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PDE Battery Model Simplification for Charging Strategy Evaluation

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***Abstract*—A safe, fast charging strategy is desired in the utilisation of rechargeable Lithium-ion batteries. Traditionally, experimental methods are used in exploring and evaluating new strategies, but these require extensive time and cost. This paper aims to establish a model-based system for quick and accurate evaluation of charging strategies. Starting from a nonlinear coupled partial differential equation (PDE) battery model that accurately captures system dynamics, simplification techniques are conducted based on the identification of separable time scales within the states. By pertinent use of a singular perturbation approach, a PDE model simplification framework containing families of simplified battery models is established. All assumptions are explicitly stated and shown to enable families of simplified models to be rigorously justified. An evaluation procedure synthesised from the simplified models and averaging theory is proposed. This procedure is implemented on several typical battery charging strategies. The benefits relative to simulation on other higher order models are assessed in terms of computational efficiency and accuracy and demonstrate significant computational savings are possible with the proposed approach.**

I. INTRODUCTION

For energy storage and mobile electrification in ubiquitous applications like communication technologies, power tools, and electrified vehicles, Lithium-ion (Li-ion) batteries are considered a critical enabling technology. Compared to alternative cell chemistries like lead-acid and nickel-mental-hydride batteries, Li-ion batteries are superior in terms of high energy/power density, long lifetime, no memory effect and low self-discharge rate, but their uptake is greatly limited by their high price [1]. To avoid high cost in maintenance or replacement, batteries have to be operated in an appropriate manner that does not rapidly degrade their state-of-health (SOH). SOH degradation is highly dependent on the charging strategies used [2]. Aside from battery's lifetime, its ability to supply requested power demand and absorb currents requested for fast charging is also very important. Developing an optimal charging strategy to address these concerns is thus necessary.

Research on battery charging strategies has experienced considerable growth with a magnitude of examples in the literature, e.g. [2]-[9]. Initially, trickle charging, whereby a small constant current is chosen during the entire process, was used to restore battery charge without unduly impacting SOH. To achieve accelerated charging rates, the constant-current constant-voltage (CCCV) protocol [3] is commonly used in current battery management systems. However, this method is recognised to be suboptimal for both charging efficiency and battery health [4], [5]. A boostcharging scheme is suggested by [6] and demonstrates that very high current rates can be used for batteries at low state-of-charge levels for a short time. Motivated by this approach, multi-stage constant current algorithms with piece-wise decreased charging rates are proposed in [7]. Through intensive experiments these algorithms have been shown to offer advantages relative to the CCCV method. Other improved charging strategies such as modified CCCV, combination of several CC and CV, and optimal control-oriented methods are presented in [2], [8], [9].

To demonstrate the advantages of new methods, experiments are typically designed and conducted. These require a battery screening test, preprocessing, charge and discharge operations, and state characterisation. These methods, however, are very costly and time-consuming. Moreover, charging performance relates to various, coupled in-situ dynamics of the battery internal states [3] like Li-ion concentration and local electrolyte current, and is potentially hard to accurately analyse in an experimental manner with only input-output data available. In this regard, a model-based simulation is essential for quickly and accurately evaluating any given charging strategy.

Physics-based battery models that describe electrochemical, thermal, and ageing behaviors are respectively presented in [4], [10], and [11]. Concatenation of all these models forms a very complex structure mathematically represented by a series of coupled nonlinear partial differential equations (PDE). The simulation of such a system is highly limited by its excessive demand on computational resources. To enable a model-based charging strategy evaluation, model simplification has to be conducted. However, to date, there is no rigorous methodology available for battery model simplification.

In this paper, we develop an efficient model-based system for battery charging strategy evaluation. A physics-based battery model that captures all the electrochemical, thermal, electrical, and ageing dynamics is proposed and used as a starting point for model simplification. By using a singular perturbation approach, a framework for PDE battery model simplification is established with families of simplified models obtained and the underlying assumptions explicitly stated and justified. Based on the simplified models from such a framework and through judicious use of averaging theory, an evaluation procedure for charging strategies is proposed and demonstrated.

II. LI-ION BATTERY MODELING

A. *Overview of the Proposed Model*

The dynamics in a Li-ion battery cell represent the Li-ion flow between the solid particles and electrolyte as well as the electrodes and separator, and the resulting normal intercalation reaction and side reaction. The starting point of this work is a complete battery model incorporating electrochemical (Σ^{ec}), thermal (Σ^{T}), electrical (Σ^{e}), and ageing dynamics (Σ^{SOH}), as summarised in Fig. 1. The nomenclature for battery states and parameters is listed in Table I.

