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NETWORKED CONTROL SYSTEMS: AN EMULATION APPROACH TO CONTROLLER DESIGN

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Abstract: We overview our recent work on a design approach for networked control systems (NCS) that resembles controller emulation for sampled-data systems. In the first step, we design a controller ignoring the network and, in the second step, we implement the designed controller over the network with sufficiently fast transmissions and a given protocol. Our results have several features: (i) they apply to general nonlinear systems with disturbances; (ii) we obtain explicit (often non-conservative) bounds on the maximal allowable transmission interval that guarantee stability; (iii) and we show that this approach is valid for a wide range of network scheduling protocols. This provides a flexible framework for design of NCS that is amenable to various extensions and modifications, such as a treatment of dropouts and stochastic protocols, combined controller/protocol design for linear plants, and so on.

Keywords: Controller design, networked control systems, nonlinear, stability.

1. INTRODUCTION

Emerging control applications, such as drive-by-wire cars, often require some control loops to be closed over a network. The network operates as a serial communication channel that transmits signals from many sensors and actuators in the control system and, possibly, some signals from other unrelated applications that are connected to the network. Motivation for using this set-up comes from lower cost, ease of maintenance, great flexibility, as well as lower weight and volume of the system. This motivates research into the emerging class of NCS. The network part of NCS consists of a serial communication channel to which many "nodes" (groups of sensors and actuators) are connected and it is assumed that only one node can transmit its value at a time. Hence, access to the channel needs to be scheduled in an appropriate manner for a proper operation of the system. We ignore the quantization effects and concentrate on the communication constraints that come from the scheduling. The network scheduling algorithm is referred to as a "protocol". NCS are currently receiving considerable attention in the literature as illustrated by recent articles [12,21–26] and references listed therein. The area of NCS is still in its infancy and we are still far from having a controller design methodology comparable to that for the classical continuous time control systems.

We follow the controller design method pioneered by Walsh et. al. in [1,22,23], in which one first designs the controller without taking into account the constraints imposed by the network. Then, in the second step, one determines a design parameter called the maximum allowable transfer interval (MATI) so that the closed loop remains stable when some control and sensor signals are

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transmitted via the network. Note that MATI determines (i.e. it is inversely proportional to) the channel bandwidth. This approach was shown to produce stabilizing controllers for linear NCS in [22] and nonlinear NCS in [23]. Similar ideas were also used to analyse mixed traffic wireless networks in [25].

In this paper, we present an overview of our recent work that further extends the NCS design approach pioneered in [1,22,23,25]. We put an emphasis on our own work and our intention is not to present a comprehensive overview of literature on NCS that is rapidly growing. We state only one of our results (Theorem 1) from [17]. This result is a good starting point for the discussion since it illustrates the approach and our contributions well and, moreover, it is possible to use it as a benchmark when discussing its further generalizations and extensions.

2. PRELIMINARIES

$\mathbb{R}$ and $\mathbb{N}$ denote, respectively, the sets of real and natural numbers. $\mathbb{R}_{\geq 0}$ denotes the set of non-negative integers. Given $t \in \mathbb{R}$ and a piecewise continuous function $f : \mathbb{R} \to \mathbb{R}^n$, we define $f(t^+) := \lim_{s \to t, s>t} f(s)$. All vector norms, denoted as $|\cdot|$, are Euclidean norms unless otherwise stated. The same notation $|\cdot|$ is used for the induced 2-norm of a matrix. Given a measurable signal $\varphi : [t_0, t] \to \mathbb{R}^n$, we denote the $\mathcal{L}_\infty$ norm as follows:

$$\|\varphi[t_0,t]\|_{\mathcal{L}_\infty} := \operatorname{ess\,sup}_{s\in[t_0,t]} |\varphi(s)| \ .$$

A function $\gamma : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to be of class $\mathcal{K}_\infty$ if it is continuous, zero at zero, strictly increasing and unbounded. A function $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to be of class $\mathcal{KL}$ if for each $s \geq 0$ the function $\beta(s, \cdot)$ is decreasing to zero in the second argument and for each fixed $t \geq 0$ the function $\beta(\cdot, t)$ is of class $\mathcal{K}$. β is said to be of class exp-$\mathcal{KL}$ if there exist $K, c > 0$ such that $\beta(s,t) = K \exp(-ct)$. To shorten notation we often use $(x, y) := (x^T \ y^T)^T$.

