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HIGH MATHEMATICAL PERFORMANCE ON CLASS TESTS IS NOT A PREDICTOR OF PROBLEM-SOLVING ABILITY: WHY?

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Drawing on my findings as a teacher and the further understandings I have developed as a researcher, I provide illustrations of confident high performing students from my research classes to explore the notion of 'enabling confidence' and 'disabling confidence'. Students with enabling confidence incline to explore unfamiliar mathematical situations and students with disabling confidence do not. The higher likelihood of many teaching practices developing disabling confidence, and our obligation to develop enabling confidence are discussed.

Introduction

Can you identify high performing students in your classes who are confident in their abilities to 'do mathematics' but need to be told: (a) what to do differently for every slight 'twist and turn' in questions in an exercise; and (b) what mathematics to use when a question is set in a different context to that in which the mathematics was learnt? Have you noticed what happens when they are asked to: (a) explore unfamiliar challenging problems; or (b) do exam questions requiring the use of familiar mathematics in unfamiliar ways? This paper examines the activity of these students in comparison to those inclined to explore mathematics.

The Nature of Confidence

My research has shown that 'resilience' in the form of Martin Seligman's (1995) 'optimism' is a characteristic of students who are inclined to explore unfamiliar mathematical ideas. It is an orientation to failures and successes which I have elaborated in terms of mathematical problem solving as 'not knowing' (failure) and 'finding out' (success). Thus, an optimistic child sees not knowing as temporary and able to be overcome through the *personal effort* of looking into the situation to identify what variables *they can control* and *which of these to vary to increase their likelihood of success—finding out more.* They perceive their *successes as permanent* (able to be achieved again) and take them on as *characteristics of self:* "I did this! I am good at this". Two of these six described facets of optimism (in italics) together characterize confidence, the "degree to which a person feels certain of her or his ability to learn and perform well in mathematics" (Hart, 1999, p. 243), and the perceived associated personal characteristic: "... one's ability to learn and to perform well on mathematical tasks" (Fenemma & Sherman, 1976, p. 326). Confident students believe they will be able to perform mathematically in the future due to personal characteristics they possess.

The Students and Their Problem Solving Responses

The four students selected, Sam, Hank, Eliza, and Patrick, illustrate the activity of students who are, or are not, inclined to explore unfamiliar mathematics. All attended the same school and were part of problem solving lessons I undertook in their classrooms during their Grade 6 year. Sam, Patrick, and Eliza were in the same class and Patrick and Eliza in the same group during the task under focus. Hank was in a different Grade 6 class in a different year. All were confident students who performed at a high level in mathematics tests in their usual mathematics classes. Although these illustrations are limited to one school and one grade, I have found such responses to tasks during my teaching (Year 7-12, see for example, Williams 1994, 1997, 2000a, 2000b, 2000c) and during my research projects (Year 12, Year 8, Grades 3-6), and can identify such activity in research undertaken by others in my classrooms (Barnes, 2000; Groves & Doig, 2004).

Sam and Hank were amongst the highest performing students in their classes on regular class tests (predominantly recall, and reproduction of taught mathematical procedures). They appeared confident in their mathematical abilities and gave their high tests marks as the reason they knew they were 'good at maths'. Each of them stated that they learnt mathematics through external means: by listening to their teacher, consulting texts, and / or finding information on the Internet.

Eliza and Patrick performed at a high level on class tests. They displayed confidence in their ability to think mathematically. They each described learning as an active process in which they participated and made sense of ideas. Patrick stated that he learnt during the problem solving lessons by working out ways to proceed where other groups had identified problems in what they had developed so far. He stated that he also learnt by thinking about why various students had made mistakes in their reports. Eliza described how concrete aides provided during the problem solving activities helped her to think as she developed an understanding of new ideas: "When I try to do things in *my mind* it is hard for me to figure it out 'till I really know how so the blocks help me to learn how to figure it out in my mind". She also described how her parents helped her when she had problems with homework: "they don't actually tell me the answer ... they sort of help me on my way". Both these students saw 'not knowing' as temporary and finding out as involving developing strategies to helped them reorganize mathematical ideas as part of sense making.

