Author/s:
Dayarathna, S;Senanayake, R;Evans, J

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Maximizing Sum-Rate via Relay Selection and Power Control in Dual-Hop Networks

Shalanika Dayarathna, Rajitha Senanayake and Jamie Evans
Department of Electrical and Electronic Engineering, University of Melbourne, Australia
Email: sdayarathna@student.unimelb.edu.au, rajitha.senanayake@unimelb.edu.au, jse@unimelb.edu.au

***Abstract*—In this paper, we focus on the sum-rate optimization problem in a general dual-hop relay network by considering the joint relay selection and power control in the presence of interference. First, we propose a new relay selection algorithm which has better sum-rate performance than the existing relay selection techniques. Then we combine relay selection and power control to propose a novel iterative algorithm based on the tight lower bound approximation which maximizes the achievable sum-rate. We also prove that for the special case of two-user networks, binary power allocation is optimum for at least two transmitting nodes. Extensive numerical examples are used to compare the performance of the proposed algorithm and to illustrate the accuracy of the analysis.**

***Index Terms*—Decode-and-forward, dual-hop networks, sum-rate optimization, relay selection, power control**

I. INTRODUCTION

Cooperative transmission has been extensively studied as a promising paradigm for next generation wireless networks. In cooperative networks, relay nodes play an important role by acting as intermediate nodes that help combat fading, path loss and interference impairments [1]. For such networks, relay selection and transmission power control are critical in gaining advantages in terms of performance, complexity and overhead. As a result, much research has focused on relay selection and power control in cooperative relay networks [2].

Relay selection in the presence of multiple source-destination (S-D) pairs is more complicated due to potential interference among S-D pairs. In the literature this problem has been approached from two directions, based on two inter-related performance measures, namely, the outage probability [3], [4] and the achievable sum-rate [5]. In [5]–[7], orthogonal channels are adopted so that the different S-D pairs do not interfere with each other. When there is no interference among S-D pairs, the relay selection problem simplifies to an assignment problem which can be solved using the well known Hungarian algorithm. However, if there are more relays than S-D pairs, then the interference in the second hop depends on the relay selection. This is considered in [8], where the second hop achievable rate is approximated as a linear combination of the upper and the lower bounds of the achievable rate derived based on the log-sum inequality and Jensen's inequality. A similar approach is used in [9], where the second hop achievable rate is approximated using the same upper bound as in [8]. Once the interference is estimated, the Hungarian algorithm has been used to solve the relay selection problem. However, for an interference limited network these approximations are only accurate in the high signal-to-interference-plus-noise-ratio (SINR) regime. Taking a more accurate approach, in [10], the authors propose a quality-of-service (QoS) aware greedy algorithm where interfering nodes are separated into clusters such that the interference of a given cluster is fixed. However, this results in a more complex algorithm because the number of clusters increases exponentially with the difference between the number of relay nodes and the S-D pairs.

Power control problem in relay networks has been approached based on three performance measures, namely, the outage probability minimization, power minimization and the achievable sum-rate maximization [11]. In [11], the authors solve the power control problem in an amplify-and-forward (AF) relay network using geometric programming in the high SINR regime. In [6], the power control problem is solved using the Lagrangian-dual approach following the use of orthogonal sub-channels to avoid interference, where as in [7], power control is performed by matching the signal-to-noise-ratio (SNR) across two hops. Power control in the presence of interference is considered in [12] for a single relay network, where the authors propose an algorithm based on the non cooperative game theory to solve power control problem in a distributed manner.

In this paper, we focus on the joint relay selection and power control problem and optimize the achievable sum-rate in a general dual-hop relay network. Since this is a non-convex optimization problem combined with binary constraints, it is an extremely hard problem to solve. To the best of our knowledge, there exists no known optimum solution. As such, we focus on finding a low-complex sub optimal solution that has better performance compared to other available reference techniques. To the best of our knowledge, a similar joint optimization problem has only been considered in [2], where the authors consider a simple minimum interference estimation based approach for both the relay selection and the power control. Different to [2], we consider a more accurate average interference estimation based relay selection and tight lower bound approximation based power control. We also propose a novel iterative algorithm to maximize the achievable sum-rate by combining relay selection and power control based on the average interference and the tight lower bound approximation, respectively. We observe that the proposed algorithm is more accurate compared to

