Is the mental number line a unique model of numerical cognition?

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Abstract

Many theories of number propose that humans possess a ‘mental number line’ (MNL) representation. The MNL is commonly measured with a number-to-position (NP) task, and this task is often used to make inferences about the MNL representation. However, most research on MNL representations to date has made assumptions about the properties of the MNL without considering the role of ordinal relationships between numbers. The research reported in this thesis examined whether the MNL metaphor should be extended to include ordinal relationships, and to examine the nature of these relationships. A pilot study tested whether a linear MNL representation could be shifted with logarithmic training. Adults completed a series of logarithmic feedback sessions, and their NP task performance was assessed at the end of each training block. Findings revealed little to no systematic effect of logarithmic training on NP task performance, despite participants successfully learning the logarithmic function. They also revealed individual differences in the overall impact and learning of the logarithmic feedback. These findings suggested that relationships between numbers may change to allow accurate task performance, which may not be reflected in the NP task. Study 1[1] tested whether the linear response profile that describes NP performance is specific to number, or whether this pattern of responding is a feature of ordered lists more generally. Adults were given NP and alphabet-to-position tasks. Findings showed that numbers and letters both displayed similar linear trends. This suggested that the linear profiles attributed to number may reflect the way in which ordinal lists of symbols are learned. Studies 2a and 2b[2] investigated whether leaning a list of novel symbols is mediated by the underlying spatial properties of the symbols (e.g., spatial complexity). Novel symbols were used to minimise the overlearned nature of Hindu-Arabic numerals. Study 2a aimed to determine the ideal novel symbol set to use in Study 2b, specifically, one which could be ordered by complexity. Participants made judgements on two novel symbol sets, and their relationship to a range of numerical stimuli. In Study 2b, a paired comparisons training method was used to teach participants the order of a list of novel symbols. Participants were allocated to either a spatial complexity or a random complexity condition, and made

[1] Podwysocki, C., Reeve, R. A., & Forte, J. D. (2019). The importance of ordinal information in interpreting number/letter line data. *Frontiers in Psychology*, *10*, Article ID 692. doi:10.3389/fpsyg.2019.00692

[2] Podwysocki, C., Reeve, R. A., Paul, J. M., & Forte, J. D. (2020). Spatial complexity facilitates ordinal mapping with a novel symbol set. *PLOS ONE*, *15*(3), e0230559. doi:10.1371/journal.pone.0230559

judgements regarding which of two symbols was numerically ‘larger’. When novel symbols were ordered by spatial complexity, learning was facilitated. These findings showed that the spatial complexity and relational information of symbols may mediate the construction of ordered representations. This suggests that a common cognitive representation underlies all ordinal lists. Overall, the findings of this research indicate that a more nuanced account of the MNL representation is required, particularly in terms of ordinal relationships between numbers. The findings also suggest that the NP task measures ordinal lists more generally. Arguably, the way in which ordered lists are learned, combined with the relative relationships between symbols, may account for performance on the NP task, and the MNL metaphor should be extended to account for these ordinal relationships.

Declaration

This is to certify that:

(i) The thesis comprises only their original work towards the Doctor of Philosophy, except where indicated in the preface;

(ii) Due acknowledgement has been made in the text to all other material used; and

(iii) The thesis is fewer than 100,000 words in length, exclusive of tables, maps, bibliographies, and appendices.

Christine Podwysocki.

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Christine Podwysocki

30/10/2019

Preface

(i) All work towards the thesis was undertaken in collaboration with A/Prof Robert A. Reeve and Dr Jason D. Forte (Chapter 4: Study 1) and also in collaboration with A/Prof Robert A. Reeve, Dr Jacob M. Paul, and Dr Jason D. Forte (Chapter 5: Studies 2a and 2b), including its original conception and editorial guidance, for which the co-authors acknowledge 80 to 90 percent of the work is attributable to Christine Podwysocki. All work is original except where acknowledgement has been made in text to all other material used;

(ii) No work towards the thesis has been submitted for other qualifications;

(iii) No work towards the thesis was carried out prior to enrolment in the degree;

(iv) No third-party editorial assistance was provided in preparation of the thesis;

(v) The contributions of all persons involved in any multi-authored publications or articles in preparation included in the thesis are as follows:

  a. Chapter 4 (Study 1): co-authors Robert A. Reeve and Jason D. Forte acknowledge that 80 to 90 percent of the work in this publication is attributable to the first author (Christine Podwysocki)

  b. Chapter 5 (Studies 2a and 2b): co-authors Robert A. Reeve, Jacob M. Paul, and Jason D. Forte acknowledge that 80 to 90 percent of the work in this publication is attributable to the first author (Christine Podwysocki);

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  c. Chapter 5 (Studies 2a and 2b): Podwysocki, C., Reeve, R. A., Paul, J. M., & Forte, J. D. (2020). Spatial complexity facilitates ordinal mapping with a novel symbol set. Published by *PLOS ONE* on 26/03/2020; and

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Chapter 1
The Research Problem

The ability to accurately estimate numerical magnitudes is an important feature of mental number representations[1] for children and adults. Some popular theories of number suggest that the mechanism underlying numerical magnitude estimation is an unconscious 'mental number line' (MNL; Moyer & Landauer, 1967; Restle, 1970), the representation whereby numbers are organized horizontally in ascending order from left-to-right in a "continuous, quantity-based, analogical format" (Zorzi, Priftis, & Umiltà, 2002, p. 138). Researchers have made many assumptions about the nature of the MNL. However, these assumptions do not explain some key properties of the MNL representation. This thesis will argue that researchers have not paid sufficient attention to certain properties of the MNL, and that a more nuanced account of the MNL that focuses on ordinal[2] relations between numbers may help explain the processes that underlie number representations. Therefore, the aim of this thesis is to clarify whether the MNL should be extended to include ordinal relationships, and to further examine the precise nature of these ordinal relationships.

The MNL representation has been described with behavioural evidence (e.g., the Spatial Numerical Association of Response Codes effect [SNARC]; Dehaene, Bossini, & Giraux, 1993; Dehaene, Dupoux, & Mehler, 1990), neuropsychological evidence (e.g., disruptions of the MNL in spatial neglect patients; Zorzi et al., 2002), and with neuroimaging research (e.g., associated brain regions for numbers and space; Dehaene, Piazza, Pinel, & Cohen, 2003), supporting the idea of a spatial representation of number in the brain. However, while many assumptions have been made regarding the nature of the MNL based on this research, the basis for some of these inferences are unclear, and some of the conclusions regarding the properties of the MNL representation may be somewhat overstated.

The MNL is often measured with a number-to-position task (NP task; Siegler & Opfer, 2003). The NP task requires participants to estimate the position of a target number (e.g., 10) on a horizontal line labelled with fixed upper and lower anchor points (e.g., 1 to 100). Magnitude estimation accuracy is measured on the NP task by calculating the mean difference

[1] Where 'mental representation' refers to an internal symbol system that represents the external environment, and can be used to inform decisions (Gallistel, 2001).
[2] Where 'ordinality' refers to the position of a number in a list, and gives information regarding how a number relates to other numbers (Lyons & Beilock, 2013).

between the estimated number location and the actual number location. Estimation profiles derived from the NP task are often used to make inferences regarding the properties of the MNL. For example, one account of magnitude estimation development suggests that with age and experience, children gradually abandon logarithmic (i.e., overestimating small numbers and underestimating large numbers) for linear (i.e., even spacing between numbers) number representations (Siegler & Opfer, 2003), implying that the MNL undergoes a 'shift' during development. However, the specific properties of the adult MNL remain unclear.

Whether or not the MNL can change to suit task demands has not been tested systematically. While some studies have addressed this issue by training adults to modify their MNL (e.g., Huber, Moeller, & Nuerk, 2014), which provide some support for the idea that the MNL is a flexible mechanism, there are some issues that need to be resolved before drawing conclusions about properties of the MNL representation. MNL training studies have used the NP task, but have not examined the long-term effects of training. Studies have also used group averages, which means the effects of training on number representations is not clear. It is unknown if adults can re-organise their number representation based on feedback. If they can, this would suggest that the MNL metaphor is a flexible representation, which can adapt to suit different task demands. If not, this would suggest that the MNL metaphor should take into account ordered relationships between numbers.

It is also not clear whether the NP task assesses the underlying representation of number specifically. It may be the case that NP task performance reflects how ordered lists are learned, rather than number representations per se. Previous research has attempted to answer this question using letters of the alphabet (e.g., Hurst, Monahan, Heller, & Cordes, 2014). However, this research did not address several issues, such as the similarities or differences in patterns of non-linearity in the task for numbers and letters. It is unclear whether performance on the NP task reflects the MNL representation. If similar patterns are found for numbers and letters, this would suggest that the MNL may reflect ordinal lists more generally. If not, this would suggest that the MNL metaphor is specific to number.

Finally, it is unclear if representation of order of a set of symbols is mediated by underlying spatial properties such as symbol complexity. It may be the case that people judge symbols as having a natural order to them, which may play some role in how people learn ordered representations (which may underlie number representations). Since the spatial complexity of symbols can only be understood *in relation to* other symbols, it is important to clarify whether the MNL representation should take into account the relational structure of symbols (especially ordinality). If learning an ordered representation can be facilitated by the property of spatial complexity, this would suggest that a more nuanced theory of the MNL metaphor that includes relative relational information is required. If not, this would suggest that ordinal information may not be an important factor for mental number representations.

Overall, the first aim of this thesis is to systematically test some of the assumptions regarding the form of the MNL, and determine whether the MNL metaphor should take into account the ordered relationships between numbers. A second aim is to examine the NP task, and determine if the task reflects ordinal representations, rather than number representations per se. A third aim is to test how ordered representations are formed in adults with the use of novel symbols and to test if this representation is influenced by underlying spatial properties of the symbols. In the next chapter (Chapter 2), research relating to the MNL metaphor will be examined. It will focus on magnitude estimation, evidence for the MNL metaphor, the developmental shift hypothesis of the MNL, training the MNL in children, training the MNL in adults, and will review the literature on number leaning, number symbols, and ordinality.

Chapter 2
Literature Review

The purpose of Chapter 2 is to present the arguments motivating the research that is reported in this thesis. The primary goal is to argue that while many assumptions have been made regarding the mental number line (MNL) metaphor, they do not consider the ordinal relationships between 'numbers', and that it would be useful to examine the nature of these ordered relationships further. Specifically, it will be suggested that the MNL metaphor should be extended to include ordinal relations, because this may be an important factor underlying representations of number. It will also be proposed that the best way to examine these issues is by considering how number is learned, because it allows an examination of whether: (1) the MNL can change to support different spatial representations, (2) performance on the number-to-position (NP) task reflects the ordinal nature of numbers, and (3) learning ordinal relations can be mediated by underlying spatial properties, such as spatial complexity.

Chapter 2 comprises seven sections. The first section (section 2.1.) considers the concept of magnitude estimation. Magnitude estimation is claimed to be a foundational property of numerical ability, which, in turn, is explained by the MNL metaphor. However, it will be argued that the inferences associated with these claims are limited, and more must be done to characterise how this ability varies in adults, and how it develops. The second section (section 2.2.) critiques the MNL. While many current empirical and theoretical studies of number rely on this metaphor to describe how number is mentally represented, it will be argued that the precise nature of the MNL metaphor is unclear, and inferences regarding it have been overstated. Furthermore, relatively little is known about the scaling properties of the MNL. The third section (section 2.3.) examines the development of the MNL construct and discusses claims that children's MNL undergoes a shift from a logarithmically compressed representation to a linear representation. It will be suggested that there are too many inconsistencies in developmental research findings for meaningful inferences to be drawn from them regarding the properties of the MNL, and that using the NP task to draw parallels with the MNL metaphor is problematic. The fourth section (section 2.4.) reviews studies that have trained the MNL in children. It will be argued that these findings provide a good starting point for testing the format of the MNL. The fifth section (section 2.5.) reviews studies that have trained the MNL in adults. It will be suggested that while these studies support the suggestion that the MNL is a flexible representation (i.e., that the MNL can adapt to different task demands), there are several methodological limitations with these studies (e.g., training and testing on the NP task), and these findings should not be used to make conclusions regarding the underlying properties of the MNL representation. The sixth section (section 2.6.) will provide an overview of studies on how number is learned, particularly in relation to number symbols and ordinality, because these fields are important to testing the nature of the MNL. The last section (section 2.7.) will summarise how the issues raised in this chapter will be addressed in the subsequent empirical studies presented in this thesis.

2.1. Magnitude estimation

A discussion of the mental number line (MNL) metaphor should start with magnitude estimation, because magnitude estimation is a fundamental math ability which the MNL is assumed to explain. Researchers claim that humans and non-human animals both possess the ability to estimate the size of very large sets of groups of objects, albeit with decreasing accuracy. Indeed, magnitude estimation may be the most used quantification process in our daily lives, because we frequently encounter situations where we lack the ability to calculate exact numerical values (Booth & Siegler, 2006). Magnitude estimation specifically refers to judging an amount in any continuous dimension, such as size, weight, density, or brightness (Brannon, 2006). These number entities are discrete, but are often linked in the environment (e.g., a more numerous item may also be heavier), and become distinct concepts with development (Newcombe, 2014). Magnitude estimation is initially typified by imprecise representations, and gains precision through formal education (Gallistel & Gelman, 1992; Piazza, 2011; Siegler & Booth, 2005; Zorzi & Butterworth, 1999). This section will outline the history of magnitude estimation, followed by a discussion of magnitude estimation ability in non-human animals and humans, the proposed link between magnitude estimation and complex math ability, and cover how magnitude estimation ability is currently measured.

2.1.1. Historical context of magnitude estimation

Historically, the notion of magnitude estimation has roots in psychophysical studies. A hallmark of magnitude representations is that they follow Weber's law (Nieder, 2005; Nieder & Miller, 2003; 2004). Originally developed by physician Ernst Weber in 1834, this law suggests that heavier weights are harder to discriminate from each other. For example, a 10 kilogram weight is harder to discriminate from a nine kilogram weight than from a two kilogram weight. Weber's law also incorporates the concept of 'just noticeable difference' (JND), which is the minimum amount a stimulus has to change in order to produce a significant change in sensory experience, where the JND is a continuous fraction of the level of stimulus strength (Brannon, 2006). Weber's law is consistent with a linear with scalar variability model of magnitude representations. Later, Weber's graduate student Gustav Fechner suggested that the difference in stimulus strength required to detect change is a constant fraction of the stimulus strength rather than a fixed amount (e.g., if an increase of one kilogram is needed to detect change in a five kilogram weight, then an increase of two kilograms would be needed to detect change in a 10 kilogram weight, see Stevens, 1970; Welford, 1960), and that there are individual differences in sensitivities to certain stimuli. Fechner's modification to Weber's law provides a better account for a wider range of sensory stimuli, including magnitude estimation. The modified Weber-Fechner law accounts for two prominent features of number discrimination, specifically, the numerical 'distance' and 'magnitude' effects. Unlike the original Weber's Law, the Weber-Fechner law is consistent with a logarithmic model of magnitude representations.

A well cited example of the distance and magnitude effects comes from seminal work by Moyer and Landauer (1967). The authors showed that when adults are asked to compare the relative magnitudes of Hindu-Arabic numerals, their reaction time is systematically influenced by the distance between, and the magnitude of, the numbers to be compared. Reaction time increases with decreasing numerical distance between two values (e.g., participants are generally faster at 1 vs. 9 than 8 vs. 9). Furthermore, if numerical distance is fixed, reaction time also increases with numerical magnitude (e.g., participants are generally faster at 1 vs. 2 than 8 vs. 9). The findings of Moyer and Landauer suggest that the speed of the number comparison process is modulated by the distance at which two numbers are separated by on a so-called 'mental number line' representation.

Performance on a wide range of number tasks follows the Weber-Fechner law, including when participants estimate the number of items in a set (Halberda, Sires, & Feigenson, 2006; Whalen, Gallistel, & Gelman, 1999), generate numbers from key presses (Cordes, Gelman, Gallistel, & Whalen, 2001), decide the larger of two dot arrays (Barth, Kanwisher, & Spelke, 2003), or estimate the product of an arithmetic problem (Pica, Lemer, Izard, & Dehaene, 2004). These results show that while number can be represented symbolically, non-symbolic magnitude estimation may be important to number representation (Buckley & Gillman, 1974; Dehaene, 1996; Hinrichs, Yurko, & Hu, 1981; Tzeng & Wang, 1983). However, it is still unclear what mechanism underlies magnitude estimation.

2.1.2. Magnitude estimation in non-human animals and humans

Research has been conducted on a wide range of species (e.g., rats, pigeons, dolphins, parrots, and non-human primates) to determine their numerical magnitude estimation abilities

(Dehaene, 2011; Dehaene, Dehaene-Lambertz, & Cohen, 1998; Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999, Gallistel, 1990; Gallistel & Gelman, 1992). Several studies have found that chimpanzees and monkeys can be taught the Hindu-Arabic numerals from 1 to 9 and use them to correctly label groups of objects (Boysen & Capaldi, 1993; Matsuzawa, 1985; Washburn & Rumbaugh, 1991). Rats have been trained to respond to tones of different lengths, which generalises to cross-modal tasks in different modalities (Meck & Church, 1983). Pepperberg (1994) also trained a parrot to verbally label collections of one to six coloured objects. After training, the parrot was able to answer complex questions such as 'How many blue keys?' when shown a group of objects with multiple colours. These findings suggest that the system for using numerical magnitudes to represent number without symbols is universal and is shared across species. Dehaene (2011) even claimed that when it comes to magnitude estimation, humans are no more accurate than rats.

It has also been shown that human infants can estimate number shortly after birth, emphasising the importance of examining this ability (Feigenson, Dehaene, & Spelke, 2004; Xu & Spelke, 2000). For instance, Wynn (1992) used a stage and screen task to test whether human infants look longer at an impossible math result. Wynn found that infants as young as five months looked longer at an impossible 2 – 1 = 2 outcome as opposed to a 2 – 1 = 1 outcome (see McCrink & Wynn, 2004 for a parallel finding with nine-month-old infants). These results imply that human infants can compute numerical magnitudes soon after birth. Furthermore, Xu and Spelke (2000) showed that the numerical discrimination profiles of six-month-old infants also obey the Weber-Fechner Law, similar to adult humans and non-human animals. The authors habituated infants to arrays with either eight or 16 dots and found that infants could distinguish the arrays when continuous variables (such as dot density and image brightness) were controlled for. However, when infants were habituated to arrays with either eight or 12 dots, the infants failed to distinguish the two arrays. The authors concluded that infants can successfully distinguish large groups of items based on numerosity when the ratio is large enough. A comparable method has been used to show that the discrimination profiles of babies continue to obey the Weber-Fechner law when they discriminate series of sounds based on number (Lipton & Spelke, 2003; 2004).

Adult humans, human infants, and non-human animals can judge the numerical equivalence of sets across sensory modalities (Barth et al., 2003). This suggests a mechanism exists for representing magnitudes that may form the basis of number representations. Overall, these findings support the idea that the ability to estimate numerical magnitudes is present early in life before the acquisition of language and formal education, and that magnitude estimation gradually acquires precision during development. However, it is not clear how well magnitude estimation is explained by the MNL metaphor.

2.1.3. Link between magnitude estimation and math ability

It has been proposed that the ability to represent numerical magnitudes may be critical to later number learning (DeWind & Brannon, 2012; Gilmore, McCarthy, & Spelke, 2010; Halberda, Mazzocco, & Feigenson, 2008; Libertus, Feigenson, & Halberda, 2011; Libertus, Odic, Halberda, 2012; Mazzocco, Feigenson, & Halberda, 2011; Siegler, Thompson, & Schneider, 2011; Thompson & Siegler, 2010). Specifically, the ability to precisely estimate sets of non-symbolic number stimuli (e.g., dot arrays) has been found to positively correlate with later math achievement. For example, Halberda, Mazzocco, and Feigenson (2008) examined 14-year-olds whose math performance had been measured longitudinally from kindergarten. Children were presented with arrays of mixed blue and yellow dots briefly presented on a computer screen and judged whether the array contained more blue or yellow dots. The authors found individual differences in estimation abilities positively correlated with standardised math achievement tests performed in kindergarten. These results persisted when individual differences in cognitive ability (e.g., intelligence, working memory, and spatial abilities) were controlled for. In a follow-up study by Halberda, Ly, Wilmer, Naiman, and Germine (2012) used an online sample of over 10,000 people aged between 11 and 85 tested how the ability to estimate numerical magnitudes changes with age. The authors found the accuracy of numerical estimation improves during formal education, and peaks at about 30 years of age. Individual differences are present for people of the same age, and these differences positively correlate with complex math ability during adolescence and adulthood.

These findings suggest that magnitude estimation may play a role in symbolic math ability, indicating magnitude estimation should be accounted for in the MNL metaphor. However, it is important to note that a correlation between magnitude estimation ability and later math skills is not always found (Holloway & Ansari, 2009; Inglis, Attridge, Batchelor, & Gilmore, 2011; Nosworthy, Bugden, Archibald, Evans, & Ansari, 2013; Price, Palmer, Battista, & Ansari, 2012; Sasanguine, Göbel, Moll, Smets, & Reynvoet, 2013; Soltész, Szűcs, & Szűcs, 2010), and that the relationship may be mediated by other factors, such as symbolic number knowledge (Lyons & Beilock, 2011), executive function (Fuhs & McNeil, 2013), or may only apply to children with number learning difficulties (Bonny & Lourenco, 2013). More research is needed to clarify the link between magnitude estimation and the MNL.

2.1.4. Assessing magnitude estimation ability in humans

The ability to estimate numerical magnitude in children and adults is often measured by a number-to-position (NP) task (other methods include measurement tasks and numerosity estimation tasks). The NP task was first popularised by Gelman and Gallistel (1986, see also Petitto, 1990) as a way of measuring children's knowledge of symbolic number, when the focus of number research was primarily on how number skills develop in children, rather than explaining the MNL. However, the version most widely used today was designed by Siegler and Opfer (2003) as a way of measuring numerical magnitude representations in children. The task(s) require either estimating the location of a given number on a horizontal line (NP task), or converting a given location on a horizontal line into a number (known as a position-to-number task). Similar patterns of aptitude should be found in both tasks, although they assess slightly different abilities. Number estimation patterns and accuracy are derived from the NP task by calculating the mean difference between the perceived number location and the true number location. While the NP task is used to make inferences regarding the properties of the MNL, there is growing evidence that the NP task may reflect the ordinal information contained in numbers more generally, meaning performance on the NP task perhaps should not be interpreted as a direct reflection of the MNL (see section 2.3.4.).

2.1.5. Summary

Overall, the capacity to estimate numerical magnitudes is a critical ability for adult humans, human infants, and non-human animals. The precision of non-symbolic magnitude representations could also assist symbolic math ability in humans. However, the nature of this relationship is not clear. Some studies have found a strong link between non-symbolic magnitude estimation and symbolic number ability (Halberda et al., 2008), while other studies have found no link between performance on a symbolic math task and non-symbolic performance (Holloway & Ansari, 2009). Furthermore, while the MNL metaphor is often invoked to account for numerical magnitude estimation, it is unclear whether this analogy is the most accurate description for this number ability.

2.2. The mental number line

The MNL is a popular metaphor to explain how humans and non-human animals represent numerical magnitudes. However, critical features of the MNL have not been determined, and certain properties of the MNL remain inadequately defined. Little is known about the ordinal relations that may comprise the MNL, and inferences regarding the MNL may be overstated in the literature. According to the most widely accepted account, the MNL is a horizontally organized representation in which numbers are represented in ascending order from left-to-right in a “continuous, quantity-based, analogical format” (Zorzi et al., 2002, p. 138). However, the MNL metaphor has also been less clearly described. For example, Case et al. (1996) characterised the MNL as a lens through which children view the world, and as a tool that children can use to create new knowledge. Variations in describing of the MNL may occur because the concept is used in several disparate research literatures. Behavioural, neuropsychological, and neuroimaging research all invoke and support the MNL metaphor as an explanation. This section will first outline the history of the MNL metaphor. It will then present a critique of the research that has been used to support the MNL metaphor. This will be followed by a discussion of a key issue in the MNL literature, that an account of the scaling properties of the MNL representation remain unclear. A selection of alternative theories to the MNL account will then be reviewed.

2.2.1. Historical context of the mental number line

The MNL was originally developed to describe distance and magnitude effects found in simple math tasks (Moyer & Landauer, 1967; Restle, 1970; see section 2.1.1.). However, the origins of the MNL representation date back to the work of polymath Sir Francis Galton (1880), who was the first to note that some individuals possess a mental map of number sequences, which are activated whenever they imagine numbers. These ‘number forms’ are thought to be spatially organised, can be used to perform number calculations, and were a precursor to what we now call the MNL. Nevertheless, while Galton’s number forms provide evidence of a spatial organisation of number, they are distinct from the MNL, as they are conscious, unique in form, and remain constant across the life of an individual.

Seron, Pesenti, Noël, Deloche, and Cornet (1992) conducted one of the few follow-up studies to Galton’s research. The authors presented case studies of idiosyncratic number representations (e.g., one individual described representing numbers as a series of curved lines and circles) and concluded that these number forms may be useful to clarifying how numbers are represented. However, while these case studies show a link between numbers and space, they do not necessarily confirm the existence of a spatially oriented MNL. Indeed, later research has identified these unique visuospatial number forms as ‘number form synaesthesia’, which is not representative of the general population (Cohen Kadosh & Henik, 2007; Sagiv, Simner, Collins, & Butterworth, 2006; Tang, Ward, & Butterworth, 2008).

2.2.2. Support for the mental number line metaphor

2.2.2.1. The SNARC effect. The first, and perhaps most widely used, line of research to support the idea of a spatially oriented MNL comes from behavioural studies of the phenomenon called the Spatial Numerical Association of Response Codes (SNARC) effect, initially observed in a series of number comparison studies conducted by Dehaene, Bossini, and Giraux (1993) and Dehaene, Dupoux, and Mehler (1990). These studies showed that humans respond faster to small numbers when responses are performed on the left side of space rather than the right, and that they respond faster to large numbers when responses are performed on the right side of space rather than the left side of space. This effect has been found in tasks that do not explicitly involve space or quantity, such as determining whether a number is even or odd (Dehaene et al., 1993), deciding whether a number contains a particular phoneme (Fias, Brysbaert, Geypens, & d'Ydewalle, 1996), and when numerical stimuli are task irrelevant (e.g., asking participants to determine if the colour of a numerical digit is red or green; Fias, Lauwereyns, & Lammertyn, 2001; Lammertyn, Fias, & Lauwereyns, 2002). The SNARC effect is not dependent on the absolute size of the numbers and is not contingent on the hand used to perform the task (and also appears when the hands are crossed over; Dehaene et al., 1993). The SNARC effect has been found in other response modalities, such as foot, eye, and head responses (see Wood, Nuerk, Willmes, & Fischer, 2008 for a review of the SNARC effect). The SNARC effect also may exist for non-symbolic number stimuli (Nemeh, Humberstone, Yates, & Reeve, 2018). Given the above results, the SNARC effect is often interpreted as unequivocal evidence for a spatially oriented MNL representation, "because it bears a natural and seemingly irrepressible correspondence with the natural left [to] right coordinates of external space" (Dehaene et al., 1993, p. 394).

The SNARC effect has also been observed in human infants and newborn chicks, providing support for the idea of an evolutionarily ancient, innate left-to-right orientation of the MNL representation. For example, de Hevia, Girelli, Addabbo, and Macchi Cassia (2014) used a habituation paradigm to show that seven-month-old pre-verbal human infants who lack any formal mathematics education show a preference for increasing magnitudes presented in a left-to-right spatial orientation. The infants did not habituate to a right-to-left orientation. The authors claim that these results suggest a very early tendency for humans to link numerical order with a left-to-right spatial orientation, prior to the development of symbolic knowledge, mathematics education, as well as reading and writing skills. Moreover, Rugani, Vallortigara, Priftis, and Regolin (2015) familiarised three-day-old domestic chicks to a target number (5), and found that the chicks spontaneously associated a smaller number (2) with the left side of space, and a larger number (8) with the right side of space. Importantly, when the target number was 20, the chicks associated 8 with the left side of space. Rugani et al. claimed that these findings provide strong support for the idea that human and non-human animals both have a disposition to order numbers increasing magnitude from left-to-right (see de Hevia and Rugani, 2017 for a review of the origins of the MNL).

However, the interpretation of the SNARC effect is not straightforward, and the claim that the SNARC effect provides direct support for the MNL may be overstated. Later research has questioned if the SNARC effect is informative about the underlying representation of numbers, since it may be a product of other processes, such as strategic problem solving (Fischer, 2006; Wood, Nuerk, & Willmes, 2006), working memory (Van Dijck & Fias, 2011; however, see Hoffmann, Pigat, & Schiltz, 2014 for an alternative account), or cognitive response selection (Keus, Jenks, & Schwarz, 2005; Keus & Schwarz, 2005; Müller & Schwarz, 2007). For example, Bächtold, Baumüller, and Brugger (1998; see also Vuilleumier, Ortigue, & Brugger, 2004) asked participants to decide whether a given number between 1 and 11 (other than 6) was smaller or larger than 6 under two different conditions ('ruler' and 'clock'). The standard SNARC effect was replaced by a reverse SNARC effect when numbers were represented on a clock face (smaller numbers were responded to faster with the right hand, and larger numbers were responded to faster with the left hand). This study suggests that the SNARC effect is vulnerable to task manipulations (see also Fischer, Shaki, & Cruise, 2009). This finding calls into question how well the SNARC effect supports the MNL metaphor, because the SNARC effect may be influenced by other factors, such as task conditions. Furthermore, it has also been found that the SNARC effect extends to vertical (and radial) dimensions (Winter, Matlock, Shaki, Fisher, 2015). For example, Schwarz and Keus (2004) found a vertical SNARC effect in a saccadic parity judgement task, and suggested that the vertical SNARC effect supports the idea of a 'number map' instead of the MNL representation. Taken together, these studies suggest evidence from SNARC effect literature does not unambiguously confirm a spatially oriented MNL representation.

The SNARC effect is also mediated by cultural factors such as reading direction. Shaki and Fischer (2008) tested bilingual Russian/Hebrew readers and found that while participants showed a prominent SNARC effect after reading Cyrillic script (from left-to-right), this effect disappeared after reading Hebrew script (from right-to-left). A follow-up study by Shaki, Fischer and Petrusic (2009) tested Canadians (who read English words and Hindu-Arabic numerals from left-to-right), Palestinians (who read Hebrew words and Hindu-Arabic digits from right-to-left), and Israelis (who read Hebrew words from right-to-left but Hindu-Arabic numerals from left-to-right). With a parity judgement task, like Dehaene et al. (1993) the authors found that the Canadian group showed a standard SNARC effect. The Palestinian group showed a reverse SNARC effect, and the Israeli group showed no SNARC effect. These results cast further doubt on the SNARC effect for inferring the MNL, because the SNARC effect is not present for some cultures that represent numbers from left-to-right.

The cognitive mechanism(s) underlying the SNARC effect is also unclear. For example, Santens and Gevers (2008) compared the mapping account of the SNARC effect (which suggests that there is a direct link between the position of a number on the MNL and the location of the response) with an alternative account (which suggests that there is a transitional period where numbers are categorised as either small or large between the number and response representations; first proposed by Gevers, Verguts, Reynvoet, Caessens & Fias, 2006; but see also Chen & Verguts, 2010). The authors conducted a number comparison task and instructed participants to judge whether a target number was smaller or larger than ‘5’ by responding with ‘near’ or ‘far’ using one hand. A SNARC effect was still found, which suggests that the SNARC effect should not be used as support for the MNL metaphor. This finding was also extended to show that participants were more likely to associate numbers more strongly with the verbal labels ‘left’ and ‘right’, irrespective of their actual physical location (Gevers et al., 2010). This finding implies the SNARC effect may be accounted for in part by verbal spatial coding instead of the MNL representation.

While the SNARC effect was originally believed to be specific to number, the SNARC effect has been observed in a range of ordinal sequences, such as letters of the alphabet, days of the week, and months of the year (Gevers, Reynvoet, & Fias, 2003; 2004; although see Dodd, Van der Stigchel, Leghari, Fung, & Kingstone, 2008 for an alternate finding). Gevers et al. (2003; 2004) found that the spatial element of non-numerical ordinal lists is involuntarily activated, implying that the association between numbers and space is not unique to number, and that caution should be taken when making inferences about the MNL metaphor from the SNARC effect. The SNARC effect can also be elicited with musical tones (Rusconi, Kwan, Giordano, Umiltà, & Butterworth, 2006) and physical weight (Holmes & Lourenco, 2013), implying that the SNARC effect may be present across magnitude representations. Research has shown that newly learned novel ordinal sequences can also lead to a SNARC effect. For example, a SNARC effect has been found in arbitrary sequences of words (Previtali, de Hevia, & Girelli, 2010), and in arbitrary visual objects (Van Opstal, Fias, Peigneux, & Verguts, 2009), again suggesting the SNARC effect may not provide useful information about the MNL metaphor.

Overall, evidence for the MNL from the SNARC effect research seems inconclusive. The SNARC effect often fails to be replicated (Wood et al., 2006 failed to replicate Dehaene et al., 1993), and is not present for members of some cultures (Shaki et al., 2009). The SNARC effect also may not be specific to number, meaning that any inferences made about an association between number and space in the form of the MNL may be inaccurate (Gevers et al., 2003; 2004). If the SNARC effect also exists for non-numerical ordered lists, then it should not be used to make the inference of the MNL representation that is number-specific.

2.2.2.2. Spatial neglect. A second area of research often used to support the MNL metaphor is neurophysiological studies of spatial neglect patients. Patients with spatial neglect after a right parietal lesion often do not detect objects in the space contralateral (i.e., opposite) to the lesion or are slow to respond to objects (Bisiach & Vallar, 2000). Spatial neglect extends to mental images actively created by the participant and is not limited to stimuli that are physically present in the environment (Bisiach & Luzzatti, 1978). Spatial neglect patients display behavioural deficits such as only shaving the right side of their face or not eating from the left side of their plate. Also, when asked to copy a picture shown to them, a spatial neglect patient may only draw the right half, or, if a clock is presented, may omit the numerals on the left side of the clock (Robertson & Marshall, 1993).

A relevant symptom of spatial neglect is when asked to bisect a horizontal line on paper, spatial neglect patients typically mark the subjective middle of the line as markedly right of the true centre (Halligan & Marshall, 1998). Analogous to this line bisection test is a mental number bisection task. Zorzi, Priftis, and Umiltà (2002) gave spatial neglect patients a mental number bisection task, and spatial neglect patients were asked to state the midpoint of two numbers (the interval could be 3, 5, or 7) without making calculations. Spatial neglect patients systematically misjudged the midpoint of the interval (e.g., responding that the midpoint of 2 and 6 is 5), and these errors mirrored errors in a bisection of physical lines (Marshall & Halligan, 1989). This bias increased as a function of number interval length, with a lesser effect being observed for the smallest intervals. This finding was interpreted by Zorzi et al. as direct evidence of a direct association between the MNL and physical lines and has been interpreted as strong support for the metaphor of a spatially oriented MNL.

The findings of Zorzi et al. (2002) have been extended in a wide range of studies to show the impact of spatial neglect on number processing (Cappelletti, Freeman, & Cipolotti, 2007; Hoeckner et al., 2008; Loftus, Nicholls, Mattingley, & Bradshaw, 2008; Rossetti et al., 2004; Vuilleumier et al., 2004; Yang, Tian, & Wang, 2009; Zamarian, Egger, & Delazer, 2007; Zorzi, Priftis, Meneghello, Marenzi, & Umiltà, 2006; for a review see Umiltà, Priftis, & Zorzi, 2009). For example, Rossetti et al. (2004) reported that bias in a mental number bisection task was improved by rightward prism adaptation (i.e., a shift of the visual field to the right induced by prismatic goggles), which suggests that space and number representations may be closely linked in the brain. Also, research has found that 'optokinetic stimulation' (for a review see Kerkhoff, 2003) has a positive effect on number interval bisection for spatial neglect patients, further supporting the idea of a close link between space and number representations (Priftis, Pitteri, Meneghello, Umiltà, & Zorzi, 2012; although see Pitteri, Kerkhoff, Keller, Meneghello, & Priftis, 2015).

