\theta_\ell) \sin\theta_v \cos\theta_v \cos\chi H_- H_0 \}, \end{aligned} \quad (5)$$

is performed to obtain an expression for the decay width in terms of the three helicity amplitudes associated with this decay, $(H_\pm, H_0)$, as a function of a set of model parameters, $\mathbf{x}$. The helicity amplitudes are defined in the following section, G_F is the Fermi constant, and η_{EW} is an electro-weak correction for the semileptonic decay [11]. These calculations are then integrated over a fixed bin width for each observable to obtain the vector for the unfolded

integrated yield, $N^{\text{int}}(\mathbf{x})$, before a detector response matrix R is applied in combination with the efficiency for reconstructing an event ϵ , to obtain an expected yield $N^{\text{exp}}(\mathbf{x})$ in each bin i :

$$N_i^{\text{exp}}(\mathbf{x}) = \sum_{j=1}^{40} R_{ij} \epsilon_j N_j^{\text{int}}(\mathbf{x}). \quad (6)$$

The expected values are compared to the data in each bin with a χ^2 minimization algorithm to determine the most likely set of parameters $\mathbf{x}$ that will model the data.

The fit is used to extrapolate measurements of the form factors for this decay to the zero-recoil point $w = 1$, where the model parameters are described and constrained by unquenched LQCD calculations.

III. FORM FACTOR PARAMETRIZATION

In the standard CLN parametrization, the three helicity amplitudes are defined as follows:

$$H_i(w) = m_{B^0} \frac{r'(1-r^2)(w+1)}{2\sqrt{1-2wr+r^2}} h_{A_1}(w) |\tilde{H}_i(w)|, \quad (7)$$

and

$$\begin{aligned} \tilde{H}_\pm &= \frac{\sqrt{1-2wr+r^2} \left(1 \mp \sqrt{\frac{w-1}{w+1}} R_1(w)\right)}{1-r}, \\ \tilde{H}_0 &= 1 + \frac{(w-1)(1-R_2(w))}{1-r}, \end{aligned} \quad (8)$$

where $r = m_{D^{*\pm}}/m_{B^0}$ and $r' = 2\sqrt{m_{B^0}m_{D^{*\pm}}}/(m_{B^0} + m_{D^{*\pm}})$.

The form factor h_{A_1} and the form factor ratios R_1 and R_2 are defined in terms of the free parameters ρ^2 , $R_1(1)$, and $R_2(1)$ as

$$\begin{aligned} h_{A_1}(w) &= h_{A_1}(1) [1 - 8\rho^2 z(w) + (53\rho^2 - 15)z(w)^2 \\ &\quad - (231\rho^2 - 91)z(w)^3], \\ R_1(w) &= R_1(1) - 0.12(w-1) + 0.05(w-1)^2, \\ R_2(w) &= R_2(1) + 0.11(w-1) - 0.06(w-1)^2, \end{aligned} \quad (9)$$

respectively, where $z(w) = \frac{(\sqrt{w+1}-\sqrt{2})}{(\sqrt{w+1}+\sqrt{2})}$. These are combined to obtain an overall expression for the form factors:

$$\begin{aligned} \mathcal{F}^2(w) &= h_{A_1}^2(w) \left(1 + 4 \frac{w}{w+1} \frac{1-2wr+r^2}{(1-r^2)}\right)^{-1} \\ &\quad \times \left[2 \frac{1-2wr+r^2}{(1-r)^2} \left(1 + R_1^2(w) \frac{w-1}{w+1}\right) \right. \\ &\quad \left. + \left(1 + (1-R_2(w)) \frac{w-1}{1-r}\right)^2\right]. \end{aligned} \quad (10)$$

Perfect heavy quark symmetry, in the limit of infinite quark mass, implies equality between all form factors and ratios: $\mathcal{F}(w) = R_1(w) = R_2(w) = 1$. The finite masses of quarks can then be accounted for in corrections at zero hadronic recoil, resulting in $h_{A_1}(1) = \mathcal{F}(1) = 0.906$ [12]. Therefore, there are four independent parameters in this model used to calculate the expected yield of $B^0 \rightarrow D^{*-}\ell^+\nu_\ell$ events: $|V_{cb}|$, ρ^2 , $R_1(1)$, and $R_2(1)$. The values of these parameters

are not calculated but are instead extracted from fits to experimental data. We note that isolating the value of $|V_{cb}|$ from fits depends on knowing the scale factor $\mathcal{F}(1)$ to a high degree of certainty. As this number is subject to change, we present all results for $|V_{cb}|$ in the form $\mathcal{F}(1)\eta_{\text{EW}}|V_{cb}|$, except where cancellation occurs in ratios.

In the BGL parametrization, the helicity amplitudes are defined as

$$\begin{aligned} H_0(w) &= \mathcal{F}_1(w)/\sqrt{q^2}, \\ H_\pm(w) &= f(w) \mp m_{B^0}m_{D^{*\pm}}\sqrt{w^2-1}g(w), \end{aligned} \quad (11)$$

where three power series

$$\begin{aligned} f(z) &= \frac{1}{P_{1+}(z)\phi_f(z)} \sum_{n=0}^{\infty} a_n^f z^n, \\ \mathcal{F}_1(z) &= \frac{1}{P_{1+}(z)\phi_{\mathcal{F}_1}(z)} \sum_{n=0}^{\infty} a_n^{\mathcal{F}_1} z^n, \\ g(z) &= \frac{1}{P_{1-}(z)\phi_g(z)} \sum_{n=0}^{\infty} a_n^g z^n \end{aligned} \quad (12)$$

are related to the CLN form factors via

$$\begin{aligned} h_{A_1}(w) &= \frac{f(w)}{\sqrt{m_{B^0}m_{D^{*\pm}}}(1+w)}, \\ h_V(w) &= g(w)\sqrt{m_{B^0}m_{D^{*\pm}}}, \\ R_1(w) &= (w+1)m_{B^0}m_{D^{*\pm}} \frac{g(w)}{f(w)}, \\ R_2(w) &= \frac{w-r}{w-1} - \frac{\mathcal{F}_1(w)}{m_{B^0}(w-1)f(w)}. \end{aligned} \quad (13)$$

In these equations, the Blaschke factors $P_{1\pm}$ are given by

$$P_{1\pm}(z) = \prod_{p=1}^n \frac{z - z_{\pm p}}{1 - z z_{\pm p}}, \quad (14)$$

where $z_{\pm p}$ is defined as

$$z_{\pm p} = \frac{\sqrt{t_+ - m_{\pm p}^2} - \sqrt{t_+ - t_-}}{\sqrt{t_+ - m_{\pm p}^2} + \sqrt{t_+ - t_-}}. \quad (15)$$

Here, $t_\pm = (m_B \pm m_{D^*})^2$ and $m_{\pm p}$ denotes the p th mass of the n $B_c^{*-} 1^\pm$ resonances available (see Table I). The functions $\phi_i(z)$ are outer functions related to these Blaschke factors [4]. We have adopted the notation from Ref. [9], where $(n_f, n_g, n_{\mathcal{F}_1})$ refers to the highest power in each of these series that has not been fixed to zero. The 2019 Belle analysis used the BGL(1,0,2) configuration, with five free parameters. This is due to instability (i.e., either a lack of convergence in fits using MINUIT [13],

TABLE I. The full set of inputs used in this study. Both the CLN and the BGL fits use the common input, in addition to the data yields and detector responses published in Ref. [4]. The uncertainties listed are incorporated into the global systematic covariance matrix.

Common input	
η_{EW}	1.0066
$h_{A_1}(1) = \mathcal{F}(1)$	0.904 ± 0.012
m_{B^0}	$5.27963 \pm 0.00015 \text{ GeV}/c^2$
$m_{D^{*\pm}}$	$2.01026 \pm 0.00005 \text{ GeV}/c^2$
τ_{B^0}	$(1.520 \pm 0.004) \times 10^{-12} \text{ s}$
$\mathcal{B}(D^{*+} \rightarrow D^0 \pi^+)$	0.677 ± 0.005
$\mathcal{B}(D^0 \rightarrow K^- \pi^+)$	0.0391 ± 0.0003
G_F	$1.16637 \times 10^{-5} \text{ GeV}^{-2} (\hbar c)^3$
$2n_{B\bar{B}f_{00}} \text{ (Belle)}$	$(750 \pm 11) \times 10^6$
BGL input	
$B_c^* 1^+ \text{ masses}$	$6.730 \text{ GeV}/c^2$
	$6.736 \text{ GeV}/c^2$
	$7.135 \text{ GeV}/c^2$
	$7.142 \text{ GeV}/c^2$
	$6.337 \text{ GeV}/c^2$
$B_c^* 1^- \text{ masses}$	$6.899 \text{ GeV}/c^2$
	$7.012 \text{ GeV}/c^2$
	$7.280 \text{ GeV}/c^2$
	2.6
n_l	2.6
$\chi^T(+u)$	$5.28 \times 10^{-4} (\text{GeV}/c^2)^{-2}$
$\chi^T(-u)$	$3.07 \times 10^{-4} (\text{GeV}/c^2)^{-2}$

parameters being returned as values within 1σ of their bounds, or from poorly defined or nonunique χ^2 minima) when more parameters were included. Through the inclusion of lattice QCD inputs, we will explore higher-order expansions in this study and test for stability in $|V_{cb}|$ as the number of free parameters is increased.