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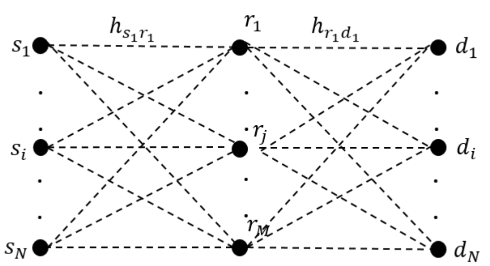

Fig. 1: A multi-user dual-hop relay network

the other techniques proposed in the literature. Subsequently, we analyze the special case of two-user relay networks and show that binary power allocation is optimum for at least two transmitting nodes.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a dual-hop wireless relay network as illustrated in Fig. 1, where N source nodes $(s_1, ..., s_N)$ send information to N corresponding destinations $(d_1, ..., d_N)$. The communication is assisted by a dual-hop relay network with M decode-and-forward (DF) relays $(r_1, ..., r_M)$ where $M \geq N$. We assume that each S-D pair is assisted by only one relay and each relay assists at most one user to minimize the synchronization requirements and to avoid too much processing complexity in any single relay. We model the channel gain between transmitter i and receiver j as a random variable denoted by $h_{i,j}$. In general, this includes small scale fading, path loss and shadowing. We also assume that each node operates in half-duplex mode with a maximum transmit power of P_{max} for each transmission. For such a network with a given relay assignment, the minimum received SINR for the S-D pair i can be expressed as,

$$\gamma_i = \min\{\gamma_i^{(1)}, \gamma_i^{(2)}\}, \tag{1}$$

where, $\gamma_i^{(1)}$ and $\gamma_i^{(2)}$ represent the received SINR of S-D pair i at the first hop and the second hop, respectively and they can be given by

$$\gamma_i^{(1)} = \frac{P_{s_i}|h_{s_i,r_{k_i}}|^2}{\sigma^2 + \sum_{j\neq i}^{N} P_{s_j}|h_{s_j,r_{k_i}}|^2}, \tag{2}$$

$$\gamma_i^{(2)} = \frac{P_{r_{k_i}}|h_{r_{k_i},d_i}|^2}{\sigma^2 + \sum_{j\neq i}^{N} P_{r_{k_j}}|h_{r_{k_j},d_i}|^2}, \tag{3}$$

where r_{k_i} is the relay node selected by S-D pair i and $P_{s_i}, P_{r_{k_i}}$ denote the transmit powers of nodes s_i and r_{k_i}, respectively. The noise power at any receiving node is denoted by σ^2.

Next, we formulate the achievable sum-rate optimization problem based on joint relay selection and power control as,

$$\begin{aligned} \max_{P_{r_{k_i}}, P_{s_i}, k_i \ \forall i} \ & \sum_{i=1}^{N} \log_2\left(1 + \min\{\gamma_i^{(1)}, \gamma_i^{(2)}\}\right) \\ \text{s.t} \quad & 0 \leq P_{s_i}, P_{r_{k_i}} \leq P_{max} \quad \forall i, \\ & k_i \neq k_j \quad \forall i \neq j, \\ & k_i \in \{1, 2, ..., M\}, \end{aligned} \tag{4}$$

where $\gamma_i^{(1)}$ and $\gamma_i^{(2)}$ are functions of k_i, P_{s_i} and $P_{r_{k_i}}$ as given in (2) and (3), respectively. The optimization problem in (4) is non-convex. This combined with the integer nature of k_i makes this an extremely hard problem to solve for a general multi-user network and to the best of our knowledge, there exists no known optimum solution.

As such, we approach this optimization problem in two steps. First we consider the relay selection problem for given power vectors $\mathbf{P_s} = [P_{s_1}, ..., P_{s_N}]$, $\mathbf{P_r} = [P_{r_{k_1}}, ..., P_{r_{k_N}}]$ as,

$$\begin{aligned} \max_{k_1, k_2, .., k_N} \ & \sum_{i=1}^{N} \log_2\left(1 + \min\{\gamma_i^{(1)}, \gamma_i^{(2)}\}\right) \\ \text{s.t} \quad & k_i \neq k_j \quad \forall i \neq j, \\ & k_i \in \{1, 2, ..., M\}. \end{aligned} \tag{5}$$