While the above studies provide support for a spatial representation of number, the extent of MNL disruption caused by spatial neglect is not always well defined. Specifically, the bias found in spatial neglect patients is not always associated with a bias towards large numbers, which would be expected according to the MNL metaphor (since the MNL locates larger numbers are on the right side of space). Doricchi, Guariglia, Gasparini, and Tomaiuolo (2005) compared the results of mental number bisection tasks and physical line bisection tasks in spatial neglect patients with right brain damage. Spatial neglect patients who displayed a strong bias in physical line bisection did not always have a comparable bias in mental number bisection, and vice versa. This finding instead implies a double dissociation between physical line bisection and mental number bisection tasks and suggests that there may be important differences in the representation of space and number in the brain. Also, Zorzi, Bonato, Treccani, Scalambrin, and Marenzi (2012) found that spatial neglect patients only exhibit biases when number is explicit (e.g., number comparison task), but not when number is implicit (e.g., parity judgment task). This finding suggests that the link between numbers and space found in studies of spatial neglect might not be strong evidence for a spatially oriented MNL, since the association between numbers and space is variable.

In contrast to the SNARC effect research findings, some spatial neglect studies indicate that the MNL could be specific to the representation of number. Zorzi, Priftis, Meneghello, Marenzi, and Umiltà (2006) tested the representation of numerical and non-numerical sequences in spatial neglect patients, using a comparable method to Zorzi et al. (2002). Spatial neglect patients completed a physical line bisection task and mental bisection tasks for number, letter, and month intervals. While the results of this study replicated those of Zorzi et al. for physical line bisection, the tasks involving letters and months revealed very different response patterns. The findings of Zorzi et al. suggest that the spatial properties of the MNL may be a unique property of number representations, rather than the MNL being a more general ordinal representation.

The disruption of the MNL observed in spatial neglect patients has been replicated in neurologically healthy participants using parietal repetitive transcranial magnetic stimulation (rTMS). Göbel, Calabria, Farne, and Rossetti (2006) applied rTMS on healthy participants while they completed a numerical interval task without calculating. Participants showed similar error patterns reported in physical line bisection studies for patients with spatial neglect (Zorzi et al., 2002). This study was interpreted as evidence that healthy individuals most likely rely on a spatially oriented MNL when completing number tasks.

A group of studies have utilised the fact that neurologically healthy individuals show a bias known as 'pseudoneglect' (a small leftward deviation when bisecting physical lines; see Jewell & McCourt, 2000 for a review) to make inferences about number representations. For example, Longo and Lourenco (2007) required neurologically healthy participants to complete the mental number bisection task as described by Zorzi et al. (2002). Participants exhibited pseudoneglect for both tasks, which Longo and Lourenco interpreted as evidence that asymmetries in bisection for physical and numerical space are analogous. Another study by Longo and Lourenco (2010) extended this finding and showed when participants are asked to bisect physical lines, or number pairs shown at different distances, pseudoneglect on the number bisection task shifts rightward as a function of distance, suggesting that number and space have overlapping mental representations.

Loftus, Nicholls, Mattingley, Chapman, and Bradshaw (2009) administered a mental number bisection task to neurologically healthy individuals. Participants were shown number triplets (e.g., 16, 36, 55), and were asked to judge if the numerical distance was larger on the left or right side of the middle number. Participants consistently overestimated the numerical length on the left, and this bias was not influenced by physical stimulus changes. These results suggest that this bias is due to the mental representation of number along a spatially oriented MNL. This left bias supports the findings of Zorzi et al. (2002) using pseudoneglect instead of spatial neglect patients, and reflects an expansion of small numbers in left space, and a contraction for large numbers in right space.

Nevertheless, the finding of parallel errors in physical line bisection and mental number bisection has not been consistently replicated in studies of pseudoneglect. For example, Rotondaro, Merola, Aiello, Pinto, and Doricchi (2015) used a similar method to Longo and Lourenco (2007) and found that when numerical anchor points are defined verbally instead of with visual spatial cues, line deviations average out across different line lengths and number intervals, and produce no systematic pattern of bias. These results led the authors to suggest that numbers may not be arranged on the MNL, and that this representation is only elicited when the numbers to be processed are shown in a left-to-right order (i.e., Longo & Lourenco, 2007), or in the case of the SNARC effect, when 'left' and 'right' codes are used in the response selection stage (i.e., Dehaene et al., 1993; although see also Stoianov, Kramer, Umiltà, & Zorzi, 2008 who suggests that visuospatial numerical interactions occur before the response selection stage). These findings suggest a spatially oriented MNL may be a function of the task being completed at the time, rather than confirming the MNL representation.

Overall, both spatial neglect and pseudoneglect studies have been used as evidence for the MNL representation. The seemingly parallel errors made by spatial neglect patients on physical line bisection tasks and number bisection tasks indicate a strong association between numbers and space (Zorzi et al., 2002). This result has been replicated in neurologically healthy participants (Göbel et al., 2006), and has also been suggested in studies of pseudoneglect (Longo & Lourenco, 2007). However, while these neglect studies provide some support for the MNL metaphor, they do not constitute solid evidence for a spatially oriented representation of number, because the link between number and space is not consistent (Doricchi et al., 2005), meaning that inferences made regarding the MNL representation from these findings may be exaggerated.

2.2.2.3. Associated brain regions for number and space. A third area that has been used to support the MNL metaphor are studies on the proposed shared brain regions for number and space. Neuroimaging research has shown that number and space both activate the intraparietal sulci (IPS; for reviews, see Fias & Fischer, 2005; Hubbard, Piazza, Pinel, & Dehaene, 2005). Dehaene, Piazza, Pinel, and Cohen (2003) tested the idea that the parietal lobe is the key brain area underlying number, based on the triple code model of number (which proposes that numbers are represented in three codes in the brain, with distinct neural substrates for each code; Dehaene, 1992; Dehaene & Cohen, 1995; 1997). The authors collected functional magnetic resonance imaging (fMRI) activations during a set of numerical tasks, and proposed a tripartite division of the parietal lobe for number processing. In this model, the horizontal segment of the IPS (HIPS) is specific to number, and it is activated when numbers are employed, independent of their format. This is the 'core quantity system', and the proposed location of the MNL representation. This system is supported by two other circuits, including a 'left angular gyrus area', which assists the processing of verbal numbers, and a 'bilateral posterior superior parietal system', which underlies attentional orientation on the MNL, like other spatial dimensions. In this theory, the MNL is located within the parietal cortex, which overlaps with spatial representations (Nieder & Dehaene, 2009).

Other studies have also shown that the HIPS has a role in quantity organisation, whether it be for calculation (Chochon, Cohen, Moortele, & Dehaene, 1999), approximation (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999), or digit detection (Eger, Sterzer, Russ, Giraud, & Kleinschmidt, 2003). Furthermore, single cell recordings in the prefrontal and parietal cortices of monkeys (Nieder, Freedman, & Miller, 2002; Nieder & Miller, 2004) have shown that this region may also be the location of number representation in non-human animals. In these studies, monkeys were trained in a delayed match to sample task, where the task required monkeys to judge if a sample dot array differed from a target dot array with regard to numerosity. Nieder et al. (2002) found that groups of neurons preferentially activated during the presentation of a certain numerosity. This 'place coding' process has been viewed as evidence of a neural basis for the MNL in non-human animals (see Ansari, 2008; Dehaene, 2011; Tzelgov, Ganor-Stern, Kallai, & Pinhas, 2015).

However, it must be noted that the HIPS may not be number-specific, which may be a limitation when making inferences for the MNL metaphor. For instance, Piazza, Izard, Pinel, Le Bihan, and Dehaene (2004) used fMRI to monitor participants when they compared Hindu-Arabic numerals for luminance, size, and numerical magnitude. The authors found the HIPS displayed an overlap for all three tasks. Simon, Mangin, Cohen, Le Bihan, and Dehaene (2002) used fMRI to test the link between calculation related activation and other spatial and language areas in the parietal lobe. Results showed that while calculation activates the HIPS, this area is also activated for manual, visuospatial, and verbal tasks. From these results, it seems the link between number and space in the brain may not be unique evidence of the MNL, because it may apply to other tasks. This may instead suggest a shared representation of magnitude rather than a representation specific to number (Walsh, 2003).

Further evidence for an overlap between number and space brain regions comes from research by de Hevia and Spelke (2009), who argued that number and space representations are linked to support the MNL representation. Using a manual line bisection design, de Hevia and Spelke examined whether number to space mappings are a product of culture, or if it reflects a link between number and space. By testing a group of young children and a group of adults, the researchers found that flankers of two and nine dot arrays distorted the midpoint towards the larger number flanker at all ages. This was interpreted to mean that numerical and spatial representations are associated before formal math education, which supports the MNL metaphor. Nevertheless, this conclusion has been challenged. Gebuis and Gevers (2011; see de Hevia, 2011 for a reply) showed that the bias observed by de Hevia and Spelke was caused by the larger area occupied by the larger dot array, rather than a special link between number and space. Gebuis and Gevers suggest that numerical information cannot be extracted from a visual scene independently of its visual cues, and that number judgements are based on combining the information from various visual cues (Gebuis & Reynvoet, 2011), which casts doubt on the study by deHevia and Spelke as evidence for a spatially-oriented MNL.

There is also evidence that the direction of body motions in space affects the representation of number, suggesting an association between space and number, possibly in the form of the MNL metaphor. Since body motions are known to influence the underlying representation of space (Amorim, Glasauer, Corpinot, & Berthoz, 1997; Klatzky, Loomis, Beall, Chance, & Golledge, 1998; Presson & Montello, 1994; Simons & Wang, 1998; Wang & Spelke, 2000), it is possible that body motions may also affect the representation of numbers. Hartmann, Grabherr, and Mast (2012) found that passive whole-body motions can influence math performance. The authors note that past work has found that active head turns affect number cognition, where small numbers are generated more often with left head turns, and large numbers are generated more often with right head turns, suggesting that active body motion directs spatial attention to a certain point of the MNL (Loetscher, Schwarz, Schubiger, & Brugger, 2008). In three studies, Hartmann et al. reported that leftward whole-body motions facilitated the generation and processing of small numbers (numbers smaller than five), while rightward motion facilitated large numbers (numbers larger than five). The authors also found that the magnitude of a number can even influence the detection of self-motion direction (e.g., body moving left or right). This study shows some link between number and space in the brain. However, the mechanism by which this interaction occurs (i.e., the MNL representation or otherwise) is not clear, because Hartmann et al. did not clarify how bodily motion was interacting with number specifically.

Overall, the link between number and space in the brain is unclear, and should not be interpreted as direct evidence for a spatially-oriented MNL representation. Another possibility is that the MNL metaphor may not adequately explain the link between number and space. While it has been found that the brain regions for number and space show some overlap in the parietal cortex (Dehaene et al., 2003), it has been shown that the parietal cortex is highly activated for all magnitudes (Simon et al., 2002), casting doubt on the assumed specificity of the MNL. Similarly, despite the finding from behavioural research that number and space are closely linked in the brain (de Hevia, Vallar, & Girelli, 2008), it is not clear whether this is specific to number, or whether this finding also applies to other magnitude representations

(Walsh, 2003). This suggests that the link between space and number should not be used as evidence for a left-to-right oriented MNL representation.

2.2.3. Scale of the mental number line

It is currently not known how the MNL representation is scaled. Specifically, there is an ongoing debate about whether the MNL is linear, logarithmically compressed, or another algebraic function. Clarifying the scale of the MNL has implications for understanding how mental operations are performed on number. Furthermore, clarifying the scale of the MNL would have important implications for learning and education programs surrounding numerical cognition. Initial attempts to define the MNL claimed it was logarithmically compressed, similar to a slide rule (comparable to estimation of other basic psychophysical phenomena, such as size, distance, and weight; Banks & Hill, 1974; Dehaene, 2003; Dehaene et al., 1990; Nieder & Miller, 2003; 2004). Dehaene (2011) claimed that humans represent symbolic numbers as analogical quantities with decreasing precision (larger quantities are represented less accurately), similar to non-human animals, and even spacing is given to the intervals between 1 and 2, 2 and 4, and 4 and 8. Dehaene claims reliance on these logarithmic representations is automatic, and cannot be supressed (Dehaene, 2007). However, other researchers have argued for a linear representation of number (Cordes et al., & Whalen, 2001; Gallistel & Gelman, 1992, 2000; Gibbon & Church, 1981; Meck & Church, 1983; Whalen, Gallistel, & Gelman, 1999), called the 'accumulator model'. These researchers propose that numbers are represented as evenly spaced, linearly increasing magnitudes with scalar variability, where scalar variability implies "the greater the magnitude, the noisier its representation" (Gallistel & Gelman, 2000, p. 59).

Support for either theory has been somewhat inconclusive. There has been some evidence for the scalar variability model from behavioural research of non-symbolic number in humans and non-human animals. Brannon, Wusthoff, Gallistel, and Gibbon (2001) trained pigeons to compare a constant number with the number remaining after a numerical subtraction (e.g., in the first testing phase of this study, one pecking key required four flashes before reinforcement was given, and another pecking key required eight minus the number of flashes for reinforcement). The authors reported that pigeons responded linearly, which does not support a logarithmic scaling of number (however, see Dehaene, 2001 for a contradictory finding also using pigeons). These findings are consistent with the research of Whalen, Gallistel, and Gelman (1999), where they used a non-verbal counting task to show humans represent numbers with scalar variability. However, the scalar variability view is not entirely supported by the literature, and evidence of logarithmic scaling is still observed. For example, neurological data locating number neurons in monkeys (Nieder & Miller, 2003; 2004) has been interpreted as support for a logarithmic model. Cohen Kadosh, Tzelgov, and Henik (2008) used synaesthesia to test the scaling of the MNL, and reported that synesthetes displayed logarithmic response patterns (however, see Verguts & Van Opstal, 2008, who claim Cohen Kadosh et al. did not directly examine the MNL representation).

Research on number representations in a cross-cultural context has also been used to make inferences about the scaling properties of the MNL. Dehaene, Izard, Spelke, and Pica (2008) tested the process of learning to associate numbers to space with the Mundurucú, a remote Amazonian indigenous tribe with a limited numerical lexicon, and little to no formal education. Dehaene et al. presented Mundurucú a line segment with one dot on the left, and ten dots on the right. Participants were shown random sets of one to ten dots, and asked to point to where they thought the number of dots should be located on the line. The researchers then moved a cursor to this point. The Mundurucú initially mapped symbolic and non-symbolic number stimuli onto a logarithmic scale regardless of age. These results are in contrast to the findings of Siegler and Opfer (2003), who found that adults from Western cultures map small or symbolic number stimuli numbers linearly, and revert to logarithmic mapping only when numbers are presented in a non-symbolic format, or in conditions that discourage counting (e.g., timed judgments). Dehaene et al. suggest the estimation patterns found in the Mundurucú supports the idea that the MNL is logarithmically scaled, and that the linear with scalar variability format of the MNL is a cultural construction that does not necessarily develop in the absence of formal Western math education.

This conclusion is also reinforced by the finding that educated Mundurucú can be taught to understand linear scaling, which is central to the Portuguese number word system. Pica, Lemer, Izard, and Dehaene (2004) found that while the Mundurucú are able to compare and add large approximate quantities that are outside the range they can name, they cannot perform precise arithmetic with numbers larger than four or five. This suggests a distinction between number approximation and culture based counting systems for exact number and arithmetic. However, research has also proposed that the Mundurucú may encode numbers linearly with scalar variability (Cantlon, Cordes, Libertus, & Brannon, 2009), casting doubt on whether the estimation patterns observed in the Mundurucú provide evidence of a logarithmic scale of the MNL representation.

Furthermore, despite the claim by Dehaene et al. (2008) that association between number and space is logarithmic in nature, Núñez (2008) claimed that since the NP task is a relatively new test, it should not be used to infer a logarithmic MNL. Núñez (2011) suggested that there is no 'innate' MNL in the brain because number to space mappings are acquired thorough formal education. Overall, neither logarithmic nor linear models sufficiently describe the scale of the MNL, because there is evidence for and against both theories present in the literature. The underlying scaling of the MNL still needs explanation.

2.2.4. Alternative accounts to the mental number line metaphor

In an attempt to account for the shortcomings of the MNL metaphor outlined above, some alternative models for number cognition have been proposed. One alternate to the MNL account is the polarity correspondence theory, proposed by Proctor and Cho (2006). This account does not only apply to number representations, but can also pertain to other tasks, such as word-picture verification or implicit association. This model also explains effects commonly attributed to the MNL, such as the SNARC effect. In this model, stimuli and responses are both assigned as either a positive ('+') or negative ('-') polarity during an intermediate processing stage (Wood, Willmes, Nuerk, & Fischer, 2008). In terms of number processing, both 'right' and 'large' have a polarity of '+', whereas 'left' and 'small' have a polarity of '-'. This can explain the SNARC effect because overlapping polarities (e.g., 'right' and 'large' or 'left' and 'small') facilitate response selection, meaning that larger numbers will be responded to faster with the right hand, and smaller numbers will be responded to faster with the left hand. While this model provides a good explanation of the SNARC effect (although see Santiago & Lakens, 2015, who found the polarity correspondence theory did not explain the SNARC effect), it does not sufficiently account for other evidence that has been explained by the MNL metaphor, such as analogous errors in mental number interval tasks and physical line bisection tasks in patients with spatial neglect.

Another theory of number representation is the numerosity code model (Zorzi & Butterworth, 1997; 1999, Zorzi, Stoianov, & Umiltà, 2005). This model specifies that the magnitudes of numbers are represented as a corresponding quantity of units (i.e., 8 represents eight units), which does not require a spatial representation. This model has been tested primarily in number comparison tasks. For example, Zorzi and Butterworth (1999) showed that the distance and magnitude effects can be replicated in this model where a decision system operates on 'numerosity codes' to select the larger of two numbers. Zorzi et al. (2005) suggest that the numerosity code accounts for findings typically attributed to logarithmic (Dehaene, 2003) or even linear scales of number (Gallistel & Gelman, 2000). Overall, the numerosity code theory may provide a reasonable account of the phenomena that typically characterise number representations.

The MNL metaphor and the numerosity code model have also been directly compared. Zhao et al. (2012) tested the two models, because while the MNL requires knowledge of spatial ordering for number representations, the numerosity code model does not. The authors used an event related potential (ERP) design with an novel number learning paradigm on adults. Participants were asked to learn associations between magnitude (represented by non-symbolic dot patterns) and a set of novel symbols, or associations between spatial order (symbols were shown within squares arranged left-to-right on the screen) while ERPs were recorded. The authors predicted that if the MNL metaphor is correct, ordinal information should be acquired as part of magnitude learning. Alternatively, if the numerosity code model is correct, magnitude learning should not involve ordinal information. Zhao et al. (2012) revealed a dissociation between the neural substrates underlying magnitude and spatial order. Specifically, the spatial order group showed a later and reversed distance effect compared to the magnitude learning group. Also, ordinal information was found not to be necessary to learning magnitude, providing support for the numerosity code model over the MNL metaphor. This suggests that features of the numerosity code model may be useful to consider for theories of number representation.

Another account of how number is represented in the brain has been proposed by Gallistel (2011), who claims both the logarithmically compressed and linear with scalar variability views are limited, and suggests an alternative ‘auto-scaling’ approach. Gallistel claims that the brain’s mechanism for representing magnitudes has a limited output, similar to measuring instruments (e.g., similar to a display screen). Gallistel claims this mechanism for representing quantities must also obey the Weber-Fechner law, adjusting the sensitivity to the input received (Ward, Gallistel, & Balsam, 2013). This proposal implies that the Weber-Fechner law is not an inability of the brain to suppress noise, but a feature of the brain’s ability to represent magnitudes. This explanation of number representations suggests the representation of number in the brain should change to suit task demands. It also stresses relationships and the relational structure between numbers, and adjusting neural weights for numbers depending on the context in which they are presented (unlike the MNL metaphor), providing an alternative account for the distance and magnitude effects.

2.2.5. Summary

There is support for the MNL metaphor from a range of behavioural and neuropsychological studies, and research on the associated brain regions for number and space. However, while many researchers have used the MNL metaphor to explain empirical findings, it is not clear whether this explanation is warranted, and claims made regarding the MNL may be overstated. It is unclear whether the MNL is specific to number, or whether it shares properties with other ordinal representations. If it is the case the MNL is not number specific, this could have repercussions for how we measure (e.g., the NP task) and describe representations of number. There are also issues regarding the scaling properties of the MNL. Specifically, it is not known whether the MNL is linear or logarithmic in form. The issue of the scale of the MNL needs to be resolved to clarify the MNL metaphor.

2.3. Change in the mental number line

While the specific characteristics of the MNL representation are still unclear, there is a growing body of evidence suggesting that the MNL undergoes a ‘shift’ during development. Many studies have been conducted to determine the characteristics and properties of this presumed shift in number representations in children. This section will first outline evidence supporting the logarithmic-to-linear representational shift claim. It will then cover evidence for two alternative theories explaining how magnitude estimation ability improves in children: the proportion judgement account and the segmented linear model. Next, the validity of the NP task as a measure of the underlying representation will be discussed, followed by a summary of the linear-to-logarithmic shift in other ordinal lists (e.g., letters of the alphabet).

2.3.1. The representational shift hypothesis

Siegler and Opfer (2003) tested how children’s number representations change with age, because while logarithmic and linear models are both plausible (see section 2.2.3.), neither sufficiently explains the representation humans use to perform math. Siegler and Opfer used a NP task (a task where participants mark the position of numerical values on a physical number line with numerical end points, see section 2.1.5.) with second, fourth, and sixth grade children in NP and PN formats in the 0 to 100 and 0 to 1,000 ranges. Second grade students’ NP estimates were better fit by a logarithmic function rather than a linear one. Fourth grade students’ NP estimates were fit equally well by both functions. Sixth grade students’ NP estimates were better fit by a linear function rather than a logarithmic one. Siegler and Opfer interpreted these results to suggest that improved magnitude estimation performance is associated with a logarithmic-to-linear shift in the MNL.

Siegler and Booth (2004) also tested the development of number estimation in younger children, and suggested that familiarity with counting may affect estimation patterns. The authors asked kindergarten, first, and second grade students to complete a 0 to 100 NP task. The kindergarten students produced estimates that were best described by a logarithmic function. The first grade students displayed a straightened curve. The second grade students produced estimates that were best fit by a linear function, with numbers spread evenly along the line. From these results, Siegler and Booth suggest that improved numerical estimation accuracy can be attributed to the increased linearity of NP estimates.

A series of different studies have also observed that younger children's NP estimates are better described by a logarithmic than a linear pattern, and older children's estimates are better fit by a linear than a logarithmic pattern, corroborating the findings of Siegler and Opfer (2003) and Siegler and Booth (2004; e.g., Booth & Siegler, 2006, 2008; Laski & Siegler, 2007; Opfer & Siegler, 2007; Opfer & Thompson, 2008; Thompson & Opfer, 2008; 2010). This trend has been observed over several tasks and number scales (see Siegler et al., 2009 for a review). This logarithmic-to-linear shift occurs from kindergarten to second grade with the 0 to 100 scale, between second and fourth grade with the 0 to 1,000 scale, and between third and sixth grade with the 0 to 100,000 scale for whole number estimation, fraction magnitude estimation, and whole number categorisation tasks. Recent research has also found that the logarithmic-to-linear shift also occurs in other number systems, such as the Chinese number system (He, Tang, Zhang, Chen, & Wang, 2016), suggesting the logarithmic-to-linear shift may also be present in other cultures.

The linearity of NP estimates has also been found to positively correlate with other math abilities (Booth & Siegler, 2006, 2008; Laski & Siegler, 2007; Sasanguie, De Smedt, Defever, & Reynvoet, 2012; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013; Sasanguie, Van den Bussche, & Reynvoet, 2012). For example, Booth and Siegler (2006) gave second, fourth, and sixth grade students a NP task. The authors found that linear number representations were gradually used more than logarithmic ones as a function of age, and that improved magnitude estimation skills were positively correlated with math ability. The NP task has also assessed children's knowledge of more complex concepts, such as fractions, decimals, and salaries (Jordan, Hansen, Fuchs, Siegler, & Gersten, 2013; Opfer & DeVries, 2008; Schneider, Grabner, & Paetsch, 2009; Siegler et al., 2011). The logarithmic-to-linear shift has also been used to help diagnose math learning disabilities in children (Geary, Hoard, Byrd-Craven, Nugent & Numtee, 2007; Geary, Hoard, Nugent, & Byrd-Craven, 2008). The above studies are often used to infer that the algebraic function (i.e., logarithmic or linear) that best describes NP task performance reflects the form of the MNL representation. However, using the NP task to make inferences about the MNL could be problematic, because there is a possibility that other processing factors may affect, or be responsible for, performance on the NP task. Furthermore, these studies assume the relationship between the NP task and the MNL is unique to number, which may not be the case.

2.3.2. The proportion judgement model

While Siegler and Opfer (2003) interpreted patterns on the NP task as reflecting the underlying number representation in children, Barth and Paladino (2011; see also Ashcraft & Moore, 2012; Barth, Slusser, Cohen, & Paladino, 2011; Barth, Lesser, Taggart, & Slusser, 2015; Slusser, Santiago, & Barth, 2013; Sullivan, Juhasz, Slattery, & Barth, 2011; and for a rebuttal see Opfer, Siegler, & Young, 2011) recently challenged this claim, suggesting that such an interpretation of NP estimation profiles should be treated with some caution. This is because developmental change in NP estimation patterns instead may reflect an increased use of 'landmarks' (i.e., the strategy of dividing lines into halves or quarters to create reference points to guide estimates) associated with a proportion judgment model, rather than a change in number representations per se. Within the proportion judgement model, NP estimates are based on a judgement of the magnitude of the target number relative to the minimum and maximum values on the NP task. Bias results in an overestimation of small numbers, and an underestimation of large numbers. Barth and Paladino (2011) asked two separate groups of children, averaging seven years of age and five years of age, to complete a NP task. Barth and Paladino found that children's NP estimates were better fit by a proportion judgment model, rather than a logarithmic-to-linear shift account. This implies that performance on the NP task may not be a direct measure of numerical representations because task strategies (such as proportion judgement) may influence NP task performance.

Longitudinal studies have also been conducted to assess which model best accounts for children's estimation performance (Friso-van den Bos, Kroesbergen, Van Luit, Xenidou-Dervou, & Jonkman, 2015). Friso-van den Bos, Kroesbergen, Van Luit, Xenidou-Dervou, and Jonkman (2015) compared a non-cyclic power model, a one-cycle power model, a two-cycle power model, a linear model, and a logarithmic model, similar to those used by Rouder and Geary (2014; who found that first grade children use anchor points to estimate magnitudes, consistent with a proportional reasoning account). A one-cycle power model fit young children best, and a two-cycle power model fit older children best, with linear placements increasing as children age. Furthermore, NP estimation accuracy and math performance were also found to reciprocally influence each other over the period from kindergarten to second grade. Nevertheless, while NP task performance may be better described by a proportion power function rather than a logarithmic-to-linear model, it is still unclear whether NP task performance provides information about the MNL metaphor.

The findings of Barth and Paladino (2011) have been challenged. Opfer, Thompson, and Kim (2016) sought to reconcile the discrepant psychophysical functions that have been fit to children's numerical estimation data. Children were assigned to either a 'free' or 'anchored' condition (where in the anchored condition, children were informed where 500 should be placed on a 1 to 1,000 NP task) with 'over' or 'even' number sampling (studies in this area tend to over-sample from the lower end of the NP task). In general, children's NP estimates in the 'free' condition were best fit by a logarithmic model, irrespective of sampling procedure (i.e., over or even). However, for children in the 'anchored' condition, a proportion judgement strategy was often used with even sampling. This means a proportion judgement model may be due to children being given anchors to guide estimates (e.g., that half 1,000 is 500), and that children would not adopt a proportion judgement strategy without this information. Opfer et al. claimed that a logarithmic-to-linear shift model may better explain children's NP task performance (see also Anobile, Cicchini, & Burr, 2012). However, it is still unclear which approach better accounts for performance on the NP task.

2.3.3. The segmented linear model

Another alternative to the developmental shift account is the segmented linear model. This theory was proposed because of past findings suggesting that two-digit numbers may not be processed holistically, but rather decomposed into tens and units (see Moeller, Fischer, Nuerk, & Willmes, 2009; Moeller, Klein, Fischer, Nuerk, & Willmes, 2011; Nuerk, Weger, & Willmes, 2004; Nuerk & Willmes, 2005). Nuerk et al. (2004) claimed that the MNL does not account for different magnitude representations and cannot adequately explain the unit-decade compatibility effect of two-digit numbers (e.g., 42 and 57 are compatible while 47 and 52 are not). Other studies have tested kindergarten, first, second, and third grade students on 0 to 100 (kindergarten, first grade and second grade) and 0 to 1,000 (third grade) NP tasks, and have found that a segmented regression model consisting of two linear portions can describe NP estimation patterns better than a logarithmic or linear model (Ebersbach, Luwel, Frick, Onghena, & Verschaffel, 2008). Ebersbach et al. found that the 'break point' between the two sections was linked to children's familiarity with that number range. Later work has supported this claim, and has found that the breakpoint of the two linear segments relates to the upper end of the number range children are familiar with (Chesney & Matthews, 2013; Stapel, Hunnius, Bekkering, & Lindemann, 2015). These studies suggest that instead of a 'shift', the range of numbers accurately represented on the MNL expands during childhood.

Research by Moeller et al. corroborates the segmented linear model (Helmreich, Zuber, Pixner, Kaufmann, & Nuerk, 2011; Moeller, Pixner, Kaufmann, & Nuerk, 2009; Moeller & Nuerk, 2011). Moeller, Pixner, Kauffman, and Nuerk (2009) tested the hypothesis that children possess two discrete linear representations for one- and two-digit numbers. First grade students were given a NP task, and the segmented linear model provided a relatively better fit than simple linear or logarithmic functions. Moeller et al. suggested that that this pattern mirrors the impact of the place value format of the Hindu-Arabic number system on number representations, which a simple linear model does not explain (see Muldoon, Towse, Simms, Perra, & Menzies, 2013 for an alternative account).

Dackermann, Huber, Bahnmueller, Nuerk, and Moeller (2015) compared the three models of estimation development (i.e., logarithmic-to-linear shift, Siegler & Opfer, 2003; proportion judgement, Barth & Paladino, 2011; segmented linear model, Moeller, Nuerk, & Willmes, 2009). The authors re-analysed NP estimation data of first grade students from Moeller et al. (2009), and concluded that the three accounts complement each other, and that perhaps a hybrid account is necessary for a better understanding of the development of numerical magnitude estimation skills in children. That is, considering what Dackermann et al. term children's 'conceptual' (familiarity with numbers and understanding of the place value system) and 'procedural' (proportion judgement strategies) number knowledge rather than focussing on the format of the MNL. While this study makes an effort to further the segmented linear account of number estimation, it is not clear how this hybrid theory would be explained by the MNL metaphor.

Moreover, researchers have questioned if the logarithmic-to-linear pattern is due to a shift, or if there are multiple classes of numerical estimation patterns that reflect individual differences in the estimation patterns of children. Bouwmeester and Verkoeijen (2012) pointed out that because Siegler and Opfer (2003) used median number estimates with set age groups and fit functions, the conclusions reached in their study may be inaccurate. The authors tested children from kindergarten, first, and second grade with a NP task. The authors found that children's NP estimates could be classed into five different patterns, suggesting that there may be individual differences in children's MNL representations. Bouwmeester and Verkoeijen also did not observe a logarithmic pattern for any of the age groups tested, casting some doubt on the conclusions reached by Siegler and Opfer (2003). Overall, it seems there currently is no one coherent theory that sufficiently explains change in children's patterns on the NP task, and it is also still unclear what underlies this change in NP task judgements.

2.3.4. The number-to-position task and the mental number line

Despite the tendency for researchers to use NP task performance to make inferences about the MNL, it seems that NP task manipulations may influence NP task judgements. For example, Cohen and Blanc-Goldhammer (2011; see also Cohen & Sarnecka, 2014) created an 'unbounded' NP task in which the number line represents a single unit distance from 0 to 1, and number estimations are made external to the bounds instead of within the bounds. On each trial, adult participants were presented with a target number (e.g., 12), and told to move the right boundary to estimate the target location. When participants completed a standard 'bounded' NP task, magnitude estimation patterns closely followed a proportion judgment account. However, when they completed the unbounded NP task (which removes proportion judgment strategies), performance was best characterised by a linear function. This research shows that NP task performance may reflect an increase in number line measurement skills, rather than a change in the underlying representation of number on the MNL.

Research on the unbounded NP task has also been extended in studies by Link et al. (Link, Nuerk, & Moeller, 2014; Link, Huber, Nuerk, & Moeller, 2014). Link, Nuerk, and Moeller (2014) asked a group of fourth grade students to complete both the bounded and unbounded NP tasks, and correlated these results with performance in addition, subtraction, and number magnitude comparison tasks. The authors found that only performance on the bounded NP task positively correlated with number ability, meaning that the bounded NP task may be reflecting other processes recruited to perform the task (such as proportion judgment), meaning that the bounded NP task may not be an accurate measure of number representations. Reinert, Huber, Nuerk, and Moeller (2015) also provided support for this claim, showing that the unbounded NP task is less influenced by proportion judgement strategies than the bounded NP task. These studies suggest that bounded and unbounded NP tasks are not accessing the same representation of number. It could be that performance on the bounded NP task is due to other external variables, such as understanding of proportion judgement strategies or addition and subtraction knowledge, meaning that the NP task should not be interpreted as a direct reflection of the MNL representation.

Karolis, Iuculano, and Butterworth (2011) also tested the claim that the nature of internal number representations cannot be inferred from responses on the NP task. Karolis et al. claim that mapping from the number representation onto behaviour may be non-linear, and distorted by response bias to perform the task correctly. To test this claim, Karolis et al. used two novel variants of the NP task. The first task was a line marking task, where participants were required to indicate the relative position of a number within a numerical interval by marking on a number line. The second task was a line construction task, where participants were asked to extend the physical length of the horizontal number line to fit the length of the number interval (similar to Cohen & Blanc-Goldhammer, 2011). These tasks were presented in left-to-right and right-to-left orientations. It was found that responses were derived from a linear scale regardless of the task. The authors also reported that participants had a linear compression bias, where small numbers were overestimated, and large numbers were underestimated in both tasks. This was attributed to a response bias that arose out of uncertain conditions, rather than a feature of number representations.

Importantly, Karolis et al. (2011) found no difference between the two different orientations of this task, which casts doubt on the idea of an 'inherent' MNL. This is because according to the MNL metaphor, the left-to-right spatial mapping associated with the MNL should be automatically activated for all number tasks. If this were the case, there should have been some additional processing cost (i.e., decreased accuracy) associated with remapping the right-to-left orientation of the NP tasks. These results instead indicate that numerical relationships may be critical in estimation, and that the amount of compression a number receives may be due to which part of the number line is currently the focus of attention. This implies that the spatial organisation of numerical magnitudes may be less straightforward than originally thought. Overall, the above studies suggest that patterns on a NP task may not be as informative about the nature of the MNL as currently assumed.