Unitarity constraints on the series coefficients require [3]

$$\sum_{n=0}^{\infty} (a_n^g)^2 < 1,$$

$$\sum_{n=0}^{\infty} [(a_n^f)^2 + (a_n^{\mathcal{F}^1})^2] < 1, \quad (16)$$

which have been enforced through a hard cutoff in the χ^2 minimization. We note that, by redefining these coefficients as $\tilde{a} = \eta_{EW}|V_{cb}|a$, we may extract a value for $|V_{cb}|$ using

$$\mathcal{F}(1)\eta_{EW}|V_{cb}| = \frac{1}{2m_{B^0}m_{D^{*\pm}}} \frac{|\tilde{a}_0^f|}{P_f(0)\phi_f(0)}. \quad (17)$$

A list of the inputs used in this analysis is given in Table I, with values chosen to remain consistent with Ref. [4], except for $\mathcal{B}(D^0 \rightarrow K^- \pi^+)$, which uses the average result

from the 2020 Particle Data Group (PDG) [1]. There have been recent changes in this value, which has an impact on the normalization of measurements in this study and, therefore, directly affects the obtained values for $|V_{cb}|$.

IV. THE CHOLSKY DECOMPOSITION METHOD

A common method of measuring free parameters is via a χ^2 fit, maximizing the likelihood of observing an obtained binned dataset. This is achieved by an algorithm that iterates through different values of a set of parameters, $\mathbf{x}$, to minimize a χ^2 variable, defined typically as

$$\chi^2 = \sum_{i,j} (N_i^{\text{obs}} - N_i^{\text{exp}}) \mathcal{C}_{ij}^{-1} (N_j^{\text{obs}} - N_j^{\text{exp}}), \quad (18)$$

where N_i^{obs} is the number of events observed in bin i of the data sample, N_i^{exp} is the number of events expected in bin i , determined from theory using $\mathbf{x}$, and $\mathcal{C}^{-1}$ is the inverse of the covariance matrix. The correlations between observed values are related to the covariance matrix by

$$\rho(i, j) = \frac{\mathcal{C}_{ij}}{\sigma_i \sigma_j}, \quad (19)$$

where σ_i is the standard deviation of the i th measurement [i.e., $\sigma_i^2 = \text{var}(i) = \mathcal{C}_{ii}$].

In evaluating χ^2 , the diagonal elements ($i = j$) must add a positive value, while those off the diagonal may add a negative value. Data samples with large off-diagonal correlations can, therefore, have a lower χ^2 than an identical data sample with correlations closer to zero. This can place a minimum χ^2 at parameter values away from those which provide the nominal fit (i.e., the expected events from the fit will appear biased away from the observed data). Bias can be introduced by incorrectly applying multiplicative systematic uncertainties to data yields rather than the theoretical expectation [14], known as the D'Agostini effect [15]. As in the original Belle analysis, this effect is correctly treated here.

To determine the uncertainty of a parameter in cases where correlations are high, we use a toy Monte Carlo (MC) method incorporating the Cholesky decomposition [16] to avoid directly including systematic uncertainties from normalization factors into the χ^2 . A positive-definite covariance matrix $\mathcal{C}$ undergoes a Cholesky decomposition such that

$$\mathcal{C} = LL^T, \quad (20)$$

where L is a lower triangular matrix. A vector u is created where each element is a random number taken from a Gaussian distribution with a mean of 0 and variance 1. The vector for the observed data is then fluctuated by

$$N'_{\text{obs}} = N_{\text{obs}} + Lu, \quad (21)$$

where Lu maintains the desired covariance properties [17]. The data are fluctuated many times by resampling the vector u , and the fit procedure is repeated to find a distribution of parameter values determined by the χ^2 minimization algorithm. This process represents the randomness in measurements of the data that are affected by systematic or statistical uncertainties and correlations incorporated in the covariance matrix. A Gaussian function can then be fit to these parameter distributions, and the mean and width can be taken to be the nominal value and uncertainty of each parameter, respectively. Assuming that all fits converged without reaching boundary conditions, the mean obtained from the fit without the toy MC method will be equal to the mean obtained from the Gaussian fit. Likewise, the standard deviation calculated from the distribution of results will be equal to the width obtained from the Gaussian fit and it is through this latter method that central values and uncertainties are calculated in this analysis.

A. Using the Cholesky decomposition method

In the Belle paper [4], the systematic uncertainties were determined through a toy MC approach and were published as a systematic uncertainty correlation matrix. Although the statistical correlations between the bins of the different observables were small, the systematic correlations were close to 1 almost everywhere, as they were dominated by scale errors. A fit using the data provided obtains the same values as Ref. [4] when only the statistical covariance matrix is used in the χ^2 function, which we will call the nominal fit. However, if systematic uncertainty is introduced into the covariance matrix in a “naive” way, where

$$\mathcal{C}_{\text{total}} = \mathcal{C}_{\text{stat}} + \mathcal{C}_{\text{sys}}, \quad (22)$$

the minimizing algorithm returns a lower χ^2 value but with a clear bias in the expected yield and fit parameters. These results are discussed in the Appendix.

To perform fits using the data provided in the Belle paper, we generate toy MC samples using the Cholesky decomposition of the systematic covariance matrix. To maintain consistency between statistical and systematic uncertainty calculations, this method was also applied to the statistical covariance matrix. Each method contained 10^4 variations of N'_{obs} , which was used to determine a set of fit parameters by minimizing the χ^2 from Eq. (18), where $\mathcal{C}$ was the statistical covariance matrix in both samples. We note that, due to limits in numerical precision of the data available, the systematic covariance matrix requires a slight correction to maintain positive definiteness; this is done by multiplying the elements along the diagonal by a factor of $(1 + 10^{-6})$.

The fit parameter distributions obtained from running from both methods are found to be Gaussian, as expected. The mean values for the parameters agree between the nominal fit without any toy MC and both toy MC methods. The width of the parameter distributions from the toy MC method using the statistical covariance matrix also agrees with the uncertainties obtained from the nominal fit. Therefore, the resulting central value for each parameter can be taken from the nominal fit without any toy MC method, while the statistical and systematic uncertainties are taken from the standard deviations of the distributions from their respective toy MC methods.

We observe in Fig. 1 that the yields obtained from the CLN and BGL(1,0,2) parametrizations are in excellent agreement with each other and model the data well. The results for CLN and BGL(1,0,2) are given in the first columns of Tables II and III, respectively, noting that the overall sign of the BGL coefficients is arbitrary. The systematic uncertainties are largest in $\mathcal{F}(1)\eta_{\text{EW}}|V_{cb}|$, as it is associated to overall normalization, but the central values are consistent between CLN and BGL(1,0,2) within statistical uncertainty alone. The reported $B^0 \rightarrow D^{*-}\ell^+\nu_\ell$ branching fractions are found to be in agreement with the previous average [18], as well as the Belle analysis.

The systematic uncertainties reported here differ to results presented in Ref. [4]. This is attributable to using a forward-folding method where fits were applied to background-subtracted data and all correlations between uncertainties were taken to be linear. This may not consistently represent the exact nature of the correlations present in the data, which were taken into account in the original study.

V. ALTERNATIVE CLN SCENARIOS

The form factor ratios for $B^0 \rightarrow D^{*-}\ell^+\nu_\ell$ are defined as ratios of the vector and axial-vector form factors:

$$\begin{aligned} R_1(w) &= \frac{h_V(w)}{h_{A_1}(w)}, \\ R_2(w) &= \frac{h_{A_3}(w) + rh_{A_2}(w)}{h_{A_1}(w)}. \end{aligned} \quad (23)$$

In the heavy quark limit, $h_{A_1} = h_{A_3} = h_V = \xi$ and $h_{A_2} = 0$, where ξ is the Isgur-Wise function [19,20]. These form factors can be expanded in powers of $\Lambda_{\text{QCD}}/m_{c,b}$ and α_s . It is convenient to parametrize deviations from the heavy quark limit using Eq. (23), which satisfy $R_{1,2}(w) = 1 + \mathcal{O}(\Lambda_{\text{QCD}}/m_{c,b}, \alpha_s)$ in the $m_{c,b} \gg \Lambda_{\text{QCD}}$ limit.

