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Coordinated Pricing Analysis with the Carbon Tax Scheme in a Supply Chain

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Abstract

The carbon tax is a cost-efficient scheme to curb emissions, and it has been implemented in Australia, British Columbia, and other places worldwide. We aim to analyze its effect on dynamic pricing in a supply chain with multiple suppliers and one manufacturer. The profit-maximizing manufacturer makes final products using raw materials from suppliers with heterogeneous prices and emission rates. A two-stage game model is built over an infinite time horizon for this issue. In the first stage, suppliers face price-dependent demand to set their prices and production rates under the constraint of inventory capacity. Then, in response to the carbon tax scheme, the manufacturer evaluates the procurement prices and emission rates of suppliers to

control its emission volumes and sets the sales price of its product. This paper predominately focuses on the optimal pricing strategies in a decentralized supply chain. The open-loop equilibrium and Markovian Nash equilibrium for the dynamic pricing game models of both suppliers and the manufacturer are derived, respectively. The equilibrium prices of suppliers and the manufacturer can be solved based on both irreversible actions and real-time states. These two types of equilibria can be regarded as the solutions of two different models in specific situations. To analyze the effect of sourcing diversity on pricing strategies and emissions control for the manufacturer, the more general equilibrium price for the manufacturer in an n -suppliers oligopoly is studied. Numerical examples are presented to illustrate the equilibrium and its monotonicity with various parameter settings.

Keywords: carbon tax, differential game, emissions control, dynamic pricing

1. Introduction

Human activities have brought huge challenges, particularly the phenomenon of global warming, to the sustainable development of Earth. Curbing greenhouse gas (GHG) emissions is considered as a significant way to develop sustainability worldwide. According to McKinsey survey, 43% of the interviewed companies in 2014 seek to align sustainability with their overall business goals, mission, or values up from 30% in 2012 (McKinsey, 2014). In practice, several actions have been implemented in different countries to develop sustainability. For example, the United States Environmental Protection Agency (EPA) has carried out a clean power plan with the aim to improve the environment, health, and the economy (EPA, 2015). Besides, as an effective financial instrument, the carbon tax scheme has been implemented in several countries to incentivize firms to improve sustainability by adopting cleaner production technologies or using environmentally friendly raw materials (Ma, Ji, Ho, & Yang, 2016). For instance, British Columbia (B.C.) implemented the carbon tax on July 1, 2008, at a rate of C\$10 per ton of CO₂. In 2015, the B.C. carbon tax was increased to C\$30 per ton of CO₂ (Murray & Rivers, 2015). In Australia, the initial price of the carbon tax was set as a fixed number. However, it was replaced by a flexible price, which is determined by the market, on July 1, 2015 (Oracle, 2015).

Constrained by the carbon tax scheme, firms are under tremendous pressure to respond to business pricing. Based on a survey from Australian Industry Group, there is a large gap between the proportion of manufacturing businesses experiencing immediate input price rises (61%) and the proportion of manufacturing businesses

planning to increase their selling prices (40%) as a result of a carbon tax (AIG, 2013). In particular, food manufacturers prefer immediate input price rises and pass on environmental-related costs through the supply chain to the end consumers of products (AIG, 2013). In addition, to produce sustainable products, manufacturers must source sustainable materials from suppliers, however, in practice, it is challenging to source sustainable inputs (Agrawal & Lee, 2016). Therefore, it is crucial for firms to identify effective methods for determining appropriate pricing and choosing environmentally friendly materials from upstream suppliers to reduce GHG emissions.

From the sustainable operations perspective, the issue of emissions control based on carbon emission regulations has been studied extensively (Hua, Cheng, & Wang, 2011; Choi, 2013; Jaber, Glockb, & El Saadany Ahmed, 2013; Chen, Benjaafa, & Elomri, 2013; Ma, Ji, Ho, & Yang, 2016). These studies focus on emissions control with classical operations research models, such as the newsvendor, dynamic programming, and economic order quantity models. However, the interaction between manufacturers and suppliers is often neglected in this context. Therefore, this paper fills this gap by using game models to establish appropriate and mutually beneficial pricing strategies for a manufacturer (or buyer) and suppliers, and studies how the manufacturer can use a reasonable pricing strategy in response to the carbon tax scheme to source from greener or traditional suppliers.

In this paper, we take the viewpoint of how to coordinate pricing in a supply chain with multiple suppliers and a manufacturer. That is, we focus on the interaction of different pricing strategies for profit-maximizing suppliers and the manufacturer under

the carbon tax scheme. In such a situation, each supplier has a fixed initial inventory setting and suppliers compete with each other by adjusting their sales price to enlarge their business volume share. With respect to the manufacturer, reasonable procurement decisions should be made with flexible ordering probabilities to satisfy demand with consideration of the carbon emissions cost. To curb its emissions amounts, the ordering probability of the manufacturer depends on the unit procurement price of raw materials, emission rates of suppliers, procurement schedule, and total emission volumes of the manufacturer.

To identify reasonable strategies for the dynamically coordinated pricing issue, two-stage differential game models are formulated for the manufacturer and suppliers. The effect of the variance of state conditions on pricing strategies is analyzed. To perform this analysis, the open-loop equilibrium and the Markovian Nash equilibrium for two sub-games of the two-stage game are derived. These solutions can be used to establish appropriate operations strategies for both suppliers and the manufacturer under different scenarios. The trade-off of game models also indicates a supplier selection issue for the manufacturer.

A comparative statics analysis is then conducted based on the equilibrium prices of both suppliers and the manufacturer. We investigate whether parameters (e.g., the market size, the production cost, and the carbon tax) can affect the coordinated pricing issue. We observe interesting results in this setting: The manufacturer prefers to cooperate with greener suppliers if facing a higher carbon tax. This result is observed particularly at the initial period of implementing the carbon tax scheme. Meanwhile,

this strategy incurs a higher price, which implies that the manufacturer is more concerned about profit loss when the carbon tax scheme is initially launched. However, in the long-run, we observe that this strategy generates a lower price, and it can bring about higher demand for the manufacturer.

The contributions of this paper are summarized as follows. First, this paper studies the coordinated pricing issue for suppliers and the manufacturer in the presence of inventory and carbon emission constraints. We characterize the open-loop equilibrium and the Markovian Nash equilibrium for both suppliers and the manufacturer over an infinite time horizon. This paper can help each individual supplier adjust its sales price and production rate to enlarge its business volume share in a timely manner, and also help the manufacturer set reasonable sales prices for their products under a carbon tax scheme and control its emissions amount effectively. Second, in a scenario of multiple suppliers, a more general model is developed to derive the general equilibrium strategies of the manufacturer. This can help the manufacturer control and adjust its emission amount and pricing, respectively, by following the variance of the unit carbon tax. Third, we further analyze the characteristics of monotonicity for equilibrium in order to investigate the impact of decision parameters (e.g., the market size, the production cost, and the carbon tax) on the equilibrium outcomes and the profit of the manufacturer.

The remainder of this paper is organized as follows. In Section 2, the relevant literature is presented. Section 3 describes the basic setting of dynamic pricing game models for both suppliers and the manufacturer. Section 4 illustrates the open-loop

equilibrium and the Markovian Nash equilibrium of two sub-games for suppliers and the manufacturer. First, some general equilibrium outcomes are identified for the sub-game model with one manufacturer and n -suppliers. Then, a special case for the sub-game model with one manufacturer and two heterogeneous suppliers is studied. In addition, the comparative statics is conducted for these equilibria of the sub-games. Section 5 discusses the managerial implications of our work. Section 6 concludes the paper. All proofs are given in the Appendix.

2. Literature Review

This paper is related to two streams of research literature on the efficacy of carbon emission regulations in operations management and pricing coordination.

The first stream of research that relates to our work focuses on the efficacy of carbon emission regulations, such as the emissions trading mechanism and the carbon tax scheme, from the perspective of operations management. In this paper, we predominantly focus on the analysis that relates to the effect of carbon tax on the pricing issue. In this context, Laffont and Tirole (1996) established a two-period model to study the interaction between a firm's pollution abatement and production decisions. Subramanian, Gupta, and Talbot (2007) developed a three-stage game model to study the behavior of abatement, production, and the bidding for emission allowances. The above two papers are anchored in the viewpoint of abatement behavior to curb emissions or pollution. This behavior presents a positive way to reduce the emission amounts of a firm or an agent and to face emission regulations. In addition, following

emission regulations, several scholars have analyzed these issues by adjusting their operation strategies. Gemechu, Butnar, Llop, and Castells (2012) studied an environmental tax based on the carbon footprint of products in the pulp and paper sector. Two individual methods, life cycle analysis (LCA) and environmentally extended input-output analysis (EIO), were established to identify the emission intensities for the product. For the scenario of multiple suppliers in the market, Choi (2013) built a multi-stage stochastic dynamic programming model based on the classical newsvendor approach to study the issue of supplier selection under the carbon tax scheme. The effects of the linear and quadratic structure of the carbon tax were discussed. Hua, Cheng, and Wang (2011) studied the optimal order size in consideration of the carbon price/tax based on the classical economic order quantity (EOQ) model. Chen, Benjaafa, and Elomri (2013) focused on the carbon-constrained EOQ model. They also studied how to reduce emissions by modifying order quantities under the influence of the carbon tax. Gong and Zhou (2013) studied the effect of the emissions trading mechanism in a multi-period production-planning problem. A dynamic programming model was developed to analyze this issue for a single firm. They focused on the structure of optimal emissions trading policy, technology selection, and production policy. Krass, Nedorezov, and Ovchinnikov (2013) mainly analyzed the effect of environmental taxes and subsidies on the choice of green technology. Drake, Kleindorfer, and Van Wassenhove (2016) studied the effects of carbon emission regulations and production technology choices on production decisions and capacity portfolio. The implications based on the above decisions were further analyzed for the

expected profit and emission allowances of a firm. Ma, Ji, Ho, and Yang (2016) developed a dynamic programming model to study the effect of the carbon tax on calculating the optimal order quantity over a finite time horizon. The effective range for the carbon tax was established to assist government in setting up a reasonable carbon tax for a certain industry. However, our paper differs from the aforementioned studies in that we aim to analyze how the carbon tax can influence the price of the final product in a dynamic scenario with the variance of control states and how to determine a reasonable sales price for the manufacturer based on traceable information in a supply chain.

The second stream of research that relates to our work is coordinated decisions for pricing. For instance, several scholars have approached this topic from the viewpoint of both discrete and continuous time. Debo, Toktay, and Van Wassenhove (2005) studied the joint pricing and production technology selected for remanufacturable products in a market that consists of heterogeneous consumers. Using the Arrow-Karlin model, Dobos (2005) studied the production and inventory strategy of a firm under the emissions trading scheme. The linear emissions procurement or selling cost was integrated into the model. Perakis and Sood (2006) analyzed a discrete-time stochastic game for pricing. The purpose of their study was to address the competitive aspect of the problem along with demand uncertainty using ideas from robust optimization and variational inequalities. Mookherjee and Friesz (2008) also focused on a discrete-time dynamic game model to study the problems of combined pricing, resource allocation, and overbooking under demand uncertainty. Martínez-de-

Albéniz and Talluri (2011) studied price competition for an oligopoly in a dynamic setting. The unique subgame-perfect equilibrium for a duopoly was presented. This structure can be extended from a marginal-value concept of bid-price control to a competitive model. In addition, Liu and Zhang (2013) considered a dynamic pricing competition between two firms offering vertically differentiated products to strategic customers. The results show that a high-end business charging constant prices is frequently a desirable market outcome for sellers.

This paper is an intersection of the two above-mentioned streams. First, this paper is the initial work on the coordinated pricing issue for suppliers and the manufacturer in the presence of inventory and carbon emission constraints. We characterize the open-loop equilibrium and the Markovian Nash equilibrium for both suppliers and the manufacturer over an infinite time horizon. Second, in a scenario of multiple suppliers, a more general model is developed to derive the general equilibrium strategies of the manufacturer. Third, we investigate the effect of carbon tax on the price setting and emissions control issues of the manufacturer. In addition, we will further analyze the characteristics of monotonicity for equilibrium.

3. The Model

In this paper, under the constraint of the carbon tax scheme, we aim to analyze a decentralized supply chain in which a manufacturer sources a raw material/component from n ($n \geq 2$) suppliers over an infinite time horizon. Both the manufacturer and suppliers independently set their sales prices. In addition, since the suppliers supply key

and valuable materials to the manufacturer, hence, the manufacturer is the price taker. Meanwhile, there exists a trade-off for both the manufacturer and suppliers because the manufacturer is constrained by the carbon tax scheme, which has impact on its sourcing decisions. A nature question to ask is: Should a supplier keep its current production technology or make improvement to supply more environmentally friendly raw materials with a higher profit? As the diverse types of raw materials (i.e., traditional and environmentally friendly) also influence the sales price of the manufacturer's final products, therefore, the sales prices of both the manufacturer and suppliers should be adjusted in each period. Focusing on our analysis in a dynamic situation, we develop a two-stage differential game model, with the manufacturer being the Stackelberg leader, to analyze the strategic interactions between the manufacturer and suppliers over time. Specifically, we focus on two separate games to analyze the strategic production, pricing, and sourcing interactions over an infinite time horizon. In the supplier's sub-game model, each individual supplier determines its sales price of diverse types of raw materials (i.e., traditional and environmentally friendly) and production rate with respect to the variance of its inventory level to the manufacturer who adopts an assemble-to-order (ATO) strategy. In the manufacturer's sub-game model, under the constraint of the carbon tax scheme, the manufacturer needs to control its emissions amount by sourcing from multiple suppliers who supply substitutable raw materials with reasonable prices and emissions rate. The sales prices of suppliers can be set or adjusted in each period to enlarge their business volume. In the following sections, two sub-game models are developed first. After that, we start by solving the supplier's sub-

game model, and then recursively solve the manufacturer's sub-game model.