2.3.5. The 'representational shift' in ordinal lists

An important issue is whether the NP task is a useful measure of the MNL. In other words, if the MNL metaphor is correct, is the NP task the best way measure it? For example, would using other ordinal sequences, such as letters of the alphabet or months of the year, elicit a parallel logarithmic-to-linear shift in children? This finding would suggest the logarithmic-to-linear representational shift reported by Siegler and Opfer (2003) can also describe non-numeric lists, and is not an unique property of number, but a general feature of ordered lists (Berteletti, Lucangeli, & Zorzi, 2012; Hurst et al., 2014). For example, Berteletti, Lucangeli, and Zorzi (2012) examined how children map numerical and non-numerical lists (e.g., letters of the alphabet or months of the year) onto a spatial scale. Their results showed that linear response patterns can extend to other ordinal sequences, suggesting that linear response patterns may not be an exclusive feature of the MNL. This finding has also been replicated with adults. Hurst et al. gave children and adults an alphabet-to-position task, with the end points 'A' and 'Z'. Children showed a logarithmic-to-linear shift with letters, and adults' performance was best characterised by a linear equation. Nevertheless, while Hurst et al. showed that mapping numbers and letters could be represented by logarithmic and linear algebraic functions, it is possible that performance on these tasks could be represented by other functions, which was not accounted for in this study (see Barth & Paladino, 2011) From this research, it seems inferences regarding a logarithmic-to-linear shift in the MNL from a NP task may be misleading, because such developmental patterns can also be elicited in ordinal lists with no inherent numerical information.

2.3.6. Summary

In general, the representational shift theory, while appearing to be a robust account of children's magnitude estimation development, has some inconsistencies. It is not clear whether children's increased accuracy on the NP task shows: a) a representational shift in the MNL (Siegler & Opfer, 2003); b) a development of proportion judgment strategies (Barth et al., 2011); or c) an increased knowledge of the unit decade structure of the Hindu-Arabic number system (Moeller et al., 2009). Moreover, it is also unclear if the NP task is a good measure of the MNL, since task manipulations can change response patterns on the NP task (Cohen & Blanc-Goldhammer, 2011), and a logarithmic-to-linear shift can also be reproduced with other ordered lists, such as letters of the alphabet (Berteletti et al., 2012; Hurst et al., 2014). It still needs to be determined whether the MNL metaphor is an appropriate explanation for change in numerical estimation ability.

2.4. Training the mental number line in children

Although researchers have observed a change in the MNL, suggesting that humans estimate magnitudes more accurately with age, the mechanism as to how this change occurs, and whether this change can be facilitated through training, is not yet clear. The majority of studies examining the MNL have described patterns of behaviour on NP tasks without attempting to manipulate NP task performance. One exception are studies training the MNL, mainly in children with number learning difficulties such as dyscalculia. This work arose because of the finding that the linearity of the MNL is linked with improved number ability in children (Booth & Siegler, 2006; 2008), giving rise to the hypothesis that training NP estimates to be more linear will improve math ability. The following section will cover training studies that have been conducted on typically and atypically developing children using board/computer games and embodied training, with the aim of training the MNL.

2.4.1. Improving magnitude estimation in children

The observation that the linearity of NP task estimates is associated with math ability later in life is consistent with the finding that individual differences in non-verbal number ability positively correlate with math test scores in adulthood (Halberda et al., 2008), and the idea that core math abilities, such as magnitude estimation, may provide the framework for more complex math skills (Feigenson et al., 2004). Support for targeting the MNL in training studies also comes from the finding that children with number learning disabilities exhibit delays in NP estimation ability (Geary , Hoard, Nugent, Byrd-Craven, 2008; Geary, Hoard, Nugent, & Bailey, 2012; Van Viersen, Slot, Kroesbergen, Van't Noordende, & Leseman, 2013; Von Aster & Shalev, 2007). This means training magnitude estimation in childhood may help improve later math ability.

Given this apparent association between the quality of the MNL and math ability, it is not surprising that researchers are invested in improving the quality of the MNL in children with targeted number intervention programs. Difficulties in math are linked to many negative life outcomes, including reduced employment, lower pay and fewer job opportunities, higher mental and physical health costs, and a lack of development in education (Cohen Kadosh, Dowker, Heine, Kaufmann, & Kucian, 2013; Duncan, Dowsett, Classens, Magnuson, Hutson, Klebanov et al., 2007). As a result, a range of MNL training programs has been developed over the years, including board games, computer training, and embodied training. However, the majority of these programs use the NP task to train young children, despite the suggestion that the NP task may not reflect the MNL (see section 2.3.4.). While an exhaustive review of the number training literature is beyond the scope of the current thesis, a selection of relevant recent training programs will be discussed and critiqued in the following section.

In an early training study, Opfer and Siegler (2007) tested the hypothesis that providing feedback on problems where the difference between two representations is the greatest (i.e., logarithmic and linear) will lead to the biggest representational shift. Second grade students were given feedback on a paper and pencil NP task. The authors found that change was abrupt, and occurred after a single trial of feedback. Furthermore, change was extensive, affecting estimates over the range of the 0 to 1,000 line, suggesting the MNL is a flexible and adaptive representation. This study implies that training in a local range of the MNL with a

NP task is successful in creating changes over the entire MNL, providing a framework to design a range of subsequent training studies.

2.4.2. Training with board games

Several studies (Ramani & Siegler, 2008, 2011; Ramani, Siegler, & Hitti, 2012; Siegler & Ramani, 2008, 2009; Whyte & Bull, 2008) have attempted to train children with linear board games. These board games are designed to be similar to the game 'Snakes and Ladders', where the numbers 1 to 10 are arranged in squares in a left-to-right format. Children are asked take turns spinning a spinner to move a token the amount of spaces as shown on the spinner (the spinner has a '1' half and a '2' half) to reach the end of the 10 square board (the board has 10 different coloured, equally spaced squares labelled from 1 to 10). Ramani and Siegler (2008) trained kindergarten students on this game for four 15- to 20-minute sessions within a two week period. Ramani and Siegler found that playing this game improved children's knowledge of numerical magnitudes and other math skills compared to children who played a comparable colour board game. These changes were also still present in a follow-up session conducted two months later, which suggests that linear board games can produce long lasting change in the MNL. This suggests that the MNL representation is flexible, and can be changed with training.

This improvement in numerical ability is not necessarily due to the use of numbers in linear board games per se. Siegler and Ramani (2009) found playing circular board games does not produce a comparable improvement in math ability, as measured by a series of counting, number line estimation, magnitude comparison, numeral identification, and arithmetic tasks. Linear board games are also effective across different socio-economic groups. Ramani and Siegler (2011) compared the influence of playing linear board games in middle income and low income children, and found playing these games improved number learning in both groups. These findings suggest that training can reduce the gap between low and middle income children's numerical knowledge when they enter primary school. Ramani, Siegler, and Hitti (2012) also found that linear games are effective when administered by briefly trained paraprofessionals, indicating that linear board games can be successfully used in a classroom context. Together, these studies suggest there is evidence that training with board games improves math skills in children, and that these benefits may be long lasting.

Nevertheless, the specific method by which linear board games produce mathematical learning is not apparent. For example, it could be that the counting strategy used in some board games has an influence on the precision of the MNL rather than the game itself. Laski and Siegler (2014) examined the effectiveness of number board games, and tested the theory that successfully encoding the numerical to spatial relations in number board games promotes learning in these games. By using a ‘Race to Space’ 0 to 100 number board game, where kindergarten children were told to adopt either a ‘count on’ or ‘count from one’ strategy. The authors reported that a ‘count on’ strategy improved not only NP magnitude estimates, but a range of other math skills, such as numerical identification, counting from numbers other than one, and encoding the structure of the board game. On the other hand, a ‘count from one’ strategy led to less learning. Improvement in NP magnitude estimation did not occur when the numbers were presented outside the context of the board game. Such observations are important, because they question how extensive the effects of training are, and highlight the need to test over a range of tasks (i.e., not just the NP task). However, the issue of how board games lead to change in the MNL more generally is still unclear.

2.4.3. Training with computer games

Several studies have attempted to increase the linearity of the MNL using computer games. These computer games are intended for the prevention or remediation of number learning difficulties, and are often used as part of a larger intervention program. For example, Kucian et al. (2011) developed a computer game named ‘Rescue Calcularis’ (see Käser et al., 2012 for a description of an updated version of this game titled ‘Calcularis’) to enhance the quality of the MNL in children diagnosed with dyscalculia[3]. This game involved children moving a rocket named Calcularis to an estimated position on a NP corresponding to a Hindu-Arabic numeral, estimated number of non-symbolic dots, and the outcome of an addition or subtraction task. Dyscalculic children and age matched controls played this game 15 minutes a day, five days a week, for five weeks. Results showed that children with and without dyscalculia benefitted from training, as measured by improved performance on a 0 to 100 NP task. The authors also reported a decreased recruitment of brain regions linked with number processing, possibly due to the related cognitive processes becoming more automatic. The results of the Calcularis program are promising, because they suggest that training can lead to change at the behavioural and neuronal levels, which implies that training with numbers can assist number learning for children with and without dyscalculia. However, the long-term

[3] Dyscalculia is generally defined as a disorder in math ability, which is assumed to be caused by a specific impairment in brain function (see Butterworth, Varma, & Laurillard, 2011 for a relatively recent review of dyscalculia).

effect of the Rescue Calcularis training program is not clear, as no follow-up test was run after the initial training period with this program.

Another computer training game is known as ‘The Number Race’ (Wilson, Dehaene, Pinel, Revkin, Cohen, & Cohen, 2006a; Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006b; Wilson, Dehaene, Dubois, & Fayol, 2009). This program trains children using a numerical comparison task, which can be adapted to suit the needs of an individual child. The Number Race involves the children playing as a dolphin character that must choose the larger of two numerosities (which can be presented in symbolic, non-symbolic, verbal number formats, or a combination of those formats, such as symbolic paired with non-symbolic). As the game progresses, children solve addition and subtraction problems to complete the comparison task. Feedback is given each time the child makes a decision (e.g., if the child wins the round, they will hear the positive phrase ‘you’ve got the most!’). Wilson et al. (2006a) found playing this game for one hour a day, four days a week, over five weeks, led to an improvement in number cognition skills for children with number learning difficulties. For instance, these children showed improvements in both symbolic and non-symbolic number comparison. However, a limitation of this game is that the training effect did not transfer to counting or arithmetic tasks in a follow-up study (see Räsänen, Salminen, Wilson, Aunio, & Dehaene, 2009). If training changes the MNL, then these changes should be present across a range of math tasks, because the MNL is automatically activated whenever numbers are shown.

2.4.4. Embodied training

These positive training outcomes are not limited to board and computer games. Recently, technology has allowed for ‘embodied’ interaction (Dourish, 2001). These embodied systems measure user movements and gestures, and can support spatial learning (Moeller, Fischer, Nuerk, & Cress, 2015). Studies have successfully used embodied training to train the MNL (Dackermann, Fischer, Huber, Nuerk, & Moeller, 2016; Link, Moeller, Huber, Fischer, & Nuerk, 2013). For example, Link et al. developed an embodied program where first grade students were told to indicate the position of a given number by walking to the estimated location of that number on a number line drawn on the ground. When compared to children who did not receive the task-specific full body training, the trained children had a larger improvement on a range of math tasks, including a 0 to 100 NP task. Also, transfer effects to an addition task were found. The embodied training literature provides more support for the claim that the MNL can be altered with training.

2.4.5. Summary

Overall, the above research provides support for the idea that the MNL can be altered by training in children. Board games, computer games, and embodied training have been used successfully to improve children's magnitude estimation accuracy. These results are important for overcoming learning disabilities in math, such as dyscalculia. These findings imply that the adult MNL may be able to be trained in a similar manner. However, some key issues remain unresolved, such as if this training is long lasting or generalisable, and if there are individual differences in the integration of this training. The issue of whether or not the MNL should be invoked as an analogy to explain these changes is also still unclear.

2.5. Training the mental number line in adults

Relatively little research has investigated whether the representation of number can be changed with training in adults. If the MNL is a flexible representation, then it should be able to change to meet specific task demands (e.g., taking on a logarithmic form when presented with a task that requires logarithmic responses). This section will outline an early MNL adult training study using dot arrays. It will then discuss MNL training with brain stimulation, followed by an outline of studies that have tried to alter the adult MNL with behavioural feedback. Next, studies testing the usefulness of the NP task in training will be critiqued, because there are several limitations that need to be addressed in order for claims to be made regarding specific properties of the underlying MNL representation.

2.5.1. Calibrating the mental number line in adults

Izard and Dehaene (2008) conducted one of the first studies to train the MNL in adults. This research contained two studies. The first study asked participants to estimate the numerosity in the absence of feedback. The stimuli were dot arrays containing 1 to 100 dots, and participants completed 600 trials each. The authors found that participants initially underestimated the numerosity of the dot arrays without feedback. However, the responses were internally consistent, and displayed scalar variability. Izard and Dehaene then asked whether the participants could be calibrated to produce more accurate responses. The second study used dot arrays from 9 to 100. Again, participants were asked to estimate the numerosity of the dot array over six trial blocks. Before each block, participants were shown an 'inducer' dot array, and were always told it contained 30 dots. However, there were three different conditions of this study. The first condition had an inducer of 25 dots ('overestimation inducer'), the second condition had an inducer of 30 dots ('exact inducer'), and the third condition had an inducer of 39 dots ('underestimation inducer'). Following these feedback trials, participants adapted their responses accurately to the inducer trial. For example, the mean response to the number 30 was 27.4 for the overestimation group, 31.5 for the exact group, and 40.1 for the underestimation group. Furthermore, the value of the inducer trial had an effect on all numerosities shown, meaning that responses were calibrated for the entire range of the numerosities tested in this study.

From these findings, Izard and Dehaene suggested that calibration of the MNL in adults is “immediate, long lasting, and global” (2008, p. 1234), and that calibration results from a combination of controlled and automatic effects (i.e., a combination of participants consciously correcting responses and automatic learning effects). This study implies that training with dot arrays may be effective in changing the MNL. Therefore, it seems possible that the NP task can be used to train the MNL of adults in a similar manner. Specifically, by providing participants with feedback on a NP task, they should be able to consciously modify their responses, or even learn to automatically alter their responses. While studies have made the inference that the MNL can be altered with training, it is not clear whether these changes are actually reflected in the MNL. For example, this study does not address the question whether the calibration of dot array estimates globally calibrates the MNL representation, because there were no other tasks used in this study to test generalisability. Furthermore, it is not clear if these changes are as long lasting as Izard and Dehaene claim, because no follow-up test was conducted. Finally, it is unclear if all adults learned the feedback in the same way.

2.5.2. Training with brain stimulation

A series of studies have tested if transcranial direct current stimulation (TDCS), a type of non-invasive brain stimulation, can be used to augment cognitive function in the math domain, and improve performance in NP tasks. Cohen Kadosh, Soskic, Iuculano, Kanai, and Walsh (2010) used TDCS to stimulate the parietal lobes of adults over a six day training period using a learning paradigm of artificial numerical symbols (the Gibson figures; see Tzelgov, Yehene, Kotler, & Alon, 2000 for the original design). This task involved participants learning the association between nine arbitrary symbols without being told the numerical value assigned to them. During the learning phase, a weak current was applied to the parietal lobes of the participants. Following this, the artificial symbols were assessed with a Stroop task (where participants were shown pairs of artificial symbols that varied in physical size and were told to pick the symbol with the greater numerical value), and a variant of the NP task with the artificial symbols (where participants placed symbols on a line according to their magnitude). The authors found that the polarity of brain stimulation (right anodal and left cathodal or left anodal and right cathodal) selectively enhanced or impaired number processing and the mapping of the numbers onto space. Improvements were present six months after the initial training period, and control tasks demonstrated that the training was specific to the representation of the artificial symbols.

Subsequent research by Iuculano and Cohen Kadosh (2013) also trained adults with the artificial Gibson figures while administering TDCS to either the posterior or dorsolateral parietal cortex. Participants were trained to link an artificial symbol with a Hindu-Arabic numeral over a six day period. Next, participants were shown pairs of the artificial symbols, and were asked to decide which was larger in magnitude. After this, participants completed a Stroop task with both the artificial symbols and Hindu-Arabic numerals. The authors were interested in testing how the application of TDCS would affect the automaticity (i.e., performing a number task quickly), and numerical learning (i.e., performing number tasks with increased facility) for the artificial symbols. The authors found that stimulation to the posterior parietal cortex (PPC) facilitated numerical learning, but impaired automaticity for the learned material. However, stimulation of the dorsolateral parietal cortex produced the opposite pattern of results, where numerical learning was impaired, but automaticity for learned material was facilitated. This study suggests that enhanced symbol learning may not facilitate number processing, and that training may not generalise to all areas of number.

Training with TDCS has also been used on adults with math disabilities. Iuculano and Cohen Kadosh (2014) used an artificial symbol leaning paradigm with TDCS for two adults with dyscalculia in an attempt to improve their math abilities. Participants were taught to associate novel symbols in a trial and error design, while TDCS was applied to the PPC. In this study, one participant received anodal stimulation to the right PPC, and cathodal stimulation to the left PPC. The other participant received the reverse configuration, anodal stimulation to the left PPC and cathodal stimulation to the right PPC. A Stroop task and NP task were used to assess the automaticity of number processing and the mapping of the numbers onto space. The authors found that only anodal stimulation to the left PPC resulted in an improvement in both measures of numerical proficiency, suggesting that more research is needed to develop training techniques for adults with numerical learning disabilities. These TDCS findings show that modulating neuronal activity with brain stimulation may produce long lasting changes in numerical competence, suggesting that the MNL may be altered with training (see Snowball, Tachtsidis, Popescu, Thompson, & Delazer, 2013, for a similar application with transcranial random noise stimulation). Overall, these studies imply that training should elicit long lasting change in the MNL of adults.

2.5.3. Training with behavioural feedback

Only a few studies have trained the MNL of adults with behavioural feedback. This work implies that the MNL is an adaptable mechanism, and should change to meet specific task demands. Opfer and DeVries (2006) tested whether the method of improving children's numerical estimation with feedback on a NP task would also change the numerical estimation skills of adults, and trained adults to have a logarithmic MNL. This task assigned participants into either '5 feedback', '150 feedback', '725 feedback', or 'no feedback' conditions. The feedback conditions were designed to provide participants with feedback in different number ranges. For example, participants in the '5 feedback' condition received feedback on the number 5 in the first block of trials, then feedback on three different numbers on the lower end of the NP task (from 2 to 42) for the second and third trial blocks. The authors predicted that feedback would be the most effective in the '150 feedback' condition, where logarithmic and linear functions diverge the most in the 1 to 1,000 range. Participants completed a 0 to 1,000 NP task with three 10 item trial blocks. Feedback was provided as if numbers were on a logarithmic scale. The authors reported that adults, similar to children, were able to quickly shift from one representation to another with minimal feedback. However, it may be that this research demonstrated that adults can treat the logarithmic training as a separate mapping task, and that training does not actually change the underlying MNL representation.

Similarly, Chesney and Matthews (2013) presented adults with a range of NP tasks, some of which encouraged participants to keep incorrect assumptions about the magnitude of the upper anchor of the NP task (e.g., when confronted with the unfamiliar upper anchor of $.999\text{x}10^{4.5}$, most participants estimated the value to be equal to 10,000, rather than the actual value of 31,623). By using different combinations of decimal and exponential stimuli as anchors, the authors were able to produce a linear-to-logarithmic shift among adults by manipulating their familiarity of the number stimuli. Chesney and Matthews suggest that differences in NP estimation performance does not reflect differences in the form of the MNL, as NP estimates that are fit well by a logarithmic function are not necessarily produced by a logarithmic representation. This claim corroborates suggestions that the NP task may not directly reflect the underlying number representation (Karolis et al., 2011). However, these two training studies relied on confusing participants, and it is not evident how widespread the transition to logarithmic representations was, if such a transition was actually present.

Nevertheless, other research has shown that the logarithmic scaling of the MNL is robust in certain situations, and cannot be altered with feedback. For example, Viarouge, Hubbard, Dehaene, and Sackur (2010) asked adult participants to listen to a series of randomly ordered numbers, which either sampled evenly across the NP task, over sampled small numbers, or under sampled small numbers. Participants then judged whether the sequences contained too many small numbers or too many large numbers (a paradigm originally developed by Banks & Coleman, 1981). A staircase method was used to decide the subjective compression of each participant. Viarouge et al. found that participants reliably judged sequences that over sampled small numbers to be more 'random' than sequences that either evenly sampled the NP task or under sampled small numbers. From these findings, Viarouge et al. concluded that adults can exhibit a logarithmic MNL in certain contexts. The authors then tested the strength of this compression with linear feedback, and found that linear feedback on a NP task did not alter the logarithmic nature of the MNL. This suggests that multiple representations of number may coexist in adults (Wood et al. 2008; a point was suggested but not explicitly tested by Siegler & Opfer, 2003). Furthermore, this finding also suggests that a single trial of feedback cannot alter the form of the MNL (Opfer & Siegler, 2007), and suggests that the format of the MNL may only change when needed to suit the specific modality or task presented.

2.5.4. The number-to-position task and training

Studies have also questioned whether the NP task uniquely reflects the MNL representation in the context of training. For example, Huber, Moeller, and Nuerk (2014) attempted to dissociate indicators of linear representations on the NP task with task requirements and strategies to achieve good task performance. In this study, adults and first grade students learned the mappings of a variety of mathematical functions (i.e., logarithmic for children and adults; exponential, sigmoid, and inverse sigmoid for adults only). The authors suggested that since correct task performance on a NP task requires a linear-fitting function, it is not clear whether the MNL develops from logarithmic-to-linear, or whether participants simply learn to correctly solve the NP task. Hence, in these non-linear NP tasks, correct performance should result in non-linear estimation patterns.

Huber et al. (2014) taught non-linear mappings with feedback, since other studies have successfully used this method a NP task (Opfer & Siegler, 2007). The authors assumed that adults would be able to learn these new mappings relatively easily, since adults are thought to have a flexible representation of number which they change depending on task demands (Thompson & Siegler, 2010). Huber et al. also assumed that children should be able to learn logarithmic estimation patterns with feedback, because research has shown children produce a logarithmic shape on a NP task with an upper bound of 100 (Siegler & Opfer, 2003; Siegler & Booth, 2004). Results showed that adults could easily learn non-linear functions in 30 trials of feedback. In comparison, children were unable to learn logarithmic functions. The authors claim these results cannot be reconciled with the notion that children's patterns on a NP task reflect their representation of numerical magnitudes (as suggested by Siegler & Opfer, 2003). This study suggests NP estimation patterns should be interpreted with caution, as they may not directly reflect the MNL representation.

While Huber et al. (2014) make a contribution to the growing literature that suggests the NP task may not be a reliable measure of the MNL[4], this study has several flaws. Firstly, Huber et al. assumed that children should be able to learn logarithmic mappings. Even though children display logarithmic estimation performance, this does not necessarily mean they have an adequate knowledge of logarithmic functions. Secondly, this study trained and tested performance on the NP task, so the claim of dissociating the task from the underlying representation was not fully achieved. A more general measure of numerical estimation performance would have been helpful. In particular, it would have been useful to include measures of 'near' and 'far' transfer[5] as described in the learning literature (Thorndike & Woodworth, 1901). To dissociate NP performance from the underlying MNL representation, near and far transfer effects should be observed, for example, by training on a general set of number tasks, and then testing on a NP task, or vice versa. Thirdly, since the authors reported group mean data without standard deviations for different function fits, individual differences were not considered. It could be the case that several types of MNL representation were present in these data, and that these differences were washed out by group averaging.

2.5.5. Summary

In general, the adult training literature provides some support for the idea that the MNL representation can change using brain stimulation and behavioural techniques. The training literature has also questioned the usefulness of the NP task in making inferences regarding change in the format of the MNL (Huber et al., 2014), suggesting that caution should be taken when using interpreting NP task performance as a direct measure of change in the MNL. However, despite the advances made by these studies, there are still shortcomings in this

[4] See Ebersbach, Luwel, and Verschaffel (2013) for a review of how different NP configurations (such as estimation type, task type, target stimuli, and estimation range) can alter NP task performance.
[5] Where 'near' transfer refers to effects on tasks similar to those trained on, while 'far' transfer refers to effects on tasks unlike those trained on (Melby-Lervåg & Hulme, 2013; Richmond, Morrison, Chein, & Olson, 2011).

research that need to be resolved to gain a better understanding of the MNL metaphor. Specifically, it is not clear whether the MNL is an appropriate analogy for describing the way number is represented, and if the MNL is the best explanation, it is not obvious whether the NP task is the best way to measure it. Considering how number is learned in terms of number symbols and ordinality specifically may help resolve some of these issues.

2.6. Learning, number symbols, and ordinality

An account of how number is learned should underpin discussion of the MNL metaphor, and how the MNL develops more generally. This section will consider some important aspects of learning number, such as number to symbol mapping and learning ordinal relationships between numbers. The neural areas that may underlie number symbol learning and ordinality will then be considered, followed by a brief discussion of number learning disability in the form of dyscalculia. It will be suggested that number symbols, ordinality, and differences in number learning are critical to consider when studying the MNL metaphor, because research to date has largely neglected to consider these factors when discussing the MNL. Indeed, there is growing evidence to suggest that these factors (ordinal relations and number symbols in particular) may be critical to understanding the MNL.

2.6.1. Learning the meaning of number symbols

Since a comprehensive review of how number is learned beyond the scope of this thesis, this section will focus on learning in relation to number symbols and ordinality, because these two components of number learning seem critical to the MNL metaphor. To represent number symbols[6], two concepts must be learned by children. Firstly, cardinality (i.e., the quantity represented by a number, cardinality asks the question of ‘How many?’), and secondly, ordinality (i.e., the location of an item in relation to other items, ordinality asks the question of ‘What position (or rank)?’; Lyons, Vogel, & Ansari, 2016). These two concepts are important for understanding number cognition, and clarifying the roles of cardinality and ordinality may help explain the form of the MNL representation.

According to current theories of number cognition (Dehaene, 2011; Piazza, 2011) humans and non-human animals have a sensitivity to numerical magnitude, regardless of modality. This ability supports magnitude estimation, which allows us to manipulate non-symbolic quantities and identify relationships between numbers (see section 2.1. for a review

[6] Where ‘number symbols’ refer to “the culturally acquired representations of numerical quantity” (Ansari, 2016, p. 29).

of magnitude estimation). Therefore, an important question is how does the ability to perceive non-symbolic numerical magnitudes relate to learning number symbols. A prevalent theory by Feigenson, Dehaene, and Spelke (2004) suggests that number symbols build upon non-symbolic approximate number representations, allowing children to perform complex math. This view is captured well by the quote “when we learn number symbols, we simply attach their arbitrary shapes to the relevant non-symbolic quantity representations” (Dehaene, 2008, p. 552). This ‘mapping account’ proposes that symbolic numerals (e.g., ‘7’) and number words (e.g., ‘seven’) gain numerical meaning by being mapped onto their matching non-symbolic number representations, which allows for the development of more complex math skills. For example, Lipton and Spelke (2005) reported that five-year-old children could map from non-symbolic number to symbolic number words, and mapping ability was directly related to children’s knowledge of the number counting range.

The strongest evidence for the mapping account comes from research demonstrating that children show a ‘distance effect’ (i.e., reaction time increasing with decreasing numerical distance between two values, see section 2.1.1.) when comparing symbolic numbers, which has also been found with non-symbolic stimuli (Buckley & Gilman, 1974). The distance effect arises because of a noisy mapping between external and internal representations of numerical magnitude, where numbers closer on the MNL overlap more in their number representation than numbers which are further apart (Holloway & Ansari, 2009). Temple and Posner (1998) asked five-year-old children and adults to perform a number comparison task, and then recorded event related potentials (ERPs) associated with the distance effect. The authors reported that the distance effect in five-year-old children and adults was similar for non-symbolic dot arrays and Hindu-Arabic numerals, suggesting that both these number notations are accessing a similar representation in the brain, and providing some support for the mapping account (however, see Zorzi & Butterworth, 1999 for an alternative account using the numerosity code model). Gilmore, McCarthy, and Spelke (2007) also found a distance effect with approximate symbolic arithmetic problems (e.g., ‘Sarah has 21 candies’, ‘She gets 30 more’, ‘John has 34 candies, who has more?’) with five-year-old children for addition, subtraction, and comparison tasks. Gilmore et al. concluded that the ability to estimate non-symbolic number present prior to math instruction, and that this ability helps children solve arithmetic problems with abstract numerical symbols.

Nevertheless, a robust relationship between symbolic and non-symbolic number is not always found. Holloway and Ansari (2009) gathered math achievement data from children between six and eight years old, and reaction time data from a number comparison task for both symbolic and non-symbolic numbers. The authors calculated the distance effect for each child, and found that both symbolic and non-symbolic number comparison exhibited the distance effect at a group level, and the distance effect lessened similarly for both tasks over time. However, no correlation was found between the non-symbolic and symbolic distance effect on an individual level, implying that the similarity of symbolic and non-symbolic number is dependent on the task used to test this effect (see also Chew, Forte, & Reeve, 2016, who found four different relationships exist between non-symbolic and symbolic number performance). Maloney, Risko, Preston, and Ansari (2010) also tested the symbolic and non-symbolic distance effects, and found no correlation between the two effects (for another measure of the distance effect using the 'numerical priming task' that supports an association between symbolic and non-symbolic number, see Defever, Sasanguie, Gebuis, & Reynvoet, 2011; Van Opstal, Gevers, De Moor, & Verguts, 2008;). These results indicate that while the ability to estimate non-symbolic magnitudes may help children learn symbolic number, the relationship between non-symbolic and symbolic number representations is not clear, which means it is uncertain how numbers are explicitly represented on the MNL.

Research has also examined the behavioural correlates of the mapping account. Mundy and Gilmore (2009) used a novel task to assess mapping between symbolic and non-symbolic number representations in six, seven, and eight-year-old children. This task involved children being shown a target quantity (either symbolic or non-symbolic), and asked to decide which of two alternate quantities (either symbolic or non-symbolic) matched the presented target quantity. Results revealed that children have the ability to translate between symbolic and non-symbolic representations of quantity, and that this ability improves with age. Mundy and Gilmore also claim that children found symbolic to non-symbolic mapping easier than non-symbolic to symbolic mapping, suggesting that symbolic and non-symbolic number representations are not mapped to each other in an exact way (this finding has been replicated in adults: see Castronovo, Crollen, & Seron, 2010). Furthermore, children's number mapping ability was positively correlated with performance on a school math test (where questions tested different aspects of math knowledge, including symbolic number knowledge and calculation skills), implying that that mapping between non-symbolic and symbolic number may be important to learning the meaning of number symbols.

An alternative explanation of the mapping account is the 'symbolic estrangement hypothesis', proposed by Lyons, Ansari, and Beilock (2012). This theory suggests that while the association between non-symbolic quantities and symbolic numerals is robust during childhood (although see Sasanguie et al., 2013 for an absence of mapping in early childhood), this link decreases as individuals gain more experience with number symbols. Lyons et al. asked participants to compare both symbolic and non-symbolic magnitudes, and also to compare numbers presented in a mixed format (i.e., comparing a dot array to either a Hindu-Arabic numeral or a number word). Lyons et al. claimed that if symbolic and non-symbolic number representations are tightly linked there should be no additional processing cost for mixed format comparison task. This study revealed adult participants found the mixed comparison task most difficult, which suggests that symbolic and non-symbolic number may not be closely mapped to each other. However, this account does not explain why a similar distance effect is found for both non-symbolic and symbolic stimuli in adults (e.g., Temple & Posner, 1998). In summary, evidence for the mapping account seems inconclusive, meaning that conclusions cannot be made regarding how non-symbolic and symbolic number interact to influence the MNL representation. The following two sections will suggest that way number symbols are learned may involve other factors (such as learning ordinal relations) which have not been considered in the above models of number cognition.

2.6.2. Role of number symbols and ordinality in number learning

While many researchers place a large emphasis on non-symbolic skills in learning math (Dehaene, 2011; Piazza, 2011), other studies have challenged this idea, suggesting that knowledge of ordinal relationships between numbers, as well as symbolic number ability, are important to number learning. Indeed, there is a relative lack of research examining ordinality knowledge when compared to cardinality knowledge in number research. However, it has been shown that understanding the ordinal sequence of number symbols and counting words, as well as an accurate knowledge of number symbols and the quantities they represent, are important skills children must develop when learning number.

Lyons and Beilock (2009) found that learning the relative order of number symbols plays a vital role in learning symbolic number representations (see also Roggeman, Verguts, & Fias, 2007; Turconi, Campbell, & Seron, 2006). The authors tested the interaction between ordinal processing of number and working memory (WM), as WM capacity is thought to underlie individual differences in a wide range of math tasks. Adult participants were trained to associate non-symbolic number stimuli with novel symbols. The study repeatedly paired the number stimuli with the symbols, and participants were required to learn these symbol pairings. Following this, participants were asked to perform pairwise judgements on the novel symbols, order the all the symbols from smallest to largest, and then asked about what strategies they employed. Participants with higher WM capacity were better able to map non-symbolic quantities to novel symbols in the training study, and this advantage extended to ordinal judgments on Hindu-Arabic numerals. Furthermore, participants who reported using an ordinal strategy performed best on numerical judgments with novel symbols. Lyons and Beilock concluded that higher WM capacity assists in learning ordinal relations between numbers, which explains why higher WM individuals perform better at math when compared to individuals with a lower WM capacity. The results of this study are supported by Lyons and Beilock (2011), who tested number ordering ability and mental arithmetic ability in adults, and found that ordinality knowledge facilitates the relationship between approximate number and more complex math skills. This suggests that knowledge of ordinal relationships between numbers may be an important factor for understanding MNL representations.

Merkley, Shimi, and Scerif (2016) also investigated the role of ordinal information in number learning with a training task. The authors used a novel symbol paradigm to compare learning via approximate numerical magnitudes and learning via numerical order. In the numerical magnitude condition, participants were trained to link novel symbols with non-symbolic magnitudes. In the numerical order condition, participants were taught the ordinal sequence of the symbols in ascending order. After the learning period, the study used electroencephalography to record brain activity while participants completed a magnitude comparison task. Merkley et al. found that ERPs of the novel symbols were similar to those of Hindu-Arabic numerals, and that ERPs did not differ between the two training conditions. Also, participants in the ordinal condition learned much faster than those in the symbol-magnitude mapping condition. These findings suggest that learning through ordinal symbolic information and learning through mapping non-symbolic quantities onto number symbols both create a similar representation of number. This highlights the importance of considering the role of ordinality in number learning.

Furthermore, Goffin and Ansari (2016) sought to delineate the numerical effects generated from cardinal and ordinal number tasks. The authors tested sixty adults on a range of number tasks, including ordinal, cardinal, visual-spatial, WM, inhibitory control, and math achievement tasks (Calculation and Math Fluency from the Woodcock Johnson III Tests of Achievement; Woodcock, McGrew, & Mather, 2001). Goffin and Ansari found a numerical distance effect in the cardinality task, and a reverse distance effect in the ordinality task. These distance effects were not statistically associated with each other, and predicted unique variance in math scores, even when controlling for WM and inhibitory control scores in a regression model. This implies that there may be distinct mechanisms underlying the cardinal and ordinal processing of number, suggesting that ordinality knowledge should be accounted for by the MNL metaphor (see also Gray & Reeve, 2016, who reported that knowledge of ordinality is not necessarily associated with knowledge of cardinality).