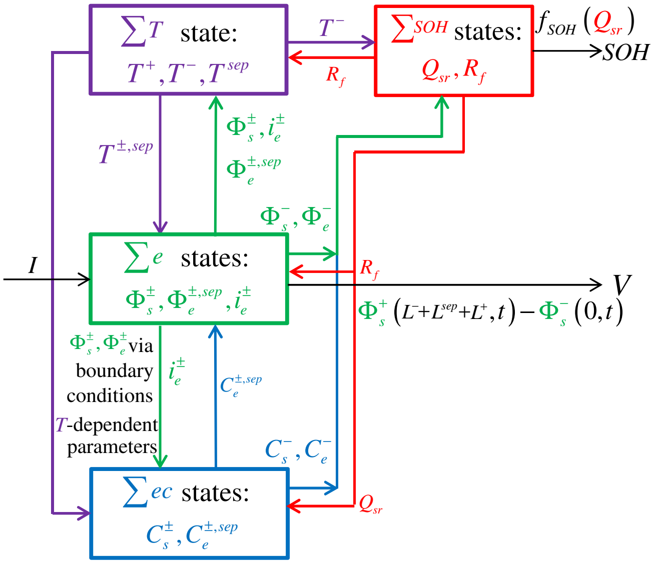

Fig. 1. Block diagram representation of a complete Li-ion battery model including the system input, outputs, state variables and coupling relationships among the electrochemical, electrical, thermal, and ageing subsystems.

During the side reaction of a battery, the formation and growth of a passive solid-electrolyte interface (SEI) film leads to capacity fade in the negative electrode, and this is accompanied by consumption of the active Li-ions in the positive electrode [11]. Based on the mass conservation law applied to the Li-ion concentrations, the lost number of Li-ions per volume, C_{loss}^{+}, in the positive electrode is given by

$$C_{\text{loss}}^{+}(x,r,t)=\frac{Q_{sr}(x,t)}{Q_{\max}}\bar{C}_s^{+}(x,r,t)\,. \tag{1}$$

Without consideration of any ageing phenomena, the mass transport in the solid particles of each electrode can be described by Fick's law of diffusion [4]

$$\frac{\partial \bar{C}_s^{\pm}(x,r,t)}{\partial t}=\frac{D_s^{\text{eff},\pm}}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \bar{C}_s^{\pm}(x,r,t)}{\partial r}\right). \tag{2}$$

In existing models of capacity fade, the initial cathode Li-ion concentration is updated in a discontinuous jump at the end of each charging operation to capture the ageing effect. To describe the continuous interaction between Li-ion loss and capacity fade, we propose the comprehensive concentration dynamics in the positive electrode based on the concentration conservation property

$$\frac{\partial C_s^{+}(x,r,t)}{\partial t}=\frac{\partial \bar{C}_s^{+}(x,r,t)}{\partial t}-\frac{\partial C_{\text{loss}}^{+}(x,r,t)}{\partial t}. \tag{3}$$

TABLE I
NOMENCLATURE

A	equivalent cross-sectional area
a	specific interfacial area at solid particle surface
$\bar{C}_s$	ideal solid-phase Li-ion concentration
C_s, C_e	Li-ion concentration in solid particles/electrolyte
C_{loss}	consumed Li-ion concentration
$D_s^{\text{eff}}, D_e^{\text{eff}}$	effective electric/ionic diffusion coefficient
F	Faraday's constant
I, i_e	input current density/local electrolyte current
i_0, i_{0sr}	exchange current density of normal/side reaction
J_I, J_{sr}	Li-ion flux of normal reaction/side reaction
L	total length of the battery cell
M_f	average molecular weight of SEI film
$Q_{sr}, Q_{\max}$	capacity fade/maximum nominal capacity
R, R_f, R_p	universal gas constant/SEI film resistance/radius
T, c	temperature/specific heat
Φ_s, Φ_e	electric potential in solid particles/electrolyte
η, η_{sr}	overpotential of normal reaction/side reaction
γ	transference number of the anion
μ_e	volume fraction of electrolyte
$\sigma^{\text{eff}}, \sigma_f$	effective electronic/SEI film conductivity
κ^{eff}	effective ionic conductivity
ρ, ρ_f	mass density of battery cell/SEI film
λ, δ_S	heat conductivity/entropy change
α_n, α_p	anodic and cathodic charge-transfer coefficient
α_{sr}	charge-transfer coefficient of side reaction
$-, +, sep$	superscript (negative/positive electrode/separator)
x, r	spatial coordinate

Combination of (1), (2), and (3) leads to

$$\begin{aligned}\frac{\partial C_s^{+}(x,r,t)}{\partial t}&=\frac{\partial \bar{C}_s^{+}(x,r,t)}{\partial t}-\frac{Q_{sr}(x,t)}{Q_{\max}}\frac{\partial \bar{C}_s^{+}(x,r,t)}{\partial t}\\&\quad-\frac{\bar{C}_s^{+}(x,r,t)}{Q_{\max}}\frac{\partial Q_{sr}(x,t)}{\partial t},\end{aligned} \tag{4}$$

where the first term of the right-hand side of (4) represents the normal Li-ion diffusion, and the last two terms denote Li-ion loss caused by battery side reaction. Inside the negative electrode, the active Li-ion loss is typically ignored [11], [12], allowing the state behavior of Li-ion concentration to be represented by (2).

