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Developing a new framework based on solid models for 3D cadastres

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Developing a new framework based on solid models for 3D cadastres

Abstract
3D cadastres aim to record, manage, and visualise spatial dimensions of volumetric legal objects in a 3D digital information environment. Definition of the geometry and topology of volumetric representations of legal objects is predicated on solid modelling approaches which are widely used in 3D modelling applications. Some solid modelling approaches are more user friendly for constructing legal objects while others enable the creation of 3D legal objects with a higher level of spatial complexity. This study aims to assess the capabilities of solid models in managing the geometry and topology of 3D legal objects with various spatial complexities. The assessment was conducted based on identification of the fundamental principles for defining geometry and topology in 3D cadastres. Various 3D legal objects with different geometric shapes were examined according to the topological principles for ensuring both internal and external validity of 3D legal objects. The identified geometric and topological principles were used to propose a new framework based on solid modelling for 3D cadastres. Our suggested framework would provide the basis for modifying land subdivision policies to adopt solid models in upgrading current cadastral systems into 3D digital environments.

Keywords: solid models; 3D cadastre; subdivision guidelines; constructive solid geometry; boundary representation

1 Introduction

The research and development in 3D cadastres have gained momentum over the last years. A trend towards 3D cadastres can especially be observed in jurisdictions with many high-rise properties and underground infrastructures that intersect with each other within a dense urban environment (e.g. Australia, China, Malaysia, Netherlands and Singapore). A very detailed and sophisticated analysis of current research and practice in 3D cadastres in a wide range of countries around the world can be found in the International Federation of Surveyors (FIG) 3D cadastres best practices book (van Oosterom, 2018).

A 3D cadastre typically considers two basic types of 3D spatial objects: legal and physical (Atazadeh et al., 2016b). A legal object is a real or virtual spatial unit with homogeneous rights, responsibilities and restrictions (RRRs) that can be represented in different forms such as text, sketch, point, lines, surface or 3D volume (Lemmen et al., 2010). There are two types of legal objects: primary and secondary (Atazadeh, 2017). Primary legal objects constitute the base map for 3D cadastral systems. Secondary legal objects affect the primary legal objects by imposing administrative conditions such as supplying benefits or introducing restrictions (Rajabifard et al., 2018).

A physical object on the other hand, is the structure of permanent construction works such as walls, ceilings, columns, windows, doors, slabs and similar architectural elements.

The legal and physical aspects of 3D cadastres are related to each other since the physical objects might define the spatial limit of a legal object in form of a boundary (e.g. when a legal space is delimited by a set of walls). In this case, legal spaces can be derived or extracted directly from the physical structures (Atazadeh et al., 2016a). However, the 3D legal objects could also be virtual spaces that are not bound by physical objects (Zlatanova et al., 2016).

In the Land Administration Domain Model (LADM) standard, "Facestring" is a type of geometry to capture cadastral data. However, a "Facestring" geometry does not have a volume and thus, does not qualify as a 3D bounded solid. Ying et al. (2015) claimed that the geometry of bounded 3D legal objects should be defined in form of a "closed polyhedron". A polyhedron is a special form of solid models which consists of four or more planar faces. In a polyhedron, a pair of faces meet along an edge and a set of three or more edges meet at a vertex (Collins Dictionary, 2019). Hence, according to this definition, a 3D legal object is not allowed to have any curvature in the edges or faces. This assumption is very popular in the 3D cadastre literature (Dimopoulou et al., 2016; Gulliver, 2015; Guo et al., 2012; Karki et al., 2010; Thompson and Van Oosterom, 2011a; Thompson and Van Oosterom, 2011b; Thompson et al., 2016; Zlatanova et al., 2004).

1.1 Scope

One important aspect of 3D cadastres is to create legally valid 3D objects. 3D objects usually are created in form of "solid geometries" which are defined as three-dimensional figures in the space or as representations that permit (at least in principle) any well-defined geometrical property of any represented object to be calculated automatically"(Allen, 1984; Mäntylä, 1988; Requicha and Voelcker, 1983). The creation of solid geometries is called solid modelling and consists of principles for mathematical and computer modelling (Shapiro, 2002). The idea was to create models that needed to be valid, unambiguous and should "support (at least in principle) *any and all geometric queries* that may be asked" (Shapiro, 2002). Similar questions that had been identified in the 1970s for mechanical objects are now important for 3D cadastres (Pickett and Boyse, 2012; Shapiro, 2002). In addition, since the created 3D objects will be legally binding, a high degree of validity is required. However, the definition of validity is not straightforward in cadastral systems. Firstly, in many jurisdictions, a 3D legal object is not restricted in terms of height and depth, which means that in theory, everything below and above the earth, would legally belong to the owner of the legal object. These unbounded 3D legal objects (also called "unrestricted parcels" or "esoteric RRRs") have been discussed in different studies (Dimopoulou et al., 2018; Jazayeri et al., 2014). In contrast to unbounded 3D legal objects, Dimopoulou et al. (2018) stated that because of mining rights as well as national and international restrictions on controlled airspaces, indeed a small layer of the Earth's surface belongs to the owner of a parcel. Therefore, every legal object is automatically bounded in height and depth. This means that the use of "bounded 3D legal objects" (=closed 3D objects) may be advisable. The use of bounded 3D legal objects is also supported by Ying et al. (2015)

