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Inverse Dynamics of Multilink Cable-Driven Manipulators with Consideration of Joint Interaction Forces and Moments

Darwin Lau, Denny Oetomo, and Saman K. Halgamuge

Abstract—Joint interaction forces and moments play a significant role within multilink cable-driven manipulators (MCDMs). In this paper, the consideration of joint interaction forces and moments in the objective function and constraints specific to the inverse dynamics of MCDMs are considered for the first time. By formulating the relationship between the joint interactions and cable forces, it is shown that the minimisation of the joint interactions results in a convex quadratic program (QP). Furthermore, the inclusion of constraints to maintain the stability of unilateral spherical joints results in a quadratically constrained quadratic program (QCQP). Simulation results of the proposed formulations on 2-link 8-cable and 8-link 76-cable manipulators are compared with the traditional 2-norm cable force minimisation. Results show that the formulations are able to take advantage of the actuation redundancy in considering the joint interactions within the inverse dynamics of MCDMs.

Index Terms—Multilink cable-driven manipulators, cable robots, inverse dynamics, redundancy resolution

I. INTRODUCTION

Cable-driven parallel manipulators (CDPMs) are mechanisms actuated by cables that are arranged in parallel configuration. The key advantages of cable-driven systems include: reduced end-effector weight and inertia compared to traditional rigid link mechanisms [1], potentially large reachable workspace [2] and high reconfigurability [3]. Furthermore, CDPMs have also been regarded as a bio-inspired mechanism [4]–[6]. The key characteristic of CDPMs is that cables can only provide forces under tension but not compression (*positive cable force*). The positive cable force constraint results in the necessity of actuation redundancy and creates challenges in workspace analysis [7]–[9] and manipulator control [10]–[12].

In the force control of CDPMs, it is essential to determine the set of cable forces required to satisfy a prescribed motion (*inverse dynamics*). For redundantly actuated CDPMs, the inverse dynamics problem has typically been formulated and solved as an optimisation problem, while considering the positive cable force constraint. Previous studies on the inverse dynamics optimisation of CDPMs have focused on two aspects. Firstly, the selection of appropriate objective functions; and secondly, the development of efficient algorithms to determine the cable forces. Common objective functions include the linear sum (1-norm) [10]–[12] and the quadratic sum (2-norm) [12]–[14] of cable forces. In [15], the objective to achieve “optimally safe” cable force distributions was also studied. In addition to the formulation of objective functions, methods to

efficiently resolve the cable forces have been studied to allow real-time implementation of inverse dynamics [14], [16], [17].

Previous analysis on CDPMs have primarily focused on single link mechanisms. For multilink cable-driven manipulators (MCDMs), it was shown that the inverse dynamics problem could be formulated and solved in the same manner as single link CDPMs [18]. MCDMs are a class of CDPMs possessing a multi-body rigid link structure and have been studied due to its anthropomorphic nature. MCDMs benefit from the compactness of serial mechanisms and the actuation advantages of cable-driven systems. Practical examples of anthropomorphic MCDMs include the ECCE robot [19], [20] and the family of musculoskeletal robots from the Kenta [21] to the Kenshiro [22]. On the theoretical side, problems related to the modelling and analysis of MCDMs have been studied [18], [23], [24].

One characteristic of MCDMs is that the actuation of the cables produces interaction forces and moments onto the manipulator joints. As a result, it may be desired to minimise the magnitude of the interactions, providing benefits such as minimising joint friction and wear during manipulator motion. Although joint friction is unavoidable in practical systems, reduction of friction is extremely advantageous for manipulator control. Minimisation of joint wear prolongs the lifetime of the joint. Additionally, it may also be desired to apply constraints on the interactions. For example, consider a spherical joint where the socket covers less than half of the ball and hence can be separated from the socket (*unilateral spherical joint*). To avoid joint dislocation (*unstable joint*), the joint interaction force must act into the socket surface.

In this paper, the inverse dynamics problem specific to MCDMs considering joint interaction forces and moments is studied. It is shown that the minimisation of the magnitudes in interaction forces and moments results in a quadratic program (QP) with respect to the cable forces. Furthermore, constraints on the interaction forces is demonstrated by ensuring the stability of unilateral spherical joints. It is proven that the resulting problem can be solved as a quadratically constrained quadratic program (QCQP). The proposed formulations are simulated on a 2-link 4 degree-of-freedom 8-cable system and an 8-link 24 degree-of-freedom 76-cable mechanism.

Comparing with the traditional minimisation of the 2-norm of cable forces, the results show that it is possible to redistribute the cable forces to satisfy the desired objective functions and constraints on the interaction forces and moments. This allows the joint interactions of MCDMs to be considered within the inverse dynamics formulation. Additionally, the measured computational times for the traditional and proposed formulations show the feasibility of using the proposed formulations on practical systems.

The remainder of the paper is organised as follows: Section II introduces the MCDM model and the inverse dynamics problem. Section III derives the expressions for the interaction forces and moments of MCDMs. Section IV formulates the inverse dynamics problem with the objective to minimise the interaction forces and moments. Section V extends this to apply constraints on the interaction forces. Section VI presents and discusses the simulation results. Finally, Section

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VII concludes the paper and presents areas of future work.

II. SYSTEM MODEL AND INVERSE DYNAMICS PROBLEM

A. Multilink Cable-Driven Manipulator Model

Consider the rigid body structure for a p link cable-driven manipulator shown in Figure 1. The inertial base is represented by body 0 and bodies 1 to p are the links of the manipulator, where link p is the outermost link. The locations G_k and P_k for $k = 1, \dots, p$ represent the centre of gravity of link k and the joint location between links k and $k - 1$, respectively.

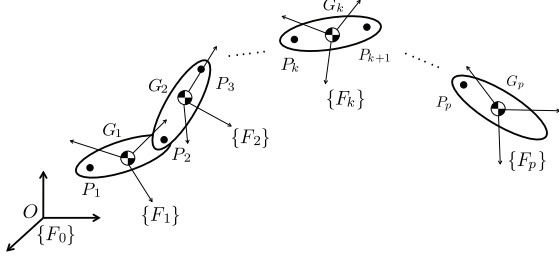


Fig. 1. Rigid body structure of MCDMs showing the coordinate frames $\{F_k\}$, centre of gravity for each link G_k , and the joint locations P_k .

In [18], a generalised model for MCDMs allowing for arbitrary cable-routing was derived through introducing the Cable-Routing Matrix C . The dynamics for an n degree-of-freedom MCDM actuated by m cables can be expressed as

$$M(\mathbf{q})\ddot{\mathbf{q}} + \boldsymbol{\eta}(\dot{\mathbf{q}}, \mathbf{q}) = -L(\mathbf{q}, C)^T \mathbf{f}, \quad (1)$$

where $M \in \mathbb{R}^{n \times n}$ and $\boldsymbol{\eta}(\dot{\mathbf{q}}, \mathbf{q}) \in \mathbb{R}^n$ represent the mass-inertia matrix and the vector containing the centrifugal, Coriolis and gravitational terms, respectively, and $\mathbf{q} \in \mathbb{R}^n$ is the system's generalised coordinates. The *cable force vector* $\mathbf{f} = [f_1 \ f_2 \ \dots \ f_m]^T \in \mathbb{R}^m$ represents the set of cable forces, where $f_i \geq 0$ denotes the force in cable i . The matrix $L^T \in \mathbb{R}^{n \times m}$ is the transpose of the Jacobian that relates the cable force vector and the resultant wrench on the manipulator.