We introduce the parametrization discussed in Ref. [21], “CLNnoR”, in which

$$\begin{aligned} R_1(w) &= R_1(1) + (w-1)R'_1(1), \\ R_2(w) &= R_2(1) + (w-1)R'_2(1), \end{aligned} \quad (24)$$

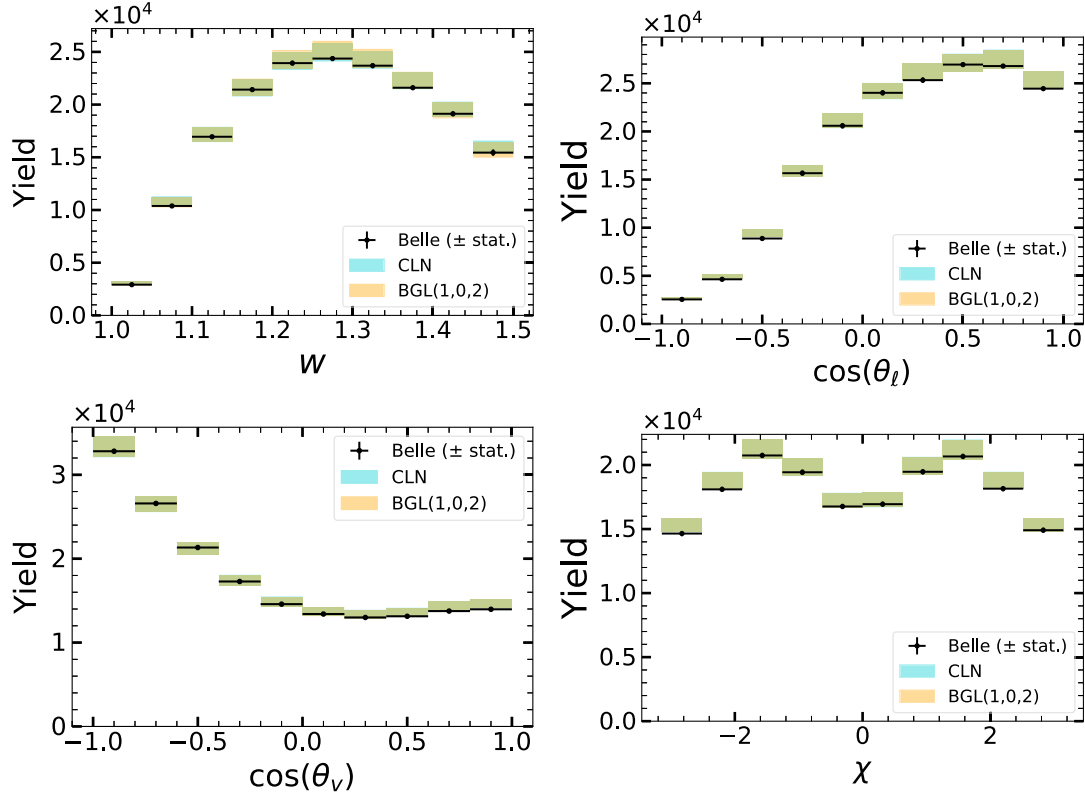


FIG. 1. The measured binned yields (data points) with statistical uncertainties for each observable of the $B^0 \rightarrow D^{*-} \ell^+ \nu_\ell$ decay overlaid with both the CLN (cyan) and BGL(1,0,2) (orange) parametrization fit results. The statistical and systematic uncertainties in the fits are determined via the toy MC method with the Cholesky decomposition and added in quadrature for these graphs. As the analysis uses a forward-folding approach, the yields predicted by the fit results are also binned. The results from the CLN and BGL parametrizations are in agreement with each other and with the data.

and fit $R'_1(1)$ and $R'_2(1)$ as additional floating parameters. CLNnoR is a simple modification of the CLN parametrization that removes QCD sum rule inputs and the condition $R_{1,2}(w) = 1 + \mathcal{O}(\Lambda_{\text{QCD}}/m_{c,b}, \alpha_s)$ but still relies on heavy quark symmetry and model-dependent input on subleading Isgur-Wise functions due to constraints on the cubic polynomial used to describe the form factor h_{A_1} . These heavy quark symmetry constraints can be further loosened to form the model “CLNnoHQS”, in which h_{A_1} from

Eq. (13) is parametrized by a quadratic polynomial in z , with unconstrained coefficients:

$$h_{A_1}(w) = h_{A_1}(1)[1 - 8\rho^2 z + (53c_{D^*} - 15)z^2], \quad (25)$$

where setting $c_{D^*} = \rho^2$ will return the original CLN parametrization truncated to its second order.

We performed fits to the $B^0 \rightarrow D^{*-} \ell^+ \nu_\ell$ data for the three different CLN scenarios, and the results are summarized in

TABLE II. Fitted parameters for the CLN, CLNnoR, and CLNnoHQS scenarios. The uncertainties listed are statistical and systematic, respectively. The branching ratios are obtained from the fit. No additional input from LQCD has been used.

Parameter	CLN	CLNnoR	CLNnoHQS
ρ^2	$1.09 \pm 0.04 \pm 0.05$	$0.89 \pm 0.09 \pm 0.15$	$0.93 \pm 0.38 \pm 0.39$
$R_1(1)$	$1.20 \pm 0.03 \pm 0.02$	$2.01 \pm 0.43 \pm 0.46$	$2.10 \pm 0.61 \pm 0.53$
$R_2(1)$	$0.86 \pm 0.02 \pm 0.01$	$0.83 \pm 0.05 \pm 0.04$	$0.77 \pm 0.42 \pm 0.38$
$R'_1(1)$	-0.12 (fixed)	$-3.50 \pm 1.77 \pm 1.85$	$-3.86 \pm 2.50 \pm 2.15$
$R'_2(1)$	0.11 (fixed)	$0.28 \pm 0.14 \pm 0.16$	$0.50 \pm 1.45 \pm 1.32$
c_{D^*}	ρ^2 (fixed)	ρ^2 (fixed)	$1.00 \pm 1.73 \pm 1.59$
$\mathcal{F}(1)\eta_{\text{EW}} V_{cb} \times 10^3$	$35.63 \pm 0.18 \pm 0.80$	$34.29 \pm 0.67 \pm 1.33$	$34.23 \pm 0.63 \pm 1.30$
$\mathcal{B}(B^0 \rightarrow D^{*-} \ell^+ \nu_\ell)$	5.04	5.04	5.04
$\chi^2/\text{n.d.f.}$	40.8/36	34.8/34	34.7/33

TABLE III. Fitted parameter values for the BGL(1,0,2) and BGL(1,1,2) configurations. The uncertainties listed are statistical and systematic, respectively. The branching ratios are obtained from the fit, and $\mathcal{F}(1)\eta_{EW}|V_{cb}|$ is calculated from Eq. (17). Configurations with higher-order coefficients do not converge without additional input from LQCD and are, therefore, not shown.

Parameter $\times 10^3$	BGL(1,0,2)	BGL(1,1,2)
$\tilde{a}_0^f$	$0.512 \pm 0.004 \pm 0.013$	$0.496 \pm 0.010 \pm 0.020$
$\tilde{a}_1^f$	$0.64 \pm 0.19 \pm 0.33$	$1.38 \pm 0.40 \pm 0.67$
$\tilde{a}_2^f$	0.0 (fixed)	0.0 (fixed)
$\tilde{a}_0^g$	$0.93 \pm 0.02 \pm 0.01$	$1.56 \pm 0.31 \pm 0.33$
$\tilde{a}_1^g$	0.0 (fixed)	$-22.63 \pm 11.03 \pm 11.62$
$\tilde{a}_2^g$	0.0 (fixed)	0.0 (fixed)
$\tilde{a}_1^{\mathcal{F}_1}$	$0.30 \pm 0.07 \pm 0.09$	$0.38 \pm 0.10 \pm 0.14$
$\tilde{a}_2^{\mathcal{F}_1}$	$-3.88 \pm 1.46 \pm 1.34$	$-3.75 \pm 1.53 \pm 1.59$
$\mathcal{F}(1)\eta_{EW} V_{cb} $	$35.34 \pm 0.27 \pm 0.87$	$34.23 \pm 0.66 \pm 1.40$
$\mathcal{B}(B^0 \rightarrow D^{*-}\ell^+\nu_\ell)$	5.04	5.04
$\chi^2/\text{n.d.f.}$	38.6/35	34.8/34

Table II. Large uncertainties are seen in the results for CLNnoR and CLNnoHQS. The contours for the standard CLN and BGL(1,0,2) models are shown in Fig. 2, where smooth curves indicate that the true minimum in the log-likelihood function was found, and, therefore, the variation in parameters with respect to each other around this minimum can be determined.

Figure 3 shows the form factor ratios R_1 and R_2 , the form factor h_{A_1} divided by its value at $w = 1$, and $\eta_{EW}^2|V_{cb}|^2\mathcal{F}^2$ for the three CLN fit configurations as a function of the hadronic recoil w . The combination of the statistical and systematic uncertainties in the fits are shown for each configuration, obtained from the toy MC method with the

Cholesky decomposition. We note that, in both CLN and BGL, the uncertainty in the plots of $h_{A_1}(w)$ reduces to zero at $w = 1$ and has been presented as a ratio to avoid assuming the value and uncertainty of $h_{A_1}(1)$. Including this value and its uncertainty would have no effect on the overall convergence of the fits and is, therefore, beyond the scope of this analysis.

The distribution of $\eta_{EW}^2|V_{cb}|^2\mathcal{F}^2$ shows that the CLNnoR and CLNnoHQS scenarios have large uncertainty at low hadronic recoil, with their central values diverging from the standard CLN value but remaining compatible within uncertainty. At larger values of hadronic recoil, this form factor becomes less sensitive to the fit parameters.

