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# Bit Modulated Frequency Permutation Waveforms for Joint Communications and Radar

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***Abstract*—In this paper, we propose the selection of a subset of waveforms based on the random stepped frequency permutation waveform to support joint radar and communication. More specifically, we solve two critical implementation problems arising from the subset selection which is motivated by the fundamental bit level operation requirements of communication systems. Noting that the practicality of any selected subset depends on the feasibility of efficient implementation, we focus on finding a specific subset for which we can design an efficient mapping process and a receiver implementation. More specifically, we propose an efficient process to map information bits to waveforms based on the factorial number system. An efficient optimal communication receiver that utilizes the Hungarian algorithm is also designed. For additive white Gaussian noise and correlated Rician fading channels, the bit error rate is analyzed in accordance with the optimum maximum likelihood detection.**

***Index Terms*—joint radar and communication, bit error rate, maximum likelihood.**

## I. Introduction

Due to the deployment of millimeter wave (mmWave) frequencies, massive multiple-input multiple-output (MIMO) and the advancements in signal processing techniques and electronics, joint radar and communication (JRC) systems have attracted a lot of attention. Compared to traditional systems, JRC systems have great advantages (eg: improved energy and spectrum efficiency, reduced cost and size) in many applications such as next generation automobile, drone surveillance, and defense applications with integrated battlefields [1].

In [2], the orthogonal frequency division multiplexing (OFDM) waveform has been considered for JRC due to its ability to maintain a good communication data rate while maintaining low sidelobe levels and high Doppler tolerance. Recently, it is shown that the novel orthogonal time frequency space (OTFS) waveform achieves a higher data rate while obtaining the same radar performance when compared to the OFDM waveform [3]. Different radar waveforms are generated in [4] where the multiplexing of the differing bit intervals across several users is used in frequency, time or code. In [5], the preamble of the data frame is utilized to develop a waveform that is virtual within the mmWave band.



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Taking a different approach, some work has incorporated communication symbols by exploiting the waveform diversity in established radar waveforms [6]–[9]. In [7], spatial index modulation is used to propose a spatial modulation based communication-radar (SpaCoR) system where the information bits are used to select the specific antennas. Taking a similar approach, the modulation of information using the selection of the waveform is proposed in [8], [9] by designing a novel waveform on the basis of permutation coding and the stepped frequency radar waveform. This is motivated by the interference reduction capability of the pseudo-random stepped frequency radar waveform in the presence of a large number of radars operating in close proximity. Although, this waveform design is proposed based on the stepped frequency radar waveform to mitigate the radar interference, it is worth noting that the usage of permutation coding in the context of pure communication has also been considered with multilevel frequency shift keying (MFSK) [10]–[12]. Therefore, the random stepped frequency permutation waveform proposed in [9] achieves a good compromise between the radar performance and the communication performance. This work is later extended in [13] to achieve a higher data rate by combining phase-shift-keying (PSK) based random phase modulation and in [14] to achieve lower communication error rates and higher radar performance by selecting a subset of waveforms.

In communication systems, information bits are encoded into the amplitude, frequency or phase of the transmitted signal via modulation schemes such as pulse amplitude modulation (PAM), PSK and quadrature amplitude modulation (QAM). When using $M$-PAM, $M$-PSK or $M$-QAM, $n=\log_2(M)$ bits can be encoded in one symbol such that $M = 2^n$ [15]. In contrast to traditional data modulation, in [9], the information bits are modulated into the new random stepped frequency permutation waveform by mapping the data symbols to the frequency sequence of the transmitted waveform. Under that method, we need $2^n$ different waveforms or permutations to encode $n$ information bits. Since there can be $M!$ potential permutations generated from $M$ frequencies, a selection of a subset is important when $\log_2(M!)$ is not an integer. However, in [9], [13], [14], this integer requirement is not considered. Motivated by the above limitation, we consider the problem of selecting $2^n$ permutations from $M!$ permutations to accommodate bit level modulation in communication networks.