B. Traditional Inverse Dynamics Problem

The inverse dynamics problem refers to the determination of the cable forces $\mathbf{f}$ required to achieve the desired motion defined by $\mathbf{q}$, $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$. Due to the actuation redundancy in cable-driven systems with $m \geq n + 1$ cables, the inverse dynamics problem has been typically formulated as an optimisation problem. Considering the positive cable force constraints, the optimal set of cable forces $\mathbf{f}^*$ can be determined by solving

$$\begin{aligned} \mathbf{f}^* &= \arg \min_{\mathbf{f}} Q(\mathbf{f}) \\ \text{s. t.} \quad &M(\mathbf{q})\ddot{\mathbf{q}} + \boldsymbol{\eta}(\dot{\mathbf{q}}, \mathbf{q}) = -L^T \mathbf{f} \\ &\mathbf{f}_{\min} \leq \mathbf{f} \leq \mathbf{f}_{\max}. \end{aligned} \quad (2)$$

The constraints for (2) are the equations of motion (1) and bounds on the cable forces. The minimum and maximum bounds on the cable forces are represented by the vectors $\mathbf{f}_{\min} \geq \mathbf{0}$ and $\mathbf{f}_{\max} > \mathbf{0}$, respectively. The objective function $Q(\mathbf{f})$ is selected to achieve a desired goal, common objectives that have been studied include $Q(\mathbf{f}) = \sum_{i=1}^m f_i$ and

$Q(\mathbf{f}) = \sum_{i=1}^m f_i^2$, where (2) results in a linear program (LP) and a quadratic program (QP), respectively.

III. INTERACTION FORCES AND MOMENTS OF MCDMS

Considering the free body diagram of link k within an MCDM as shown in Figure 2, the forces and moments acting on link k is comprised of: the interaction at joints P_k and P_{k+1} , the cable forces and the gravity force.

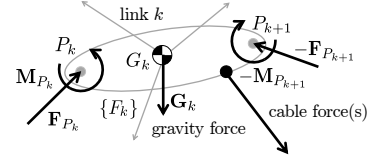


Fig. 2. Free body diagram of link k showing the forces acting on the body.

The interaction force $\mathbf{F}_{P_k}$ and moment $\mathbf{M}_{P_k}$ acting on joint P_k can be denoted by the interaction wrench $\mathbf{p}_k = [\mathbf{F}_{P_k}^T \ \mathbf{M}_{P_k}^T]^T$, expressed with respect to frame $\{F_k\}$. The gravity wrench acting on link k in frame $\{F_k\}$ can be denoted by $\mathbf{w}_{G_k} = [\mathbf{G}_k^T \ 0]^T$, where $\mathbf{G}_k$ is the gravity force acting on link k . Denoting the wrench exerted by cable i on link k in frame $\{F_k\}$ as $\mathbf{w}_{T_{ik}}$, the resultant wrench exerted by all cables is $\mathbf{w}_{T_k} = \sum_{i=1}^m \mathbf{w}_{T_{ik}}$. From Figure 2, Newton's second law for link k can be expressed as

$$\begin{aligned} \mathbf{a}_k &= \mathbf{w}_{G_k} + \mathbf{w}_{T_k} + \begin{bmatrix} I_3 & 0_3 \\ [{}^k\mathbf{r}_{G_k P_k}]^\times & I_3 \end{bmatrix} \mathbf{p}_k \\ &\quad - \begin{bmatrix} {}^k R_{k+1} & 0_3 \\ [{}^k\mathbf{r}_{G_k P_{k+1}}]^\times & {}^k R_{k+1} \end{bmatrix} \mathbf{p}_{k+1}, \end{aligned} \quad (3)$$

where I_3 and 0_3 are 3×3 identity and zero matrices, respectively. The vector $\mathbf{a}_k$ contains the derivatives of the linear and angular momentums of link k in frame $\{F_k\}$. The notation ${}^k\mathbf{r}_{AB}$ represents the vector from point A to point B in frame $\{F_k\}$ and ${}^a R$ represents the rotation matrix from $\{F_k\}$ to $\{F_a\}$.

For the outermost link p , it can be assumed that $\mathbf{F}_{P_{p+1}} = \mathbf{M}_{P_{p+1}} = \mathbf{0}$. Hence, the interaction wrench for joint a can be derived recursively from the relationship (3) and expressed as

$$\mathbf{p}_a = \sum_{k=a}^p P_{ka}^T (\mathbf{a}_k - \mathbf{w}_{G_k} - \mathbf{w}_{T_k}), \quad (4)$$

where

$$P_{ka}^T = \begin{bmatrix} {}^a R_{k+1} & 0_3 \\ {}^a R [{}^k\mathbf{r}_{P_a G_k}]^\times & {}^a R_{k+1} \end{bmatrix}. \quad (5)$$

In [18], it was shown for MCDMs that the resultant wrench exerted by the cable forces on the system $\mathbf{w}_T = [\mathbf{w}_{T_1}^T \ \dots \ \mathbf{w}_{T_p}^T]^T$ can be expressed in the form $\mathbf{w}_T = V^T \mathbf{f}$, where $V^T \in \mathbb{R}^{6p \times m}$ is a Jacobian matrix relating the cable forces to the wrenches acting on the rigid bodies of the system. From (4), the set of interaction forces and moments for the system $\mathbf{p} = [\mathbf{p}_1^T \ \dots \ \mathbf{p}_p^T]^T$ can be expressed in the form

$$\mathbf{p} = \mathbf{u} + R^T \mathbf{f}, \quad (6)$$

where $R^T = P^T V^T$, $\mathbf{u} = [\mathbf{u}_1^T \ \dots \ \mathbf{u}_p^T]^T$ and

$$\mathbf{u}_a = \sum_{k=a}^p P_{ka}^T (\mathbf{a}_k - \mathbf{w}_{G_k}) .$$

The matrix $P^T \in \mathbb{R}^{6p \times 6p}$ is comprised of the terms from (5) and can be expressed as

$$P^T = \begin{bmatrix} P_{11}^T & P_{21}^T & \dots & P_{p1}^T \\ 0_3 & P_{22}^T & \dots & P_{p2}^T \\ \vdots & & & \vdots \\ 0_3 & 0_3 & \dots & P_{pp}^T \end{bmatrix} .$$

From (6), it is shown that the interaction wrench can be expressed linearly with respect to the cable forces.