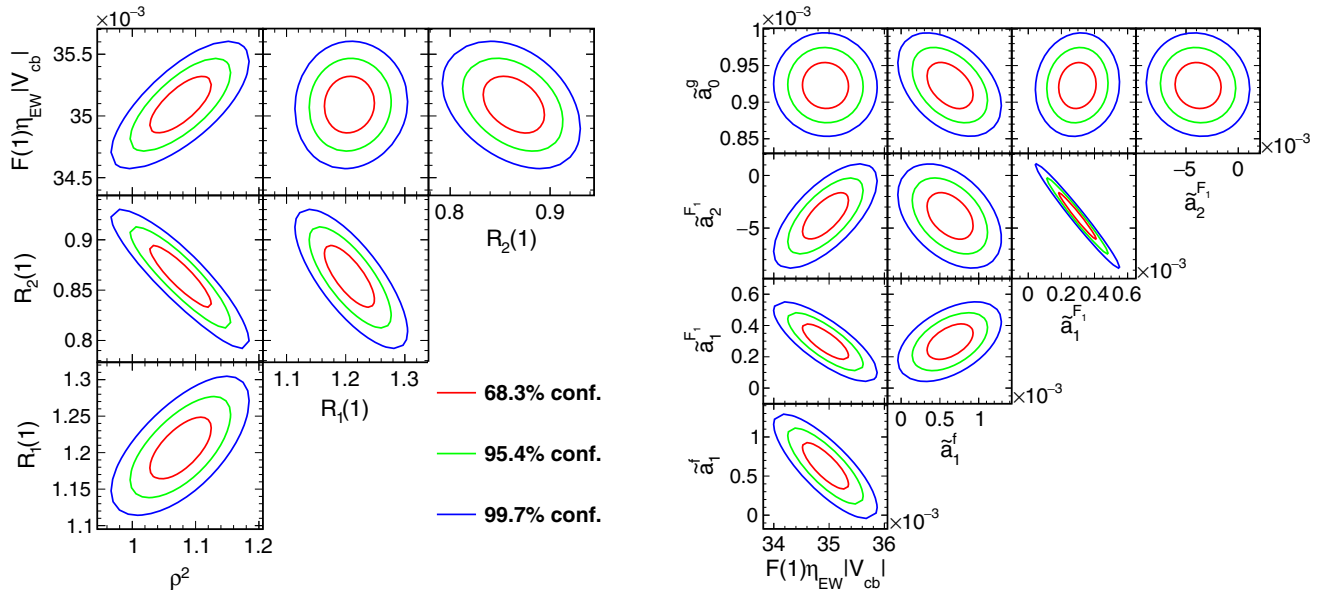


FIG. 2. 2D contours of pairs of fit parameters for CLN (left) and BGL(1,0,2) (right) at 68.3%, 95.4%, and 99.7% confidence levels. Note that, from Eq. (17), the correlations for $\tilde{a}_0^f$ and $\mathcal{F}(1)\eta_{EW}|V_{cb}|$ are equal in BGL, and, therefore, only the latter is shown.

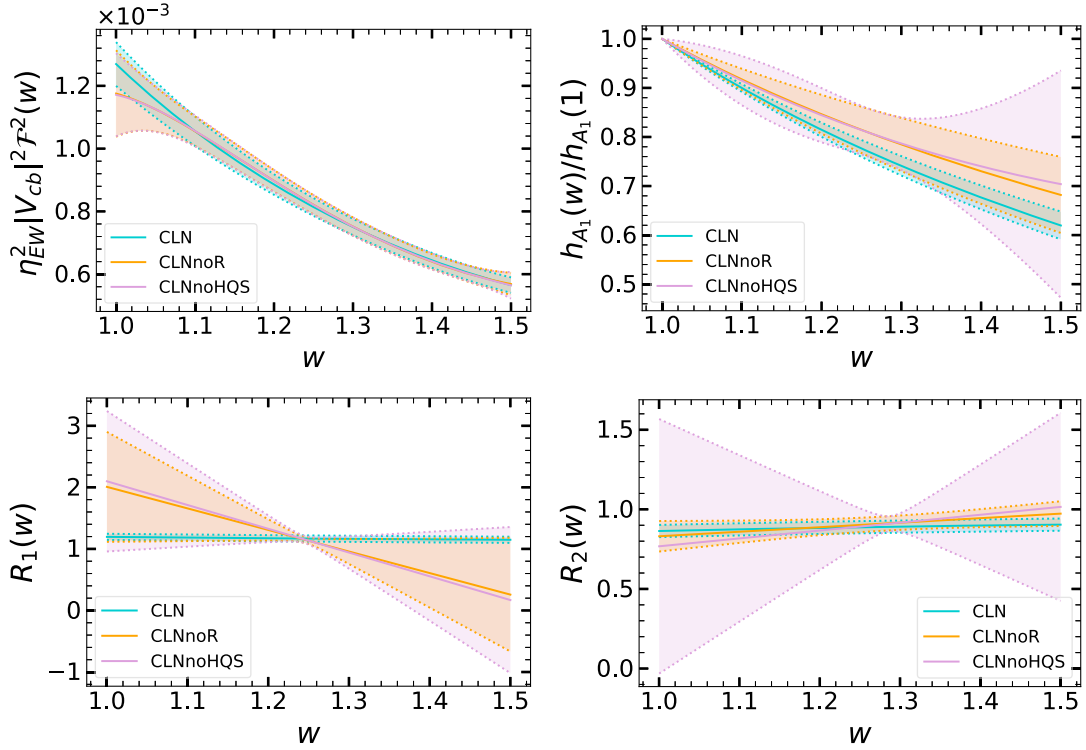


FIG. 3. $\eta_{EW}^2 |V_{cb}|^2 \mathcal{F}^2$, $h_{A_1}(w)/h_{A_1}(1)$, R_1 , and R_2 as a function of the hadronic recoil w for the CLN, CLNnoR, and CLNnoHQS scenarios. The solid lines correspond to the central values from the fit with no toy MC, and the uncertainty bands correspond to the combination of the statistical and systematic uncertainties obtained from the standard deviation of the distribution of the function values at each point in w .

The results of CLNnoR and CLNnoHQS for the form factor ratios $R_1(w)$ and $R_2(w)$ show large uncertainties near zero and maximal recoil and remain consistent with CLN. The compatibility between these two configurations and the more model-dependent CLN are an indication that the heavy quark symmetry assumed in the structure of CLN remains present when these assumptions are no longer included. This is in contrast to the study performed in Ref. [21] using a preliminary tagged Belle dataset [5]. Our results are found to be more similar to those from a study based on BGL [9], where the plots of form factor ratios are consistent with expected theoretical results for heavy quark symmetry.

VI. HIGHER-ORDER BGL

The form factors f , g , and $\mathcal{F}_1$ in the BGL parametrization are defined as a power series of $z(w)$. We can perform fits to the data for truncations at varying order to search for a plateau in fit results at increasing order and, therefore, reduce the model dependence. The results of the BGL fit with expansions (1,0,2) and (1,1,2) are given in Table III, where fits to expansions with more free parameters do not converge with the given dataset using MINUIT. From Table III, we note that increasing the power in the z expansions also has an impact on the results, including

shifting the value obtained for $|V_{cb}|$ lower by 3% but remaining compatible within systematic uncertainties. This shift can also be seen in Fig. 4, where the introduction of additional parameters to the fit leads to large deviations and large uncertainties in form factors, similar to the effects seen in CLNnoR and CLNnoHQS. These shapes are consistent with the results seen in Ref. [9], where uncertainties dominate the hadronic recoil end points. Form factors obtained by LQCD calculations [22] have a shape that appears compatible with the fits using the BGL(1,0,2) and the standard CLN configurations but also those with more free parameters due to large uncertainties. We conclude that adding more free parameters into the fits to the current dataset without further constraints introduces instability in both parametrization models. Although most results remain consistent within large uncertainties, the fitting algorithm does not provide a good χ^2 minimum for all fit parameters.

VII. ADDITIONAL DATA FROM LQCD

The use of data from LQCD calculations of form factors at nonzero recoil is an important recent development in the measurement of $|V_{cb}|$ [23]. By providing precise input near zero recoil, form factors can be much better constrained than from experimental information alone, owing to the