who highlighted that geometrically bounded 3D volumes are essential for implementing a 3D cadastral system. We also support this view and therefore will only consider bounded 3D legal objects in this article.

1.2 Approach

To create bounded 3D legal objects, several solid modelling approaches exist. The most well-known approaches are Primitive Instancing (PI), Sweep Representations (SWP), Boundary Representation (B-rep), Spatial-Partitioning Representations (SPRs) and Constructive Solid Geometry (CSG).

In this study, we limited our approach slightly as we only consider bounded 3D legal objects with planar faces. The whole range of suggested models may be much broader; however, we think that this assumption is necessary for a meaningful 3D legal object creation in most cases. Additionally, in previous publications, there has often been a focus on the identification of one solid modelling approach for the creation of 3D legal objects: CSG in (Zhang et al., 2016); SWP in (Ding et al., 2016), (Seddiki, 2016), (Griffith-Charles et al., 2016) and (Bydłosz, 2016); B-Rep in (Karki et al., 2010). However, the real-world situation may be more complex, in which there is not just one "best" modelling approach, but a combination of several approaches to construct 3D legal objects. Nevertheless, to reduce time and cost and to make an approach as "fail-safe" as possible in terms of validity, only the "easiest" approach should be used. "Easiest" means that the approach produces almost certainly valid objects. At the same time, the approach needs to be flexible enough to produce the desired 3D shape or the easiest option is just not possible to use because it is constrained by the existing method of field data capture and representation. In this study, we suggest a framework as a guideline for an approach that helps to find the easiest approach for modelling 3D legal objects based on their shape complexity.

1.3 Structure

First, we will review the fundamental geometric and topological concepts for creating valid 3D legal objects (see Section 2). This includes a categorization of possible 3D legal object shapes. Furthermore, we will review the criteria that are considered to define if a single 3D legal object is valid (intrinsic topology) and if several 3D legal objects in relation to each other are valid (extrinsic topology). For example, they do not overlap each other. Section 3 discusses 3D solid modelling approaches, which is followed by using them to create 3D legal objects in the current literature (see Section 4). In Section 5, we perform an assessment of these modelling approaches in terms of several criteria which are important for the creation of 3D legal objects. From this assessment, we link our assessment back to the fundamental concepts to provide a framework that helps to choose the most suitable approach based on the shape complexity as well as internal and external validation for 3D legal objects (see Section 6). In Section 7, we will also discuss how our proposed framework could relate to changing regulations and guidelines to support 3D digital subdivision and examination of legal objects.

2 Fundamentals of Geometry and Topology in 3D Cadastres

Geometry and topology concepts play underpinning roles in creation and validation of 3D legal objects. Geometry is about the size, shape and spatial position of an object. Topology refers to the study of properties of spatial objects including adjacency, connectivity and containment (Ellul and Haklay, 2006; McDonnell and Kemp, 1995). In 3D cadastres, topology consists of two concepts: intrinsic and extrinsic (Frédéricque et al., 2011). In this section, we will first start with the categorization of 3D legal objects with different geometries. Next, we will describe the rules for ensuring for a valid intrinsic topology of 3D legal objects. Finally, we end this chapter with explaining rules for the validity of the extrinsic topology of 3D legal objects.

2.1 Geometry of 3D legal objects

3D legal objects can have very different shapes. While some are more complex, others have simpler geometries. A comprehensive study on the taxonomy of spatial units was conducted by Thompson et al. (2015). They proposed a framework to categorize spatial units with the simplest to the most complex shapes. Additionally, the use of simplifications is suggested, when it is required, such as using simpler primitives with planar surfaces instead of curved surfaces. The proposed framework was developed based on various cases in different jurisdictions, which makes it a cross-jurisdictional categorisation of legal objects. The identified types of 3D legal objects include Thompson et al. (2015):