IV. MINIMISATION OF INTERACTION FORCES/MOMENTS

For MCDMs, the objective function to achieve the minimisation of interaction forces and moments can be expressed as

$$Q(\mathbf{f}) = \sum_{a=1}^p \alpha_a |\mathbf{F}_{P_a}|^2 + \beta_a |\mathbf{M}_{P_a}|^2 , \quad (7)$$

where $\alpha_a, \beta_a \geq 0$ are weights that prioritise the different joints of the system. The weights can be normalised by ensuring that

$$\sum \alpha_a + \sum \beta_a = 1 . \quad (8)$$

Expressing $R^T \in \mathbb{R}^{6p \times m}$ from (6) in the form

$$R^T = \begin{bmatrix} R_{11}^T & \dots & R_{m1}^T \\ \vdots & & \vdots \\ R_{1p}^T & \dots & R_{mp}^T \end{bmatrix} , \quad (9)$$

the vector $R_{ia}^T \in \mathbb{R}^6$ represents the relationship between the force of cable i and the interaction wrench of joint a , where

$$\begin{bmatrix} \mathbf{F}_{P_a} \\ \mathbf{M}_{P_a} \end{bmatrix} = \mathbf{u}_a + \sum_{i=1}^m R_{ia}^T f_i . \quad (10)$$

From (10), the interaction force $\mathbf{F}_{P_a} = [F_{P_{ax}} \ F_{P_{ay}} \ F_{P_{az}}]^T$ and moment $\mathbf{M}_{P_a} = [M_{P_{ax}} \ M_{P_{ay}} \ M_{P_{az}}]^T$ of joint a can be expressed as

$$\begin{aligned} F_{P_{ax}} &= u_{ax} + \mathbf{r}_{ax}^T \mathbf{f}, \quad F_{P_{ay}} = u_{ay} + \mathbf{r}_{ay}^T \mathbf{f} \\ F_{P_{az}} &= u_{az} + \mathbf{r}_{az}^T \mathbf{f}, \quad M_{P_{ax}} = u_{a\theta} + \mathbf{r}_{a\theta}^T \mathbf{f} \\ M_{P_{ay}} &= u_{a\phi} + \mathbf{r}_{a\phi}^T \mathbf{f}, \quad M_{P_{az}} = u_{a\psi} + \mathbf{r}_{a\psi}^T \mathbf{f}, \end{aligned} \quad (11)$$

where $\mathbf{u}_a = [u_{ax} \ u_{ay} \ u_{az} \ u_{a\theta} \ u_{a\phi} \ u_{a\psi}]^T$, and the vectors $\mathbf{r}_{ax}, \mathbf{r}_{ay}, \mathbf{r}_{az}, \mathbf{r}_{a\theta}, \mathbf{r}_{a\phi}, \mathbf{r}_{a\psi} \in \mathbb{R}^m$ can be determined from $R_{ia}^T \ \forall i$.

From (11), the magnitude of the interaction force and moment at joint a can be expressed as

$$\begin{aligned} |\mathbf{F}_{P_a}|^2 &= \mathbf{f}^T \Lambda_a \mathbf{f} + \boldsymbol{\lambda}_a^T \mathbf{f} + w_a \\ |\mathbf{M}_{P_a}|^2 &= \mathbf{f}^T \Gamma_a \mathbf{f} + \boldsymbol{\gamma}_a^T \mathbf{f} + v_a , \end{aligned} \quad (12)$$

where

$$\begin{aligned} \Lambda_a &= \mathbf{r}_{ax} \mathbf{r}_{ax}^T + \mathbf{r}_{ay} \mathbf{r}_{ay}^T + \mathbf{r}_{az} \mathbf{r}_{az}^T \\ \boldsymbol{\lambda}_a &= 2u_{ax} \mathbf{r}_{ax} + 2u_{ay} \mathbf{r}_{ay} + 2u_{az} \mathbf{r}_{az} \\ w_a &= u_{ax}^2 + u_{ay}^2 + u_{az}^2 \\ \Gamma_a &= \mathbf{r}_{a\theta} \mathbf{r}_{a\theta}^T + \mathbf{r}_{a\phi} \mathbf{r}_{a\phi}^T + \mathbf{r}_{a\psi} \mathbf{r}_{a\psi}^T \\ \boldsymbol{\gamma}_a &= 2u_{a\theta} \mathbf{r}_{a\theta} + 2u_{a\phi} \mathbf{r}_{a\phi} + 2u_{a\psi} \mathbf{r}_{a\psi} \\ v_a &= u_{a\theta}^2 + u_{a\phi}^2 + u_{a\psi}^2 . \end{aligned}$$

From (12), the objective function in (7) can be expressed in the quadratic form

$$Q(\mathbf{f}) = \mathbf{f}^T H \mathbf{f} + \mathbf{c}^T \mathbf{f} + y , \quad (13)$$

where

$$\begin{aligned} H &= \sum \alpha_a \Lambda_a + \beta_a \Gamma_a \\ \mathbf{c} &= \sum \alpha_a \boldsymbol{\lambda}_a + \beta_a \boldsymbol{\gamma}_a \\ y &= \sum \alpha_a w_a + \beta_a v_a . \end{aligned}$$

Using (13), the inverse dynamics problem for MCDMs with the objective to minimise the interaction forces and moments within the joints can be expressed in the form (2) as

$$\begin{aligned} \mathbf{f}^* &= \arg \min_{\mathbf{f}} \mathbf{f}^T H \mathbf{f} + \mathbf{c}^T \mathbf{f} \\ \text{s. t.} \quad & M(\mathbf{q}) \ddot{\mathbf{q}} + \boldsymbol{\eta}(\dot{\mathbf{q}}, \mathbf{q}) = -L^T \mathbf{f} \\ & \mathbf{f}_{min} \leq \mathbf{f} \leq \mathbf{f}_{max} . \end{aligned} \quad (14)$$

From (14), it is observed that the minimisation of interaction forces and moments results in a quadratic programming (QP) problem. It will now be shown that the problem is convex.

Lemma 1. *Given any vector $\mathbf{r} = [r_1 \ \dots \ r_m] \in \mathbb{R}^m$, the matrix $\Omega = \mathbf{r} \mathbf{r}^T \in \mathbb{R}^{m \times m}$ is a positive semidefinite Hermitian matrix.*

Proof. The matrix Ω is Hermitian since the vector $\mathbf{r}$ is real and the product $\mathbf{r} \mathbf{r}^T$ results in a symmetric matrix. The value of $\mathbf{f}^T \Omega \mathbf{f}$ is non-negative since

$$\mathbf{f}^T \mathbf{r} \mathbf{r}^T \mathbf{f} = (\mathbf{r}^T \mathbf{f})^T (\mathbf{r}^T \mathbf{f}) \geq 0$$

Since Ω is Hermitian and $\mathbf{f}^T \Omega \mathbf{f} \geq 0$, then Ω is a positive semidefinite matrix. $\square$

Theorem 1. *The matrix H from the problem (14) is positive semidefinite.*

Proof. By Lemma 1, the matrices $\Lambda_{ax} = \mathbf{r}_{ax} \mathbf{r}_{ax}^T$, $\Lambda_{ay} = \mathbf{r}_{ay} \mathbf{r}_{ay}^T$, $\Lambda_{az} = \mathbf{r}_{az} \mathbf{r}_{az}^T$, $\Gamma_{a\theta} = \mathbf{r}_{a\theta} \mathbf{r}_{a\theta}^T$, $\Gamma_{a\phi} = \mathbf{r}_{a\phi} \mathbf{r}_{a\phi}^T$ and $\Gamma_{a\psi} = \mathbf{r}_{a\psi} \mathbf{r}_{a\psi}^T$ are also positive semidefinite. Hence, the matrices Λ_a and Γ_a from (12) are positive semidefinite since

$$\begin{aligned} \mathbf{f}^T \Lambda_a \mathbf{f} &= \mathbf{f}^T \Lambda_{ax} \mathbf{f} + \mathbf{f}^T \Lambda_{ay} \mathbf{f} + \mathbf{f}^T \Lambda_{az} \mathbf{f} \geq 0 \\ \mathbf{f}^T \Gamma_a \mathbf{f} &= \mathbf{f}^T \Gamma_{a\theta} \mathbf{f} + \mathbf{f}^T \Gamma_{a\phi} \mathbf{f} + \mathbf{f}^T \Gamma_{a\psi} \mathbf{f} \geq 0 . \end{aligned} \quad (15)$$