Next, we consider the power control problem for a given relay assignment $[r_{k_1}, r_{k_2}, ..., r_{k_N}]$ as,

$$\begin{aligned} \max_{P_{r_{k_i}}, P_{s_i} \ \forall i} \ & \sum_{i=1}^{N} \log_2\left(1 + \min\{\gamma_i^{(1)}, \gamma_i^{(2)}\}\right) \\ \text{s.t} \quad & 0 \leq P_{s_i}, P_{r_{k_i}} \leq P_{max} \quad \forall i. \end{aligned} \tag{6}$$

It is important to note that, for a general multi-user network, solving each optimization problem in (5) and (6) separately is still a challenging problem [8], [11]. In the following, we solve the optimization problems in (5) and (6), separately, and then propose an iterative algorithm that combines the proposed solutions in order to provide a novel joint solution.

III. RELAY SELECTION

In this section we focus on the optimization problem formulated in (5) and present a novel and more accurate relay selection algorithm. First, we note that the second hop SINR, which is caused by the relay network, changes depending on the relay selection. In the literature, several approximation techniques are proposed to estimate the second hop SINR. For example, [8], [9] use approximations based on upper and lower bounds to re-write the interference as a linear summation of penalty terms, which are only accurate in the high SINR regime. In the following, we take a different approach and approximate the second hop interference $\gamma_i^{(2)}$ by the average interference caused by $N-1$ relays and write

$$\gamma_i^{(2)} \approx \hat{\gamma}_i^{(2)} = \frac{P_{r_{k_i}}|h_{r_{k_i},d_i}|^2}{\sigma^2 + \dfrac{(N-1)}{(M-1)}\sum_{j\neq i}^{M} P_{r_j}|h_{r_j,d_i}|^2}, \tag{7}$$

We can compute this average interference for each S-D pair and relay combination respectively by computing the average interference with assumption that all $M-1$ relays transmit and then multiplying it by the actual number of active and interfering relays. Then, based on (7), we can derive the achievable rate of S-D pair i as,

$$G_{i,k_i} = \log_2(1 + \min\{\gamma_i^{(1)}, \hat{\gamma}_i^{(2)}\}).$$

Since I_{i,k_i} does not depend on the relay selection, we can simplify the optimization problem in (5) to an assignment problem where a column of matrix $\mathbf{G}$ needs to be assigned

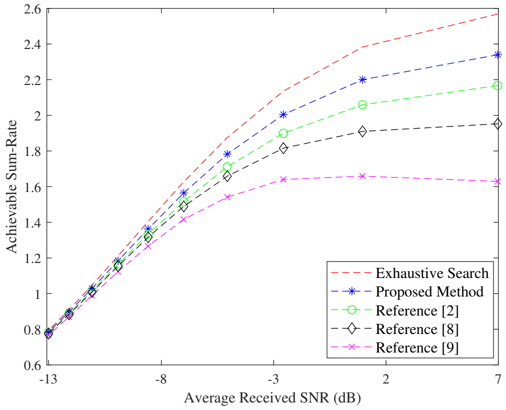

Fig. 2: The achievable sum-rate versus the received SNR

to each row such that the sum is maximized. This problem can be solved for the optimum solution using the well known Hungarian algorithm. Therefore, we can consider that our proposed relay selection method as well as the techniques given in [2], [8], [9] have a complexity of $O(M^3)$ while the exhaustive relay selection has a complexity of M^N. In the following example, the performance of our proposed relay selection method is compared against existing techniques.

Example 1: Consider a dual-hop relay network where M=10, N= 6 and the channels between nodes follow a Rayleigh distribution with zero mean and unit variance. For such a network, Fig. 2 plots the achievable sum-rate versus the received SNR when the relay selection is performed based on the proposed average interference based relay selection. The performance of our proposed method is compared against the two approximation based techniques given in [8], [9], the minimum interference based relay selection given in [2] and the optimum relay selection found via exhaustive search. In this example, the average received SNR is computed as the average of the minimum SNR across two hops for each S-D and relay combination. In the low SNR regime, the network is noise limited and the effect of interference is negligible. As such, all the techniques have similar performance. However, as the SNR increases, all techniques start to deviate from the optimum solution with the proposed method having comparatively better performance. As such, we can conclude that in the high SNR regime, the average interference provides a better approximation when compared to the available approximation techniques in [2], [8], [9].

It is important to note that for the special case of $M = N$, interference in the second hop remains constant. Therefore, $\gamma_i^{(2)}$ remains fixed irrespective of the relay selection, thus, we can directly define $\mathbf{G}$ and find the optimum relay selection via the Hungarian algorithm.