The efficient mapping process and the receiver implementation proposed in [9] are only feasible when the entire set of waveforms is used to modulate communication data. Further, subset selection usually results in a complex information mapping process and receiver implementation, thus, significantly reducing the practicality of the random stepped frequency permutation waveform [14]. However, a selection of a subset of waveforms is necessary when $\log_2(M!)$ is not an integer. Noting that many subsets are infeasible in a practical setting, we focus on finding a specific subset of $2^n$ permutations where we can design an efficient mapping process and a receiver implementation. Thus, we make the following contributions,

- In contrast to [14], we solve critical implementation problems when the subset selection problem is motivated by the fundamental bit level operation requirements of communication systems. More specifically, we propose the selection of the first $2^n$ permutations in the lexicographic order as a method to ensure that the implementation of the system remains practically feasible. Further, we show that the proposed subset benefits from utilizing the factorial number system and Lehmer code based efficient mapping.
- We explore the special properties of the proposed subset and propose a practically feasible novel optimal receiver implementation. More specifically, we show that our proposed receiver implementation has a significant increase in efficiency compared to the integer programming (IP) based optimal receiver proposed in [14].
- The available analytical work on the communication performance of the random stepped frequency permutation waveform is limited to the block error rate. Taking a step further, in this work, we analyze the bit level performance by deriving the bit error rate (BER) based on the optimal maximum likelihood detector.

*Notation:* Vectors and matrices are represented by lowercase and uppercase bold letters, respectively. The subscripts denote the elements in vectors and matrices. The superscripts $(.)!$, $||.||$, $(.)^*$ and $(.)^H$ denote the factorial operator, absolute value, complex conjugate and Hermitian operator, respectively. $\mathcal{N}(0,\sigma^2)$ and $\mathcal{CN}(0,\sigma^2)$ denote the Gaussian and complex Gaussian distributions with mean zero and variance $\sigma^2$. $\mathbb{E}[.]$, Pr[.] and $|.|$, denote the expectation operator, probability and cardinality, respectively and $\mathbb{R}$ defines the set of real numbers.

## II. Proposed JRC Waveform

In this work, we consider the random stepped frequency permutation waveform in [9]. For completeness, we briefly present some important details of this waveform as follows. A random stepped frequency permutation waveform composed of $M$ evenly spaced frequencies given by $f_0,\dots,f_{M-1}$ such that any frequency is utilized just once. A given waveform is generated over $MT$ seconds using $M$ pulses. The orthogonality between frequencies is ensured by setting the frequency separation to $\Delta f = q/T$ such that $q$ is an integer. For the $i$-th waveform, the complex baseband signal can be written as

$$s_i(t) = \sqrt{\frac{E}{MT}}\sum_{m=0}^{M-1} p(t-mT)\exp\left(2\pi f_m^{(i)}(t-mT)\right), \quad (1)$$

where $p(t)$ is the unit pulse function which is zero outside $0 \le t \le T$ with $f_m^{(i)}$ denoting frequency of the $m$-th pulse in the $i$-th waveform and $E$ denoting the energy of the signal satisfying $\int_0^{MT}||s_i(t)||^2 dt = E$. As commonly assumed in stepped frequency radar waveforms, we assume that the speed of the synthesizer at the transmitter side is sufficiently fast such that the impact of frequency settling time on data rate and radar performance is negligible [16]. Further, we have avoided the abrupt switching from one frequency to another by assuming a carrier whose frequency is continuously changing [17]. Considering all the permutations that can be constructed from $M$ frequencies, $M!$ waveforms are available and the selection of the waveform is used to modulate the information. In [14], the authors develop three interesting subsets in a systematic approach to improve the communication and radar performances. In these existing works, it is assumed that $\log_2(L)$ bits are modulated into each waveform, where $L$ is the total number of waveforms selected for the modulation purpose. We note that the communication data rate is proportional to the number of selected waveforms, $L$. However, in the implementation of communication networks, the number of bits should be an integer, thus, restricting the number of selected waveforms to a power of two. Therefore, a selection of a subset of waveforms is required irrespective of the potential reduction in the communication data rate. We further note that by selecting the largest possible $L \le M!$ such that $\log_2(L)$ is an integer, we can ensure the achievability of the highest feasible data rate under the random stepped frequency permutation waveform.

