GAYNOR WILLIAMS EXPLAINS AN APPROACH THAT CAPTURES PEDAGOGICAL COMPLEXITY IN ORDER TO ENGINEER OPPORTUNITIES FOR STUDENTS TO FIND THE 'SPACE TO THINK.'

Space to think

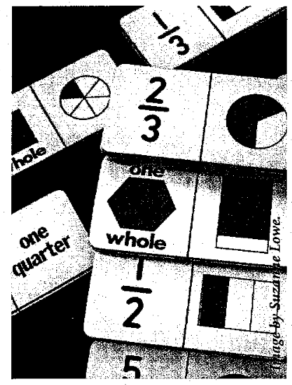

Image by Suzanne Lowe.

When I was invited to speak at the Deakin University Charting Futures Forum, held in Melbourne back in May, I was asked to speak as an innovative teacher of mathematics who had researched my own practice and the practice of 'good' teachers internationally. My long-term passion has been to improve the quality of the mathematics learning experiences for students. For too long, many students have been afraid of mathematics as a subject. Many have erected 'psychological barriers' that have inhibited their mathematics learning, or have been so bored by the repetitive nature of the learning experience that they have just stopped listening.

We need to find ways to change this situation before we have another generation of students who have not had the opportunity to experience success and pleasure in the learning of mathematics. The absence of high-quality learning experiences where complex thinking and positive affect co-exist in mathematics classrooms in the United States and Australia has been associated with the provision of tasks lacking complexity, student alienation from school, and increased adolescent depression. My multi-domain approach to teaching and research – including the domains of mathematical problem solving, psychological, cognitive and social elements of the problem-solving process, and resilience associated with mental well-being – helps to unravel the interconnections between these diverse research domains and to plot our educational future.

THE IMPETUS TO EXPERIMENT WITH MY TEACHING APPROACH

As a secondary mathematics teacher in the late 1970s, I remember rushing to the local rural primary school at recess to see my young daughter displaying her creation in the Easter Hat parade. The primary school library was filled to bursting point with students seated on the floor waiting in anticipation for the show to begin. Suddenly there was a ripple of warm laughter as the students craned their necks towards the back of the library to see the Year Five teacher entering the room resplendent in his hat. This teacher was considered to be a mathematics expert by his school community. From the front of the room I couldn't see his hat clearly but there was obviously something about it that had created a surge of energy that reverberated throughout the library. I wondered what had created that. As he approached, the sign on his hat became clear: 'I Hate Maths.' This was a turning point for me. I began to wonder how we could turn all this negative energy associated with learning mathematics into positive energy. (Williams, 2003)

CLASS COLLABORATION

Fifteen years later I was reasonably satisfied with what I had achieved – a teaching approach I called 'class collaboration' because students collaborated at two levels: in small groups to develop ideas, and in whole-class settings to report and interconnect their findings. (Williams, 2002) I developed complex tasks, which generally extended between two and six lessons that had less than ten minutes of 'teaching' beforehand. Students focused on using the task to explore new mathematics rather than rushing to completion.

During the small-group and whole-class collaborative cycles I asked questions to assist students to clarify the problem, decide upon a plan of action, select useful specific examples, search for patterns, consider whether their results were reasonable, look for reasons why patterns worked, and develop clear mathematical arguments. Examples of these questions appear in research undertaken by others and by me in my classrooms. Here are a few examples:

Oh! That's an interesting idea, I wonder does that work?

Can they really say they have a pattern when they have only worked with one triangle? . . . I hope they looked at some really unusual triangles. (Groves and Doig, 2004)

I wonder why both ways are giving the same answer . . . ?

Why does it have to be . . . ? (Barnes, 2000: 38)

Well if it really is so, why is it so? (Groves and Doig, 2004)

That's neat! I wonder is it useful for anything else?

These questions encouraged students to employ more complex thinking.