The Task 'How Many Boxes'

Properties of 'boxes' as the term was used in this task (rectangular prisms/cuboids) were explored initially with students using common language to describe them. Students were then asked to find as many different solid boxes as they could that each contained 24 'little' cubes (cubic centimetres: term not provided). As they worked with this task, they were asked questions like: How many can you make? Can you see any patterns to the boxes that can be made? Why do you get that pattern? Any ideas? Have you found them all? How do you know? Can you make a mathematical argument for how you know you have them all? Once these boxes had been explored, groups were told they were going to participate in a game. They would be told how many little cubes were used to make a 'hidden box' and asked to find its dimensions (including its orientation). To do so, each group could ask one 'yes/no' question. All groups would have access to all questions and their answers, and the winning group would be the first group to give the correct dimensions of the box with an appropriate argument as to why these were the dimensions. Before the game commenced, groups had five minutes to brainstorm about what question to ask. During that time they had access to twenty-four little cubes.

Student Activity

Student task activity was captured on video with audio record of talk in each group, group reports, and group discussion of these reports. Each student took part in a post lesson video stimulated interview in which they found parts of the video they wanted to talk about, and discuss anything new they learnt and what helped them to learn it.

Sam and Hank

These two students knew the volume of a cuboid formula before the task commenced and each stated they did not learn anything new during the three-session task.

Sam, in his post-lesson interview, knew the formula included the side lengths of the box but not why multiplying them together gave the number of little cubes in the box. Sam's lack of understanding of the structure of cubes within the box was demonstrated during the task: he did not recognize there was something the matter with a 3×3×3 box presented by a member of his group. It appeared Sam did not know this box contained 27 little cubes rather than 24. As I moved around the groups, and in whole class settings, I asked him questions across more than a twenty-minute interval before Sam even began to realize there could be something the matter with the box his group member had presented. Sam's post lesson interview showed he was still unaware of the structure of a rectangular prism at the end of the task. This lack of awareness of the structure limited the ways in which he could evaluate the reasonableness of the 3 × 3 × 3 box. He would have been unable to find the number of cubes in the box by considering layers of cubes. Sam considered the task boring stating he already knew everything about this topic. This raises questions about whether Sam's confidence in his ability to know and reproduce information about volumes of cuboids limited his likelihood of recognizing task complexities that could help develop understanding of the structure that led to the rule?

Hank's activity was similar to Sam's in some regards in that he continued to work only with the rule and numerical representations through out the task. Hank quickly recognized number patterns: factors were involved. He could see that this fitted with the multiplicative nature of the volume of a rectangular prism rule. Like Sam though, although he was repeatedly asked questions about why these factors with the rule related to finding the number of cubes in the boxes, he was unable to extend his thinking beyond identifying the factors as the dimensions of the 'boxes'. One of the final questions I asked Hank was:

How does that number pattern, which you beautifully explained yesterday, fit with those actual cubes in that box- and I don't just mean length width and height. Why, when you multiply those together, do you get the total number in that box?

I then shifted away from the group and Hank repeated again what he had already said to his group (without extending his thinking further): "[fast and soft] factors are numbers that are multiple- you can multiply factors to get the number". This was a cyclical argument based around Hank's knowledge that the volume was found by multiplying the length by the width by the height, and his knowledge of the meanings of factors and multiples. Hank was thinking numerically without linking what he had identified to the internal structure of the boxes. His lack of understanding of that structure and how it related to the factors he had generated was apparent when I gave an additional clue about the hidden box during the game: "the cross section to it has nine little squares in it". Many groups were able to make sense of this clue because they had become aware of the arrangement of cubes in boxes (the structure), and the term 'cross section' had already been explained. Hank though, exclaimed: "Hu … that's weird!" as he tried to make sense of the clue linking the structure of the cubes within the box with the dimensions and the volume.

In each of these classes, by the end the task, many groups who did not know the volume formula at the start of the task had developed an understanding of the structure of a cuboid and had developed some way of finding the number of cubes within using this structure. Hank and Sam on the other hand continued to know only the volume rule and were unable to give any explanation for why it worked.

Eliza and Patrick

These two students both actively thought about relationships between the numbers, the cubes, and the boxes during the task rather than only focusing on numbers and number patterns. Eliza reported changes in her understandings over time that demonstrated her group had progressively developed new ideas, and that she had contributed to this development. When her group (including Patrick) tried to work out whether a box could be made with 32 cubes, they only had 24 little cubes available. The group started to make a cube stack (2×2 square cross section) counting the cubes one by one. In her interview Eliza described the strategy the group used to cope with the limited number of cubes they had available: "I can't remember how many we had stacked up- but then we had (pause) a drawing on a piece of paper [two 2×2 grids]". She went on to explain the purpose of the drawing: "we needed that to pretend there was another bit of eight [two layers of four to add to the stack]". At that stage, the group did not know how many layers they needed. It was Eliza's idea to represent the rest of the cubes on paper. In her interview, Eliza identified that it was during this time of making extra layers on paper that the group realized they could count by fours (layers) to find the number of cubes in the box. Eliza displayed her new understandings in her report to the class in which she paused / hesitated at times over terms she and her group had recently begun to use. By the time she reported, she had realized she could change the orientation of the box to four layers of eight cubes:

