

DESIGNING AND IMPLEMENTING AN INTERVENTION PROGRAM TO IMPROVE
UNDER-ACHIEVING YEAR 8 STUDENTS' UNDERSTANDING OF MULTIPLICATIVE
THINKING AND ASSESS ASSOCIATED GAINS IN MOTIVATION AND ENGAGEMENT.

By

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Abstract

A fifteen-week intervention was carried out in order to examine the efficacy of the Scaffolding Numeracy in the Middle Years (SNMY) (Education Victoria, 2013b) instructional resources in improving the multiplicative thinking of eight under-achieving Year 8 students. The eight students participating in the intervention were contrasted against a similarly profiled control group of ten students. Participants were drawn from a co-educational high school on the lands of the People of the Kulin Nation in Northern Metropolitan Melbourne. Changes in students' multiplicative thinking, as well as associated changes in motivation and engagement, were measured in pre- and post-intervention assessments. Multiplicative thinking was measured with the SNMY Assessment Booklets 1 and 2, with motivation and engagement examined by use of the High School Motivation and Engagement Scale (MES-HS) (Martin, 2003), also prior-to and following the intervention. These quantitative data sources were complemented with qualitative sources in the form of researcher's notes and student work samples.

Whilst analysis of quantitative data did not indicate significant changes in multiplicative thinking for either the intervention or the control groups, qualitative data sources indicated that students within the intervention group demonstrated modest gains in multiplicative thinking. No statistically significant changes to motivation and engagement were recorded for students in the control group, whereas those in the intervention group showed a significant decrease in both failure avoidance and anxiety, and a significant increase in disengagement between pre- and post-testing. Findings point to several barriers to achievement for mathematically under-achieving students in the middle years, as well as directions for the improvement of similar interventions in future.

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It is proposed that additional years of mathematical failure, as well as motivation and engagement challenges particular to students in the middle years, act as significant barriers to improving the multiplicative thinking abilities of under-achieving students in this period of their schooling. In addition, it is suggested that the SNMY assessment resources could be improved by greater conceptual alignment between the pre- and post- assessment booklets, a reduction in the marking scheme's reward for effort, and greater consistency between pre- and post-tests in regards to conversion of raw scores to levels on the SNMY's multiplicative thinking conceptual framework.

Suggestions are made for future interventions based on the SNMY resources. These suggestions include more explicit instruction in problem solving strategies, as well as in the identification of similarities between novel problems and those previously encountered.

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Dedication

For my Grandfather Bob.

1. Introduction

1.1. Introduction

For many students, the transition into the middle years of school occurs in tandem with a decrease in achievement in mathematics (Martin, Way, Bobis, & Anderson, 2014a; Siemon, Virgona, & Corneille, 2001). The drivers for such an occurrence are complex and interlinked, however, two factors appear in the literature as significant contributors. The first is low multiplicative thinking abilities (Mulligan, 2011; Siemon, Breed, & Virgona, 2005), and the second is reduced mathematical motivation and engagement (Chouinard & Roy, 2008; Martin et al., 2014a; Middleton & Spanias, 1999). The current project implemented a small group tutoring program based upon the Scaffolding Numeracy in the Middle Years (SNMY) instructional resources (Education Victoria, 2013b), in an attempt to address these two key middle years challenges to students' mathematical performance.

1.2. Research Questions

The present study set out to answer two questions:

- To what extent can a small group mathematics intervention program based upon the Scaffolding Numeracy in the Middle Years framework improve the multiplicative thinking of low-achieving students in the middle years of schooling?
- To what extent does the motivation and engagement of participating students influence student growth throughout the intervention, or how is motivation and engagement influenced by student growth?

1.3. Key Terms

1.3.1. Multiplicative Thinking

Multiplicative thinking is a students' ability to effectively and efficiently work with concepts such as fractions, ratios, large whole numbers, percentages, and multiplication and division problems in their wide variety of representations (Education Victoria, 2013a).

1.3.2. Motivation and Engagement

Engagement refers to behaviours and attitudes ranging from motivation and participation to identification, membership, and attachment (Cavanagh, Kennish, & Sturgess, 2008). Fredricks and colleagues (Fredricks, Blumenfeld, & Paris, 2004) proposed a definition for engagement as consisting of behavioural, cognitive, and emotional components. These span student displays of participation and rule following (behavioural), intrinsic/extrinsic motivations and challenge orientations (cognitive), and curiosity, senses of belonging, and interest (emotional).

Motivation is the reasons that an individual has or gives for displaying a certain behaviour in a given situation (Ames, 1992).

1.4. Significance and Justification

In Australian mathematics classrooms, in any one grade level, students exhibit a large variety of mathematical competency (Sullivan, 2011). This is an achievement gap that grows over time, with the typical two and a half year range in Year 3 growing to five and a half years by Year 9 (Goss & Sonnemann, 2016). The expectation that classroom teachers are able to close such gaps during normal class time is unrealistic (Sullivan & Gunningham, 2011). It is therefore necessary to explore mathematics interventions targeted at providing more learning time to under-achieving students, to the end of bridging such gaps. The small group intervention approach has been shown to offer significant increases in teacher time per student, with one study reporting over 100 mathematical idea focused interactions in a single session (Gerasoni, 2004).

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Previous small group tutoring programs have reported significant gains in student achievement (Breed, 2012; Gerasoni, 2004; Pegg, Graham, & Bellert, 2005). A large scale study by Siemon, Virgona, and Corneille (2001) demonstrated that at the heart of students' challenges with middle years mathematics is a lack of multiplicative thinking. As such, interventions targeting multiplicative thinking in the middle years pose some promise. There is a lack of research in this area with Breed (2011), in her study on promoting multiplicative thinking in upper primary school, asserting that "Further research is also needed to explore whether the approach to intervention reported in this study can be usefully applied with students in Years 7-9" (p. 263). This is against a backdrop of fewer mathematical intervention research projects focusing on the middle years when compared to those in the early years (Meiers, Reid, McKenzie, & Mellor, 2013), as well as evidence suggesting that such middle years interventions may have larger impacts on students' future educational achievement than is widely considered to be the case (Pogrow, 1998).

Student motivation and engagement is a key precursor to academic effort and subsequent mathematical success (Farrington et al., 2012; Hattie, 2012; Stephens, 2011). Marsh and Craven (2006) argue that "Enhancing skills alone is not enough; people also need to hold positive self-concepts of their abilities in specific areas." (p. 158), with O'Mara, Marsh, Craven, and Debus' (2006) meta-analysis finding that interventions addressing skills without specific focus on praise/feedback, with associated impacts upon motivation and engagement, were less effective than those that did. As such, it is highly relevant to also foster and track increases in students' motivation and engagement, in conjunction with skills interventions, as is done in the present study.

The paired tracking of growth in both multiplicative thinking and motivation and engagement resulting from an intervention based on the SNMY (Education Victoria, 2013b) resources represents an original undertaking.

1.5. Thesis Structure

The following chapter presents an overview of the literature pertaining to motivation and engagement, as well as multiplicative thinking in the middle years of schooling. This literature review examines past interventions that have focused upon multiplicative thinking, as well as previous studies that have explored student motivation and engagement. Chapter 3 explicates the theoretical framework and methodological basis of the present study, outlining the instruments used and relevant considerations for the pre-intervention, intervention, and post-intervention phases of the project. Following this, chapter 3 outlines the data types used, and analysis procedures undertaken, in order to interpret the study's results. Chapter 4 presents results from the mix of quantitative and qualitative sources, going into detail regarding the performance of several students in particular. The final chapter presents a discussion of the limitations of the study, implications, and key findings in line with the two research questions posed.

2. Literature Review

2.1. ‘Under-achieving’ Students and the Mathematical Decline in the Middle Years

The transition from primary to secondary school, for many students, heralds a significant decrease in both mathematical achievement and engagement. Researching one group of students, Martin, Way, Bobis, and Anderson (2014a) found that as these students moved from Year 6 to 7 their engagement in mathematics decreased, followed by a further decrease from Year 7 to 8. These decreases were correlated in particular with a decrease in mathematical self-efficacy and valuing of mathematics. In the realm of multiplicative thinking ability, Siemon and colleagues (Siemon, Breed, Dole, Izard, & Virgona, 2006) found that this dip was so pronounced that multiplicative thinking scores fell significantly enough from Years 6 to 7 such that they did not return to Year 6 levels until Year 9. Such dips have the potential to reduce the efficacy of mathematics interventions in the middle years when compared to their primary school equivalents.

2.2. Multiplicative Thinking

Multiplicative thinking is characterised by an ability to work efficiently with a range of concepts such as large whole numbers, fractions, ratios, percentages, and other problems involving multiplication and division in a variety of representations (Education Victoria, 2013a). Multiplicative thinking has been found to be a critical pre-requisite for students wishing to move beyond primary mathematics and into a study of more advanced concepts (Siemon, Bleckly, & Neal, 2012).

There have been several developmental progressions proposed regarding the development of multiplicative thinking (Carrier, 2014; Siemon & Breed, 2006; Tzur et al., 2013; Vergnaud, 1994), each of which sheds a different light on students’ multiplicative thinking processes. Carrier (2010; 2014) broke down the development of multiplicative thinking into three categories: pre-multiplicative, emergent, and multiplier

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thinking. Vergnaud (1994) proposed multiplicative thinking as a conceptual field containing concepts such as functions, ratio, rate, and fraction, a framework that has been used in subsequent research on multiplicative thinking (Caddle & Brizuela, 2011). Tzur and colleagues (2013) suggest a multiplicative thinking developmental framework of six schemes, and propose teaching practices and activities appropriate for each of the six levels. The present study adopts the framework set forth by Siemon, Breed, Dole, Izard, & Virgona (2006), the 'Learning Assessment Framework' (LAF). The framework was developed through Rasch (1993) analysis of the assessment of approximately 3400 Australian school students ranging in Year level from 4 to 8. A summary of the adopted derived framework is displayed as Figure 1.

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Summary of Learning Assessment Framework (LAF) (<i>Siemon et al., 2006</i>)	
Zone Description	
1	Solves simple multiplication and division problems involving relatively small whole numbers but tends to rely on drawing, models and count-all strategies. May use skip counting for groups less than 5. Makes simple observations from data and extends simple number patterns. Multiplicative thinking (MT) not yet apparent as no indication that groups are perceived as composite units, dealt with systematically, or that the number of groups can be manipulated to support more efficient calculation.
2	Counts large collections efficiently, keeps track of count but needs to see all groups. Shares collections equally. Recognises small numbers as composite units (e.g., can count equal groups, skip count by twos, threes and fives. Recognises multiplication needed but tends not to be able to follow this through to solution. Lists some of the options in simple Cartesian product situations. Some evidence of MT as equal group/shares seen as entities that can be counted.
3	Demonstrates intuitive sense of proportion. Works with useful numbers such as 2 and 5 and intuitive strategies to count/compare groups (e.g., doubling, or repeated halving to compare simple fractions). May list all options in a simple Cartesian production, but cannot explain or justify solutions. Beginning to work with larger whole numbers and patterns but tends to rely on count all methods of additive thinking (AT).
4	Solves simple multiplication and division problems involving two-digit numbers. Tends to rely on AT, drawings and/or informal strategies to tackle problems involving larger numbers, decimals and/or less familiar situations. Tends not to explain thinking or indicate working. Partitions given number or quantity into equal parts and describes part formally. Beginning to work with simple proportion.
5	Solves whole number proportion and array problems systematically. Solves simple, 2-step problems using recognised rule/relationship but finds this difficult for larger numbers. Determines all options in Cartesian product situations involving relatively small numbers, but tends to do this additively. Beginning to work with decimal numbers and percent. Some evidence MT being used to support partitioning. Beginning to approach a broader range of multiplicative situations systematically.
6	Systematically lists/determines the number of options in Cartesian product situation. Solves a broader range of multiplication and division problems involving 2-digit numbers, patterns and/or proportion but may not be able to explain or justify solution strategy. Renames and compares fractions in the halving family, uses partitioning strategies to locate simple fractions. Developing sense of proportion, but unable to explain or justify thinking. Developing capacity to work mentally with multiplication and division facts.
7	Solves and explains one-step problems involving multiplication and division with whole numbers using informal strategies and/or formal recording. Solves and explains solutions to problems involving simple patterns, percent and proportion. May not be able to show working and/or explain strategies for situations involving larger numbers or less familiar problems. Constructs/locates fractions using efficient partitioning strategies. Beginning to make connections between problems and solution strategies and how to communicate this mathematically.
8	Uses appropriate representations, language and symbols to solve and justify a wide range of problems involving unfamiliar multiplicative situations, fractions and decimals. Can justify partitioning, and formally describe patterns in terms of general rules. Beginning to work more systematically with complex, open-ended problems.

Figure 1. Summary of the LAF (Mulligan, 2011b; Siemon et al., 2005)

This framework was selected for the present study as it is based upon the largest sample size and it has accompanying instructional resources (Education Victoria, 2013b) and assessment materials that have been validated (Siemon & Breed, 2006). This will allow students to be accurately located along the developmental continuum, after which the researcher will be able to target teaching directly to students' 'zone of proximal development' (Vygotskiĭ & Cole, 1978).

2.2.1. *Mathematical Intervention Programs Targeting Multiplicative Thinking*

Several small group interventions have reported increased understanding of multiplicative thinking for students in the upper years of primary school (Breed, 2011; Brickwedde, 2011; Norton, 2005; Tobias & Andreasen, 2013). Despite the existence of some informal school-based research (Epstein, 2015), there is a shortage of research examining multiplicative thinking-based interventions in the middle years of schooling. As such, the achievement gains produced in primary school multiplicative thinking interventions are yet to be replicated in similar experiments involving students in the middle years.

2.3. Motivation and Engagement

There is now a broad and expansive literature on motivation and engagement, with such writing now well and truly in the public realm (see Pink (2010) and Sinek (2009)). The words ‘motivation’ and ‘engagement’ are used loosely, interchangeably, and together in this expansive literature. *Noncognitive factors*¹ is a more encompassing term including these and other affective factors that interact with, and impact upon, student academic achievement (Farrington et al., 2012). The words ‘motivation’ and ‘engagement’ have been specifically chosen here due to their alignment with the eleven factor framework, proposed by Martin (2007), and adopted in the current study. This framework has been chosen for two main reasons. Firstly, it is accompanied by a validated instrument entitled ‘The Motivation Engagement Scale’ (MES) (Martin, 2003) that will allow student progress to be measured in line with examined constructs. Secondly, Martin’s model aims to make such constructs “readily accessible to practitioners, parents and students” (p. 415), helping to locate the current project in the practical realm of research, Pasteur’s quadrant (Berliner, 2014). A summary of Martin’s model can be seen in Figure 2.

¹ For a detailed critique of the use of this term, and its alternatives, see (Duckworth & Yeager, 2015).

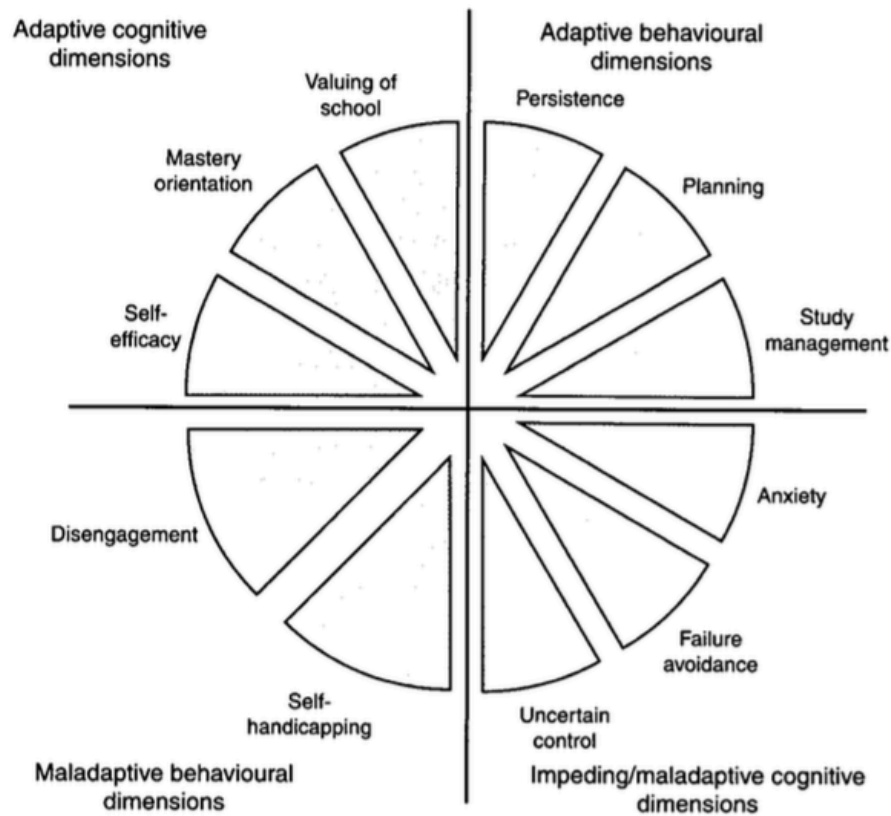


Figure 2. Andrew Martin's (2007) 11 factor framework

2.3.1. Motivation, Engagement, and Mathematics

A necessary precursor to all learning is engagement (Hattie, 2012) with studies indicating that reduced motivation is a correlate of school dropout, aggression, and other negative academic outcomes (Barkoukis, Tsorbatzoudis, Grouios, & Sideridis, 2008), and increased motivation a correlate of higher academic achievement (A. E. Gottfried, Marcoulides, Gottfried, & Oliver, 2013; A. E. Gottfried, Marcoulides, Gottfried, Oliver, & Guerin, 2007; Martin, 2003; Schunk, Meece, & Pintrich, 2012). In relation to mathematics, differences in motivation have been revealed as explanatory variables in both inter- and intra-cultural contexts (Zhao & Singh, 2011).

2.3.1.1. Motivation and engagement, causality, and interventions

Students' mathematical success, or otherwise, has a large effect on their motivational orientations (Middleton & Spanias, 1999). Middleton and Spanias (1999), through their review of the literature, found that

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motivations are relatively stable after their development in a student's early stages of schooling. Despite this, student motivations are sensitive to both teacher actions and attitudes, and 'can be affected through careful instructional design' (Middleton & Spanias, 1999, p. 82).

Farrington and colleagues (2012), in their meta-analysis, found self-efficacy (equivalent to Martin's (2007) self-efficacy) and learning strategies (equivalent to Martin's (2007) 'Planning and Study Management') to be the two non-cognitive factors most strongly influencing academic achievement. Self-efficacy is the "beliefs in one's capabilities to mobilize the motivation, cognitive resources, and courses of action needed to meet given situational demands." (Wood & Bandura, 1989, p. 408)

The non-cognitive factor of self-efficacy is a key correlate of both mathematical achievement (Shores & Shannon, 2007; Steinmayr & Spinath, 2009) and engagement (Chouinard, Karsenti, & Roy, 2007). Notably, motivation has been shown to be mediated via students' self-efficacy (Chouinard et al., 2007; Levpušek & Zupancic, 2008), as well as self-efficacy also mediating skills and approaches and associated potential increases in performance (Gist & Mitchell, 1992). It is important to note that self-efficacy and mathematical achievement appear to exhibit a bi-directional relationship (Clarence Ng, 2014), each factor impacting the other, with this effect relevant in both high (Lim & Chapman, 2015) and low (Borman & Overman, 2004) socio-economic settings as well as across cultures (Klassen, 2004; Stevens, Olivarez, Lan, & Tallent-Runnels, 2004).

Self-efficacy not only contributes to students' academic achievement but also their resilience to setbacks (Martin & Marsh, 2008; Sommer & Baumeister, 2002).

3. Design and Methods

3.1. Introduction

The aim of this chapter is to explain the basis for the intervention, the intervention's content, and how the impacts of the intervention will be evaluated. It begins by introducing the present research project as a design experiment (Romberg, 1992). Following on from this theoretical overview, it explains the instruments and methods employed. The intervention's participants, basis, structure, and content is then explicated. The final section details the types of data collected as well as selection and justification of methods used in the analysis of data.

3.2. Theoretical Framework

The assumption that non-cognitive factors such as motivation and engagement can be studied in a quantitative fashion relies upon the acceptance of a positivistic epistemology, asserting that it is valid to apply the principles of the natural sciences to the study of humans (May & Williams, 2002).

The methodology of the present study is a design experiment. Romberg (1992) describes design experiments as a process whereby:

hypotheses are first formed concerning the learning process, a teaching strategy is developed that involves systematic intervention and stimulation of the student's learning, and both the effectiveness of the teaching strategy and the reasons for its effectiveness are determined. (p. 57)

Cobb and colleagues (Cobb, Confrey, Lehrer, & Schauble, 2003) further deconstruct this approach by describing 'five crosscutting features' (p.9) of educational design experiments, each of which is present in the current intervention. Firstly, the experiment must develop or espouse theories regarding learning and link these theories to processes proposed to support learning. Secondly, methodologies are to be highly interventionist, as with the present small-group tutoring program. Thirdly, design experiments must have both prospective and

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reflective facets, that is, they ‘place these theories in harm’s way’ (Cobb et al., 2003) to provide information as to their efficacy. Fourth, design experiments are inherently iterative in nature, featuring ‘cycles of intervention and revision’ (p. 10). This is the natural process of teaching, whereby student responses and progress will be used to inform future planning and instruction. Finally, ‘the theory must do real work’ (p. 10). Fundamentally, this is a primary motivation behind the present study. This topic of multiplicative thinking was selected on the basis of its promise to scaffold and support low-achieving students in their learning of mathematics. The goal of this experiment is not only to test a theory, but to directly advance the learning of both researcher and Participants.

Design experiment was chosen over other similar options such as case study (Merriam, 1998) or action research (Kemmis & McTaggart, 1988) as the researcher is also the teacher in this context.

3.3. Methodological Basis

The purpose of this study is to explore the extent to which an intervention based upon the SNMY framework could enhance the multiplicative thinking skills of low-achieving students in Year 8. In addition, it intends to explore the ways in which students’ motivation and engagement influences or is influenced by such changes. In order to answer these questions, multiplicative thinking is measured with the SNMY assessment booklets (DEECD, 2008) and student motivation and engagement measured with the High School Motivation and Engagement Scale (MES-HS) (Martin, 2003) both pre- and post-intervention.

3.4. Instruments

3.4.1. Measuring Multiplicative Thinking

The SNMY assessment booklets (DEECD, 2008) were developed through the SNMY project (Siemon et al., 2006) and have subsequently been used in numerous schools and assessment contexts to locate students on a multiplicative thinking Learning Assessment Framework (LAF) (Education Victoria, 2008). In a more in-depth

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study, Breed (2011) used an early version of these assessment tools in conjunction with face-to-face interviews to assess student growth. These tools have also been used independently in the school context to assess growth associated with SNMY resource based instruction (Epstein, 2015).

The assessment resources are a pair of booklets (Booklet 1 and Booklet 2) that are both mapped onto the LAF with explicit marking schemes. Booklet 1 has one extended task entitled ‘Tables and Chairs’ and four short tasks: Butterfly house, Tiles, Tiles, Tiles, Adventure Camp, and Stained Glass Windows. Booklet 2 has one extended task that is an extension of the short task ‘Butterfly House’ from Booklet 1, and five short tasks: Tables and Chairs, Packing Pots, Speedy Snail, Adventure Camp, and Canteen Capers. In Booklet 2, both Tables and Chairs and Adventure Camp are identical to the questions of the same name in Booklet 1, albeit Tables and Chairs is shortened.

Both booklets provided numerous opportunities for students to demonstrate multiplication and division skills in a variety of contexts from pictorial representations (arrays) and ‘groups of’ approaches with whole numbers (with and without remainders), as well as proportional reasoning skills.

Despite some discrepancies between the two assessment instruments (see section 5.2.2.1 for a detailed discussion of these discrepancies), they were used both because of their alignment with the multiplicative thinking learning assessment framework (Siemon et al., 2005), and for ease of comparison with previous studies (Breed, 2011; Epstein, 2015). The goal of the present project was to explore the efficacy of an SNMY resource-based intervention, thus, the use of the SNMY assessment booklets was deemed a relevant component of this exploration.

3.4.2. *Measuring Students’ Motivation and Engagement*

In order to measure students’ motivation and engagement at the two time points, and explore interactions with growth and achievement, the High School Motivation and Engagement Scale (MES-HS) was used (Martin, 2003). This is a robust instrument that examines eleven motivation and engagement related constructs through 44 questions (four questions pertaining to each construct). Questions took the form of a Likert scale, each with seven options (see for Figure 3 for full scale and Figure 4 for example questions).