The steady-state behaviors of local potentials and currents are provided in [4]. Their dynamic equations for the electrodes or separator j ($j\in\{+,-,sep\}$) can be explicitly formulated as

$$\epsilon_{\Phi s}\frac{\partial \Phi_s^{\pm}(x,t)}{\partial t}=\frac{\partial \Phi_s^{\pm}(x,t)}{\partial x}+\frac{I(t)-i_e^{\pm}(x,t)}{\sigma^{\text{eff},\pm}}, \tag{5}$$

$$\begin{aligned}\epsilon_{\Phi e}\frac{\partial \Phi_e^{j}(x,t)}{\partial t}&=\frac{\partial \Phi_e^{j}(x,t)}{\partial x}+\frac{i_e^{j}(x,t)}{\kappa^{\text{eff},j}}\\&\quad-\frac{2R\gamma^{j}T^{j}(x,t)}{F}\frac{\partial \ln C_e^{j}(x,t)}{\partial x},\end{aligned} \tag{6}$$

$$\epsilon_{ie}\frac{\partial i_e^{j}(x,t)}{\partial t}=\frac{\partial i_e^{j}(x,t)}{\partial x}-Fa^{j}J^{j}(x,t)\,, \tag{7}$$

where $\epsilon_{\Phi s}, \epsilon_{\Phi e}, \epsilon_{ie}$ are small positive parameters and their reciprocals are respectively related to the transmission speeds of electric potential, ionic potential, and electrolyte current. The total molar Li-ion flux J consists of the normal intercalation reaction flux and side reaction flux

$$J_I^{\pm}(x,t) = \frac{i_0^{\pm}(x,t)}{F}\left(e^{\frac{\alpha_n F\eta^{\pm}(x,t)}{RT^{\pm}(x,t)}} - e^{-\frac{\alpha_p F\eta^{\pm}(x,t)}{RT^{\pm}(x,t)}}\right), \quad (8)$$

$$J_{sr}(x,t) = \frac{-i_{0sr}}{F}e^{\frac{-F\alpha_{sr}\eta_{sr}(x,t)}{RT^{-}(x,t)}}. \quad (9)$$

Battery degradation is evaluated by the capacity fade and internal resistance rise in the negative electrode during charging operations [11]. To model the spatially distributed SOH variations accurately, we propose the dynamic equations of capacity fade and internal resistance in the following form

$$\frac{\partial Q_{sr}(x,t)}{\partial t} = -Fa^{-}A^{-}L^{-}J_{sr}(x,t), \quad (10)$$

$$\frac{\partial R_f(x,t)}{\partial t} = -\frac{M_f}{\rho_f\sigma_f}J_{sr}(x,t). \quad (11)$$

The dynamic equations for electrolyte concentration and temperature are provided by [4], [10]

$$\begin{aligned}\frac{\partial C_e^j(x,t)}{\partial t} =& \frac{\partial}{\partial x}\left(\frac{D_e^{\text{eff},j}}{\mu_e^j}\frac{\partial C_e^j(x,t)}{\partial x}\right) + \frac{\gamma^j}{F\mu_e^j}\frac{\partial i_e^j}{\partial x}, \quad (12)\\ \rho^j c^j\frac{\partial T^j(x,t)}{\partial t} =& \lambda^j\frac{\partial^2 T^j(x,t)}{\partial x^2} - (I(t) - i_e^j(x,t))\frac{\partial \Phi_s^j(x,t)}{\partial x}\\ & - i_e^j(x,t)\frac{\partial \Phi_e^j(x,t)}{\partial x} + Fa^jJ^j(x,t)\eta^j(x,t)\\ & + Fa^jJ^j(x,t)T^j(x,t)\delta_S^j. \quad (13)\end{aligned}$$

For space reason, the definition for overpotentials and all the initial and boundary conditions are not presented, but readers are directed to [4], [10], [13] for a comprehensive introduction.

In summary, the battery model including electrochemical, thermal, electrical, and ageing dynamics is proposed with 17 coupled nonlinear PDEs (4)-(13) over x and r spatial dimensions. For this dynamic system, the applied current density, $I(t)$, is the input, and the outputs of interest in this work are the terminal voltage, $V(t)$, and SOH:

$$V(t) = \Phi_s(L,t) - \Phi_s(0,t), \quad (14)$$

$$SOH(t) = 1 - \int_0^{L^-}\frac{Q_{sr}(x,t)}{Q_{\max}}dx. \quad (15)$$

B. Model Simplification

The initial model presented in previous section is known to capture battery system dynamics very well [4], [10], [11] but is too complex for most model-based applications. In the following we use a systematic procedure to develop a battery model simplification framework. Assumptions imposed on battery physical and chemical properties are gradually introduced, justified, and remarked. After the identification of separable time scales within battery state variables, a singular perturbation approach is used in the model simplification.