For the manufacturer's decision, a general model is first developed for the manufacturer sourcing from multiple suppliers ($n > 2$). In addition, a basic model is formulated to study a special case for the manufacturer sourcing from two types of suppliers: traditional and environmental ($n = 2$). A traditional supplier provides raw materials with a lower sales price, but its emissions rate of materials is high; an environmental supplier has lower emissions rate but a higher sales price. Based on the price setting of suppliers, the manufacturer determines its ordering probability. Then, under the carbon tax scheme, the manufacturer makes dynamic adjustment regarding its sales price and emissions amount. To achieve both profit maximization and emissions minimization, the manufacturer can source from traditional suppliers, environmental suppliers, or both. The cost structure of the manufacturer is predominantly determined by the procurement cost and the carbon tax. With respect to the demand function, we assume additive demand where $D(p) = a + bp$. The additive demand function has been widely used in the literature (e.g., Chen & Simchi-Levi, 2004; Chou & Parlar, 2006). The notation used in the model formulations is summarized in Table 1.

<Please insert Table 1 around here>

3.1 The Supplier's Decision Issue

For each individual supplier, the emission rate of its product (raw material) is directly determined by its production technology. Therefore, the value of the emission rate can

be taken as a fixed constant. In such a scenario, a supplier predominately focuses on production planning, inventory control, and price setting to maximize its profit. With respect to the demand function of the supplier, $D_s(t)$, the additive case is adopted, $D_s(t) = \alpha_s - \beta_s p_s(t)$, where α_s is the vertical intercept and β_s is the slope of the demand curve; both α_s and β_s are nonnegative constants. The demand of a supplier, $D_s(t)$, is predominately influenced by the sales price of a supplier, $p_s(t)$. The suppliers, as the leader, determine their sales price first. Then, the manufacturer makes the sourcing decision and sets its sales price under the carbon tax scheme. Similar model settings are commonly used in the literature in this domain, e.g., Camdereli and Swaminathan (2010), Dai et al. (2012), and Chiang (2012). Besides, we assume that the demand of each individual supplier is determined by its sales price which changes over time. This assumption is commonly used in pricing situations, e.g., Chou and Parlar (2006). However, to describe the competition behavior of suppliers regarding their sales prices, the multinomial logit model was applied in the following Section 3.2. That is, the sales price of a supplier not only affects its market demand but also influences the ordering probability of the manufacturer. In addition, due to the mutual influence between production planning and inventory control, we model the inventory dynamics of the individual supplier with the following kinematic equation.

$$\dot{x}_s(t) = q_s(t) + x_s(t) - D_s(t) \quad (1)$$

where $x_s(t)$ is the inventory level of the supplier, and $q_s(t)$ is the production rate of the supplier. The available inventory level is the summation of the production quantity and the leftover inventory. As shown in Equation (1), for a certain period t , the dynamics

of the inventory level is the difference between the available inventory level and the demand. Assume the initial inventory level of the supplier is zero, i.e., $x_s(0) = 0$.

Given the dynamic process of inventory as shown in Equation (1), the objective of an individual supplier is to maximize its net discounted profit over an infinite time horizon with discount factor $e^{-r_s t}$, here, r_s is the continuous discount rate, which is an exogenous variable.

$$\Pi_s = \int_0^\infty e^{-r_s t} [p_s(t)D_s(t) - \frac{1}{2}h_s x_s^2(t) - \frac{1}{2}c_s q_s^2(t)]dt, \quad (2)$$

In Equation (2), the supplier's sales revenue is given by $p_s(t)(\alpha_s - \beta_s p_s(t))$. Backlogging is allowed in this model, that is, the holding cost is incurred by the leftover raw materials at the end of each period. The holding cost is modeled as $h_s x_s^2(t)/2$, where h_s is the unit holding cost for each raw material, as the level of inventory increases, so does the labor force or time spent in inventory, which in turn increases the risk of obsolescence (Choi, 2013). The production cost is given by $c_s q_s^2(t)/2$, where c_s is the unit production cost, that is, the production cost is a convex function increasing with the production rate. Both the holding cost and the production cost are commonly modeled using the quadratic function in the literature (e.g., Jørgensen, 1986; Dobos, 2005; Ferguson & Toktay, 2006; Galbreth & Blackburn, 2006; Erickson, 2011). Note that the quadratic cost function is used in this paper for all the holding cost, production cost, and emission cost because: (1) there exists diminishing returns to variables, such as inventory level, production rate, and emissions rate; (2) the property of concavity can be imposed on payoff functions for both suppliers and manufacturer in a simplified way, while the flexibility of the functional form can still be maintained.

Based on the above analysis, each individual supplier seeks to maximize its long-run total discounted payoff over an infinite time horizon subject to inventory capacity at any time, i.e., Problem P_s (Supplier's sub-game):

$$\begin{aligned} \max \Pi_s = & \max_{p_s(t), q_s(t)} \int_0^{\infty} e^{-r_s t} [p_s(t) D_s(t) - \frac{1}{2} h_s x_s^2(t) - \frac{1}{2} c_s q_s^2(t)] dt, \\ \text{s.t. } & \dot{x}_s(t) = q_s(t) + x_s(t) - D_s(t) \end{aligned}$$

where the decision variables in Problem P_s are $p_s(t)$ and $q_s(t)$.

3.2 The Manufacturer's Decision Issue

Under the carbon emission regulation, i.e., the carbon tax scheme, the emissions-related cost is the vital component of the manufacturer's total cost. Let $E(t)$ be the emissions amount of the manufacturer in period t , which is predominantly determined by the emission rates of raw materials and the demand of the manufacturer. We denote ξ_i by the ordering probability of the manufacturer from supplier i , and we use multinomial logit function to describe the ordering probability (Lin & Sibdari, 2009).

$$\xi_i = \frac{e^{\tau_i - \rho p_i}}{\sum_{i=1}^N e^{\tau_i - \rho p_i}}, \quad (3)$$

where τ_i is the manufacturer's expected utility of reducing emissions by using the raw materials from supplier i , and ρ is the price sensitivity parameter. For manufacturer's decision, when $n \geq 2$, it is no longer appropriate to use the subscript 's' for a single supplier. Instead, we employ subscript 'i' to represent the variables related to suppliers when $n \geq 2$, such as, the sales price p_i . The unit emissions rate and the sales price of supplier i are denoted by ε_i and p_i , respectively. The summation of ξ_i equals one,

that is, $(\xi_1, \xi_2, \dots, \xi_n)$ is the sourcing profile of the manufacturer. Analogously, we use additive demand function to describe the manufacturer's demand, $D_m(t) = \alpha_m - \beta_m p_m(t)$, where $p_m(t)$ is the unit sales price of the manufacturer, α_m is the vertical intercept, and β_m is the slope of demand curve. Both α_m and β_m are nonnegative constants. Therefore, $E(t)$ can be formulated as the following kinematic equation.

$$\dot{E}(t) = \sum_{i=1}^n \varepsilon_i \xi_i (\alpha_m - \beta_m p_m(t)) + zE(t) \quad (4)$$

The first part of Equation (4) represents the total emissions amount incurred using raw materials from multiple suppliers. The second part is the amount of environmental absorption, where z ($z < 0$) is the absorption rate. In particular, this model can be applied to study a special scenario of procurement management from two suppliers, which will be discussed in the following section. At the initial period, assume the emission amounts of the manufacturer is zero, i.e., $E(0) = 0$.

The objective of the manufacturer is to maximize its profits over an infinite time horizon. The discounting of profits is accomplished through discount factor $e^{-r_m t}$, where r_m is the discount rate (an exogenous variable). It is particularly important to discount returns if the time horizon is infinite, so the integral in Equation (5) can be finite (Erickson, 2011). Following the emissions dynamics in Equation (4), the payoff function of the manufacturer can be formulated as follows.

$$\Pi_m = \int_0^{\infty} e^{-r_m t} [(\alpha_m - \beta_m p_m(t)) \sum_{i=1}^n \xi_i (p_m(t) - p_i(t)) - \frac{1}{2} \omega E^2(t)] dt \quad (5)$$

Regarding the integral function in Equation (5), the first part represents the revenue using the raw materials from suppliers. The purchasing quantity is determined

by ordering probability ξ_i . The second term is the emissions-related cost incurred by the carbon tax scheme. In this paper, the emissions cost is established by using an increasing convex function with the quadratic form, the unit carbon tax, ω , can be considered as the cost coefficient associated with the manufacturer's emissions amount. The quadratic form is commonly used in the literature of emissions control (e.g., Subramanian, Gupta, & Talbot, 2007; Choi, 2013; Bertinelli, Camacho, & Zou, 2014; Li, 2014). Besides, the reasons of using the quadratic function for the emission cost were mentioned above as the case with the holding cost and production cost in Section 3.1. The difference in the total revenue and the emissions cost is the profit of the manufacturer. In this model, in consideration of dynamic pricing with the constraint of the carbon tax scheme, we aim to analyze a manufacturer that adopts the ATO strategy; thus, the inventory cost can be ignored.

Based on the above analysis, the manufacturer's objective is to maximize its long-run total discounted payoff over an infinite time horizon subject to emissions amount at any time, i.e., Problem P_m (Manufacturer's sub-game):

$$\begin{aligned} \max \Pi_m &= \max_{p_m(t)} \int_0^\infty e^{-rt} [(\alpha_m - \beta_m p_m(t)) \sum_{i=1}^n \xi_i (p_m(t) - p_i(t)) - \frac{1}{2} \omega E^2(t)] dt, \\ \text{s.t. } \dot{E}(t) &= \sum_{i=1}^n \varepsilon_i \xi_i (\alpha_m - \beta_m p_m(t)) + zE(t), \end{aligned}$$

where the decision variable in this model is $p_m(t)$.

4. Equilibrium Analysis

This section focuses on the equilibrium analysis of three sub-games, including the

supplier's sub-game, the manufacturer's sub-game with multiple suppliers (the general case), and the manufacturer's sub-game with two suppliers (the special case), under the carbon tax scheme. Both suppliers and the manufacturer seek to maximize their payoff functions (the present value of the profit functions) by adopting appropriate pricing strategies over an infinite time horizon. Based on the two sub-game model settings in Section 3, we derive equilibria, including the open-loop equilibrium and the Markovian Nash equilibrium, for the individual supplier and manufacturer, respectively.

Both the open-loop and the Markovian Nash equilibria can be regarded as the solutions of two different models under specific situations, where the former depends only on time and the latter based on the state variable at that particular time (Chiang, 2012). Note that the open-loop equilibrium presents the original best decisions, which make it easier to derive tractable strategies. If the manufacturer and suppliers adopt the open-loop strategy, they can make an irreversible pre-commitment decision only at the beginning of the game (Gallego & Hu, 2014). However, the open-loop equilibrium is time-inconsistent. That is, the original best decision for some future period is inconsistent with what is preferred when that future period arrives (Chiang, 2013). Therefore, it is worth examining another strategy, i.e., the Markovian Nash equilibrium, to analyze the game model. Based on the outcomes of the Markovian Nash equilibrium, both the manufacturer and suppliers can make timely adjustments based on the changing of states to pursue the maximized total profit. The characteristics and practical implications of two types of equilibria in our models are further discussed in the following sub-sections.

4.1 Supplier's Sub-game Division

This subsection aims to address issues of optimal production and inventory management for individual suppliers. The decision issues are the optimization of the inventory level, which is a state variable, and the sales price and the production rate, which are control variables.

Based on the standard procedure from differential game theory (Dockner, Jorgensen, Long, & Sorger, 2000), the Hamiltonian function of the supplier sub-game can be established as follows:

$$H_s = p_s(t)(\alpha_s - \beta_s p_s(t)) - \frac{1}{2} h_s x_s^2(t) - \frac{1}{2} c_s q_s^2(t) + \lambda[q_s(t) + x_s(t) - D_s(t)], \quad (6)$$

where λ is a co-state variable (or the shadow price).

The supplier's necessary conditions for optimality are

$$\frac{\partial H_s}{\partial p_s} = \alpha_s - 2\beta_s p_s(t) + \lambda\beta_s = 0, \quad (7)$$

$$\frac{\partial H_s}{\partial q_s} = -c_s q_s + \lambda = 0. \quad (8)$$

Then, we can obtain $p_s = \frac{1}{2}(\frac{\alpha_s}{\beta_s} + \lambda)$ and $q_s = \frac{\lambda}{c_s}$.

Proposition 1. *There exists the Nash equilibrium for the supplier's sub-game.*

Proposition 1 indicates the existence of the Nash equilibrium for the supplier's sub-game model. In addition, the maximized Hamiltonian function of the supplier's sub-game is a concave function with respect to $x_s(t)$, $p_s(t)$, and $q_s(t)$. Therefore, we need to further analyze how these three variables influence the decisions of the supplier. The

equilibrium analyses for three variables over time are summarized in the following propositions.

Proposition 2. (*The Open-loop Equilibrium*) *The open-loop inventory level, the production rate, and the sales price, respectively, are given by*

$$x_s(t) = \frac{(1-r_s)\alpha_s}{2(bh_s - r_s + 1)}(1 - e^{gt}), \quad (9)$$

$$q_s(t) = \frac{1}{c_s} \left[\frac{\alpha_s h_s}{2(bh_s - r_s + 1)} - \frac{(1-r_s)\alpha_s f}{2(bh_s - r_s + 1)} e^{gt} \right], \quad (10)$$

$$p_s(t) = \frac{1}{2} \left[\frac{\alpha_s}{\beta_s} + \frac{\alpha_s h_s}{2(bh_s - r_s + 1)} - \frac{(1-r_s)\alpha_s f}{2(bh_s - r_s + 1)} e^{gt} \right], \quad (11)$$

$$\text{where } b = \frac{1}{c_s} + \frac{\beta_s}{2}, \quad f = \frac{rb - \sqrt{r_s^2 - 4r_s + 4bh_s + 4} - 2}{2b} \quad \text{and } g = \frac{1}{2}(r_s - \sqrt{r_s^2 - 4r_s + 4bh_s + 4}).$$

Proposition 2 illustrates the optimal operation trajectories, including the sales price, the production rate, and the inventory level of a supplier. When the supplier makes its individual decision at the initial state, both the price trajectory and the trajectory of the production rate demonstrate the decreasing trends. This is because the first-order conditions of $p_s(t)$ and $q_s(t)$ with respect to t are smaller than zero. The decreasing phenomenon of the inventory level is particularly apparent at the beginning phase. Then, with the increase in the production rate, this phenomenon will incur an increase in the inventory level. In addition, the way other parameters involved in control and state variables influence suppliers' decision is worth addressing. Based on the results in Proposition 2, closed forms of stable states ($t \rightarrow \infty$) and their monotonicity are further analyzed.