Disagree Strongly	Disagree	Disagree Somewhat	Neither Agree nor Disagree	Agree Somewhat	Agree	Agree Strongly
1	2	3	4	5	6	7

Figure 3. The Likert's scale as appeared in the MES-HS booklet

PLEASE CIRCLE ONE NUMBER FOR EACH STATEMENT	Disagree Strongly						Agree Strongly
1. If I can't understand my schoolwork at first, I keep going over it until I do	1	2	3	4	5	6	7
2. I feel very pleased with myself when I really understand what I'm taught at school	1	2	3	4	5	6	7
3. When I study, I usually study in places where I can concentrate	1	2	3	4	5	6	7

Figure 4. Three questions from the MES-HS assessing 1. persistence, 2. learning focus, 3. task management

The MES-HS has been independently validated in rural Australia (Plenty & Heubeck, 2011), used to assess motivation similarities and differences between suburban Australian and East Asian contexts (Martin & Hau, 2010; Martin, Yu, & Hau, 2014b) and, more recently, extended to a large-scale analysis of Motivation and Engagement in Canada, the United Kingdom, the United States, China, and Australia (Martin, Yu, Papworth, Ginns, & Collie, 2015).

3.5. Pre-intervention Phase

The pre-intervention phase stretched from initial school contact in September, 2015, until the commencement of the project in April, 2016.

3.5.1. Sampling Procedures

In September, 2015, the researcher contacted the principal of a co-educational public high school situated on the lands of the People of the Kulin Nation in metropolitan Melbourne's northern suburbs. The school is located in a low socioeconomic area with an ICSEA value of approximately 930 (ACARA, 2016).

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Approximately 70 percent of the students are male and 70 percent from language backgrounds other than English. This school was approached in part due to the student demographic, in addition to a high percentage of low-achieving students in attendance. The principal indicated an interest in a mathematics tutoring intervention for low-achieving students and, following the researcher's presentation to the school's leadership, indicated a willingness to take part in the study by subsequently reading the plain language statement for principals and signing the principal's consent form (see Appendix A).

Throughout Term 1 of 2016 the researcher worked closely with the school's mathematics department in order to identify a group of low-achieving Year 8 students from which an intervention and a control group could be formed. The following necessary participant characteristics and requirements were communicated to the school (See Appendix A for further documentation)

- Parental permission and student's willingness to participate
- Low achievement in mathematics (based upon school recommendations and assessment data)
- No diagnosed learning/behavioural difficulties
- Consistent school attendance

The Mathematics department reviewed their records and, though consideration of On-Demand test results (a state-wide adaptive test that aims to place students at an approximate year level of achievement) and teacher judgment, identified a total of approximately 30 students who would be suitable for the program. Of these 30 students the school distributed plain language statements, gaining consent (see Appendix B) from a total of 27 students who were chosen to participate in the project. These students were drawn from the three Year 8 classes at the school. The average age of participants at the time of pre-testing was 13.4 years.

3.5.1.1. Initial Testing and formation of control and intervention groups

SNMY Assessment Booklet 1 and MES-HS were administered to the 27 participants during Weeks 1 and 2 of Term 2, 2016 (early April). Table 1 shows student results from these tests as LAF level achievement, as well as final student groupings prior to the commencement of the intervention. It details each student's

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(pseudonyms used) class, group (intervention or control), gender (no students identified themselves as genderqueer) and LAF level achievement at the time of pre-testing.

Table 1. Pre-intervention groupings and LAF Levels (pseudonyms used)

	Control	Gender	LAF Level	Intervention	Gender	LAF Level
Class A	Abdullah	M	2	Rayyan	M	1
	Hamza	M	2	Haziq	M	2
	Adam	M	2	Daniel	M	2
	Zikri	M	2	Steve	M	3
	Max	M	3			
	Ash	M	3			
	Kiaan	M	4			
	Ahmad	M	4			
Class B	Kath	F	2	Wendy	F	3
	Alya	F	3	Adam	M	4
	Samar	M	4	Umar	M	4
Class C	Alice	F	3	Syed	M	1
	Zara	F	3	Peter	M	2
	Sophie	F	4	Mike	M	2
	Zikri	M	5	Nour	F	2
				Sharon	F	3

It was initially intended that students in the intervention and control groups might be matched such that for each intervention student at Level 1 of the LAF there was a corresponding student from the same class at the same level. Given test results, student preferences, and teacher recommendations, this was not possible.

Although grouping students by LAF level would have been preferable for instructional purposes, the decision was made to group students by class group for ease of timetabling and replicability should the school decide to run the program the following year. Thus, group composition was based primarily on classes, teachers' recommendations, and students' expressed desire to participate.

MES-HS results were not considered during the process of group formation.

3.6. The Intervention Phase

The goal of the intervention phase was to explore the research question: "To what extent can a small group mathematics intervention program based upon the Scaffolding Numeracy in the Middle Years framework improve the multiplicative thinking of low-achieving students in the middle years of schooling?"

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The intervention program was carried out over a total of 15 weeks in Terms 2 and 3, commencing on May 10th and concluding on September 2nd, 2016. Students attended sessions twice per week for one hour and were drawn out of classes other than mathematics in order to attend. Each session went for a total of 60 minutes, after which students were returned to their usual class for the remaining 15 minutes of the 75 minute period. Over the 15 weeks this amounted to a total of 24 sessions as 6 sessions did not go ahead due to public holidays, school timetable anomalies, and various other disruptions.

Instruction was given to groups (Groups A, B and C with 4, 3, and 5, students respectively) and took place in a small room adjoined to the numeracy room in the participating school. The room was chosen for its proximity to a usual classroom and associated mathematical teaching resources.

The sessions were timetabled by the school's head of mathematics in co-ordination with the students' other classroom teachers (from whose classes the students would be withdrawn), and the school's leadership team.

3.6.1. Content of the Intervention Program

Sessions were referred to by intervention week, followed by the lesson number for that week. For example, session 5.1 was the first session in the fifth week of the intervention. Each session of the intervention program was planned based upon student achievement in the preceding sessions, as captured in the researcher's notes. An example of these notes, taken from session 8.1, can be seen below:

Boys (Group A) in particular have lost the mental model for our multiplication strategies: (Need to do some exercises to re-visit these strategies). With fractions, really need to spend more time on naming fractions and parts. Need to do in lots of different contexts/situations, and get students to also create their own shapes and shade. Especially for group A. Group C boys also struggling. Perhaps get students to draw strategies? Make it concrete. Also, need to do some work on subtraction strategies.

Progression of the intervention was guided by the 'big ideas' (Siemon, 2006) in mathematics from Foundation level to Year 8. These 'big ideas' lie at the heart of the SNMY framework and related resources. The 'big idea' associated with each of the first six levels of the LAF are summarised in Table 2.

Table 2. The 'big ideas' in mathematics from Foundation to Year 8 (Siemon, 2006)

	Big Idea	Description of the idea	Success Indicators
Level 1	Trusting the Count	Developing flexible mental objects for the numbers zero to ten.	Students can recognise collections of the numbers from zero to ten without counting, that is, they can subitise.
Level 2	Place-value	Moving beyond counting by ones to recognizing the underlying base ten system.	Students can efficiently count large collections, make, name, record, compare, order, sequence, count forward and backwards in place value parts, and rename 2 and 3 digit numbers by place-value.
Level 3	Multiplicative thinking	Forming a basis for effective subsequent mental and computational strategies.	Students can manipulate both the size of a group and the number of groups to meet specific needs. For example, recognise that 6 eights is the same as 5 eights and 1 more eight, or 3 eights doubled, rather than committing a number fact to memory.
Level 4	Partitioning	The missing link between common fraction and decimal knowledge.	Students can construct their own fraction models and diagrams, and name, record, compare, order, sequence, and rename, common and decimal fractions.
Level 5	Proportional Reasoning	Extension of multiplicative thinking beyond rule-based procedures to contexts involving fractions, decimals, per cent, ratio, rate, and proportion.	Students can work flexibly with measures, rates and/or ratios, and recognise multiplicative relationships in a range of problem contexts. For example, understanding that $\frac{2}{3} \times \$24$ as two thirds of 24 or 3.5×68 as three and a half times 68.
Level 6	Generalising	Expanding previous skills and strategies to recognise number properties and patterns, apply to situations involving algebraic text, and broader curricula.	Students can deal with equivalent forms of expressions, recognise and describe number properties and patterns, and work with the complexities of algebraic text.

Sessions followed a consistent structure. At the start of the lesson students would be greeted, handed their workbook, and encouraged to read comments that had been written by the researcher since last session (see Figure 1, approximately five minutes).

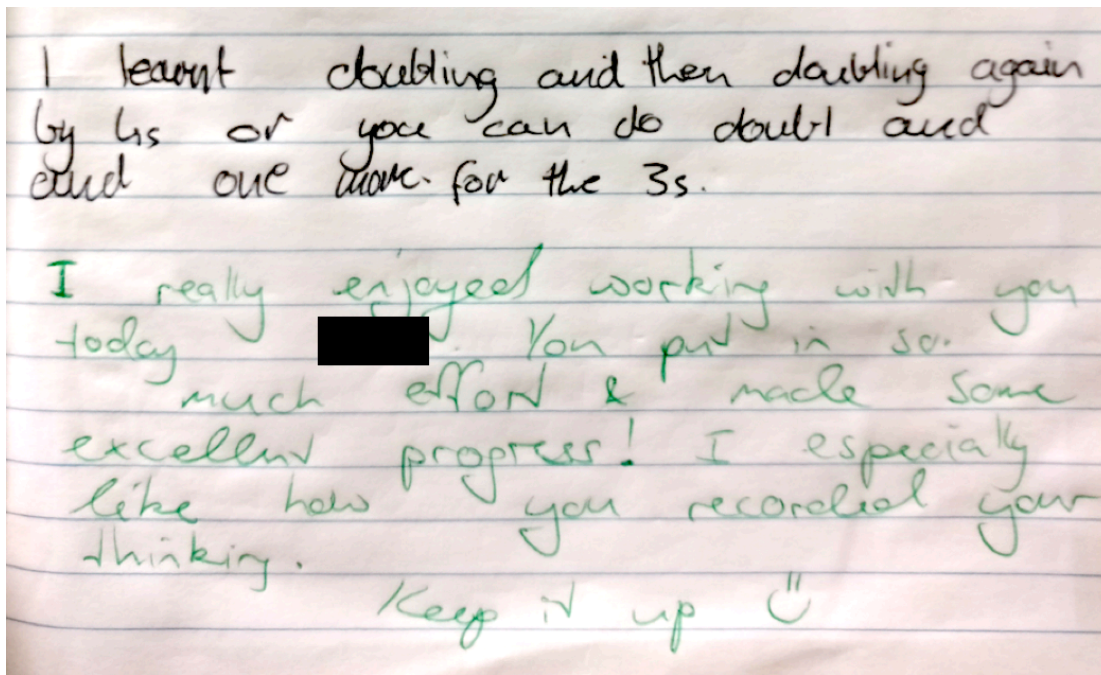


Figure 5. An example of the researchers' comment to Rayyan

This would be followed by a ‘daily’, a short activity targeted to address fundamental skills such as subitising², addition, subtraction, doubling, or halving strategies (five to twenty minutes). The main body of the lesson would consist of one to two activities targeting the ‘big idea’ for that lesson (twenty to forty minutes). The lesson concluded with a student reflection and summary of concepts learnt (ten to fifteen minutes). Due to the spread of ability levels of students in each group, it was necessary for most sessions to address more than one big idea. A week-by-week summary of big ideas and related activities included in the intervention is presented in Table 3. This is a retrospective summary of the intervention’s content. An overall outline was not developed prior to the intervention due to the adaptive nature of the planning process, using student achievement to inform future sessions.

² The ability to recognise the number of items presented without explicit counting.

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Table 3. Week by week summary of the researcher's intervention program

	Big Ideas	Related Activities
Week 1	Trusting the count and place-value	Subitising, estimating and counting large collections, counting in place value parts, trading between ones, tens and 100s, problem solving with addition
Week 2	Trusting the count and place-value	Subitising, counting in place value parts, trading between ones, tens, and hundreds, problem solving with addition,
Week 3	Trusting the count, place-value and multiplicative thinking	Subitising, doubling strategies, problem solving with addition, array method of multiplication, addition strategies
Week 4	Trusting the count, place-value and multiplicative thinking	Subitising, array method of multiplication, multiplication strategies, problem solving with multiplication
Week 5	Trusting the count, place-value and multiplicative thinking	Subitising, doubling strategies, multiplication strategies
Week 6	Trusting the count, place-value and multiplicative thinking	Subitising, multiplication strategies, addition strategies
Week 7	Trusting the count, place-value and multiplicative thinking	Subitising, place value parts, multiplication based upon place value parts, doubling strategies.
Week 8	Trusting the count, place-value, multiplicative thinking, and partitioning	Subitising, Introduction to fractions, fractions wall, array method of multiplication
Week 9	Trusting the count, place-value and multiplicative thinking	Subitising, array method of multiplication, doubling strategies
Week 10	Trusting the count, place-value, multiplicative thinking, and partitioning	Doubling strategies, array method of multiplication, addition and subtraction strategies, identifying fractions
Week 11	Place-value, multiplicative thinking, and partitioning	Addition and subtraction strategies, identifying fractions, array method of multiplication, visual representations of fractions
Week 12	Place-value, multiplicative thinking, and partitioning	Addition and subtraction strategies, doubling strategies, locating fractions on a number line, array method of multiplication
Week 13	Place-value, multiplicative thinking, and partitioning	Doubling strategies, array method of multiplication, locating fractions on a number line, partitioning strategies
Week 14	Place-value, multiplicative thinking, and partitioning	Halving strategies, comparing fractions, thirding strategies, identifying fractions on a number line, array method of multiplication.
Week 15	Place-value, multiplicative thinking, proportional reasoning	Halving and doubling strategies, problem solving with multiplication and proportional reasoning

The fact that each week covers a number of the 'big ideas' is a reflection of the diversity of achievement levels present in each group. The three groups progressed at different rates, with different groups (and sometimes individuals) often undertaking different tasks in sessions. An example of the variety exhibited in any one lesson is captured in

Table 4. Activities from selected weeks are presented in Appendix C.

Table 4. Lesson 14.1 overview: different groups, different tasks

Group A	Group B	Group C
1. Daily	1. Daily	1. Daily
2. Continuous Quantities (reinforce fraction understanding)	2. Who could I be? (reinforce fraction understanding)	2. Who could I be? (reinforce fraction understanding)
3. Tricky times (reinforcing place value and multiplication)	3. Surprise strategies, incorporating thirding (division strategies)	3. Folding for fractions (divisions strategies extended)

In addition to the main goal of promoting growth in multiplicative thinking abilities, the intervention also aimed to support participants to feel safe in the program, and to better communicate their mathematical thinking. These goals in tandem were captured in the introduction to session one, as outlined in Figure 6.

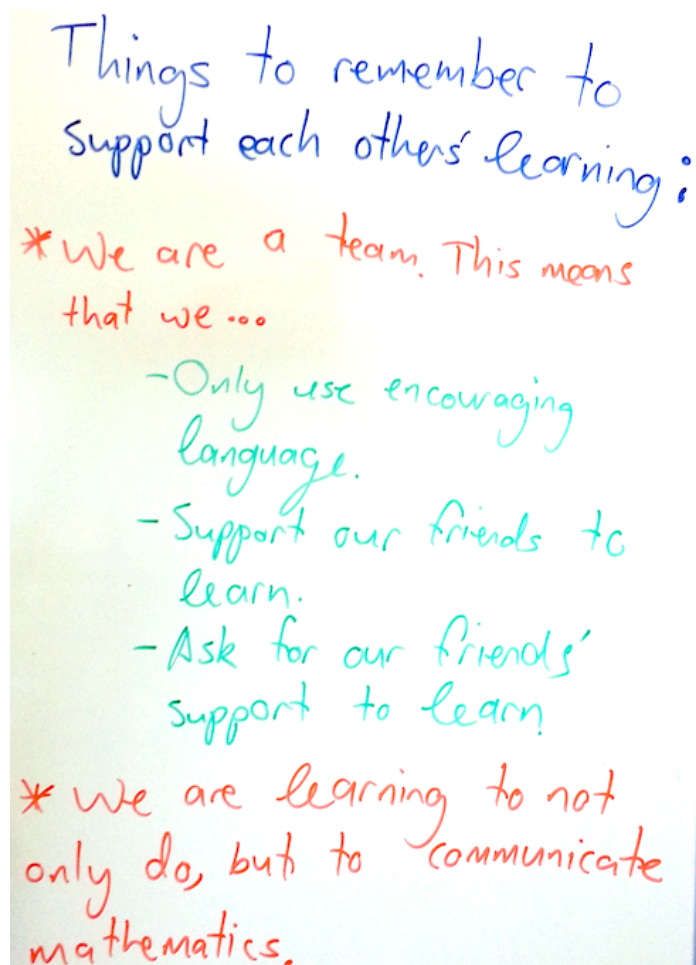


Figure 6. Note for students on whiteboard during session 1.1

3.7. Post-intervention Phase

The post-intervention phase was directed to answering the following research questions:

- To what extent can a small group mathematics intervention program based upon the SNMY framework improve the multiplicative thinking of low-achieving students in the middle years of schooling?
- To what extent does the motivation and engagement of participating students influence student growth throughout the intervention, or how is motivation and engagement influenced by it?

The purpose of the post-intervention phase was to administer the SNMY Assessment Booklet 2 and MES-HS to all participating students. This was intended to facilitate the examination of any changes that took place throughout the duration of the program.

The post-intervention phase occurred in the penultimate and final weeks of Term 3 (September, 2016). Classroom teachers decided that it would be least disruptive to their classes if all students completed the SNMY Assessment Booklet 2. Students were given two 75 minute lessons for this task, with the MES-HS administered to participating students directly following the multiplicative thinking assessment. Due to truancy, moving schools, or refusal to participate, nine students were excluded from the final analysis (see Table 5). A total of eighteen students were included in the final analysis.

Table 5. Students excluded from final analysis and associated justifications

Student	Group	Reason for exclusion from final analysis
Abdullah	Control	Refused post-intervention MES-HS
Ahmad	Control	Refused post-intervention MES-HS
Haziq	Intervention	Exhibited excessive desirability bias ³ on post-intervention MES-HS
Wendy	Intervention	Incomplete SNMY Assessment Booklet 2 due to truancy
Kath	Intervention	Incomplete SNMY Assessment Booklet 2 due to truancy
Sophie	Control	Incomplete SNMY Assessment Booklet 2 due to truancy
Nour	Intervention	Incomplete SNMY Assessment Booklet 2 due to leaving the school
Zara	Control	Incomplete SNMY Assessment Booklet 2 due to truancy
Sharon	Intervention	Incomplete SNMY Assessment Booklet 2 due to truancy

3.7.1. *Summary of data sources*

The quantitative data collected from the SNMY assessment booklets and the MES-HS was complemented by a variety of qualitative sources. These qualitative sources were researcher's notes of student comments and behaviours, student work samples, and researcher notes from discussions with teachers. Data sources associated with the three phases of the intervention are summarised in Table 6.

Table 6. Summary of intervention phases and associated data sources

Phase	Data Sources
Pre-intervention Phase	MES-HS, control and intervention groups
	SNMY Assessment Booklet 1, control and intervention groups
	MES-HS-Short, teachers
Intervention Phase	Researcher notes of students' comments and behaviours
	Student work samples
	Researcher notes of discussions with teachers
Post-intervention Phase	MES-HS, control and intervention groups
	SNMY Assessment Booklet 2, control and intervention groups

3.8. Analysis procedures appropriate to the data

This section outlines the procedures chosen in order to analyse the data obtained, along with justification of choices.

3.8.1. *The Types of Data Collected and Reported*

The Motivation and Engagement Scale (Martin, 2014b) required students to self-report on a seven-point Likert scale. Four questions related to each of the eleven non-cognitive constructs in the MES model, creating a total of 44 questions. In line with Martin's (2008) previous research, these four scores were then added and multiplied by 3.575 (or 4.762 in the case of only three items being answered³) to generate a raw score out of

³ In the case of less than three items being answered, this student was excluded from the final analysis.

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100. This allowed for appropriate comparisons between the present study and previous studies examining the motivation and engagement of students in the middle years (See Martin, 2008; 2014b).

Data used to assess changes in multiplicative thinking was based on LAF levels, as specified by the assessment booklet marking schemes, rather than SNMY assessment booklet raw scores. This use of LAF levels rather than raw scores reduced the amount of variation in scores (as the maximum number of levels changed was 3) and thus made examination of variations in student achievement more difficult, however, this was nonetheless taken for three reasons.

Firstly, as the construct under examination was multiplicative thinking, with Siemon's (2005) Learning Assessment Framework adopted as the developmental construct, reporting growth as changes in LAF levels appeared to be a relevant approach.

Secondly, this reporting on LAF levels rather than raw scores is consistent with previous studies using these instruments (Breed, 2011; Epstein, 2015). Using LAF levels in this instance facilitated ease of comparison between the present study and previous related projects.

Finally, the SNMY Assessment Booklets 1 and 2 were out of a different total number of marks (47 and 48 marks respectively), and in each case, percentage scores were not proportionally converted to levels on the LAF. For example, in Booklet 1 the LAF Level 2 threshold was 23% whereas for Booklet 2 it was 27%. Similarly, in Booklet 1 the Level 4 threshold was 43%, whereas in Booklet 2 it was 48%. This is a significant barrier to parsimonious statistical analysis of growth based upon assessment booklets' raw scores.

At this point it is important to note that Breed (2011) and Epstein (2015) both refrained from applying statistical analysis procedures to the results from their SNMY instruction based studies, with both reporting the number of students in each LAF level growth interval. However, Breed (2011) attempted to improve the accuracy of assessments by supplementing the SNMY assessment booklets this with a host of teacher assessed tasks and Epstein (2015) offered additional analysis by means of calculating effect size (though her study included no control group). The present study's use of statistical techniques (see sections 3.8.2 and 3.8.3) is an attempt to statistically quantify the effect of the present intervention in a more rigorous fashion.

3.8.2. *Selecting the type of statistical tests*

Given the small sample size it was assumed that non-parametric tests, such as the Mann-Whitney U test and the Wilcoxon Signed-Ranks test, would be more suitable than parametric ones. Parametric tests are based upon the assumptions of normality and homogeneity of variances, which are often violated in small samples (Field, 2013).

The assertion that the parametric assumptions would be violated in this small sample was confirmed by conducting both the Kolmogorov-Smirnov and the Shapiro-Wilk tests for normality, and Levene's test for homogeneity of variances. Of the twenty four data sets tested (the eleven factor MES-HS pre- and post-tests, as well as the pre- and post-intervention LAF results) nine of the data sets violated the assumptions of at least one of these tests. These results confirmed the suitability of the use of non-parametric tests (Field, 2013).

3.8.3. *The Tests Selected*

The Mann-Whitney U test is a rank-based test, also known as an assumption-free test, used to make a comparison between two groups. It is suitable in the case of small samples with small amounts of variation (as in the present study). Ranking the outcomes from two different groups (in this case, control and intervention) enables researchers to examine whether the participants from either group converge disproportionately towards the top or the bottom of the rankings. By carrying out statistical analyses on the ranks, rather than the initial scores, researchers can see trends that would not be revealed with more frequently used mean-based techniques. The Mann-Whitney U test assumes the null hypothesis that if a value is randomly selected from one group it is no more likely to be above the value of a randomly selected value from the other group, than below it. It was used to examine whether there existed statistically significant differences between control and intervention groups, prior-to or after the intervention, on any of the MES-HS or LAF measures.

Changes in student outcomes, based upon results captured with the MES-HS and the SNMY assessment booklets, were examined using another rank-based test, the Wilcoxon Signed-Ranks test. The Wilcoxon Signed-Ranks test differs from the Mann-Whitney U test in that it is a mean difference based ranking procedure, rather than a procedure based upon the ranking of scores from different subjects at one time interval. This makes it a

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within-subject rather than a between-subject test, and appropriate for the examination of growth of a participant over time, or the comparison of two ratings of the same individual (such as student self-ratings and a teacher's rating of that same student). This test examines the null hypothesis that a mean difference score (in this case, the difference between pre- and post-testing LAF) is no more likely to be positive than it is negative. The Wilcoxon Signed-Ranks test has the test statistic 'T', which is the absolute value of the smallest sum of the ranks. This test was used to explore the questions:

- To what extent can a small group mathematics intervention program based upon the scaffolding numeracy in the middle years framework improve the multiplicative thinking of low-achieving students in the middle years of schooling?
- 'How is motivation and engagement influenced by it [the intervention]?' (one half of question two)

For both the Mann-Whitney and Wilcoxon Signed-Rank tests, effect sizes were calculated by dividing z-score outputs by the square root of the total number of students in the sample (Field, 2013).

In order to explore the question, "To what extent does the motivation and engagement of participating students influence student growth throughout the intervention?" (the other half of question two), the non-parametric analysis of correlation, Spearman's Rho, was used to explore correlations between student self-ratings of motivation and engagement at the time of pre-testing and pre-to-post-intervention growth on the SNMY LAF.

Researcher's notes of students' comments and behaviour, work samples, and researcher's notes of discussions with teachers were used to supplement quantitative data.

3.9. Summary

This chapter began by outlining the current project as a design experiment. It then introduced the participants and explained how the SNMY assessment booklets were used in tandem with the MES-HS and the MES-Short to collect information as to how their multiplicative thinking, and motivation and engagement

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changed throughout the intervention. The basis of the intervention, the SNMY resources and the ‘big ideas’ in school mathematics, were then outlined along with a schedule of the intervention’s phases. The chapter concluded with an explanation and justification of the data collected, and analysis procedures undertaken, in order to answer this project’s research questions.