> Start by making four flat boxes out of eight one-centimetre cubes. Stack the four to make 32. Count them- there should be four in the height and eight in the length so you use four times eight, which is thir- so you have 32 in the box

Eliza gave the length as eight but her explanations to a student questioning this use of the term 'length' showed she meant the base contained eight little cubes—the group were still in the process of developing appropriate terminology. Eliza contributed several times to her group's construction of new knowledge. Through this process she and other group members developed an understanding of the structure of the cubes in the 'box' to the extent of 'seeing' layers but not yet 'seeing' the base as an array (see Williams, 2010 for more about problems students have 'seeing' these arrays).

Eliza's responses show that trying to work out what is unfamiliar ('not known') is something she does frequently, and that she expects to be able to gain some success in doing so. She perceived that the personal effort of using her mind while using the blocks would help her. When asked how she thought she was 'going in maths' and how she decided that, Eliza reflected for some time before saying: "I know it's not because (pause) I get things right" and after further consideration she continued: "I think it's because I contribute to the question and things (pause) rather than (pause) just agree and disagree".

Patrick explained that he learnt by thinking about other group reports including: how to do something another group was still trying to work out, and thinking about why another group had developed an answer that he could see was incorrect. He was willing to look in to situations to see what could be changed to increase the likelihood of finding out about something that 'was not yet known'. For example, he used his newly developed understanding of layers to find a way to solve what another group was puzzling about. That group had intended to make a box containing 12 cubes and found the box they produced contained 24 cubes. In his interview, Patrick reported his thinking on how they could solve their problem: "You know how they got it wrong- it made me think about (pause) how they could get it right":

> It [the shape they made] was 2 2 6 [dimensions two by two by six]. They got 24 and they have to get 12 what if they changed the 6 to 3 and that would just halve it and instead of 24 you get 12.

Patrick considered the shape as six layers of four and halving the number of layers giving half the number of little cubes. Like Eliza, Patrick enacted optimistic problem solving activity.

Discussion and Conclusions

All four students displayed the pair of optimistic indicators associated with confidence, but when considering other features of optimism, Sam and Hank perceived 'finding out' as requiring the assistance of external sources (teacher, books, internet), which is a non-optimistic indicator. Each considered they knew the topic well and did not need to know more about it. Eliza and Patrick, on the other hand, perceived not knowing as temporary and were inclined to expend personal effort in looking in to situations because they knew they would be able to find out more. These are optimistic indicators. The activity of these four students shows some confident high performing students (Sam, Hank) judge their ability to do mathematics through external means like the marks they achieve, and others (Patrick, Eliza) judge their ability to do mathematics internally, by the actions they take to find out what they still do not know. Where Sam and Hank considered learning mathematics as 'taking in' rules and procedures from external sources, memorizing them and substituting into them without thinking about their meanings, Patrick and Eliza considered learning mathematics was primarily about puzzling over unfamiliar mathematical ideas and making meaning from them. Eliza and Patrick developed their confidence through the success they gained from working things out for themselves (enabling confidence), and Hank and Sam developed their confidence from their high marks on tests that were primarily about reproducing mathematics they had been taught (disabling confidence). These illustrations show what I have found more generally across upper elementary and secondary school classes: the way students develop confidence can influence their inclination to problem solve.

Research has linked confidence with ability to do mathematics, but ability to do mathematics was measured by performance on multiple choice items not open-ended tasks (Pajares & Miller, 1997). This raises several questions: What is the nature of the ability to do mathematics? Do we need to look more deeply at the nature of confidence and mathematical performance to be able to interpret associations between them?

Given many mathematics classes across the state (and internationally) still use teaching approaches that focus primarily on memorizing rules, reproducing rules, and using procedures to answer questions that students have already been taught how to solve, there is a high likelihood that many students in our classes are developing disabling confidence. Where assessment *only* values such mathematical performances, the problem is amplified. If we want to build the problem solving capacity of students, we need to give them opportunities to think for themselves and reward them for doing so. In this way, the ability to think mathematically rather than just repeat rules and procedures becomes the intended goal and our students become better problem solvers who understand where it is appropriate to use what mathematics. This is our challenge! The shaping paper for the Australian Mathematics Curriculum highlights the need for students to develop deep mathematical understandings (see for example, Williams, 2010). We must not lose sight of this intention as we plan the future of mathematics education in Victoria.

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