### A. Subset selection

Taking a different approach, we examine the process of selecting a subset that accommodates the bit modulation requirement in communication. The set of waveforms acquired based on the chosen subset of permutations is defined as $\mathcal{S}$ such that $|\mathcal{S}| = 2^n$, where $|\mathcal{S}|$ denotes the cardinality of $\mathcal{S}$ and the integer $n$ is the number of bits modulated into the waveforms designed with $M$ frequencies. We note that when a subset of waveforms is selected in general, a massive look-up table is necessary to perform the mapping between the information symbols and the corresponding waveforms. In addition, the optimal communication receiver takes the form of an IP optimization problem with a worst case complexity that is exponential in $M$ [14]. This creates a major implementation problem for any JRC system that considers the random stepped frequency permutation waveform. Therefore, we ask the question whether there exists any specific subset for which we can design an efficient mapping process and a low-complexity optimal receiver implementation such that $|\mathcal{S}| = 2^n$. In this paper, we propose the selection of the first $2^n$ permutations in the lexicographic order. This is motivated by the ability of this specific subset to utilize the factorial number system and Lehmer code based efficient mapping at the communication transmitter. The proposed subset also allows the design of a more efficient optimal receiver. In the following, we first outline the efficient mapping process and then provide the proposed optimal receiver implementation in Section III.

### B. Mapping between information symbols and waveforms

At the transmitter, the factorial number system is used to first compute the natural number related to the information symbol to be transmitted. For $n$ information bits, any computed natural number $i$ belongs to $\{0, 1, ..., 2^n - 1\}$. Therefore, following the lexicographic permutation order produced by the set $\{1, 2, ..., M\}$, we select the $i$-th permutation denoted by $\chi_i$ and transmit the corresponding waveform $s_i(t)$. At the communication receiver, we follow the opposite process. First, the closest permutation to the received signal is detected from the selected permutation subset, $[\chi_0, \chi_1, \ldots, \chi_{2^n-1}]$. If $\chi_j$ is detected, the factorial number system can be used to decode the received information symbol by taking $j$ as the natural number of the received information symbol. In contrast to using a large look-up table, this factorial number system and Lehmer code based mapping can be implemented with a linear complexity in $M$ [9].

### C. Radar Sensing

From the radar sensing perspective, the operation of the proposed bit modulated frequency permutation waveform is similar to a traditional radar system, where the radar receiver estimates the measures of interest using the received echos of the transmitted waveform [18]. We consider a traditional mono-static full-duplex radar system where the radar transmitter and the radar receiver are co-located but with sufficient isolation to prevent the radar detector saturation [3]. Therefore, the radar receiver knows the randomized transmitted waveform and as a result, the radar operation would not be affected by the random nature of the modulated data.

We note that the analysis of the radar performance for the proposed subset would be similar to that of a general permutation subset under the random stepped frequency permutation waveform. Thus, in the current paper, we have limited our scope to selecting a permutation subset to accommodate the bit modulation requirement in data communication as well as designing an efficient mapping process and an optimal receiver for the selected subset. The radar performance of the random stepped frequency permutation waveform is discussed in detail in [9], [14]. Using the mainlobe properties of the ambiguity function, in [9], it is shown that the local accuracy of radar sensing does not depend on the subset selection. Taking a step further, using the peak-to-sidelobe ratio (PSLR), in [14], it is shown that subset selection based on the communication performance has a negligible affect on the radar performance compared to the universal set. The Cramer-Rao lower bounds (CRLBs) on the delay and velocity estimation error are also computed in [9] to illustrate the effect of different parameters on radar range and Doppler resolution. Further, it is shown in [9] that the overall structure of the ambiguity function, resulting from the random stepped frequency permutation waveform, is capable of achieving good radar performance in the presence of clutter due to the averaging effect on sidelobes caused by the randomness in communication data. Whilst not included here due to page limitations, these observations can be easily extended to the current model.