4. Results

4.1. Introduction

This chapter explores changes in multiplicative thinking, motivation and engagement that took place in both intervention and control group students during the period over which the intervention was carried out. It begins by presenting changes in multiplicative thinking, as captured by the Assessment Booklets 1 and 2, examines changes in motivation and engagement in students, then compares changes in multiplicative thinking to teacher ratings of student motivation and engagement. At all stages, qualitative data in the form of student work samples and notes from the researcher's journal will be used in order to provide context for, and colour to, the quantitative results.

4.2. Changes in Multiplicative Thinking

This section aims to answer the question, "To what extent can a small group mathematics intervention program based upon the Scaffolding Numeracy in the Middle Years framework improve the multiplicative thinking of low-achieving students in the middle years of schooling?".

For students in the intervention group, LAF levels were not significantly higher at the time of post-testing ($Mdn=1$) than at the time of pre-testing ($Mdn=2$), $T = 5$, $p = 0.496$, $r = -0.16$. For students in the control group, LAF levels were also not significantly higher at the time of post-testing ($Mdn=3$) than at the time of pre-testing ($Mdn=3$), $T = 18$, $p = 0.564$, $r = -0.14$. This suggests that the intervention generated no significant improvements in multiplicative thinking, as captured by the SNMY assessment booklets. However, considering the results captured by the assessment booklets in isolation paints an incomplete picture of learning that occurred throughout the intervention, these quantitative results are complemented throughout this section by qualitative results by means of researcher's notes and student work samples. The changes in LAF level between

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pre- and post-testing for each student, with some students highlighted, are presented in Table 7 (see Table 5 in section 3.7 for a discussion of exclusion criteria for the results reported N/A).

Table 7. Growth in multiplicative thinking from Pre- to Post-testing

	Control	LAF Level	LAF Level	Intervention	LAF Level	LAF Level
	Group	(Pre-test)	(Post-test)	Group	(Pre-test)	(Post-test)
Class A	Abdullah	2	N/A	Rayyan	1	1
	Hamza	2	3	Haziq	2	N/A
	Adam	2	3	Daniel	2	1
	Zikri	2	2	Steve	3	1
	Max	3	2			
	Ash	3	4			
	Kiaan	4	3			
	Ahmad	4	N/A			
Class B	Kath	2	N/A	Wendy	3	N/A
	Alya	3	5	Adam	4	1
	Samar	4	5	Umar	4	4
Class C	Alice	3	2	Syed	1	1
	Zara	3	N/A	Peter	2	4
	Sophie	4	N/A	Mike	2	3
	Zikri	5	4	Nour	2	N/A
				Sharon	3	N/A

The results from Table 7 are summarised in Figure 7, with intervention students highlighted and factors relevant to LAF level change noted in some cases. These relevant factors are elaborated upon in following sections.

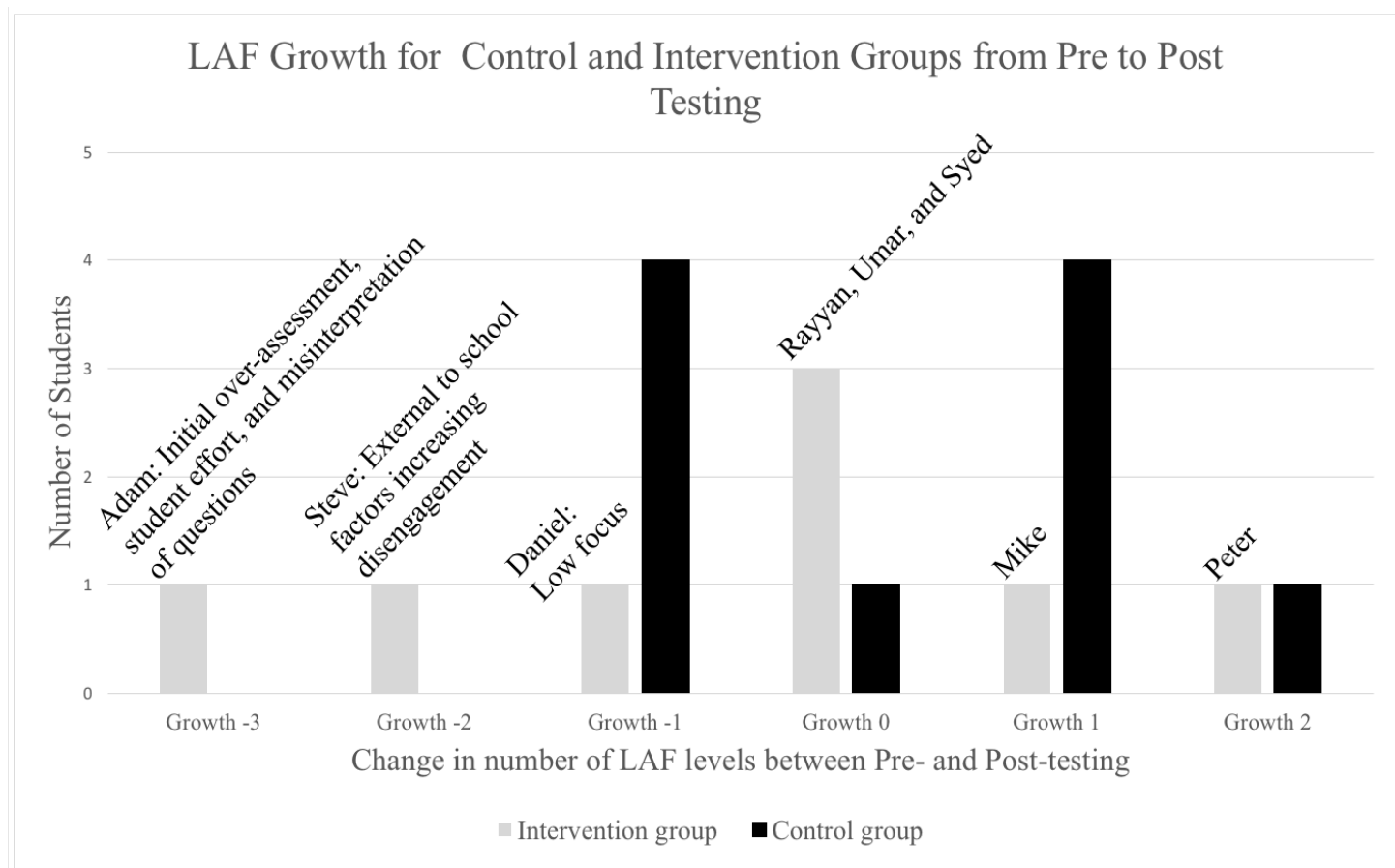


Figure 7. Annotated chart of student growth in multiplicative thinking

Given the aim of the present study, the results presented above are not encouraging. Whilst growth was exhibited by two of the intervention participants in the study, this growth didn't exceed that of the control group. These results warrant significant exploration and discussion, especially in light of much more encouraging results reported by Breed (2011) and Epstein (2015) resulting from similar instructional approaches.

The growth that was reported, such as for Mike (1 level) and Peter (2 levels), is discussed in sections 4.2.1 and 4.2.2. Adam's significant decrease is outlined in section 4.2.3. Growth not captured in these written assessments, as well as barriers to growth, will be outlined in section 4.2.4, with some additional comments on the remaining students in section 4.2.5.

4.2.1. Mike's Growth

Mike's performance on Assessment Task Booklet 2 demonstrated a firm grasp of the array method of multiplication, extending it to a recognition of place-value-part multiplication techniques (see Figure 8). This was in contrast to Booklet 1 in which he did not demonstrate an ability to multiply two 2-digit numbers.

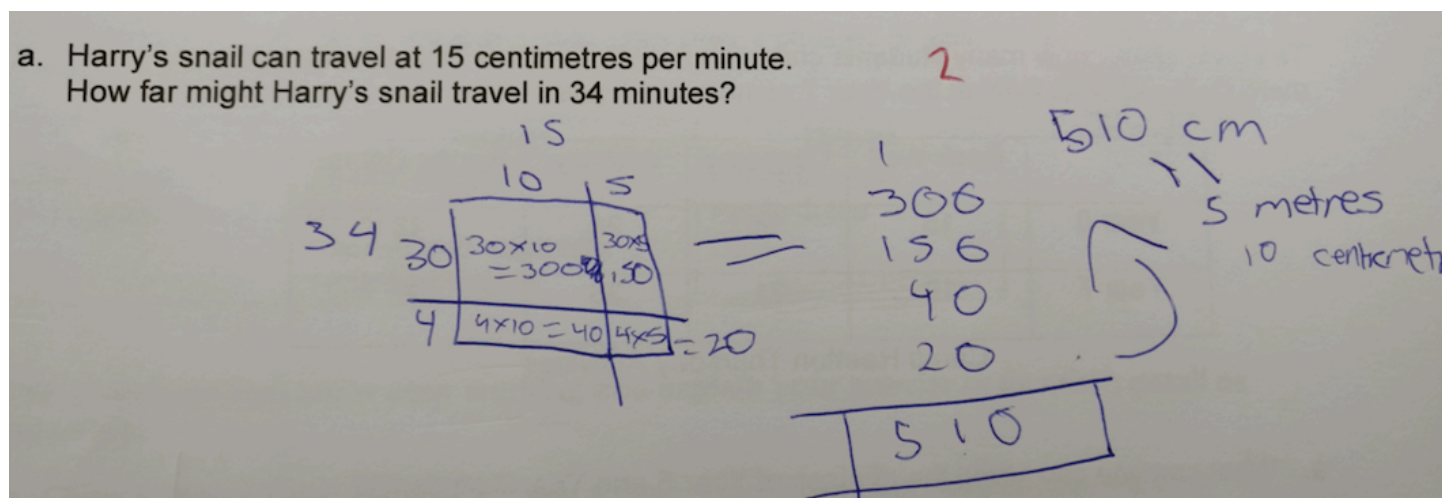


Figure 8. Mike combines place value and the array method of multiplication, Booklet 2 (post-test)

Furthermore, in Booklet 2, Mike showed a strong grasp of doubling strategies, applying his knowledge of place-value in order to double a decimal (see Figure 9). Again, this contrasts with a lack of ability to perform this same task in Booklet 1, where he suggested that $2 \times 99 = 1998$ (see Figure 10).

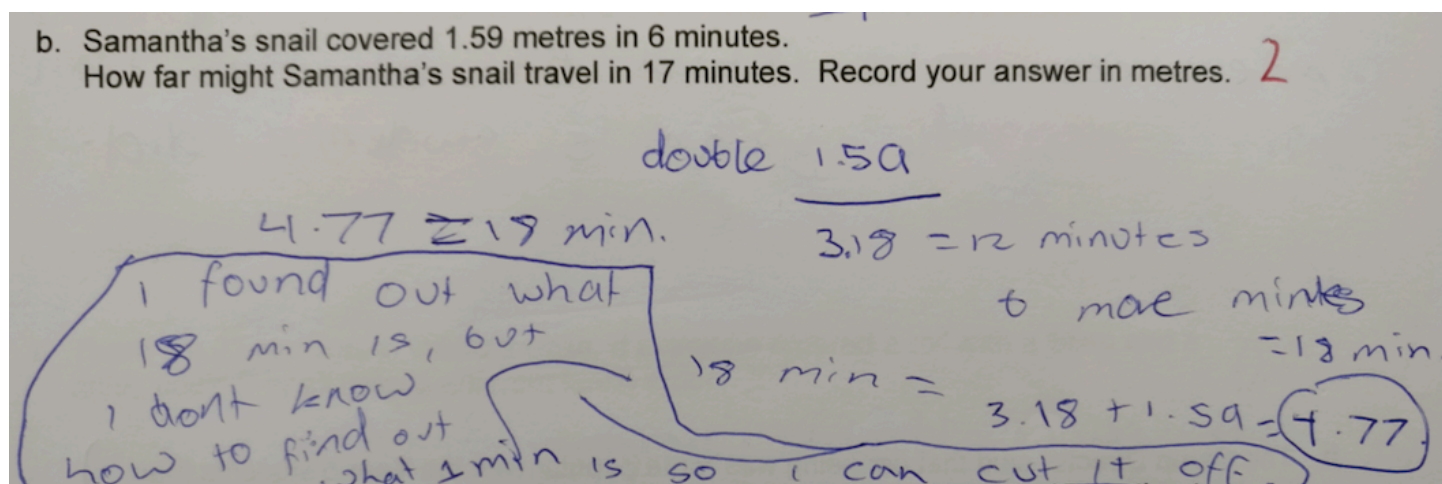


Figure 9. Mike's successful doubling of 1.59, Booklet 2 (post-test)

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h. The community can borrow 99 tables. How many people could they seat using 99 tables placed end-to-end? **Show your working and explain your answer in as much detail as possible.**

2000 people.
 if you cut off the end bit, you have 2.
 ~~$2 \times 99 = 198$~~
 ~~$2 \times 99 = 198$~~
 $2 \times 99 = 1998$
 then add the 2 ends.
 $1998 + 2 = 2000$

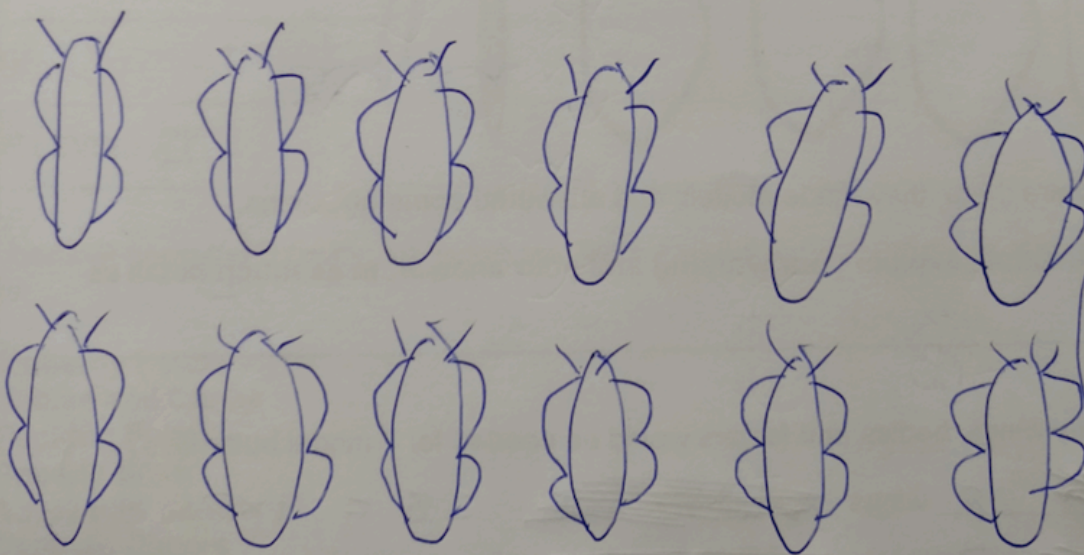
Figure 10. Mike's unsuccessful doubling of 99 in Booklet 1 (pre-test)

This growth is contrasted by Mike's difficulty in interpreting questions in order to apply his demonstrated multiplicative skills. The question depicted in Figure 11 (Booklet 2, Butterfly house part c.) required him to multiply 98 by 4, 1, and 2 respectively. His inability to uncover the underlying structure of the question is a consistent theme throughout the results and will be explored further in the discussion.

c. How many wings, bodies and feelers will be needed to make 98 model butterflies. **Show all your working and explain your answer in as much detail as possible.**

72 wings
36 bodies
72 feelers

half of 98 = 49
 half of 49 = 24.5
 half of 24.5



12 - 3.
 $24 \times 3 = 72$
 $12 \times 3 = 36$
 $24 \times 3 = 72$
 double
 1 $\times 3$
 24 wings
 12 bodies
 24 wings

Figure 11. Mike fails to correctly identify the underlying multiplicative structure of the question

4.2.2. *Peter's Growth*

The sources of Peter's growth are less clear than Mike's. Like Mike, in Butterfly house, part c. he also failed to identify the underlying structure of the question in both Booklet 1 and Booklet 2. He displayed similar answers, with similar misconceptions in both booklets. This is apparent in Figure 12 and Figure 13.

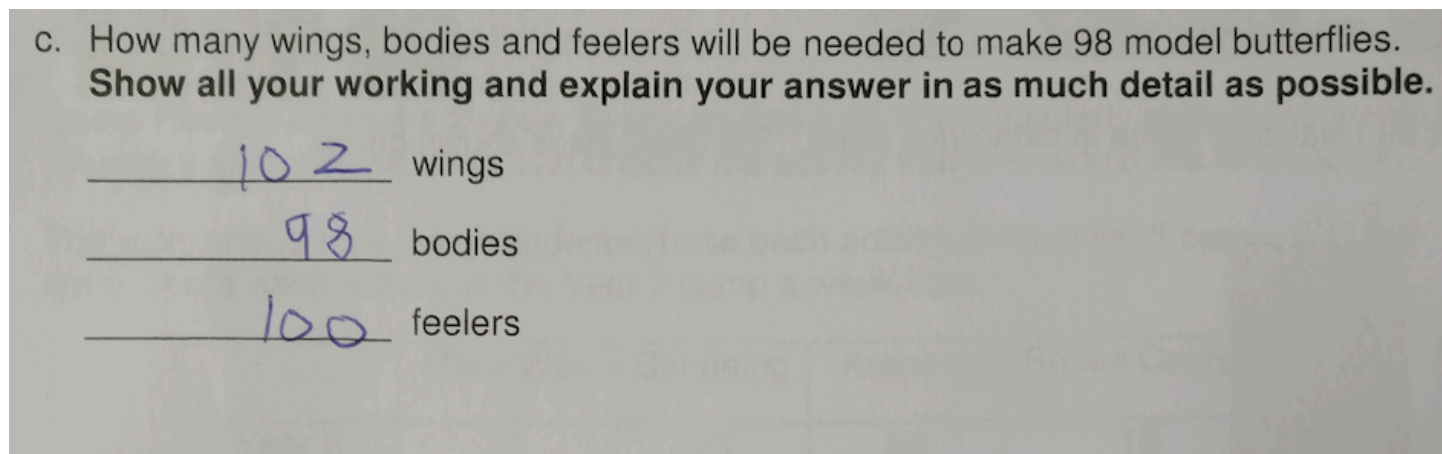


Figure 12. Peter's response to Butterfly house, part c. Booklet 1

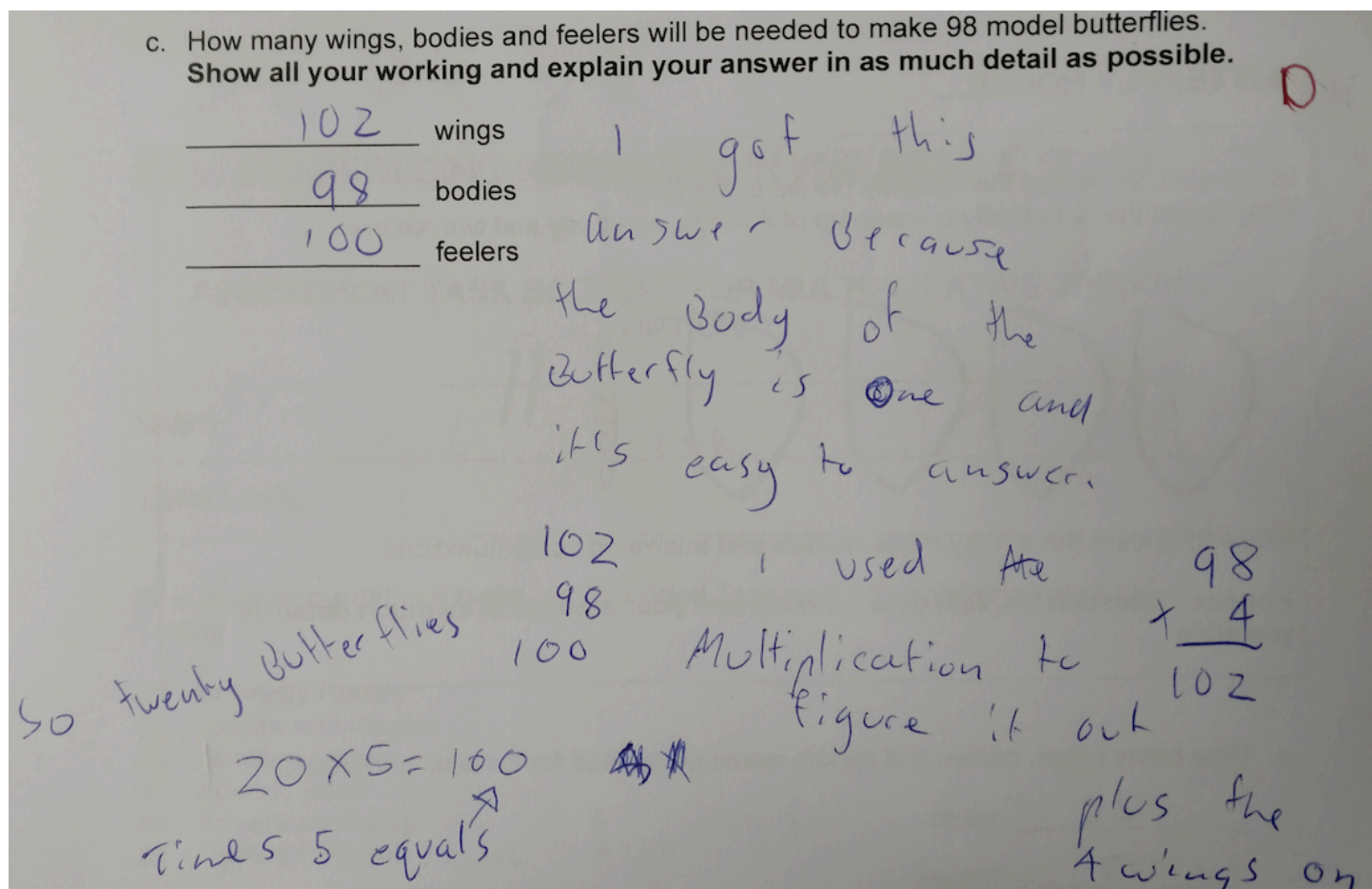


Figure 13. Peter's response to Butterfly house, part c. Booklet 2

It would appear that a significant portion of Peter's reported growth from pre- to post-testing originated from his ability to interpret Cartesian product ideas, despite this concept not being taught as part of the intervention. As will be expanded upon in section 5.2.2.1, this is a concept that was not included in Assessment Task Booklet 1. From the questions pertaining to Cartesian products, Peter obtained 4 marks in total, constituting 15% of his overall mark.

4.2.3. *Adam's Negative Growth*

Adam's results demonstrate a three level LAF decrease. This is a decrease not matched by any participants of intervention groups in previous similar studies (Breed, 2011; Epstein, 2015). It is proposed that this is due to three interrelated factors: initial over-assessment, student effort, and misinterpretation of questions. Each of these factors embody a paradigmatic example of three challenges present in the intervention more broadly, all three of which will be elaborated upon in Section 5.

4.2.3.1. *Initial over-assessment*

Though Adam was scored at level 4 in the LAF at the time of pre-testing, examination of his answers suggests that he did not possess the skills associated with the 'big idea' of levels 3 (Multiplicative Thinking), or 4 (Partitioning). In the pre-intervention assessment, Adam did not use multiplication in the answering of any questions in which it would have been appropriate, instead reverting to additive strategies (see Figure 14).

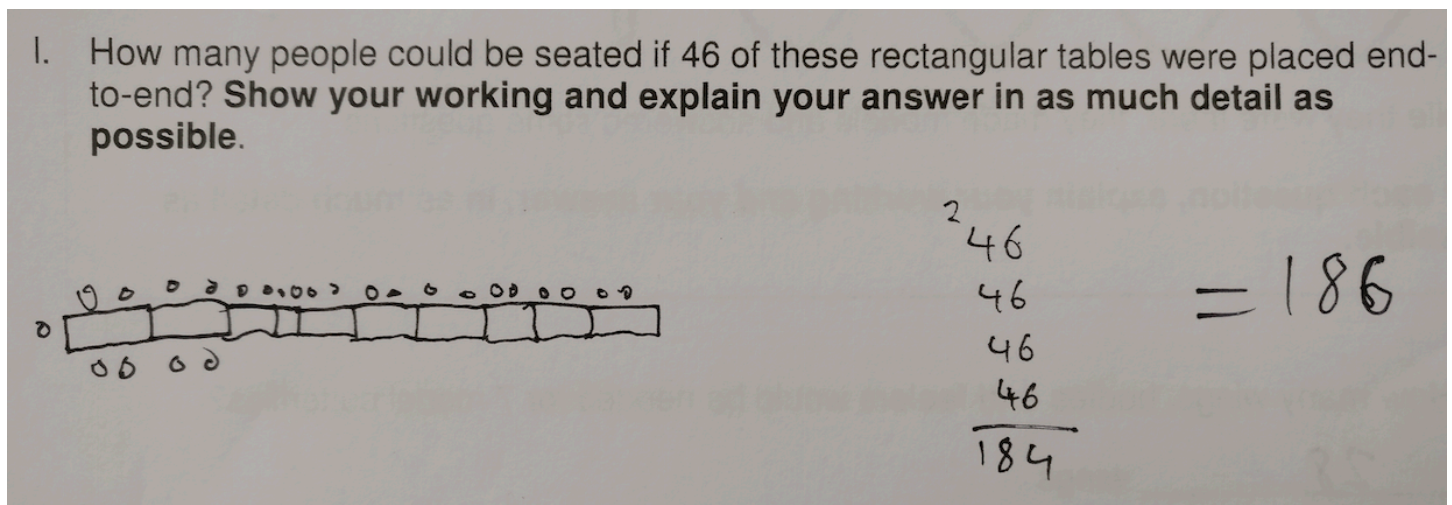


Figure 14. Adam demonstrating additive rather than multiplicative approaches in the pre-test

A MIDDLE YEARS MULTIPLICATIVE THINKING INTERVENTION

Also in the pre-test, rather than demonstrating a partitioning approach (associated with LAF Level 4), Adam instead used make-all and count-all strategies associated with LAF Level 1 (see Figure 15).