Proposition 3. *The stable states of sales price, production rate, and inventory level are*

given as follows:

(i) The stable state of the inventory level is

$$\tilde{x} = \frac{(1-r_s)\alpha_s}{2(bh_s - r_s + 1)} \quad (12)$$

which is a submodular function in (h_s, α_s) ;

(ii) the stable state of the production rate is

$$\tilde{q} = \frac{\alpha_s h_s}{2c_s(bh_s - r_s + 1)} \quad (13)$$

which is a submodular function in (α_s, c_s) ;

(iii) the stable state of the sales price is

$$\tilde{p} = \frac{\alpha_s}{2} \left[\frac{1}{\beta_s} + \frac{h_s}{2(bh_s - r_s + 1)} \right] \quad (14)$$

which is a supermodular function in (h_s, c_s) .

As we discussed above, the increasing phenomenon is particularly apparent at the beginning phase. Therefore, controlling and adjusting these factors for maximization of supplier profit is critical. The results of Proposition 3 present stable states of the inventory level, the production rate, and the sales price. In the long run, their trajectories change from a trend to constant values, which can assist suppliers in coordinating their operational behaviors at the initial period. The results in Propositions 2 and 3 are presented in the following three numerical examples.

EXAMPLE 1. Suppose $c_s = 100$, $\beta_s = 0.1$, and $r_s = 0.5$. The inventory trajectory with the variance of h_s and α_s is shown in Figure 1 (a), and the submodularity of $\tilde{x}$ is shown in Figure 1 (b). As shown in Figure 1 (a), $x_s(t)$ will converge to a constant value, which can be described by a closed-form, as shown in Equation (12). With the same initial

setting, the supplier is willing to increase its inventory level to meet the large-scale market demand, even when facing a higher inventory holding cost. In addition, the submodularity of $\tilde{x}$ implies that the individual supplier needs to adjust its stock level if the holding cost is continuously increasing, even with a larger market size; this is because the supplier should spend more sales revenue to cover the higher inventory holding cost.

<Insert Figure 1 (a) and Figure 1 (b) around here>

EXAMPLE 2. Suppose $h_s = 0.8$, $\beta_s = 0.1$, and $r_s = 0.5$. The trajectory of the production rate with the variance of α_s and c_s is shown in Figure 2 (a), and the submodularity of $\tilde{q}$ is shown in Figure 2 (b). The production rate converges to a constant value, illustrated in Equation (13). With respect to $\tilde{q}$, it shows an intuitive result; that is, $\tilde{q}$ decreases with c_s and increases with α_s . The submodularity of $\tilde{q}$ shows that a supplier will reduce its production rate when facing a smaller market size and a higher unit production cost. In practice, this result can occur when a supplier is preparing for adjusting its production technology to meet sustainability requirements.

<Insert Figure 2 (a) and Figure 2 (b) around here>

EXAMPLE 3. Suppose $\alpha_s = 10$, $\beta_s = 0.1$, and $r_s = 0.5$. Figure 3 (a) presents the price trajectories of three suppliers with the variance of c_s and h_s , and the supermodularity of $\tilde{p}$ is shown in Figure 3 (b). Considering an increase in c_s and h_s , a greener supplier (supplier 3) with the highest production cost increases its sales price to cover production and inventory costs. In this scenario, the greener supplier has to increase the sales price, that is, part of cost will shift to the manufacturer. However, with respect to the suppliers,

the increase in sales prices could induce the loss of market share. Therefore, there exists a trade-off for the suppliers.

<Insert Figure 3 (a) and Figure 3 (b) around here>

Note that the open-loop equilibrium presents the original best decisions, which make it easier to derive tractable strategies. However, the open-loop equilibrium is time-inconsistent. If a supplier adopts the open-loop strategy, it cannot observe the change in the state variable (Dockner, Jorgensen, Long, & Sorger, 2000). That is, the supplier can make an irreversible pre-commitment decision only at the beginning of the game using the open-loop strategy (Gallego & Hu, 2014). Therefore, it is worth examining another strategy, i.e., the Markovian Nash equilibrium, to analyze the game model. Based on the state equation of a supplier, the inventory level is influenced by the interaction between the sales price and the production rate. In this scenario, the supplier needs to design appropriate pricing and production strategies that depend on the state of the inventory level. That is, the supplier can make timely adjustments based on the changing of states to pursue the maximized total profit.

Proposition 4. (Markovian Nash Equilibrium) *The Markovian Nash equilibrium of the sales price and the production rate are characterized by*

$$\hat{p}_s(t) = \frac{1}{2} \left(\frac{\alpha_s}{\beta_s} + B + 2Dx(t) \right), \quad (15)$$

$$\hat{q}_s(t) = \frac{1}{c_s} (B + 2Dx(t)), \quad (16)$$

and the corresponding inventory level over time is given by

$$\hat{x}(t) = \frac{c_s(\alpha_s - \beta_s B) - 2B}{4D + 2c_s(1 + \beta_s D)} [1 - e^{\left(\frac{2D}{c_s} + \beta_s D + 1\right)t}], \quad (17)$$

$$\text{where } B = \frac{\alpha_s D}{1 + \beta_s D - r_s + 2D / c_s} \text{ and } D = \frac{(r_s - 2) - \sqrt{(2 - r_s)^2 + 2h_s(\beta_s + 2 / c_s)}}{2(\beta_s + 2 / c_s)}.$$

The Markovian Nash equilibria of the supplier are presented in Proposition 4, which indicate the more general results because each Markovian Nash equilibrium of a differential game is time consistent (Dockner, Jorgensen, Long, & Sorger, 2000). These solutions allow suppliers to control/adjust their production and pricing rates contingent upon the state of the game, which are more realistic and tractable results. In addition, the Markovian Nash equilibria of both price and production are dependent on and non-decreasing in the state variable. The characteristics of the monopolist for price and production rate are similar to the results in Proposition 2 because the Markovian Nash equilibrium takes into full consideration the strategic interactions through the evolution of cumulative demand, which is sub-game perfect (Chiang, 2012). Compared with the results in Propositions 2 and 4, we can observe the differences between two types of equilibria. That is, these results provide two types of decision modes for suppliers. However, unlike the open-loop equilibrium, the Markovian Nash equilibrium takes time and inventory into consideration jointly.

4.2 Manufacturer's Sub-game Division (A general case for $n > 2$)

In this section, we aim to study the sub-game model of the manufacturer with multiple independent suppliers ($n > 2$). The objective is to analyze the robustness of pricing strategies and to study the more general observations for the pricing strategies of the manufacturer. With respect to the effect of the carbon tax, the manufacturer needs to determine the optimal price to maximize its total profit. In this sub-game model, the

emissions amount is a state variable, and the sales price is a control variable. First, the decision issue is analyzed by considering the time factor. Then, the optimal sales price strategy with the state of emission amounts is discussed.

Based on the model P_m in Section 3.2, the Hamiltonian function of the payoff function of the manufacturer can be developed as follows.

$$H_m^n = (\alpha_m - \beta_m p_m(t)) \sum_{i=1}^n \xi_i(p_m(t) - p_i(t)) - \frac{1}{2} \omega E^2(t) + \mu \left[\sum_{i=1}^n \varepsilon_i \xi_i (\alpha_m - \beta_m p_m(t)) + zE(t) \right] \quad (18)$$

where μ is the co-state variable. The manufacturer's necessary condition for optimality should satisfy the following equation:

$$\frac{\partial H_m^n}{\partial p_m} = \alpha_m - 2\beta_m p_m + \beta_m \sum_{i=1}^n p_i \xi_i - \mu \beta_m \sum_{i=1}^n \varepsilon_i \xi_i = 0. \quad (19)$$

Proposition 5. *There exists the Nash Equilibrium for the manufacturer's sub-game with multiple suppliers ($n > 2$).*

This proposition indicates the existence of the equilibrium of the manufacturer's sub-game with multiple suppliers. Therefore, the maximized Hamiltonian function can be achieved. The optimal pricing strategy of the scenario with multiple independent suppliers ($n > 2$) is illustrated in the following propositions.

Proposition 6. (Open-loop Equilibrium) *For $n > 2$, the open-loop equilibrium of the sales price and the emissions amount of the manufacturer are*

$$p_m^n = \frac{1}{2}V + \frac{(\alpha_m - \beta_m S / 2)J^2}{4\sqrt{k}} [(r_m - 2z) + \sqrt{k} - (r_m - 2z - \sqrt{k})y_1 e^{\bar{a}t}], \quad (20)$$

$$E_n(t) = (\alpha_m - \frac{1}{2}\beta_m A)J \frac{\sqrt{K_1} - r_m + 2z}{2\sqrt{K_1}} (1 - e^{w_1 t}), \quad (21)$$

$$\text{where } V = \frac{\alpha_m}{\beta_m} + \sum_{i=1}^n p_i \xi_i, \quad S = \frac{\alpha_m}{\beta_m} + \sum_{i=1}^n c_i \xi_i, \quad J = \sum_{i=1}^n \varepsilon_i \xi_i, \quad \bar{a} = \frac{r_m - \sqrt{(r_m - 2z)^2 + 2\omega\beta_m J}}{2},$$

$$\bar{k} = (r_m - 2z)^2 + 2\beta_m \omega J^2, \text{ and } y_1 = \frac{r_m \beta_m J^2 - 4z - 2\sqrt{(r_m - 2z)^2 + 2\omega\beta_m J^2}}{2\omega\beta_m J^2}.$$

Since the manufacturer has to face the conflicting issue of sourcing from the supply pool with a large number of suppliers, Proposition 6 presents a more general result of the open-loop equilibrium strategies for the manufacturer. With respect to the manufacturer, the price is predominately determined by the ordering probability and unit procurement cost from each individual supplier. This also indicates that the carbon tax has less effect on the pricing issue for the manufacturer. The shadow price shows a positive correlation with the carbon tax. Accordingly, the manufacturer can adjust the price based on the initial setting to maximize its profit.

Proposition 7. (Markovian Nash Equilibrium) *For $n > 2$, the Markovian Nash equilibrium of the sales price and emission volumes of the manufacturer are given as*

$$\hat{p}_m^n(t) = \frac{1}{2}[V - JN_n - 2JQ_n \hat{E}(t)], \quad (22)$$

$$\hat{E}_n(t) = \frac{\alpha_m - \beta_m V / 2 + \beta_m JN_n / 2}{\beta_m Q_n J} (e^{\beta_m Q_n J^2 t} - 1), \quad (23)$$

$$\text{where } V = \frac{\alpha_m}{\beta_m} + \sum_{i=1}^n p_i \xi_i, \quad J = \sum_{i=1}^n \varepsilon_i \xi_i, \quad N_n = \frac{2Q_n U}{r - 4Q_n I}, \quad Q_n = \frac{r_m - \sqrt{r_m^2 + 8\omega I}}{8I}, \quad I = \frac{1}{4} \beta_m J^2,$$

$$\text{and } U = \frac{1}{2} J (\alpha_m - \beta_m \sum_{i=1}^n \xi_i p_i) + z \hat{E}(t).$$

In contrast to the sales price in the open-loop equilibrium, Proposition 7 illustrates that the Markovian Nash equilibrium of the sales price decreases over time with a relatively higher initial setting, and it converges to a fixed constant. This convergent

tendency is observable because Markovian Nash equilibrium takes full consideration of the interaction between the sales price and the emissions amount of the manufacturer over the time horizon. The result in Proposition 7 is also sub-game perfect and time consistent. In addition, the limit value of $E_n(t)$ is a fixed constant that is negatively correlated with carbon tax ω . This further shows that the carbon tax scheme shows a cost effective way to curb the carbon emissions. The manufacturer can follow the variance of the unit carbon tax to control and adjust its emissions amount and pricing, respectively. Based on the outcomes of numerical example 3, the following example is presented to describe the pricing trajectory and the trajectory of emissions volume of the manufacturer with the variance of ξ_i and ω when the manufacturer sources from three suppliers.

EXAMPLE 4. Suppose $\alpha_m = 500$, $\beta_m = 2$, $\varepsilon_1 = 0.9$, $\varepsilon_2 = 0.5$, $\varepsilon_3 = 0.1$, $z = -0.05$, $\omega = 12$, and $r_m = 0.5$. The price trajectory of the manufacturer sourcing from three suppliers with the variance of ξ_i is shown in Figure 4. Using the same data set, the trajectory of emission amounts and the profit of the manufacturer are shown in Figure 5 and Figure 6, respectively, with the variance of ω . The sales price of the manufacturer presents a concave trend with the variance of ξ_i in Figure 4. At the initial period, the sales price is lower if the manufacturer sources more from supplier 1 with the lowest sales price and the highest emissions rate. Meanwhile, the emissions amount of the manufacturer increases by using raw materials from supplier 1. The manufacturer must pay higher emissions cost, which is incurred by the carbon tax, and to increase its sales price to gain profit, which will reduce the demand of the manufacturer. Over time, the

manufacturer should make adjustments to its order profile. Then, as is evident from Figure 4, the sales price of the manufacturer shows a decreasing trend by increasing its sourcing quantity from a greener supplier (supplier 3) to reduce its emissions cost. On one hand, the manufacturer can source from a traditional supplier, such as supplier 1, with a lower procurement cost; on the other hand, the manufacturer can source from a greener supplier, such as supplier 3, to control its emissions amount. In the same situation, Figure 5 implies that the emissions volume of the manufacturer can also be controlled effectively under the carbon tax scheme. Figure 6 shows that the profit of the manufacturer follows an increasing trend, that is, the manufacturer benefits from adopting mixed sourcing strategy from multiple suppliers. In addition, this flexible or mixed sourcing strategy can benefit the manufacturer even when facing a higher value of the carbon tax.

<Insert Figure 4, Figure 5, and Figure 6 around here>

4.3 Further Discussions on the Equilibria Outcomes

In this section, we further discuss the differences and implications of both the open-loop and the Markovian Nash equilibria. With respect to the open-loop equilibrium, both the manufacturer and suppliers are forward looking and plan ex ante their decisions, which are dependent only on time. Specifically, at the beginning of the planning horizon, the manufacturer announces its decisions regarding the sales price, $p_m(t)$, ordering probability, ζ_i , before the supplier makes its production and sales decisions for each period t . To obtain the equilibrium outcomes, we start by solving the supplier's problem,

and then recursively solve the manufacturer's problem. In practice, the outcomes of the open-loop equilibrium can be used to assist the manufacturer in designing an original best sourcing contract. The manufacturer sticks to the preannounced sourcing contract for the entire duration of the game. However, the open-loop equilibrium is time-inconsistent as mentioned in Section 4.