As a result, H is positive semidefinite for nonnegative constants $\alpha_a, \beta_a \geq 0 \ \forall a$, since

$$\mathbf{f}^T H \mathbf{f} = \sum_a \alpha_a \mathbf{f}^T \Lambda_a \mathbf{f} + \beta_a \mathbf{f}^T \Gamma_a \mathbf{f} \geq 0$$

is true as a result of (15). $\square$

The inverse dynamics problem (14) is convex since H is positive semidefinite (Theorem 1). Compared with the well studied objective $Q(\mathbf{f}) = \mathbf{f}^T \mathbf{f}$, the terms H and $\mathbf{c}$ distribute the weighting of cable forces such that the actuation of cables that produces larger interaction forces and moments are penalised.

V. CONSTRAINTS TO MAINTAIN STABILITY OF UNILATERAL SPHERICAL JOINTS

Section IV introduced the minimisation of joint interactions within the inverse dynamics objective function. In this section, constraints with respect to the interaction forces and moments for unilateral spherical joints, as shown in Figure 3, are considered. The key characteristic of unilateral joints is that the socket covers less than half of the surface of the ball, and hence it is possible for the ball to dislocate from the socket.

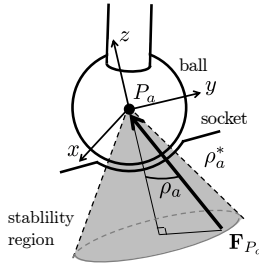


Fig. 3. Unilateral spherical joint showing the interaction force angle ρ_a and the region of joint stability. The joint is considered stable if the interaction force is within the shaded region.

Unilateral joints appear in both engineered and biological systems. For example, the glenohumeral joint of the human shoulder complex is a type of such joint, where the joint can dislocate depending on the direction of the interaction force. For joint a , the angle between the direction of the interaction force and the joint can be described as the *interaction force angle* ρ_a , as shown in Figure 3. For unilateral spherical joints, if the interaction force angle exceeds ρ_a^* , the joint would dislocate and can be considered as *unstable*.

Mathematically, joint a can be regarded as stable if: 1) the contact force is applied to the area of the ball joint that is covered by the socket

$$\rho_a = \tan^{-1} \left(\frac{\sqrt{F_{P_{ax}}^2 + F_{P_{ay}}^2}}{F_{P_{az}}} \right) \leq \rho_a^*, \quad (16)$$

and 2) the ball of the joint is pushed into the socket

$$F_{P_{az}} > 0. \quad (17)$$

The joint angle constraint (16) can be alternatively expressed as

$$F_{P_{ax}}^2 + F_{P_{ay}}^2 \leq \mu_a F_{P_{az}}^2, \quad (18)$$

where $\mu_a = \tan^2 \rho_a^*$. Substituting the expressions from (11) into (18), the condition for the stability of joint a results in the quadratic form

$$\mathbf{f}^T G_a \mathbf{f} + \mathbf{b}_a^T \mathbf{f} + g_a \leq 0, \quad (19)$$

where

$$\begin{aligned} G_a &= \mathbf{r}_{ax}^T \mathbf{r}_{ax} + \mathbf{r}_{ay}^T \mathbf{r}_{ay} - \mu_a \mathbf{r}_{az}^T \mathbf{r}_{az} \\ \mathbf{b}_a &= 2(u_{ax} \mathbf{r}_{ax} + u_{ay} \mathbf{r}_{ay} - \mu_a u_{az} \mathbf{r}_{az}) \\ g_a &= u_{ax}^2 + u_{ay}^2 - \mu_a u_{az}^2. \end{aligned}$$

Incorporating the constraints (17) and (19) into the inverse dynamics problem (2) results in the optimisation problem

$$\begin{aligned} \mathbf{f}^* &= \arg \min_{\mathbf{f}} Q(\mathbf{f}) \\ \text{s. t. } & M(\mathbf{q}) \ddot{\mathbf{q}} + \boldsymbol{\eta}(\dot{\mathbf{q}}, \mathbf{q}) = -L^T \mathbf{f} \\ & \mathbf{f}_{min} \leq \mathbf{f} \leq \mathbf{f}_{max} \\ & \mathbf{f}^T G_a \mathbf{f} + \mathbf{b}_a^T \mathbf{f} + g_a \leq 0 \quad \forall a \\ & u_{az} + \mathbf{r}_{az}^T \mathbf{f} > 0 \quad \forall a. \end{aligned} \quad (20)$$

Compared with (2) and (14), the problem in (20) consists of both linear and quadratic constraints. Hence, if the objective function $Q(\mathbf{f})$ is either linear or quadratic, the resulting inverse dynamics problem is a quadratically constrained quadratic program (QCQP). The QCQP problem is convex if and only if the objective function is convex and also the matrices G_a are all positive semidefinite. Due to the subtraction of $\mu_a \mathbf{r}_{az}^T \mathbf{r}_{az}$ in G_a , the convexity of (20) cannot be guaranteed. Finally, note that the additional constraints on the interaction forces in (20) may lead to no solutions to the inverse dynamics problem.

VI. SIMULATION RESULTS AND DISCUSSION

Two example systems were selected to demonstrate the proposed inverse dynamics schemes formulated in Sections IV and V. Section VI-A presents the simulation for a simple 2-link 4 degree-of-freedom system actuated by 8 cables and in Section VI-B a more complex 8-link 24 degree-of-freedom mechanism actuated by 76 cables.

For each example trajectory, simulations using three different inverse dynamics formulations were performed:

- 1) **Case 1:** Minimisation of the well studied 2-norm of cable forces, $Q(\mathbf{f}) = \mathbf{f}^T \mathbf{f}$, from Section II-B (*QP problem*) to serve as a comparison *baseline*.
- 2) **Case 2:** Minimisation of interaction forces using (14) formulated in Section IV (*QP problem*).
- 3) **Case 3:** Minimisation of interaction forces with constraints on the interaction angle using (20) formulated in Section V to maintain joint stability (*QCQP problem*).

Case 1 is a well studied method in the literature and is used as a baseline to compare against the proposed approaches.

A. 2-link 4-DoF Spherical-Revolute Manipulator

The rigid body structure and cable attachments for the 2-link example are shown in Figure 4. The system consists of a unilateral spherical joint connecting link 1 to the base and a revolute joint connecting links 2 and 1. The generalised coordinates for the system can be denoted by $\mathbf{q} = [q_1 \ q_2 \ q_3 \ q_4]^T$, where q_1 , q_2 and q_3 represent the XYZ-Euler angle rotations of the spherical joint and q_4 represents the relative rotation in the X-axis between links 2 and 1. The mass and principal moments of inertia are $m = 0.1$ kg and $I_x = I_y = I_z = 1$ kg-m², respectively, for both of the links.