To deal with the infinite dimensional battery problem, we first reformulate the battery model (4)-(15) in Hilbert space to precisely present its mathematical structure. The state variables are defined as a vector $\bar{\mathbf{x}}{:=}[C_s^{\pm}, C_e^{\pm,sep}, \Phi_s^{\pm}, \Phi_e^{\pm,sep}, i_e^{\pm}, T^{\pm,sep}, Q_{sr}, R_f]^T$, and the system input is $\mathbf{u}{:=}I(t)$. Then the initial PDE-based battery system may be written in the general form

$$\frac{\partial\bar{\mathbf{x}}}{\partial t} = F_1\frac{\partial\bar{\mathbf{x}}}{\partial r} + S_1\frac{\partial\bar{\mathbf{x}}}{\partial x} + F_2\frac{\partial^2\bar{\mathbf{x}}}{\partial r^2} + S_2\frac{\partial^2\bar{\mathbf{x}}}{\partial x^2} + h(\bar{\mathbf{x}},\mathbf{u}), \quad (16)$$

subject to boundary conditions

$$\begin{aligned} C_1\bar{\mathbf{x}}\big|_{x=0} &= 0, & C_2\bar{\mathbf{x}}\big|_{x=L} &= 0,\\ C_3\frac{\partial\bar{\mathbf{x}}}{\partial x}\bigg|_{x=0} &= D_1, & C_4\frac{\partial\bar{\mathbf{x}}}{\partial x}\bigg|_{x=L} &= D_2,\\ C_5\frac{\partial\bar{\mathbf{x}}}{\partial r}\bigg|_{r=0} &= 0, & C_6\frac{\partial\bar{\mathbf{x}}}{\partial r}\bigg|_{r=R_p} &= 0. \quad (17)\end{aligned}$$

The initial conditions are given as

$$\bar{\mathbf{x}}(t=t_0) = \bar{\mathbf{x}}_0. \quad (18)$$

The domain of definition of the state variables is $\mathcal{D}(x,r) = \{(x,r)|x \in [0,L], r \in [0,R_p]\}$; the matrices $C_1,\cdots,C_8, D_1,\cdots,D_8$ and the vector functions F_1, F_2, S_1, S_2, and h can be derived from the dynamic equations (4)-(13); all the state variables are uniformly initialised at constant values as given in (18).

Define the state function $\mathbf{x}$ on the Hilbert space $\mathscr{H}(\mathcal{D},\mathbb{R}^n)$:

$$\mathbf{x}(t) = \bar{\mathbf{x}}(x,r,t),\ t>0,\ (x,r)\in\mathcal{D}(x,r),$$

and the operator $\mathcal{F}$ in $\mathscr{H}(\mathcal{D},\mathbb{R}^n)$ as:

$$\begin{aligned}\mathcal{F}\mathbf{x} &= F_1\frac{\partial\bar{\mathbf{x}}}{\partial r} + S_1\frac{\partial\bar{\mathbf{x}}}{\partial x} + F_2\frac{\partial^2\bar{\mathbf{x}}}{\partial r^2} + S_2\frac{\partial^2\bar{\mathbf{x}}}{\partial x^2},\\ \mathbf{x} \in D(\mathcal{F}) &= \{\mathbf{x}\in\mathscr{H}(\mathcal{D},\mathbb{R}^n);\ (17)\}.\end{aligned}$$

Then, the original battery system (16)-(18) may be represented compactly into the form

$$\dot{\mathbf{x}} = \mathcal{F}\mathbf{x} + h(\mathbf{x},\mathbf{u}),\ \ \mathbf{x}(0) = \mathbf{x}_0, \quad (19)$$

where $h(\mathbf{x}(t),\mathbf{u}(t)){=}h(\bar{\mathbf{x}}(x,r,t),\mathbf{u}(t))$ and $\mathbf{x}_0{=}\bar{\mathbf{x}}_0$.