- Unbounded legal object: This refers to a land parcel restricted by vertical surfaces but there is no top or bottom boundary surface.
- Unbounded legal object above or below a specific elevation: It is a land parcel with defined vertical sides restricted by a horizontal surface in either depth or height, which is also restricted on one side only. This legal object is typically presented in a textual form.
- Polygonal Slice (PS): A 3D legal object with a specified height and depth limitation. It is a volumetric shape defined by extruding a polygon based on height and depth values.
- Single-Valued Stepped Slice (SVSS): This 3D legal object is defined by a set of horizonal and vertical faces with a constraint that the volume must be single valued in the vertical dimension. The constraint means that all the points between any pair of points with the same horizonal coordinates (x, y) must not be in the exterior region of the legal object. Therefore, holes or tunnels are not allowed in this case.
- Multi-Valued Stepped Slice (MVSS): Similar to SVSS, it is geometrically defined by a set of horizonal and vertical faces, but cavities and tunnels are allowed in this type of 3D legal objects.
- General 3D Parcels (G3D): Any 3D legal object that does not belong to the previous categories are considered as G3D. Specific subtypes of G3D include non-manifold objects, open/closed volumes, wedge and curved shaped objects. One particular type of G3D parcels is called Balance of Parcels. This type of parcels are defined when

jurisdictions are creating volumes, the balance airspace needs to be specified so that it is available for other developers

While the first two categories describe unbounded 3D legal objects, the last four categories (PS, SVSS, MVSS, G3D) can be considered classes of closed 3D legal objects. While one would expect the majority of legal objects are of type G3D, Thompson et al. (2016) stated that SVSS is a "surprisingly common form of volumetric spatial unit". This is also supported by Dimopoulou et al. (2018) who highlighted that G3D are hard to be captured or visualized but these 3D legal objects tend to be relatively few in number. Therefore, while some 3D legal objects have complex shapes, most of them are defined by simple shapes.

The taxonomy suggested by Thompson et al. (2016) has been used by Soon et al. (2016) to also relate to the Level of Detail (LoD) adopted in the CityGML standard. While the unbounded 3D objects relate to an LoD0 representation, PS corresponds to LoD1 geometries. SVSS would be considered as LoD2. MVSS as well as G3D can be aligned with LoD3 and LoD4 models (Soon et al., 2016).

2.2 Intrinsic topology and additional properties of valid 3D legal objects

Intrinsic topology in the definition of 3D objects is generally the concept of the object's internal connectivity and behaviour, or what criteria should be met for ensuring the validity of a single object. While this is a very broad definition, Ying et al. (2015) narrowed it down and claimed that the geometry of 3D legal objects should be represented by one closed polyhedron., A polyhedron is a special form of a 3D legal object that is only allowed to have planar surfaces.

A very important step in the definition of 3D legal objects is the precise development of criteria that determine if a created 3D legal object can be considered valid. According to Kazar et al. (2008), objects are typically considered valid if they have planar faces, they are closed, their volume is connected, they have a correct orientation, their surfaces are valid and their polygons have no inner rings. In contrast, Karki et al. (2010) only consider closedness and connectedness as the required rules for valid 3D legal objects. Further intrinsic topology assumptions have been made by Kazar et al. (2008), Thompson and Van Oosterom (2011b), Ying et al. (2011), Guo et al. (2012), Ying et al. (2015), Jaljolie et al. (2016), Ding et al. (2016) and Gulliver et al. (2016). The key assumptions suggested for intrinsic topology properties and additional properties of valid 3D legal objects are summarized as:

- **Closedness**: The object should have a closed boundary (thus, be watertight). This also means that polygons forming the boundary do not have any inner rings.
- **Connectedness** of interior: Holes and voids are allowed if the solids' interior stays connected.
- **Orientable**: The object should have a clearly definable inside and outside. This means that objects such as a Moebius band or a Klein bottle are not allowed.
- **Homogeneity:** The object should have no dangling lower geometrical parts, such as dangling faces, edges or vertices.

* **No intersection** of edges or faces. This means that the object is not allowed to have any intersecting parts.
* **No duplicated** geometry at the same position (must be correctly snapped).
* **No self-intersection** of the solid's interior.

While all the investigators agree with the characteristics mentioned above, there is a discussion about the topic of self-touching objects with connected volumes. Most investigators agree with the assumption that the volume of a solid needs to stay internally connected whereas self-touching (which creates a so-called non-manifold object), is often considered a valid 3D legal object For example, Thompson and Van Oosterom (2011b) indicated that 3D legal objects can self-touch as long as their volumes stay connected. The main question here is if the 3D legal object represented by a closed solid should be a 2-manifold (no self-touching allowed) or if a non-manifold (self-touching, but non-overlapping) object should be also a valid object.
Mathematically defined, a manifold needs to fulfil the characteristic of being "topological smooth", which means that every point in it is homeomorphic to a disk or half-disk (Shene, 2014). In other words, the object is manifold, if any vertex is enclosed by faces that together form one closed (internal point) or open (boundary point) polygon and every edge belongs to exactly 2 faces. This proposition can be checked using the principle of "stars" and "links". In Figure 1a, point x shows the situation for an internal point with the surrounding polygons (star) on the left and the link (red lines) on the right. All the faces (red on the left) can be surrounded with one closed line (link). In Figure 1b, point y shows the situation for a border point (y), where the link forms a connected, but open polygon (connected lines that do not close the polygon). In Figure 1c, point z shows a non-manifold situation where the link is not just one connected line, but two non-connected ones. This means that as long as there is only one connected link, the object is a manifold object.