## III. Optimal Receiver Implementation

Next, we concentrate on the implementation of the communication receiver for the proposed system. Similar to [14], we consider a JRC system model consisting of one transmitter, one communication receiver with $N$ antennas and a geographically separated single moving target. Thus, the received signal vector corresponding to the transmitted waveform, $s_i(t)$, is

$$\mathbf{r}(t) = \mathbf{h}\, s_i(t) + \mathbf{w}(t), \tag{2}$$

where the fast fading channel and the AWGN vectors at the receiver are denoted by $\mathbf{h}$ and $\mathbf{w}(t) \sim \mathcal{CN}(0, \sigma^2)$, respectively. Consideration of multi-user interference due to multiple JRC systems operating in close proximity would be an interesting future extension. As is commonly used in the literature, we assumed that $\mathbf{h}$ is known at the receiver and consider a coherent maximum likelihood (ML) detector [3], [7], [9]. Then, the detected symbol is expressed as,

$$\hat{s_i}(t) = \underset{s_j(t) \in \mathcal{S}}{\arg\max}\, \mathrm{Re}\bigg( \int_0^{MT} s_j^*(t) \mathbf{h}^H \mathbf{r}(t)\, dt \bigg), \tag{3}$$

where the real part of the argument is denoted by Re(.). Under the universal permutation set, the implementation of the optimal receiver reduces to an assignment problem. Therefore, the optimal solution can be obtained using the Hungarian algorithm [9]. On the other hand, when a subset of permutations are chosen for modulation, the waveform corresponding to the solution of the Hungarian algorithm may not be a part of the selected set and the optimal receiver takes the form of an IP optimization problem [14]. In this work, we show that when the first $2^n$ permutations in the lexicographic order are selected, a more efficient optimal receiver can be implemented with the operation of a set of Hungarian algorithms in parallel.

First, we consider the received signal and compute the correlation matrix as,

$$\mathbf{R} = (r_{vu}) \in \mathbb{R}^{M \times M}, \tag{4}$$

where the correlation of the processed received signal, $\mathbf{h}^H \mathbf{r}(t)$, with the basis function $\psi_u(t-(v-1)T)$ is the $vu$-th entry of $\mathbf{R}$, given by $r_{vu}$, with $\psi_u(t) = \sqrt{2E/T}\, p(t) \cos(2\pi f_u(t))$. Thus,

$$r_{vu} = \mathrm{Re}\bigg( \int_{(v-1)T}^{vT} \mathbf{h}^H \mathbf{r}(t) \psi_u(t - (v-1)T)\, dt \bigg). \tag{5}$$

Next, we split the $2^n$ permutations into $(M-1)$ blocks such that the $m$-th block contains the permutations from $\sum_{l=1}^{m-1} a_l (M-l)! + 1$ to $\sum_{l=1}^{m-1} a_l (M-l)! + a_m (M-m)!$. To do this, we define the integer vector $\mathbf{a} = [a_1, \ldots, a_{M-1}]$ such that $\sum_{m=1}^{M-1} a_m (M-m)! = 2^n$ where $a_m$ takes the largest possible integer value after selecting $a_l, \forall l < m$. Then we define a new vector $\bar{\mathbf{a}} = \mathbf{a}$ and update the $m$-th entry, $\bar{a}_m$, with the smallest integer such that there are $a_m$ values without including any of the values $\bar{a}_l + 1, \forall l < m$. In the first iteration, we define a new cost matrix $\mathbf{R}^1 = (-\mathbf{R})$ and then update it by setting all the rows greater than $\bar{a}_1$ in the first column to infinity. As such, the solution of the Hungarian algorithm for the updated cost matrix $\mathbf{R}^1$ provides the permutation with the highest correlation among the first $a_1(M-1)!$ permutations. In the second iteration, we define a new cost matrix $\mathbf{R}^2 = (-\mathbf{R})$ and update it by setting all the rows in the first column except the $(\bar{a}_1 + 1)$-th row and all the rows in the second column greater than $\bar{a}_2$ as infinity. As such, the solution of the Hungarian algorithm for the updated cost matrix $\mathbf{R}^2$ provides the permutation with the highest correlation among the permutations from $a_1(M-1)!+1$ to $a_1(M-1)!+a_2(M-2)!$. We continue until all the permutations up to the first $2^n$ permutation are considered and this requires at most $M-1$ iterations. Let us define the corresponding waveform for the permutation given by the solution of Hungarian algorithm for the cost matrix $\mathbf{R}^m$ in the $m$-th iteration as $s_m(t)$. Next, the correlation of each waveform $s_m(t)$ with the processed received signal, $\mathbf{h}^H\mathbf{r}(t)$, is computed and consequently the waveform that leads to the maximum correlation is chosen as the detected symbol, $\hat{s}_i(t)$, corresponding to the transmitted waveform, $s_i(t)$. The key steps related to the implementation of our proposed optimal receiver are summarized in Algorithm 1. As we have considered all the permutations in the selected set by considering the first $2^n$ permutations, there cannot exist another waveform that leads to a better correlation with the received signal. Therefore, Algorithm 1 provides the optimal solution for the ML detector given in (3)[1].