To state our stability definitions, we consider the class of hybrid systems with disturbances:

$$\begin{aligned} \dot{x} &= f(t,x,w) \qquad t \in [t_{s_i}, t_{s_{i+1}}] \\ x(t_{s_i}^+) &= h(i, x(t_{s_i})) \ . \end{aligned} \tag{1}$$

Definition 1. Let $\gamma \in \mathcal{K}$ and $\beta \in \mathcal{KL}$ be given. The system (1) is Input-to-State Stable (ISS) from w to x if for all $t_0 \geq 0$, $x_0 \in \mathbb{R}^{n_x}$, $w \in \mathcal{L}_\infty$ and each corresponding solution $x(\cdot)$, we have for all $t \geq t_0 \geq 0$ that

$$|x(t)| \leq \beta(|x_0|, t - t_0) + \gamma(\|w\|_{\mathcal{L}_\infty}) \ . \tag{2}$$

Moreover, if $\gamma(\cdot)$ is a linear function, $\beta(\cdot,\cdot)$ is an exp-$\mathcal{KL}$ function and the system (1) is ISS, then we say that the system (1) is ISS with a linear gain and an exp-$\mathcal{KL}$ function.

Consider now (1) with $w \equiv 0$ and suppose that for all $t \geq t_0 \geq 0$ we have that

$$|x(t)| \leq \beta(|x_0|, t - t_0) \ . \tag{3}$$

Then ,the system (1) is said to be uniformly globally asymptotically stable (UGAS). If, moreover, β is an exp-$\mathcal{KL}$ function, then the system (1) is said to be uniformly globally exponentially stable (UGES). ■

3. DEFINITION OF NETWORKED CONTROL SYSTEMS

In this section we introduce a class of models with jumps from [17] that we will use to describe NCS. Let the sequence $t_{s_i}, i \in \mathbb{N}$ of monotonically increasing transmission times satisfy $\epsilon \leq t_{s_{j+1}} - t_{s_j} \leq \tau$ for all $j \in \mathbb{N}$ and some fixed $\epsilon, \tau > 0$. At each t_{s_i}, the protocol gives access to the network to one of the nodes $i \in \{1, 2, \ldots, \ell\}$. We adopt terminology from [23] and refer to τ as the *maximum allowable transmission interval* (MATI). We consider general nonlinear NCS with disturbances of the following form

$$\begin{aligned} \dot{x}_P &= f_P(t, x_P, \hat{u}, w) && t \in [t_{s_{i-1}}, t_{s_i}] \\ y &= g_P(t, x_P) \\ \dot{x}_C &= f_C(t, x_C, \hat{y}, w) && t \in [t_{s_{i-1}}, t_{s_i}] \\ u &= g_C(t, x_C) \\ \dot{\hat{y}} &= \hat{f}_P(t, x_P, x_C, \hat{y}, \hat{u}, w) && t \in [t_{s_{i-1}}, t_{s_i}] \\ \dot{\hat{u}} &= \hat{f}_C(t, x_P, x_C, \hat{y}, \hat{u}, w) && t \in [t_{s_{i-1}}, t_{s_i}] \\ \hat{y}(t_{s_i}^+) &= y(t_{s_i}) + h_y(i, e(t_{s_i})) \\ \hat{u}(t_{s_i}^+) &= u(t_{s_i}) + h_u(i, e(t_{s_i})) \end{aligned} \tag{4}$$

where x_P and x_C are respectively states of the plant and the controller; y is the plant output and u is the controller output; $\hat{y}$ and $\hat{u}$ are the vectors of most recently transmitted plant and controller output values via the network; e is the network induced error defined as

$$e(t) := \begin{pmatrix} \hat{y}(t) - y(t) \\ \hat{u}(t) - u(t) \end{pmatrix} = \begin{pmatrix} e_y \\ e_u \end{pmatrix} \ . \tag{5}$$

Typically, we have that $\hat{f}_P = 0$ and $\hat{f}_C = 0$ (this is similar to a zero order hold assumption in sampled-data systems) but this is not necessary and, moreover, sometimes it is useful to have non-zero $\hat{f}_P$ and $\hat{f}_C$ (see [6]). Also, $\hat{f}_P$ and $\hat{f}_C$ never depend on the whole vectors x_C, x_P but they may depend on parts of y and u that are respectively functions of x_P and x_C. Note that if NCS has ℓ nodes, then the error vector can be partitioned as follows $e = [e_1^T \ e_2^T \ \ldots \ e_\ell^T]^T$ where each e_i in the partition corresponds to the group of sensor and actuator signals in the node i. The functions h_u and h_y are typically such that, if the jth node gets access to the network at some transmission time t_{s_i} we have that the corresponding part of the error vector has a jump. For several protocols, such as the RR and TOD protocols (see Examples

1 and 2), we typically assume that e_j is reset to zero at time $t_{s_j}^+$, that is $e_j(t_{s_i}^+) = 0$. However, we emphasize that this assumption is not needed in general. If a node j does not transmit at t_{s_i} then we write $e_j(t_{s_i}^+) = e_j(t_{s_i})$.