Since periodically revisiting (adjusting) the sales price can help the manufacturer to exhaust the residual market (Chiang, 2013), that is, an ex post decision, hence making reasonable adjustment for both the manufacturer and suppliers is likely to be mutually beneficial and to fully capture the strategic interactions. Following the outcomes of the Markovian Nash equilibrium, the manufacturer may re-contract if it would bring extra profit by updating the state information. Nevertheless, in practice, re-contracting is undesirable for two reasons. First, the time value of money may erode because there exists processing time to deal with the latest state information and to redesign the contract following the Markovian Nash equilibrium strategy. Second, re-contracting can derogate the reputation of the manufacturer. Therefore, for a risk-averse manufacturer, the open-loop equilibrium strategy is still an excellent choice. Conversely, for a strong manufacturer being the leader in a supply chain, the Markovian Nash equilibrium strategy can be adopted to exhaust the residual market.

4.4 Manufacturer's Sub-game Division (A special case for $n = 2$)

In this subsection, we focus on analyzing the special case for the manufacturer's sub-game model with two types of suppliers: a greener supplier and a traditional supplier.

Based on the general case in Section 3.2, the sub-game model of the manufacturer with two suppliers can be developed as follows.

$$\max \Pi_m = \max_{p_m(t)} \int_0^\infty e^{-r_m t} [\xi(p_m(t) - p_1(t))D_m(t) + (1 - \xi)(p_m(t) - p_2(t))D_m(t) - \frac{1}{2}\omega E^2(t)]dt, \quad (24)$$

$$s.t. \dot{E}(t) = \varepsilon_1 \xi(\alpha_m - \beta_m p_m(t)) + \varepsilon_2 (1 - \xi)(\alpha_m - \beta_m p_m(t)) + zE(t). \quad (25)$$

With respect to the integral function in Equation (24), the first two parts represent the revenue using the raw materials from suppliers 1 and 2, respectively. The sourcing quantity is determined by ordering probability ξ . The existence of the equilibrium of models (24) and (25) is straightforward, it is because the second-order condition of the Hamiltonian function with respect to p_m is smaller than zero; that is, it is strictly concave in p_m , and the maximized Hamiltonian function can be achieved. In addition, the maximized Hamiltonian function is a concave function with respect to $E(t)$. The optimal pricing strategy of the manufacturer is summarized in the following propositions.

Proposition 8. (Open-Loop Equilibrium). *The open-loop sales price and emissions amount of the manufacturer are given by*

$$p_m(t) = G + T^2(\alpha_m - \beta_m G) \left[\frac{r_m - 2z + \sqrt{K}}{2\sqrt{K}} e^{m_1 t} + \frac{[\sqrt{K} - (r_m - 2z)]^2}{2(\beta_m B)^2 \sqrt{K}} \right], \quad (26)$$

$$E(t) = T(\alpha_m - \beta_m G) \frac{r_m - 2z - \sqrt{K}}{2\sqrt{K}} (1 - e^{m_1 t}), \quad (27)$$

where $G = [\alpha_m + p_2 \beta_m + \beta_m \xi(p_1(t) - p_2(t))] / 2\beta_m$, $T = \xi(\varepsilon_1 - \varepsilon_2) + \varepsilon_2$, $K = (r_m - 2z)^2 + 2\omega(\beta_m T)^2$ and $m_1 = (r_m - \sqrt{K}) / 2$.

Proposition 8 presents the equilibrium of the sales price and emissions amount of a manufacturer purchasing from two independent suppliers. The sales price of the manufacturer is jointly determined by the carbon tax and its procurement cost (the sales

prices of suppliers). Facing a specific carbon tax ω , when the manufacturer orders more from the greener supplier 2 (ξ is decreasing), its sales price presents the decreasing trend. In addition, the emissions volume of the manufacturer shows the decreasing trend with the increasing of the carbon tax. Further analysis is conducted when t moves towards infinity, the stable states of $p_m(t)$ and $E(t)$ ($t \rightarrow \infty$) and their monotonicity are summarized in the following proposition.

Proposition 9. *The stable states of the sales price and emissions amount of the manufacturer are given as follows.*

(i) *The stable state of sales price of the manufacturer is*

$$\tilde{p}_m = G + T^2(\alpha_m - \beta_m G) \frac{[\sqrt{K} - (r_m - 2z)]^2}{2(\beta_m B)^2 \sqrt{K}} \quad (28)$$

which is a supermodular function in both (ω, p_1) and (ω, p_2) ;

(ii) *the stable state of the emissions amount of the manufacturer is*

$$\tilde{E}(t) = T(\alpha_m - \beta_m G) \frac{r_m - 2z - \sqrt{K}}{2\sqrt{K}} \quad (29)$$

which is a submodular function in both (ω, p_1) and (ω, p_2) .

This proposition describes the main results for the manufacturer's pricing issue. The characteristic of the stable price value presents a monotone trend with respect to the carbon tax and procurement prices. The increase in p_m reduces the demand of the manufacturer. That is, the total cost will be transferred to customers. However, the amounts of carbon emissions of the manufacturer present a decreasing trend with respect to the carbon tax and procurement prices. In addition, the above pricing strategy focuses only on the one-shot decision based on the initial setting. The submodularity of

$\tilde{E}(t)$ indicates that the manufacturer would select a greener supplier to reduce the emissions amount even when facing a higher procurement price and carbon tax. The manufacturer can also adjust its pricing strategy based on the real-time emissions level and the value of carbon tax to make a dynamic decision. In the following content, we aim to study the time-consistent equilibrium. The Markovian Nash equilibrium for the special case is shown as follows.

Proposition 10. (Markovian Nash Equilibrium) The Markovian Nash equilibrium strategies of the sales price and the emissions amount of a manufacturer are given as

$$\hat{p}_m(t) = A - \frac{1}{2}TM + TQ\hat{E}(t), \quad (30)$$

$$\hat{E}(t) = \frac{T(\alpha_m - \beta_m M)}{z - 2\beta_m QT} [e^{(z - 2\beta_m QT)t} - 1], \quad (31)$$

where $A = \frac{\alpha_m + \beta_m p_2 + \xi \beta_m (p_1 - p_2)}{2\beta_m}$, $T = \xi(e_1 - e_2) + e_2$, $M = \frac{\alpha_m TQ}{r_m - z - \beta_m QT^2}$, and

$$Q = \frac{r_m - 2z - \sqrt{(r_m - 2z)^2 + 2\omega\beta_m T^2}}{2\beta_m T^2}.$$

In contrast to the static outcomes in the open-loop equilibrium, Proposition 10 characterizes the trajectory of the price with respect to the changing of emissions amount. That is, the manufacturer can dynamically adjust the pricing strategy based on the observation of the current emissions level. The emissions level of the manufacturer shows a decreasing trend with respect to the carbon tax. In addition, in the Markovian Nash equilibrium strategy, the sales price of the manufacturer is decreasing with regards to reducing of its emissions amount. Furthermore, Equation (31) shows the effect of emissions on the long-run values of the manufacturer's sales price. That is, the carbon

tax scheme is an effective way to curb the emissions amount of the manufacturer by flexibly sourcing from multiple suppliers.

5. Managerial Implications

The research results provide meaningful managerial implications for both suppliers and the manufacturer. The suppliers can have a better understanding of setting appropriate prices in consideration of production planning and inventory control. The manufacturer can also understand how to determine a reasonable price to jointly maximize profit and minimize emissions-related cost.

For the manufacturer, the results of the comparative statics analysis for emission volumes indicate that the manufacturer would select a greener supplier to reduce the emissions amount even when facing a higher procurement price and a carbon tax. In addition, the characteristic of the sales price of the manufacturer presents a monotone trend with respect to the carbon tax and procurement cost (sourcing prices of raw materials). With the increasing sales price, the total cost will be transferred to customers. The manufacturer would cooperate with the traditional suppliers in the initial phase of implementing the carbon tax scheme. However, this phenomenon will change in the long run. The sales price shows a significant convergence trend. This implies that cooperating with a greener supplier incurs the lowest sales price, which in turn, contributes to expanding the market demand for the manufacturer. Therefore, to avoid losing business volumes, the manufacturer needs to adjust its pricing strategy and make a dynamic decision based on the real-time emissions level and the price of carbon tax.

For both traditional and greener suppliers, increasing the inventory level to meet the large-scale market demand is a good choice, even when facing a higher inventory holding cost. Nevertheless, each supplier needs to adjust its stock level dynamically if the holding cost is continuously increasing, even with a larger market size. This is because the increasing inventory holding cost can affect the supplier's sales revenue significantly. As mentioned above, the manufacturer would prefer cooperating with a greener supplier in the long run. If a supplier is preparing to transfer its current production technology from traditional to environmental, the supplier has to increase its sales price, and part of the cost will shift to the manufacturer. The increase in sales prices could induce the loss of market share. Nevertheless, the buyers are willing to pay higher for the environmentally friendly materials in the future (Agrawal & Lee, 2016). A greener supplier takes over the market share from its competitors and also sets up a high barrier for new entrants as the technology transition process is lengthy. Therefore, to achieve the goal of profit maximization, both types of suppliers should adopt inter-temporal strategies based on the original best decisions. In addition, to avoid time-inconsistent, the suppliers should make adjustment dynamically based on the changing inventory states which are influenced by the interaction between the production and pricing strategies.

To summarize, the pre-committed or contingent pricing strategies presented in this paper can jointly achieve the goals of profit maximization and carbon emissions minimization for both suppliers and manufacturers simultaneously, and can facilitate sustainable operations of the upstream supply chains.

6. Conclusions

Motivated by the influence of global climate change in recent years and the research trend of sustainable supply chain management, this paper formulated a two-stage differential game model to study pricing issues between a manufacturer and suppliers under the carbon tax scheme. The contributions and managerial insights of this paper can be summarized as follows.

First, a sub-game model for each independent supplier was developed with the constraint of inventory capacity. The open-loop strategies for the price, the production, and the inventory level were derived. In addition, with respect to the interactions among price, production rate, and inventory level, the Markovian Nash equilibrium was identified to illustrate the characteristic of time consistency for these strategies. The managerial insight of this sub-game is that each individual supplier can adjust its pricing and production strategies with respect to the timely information of its inventory level. That is, each individual supplier can maximize total profit by controlling the current inventory level and can update its price and production rate to enlarge its business volume share in a timely manner.

Second, starting from the view point of the manufacturer that adopts the ATO strategy, a sub-game model was developed under the carbon tax scheme. In this model, the manufacturer has to consider the constraint of its emissions amount. This factor will affect the pricing issue for the manufacturer. The results indicate that the carbon tax and the unit procurement price are two predominant factors. The manufacturer might

be inclined to allocate the order to those traditional suppliers when facing a lower carbon tax at the beginning period. In the long-run, the manufacturer can benefit from cooperation with greener suppliers. In addition, the Markovian Nash equilibrium further presented the interaction between the price setting of the manufacturer and its emissions level.

Third, the basic pricing model for the manufacturer was extended by cooperation with multiple suppliers. Due to the selection preference of the manufacturer, enlarging the pool of potential suppliers could benefit the manufacturer and curb its emissions amount. The more general setting for both the open-loop equilibrium and the Markovian Nash equilibrium were derived. The latter aimed to overcome the time inconsistency of the open-loop equilibrium. The Markovian Nash equilibrium indicates that the manufacturer can follow the variance of the unit carbon tax to control and adjust its emission amounts and pricing, respectively.

In addition, the outcomes of the open-loop equilibrium and the Markovian Nash equilibrium provide managerial implications for both the manufacturer and suppliers. First, both players can make reasonable decisions under different states of information. The open-loop equilibrium can be used by both players to make reasonable decisions only based on their initial information, which generates the static outcomes for the entire duration of the game. In order to respond to all possibilities of state information and to fully capture strategic interactions, the Markovian Nash equilibrium can be adopted. Second, because of the limited information at the beginning of the planning horizon, the open-loop equilibrium can be used to design an original best sourcing

contract. As the market conditions and the customer demand change over time, the manufacturer may make timely adjustment to its sourcing contract based on the outcomes of the Markovian Nash equilibrium. Facing the outcomes of two types of equilibria, one conceivable way for the manufacturer in designing a reasonable sourcing contract is to consider the product characteristics and the duration of a contract simultaneously. The following two research questions are worth studying in the future research: How can the key turning point be identified for the manufacturer to re-contract and re-negotiate with its suppliers? Is compensation to suppliers a cost-effective way to solve the re-contracting issue?

The developed models can be extended in several directions. The manufacturer can purchase the components from each supplier with different ratios, which can be determined by the bill of material of the final product, i.e., different ratios could affect both the inventory level of suppliers and the emissions amount of the manufacturer. In addition, the manufacturer also could invest in other technology, such as carbon emissions storage, to curb its emissions amount. Adopting other economic means, such as the emissions trading scheme, is also an alternative way to optimize the emissions amount. We hope that our work can provide a solid foundation for future research in this domain.