**Algorithm 1:** Proposed optimal receiver design

```
rank = 2^n
for m = 1 : M − 1 do
    a_m = ⌊rank/(M − m)!⌋
    rank = rank − a_m(M − m)!
    ā_m = Smallest integer such that there are a_m
      values not including ā_l + 1, ∀l < m
    if a_m > 0 then
        R^m = (−R)
        R^m_{vm} = ∞, ∀v > ā_m
        R^m_{vl} = ∞, ∀v ≠ ā_l + 1, l < m
        s_m(t) ← corresponding waveform to the
          solution of the Hungarian algorithm with R^m
    end
    if ā_m == ā_l for any l < m then
        increase ā_m until ā_m ≠ ā_l, ∀l < m
    end
end
ŝ_i(t) ← s*_m(t) which is the highest correlated
  waveform with h^H r(t)
```

Note that our proposed optimal receiver implementation involves the parallel use of the Hungarian algorithm for at most $M-1$ times and a comparison of the correlation of the resulting $M-1$ waveforms with the received signal. Thus, the worst case complexity of our proposed optimal solution is $O(M^4)$. We note that this complexity is significantly lower than the IP based optimal receiver proposed in [14], which has a worst case complexity of $O(2^M)$.

## IV. Communication Bit Error Rate Analysis

In this section, we focus on the communication performance in terms of the BER of the proposed system under the optimal ML detector given in (3). Thus, while the current work provides an upper bound on the BER under any practical receiver implementation, the BER analysis of a more practical non-coherent receiver is left as interesting future work.

Assuming equiprobable information symbols, the BER of detecting the received signal can be expressed as,

$$P_e = \frac{1}{n2^n}\sum_{i=0}^{2^n-1}\sum_{k=0}^{n-1}[1-P_c(i,k)], \tag{6}$$

where the probability of making a correct decision for the $k$-th bit of the transmitted waveform, $s_i(t)$ is given by $P_c(i,k)$. Since, the exact computation of $P_e$ requires complex multi-dimensional integrals, we adopt the union bound giving

$$P_e \le P_e^{UB} = \frac{1}{n2^n}\sum_{i=0}^{2^n-1}\sum_{j=0}^{2^n-1}\sum_{k=0}^{n-1}\left(b_k^{(i)}-b_k^{(j)}\right)^2 P(i,j), \tag{7}$$

where the $k$-th bit of the information symbol corresponding to $s_i(t)$ is given by $b_k^{(i)}$ and the pairwise error probability of detecting $s_j(t)$ when $s_i(t)$ is transmitted is given by $P(i,j)$. Using the ML detection rule, $P(i,j)$ can be written as [14],

$$P(i,j) = \Pr\left[\sqrt{\mathbf{h}^H\mathbf{h}} < Z\sqrt{\frac{M\sigma^2}{Ed(i,j)}}\right], \tag{8}$$

where $Z \sim \mathcal{N}(0,1)$ and the Hamming distance between the permutations relating to $s_j(t)$ and $s_i(t)$ is denoted by $d(i,j)$.