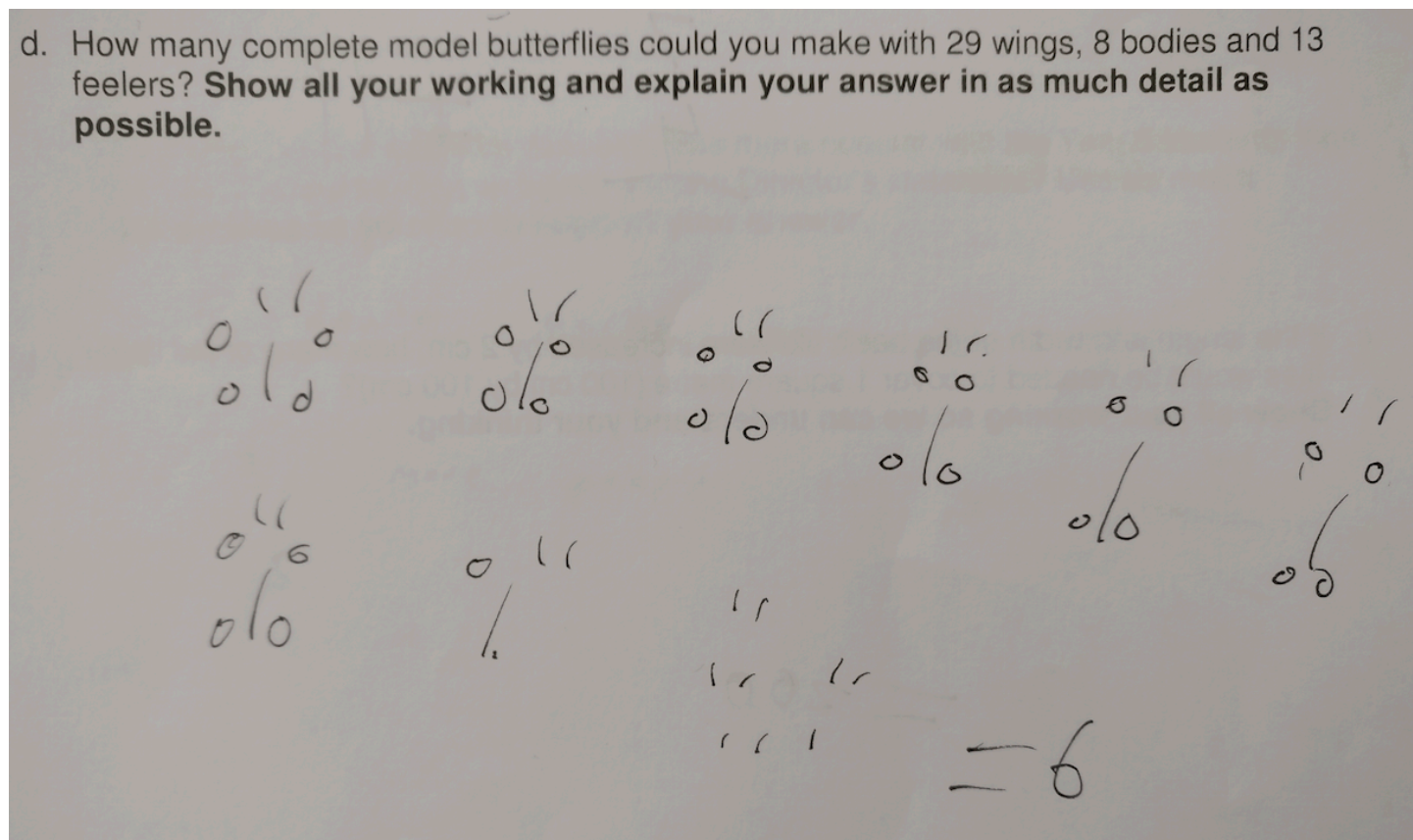


Figure 15. Adam's make-all and count-all approaches in the pre-intervention assessment

Taken together, this evidence suggests that the assessment of Adam's multiplicative thinking skills from the SNMY Task Assessment Booklet 1 and associated marking scheme produced an over-rating at the time of the pre-intervention assessment.

4.2.3.2. Student effort

On several of the questions, Adam's responses suggest a higher level of effort on the pre-test when compared to the post-test. One example of this is demonstrated by contrasting Figure 16 with Figure 18. In these figures, it can be seen that attempting the second question in Booklet 1 (with an incorrect answer) resulted in credit, as dictated by the marking scheme (see Figure 17).

A MIDDLE YEARS MULTIPLICATIVE THINKING INTERVENTION

Camp Reefton offers 4 activities. Everyone has a go at each activity early in the week. On Thursday afternoon students can choose the activity that they would like to do again.

The table shows how many students chose each activity at the Year 5 camp and how many chose each activity at the Year 7 camp a week later.

	Rock Wall	Canoeing	Archery	Ropes Course
Year 5	15	18	24	18
Year 7	19	21	38	22

Camp Reefton Thursday Activities

- a. What can you say about the choices of Year 5 and Year 7 students?

there are more people in
year 7 than year 5
that go to the camp

2

- b. The Camp Director said that canoeing was more popular with the Year 5 students than the Year 7 students. Do you agree with the Director's statement? **Use as much mathematics as you can to support your answer.**

I don't agree because there are
3 more people in year ~~seven~~ 7 than
5

Figure 16. Adam's incorrect but credited response to Adventure Camp, part b. in the pre-test

A MIDDLE YEARS MULTIPLICATIVE THINKING INTERVENTION

b.	No response	0
	Incorrect (No), argument based on numbers alone, eg, "There were 21 Year 7s and only 18 Year 5s"	1

Figure 17. Part of the Marking Scheme for SNMY Booklet 1, Adventure Camp, Part b

This is contrasted with Adam's lack of effort on the same question in Booklet 2 (see Figure 18).

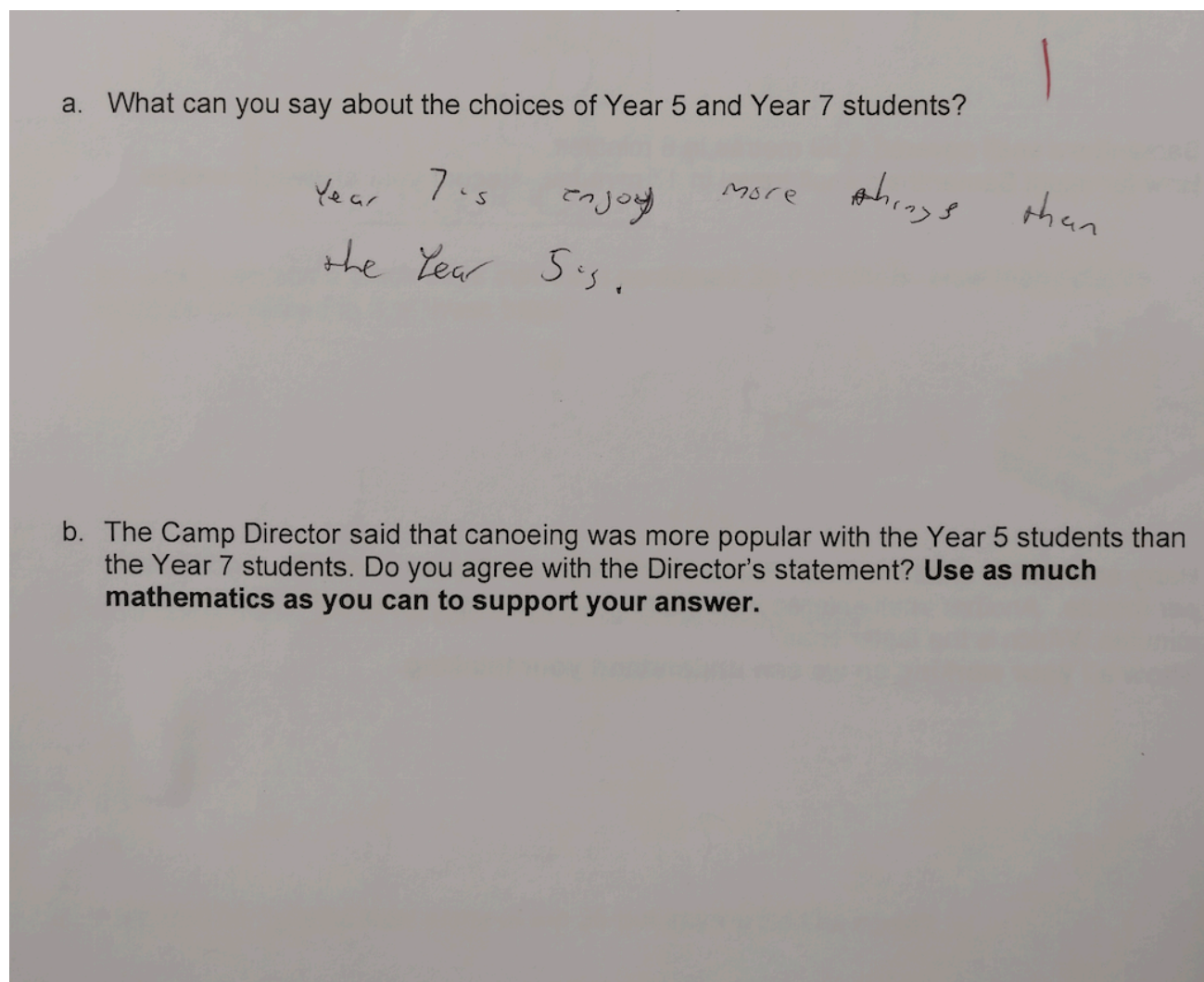


Figure 18. Adam's absence of response to Adventure Camp, part b. in the post-test

Further examples evincing this lack of effort are Adam's attempts at the final three questions of Booklet 2, as depicted in Figure 19, Figure 20, and Figure 21.

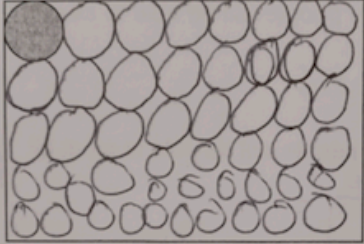
www.education.vic.gov.au/studentlearning/teachingresources/maths

PACKING POTS

Jim works in a plant nursery.
He has been asked to put all the small pot plants into trays to take to the market.

For each question, show all your working and explain your answer in as much detail as you can.

a. To stop the pots from moving around Jim packs them into the tray as tightly as possible.
How many pots will he be able to fit into this tray?

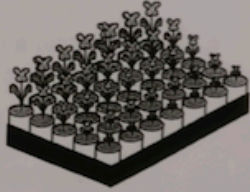


9 per row 1
45
9 x 5

b. Jim was given some black trays that each contained 35 pot plants. How many plants would be contained in 3 of these trays?

?

105
1 added



c. How many plants would be contained in 14 of the 35-pot trays?

?

d. If Jim had 560 plants, how many of the 35-pot trays would he need?

?

Figure 19. Adam's lack of effort on 'Packing Pots'

SPEEDY SNAIL...



a. Harry's snail can travel at 15 centimetres per minute.
How far might Harry's snail travel in 34 minutes?

?

b. Samantha's snail covered 1.59 metres in 6 minutes.
How far might Samantha's snail travel in 17 minutes. Record your answer in metres.

?

c. Harry entered his snail in a race. Remember Harry's snail can travel at 15 centimetres per minute. Another snail entered in the same race, covered 3.71 metres in 24 minutes. Which is the faster snail?
Show all your working so we can understand your thinking.

?

Figure 20. Adam's lack of effort on 'Speedy Snail'

CANTEEN CAPERS...

The school canteen makes great lunches. They offer 2 choices of rolls, 4 choices of filling and 3 choices of drink.



Rolls	Fillings	Drinks
White	Peanut butter	Orange Juice
Multi-grain	Salad	Apple Juice
	Cheese and vegemite	Cola
	Tuna and lettuce	

For each question, show your working and explain your answer in as much detail as possible.

a. Claire ordered a peanut-butter roll and a drink. What might she have ordered? List all possibilities.

white peanut butter and a cola

b. There are 23 students in Claire's class. If everyone ordered a roll with a filling and a drink, could they all have a different lunch order?

?

Figure 21. Adam's lack of effort on 'Canteen Capers'

Adam wrote no such question marks in Booklet 1, instead attempting all questions, and taking significantly more time on Booklet 1 than Booklet 2. Taken together, these examples suggest a higher level of effort, and therefore more marks obtained, by Adam in the pre-test when compared to the post-test.

4.2.3.3. Misinterpretation of questions

A differential in Adam's interpretation of the question 'Tables and Chairs' resulted in Adam achieving two marks from parts a. to e. on the post-test in comparison to five marks in the pre-test. Many students struggled to correctly interpret this question, a point that will be expanded upon in section 5.

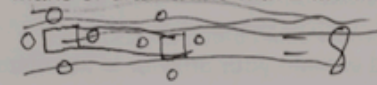
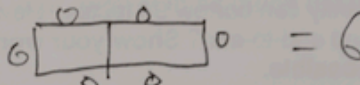
TABLES AND CHAIRS ...

Dean's community is planning a street party. They have lots of small square tables. Each table seats 4 people like this:

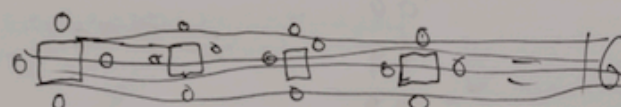
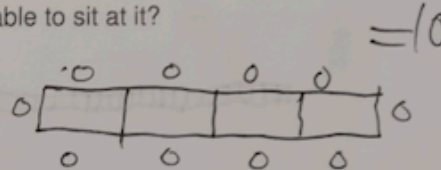


The community decides to put the tables in an end-to-end line along the street to make one big table.

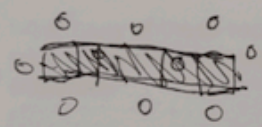
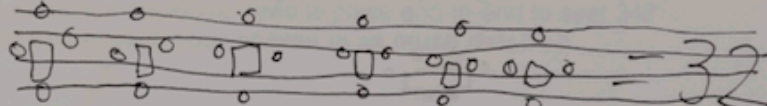
a. Make or draw a line with 2 tables. How many people will be able to sit at it?

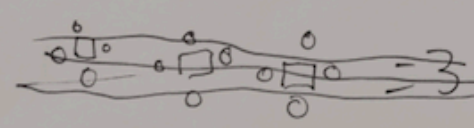
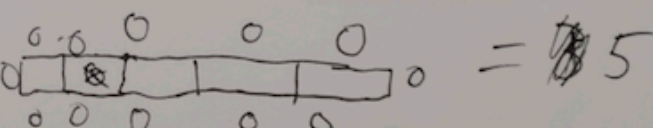
b. Make or draw a line of 4 tables. How many people will be able to sit at it?

c. Make or draw a line of tables that would seat 8 people. How many tables are needed?

d. Make or draw a line of tables that would seat 12 people. How many tables are needed?

e. Make or draw a line of tables that would seat 20 people. How many tables are needed?

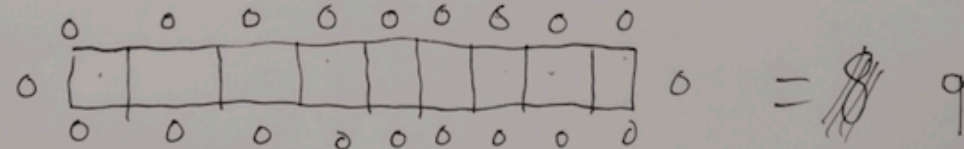


Figure 22. Adam's correct interpretation of Tables and Chairs in the pre-test

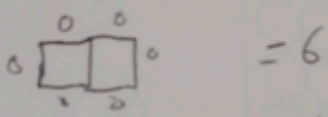
TABLES AND CHAIRS ...

Dean's community is planning a street party. They have lots of small square tables. Each table seats 4 people like this:

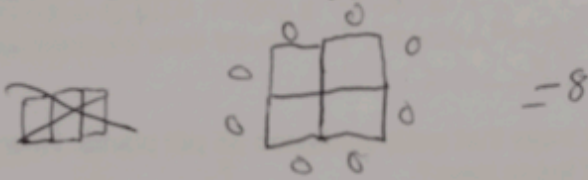


The community decides to put the tables in an end-to-end line along the street to make one big table.

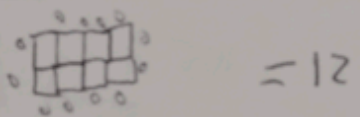
a. Make or draw a line with 2 tables. How many people will be able to sit at it?



b. Make or draw a line of 4 tables. How many people will be able to sit at it?



c. Make or draw a line of tables that would seat 8 people. How many tables are needed?



d. Make or draw a line of tables that would seat 12 people. How many tables are needed?



e. Make or draw a line of tables that would seat 20 people. How many tables are needed?

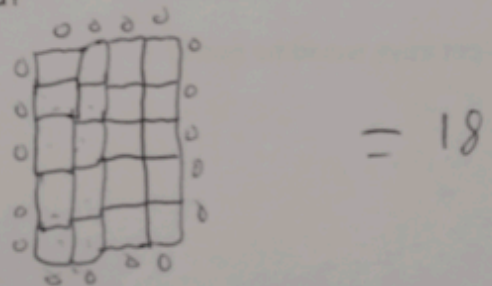


Figure 23. Adam's incorrect interpretation of Tables and Chairs in the post-test

4.2.3.4. Concluding remarks upon Adam's results

Whilst Level 4 on Booklet 1 may be an overestimate of Adam's multiplicative thinking, Level 1, based upon Booklet 2 results, may be an underestimate. This is especially the case given that in 'Packing Pots', part a. Adam clearly demonstrated an understanding of the array method of multiplication (see Figure 19), a skill associated primarily with level 3 of the LAF.

4.2.4. Growth Not Captured by the SNMY Assessment Booklets

For some students such as Rayyan, Adam, and Umar, growth occurred that was not captured in the assessment booklets.

Rayyan came into the intervention unable to double single-digit numbers, or halve the even numbers to 20. He openly made comments such as 'I don't know my times tables', and clearly had very low self-efficacy with regards to mathematics. Despite this, by the end of the intervention he was able to double all of the numbers up to 100 by considering place-value. Evidence for this is provided in the excerpt from the researcher's journal from lesson session 12.1, included below, in which the students were playing 'Double trouble bingo', a game where students are required to double the numbers that are drawn from a selection of two digit numbers and cross off the corresponding answer on their bingo board:

we got to an interesting point where Rayyan mentioned that he needed 110 to get the first bingo line... I asked him 'Ok, so what's the number you're looking for?', he said '110', and I said, 'yeah, but what do you need to double to get to 110?'. He knocked my socks off by straight out saying 'You need fifty to make the hundred, and five to make the 10, so fifty five.' A clear demonstration of a strong understanding of place value. Amazing!

This was a significant achievement for Rayyan and one for which he overtly demonstrated a sense of pride.

A MIDDLE YEARS MULTIPLICATIVE THINKING INTERVENTION

Adam's enhanced capacity for multiplicative thinking (pictured in Figure 19) was also demonstrated throughout the intervention. The reflection depicted in Figure 24 demonstrates Adam's revelation regarding a five times strategy.

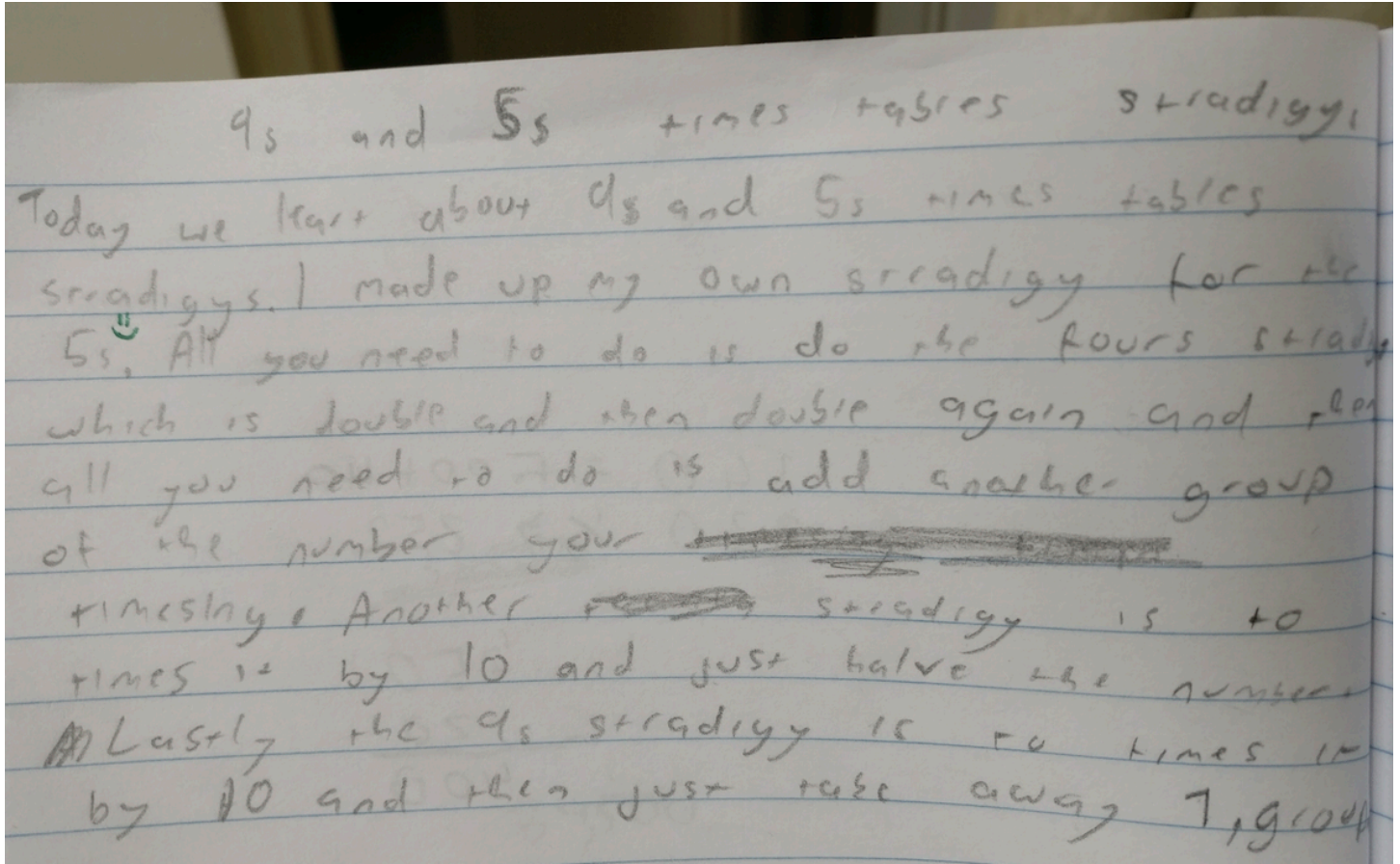


Figure 24. Adam using part-part-whole understanding to build multiplication strategies

The above reads:

Today we learnt about 9s and 5s times tables stradigys [*sic*]. I made up my own stradigy for the 5s. All you need to do is do the fours stradigy, which is double and then double again, and then all you need to do is add another group of the number your timesing. Another stradigy is to times it by 10 and just halve the number. Lastly, the 9s stradigy is to times it by 10 and then just take away 1 group.

Adam's comment, "do the fours stradigy [*sic*], which is double and then double again, and then all you need to do is add another group", is analogous to the example given by Siemon (2006) as a key indicator of the big idea of multiplicative thinking; 'instead of committing 6 eights to memory in a meaningless or rote way, recognise that this can be thought of as 5 eights and 1 more eight,' (p. 6).

A MIDDLE YEARS MULTIPLICATIVE THINKING INTERVENTION

Umar displayed growth in his understanding of fractions, especially in relation to the number line. This is not something that was explicitly tested in the SNMY Assessment Booklets 1 or 2. He summarised and reflected upon this growth through two different reflections (see Figure 25 and Figure 26) and the full time reflection (see Figure 27).