Having presented the full order system in Hilbert space, we now commence the process of model simplification. We begin by defining $\epsilon_1 := \min\{\epsilon_{\Phi s}, \epsilon_{\Phi e}, \epsilon_{ie}\}$ and $\epsilon_2 := i_{0sr}$. The system (19) can be equivalently written as

$$\begin{aligned}\Sigma: \dot{\mathbf{x}}_s &= \epsilon_2\mathcal{F}_s\mathbf{x}_s + \epsilon_2 h_s(\mathbf{x}_s,\mathbf{x}_m,\mathbf{x}_f,\mathbf{u}),\ \mathbf{x}_s(0) = \mathbf{x}_{s0}, \quad (20)\\ \dot{\mathbf{x}}_m &= \mathcal{F}_m\mathbf{x}_m + h_m(\mathbf{x}_s,\mathbf{x}_m,\mathbf{x}_f,\mathbf{u}),\ \mathbf{x}_m(0) = \mathbf{x}_{m0}, \quad (21)\\ \epsilon_1\dot{\mathbf{x}}_f &= \mathcal{F}_f\mathbf{x}_f + h_f(\mathbf{x}_s,\mathbf{x}_m,\mathbf{x}_f,\mathbf{u}),\ \mathbf{x}_f(0) = \mathbf{x}_{f0}, \quad (22)\end{aligned}$$

where the state function $\mathbf{x}$ of system (19) is decomposed into $\mathbf{x}{=}[\mathbf{x}_s,\mathbf{x}_m,\mathbf{x}_f]^T$, with $\mathbf{x}_s{:=}[C_{s,s}, Q_{sr}, R_f]^T$, $\mathbf{x}_m{:=}[C_{s,m}^{\pm}, C_e^{\pm,sep}, T^{\pm,sep}]^T$, and $\mathbf{x}_f{:=}[\Phi_s^{\pm}, \Phi_e^{\pm,sep}, i_e^{\pm}]^T$. Since the state C_s simultaneously relates to concentration diffusion and Li-ion consumption, it becomes two states after separation, *i.e.* $C_s{=}C_{s,m}{+}C_{s,s}$. The operator $\mathcal{F} = \text{diag}(\mathcal{F}_s,\mathcal{F}_m,\mathcal{F}_f)$, and the function $h = [h_s,h_m,h_f]^T$. The functions h_s,h_m,h_f are continuous and bounded in their arguments for $(\mathbf{x}_s,\mathbf{x}_m,\mathbf{x}_f,\mathbf{u}){\in}\mathscr{H}_s{\times}\mathscr{H}_m{\times}\mathscr{H}_f{\times}\mathbb{U}$, where $\mathscr{H}_s$, $\mathscr{H}_m$, and $\mathscr{H}_f$ are subspaces of $\mathscr{H}$.

The model simplification is motivated by the insights into the battery dynamics and parameters. The following assumptions are imposed on battery parameters.

Assumption 1: Within battery operating processes, the parameter ϵ_1 satisfies $\epsilon_1 \ll 1$.

Justification for Assumption 1. Typically, a charge or discharge operation associated with diffusion process and chemical reaction is completed within several minutes or hours. While, the signal transmission speeds of potentials and currents are very large quantities leading to time constants of order microseconds [14]. Thus, in comparison with the electrochemical and thermal states which are nominally of order 1 in Σ, the dynamics of potentials and currents result in a sufficiently small ϵ_1.

Remark 1: The mass and thermal diffusion progress is negligibly slow in the fast time scale measured from the state dynamics of potentials and currents. In view of this, the states of Li-ion concentration and temperature can be considered in a compact set that includes the equilibrium point. The small ϵ_1 can be considered as a parasitic parameter allowing for the effective use of singular perturbation methods as per [15] to develop simplified battery models.

Based on *Assumption* 1, ϵ_1 is approximated by zero, leading to an abrupt change in the battery dynamic properties. The differential equation of the fast state $\mathbf{x}_f$ in (22) is degenerated into an algebraic equation:

$$0 = \mathcal{F}_f \mathbf{x}_f + h_f(\mathbf{x}_s, \mathbf{x}_f, \mathbf{u}). \tag{23}$$

where $\mathbf{x}_f^0$ is defined as the quasi-steady state of the fast dynamics and is solved from (23) to be $\mathbf{x}_f^0(t) = H(\mathbf{x}_s, \mathbf{u})$. As a result, the fast electrical dynamics are eliminated from the original battery system. The full order battery model Σ is simplified to the simplified (reduced) model containing the states $\mathbf{x}_m$ and $\mathbf{x}_s$

$$\Sigma_s^2 : \dot{\mathbf{x}}_s = \epsilon_2 \mathcal{F}_s \mathbf{x}_s + \epsilon_2 h_s(\mathbf{x}_s, \mathbf{x}_m, \mathbf{x}_f^0, \mathbf{u}), \mathbf{x}_s(0) = \mathbf{x}_{s0}, \tag{24}$$

$$\dot{\mathbf{x}}_m = \mathcal{F}_m \mathbf{x}_m + h_m(\mathbf{x}_s, \mathbf{x}_m, \mathbf{x}_f^0, \mathbf{u}),\ \mathbf{x}_m(0) = \mathbf{x}_{m0}. \tag{25}$$

Assumption 2: The parameter ϵ_2 involved in battery SOH dynamic characteristics satisfies $\epsilon_2 \ll 1$.