IV. TRANSMIT POWER CONTROL

In this section we focus on the optimization problem formulated in (6) and present an iterative power control algorithm. In general, the achievable sum-rate optimization problem given in (6) is non-convex with respect to P_{s_i} and $P_{r_{k_i}}$ [11]. Therefore, we consider the tight lower bound approximation given in [13] and approximate the objective function in (6) as,

$$\sum_{i=1}^{N} a_i \log(\min\{\gamma_i^{(1)}, \gamma_i^{(2)}\}) + b_i, \tag{8}$$

that is tight at a chosen value $\bar{z} = [\bar{z}_1, ..., \bar{z}_N]$ when the constants a_i and b_i are chosen as,

$$a_i = \frac{\bar{z}_i}{1+\bar{z}_i}, \qquad b_i = \log(1+\bar{z}_i) - \frac{\bar{z}_i}{1+\bar{z}_i}\log(\bar{z}_i).$$

By selecting $\bar{z}_i$ as the minimum SINR for S-D pair i achieved using the initial solution or the solution achieved via the previous iteration, we can re-write the achievable sum-rate optimization problem given in (6) as,

$$\begin{aligned} &\max_{P_{r_{k_i}}, P_{s_i}\ \forall i} \sum_{i=1}^{N} a_i \log(\min\{\gamma_i^{(1)}, \gamma_i^{(2)}\}) + b_i \\ &\text{s.t} \quad 0 \le P_{s_i}, P_{r_{k_i}} \le P_{max}, \forall i. \end{aligned} \tag{9}$$

To convert this non-convex objective function into a concave function we use variable transformations $P_{s_i} = e^{y_{s_i}}, P_{r_{k_i}} = e^{y_{r_{k_i}}}$ and $t_i = \log(\min\{\gamma_i^{(1)}, \gamma_i^{(2)}\})$ and reformulate (9) as,

$$\begin{aligned} &\max_{y_{r_{k_i}}, y_{s_i}\ \forall i} \sum_{i=1}^{N} a_i\, t_i + b_i \\ &\text{s.t} \\ &t_i \le y_{s_i} + \log(|h_{s_i,r_{k_i}}|^2) - \log\left(\sigma^2 + \sum_{j \ne i}^{N} e^{y_{s_j}} |h_{s_j,r_{k_i}}|^2\right) \forall i, \\ &t_i \le y_{r_{k_i}} + \log(|h_{r_{k_i}}, d_i|^2) - \log\left(\sigma^2 + \sum_{j \ne i}^{N} e^{y_{r_{k_j}}} |h_{r_{k_j}}, d_i|^2\right) \forall i, \\ &y_{s_i}, y_{r_{k_i}} \le \log(P_{max}),\ \forall i. \end{aligned} \tag{10}$$

For a given relay assignment, the optimization problem (10) is concave. Therefore, in each iteration, we can compute the coefficients a_i and b_i based on the solution of the previous iteration and solve the above problem using any existing convex solver or by implementing a gradient decent algorithm.

V. JOINT RELAY SELECTION AND POWER CONTROL

In this section, we focus on the joint relay selection and power control problem and propose a novel iterative algorithm that can be implemented in a central controller to maximize the achievable sum-rate.

First, we consider the case where $M = N$ and propose Algorithm 1 where we apply the Hungarian algorithm for relay selection and (10) for power optimization. At the start of Algorithm 1, we initialize all transmit powers to P_{max}. In each iteration, we first solve the relay selection problem for a given transmit power vector and assign the selected relays to a vector denoted by $\mathbf{X}$. After the first iteration, the relay selection is changed only if the resultant achievable sum-rate is higher for a different relay selection under the new transmit power vector. Then we proceed to iteratively solve the power control problem. In the n^{th} iteration of the inner loop, we solve the optimization problem (10) and assign the solution to vectors $\mathbf{Y_s}^{(n)} = [y_{s_1}, \dots, y_{s_N}]$, and $\mathbf{Y_r}^{(n)} = [y_{r_{k_1}}, \dots, y_{r_{k_N}}]$. Then the calculated error e is compared against a user defined threshold e_{th}. The tight lower bound approximation monotonically improves the objective function and always converges [13]. As such, the achievable sum-rate improves within each iteration of the inner loop. After the first iteration of the outer loop, the transmit power vectors are changed only if the resultant achievable sum-rate is higher for a different transmit power allocation under the relay assignment. Therefore, in each iteration in the outer loop, the achievable sum-rate improves monotonically until it converges to a solution. Since, the optimization problem (4) is non-convex, there might exist multiple local optima. The objective function eventually converges to one of the local solutions as it monotonically improves in each iteration. We consider the proposed algorithm to be sub-optimal as we cannot guarantee that the converged solution is the global optimum solution.