In terms of number symbols, Merkley and Ansari (2016) propose that knowledge of numerical symbols is critical to learning math, as this skill is often linked to math ability. Indeed, several studies support this claim. Göbel, Watson, Lervåg, and Hulme (2014) found that performance on a symbolic number identification task better predicted the arithmetic ability (measured using the Numerical Operations subtest of the Wechsler Individual Achievement Test-Second UK Edition; Wechsler, 2005) of six-year-old students over an 11-month testing period than performance on a non-symbolic number comparison task. A positive relationship has also been found between symbolic magnitude processing acuity and children's ability to learn arithmetic facts (Vanbinst, Ghesquière, & De Smedt, 2015), showing the importance of number symbol knowledge. Knowledge of number symbols has also been found to be a better predictor of non-symbolic number comparison ability than the opposite relationship between non-symbolic number comparison and number symbol knowledge, implying that symbolic number knowledge supports non-symbolic number discrimination (Mussolin, Nys, Content, & Leybaert, 2014; see also Sarnecka, 2015 for a review of exact number representation). These findings suggest that learning the features of number symbols (including identification, cardinality, and ordinality), should be promoted early in childhood, because this may help children learn complex math more successfully.

The role of number symbols in learning has also been examined in different cultures. Zhou et al. (2006) tested a group of Chinese four and five-year-old children at two different time points in a year, where children's representation and knowledge of written number symbols were tested across a range of tasks. The authors reported that the majority of Chinese children could represent written number symbols by the age of five, and that the ability to represent number symbols was also positively correlated with their knowledge of cardinality. This suggests that number symbol knowledge helps improve understanding of more complex math concepts, such as addition and subtraction. Overall, this research suggests that while knowledge of non-symbolic number may be important in giving number symbols meaning, number symbol knowledge should also be considered in this process. These studies also indicate that symbolic number knowledge may predict math skills better than magnitude estimation ability, suggesting that the MNL metaphor should consider the role of ordinal relationships and number symbols, rather than focusing on cardinality.

2.6.3. Neural basis of number symbols and ordinality

A large amount of research on the role of number symbols and ordinal relationships between numbers has focused on the brain regions involved in these processes. Recently, brain imaging studies have identified the IPS as the likely neural basis of symbolic number (e.g., Ansari, Fugelsang, Dhital, & Venkatraman, 2006; Kaufmann et al., 2005; 2006; Pinel, Dehaene, Riviere, & Le Bihan, 2001; Pinel et al., 1999). This link between the IPS and symbolic number is observed controlling for the possible confound of response selection (it has been found that the IPS is activated in response selection even when the stimuli are not numeric; Göbel, Johansen-Berg, Behrens, & Rushworth, 2004) using fMRI adaptation (Reynvoet, Notebaert, & Nelis, 2010). It has also been found that similar areas of the IPS are activated for symbolic and non-symbolic number representations, and that the right IPS may link symbolic number representations with non-symbolic number representations, suggesting a close connection between these two forms of number representation (Holloway & Ansari, 2010; Holloway, Price, & Ansari, 2010). Research has also tested if showing one format leads to activation of another (i.e., does showing a non-symbolic representation lead to the activation of the corresponding symbolic number), and found that both modalities activate similar networks, implying that symbolic and non-symbolic representations may be mapped onto the same area in the brain (Piazza, Pinel, Le Bihan, & Dehaene, 2007).

Lyons and Ansari (2009) conducted a fMRI training study, and found the left parietal cortex may play an important role in mapping novel symbols with non-symbolic numerical magnitudes, providing support for the symbolic mapping hypothesis. However, research comparing the similarity between non-symbolic and symbolic (Hindu-Arabic numerals) often produces different results. For example, Cohen Kadosh, Bahrami, Walsh, Butterworth, Popescu, and Price (2011) found no evidence of activation across number modalities in the IPS using fMRI, and other research has shown that symbolic numbers may not be associated with a non-symbolic numbers (Bulthé, De Smedt, & De Beeck, 2014). Ansari (2016) notes that these findings imply that while small symbolic numbers (~1 to 3) in the subitizing range (the rapid and accurate judgment of small numbers without counting; Trick & Pylyshyn, 1994) may be closely linked to non-symbolic quantities, larger numbers may not be related to non-symbolic quantities in a comparable way. Taken together, this research suggests the link between symbolic and non-symbolic number in the brain is not always straightforward, casting some doubt on the mapping account of number symbols (see section 2.6.1).

The involvement of the IPS in processing number symbols has also been considered developmentally. This is because while humans can estimate non-symbolic magnitudes from birth, we are not born with specific brain regions to process culturally specific number symbols. Indeed, humans have only been using number symbols for approximately the past 5,000 years, not enough evolutionary time to develop a specific brain region for number symbols (Ansari, 2016; Dehaene & Cohen, 2007). Therefore, there must be changes in the brain and the re-wiring of existing brain regions to support symbolic number processing. Research has reported that the activation of the left IPS in symbolic number tasks increases with age, and that the prefrontal brain regions are more active in children than adults (Ansari, Garcia, Lucas, Hamon, & Dhital, 2005; Kaufman et al., 2005), implying the parietal cortex becomes dedicated to support the processing of numerical symbols during development. This suggests that it is important to consider the role of number symbols in number learning.

The differences and similarities between ordinal and cardinal number representations has also been examined using neuroimaging methods. Lyons and Beilock (2013) investigated the neural correlates of cardinality and ordinality for symbolic and non-symbolic number to test whether symbolic and non-symbolic number are actually closely connected to each other. Participants decided which of two numbers was larger (cardinal judgment), and also if a series of numbers was in increasing, decreasing, or random order (ordinal judgment). Two formats were used for both questions, Hindu-Arabic numerals and non-symbolic dot arrays. Lyons and Beilock found similar brain regions were involved in cardinality for both symbolic and non-symbolic stimuli, but different brain regions were involved for the processing of ordinality for symbolic and non-symbolic stimuli. The finding that there was no overlap for symbolic and non-symbolic stimuli for ordinal processing suggests that accounting for ordinality is critical for understanding how number is represented along the MNL.

Jacob and Nieder (2008) also noted the likelihood of a link between cardinal and ordinal number processing in the IPS because research that has reported a close overlap between activations for number and letter stimuli measuring neural activity using fMRI. For example, Fias, Lammertyn, Caessens, and Orban (2007) asked neurologically healthy adults to compare which of two letters came later in the alphabet, and blood oxygenation level-dependent (BOLD) activations were compared with BOLD levels when determining which of two numbers was larger. These results showed that comparisons of letters and numbers are associated with similar regions clustered around the IPS. Furthermore, Zorzi, Di Bono and Fias (2011) used data from Fias et al., which found overlapping regions for number and letter comparison tasks. However, unlike Fias et al., Zorzi et al. were able to distinguish between number and letter trials in the IPS, implying that number and letter stimuli can be separated depending on the analysis used. Overall, it seems to learn numerical symbols, new brain representations must be created in the course of learning number, and the IPS is the most likely basis of this process. This suggests that ordinality is important to successfully learning numerical symbols, and that ordinality should be considered when studying the MNL.

2.6.4. Individual differences in number learning

It may be possible to gain further insight into number learning by observing situations where learning has not properly occurred in children and adults. Research has focused mainly on the number learning disability known as dyscalculia (see section 2.4.2.). Dyscalculia is a specific learning disability in math, and was originally defined by Kosc (1974) as a disability in learning math as a result of damage to areas of the brain involved in number cognition, but with no loss of general cognitive function. Butterworth (2005) notes that while multiple definitions of dyscalculia exist, it is generally agreed upon that dyscalculia entails a difficulty in math which is specific to number, and this disorder is caused by neurological dysfunction. In terms of the common characteristics of dyscalculia, dyscalculic children have difficulty learning arithmetic facts (e.g., Geary & Hoard, 2001), and performing simple calculations (e.g., Landerl, Bevan, & Butterworth, 2004 found that dyscalculic children were less accurate in single digit subtraction and multiplication than control children, and slower on addition, subtraction, and multiplication tasks). Children diagnosed with dyscalculia rely on what Butterworth (2005, p. 459) terms “immature strategies”, such as counting on fingers to solve simple arithmetic problems (e.g., Butterworth, 1999).

Dyscalculic children also have difficulty in retrieving arithmetic facts and procedures (Russell & Ginsburg, 1984). Children with dyscalculia struggle with tasks that require a knowledge of simple number concepts such as cardinality, which can hinder performance in numerical comparison or counting tasks (Butterworth, 2005). Research has also found that instead of being able to automatically ‘see’ the number represented by groups of dots under four without counting (i.e., subitizing), children with dyscalculia often count up to 3 instead (Koontz,1996). Also, an extreme case detailed by Butterworth (1999) illustrates how reaction time can vary between typically developing children and dyscalculic children for tests of dot counting and magnitude comparison. Here, a child with dyscalculia showed a reverse distance effect, where the child took longer to decide that 9 was larger than 2, than they did to decide 9 was larger than 8. This suggests that the dyscalculic child may have used an inefficient counting strategy to count from 2 to 9 then counted again from 8 to 9. Studying the characteristics of dyscalculia provides an example of when accurate number learning does not occur, and suggests there may be individual differences in MNL representations.

Regarding the neural basis of dyscalculia, children with dyscalculia tend to have lower activation of the IPS when performing a variety of math tasks (e.g., Mussolin, De Volder, Grandin, Schlögel, Nassogne, & Noël, 2010 reported low activation of the IPS on a symbolic number comparison task). Children with dyscalculia have also been observed to have lower gray matter levels in various areas of the IPS (e.g., Isaacs, Edmonds, Lucas, & Gadian, 2001 found lower activation in the left IPS). Indeed, the finding that differences in brain regions can impact number processing so severely suggests that math processing ability falls on a spectrum, with dyscalculia being on one end of that spectrum. It seems reasonable to assume that there may be variation in the way children learn and represent numbers, which imply it is important to look at individual variation when studying the MNL.

2.6.5. Summary

Overall, it seems an accurate knowledge of numerical symbols, combined with an understanding of the ordinal relationships between numbers, is critical to learning and representing number. Neuroimaging research has found the IPS to be a brain region critical for learning number symbols and ordinality. Considering these two factors of number learning may help clarify the MNL metaphor. It is possible that the MNL may consist of ordered relations between pairs of numbers, and that number symbols themselves may help to shape underlying number representations. Studies of number learning disabilities also provide a case study of what an inability to learn math looks like, stressing the importance of studying individual differences when examining the MNL. Taken together, this evidence suggests considering how number is learned may be the ideal way to study the features of the MNL.

2.7. Summary and research agenda

To conclude, the main goal of this thesis is to test some of the assumptions that have been made in the literature regarding the form of the MNL, and to test whether the MNL metaphor should be extended to include ordinal relationships between numbers. While the literature reviewed above often invokes the MNL metaphor to explain experimental findings, some of these inferences may be overstated. Another aim of the current thesis is to critically examine the NP task, because the NP task may reflect ordinal lists in general, rather than the MNL. The last aim of this thesis is to examine how ordinal representations are initially created in adults with the use of novel symbols, because there could be underlying spatial information contained in the symbols that mediates how ordinal representations are learned (which could extend to number representations). Determining the role of ordinal relationships in number representations may help clarify the MNL metaphor.

While the MNL is a popular metaphor which is assumed to explain how humans and non-human animals represent number, Chapter 2 suggested that the MNL metaphor is poorly defined, and that little is known about the underlying properties of the MNL representation. It was also argued in Chapter 2 that while the representational shift hypothesis of number initially appears to be a robust account of magnitude estimation development, there are discrepancies with these research findings (e.g., a logarithmic pattern of responding is not always found in the NP task) and issues with the NP task (e.g., the unbounded NP task produces different patterns of response), meaning that inferences made regarding the format of the MNL representation from this research may be exaggerated.

It was also argued in Chapter 2 that while training studies for both children and adults have started to address the issue of how learning influences the MNL, there are several issues with these studies, meaning that inferences should not yet be drawn about the flexibility of the MNL, which is important to clarifying the MNL metaphor. For example, these studies have not yet determined whether training is widespread, whether training is long lasting, or whether the integration of training varies between individuals. These issues are all important to address to get a better understanding of the MNL. If the MNL cannot be altered with feedback, then this may mean a more nuanced account of the MNL metaphor is required.

Moreover, the majority of research on the MNL has not considered the role of how number is learned, and how this may influence the MNL. It could be the case that underlying spatial properties of number symbols influence how ordinal representations are formed. Indeed, research is starting to suggest that the ordered relationships between numbers may influence number representations. This means estimation patterns on the NP task may be due to the ordinal structure of numbers rather than a reflection of the MNL specifically. It also suggests that the MNL metaphor should be expanded to include relational information (especially ordinality). Including ordinality is important to facilitate our understanding of the MNL metaphor because it is likely the case that the MNL is composed of sets of ordered relationships, and these relationships are what underlies the MNL. Chapter 2 proposed that adopting learning approach to study the MNL will help clarify the MNL metaphor, and that grounding the MNL in ordinality will allow the MNL metaphor to provide more of a conceptual contribution to the numerical cognition literature.

The specific research agenda that results from the above arguments is designed to achieve three main objectives via two converging lines of training studies. Firstly, to test if the MNL can change to accommodate task demands as claimed by literature training the MNL. Secondly, to examine if non-numerical ordered lists (e.g., the letters from the alphabet) can elicit an analogous linear pattern in adults on an alphabet-to-position task. Thirdly, to unpack the MNL metaphor, and to examine how ordinal representations may be initially constructed more generally, the last aim of this research is to examine if learning a list of novel symbols can be facilitated by ordering symbols by their underlying spatial properties. These studies aim to answer the question of whether the MNL metaphor is going too far beyond the information given, and whether this metaphor applies to ordinal lists in general.

Chapter 3
Pilot study

Chapter 3 (pilot study) details a study that investigated if the adult MNL could be changed with logarithmic training. Previous studies have suggested that a widespread transition from logarithmic-to-linear representations of the MNL leads to an improvement in children's ability to accurately estimate numerical magnitudes (Booth & Siegler, 2006; Booth & Siegler, 2008; Laski & Siegler, 2007; Opfer & Siegler, 2007; Siegler & Booth, 2004; Siegler & Opfer, 2003; Siegler et al., 2009; Thompson & Opfer, 2010). Furthermore, research has also found that feedback can lead to logarithmic number representations in adults (Chesney & Matthews, 2013; Huber et al., 2014; Matthews & Chesney, 2011), suggesting that the MNL can be altered with training. The inherent assumption is that the NP task directly reflects the underlying MNL. If this is the case, then an extended logarithmic training program should theoretically lead to change in NP task performance and therefore the MNL. While this assumption is often made in the literature (e.g., Siegler & Opfer, 2003), a training study has not been conducted to determine several empirical issues, such as whether the MNL is actually adaptable, and if it is, whether change is long lasting and the degree of individual differences in the integration and learning of feedback.

In this pilot study, participants were trained over a time period of either one, two, or four weeks, then were tested approximately one week after training concluded to determine whether training could produce long lasting change. This study also employed a small sample design, so individual profiles could be examined. This ensured any unique response patterns between and within participants could be identified. In sum, this pilot study is designed to test an assumption in the numerical cognition literature, specifically, that the MNL is a flexible representation that can be altered with a training program.

3.1. Aims of Chapter 3

The aim of the pilot study was to address some issues raised in Chapter 2, primarily, whether the MNL is a flexible representation that can be altered with training, which has been suggested by previous research. There are two possible outcomes for the pilot study. Firstly, participants could show a substantial effect of training on NP estimation (Figure 1 shows a logarithmic training effect with hypothetical data). This result would suggest that the MNL is a flexible structure that can change to meet task demands, and would provide support for the idea that the development of magnitude estimation is due to a shift of the MNL (Siegler &

Opfer, 2003; Siegler & Booth, 2004). This inference could be made if participants showed an influence of logarithmic training across the range of number tasks. Secondly, participants could only show a minimal influence of training on NP estimation. This result would instead suggest that the MNL is not an adaptable structure, and would suggest a more nuanced account of the MNL is required. This interpretation could be made if adults successfully learned the logarithmic function, but failed to show a transfer effect to the NP task.

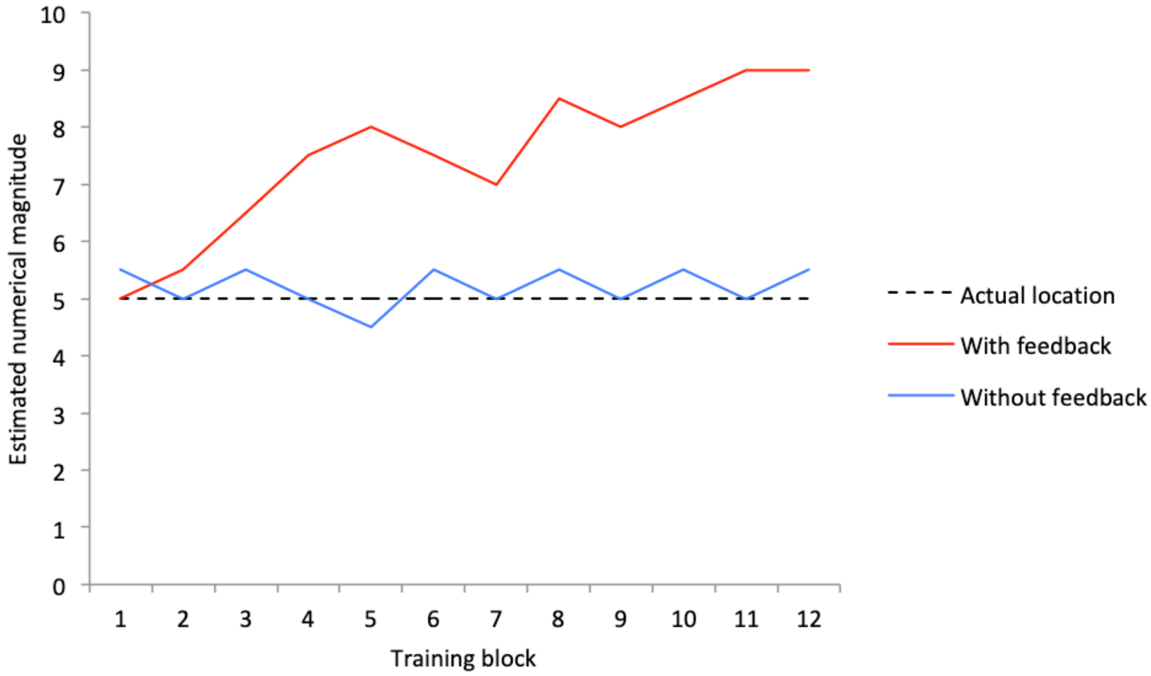

Figure 1. Hypothetical data for a logarithmic training effect on the number five. Plotted as a function of training block (x-axis) by estimated number magnitude (y-axis). The solid blue line corresponds to estimates with no training, the red line corresponds to estimates with logarithmic training, and the dashed line represents linear performance.

3.2. Method

3.2.1. Participants

A total of six participants (three male, three female) participated in this study, including the author (CP), and five volunteers who were naïve to the specific aims of this research (AT, FN, JP, LP, and SL). Participants ranged in age from 23 to 37 years (M = 27.17, SD = 4.99), and all had normal or corrected to normal vision. Written and informed consent was obtained by asking participants to read a plain language statement and then sign a consent form. All procedures involved were approved by, and in accordance with, the University of Melbourne Human Ethics Advisory Group (HREC number 1441499).

3.2.2. Apparatus

Stimuli were created on an Apple Power Mac computer running OS X 10.6 with MATLAB software and Psychophysics Toolbox routines (Brainard, 1997; Kleiner, Brainard, & Pelli, 2007; Pelli, 1997), shown on a 23-inch Apple Cinema Display operating at a spatial resolution of 1,920 by 1,200 pixels at a refresh rate of 60Hz.

3.2.3. Stimuli

The logarithmic training paradigm was based on a 1 to 100 NP task (Siegler & Opfer, 2003). Existing studies attempting to train the MNL in adults have also used a NP task, and have successfully used feedback. For example, Huber et al. (2014) provided feedback by placing a mark on the NP where the number 'should' have gone according to the numerical scale participants were being trained on (in Huber et al., linear, exponential, logarithmic, sigmoid, and inverse sigmoid algebraic functions were used), and Opfer and DeVries (2006) also provided logarithmic feedback by placing a mark on the NP task.

For this study, electronic logarithmic feedback arrows were used (1.5 degree long, 0.15 degree wide, arrowhead 1 degree long with an angle of 45 degrees at a viewing distance of 60cm, where 1 degree = 1cm). These arrows were graded, and designed to indicate how far participants' responses were from the 'correct' logarithmic answer. Arrows appeared after every trial (i.e., even if participants were correct, a green right arrow appeared to encourage overestimation). Participants could receive one of six feedback arrows each trial: (1) green left or right arrow (responses should be slightly further to the left or right, respectively); (2) yellow left or right arrow (responses should be further to the left or right, respectively); and (3) red left or right arrow (responses should be much further to the left or right, respectively). A green arrow appeared when a participant responded within a ratio of 0 and 1.5 of the correct logarithmic answer; a yellow arrow appeared when a participant responded within a ratio of 1.5 and 3 of the correct logarithmic answer; and a red arrow appeared when a participant responded beyond a ratio of 3 of the correct logarithmic answer.

Participants were allocated to a 'low bias' or a 'no bias' feedback condition. The 'low bias' condition aimed to train participants more on the lower end of the NP task (i.e., from 2 to 50), and contained 30 target numbers: 2, 2, 3, 3, 3, 4, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 17, 20, 22, 26, 29, 33, 38, 44, 50, 57, 66, 75, 86, and 98. The 'no bias' condition trained participants on the whole NP task (i.e., from 2 to 98), and contained 30 target numbers: 2, 5, 9, 12, 15, 19, 22, 25, 28, 32, 35, 38, 42, 45, 48, 52, 55, 58, 62, 65, 68, 72, 75, 78, 81, 85, 88, 91, 95, and 98. Participants were naïve regarding which condition they had been allocated to.

3.2.4. Procedure

All testing was conducted in a quiet office space with tinted windows to control room illumination, and the screen was oriented to lessen reflection from windows. Participants were seated in front of the computer monitor on a chair with arm rests. Participants were told that they would be asked to complete a series of basic tasks with numbers, and that they would be able to take frequent rest breaks to prevent fatigue if required.

Before training, participants completed a 1 to 100 NP task without feedback to get a baseline measure of NP task performance. Participants were shown a NP with a marker, where the marker had a randomly generated starting point along the line on each trial. Participants moved the marker with a mouse to indicate where they believed a target number went on the NP, and click when they were satisfied with their response to proceed to the next trial. This task appeared in either a 1 to 100 or a reversed 100 to 1 orientation. Participants completed 132 trials in total for the NP task.

The training phase involved participants completing a total of 600 trials of a modified NP task with logarithmic feedback for eight sessions, making for a total of 4,800 trials. Participants were shown a modified NP task from 1 to 100, with a range of target numbers to position on the line. The marker started on a random location on the line, and participants moved the marker with the mouse and clicked to indicate where they believed the target number was on the line. The stimuli remained on screen until participants responded. After the click, a feedback arrow was provided, which stayed on screen for 500ms.

Three participants were allocated to the 'low bias' condition, where the target numbers presented sampled the low end of the NP task more (i.e., 2 to 50). The other three participants were allocated to the 'no bias' condition, where the target numbers presented sampled evenly across the entire NP task (i.e., 2 to 98). Participants were instructed to pay attention to the feedback arrows provided, and adjust their future responses accordingly, to minimise the difference between their responses and the feedback given. Participants were advised that feedback may not necessarily correspond to what they would normally expect for a NP task. After 300 trials, participants were given the option to take a rest break.

Participants were placed into one of three time groups for completing this training program, subject to their availability. The first group had one week of training, where participants completed two training sessions, four days a week. The second group had two weeks of training, where participants completed one training session, four days a week. The third group had four weeks of training, where participants completed one training session, two days a week. Two participants were allocated to each group, with one in each group completing the ‘low bias’ training, and the other completing the ‘no bias’ training. After each training session, participants completed the 1 to 100 NP task. The NP task was given after every session to measure any changes that occurred over the course of one training session, rather than only measuring after training had ended.

After completing the final training session, participants also completed a final 1 to 100 NP task without feedback. This task was the critical test to examine whether logarithmic training on the NP task had an over performance on a standard NP task. Participants were shown a NP with a marker, where the marker had a randomly generated starting point along the line on each trial. Participants moved the marker with a mouse to indicate where they believed a target number went on the NP, and click when they were satisfied with their response to proceed to the next trial. This task randomly appeared in either a 1 to 100 or a reversed 100 to 1 orientation. Participants completed 132 trials in total for the NP task.

3.3. Results and discussion

Deviations from accurate linear NP performance were plotted for each participant for each training session as a function of actual number magnitude against deviation from actual number magnitude. From these plots, performance seemed remarkably consistent between the ten training sessions within participants. Since there appeared to be no large differences between the training sessions for any one participant, it was decided to pool the training sessions, and plot the mean linear deviations for each participant. Doing this uncovered that several participants had ‘extreme’ values, which occurred early in the task, and were likely to have been response coding errors (e.g., participants may have not realised the direction of the NP task had switched from 1 to 100 to 100 to 1). As such, to ensure that these values did not influence the means of each individual estimation point, the z-scores were calculated for each target number, and outliers that were beyond 2.5 standard deviations from the mean for each number were removed and plotted separately.

Figures 2 to 7 below display the mean deviations from linear performance for all participants. Blue data points denote the 1 to 100 NP task, and red data points denote the 100 to 1 NP task for the 33 target numbers as a function of actual number magnitude (x-axis) by the estimated number magnitude deviation (y-axis). Error bars represent 2 standard errors around the mean deviation of each number location. The dotted black line represents linear performance on the NP task. Outliers were defined as 2.5 standard deviations from the mean, and are denoted with asterisks. ‘Extreme’ outliers beyond 40 points from the true number location are denoted by solid circles on the boundaries of the plot. Together, these data suggest that logarithmic training exerted a minimal influence over NP task performance, and that the effect of training varied between participants.

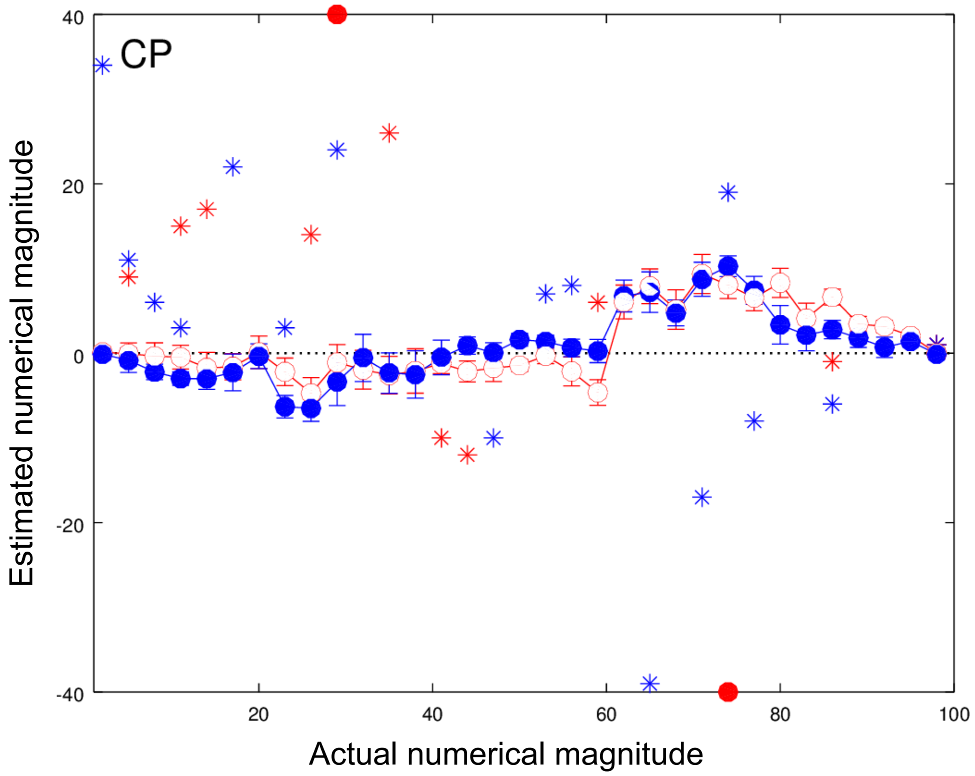

Figure 2. Mean deviations from linear performance for the 1 to 100 NP task (filled blue circle) and 100 to 1 NP task (open red circle) for participant CP in the no bias training condition. The circles represent mean number estimations (excluding outliers) of the 33 target numbers presented in the NP task, plotted as a function of actual number magnitude (x-axis) by estimated number magnitude deviation (y-axis). Error bars represent 2 standard errors around the mean deviation of each number location. The dotted black line represents linear performance. Outliers were defined as 2.5 standard deviations from the mean, and are denoted with asterisks. ‘Extreme’ outliers beyond 40 points from the true number location are denoted by solid circles on the boundaries.

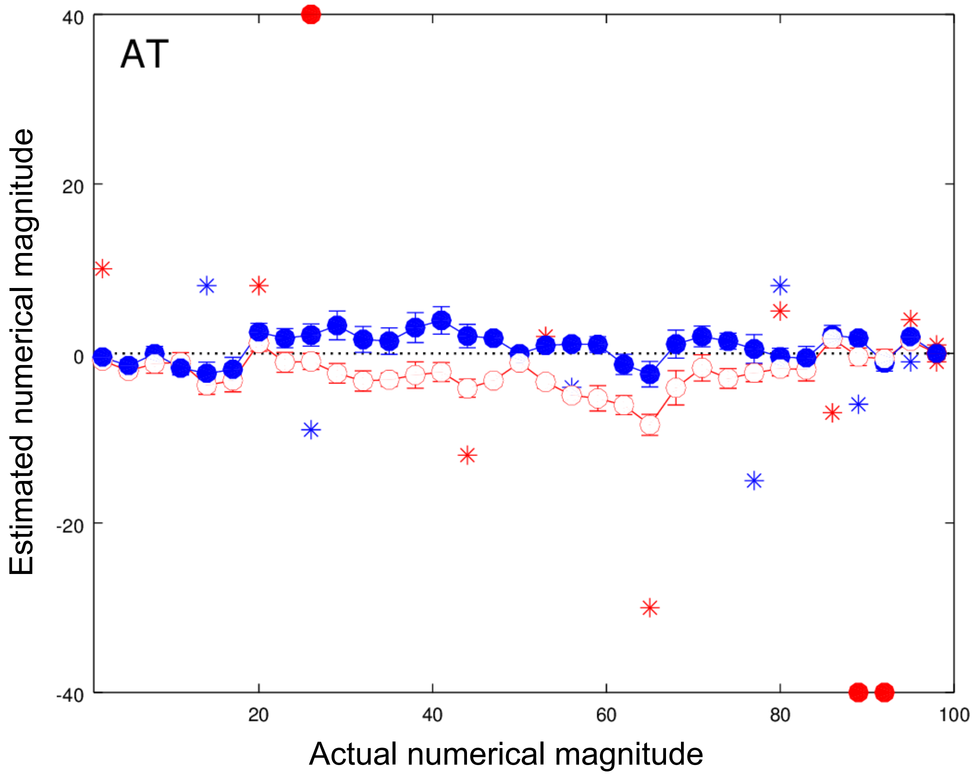

Figure 3. Mean deviations from linear performance for the 1 to 100 NP task (filled blue circle) and 100 to 1 NP task (open red circle) for participant AT in the no bias training condition. The circles represent mean number estimations (excluding outliers) of the 33 target numbers presented in the NP task, plotted as a function of actual number magnitude (x-axis) by estimated number magnitude deviation (y-axis). Error bars represent 2 standard errors around the mean deviation of each number location. The dotted black line represents linear performance. Outliers were defined as 2.5 standard deviations from the mean, and are denoted with asterisks. 'Extreme' outliers beyond 40 points from the true number location are denoted by solid circles on the boundaries.

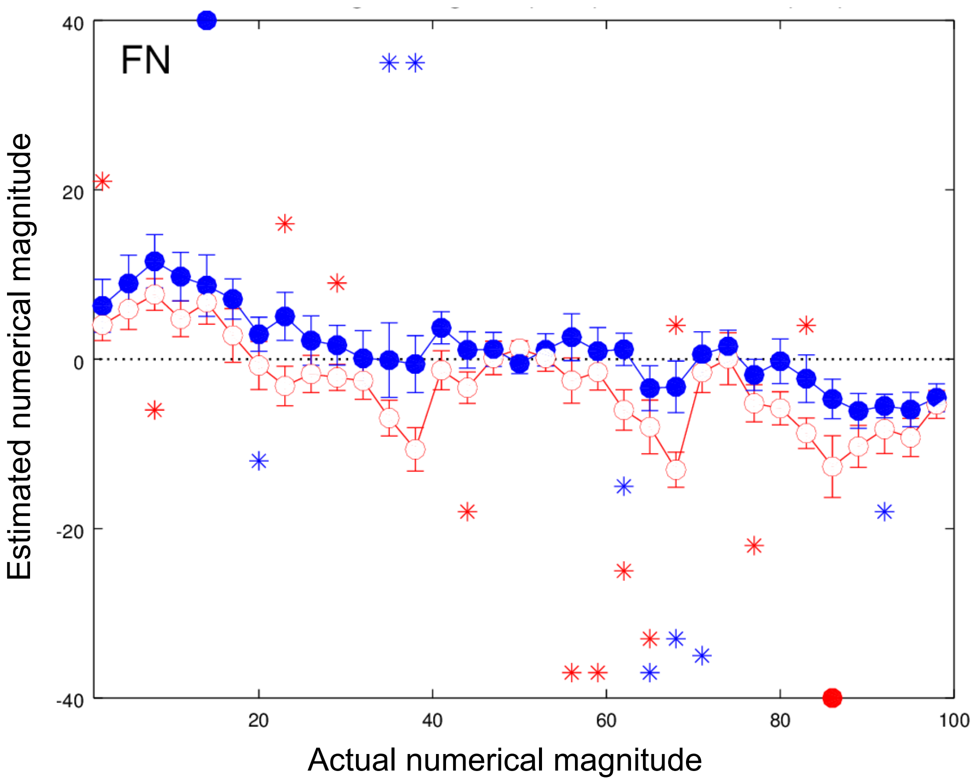

Figure 4. Mean deviations from linear performance for the 1 to 100 NP task (filled blue circle) and 100 to 1 NP task (open red circle) for participant FN in the no bias training condition. The circles represent mean number estimations (excluding outliers) of the 33 target numbers presented in the NP task, plotted as a function of actual number magnitude (x-axis) by estimated number magnitude deviation (y-axis). Error bars represent 2 standard errors around the mean deviation of each number location. The dotted black line represents linear performance. Outliers were defined as 2.5 standard deviations from the mean, and are denoted with asterisks. 'Extreme' outliers beyond 40 points from the true number location are denoted by solid circles on the boundaries.