Whilst not included due to page limitations, applying the same strategy as in [14], the union bound in (7) is re-expressed under the AWGN channel as,

$$P_e^{UB} = \frac{1}{n2^n}\sum_{i=0}^{2^n-1}\sum_{j=0}^{2^n-1}\sum_{k=0}^{n-1}\left(b_k^{(i)}-b_k^{(j)}\right)^2 \mathcal{Q}\left(\sqrt{\frac{NEd(i,j)}{\sigma^2 M}}\right), \tag{9}$$

where $\mathcal{Q}(.)$ is the Gaussian Q-function and $\mathbb{E}[\mathbf{h}^H\mathbf{h}] = N$ corresponding to a unit channel gain.

Next, we focus on the correlated fading channel given by $\mathbf{h} = \sqrt{K/(K+1)}\Delta + \sqrt{1/(K+1)}\mathbf{C}^{1/2}\mathbf{g}$, where $K$ is the Rician factor that governs the strength of the line-of-sight (LoS) path relative to the scattered path. Further, the $l$-th entry of the complex LoS phase vector, $\Delta$, satisfies $||\Delta_l||^2 = 1$ and for the scattered components, the $N \times N$ correlation matrix is given by $\mathbf{C}$ with $\mathbf{g} \sim \mathcal{CN}(0, \mathbf{I})$. Under this channel model, (7) can be re-expressed as (10), given at the top of the next page, where $\alpha(i,j) = 2\sigma^2 M(K+1)/Ed(i,j)$ and $(\mathbf{V}^H\Delta)_l$ is the $l$-th entry of the vector $\mathbf{V}^H\Delta$ where $\mathbf{V}$ represents a unitary matrix such that $\mathbf{C} = \mathbf{V}\mathbf{\Omega}\mathbf{V}^H$ with $\mathbf{\Omega}$ denoting a diagonal matrix comprising of the eigenvalues of $\mathbf{C}$ .

The union bound, which is computed using the sum of all pairwise error probabilities, can be loose, particularly for large $M$. As a result, we consider the Hamming distance 2 neighbors to derive the nearest neighbor (NN) approximation [19]. Under this approximation, we only consider the pairwise error probabilities corresponding to those nearest neighbors and obtain (11) and (12) under the AWGN and the correlated Rician fading channels, respectively. The indicator function, $\delta(i,j)$, in (11) and (12) can be expressed as 1 if $d(i,j) = 2$ or 0 otherwise. These expressions provide a more accurate approximation for the BER in the high signal-to-noise ratio (SNR) regime and we further illustrate the accuracy of these expressions in the next section using numerical examples.

[1]The proposed Algorithm 1 can also be used with non-coherent receivers by replacing $\mathbf{R}$ with the detected energy matrix following the envelop detection.

$$P_e^{UB} = \frac{1}{n\pi 2^n} \sum_{i=0}^{2^n-1} \sum_{j=0}^{2^n-1} \sum_{k=0}^{n-1} \left(b_k^{(i)} - b_k^{(j)}\right)^2 \int_0^{\pi/2} \left[\prod_{l=1}^{N} \left(\frac{\alpha(i,j)\sin^2\theta}{\lambda_l + \alpha(i,j)\sin^2\theta}\right) \exp\left(\sum_{l=1}^{N} \frac{-K||(\mathbf{V}^H\Delta)_l||^2}{\lambda_l + \alpha(i,j)\sin^2\theta}\right)\right] d\theta. \tag{10}$$

$$P_e^{NN} = \frac{1}{n 2^n} \sum_{i=0}^{2^n-1} \sum_{j=0}^{2^n-1} \sum_{k=0}^{n-1} \left(b_k^{(i)} - b_k^{(j)}\right)^2 \delta(i,j) \mathcal{Q}\left(\sqrt{\frac{NEd(i,j)}{\sigma^2 M}}\right). \tag{11}$$

$$P_e^{NN} = \frac{1}{n\pi 2^n} \sum_{i=0}^{2^n-1} \sum_{j=0}^{2^n-1} \sum_{k=0}^{n-1} \left(b_k^{(i)} - b_k^{(j)}\right)^2 \delta(i,j) \int_0^{\pi/2} \left[\prod_{l=1}^{N} \left(\frac{\alpha(i,j)\sin^2\theta}{\lambda_l + \alpha(i,j)\sin^2\theta}\right) \exp\left(\sum_{l=1}^{N} \frac{-K||(\mathbf{V}^H\Delta)_l||^2}{\lambda_l + \alpha(i,j)\sin^2\theta}\right)\right] d\theta. \tag{12}$$