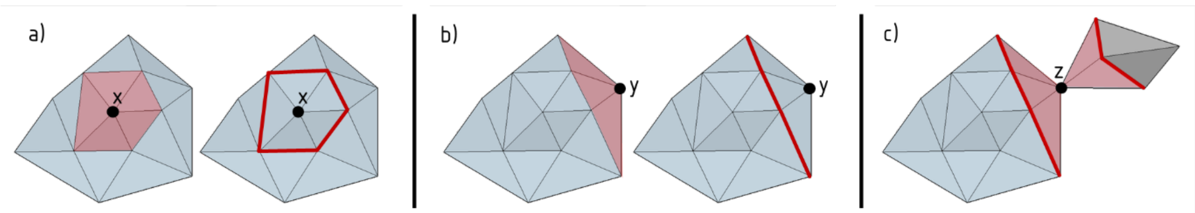

Figure 1: a) Stars and links for an internal point; b) stars and links for a border point; c) non-manifold situation (adapted from De Floriani et al. (2010))

In a 3D space, a non-manifold object is any object which has at least one point whose link does not form one closed polygon. These non-manifold parts are called "singularities" (Hui, 2008). For a 3D object, there are three non-manifold conditions: 1) non-manifold point, 2) non-manifold edge and 3) non-manifold face. These conditions arise when one object is touching or intersecting itself or if lower-dimensional elements are present. Figure 2 gives examples for non-manifold conditions.

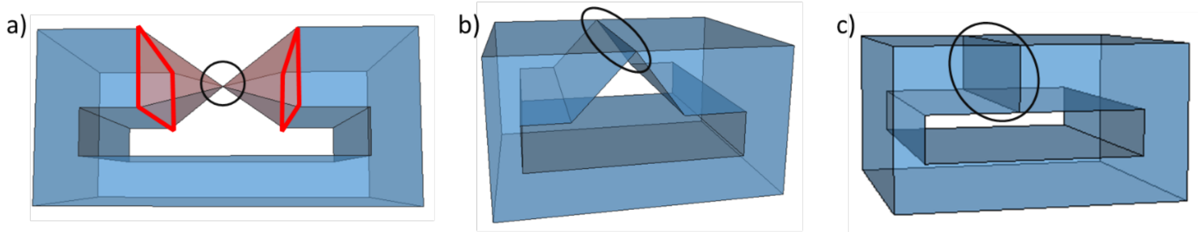

Figure 2: Examples of a) non-manifold point with star (red faces and link (red lines), b) non-manifold edge and c) non-manifold face conditions

Some non-manifold objects may be considered valid 3D legal objects. However, non-manifold objects may lead to some geometric and topological problems such as:

1. **Dangling pieces:** The non-manifold may contain dangling edges, faces or vertices or non-connected volumes.
2. **Self-intersection/self-touching:** The intersection/self-touching can be a modelling error which is not intended by the user (Zhao et al., 2013).
3. **Non-orientable:** Inside and outside of a 3D legal object may not be distinguishable. The object may not be observable and well-defined (Tse and Gold, 2001).
4. **Calculation errors:** Wrong calculation results may occur when we compute quantities such as volumes and distances.
5. **Software restrictions:** Non-manifold 3D representations are not well-supported by the existing 3D modelling tools and generic ISO standards such as ISO 19107 (Hui, 2008; van Oosterom, 2013; Ying et al., 2015).

2.3 Extrinsic topology

Extrinsic topology is about the spatial relations between a set of two objects (Billen and Zlatanova, 2003; Zlatanova et al., 2004). While a specific spatial relation may be desired in one use case, it may not be allowed in another use case. For 3D legal objects, strict rules must be applied to avoid ambiguities. For example, a volume of space must be only occupied by exactly one legal object (Figure 3b).