We combine the controller and plant states into a vector $x := (x_P, x_C)$ and using the defined error vector e, we can rewrite (4) as a system with jumps that is more amenable for analysis (see [17] for more details):

$$\dot{x} = f(t, x, e, w) \qquad \forall t \in [t_{s_{i-1}}, t_{s_i}] \quad (6)$$

$$\dot{e} = g(t, x, e, w) \qquad \forall t \in [t_{s_{i-1}}, t_{s_i}] \quad (7)$$

$$e(t_{s_i}^+) = h(i, e(t_{s_i})) \ , \quad (8)$$

$$\epsilon \leq t_{s_{i+1}} - t_{s_i} \ \leq \ \tau \ , \quad (9)$$

where $x \in \mathbb{R}^{n_x}$, $e \in \mathbb{R}^{n_e}$, $w \in \mathbb{R}^{n_w}$. Further assumptions on functions f, g and h can be found in [17].

The jump equation (8) models the operation of the network scheduling "protocol" (the algorithm that schedules access of different nodes to the network). This model of protocols was introduced in Nešić and Teel in [17]. The results of this paper are applicable to a large class of protocols that is characterized in the next section. We next present two protocol examples that have been studied in [23] and shown in [17] that their models can be written in the form (8).

Example 1. [Round Robin (RR) protocol]

Let there be $\ell \geq 1$ nodes in NCS and let the protocol grant access to the network to the node $i \in \{1, \ldots, \ell\}$ at $t_{s_{i+j\ell}}$, for all $j \in \mathbb{N}$, that is $e(t_{s_{i+j\ell}}^+) = 0, \forall j \in \mathbb{N}$. In this case we can write

$$h(i, e) = (I - \Delta(i))e \ , \quad (10)$$

where $\Delta(i) = \text{diag}\{\delta_1(i)I_{n_1}, \ldots, \delta_\ell(i)I_{n_\ell}\}$, $\sum_{k=1}^{\ell} n_k = n_e$, I_{n_k} are identity matrices of dimension n_k and

$$\delta_k(i) = \begin{cases} 1, & \text{if } i = k + j\ell, \ j \in \mathbb{N} \\ 0, & \text{otherwise.} \end{cases}$$

■

Example 2. [Try-Once-Discard (TOD) protocol]

Suppose that there are ℓ nodes competing for access to the network. The node i with the greatest weighted error at time t_{s_j} will be granted access to network at $t_{s_j}^+$ and hence we have that $e_i(t_{s_j}^+) = 0$. We assume that the weights are already incorporated into the model. If a data packet fails to win access to the network, it is discarded and new data is used at the next transmission time $t_{s_{j+1}}$. If two or more nodes have equal priority, a pre-specified ordering of the nodes is used to resolve the collision. This verbal description can be converted into the model of the form (8) where

$$h(e) = (I - \Psi(e))e \ ,$$

and $\Psi(e) := \text{diag}\{\psi_1(e)I_{n_1}, \psi_2(e)I_{n_2}, \ldots, \psi_\ell(e)I_{n_\ell}\}$. I_{n_j} are identity matrices of dimension n_j with $\sum_{j=1}^{\ell} n_j = n_e$ and

$$\psi_j(e) := \begin{cases} 1, & \text{if } j = \min\left(\arg\max_j |e_j|\right) \\ 0, & \text{otherwise.} \end{cases}$$

It was pointed out in [22,24] that one can implement TOD in CAN (control area network) as the local node errors can be encoded into an arbitration field in a frame. This is not possible in wireless and many other wireline networks where such arbitration is not available.

4. LYAPUNOV UGES PROTOCOLS

In this section we define a large class of protocols and show that the RR protocol in Example 1 and the TOD protocol in Example 2 belong to this class. We will show that all protocols that belong to this large class will preserve important $\mathcal{L}_p$ stability properties of the system for sufficiently small values of MATI. Therefore, our results provide a *framework* for generating genuinely new classes of protocols.