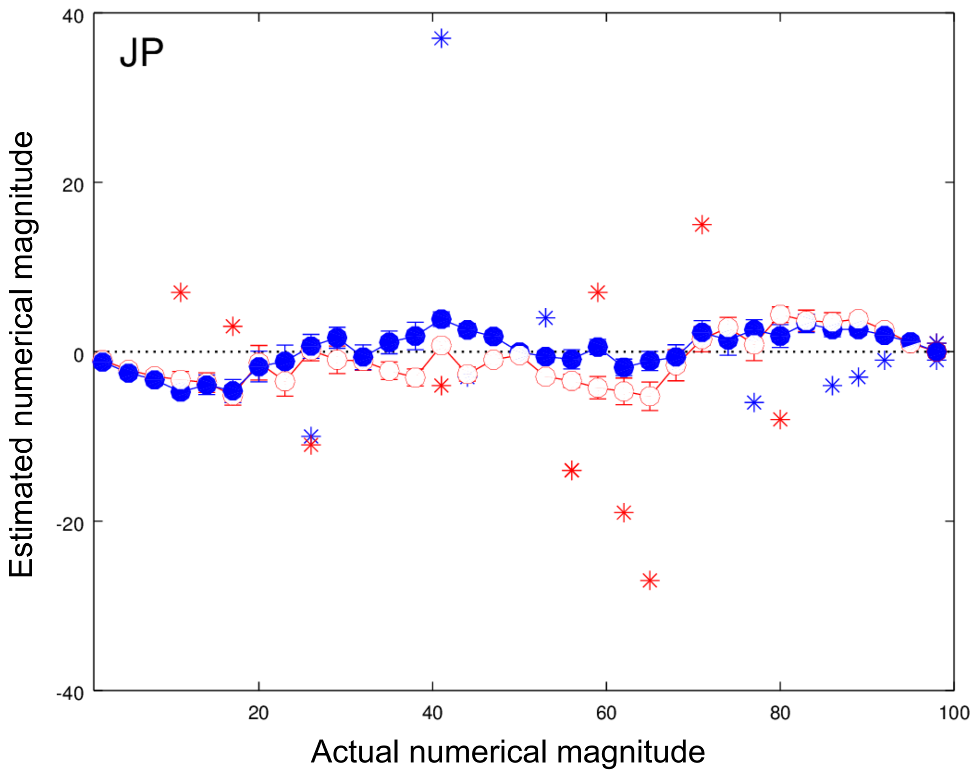

Figure 5. Mean deviations from linear performance for the 1 to 100 NP task (filled blue circle) and 100 to 1 NP task (open red circle) for participant JP in the low bias training condition. The circles represent mean number estimations (excluding outliers) of the 33 target numbers presented in the NP task, plotted as a function of actual number magnitude (x-axis) by estimated number magnitude deviation (y-axis). Error bars represent 2 standard errors around the mean deviation of each number location. The dotted black line represents linear performance. Outliers were defined as 2.5 standard deviations from the mean, and are denoted with asterisks. 'Extreme' outliers beyond 40 points from the true number location are denoted by solid circles on the boundaries.

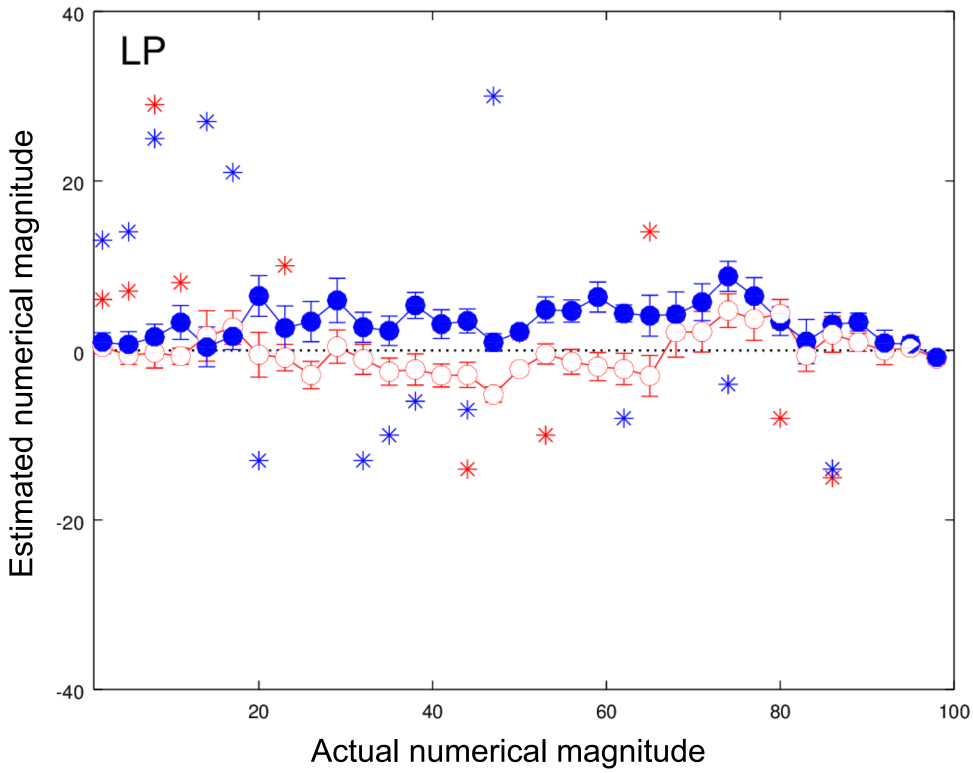

Figure 6. Mean deviations from linear performance for the 1 to 100 NP task (filled blue circle) and 100 to 1 NP task (open red circle) for participant LP in the low bias training condition. The circles represent mean number estimations (excluding outliers) of the 33 target numbers presented in the NP task, plotted as a function of actual number magnitude (x-axis) by estimated number magnitude deviation (y-axis). Error bars represent 2 standard errors around the mean deviation of each number location. The dotted black line represents linear performance. Outliers were defined as 2.5 standard deviations from the mean, and are denoted with asterisks. 'Extreme' outliers beyond 40 points from the true number location are denoted by solid circles on the boundaries.

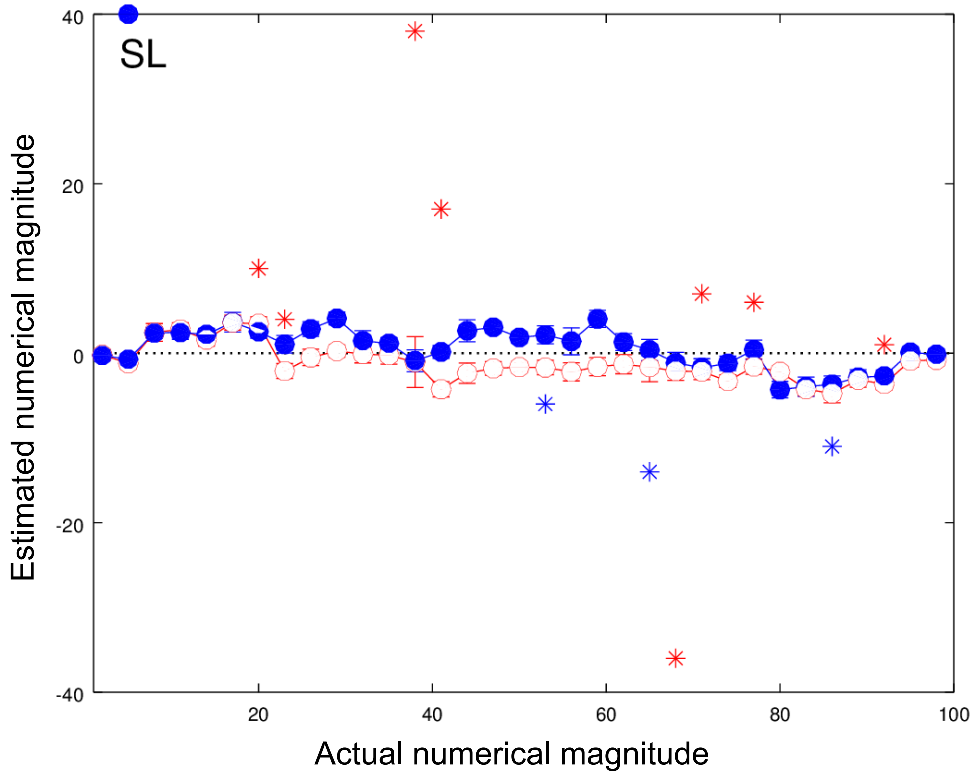

Figure 7. Mean deviations from linear performance for the 1 to 100 NP task (filled blue circle) and 100 to 1 NP task (open red circle) for participant SL in the low bias training condition. The circles represent mean number estimations (excluding outliers) of the 33 target numbers presented in the NP task, plotted as a function of actual number magnitude (x-axis) by estimated number magnitude deviation (y-axis). Error bars represent 2 standard errors around the mean deviation of each number location. The dotted black line represents linear performance. Outliers were defined as 2.5 standard deviations from the mean, and are denoted with asterisks. 'Extreme' outliers beyond 40 points from the true number location are denoted by solid circles on the boundaries.

From these graphs, it can be seen that there were no differences between the time groups for the study (i.e., one, two, or four weeks). For example, participants CP and JP completed the one-week session, and did not show more logarithmic responses than those in the two-week condition (LP and AT), or the four-week condition (FN and SL). If training had an influence over NP responses, then it would have been expected that an increased effect would be found for the one-week condition, since these participants completed a more intensive training program. Also, there was no difference between the 'low bias' and 'no bias' logarithmic feedback conditions. For example, participants JP, LP, and SL completed the low bias condition, and did not show more bias towards overestimating the lower end of the NP than the participants who completed the no bias condition (AT, CP, and FN).

It can also be seen that there were both systematic and unique individual variations present in the NP patterns. Some participants seemed to be ‘attracted’ to or ‘repulsed’ by the endpoints of the NP task; that is, placing estimates on the extreme ends of the line, or avoiding placing estimates on the extreme ends of the line, respectively. For example, participant JP showed a slight attraction to the endpoints by underestimating the target numbers at the beginning of the line (1 to 20), and overestimating numbers and the end of the line (80 to 100). Participants FN and SL both demonstrated a slight repulsion from the endpoints, by overestimating the beginning of the line (1 to 20), and underestimating the end of the line (80 to 100). Participants CP, AT, and LP seemed to be neither attracted nor repulsed by the endpoints of the number line.

Participants also varied in whether they estimated according to the spatial location or the numerical magnitude or of the numbers. Specifically, if participants estimated number according to spatial location, there should be an opposite pattern of responses between the 1 to 100 and 100 to 1 orientations of the NP task; and if participants estimated according to numerical magnitude, there should be a similar pattern of responses between the 1 to 100 and 100 to 1 orientations of the NP task. For example, participant AT estimated according to both the spatial location (for numbers 20 to 40) and the numerical magnitude (for numbers 60 to 80) of the target numbers. Participants CP, FN, and LP seemed to estimate according to the numerical magnitude of the target numbers, since estimates were all made in a similar direction irrespective of NP direction. Participants JP and SL made estimates based on numerical magnitude, but seemed to base estimates on spatial location around the midpoint of the NP (from 30 to 40 and from 60 to 70).

Additionally, participants differed in how they estimated the target numbers based on the orientation of the task. For example, participants AT, FN, and LP all showed a slight trend towards overestimation in the 1 to 100 orientation and a slight trend towards underestimation in the 100 to 1 orientation. However, participants CP, JP, and SL all showed less variability depending on the orientation of the task. All participants had relatively small error bars across the ten training sessions, meaning individuals had high stability in their estimates. This shows that the errors found in the above graphs are systematic deviations from linearity for each participant. If training had caused participants to adopt a logarithmic manner of responding, large error bars would have been expected for all target numbers tested. Overall, it can be inferred from these data that the deviations from linearity were likely not due to training, as they remained consistent over the ten training assessment sessions.

Performance on the NP task showed little evidence that training causes a shift in the MNL. Most of the participants did not exhibit an influence of logarithmic training on NP estimation, with NP estimates staying relatively linear across the ten testing periods. The effect also did not transfer for either orientation of the task (i.e., 1 to 100 and 100 to 1). The time period of logarithmic training implemented also did not have an effect (i.e., one, two, or four weeks), and neither did the high or no bias training conditions (i.e., oversampling the upper end or evenly sampling the NP task). While some participants showed a trend to overestimate in the NP, this effect was small, and varied between participants.

Large individual differences were observed in the effect of logarithmic training on NP task performance, but this seemed to be due to local spatial effects. For example, some participants seemed to overestimate or underestimate the spatial location of the target numbers, while others overestimated or underestimated the numerical magnitude of the target numbers. Several participants also showed no bias towards overestimation (which would have been expected if logarithmic training was effective), even in the final assessment session. Finally, no participants transitioned from a linear response to a logarithmic response as predicted by the existing adult NP training literature.

The failure to find a strong effect of logarithmic NP training could be due to participants not sufficiently learning the logarithmic function. However, by examining the logarithmic training data from the NP task with feedback, it is clear that participants learned the logarithmic function very quickly, often within the first session of training. To quantify the degree of learning for the logarithmic training task, R squared values were calculated for the last training session on the logarithmic training task for each participant. The R squared values ranged between 0.87 and 0.99 (M = 0.95, SD = 0.05), showing that all participants successfully learned the logarithmic function on the training task. Therefore, it seems that the null effect of training found in this study is not due to the training technique employed. These results indicate that participants could separate the new logarithmic representation from their existing linear representation of number to perform the NP task, and suggest that participants may have treated the training as an unrelated mapping exercise. This puts into question the idea that the MNL is a flexible representation, or alternatively, may suggest that the NP task is resistant to feedback.

Nevertheless, the lack of a shift of the form of the MNL is perhaps not surprising, since participants were trained on a logarithmic function for only a short period of time each day before returning to the outside world, where a linear representation of numerical magnitude is constantly being reinforced and encouraged. These data cannot be reconciled with prior claims that a single trial of feedback is enough to modify the MNL (Opfer & Siegler, 2007), and suggest that learning a new algebraic function is not sufficient to change or shift the underlying representation of number. Previous studies training the adult MNL have used drastically fewer trials (e.g., the training study by Opfer & DeVries, 2006 used three sets of ten trials to train adults on a logarithmic function). By giving participants 4,800 feedback trials, strong enough feedback was provided to observe a training effect if one was present. However, it is clear in this study that NP task performance was not influenced by training. There are many possible explanations for what these results mean for the MNL. It could be that the NP task does reflect the MNL, but the MNL is not adaptable to training. This could be due to linear mapping being over-learnt, or a bias by the MNL against logarithmic scaling. Another explanation could be that performance on the NP task is not flexible, at least in regards to biasing participants to respond in a non-linear manner. This could be due to the fact that participants knew the feedback was incorrect, and did not deviate from linearity. If this is the case, then no strong conclusions can be made from this study regarding the nature of the MNL. This is because in this study, the NP task was not necessarily reflecting the MNL. Therefore, while the NP task may be a helpful tool to assess math ability, caution should be taken when using the NP task to directly infer properties of the MNL in adults.

3.4. Summary of Chapter 3

The pilot study tested whether adult participants could adopt a logarithmic manner of responding on a NP task with extended training, as previous literature has stated that adults can shift their representation of number in a single trial of feedback (e.g., Opfer & Siegler, 2007). The results of this study demonstrated that participants remained relatively linear before, during, and after training, suggesting either that the NP task measures the MNL and MNL cannot be altered with training; or that performance on the NP task cannot be altered with training, and that the NP task may not measure the MNL. There were large individual differences found in these data which could not be attributed to the training sessions. These local deviations from linearity remained consistent over time, and were either associated with numerical magnitude or spatial location, depending on the individual participant. Since the pilot study suggests that a more nuanced account of the MNL and the NP task is required, the following chapter presents Study 1, in which a spatial mapping task with numbers and letters of the alphabet were run to determine if the supposed linear nature of the MNL is an exclusive property of number, or if linear performance is a shared property of all ordinal lists in general.

Chapter 4

Study 1

Chapter 4 (Study 1) presents a study that explored whether numbers and letters show different responses on a symbol mapping task. Previous studies have argued that findings from the number-to-position task can be used to make direct inferences about the representation of number (Siegler & Opfer, 2003). However, there is growing evidence to suggest that NP task performance may reflect proportion judgement strategies (Barth & Paladino, 2011), or that the MNL may reflect ordered lists more generally, and may not be unique to number (Hurst et al., 2014). While some studies have suggested numbers and letters may produce similar response patterns on the NP task (Berteletti et al., 2012; Hurst et al., 2014), Study 1 aimed to clarify whether: (1) systematic deviations from linearity exist as a function of number or letter symbol set; (2) the dissociation of spatial position by requiring either left-to-right or right-to-left judgments has the same effects on spatial mapping for numbers and letters; and (3) response time patterns for numbers and letters are similar for judgments when the directions of space and symbol order are matched or mismatched.

Chapter 4 will also help clarify the results of the pilot study, which questioned the accuracy of the NP task (if the NP task is actually measuring the MNL, there should have been a greater effect of logarithmic training). This study will also help clarify the MNL metaphor, because it is currently unclear whether the MNL is specific to number, or whether it represents ordinal lists more generally. Overall, Study 1 uses a number-to-position task and an alphabet-to-position task to test if numbers and letters produce the same patterns of response on a spatial mapping task. If it is the case that numbers and letters produce the same response patterns on this task, then it would indicate that the NP task is not a unique measure of number, and that there may be a more general representation for ordinal lists.

4.1. Aims of Chapter 4

Study 1 aims to resolve an issue raised in Chapter 2, and address the issue of whether the NP task is an accurate measure of the MNL. This may help explain why the pilot study failed to measure any changes in participants' MNL representations when using the NP task. It may also provide information about the role of ordinality in number representations. There are two possible outcomes for Study 1. Firstly, it may be the case that numbers and letters produce similar response patterns. This would indicate that the NP task, and the MNL are not specific to number. Secondly, it may be found that numbers and letters produce different patterns of response. This would indicate that the NP task is a measure of number. The study presented in Chapter 4 (titled ‘The importance of ordinal information in interpreting number/letter line data’) is published in *Frontiers in Psychology* in the section Cognition as part of the research topic ‘Towards an Understanding of the Relationship between Spatial Processing Ability and Numerical and Mathematical Cognition’ (Podwysocki, Reeve, & Forte, 2019), and will be presented on the following pages in its published format.

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The Importance of Ordinal Information in Interpreting Number/Letter Line Data

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The degree to which the ability to mark the location of numbers on a number-to-position (NP) task reflects a mental number line (MNL) representation, or a representation that supports ordered lists more generally, is yet to be resolved. Some argue that findings from linear equation modeling, often used to characterize NP task judgments, support the MNL hypothesis. Others claim that NP task judgments reflect strategic processes; while others suggest the MNL proposition could be extended to include ordered list processing more generally. Insofar as the latter two claims are supported, it would suggest a more nuanced account of the MNL hypothesis is required. To investigate these claims, 84 participants completed a NP and an alphabet-to-position task in which they marked the position of numbers/letters on a horizontal line. Of interest was whether: (1) similar judgment deviations from linearity occurred for number/letter stimuli; (2) left-to-right or right-to-left lines similarly, affected number/letter judgments; and (3) response times (RTs) differed as a function of number/letter stimuli and/or reverse/standard lines. While RTs were slower marking letter stimuli compared to number stimuli, they did not differ in the standard compared to the reverse number/letter lines. Furthermore, similar patterns of non-linear RTs were found marking stimuli on the number/letter lines, suggesting that similar strategic processes were at play. These findings suggest that a general mental representation may underlie ordered list processing and that a linear mental representation is not a unique feature of number *per se*. This is consistent with the hypothesis that number is supported by a representation that lends itself to processing ordered sequences in general.

Keywords: ordered sequences, non-numerical order, ordinal information, number representation, number line, numerical estimation

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INTRODUCTION

Francis Galton (1880) was an early advocate of the position that space and number are related. More recently, it has been proposed that the brain represents numerical magnitude information on something akin to a mental number line (MNL; Dehaene, 2011). The MNL model suggests numbers are represented in ascending order from left-to-right in a "continuous, quantity-based, analogical format" (Zorzi et al., 2002, p. 138). Inferences about the nature of the MNL have been mostly based on data from a number-to-position (NP) task (see Siegler and Opfer, 2003), as well as studies on the spatial-numerical association of response codes (Dehaene et al., 1990, 1993), and spatial neglect (Zorzi et al., 2002).

In the NP task, participants mark the position of numerical values on a physical number line on which only the numerical end points are typically specified (e.g., '1' and '100'). Findings show a close alignment (i.e., a linear algebraic function) between the marked positions of numerical values and their actual positions in older children and adults, while younger children typically overestimate small numbers and underestimate large numbers, and whose performance is best characterized by a logarithmic algebraic function. These findings are often used to suggest that the logarithmic/linear function that best fits NP task performance also characterizes the underlying form of the MNL. However, this interpretation should be treated with caution for at least two reasons. Firstly, it ignores the possibility that other processing factors affect, or may be responsible for, NP judgments. Secondly, it assumes that the NP/MNL relationship is unique to number. Clarifying these two possibilities may provide a more nuanced account of the relationship between number and space.

Evidence, albeit developmental evidence, suggests that performance on the NP task may not be a direct measure of numerical representations, and may depend on other processing factors (Barth and Paladino, 2011). NP estimation patterns instead may reflect an increased use of landmarks (the strategy of dividing lines into halves or quarters to create reference points to guide estimates) associated with a proportion judgment model, rather than a change in number representations *per se*. Barth and Paladino (2011) showed that children's NP estimates were better fit by a proportion judgment model, rather than a "logarithmic-to-linear shift" account. This finding implies that performance on the NP task is not a direct measure of numerical representations because task strategies may influence NP task performance. While Barth and Paladino (2011) suggest NP task performance may be better described by a proportion power function rather than logarithmic/linear functions, they do not address whether NP task performance provides information about the representation of number *per se*.

It is clear that NP task manipulations may affect NP judgments. Cohen and Blanc-Goldhammer (2011; see also Cohen and Sarnecka, 2014), for example, used an "unbounded" NP task in which the number line represents a single unit distance from 0 to 1, and number estimations are made external to the bounds instead of within the bounds. On each trial, adult participants were presented with a target number (e.g., 12), and told to move the right boundary to estimate the target location. When participants completed a standard "bounded" NP task, estimation patterns closely followed a proportion judgment model, however, when they completed the unbounded NP task (which removes proportion judgment strategy opportunities), performance was best characterized by a linear function. This shows that task performance may reflect an increase in number line measurement skills, rather than a change in the underlying representation of number on the MNL (see also Huber et al., 2014). It also raises the issue of whether performance on a NP task reflects the way in which ordered lists, not just ordered number lists, are learned. Indeed, there is evidence that the pattern of mapping on the NP task may not be unique to number.

If number/letter line tasks, for example, produced the same mapping relationship to space, it would imply that linearity on the NP task may reflect a general mapping between spatial direction and list order. Several studies have investigated this possibility (Gevers et al., 2003, 2004; Berteletti et al., 2012; Hurst et al., 2014). For example, Hurst et al. (2014) gave children and adults an alphabet-to-position (AP) task, with the end points 'A' and 'Z'. Children showed a logarithmic-to-linear shift with letters, and adults' performance was characterized by a linear equation. While Hurst et al. (2014) were the first to show that numbers/letters spatial mapping abilities could be represented by linear/logarithmic functions, it is possible that performance on their task could be also represented by other algebraic functions (see comments on Barth and Paladino, 2011, above). Insofar as number/letter symbols display similar patterns on the spatial mapping task, it would argue against the claim that judgment patterns in the NP task are specific to number. Nevertheless, the issue of how best to compare similar/different number/letter judgment patterns, and ipso facto potential indices of different mental representations, remains an open question. In the present paper we address this issue by examining similarities/differences in non-linear deviations in NP and AP task performance.

In short, it is unclear whether NP judgment patterns are unique to number, or a general property of ordered lists that can be arranged spatially (e.g., the position of letters in an alphabet). To investigate this issue, we compared adults' responses on NP and AP tasks, the aim of which was to determine whether the hypothetical mental representation of these two types of ordered lists differed. The research was designed to answer whether: (1) similar judgment deviations from linearity occurred for number/letter stimuli; (2) left-to-right or right-to-left lines similarly, affected number/letter judgments; and (3) response times (RTs) differed as a function of number/letter stimuli and/or reverse/standard lines. If patterns of spatial mapping and RT are similar for NP and AP tasks, it would suggest that NP task performance may reflect the way ordinal lists are processed, rather than anything specific about how numbers are represented. Such evidence would cast doubt on the value of drawing unique inferences about the underlying representation of number from spatial mapping tasks. Alternatively, if numbers/letters are found to have independent spatial mapping or RT patterns on NP and AP tasks, this would suggest that number symbols involve unique representations that can be studied using the NP task.

MATERIALS AND METHODS

Participants

Eighty-four (M = 19.05 years, SD = 2.72 years; 30 males, 54 female) undergraduate students from an Australian university participated in the research for course credit (our sample size was determined by how many participants we were able to recuit via our first year research participation program, and is a similar size to the sample used by Berteletti et al., 2012). All participants had normal or corrected-to-normal vision. Written and informed consent was obtained by asking participants to read a plain language statement and sign a consent form. All procedures involved were approved by the University of Melbourne Human Ethics Advisory Group (HREC number 1441499).

Apparatus

Stimuli were created on a Dell OptiPlex 9020 computer running Ubuntu with MATLAB software and Psychophysics Toolbox routines (Brainard, 1997; Pelli, 1997; Kleiner et al., 2007), and displayed on a 23 inch Dell E2314H LED monitor operating at a spatial resolution of 1,920 by 1,080 pixels at a refresh rate of 60 Hz.

Stimuli

Participants completed a horizontal NP estimation task using integers between '1' and '26', and an alphabet-to-position (AP) task using letters between 'A' and 'Z' (with list positions that matched the numbers). In the NP task, the line endpoints were anchored with '1' on the left and '26'on the right, or '26' on the left and '1' on the right (i.e., a reversed NP). Participants positioned a target number (2 to 25) along the line to indicate a location in space that corresponded to the number's numerical position relative to the numerical endpoints. In the AP task, the lines were anchored with 'A' on the left and 'Z' on the right, or 'Z' on the left and 'A' on the right (i.e., a reversed AP). Participants positioned a target letter (B to Y) along the line to indicate a location in space that corresponded to the letter's alphabetical position relative to the letter endpoints.

Participants were tested on half the possible items in the number/letter lists to minimize the total number of trials. Numbers/letters were chosen by selecting every second number and the corresponding list position letter between 2 and 25 (e.g., '3' and 'C'). Participants were randomly assigned to even number/letter or odd number/letter target symbol conditions. The even condition set used the twelve target numbers: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, and 24, and the corresponding letters: B, D, F, H, J, L, N, P, R, T, V, and X. The odd condition participants were shown twelve target numbers: 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, and 25, and the corresponding target letters: C, E, G, I, K, M, O, Q, S, U, W, and Y. The target letter was displayed 8 cm above the middle of the horizontal line. The horizontal line was 15.5 cm long and 0.1 cm wide. The vertical marker used to indicate the target spatial position was 0.8 cm long and 0.1 cm wide. All stimuli were presented in black on a white background. Number/letter stimuli were presented in Arial font about 1 cm high.

Procedure

Testing was conducted in a quiet testing space with participants seated about 60 cm in front of the computer monitor. Participants were presented with a series of horizontal lines on the screen, anchored by a number/letter on the left and right ends of the line. On each trial a number/letter appeared in the middle of the screen, above the line. Participants were instructed to move the computer mouse (i.e., vertical marker) left or right to the place where they thought the symbol should be on each line. Participants were asked to move the mouse marker and click when they thought they had positioned the symbol correctly.

Trials were randomized within a block of 48 trials (numbers/letters, left-to-right or right-to-left, 12 different targets), and participants completed nine blocks (i.e., 432 trials in total). Participants had as long as they wished on each trial to make their response. Trials were separated by 500 ms to reduce the potential for outlier bias in calculating RTs, participants with fewer than seven repeats for any condition were excluded from the analyses. Eighteen participants were excluded using this criterion, resulting in a final sample size of 66 participants, 79.57% of whom completed the testing.

Analytic Approach

The overall aim of the analyses was to investigate similarities/differences in possible nonlinearities in the NP and AP data. As a first step though, we report the linear/logarithmic equation fits to our data. Specifically, we report the mean logarithmic and linear fit residuals for the NP and AP tasks to provide an overview of the data. To determine whether performance differed for the NP and AP tasks, we compared the spatial mapping RTs for number/letter symbols for the left-to-right version of the task, as well as right-to-left version of the task. The aim was to identify possible nonlinearities in the different tasks.

To more closely examine possible nonlinearities in these data, we plotted the average residuals from a linear fit for numbers/letters for both directions of the task. And to determine whether nonlinearities in the residuals depended on symbol type or task direction, we compared performance across numbers/letters and task direction. We also compared RT data for numbers/letters in both task directions to examine strategies used for NP/AP tasks.

To achieve the last three steps, we used a nested bootstrapping method to generate 95% confidence intervals. A distribution of group mean data was generated by calculating the mean of 10,000 samples of participants (with replacement) using resampled raw mapping data (with replacement) for each participant. To get a better idea of the underlying patterns in these data, we also fit polynomial curves to the data using the polyfit function in MATLAB.

RESULTS

Overall, the data are consistent with a linear mapping function. For each participant in each condition, we calculated the mean of the residuals from both a linear and a logarithmic mapping function as a proportion of the line length. In every case, the mean linear residual was lower than the mean logarithmic residual. The group average mean linear residual was 0.005, whereas the group average mean logarithmic residual was 0.064. This shows that a logarithmic model was not a good fit for any participant in any of the conditions, and that a linear model was a better general characterization of task performance.

Mapping Between Symbols and Space

The first step was to determine whether there was a difference between number/letter symbols in the spatial mapping task.

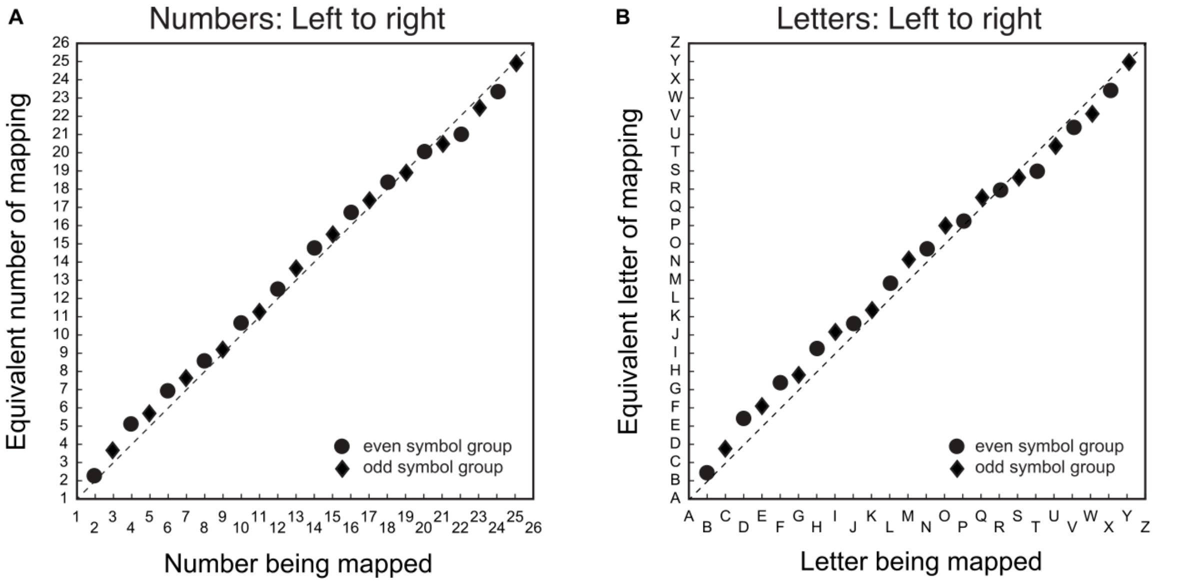

FIGURE 1 | Group average symbol to line position matching for number/letter items. The *X*-axis is the symbol being matched, and the *Y*-axis is the equivalent symbol of the match. Panel **(A)** is the mapping data for number symbols from left-to-right and panel **(B)** is the corresponding data for the letter symbols from left-to-right. Circular data points are for the group that mapped even item position symbols. The diamond data points are for the group that mapped odd item position symbols. The dashed diagonal line is a veridical linear mapping between symbols and line position.

Figure 1 shows the group mean average symbol to line position matching for number (**Figure 1A**) and letter (**Figure 1B**) items. The solid symbols are the group averages of individual mapping data, which is the mean of 6 to 9 repetitions of each item. The data deviated minimally from the diagonal (the average residuals were typically less than 5% of the spatial distance between items).

It is evident that there is a general linear mapping between symbols and space that does not depend on symbol type. While the pattern of the mapping between symbol and space is similar for both numbers/letters, the systematic deviations from the diagonal suggest that these mapping functions both have non-linear components. Specifically, there was a tendency to mark the line further to the center of the veridical linear location of the symbol for items proximal to, but not immediately adjacent to, the end points. There was also a tendency for items in the middle of the list to be positioned toward the right of the veridical location.

Non-linear mappings of number to space have been reported previously (e.g., Siegler and Opfer, 2003). However, these nonlinearities are typically characterized as logarithmic or quasi-logarithmic patterns, where symbols are mapped toward the right of the linear prediction for all items. The present results differ from previously reported logarithmic patterns in two ways. Firstly, the nonlinearities are small. Secondly, the patterns of non-linear residuals, both above and below a veridical linear function, are not consistent with a mapping model based on a power or quasi-logarithmic function. It is possible the systematic nonlinearities in the spatial mapping function for numbers/letters may reflect a bias to either under- or over-estimate the position of number/letter items due to the properties of the task. Alternatively, the pattern of nonlinearities may reflect list order effects common to both numbers/letters when they are mapped to space.

Mapping Between Symbols and Space With Reversed Mapping Direction

The second step was to determine whether there was a difference between number/letter symbols in the spatial mapping task when the mapping direction was switched from left-to-right to right-to-left. **Figure 2** displays the group mean average symbol to line position matching in the mapping task for number (**Figure 2A**) and letter (**Figure 2B**) items, where the spatial direction of the line was reversed by anchoring the left side of the line with '26' or 'Z', and the right side with '1' or 'A', respectively. The data are displayed in the item order rather than the spatial direction, because consistent patterns in the data depend on item position. As with **Figure 1**, the mapping between number symbols and line position deviates minimally from the diagonal, consistent with a linear mapping from number symbols to space.

Also similar to **Figures 1**, **2** shows systematic deviations above and below the diagonal that are qualitatively similar for both number/letter symbols. There is a tendency to map symbols onto the line further to the center of the veridical linear location of the symbol. Since the direction of the line was dissociated from item order, nonlinearities in the same direction for **Figures 1**, **2** reflect nonlinearities that are related to item order, whereas nonlinearities in the opposite direction are nonlinearities that depend on spatial direction.

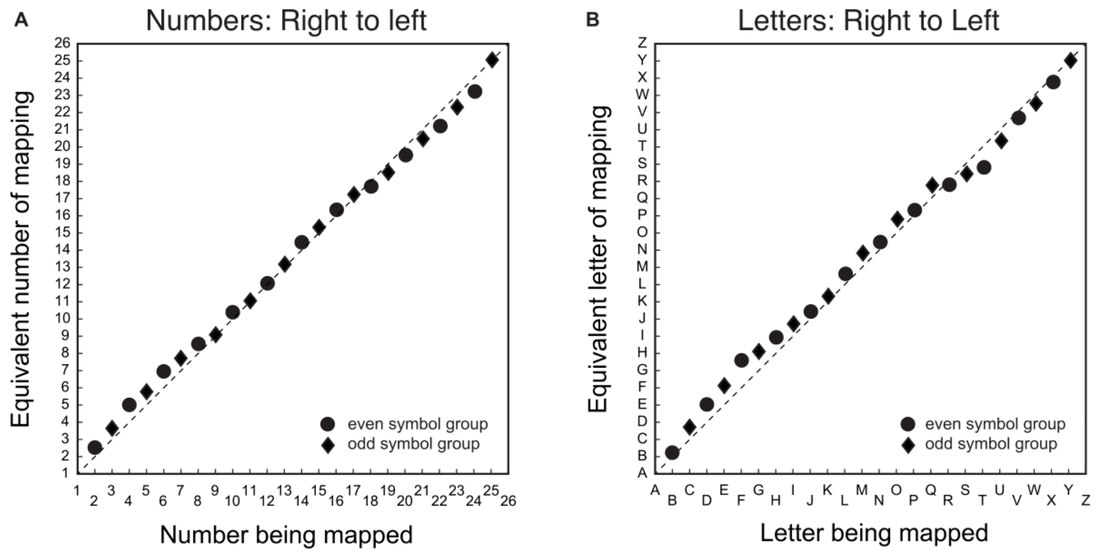

FIGURE 2 | Group average symbol to line position matching for number/letter items when the left-to-right spatial direction was mismatched with symbol order. The *X*-axis is the symbol being matched, and the Y-axis is the equivalent symbol of the match. Panel **(A)** is the mapping data for number symbols from right-to-left and panel **(B)** is the corresponding data for the letter symbols from right-to-left. In the actual experiment, the left side of the line was '26' or 'Z', and the right side was '1' or 'A'. Circular data points are for the group that mapped even item position symbols. The diamond data points are for the group that mapped odd item position symbols. The dashed diagonal line is a veridical linear mapping between symbols and line position.