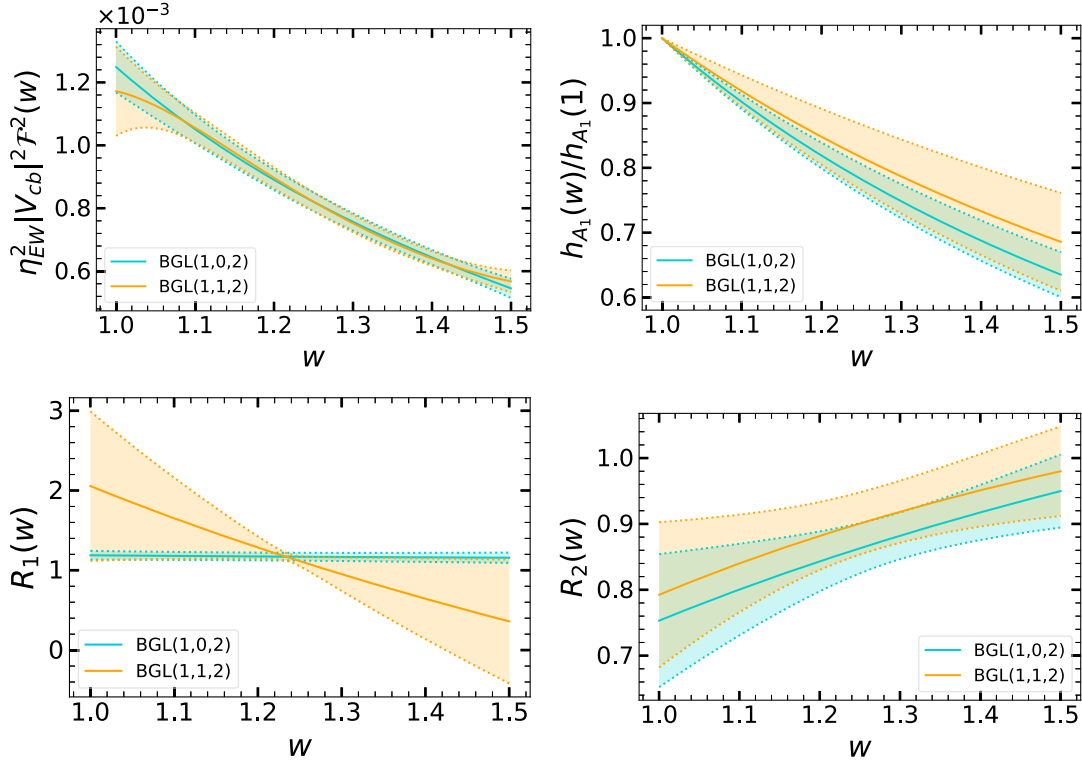


FIG. 4. The form factors and ratios $\eta_{EW}^2 |V_{cb}|^2 \mathcal{F}^2$, $h_{A_1}(w)/h_{A_1}(1)$, R_1 , and R_2 as a function of the hadronic recoil w for BGL(1,0,2) and BGL(1,1,2). The central values and uncertainty bands are calculated with the same method as Fig. 3.

presence of low-momentum (slow) pions affecting efficiencies in this region of phase space. Results have been presented in Ref. [24] (Fermilab/MILC Collaboration), which has a blinded normalization factor, and Ref. [22] (JLQCD Collaboration), which presents preliminary unblinded results.

This paper examines the impact of LQCD inputs on constraining higher-order parametrizations (BGL) and the impact of removing theoretical constraints (CLN), leading to reductions in model dependence in the evaluation of $|V_{cb}|$. Although $\mathcal{F}(1)\eta_{EW}|V_{cb}|$ values are quoted, the results are based on preliminary LQCD inputs that are subject to change. While other studies have used form factors evaluated at nonzero recoil in discussions on implications for $R(D^*)$ [25] or have included additional form factor constraints from light cone sum rules [26], this analysis features a novel approach for including data from LQCD as additional constraints for determining $B^0 \rightarrow D^{*-}\ell^+\nu_\ell$ form factors in various parametrization scenarios. The study described in Ref. [25] performed fits on non-zero-recoil LQCD points alone, as calculated by JLQCD, and found that the decay rate of $B^0 \rightarrow D^{*-}\ell^+\nu_\ell$ as a function of w was consistent with the Belle measurement [4], with and without normalizing by $|V_{cb}|$. This further highlights the compatibility of the JLQCD calculations for as additional constraints in this analysis. Four points related to h_V and h_{A_1} at different values of hadronic recoil were used:

$$\begin{aligned}
 h_{A_1}(1.04)/h_{A_1}(1) &= 0.9534 \cdot (1 \pm 1.3\%_{-1.8\%}^{+3.4\%}), \\
 h_{A_1}(1.08)/h_{A_1}(1) &= 0.9093 \cdot (1 \pm 1.3\%_{-1.8\%}^{+3.5\%}), \\
 h_V(1.04)/h_V(1) &= 0.9403 \cdot (1 \pm 2.4\%_{-3.7\%}^{+6.8\%}), \\
 h_V(1.08)/h_V(1) &= 0.9040 \cdot (1 \pm 2.4\%_{-3.7\%}^{+6.9\%}),
 \end{aligned} \tag{26}$$

with $h_{A_1}(1) = 0.906$ and $h_V(1) = 1.18$ [22]. The systematic uncertainties have been symmetrized by conservatively taking the larger of the upper and lower errors that were provided. These points were appended to the N_{obs} vector from Eq. (18), expanding it to size $40 + 4$, while the correlations between these values (obtained from preliminary estimates via private correspondence with the JLQCD group and listed in Table VI) were appended to the original statistical covariance matrix in a block-diagonal manner. The N_{exp} vector was expanded to include the calculated values for the corresponding form factors given a set of model parameters.

A. Impact of LQCD constraints on fits

The χ^2 minimization approach was applied with the Cholesky toy MC method to obtain results for various BGL and CLN parametrizations presented in Tables IV and V.

Most fit parameters are reasonably consistent between the different configurations, while the values of $\mathcal{F}(1)\eta_{EW}|V_{cb}|$ are in agreement in all scenarios. Overall, we find that all fit

TABLE IV. Fitted parameters for the CLN, CLNnoR, and CLNnoHQS scenarios using data from LQCD calculations of form factors at nonzero recoil as additional constraints. The uncertainties listed are statistical and systematic, respectively. The branching ratio is obtained from the fit.

Parameter	CLN	CLNnoR	CLNnoHQS
ρ^2	$1.10 \pm 0.04 \pm 0.05$	$1.05 \pm 0.04 \pm 0.06$	$0.80 \pm 0.18 \pm 0.23$
$R_1(1)$	$1.21 \pm 0.03 \pm 0.02$	$1.29 \pm 0.02 \pm 0.02$	$1.25 \pm 0.02 \pm 0.02$
$R_2(1)$	$0.86 \pm 0.02 \pm 0.01$	$0.80 \pm 0.05 \pm 0.04$	$0.99 \pm 0.17 \pm 0.16$
$R_1'(1)$	-0.12 (fixed)	$-0.45 \pm 0.10 \pm 0.05$	$-0.26 \pm 0.11 \pm 0.10$
$R_2'(1)$	0.11 (fixed)	$0.26 \pm 0.13 \pm 0.13$	$-0.39 \pm 0.58 \pm 0.52$
c_D^*	ρ^2 (fixed)	ρ^2 (fixed)	$-0.06 \pm 0.71 \pm 0.77$
$\mathcal{F}(1)\eta_{EW} V_{cb} \times 10^3$	$35.63 \pm 0.19 \pm 0.76$	$35.29 \pm 0.23 \pm 0.85$	$34.96 \pm 0.32 \pm 0.96$
$\mathcal{B}(B^0 \rightarrow D^{*-}\ell^+\nu_\ell)$	5.04	5.04	5.04
$\chi^2/\text{n.d.f.}$	42.3/40	38.8/38	37.5/37

TABLE V. Fitted parameters for the BGL(1,0,2), BGL(1,1,2), and BGL(2,2,2) configurations using data from LQCD calculations of form factors at nonzero recoil as additional constraints. The uncertainties listed are statistical and systematic, respectively. The branching ratio is obtained from the fit and $\mathcal{F}(1)\eta_{EW}|V_{cb}|$ is calculated from Eq. (17).

Parameter $\times 10^3$	BGL(1,0,2)	BGL(1,1,2)	BGL(2,2,2)
$\tilde{a}_0^f$	$0.512 \pm 0.004 \pm 0.013$	$0.511 \pm 0.004 \pm 0.013$	$0.507 \pm 0.004 \pm 0.013$
$\tilde{a}_1^f$	$0.62 \pm 0.18 \pm 0.30$	$0.67 \pm 0.17 \pm 0.30$	$1.43 \pm 0.64 \pm 0.84$
$\tilde{a}_2^f$	0.0 (fixed)	0.0	$-19.81 \pm 17.52 \pm 18.50$
$\tilde{a}_0^g$	$0.94 \pm 0.02 \pm 0.01$	$1.00 \pm 0.02 \pm 0.02$	$0.98 \pm 0.02 \pm 0.05$
$\tilde{a}_1^g$	0.0 (fixed)	$-2.35 \pm 0.61 \pm 0.66$	$0.01 \pm 0.97 \pm 1.74$
$\tilde{a}_2^g$	0.0 (fixed)	0.0 (fixed)	$-38.62 \pm 1.19 \pm 3.81$
$\tilde{\mathcal{F}}_1^{\mathcal{F}_1}$	$0.31 \pm 0.06 \pm 0.08$	$0.30 \pm 0.06 \pm 0.08$	$0.29 \pm 0.07 \pm 0.08$
$\tilde{\mathcal{F}}_2^{\mathcal{F}_1}$	$-4.01 \pm 1.13 \pm 1.01$	$-3.68 \pm 1.26 \pm 1.20$	$-3.06 \pm 1.48 \pm 1.24$
$\mathcal{F}(1)\eta_{EW} V_{cb} $	$35.32 \pm 0.24 \pm 0.87$	$35.28 \pm 0.24 \pm 0.87$	$35.02 \pm 0.29 \pm 0.88$
$\mathcal{B}(B^0 \rightarrow D^{*-}\ell^+\nu_\ell)$	5.04	5.04	5.04
$\chi^2/\text{n.d.f.}$	40.3/39	38.9/38	37.7/36

scenarios model the data well, where the CLN configuration has a p value of 0.37 and all other scenarios have similar values. We find that in the BGL(2,2,2) scenario the value for $\tilde{a}_2^g$ will often reach the boundary condition set by Eq. (16), and so its uncertainties cannot be considered Gaussian shaped. Instead, they are calculated in an asymmetric manner where the mean of the difference between each value of $\tilde{a}_2^g$ and its lower bound is taken as the lower uncertainty, and the upper uncertainty is taken as the standard deviation minus this number. The measurement and uncertainty of $\mathcal{F}(1)\eta_{EW}|V_{cb}|$ remains unaffected.