Justification for Assumption 2. The degree of battery degradation observed from typical Li-ion batteries is quantified on the magnitude of months or years and it is significantly slower than the heat and mass transport progress which corresponds to several minutes or hours. These different time scales within battery state variables of the nominal intercalation reaction and side reaction are essentially attributed to $J_I \gg J_{sr}$. Studies on (8) and (9) further evince that $i_0 \gg i_{0sr}$, in which $\epsilon_2 = i_{0sr}$ is a positive parameter on the order of $1e{-}6$, in contrast the value of i_0 is as large as 1 [11]. Thus, in the time scale of the nominal intercalation reaction ϵ_2 is negligibly small.

Remark 2: The smallness of ϵ_2 permits the SOH to work as a small perturbation for the normal intercalation reaction. Within several minutes or hours, the change of battery SOH is considered negligible such that the battery dynamics can be approximately studied in the absence of SOH states. In this regard, a singular perturbation technique can be further implemented for battery model simplification.

In deriving Σ_s^2, the quasi-steady state model was obtained through elimination of the fast state dynamics. Following this thread, the slow state $\mathbf{x}_s$ can be separated from the model of Σ_s^2 by eliminating the medium dynamics. However, the dynamics in the medium time scale do not approach a constant steady state, and this requires an alternative approach to analyse them. Thus, here we pursue the extraction of the medium states by excluding the slow state.

Based on *Assumption* 2, ϵ_2 is set equal to 0, and the dynamic equations in terms of $\mathbf{x}_s$ in Σ_s^2 are approximated by:

$$\dot{\mathbf{x}}_s = 0 \Rightarrow \mathbf{x}_s(t) = \mathbf{x}_{s0},$$

where $\mathbf{x}_{s0}$ is the initial values of $C_{s,s}, Q_{sr}, R_f$ at the beginning of each operating cycle. The state $\mathbf{x}_m$ in model Σ_s^2 is then approximately captured by the following system

$$\Sigma_m^3 : \dot{\mathbf{x}}_m = \mathcal{F}_m \mathbf{x}_m + h_m(\mathbf{x}_{s0}, \mathbf{x}_m, \mathbf{x}_f^0, \mathbf{u}), \mathbf{x}_m(0) = \mathbf{x}_{m0}. \tag{26}$$

So far, three time scales, corresponding to electrical, electrochemical and thermal, and ageing phenomena respectively, have been identified in the battery system. When coupled with appropriate convergence results, for each of the individual subsystems, there are parallels with the time scale separation results for ordinary differential equation (ODE) systems. By relying on this analogy, a systematic procedure is proposed for developing simplified battery models with interconnections based on clearly specified assumptions.

III. THE PROPOSED EVALUATION PROCEDURE

For charging strategy evaluation, although the SOH can be solved by using (24) and (26) that resulted from model simplification, such implementations involving both the medium and slow time scales are still computationally expensive. In this section, model simplification is further conducted by using averaging theory.

Assumption 3: There exists a constant $\mathbf{u}_{av}$ such that the input current satisfies:

$$\frac{1}{T_c} \int_{t+kT_c}^{t+(k+1)T_c} \mathbf{u}(\tau) d\tau = \mathbf{u}_{av}, \forall t \in [0, T_c], \forall k = 0, 1 \ldots N,$$

where T_c is the time period for a complete cycle composed of charging, discharging, and relax modes, and N is the cycle number representing battery's lifetime.

Remark 3: Assumption 3 implies the periodic input currents are used for battery successive operating cycles. Under this condition the solution of system Σ_m^3 is almost periodically varying over multiple cycles, and its trajectories at the quasi-steady state converge to a family of limit cycles parameterised by the slow state $\mathbf{x}_s$. In view of this, by using averaging theory [16], [17], a static, *average* mapping can be developed to capture the averaged slow dynamics.

Based on *Assumption* 3 and *Remark* 3, the solution of $\mathbf{x}_s$ in the model Σ_s^2 is derived by averaging the steady-state behavior of the system Σ_m^3. Then $\mathbf{x}_s$ from (24) is approximated by the following averaging system

$$\Sigma_s^3 :\ \dot{\mathbf{x}}_s = \epsilon_2 \mathcal{H}_{av}(\mathbf{x}_s),\ \mathbf{x}_s(0) = \mathbf{x}_{s0}, \tag{27}$$

where the static mapping $\mathbf{x}_s \to \mathcal{H}_{av}(\mathbf{x}_s)$ is defined as:

$$\mathcal{H}_{av}(\mathbf{x}_s) := \lim_{T_s\to\infty} \frac{1}{T_s}\int_0^{T_s} \mathcal{F}_m \mathbf{x}_m^* + h_m(\mathbf{x}_{s0}, \mathbf{x}_m^*, \mathbf{x}_f^0, \mathbf{u}^*)dt. \tag{28}$$

In (28), $\mathbf{x}_m^*$ is the solution of Σ_m^3 under the specified system input $\mathbf{u}^*$, and the static mapping $\mathcal{H}_{av}$ is independent of ϵ_2 and $(\mathbf{x}_m(0), \mathbf{u}(0)) \in \mathscr{H}_m \times \mathbb{U}/\{0\}$.