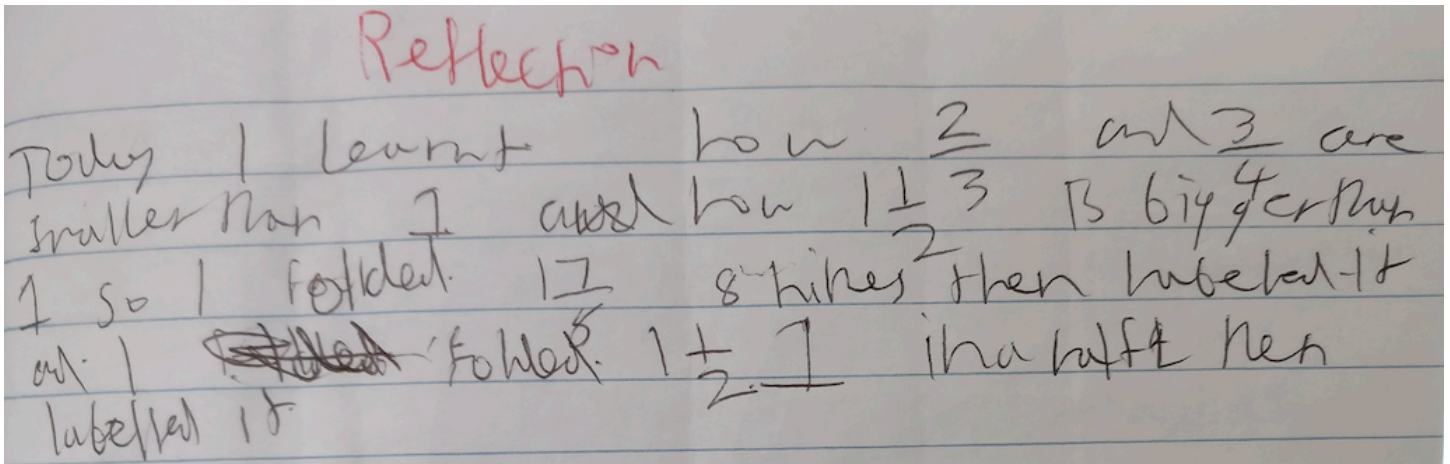


Figure 25. Umar's first reflection relating fractions to the number line

"Today I learnt how $\frac{2}{3}$ and $\frac{3}{4}$ are smaller than 1 and how $1\frac{1}{2}$ is bigger than 1".

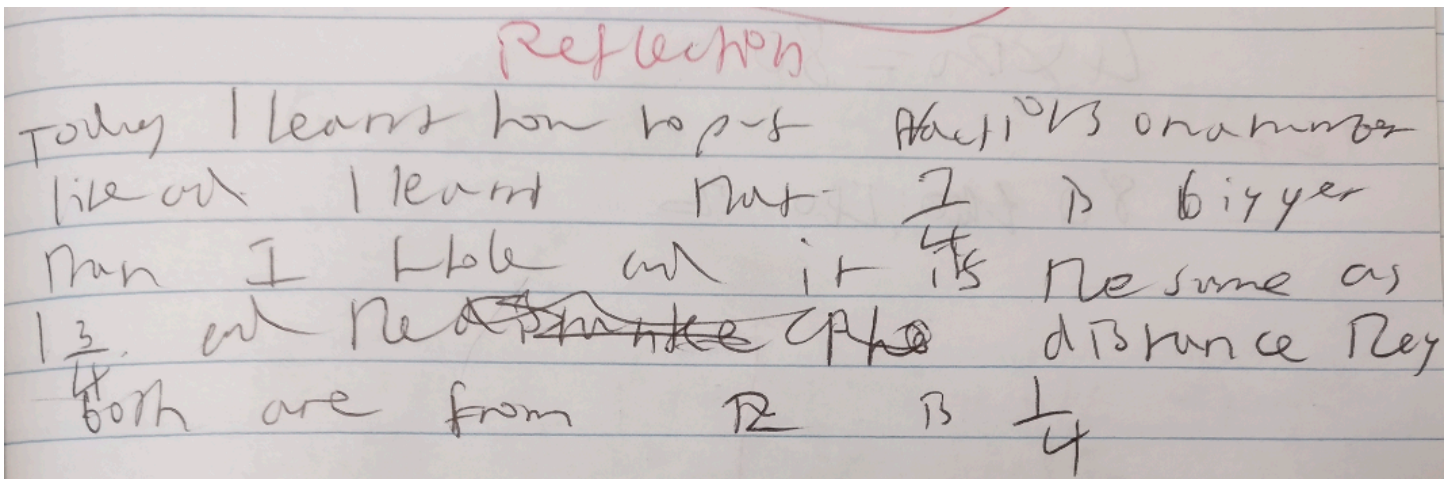


Figure 26. Umar's second reflection relating fractions to the number line

"Today I learnt how to put fractions on a number line and I learnt that $\frac{7}{4}$ is bigger than 1 whole and it is the same as $1\frac{3}{4}$ and the distance they both are from 2 is $\frac{1}{4}$."

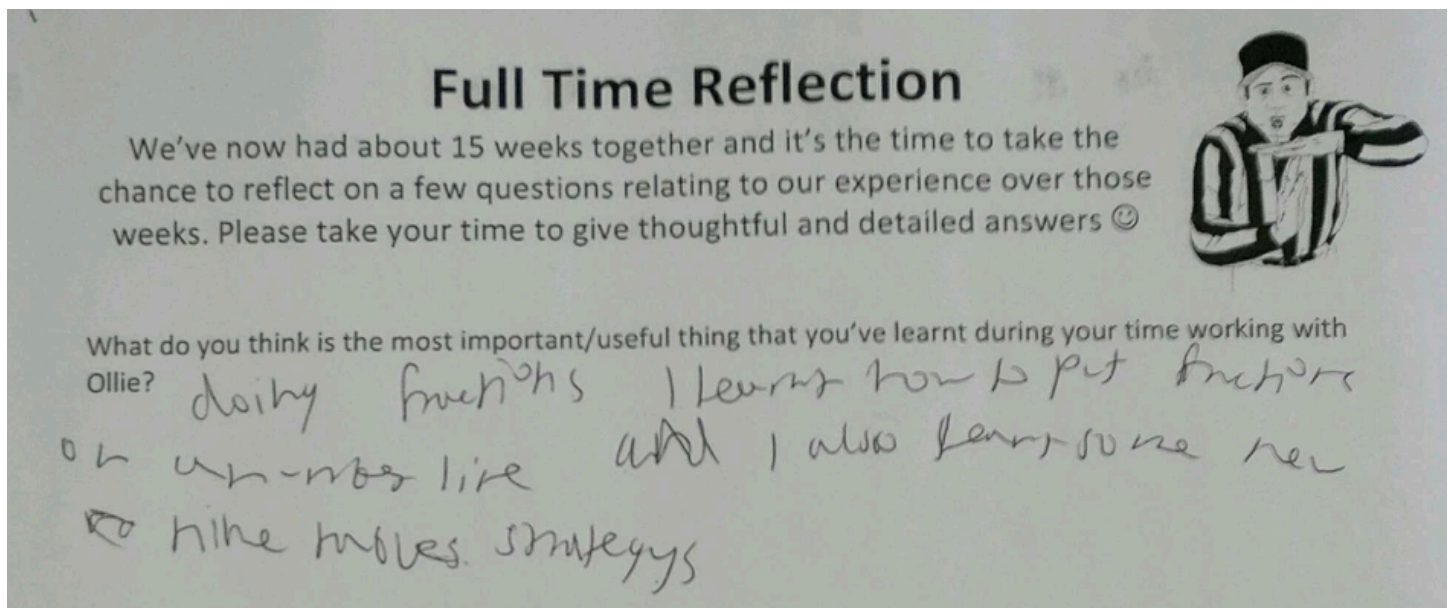


Figure 27. Umar reflects upon learning from the intervention

“Doing fractions I learnt how to put fractions on a number line and I also learnt some new nine tables strategys [sic].”

4.2.5. *The Other Students*

Factors leading to a lack of growth demonstrated by the remaining students (Daniel, Steve, and Syed) will be explored in depth in section 5. These relate particularly to out-of-school factors (Steve, sections 5.2.1.2 and 5.2.1.4), attendance and focus (Daniel, sections 5.2.1.1 and 5.2.1.2), and potential learning difficulties (Syed, section 5.2.1.1).

4.2.6. *Summarising Growth in Multiplicative Thinking*

The intervention was not associated with any significant increase in multiplicative thinking, as captured by the SNMY assessment booklets. Notwithstanding the barriers to reported growth highlighted in this section, such as over-rating at the time of pre-testing, student effort at the time of post-testing, and misinterpretation of questions, it is still the case that similar interventions (Breed, 2011; Epstein, 2015) using the same approaches and techniques produced more promising results. Within this context, the results produced by the present study are far from encouraging. The aforementioned barriers to success, in tandem with additional factors and

considerations relating to the outcomes presented above, will be examined in more detail in section 5, discussion.

4.3. Motivation and Engagement

This section aims to address the question: ‘To what extent does the motivation and engagement of participating students influence student growth throughout the intervention, or how is motivation and engagement influenced by it?’. Following an examination of the internal consistency reliability of the data, it examines differences in pre- and post- intervention levels of motivation and engagement between intervention and control groups, as well as comparison of these levels with a large sample of students of the same age from a previous study. It then explores whether motivation and engagement changed throughout the study, and concludes with an examination of whether any of the motivation or engagement factors predicted student growth in multiplicative thinking.

4.3.1. *Reliability of Data*

Several students expressed confusion regarding interpretation of some of the questions included in the MES-HS. Given this confusion, and as has been recommended for researchers using Likert scales (R. R. Gliem & Gliem, 2003), it was deemed desirable to test for internal consistency reliability for each of the four items relating to each of the eleven MES-HS constructs. Cronbach’s α was used for this purpose. The results of analysis are presented in Table 8.

Table 8. Reliability statistics for intervention, control, and a comparison sample

	Internal consistency reliability		
	Pre	Post	Comparison ^a
	Cronbach's α	Cronbach's α	Cronbach's α
<i>Adaptive</i>			
Self-efficacy	0.88	0.91	0.77
Persistence	0.90	0.87	0.81
Learning Focus	0.80	0.83	0.81
Valuing	0.94	0.83	0.77
Task Management	0.90	0.83	0.82
Planning	0.78	0.62	0.77
<i>Maladaptive</i>			
Disengagement	0.62	0.78	0.81
Self Sabotage	0.59	0.30	0.81
<i>Impeding/maladaptive</i>			
Uncertain Control	0.49	0.71	0.79
Failure Avoidance	0.76	0.72	0.79
Anxiety	0.69	0.69	0.77

^a External comparison is weighted sample of $n = 12,159$ 12-13 year old High School students (Martin, 2014a).

Following George & Mallery's (2003, p. 231) rules of thumb, uncertain control and self-sabotage presented as of 'questionable' reliability at the time of pre-testing, and self-sabotage presented as of 'unacceptable' reliability at the time of post-testing. These data suggest that conclusions drawn regarding students' uncertain control or self-sabotage are unlikely to be reliable. Reasons for the lack of internal consistency reliability on these questions is discussed in section 5.2.2.2.

4.3.2. *Motivation and Engagement Results*

Pre- and post-testing motivation and engagement levels for both intervention and control groups, as well as a comparison group (Martin, 2014a), are presented in Table 9.

A MIDDLE YEARS MULTIPLICATIVE THINKING INTERVENTION

Table 9. Pre- and post- treatment and control group means and SDs and external comparison data

	Intervention		Control		Comparison ^a
	Pre	Post	Pre	Post	
	M (SD)	M (SD)	M (SD)	M (SD)	
<i>Adaptive</i>					
Self-efficacy	64* (19)	68 (21)	90 (16)	89 (12)	82 (15)
Persistence	53 (20)	57 (23)	73 (26)	68 (19)	73 (16)
Learning Focus	78* (18)	76 (15)	93 (8)	94 (5)	83 (14)
Valuing	78 (22)	77 (18)	81 (26)	79 (24)	85 (14)
Task Management	52 (13)	50 (19)	75 (28)	73 (20)	72 (19)
Planning	45 (11)	48 (12)	61 (20)	61 (16)	64 (18)
<i>Maladaptive</i>					
Disengagement	39 (16)	46 (21)	38 (18)	41 (19)	31 (16)
Self Sabotage	44 (17)	44 (14)	47 (15)	46 (13)	38 (18)
<i>Impeding/maladaptive</i>					
Uncertain Control	57 (15)	51 (12)	61 (15)	70 (18)	49 (19)
Failure Avoidance	59 (21)	41 (14)	52 (25)	56 (25)	45 (21)
Anxiety	72 (16)	55 (19)	80 (15)	84 (11)	58 (20)

^a External comparison is sample of $n = 12,159$ 12-13 year old High School students (Martin, 2014a).

*A significant difference existed between intervention and control groups at the time of pre-testing

4.3.2.1. Benchmarking motivation and engagement at the time of pre-testing

The only statistically significant differences between the motivation and engagement of students in the intervention and control groups at the time of pre-testing were in *self-efficacy* and *learning focus*. Self-efficacy levels in the intervention group ($Mdn^4=70$) significantly differed from those in the control group ($Mdn=100$) at the time of pre-testing, $U = 68.5$, $z = -2.58$, $p = 0.01$, $r = -0.61$. Learning focus levels in the intervention group ($Mdn=82$) also significantly differed from those in the control group ($Mdn=95$) at the time of pre-testing, $U = 63$, $z = -2.065$, $p = 0.039$, $r = -0.49$.

⁴ Readers will note that whilst mean scores were reported in Table 9, median scores are reported here. Mean scores were reported in Table 9 for ease of comparison with the comparison sample. Medians are reported here as, in rank-based statistical testing (such as the present Mann-Whitney U test), the ranking approach is based upon the relative position of data points rather than an analysis of means.

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Farrington and colleagues' (2012) meta-analysis found self-efficacy to be one of the two strongest factors influencing academic achievement. The initial disparity between intervention and control groups on this factor indicated a potential influencing factor on student growth.

Comparing both groups in this study to the comparison sample (Martin, 2014a) suggests coherence with their classification as disengaged and low-achieving. At the time of pre-testing both groups rated higher on all maladaptive factors than did the comparison sample, and lower on all adaptive constructs (excepting task management and learning focus, and persistence for the control group).

4.3.2.2. Significant changes to motivation and engagement

In an attempt to answer the second half of the second research question 'How is motivation and engagement influenced by it [the intervention]?', The Wilcoxon Signed-Rank test was applied to pre- and post-MES-HS results. Whilst the following results do not definitively answer this question, they do highlight what changed over the time period of the intervention.

The only significant changes to motivation and engagement occurred in the intervention group. For this group, disengagement was significantly higher at the time of post-testing ($Mdn=43$) than at the time of pre-testing ($Mdn=38$), $T = 0$, $p = 0.027$, $r = -0.52$; failure avoidance was significantly lower at the time of post-testing ($Mdn=59$) than at the time of pre-testing ($Mdn=45$), $T = 3.5$, $p = 0.042$, $r = -0.48$; and anxiety also significantly decreased from the pre-test ($Mdn=73$) to the post-test ($Mdn=52$), $T = 1$, $p = 0.017$, $r = -0.56$. These quantitative results are coloured with qualitative examples in the following.

Disengagement

The statistically significant increase in disengagement stemmed primarily from Adam and Syed's changes in disengagement ratings. These students move from 64 to 82, and 54 to 68 respectively. In both cases, both students started well above the group median at the time of pre-testing (38) and moved further from the post-testing group mean (43). Adam's increase in disengagement matches his decrease in performance and Syed's increased disengagement was explicitly communicated at the time of post-testing. See the following excerpt from the researchers' journal:

A MIDDLE YEARS MULTIPLICATIVE THINKING INTERVENTION

I asked 'how are you feeling today', he said 'miserable', I said 'How do you think you'd be going on this assessment if you were feeling a bit better today?' he said 'I'm feeling bad BECAUSE of this, I hate math'.

Failure avoidance

Decreased failure avoidance was particularly encapsulated by a diary entry by Rayyan, as depicted in Figure 28.

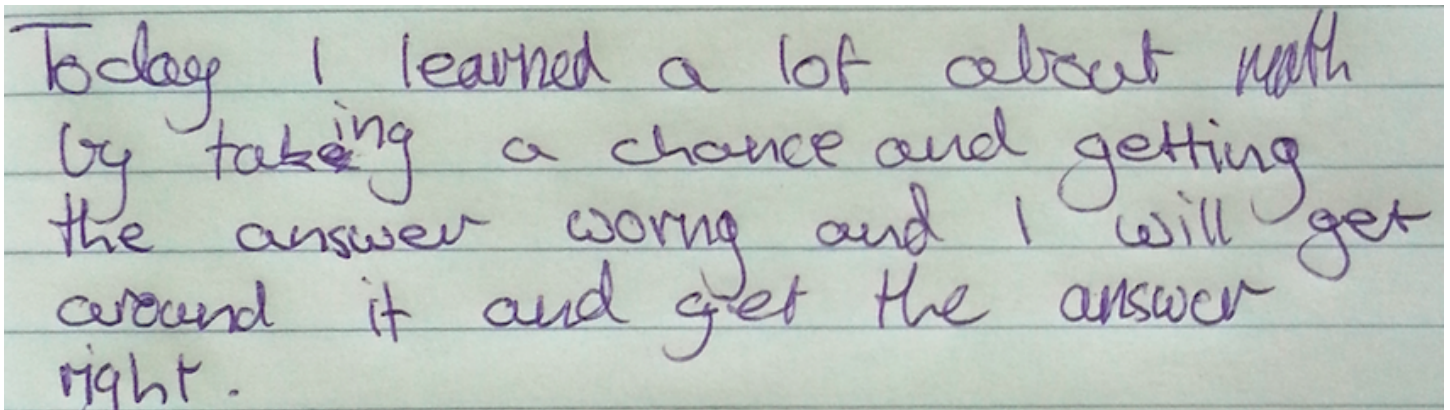


Figure 28. Rayyan exhibiting a growth mindset

This attitude was a significant departure from Rayyan's initial attitude which was captured in the researcher's notes from session 4.1:

I think that here what we're dealing with is battling with students' PMSD⁵. I.e., they have had so many negative experiences towards mathematics (I have Rayyan and Nour in mind here...)...at the slightest hint of challenge or failure, their brain will use the 'This is maths, it's a bit hard, therefore I won't solve this problem, therefore I may as well give up', heuristic and throw in the towel straight away.

This is a promising result given the way in which the researcher stressed the importance of seeing mistakes as an opportunity for learning and growth (Dweck, 2006).

Anxiety

The source of the students' reduction in anxiety is not immediately obvious. It is tempting to argue that a decrease in engagement, as was also exhibited, means that students don't care about their results as much, and

⁵ Post Mathematic Stress Disorder, a term used jokingly here by the researcher.

thus are less anxious. This seemingly common sense reasoning is, however, contradicted by Martin's (2014a) confirmatory factor analysis which suggested a correlation between disengagement and anxiety of 0.06. Given the number of statistical tests carried out (24) at the selected significance level ($p < 0.05$), this result could be a statistical phenomena, as is further discussed in the sub section of 5.2.2.2 entitled 'Probability of type 1 errors when administering multiple tests with related hypotheses (Bonferroni correction)'.

4.3.2.3. Motivation and engagement's influence on growth in multiplicative thinking throughout the intervention

In an attempt to answer the first half of the second research question, 'To what extent does the motivation and engagement of participating students influence student growth throughout the intervention?', Spearman's rho, a measure of correlation between ranked data, was used to examine correlations between growth in multiplicative thinking and pre-intervention motivation and engagement ratings. Each student's growth was compared to their pre-intervention rating for each of the constructs. None of the eleven factors of the MES-HS at the time of pre-testing were significantly correlated with growth. This is unsurprising given that the maximum LAF level change for students engaged in this intervention was three levels (Adam's result), with a median level change of zero. With a sample of this small size, and the non-significant changes in multiplicative thinking achievement for both control and intervention groups, any correlations would have to be significantly pronounced in order to be detected.

4.4. Summary

This chapter explored changes in multiplicative thinking, motivation and engagement that occurred throughout the intervention. It began by presenting the quantitative growth in multiplicative thinking, as captured by the SNMY assessment booklets, adding qualitative results in the form of researcher's notes and student work samples. It then highlighted how, by comparison with a large-scale sample, these students were accurately classified as low-achieving and of low motivation and engagement. Finally, explored motivation and engagement factors for which student self-ratings significantly changed throughout the intervention

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(disengagement, failure avoidance, and anxiety), as well as highlighting that none of the MES factors were significantly correlated with growth. The following section expands upon these findings, offering explanations and implications for future research.

5. Limitations, Implications, and Conclusions

5.1. Introduction

Beginning with limitations, this chapter outlines the ways in which participant considerations, assessment, and intervention approaches limited the ability of the present study to definitively answer the questions posed. From these limitations it suggests implications for future research in this area. It concludes with the main findings of the study.

5.2. Limitations

5.2.1. *Participant and Participation factors*

5.2.1.1. *Duration and frequency*

The present study, carried out over 15 weeks with a total of 24 sessions, significantly differed in duration from both Breed's (2011) study, which ran for 43 sessions, and Epstein's (2015) study, which took place over an entire school year. Breed's (2011) study was also delivered at a higher frequency, at three sessions per week. In the case of Breed, Epstein's and the present intervention, session duration was approximately one hour. It is possible that the reduced frequency and duration of this study contributed to its relatively reduced efficacy, as measured by the SNMY assessment booklets. This would be in line with results from a recent study (Duhon, Mesmer, Atkins, Greguson, & Olinger, 2009) in which it was found that differing intervention frequencies were required for different participants to achieve the same result, with some participants requiring ten times the intervention frequency than others in order to achieve a class benchmark level of achievement. In the present study, these factors were compounded by student truancy to the point that the average number of sessions attended by intervention students was less than 17.5.

5.2.1.1. *Undiagnosed learning difficulties*

It is suspected that Syed has a significant learning difficulty in learning mathematics. Syed consistently demonstrated a compromised ability to take in and retain mathematical information, as well as complaining from headaches when he began thinking about mathematics. This was particularly evident when contrasted with Syed's advanced linguistic skills, with Syed displaying the most advanced vocabulary and grammatical structures of all of the students involved in the intervention. Syed's mathematical abilities were discussed with his teacher who also expressed confusion at Syed's engagement with mathematical content. For example, one of Syed's responses in Booklet 2 demonstrated an understanding that arrays are associated with the operation of multiplication, but the multiplication chosen in this case was 5×4 , which had little relation to the array drawn (with sides of four, eight, five, and six pots respectively. See Figure 29). Given the amount of time spent on array based multiplication throughout the intervention, this lack of coherence between model and equation is very surprising.

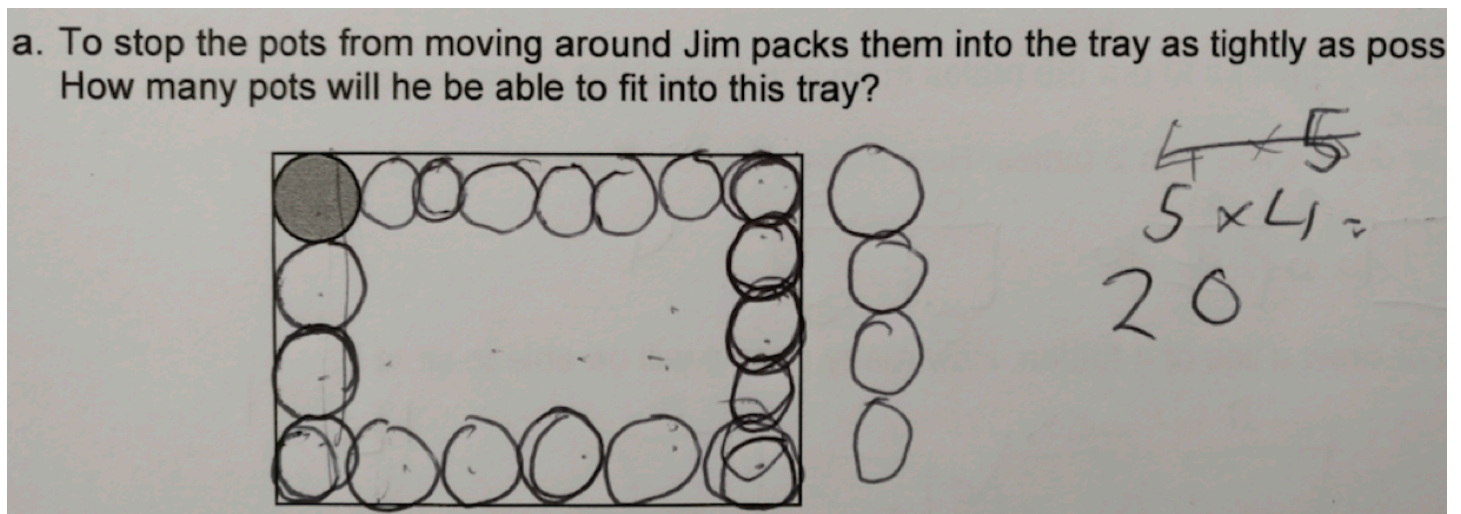


Figure 29. Syed's developing understanding of the array method of multiplication, Booklet 2 (post-test)

It is common for individuals to reach adulthood without their learning disabilities being diagnosed (Kaplan & Shachter, 1991). In the final report to the school, the researcher recommended additional testing for Syed in order to explore possible learning difficulties.