Algorithm 1: The Proposed Iterative Joint Relay Selection and Power Control Algorithm

```
Input  : Channel state information (CSI), $P_{max}$
Output: Achievable sum-rate $R^*$, Relay assignment
        $\mathbf{X_{opt}}$ and Power allocation $\mathbf{P_s}$ and $\mathbf{P_r}$
$\mathbf{X_{opt}} \leftarrow \{\}$, $\mathbf{P_s}, \mathbf{P_r} \leftarrow P_{max}$, $R^* \leftarrow 0$
while true do
    $G_{ij} = \log_2(1 + \min\{\gamma_i^{(1)}, \gamma_i^{(2)}\})\ \forall i, j$
    $\mathbf{X} \leftarrow$ solution to relay selection problem using the
      Hungarian algorithm
    $\mathbf{k}_i \leftarrow X(i)\ \forall i, \quad n = 1$
    while true do
        $\mathbf{Y_s}^{(n)}, \mathbf{Y_r}^{(n)} \leftarrow$ solution to problem (10)
        $e \leftarrow |\, [\mathbf{Y_s}^{(n)}, \mathbf{Y_r}^{(n)}] - [\mathbf{Y_s}^{(n-1)}, \mathbf{Y_r}^{(n-1)}]\, |$
        if $e < e_{th}$ then
            break
        n ← n+1
    end
    $\mathbf{P_s} = e^{\mathbf{Y_s}^{(n)}}$, $\mathbf{P_r} = e^{\mathbf{Y_r}^{(n)}}$
    $R \leftarrow$ achievable sum-rate for $\mathbf{P_s}, \mathbf{P_r}$ and $\mathbf{k}_i\ \forall i$
    if $R > R^*$ then
        $R^* \leftarrow R$, $\mathbf{X_{opt}} \leftarrow \mathbf{X}$
    else
        break
end
```

The performance of our proposed algorithm is illustrated in the following example.

Example 2: Consider a dual-hop relay network where $M = N$ and the channels between nodes follow a Rayleigh distribution with zero mean and unit variance. For such a network, Fig. 3 plots the achievable sum-rate versus N with $P_{max} = 1$, when Algorithm 1 based joint optimization is employed. We compare the performance of the proposed algorithm with three reference techniques - the relay selection only (with no power control), the relay selection with SINR matching[1] and the iterative relay selection with SINR matching. From the plot, we observe that the achievable sum-rate increases with N while the gap between the proposed algorithm and other methods increases as well. We also note that there is no significant difference between the relay selection with SINR matching and the iterative relay selection with SINR matching.

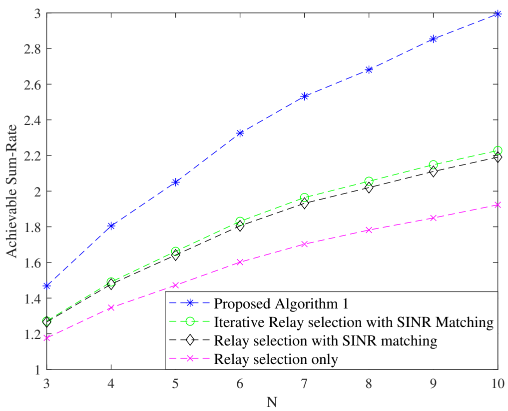

Fig. 3: The achievable sum-rate versus N when $M = N$

Next, we extend our proposed Algorithm 1 such that it can be used for joint relay selection and power control even when $M > N$. First, we initialize all transmit powers to P_{max} and use the average interference based relay selection given in section III. Next, the transmit powers of non-active relays are set to zero thus, resulting in a dual-hop relay network with $M = N$. As such, Algorithm 1 can be used for power optimization and further relay selection in the resulting network. We note that even though, this proposed algorithm is sub-optimal, it has better performance compared to the reference techniques. The performance of this proposed algorithm is illustrated in the following example.