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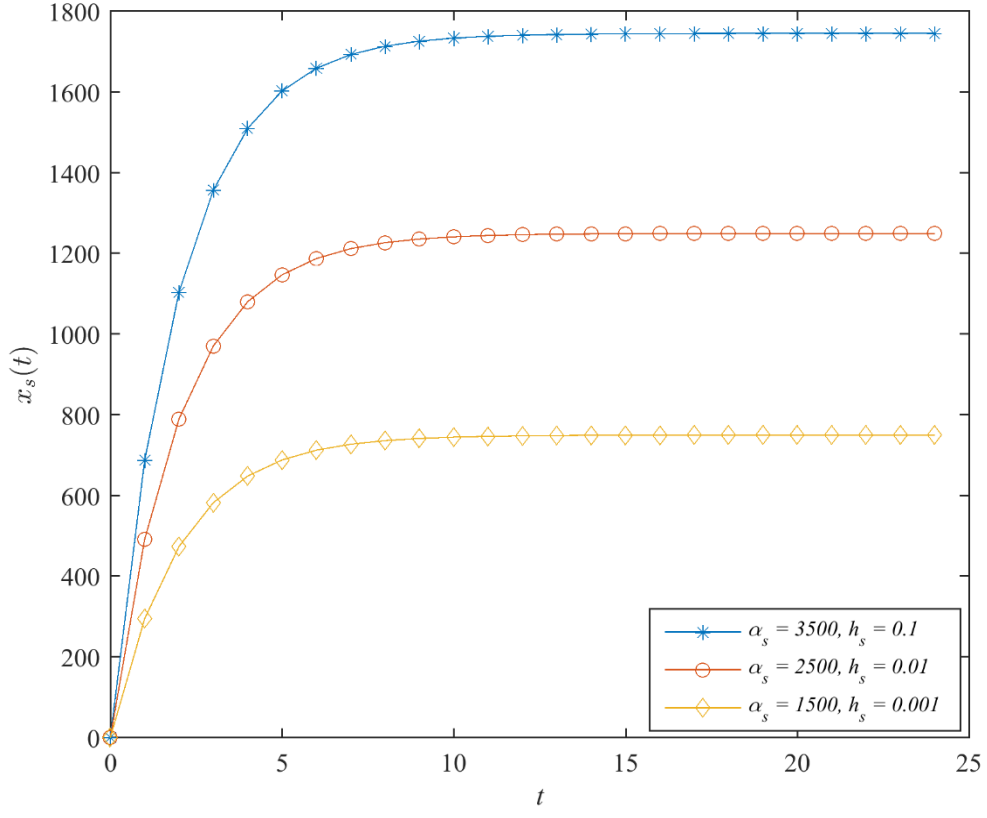
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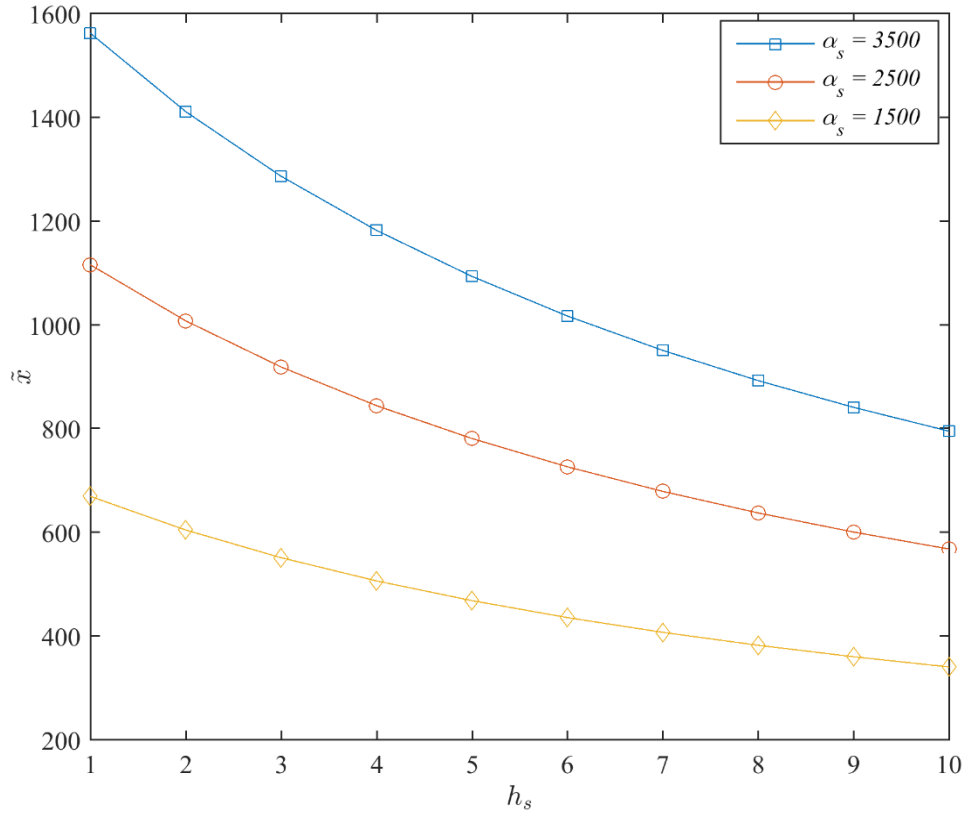
plants#additional-resources.

Table 1: Notation

Variable	Description
n	The number of suppliers, $n \geq 1$
t	The decision time period, $t \in [0, \infty)$
$D_s(t)$	The demand of a supplier in period t
α_s	The intersection of a supplier's demand function
β_s	The slope of a supplier's demand function
$p_s(t)$	The sales price of a single supplier in period t
$p_i(t)$	The sales price of supplier i with $n \geq 2$
$x_s(t)$	The inventory level of a supplier in period t
$q_s(t)$	The production rate of a supplier in period t
r_s	The continuous discount rate of a supplier
h_s	The unit holding cost of a supplier
c_s	The unit production cost of a supplier
$E(t)$	The emissions amount of a manufacturer in period t
ζ_i	The ordering probabilities of a manufacturer from supplier i
τ_i	The manufacturer's expected utility of reducing emissions by sourcing from supplier i
ρ	The price sensitivity parameter
$D_m(t)$	The demand of a manufacturer in period t
α_m	The intersection of a manufacturer's demand function
β_m	The slope of a manufacturer's demand function
$p_m(t)$	The unit sales price of a manufacturer in period t
ε_i	The unit emissions rate by using materials from supplier i
z	The absorption rate of emissions
r_m	The continuous discount rate of a manufacturer
ω	The unit carbon tax

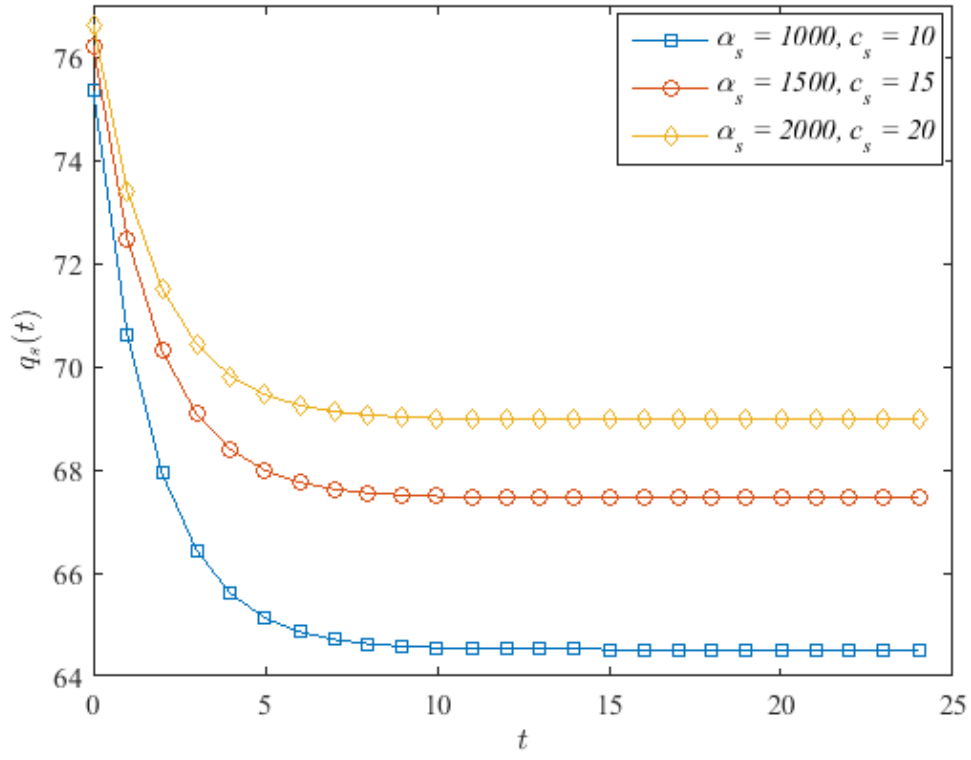


(a)

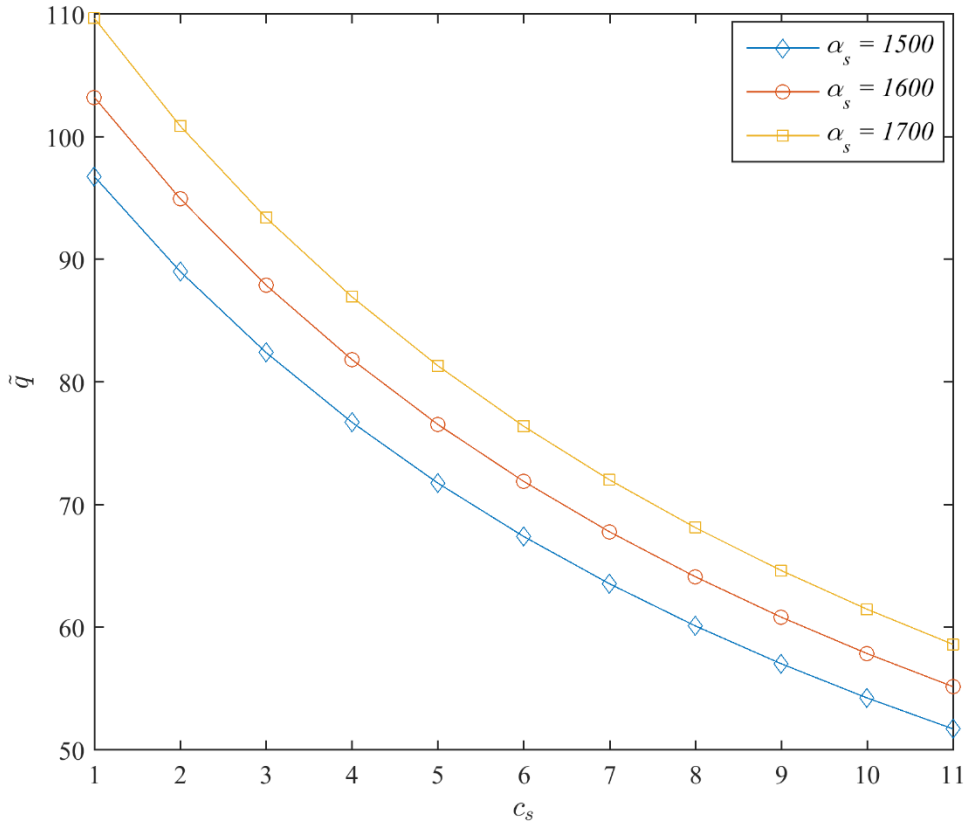


(b)

Figure 1: Inventory trajectory with variances of the unit holding cost of a supplier (h_s) and the intersection of a supplier's demand (α_s) and its submodularity

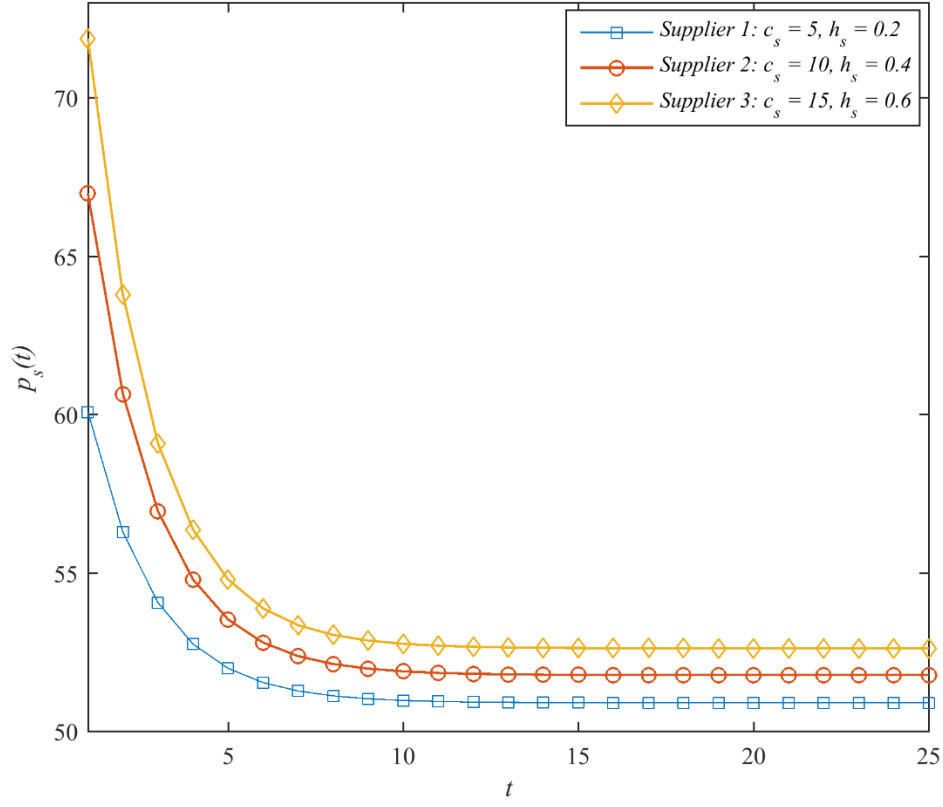


(a)

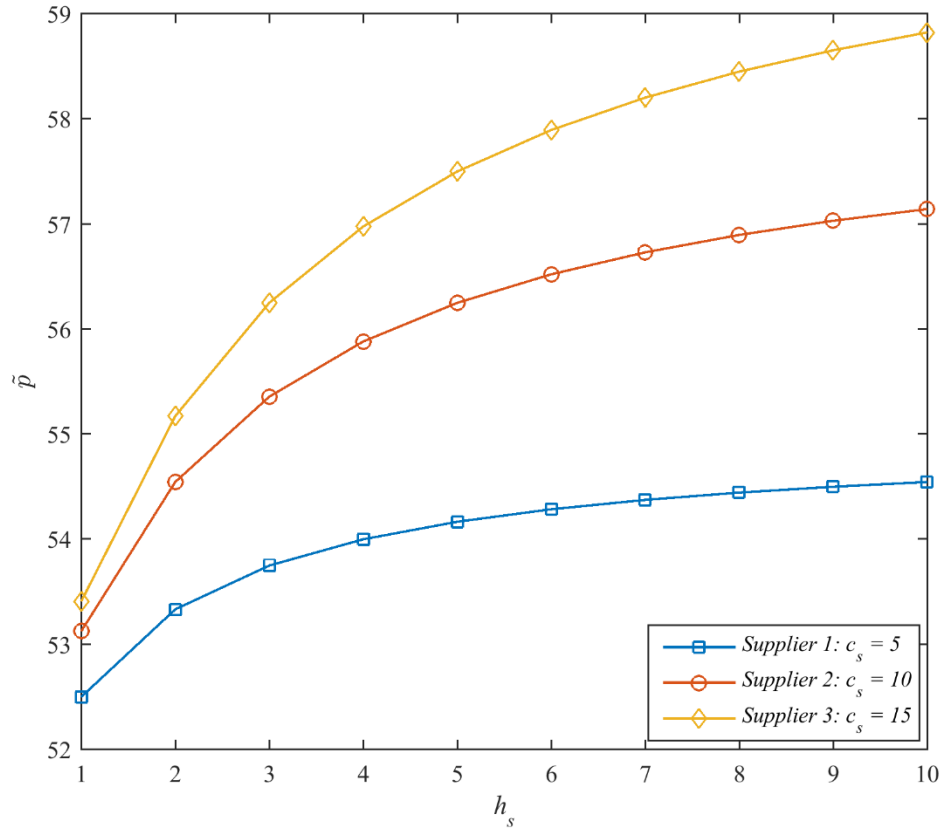


(b)