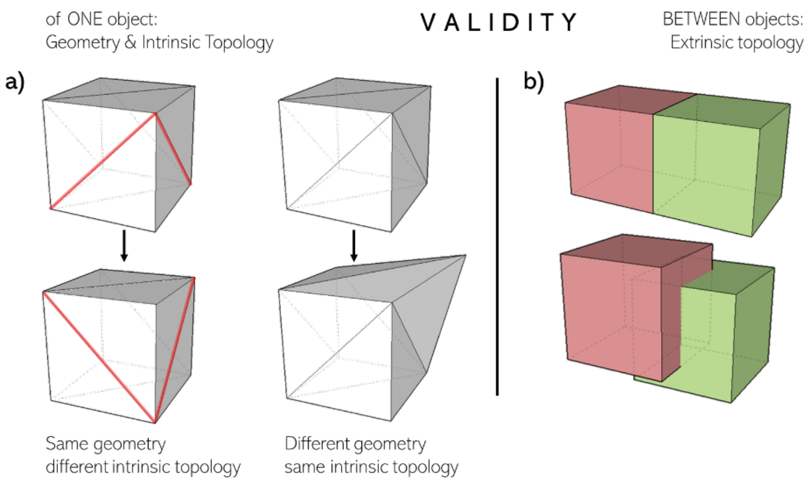

Figure 3: Visual examples of the principles of a) "geometry" and "intrinsic topology" as well as b) "extrinsic topology" (adapted after Ng (2017))

For the extrinsic topology, Zlatanova (2000) and Zlatanova et al. (2004) reviewed topological frameworks that describe the spatial relationships between 3D objects. The most well-known amongst them is the 9-intersection model (9IM) proposed by Egenhofer and Herring (1990). Zlatanova (2000) catalogued all relationships between objects with zero to three dimensions. They found a total of 69 possible relationships after reducing the total number with several conditions. For 3D object relationships, Zlatanova et al. (2004) follow the relationships given by the 9IM, which results in eight different relationships namely "disjoint", "meet", "contains", "covers", "inside", "coveredBy", "equal" and "overlap".

Zlatanova (2000) further stated that topological relationships are especially relevant for checking of the validity and integrity of 3D spatial models. For 3D cadastres, all of these relationships are very important. As every single part of the world needs to belong to somebody, every single piece of land must be a 3D object. It is up to further discussion on which relationships are allowed for one piece of land and if further sub parcels within land parcels are allowed and what rules these sub parcels have to fulfil. The framework was then further developed in Billen and Zlatanova (2003) as the "dimensional model" (DM). The DM is based on so-called "dimensional elements", which is defined by the "order of points", such as that a 2D-object can be separated into the points in its "inside" (order 2), the points in its boundary (order 1) and the vertices (order 0) (Billen and Zlatanova, 2003).

Peres and Benhamu (2009) defined the following rules for 3D shapes & polygons namely, that (1) spatial parcels might vertically overlap, but cannot penetrate, (2) they cannot overlap horizontally, (3) sub-parcels are contained in a primary parcel, (4) gaps between sub parcels of the same primary parcel are not allowed, but (5) sub-parcels can overlap vertically.

Zhao et al. (2012) identified that touching between two solids is the main important relationship between two cadastral objects. Thus, they developed the "virtual primitives method" to reducing redundancy through the use of shared faces and splitting shared part-faces into smaller objects (virtual primitives) for relationships like equal, intersect, cover and contain.

To sum up, it can be stated that:

- 3D legal objects share boundaries (touch)
- 3D legal objects cannot penetrate/intersect each other. In other words, every point in a 3D space must only belong to exactly one 3D legal object

3 Solid Modelling Approaches

In this section, we will review the widely used solid modelling approaches. These include Primitive instancing (PI), Sweep Presentations (SWP), Boundary Representation (B-rep), Spatial-Partitioning Representations (SPRs) and Constructive Solid Geometry (CSG).

In **primitive instancing (PI)**, a certain number of possible shapes is predefined (primitives). These primitives mostly include blocks/cubes, cylinders, spheres, cones and tori. To instantiate them, values must be assigned to certain parameters (Arroyo Ohori et al., 2017; Hoffmann, 1989; Jarroush and Even-Tzur, 2004). For a sphere, the parameter might be the diameter, whereas for a cube, the parameters are the side lengths (width, length, height). Figure 4 shows an example of primitive instancing for a block and a cylinder with each 3 instances with different parameters.