The suggestions I developed to stimulate the whole-class reporting process – unbeknownst to myself at the time – contained resilience-building features. (Seligman, 1995) I've since found resilience to be associated with student inclination to explore the unfamiliar. These resilience-building features are identified below in italics after each suggestion of a topic for brief reports occurring after each ten to twenty minutes of group work:

- Something you can't work out and are hoping others might help with. *Failure is temporary.*
- Something you didn't know before and you do know now. *You turned failure into success.*
- A pattern you have found. *You attained success.*
- A reason to explain why this pattern works. *Increased challenge once smaller challenge was overcome.*
- Or anything else – you choose. *I have faith in your ability.*

STUDENT RESPONSES TO CLASS COLLABORATION

I was surprised and delighted when students began to express their enjoyment of the class-collaboration process, saying they were learning and understanding far more than they ever had before. Here are some student responses:

The way we learned to solve problems in Specialist Maths has helped me more in my medical studies than anything else I learned at school. Year Twelve Specialist Maths female student when a third year Medical student.

It is so exciting when you know you can report something that you know no one else has found. Year Twelve Maths Methods male student.

I've really enjoyed seeing how other people think about a problem. I had always thought we all did it the same way. Year Nine male student.

We can stay at recess and finish our reports. Year Seven low-achieving male student who was generally uninterested in mathematics. It was his turn to report when the bell went.

I found there were times when students were so involved that they continued their intense discussions oblivious to our preparations for the next reporting session. There were other times when students remained after the final bell for the day to continue their mathematical discussions. I began to wonder what made it all so special.

SHAPING RECTANGLES

You are going to think about the rectangles you can make with square tiles. In each case you have to use all of the number of tiles stated in making a rectangle.
Always give all the answers your group can think of for each question.
Insides of rectangles must be filled with tiles.

Question 1. What is a rectangle? List all the things your group knows about what a rectangle is.

Question 2. If you use fourteen tiles, what rectangle can you make?

Question 3. If you use twelve tiles, what rectangle can you make?

Question 4. What rectangle could you make if you had sixteen tiles?

Question 5. Can you make the same number of rectangles with each number of tiles? Why? Explain your group's thinking.

Question 6. Can you give a mathematical argument to show you have found all possible rectangles that can be made using sixteen tiles?

Question 7. Your group is going to choose how many tiles to use for this question. You can choose any number of tiles between 1 and X (The number has been replaced with X for this article).

a) Choose the number of tiles that will make the most rectangles. Is there another number of tiles that will also make this many rectangles?

b) Explain how you know there is not a number of tiles that will make more rectangles than that?

c) Think about the answer you have given in 7b. Can you rewrite your thinking so it sounds more like a mathematical argument? Or, can you explain why you don't need to rewrite it because you are satisfied that it already sounds like a mathematical argument.

THEORISING PRACTICE

I returned to university and undertook research in my Specialist Mathematics classes to find out. I found the state of 'flow' was consistent with what I had observed. Flow (Csikszentmihalyi and Csikszentmihalyi, 1992) is a state of total involvement where all energies are directed to the task at hand and feelings of pleasure are experienced. This state can be achieved when people work just above their present skill level on a self-set challenge that is almost out of reach. The state of flow cannot be sustained for long unless complexity is increased. I developed the conditions for flow specific to mathematical problem solving and called this process 'discovering complexity.'

I found it was not the actual task that created the engagement, it was the mathematical complexities the students discovered for themselves and wondered about and experimented with as they worked with the task that did the trick. Different groups discovered different complexities so when they combined their ideas in the reporting sessions they began to see the mathematics in the task from a variety of perspectives. Each group was setting its own intellectual challenge that was just beyond the present conceptual level of the group members. The task above is used to explain this process.

COMPLEX TASKS

It's important that tasks are developed in such a way that they can be accessed through numerical examples, developing patterns and thinking about why patterns work. Consider the task above. This is presented without teaching the students about arrays, areas of rectangles or factors just prior to the task. The task is accessible whether or not this work has been covered in the past. By not identifying these mathematical topics as relevant, students have the opportunity to recognise their relevance for themselves or to develop these concepts as they explore. This contributes to the spontaneity of their exploration, spontaneity of exploration being a condition for flow.

My tasks are designed with the intention to surprise. Some students are surprised to find twelve tiles make more rectangles than fourteen tiles – Questions 2 and 3 – and begin to wonder why. Other students may not become engaged until they reach Question 6 where they need to show they have found all possibilities. At this stage, students can begin to develop systematic ways to find factors. It could be the first time they have had to think about developing an argument to show they have exhausted all possibilities.