Group Average Residuals of Symbol to Line Mapping

The third step was to examine the group average residuals from the veridical location for the number/letter symbols in the spatial mapping task. **Figure 3** shows that the residuals from the veridical linear are qualitatively similar for symbol type and mapping direction. **Figures 3A,B** show the data for numbers. **Figures 3C,D** show the data for letters. **Figures 3A,C** show the data for left-to-right item direction. **Figures 3B,D** show the data for right-to-left item direction. To determine if the residuals systematically deviate from a linear mapping function, a sixth order polynomial was fit to the 95% confidence intervals (CIs) using polyfit in MATLAB.

These data show that there is a tendency to map symbols that are a fourth or three-fourths of the way through the item list length toward the center item. The pattern appears to be systematic, given that all data display this trend regardless of symbol or mapping direction. Furthermore, given that the polynomial fit to the 95% CIs do not include 0, the non-linear pattern near the end points is unlikely to have occurred by chance.

The data in **Figure 3** show a small systematic tendency for items toward the middle of the list to be positioned away from the start of the list. This is evident for both numbers/letters. The bias is larger for left-to-right ordered items, suggesting that this effect is strongest when the item order and spatial mapping direction are matched. The polynomial fit to the 95% CIs for the left-to-right mapping data provides support for the proposition that the bias is unlikely to have occurred by chance. The polynomial fit to the 95% CIs for the right-to-left letter data does not include 0, meaning the residuals represents a systematic bias. While the polynomial fit to 95% CIs for the right-to-left number data does not provide evidence of systematic bias, there is a trend in the same direction as the other conditions. The pattern of results suggests there are few differences in the nonlinearities across symbol type or mapping direction.

Comparison of Mapping for Symbol Type and Mapping Direction

The fourth step was to compare spatial mapping across number/letter symbols and mapping direction. **Figure 4** displays the *t*-score for each item position. **Figure 4A** shows mapping direction for number symbols. **Figure 4B** shows mapping direction for letter symbols. **Figure 4C** shows symbol type for left-to-right mapping. **Figure 4D** shows symbol type for right-to-left mapping. To address how much the residual patterns depend on symbol type and the congruency between symbol item order and spatial direction, we performed a series of permutation *t*-tests for each item position, comparing mapping direction and symbol type. Comparisons of item direction were performed separately for each possible condition. In the permutation *t*-test, the 95% CI of the *t*-statistic for each data set was calculated using a bootstrapping procedure. The distribution of *t*-statistics was obtained by re-computing the *t*-value 10,000 times with random assignment of the data to each group. If the *t*-value of the data is outside the 95% CI for the bootstrap *t*-distribution, then we can infer that the data are different for the condition.

The trend in *t*-scores suggests that number mapping differs for left-to-right versus right-to-left direction for symbols in the middle of the list. There is a similar trend for letters, but the

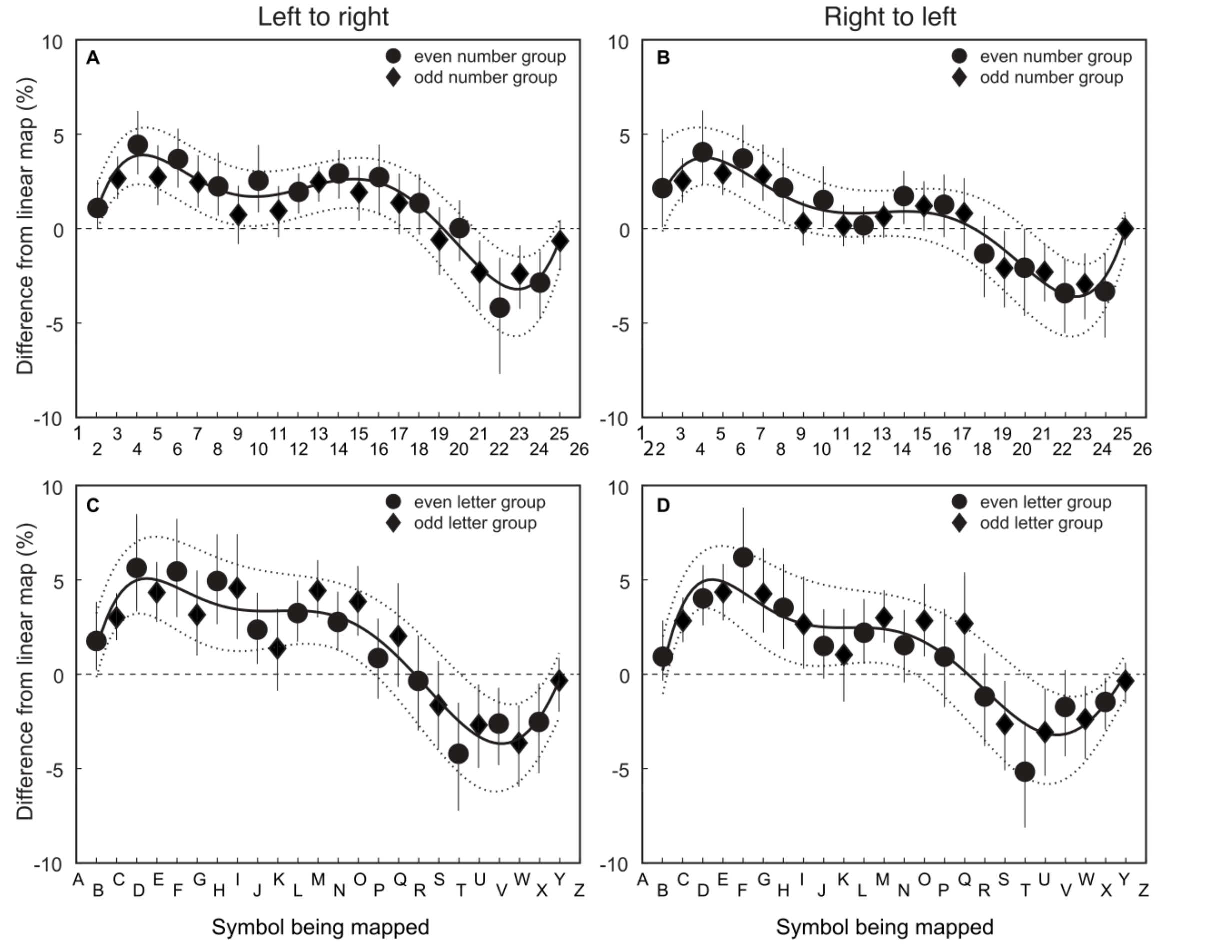

FIGURE 3 | Group average residuals from veridical linear fit of symbol to line mapping for number/letter items for left-to-right and right-to-left spatial directions. The *X*-axis is the symbol being matched, and the *Y*-axis is the deviation from linear for the match. The top row **(A)** and **(B)** shows the data for numbers and the bottom row **(C)** and **(D)** shows the data for letters. The left column **(A)** and **(C)** shows the data for left-to-right item direction. The right column, **(B)** and **(D)** have the data for right-to-left item direction. Circular data points are for the group that mapped even item position symbols (n = 34). The diamond data points are for the group that mapped odd item position symbols (n = 32). Error bars are 95% CIs of the mean calculated using a bootstrapping procedure. The solid line is the best fitting sixth order polynomial to the group mean data. The dashed lines are the best fitting sixth order polynomial to the 95% CI estimates from the bootstrapped calculation.

t-scores for letters are not as large as the number t-scores, or as consistently outside the 95% CIs. Although the t-statistics show that there are systematic differences in number mapping for left-to-right versus right-to-left, the magnitude of the bias is small. The large t-values for middle items are partly due to less variance (or greater consistency) of position data across individuals for middle number items. As such, there may be more precision in the estimates for numbers (and less so for letters) in the middle of the list.

In sum, the pattern of t-statistics suggests that left-to-right and right-to-left mapping is similar for numbers/letters. There is a small trend for larger differences between numbers/letters in position mapping for symbols at the top of the item list. However, there is little in the position mapping data that reveals differences in the spatial mapping of numbers/letters. Nevertheless, there are differences in the response times (RTs) taken to perform symbol to space mapping which may be clarified with further analysis.

Response Times for Symbol Type and Mapping Direction

The fifth step was to compare RTs across number/letter symbols and mapping direction. **Figure 5** shows group average RTs for matching symbols to line position. **Figure 5A** shows the RT to map numbers that are ordered left-to-right. **Figure 5B** shows the RT to map numbers that are ordered right-to-left. **Figure 5C** shows the RT to map letters that are ordered left-to-right. **Figure 5D** shows the RT to map letters that are ordered right-to-left.

The RT to map a symbol onto a line has a distinct pattern as a function of item position that does not depend on symbol type

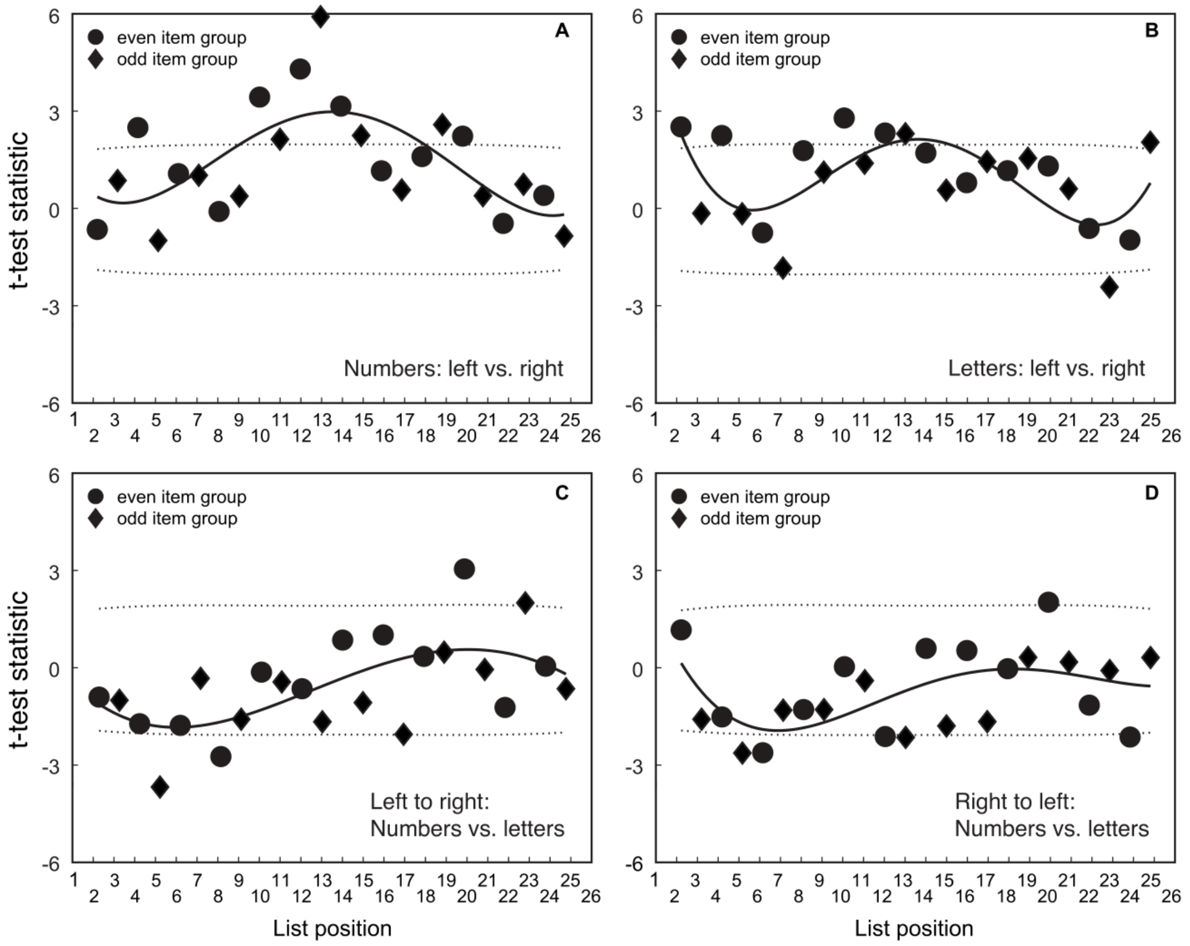

FIGURE 4 | Comparison of mapping position across symbol type and mapping direction. Panel **(A)** shows data for the comparison of mapping direction for number symbols. Panel **(B)** shows data for the comparison of mapping direction for letter symbols. Panel **(C)** shows data for the comparison of number versus letter symbols for left-to-right mapping. Panel **(D)** shows data for the comparison of number versus letter symbols for right-to-left mapping. Solid symbols are *t*-scores computed for each comparison at each item position. The dashed line represents the 95% CIs of the distribution of *t*-scores obtained from randomly assigning data to groups. Solid symbols outside the dashed line indicate there is a significant difference between mapping position for conditions on a particular item. The solid line showing the trend in *t*-scores across items for each comparison is a third order polynomial. Circular data points are for the group that mapped even item position symbols. The diamond data points are for the group that mapped odd item position symbols.

or mapping direction. Mapping is fastest for items at either end of the list, becoming slower for items further from the list end, with RTs being slowest for items about a third from the end. RTs then decrease for symbols in the middle of the list, with the 13th element of the list ('13' or 'M') being mapped almost as quickly as items at the ends of the list. Given that '13' is half the list length of '26' if counting from the top of the list, the RT data suggest that symbol list order is important for understanding the process of how symbols are mapped onto the line position.

The pattern of RTs does not seem to depend strongly on mapping order. The magnitude and pattern of change in RTs with item position in panel 5A is very similar to panel 5B. Panel 5C is also similar to panel 5D. The importance of the 13th item for numbers/letters suggests that item order is processed in a similar way regardless of the symbol type. However, the variability in letter RTs is much larger than that for number RTs, so a small effect would be less evident in these data.

Other patterns of RTs may provide information about how participants map number/letter symbols to spatial locations on a line. The RTs for 'B', 'C' and 'D' are similar to '2', '3' and '4'. Similarly, the speed for mapping 'X' and 'Y' is similar to '24' and '25'. The RT variance is also smaller for these letters compared to those far from the end of the list. This is good evidence of a processing speed advantage when item positions are close to the beginning or end of the list. Such positions are also easier to calculate for letters. The letter items toward the middle of the list take longer to calculate, and this may be reflected in the longer RTs for letters toward the middle of the list compared to numbers.

Overall, the RT patterns indicate that the strategies used to map numbers to spatial position are similar to that used to map

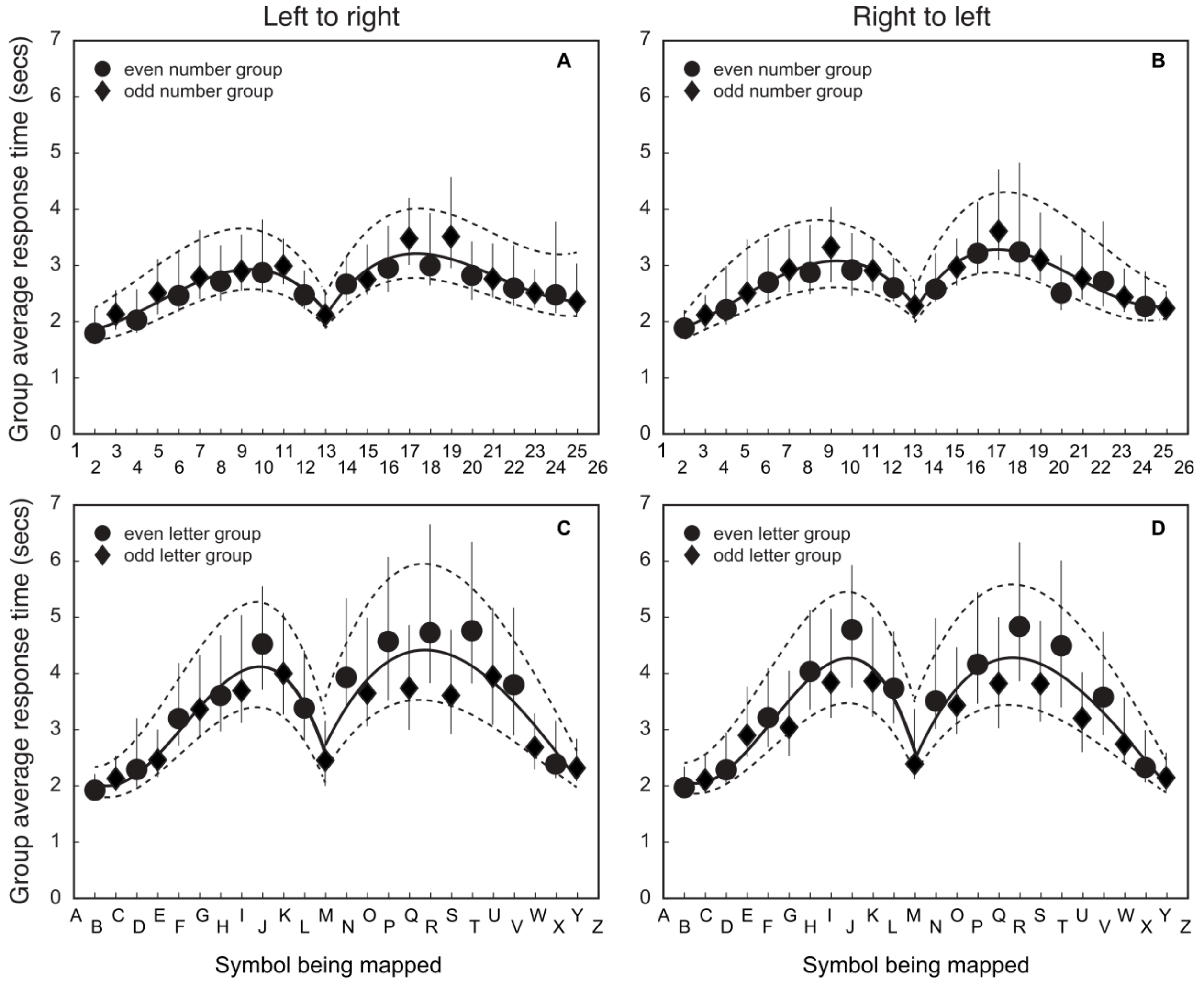

FIGURE 5 | Group average RTs of matching a symbol to line position for number/letter items for left-to-right and right-to-left spatial directions. The *X*-axis is the symbol being matched, and the *Y*-axis is the group average response time for the match. The top row **(A)** and **(B)** shows the data for numbers and the bottom row **(C)** and **(D)** shows the results for letters. The left column **(A)** and **(C)** shows the data for left-to-right item direction. The right column, **(B)** and **(D)** have the data for right-to-left item direction. Circular data points are for the group that mapped even item position symbols (n = 34). The diamond data points are for the group that mapped odd item position symbols (n = 32). Error bars represent 95% CIs of the mean calculated using a bootstrapping procedure. The solid line is the best fitting third order polynomial to the group mean data for items in the upper and lower half of the symbol list (both fits include the 13th item). The dashed lines are the best fitting third order polynomial to the 95% CI estimates from the bootstrapped calculation (separately for the upper and lower halves of the data).

letters to spatial position. The mapping process requires finding the item position, converting that to a proportion of list length, and mapping that into visual space. Such a process does not require there to be a spatial arrangement of numbers along a number line. However, we now have evidence that numerical ability is necessary to complete a spatial mapping task with numbers/letters. This numerical requirement supports the idea that changes in the spatial mapping of numbers onto a line in the NP task may reflect increased familiarity with ordered lists, the speed an item position can be determined, and the ability to calculate a proportion from the item position and total list lengths.

DISCUSSION

We compared the pattern of responses for NP and AP tasks. Our aim was to examine whether mapping of number to space, as indexed by the pattern of responses in a NP task, is unique to number, or reflects the way in which ordered lists are spatially represented (i.e., the AP task). Of interest was whether: (1) similar judgment deviations from linearity occurred for number/letter stimuli; (2) left-to-right or right-to-left lines similarly, affected number/letter judgments; and (3) RTs differed as a function of number/letter stimuli and/or reverse/standard lines. Three findings are of note. Firstly, similar deviations from linearity were found for numbers/letters, suggesting that participants used similar methods for judging the position of symbols in the two tasks. Secondly, end point symbols did not affect performance. Thirdly, participants took longer to make AP compared to NP task judgments. These findings support the view that NP judgments may not be uniquely informative about the MNL because the pattern of performance was similar for number/letter symbols.

Our findings are consistent with those of other researchers who have argued that changes in NP judgment patterns reflect an increased use of proportion judgment strategies (Barth and Paladino, 2011), or increased list processing automaticity (Hurst et al., 2014). We also found systematic nonlinearities in both number/letter mapping patterns that may reflect a small bias for marking lines away from end points. These small linear deviations are not explained by combinations of linear and/or logarithmic mapping functions, which would produce only positive residuals. Our data do not support the view that mapping symbols onto a horizontal line depends uniquely on the numerical properties of the symbols associated with their location on a hypothetical MNL. Rather, our findings are consistent with a model in which the symbol position within an ordered list is used to map a symbol to a location on a horizontal line.

We found little evidence that mismatches in number/letter symbol position order, or task direction, alter the shape of the mapping function between symbols and space. Specifically, the patterns of non-linear residuals were similar regardless of symbol type and regardless of whether the spatial direction of the horizontal line was matched with numerical magnitude direction. We found a small trend for middle items to be estimated toward the right side of space (an effect that was more evident for numbers than letters), but the t-statistics for combinations of symbol type and direction congruence show that the patterns were similar across conditions. These findings are consistent with the claim that participants use a single method to map the position of number/letter symbols to the location along the horizontal line.

The RT data showed that responses were faster for items near the end of the list and close to the middle of the item sequence. This indicates participants were most efficient judging the position of symbols that reflected a fraction of a half. While letter judgments were slower than number judgments, when the item position of the letter is known, RTs for the comparable symbol sets was similar. For example, the relatively fast RTs of '13' and 'M' suggest that it is more efficient to map symbols that map onto simple fractions. This is consistent with the view that mapping the line position of symbols requires the use of list length to calculate the position of an item as a proportion. This calculation does not depend on the line end points. In other words, our findings are consistent with a model that suggests that the ability to judge proportions is critical to performing the NP task.

We suggest that the linearity of judgments on NP tasks may reflect how ordered lists are learned (items in the beginning of number/letter sequences are learned before elements later in the list), rather than unique information about the MNL. This is because there is no numerical requirement for letters, which are items that are ordered without inherent magnitude (e.g., 1 + 2 is meaningful, whereas A + B is not), to be spaced linearly with a fixed interval between each item. Our findings support those of Hurst et al. (2014) and Berteletti et al. (2012), who show that older children and adults mark letters linearly on an AP task. They cast doubt on the value of making inferences about NP task performance as a measure of the underlying number representation. Specifically, the mapping process in NP judgments may not involve decoding numerical information from a MNL and suggests that NP judgments may reflect list order learning of numbers in formal education.

It is important to note that NP judgments may require "number" precision, irrespective of the elements being mapped. A participant must know where an item is in a list, be able to calculate the relative position in the list, and be able to match the relative proportionate location of items on the line. In other words, participants must be able to calculate relative proportions of items on the line. For numbers/letters, participants were faster and more accurate for items at the beginning and end of lines, and for items in the middle of the line, most likely because it was easier to calculate these proportions. If a participant diagnosed with dyscalculia (Butterworth et al., 2011) were to complete this task, for example, we would expect equally poor performance for the number/letter elements, since this participant might be incapable of calculating the relative proportions of the line, even if they knew the order of the letters. The fact that letter responses were slower overall suggests participants may be mentally mapping letters onto numbers to complete the task.

A possible limitation of our study is that we analyzed group data without regard to the possibility of individual differences. However, given that the standard errors were low, we think there were probably few systematic differences in mapping patterns. Nevertheless, future work should investigate whether individual differences are related to the ability to estimate item position and make proportion calculations. Furthermore, evidence that familiar letters near the end of the list were processed as quickly as numbers suggest that list familiarity may be important in shaping the methods participants use in line judgment tasks. Future work with artificial non-numerical symbols could reveal learning processes that are relevant to numerical learning more generally.

Overall, our findings support the claim that the NP task should not be taken as a unique measure of number along a hypothetical MNL. Our results are consistent with the idea that ordinality plays an important role in explaining the mapping function on the NP task, rather than the magnitude of number symbols being matched. It seems possible that instead of a number specific representation, number may be supported by a representation that is able to process ordered lists in general.

AUTHOR CONTRIBUTIONS

CP contributed to investigation, methodology, data collection, and editing the original draft. RR contributed to draft editing and supervision. JF prepared the methodology, coding, data analysis, draft editing, and supervision.

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Conflict of Interest Statement: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

4.3. Summary of Chapter 4

Study 1 (Podwysocki et al., 2019) tested whether adult participants would respond in a similar manner (i.e., linearly) on a number-to-position task and on an alphabet-to-position task. Previous research has found that numbers and letters are treated similarly on a spatial mapping task; however, the extent of this similarity was not entirely clear. The results of this study showed not only that participants responded linearly on both number and letter tasks, but that the patterns of non-linearities were similar, that responses were similar in both directions of the task (i.e., left-to-right and right-to-left), and that response times also followed a similar pattern for both number and letter stimuli. These similar response patterns for number and letter stimuli on the spatial mapping task suggest that the NP task should not be taken as a direct measure of the MNL, because similar patterns of response can be elicited with non-numerical stimuli. Furthermore, these results suggest that number representations may be supported by a structure that lends itself to representing ordered lists more generally. As the results of this study and the previous study question the NP task and the MNL metaphor in general, the next chapter presents Studies 2a and 2b, which were two studies testing the construction of ordinal representations with a novel symbol training study.

Chapter 5
Studies 2a and 2b

Chapter 5 (Studies 2a and 2b) reports a study that investigated whether a set of novel symbols would be learned more quickly and accurately when the spatial complexity of the symbols (i.e., how many curves or lines were in the symbol) was associated with the list order of the symbols. While research currently assumes that symbol objects are arbitrary, it could be the case that symbols have a 'natural order' to them in terms of spatial complexity, which may influence learning ordered relationships. However, it is unclear whether learning number symbols also involves this ordinal dimension. In Chapter 3, adult representations of number were unable to be shifted with extensive logarithmic training. This indicates that it may be difficult to study number representations with Hindu-Arabic numerals in adults because they are too familiar. Therefore, the study reported in Chapter 4 examined the MNL with letters from the alphabet, because they are an ordered list without magnitude information. The study showed familiar ordinal lists also produce linear response patterns, and that number and letter symbols share similar patterns of non-linearity and response time patterns, indicating that a set of novel symbols should be used to test the internal representation of ordinal lists.

The aim of Chapter 5 was to identify a novel symbol set with a consistent relationship to spatial complexity, and then to test how participants learned the order of this novel symbol set when the relationship between spatial complexity and list order was manipulated (the novel symbols had either a positive relationship between spatial complexity and list order or a random relationship between spatial complexity and list order). This may provide a better understanding of how ordinal lists are internally represented in adult participants. Overall, Chapter 5 tests two novel symbol sets (Gibson and Sunúz) to determine which set has the most consistent relationship to complexity, and then tests the most consistent set to determine whether the property of spatial complexity affects ordinal learning in general.

5.1. Aims of Chapter 5

Studies 2a and 2b aim to help resolve whether the underlying spatial properties of numerical symbols influence ordinal learning. This may help explain the findings of both the pilot study and Study 1. Specifically, this study wanted to answer what role (if any) spatial complexity has in learning the order of a set of symbols, and to clarify whether the MNL metaphor should be extended to include all ordinal lists. There are two possible outcomes for Study 2b. Firstly, if it is found that the spatial complexity of novel symbols has an influence on list learning, then this may indicate that there are underlying spatial properties (such as spatial complexity) that affect how the order of symbols in general are learned. Secondly, if it is found that the spatial complexity of the symbols does not influence list learning, then this suggests symbols are abstract, and that ordinality plays a minimal role in the representation of number. The study presented in Chapter 5 (titled 'Spatial Complexity Facilitates Ordinal Mapping with a Novel Symbol Set') is the manuscript of a publication in *PLOS ONE* (Podwysocki, Reeve, Paul, & Forte, 2020), and will be presented on the following page in its format at the time of thesis submission[7].

[7] Please note the final revised version (published 26/03/2020) of this article can be viewed online at https://doi.org/10.1371/journal.pone.0230559

5.2. Research paper

Spatial Complexity Facilitates Ordinal Mapping with a Novel Symbol Set

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Running head: Spatial Complexity and Ordinal Mapping

Abstract

The representation of number symbols is assumed to be unique, and not shared with other ordinal symbol lists. However, little research has examined if this is the case, or if the properties of symbols affect learning (such as spatial complexity). A study was conducted to investigate if the property of spatial complexity affects ordered list learning. In Study 1, 46 adults made a series of judgements about two novel symbol sets (Gibson and Sunúz). The goal was to select a novel symbol set that could be ordered by complexity to use in Study 2. In Study 2, 84 adults learned to order nine novel symbols (Sunúz) via a paired comparison procedure, judging which symbol was 'larger' (where the larger symbol became physically larger as feedback). Participants were assigned to either a condition where there was a relationship between spatial complexity and symbol order, or a condition where there was a random relationship. Of particular interest was if symbol list learning would be facilitated by the spatial complexity of the novel symbols. Findings suggest spatial complexity affected learning speed and accuracy, and that spatial complexity with relational information can facilitate learning ordinal sequences. This suggests that the implicit cognitive representation of number may be a more general feature of ordinal lists.

Keywords: Numerical symbols; Gibson symbols; Sunúz symbols; numerical representation; ordinal representation; relational information

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Introduction

It is unclear if different ordinal lists of symbols (e.g., numbers, letters, novel symbols) share similar representational formats, or are represented/learned differently. This is an important issue, since it is claimed that numbers are represented uniquely on a so-called mental number line (MNL); and a large body of research supports this interpretation of number representation [1]. Evidence for the MNL hypothesis, however, should not be taken as evidence of its representational uniqueness. Indeed, recent research found that non-numerical ordered lists (i.e., letters of the alphabet) are represented in a similar manner to numerical information [2]. The authors interpreted this finding as implying ordinal lists share a common representational format. Nevertheless, it could be argued that familiarity with the position of letters in the alphabet lends itself to a representational format analogous to the MNL representation. Potentially spurious claims about similar/different representational formats could be avoided by examining the ways unfamiliar ordinal symbol lists are represented and/or learned. To this end, the current paper investigated if the property of spatial complexity could be used to learn the ordinal representation of a novel symbol set. Insofar as spatial complexity could be used for such an endeavour, it would suggest a common representational format supports ordinal lists more generally.

The MNL is a popular metaphor for how natural numbers are mentally represented. The MNL model suggests numbers are represented in ascending order from left-to-right in a "continuous, quantity-based, analogical format" [3, p. 138]. However, an issue for the MNL is that inferences about the underlying form of the MNL are typically gained from the number-to-position (NP) task [1], and responses on the NP task may be affected by other factors. For example, Barth and Paladino [4] showed that responses on the NP task may reflect an increase in proportion judgement strategies, rather than the form of the underlying MNL. Furthermore, Cohen and Blanc-Goldhammer [5] showed that NP task judgements can be manipulated by altering the form of the NP task (e.g., using an 'unbounded' NP task produces linear responses, whereas using a 'bounded' NP task produces responses consistent with proportion judgement). This suggests that measurement of the MNL representation may be affected by task demands, and also leaves unanswered the issue of whether the putative representation of the MNL is unique to number or shares properties with other ordinal lists.

Researchers have examined the MNL model using letters of the alphabet, or other ordinal lists, such as months of the calendar year [2,6-9]. For example, Podwysocki, Reeve, and Forte [2] compared judgements of numerical stimuli and letter stimuli on a NP task for letters, and an alphabet-to-position task for letters. The authors found that numbers and letters both produced linear patterns of response for these tasks, and that the patterns of non-linear responses were also similar for the number and letter stimuli. These results support the notion that a more general ordinal representation underlies number processing. Similar findings are supported by researchers who have found similar patterns of response for numbers and letters on a spatial mapping task [6,9]. However, a possible issue using letters to make inferences about the MNL is that letters of the alphabet and numbers are often learned in a similar way (i.e., 1, 2, 3, and A, B, C are learned first in the number and letter sequences), and at a similar developmental stage, which may result in a similar representation for both number and letter stimuli. One way to overcome this interpretive limitation is to use novel symbols.

Novel symbols have often been used in research to study ordinal list learning. For example, Lyons and Ansari [10] used novel visual shapes to test the cerebral basis of mapping non-symbolic number onto abstract symbols. Lyons and Beilock [11] also used the novel visual shapes to test whether working memory capacity affects the ability to infer ordinal relationships between symbols. The Gibson symbol set [12-14] is also widely used to test magnitude relations and representations in neurologically healthy adults [15,16] and in adults with math learning disabilities [17,18]. However, a potential issue with the Gibson symbols is that they were created by transforming letters of the Roman alphabet [14]. Another symbol set that has been used more recently that was not made from letters of the alphabet are the fictional numbers from the script of the Sunúz, from the fantasy world of *Tékumel: Empire of the Petal Throne* [19]. Research by Merkley et al. has used the Sunúz symbol set to test the role of magnitude information and ordinality in number representations in adults to avoid existing knowledge of ordinality in Hindu-Arabic numerals [20,21].

While novel symbols have not been used to test whether the representation that underlies number is common to ordinal lists, they offer a means of avoiding the over-learned and over-familiar Hindu-Arabic numerals, letters of the Roman alphabet, or other common everyday symbol lists [22]. They also allow for an examination of whether some property of the symbols (such as spatial complexity) affects ordinal list learning, an impossibility with more common everyday symbols as they contain information from prior learning. For novel symbols to be useful for such a task, it would be beneficial if they could be ordered by the property of spatial complexity (i.e., the number of curves/lines in a symbol), since there is an association between the numerosity a symbol represents and the spatial complexity of the symbol in a range of different cultures [23-25]. This suggests that ordering a novel symbol set by spatial complexity could enhance ordinal symbol mapping.

One method for studying the efficacy of ordinal list learning with novel symbols (and used in the current research) is with a paired comparison relational learning procedure. A participant would learn the order of a list of symbols by making ‘greater than’ judgements for a series of novel symbols pairs. With a series paired comparison judgment trials, a mental representation of the ordered list of novel symbols could be inferred. This procedure would then allow for an assessment of whether the ordinal representation of novel symbols resembles the ordinal nature of the MNL representation.

The current study

This study investigated whether the spatial complexity of a list of novel symbols would facilitate ordinal list learning. Study 1 evaluated whether the Gibson and Sunúz symbol sets could be ordered by complexity by asking participants to rank the symbols according to their numerical value (e.g., ‘which symbol is best suited to represent a small number?’) and their complexity (e.g., ‘which symbol do you think is the most complex?’). This was done so that a condition could be created that randomised the spatial complexity of the novel symbols in Study 2. Study 2 tested the hypothesis that learning a novel symbol set ordered by spatial complexity would facilitate ordinal list learning when compared to learning a novel symbol list ordered by random complexity. It was expected in Study 2 that learning a list of novel symbols would be facilitated when symbols were ordered by spatial complexity, compared with spatial complexity not being an indicator of ordinality.