Example 3: Consider a dual-hop relay network where $N = 8$, $M > N$ and the channels between nodes follow a Rayleigh distribution with zero mean and unit variance. For such a network, Fig. 4 plots the achievable sum-rate versus M with $P_{max} = 1$, when the proposed sub-optimal algorithm based joint optimization is employed. We compare the performance of our proposed algorithm with three reference techniques - the exhaustive search based relay selection in the first iteration followed by Algorithm 1 (reference technique 1), the exhaustive search based relay selection with SINR matching (reference technique 2) and the SNR based relay selection with SINR matching (reference technique 3). From the plot, we observe that the achievable sum-rate increases with M and Algorithm 1 based methods have better performance compared to reference techniques 2 and 3. We also note that the proposed sub-optimal algorithm and the reference technique 1 have very similar performance. This implies that

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[1] By SINR matching we simply mean that for each user the SINR of any given hop is matched to the minimum between the SINRs in the two hops.

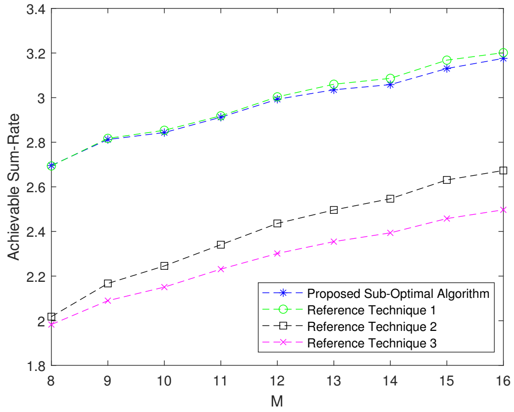

Fig. 4: The achievable sum-rate versus M when $N = 8$

there is no significant difference between the use of average interference based relay selection compared to the exhaustive search based relay selection at the first iteration, when subsequent relay selection is performed iteratively. Thus, the accuracy of the proposed algorithm is not compromised by the use of proposed low-complexity relay selection method given in section III.

VI. SPECIAL CASE OF TWO-USER NETWORK

When two S-D pairs share the same OFDMA carriers, power control in multi-user orthogonal relay selection simplifies to a two-user network. As such, let us now consider a special case of two-user network. For a such network, we prove the following Lemma.

Lemma 1: For a two user dual-hop relay network, there exists an optimum power vector which maximizes the overall achievable sum-rate, such that for each user the resulting SINRs in the two hops are equal.

Proof: See Appendix A

Based on Lemma 1, we can develop the following theorem.

Theorem 1: For a dual-hop DF relay network with two users, binary power allocation is optimum for at least two of the four transmitting nodes. Transmit powers of the remaining nodes can be found via SINR matching.

Proof: According to Lemma 1, at the optimum achievable sum-rate, each user can have equal SINRs in the two hops. Therefore, the achievable sum-rate optimization problem can be expressed as

$$\begin{aligned}&\max_{\mathbf{P_s},\mathbf{P_r}} \left(1+\frac{P_{s_1}|h_{s_1,r_{k_1}}|^2}{\sigma^2+P_{s_2}|h_{s_2,r_{k_1}}|^2}\right)\left(1+\frac{P_{s_2}|h_{s_2,r_{k_2}}|^2}{\sigma^2+P_{s_1}|h_{s_1,r_{k_2}}|^2}\right)\\ &\text{s.t}\quad \frac{P_{s_1}|h_{s_1,r_{k_1}}|^2}{\sigma^2+P_{s_2}|h_{s_2,r_{k_1}}|^2}-\frac{P_{r_{k_1}}|h_{r_{k_1},d_1}|^2}{\sigma^2+P_{r_{k_2}}|h_{r_{k_2},d_1}|^2}=0,\\ &\qquad \frac{P_{s_2}|h_{s_2,r_{k_2}}|^2}{\sigma^2+P_{s_1}|h_{s_1,r_{k_2}}|^2}-\frac{P_{r_{k_2}}|h_{r_{k_2},d_2}|^2}{\sigma^2+P_{r_{k_1}}|h_{r_{k_1},d_2}|^2}=0,\\ &\qquad 0\le P_{s_1},P_{s_2},P_{r_{k_1}},P_{r_{k_2}}\le P_{max}. \qquad (11)\end{aligned}$$