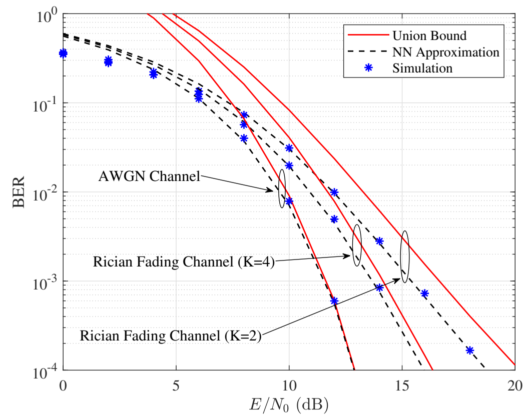


Fig. 1: The BER versus the average received SNR with $M = 5, N = 2, K = 2, 4, \rho = 0.5$ and $n = 6$.

## V. Numerical Examples

In this section, we provide numerical examples illustrating the performance of the proposed JRC system and the proposed optimal receiver implementation. Orthogonality between frequencies is maintained by setting $\Delta f = 1/T$ where $T = 1$. Waveform energy is fixed to unity. Under the correlated Rician fading, the exponential correlation model in [13] is considered. Thus, for a correlation coefficient $\rho \in \{0, 1\}$, the $(i,j)$-th element in $\mathbf{C}$ can be computed as $\rho^{||i-j||}$. However, we note that the analysis in Section IV holds for all $\mathbf{C}$.

We consider a communication receiver under the AWGN and correlated Rician fading channels and set $M = 5, N = 2, K = 2, 4$ and $\rho = 0.5$. The BER performance is plotted against the received SNR in Fig. 1 under both channel models. We consider the selection of the first $2^n = 64$ permutations in the lexicographic order from the total of $M! = 120$ permutations to communicate information symbols corresponding to $n = 6$ bits. The analytical approximations are obtained through the union bounds in (9), (10) and the NN approximations in (11), (12). As illustrated in the figure, the NN approximation has a higher precision than the union bound through the entire range of SNRs under both channels. On the other hand, in the high SNR regime, the union bound closely tracks the simulation results under the AWGN channel while the accuracy of the union bound increases with the Rician factor $K$ under the Rician fading channel.

Next, we consider a communication receiver under the correlated Rician fading channel and set $M = 4, 5, 6, 7, N = 2$, $K = 2$ with $\rho = 0.5$. The first $2^n$ permutations in the lexicographic order are selected to communicate information symbols corresponding to $n = \lfloor \log_2(M!) \rfloor$ bits with the intention of obtaining the maximum possible data rate. In Fig. 2, we compare the performance of our Algorithm 1 against the IP based optimal (IPO) receiver and the sub-optimal receiver with $d = 2$ proposed in [14]. We solved the IPO receiver using the mixed-integer linear programming (MILP) toolbox in Matlab. From the plot, we can observe that as we increase $M$, the BER increases. This can be explained by the constant waveform energy considered in this work. With increasing $M$, the number of bits increases, thus decreasing the SNR per bit and resulting in an increase in the BER. We also observe that Algorithm 1 obtains practically the same BER results compared to the IPO receiver thus validating the optimality of Algorithm 1 as discussed in Section III. Further, we can observe that the sub-optimal receiver in [14] performs close to the optimal receivers for this specific example. Whilst not included due to page limitation, our extended simulation studies have shown that the gap between the sub-optimal receiver and the optimal receivers increases significantly when the selected subset size is reduced relative to $M!$. This is also indicated by the plots for $M = 5$, where we only select 64 permutations, which is approximately half of the total number of permutations. Therefore, a simple search of Hamming distance 2 neighbors is not sufficient. In such situations, the slightly increased complexity of Algorithm 1, which is $O(M^4)$, over the complexity of $O(M^3)$ in the sub-optimal receiver is justified given the performance gap.