A smaller uncertainty is seen for higher-order expansions in BGL with LQCD input, compared to fits without. Figure 5 emphasizes this, where fits using the BGL parametrization with constraints from LQCD are compatible with the additional h_V data points at nonzero recoil. Therefore, we find that lattice input has brought stability to the higher-order BGL configurations. As discussed in Ref. [10], the BGL(1,0,2) configuration can be considered overconstrained with limited flexibility and, therefore, not the optimal fit configuration in BGL. Using LQCD

constraints, we find little difference between BGL configurations for the CLN-equivalent form factors and form factor ratios at low hadronic recoil, while, at larger values, the order of the power series for the three BGL form factors causes a divergence in R_1 . This is depicted in Figs. 6 and 7, which show a significant reduction in uncertainties when compared to Figs. 3 and 4. Similar behavior is seen in the CLNnoR and CLNnoHQS scenarios in R_1 .

The inclusion of LQCD constraints has not been found to introduce a bias in the normalization of the fits as the measured $\mathcal{B}(B^0 \rightarrow D^{*-}\ell^+\nu_\ell)$ values are consistent with previous measurements without LQCD, and the yields, as seen in Fig. 8, are compatible with the data.

The results clearly show that obtaining a model-independent value of $|V_{cb}|$ is possible with the use of only four additional constraints from LQCD.

B. Sensitivity to variation in LQCD inputs

The LQCD inputs used in this paper are preliminary and subject to change. We therefore explore the sensitivity of

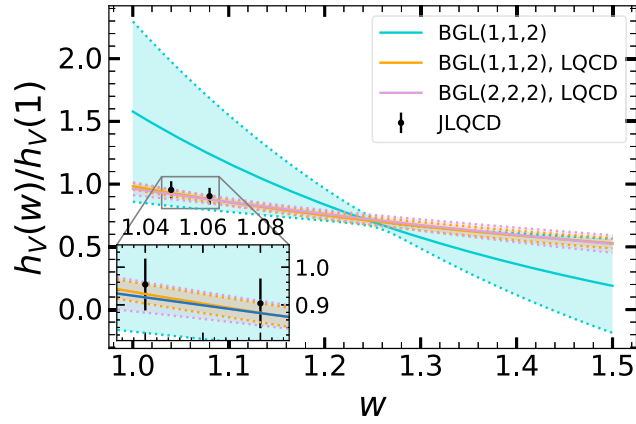


FIG. 5. The normalized form factor $h_V(w)/h_V(1)$ as a function of the hadronic recoil w for BGL(1,1,2) with (orange) and without (cyan) LQCD constraints and BGL(2,2,2) with LQCD constraints (pink). The enlarged window within the plot is used to emphasize the region around the LQCD inputs for h_V . The central values and uncertainty bands are calculated with the same method as Fig. 3.

$\mathcal{F}(1)\eta_{EW}|V_{cb}|$ to variations in these inputs. Two values were taken at $w = 1.04, 1.08$ for each of $h_{A_1}(w)$ and $h_V(w)$, as this was found to be the minimum amount of extra information required for the BGL(2,2,2) parametrization to

converge. To measure what effect these values have on the fit results, we establish a toy MC analysis using the standard CLN parametrization. The four non-zero-recoil values for this toy MC study are set to

$$\begin{aligned} h_{A_1}(1.04)/h_{A_1}(1) &= (0.95 \pm 0.05), \\ h_{A_1}(1.08)/h_{A_1}(1) &= (0.91 \pm 0.05), \\ h_V(1.04)/h_V(1) &= (0.95 \pm 0.05), \\ h_V(1.08)/h_V(1) &= (0.90 \pm 0.05). \end{aligned} \quad (27)$$

The correlations between these are given in Table VI. These values have been rounded off from Eq. (26), with inflated uncertainties to account for the inclusion of any additional systematic errors. The sensitivity to form factor values was explored in two ways; each form factor pair was varied together or each value was varied individually. The inputs were allowed to vary to cover a range of different possible slopes as follows:

$$\begin{aligned} h_{A_1}(1.04)/h_{A_1}(1) &\in [0.9, 1.0], \\ h_{A_1}(1.08)/h_{A_1}(1) &\in [0.85, 0.95], \\ h_V(1.04)/h_V(1) &\in [0.9, 1.0], \\ h_V(1.08)/h_V(1) &\in [0.85, 0.95]. \end{aligned} \quad (28)$$

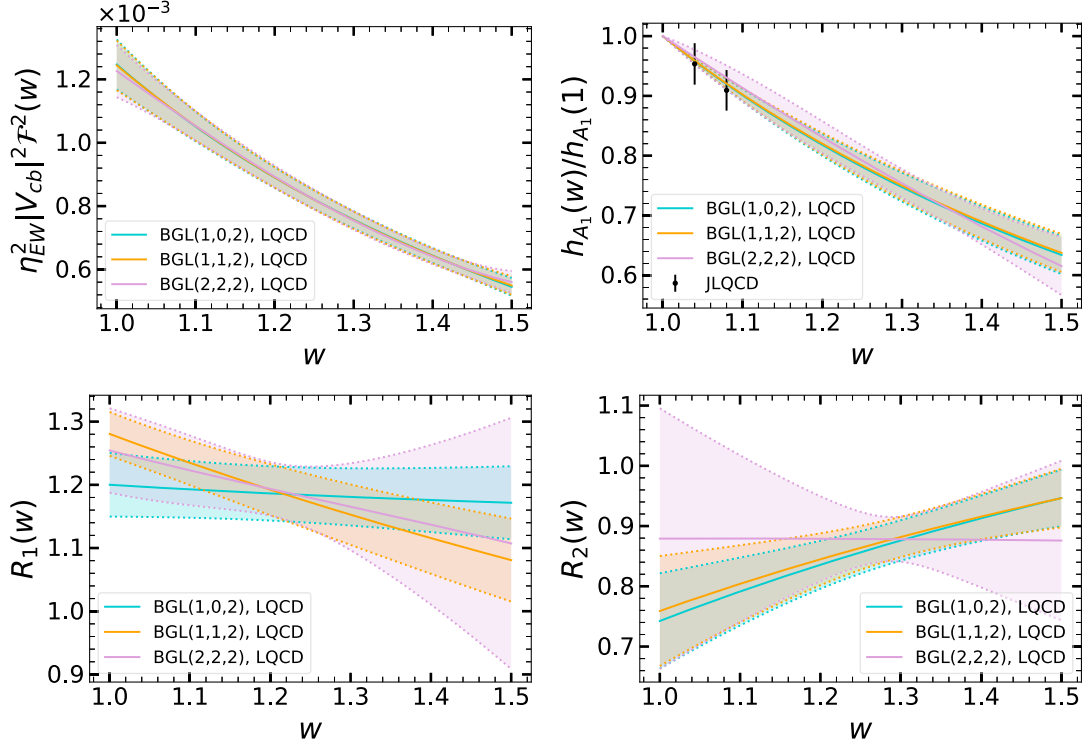


FIG. 6. The form factors and ratios $\eta_{EW}^2|V_{cb}|^2\mathcal{F}^2$, $h_{A_1}(w)/h_{A_1}(1)$, R_1 , and R_2 as a function of the hadronic recoil w for BGL(1,0,2), BGL(1,1,2), and BGL(2,2,2) using additional LQCD constraints. The LQCD input has been overlaid in the plot for $h_{A_1}(w)/h_{A_1}(1)$. The central values and uncertainty bands are calculated with the same method as Fig. 3.