Since the objective function and the equality constraints are twice differentiable with respect to $\mathbf{P_s}$ and $\mathbf{P_r}$, we can rewrite (11) as an unconstrained optimization problem using the Lagrangian dual as,

$$\begin{aligned}\max_{\lambda_1,\lambda_2}\Bigg[\min_{\mathbf{P_s},\mathbf{P_r}}\ &\frac{(\lambda_1-1)P_{s_1}|h_{s_1,r_{k_1}}|^2}{\sigma^2+P_{s_2}|h_{s_2,r_{k_1}}|^2}-\frac{\lambda_1 P_{r_{k_1}}|h_{r_{k_1},d_1}|^2}{\sigma^2+P_{r_{k_2}}|h_{r_{k_2},d_1}|^2}\\ &+\frac{(\lambda_2-1)P_{s_2}|h_{s_2,r_{k_2}}|^2}{\sigma^2+P_{s_1}|h_{s_1,r_{k_2}}|^2}-\frac{\lambda_2 P_{r_{k_2}}|h_{r_{k_2},d_2}|^2}{\sigma^2+P_{r_{k_1}}|h_{r_{k_1},d_2}|^2}\\ &-\frac{P_{s_1}P_{s_2}|h_{s_1,r_{k_1}}|^2|h_{s_2,r_{k_2}}|^2}{(\sigma^2+P_{s_2}|h_{s_2,r_{k_1}}|^2)(\sigma^2+P_{s_1}|h_{s_1,r_{k_2}}|^2)}-1\Bigg]\\ \text{s.t}\ \ 0\le P_{s_1},P_{s_2},P_{r_{k_1}},P_{r_{k_2}}\le P_{max}, \qquad (12)\end{aligned}$$

where λ_1 and λ_2 are the Lagrangian multipliers. Note that in (12), the minimization of the negative achievable sum-rate is considered. Therefore, the maximum achievable sum-rate is achieved when the objective function of (12) is minimized. In the following, we denote the objective function of (12) by f, which is a variable of $P_{s_1}, P_{s_2}, P_{r_{k_1}}$ and $P_{r_{k_2}}$.

Since the Lagrangian multipliers λ_1 and λ_2 are related to the equality constraints, at the optimum solution of (12) they can have any real value. Whilst not given here due to page limitations, by analyzing the first and the second derivatives of f with respect to these variables, we can show that at least for two variables either the first derivative cannot be zero (i.e., they are either increasing or decreasing functions, indicating that the achievable sum-rate is maximized at the corner points) or the second derivative is not positive (indicating that any existing critical point would be a local maximum of f). This implies that the achievable sum-rate is maximized at the corner points for at least two of the variables out of $P_{s_1}, P_{s_2}, P_{r_{k_1}}$ and $P_{r_{k_2}}$. Therefore, we can conclude that irrespective of the value of λ_1 and λ_2, at least for two transmitting nodes binary power allocation is optimum. Since, the two equality constraints in (11) connect all four power values, the other two can be obtained solving those two equations. This concludes the proof of Theorem 1.

Based on Theorem 1, we can find the optimum power allocation for the special case of two-user networks, by comparing the eight resulting power allocations with at least two nodes having binary power allocation. Therefore, for a two user network, it is possible to find the optimum solution of the joint optimization problem given in (4). As such, the performance of our proposed algorithm with respect to the optimum solution is illustrated in the following example for the special case of two user network.

Example 4: Consider a dual-hop relay network where N=2, $M > 2$ and the channels between nodes follow a Rayleigh distribution with zero mean and unit variance. For such a network, Fig. 5 plots the achievable sum-rate versus M with the average received SNR of 10 dB, when the proposed sub-optimal algorithm based joint optimization is employed. We compare the performance of our proposed algorithm with the optimum solution achieved via the exhaustive search based relay selection and Theorem 1 based power control and the SNR based relay selection with SINR matching. From the plot, we observe that the proposed algorithm has better performance compared to SNR based join optimization.