Finally, we evaluate the efficiency of our proposed receiver implementation in comparison to the IPO receiver proposed in [14]. We define the efficiency of our proposed algorithm as how fast it is compared to the IPO receiver. Thus, we compute the efficiency as the fraction of the computational time taken by the IPO receiver over that of Algorithm 1. Fig. 3 plots a zoomed section of the histogram of the efficiency distribution of Algorithm 1, after removing the right side tail. Therefore, Fig. 3 illustrates the speed of Algorithm 1 compared to the IPO receiver. The plot is generated with $M = 6, N = 2, K = 2$ and $\rho = 0.5$ for a correlated Rician fading channel. From the plot, it can be observed that the computational time of the IPO receiver is significantly greater compared to the proposed solution with the mean value of 38.29 as marked in Fig. 3. Further, we note that the the smallest and the largest values in the histogram are 0.09 and 22942, respectively. This indicates

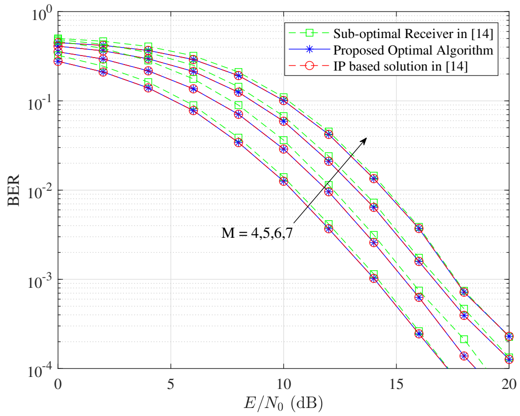


Fig. 2: The BER versus the average received SNR under Rician fading channels with $N = 2, K = 2$ and $\rho = 0.5$.

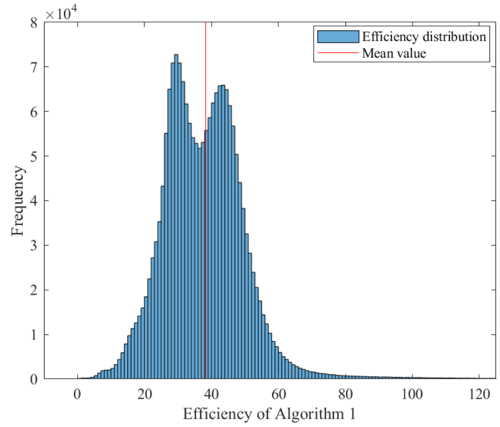


Fig. 3: Efficiency distribution of Algorithm 1 under Rician fading channels with $M = 6, N = 2, K = 2$ and $\rho = 0.5$.

that while at times the IPO receiver is more efficient than Algorithm 1, on average Algorithm 1 is 38.29 times faster than the IPO receiver and it can increase up to a factor 22942 in the worst case[2]. As such, we observe that the proposed algorithm is far more computationally efficient than the IPO receiver proposed in [14].

## VI. Conclusion

We presented a novel approach to obtain a subset of waveforms for the JRC problem that alleviates the bit modulation related implementation limitations associated with the general random stepped frequency permutation waveforms. The proposed approach selects a subset of permutations by keeping the subset size to a power of two in order to meet the modulation requirements. The proposed subset selection enables an efficient mapping process based on the factorial number system and Lehmer code. For the communications receiver, we used a set of Hungarian algorithms in parallel and presented an efficient implementation of the optimal receiver. We showed that the proposed receiver has reduced computational complexity compared to the existing IPO receiver. Overall, our proposed approach reduces the complexity of the mapping process at the transmitter and the detection process at the receiver to a reasonable level. The BER performance analysis is carried out under the ML detector. By considering both AWGN and correlated Rician fading channels, the analytical expressions are derived for the union bound and the NN approximation. While the proposed subset of permutations and mapping rule ensure efficient implementation at the communication receiver, an optimization of the mapping process to further improve the BER is an interesting future extension.

[2]We note that the bimodal distribution in Fig. 3 can be explained by the two operation processes of the Matlab MILP toolbox which first solves the relaxed problem and then uses heuristic algorithms to obtain the integer solution.

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