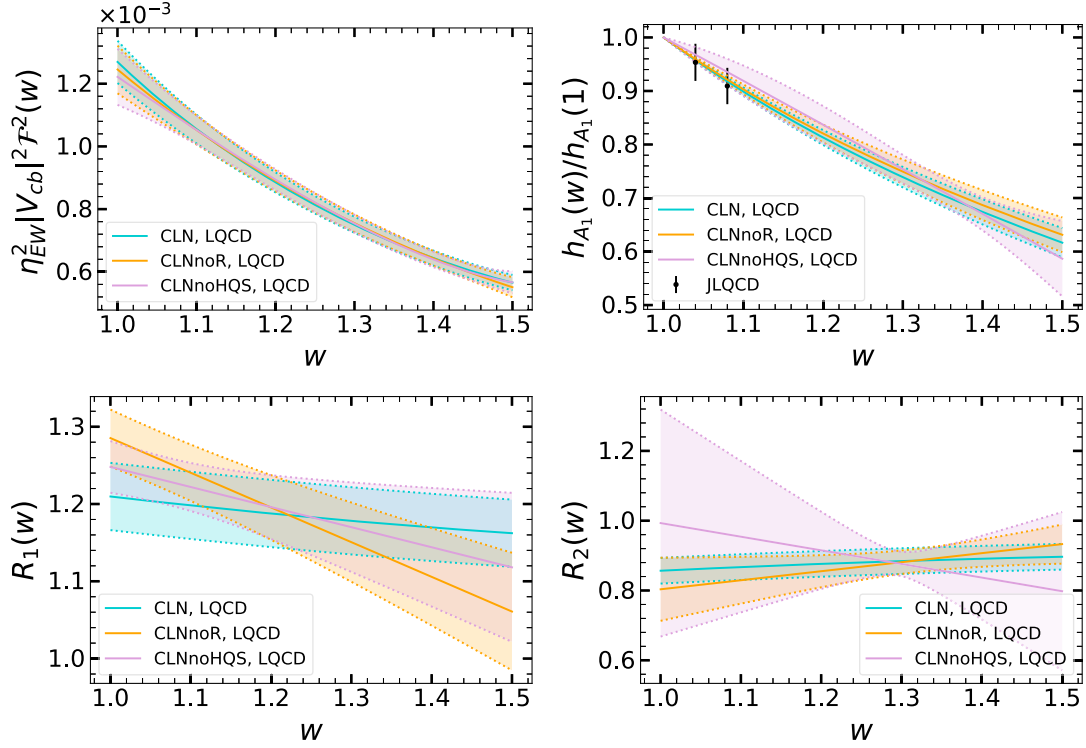


FIG. 7. The form factors and ratios $\eta_{EW}^2 |V_{cb}|^2 \mathcal{F}^2$, $h_{A_1}(w)/h_{A_1}(1)$, R_1 , and R_2 as a function of the hadronic recoil w for CLN, CLNnoR, and CLNnoHQS using additional LQCD constraints. The LQCD input has been overlaid in the plot for $h_{A_1}(w)/h_{A_1}(1)$. The central values and uncertainty bands are calculated with the same method as Fig. 3.

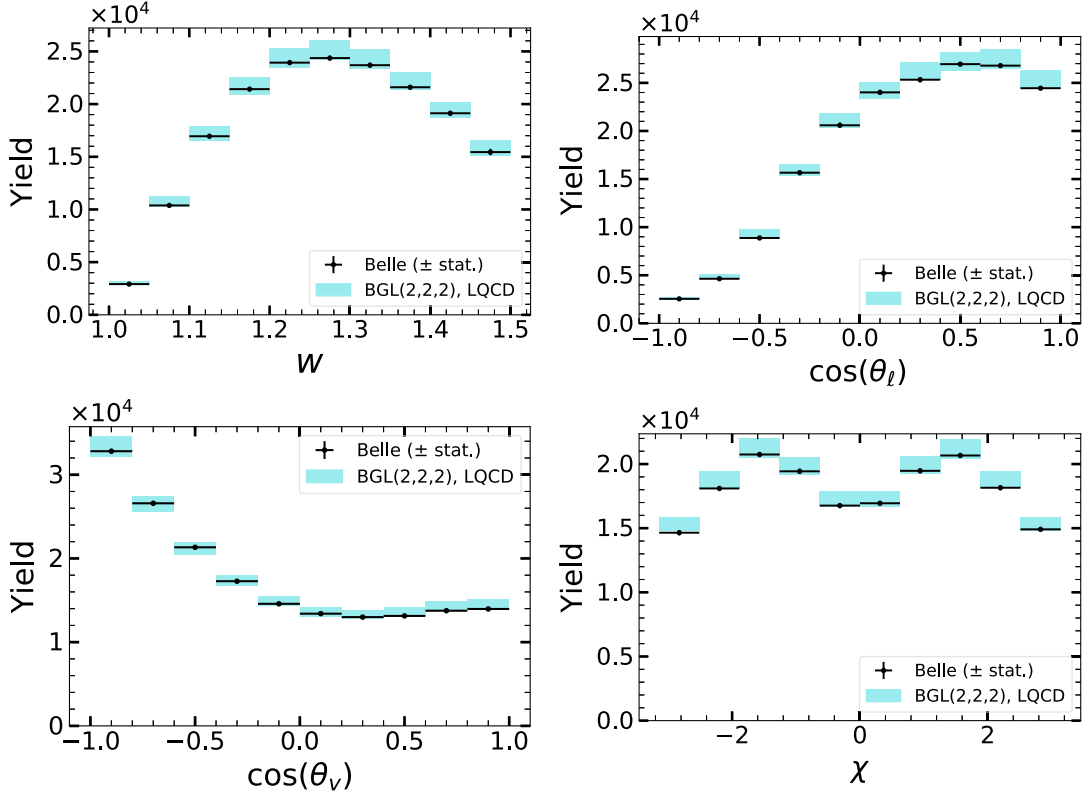


FIG. 8. The measured binned yields (data points) with statistical uncertainties for each observable of the $B^0 \rightarrow D^{*-}\ell^+\nu_\ell$ decay overlaid with the BGL(2,2,2) (cyan) parametrization fit results, using additional LQCD constraints. The statistical and systematic uncertainties in the fits are determined in the same way as Fig. 1. The results from the fit are in agreement with the data.

TABLE VI. Correlations between the form factor values at nonzero recoil, $h_X(w)/h_X(1)$, where $X = A_1, V$ and $w = 1.04, 1.08$. The entries in this table are approximated from preliminary results from JLQCD. This correlation matrix is multiplied by the uncertainties for each LQCD point and then appended to the 40×40 statistical covariance matrix in a block-diagonal manner.

	$h_{A_1}(1.04)/h_{A_1}(1)$	$h_{A_1}(1.08)/h_{A_1}(1)$	$h_V(1.04)/h_V(1)$	$h_V(1.08)/h_V(1)$
$h_{A_1}(1.04)/h_{A_1}(1)$	1.0	0.85	0.38	0.49
$h_{A_1}(1.08)/h_{A_1}(1)$	0.85	1.0	0.18	0.44
$h_V(1.04)/h_V(1)$	0.38	0.18	1.0	0.93
$h_V(1.08)/h_V(1)$	0.49	0.44	0.93	1.0

In both treatments of the variation, the range of values obtained for $|V_{cb}|$ changed by at most 0.1%, while the largest change over any other fit parameter was 2%, which is smaller than the systematic uncertainty calculated in the nominal analysis.

Each correlation was then unfixed individually and set to 100 different values spanning 0 to 1 while the others remained fixed, and then the fit was repeated. The fit parameters as a function of each correlation were also found to vary by at most 1%, except around finely tuned points where the correlation matrix has eigenvalues nearing zero, at which point the entries in $\mathcal{C}^{-1}$ diverge to infinity. From this, we conclude that any results for $|V_{cb}|$ are not highly sensitive to the actual values of the additional form

factor constraints at nonzero recoil, under the assumption that the correlation matrix remains invertible.

VIII. CONCLUSION

We have performed a study of fits to exclusive $B^0 \rightarrow D^{*-}\ell^+\nu_\ell$ measurements for the determination of $|V_{cb}|$, based on the Belle 2019 untagged measurement. We used preliminary results from the JLQCD group for form factor calculations at nonzero hadronic recoil as additional constraints and find that fits with higher-order parametrizations or less theoretical constraints reliably converge. The $\mathcal{F}(1)\eta_{EW}|V_{cb}|$ results obtained from each of the different methods described are shown in Fig. 9.

We have developed a toy MC approach making use of the covariance properties of the Cholesky decomposition to take into account scale-error-dominant systematic uncertainties to avoid bias. Fits to the CLN parametrization without the requirement of heavy quark symmetry on one or both of the form factor ratios and h_{A_1} have compatible results for the form factors and $\mathcal{F}(1)\eta_{EW}|V_{cb}|$. This demonstrates that the heavy quark symmetry assumptions used in the modeling of CLN are valid and that the results are not indicative of any breakdown in heavy quark symmetry or the use of subleading Isgur-Wise functions.

With the inclusion of JLQCD non-zero-recoil constraints, we achieve a model-independent result for $\mathcal{F}(1)\eta_{EW}|V_{cb}|$, $(35.02 \pm 0.29 \pm 0.88) \times 10^{-3}$ in BGL(2,2,2) and $(34.96 \pm 0.32 \pm 0.96) \times 10^{-3}$ in CLNnoHQS. Assuming a value of $\mathcal{F}(1)$ from Table I, we find $|V_{cb}| = (38.48 \pm 0.32 \pm 0.97 \pm 0.46) \times 10^{-3}$ and $(38.42 \pm 0.35 \pm 1.05 \pm 0.46) \times 10^{-3}$ in BGL(2,2,2) and CLNnoHQS, respectively, where the additional uncertainty is from $\mathcal{F}(1)$. These results remain in tension with the inclusive determination.