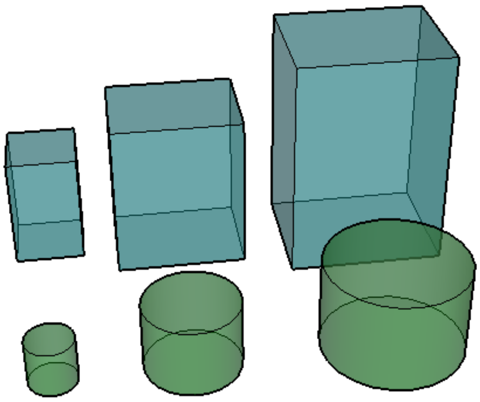

Figure 4: Example of 3 different instances of a block and a cylinder

Sweep representations (SWP) are volumes that are created by objects that are moved along a path/2D profile (Arroyo Ohori et al., 2017; Hoffmann, 1989; Jarroush and Even-Tzur, 2004) using translation or rotation. The moving object can be a 2D or 3D shape and the path can be any kind of space curve. Figure 5 shows examples of sweeping with a circle moved along a line to form a cylinder (see Figure 5a), a rotational sweeping form with a profile rotated around a circle (see Figure 5b) and a circle moved along a freeform curve to create a pipe (see Figure 5c). While all of these are examples for sweeping, Figure 5a provides a special form of sweeping, called “extrusion” or, according to (Requicha, 1980), “translational sweeping”. The principle of sweeping has been used by Ding et al. (2016), Jaljolie et al. (2016) and Seddiki (2016) to create a 3D cadastre from 2D data.

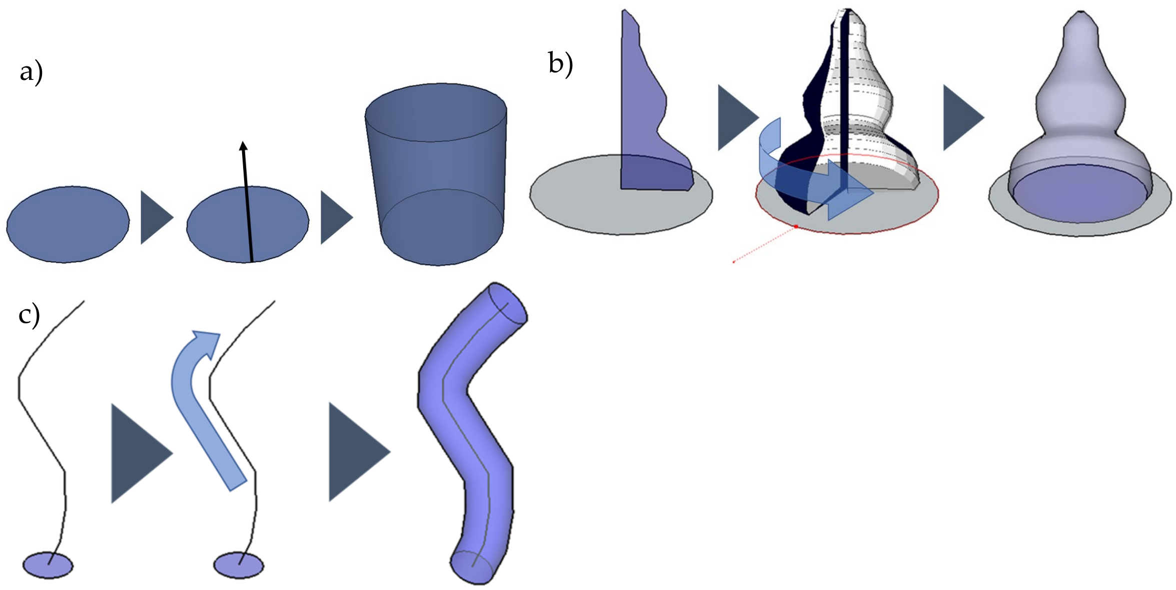

Figure 5: a) Extrusion of a circle into a cylinder, b) Sweeping of a figure around an axis, c) Sweeping of a circle along an irregular path to create a 3D solid object

A widely adopted solid model in 3D geospatial applications is called **Boundary representation (B-rep)**. In B-rep, an object is represented by its bounding surfaces, either triangulated meshes, polygonal meshes or topological arrangements of free-form surfaces (Arroyo Ohori et al., 2017). The interior of a B-rep solid is not defined explicitly but it is identified by the definition of its boundary surfaces and boundary orientations (outside/inside). An example of B-rep solid model is represented in Figure 6.

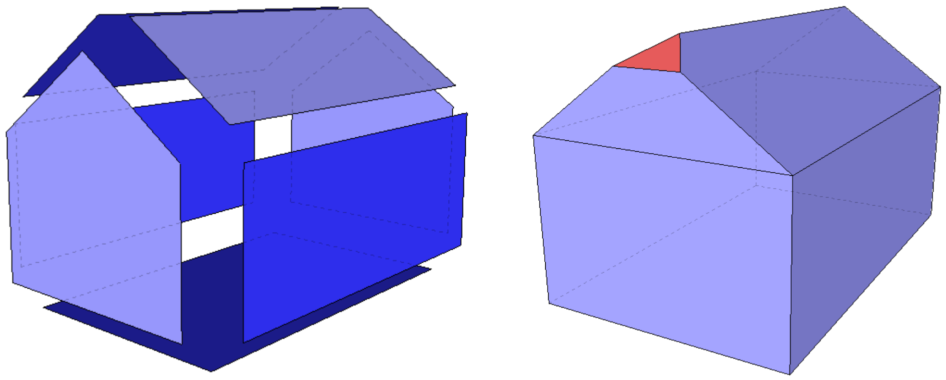

Figure 6: A) Faces that make up a boundary representation model. B) A possible local modification shown in red

Spatial-partitioning representations (SPRs) are a subgroup of cell decomposition, which is the decomposition of an object into smaller solid objects which must either be disjoint or meet precisely at a common face (Requicha, 1980). In Spatial-Occupancy Enumeration/SPR, all of the smaller objects (cells) in the scheme must be cubical and lie in a fixed grid (Requicha, 1980), which basically means using voxels. Like pixels, voxels are same-sized cubes created to approximate an object. In the case of "octrees", different sized voxels are used to provide more detail where needed.