Note that the protocol (8) is a mapping that specifies how errors at transmission times t_{s_j} are mapped to errors at times $t_{s_j}^+$. We find it very convenient to use h in (8) to introduce an auxiliary discrete time system of the form:

$$e^+ = h(i, e) \qquad i \in \mathbb{N} \ , \quad (11)$$

and refer to it as a *discrete time system induced by the protocol (8)*. Sometimes we abuse the terminology and refer to (11) simply as a protocol. The following class of protocols are used to state our main results:

Definition 2. Let $W : \mathbb{N} \times \mathbb{R}^{n_e} \to \mathbb{R}_{\geq 0}$ be given and suppose that there exist $\rho \in [0, 1)$ and $a_1, a_2 > 0$ such that the following conditions hold for the discrete time system (11) for all $i \in \mathbb{N}$ and all $e \in \mathbb{R}^{n_e}$:

$$a_1 |e| \leq W(i, e) \ \leq \ a_2 |e| \quad (12)$$

$$W(i + 1, h(i, e)) \leq \rho W(i, e) \ . \quad (13)$$

Then, we say that the protocol (8) (or equivalently (11)) is *uniformly globally exponentially stable (UGES) with Lyapunov function W.* ■

Remark 1. Note that the system (11) and Examples 1 and 2 motivate the classification of protocols into *linear* and *nonlinear*. Indeed, it is easy to see that the RR protocol induces a time-varying linear system (11), whereas for the TOD protocol we have that (11) is time-invariant and nonlinear. ■

Remark 2. Definition 2 does not make any reference to the NCS (6), (7) and, hence, it captures

the intrinsic properties of the protocol itself. This novel approach to viewing protocols was first proposed in [17]. This approach turned out to be very useful for many other problems in this area, as discussed in the sequel. ■

It is a well known fact in the literature that the conditions (12), (13) are equivalent to uniform global exponential stability of the system (11). It was shown in [17] that the protocols considered in Examples 1 and 2 are UGES with appropriate Lyapunov functions. Ineed, we can state:

Proposition 1. The RR protocol in Example 1 is UGES with the Lyapunov function $W(i,e) := \sqrt{\sum_{k=i}^{\infty} |\phi(k,i,e)|^2}$, where ϕ denotes the solutions of the system (11) induced by the RR protocol. In particular, we can take $a_1 = 1$, $a_2 = \sqrt{\ell}$ and $\rho = \sqrt{\frac{\ell-1}{\ell}}$. ■

Proposition 2. The TOD protocol in Example 2 is UGES with the Lyapunov function $W(i,e) := |e|$. In particular, we can take $a_1 = a_2 = 1$ and $\rho = \sqrt{\frac{\ell-1}{\ell}}$. ■

5. $\mathcal{L}_P$ STABILITY PROPERTIES OF NCS WITH UGES PROTOCOLS

In this section, we present a consequence of the main results in [17] that illustrates our approach. We are not aiming here for generality but rather we select a version of results that are easy to state and understand. Various generalizations and extensions are discussed in the next section.

Theorem 1. Consider NCS (6)-(9). Suppose that the following conditions hold:

(1) System (6) is ISS from (e,w) to x with linear gain γ.
(2) $h(i,e)$ in equation (8) is such that inequalities (12), (13) hold, i.e. the protocol (8) is UGES.
(3) There exist $L, c \geq 0$ such that for all i,t,e,x,w we have that g in (7) satisfies:

$$\left\langle \frac{\partial W}{\partial e}, g(t,x,e,w) \right\rangle \leq LW(i,e) + c(|x| + |w|) \,. \tag{14}$$

(4) MATI in (9) satisfies $\tau \in (\epsilon, \tau^*)$ where

$$\tau^* := \frac{1}{L} \ln \left(\frac{a_1 L + c\gamma}{a_1 \rho L + c\gamma} \right) \tag{15}$$

$\epsilon \in (0, \tau^*)$ is arbitrary, $L, c \geq 0$ come from (14), $a_1 > 0$ comes from (12) and $\rho > 0$ comes from (13).

Then, the NCS is ISS from w to (x,e) with linear gain. In particular, when $w \equiv 0$, the NCS is UGAS. If all the above properties hold with exp-$\mathcal{KL}$ functions and linear gains, then the NCS is exponentially stable when $w \equiv 0$. ■

It is instructive to discuss the conditions of Theorem 1 in more detail.

Condition (1) states that the subsystem (6) should be robust with respect to exogenous disturbances w, as well as e which is treated as a disturbance. This condition can be ensured by carefully designing a continuous time controller for the continuous time plant that is perturbed by e. In [22], a simpler version of this condition was considered for systems without disturbances (i.e. $w \equiv 0$), requiring that

$$\dot{x} = f(t,x,0,0)$$

is stable. Hence, the controller is designed completely ignoring the network, which is typical in controller emulation. Our design takes effects of the network into account through e but we still refer to it as an emulation approach for the above reasons.