5.2.1.2. *Student focus, external to school factors, and group composition*

Several affective factors significantly impacted upon the students' learning throughout the intervention; these were student focus, external to school factors, and group composition.

Students' focus was clearly linked to their motivations for involvement in the program. This was evident from the first session in which the researcher recorded in their notes: "There was a lot of talk about 'we don't have to go to Indonesian'". A desire to participate based on a preference to avoid other classes is not a strong foundation for engagement and learning. Lack of focus was a particular problem for Daniel, as was captured in the researcher's notes from session 13.1:

Teacher A said that she needed to keep them in her English class for some literacy testing. I went in to observe how they performed for her and, particularly, how they listened to instructions.... Daniel was completely tuned out, he was wandering around apparently aimlessly looking in different directions.

External to school events also had an impact. Throughout the course of the intervention, one of the intervention participants experienced disruptions in his home situation that impacted upon his schooling more generally. This individual, Steve, was suspended at this time and, upon return to the school, refused to continue to participate in the intervention program. Steve displayed a lack of effort at the time of post-testing (disengagement score of 36 as compared to pre-test score of 32), with his post-test score two LAF levels below his pre-test score. Another student, Nour, suddenly moved to another state without warning.

Despite being composed by the students' teachers, the groups were not optimally structured. This was captured in the researcher's notes such as the following from the first session:

Well, that was messy.. They were super rowdy. Could barely get them to focus. It's going to be an uphill battle to reign in attitudes... They're the biggest group, and the rowdiest too. Those two things combined means that it was hard work, esp at end of the day. I don't want to be a task master, but gosh, lacking other options really.

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Such challenges continued well into week nine (even after a group re-structuring): “These boys came in rowdy as ever. Were super loud whilst playing stop or dare, to the point that (teacher in adjoining class) stuck her head in and told us to quieten down!!!!”.

This proved a challenge to the point that the groups were eventually re-structured. Sullivan and Gunningham (2011) commented upon how, in their study, careful attention was paid to the mix of students in each group in order to ameliorate potential personality clashes. This challenge had an undeniable impact upon Students’ capacity to learn throughout the intervention.

5.2.1.3. *Conceptions of learning mathematics*

It is also relevant to consider the ways in which students’ pre-intervention conceptions of what mathematics learning is, and how it occurs, influenced how they engaged in the intervention. Some of these concepts were captured in their drawings at the commencement of the intervention (see Figure 30).

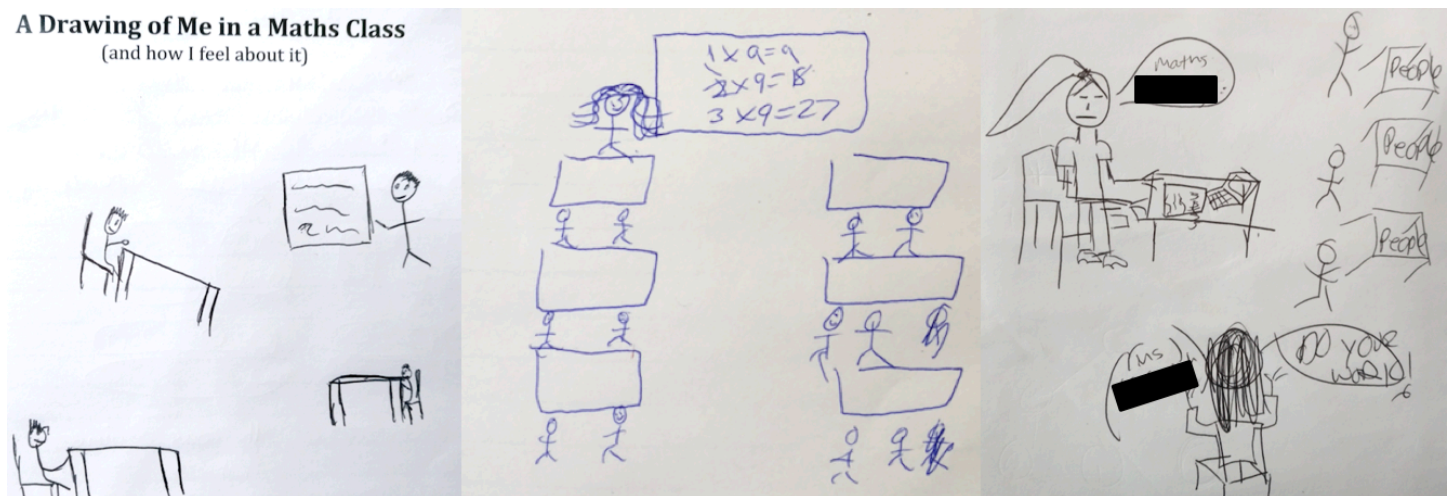


Figure 30. Rayyan, Umar, and Sharons' responses to the prompt 'A Drawing of Me in a Maths Class (and how I feel about it)' at the time of pre-testing

The first two images of Figure 30 depict students’ passive involvement in the mathematics classroom. A teacher stands at the front presenting some explanation on the board, whilst students sit at desks. At the school in which the present intervention took place, the majority of time in class was taken up by the use of a digital (computer-based) mathematics program. Sharon’s drawing (right-most image in Figure 30) captures this, as well as a teacher yelling at students to ‘do your work’. The activities present in the current intervention, many of which were interactive and based around discussion, represented a stark contrast to these pre-existing passive

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and computer-based conceptions of a mathematics class. This contrast meant that, as the program was delivered, time had to be allocated to building a different culture around the teaching and learning of mathematics.

5.2.1.4. Persistent limiting factors

Thinking beyond the post-test, a limitation present in intervention programs targeting low-achieving students, such as in the present study, is the tendency for gains associated with such programs to fade over time (Leak et al., 2010). In the early schooling context, the majority of this fadeout following mathematics interventions has been attributed to pre-existing and relatively stable differences between students identified as at-risk and those who are not (Bailey et al., 2016). These differences could be in the areas of domain-general cognitive abilities, home and environmental conditions, or academic skills and motivations stemming from outside of school factors. One such easily observable factor in the current intervention was student truancy, with another being the unstable home environments of Steve and Nour. Interventions such as the one presented here are carried out against the backdrop of these longer-term and more stable challenges to student achievement.

5.2.2. Instruments

5.2.2.1. Measuring Multiplicative Thinking

LAF levels vs. raw scores

One limitation of the SNMY assessment booklets was the reduced granularity of data resulting from the requirement to measure Students' growth by LAF level change. This approach was mandated, in part, by the disproportionate mapping of raw scores to LAF levels in Booklets 1 and 2 (see Section 3.8.1) by the respective marking schemes of Booklets 1 and 2.

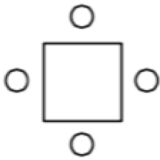
Student interpretations of questions

Several issues pertaining to SNMY assessment booklets' questions and question contexts appear to have impacted on the results. Primarily, the ambiguity of some questions (especially when viewed through the eyes of a student with a language background other than English), as well as considerations regarding the spatial contiguity effect (Ginns, 2006).

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Student misinterpretation of questions was particularly apparent in the question Tables and Chairs (Booklet 1 and Booklet 2) in which the question text read as depicted in Figure 31.

Dean's community is planning a street party. They have lots of small square tables. Each table seats 4 people like this:



The community decides to put the tables in an end-to-end line along the street to make one big table.

Figure 31. Tables and Chairs

This question was significantly misinterpreted by many students, with several students explicitly asking the researcher what 'end-to-end' meant. The correct interpretation in Figure 32, is contrasted with several alternate interpretations presented in Figure 33. Applying the marking scheme, this resulted in marks lost for the students presenting alternate interpretations, even in cases when multiplicative thinking was demonstrated.

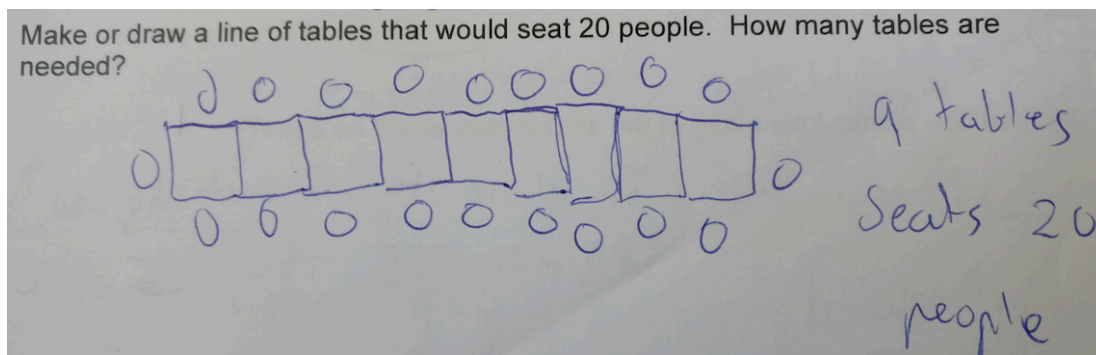


Figure 32. Correct interpretation of Tables and Chairs, part e.

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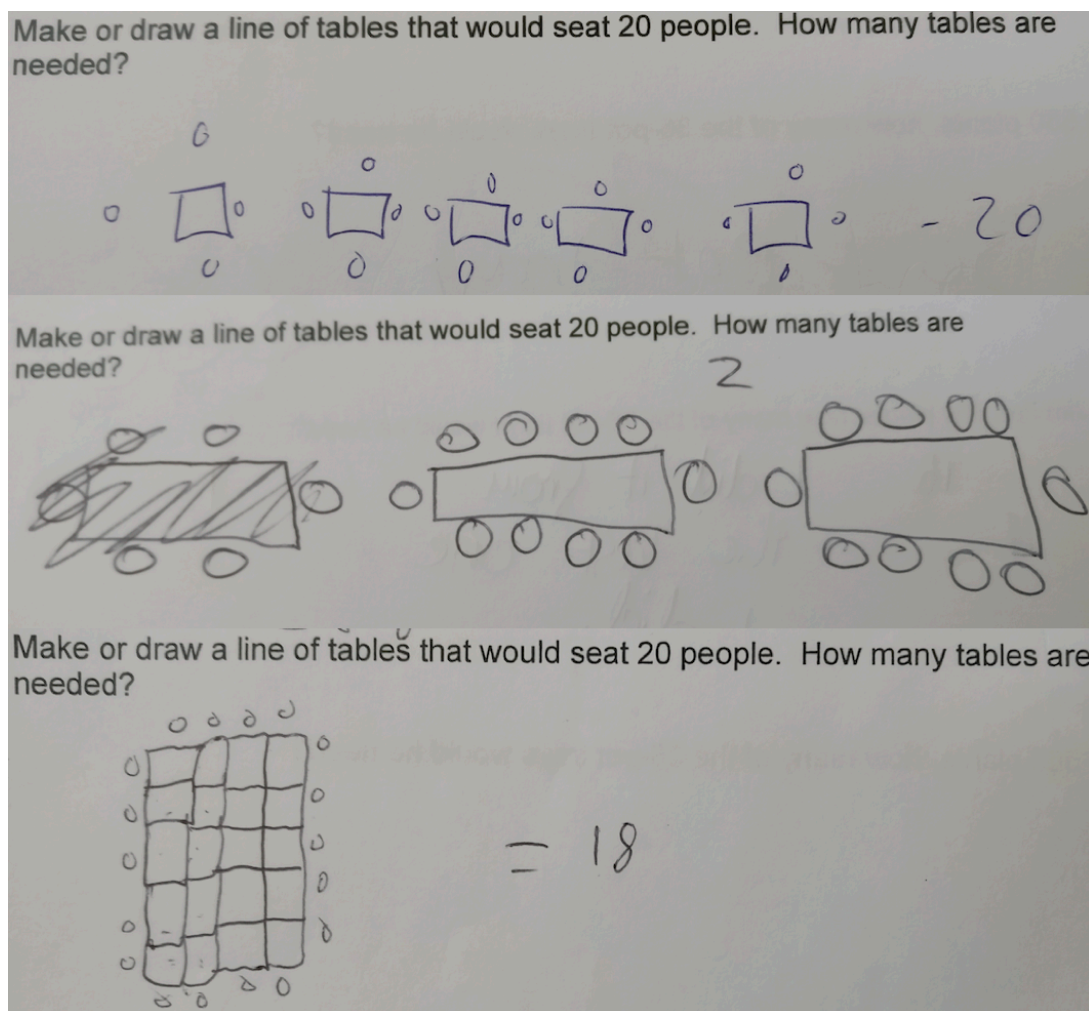


Figure 33. Several alternate interpretations of the question 'Tables and Chairs, part e.'

A Lack of Alignment between Assessment Booklets 1 and 2

There existed some discrepancy between the SNMY Task Assessment Booklets 1 and 2 in that some concepts that were tested in one that were not present in the other. Primarily, Booklet 2 included a question (Speedy Snail) that explicitly required students to work with decimal multiplication (see Figure 34), a concept not tested in Booklet 1.

SPEEDY SNAIL



a. Harry's snail can travel at 15 centimetres per minute.
How far might Harry's snail travel in 34 minutes?

b. Samantha's snail covered 1.59 metres in 6 minutes.
How far might Samantha's snail travel in 17 minutes. Record your answer in metres.

c. Harry entered his snail in a race. Remember Harry's snail can travel at 15 centimetres per minute. Another snail entered in the same race, covered 3.71 metres in 24 minutes. Which is the faster snail?
Show all your working so we can understand your thinking.

Figure 34. Speedy Snail, SNMY Assessment Booklet 2

Booklet 2 also differed from Booklet 1 in that it included two questions that included Cartesian product ideas (Parts h. and i. of 'Butterfly House', and 'Canteen Capers'. See Figure 35 and Figure 36).

- h. Model butterflies can be made with wings, grey, brown or black bodies and either long or short feelers. How many different model butterflies are possible? **Show all your working and explain your answer in as much detail as possible.**

Figure 35. Part h. of Butterfly House, SNMY Assessment Booklet 2

CANTEEN CAPERS

The school canteen makes great lunches. They offer 2 choices of rolls, 4 choices of filling and 3 choices of drink.

Rolls	Fillings	Drinks
White	Peanut butter	Orange Juice
Multi-grain	Salad	Apple Juice
	Cheese and vegemite	Cola
	Tuna and lettuce	

For each question, show your working and explain your answer in as much detail as possible.

a. Claire ordered a peanut-butter roll and a drink. What might she have ordered? List all possibilities.

b. There are 23 students in Claire's class. If everyone ordered a roll with a filling and a drink, could they all have a different lunch order?



Figure 36. Canteen Capers. SNMY Assessment Booklet 2

Due to the brevity of the intervention, Cartesian products were not covered. Considering the relatively high number of marks available for the above questions (7 for speedy snail and 8 for the Cartesian product related questions), it is highly likely that their inclusion in Booklet 2, and not in Booklet 1, impacted negatively upon Students' reported growth.

The marking scheme's reward for effort

The SNMY assessment booklets' allocation of marks for 'attempted' questions meant that student reported achievement was a measure of effort as well as a measure of multiplicative thinking. The mark range for each of the LAF levels was quite narrow (often only 5 marks) meaning that simply writing something, or making a guess on a question, was sufficient in some cases to push students over an LAF level threshold. In particular, there were several yes/no questions for which students would receive credit for guessing the correct answer with little to no working (see Figure 39, Figure 40, and Figure 41), This is demonstrated when Daniel's

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response to Speedy Snail, part c. (Booklet 2, see Figure 37) is contrasted with that of Adam, who refrained from guessing or writing any answer to this question (see Figure 38).

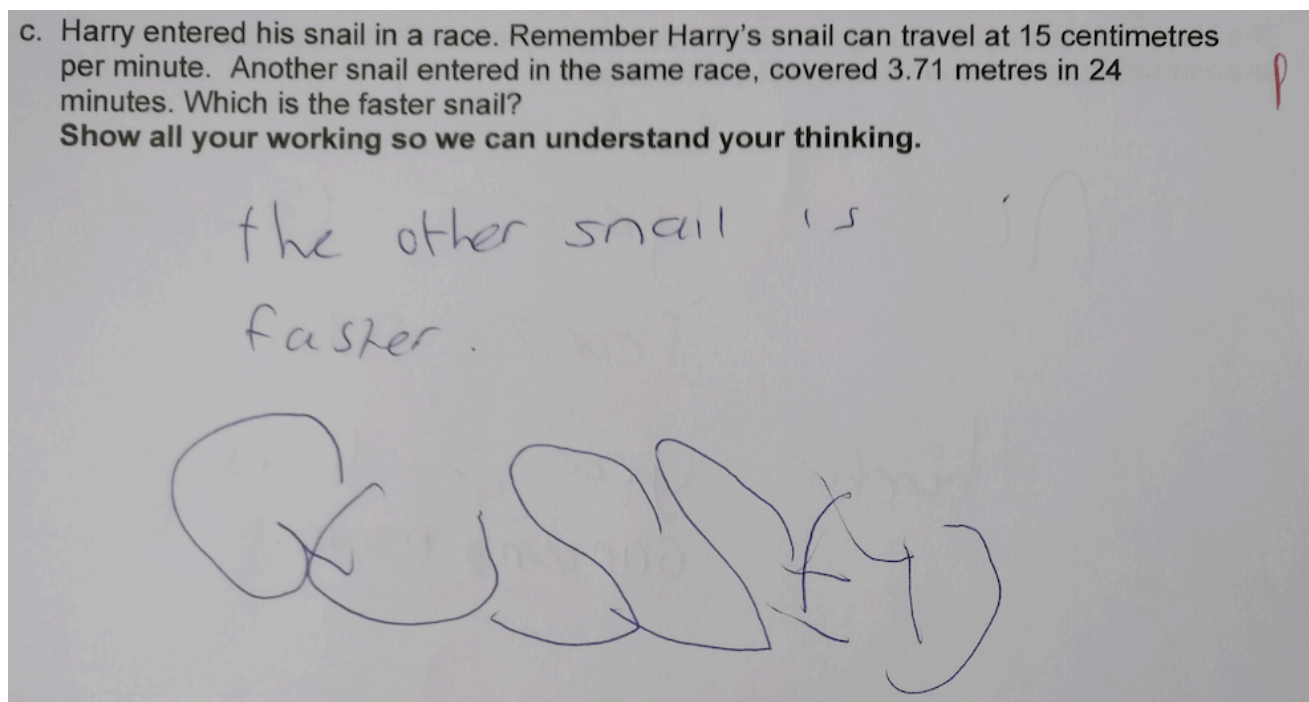


Figure 37. Daniel receives a mark for a guess, part c. of Speedy Snail (Booklet 2)

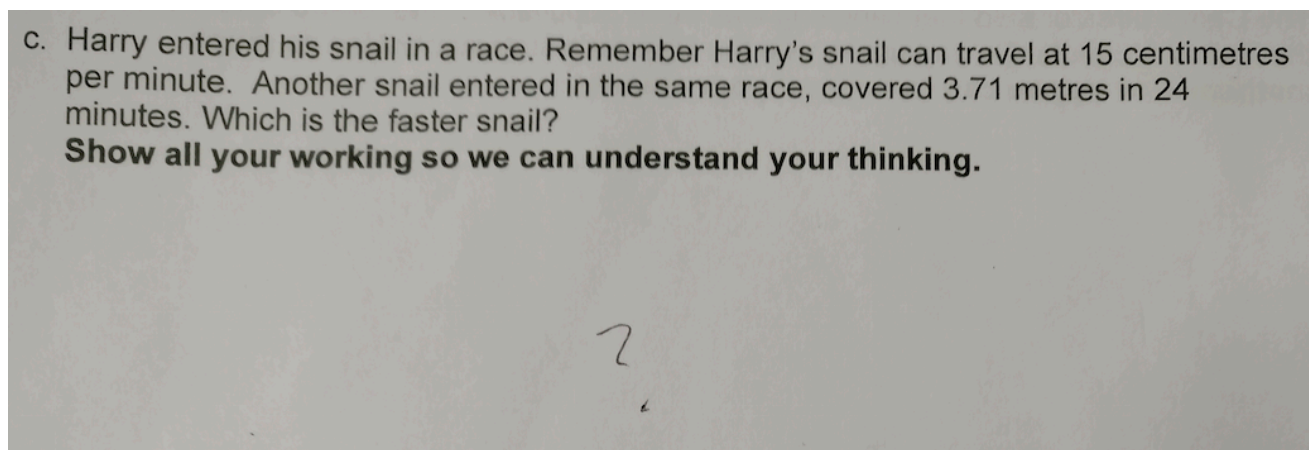


Figure 38. Adam refrains from guessing at part c. of Speedy Snail and thus forgoes a 50% chance of a mark

The marking scheme for part c. of Speedy Snail is depicted in Figure 39. This 'mark for a guess' scheme is also shown to be the case for other questions in Figure 40 and Figure 41.

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Incorrect/incomplete but working and/or explanation recognises multiplication needed, or correct (other snail), with little/no explanation or inappropriate use of units	1
--	---

Figure 39. Marks awarded for correct guess with no justification

Correct (Yes), but little/no working or explanation to support conclusion	2
---	---

Figure 40. Marks awarded for correct guess with no justification

Correct (Yes), but little/no working or explanation to support conclusion, eg, drawing or list not systematic, not clear that Cartesian Product idea seen as relevant	1
---	---

Figure 41. Marks awarded for correct guess with no justification

In some cases, the marking scheme stipulated that marks could be assigned to responses that were a simple observation. For an example of this, see the marking scheme in Figure 42 and a response qualifying of a mark (Daniel’s) in Figure 43.

One or two relatively simple observations based on numbers alone, eg, "Archery was the most popular activity for both Year 5 and Year 7 students", "More Year 7 students liked the rock wall than Year 5 students"	1
--	---

Figure 42. Marks awarded for simple statement

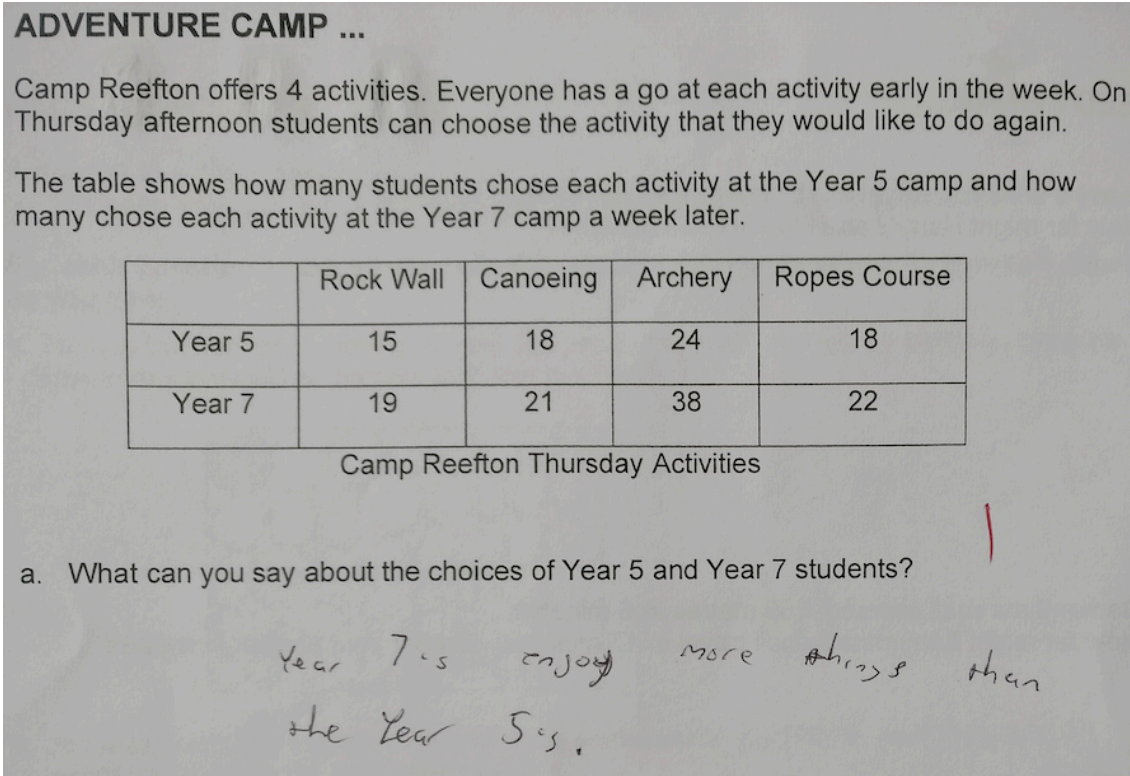


Figure 43. Daniel's qualifying response

This section demonstrates how the marking scheme was sensitive to student effort at the time of testing. Such confounding factors decrease the accuracy to which instruments are able to capture students' multiplicative thinking abilities.