The state of $\mathbf{x}_s$ is decoupled from the original highly nonlinear system. To solve the developed mapping $\mathbf{x}_s \to \mathcal{H}_{av}(\mathbf{x}_s)$, only the behavior of $\mathbf{x}_m$ governed by the system Σ_m^3 is required. Once $\mathcal{H}_{av}$ is obtained, the averaged model becomes very straightforward from (27). With this theoretical result, we are now in the position to implement a SOH prediction for battery charging strategy evaluation.

Charging strategies are defined in terms of current, $I(t)$, and terminal voltage, $V(t)$. For example, the CCCV strategy specifies a current level for a period until $V(t){=}V_{\max}$, and remains the maximum voltage until $I(t){=}I_{\min}$. The combination of current and voltage levels and their thresholds defines the charging strategy, u. To analyse and compare various charging strategies, the following evaluation procedure using the averaged system Σ_s^3 and the simplified model Σ_m^3 from the established framework is proposed. Here, degradation to 80% of the initial SOH is used to quantify battery's lifetime.

Procedure 1:

1) Specify a battery charging strategy, u, initial conditions of the states, $[\mathbf{x}_{s0}, \mathbf{x}_{m0}]$, and the cycle number $k{=}1$.
2) Run the battery model (26) under the specified charging strategy.
3) Calculate the static mapping $\mathbf{x}_s{\to}\mathcal{H}_{av}(\mathbf{x}_s)$ using (28). For arbitrary initial conditions, a sufficiently large value of T_s can be chosen to approach the infinite time interval. If the initial values $\mathbf{x}_{s0}$ and $\mathbf{x}_{m0}$ are within the limit cycles of the steady state of the $\mathbf{x}_m$, then in practice, T_s can be set to T_c.
4) Run the averaged battery system (27) and record $\mathbf{x}_s$ at the time $t = kT_c$.
5) Calculate SOH using (15).
6) If $SOH > 80\%$, then increment k, update $\mathbf{x}_{s0}$ and go to Step 2, else go to Step 7.
7) Output the battery lifetime, k.

Though the proposed procedure, the SOH evolution of a battery under a given charging strategy can be derived. The performance of different charging strategies can be evaluated based on the life cycles they can support.

IV. SIMULATION RESULTS

In this section, simulation results are presented which demonstrate the effectiveness of the proposed evaluation procedure for different charging strategies.

All the simulations are implemented in Modelica. The battery parameters being simulated are adopted from [11], [18] and related to a battery with 1.8Ah normal capacity.

The CCCV method and multistage CC charging of [7] are considered. Based on the multistage CC and the boostcharging algorithm of [6], we propose a new five-stage CC charging scheme as well for the purpose of comparison, which has higher current rates and voltage limits in the beginning compared to its counterparts.