Condition (2) requires the protocol (8) to be UGES. Propositions 1 and 2 state respectively that RR and TOD protocols are UGES. We showed in our work that a range of other protocols are UGES and, hence, our approach can be applied in all such cases.

Condition (3) is used to quantify the growth of $W(i,e(t))$ on the transmission intervals $[t_{s_i}, t_{s_{i+1}}]$. The idea is that the value of $W(i,e(t))$ decreases at transmission times because the protocol is UGES, while between transmission times $W(i,e(t))$ grows exponentially with time. However, for small MATI we can show that the the system (7), (8) is ISS from (x,w) to e with linear gain and exp-$\mathcal{KL}$ function. Moreover, the smaller the MATI, the smaller the gain.

Condition (4) requires that MATI in (9) is small enough. The MATI bound relates Condition (1) (through the gain γ), Condition (2) (through ρ and a_1) and Condition (3) (through constants L, c) so that stability is guaranteed. This condition is obtained by applying an appropriate small gain theorem that guarantees that the feedback interconnection of the system (6) and the system (7), (8) is ISS from w to (x,e). It is quite easy to analyze how properties of the protocol affect the MATI bound. For instance, consider the values of $a_1 = 1$ and $\rho = \sqrt{\frac{\ell-1}{\ell}}$ obtained for the RR and TOD protocols in Propositions 1 and 2, where ℓ is the number of nodes. Substituting these values in (15), we obtain: $\tau^* := \frac{1}{L} \ln \left(\frac{L + c\gamma}{\sqrt{\frac{\ell-1}{\ell}} L + c\gamma} \right)$. Then, for any fixed values of L, c, γ we have that $\tau^* \to 0$ as $\ell \to \infty$. In other words, the more nodes we have, the smaller the MATI required to preserve stability of the system. This is consistent with the intuition. We note that although the above MATI bound appears to be the same for RR and TOD protocols, the values of L, c would in general be different for the two protocols and the MATI bounds would differ.

Remark 3. Theorem 1 provides the framework for design of NCS. In the first step of this approach, we design the controller so that the subsystem (6) is ISS from (e, w) to x. Note that in this step, we can use the standard continuous time design methods to design the controller. In the second step, we choose a UGES protocol and make sure that MATI is sufficiently small, as given by Condition (4) in Theorem 1.

6. RELATED RESULTS AND EXTENSIONS

Theorem 1 is a special case of results in [17] that is easy to state and understand but is general enough to illustrate the flexibility and generality of our approach. Various extensions and modifications of this basic result have been considered in the literature. Next, we discuss some of these extensions and comment on various aspects of this approach.

6.1 Stability properties of NCS

In [17] we have considered more general versions of Theorem 1, as well as different stability properties not considered here. Indeed, we proved $\mathcal{L}_p$ stability theorems for arbitrary $p \in [1, \infty]$ that may be more appropriate to use in certain situations. Moreover, instead of using input-to-state stability bounds, we presented weaker versions of results that involve input-to-output stability (IOS) properties and input-output-to-state stability (IOSS) properties (detectability) to state our results. These generalizations are quite useful when addressing various examples and they may lead to less conservative bounds on MATI than the one presented in Theorem 1. Moreover, ISS of NCS with a larger class of UGAS protocols was considered in [18].

Several other stability properties of NCS were considered in the literature. In [11], instead of the deterministic model (6), (7), the authors consider a stochastic NCS model of the form:

$$\begin{aligned} dx_P &= f_P(x_P, \hat{u}) + g_P(x_P, \hat{u})dw \qquad y = h_P(x_P) \\ dx_C &= f_C(x_C, \hat{y}) + g_C(x_C, \hat{y})dw \qquad u = h_C(x_C) \ , \end{aligned}$$

where w is a standard Wiener process. Moreover, in this paper the transmission times are randomly distributed. For this class of NCS models, the authors prove mean square stability of NCS by stating a result similar to Theorem 1. In [20] deterministic plants with stochastic protocols and disturbances are considered and $\mathcal{L}_p$ stability in expectation proved.

6.2 UGAS and PE protocols

There are several situations where it is useful to relax the definition of UGES protocol. We consider two such generalizations that are respectively reported in [19] and [18]. Examples 1 and 2 illustrate that many protocols can be written in the following form [2]:

$$e^+ = (I - \Psi(i, e))e \ , \tag{16}$$

where we can think of $\Psi(i, e)e$ as the "transmitted value" and $\Psi(i, e)$ as a scheduling function that determines which node gets to transmit.