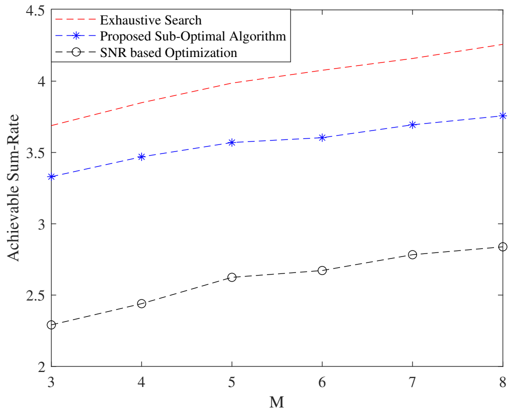

Fig. 5: The achievable sum-rate versus M when $N = 2$

VII. CONCLUSION

We considered the achievable sum-rate optimization problem in a general dual-hop relay network with multiple S-D pairs and multiple relays. First, we investigated the relay selection problem when transmit powers are fixed. We identified that when the number of relays is larger than the number of S-D pairs, the average interference based relay selection provides a good approximation for the optimum relay selection. Then, we combined relay selection and power control and presented a novel iterative algorithm based on the tight lower bound approximation. Our proposed algorithm performs joint relay assignment and power control in a such a way that the achievable sum-rate is maximized. We also proved that for the special case of two-user networks, binary power allocation is optimum for at least two nodes.

APPENDIX A
PROOF OF LEMMA 1

In this section we provide the proof of Lemma 1. Let R^* denotes the optimum achievable sum-rate that results from the transmit powers of s_1, s_2, r_{k_1} and r_{k_2} denoted by $P^*_{s_1}$, $P^*_{s_2}$, $P^*_{r_{k_1}}$ and $P^*_{r_{k_2}}$, respectively. Let the resulting optimum SINR of s_1 and s_2 for the first hop and the second hop be denoted by $\gamma_1^{(1)*}, \gamma_2^{(1)*}, \gamma_1^{(2)*}$ and $\gamma_2^{(2)*}$, respectively. We start the proof by assuming that the two users do not have equal SINRs in both hops at the same time, i.e, $\gamma_1^{(1)*} = \gamma_1^{(2)*}$ and $\gamma_2^{(1)*} = \gamma_2^{(2)*}$ does not happen simultaneously. In the following, we consider three possible situations resulting from the above assumption.

Case 1 - Two users have the minimum SINR in the same hop

Let us assume that both users have minimum SINR in the second hop, which gives $R^* = \log_2(1+\gamma_1^{(2)*}) + \log_2(1+\gamma_2^{(2)*})$. Next, we change the power values of s_1, s_2 as $P_{s_1} = P^*_{s_1} - x_1$ and $P_{s_2} = P^*_{s_2} - x_2$ such that $\gamma_1^{(1)} = \gamma_1^{(2)*}$ and $\gamma_2^{(1)} = \gamma_2^{(2)*}$. Based on the values of x_1, x_2 and considering the fact that $\gamma_1^{(1)*} > \gamma_1^{(2)*}$ and $\gamma_2^{(1)*} > \gamma_2^{(2)*}$, we can show that P_{s_1}, P_{s_2} falls within 0 and P_{max}. Therefore, we can achieve $\gamma_1^{(1)} = \gamma_1^{(2)*}$ and $\gamma_2^{(1)} = \gamma_2^{(2)*}$ for same optimum achievable sum-rate R^*.

Case 2- One user has equal SINRs in the two hops

Let us assume that the first user has the minimum SINR in the second hop and the second user has equal SINRs in both hops, which gives $R^* = \log_2(1 + \gamma_1^{(2)*}) + \log_2(1 + \gamma_2^{(2)*})$. Similar to scenario 1, it can be shown that there exist new transmit power values such that we can achieve $\gamma_1^{(1)} = \gamma_1^{(2)*}$ and $\gamma_2^{(1)} = \gamma_2^{(2)*}$ for same optimum achievable sum-rate R^*.

Case 3- Two users have the minimum SINR in different hops

Let us assume that the first user has the minimum SINR in the second hop and the second user has the minimum SINR in the first hop, which gives $R^* = \log_2(1+\gamma_1^{(2)*}) + \log_2(1+\gamma_2^{(1)*})$. If we change the power value of s_1 similar to scenario 1, it can be shown that the achievable sum-rate would increase as $\gamma_2^{(1)} > \gamma_2^{(1)*}$. Therefore, this scenario cannot happen. The fact that the third scenario is not possible and in the first two scenarios, we can achieve R^* such that $\gamma_1^{(1)} = \gamma_1^{(2)*}$ and $\gamma_2^{(1)} = \gamma_2^{(2)*}$ completes the proof of Lemma 1.

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