Figure 2: Production rate trajectory with variances of the unit production cost of a supplier (c_s) and the intersection of a supplier's demand (α_s) and its submodularity



(a)



(b)

Figure 3: Price trajectory with variances of the unit production cost of a supplier (c_s) and the unit holding cost of a supplier (h_s) and its supermodularity

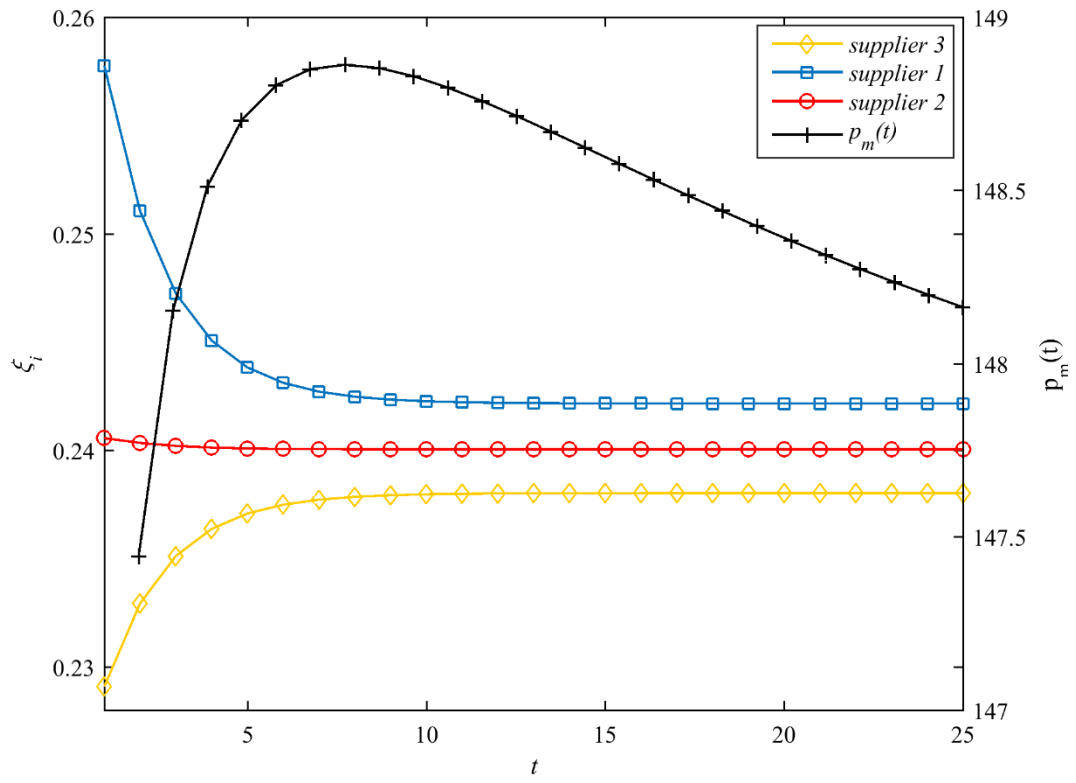


Figure 4: Price trajectory of the manufacturer sourcing from three suppliers

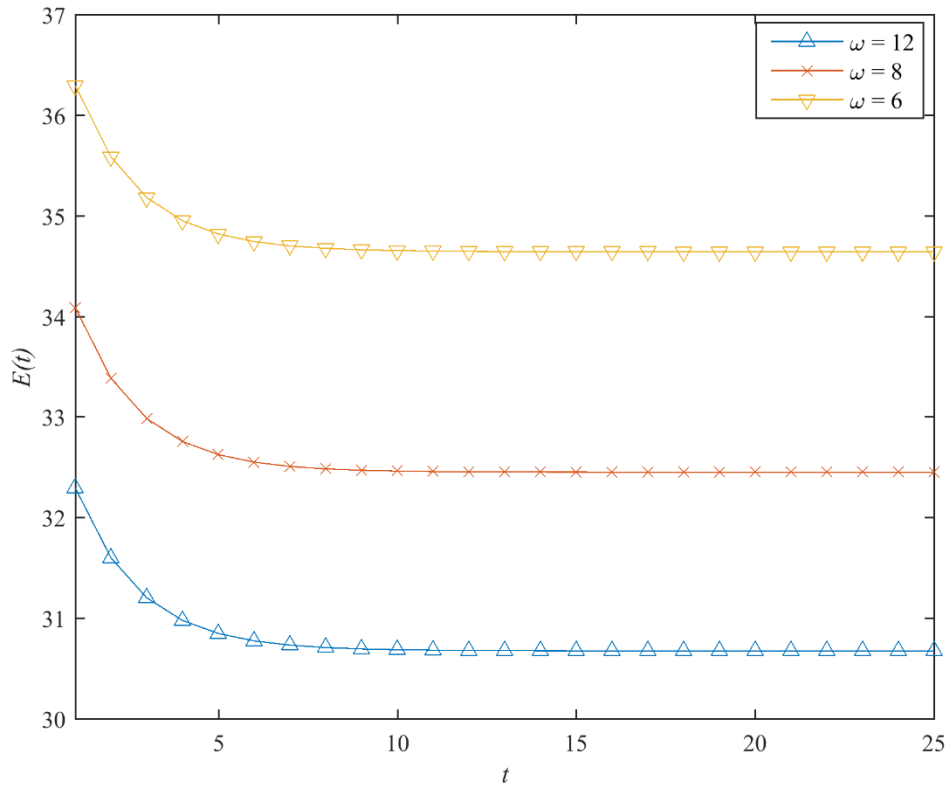


Figure 5: Emission volumes of the manufacturer sourcing from three suppliers with the variances of the unit carbon tax (ω)

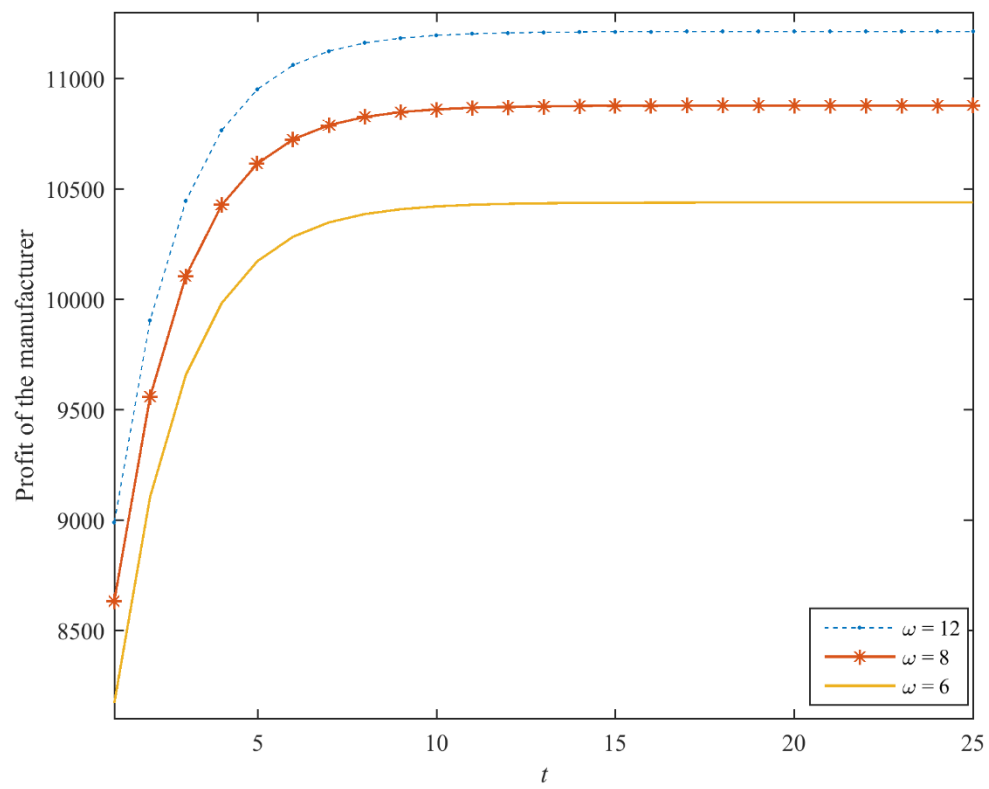


Figure 6: The profit of the manufacturer sourcing from three suppliers with the variance of the unit carbon tax (ω)

APPENDICES

PROOF OF PROPOSITION 1.

Based on the Hamiltonian function of the supplier Equation (6), the Hessian matrix for both p_s and q_s can be formulated as follows.

$$HM_s = \begin{bmatrix} \frac{\partial^2 H_s}{\partial^2 q_s} & \frac{\partial^2 H_s}{\partial q_s \partial p_s} \\ \frac{\partial^2 H_s}{\partial p_s \partial q_s} & \frac{\partial^2 H_s}{\partial^2 p_s} \end{bmatrix} = \begin{bmatrix} -c_s & 0 \\ 0 & -2\beta_s \end{bmatrix}, \quad (\text{A.1})$$

where c_s and β_s are nonnegative variables.

The Hamiltonian function of the supplier is concave in (p_s, q_s) , because the Hessian matrix is negative definite and the Legendre-Clebsch condition (Grass et al., 2008) can be satisfied.

Q.E.D.

PROOF OF PROPOSITION 2.

Substituting p_s and q_s into Equation (1), then, the inventory dynamics can be obtained as

$$\dot{x}_s(t) = \left(\frac{1}{c_s} + \frac{\beta_s}{2}\right)\lambda(t) + x(t) - \frac{\alpha_s}{2}. \quad (\text{A.2})$$

A non-homogeneous linear system with constant coefficients can be developed as follows, by combining the co-state equation

$$\dot{\lambda}(t) = (r_s - 1)\lambda(t) + h_s x(t). \quad (\text{A.3})$$

Note that

$$\begin{bmatrix} \dot{\lambda} \\ \dot{x} \end{bmatrix} = A \begin{bmatrix} \lambda \\ x \end{bmatrix} + B, \quad (\text{A.4})$$

where $A = \begin{bmatrix} r_s - 1 & h_s \\ b & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ -.5\alpha_s \end{bmatrix}$, and $b = \frac{1}{c_s} + \frac{\beta_s}{2}$.

Two eigenvalues of A are denoted by a_1, a_2 , and the eigenvector of A is denoted by H as follows.

$$a_1 = .5(r_s - \sqrt{r_s^2 - 4r_s + 4bh_s + 4}), \quad (\text{A.5})$$

$$a_2 = .5(r_s + \sqrt{r_s^2 - 4r_s + 4bh_s + 4}), \quad (\text{A.6})$$

$$H = \begin{bmatrix} \frac{r_sb - \sqrt{r_s^2 - 4r_s + 4bh_s + 4} - 2}{2b} & \frac{r_sb + \sqrt{r_s^2 - 4r_s + 4bh_s + 4} - 2}{2b} \\ 1 & 1 \end{bmatrix}. \quad (\text{A.7})$$

Therefore, λ and x can be specified as

$$\begin{aligned} \begin{bmatrix} \lambda \\ x \end{bmatrix} &= H \begin{bmatrix} e^{a_1 t} & 0 \\ 0 & e^{a_2 t} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} - A^{-1}B \\ &= \begin{bmatrix} e^{a_1 t} w_1 & e^{a_2 t} w_2 \\ e^{a_1 t} & e^{a_2 t} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} - \frac{\alpha_s}{2(bh_s - r + 1)} \begin{bmatrix} -h_s \\ r - 1 \end{bmatrix}, \end{aligned} \quad (\text{A.8})$$

where $w_1 = \frac{r_sb - \sqrt{r_s^2 - 4r_s + 4bh_s + 4} - 2}{2b}$, $w_2 = \frac{r_sb + \sqrt{r_s^2 - 4r_s + 4bh_s + 4} - 2}{2b}$.

The two boundary conditions $x(0) = 0$ and $\lim_{t \rightarrow \infty} e^{-rt} \lambda(t) x(t) = 0$ imply $k_1 = \frac{\alpha_s(r-1)}{2(bh_s-r+1)}$, and $k_2 = 0$. Thus, the optimal inventory level path, the production path, and the path of price can be obtained as follows:

$$x(t) = \frac{(1 - r_s)\alpha_s}{2(bh_s - r_s + 1)}(1 - e^{gt}), \quad (\text{A.9})$$

$$q(t) = \frac{1}{c_s} \left(\frac{\alpha_s h_s}{2(bh_s - r_s + 1)} - \frac{(1 - r_s)\alpha_s f}{2(bh_s - r_s + 1)} e^{gt} \right), \quad (\text{A.10})$$

$$p(t) = \frac{1}{2} \left(\frac{\alpha_s}{\beta_s} + \frac{\alpha_s h_s}{2(bh_s - r_s + 1)} - \frac{(1 - r_s)\alpha_s f}{2(bh_s - r_s + 1)} e^{gt} \right). \quad (\text{A.11})$$

To avoid confusion, we use f and g to replace w_1 and a_1 . Q.E.D.

PROOF OF PROPOSITION 3.