To motivate the first generalization, we note that TOD protocol can be implemented in CAN but not in wireless and many wireline channels (see Example 2). Hence, in many wireless and wireline channels it is not possible to implement protocols in which the scheduling function Ψ depends on the error vector e. Instead, in [25] several protocols were introduced where the scheduling function Ψ does not depend on the error vector but rather on its "estimate". Furthermore, in [19] a more general class of protocol models was introduced that contains as special cases all protocols considered in [25]:

$$\begin{aligned} e^+ &= (I - \Psi(i, \hat{e}))e \\ \hat{e}^+ &= \Lambda(i, \Psi(i, \hat{e})e, \hat{e}) \ , \end{aligned}$$

where we can think of $\hat{e}$ as the scheduling variable. Note that as $\Psi(i, \hat{e})e$ is the error of the transmitted node, this information is broadcast to all nodes in the network and hence the scheduling variable is allowed to depend on it. It was shown in [19] that if the scheduling function $\Psi(i, \hat{e})$ has an appropriate robust persistency of excitation (PE) property then we can state similar stability results like in Theorem 1. A protocol is PE if it regularly visits every NCS node within a fixed period of time $T > 0$. RR in Example 1 is the simplest PE protocol but many more PE protocols were considered in [25] and [19]. TOD protocol is not PE. General results on $\mathcal{L}_p$ stability of NCS with PE protocols were reported in [19].

Next, we discuss another possible generalization for protocol models given by (11). It was shown in [18] that instead of (12), (13), we can use the following inequalities to characterize protocols:

$$\begin{aligned} \alpha_1(|e|) \ \leq \ W(i, e) &\leq \alpha_2(|e|) \\ W(i + 1, h(i, e)) - W(i, e) &\leq -\alpha_3(W(i, e)) \ . \end{aligned}$$

where $\alpha_1, \alpha_2, \alpha_3$ are class $\mathcal{K}_\infty$ functions. This class of protocols was referred to as uniformly globally asymptotically stable (UGAS) protocols for obvious reasons. Several examples of UGAS protocols can be found in [18]. It was shown in [18] that if (6) is ISS from (e, w) to x and the protocol is UGAS, then the NCS is semi-globally practically ISS from w to (x, e) in MATI. This means that one can achieve practical ISS on arbitrarily large sets of initial conditions and disturbances by reducing MATI.

[2] We actually write formulas for the system induced by the protocol (11).

6.3 MATI bounds

The emulation approach that we presented here was pioneered by Walsh et. al. in a series of papers [22,24]. Analytical MATI bounds in these papers were very conservative and, hence, not very useful for design. Moreover, these results do not distinguish between different protocols and, for example, the MATI bounds are identical for RR and TOD protocols.

These problems were partially resolved by Nešić and Teel in [17] where less conservative MATI bounds that distinguish between different protocols were presented. For instance, it was shown in [17, Section IX] that the analytic MATI bounds for a linearized model of a batch reactor controlled over a network are of the same order of magnitude as the critical value of MATI at which instability occurs [3] . On the other hand, the MATI bounds from [22,24] are about 1000 times smaller than the critical value of MATI. More importantly, it was shown in [17] that the following holds for RR protocols

$$\frac{\tau^{RR}_{[17]}}{\tau^{RR}_{[22,24]}} \geq K\frac{(\ell+1)}{\sqrt{\ell}}$$

and for TOD protocols we have:

$$\frac{\tau^{TOD}_{[17]}}{\tau^{TOD}_{[22,24]}} \geq K(\ell+1) \ ,$$

where ℓ is the number of nodes, $K > 1$ depends on the Lyapunov function that is used to verify stability of the continuous time closed loop system without network and $\tau^{RR}_{[17]}$ denotes the analytic MATI bound for RR protocol that was obtained in [17] (similar notation is used for TOD protocol). The above formulas show that as the number of nodes ℓ grows, the analytic MATI bounds in [17] are increasingly less conservative than the analytic MATI bounds in [22,24].