Study 1

The goal of Study 1 was to select a novel symbol set where symbols were consistent in terms of perceived complexity ratings, but did not evoke other symbol sets (such as Hindu-Arabic digits or letters from the alphabet). Nine Gibson symbols and nine Sunúz symbols were selected for testing. To test the novel symbol sets, participants were required to order each symbol set from small to large, and from simple to complex. Participants also mapped the symbols directly onto a list of symbolic (i.e., Hindu-Arabic digits) and non-symbolic (i.e., dot arrays) number stimuli. The purpose was to identify which novel symbol set had the most consistent pattern between perceived numerosity and complexity judgements (i.e., that participants would reliably rate specific symbols as ‘large’, ‘small’, ‘complex’, and ‘simple’). In summary, the novel symbol set that is determined to have the most consistent perceived complexity ratings would make a good candidate for testing in Study 2.

Method

Participants

Forty-six (*M* = 19.34 years, *SD* = 2.99 years; 24 male, 22 female) undergraduate students from an Australian university participated in the research for course credit. All participants had normal or corrected-to-normal vision. Written and informed consent was obtained by asking participants to read a plain language statement and then sign a consent form. All procedures involved were approved by, and in accordance with, the University of Melbourne Human Ethics Advisory Group (HREC number 1441499).

Apparatus

Stimuli were created on a Dell OptiPlex 9020 computer running Ubuntu 12.04 with MATLAB software and Psychophysics Toolbox routines [26-28] and displayed on a 23 inch Dell E2314H LED monitor operating at a spatial resolution of 1,920 by 1,080 pixels at a refresh rate of 60Hz.

Stimuli

Two sets of novel symbols were chosen to test in this study. The first symbol set were a subset of Gibson figures (Fig 1a). The second symbol set were fictional numbers of the script of the Sunúz from the fantasy world of *Tékumel: Empire of the Petal Throne* (Fig 1b).

Fig 1. The nine a) Gibson symbols and b) Sunúz symbols tested in Study 1. The ideal symbol set would be orderable by complexity.

The first task required participants to make four sets of judgments regarding the Gibson symbols and the Sunúz symbols. Symbols were presented on the screen in a circular arrangement. Judgements included: a) 'symbol you would choose to represent a small number?'; b) 'which symbol you would choose to represent a large number?'; c) 'which symbol do you think is the simplest?'; and d) 'which symbol do you think is the most complex?'. At a viewing distance of 60cm, the symbols were 2 degrees of a visual angle high (1 degree = 1cm). Instructions remained on the screen for the duration of the testing period. All stimuli were presented in black on a white background. A small black circle appeared around each symbol as the cursor moved over it. The cursor returned to the centre of the screen after every trial to re-orient participants. Each time participants clicked a symbol, it disappeared, resulting in one fewer symbol to select from. This process continued until all symbols were removed from the screen. For all tasks, the Gibson symbols were presented first, followed by the Sunúz symbols for consistency. The second task required participants to make four different sets of mapping judgements regarding the Gibson and the Sunúz symbols. The stimuli were presented in an identical format to the previous task. Participants were asked to choose the symbol that was most similar to the number in the middle of the circle. These questions were presented in order (from 1 to 9) and reverse order (from 9 to 1) for the symbolic (i.e., Hindu-Arabic digits) and non-symbolic (i.e., dot arrays) number sets.

Procedure

Testing was conducted in a quiet testing space. Participants were told that they would be asked to complete a series of basic tasks on some novel symbols. The study began with instructions informing participants that they would be asked to click some symbols, and that there were no correct or incorrect answers. The first task was the symbol judgment task, where participants were presented with a series of questions for both symbol sets. There were four different questions, with one trial for each of the nine stimuli, with a total of 72 trials. The second task was the symbol mapping task, where participants were presented with mapping questions for both symbol sets. There were four mapping questions, with one trial for each of the nine stimuli, resulting in 72 trials.

Results and discussion

To determine whether the novel symbols had a consistent relationship between perceived number rakings and complexity ratings, the median ratings were calculated for each task for both symbol sets. Median ratings were used instead of mean ratings, since outliers were present in these data. Fig 2 displays box plots of the smallest to largest number rating (y-axis) ordered by the median simplest to most complex rating (x-axis) for the Gibson (top) and Sunúz (bottom) symbols, respectively.

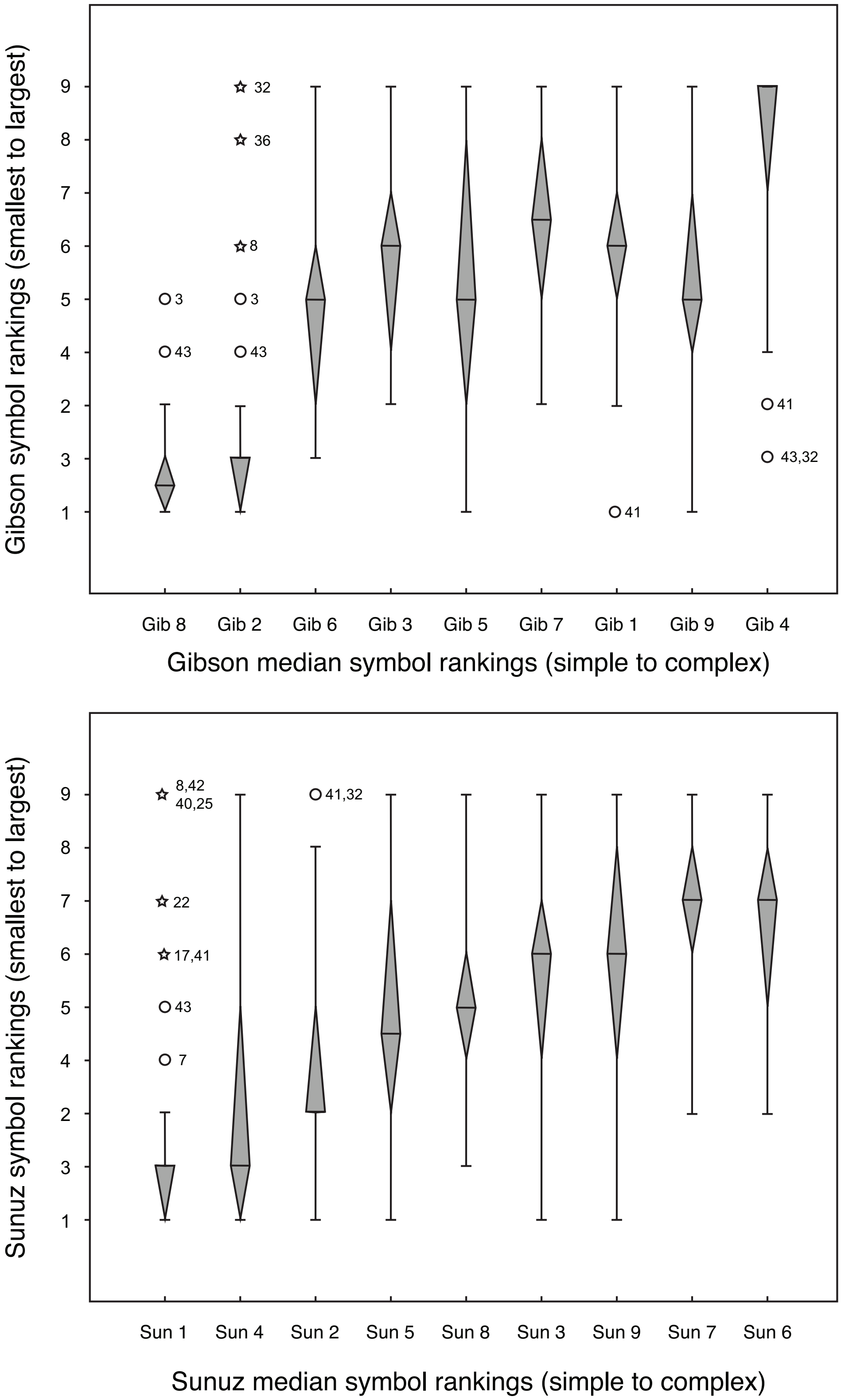

Fig 2. Box plots displaying median smallest to largest symbol ratings (y-axis) ordered by the median simplest to most complex rating (x-axis) for the Gibson symbols (top) and Sunúz symbols (bottom). Regular outliers are denoted by open circles and represent 1.5 times the interquartile range. 'Extreme' outliers are denoted by stars and represent 3 times the interquartile range. The specific numbers next to outliers correspond to individual participants.

In these box plots, an ascending monotonic relationship between the simple to complex and smallest to largest ratings would suggest a relationship between number and complexity. The Gibson and Sunúz symbol sets have a similar relationship between perceived complexity and number, displaying an ascending relationship between number and complexity ratings. Also, for both symbol sets, there is a consistent trend where the symbols ranked as simpler are also ranked as a smaller number and the symbols ranked as more complex are also ranked as a larger number. This indicates that either symbol set could be used for the next study. However, the boxplots show that the Sunúz symbols had a clearer monotonic relationship between complexity ratings and fewer outliers than the Gibson symbols. Furthermore, because many of the Gibson symbols resemble common everyday symbols, and were created by transforming letters of the alphabet [14], it was decided that the Sunúz symbols were better suited to the purpose of Study 2.

Study 2

The aim of Study 2 was to determine if learning a set of novel symbols would be facilitated by associating list position with spatial complexity. Prior research has claimed that the symbols are abstract [29], implying that the format of the symbols is irrelevant. However, research has not yet tested whether the underlying spatial properties in symbols affect learning ordinal relationships. The aim of this study was to test if spatial complexity could facilitate learning a novel symbol set. If it is found that spatial complexity enhances learning novel symbols, it would suggest that ordered lists can be learned with only relational information and the property of spatial complexity.

Participants were presented with pairs of Sunúz symbols, and were required to learn the order of the symbols through relational information (i.e., participants were asked 'which symbol do you think is the largest?'). Participants were provided with visual feedback (the symbols either increasing or decreasing in size to indicate whether the symbol is 'larger' or 'smaller', respectively). To test if spatial complexity facilitates learning the novel symbol list, participants were randomly allocated to one of two learning conditions. Participants in the first group learned the symbols ordered by the median perceived complexity ratings from Study 1. Participants in the second condition learned the symbols in a random complexity order, where the spatial complexity of the symbols was not associated with the ordinal position of the symbol. Participants were then shown symbols in a circular arrangement, and clicked on the symbols according to the order they learned during the training phase. If participants in the spatial complexity group outperformed participants in the random complexity group, then this would suggest properties such as spatial complexity act as a mediating construct when learning internal ordinal representations.

Method

Participants

Eighty-four (*M* = 19.05 years, *SD* = 2.72 years; 30 male, 54 female) undergraduate students from an Australian university participated in the research for course credit. All participants had normal or corrected-to-normal vision. Written and informed consent was obtained by asking participants to read a plain language statement and then sign a consent form. All procedures involved were approved by, and in accordance with, the University of Melbourne Human Ethics Advisory Group (HREC number 1441499).

Apparatus

The apparatus were the same as Study 1.

Stimuli

The nine Sunúz symbols from Study 1 were used in Study 2. There were two groups. In the first group, the symbols were ordered according to their spatial complexity, which was determined by the median complexity ratings of participants in Study 1 (order: 1, 2, 3, 4, 5, 6, 7, 8, 9; Fig 3). In the case of a tied median rating, the mean was used to decide the order. In the second group, the symbols were ordered randomly, where both halves of the symbol list were matched for complexity (i.e., both halves were equally complex; order: 4, 6, 1, 9, 2, 7, 5, 8, 3; Fig 3). This study had two phases. The first phase presented participants with pairs of Sunúz symbols (the paired comparison task), and they were asked to judge which symbol represented a 'large' value to learn the order of the symbols.

At a viewing distance of 60cm, the symbols were 2 degrees high. As feedback, the 'larger' symbol appeared physically larger on the screen (4 degrees high), and the 'smaller' symbol appeared physically smaller on the screen (1 degree high). The cursor returned to the centre of the screen after every trial to re-orient participants. Symbols remained on the screen until a decision was made. The second phase presented participants with the nine symbols on the screen in a circular arrangement to assess how well participants learned the order of the symbols in the training phase. Participants ranked the symbols from smallest to largest, corresponding to the order they learned during training. Each time participants clicked a symbol, it disappeared from the screen, resulting in one fewer symbol to select from. This continued until all symbols were removed from the screen.

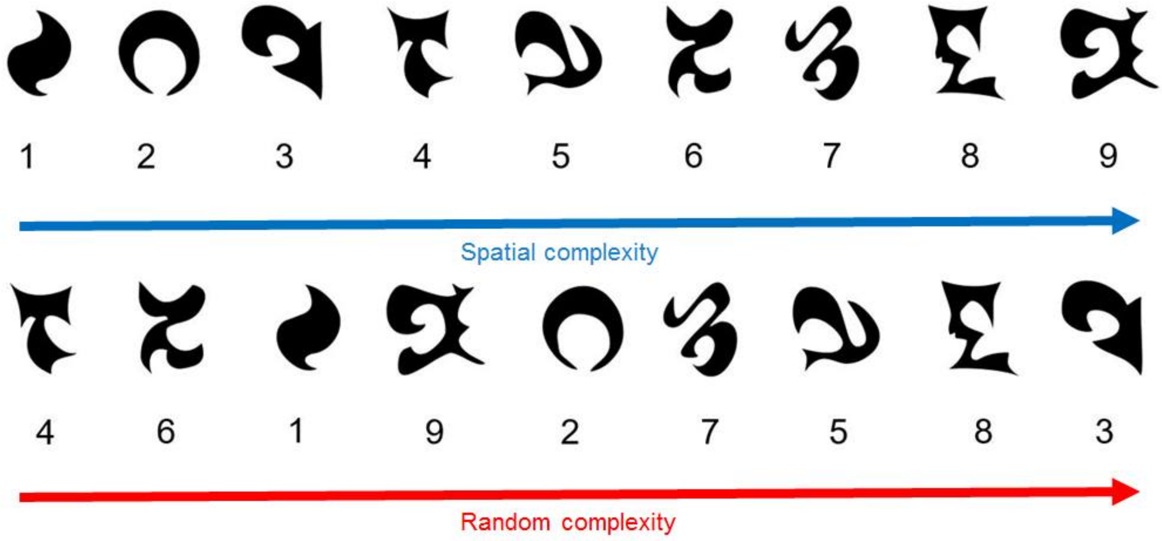

Fig 3. The Sunúz symbols ordered spatial complexity (top) and random complexity (bottom). Complexity was determined by perceived complexity ratings in Study 1.

Procedure

Testing was conducted in a quiet testing space. Participants were told that they would be asked to complete a series of basic tasks on some novel symbols. The study began with instructions informing participants that they would be asked to determine the numerical order of some symbols in a paired comparison procedure. Participants were first told that two unfamiliar symbols would appear on the screen, and that one symbol was 'larger' than the other one. Participants were also instructed that we wanted them to learn the order of the symbols, and to click on the symbol that was 'larger'. Finally, participants were told that feedback would be provided by the size of the symbols changing to reveal the larger symbol. Each trial presented a pair of symbols, where every symbol was paired with every other symbol excluding itself in both left-to-right and right-to-left orientations. There was a 500ms gap between each trial. After each training block, participants were shown a new instruction screen, which informed them that they were now required to order all symbols from smallest to largest. This task was designed to assess global symbol list learning. Participants were then presented with the symbols in a circular arrangement, and ranked the symbols according to the order in which they were learned during training. There were a maximum of 6 blocks consisting of 72 feedback trials each, with a total of 432 trials.

Results and discussion

To determine whether there was a difference in how accurately and quickly the spatial complexity group and the random complexity group learned the order of the novel symbols, the mean group Spearman's rank order correlations were calculated for both groups for each training session (Fig 4). Several participants did not complete the six training blocks within the 25 minute time allocated for the session (*n* = 2 for the spatial complexity group, and *n* = 8 for the random complexity group). For these participants, their last correlation they achieved was used to replace missing blocks. Spearman's rank order correlation coefficients were calculated for each participant between their ordering in each block and the 'correct' order. The 95% confidence intervals were also calculated for the average of each group in each training block. The spatial complexity group (blue) outperformed the random complexity group (red). The spatial complexity group learned the correct order quickly and consistently, often achieving near perfect performance from the second training block, with an average correlation coefficient of almost 1.0. In contrast, the random complexity group displayed much slower learning, and achieved an average of around 0.7 by the sixth and training block.

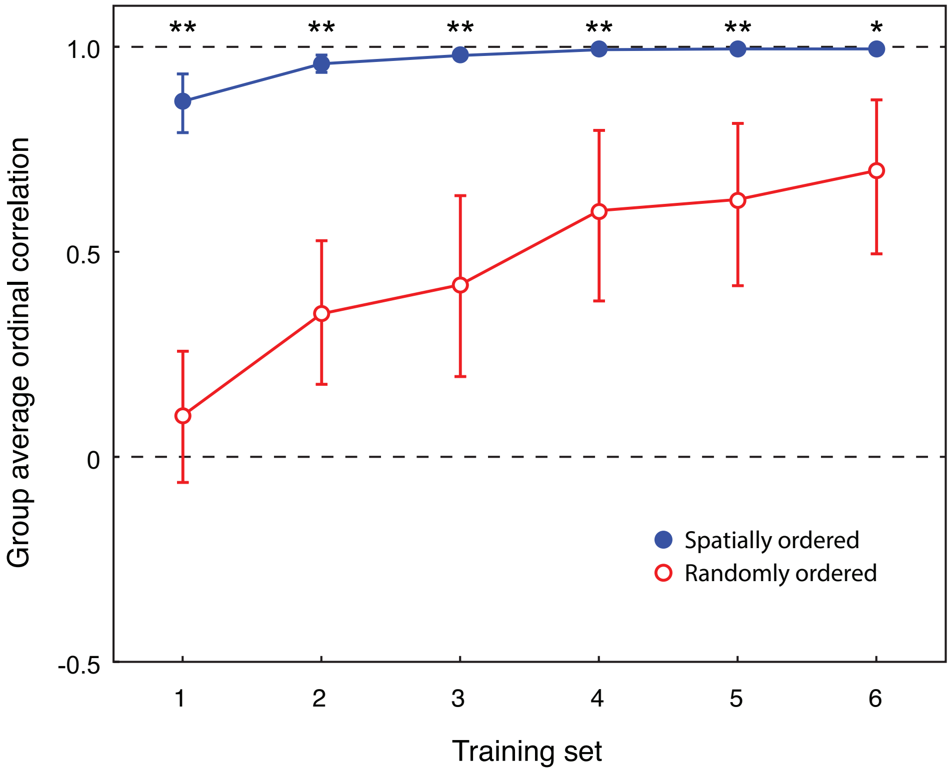

Fig 4. Group average Spearman's rank order correlations (y-axis) for the spatially ordered group (blue filled circle) and the randomly ordered group (open red circle), as a function of training session (x-axis). The error bars represent the 95% confidence interval. The dashed lines are for correlations of 0 (no correlation) and 1.0 (perfect correlation). Two asterisks is significance at the $p < .01$ level, and one asterisk is significance at the $p < .05$ level.

Examining the error bars for the two groups, there is little variation in the rate of learning for the spatially ordered group. There is a small amount of variation at the first training block, which reduces substantially after the second training block. However, the random complexity group displayed much more variation in learning the correct order, with large error bars for every training session. The variability in these data suggested that it would be useful to split the two groups into different levels of learning performance, to better examine the differences in learning within the spatially ordered group (Fig 5) and within the random complexity group (Fig 6).

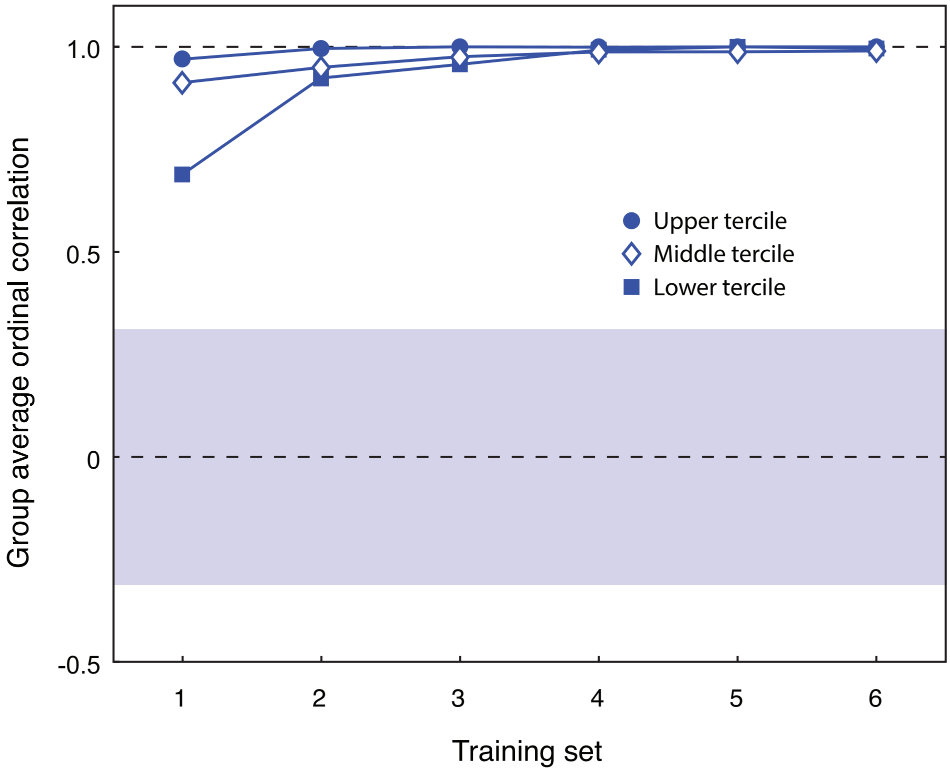

Fig 5. Spatial complexity group split into fastest (filled circle), middle (open diamond), and slowest (filled square) learning terciles as a function of training session (x-axis) by mean Spearman rank order correlation (y-axis). The blue bar is the bootstrap 95% confidence interval for the spatially ordered group. The dashed lines are for correlations of 0 (no correlation) and 1.0 (perfect correlation).

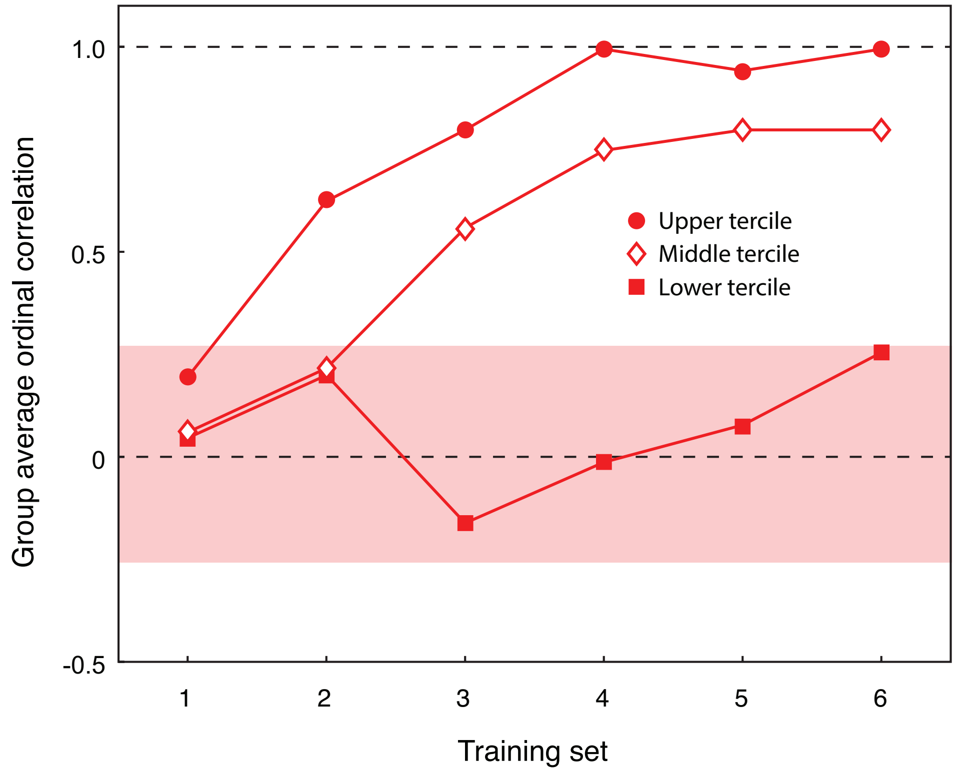

Fig 6. Random complexity group split into fastest (filled circle), middle (open diamond), and slowest (filled square) learning terciles as a function of training session (x-axis) by mean Spearman rank order correlation (y-axis). The red bar is the bootstrap 95% confidence interval for the spatially ordered group. The dashed lines are for correlations of 0 (no correlation) and 1.0 (perfect correlation).

The learning profiles for the spatial complexity group are similar in structure. The slowest learning tercile display lower performance in the first training block compared to the middle and fastest terciles. However, by the second training block, the performance for all three terciles is comparable. This difference could be possibly explained by the fact several of the symbols received similar complexity ratings particularly towards the middle of the symbol list in Study 1. The 95% bootstrap confidence interval for the spatially ordered group confirms that these data cannot be explained by chance, and that the participants successfully learned the correct order of the novel symbols in this condition.

In comparison, in the random complexity group, the learning profiles of the fastest, middle, and slowest learning terciles were different from each other. While the fastest and the middle learning terciles are comparable (with the middle learning tercile having a similar format, albeit slower), the slowest learning tercile shows little evidence of having learnt the correct order of the symbols. The 95% bootstrap confidence interval for the random complexity group confirms this, and suggests that performance of this tercile is not distinguishable from chance performance. Indeed, the final correlation achieved for the slowest learning tercile is only around 0.2. Overall, while the fastest and middle terciles for the random complexity group display some evidence of successfully learning the correct order, the slowest learning tercile failed to learn the correct order of the novel symbols.

There are various possible reasons why some participants failed to learn the randomly ordered novel symbols. Some participants could have failed to recognise the different symbols. Alternatively, other participants could have failed to recognise the correct position of the symbols, or did not associate the symbols with a particular order. From looking at the fastest and middle learning terciles, it can be seen that learning this symbol list was not impossible, because some participants still learned the correct order of this list even without the spatial complexity cue. Therefore, it seems that some participants may have difficulty learning a list of novel symbols, such that six training blocks was not enough time to learn the correct order. A possibility is that participants could have learned ‘chunks’ of the correct order, but failed to learn other parts of the list, giving them a low correlation (e.g., learning 1, 2, 3 but not 4, 5, 6, and then learning 7, 8 9). Another possibility is that participants in the random complexity group could have had an expectation that the novel symbols would be ordered by spatial complexity, and when this was not the case, learning the correct order of these symbols became very difficult. It is important to note that the sample of the current study were educated university students. If this task was given to younger children with less education, we would expect performance in the random condition to be even worse, perhaps with no participants learning the correct order.

General discussion

The aim of this research was to determine whether learning a set of novel symbols would be facilitated if symbol list order was associated with spatial complexity. This was done to investigate the ordinal relations that may underlie number representations, and to determine if ordinal lists are served by a common cognitive representation. Specifically, the goal was to determine if ordering symbols according to spatial complexity improves list learning in terms of speed and accuracy. The key finding of this research was that participants in the spatial complexity group learned the order of the new symbols more quickly and accurately than participants in the random complexity group. This finding support the view that underlying information contained in symbols may provide relational information that influences learning representations of ordered lists.

This study found that participants who learned the novel symbol set where list order was associated with spatial complexity often learned the correct order in the first training block, and performance thereafter was almost at ceiling level for most participants. In comparison, in the random complexity condition, learning the relationships between novel symbols was more difficult. In this condition, most participants never reached ceiling performance, while many also failed to score above chance level performance after the six training blocks. This suggests that participants can easily use the spatial complexity of novel symbols to assist them in making paired comparison judgements, and to reconstruct the global order of the novel symbol list. A possible caveat is that it is not entirely clear whether symbol complexity facilitates learning, or whether randomised spatial complexity inhibits list learning. In either case, it seems there is a relationship between learning an ordered list of novel symbols and the spatial complexity of those novel symbols.

Symbols are often thought to be abstract objects [29]. However, if this was the case, then we should not have found any difference between the two conditions, especially since this study accounted for differences in recognition of the symbols by including the random complexity condition (which also used Sunúz symbols). If participants were not attending to the underlying spatial properties of the novel symbols when learning the ordinal lists, there should have been no advantage for the spatial complexity condition over the random complexity condition. This casts doubt on the claim that numerical symbols are abstract, and suggests the spatial complexity of number symbols and the relational information they convey should be considered when describing internal representations of number. This research suggests that when spatial complexity is paired with relational information, this is sufficient to create an internal ordinal representation similar to the putative MNL.

The finding that the property of spatial complexity had an effect on learning has implications for thinking about the way ordinal lists are represented. In relation to the MNL metaphor [29], which places large emphasis on the numerical values of number symbols (cardinality), the findings of this study suggest that ordinality and the underlying spatial properties of symbols should be highlighted more when learning numbers, because number symbols may provide information which assists with learning. For instance, the ease with which the first three numbers in a range of cultures are learned may have something to do with their spatial complexity [23-25], which could inform how other numbers are mentally represented. This is highlighted by looking at the correlations for Study 2. While the spatial complexity group performed well overall, the random complexity group displayed unpredictable responses, with some participants failing to learn the correct order, even after six training blocks. This is may be because the random complexity group had no consistent spatial complexity information in the symbols to assist them learning the correct order.

This research found that ordering symbols by spatial complexity enhanced novel symbol learning. This suggests that learning number symbols could also involve this relational component. The property of spatial complexity may also be a relevant dimension to consider in creating number representations, because it strongly relates to how ordered sequences or patterns were generated in this study. Indeed, as mentioned above, relational information and spatial complexity seem to be sufficient to derive an internal ordered representation of a novel symbol set. Furthermore, since the spatial complexity of a symbol can only be understood *in relation to* other symbols, discussing number representations without considering the relational structure of symbols means important factors of number representations are likely being overlooked. Indeed, research is beginning to highlight the importance of symbolic number competence and knowledge of ordinality with formal math achievement [30], and this study adds to this body of research by suggesting that underlying spatial properties of symbols (e.g., spatial complexity) may be important to consider when thinking about number representations.

Beyond symbols containing relational information, an important point to make is that the perceived spatial complexity of the symbols was fairly consistent across all participants in Study 1. When asked to order a new set of symbols by complexity, most participants ordered the symbols in the exact same way, despite not being told what to use to perform these complexity ratings. This suggests that people may spontaneously order lists by spatial complexity, which may further explain why the random complexity group performed so poorly, because the relational structure of the symbols was incongruent with the feedback provided. Extending this finding to number, it suggests that the Hindu-Arabic numerals may be a difficult symbol set to learn, especially for those with maths difficulties such as dyscalculia [31]. For example, if a child is struggling to learn number, they may have a poor knowledge of numerical symbols, similar to the random complexity group in this study. From these findings, it seems that it may be beneficial to emphasise the spatial properties of numerical symbols more during number learning in an educational setting.

A limitation of this study is that it did not test whether the novel symbols in this study acquired mathematical properties, which may limit the inferences that can be made from this research. The next step for future research would be to create mathematical relationships between these symbols (such as with addition or multiplication tasks), to examine whether these symbols can take on the computation properties of number. It would also be interesting to ask participants to complete a 'symbol-to-position task' (similar to a number-to-position task) for these novel symbols for both the spatial complexity and random complexity conditions. If it was found that the spatial complexity condition produced more accurate responses, then this would suggest that spatial complexity may be used when completing numerical tasks, which means metaphors of number representation should take this into account. For now, this research can be interpreted as a 'proof of concept' that the spatial complexity of symbols can influence the learning of ordinal lists of symbols, which most likely also underlies number representations. This also suggests that the MNL metaphor may be better thought of as a 'mental ordinal line' that underlies ordinal lists more generally.

Overall, this research shows that underlying spatial properties of symbols (such as spatial complexity) may be important for learning internal ordinal relationships between symbols in a list. By asking participants to learn a novel symbol set, this study was able to manipulate learning in adult participants. This research found that when spatial complexity was linked with list order, this aided learning. In this case, spatial complexity was used as a proxy for ordinal relationships between symbols to help participants learn the correct order of the symbols. This suggests that spatial complexity of symbols should be accounted for in models of number representation, and that research should consider the relational information between number symbols. Since an ordinal representation was created with novel symbols, it seems likely that a more general cognitive representation underlies ordinal lists.

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5.3. Summary of Chapter 5

Studies 2a and 2b (Podwysocki et al., 2020) aimed to achieve two key objectives. Firstly, Study 2a found a novel symbol set with a consistent relationship to complexity to use in Study 2b. Secondly, Study 2b conducted a relational training study, where participants were randomly assigned to learn a list of symbols in order of either spatial complexity, or random complexity (using the ratings from Study 2a) via a paired comparison procedure. The results of this study showed that when spatial complexity was associated with list order, participants learned the order of the symbols much more quickly and accurately when compared to participants who learned the random order. This suggests there is relational information present in symbols that may influence the form of internal ordinal cognitive representations, and that this may also extend to number representations. This suggests that the MNL metaphor should be extended to include these ordinal relationships, and that there may be a more general representation that encompasses all ordinal lists. The next chapter, the General Discussion, attempts to bring together all the findings from the last three chapters in terms of theoretical and practical implications.

Chapter 6
General Discussion

The aim of this thesis was to examine whether the mental number line (MNL) metaphor should be extended to include ordinal relations between numbers, and to test the nature of these ordered relationships. This thesis was motivated by the claim that through formal education, children undergo a shift from logarithmic to linear number representations, which is often explained by a parallel 'shift' in the MNL representation (Siegler & Opfer, 2003). It was suggested that some of these claims may have been overstated in the literature. The first goal was to empirically test claims that have been made regarding the scaling properties of the MNL, and to clarify whether the MNL is a flexible representation. The second goal was to further investigate the number-to-position (NP) task, and determine whether this task reflects ordinal representations more generally. While the NP task has been interpreted as a direct measure of the MNL, the NP task may not be measuring number representations specifically, and may instead reflect the structure of ordinal lists. The third goal was to test how ordinal lists are represented and learned with the use of novel symbols, and to clarify whether the representation that underlies number is shared by ordinal lists more generally.

To address these issues, four studies were conducted via two converging lines of learning studies. The purpose of the pilot study (Chapter 3) was to examine whether the MNL representation could be changed from linear to logarithmic by providing logarithmic feedback on a NP task. Study 1 (Chapter 4; Podwysocki et al., 2019) aimed to test whether the linear properties of the MNL (as measured with the NP task) is unique to number, or whether it is a feature of ordered lists more generally. This was done by comparing NP task performance to performance on an alphabet-to-position (AP) task in standard and reverse conditions. Study 2a (Chapter 5; Podwysocki et al., 2020) assessed two novel symbol sets for possible use in Study 2b. The goal of Study 2b (Chapter 5; Podwysocki et al., 2020) was to test whether ordering novel symbols by spatial complexity would facilitate learning ordinal relations between pairs of symbols, to help answer the question of whether a common representation underlies ordinal lists. In this chapter, the main research findings for each empirical study are reiterated, followed by a discussion of the theoretical and practical implications of this thesis for understanding number representations and numerical learning more generally. Finally, directions are suggested for future research in this area of numerical cognition, followed by conclusions for the current research.