Limiting the price path, the path of production rate, and the path of inventory level with respect to time t , enable the stable states of price, production rate, and inventory level, respectively, to be obtained. In view of Topkis (1998), it is equivalent to verifying that the cross-partials of $\frac{\partial^2 \tilde{x}}{\partial \alpha_s \partial h_s}$, $\frac{\partial^2 \tilde{q}}{\partial \alpha_s \partial c_s} \leq 0$, and $\frac{\partial^2 \tilde{p}}{\partial h_s \partial c_s} \geq 0$, respectively.

(i) The cross partial derivative of $\tilde{x}$ with respect to h_s and α_s is

$$\frac{\partial^2 \tilde{x}}{\partial \alpha_s \partial h_s} = -\frac{2(1 - r_s)b}{[2(bh_s - r_s + 1)]^2} < 0. \quad (\text{A.12})$$

Thus, $\tilde{x}$ is submodular in (α_s, h_s) .

(ii) The cross partial derivative of $\tilde{q}$ with respect to α_s and c_s is

$$\frac{\partial^2 \tilde{q}}{\partial \alpha_s \partial c_s} = -\frac{h_s/\beta_s + 2(1 - r_s)}{[2c_s(bh_s - r_s + 1)]^2} < 0. \quad (\text{A.13})$$

Thus, $\tilde{q}$ is submodular in (α_s, c_s) .

(iii) The cross partial derivative of $\tilde{p}$ with respect to h_s and c_s is

$$\frac{\partial^2 \tilde{p}}{\partial h_s \partial c_s} = \frac{(1 - r_s)h_s}{c_s^2(bh_s - r_s + 1)^3} > 0. \quad (\text{A.14})$$

Thus, $\tilde{p}(t)$ is supermodular in (h_s, c_s) .

Q.E.D.

PROOF OF PROPOSITION 4.

The Markov perfect equilibrium is derived by using the Hamilton-Jacobi-Bellman equations.

$$\begin{aligned} r_s V_s = \max_{p_s, q_s} \{ & p_s(t)(\alpha_s - \beta_s p_s(t)) - \frac{1}{2} h_s (x_s(t))^2 - \frac{1}{2} c_s (q_s(t))^2 \\ & + \frac{\partial V_s}{\partial x} [q_s(t) + x_s(t) - (\alpha_s - \beta_s p_s(t))] \} \end{aligned} \quad (\text{A.15})$$

Taking the first order derivative of Equation A.15 with respect to p_s and q_s , respectively, for maximization of Equation A.15, we get

$$\frac{\partial V_s}{\partial p_s} = \alpha_s - 2\beta_s p_s + \frac{\partial V_s}{\partial x} \beta_s = 0, \quad (\text{A.16})$$

$$\frac{\partial V_s}{\partial q_s} = -c_s q_s + \frac{\partial V_s}{\partial x} = 0. \quad (\text{A.17})$$

Then, we can obtain $p_s = \frac{1}{2}(\frac{\alpha_s}{\beta_s} + \frac{\partial V_s}{\partial x})$ and $q_s = \frac{1}{c_s} \frac{\partial V_s}{\partial x}$. Substituting p_s and q_s into Equation A.15, gives

$$r_s V_s = \frac{\alpha_s^2}{4\beta_s} - \frac{1}{2} h_s x^2 + (x - \frac{\alpha_s}{2}) \frac{\partial V_s}{\partial x} + (\frac{\beta_s}{4} + \frac{1}{2c_s}) (\frac{\partial V_s}{\partial x})^2. \quad (\text{A.18})$$

Conjecture the functional form for the value function: $V_s = A + Bx + Dx^2$, where A , B , and D are determined by A.18. Substituting $\frac{\partial V_s}{\partial x} = B + 2Dx$ into A.18, gives

$$\begin{aligned}
r_s V_s &= r_s (A + Bx + Dx^2) \\
&= \frac{\alpha_s^2}{4\beta_s} - \frac{1}{2}h_s x^2 + (x - \frac{\alpha_s}{2})(B + 2Dx) + (\frac{\beta_s}{4} + \frac{1}{2c_s})(B + 2Dx)x^2 \\
&= [(\beta_s + \frac{2}{c_s})D^2 + 2D - \frac{h_s}{2}]x^2 + [B(1 - \beta_s D + \frac{2D}{c_s}) - \alpha_s D]x \\
&\quad + \frac{\alpha_s^2}{4\beta_s} - \frac{\alpha_s B}{2} + \frac{\beta_s B^2}{4} + \frac{B^2}{2c_s},
\end{aligned} \tag{A.19}$$

which implies

$$\begin{aligned}
(\beta_s + \frac{2}{c_s})D^2 + 2D - \frac{h_s}{2} &= 0, \\
B(1 - \beta_s D + \frac{2D}{c_s}) - \alpha_s D &= 0, \\
\frac{\alpha_s^2}{4\beta_s} - \frac{\alpha_s B}{2} + \frac{\beta_s B^2}{4} + \frac{B^2}{2c_s} - rA &= 0.
\end{aligned} \tag{A.20}$$

Giving

$$\begin{aligned}
D &= \frac{(r_s - 2) \pm \sqrt{(2 - r_s)^2 + 2h_s(\beta_s + 2/c_s)}}{2(\beta_s + 2/c_s)}, \\
B &= \frac{\alpha_s D}{1 + \beta_s D - r_s + 2D/c_s}, \\
A &= \frac{1}{r} \left(\frac{\alpha_s^2}{4\beta_s} - \frac{\alpha_s B}{2} + \frac{\beta_s B^2}{4} + \frac{B^2}{2c_s} \right).
\end{aligned} \tag{A.21}$$

Substituting p_s and q_s into $x(t)$, the inventory dynamics equation can be specified as

$$\dot{x}(t) = \left(\frac{2D}{c_s} + \beta_s D + 1 \right) x + \frac{B}{c_s} + \frac{\beta_s B - \alpha_s}{2}, \tag{A.22}$$

since $x(t)$, which is specified above, has to converge in t , the solution of D must satisfy

$D < 0$ (Erickson, 2011). Only one of two roots of D is eligible.

The solution of $x(t)$ is

$$x(t) = \frac{c_s(\alpha_s - \beta_s B) - 2B}{4D + 2c_s(1 + \beta_s D)} [1 - e^{(\frac{2D}{c_s} + \beta_s D + 1)t}]. \quad (\text{A.23})$$

Therefore, the Markov perfect equilibria can be obtained as follows.

$$p_s(t) = \frac{1}{2} \left[\frac{\alpha_s}{\beta_s} + B + 2Dx(t) \right], \quad (\text{A.24})$$

$$q_s(x(t)) = \frac{1}{c_s} (B + 2Dx(t)). \quad (\text{A.25})$$

Q.E.D.

PROOF OF PROPOSITION 5.

The second order derivative of the manufacturer's Hamiltonian function is $-2\beta_m$, where β_m is a nonnegative variable because the manufacturer's Hamiltonian function is strictly concave in p_m . Thus, the existence of the Nash equilibrium is proven.

Q.E.D.

PROOF OF PROPOSITION 6.

The manufacturer's Hamiltonian function can be developed as

$$H_m^n = (\alpha_m - \beta_m p_m) \sum_{i=1}^n \xi_i (p_m - p_i) - \frac{1}{2} \omega E^2 + u \left[\sum_{i=1}^n \varepsilon_i \xi_i (\alpha_m - \beta_m p_m) + zE \right], \quad (\text{A.26})$$

The manufacturer's necessary condition for optimality is

$$\frac{\partial H_m^n}{\partial p_m} = \alpha_m - 2\beta_m p_m + \beta_m \sum_{i=1}^n p_i \xi_i - u \beta_m \sum_{i=1}^n \varepsilon_i \xi_i = 0, \quad (\text{A.27})$$

then

$$p_m = \frac{1}{2}(V - uJ), \quad (\text{A.28})$$

where $V = \frac{\alpha_m}{\beta_m} + \sum_{i=1}^n p_i \xi_i$, and $J = \sum_{i=1}^n e_i \xi_i$. Substituting p_m into Equation (4), the dynamics of the emission volumes can be obtained as

$$\dot{E}_n(t) = \frac{1}{2}\beta_m u J^2 + zE(t) + (\alpha_m - \frac{1}{2}\beta_m V)J \quad (\text{A.29})$$

A non-homogeneous linear system with constant coefficients can be developed as follows by combining the co-state equations

$$\dot{u}(t) = r_m u - \frac{\partial H_m^n}{\partial E} = (r_m - z)u + wE. \quad (\text{A.30})$$

Note that

$$\begin{bmatrix} \dot{u} \\ \dot{E} \end{bmatrix} = W \begin{bmatrix} u \\ E \end{bmatrix} + R, \quad (\text{A.31})$$

where $W = \begin{bmatrix} r_m - z & w \\ \beta_m J^2/2 & z \end{bmatrix}$, $R = \begin{bmatrix} 0 \\ Y(\alpha_m - \frac{1}{2}\beta_m V) \end{bmatrix}$, and $Y = \sum_{i=1}^n e_i \xi_i$. The two eigenvalues of W are w_1, w_2 , and the eigenvector of R is I .

$$w_1 = \frac{r_m - \sqrt{(r_m - 2z)^2 + 2\omega\beta_m J^2}}{2} \quad (\text{A.32})$$

$$w_2 = \frac{r_m + \sqrt{(r_m - 2z)^2 + 2\omega\beta_m J^2}}{2} \quad (\text{A.33})$$

$$I = \begin{bmatrix} \frac{r_m \beta_m J^2 - 4z - 2\sqrt{(r_m - 2z)^2 + 2\omega \beta_m J^2}}{2\beta_m J^2} & \frac{r_m \beta_m J^2 - 4z + 2\sqrt{(r_m - 2z)^2 + 2\omega \beta_m J^2}}{2\beta_m J^2} \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} y_1 & y_2 \\ 1 & 1 \end{bmatrix} \quad (\text{A.34})$$

Therefore, u and E_n can be expressed as

$$\begin{aligned} \begin{bmatrix} u \\ E_n \end{bmatrix} &= I \begin{bmatrix} e^{w_1 t} & 0 \\ 0 & e^{w_2 t} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} - I^{-1} R \\ &= \begin{bmatrix} y_1 e^{w_1 t} & y_2 e^{w_2 t} \\ e^{w_1 t} & e^{w_2 t} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} - J(\alpha_m - \beta_m V) \begin{bmatrix} \frac{r_m - 2z + \sqrt{(r_m - 2z)^2 + 2\omega \beta_m J^2}}{2\sqrt{(r_m - 2z)^2 + 2\omega \beta_m J^2}} \\ \frac{r_m - 2z - \sqrt{(r_m - 2z)^2 + 2\omega \beta_m J^2}}{2\sqrt{(r_m - 2z)^2 + 2\omega \beta_m J^2}} \end{bmatrix} \end{aligned} \quad (\text{A.35})$$

The two boundary conditions $E(0) = 0$ and $\lim_{t \rightarrow \infty} e^{-r_m t} u(t) E(t) = 0$ imply that $k_1 = J(\alpha_m - \beta_m V) \frac{r_m - 2z - \sqrt{(r_m - 2z)^2 + 2\omega \beta_m J^2}}{2\sqrt{(r_m - 2z)^2 + 2\omega \beta_m J^2}}$ and $k_2 = 0$. Therefore, the open-loop equilibrium strategies of the sales price and the emissions amount of the manufacturer can be found as follows.

$$p_m^n = \frac{1}{2}V + \frac{(\alpha_m - \frac{1}{2}\beta_m S)J^2}{4\sqrt{\bar{k}}}[(r_m - 2z) + \sqrt{\bar{k}} - [(r_m - 2z) - \sqrt{\bar{k}}]y_1 e^{\bar{a}t}], \quad (\text{A.36})$$

$$E_n(t) = (\alpha_m - \frac{1}{2}\beta_m S)J \frac{\sqrt{\bar{k}} - (r_m - 2z)}{2\sqrt{\bar{k}}}(1 - e^{\bar{a}t}), \quad (\text{A.37})$$

where $S = \frac{\alpha_m}{\beta_m} + \sum_{i=1}^n c_i \xi_i$, $J = \sum_{i=1}^n e_i \xi_i$, $w_1 = \frac{r_m - \sqrt{(r_m - 2z)^2 + 2\omega \beta_m B}}{2}$, $K_1 = (r_m - 2z)^2 + 2\beta_m \omega B^2$, and $y_1 = \frac{r_m \beta_m J^2 - 4z - 2\sqrt{(r_m - 2z)^2 + 2\omega \beta_m J^2}}{2\beta_m J^2}$. To avoid confusion, we use $\bar{a}$ and $\bar{k}$ to replace w_1 and K_1 .

Q.E.D.

PROOF OF PROPOSITION 7.

For the case of multiple sourcing, the Hamilton-Jacobi-Bellman equation can be established as follows.

$$r_m V_m^n = \max_{\hat{p}_m^n} \{ (\alpha_m - \beta_m \hat{p}_m) \sum_{i=1}^n \xi_i (\hat{p} - p_i) - \frac{1}{2} \omega \hat{E}_n^2 + [\sum_{i=1}^n \varepsilon_i \xi_i (\alpha_m - \beta_m \hat{p}_m) + z E] \frac{\partial V_m^n}{\partial \hat{E}_n} \} \quad (\text{A.38})$$

Taking the first order derivative of Equation A.66 with respect to $\hat{p}_m$, we can obtain

$$\hat{p}_m = \frac{1}{2} (V - J \frac{\partial V_m^n}{\partial \hat{E}_n}), \quad (\text{A.39})$$

where $V = \frac{\alpha_m}{\beta_m} + \sum_{i=1}^n p_i \xi_i$ and $J = \sum_{i=1}^n e_i \xi_i$. Substituting $\hat{p}_m$ into Equation A.66, $r_m V_m^n$ can be rewritten as

$$r V_m^n = I \left(\frac{\partial V_m^n}{\partial \hat{E}_n} \right)^2 + U \frac{\partial V_m^n}{\partial \hat{E}_n} + C, \quad (\text{A.40})$$

where $I = \frac{1}{4} \beta_m J^2$, $U = \frac{1}{2} J (\alpha_m - \beta_m \sum_{i=1}^n \xi_i p_i) + z \hat{E}_n$, and $C = \frac{1}{4} V (2\alpha_m - \beta_m V) - (\alpha_m + \frac{1}{2} \beta_m V) \sum_{i=1}^n \xi_i p_i$. Conjecture the form of $V_m^n = M_n + N_n E + Q_n \hat{E}_n^2$, then, substituting $\frac{\partial V_m^n}{\partial \hat{E}_n} = N_n + 2Q_n \hat{E}_n$ into Equation A.38, we can obtain

$$r_m V_m^n = 4I Q_n^2 \hat{E}_n^2 + (4N_n Q_n I + 2Q_n U) E + I N_n^2 + U N_n + C. \quad (\text{A.41})$$

Then, M_n , N_n , and Q_n can be derived by the following equations.

$$4I Q_n^2 - r_m Q_n - \frac{1}{2} \omega = 0, \quad (\text{A.42})$$

$$4N_n Q_n I + 2Q_n U - r_m N_n = 0, \quad (\text{A.43})$$

$$I N_n^2 + U N_n + C - r_m M_n = 0, \quad (\text{A.44})$$

That is,

$$Q_n = \frac{r_m \pm \sqrt{r_m^2 + 8\omega I}}{8I} \quad (\text{A.45})$$

$$N_n = \frac{2Q_n U}{r_m - 4Q_n I}, \quad (\text{A.46})$$

$$M_n = (IN_n^2 + UN_n + C)/r_m, \quad (\text{A.47})$$

Because $\hat{E}_n$ has to converge in t and is larger than zero, thus, Q must satisfy $Q < 0$, i.e.