Despite considerable improvements reported in [17], several other subsequent improvements of MATI bounds were reported in [19] and [4]. For instance, a novel proof technique pursued in [19] for PE protocols, such as the RR protocol, produced an analytic bound for MATI that satisfies in general:

$$\frac{\tau^{RR}_{[19]}}{\tau^{RR}_{[17]}} \geq K\frac{1}{\sqrt{\ell}\ln\left(\frac{\ell}{\ell-1}\right)} \ ,$$

which for very large ℓ can be approximated as follows:

$$\frac{\tau^{RR}_{[19]}}{\tau^{RR}_{[17]}} \geq K\sqrt{\ell} \ ,$$

where $K > 1$ is a number that depends on the model of the system. We note that results of [19] do not apply to TOD protocol since it is not PE.

[3] This critical value of MATI can be obtained analytically for linear plants and RR protocols or via numerical simulations for TOD protocol.

Yet another improvement of MATI bounds was reported in [4] where a novel Lyapunov based proof was presented to compute the MATI bound for NCS with protocols (11). It was shown in [4] that

$$\frac{\tau^{RR,TOD}_{[4]}}{\tau^{RR,TOD}_{[17]}} \geq K \ ,$$

where $K \in (1, 1.5]$, depending on the type of protocol and the system.

For linear NCS, it is tractable to design controller and protocol simultaneously and such designs typically produce larger bounds on MATI (see [5,6,15,16]). Finally, we note that by considering stochastic protocols and/or plants it was shown in [11] and [20] that MATI bounds can further be relaxed. However, in this case the considered stochastic stability properties are different from the deterministic ones considered in Theorem 1 and, hence, we do not present those comparisons.

6.4 Dropouts

Two very important issues in NCS are dropouts and network induced delays. A dropout occurs whenever transmitted data are lost or received in error. Data packet dropout may occur in NCS for various reasons, such as malfunction of the node (sensor or smart actuator) or data corruption in noisy channels and collisions. While most protocols are equipped with transmission retry mechanisms, they can only retry for a limited time. After this time has lapsed, the data is "dropped" and a new data is collected. Dropping old data may be advantageous for the operation of NCS since the delays in the network can be potentially reduced in this way. However, NCS can cope with limited data loss and quantifying the effect that data loss has on stability and performance is an important issue in analysis of NCS.

In [17], we model data packet dropout by slightly changing the condition (13) in the definition of Lyapunov UGES protocols. In particular, we use the following inequality:

$$W(i+1, h(i,e)) \leq \rho_i W(i,e) \ , \qquad (17)$$

instead of (13), which holds for all $e \in \mathbb{R}^{n_e}$ and $i \in \mathbb{N}$, where $\rho_i \geq 0$ for all $i \in \mathbb{N}$. If there is a packet dropout at transmission time t_{s_i}, then typically we set $\rho_i = 1$. In the ideal situation without dropouts, $\rho_i = \rho \in [0, 1)$ for all $i \in \mathbb{N}$ and, hence, we recover (13). For the purpose of generality, we also allow situations where ρ_i are different from 1 and ρ. Deterministic conditions on dropouts that guarantee stability of the NCS similar to Theorem 1 are presented in [17]. While these deterministic conditions confirm the intuition that "if the dropouts occur rarely, then the stability of NCS is preserved", they are hard to use since a deterministic characterization of dropouts is not possible.

This provides a motivation for considering a stochastic model for the protocols [11,20]. We concentrate on the approach taken in [20] where we consider protocols of the form:

$$e(t_{s_i}^+) = Q_i e(t_{s_i}) \ ,$$

for contention protocols[4]. $Q_i \in \mathcal{P}$ is an iid[5] sequence of diagonal matrices and $\mathcal{P}$ is defined as follows

$$\mathcal{P} := \{\Psi_1, \ldots, \Psi_\ell\} \ , \qquad (18)$$

where ℓ is the number of nodes and $\Psi_j = \text{diag}\{\psi_j^1 I_{n_1}, \ldots, \psi_j^\ell I_{n_\ell}\}$ and for all $i, j \in \{1, 2, \ldots, \ell\}$

$$\psi_j^i := \begin{cases} 0, & \text{if } i = j \\ 1, & \text{if } i \neq j \end{cases} \ .$$

For contentionless protocols in the presence of dropouts, we considered in [20] jump maps of the form (similar models were first considered in [11]):

$$e(t_{s_i}^+) = q_i h(i, e(t_{s_i})) + (1 - q_i) e(t_{s_i}) \ ,$$

where h is a deterministic jump map (e.g., as in (8)) and q_i is an iid sequence of Bernoulli random variables. The above jump equation (together with the random sequence of transmission instants t_i) models "stochastic protocols". It was shown in [20] that the above models can be used to model several protocols commonly found in practice, such as the slotted p-persistent CSMA (carrier sense multiple access) protocol. For the above protocols, we used the notion of almost surely UGES, that satisfy (12), (17), as well as:

$$\mathbf{E}[\rho_i] = \rho \in [0, 1) \ .$$

Results in [20] use exactly the same deterministic model of the continuous part of NCS (6), (7), whereas (9) was replaced by transmission times t_{s_i} that were modelled using a Poisson process with intensity λ. Results that guarantee $\mathcal{L}_p$ stability in expectation are proved for this class of NCS models.