ACKNOWLEDGMENTS

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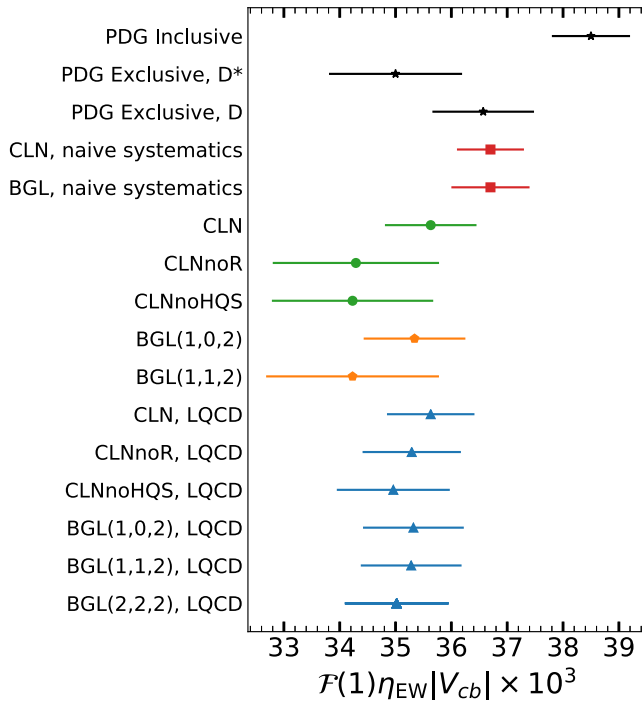


FIG. 9. Summary of the $\mathcal{F}(1)\eta_{EW}|V_{cb}|$ values from the various fit scenarios covered in this paper, where the PDG values of $|V_{cb}|$ from Sec. I have been multiplied by the relevant values listed in Table I.

APPENDIX: VERIFYING THE CHOLESKY DECOMPOSITION METHOD

It is well established that normalization uncertainties for measurements will introduce a bias in χ^2 fits [15,27]. If the systematic uncertainty from the Belle publication is added to the covariance matrix, the minimizing function provides a biased fit due to high correlations and scale-dominant uncertainties, returning a lower χ^2 for expectation values that do not agree with measurements. Figure 10 shows a demonstration of this effect. Fit values for the free parameters with and without the addition of the systematic covariance matrix are listed in Table VII and show inconsistency, particularly in the normalization. The data is modeled in a forward-folding approach, and this leads to a result that overestimates the yield and, therefore, also the $\mathcal{B}(B^0 \rightarrow D^{*-}\ell^+\nu_\ell)$ branching ratio and $\mathcal{F}(1)\eta_{EW}|V_{cb}|$.

To avoid this effect, we use a Cholesky decomposition and toy method to propagate systematic uncertainties that are dominated by scale errors. The Belle analysis was repeated with a false data sample generated from arbitrarily chosen CLN parameters:

$$\begin{aligned}\rho^2 &= 1.1, \\ R_1(1) &= 1.2, \\ R_2(1) &= 0.8, \\ \eta_{EW}|V_{cb}| &= 0.04.\end{aligned}$$

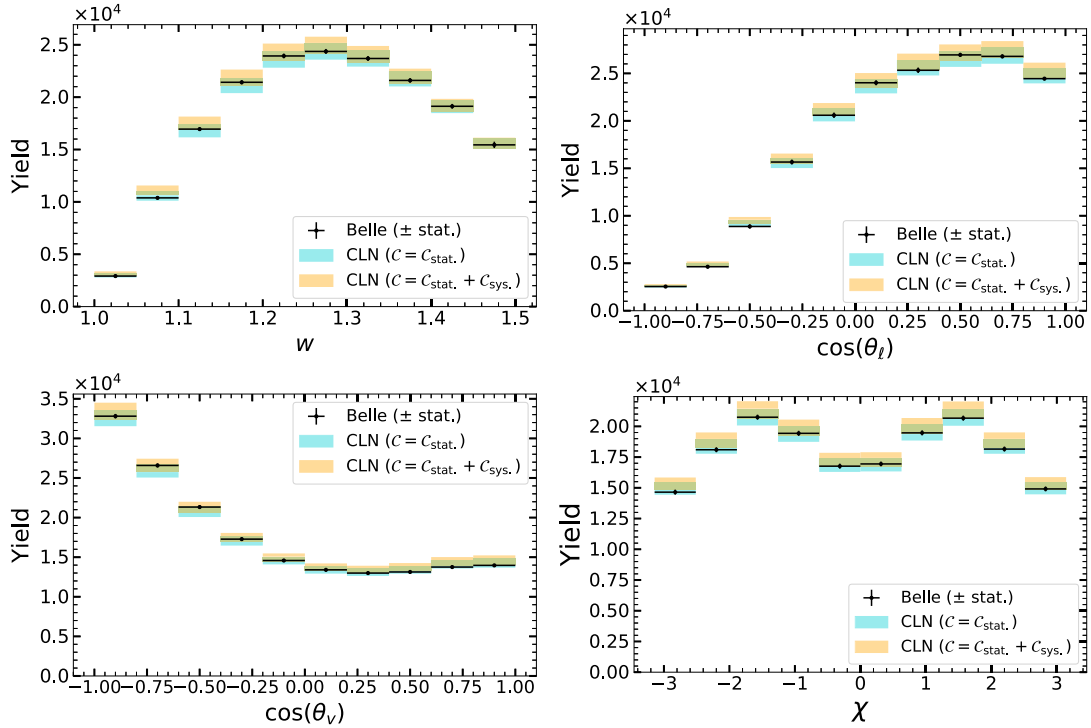


FIG. 10. The measured binned yields (data points) with statistical uncertainty for each observable of the $B^0 \rightarrow D^{*-}\ell^+\nu_\ell$ decay overlaid with CLN fit results for different covariance matrices $\mathcal{C}$, used in Eq. (18). Results are shown for the total covariance matrix being equal to the statistical covariance matrix alone (cyan) and the sum of both the statistical and systematic covariance matrices in a naive way (orange).

The statistical uncertainties in each bin were defined as the square root of the number of events in that bin, while the statistical correlations were defined by the following:

$$\rho_{\text{stat}}(i, j) = \begin{cases} 1 & i = j, \\ 0.01 & i \neq j, i, j \text{ block diagonal}, \\ 0 & \text{otherwise}, \end{cases} \quad (\text{A1})$$

where “ i, j block diagonal” refers to entries within the 10×10 submatrix for each observable along the diagonal of the full 40×40 statistical correlation matrix.

The systematic uncertainty is given by $\sigma_i = N_i^{\text{exp}} \epsilon_i$, where $\epsilon_i = 0.01$ in each bin and

$$\rho_{\text{sys}}(i, j) = \begin{cases} 1 & i = j, \\ 0.99 & \text{otherwise}. \end{cases} \quad (\text{A2})$$

The pull obtained from the fits from 10^4 iterations was measured, where the pull for the i th fit, p_i , of each parameter measurement x_i is given by

$$p_{i,x} = \frac{x_i - \hat{x}}{\sigma_x}. \quad (\text{A3})$$

Here, $\hat{x}$ is the nominal value of the parameter, defined earlier, and $\sigma_x^2 = \sum_i (x_i - \hat{x})^2 / n$ for n fits. The results are

TABLE VII. Fitted parameters in the CLN and BGL(1,0,2) configurations comparing different total covariance matrices. The results with only statistical uncertainties are obtained by setting $\mathcal{C}$ from Eq. (18) to be equal to $\mathcal{C}_{\text{stat}}$, and the results from combining statistical and systematic uncertainties in a naive way are obtained using $\mathcal{C} = \mathcal{C}_{\text{stat}} + \mathcal{C}_{\text{sys}}$. The branching ratio is obtained from the fit, and $\mathcal{F}(1)\eta_{\text{EW}}|V_{cb}|$ is calculated from Eq. (17).

Covariance	Stat	Sys + stat
CLN		
ρ^2	1.09 ± 0.04	1.16 ± 0.04
$R_1(1)$	1.20 ± 0.03	1.18 ± 0.04
$R_2(1)$	0.86 ± 0.02	0.85 ± 0.03
$\mathcal{F}(1)\eta_{\text{EW}} V_{cb} \times 10^3$	35.6 ± 0.2	36.7 ± 0.6
$\mathcal{B}(B^0 \rightarrow D^{*-}\ell^+\nu_\ell)$	5.04%	5.18%
$\chi^2/\text{n.d.f.}$	41/36	25/36
BGL(1,0,2)		
$\tilde{a}_0^f \times 10^3$	0.512 ± 0.004	0.532 ± 0.010
$\tilde{a}_1^f \times 10^3$	0.6 ± 0.2	0.2 ± 0.3
$\tilde{a}_1^{f_1} \times 10^3$	0.30 ± 0.08	0.19 ± 0.10
$\tilde{a}_2^{f_1} \times 10^3$	-3.8 ± 1.4	-2.6 ± 1.8
$\tilde{a}_0^g \times 10^3$	0.93 ± 0.02	0.93 ± 0.03
$\mathcal{F}(1)\eta_{\text{EW}} V_{cb} \times 10^3$	35.3 ± 0.3	36.7 ± 0.7
$\mathcal{B}(B^0 \rightarrow D^{*-}\ell^+\nu_\ell)$	5.04%	5.18%
$\chi^2/\text{n.d.f.}$	39/35	24/35

found to have no bias, with the pull for each fit parameter being consistent with zero. Therefore, we consider this method of producing a toy MC sample via the Cholesky

decomposition as suitable for determining the nominal values and uncertainties of parameters in the case of dominant scale errors with high correlations.

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