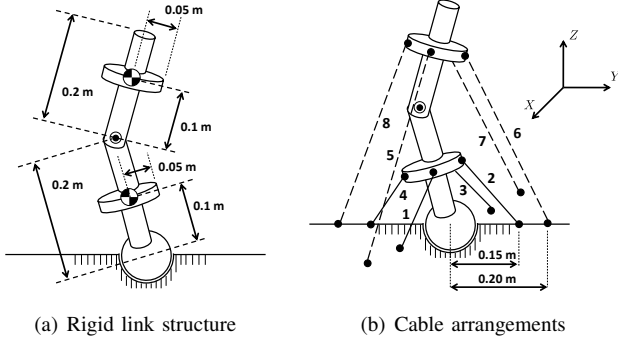


Fig. 4. Model of the 2-link 4 degree-of-freedom SR system.

The cables 1 to 4 were connected from the base link to link 1 symmetrically about the ball joint. Cables 5 to 8 were similarly connected from the base link to link 2. The minimum and maximum cable forces for all cables were set at $f_{min} = 0.001$ N and $f_{max} = 1000$ N, respectively. Details of the inverse dynamics cases for the 2-link example are as follows:

- 1) Minimisation of $Q(\mathbf{f}) = \mathbf{f}^T \mathbf{f}$ subject to $\mathbf{f}_{min} \leq \mathbf{f} \leq \mathbf{f}_{max}$ (traditional method).
- 2) Minimisation of the magnitude of the interaction force at the spherical joint $Q(\mathbf{f}) = |\mathbf{F}_{P_1}|^2$, where $\alpha_1 = 1, \alpha_2 = \beta_1 = \beta_2 = 0$ from (7), subject to $\mathbf{f}_{min} \leq \mathbf{f} \leq \mathbf{f}_{max}$.
- 3) Minimisation of the magnitude of the interaction force at the spherical joint $Q(\mathbf{f}) = |\mathbf{F}_{P_1}|^2$, subject to $\mathbf{f}_{min} \leq \mathbf{f} \leq \mathbf{f}_{max}$ and a constraint of $\rho_1 \leq 30^\circ$ on the interaction angle of the spherical joint.

The constraint on the interaction angle of the unilateral spherical joint is required to avoid large interaction angles that may lead to instability and dislocation of the joint. Two trajectories were chosen to illustrate the proposed inverse dynamics approaches.

1) *Trajectory 1*: A simple trajectory was first chosen to illustrate the inverse dynamics for the 2-link manipulator. The motion of the manipulator was purely in the YZ -plane with rotations in X for both the spherical and revolute joints. The 1 second trajectory $\mathbf{q}(t)$ was generated by fitting a quintic spline to the initial conditions $\mathbf{q}(0) = [\frac{\pi}{6} \ 0 \ 0 \ -\frac{\pi}{10}]^T$, $\dot{\mathbf{q}}(0) = \mathbf{0}$ and final conditions $\mathbf{q}(1) = [-\frac{\pi}{6} \ 0 \ 0 \ \frac{\pi}{10}]^T$, $\dot{\mathbf{q}}(1) = \mathbf{0}$. The generated trajectory $\mathbf{q}(t)$ and its derivative $\dot{\mathbf{q}}(t)$ are shown in Figure 5.

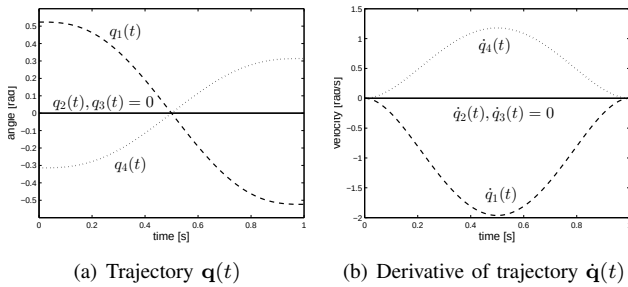


Fig. 5. Trajectory 1 for the 2-link manipulator example.

The inverse dynamics solutions to trajectory 1 are shown in Figure 6. For this example, it was found that the minimisations

of 2-norm of cable forces $Q(\mathbf{f}) = \mathbf{f}^T \mathbf{f}$ and the interaction force on the spherical joint $Q(\mathbf{f}) = |\mathbf{F}_{P_1}|^2$ produced the same resulting cable forces. This is due to the fact that since the trajectory motion is purely in the YZ -plane, cables 1, 3, 5 and 7 were not used in generating the motion. As a result, there is less actuation redundancy available for the minimisation of the joint interaction force. Furthermore, it should be noted that the minimisation of the 2-norm of cable forces also has an indirect impact to minimise the interaction forces. As such, the magnitude of interaction forces that can be further minimised depends on the manipulator, cable arrangement and trajectory.

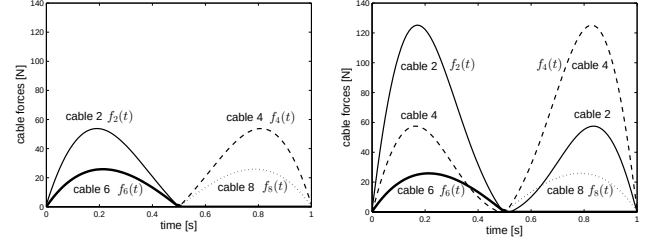


Fig. 6. Force profiles $\mathbf{f}(t)$ of different inverse dynamics problems for trajectory 1 of the 2-link manipulator example.

The resulting interaction angle of the spherical joint produced by cable force profile from Figure 6(a) is shown in Figure 7(a). It can be observed that the interaction angle ρ_1 exceeded the limit of $\rho_1^* = 30^\circ$ for nearly the entire trajectory. The cable forces solution to the inverse dynamics problem considering the interaction angle constraint (case 3) are shown in Figure 6(b). The constraint of $\rho_1 \leq 30^\circ$ was satisfied, as shown in Figure 7(b), through a redistribution of the cable forces. As only cables 6 and 8 is able to produce motion on link 2 for trajectory 1, the force profiles for cables 6 and 8 as shown in Figures 6(a) and 6(b) are the same. However, for $0 \leq t \leq 0.5$ it can be observed that cable 4 was used to lower the interaction angle such that the constraint is satisfied, resulting in an increased force in cable 2. This effect is similar in the second part of the trajectory $0.5 \leq t \leq 1$.

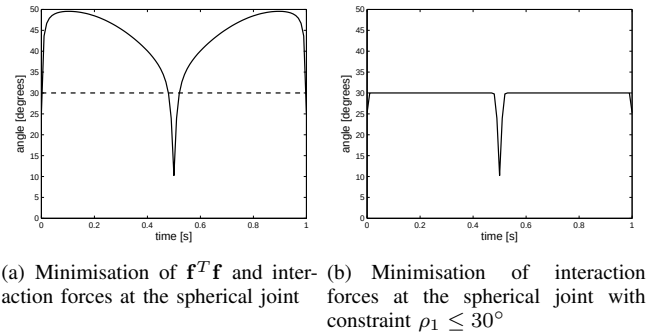


Fig. 7. Profiles of interaction angles $\rho_1(t)$ for the spherical joint of different inverse dynamics problems for trajectory 1 of the 2-link manipulator example.