6.5 LMIs for linear NCS design

The emulation approach is attractive for its simplicity since the controller design is decoupled from the choice/design of the network protocol (e.g. the protocol may be fixed prior to controller design). The price we pay is that we need sufficiently small MATI for this approach to be feasible.

An interesting alternative to the emulation approach is a simultaneous controller/protocol design that we pursued in [5] for linear plants to achieve quadratic stabilization[6] under the assumption that transmission instants are equidistant. In particular, we considered:

$$\begin{aligned}
\dot{x}_p &= A_p x_p + B_p \hat{u} \qquad t \in [t_{s_i}, t_{s_{i+1}}] \\
\dot{x}_c &= 0 \qquad t \in [t_{s_i}, t_{s_{i+1}}] \\
\dot{\hat{u}} &= 0 \qquad t \in [t_{s_i}, t_{s_{i+1}}] \\
\dot{\hat{y}} &= 0 \qquad t \in [t_{s_i}, t_{s_{i+1}}] \\
x_c(t_{s_i}^+) &= A_c x_c(t_{s_i}) + B_c(\Psi_y \hat{y}(t_{s_i}) + (I - \Psi_y) y(t_{s_i})) \\
\hat{u}(t_{s_i}^+) &= \Psi_u \hat{u}(t_{s_i}) + (I - \Psi_u) u(t_{s_i}) \\
\hat{y}(t_{s_i}^+) &= \Psi_y \hat{y}(t_{s_i}) + (I - \Psi_y) y(t_{s_i}) \ ,
\end{aligned}$$

where $y = C_p x_p$, $u = C_c x_c + D_c \hat{y}$ and $t_{s_{i+1}} - t_{s_i} = T$, for all $i \in \mathbb{N}$ and some fixed $T > 0$. We design the controller (i.e. find the matrices (A_c, B_c, C_c, D_c)) and the protocol (i.e. find how the scheduling function $\Psi := \text{diag}\{\Psi_u, \Psi_y\}$ should depend on the error vector e) to achieve quadratic stability of the discrete time closed loop system. A class of TOD-like protocols[7] present themselves as a natural solution to this problem. The resulting closed loop discrete time system is a switched linear system and the controller/protocol pair are obtained by solving appropriate bilinear matrix inequalities. Moreover, we showed in an example that the combined controller/protocol design achieves stabilization of the system with larger MATI than the one required in the emulation approach. Hence, we believe that exploring further simultaneous controller/protocol design is an important direction for further research.

A similar approach was pursued in [6] for observer design of linear systems without control inputs. Using switched Luenberger-like observers, we showed that TOD-like protocols arise naturally when ensuring quadratic stability of the error dynamics. The discrete time model of error dynamics becomes a switched linear system and the observer/protocol pair are obtained via appropriate linear and bilinear matrix inequalities.

7. SUMMARY AND FUTURE WORK

We presented an overview of our recent results on an emulation like approach for design of NCS that was pioneered in [21–25]. This approach benefits significantly from the modelling framework for NCS proposed in [17]. Using this modelling framework, there are many directions in which the presented results can be extended: (i) in the emulation approach it would be interesting to determine ways of choosing UGES protocols to achieve certain objectives (e.g. robustness or optimality of the system); (ii) developing optimal control and estimation theory for NCS appears

[4] Contention protocols allow nodes to contend for network access, i.e. two or more nodes may try to send messages across the network simultaneously and the contention protocol defines what happens when this occurs. The most widely used contention protocol is CSMA/CD, used by Ethernet. Contentionless protocols do not allow nodes to contend for network access. Round robin or token ring protocol is an example of such protocols.

[5] Independent and identically distributed.

[6] There exists a quadratic Lyapunov function verifying uniform exponential stability of the closed loop system.

[7] These TOD-like protocols are implementable in CAN.

to be an interesting open problem; (iii) simultaneous controller/protocol design is an interesting problem for linear and general nonlinear systems; (iv) design of controller/protocol for fixed (and, potentially, large) MATI is an interesting problem that would require very different design techniques; (v) incorporating pure delays within our modelling framework is important as delays are always present in real networks; and so on.

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