While Nourian et al. (2016) presented methods to create voxels from point clouds, lines and surfaces for different urban applications, voxel modelling and its visualization have not yet been used in 3D cadastral applications (Pouliot et al., 2018).

Constructive solid geometry (CSG) is a construction paradigm that was developed for 3D modelling tools such as CAD (computer-aided design) and CAM (computer-aided manufacturing). In CSG, every object consists of simple primitives (as in PI), which can be combined using Boolean operations (Jarroush and Even-Tzur, 2004) such as union, intersection and difference (Chen and Tucker, 2000; Requicha, 1980). According to buildingSMART (2017), a CSG model consists of geometric and structural information. The geometric information stores the information on the primitives, whereas the structural information contains the creation tree ('recipe') of the solid. An example of a CSG tree defined by a union operation is shown in Figure 7.

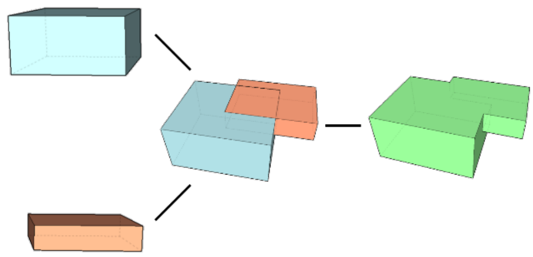

Figure 7: Exemplary CSG tree combining two cubes into one object

4 Related works for modelling and validation in 3D Cadastres

In the previous sections, we presented the fundamentals of geometry and topology of 3D legal objects and approaches how to model valid 3D legal objects. Table 1 provides an overview of the existing literature in terms of what solid modelling approach has been used, what restrictions have been defined for the intrinsic and extrinsic topology and the shape complexities of the created 3D legal objects:

Table 1: Approaches, topologic restrictions and shape complexity in selected studies on 3D cadastres

Study	Used approach	Intrinsic topology restrictions	Extrinsic topology restrictions	Shape complexity
Ding et al. (2016)	SWP-EXT	• Faces horizontal or vertical • Only two horizontal faces	• No overlaps, but touch, no sub-parcels	MVSS
Seddiki (2016)	SWP-EXT	-	-	MVSS
Griffith-Charles et al. (2016)	SWP-EXT	-	-	SVSS
Bydłosz (2016)	SWP-EXT	• "Right prism"	-	SVSS
Karki et al. (2010)	B-rep	• complete boundary (watertight) • closedness • connectedness	• No overlaps or gaps among other parcels	-
Jarroush and Even-Tzur (2004)	CSG Simple primitives (Cube, cylinder, ellipsoid, cone and two-heads pyramid) + SWP as primitive Use of intersection, substraction, union and slicing	-	-	SVSS
Peres and Benhamu (2009)	B-rep & CSG	• Finite volume (closed)	• Parcel: no intersection or penetration, vertically: overlaps and gaps allowed, can contain one or more subparcels, no intersection between parcel and subparcel • No intersection between subparcels, no gaps between sub parcels or same parcel	-
Jaljolie et al. (2016)	-	• Non-planar geometry allowed	• "safe" distance • Sub-parcels allowed • No intersection (with other 3D volumes) and with 2D volumes • Vertical overlap allowed	-
Gulliver et al. (2016)	Several sources	• 3D polyhedron • "correctly formed shape and meets definition of spatial object	• 3D spatial secondary parcel within primary parcel • No collisions with other spatial objects	-