2) *Trajectory 2*: A more general spatial trajectory was selected for this example. In the same manner as the first

trajectory, the 1 second trajectory was generated by fitting a quintic spline to the initial conditions $\mathbf{q}(0) = [0.2 \ -0.2 \ -0.1 \ 0.2]^T$, $\dot{\mathbf{q}}(0) = \ddot{\mathbf{q}}(0) = \mathbf{0}$ and final conditions $\mathbf{q}(1) = [-0.5 \ 0.5 \ 0.2 \ -0.2]^T$, $\dot{\mathbf{q}}(1) = \ddot{\mathbf{q}}(1) = \mathbf{0}$. The generated trajectory $\mathbf{q}(t)$ and its derivative $\dot{\mathbf{q}}(t)$ for this scenario are shown in Figure 8. For the described trajectory, the solution forces for the three inverse dynamics cases are shown in Figure 9.

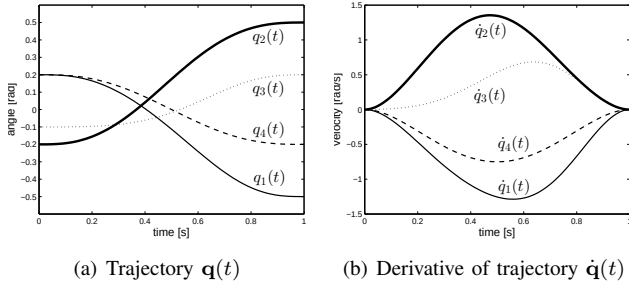


Fig. 8. Trajectory 2 for the 2-link manipulator example.

One known characteristic of the minimisation of $Q(\mathbf{f}) = \mathbf{f}^T \mathbf{f}$, as observed in Figure 9(a), is that it aims to distribute the use of cables more evenly as the quadratic sum penalises excessive force in a single cable. However, this resulted in high interaction forces at the joint as shown in Figure 10(a).

By performing minimisation of the interaction force $|\mathbf{F}_{P_1}|$, the cable forces solution from Figure 9(b) produced a lower interaction force at the spherical joint (Figure 10(b)). This is most significant at $t \approx 0.5$ s where the peak magnitude decreased from 572 N to 392 N. Comparing $\mathbf{f}(t)$ for case 1 (Figure 9(a)) and case 2 (Figure 9(b)) in more detail, it can be observed that the reduction in interaction force at the spherical joint is achieved by decreasing the forces in cables 5 to 8 attached to link 2 of the manipulator. At $t \approx 0.5$ s and $0.6 \leq t \leq 1$ s, the forces in cables 1 to 4 were increased while the forces in cables 5 to 8 significantly lowered. This can be explained by the fact that the moment arms produced by the cables attached to link 2 are larger than that of link 1, and hence producing higher interaction forces at the spherical joint.

The interaction angle at the spherical joint for cases 1 to 3 are shown in Figure 11. It can be seen in Figures 11(a) and 11(b) that for both cases 1 and 2, respectively, the interaction angle exceeded the maximum angle $\rho_1^* = 30^\circ$. With the inclusion of the interaction angle constraint (case 3), it is shown in Figure 11(c) that the constraint was satisfied. For the time period $0 \leq t \leq 0.6$ s, the cable forces for case 3 (Figure 9(c)) were the same as that with case 2 (Figure 9(b)) as the interaction angle constraint was not violated. However, for $0.6 < t \leq 1$ s it can be observed that the cable forces were redistributed in order to satisfy the interaction angle constraint. Compared with case 2, the solution to case 3 resulted in both higher cable and interaction forces, as seen by Figures 9(c) and 10(c), respectively.

B. 8-link 24-DoF 8-Spherical Neck-Inspired Manipulator

To demonstrate the scalability of the proposed inverse dynamics formulations, the analysis was performed on the human neck inspired 8-link 24 degree-of-freedom mechanism actuated by 76 cables [18], [25] as shown in Figure 12.

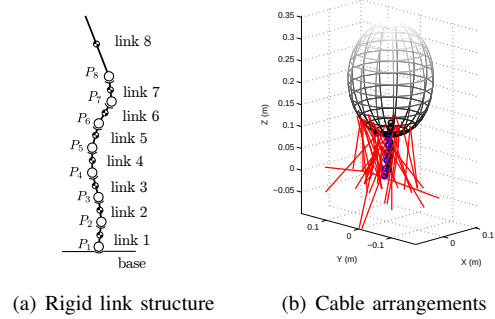


Fig. 12. Model of the 8-link 24 degree-of-freedom system.

As displayed in Figure 12(a), the 8-link system is connected by unilateral spherical joints. The joint location P_k denotes the location that link k is connected to link $k - 1$. The generalised coordinates for the system $\mathbf{q} = [\mathbf{q}_1^T \dots \mathbf{q}_8^T]^T$ can be represented by 8 sets of Euler angles, where $\mathbf{q}_k = [\theta_k \ \phi_k \ \psi_k]^T$ are the xyz -Euler angles of joint k . The cable-routing and attachment locations were obtained from that of a human neck described in [25] and are visualised in Figure 12(b).

Using the generalised model presented in [18], the interaction forces acting on the system joints were determined by (11). The simulated trajectory for this example was a roll motion trajectory (left to right tilting of the head). The trajectory (pure rotation in the x axis) was generated by interpolating from the initial pose $\mathbf{q}_1 = \dots = \mathbf{q}_7 = [-\frac{\pi}{45} \ 0 \ 0]^T$, $\mathbf{q}_8^T = [-\frac{\pi}{30} \ 0 \ 0]^T$ to the final pose $\mathbf{q}_1 = \dots = \mathbf{q}_7 = [\frac{\pi}{45} \ 0 \ 0]^T$, $\mathbf{q}_8^T = [\frac{\pi}{30} \ 0 \ 0]^T$ with zero initial velocities and accelerations. The trajectories were generated in the same manner as that for the 2-link manipulator example in Section VI-A.

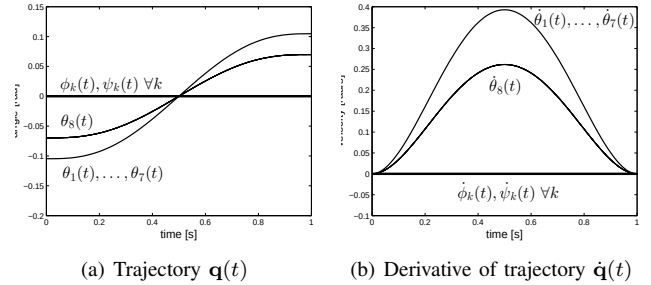


Fig. 13. Trajectory for the 8-link manipulator example.