6.1. Key research findings

6.1.1. Pilot study

The purpose of the pilot study was to test whether a small group of adult participants could be trained to respond logarithmically on a NP task with logarithmic feedback. Prior literature has claimed that with age and experience, children transition from responding logarithmically to responding linearly on the NP task (Siegler & Opfer, 2003). Studies have also shown that adults can adopt a logarithmic MNL with feedback (Chesney & Matthews, 2013; Huber, et al., 2014; Matthews & Chesney, 2011), and that a single trial of feedback can alter the scale of the MNL (Opfer & Siegler, 2007). Therefore, if the existing MNL metaphor is correct, then the MNL should be able to change with logarithmic feedback.

In the pilot study, participants completed a series of training sessions, where the time course of the training and the type of training were varied between participants. For training, participants completed approximately 5,000 trials of logarithmic feedback on a NP task. This was done over a one week, two week, or four week period. Participants were assigned to either a 'no bias' training condition (where the numbers to be estimated evenly sampled the number line), or a 'low bias' training condition (where the numbers to be estimated oversampled from the lower end of the number line). While previous studies have shown training the MNL to be successful, they have not considered whether training is long lasting and whether there are individual differences in the integration and learning of logarithmic feedback. The pilot study's purpose was to clarify these issues and determine whether a group of adults could adapt to logarithmic feedback and change their MNL to be more logarithmic.

The findings of the pilot study revealed that, while all participants successfully learned the logarithmic function, participants did not respond logarithmically on the NP task, even after extensive logarithmic training. Looking at participants individually, this study revealed small but systematic deviations from linearity in NP task performance, as well as unique variations in NP task performance. Despite these variations in NP task responses, the results showed that participants were relatively linear before, during, and after logarithmic training. This finding suggests that the MNL is not flexible in adult participants, at least when measured with the NP task (although this could be a product of using Hindu-Arabic numerals). This may go against the claims made in the literature that the MNL is a "dynamic, continually changing structure" (Siegler & Lortie-Forgues, 2014, p. 145), and suggest a more nuanced theory of the MNL metaphor is required. Indeed, if the MNL cannot change to meet task demands, the current MNL metaphor may have limited usefulness in describing how number is represented, and may not sufficiently explain how humans perform mathematics.

6.1.2. Study 1

The aim of Study 1 was to test if number representations are unique, or whether they share properties with other ordinal lists. While past research has assumed that patterns on the NP task directly reflect the form of the MNL (Siegler & Opfer, 2003), there is evidence that responses on the NP task may be influenced by other factors. As mentioned in Chapter 2, Barth and Paladino showed that responses could reflect an increase in proportion judgment strategies, rather than the underlying structure of the MNL representation. Also, Cohen and Blanc-Goldhammer showed that responses on the NP task can be altered by changing the form of the NP task (the authors used an 'unbounded' NP task and found this produced linear responses, whereas a traditional 'bounded' NP task produces patterns of response consistent with a proportion judgement model). There is also a growing body of literature suggesting that the linear pattern of performance attributed to the MNL can be elicited with other ordered lists with no inherent numerical value, such as letters of the alphabet (Berteletti et al., 2012; Hurst et al., 2014). Taken together, these studies suggest that the NP task is not a direct measure of the underlying MNL, but instead may reflect ordered lists more generally.

In Study 1, participants completed a series of NP and AP tasks to determine whether performance on a spatial mapping task is unique to number, or if letters of the alphabet can elicit the same linear pattern of responses as numbers. While previous research has attempted to address this issue, it had not considered whether (1) systematic deviations from linearity exist as a function of number or letter symbol set; (2) the dissociation of spatial position by requiring either left-to-right or right-to-left judgments has the same effects on spatial mapping for numbers and letters; and (3) response time patterns for numbers and letters are similar for judgments when the directions of space and symbol order are matched or mismatched. If estimation patterns and response times are similar for NP and AP tasks, it would suggest that the NP task may reflect the way ordinal lists are learned more generally, rather than anything specific about the representation of numbers along a putative MNL.

The findings of Study 1 showed that while response times were slower when estimating alphabet line stimuli compared to number line stimuli, they did not differ in the standard left-to-right compared to the reverse right-to-left orientations for number and letter line tasks. Furthermore, similar patterns of response times were found when marking stimuli on the number and letter lines. These findings imply that a general mental spatial representation may underlie ordered list processing (i.e., a representation that is not unique to number), and that a linear mental representation is not a unique feature of number. The results of this study thus suggest that number estimation data from the NP task should be interpreted with caution, as the NP task may not be a direct measure of the MNL, as currently assumed in much of the literature (e.g., Siegler & Opfer, 2003; Siegler & Booth, 2004; Booth & Siegler, 2006; 2008).

6.1.3. Study 2a

The goal of Study 2a was to determine which of two pre-existing novel symbol sets would have the most consistent complexity ratings to create a random complexity condition for testing in the next study (Study 2b). Since the results of Study 1 showed that letters have a consistent relationship with number (it is plausible that participants were translating letters into numbers to perform the AP task), and the results of the pilot study did not show any logarithmic training effect with Hindu-Arabic numerals (most likely because these number symbols are constantly being reinforced in everyday life), it was reasoned that a novel symbol set was required for the next stage of testing to reduce the possible influence of pre-existing symbols when studying the nature of number representations.

For Study 2a, two commonly used novel symbol sets in the ordinal learning literature were tested. The first set comprised nine Gibson symbols (Gibson, Gibson, Pick, & Osser, 1962), and the second set consisted of nine Sunúz symbols (Ager, 1998). In Study 2a, participants were asked to answer a series of mathematical questions relating to the two symbol sets to determine if there was a relationship between perceived number and complexity. Two examples of the questions asked are ‘Which symbol is best suited to represent a small number?’ and ‘Which symbol do you think is the most complex?’. Participants completed these questions for both Hindu-Arabic numerals and non-symbolic dot arrays for both the Gibson and Sunúz novel symbol sets, resulting in four different conditions.

The findings of Study 2a showed that while both novel symbol sets were relatively consistent in their complexity ratings (e.g., the most ‘complex’ symbols were also judged as larger in terms of numerosity, and the most ‘simple’ symbols were also judged as smaller in terms of numerosity fairly consistently), it was decided that the Sunúz symbol set had the clearest monotonic relationship with fewer outliers, so this symbol set was selected for examination in Study 2b. Another advantage of using the Sunúz symbol set is that the Gibson symbols were derived from letters of the alphabet (see Gibson et al., 1962, for a description of how this was achieved), and many of the Gibson symbols resemble symbols encountered in everyday life, which may influence the learning of these novel symbols.

6.1.4. Study 2b

The objective of Study 2b was to determine whether ordering a novel symbol set by the mediating property of spatial complexity could facilitate novel symbol learning. Insofar as spatial complexity can be used to represent or learn an ordinal list of novel symbols, this study aimed to determine whether a common representational format underlies ordinal lists. Study 2a showed that there may be a relationship between perceived number and complexity ratings. Study 2b aimed to extend this finding, and determine whether the property of spatial complexity has an impact on ordinal list learning. Furthermore, while Study 1 showed that letters of the alphabet and numbers may share a common representational format, the implications of this finding may be limited by the fact that numbers and letters are learnt in a similar way, and at a similar developmental stage. If novel symbols are learned more easily when they are ordered by spatial complexity, then this would imply that there may be underlying spatial information in symbols that could affect how ordinal representations are formed. This would suggest that the representation assumed to underlie number may be a more general property of ordinal lists.

For Study 2b, two different training conditions were devised for learning the order of the novel symbols. In one condition, participants were required to learn the symbols where the symbols were ordered by spatial complexity (determined by median perceived complexity ratings in Study 2a). In the second condition, participants were required to learn the symbols where the symbols were ordered randomly, such that the spatial complexity of the symbols was not informative of their order in the list. To achieve this, participants completed a paired comparison task whereby pairs of symbols were shown on the screen, and participants were asked to select which symbol was numerically ‘larger’. Once participants had made their selection, the larger symbol would appear larger on the screen, and the smaller symbol would appear smaller on the screen, in order to teach participants the order of the symbols via relational feedback. The aim of this feedback was to avoid participants learning these symbols by a direct mapping association (e.g., ‘this novel symbol = 1’). If underlying spatial properties (such as spatial complexity) facilitate list learning, then participants who learned the symbols ordered by spatial complexity should perform better than participants who learned the symbols where there was a random relationship between spatial complexity and list order.

The findings of Study 2b revealed that participants who learned the novel symbols ordered by spatial complexity vastly outperformed participants who learned the symbols with a random order. Participants who learned the symbols ordered by spatial complexity all learned the correct order by the third training block and performed at almost ceiling level in subsequent blocks. In comparison, participants in the random complexity condition often failed to learn the correct order, even by the sixth (and final) training block. This study found that symbols may convey relational information through underlying spatial properties such as spatial complexity when presented in pairs, suggesting that spatial complexity may be a factor that influences the way ordinal representations are learned. This implies the MNL metaphor should be extended to include ordinal representations more generally. Overall, these findings suggest that the MNL is composed of ordered relations, and that these relationships may be facilitated by the underlying spatial information contained in symbols. This suggests that a more general ordinal representation underlies number processing.

6.1.5. Summary of the research findings

There are four main findings of note. Firstly, the pilot study showed little evidence that logarithmic feedback changed the way participants performed on a NP task, even though all participants successfully learned the logarithmic function. Secondly, Study 1 found that both numbers and letters were responded to linearly on a spatial mapping task (even in reverse list order), suggesting that the NP task should not be used to make direct inferences regarding the MNL. Thirdly, Study 2a demonstrated that the Gibson and Sunúz symbol sets could both be ordered by perceived complexity. However, the Sunúz symbols were chosen for Study 2b as they had a clearer monotonic relationship, had fewer outliers, and were not derived from letters of the alphabet (unlike the Gibson symbols). Fourthly, Study 2b found that participants who learned the Sunúz symbols ordered by spatial complexity outperformed those in the random complexity condition, suggesting that the underlying spatial properties of symbols may play a role in shaping ordinal representations similar to the MNL. This finding suggests that the representation thought to underlie number may be a general property of ordinal lists, because an ordinal representation comparable to the MNL was derived from relational information and spatial complexity only. This finding cannot be explained by a cognitive representation that is specific to number. In the next two sections (sections 6.2 and 6.3), the theoretical and practical implications of the research findings of the current research are discussed in relation to the broader literature.

6.2. Theoretical implications of the research findings

The goal of this research was to clarify the MNL metaphor, and to examine the role of ordinal relations in the MNL metaphor from a learning perspective. In this section, the broader implications of the findings of the four studies will be discussed in relation to previous research. Firstly, the implications of the research for the NP task as a measure of the MNL representation will be reviewed. Secondly, what this research reveals about the importance of considering number symbols and relational information in terms of the MNL metaphor will be discussed. Thirdly, what number training and novel symbol learning reveals about the MNL metaphor and number representations more generally will be considered.

6.2.1. The number-to-position task

In Chapter 2, it was suggested that one of the main issues with the MNL metaphor is past research drawing a parallel between estimation patterns on the NP task and the MNL. These studies state that the NP task directly reflects the MNL, and as a result, the NP task is now widely used in educational settings to assess and 'improve' mathematical ability via linear training programs (e.g., Booth & Siegler, 2006, 2008; Laski & Siegler, 2007; Sasanguie et al., 2012; Sasanguie, Van den Bussche, & Reynvoet, 2012; Sasanguie et al., 2013). Because of the link between the NP task and the MNL, the NP task is also used to make direct inferences regarding the scaling properties of the MNL, for example, that the MNL shifts from being logarithmic in scale to linear in scale with age and education (Siegler & Opfer, 2003). However, there is growing evidence that the linearity observed on the NP task may also be found with other non-numerical lists, such as letters of the alphabet or months of the year (Berteletti, Lucangeli, & Zorzi, 2012; Gevers, et al., 2003; 2004; Hurst et al., 2014). The current research provides further support for the claim that the NP task should not be interpreted as a direct reflection of the MNL. These results suggest that the NP task is measuring the ability to judge proportions along a horizontal line regardless of the symbols being used, and therefore cannot tell us anything specific about the MNL representation.

The NP task was examined from several different perspectives. To begin, the pilot study cast some doubt upon the MNL metaphor, because an extensive logarithmic training program failed to elicit significant changes in performance on a NP task. However, a problem here may have been with the NP task itself. It can be assumed that if all participants successfully learned the logarithmic function (which they did), then the MNL of these participants would have been logarithmic, which, according to the number training literature, should have manifested as logarithmic performance on the NP task. Nevertheless, this was not the case. All participants remained relatively linear in terms of NP task performance, even at the end of almost 5,000 logarithmic training trials (participants completed a NP task before training, after each of the six training blocks, and finally a week after training. They also completed many more logarithmic training trials than other similar studies, e.g., Opfer & DeVries, 2006 asked participants to complete three blocks of ten trials). This implies that the NP task may not be an accurate way of measuring the number representation participants are using to complete a specific number task (such as responding logarithmically on a NP task in the pilot study). It seems more likely that the NP task was treated as a separate spatial mapping task, and that the NP task does not necessarily reflect the underlying MNL.

The primary evidence casting doubt on the NP task was gained from Study 1, which compared how participants responded on an alphabet-to-position (AP) task and a standard NP task. The rationale for this study was if similar patterns of responding are present on both tasks, then this would suggest that the NP task does not tell us anything specific about number representations per se, but instead reflects the way that ordinal lists are learned. This is because there is no advantage to letters of the alphabet (a list with no inherent magnitude) being placed linearly on an AP task. The results of Study 1 showed that numbers and letters were not only both linear, but that both symbol sets remained linear for both directions of the task (i.e., both left-to-right and right-to-left orientations), and also had similar response times. As such, while 'number' is necessary to complete the NP task (i.e., individuals must be able to translate symbols to a position on the line in terms of proportion judgment), the NP task should not be used to make any specific inferences about the nature of the representation of number specifically, because it seems that the NP task may elicit similar patterns of response for ordinal lists more generally. This means we should not use a task that relies on prior learning (the NP task) to make inferences about the representation of number. The findings of Study 1 put into question studies that make strong assumptions regarding the MNL from NP task performance (e.g., Booth & Siegler; 2006, 2008; Laski & Siegler, 2007; Opfer & Siegler, 2007; Opfer & Thompson, 2008; Thompson & Opfer, 2008; 2010), and suggest conclusions from these studies should be interpreted with a degree of caution.

Since the results of Study 1 found that the NP task most likely does not reflect the MNL, given that letters from the alphabet produce similar linear response patterns to numbers, another study was conducted to examine if number is served by a more general ordinal representation, and whether underlying symbol properties play a role in shaping ordinal representations. Studies 2a and 2b examined the MNL from a different perspective via a relational learning study with novel symbols. Novel symbols were employed to avoid the influence of over-learned and over-familiar Hindu-Arabic numerals. This is because the results from the pilot study may have been due to using familiar symbols, and it also could be argued that because numbers and letters of the alphabet are learnt in a similar manner, and at a similar developmental stage, letters of the alphabet may lend themselves to a representation similar to the MNL, especially when measured with a spatial mapping task. Studies 2a and 2b showed that a representation similar to the MNL can be created with novel symbols, which suggests that the MNL is not specific to number, and cannot be measured with a NP task. In Study 2b it was shown that when novel symbols are ordered by the mediating property of spatial complexity, they are learned more effectively than if they are ordered by random complexity. This finding may correspond to the observation that the first three numbers of the Hindu-Arabic number sequence are also ordered by spatial complexity (Dehaene, 2011), which could influence the way other numbers in the numerical sequence are learned. As such, the way in which ordered lists are learned may be what determines performance on a NP task, rather than the MNL. Overall, if the NP task reflects relative position or other kinds of information found in number symbols, then this suggests that the NP task probably cannot tell us anything about the abstract representation of numbers in the form of the MNL.

6.2.2. Number symbols and ordinality

In Chapter 2, the concepts of number symbols and ordinality were considered as key factors potentially underlying the MNL representation. While the pilot study and Study 1 sought to test the MNL via the training methods set out in the literature (i.e., the NP task) the results of these two studies made it clear a different approach was needed to test the MNL to avoid interference from Hindu-Arabic numerals. In the context of this thesis, this was achieved by examining the underlying spatial properties of novel symbols, and examining the role of relational information between novel symbols in Studies 2a and 2b. These studies suggest that the underlying spatial properties of symbols may play a greater role in forming the representation of ordered lists than is currently assumed in the literature, and that there may be a representational format common to ordinal lists more generally. As noted in Chapter 2, most MNL research has primarily been concerned with cardinality, and there is a relative lack of research on ordinal relations or ordinality, which most likely is the key factor underlying the MNL. Also, while prior research has claimed that numerical symbols are abstract and do not influence number representations, this thesis suggests that symbols may contain underlying spatial information that can assist the formation of internal ordinal representations more generally, especially when presented with relational feedback.

The findings of Study 2a suggested there may be a link between the underlying spatial properties of symbols and their ordinality, and that humans may have a tendency to order novel symbols according to spatial complexity. In this study, two novel symbol sets were compared to determine if there was a link between perceived number (both symbolic and non-symbolic) and the perceived complexity of the novel symbols. Study 2a found that symbols which were simpler in form (i.e., fewer lines or curves) were reliably ranked as representing a smaller number, and symbols which were more complex in form (i.e., more lines or curves) were reliably ranked as representing a larger number. Another interesting finding of note is that despite not being told how to order the novel symbols by complexity, most participants ordered the symbols by complexity in the same way. This means that humans may pay special attention to the underlying spatial properties of symbols during learning, rather than treating the symbols as purely abstract objects. The findings of Study 2a suggested that ordering a novel symbol set by spatial complexity may influence how these novel symbols are learned, which was empirically tested in Study 2b.

Study 2b examined the role of spatial complexity in ordering a list of novel symbols more closely, and directly tested whether the spatial complexity of novel symbols facilitates ordinal list learning. One novel symbol set was tested (the Sunúz symbols), and two groups of participants were asked to learn the symbols either ordered by spatial complexity or ordered randomly. The findings of Study 2b showed a clear advantage to learning the order of the symbols when there was an association between the ordinal position of the symbol and the spatial complexity of the symbol. This suggests that a representation akin to the MNL can be derived when symbols are ordered by spatial complexity, and that novel symbols may be represented in a similar way to number and letter symbols. The implication is that the MNL should be thought of as a more general ordinal representation. These results also suggest when provided with relational feedback, participants can use the underlying spatial properties of the symbols to assist with learning the symbols, most likely because this is the most salient information available to participants at the time.

Perhaps even more interesting is the relatively large disadvantage to the participants who were assigned to the condition that had a random association to spatial complexity. In the random complexity condition, learning was much more difficult. It almost seemed that there was interference to learning when there was a mismatch between spatial complexity and ordinality. Indeed, the poor performance of the random complexity group could have been more obvious if we used a younger sample with less formal education. A possibility is that instead of spatial complexity facilitating learning, having a random complexity condition actively impairs learning. However, in either case, it seems that spatial complexity has an impact on symbol list learning. Also, as there was no overlap between participants assigned to each condition, this decrease in performance was not due to poor recognition of the symbols. Overall, it appears humans may be able to generate ordinal representations similar to the

MNL based on relatively isolated pieces of information (pairwise comparisons) when a mediating judgement framework such as spatial complexity is present. In the absence of this mediating construct (i.e., the condition where there was a random link between spatial complexity and list order), this process becomes difficult, and learning is impaired.

Linking these results back to the nature of the MNL representation, the findings of Studies 2a and 2b suggest that ordinal relationships between number symbols may form the basis of the MNL, and that there may be a more general representation underlying ordinal lists. This also may be related to the observation that when first learning number, there is a link between the spatial complexity of number symbols and the numerical quantity that number symbol represents for a variety of different cultures (Dehaene, 2011; e.g., in Hindu-Arabic numerals 'one' is one vertical line, in Chinese numerals 'one' is one horizontal line, and in Roman numerals 'one' is one vertical line, etc.). Indeed, while the MNL metaphor assumes numbers are purely abstract, the results of Studies 2a and 2b suggest that underlying spatial properties in symbols may play a role in forming internal ordinal representations. For example, if the current MNL metaphor is correct, and numbers are abstract, then why should it matter which symbols you use? It may be the case that instead of a fixed MNL, or even a flexible MNL, there may just be 'stuff' in the brain to do number with, or even a kind of MNL that is created on the spot to do number tasks with, and that relational information underlies this process. This makes sense in the context of Study 2b, because the spatial complexity of one symbol can only be understood *in relation to* other symbols, stressing the importance of relational information in number representations. As such, a more nuanced account of number representations is needed that includes ordinal relations between symbols.

6.2.3. The mental number line metaphor

At the beginning of Chapter 2, it was argued that support for the MNL metaphor is lacking, because it heavily relies on evidence from the NP task to infer its form, despite the fact the NP task may not be specific to number (Hurst et al., 2014). This implies that the MNL (as it is currently defined) may not be the ideal metaphor for describing number representations. The studies presented in this thesis indicate that the MNL may not be an accurate metaphor for number representation, because as mentioned in section 6.2.1., it seems that the NP task does not tell us anything specific about the MNL; instead, this task may reflect ordinal lists more generally. Also, as noted in section 6.2.2., this thesis suggests that ordinality and numerical symbols may play a more important role in number representations than the MNL metaphor currently suggests. Taken together, the current research suggests that the MNL metaphor needs to be extended to account for ordinal lists more generally.

In the pilot study, some assumptions of the MNL were tested empirically with a logarithmic training study. If it is the case that the NP task directly reflects the MNL, then a shift in performance should have theoretically been observed when the MNL was trained according to methods used in the literature (Chesney & Matthews, 2013; Huber et al., 2014; Matthews & Chesney, 2011). However, the pilot study showed that while new algebraic functions can be learned quickly and accurately, this change was not reflected in the NP task, meaning that the MNL was also likely not changing to reflect these new algebraic functions. The pilot study showed that training to induce a logarithmic shift in the MNL only resulted in limited changes in NP task performance, with a high level of individual variation, even though all participants learned the new logarithmic function. This suggests that instead of encompassing all of math, the MNL may not be able to readily adapt to different number tasks. Instead, children and adults learn particular number to space mappings (or relationships) for particular tasks, which does not require the MNL representation. Indeed, a singular number representation may not be sufficient to reflect how numbers are internally represented, and it may be the case instead that relationships between numbers change to allow accurate task performance. Furthermore, the logarithmic-to-linear shift reported in the literature (Siegler & Opfer, 2003) may reflect developmental changes associated with a formal educational setting, where there is no practical use for logarithmic functions, rather than a change in the scale of the MNL representation per se.

The findings of Study 1 also made clear that the results of NP task studies are not specific to number, but apply to all ordered lists. This puts into question the degree to which linear performance reflects a linear numerical relationship in the form of the MNL. Study 1 suggests that linear performance may be an artifact of NP task performance rather than informative of the nature of the MNL representation, because linear performance can also be produced with letters from the alphabet. This may imply that instead of making the MNL linear, experience and formal education may change the understanding of relations between numbers. If this is the case, then completing number tasks may not require the MNL, because even if certain ranges of numbers can be changed for a task in a particular way, then this does not mean that the entire representation of number has to change in the same way. Moreover, if the NP task is ruled out as a key index of the MNL, then SNARC effect studies, spatial neglect studies, and studies examining the overlapping brain regions between number and space are all that remain as confirmation of a left-to-right oriented MNL. However, this is insufficient evidence for the MNL metaphor, because none of these effects is specific to number. Overall, Study 1 suggests that if the MNL representation exists, it is more likely a ‘mental ordinal line’ that is not specific to number, and extends to ordinal lists.

Further, evidence from Studies 2a and 2b showed that relational information, or relationships between symbols (e.g., as provided by the underlying property of spatial complexity), should be considered when discussing number representations. This is because these studies suggest that novel symbols contain underlying information that can influence the representation of ordinal lists. This finding cannot be accounted for by the current MNL metaphor, which assumes symbol objects are abstract. These results suggest that there may be a more general cognitive representation underlying ordinal list processing, including numbers and letters. This is because in Study 2b, a representation akin to the MNL was derived from relational information and spatial complexity. As the current MNL metaphor primarily focuses on the role of cardinality, the current research suggests that the MNL metaphor should be extended to consider the role of ordinality in number representations, and to encompass ordinal list processing more generally.

6.2.4. Summary of the theoretical implications

Overall, the findings of this thesis suggest that relational information and the underlying spatial properties of number symbols should be considered in future theories of number representation. The main theoretical findings of this thesis can be recapped as follows. Firstly, researchers should be cautious when using the NP task to make assumptions about the underlying representation of number. The NP task is not specific to number, and produces similar results for other ordered lists, such as letters of the alphabet. Secondly, the representation of ordered lists may not be entirely abstract, as is currently assumed in the literature. It may be the case that there are underlying spatial properties in symbols that shape ordinal representations (such as spatial complexity), and that ordinality is most likely the key component underlying the MNL. Thirdly, the current MNL metaphor does not appear sufficient to account for the representation of number. The MNL is not reflected in the NP task, as is often assumed, and does not take into account the importance of ordinality and number symbols in number representations. It seems likely that number processing is served by a representation that deals with ordinal lists more generally.

6.3. Practical implications of the research findings

Beyond this research contributing to the theoretical framework on the MNL and number representations more generally, it also provides some useful information regarding number learning and assessing number ability in an educational and developmental context. This thesis provides some clarification regarding the usefulness of the NP task, and also highlights the importance of learning ordinal relationships between number symbols in math education. These two ideas will be considered below, together with suggestions for implementation.

6.3.1. The number-to-position task in education

Firstly, the findings of this thesis suggest that data gained from the NP task should be interpreted with caution. While the NP task may be a useful diagnostic tool for identifying individuals with number processing deficits (e.g., dyscalculia; see Butterworth et al., 2011), the findings of this research suggest that the NP task is not informative regarding the underlying representation of number per se, and that the NP task reflects the properties of ordinal lists more generally. It is also arguable that the NP task simply measures the ability to judge proportions on a horizontal line. When testing number development with the NP task in an educational context, it is likely that what is being assessed is an individual's ability to make proportion judgments, and the symbols being used on the task are irrelevant. Indeed, all the number training studies using the NP task mentioned in Chapter 2 (such as Siegler & Opfer, 2003) should be read with this caveat in mind, as improvement in NP task performance does not necessarily mean that the underlying representation of number has changed or 'improved' (i.e., become more linear rather than logarithmic in form).

While research reviewed in Chapter 2 has reported that the linearity of NP task performance correlates with later math ability (e.g., Booth & Siegler, 2006), it seems that the ability to judge proportions along a horizontal line may relate to improvement in mathematical skills over and above the linearity of NP task estimates. It seems likely that when performing the NP task repeatedly via a training program, individuals learn how to master the NP task, which does not mean that the underlying representation of number is improving. Therefore, in future number training programs, the NP task should not be used to train number ability in isolation from other numerical tasks. Other number tasks, especially those that are not spatial in nature, should be used in conjunction with the NP task when trying to improve math skills. This research suggests that while the NP task could be used in math education to teach students the order of number symbols and to reinforce a linear ordering of number symbols, this may be where the utility of this task ends.

6.3.2. Number symbols and ordinality in education

Secondly, as mentioned towards the end of Chapter 2, the number cognition literature is beginning to uncover the importance of numerical symbols and ordinality in performing math tasks (e.g., Lyons & Beilock, 2011; Merkley, Shimi, & Scerif, 2016). The findings of the current research contribute to this literature by suggesting that more emphasis should be placed on learning numerical symbols in education in order to succeed at math. This research also highlights the importance of accurately learning the ordinal relationships between number symbols. While past research has placed major importance on learning the numerical magnitude each number symbol represents (i.e., cardinality), which has informed not only metaphors of number such as the MNL, but also how math is taught in an educational setting, the current research contributes to the claim that more emphasis should be placed on learning how number symbols are related to each other in terms of their order in the counting sequence (i.e., ordinality). It seems likely that when we represent numbers, we represent them in terms of how they relate to each other in an ordered sequence.

The current research suggests that our Hindu-Arabic numerals may be more difficult to learn because the link between the form of the symbols (in terms of spatial complexity) and the order of the numerical symbol in the list only exists for the first three numbers (i.e., 1, 2, and 3; Dehaene, 2011). As shown in Study 2b, constructing ordinal representations is assisted when there is a mediating judgment framework present (such as spatial complexity), which could apply to number representations. Indeed, when this framework is absent, learning becomes much more difficult, which means it is particularly important during education to ensure number symbols, and how they relate to each other in terms of ordinality, is accurately learned. Focusing on number symbols and the ordinal relationships between number symbols may assist individuals to better understand numbers. This could possibly be achieved in a manner similar to Study 2b, where participants learned novel symbols via 'greater than' relationships, instead of a direct mapping task. This could also involve highlighting the importance of the ordinal relations between number symbols in the counting sequence as well as the magnitude symbols represent. This may help give numerical symbols more meaning and assist math learning in an educational setting, which would be particularly useful to individuals suffering from mathematical learning difficulties.

6.3.3. Summary of the practical implications

Overall, from the research conducted in this thesis, it seems that the MNL may be composed of ordered relationships between numbers, and that instead of the MNL per se, there may be a more general ordinal representation. If this is the case, then there are two important implications of this thesis in a practical sense. Firstly, it seems that reliance on the NP task as a measure of general mathematical ability and a measure of the MNL should be decreased, because this thesis showed that the NP task most likely reflects the structure of ordinal lists more generally. Secondly, it also seems that numerical symbols and the ordinal information present between symbols should be stressed in early math education, because it is likely that ordinal relationships between symbols form the basis of the MNL representation.

6.4. Future research agenda

In this section, the limitations and implications the present research for future studies into the representation of number will be discussed. The plot study tested if the MNL was resistant to change via an extended training study, and Study 1 found that the NP task most likely reflects ordinal list processing more generally by using letters of the alphabet with a task similar to the NP task. However, there are opportunities for future research to expand the findings of Study 2b. This study tested the possible ordinal basis of number representations by using a relational learning paradigm with adult participants and novel symbols, and these findings could be expanded in several ways, which will be described below.

A possible limitation of Studies 2a and 2b is that it was unclear what mental strategies participants were using to complete the task. For example, reaction times were not studied, and cognitive issues such as working memory capacity and general intelligence were not measured. Another potential issue is that the nature of how participants ordered things along a scale (e.g., ordinal, interval, ratio, etc.) was not known, which could have impacted responses. Therefore, future studies could include a cognitive ability test in this experiment, or a post-experiment questionnaire, to determine what strategies participants were using.

It also may be useful to extensively train a smaller number of participants on a set of novel symbols over weeks or months, to more closely approximate learning Hindu-Arabic numerals. This would provide an opportunity to do more research on these symbols, and go beyond the initial creation of a representation similar to the MNL. It would be interesting to do this task with participants who are younger or have less formal education, to examine how performance varied as a function of age and education level. When deciding on a novel symbol set, there are several resources available, such as the symbols used in this thesis, or symbols such as the Brussels Artificial Character Sets, which were specifically designed to avoid interference from everyday symbols (Vidal, Content, & Chetail, 2017). In any case, this thesis found it is not enough to use existing number systems, or even familiar ordinal lists, such as letters of the alphabet, because these symbol sets contain information that interferes with learning new relationships. Furthermore, this thesis also found that symbols should be ordered such that the spatial complexity of the symbols is not informative of their list order, because Study 2b found that participants latch on to the underlying spatial properties of the symbols to make inferences regarding their ordinal properties. Future studies could even examine how participants learn different algebraic functions besides the standard linear function, such as logarithmic or reverse logarithmic, or train and test on different symbol sets.

The novel symbols could also be taught to participants in a variety of ways. In Study 2b, participants learned the novel symbols via relational information (participants learned which symbol was 'larger' in a series of pairwise judgments). Another way symbols could be taught to participants is in a more traditional one-to-one correspondence method (or 'mapping' task), such as indicating which symbol corresponds a Hindu-Arabic numeral. Testing whether there are differences in the eventual representation of these novel symbols that is dependent on the learning method used could provide important information regarding the most effective technique to teach number symbols in education. Another variation of training that could be employed would be to train participants on different sections on the list at different times (e.g., training the novel symbol equivalent of 4, 5, 6 before 1, 2, 3), and looking at whether this affects list learning, or changes the underlying representation.

There are also several interesting tests that could be run on participants once they have successfully 'learned' the novel symbol list. The novel symbols could be tested on a 'symbol-to-position' task, similar to the 'alphabet-to-position' task variation of the NP task used in Study 1. If novel symbols are estimated in a linear fashion on this task, or if there was a logarithmic-to-linear shift of estimation performance on this task, then this would suggest that novel symbols may be represented in a similar manner to symbols with an ordinal structure, such as numbers, or letters of the alphabet. This would then further suggest that ordinality is the property that underlies the mental representation of number, and that all symbols may have some sort of 'natural order' to them. Another test that could be performed once the novel symbol set has been learned is whether mathematical relationships can be elicited in these novel symbols, or whether the novel symbols can take on the computational properties of number symbols. This could be achieved by asking participants to complete a series of addition or multiplication tasks with the novel symbols (e.g., symbol A + symbol B = ?). If mathematical relationships can be created in novel symbols, then this would provide further evidence that the MNL metaphor needs to account for ordinal relations between symbols, and that the MNL may be composed of pairs of ordered relations between symbols.

Overall, the current research provides an avenue forward for future studies along the lines of novel symbol learning and ordinality. Future research would need to use symbols that were devoid of numerical information (or create a novel symbol set that achieves this), and conduct training studies to clarify the role of ordinality and symbols in mental number representations. This training could use different time periods, different learning methods, and different post-training assessment tasks to test how number representations are constructed in adults. Continuing the research started in this thesis will further our understanding of the nature of number representations, and clarify the roles of numerical symbols and relational information in creating these representations. Doing this may allow for a more nuanced MNL metaphor which could help further the number cognition literature.

6.5. Conclusions

This thesis examined whether the MNL metaphor should be thought of as comprising a series of ordinal relations, and to examine the nature of these relationships. It began by testing whether the MNL could be changed with training. This was achieved by conducting a logarithmic training study (the pilot study), which, if the assumptions in the literature were correct, should have elicited a logarithmic manner of responding on the NP task. However, this was not the case, and NP task performance was not altered by logarithmic training. The next step was to test the validity of the NP task by using letters of the alphabet (Study 1). This revealed that the NP task may not reflect number specifically, and that the NP task may reflect how ordinal lists are structured. This implies that the MNL metaphor needs to be expanded to account for ordinal relations, and that numbers may be supported by a representational structure that lends itself to processing ordinal lists more generally.

As the results of the first two studies suggested that ordinality may be a key factor underlying the MNL, the remainder of this thesis approached the issue of the MNL metaphor from a different view, specifically, investigating how ordinal representations are created. To do this, a novel symbol set was found to test the formation of ordinal representations (Study 2a), where the symbol set with the most consistent complexity ratings would be the best symbol set to use in Study 2b. The next step in this approach was to test if the spatial complexity of novel symbols affects learning ordinal representations. To do this, the next study tested whether learning novel symbols would be facilitated when symbol spatial complexity and list order were linked (Study 2b). This showed that participants learn the order of symbols more easily when the order of the symbol is linked to the spatial complexity, suggesting that there could be underlying information in symbols that influences how ordinal representations are learned, which most likely also extends to number representations.

Overall, this thesis shows that direct inferences about the MNL should not be drawn from the NP task. It also shows that the MNL metaphor needs to be more nuanced, and better account for ordinal relationships between symbols. Indeed, it seems the MNL metaphor comprises ordinal list processing more generally, and may be better thought of as a 'mental ordinal line'. Furthermore, the work with novel symbols in this thesis suggests that underlying spatial properties in symbols may influence how ordered lists are learned. This cannot be sufficiently captured by the current MNL metaphor, which assumes the MNL is an abstract representation. The findings of the current thesis provide a starting point for future research to continue testing the MNL metaphor to achieve a more complete account of underlying number representations, which should account for ordinal relationships between symbols.

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