$Q_n = \frac{r_m - \sqrt{r_m^2 + 8\omega I}}{8I}$. Therefore, the Markov perfect equilibrium can be derived.

$$\dot{p}_m^n(t) = \frac{1}{2}[V - JN_n - 2JQ_n \hat{E}(t)], \quad (\text{A.48})$$

$$\hat{E}_n(t) = \frac{\alpha_m - \frac{1}{2}\beta_m V + \frac{1}{2}\beta_m JN_n}{\beta_m Q_n J} (e^{\beta_m Q_n J^2 t} - 1), \quad (\text{A.49})$$

where $V = \frac{\alpha_m}{\beta_m} + \sum_{i=1}^n p_i \xi_i$, $J = \sum_{i=1}^n e_i \xi_i$, $N_n = \frac{2Q_n U}{r - 4Q_n I}$, $Q_n = \frac{r_m - \sqrt{r_m^2 + 8\omega I}}{8I}$, $I = \frac{1}{4}\beta_m J^2$,
 $U = \frac{1}{2}J(\alpha_m - \beta_m \sum_{i=1}^n \xi_i p_i) + z\hat{E}_n$.

Q.E.D.

PROOF OF PROPOSITION 8.

The manufacturer's Hamiltonia function can be developed as

$$\begin{aligned} H_m = & \xi(p_m - p_1)(\alpha_m - \beta_m p_m) + (1 - \xi)(p_m - p_2)(\alpha_m - \beta_m p_m) \\ & - wE^2/2 + \mu[\varepsilon_1 \xi(\alpha_m - \beta_m p_m) + \varepsilon_2(1 - \xi)(\alpha_m - \beta_m p_m) + zE] \end{aligned} \quad (\text{A.50})$$

and the necessary condition for optimality is

$$\frac{\partial H_m}{\partial p_m} = \alpha_m - 2\beta_m p_m + p_2 \beta_m + \xi \beta_m (p_1 - p_2) - \mu[\xi \beta_m (\varepsilon_1 - \varepsilon_2) + \varepsilon_2] = 0, \quad (\text{A.51})$$

Leading to

$$p_m = [\alpha_m + p_2\beta_m + \beta_m\xi(p_1 - p_2)]/2\beta_m - \mu\beta_m[\xi(\varepsilon_1 - \varepsilon_2) + \varepsilon_2]/2 = G - \mu\beta_m T/2, \quad (\text{A.52})$$

where $G = [\alpha_m + p_2\beta_m + \beta_m\xi(p_1 - p_2)]/2\beta_m$, $T = \xi(e_1 - e_2) + e_2$. Substituting p_m into Equation (25), the dynamics of the emissions amount can be obtained as

$$\dot{E}(t) = T(\alpha_m - \beta_m G) + u\beta_m T^2/2 + zE, \quad (\text{A.53})$$

A non-homogeneous linear system with constant coefficients can be developed as follows by combining the co-state equations.

$$\dot{u}(t) = r_m u - \frac{\partial H_m}{\partial E} = (r_m - z)u + \omega E. \quad (\text{A.54})$$

Note that

$$\begin{bmatrix} \dot{u} \\ \dot{E} \end{bmatrix} = M \begin{bmatrix} u \\ E \end{bmatrix} + N, \quad (\text{A.55})$$

where $M = \begin{bmatrix} r_m - z & \omega \\ (\beta_m T)^2/2 & z \end{bmatrix}$, $N = \begin{bmatrix} 0 \\ T(\alpha_m - \beta_m G) \end{bmatrix}$. The two eigenvalues of M (m_1 and m_2) and the eigenvector of M (L) are as follows.

$$m_1 = \frac{r_m - \sqrt{(r_m - 2z)^2 + 2\omega(\beta_m T)^2}}{2} \quad (\text{A.56})$$

$$m_2 = \frac{r_m + \sqrt{(r_m - 2z)^2 + 2\omega(\beta_m T)^2}}{2} \quad (\text{A.57})$$

$$L = \begin{bmatrix} \frac{r_m - 2z - \sqrt{(r_m - 2z)^2 + 2\omega(\beta_m T)^2}}{2\beta_m T^2} & \frac{r_m + 2z + \sqrt{(r_m - 2z)^2 + 2\omega(\beta_m T)^2}}{2\beta_m T^2} \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ 1 & 1 \end{bmatrix} \quad (\text{A.58})$$

Therefore, u and E can be expressed as

$$\begin{aligned} \begin{bmatrix} u \\ E \end{bmatrix} &= L \begin{bmatrix} e^{m_1 t} & 0 \\ 0 & e^{m_2 t} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} - L^{-1}N \\ &= \begin{bmatrix} x_1 e^{m_1 t} & x_2 e^{m_2 t} \\ e^{m_1 t} & e^{m_2 t} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} - T(\alpha_m - \beta_m G) \begin{bmatrix} \frac{r_m - 2z + \sqrt{(r_m - 2z)^2 + 2\omega(\beta_m B)^2}}{2\sqrt{(r_m - 2z)^2 + 2\omega(\beta_m T)^2}} \\ \frac{\sqrt{(r_m - 2z)^2 + 2\omega(\beta_m T)^2} - (r_m - 2z)}{2\sqrt{(r_m - 2z)^2 + 2\omega(\beta_m T)^2}} \end{bmatrix} \end{aligned} \quad (\text{A.59})$$

Two boundary conditions $E(0) = 0$ and $\lim_{t \rightarrow \infty} e^{-rt}u(t)E(t) = 0$ imply that $k_1 = T(\alpha_m - \beta_m G) \frac{2z - r_m + \sqrt{(r_m - 2z)^2 + 2\omega(\beta_m T)^2}}{2\sqrt{(r_m - 2z)^2 + 2\omega(\beta_m T)^2}}$ and $k_2 = 0$. Therefore, the open-loop sales price and emissions amount of the manufacturer can be derived as follows.

$$\begin{aligned} p_m &= G + T^2(\alpha_m - \beta_m G) \left[\frac{r_m - 2z + \sqrt{(r_m - 2z)^2 + 2\omega(\beta_m T)^2}}{2\sqrt{(r_m - 2z)^2 + 2\omega(\beta_m T)^2}} e^{m_1 t} \right. \\ &\quad \left. + \frac{[\sqrt{(r_m - 2z)^2 + 2\omega(\beta_m B)^2} - (r_m - 2z)]^2}{2(\beta_m T)^2 \sqrt{(r_m - 2z)^2 + 2\omega(\beta_m B)^2}} \right], \end{aligned} \quad (\text{A.60})$$

$$E(t) = T(\alpha_m - \beta_m G) \frac{r_m - 2z - \sqrt{(r_m - 2z)^2 + 2\omega(\beta_m B)^2}}{2\sqrt{(r_m - 2z)^2 + 2\omega(\beta_m B)^2}} (1 - e^{m_1 t}), \quad (\text{A.61})$$

Q.E.D.

PROOF OF PROPOSITION 9.

With limitation of the sales price path, and the path of the emissions amount with respect to time t , we can find the stable states for the sales price and emissions amount, respectively.

(i) The cross partial derivatives of $\tilde{p}_m$ with respect to ω , p_1 and p_2 are

$$\frac{\partial^2 \tilde{p}_m}{\partial \omega \partial p_1} = \frac{\beta_m^5 T^4 \xi (r_m - 2z)}{4[(r_m - 2z)^2 + 2\omega(\beta_m T)^2]^{\frac{3}{2}}} \geq 0, \quad (\text{A.62})$$

$$\frac{\partial^2 \tilde{p}_m}{\partial \omega \partial p_2} = \frac{\beta_m^5 B^4 (1 - \xi) (r_m - 2z)}{4[(r_m - 2z)^2 + 2\omega(\beta_m T)^2]^{\frac{3}{2}}} \geq 0, \quad (\text{A.63})$$

because $0 \leq \xi \leq 1$ and $z \ll r$.

Thus, $\tilde{p}_m$ is a supermodular function in both (ω, p_1) and (ω, p_2) .

(ii) The cross partial derivatives of $\tilde{E}$ with respect to w , p_1 and p_2 are

$$\frac{\partial^2 \tilde{E}}{\partial \omega \partial p_1} = -\frac{\beta_m^3 B^2 \xi (r_m - 2z)}{2\sqrt{(r_m - 2z)^2 + 2\omega(\beta_m T)^2}} \leq 0, \quad (\text{A.64})$$

$$\frac{\partial^2 \tilde{E}}{\partial \omega \partial p_2} = -\frac{\beta_m^3 B^2 (1 - \xi) (r_m - 2z)}{2\sqrt{(r_m - 2z)^2 + 2\omega(\beta_m B)^2}} \leq 0, \quad (\text{A.65})$$

Thus, $\tilde{E}$ is a submodular function in both (ω, p_1) and (ω, p_2) .

Q.E.D.

PROOF OF PROPOSITION 10.

The Markov perfect equilibrium is derived by using the Hamilton-Jacobi-Bellman (HJB) equations.

$$\begin{aligned} r_m V_m = \max_{p_m} \{ & \xi(p_m - p_1)(\alpha_m - \beta_m p_m) + (1 - \xi)(p_m - p_2)(\alpha_m - \beta_m p_m) \\ & - \omega E^2/2 + \frac{\partial V_m}{\partial E} [T(\alpha_m - \beta_m p_m) + zE] \}, \end{aligned} \quad (\text{A.66})$$

where $T = \xi(\varepsilon_1 - \varepsilon_2) + \varepsilon_2$. Taking the first order derivative of Equation A.66 with respect to p_m for maximization of Equation A.66, we can obtain

$$p_m = A - \frac{T}{2} \frac{\partial V_m}{\partial E}, \quad (\text{A.67})$$

where $A = [\alpha_m + \beta_m p_2 + \beta_m \xi(p_1 - p_2)]/2\beta_m$. Substituting Equation A.67 into Equation A.66, rV_m can be rewritten as

$$\begin{aligned} r_m V_m = & \frac{1}{4} \beta_m T^2 \left(\frac{\partial V_m}{\partial E} \right)^2 + \left(\frac{1}{2} \alpha_m T + zE \right) \frac{\partial V_m}{\partial E} \\ & + \alpha_m A + \alpha_m [\xi(p_2 - p_1) - p_2] - \beta_m A^2 - \frac{1}{2} \omega E^2 \end{aligned} \quad (\text{A.68})$$

Conjecture the functional form for the value function $V_m = N + ME + QE^2$, where M , N , and Q are determined by Equations A.66 and A.67. Substituting $\frac{\partial V_m}{\partial E} = M + 2QE$ into Equation A.67, then

$$\begin{aligned} r_m V_m = & (\beta_m T^2 Q^2 + 2Qz - \frac{1}{2} \omega) E^2 + [\beta_m M Q T^2 + zM + 2\alpha_m T Q] E \\ & + \frac{1}{4} \beta_m T^2 M^2 + \frac{1}{2} \alpha_m T M + \alpha_m A + \alpha_m [\xi(p_2 - p_1) - p_2] - \beta_m A^2 \end{aligned} \quad (\text{A.69})$$

Then, Q , N , and X can be derived by the following equations.

$$\beta_m T^2 Q^2 + (2z - r)Q - \frac{1}{2} \omega = 0, \quad (\text{A.70})$$

$$\beta_m M Q T^2 + zM + 2\alpha_m T Q - rM = 0, \quad (\text{A.71})$$

$$X - rM = 0, \quad (\text{A.72})$$

where $X = \frac{1}{4} \beta_m T^2 M^2 + \frac{1}{2} \alpha_m T M + \alpha_m A + \alpha_m [\xi(p_2 - p_1) - p_2] - \beta_m A^2$. Then, we can observe

$$Q = \frac{r_m - 2z \pm \sqrt{(r_m - 2z)^2 + 2\omega\beta_m T^2}}{2\beta_m T^2}, \quad (\text{A.73})$$

$$M = \frac{\alpha_m T Q}{r_m - z - \beta_m Q T^2}, \quad (\text{A.74})$$

$$M = X/r_m, \quad (\text{A.75})$$

This is because $E(t)$ has to converge in t and is larger than zero, so the solution of Q must satisfy $Q < 0$. Only one if the two roots of Q is eligible (Erickson, 2011). That is, $Q = \frac{r_m - 2z - \sqrt{(r_m - 2z)^2 + 2\omega\beta_m T^2}}{2\beta_m T^2}$. Therefore, the Markov perfect equilibrium can be obtained as follows.

$$\hat{p}_m(t) = A - \frac{1}{2}TM + TQE(t), \quad (\text{A.76})$$

$$\hat{E}(t) = \frac{T(\alpha_m - \beta_m M)}{z - 2\beta_m QT} [e^{(z - 2\beta_m QT)t} - 1], \quad (\text{A.77})$$

where $T = \xi(e_1 - e_2) + e_2$, and $A = [\alpha_m + \beta_m p_2 + \beta_m \xi(p_1 - p_2)]/2\beta_m$.

Q.E.D.