For this article I have intentionally replaced one of the numbers in the task with an 'X' to provide you with the opportunity to construct part of the task by considering the nature of a number that could replace the 'X.' We need a number that:

- is lower than 50 – so the working is not tedious
- makes quite a few different rectangles – to entice students to select this endpoint
- does not make the largest number of rectangles – to demonstrate that careful thinking and checking is required.

By selecting 'X' carefully, some groups think they have reached closure too early. Their surprise arises when they find that a group they had not considered to be 'good at maths' finds something they did not. Such experiences entice students into analysis, synthesis and evaluation (Williams, 2002) as they search for ways to check the mathematics they have generated.

To retain the challenge and excitement, it is important that no one who already knows has the opportunity to 'tell' those who don't during small-group work. Instead of composing groups for peer tutoring, students in the same group need to have similar mathematical backgrounds and think at similar paces.

What is the role of the teacher during such a task? My PhD research within the Learners' Perspective Study (Clarke, 2001) provides an opportunity to reflect upon this.

FACTORS THAT PROMOTE HIGH-QUALITY LEARNING SITUATIONS

Of five teachers identified by their school communities in Australia and the US as 'good' teachers, none specifically focused upon providing opportunities for the type of high-quality learning associated with flow. These teachers were expert in providing safe emotional environments, but students in these classrooms rarely engaged in thinking more complex than analysis. It appeared that most of these teachers overgeneralised their students' need for emotional security to a need for 'cognitive protection.' These teachers said their efforts were aimed at ensuring students did not have to struggle with their mathematics. In spite of this, five of their eighty-six students spontaneously manoeuvred their own 'space to think,' creating high-quality learning where complex thinking and high positive affect co-existed.

The inclination to explore

Each of the five students – of varying achievement levels – who created 'space to think' was found to be resilient, inclined to explore the unfamiliar and bounce back from problems encountered. This finding raises questions about whether we should be building students' resilience to improve their mathematical problem-solving ability rather than just focusing on the mathematical problem solving process! If resilience is important, we need to broaden our research focus to study resilience building through mathematics learning – such as occurs during class collaboration.

Questions to structure the exploration

The structuring of questions to propel exploration forward was undertaken by these students but not by their teachers. Examples of these questions follow, with the type of thinking involved in answering the question in italics.

Can I use this for anything else? Kerri. *Evaluation.*

How can we express this relationship (generally)? Leon and Pepe. *Synthesis.*

Which way is better? Leon. *Evaluative analysis.* (Williams, 2004)

Is there another way? Leon. *Synthetic analysis.* (Williams, 2004)

How could that help? Leon. *Analysis.* (Williams, 2004)

These questions parallel the questions asked during class collaboration by the teacher and students. Development of these questioning techniques could reduce the desire felt by teachers to provide cognitive protection.

SURVEYING THE FUTURE

Intellectual quality has been recognised as a feature of schools in which higher learning gains are achieved. (Luke, Elkins et al., 2003) Intellectual quality when associated with high positive affect has the potential to assist students to interconnect mathematical concepts, decrease alienation from school, and build resilience to improve mathematical problem solving and buffer against depression. Such opportunities are too good to miss, and class collaboration illustrates one way this could be achieved. We need to broaden our research focus to explore resilience in the context of mathematics learning. We must raise teacher awareness of the differences between emotional security and 'cognitive protection' and scaffold teacher realisation that, although it is productive to promote the former, the latter should be replaced by opportunities for students to focus spontaneously on appropriate intellectual challenges. When teachers are supported through the provision of structured questions relating to their pedagogical explorations as they develop pedagogical approaches that promote high-quality learning, they are likely to build their own resilience and thus be better equipped to build the resilience of their students. How could we decide to do anything other than pursue such learning opportunities? ▮

Gaynor Williams works with the Brunswick Cluster of the Victorian Department of Education and Training, associated with the University of Melbourne, and is a recipient of a National Excellence in Teaching Award, an Australian Association of Mathematics Teachers Innovative Teaching Award and a Mathematics Education Research Group of Australasia Early Career Award. This article is a version of her address at the Charting National Futures Forum convened by Deakin University in Melbourne in May.

For references go to www.educare.com.au/news_references.html