It is of note that, had students' motivation and engagement been equal at the time of pre- and post-testing, these factors would have likely impacted upon students' reported multiplicative thinking achievement in a similar way at both times. Looking at those students who demonstrated a decrease in performance (for example Steve and Adam⁶), this did not appear to be the case. For these students, it is possible that results from pre- and post-tests were differentially affected by these affective-sensitive elements of the marking scheme.

⁶ Steve was suspended during the period of the intervention and his perseverance decreased from 32 to 29, with disengagement increasing from 32 to 36. These figures were similar for Adam, whose perseverance decreased from 57 to 54, and disengagement increased from 64 to 84. These changes can be considered in the context of each construct being measured by four questions, thus, a four point change represents an average shift of one point on the 7-point Likert scale for each of the four questions.

The SNMY Assessment Task Booklets and Transfer

As was discussed in section 4.2.4, students' growth in multiplicative thinking occurred that was not captured by the SNMY assessment booklets. Coupled with the challenges outlined above, it is asserted that the inability of students to apply conceptual understanding (such as doubling, halving, and array based multiplication strategies) to unfamiliar problem contexts resulted in an under-reporting of growth in multiplicative thinking. This challenge relates to the fundamental concept of *knowledge transfer*, which has been argued by some as the ultimate goal of education (McKeough, Lupart, & Marini, 2013), and which proves a challenge for many students when attempting to apply mathematical knowledge to problem solving contexts (Rebello, Cui, Bennett, Zollman, & Ozimek, 2007). It also drives at the core of the affordances of the assessment booklets. From the present study, it emerges as important to make distinct two highly interrelated concepts: one being a student's multiplicative thinking abilities (such as those captured by the 'big ideas', see Table 2), and the other being their ability to recognise multiplicative structures in problem scenarios, and apply appropriate multiplicative processes. Such a distinction has implications for multiplicative thinking interventions, as is further discussed in section 5.3.2.

A comparison with growth reported in related studies

Despite the aforementioned limitations to the instruments used to measure multiplicative thinking, it is necessary to explore how similar studies using only these SNMY assessment booklets (Epstein, 2015), or complemented with additional tasks (Breed, 2011), reported more significant gains.

Contrasting the present intervention to that of Epstein (2015), there exist several key differences to note. Epstein's intervention spanned from February 2015 to November 2015. Whilst the intervention was less frequent (once per week), students also benefitted from additional learning time in their usual mathematics classroom over this time period. Epstein's results are summarised in

Table 10.

Table 10. Epstein's (2015) results from an SNMY intervention

LAF Growth	Percentage of students by Level Growth (<i>n</i> = 101)
- 1 Level	3%
+0 Levels	30%
+1 Level	42%
+2 Levels	10%
+3 Levels	12%
+4 Levels	2%
+5 Levels	1%

Other notable differences between Epstein's and the present intervention are that the former intervention was carried out in a full sized class with the usual class teacher, a numeracy coach, and several helpers. Students worked independently on SNMY tasks (similar, and in some cases identical to, those used in the present study) as well as explicitly anticipating and reflecting upon problem solving strategies. It is feasible that if the current intervention was carried out over an equivalent length of time as that of Epstein's, and directly using the SNMY, it could have produced comparable results.

Breed (2011) reported much larger gains from her study. Key differences were that Breed's intervention took place three times a week, for a total of 43 sessions. Thus, it was more intense, and longer in duration than the present study. It is also of note that Breed was one of the researchers involved in the initial SNMY research project (Siemon et al., 2006), and so may have had a deeper understanding of the resources and approach than either Epstein or the author of this current study. Additionally, all students in Breed's study began at LAF level 1 (a condition for participation in Breed's study), which was not the case for Epstein's study or the current study. The growth of the students in Breed's intervention is summarised in

Table 11.

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Table 11. Breed's (2011) results from an SNMY intervention

LAF Growth	Number of Intervention Group students by Level	Number of Control Group students by Level Growth
	Growth ($n = 9$)	($n = 5$)
+0 Levels	0	2
+1 Level	0	2
+2 Levels	0	1
+3 Levels	7	0
+4 Levels	2	0

As can be seen above, all students in Breed's (2011) intervention group were reported to have grown by either three or four levels. This was in significant contrast to the control group who exhibited a maximum growth (one student) of two levels.

In an attempt to identify the source of the large discrepancy between Breed's (2011) results, and those reported by Epstein (2015), as well as this study, Breed's intervention notes were explored in detail. Such an examination revealed several records that were hard to interpret. Figure 44 is taken from the third to last session of the intervention.

LEARNING FOCUS ~ Identify the type of problem

(SCHOOL B) Identify the multiplication problem

A packet of stickers has 5 sheets. Each sheet has 14 stickers. How many stickers are there in total?

A furniture maker needs to make 25 stools. A stool has 3 legs. How many legs will need to be made?

Jane brings 32 freddo frogs to school. She gives out 27. How many does she have left?

I have 24 textas. My friend has 14. How many do we have altogether?

Students are presented with these problems, two are multiplicative. Have a variety of materials at hand, for students to access if necessary. Their task is to determine which of the four problems are multiplicative. Students articulate their thinking and the teacher pushes the students to “be convincing!”

Figure 44. An activity from Breed's (2011) session number 40 (Appendix D, p. 95)

In her researcher's notes, Breed (2011) noted the following in relation to the activity outlined in Figure 44; “When the two multiplication problems were identified, I asked, ‘how can we solve them?’ eg, stool problem Cansu suggested drawing the stool 25 times. The efficiency of this strategy was discussed.” (Appendix D, p. 96).

The solution approach suggested by Cansu in the above excerpt (draw-all) is consistent with Level One of the LAF. Breed (2011) quotes Siemon and colleagues (2006) when she suggests that a student in Level One “tends to rely on drawing, models and count-all strategies” (p. 55). Given that after an additional three sessions of Breed's intervention, Cansu tested at level 5 of the LAF (Breed, 2011, p. 184), these results are quite surprising.

The above excerpt is not an isolated case of students in the final stages of the intervention displaying significant challenge in applying multiplicative thinking concepts. In session 39 of 43, Breed indicated that Yousif (post-test level 4) “still had trouble conceptualising his array situation.” (Appendix D, p. 95).

In session 37 of 43, Breed noted that “In describing arrays, students unable [*sic*] to confidently describe their situation.” (2011, Appendix D, p. 91)

Looking at the activities undertaken by Breed (2011), as well as the recorded student responses towards the end of the intervention, it is unclear how, such a short time later, all students tested at level 4 or above.

5.2.2.2. *Measuring Motivation and Engagement*

Domain specificity and calibration

The accuracy of the assessment of student's motivation and engagement based on the MES-HS was limited primarily by domain specificity of the instrument and its calibration (Hattie, 2013).

Whilst the MES-HS that was used is a validated instrument, it is not domain specific to mathematics, nor to multiplicative thinking. The MES-HS was used over a more domain specific version as, at the time of ethics application, the researcher was not aware of the existence of a mathematics domain specific MES. Information related specifically to multiplicative thinking motivation and engagement could have been obtained by adjusting questions from, for example: 'If I try hard, I believe I can do my schoolwork well' (MES, Question 13.) to 'If I try hard, I believe I can do multiplication and division questions well'. There is no such multiplicative thinking domain specific MES currently available.

Inaccuracies arising from the lack of domain specificity of the MES-HS employed are compounded when considered in tandem with the fact that participating students were low-achievers. Low-achievers have been shown to exhibit less accurate 'calibration', the alignment of confidence and accuracy of self-assessments with genuine ability (Hattie, 2012). Thus, the fact that the participants were low-achievers may correlate with less accurate self-assessments.

Linguistic factors

As reported in section 4.3.1, the items presenting 'questionable' and 'unacceptable' (George & Mallery, 2003) levels of reliability were *uncertain control* and *self-sabotage* respectively.

Examination of the MES questions related to each construct, and associated student responses, suggests that confusion could have arisen from the use of double negatives in questions. For example, the questions relating to self-sabotage were questions 5, 24, 35, and 42. These read:

"5. Sometimes I don't try hard at assignments so I have an excuse if I don't do so well"

"24. I sometimes don't study very hard before exams so I have an excuse if I don't do so well"

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“35. I sometimes do things other than study the night before an exam so I have an excuse if I don’t do so well”

“42. I sometimes put assignments and study off until the last moment so I have an excuse if I don’t do so well.”

Several students appeared to be confused by these measures with one student in the control group, Zikri, answering questions 5 and 24 with the rating of one (Disagree Strongly) and 35 and 42 with the rating of seven, (Agree Strongly).

Similar use of double negatives was present in the questions relating to uncertain control such as “When I don’t do so well at school I’m often unsure how to avoid that happening again”, and, “When I get a bad mark I’m often unsure how I’m going to avoid getting that mark again”.

Probability of type 1 errors when administering multiple tests with related hypotheses (Bonferroni correction)

The significance level chosen in the present study, $p < 0.05$, represents there being a less than five percent chance that the results conform to a null hypothesis. A five percent chance is equivalent to one in twenty, thus, it is likely that if a researcher conducts twenty tests at the $p < 0.05$ significance level, one of these tests will incorrectly reject the null hypothesis (A type 1 error) and falsely report significance. In such cases, it can be helpful for researchers to conduct a Bonferroni correction (Cabin & Mitchell, 2000). Such a correction stipulates that if the researcher intends to test twenty related hypotheses at the 5 percent significance level, each individual test must have its significance level reduced to $0.05/20 = 0.0025$ to avoid such type 1 errors. There is, however, a tradeoff to this approach; such a correction conversely increases the probability of type 2 errors (accepting the null hypothesis when it should be rejected) (Cabin & Mitchell, 2000).

Given this tradeoff, the present study has refrained from conducting a formal Bonferroni correction. It is suggested that in the present context it is sufficient to note that there were 33 motivation and engagement related statistical tests carried out at the 5% significance level and, as such, results reported as significant should be interpreted with the increased probability of type 1 errors in mind.

5.2.2.3. *The role of the teacher*

With teachers' beliefs and commitments being the factor with the greatest influence on student achievement over which educators have some control (Hattie, 2012), it would be an oversight not to mention the role of teacher experience in the present intervention. The researcher carrying out the intervention to which this thesis pertains was a teacher in his first year, with very limited prior experience from which to form such beliefs and commitments. This was not the case for Breed (2011, p. 1), who graduated from her teacher training over 20 years prior to implementing her intervention. It is highly likely that additional years of teacher experience would have added to the efficacy of her program.

5.3. Implications for Future Research

5.3.1. *Using teacher-student rating agreements to inform teaching*

Additional data that were collected as part of the present project was teacher ratings of anticipated student motivation and engagement ratings. This was done by means of the MES-HS-Short (Martin, 2014a), a battery of eleven questions, each relating to one of the MES constructs. Each of the teachers from the three classes was asked to rate students in both the control and intervention groups at the beginning of the intervention. Whilst this information did not aid in answering the primary research questions of this project, it can be used to aid future teaching.

Without embarking upon an exhaustive analysis of these results, the general findings were that teachers rated students significantly lower than the students rated themselves on *self-efficacy* ($Mdn=64$ vs. 80), *learning focus* ($Mdn=72$ vs. 91) and *valuing of schooling* ($Mdn = 57$ vs. 89), whereas students rated themselves significantly higher on *anxiety* than teachers anticipated ($Mdn = 79$ vs. 57). These results could be interpreted as students having a greater desire to learn, and a greater belief that they can learn than teachers anticipated, but experiencing more anxiety in relation to that learning than their teachers realized. These findings are consistent with previous research which found that self-other agreement tends to be more coherent on factors that are

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accompanied by an observable behaviour (such as perseverance, disengagement, and task management) than for factors not externally expressed (such as self-efficacy, learning focus, and value of schooling) (McCrae & Costa, 1989; Shavelson, Hubner, & Stanton, 1976).

These findings allude to the importance of teachers explicitly checking in with students and asking how they are feeling about their learning and learning tasks, rather than assuming that they know what is occurring in those students' internal worlds. Future research could examine the factors, other than observable/non-observable factors, that influence such discrepancies, as well as looking more closely at how knowledge of such discrepancies can be used to inform teaching.

5.3.2. *Intervention approach considerations*

The results from this study raise three main considerations regarding mathematical interventions with under-achieving students in the middle years of schooling: the extent to which additional years of student failure limit the ability of students to learn from such a program, the efficacy of the SNMY program and similar 'conceptual-understanding' related interventions, and approaches to increase the transfer of any conceptual-understanding gained.

5.3.2.1. *The impact of additional years of failure*

When comparing middle years students to those in upper primary school, the extent to which the additional years of student failure and challenge in the mathematics classroom contributes to student resistance to learning are unclear. The results presented above suggest that the longer the period of time over which mathematical challenge is faced by such students, the harder it is for them to overcome academic difficulties and negative self-concepts in relation to mathematics. Through their review of the literature, Middleton and Spanias (1999) found that whilst student motivations are relatively stable after initial establishment in the early years of schooling, they 'can be affected through careful instructional design' (Middleton & Spanias, 1999, p. 82). Additional to this, it could be the case that the later the intervention, the harder it becomes for student beliefs and achievement to shift. This could relate to the reason why the majority of mathematics interventions

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are focused on students in primary school (Meiers et al., 2013). Additional research comparing similar programs in both the primary and high school contexts would shed further light on this question.

5.3.2.2. Interventions targeting conceptual-understanding

The ‘conceptual-understanding’ nature of the SNMY program is in contrast to other mathematics interventions that have also been reported as efficacious. Contrary to the SNMY approach, in which students are encouraged to model, decompose and understand the origins of multiplicative operations, Gersten, Jordan, and Flojo (2005) argue for mathematics interventions that target fluency of arithmetic combinations. This is in line with the approach of the Quicksmart intervention (Graham, Bellert, Thomas & Pegg, 2007), which aims to improve the automaticity of basic mathematics skills.

From a cognitive science standpoint, there is a further argument to be made for fluency-based interventions. Load reduction instruction, LRI (Martin, 2016), acknowledges the constraints of a student’s working memory, positing that in order for students to access higher order thinking and problem solving, they must first free up working memory capacity by learning and storing lower order processes (such as multiplication tables) in their long term memory (Winne & Nesbit, 2010). Such an approach advocates for more direct instructional approaches when students are in the ‘novice’ stage (the stage of under-achieving students, such as the participants of this study), allowing for guided-discovery learning approaches as students progress beyond ‘novice’. Not to discard conceptual-based or guided-discovery learning altogether, Martin (2016) writes:

following sufficient explicit input, guided practice and demonstration of independent learning, there is an important place for guided discovery learning...Hence, students’ academic development relies on both load reduction approaches and guided discovery-oriented learning. (p. 6)

The above is consistent with the researcher’s observations throughout the intervention. In line with the widely cited research on the importance of prior knowledge (Ausubel, Novak, & Hanesian, 1968), the students in the present study who appeared to possess a stronger grasp of number facts prior to the intervention (for example, Mike and Peter), were able to more quickly grasp the decomposition of these number facts than were

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the students who did not possess such prior knowledge. They were also able to apply the conceptual processes more fluently and competently in a broader range of situations. For the students (for example, Rayyan and Daniel) who came into the program unable to recognise the complements of 10 (for example, ‘2 and *what* makes 10?’), or the result of doubling each of the single digit numbers, such a conceptual approach may have placed undue strain on their working memory.

A load reduction approach, especially for under-performing students, acknowledges the presence of these cognitive constraints and would potentially be a more effective approach for such students. Further research contrasting a direct instruction (Adams & Engelmann, 1996) approach with the current SNMY approach, as well as a control condition, would help to elucidate the relative efficacy of these approaches.

5.3.2.3. Increasing transfer of conceptual-understanding

In cases in which teachers and educators deem a conceptual-understanding approach most suitable, this could be complemented by instructional practices that have been shown to increase transfer of such conceptual-understanding. In his work relating to mathematical transfer, Schoenfeld (1985) found that explicit instruction in recognizing similarities between novel and previously encountered problems improved students’ transfer. This could be complemented by process-oriented instruction that explicitly supports students to recognise which problem solving approaches they have at their disposal. This is an addition that Epstein (2015) made to the SNMY instructional approach when implementing the program (see Figure 45), and is in line with prior research reporting the efficacy of such process-oriented problem solving instructional approaches (Charles & Lester, 1984). Further to this, combining both of these approaches (instruction in problem solving processes as well as searching for similarities) has been shown to be an effective method for increasing transfer (Fuchs et al., 2003). Accompanying the online and freely available SNMY resources with such resources and recommendations could increase the efficacy of the program when implemented in similar interventions, as well as broader school contexts.

STRATEGIES:

4. What mathematics did you use to solve this task?

☐ Addition/Subtraction

☐ Number patterns

☐ Algebra

☐ Measurement
(e.g. perimeter/area/volume)

☐ Other

☐ Multiplication/Division

☐ Place value

☐ Statistics

☐ Probability

☐ Shape/Space

5. What strategy/strategies did you use to solve this problem?

☐ Guess & Check Try a solution, Test it, Try again.	☐ Act it out Try to physically work through it.
☐ Make a Model Build something like it.	☐ Solve a simpler problem Try an easier similar problem.
☐ Find out what it is NOT Eliminate	☐ Draw Diagrams Visualise the Problem with a drawing.
☐ Make an Organised List Use logic to make a list	☐ Work backwards Start at the answer
☐ Try ALL possibilities Useful if only few options	☐ Look for a pattern Most things have patterns. Find it.

Figure 45. Excerpt from Epstein's (2015) problem solving strategy resource used to accompany the SNMY instructional approach

5.3.3. *Assessing student growth*

Analysis of the SNMY assessment booklets indicates room for several potential improvements. An alignment of the marking schemes to facilitate equivalent raw-score-to-LAF-level conversion for Booklets 1 and 2 (as mentioned in section 3.8.1) would facilitate statistical analysis based upon raw scores, and allow for finer grained appraisal of student growth. Ensuring that questions are worded in a fashion so as to not disadvantage students from language backgrounds other than English would reduce bias in the results also. Modifying marking schemes in such a way as to reward multiplicative thinking despite misinterpretation of questions (see Figure 33) would further ameliorate literacy related disadvantages. Greater conceptual alignment between Booklets 1 and 2 would help facilitate a more accurate appraisal of student growth. Modifying marking schemes to disaggregate multiplicative thinking growth from the impacts of student effort would also help to isolate this construct. Finally, perhaps the most accurate method of assessing multiplicative thinking (outside of

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resource-intensive student interviews) would be to employ an adaptive testing procedure that allocates questions to students based upon previous responses. Such a procedure has the potential to facilitate a higher level of accuracy of assessment than the current booklet approach.

Building upon the limitations outlined in section 5.2.2.2 , assessments of motivation and engagement could be improved in future studies by ensuring greater domain specificity of the MES employed. It would also be beneficial to ensure that questions are at a linguistic level appropriate to the subjects. For example, in a high school study where there are many students of language backgrounds other than English (such as in the present case), perhaps the primary school version of the MES would be more appropriate.

5.4. Conclusions

This study has examined the impact of a small group mathematics intervention program based upon the Scaffolding Numeracy in the Middle Years resources on a sample of under-achieving Year 8 students. It set out to explore the effects of such an intervention on students' multiplicative thinking abilities, as well as interrelations between the intervention and student motivation and engagement. The results of this study relate back to these initial questions both at the micro and the macro level, raising further questions, as well as offering ways forward.

Primary to the study's findings is the importance of considering the 'Who?' of such programs. An abstract and quantitative appraisal of results, if taken in isolation, would have provided a less than informative picture of the impact of the present intervention. What would be the meaning of an unknown student's three level drop without meeting Adam and examining how his initial over-assessment, effort, and misinterpretation of questions impacted his results? How would one interpret the zero LAF level growth result of an unnamed student without researcher's notes highlighting how Rayyan grasped place value and applied it in subsequent halving strategies? In a climate of increasing national and international narrative regarding the primacy of standardized test scores, the design experiment approach taken here affords a glimpse into the complexity of the learning process that is often reduced to a mere number. Additional to the 'Who?' is the 'When?'. The time in

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the participants' lives at which the present intervention was conducted appeared to have a significant impact upon its efficacy. Here the impacts of an increased number of years of mathematical challenge and apparent failure, student focus and behaviour, group dynamics, and home environments, come to the fore in a way that could easily have been missed in larger studies.

As well as the present study highlighting the complexities of the 'When?' and the 'Who?', it also offers clear direction in response to the question: 'What to do?'. These directions relate to improved measurement of the impact of interventions, as well as improvements to multiplicative thinking interventions themselves.

In relation to measuring motivation and engagement, the study highlighted how in linguistically diverse high school contexts, the primary school MES could be a more appropriate choice than the high school version. Additionally, domain specificity of the instrument used is a key consideration in the design phase of such an experiment.

Whilst providing some insight into students' levels of multiplicative thinking, use of the SNMY assessment booklets to gauge students' multiplicative thinking abilities is best done in tandem with an appraisal of student motivation and engagement at the time of testing. This is a finding that was only made possible by the concurrent use of both the SNMY assessment booklets and the MES. Teachers and researchers would do well to supplement the use of these assessment booklets with teacher observations, with additional room for improvement of these assessment resources to both disaggregate student effort from ability, reduce the ambiguity of questions, and better align the marking schemes between the two booklets.

Regarding interventions, if students are to transfer lessons learnt in conceptual-understanding based programs to broader problem solving contexts, interventions would benefit from a component directly addressing problem solving strategies, as well as the identification of similarities between novel questions and those previously encountered. Despite these clear directions, the ongoing rivalry between explicit and conceptual-understanding based instruction approaches remains unresolved. It is likely that, in line with the above, reconciling these competing claims is not a matter of one or the other, but rather context dependent, also requiring a consideration of the 'Who?' as well as the 'When?'.

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The present intervention based upon the Scaffolding Numeracy in the Middle Years resources, and participated in by under-achieving year 8 students, produced modest gains in multiplicative thinking. Some of these gains were captured by the SNMY assessment resources, and some of them were captured by complementary qualitative data sources. In considering the mathematical motivation and engagement of under-achieving in the middle years, the SNMY framework does not, in and of itself, offer a solution to such students' mathematical challenges, nor their related attitudes and dispositions. This study shows that, for under-achieving students, the bridge from mathematical challenge and disengagement to success and motivation is a fragile one, and the journey across it becomes more perilous the older a student gets. The ongoing challenge for teachers is to shore up and scaffold this fragile bridge's structure, and to ensure that the scaffolding provided is appropriate to both the 'who' that is crossing, and the 'when' of their traverse.

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Appendix A - Principal Plain Language Statement and Consent

Forms

PLAIN LANGUAGE STATEMENT (for Principals of participating schools)

Dr Max Stephens (supervisor)
Melbourne Graduate School of Education
Ph. (03) 9035 4976
m.stephens@unimelb.edu.au

Mr Oliver Lovell (researcher)
Melbourne Graduate School of Education
Ph. 0417116931
oliverlovelltas@gmail.com

Project: Designing and implementing an intervention program to improve under-achieving Year 8 students' understanding of multiplicative thinking and assess associated gains in motivation and engagement.

Introduction

Your school is invited to participate in our research project which aims to determine the effect of a small group mathematical intervention on under-achieving Year 8 students' understanding of multiplicative thinking and assess associated gains in motivation and engagement. This project has been approved by the Human Research Ethics Committee of the University of Melbourne.

What is the school being asked to do?

The participating school is being asked to help identify 10 students to participate in an intervention group, and 10 similarly profiled students in a non-intervention group, to take part in the research project as outlined herein. The participating school will aid in adjusting intervention group students' timetables to facilitate their participation in 3 tutorial sessions per week, each of approximately one hour, for the duration of the tutoring program. Students are to be selected based upon the following criteria:

- Parental permission and student's willingness to participate
- Low achievement in mathematics (based upon school recommendations and assessment data)
- No diagnosed learning/behavioural difficulties
- Consistent school attendance

The participating school is also being asked to aid the researcher in contacting parents of selected students for the purpose of obtaining parental permission for student participation.

What are participating students being asked to do?

Participating students in the intervention group (approx. 10) will receive small group tutoring (Mr Lovell will conduct these mathematics tutoring sessions) three times a week for between 12 and 18 weeks in the 2016 school year. All participating students will continue to attend conventional mathematics classes. Students will be asked to complete a series of short tests at three different times in order to measure any changes in multiplicative thinking. These tests will be conducted three times; at the program's commencement, the end of the 12 to 18 week tutoring period, and again between 5 and 15 weeks after the final tutoring session. Tests will be paper and pencil tests along with the tutor asking questions to

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try to get a clearer picture of students' multiplicative thinking skills. The tutor will be available to provide any clarification necessary to students whilst they are completing the tests.

Participating students in the intervention group will also be asked to answer a series of questions relating to their motivation and engagement in mathematics. These questions will be on a printed sheet and students will be asked to rate themselves on a scale from one (disagree strongly) to seven (agree strongly) on questions such as 'I feel very pleased with myself when I really understand what I'm taught at school'. The tutor will be available to provide any clarification necessary to students whilst they are completing the questionnaire. This questionnaire will take approximately 20-40 minutes. These questionnaires would be administered at similar times to the multiplicative thinking tests, but slightly offset so as to not unduly overload students.