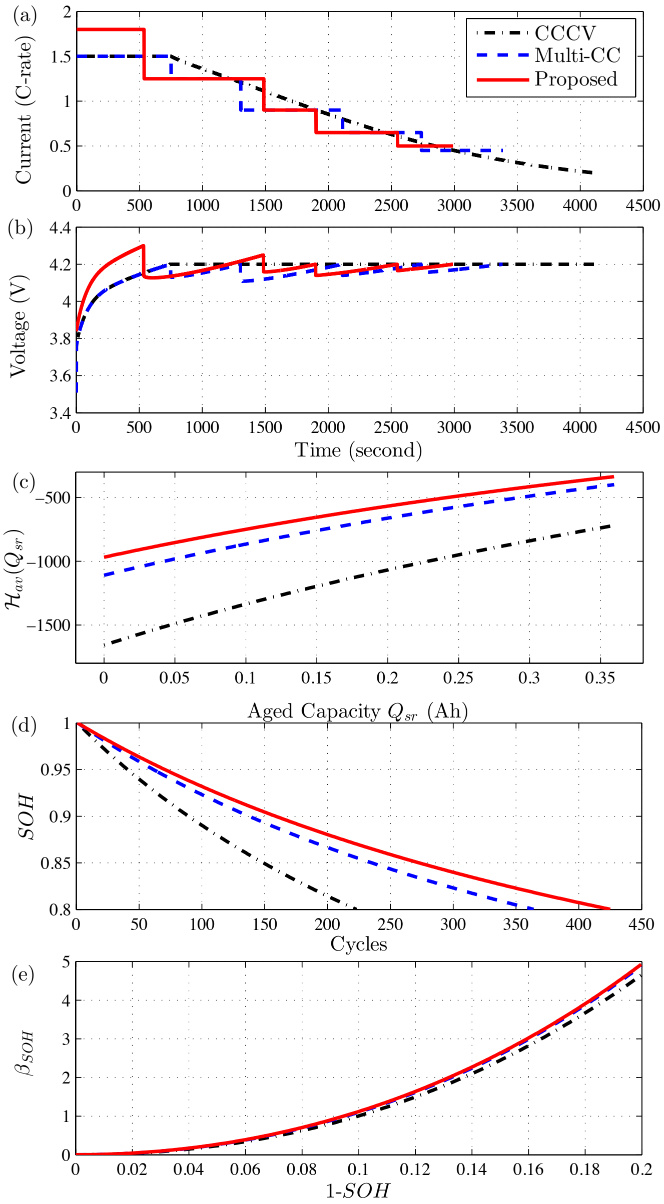

Fig. 2. Comparisons of the CCCV, multistage constant current charging, and proposed charging method in terms of the input current, the terminal voltage, and their terminal effect on battery SOH.

The integrator DASSL with flexible integration stepsize is chosen for these simulations. And the integration tolerance is setup as 0.0001. The spatial discretisation method detailed in [19] is adopted in solving the PDE system with each electrode and separator discretised into 15 elements. To ensure the maximum error in state prediction introduced by temporal discretisation within 0.01%, the sampling time is chosen as 1 second for the models that contain the medium states while as 1000 seconds for the averaged battery system. In the simulation of Σ_s^3, the mapping $\mathbf{x}_s \to \mathcal{H}_{av}(\mathbf{x}_s)$ is assumed to be already available. The CPU-time for integration is recorded

TABLE II

COMPARISON OF COMPUTATIONAL TIME FOR 400 CYCLES ASSOCIATED WITH DIFFERENT BATTERY MODELS.

Models	Sampling time (s)	Simulation time (mins)
Σ_s^2	1	171
$\Sigma_s^3 + \Sigma_m^3$	1	120
Σ_s^3	1000	$1.8{\times}10^{-4}$

for the assessment of computational efficiency.

The estimate error in SOH prediction is defined as $\beta := |\mathbf{x}_r - \mathbf{x}|/\mathbf{x} \times 100\%$, where $\mathbf{x}_r$ and $\mathbf{x}$ are respectively the solutions of the tested model and the benchmark.

The CCCV, multistage CC, and the proposed charging method are simulated and comprehensively compared with respect to their required charging times and terminal effects on battery SOH, as provided in Fig. 2. The accuracy of the simplified model from the established framework is first examined. Given the fast electrical dynamics exhibit in microseconds, the full model that includes all the dynamics is impractical to simulate. The model Σ_s^2 that well approximates the original model in the slow and medium time scales is used as a test bed. From Fig. 2(e), it can be seen the percentage deviations in SOH prediction by using the proposed procedure are less than 5% for all the considered charging strategies. These results illustrate the obtained simplified models are capable of accurately capturing system dynamics, and the *Procedure 1* is reliable in evaluating charging strategies.

Table II compares the computational times for simulating 400 cycles associated with different battery models, where all the simulations are conducted on a desktop computer with 3.4GHz processor and 8GB RAM. It is found the time of 29.8% can be saved by using Σ_m^3 and Σ_s^3 as proposed in *Procedure 1* rather than the high order model Σ_s^2.

For SOH prediction, if the static mapping as shown in Fig. 2(c) is available a priori, simulations using Σ_s^3 becomes extremely fast. Thus, this proposed evaluation system based on simplified models is able to significantly improve the computational efficiency.

In addition, simulation results of Fig. 2 show the multistage CC can support 60% more cycles and save 17.6% charging time relative to the CCCV. This is consistent with the experimental outputs observed from [7]. Meanwhile, the proposed method can transfer similar capacity to the battery, but achieves 13% more lifetime and 11.6% less charging time compared to the multistage CC. This may be useful information for developing optimal charging strategies.

V. CONCLUSION AND FUTURE WORK

In this paper, a novel framework for PDE Li-ion battery model simplification was developed, with the novelty arising from the systematic simplification procedure through which the underlying assumptions imposed on simplified models can be explicitly stated. Families of simplified models in the developed framework can be used for various applications.

From a sound theory basic, a model-based evaluation procedure for battery charging strategies is proposed. This procedure was found to offer a substantial advantage on computational efficiency but still keep reasonable accuracy. Furthermore, this proposed evaluation procedure is sufficiently general covering different charging strategies and battery types.

Instead of evaluating existing strategies, future work will focus on optimal fast charging strategy design using battery models from the developed framework. Also, potential future research opportunities exist in the application of simplified models for real-time battery parameter and state estimation.

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