Details for the three inverse dynamics cases that were described at the beginning of Section VI are:

- 1) Minimisation of $Q(\mathbf{f}) = \mathbf{f}^T \mathbf{f}$ subject to $\mathbf{f}_{min} \leq \mathbf{f} \leq \mathbf{f}_{max}$ (traditional method).
- 2) Minimisation of the magnitudes of the interaction forces $Q(\mathbf{f}) = \sum_{a=1}^8 |\mathbf{F}_{P_a}|^2$, where $\alpha_a = \frac{1}{8}, \beta_a = 0$ satisfies (8), subject to $\mathbf{f}_{min} \leq \mathbf{f} \leq \mathbf{f}_{max}$.

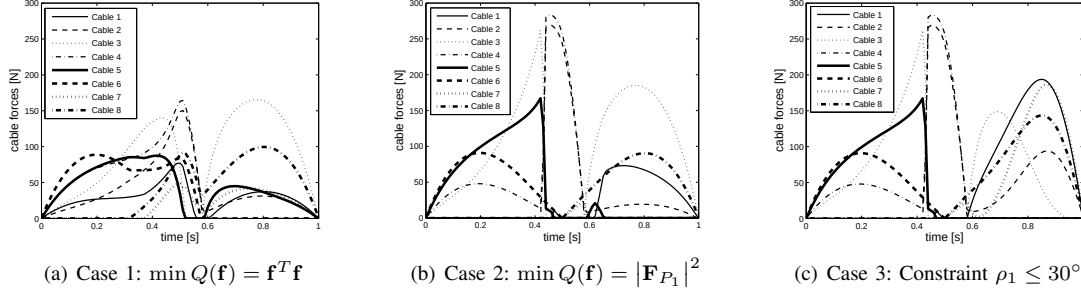


Fig. 9. Profiles of the cable forces $\mathbf{f}(t)$ of different inverse dynamics problems for trajectory 2 of the 2-link manipulator example. Compared with the forces shown in (a) for $Q(\mathbf{f}) = \mathbf{f}^T \mathbf{f}$, (b) and (b) show a redistribution of cable forces in order to minimise the interaction forces at the spherical joint. From $t = 0.6$ s to $t = 1$ s, the cable forces in (c) are redistributed from (b) to ensure that the interaction angle constraint is satisfied.

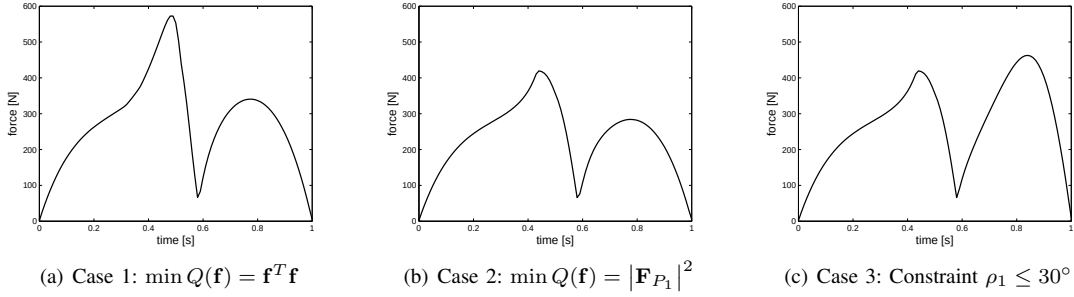


Fig. 10. Profiles of the interaction force on the spherical joint $|\mathbf{F}_{P_1}|$ of different inverse dynamics problems for trajectory 2 of the 2-link manipulator example.

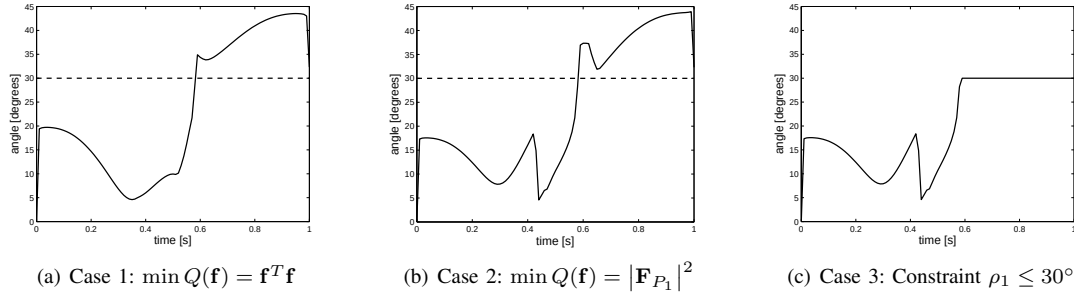


Fig. 11. Profiles of the interaction angle on the spherical joint $\rho_1(t)$ of different inverse dynamics problems for trajectory 2 of the 2-link manipulator example.

- 3) Minimisation of the magnitude of the interaction forces, subject to $\mathbf{f}_{min} \leq \mathbf{f} \leq \mathbf{f}_{max}$ and a joint interaction angle constraint of $\rho_a \leq 15^\circ \forall a$ on all joints.

Figure 14 shows the resulting cable forces for the three different inverse dynamics problems. Similar to the 2-link examples in Section VI-A, it can be clearly observed that a redistribution of cable forces occurred to satisfy the objective and constraints. The resulting interaction forces at the joints from cases 1 and 2 are shown in Figure 15. Comparing Figure 15(b) with Figure 15(a), it can be seen that the increased cable forces in Figure 14(b) resulted in lower magnitudes of interaction forces across all of the joints.

For both cases 1 and 2, Figures 16(a) and 16(b) show that the interaction angle constraint of $\rho_a \leq 15^\circ$ was violated by at least some of the manipulator joints. However, if the constraint was incorporated (case 3), it could be observed in Figure 16(c) that it was possible satisfy the constraint by the redistribution of the cable forces. However, as with the 2-link example, the

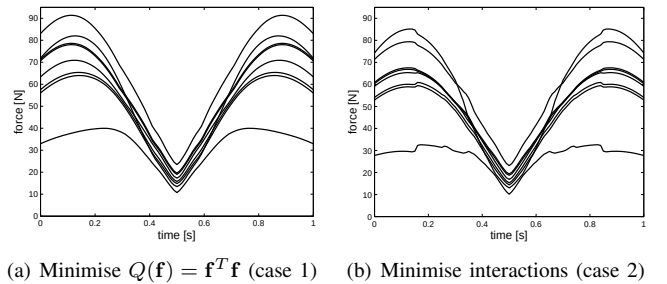


Fig. 15. Profiles of the interaction forces $|\mathbf{F}_{P_a}| \forall a = 1, \dots, 8$ at the joints of the 8-link manipulator example.

magnitude of the cable forces required to generate the motion (Figure 14(c)) significantly increased.