Participating students in the control group (approx. 10) will be given the same tests and questionnaires as those in the treatment group. Students in the control group will receive normal classroom lessons.

Where possible, Mr. Lovell will speak briefly (by phone or in person) with intervention group students' parents at the commencement of the tutoring period and at its conclusion. Parents will be asked their impressions of their child's motivation and engagement with mathematics.

Only the work of those children who return permission forms will be used for data collection and analysis.

What are teachers being asked to do?

Mr Lovell will contact teachers of participating students in order to assess teacher's impressions of changes in student motivation and engagement during the time period over which the tutoring occurs, as well as over the 5-15 weeks following. These contacts will occur on three occasions for research purposes. Other contact between Mr. Lovell and students' teachers will follow normal correspondence norms between teaching colleagues at a school.

How will your school's confidentiality and students' confidentiality be protected?

We intend to protect your anonymity and the confidentiality of students' responses to the fullest possible extent and within the limits of the law. Students' names and the school's name will be kept in a password-protected computer file separate from any data derived from their test and questionnaire responses. This information will only be able to be linked to actual responses by the researchers. In the final report and associated publications, your school will not be named (even with a pseudonym) while students will be referred to by pseudonyms. As far as possible we will remove any references to information that might allow someone to guess your school's identity. The data will be kept securely in the Melbourne Graduate School of Education for five years from the date of publication, before being destroyed.

How will I receive feedback?

Once the thesis arising from this research has been completed, a brief summary of the findings will be sent to the participating school. The results will be shared through professional and academic conferences and journals.

Will participation prejudice my school in any way?

Please be advised that your school's participation in this study is completely voluntary. Should you wish to withdraw your school at any stage, or to withdraw any unprocessed data you have supplied, you are free to do so without prejudice.

Where can I get further information?

Should you require any further information, or have any concerns, please do not hesitate to contact either of the researchers on the numbers given above. Should you have any concerns about the conduct of the project, you are welcome to contact the Executive Officer, Human Research Ethics, The University of Melbourne, on ph: 8344 2073, or fax: 9347 6739.

How do I agree to participate?

If you would like to participate, please indicate that you have read and understood this information by signing the accompanying consent form and returning it in the envelope.

Melbourne Graduate School of Education

Consent form for Principals of participating schools

PROJECT TITLE: *Designing and implementing an intervention program to improve under-achieving Year 8 students' understanding of multiplicative thinking and assess associated gains in motivation and engagement.*

Name of participating school: _____

Name of investigator(s): Dr Max Stephens & Mr Oliver Lovell _____

1. I consent to School's participation in this project, the details of which have been explained to me in a written plain language statement to keep.
2. I understand that after I sign and return this consent form it will be retained by the researcher.
3. I understand that School's participation will involve completion of three sets of paper and pencil multiplicative thinking tests by students, three motivation and engagement questionnaires, brief interviews with students' teachers and, where possible, parents. I also agree that the researchers may use the results as described in the plain language statement.
4. I acknowledge that:
 - a) I have been informed that I am free to withdraw my school from the project at any time without explanation or prejudice and to withdraw any unprocessed data that has been provided;
 - b) The project is for the purpose of research;
 - c) I have been informed that the confidentiality of the information provided will be safeguarded subject to any legal requirements;
 - d) Students will be referred to by pseudonyms and my school will not be referred to (even by pseudonym) in any publications arising from the research;
 - e) I have been informed that a copy of the research findings will be made available to me upon the project's conclusion
 - f) I understand that all data collected will be stored at University of Melbourne and will be destroyed after five years

I consent to my school participating in this research

☐ yes ☐ no
(please tick)

Principal's signature: _____

Date: _____

Appendix B - Participant Plain Language Statements and Consent Forms

Plain Language Statement for Parents and Students

PROJECT TITLE: *Designing and implementing an intervention program to improve under-achieving Year 8 students' understanding of multiplicative thinking and assess associated gains in motivation and engagement.*

Introduction

Your child is invited to participate in the above research project, which is being conducted by Dr Max Stephens (supervisor) and Mr Oliver Lovell (Masters research student) of the Melbourne Graduate School of Education at The University of Melbourne. The study will investigate changes in students' mathematics skills, motivation, and engagement associated with participation in a tutoring program. The findings of this project will form Mr Lovell's Masters thesis. The project has been approved by the Human Research Ethics Committee.

What are students being asked to do?

Should you agree to your child's participation, your child will receive small group tutoring (Mr Lovell will conduct these mathematics tutoring sessions for your child and one other student) three times a week for between 12 and 18 weeks in the 2016 school year. Your child will be asked to complete a series of short paper and pencil questionnaires at three different times in order to measure their improvement in mathematics. Mr Lovell will be there to clarify any questions that your child has during the process of completing the questionnaire.

In addition, your child will also be asked to answer a series of questions relating to their motivation and engagement with mathematics. These questions would be on a printed sheet and your child will be asked to rate themselves on a scale from one (disagree strongly) to seven (agree strongly) on questions such as 'I feel very pleased with myself when I really understand what I'm taught at school'. These questions will be asked at three different times during the study. This task will be completed with the support of the tutor should students need help. These questions will take approximately 20-40 minutes on each of the three occasions.

Only the work of those children who return permission forms will be used for data collection and analysis.

Permission is also being sought for Mr. Lovell to have access to information relating to your child's performance in mathematics that has been collected by the school or through other testing (for example, NAPLAN).

What are parents being asked to do?

With your permission, Mr. Lovell would like to briefly speak with you (by phone or in person) at the commencement of the project to ask your opinions regarding your child's levels of motivation and engagement towards mathematics. At the end of the study Mr. Lovell will also contact you to ask you whether you believe your child's levels of motivation and engagement towards mathematics have changed during the time period over which they have participated in the tutoring program.

How will your child's identity and confidentiality be protected?

We intend to protect your child's anonymity and the confidentiality of his/her responses to the fullest possible extent, within the limits of the law. Your child's name and school details will be kept in a password-protected computer file separate from any data that is supplied. Data will be aggregated but some illustrative examples will be used in the final report and associated publications. Any child mentioned will be referred to by a pseudonym (a made up name). We will remove any references to information that might allow someone to easily guess a child's identity, though any pictures of student work are likely to include students' handwriting. The data will be kept securely in the Melbourne Graduate school of Education for five years from the date of publication, before being destroyed.

How will I receive feedback?

A brief report of results will be available upon request after the study is completed. The results will be shared with teachers and researchers, through journal articles and conferences.

Participation is completely voluntary

A MIDDLE YEARS MULTIPLICATIVE THINKING INTERVENTION

Your child's participation in this study is completely voluntary and your child is free to withdraw at any time. Should you wish to withdraw your child at any stage, or to withdraw any unprocessed data supplied, you are free to do so without prejudice.

If you are willing for your child to participate, please indicate that you have read and understood this information by signing the accompanying consent form and returning it in the envelope provided.

Should you require any further information, or have any concerns, please do not hesitate to contact either of the researchers; Dr Max Stephens: (03) 9035 4976, Mr Oliver Lovell: 0417116931.

Should you have any concerns about the conduct of the project, you are welcome to contact the Executive Officer, Human Research Ethics, The University of Melbourne, by phone: 8344 2073, or fax: 9347 6739.

Melbourne Graduate School of Education
Consent form for persons participating in a research project (intervention group)

PROJECT TITLE: *Designing and implementing an intervention program to improve under-achieving Year 8 students' understanding of multiplicative thinking and assess associated gains in motivation and engagement.*

Name of participant: _____

Name of investigator(s): Dr Max Stephens & Mr. Oliver Lovell _____

1. I consent to my child's participation in this project, the details of which have been explained to me in a written plain language statement to keep.
2. I understand that after I sign and return this consent form it will be retained by the researcher.
3. I understand that my child's participation will involve them receiving mathematics tutoring.
4. I understand that my child's participation will involve pencil & paper questionnaires on both mathematics and motivation and engagement.
5. I understand that the researcher would like to briefly speak to me (by phone or in person) at the commencement of the tutoring and at the end of the tutoring period to ask my impressions on my child's motivation and engagement with mathematics.
6. I agree that the researcher may use the results as described in the plain language statement.
7. I acknowledge that I have been informed that I am free to withdraw my child from the project at any time without explanation or prejudice and to withdraw any unprocessed data that has been provided.
8. I acknowledge that the project is for the purpose of research.
9. I acknowledge that I have been informed that the confidentiality of the information provided will be safeguarded subject to any legal requirements and, if necessary, my child's name will be referred to by a pseudonym in any publications arising from the research.

Participant signature (Parent/ Guardian): _____

Date: _____

Participant signature (Child): _____

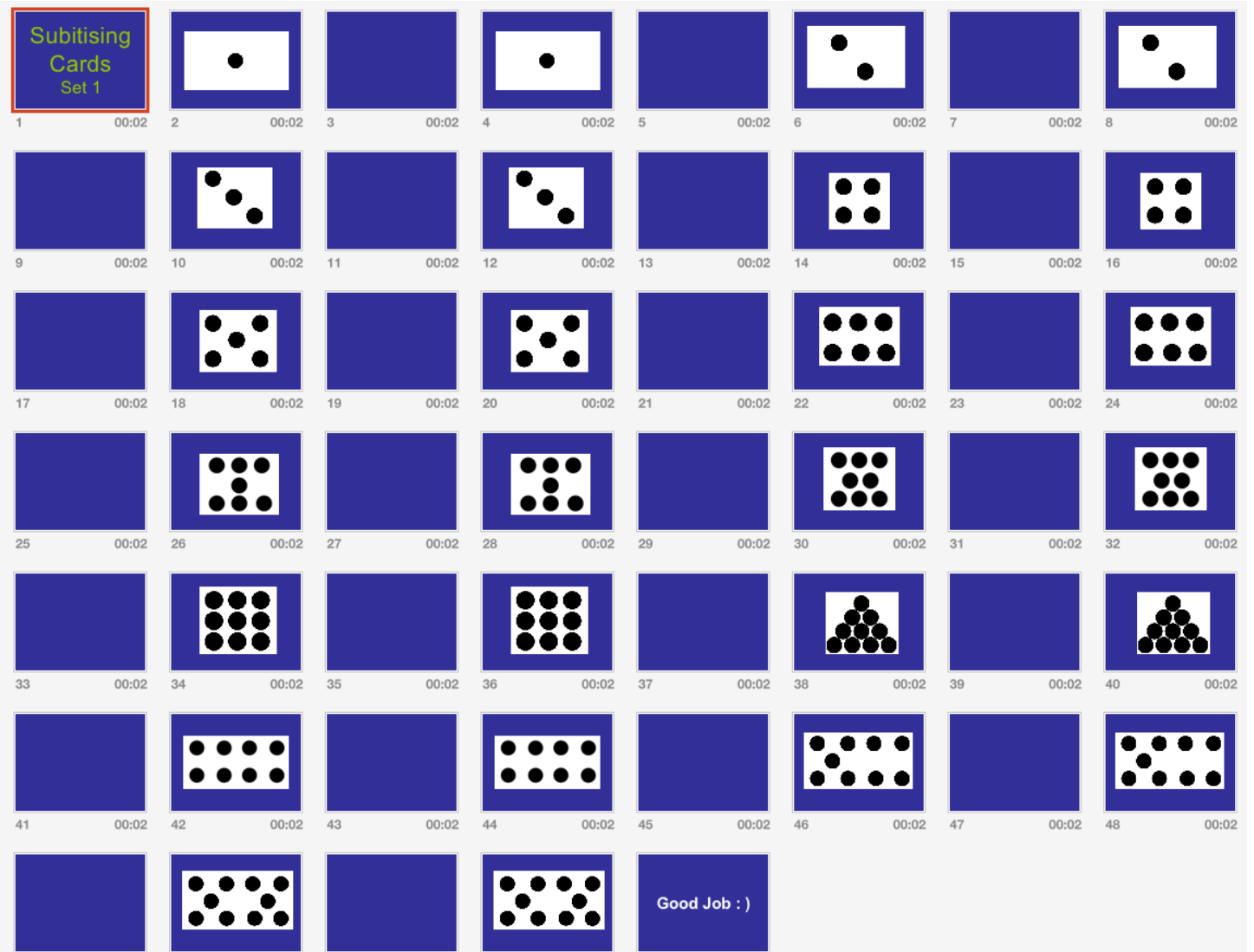
Date: _____

Appendix C - Activities from Selected Lessons

Daily activities

The following activities were used briefly at the commencement of sessions.

Session 1.1, Subitising. (A slideshow)



A MIDDLE YEARS MULTIPLICATIVE THINKING INTERVENTION

Session 4.1, Subitising.

Subitising Cards Set 6	Ready		Check		Ready		Check
10:02	00:02	00:02	00:02	00:02	00:02	00:02	00:02
	Ready		Check		Ready		Check
9	10	11	12	13	14	15	16
00:02	00:02	00:02	00:02	00:02	00:02	00:02	00:02
	Ready		Check		Ready		Check
17	18	19	20	21	22	23	24
00:02	00:02	00:02	00:02	00:02	00:02	00:02	00:02
	Ready		Check		Ready		Check
25	26	27	28	29	30	31	32
00:02	00:02	00:02	00:02	00:02	00:02	00:02	00:02
	Ready		Check		Ready		Check
33	34	35	36	37	38	39	40
00:02	00:02	00:02	00:02	00:02	00:02	00:02	00:02
	Ready		Check		Good Job :)		
41	42	43	44	45	46		
00:02	00:02	00:02	00:02	00:02	00:02		

Session 8.1, A variation on subitising

105

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Multiple sessions from session 10.1 onwards, Double trouble Bingo, levels B

Double Trouble Bingo!

30	32	34	36	38
40	42	44	46	48
50	52	54	56	58
60	62	64	66	68
70	72	74	76	78
80	82	84	86	88
90	92	94	96	98
100	102	104	106	108
110	112	114	116	118
120	122	124	126	128
130	132	134	136	138
140	142	144	146	148
150	152	154	156	158
160	162	164	166	168
170	172	174	176	178
180	182	184	186	188

50

36

54

83

26

62

15

48

70

61

18

71

49

52

84

80

35

60

93

19

17

73

29

90

74

64

37

25

45

39

81

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28

91

94

72

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16

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38

63

46

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A MIDDLE YEARS MULTIPLICATIVE THINKING INTERVENTION

Student's double trouble bingo board

Double Trouble Bingo

Number options

2	4	6	8	10
12	14	16	18	20
22	24	26	28	40
42	44	46	48	60
62	64	66	68	80
82	84	86	88	

From the number options above, select 9 different numbers and place them in the boxes below. As a card gets drawn, double the number shown on the card and see if it matches one of your numbers. If one of your numbers comes off, cross it off! The first person to get a vertical (↑ up and down ↓), horizontal (←left and right→), or a diagonal (corner to corner) line of 4 numbers all crossed off 😊 Then you can shout 'Bingo!'

Round 1

2	6	80	16
4	22	8	10
40	44	84	66
62	88 46	24	28

Round 2

2	6	48	88
10	8	24	4
46	12	14	44
26	88	18	80

A MIDDLE YEARS MULTIPLICATIVE THINKING INTERVENTION

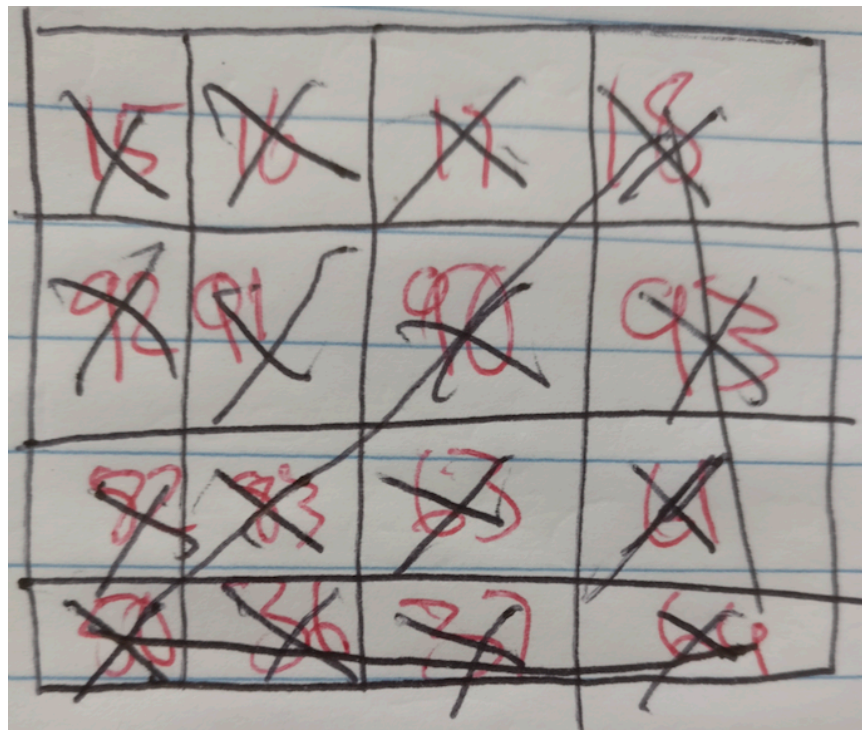
Halving Bingo, Level B

Double Trouble Bingo!

15	16	17	18	19
20	21	22	23	24
25	26	27	28	29
30	31	32	33	34
35	36	37	38	39
40	41	42	43	44
45	46	47	48	49
50	51	52	53	54
55	56	57	58	59
60	61	62	63	64
65	66	67	68	69
70	71	72	73	74
75	76	77	78	79
80	81	82	83	84
85	86	87	88	89
90	91	92	93	94

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	

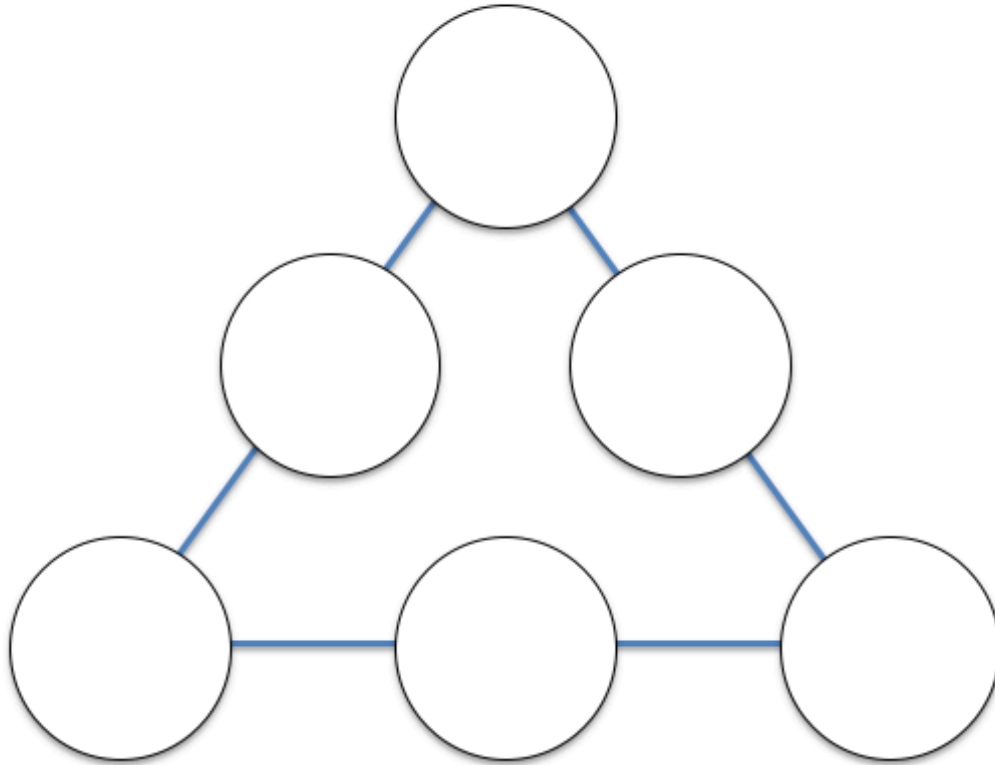
Student's halving bingo board



Selected activities forming the body of the lessons

Session 1.1, Circle Puzzles. Students placing numbers 1-6 in the circles such that each edge sums to the same number. Helping students to identify patterns within number.

Circle puzzle template



Session 1.2, Number charts. Students develop understanding of the base 10 number system.

Number Charts

[illegible]

5		

	15	

		67

	98	

Name: _____ Date: _____

World's biggest chocolate bar

In 2010, the Grand Candy Factory in Armenia produced the world's largest chocolate bar! This giant chocolate bar weighed 4400kg (about two and a half times as much as a Holden Commodore!), and was made up of a total of 24 squares of chocolate.



Your task is to work out how many different ways these 24 squares of chocolate could have been arranged in the bar, and to make a guess at which arrangement the bar was in! Use your tiles to make the different arrangements, and record the different **arrays** (arrangements) below. Circle the arrangement that you think the bar was in, and then we will look at a picture of it!!!

Session 5.2, Students modelled multiplication word problems with arrays.

Mult by 5, easier	Mult by 5, harder	Mult by 9, easier	Mult by 9, harder
Family value meal: Family value meals come with 5 serves of chips. Hungry Harry orders 12 family meals. How many serves of chips does he get?	How much does Harry pay? If a family value meal is \$32 and Harry buys 5 meals, how much does he pay?	Minivan: The minivan seats 9 students. 11 minivans are taking students to the football game. How many students are going to the football game?	Delivering presents: If Santa takes 9 seconds to deliver presents to each house. How many seconds does it take him to deliver presents to 1230 houses?
Family value meal: Family value meals come with 5 serves of chips. Hungry Harry orders 16 family meals. How many serves of chips does he get?	How much does Harry pay? If a family value meal is \$36 and Harry buys 5 meals, how much does he pay?	Minivan: The minivan seats 9 students. 13 minivans are taking students to the football game. How many students are going to the football game?	Delivering presents: If Santa takes 9 seconds to deliver presents to each house. How many seconds does it take him to deliver presents to 2340 houses?
Family value meal: Family value meals come with 5 serves of chips. Hungry Harry orders 24 family meals. How many serves of chips does he get?	How much does Harry pay? If a family value meal is \$54 and Harry buys 5 meals, how much does he pay?	Minivan: The minivan seats 9 students. 25 minivans are taking students to the football game. How many students are going to the football game?	Delivering presents: If Santa takes 9 seconds to deliver presents to each house. How many seconds does it take him to deliver presents to 3450 houses?

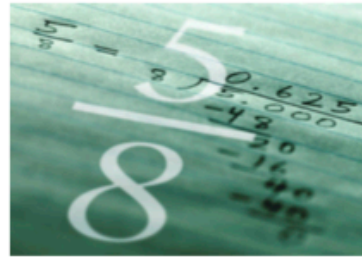
IDENTIFYING FRACTIONS WORKSHEET

The following rectangles should be partitioned into the fractions listed in order, without using a ruler.

The fractions are:

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{3}, \frac{3}{5}, \frac{5}{6}, \frac{3}{4}, \frac{2}{4}, \frac{1}{8}, \frac{3}{7}$$

(10 fractions, 10 rectangles)



Write the name of the fraction in a different way under the rectangle.

1.

2.

3.

Adapted from the activity developed by Michaela Epstein for Hume Secondary College

A MIDDLE YEARS MULTIPLICATIVE THINKING INTERVENTION

Session 13.1, Fraction Action, identifying fractions larger than one whole.

Equipment needed:

- Lengths of rope about 2-3 metres
- Sets of 6-8 pegs
- Cards labelled with various simple common fractions as appropriate (e.g. $1\frac{1}{2}$, $\frac{3}{4}$)

What to do:

TASK B

1. Stretch out a length of rope (held between two students or ask a teacher) and place a peg where you think half is (the halfway point of the rope).
2. How can you check that you are correct? What strategies have you used to check if your answer is correct?
3. Now using the pegs place the cards with the simple common fractions along the rope labelled 0 to 2

Session 13.2, Continuous Quantities, extending knowledge of fractions larger than one whole.

Equipment needed:

- Rope
- Coloured clothes pegs

What to do:

1. Label one end of your rope 3 and the other end 5.
2. Show the halfway point using a peg. What number would be placed at this point on the rope?
3. How can you check to see if this is correct? What strategies have you used to check if your answer is correct?
4. Now, place a peg where you think $3\frac{1}{2}$ would be. What strategies did you use?
5. Repeat the above for $3\frac{1}{4}$, $4\frac{2}{3}$, $4\frac{2}{5}$ doing **one** fraction at a time. What are you thinking about, when deciding where to place the peg?
How do you know that your spot is about right?
How could you check?

RECIPE FOR FRUIT SALAD

For TWELVE (12) people you need:

- 6 Bananas
- 12 oranges
- 12 peaches
- 24 strawberries



Chop into bite size pieces and place in a large bowl.

What you need to do:

1. The above recipe for Fruit Salad is enough to feed 12 people.
2. However, you only have 10 people coming over.
3. **Question:** How much of each ingredient would you need for only 10 people? How could you work it out?
4. Explain your thinking and the strategies you used in your own words.
5. How much of each ingredient would you need for 25 people? Explain and record your thinking.

HINT: If you are not sure where to begin read below.
(How much would you need for 1 person?)