From the cable force profiles for cases 2 and 3 in Figures 14(a) and 14(b), respectively, abrupt changes in cable forces

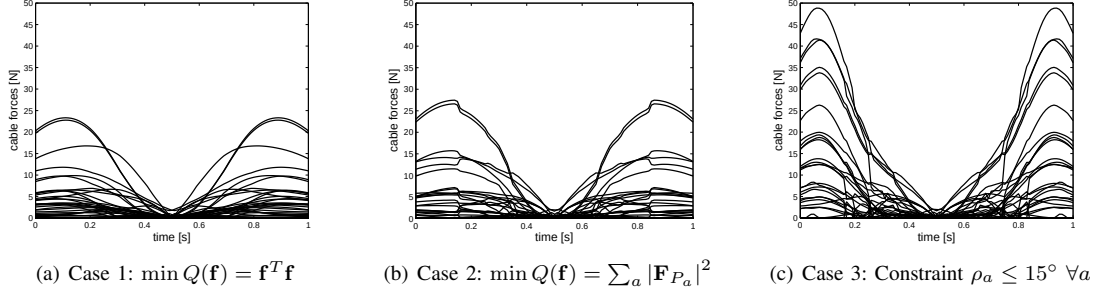


Fig. 14. Profiles of the cable forces $\mathbf{f}(t)$ of different inverse dynamics problems for the 8-link manipulator example. Compared with the forces shown in (a) for $Q(\mathbf{f}) = \mathbf{f}^T \mathbf{f}$, (b) and (b) show a redistribution of cable forces in order to minimise the interaction forces. Furthermore, (c) shows that increased cable forces have been used to ensure that the interaction angle constraint is satisfied.

$\frac{\partial f_i}{\partial t} \approx 0$ can be observed in the solution. This is due to the fact that the minimisation of interaction forces (14) is a convex problem and is not strictly convex, since the matrix H is positive semidefinite and not positive definite. As such, multiple global minima may exist resulting in possible abrupt changes in cable forces. In practical implementation, one possible approach to resolve this issue is to minimise or constrain the change in cable forces.

C. Computational Speed

A comparison of the computational speed for the proposed formulations with the traditional 2-norm minimisation was performed to assess the viability for implementation in practical applications. All cases of the inverse dynamics problems were performed on the same hardware with the CPU Intel®Core™ i7-3520M @ 2.90Hz and 8 GB of RAM. Two different QP solvers, object orientated quadratic programming (OOQP) [26] and interior point optimiser (IPOPT) [27], were used through the OPTI Toolbox [28] software running on MATLAB 2014a 64-bit version. Table I displays the computational times used to resolve one instance of the problem for the different trajectories and inverse dynamics formulations.

The mean computational time μ_{time} for each example trajectory was computed by taking the average time required to compute the inverse dynamics problem at each instance in time for the trajectory over 5 repeated trajectory runs, a total of 505 instances of the inverse dynamics problem. The table also includes the standard deviation of the computational time and average number of iterations for each problem. Since cases 1 and 2 are convex, as shown in Section IV, OOQP could be applied to such problems. However, only IPOPT could be applied to case 3 as the quadratic constraint had the possibility to be non-convex.

From the resulting computational times, it can be observed that the OOQP solver was able to resolve the cable forces much faster than IPOPT for the same problem. For cases 1 and 2 for the 2-link example with 8 cables, the mean time μ_{time} for OOQP was less than 1 ms and for IPOPT was approximately 5 ms. As expected, no observable difference in computational time was found in the minimisation of the magnitudes of interaction forces and moments (case 2) compared with the 2-norm minimisation (case 1) as both problems are convex QPs. However, since the consideration of interaction constraints

(case 3) results in a possibly non-convex problem, IPOPT was needed to be used. This resulted in longer computational times to resolve the inverse dynamics at approximately 5 ms, similar to the time required to resolve cases 1 and 2 using IPOPT.

For the complex 8-link example with 76 cables, the computational times were significantly higher. The mean computational times for cases 1 and 2 using OOQP increased from less than 1 ms to approximately 6 and 9 ms, respectively. Furthermore, IPOPT for cases 1 and 2 required an average of approximately 9 and 16 ms, respectively. For this example, the interaction angle constraint QCQP problem using IPOPT increased the computational effort to an average of 39 ms. From the measured computational times, for simpler problems of lower dimensions the proposed approaches could be used in practical applications. As the number of cables and number of degrees of freedom increases, both the proposed formulations and the traditional QP approach suffer from increased computational costs. However, the computational effort required for the proposed approaches are comparable to that of the traditional minimisation of the 2-norm of cable forces.

VII. CONCLUSION

The inverse dynamics problem for MCDMs with the consideration of interaction forces and moments was formulated. The minimisation in the magnitude of the interaction forces and moments was shown to result in a convex quadratic program. Furthermore, the inclusion of interaction force constraints was formulated. These formulations were simulated for two MCDM examples: a 2-link 8-cable system and an 8-link 76 cable system. It was shown that the redistribution of cable forces can result in the minimisation or constraint satisfaction on the joint interaction forces and moments. Future work would focus on increasing the computational efficiency in resolving the inverse dynamics of MCDMs.

VIII. ACKNOWLEDGEMENT

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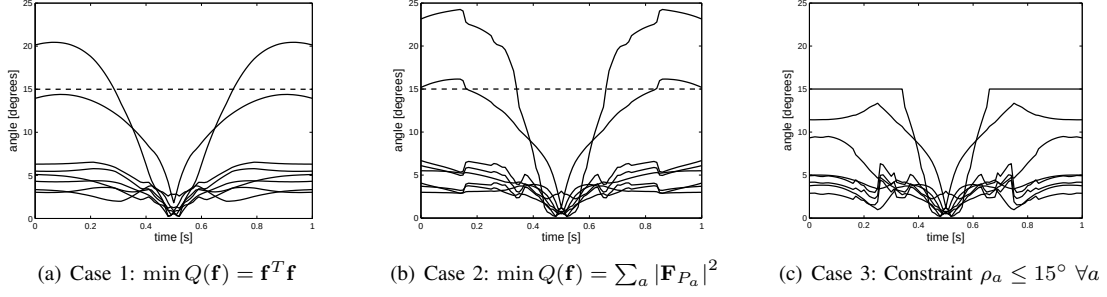


Fig. 16. Profile of the interaction angles on the joints for different inverse dynamics formulations. Compared with the interaction angles for $Q(\mathbf{f}) = \mathbf{f}^T \mathbf{f}$ as shown in (a), the interaction angles in (b) increased when minimising for the joint interaction forces. (c) shows the interaction angles when a constraint of $\rho_a \leq 15^\circ \forall a$ was applied. Without the constraint, the interaction angles exceeded 15° as shown in (a) and (b).

TABLE I
COMPUTATIONAL COSTS FOR THE DIFFERENT INVERSE DYNAMICS PROBLEMS AND EXAMPLES

Example	Case 1						Case 2						Case 3		
	OOQP [26]			IPOPT [27]			OOQP [26]			IPOPT [27]			IPOPT [27]		
	μ_{time} [s]	σ_{time} [s]	μ_{iter}	μ_{time} [s]	σ_{time} [s]	μ_{iter}	μ_{time} [s]	σ_{time} [s]	μ_{iter}	μ_{time} [s]	σ_{time} [s]	μ_{iter}	μ_{time} [s]	σ_{time} [s]	μ_{iter}
2-link trajectory 1	0.00043	0.00043	17	0.00576	0.00072	12	0.00044	0.00046	17	0.00375	0.00044	6	0.00578	0.00137	8
2-link trajectory 2	0.00041	0.00065	11	0.00464	0.00316	10	0.00074	0.00339	23	0.00981	0.05007	25	0.00662	0.00317	9
8-link example	0.00592	0.00078	15	0.00935	0.00186	13	0.00993	0.00133	13	0.01586	0.00297	20	0.03867	0.03170	21

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