Thompson and Van Oosterom (2011b)	-	• Very detailed axiomatic definition of internal topology	• LADM topology: no overlaps of spatial units	G3D
Ying et al. (2015)	-	• Closed volumes, normal vector to determine "inside/outside", internally connected (self-touching allowed, but with interior staying connected): • Closedness • Interior connection • Face-construction • Proper orientation • Manifold/non-manifold • No curved surfaces • No dangling pieces	• 3D parcels cannot intersect and penetrate mutually • Each has precise geometric boundaries and is a closed spatial occupation	G3D
Guo et al. (2012); Ying et al. (2011)	Using methods from sketchup	• Closed volume bounded by at least 4 faces, • Connectedness of volumes	• 3D parcels must be disjoint/mutually exclusive in the interior • Intersection between bodies is not allowed • disjunction and connection are allowed (at common node, edge or face)	G3D
Kazar et al. (2008)	-	• Planarity of faces • Closedness (vector sum of the edges in the boundary traversal =0) • Connectedness (volume has to be connected • Inner-outer check: inner boundaries can only touch the outer boundary • Orientation: polygon normal point outwards (right-hand thumb rule) • Element-check: surfaces need to be valid • No inner-ring in in polygons	-	G3D
Thompson and Van Oosterom (2011b)	-	• May not be fully bounded • If bounded, very detailed axioms, e.g. distances between nodes and edges, planarity, no self-intersections • Parcel does not have to be contiguous • The parcel is at least weakly connected, but is strongly connected (must not be 2-manifold) • Triangles/faces must be planar • Bounded by orientable surfaces	• No point of a parcel may be within the volume of another parcel • Each face is paired with exactly one anti-equal face	G3D

It is observable in Table 1 that SWP is the main solid modelling approach adopted for creating 3D legal objects. . In only one case, B-rep was used for 3D modelling of legal objects. In two investigations, two modelling approaches have been used together (CSG+SWP; B-rep+CSG). In other studies, the approach was not clearly described. In five studies, simpler objects have been created (SVSS & MVSS), which have also been the publications with one or two methods used. In studies looking at 3D legal objects with G3D complexity, the used approaches have usually not been described or they used entirely different methods (like SketchUp). At the same time, most of the investigations looking at G3D models have focused on the definition of topology rules on both intrinsic and extrinsic levels.

5 Assessment of Solid Modelling Approaches

To assess the different solid modelling approaches introduced in section 3, we perform an assessment in terms of the specific advantages and disadvantages for manipulating 3D legal objects. To do so, the first part of this section will discuss the assessment criteria we have chosen in order to perform the assessment, followed by the assessment itself.

5.1 Assessment criteria

To perform an assessment of solid modelling approaches, different criteria have been presented. Since there are multiple steps in creating 3D legal objects, we tried to include these steps into the criteria to evaluate different parts of the shape creation. The criteria have been defined based on criteria found during the literature review and should help to identify advantages and disadvantages of the solid modelling approaches. We considered five assessment criteria:

Ease of creation: For a typical user to create the desired shapes, it is important that 3D data creation process is easy and straightforward. Experts are able to create very complex 3D modelling techniques, but the typical user may not be able to use complex 3D modelling techniques. Therefore, in terms of usability, the easier the process the more likely is its use.
Shape complexity: Since 3D legal objects can have very different shapes, the shape complexity that is possible with a certain modelling approach is very important. Some of the approaches may be easy to implement and validate but they can only create very simple objects. Thus, this criterion is adopted to evaluate the possible shape complexity.
Local modifications: The fourth criterion is defined to assess the possibility of local modifications of the created geometries using one or another approach. While the use of some approaches can create very complex shapes, it may only be possible to create shapes with a certain symmetry or some limitations. As 3D legal objects can be very complex and special, local modifications would be necessary in order to create a valid and realistic object that can represent legal spaces.
Ease of validation/likelihood of erroneous geometry: For some solid models, due to their definition, it is very unlikely or even impossible to create erroneous geometries. Thus, if one of those approaches can be chosen, it probably also should be chosen in order to create valid geometries. However, one drawback that comes with easy validation is that the created objects may not be very complex.
Other strengths/limitations: This criterion has been defined in order to present additional strengths/limitations of solid models which have not been covered by the previous criteria.
While some criteria have a certain influence on each other, each criterion is rather independent from the others. For example, if the ease of creation is met, it is likely that the shape complexity may not be very high. However, some solid models may meet both criteria simultaneously. Therefore, we used this specific set of criteria. The assessment has been performed using a scale ranging from "very low" as performing very poor for the specific criterion up to "very high" which represents a very good performance.

5.2 Assessment

We assessed solid modelling approaches for manipulating 3D legal objects based on the criteria presented in section 5.1. The assessment is mainly based on the literature review. Some approaches were tested with Trimble SketchUp. However, we mainly relied on the literature findings. Table 2 provides a summary of the advantages and drawbacks of solid models according to the assessment criteria.

Primitive instancing (PI) is relatively easy to implement as only predefined shapes can be used with parameters, which makes it unambiguous, unique, easy to validate and concise. Additionally, this also makes PI good for standardization as elements can be predefined if the domain is small enough. On the other hand, this method is very restrictive since only the predefined objects are allowed. Furthermore, it is impossible to perform local modifications and create complex objects. In general, this method can be analogous to playing with toy blocks: only given shapes can be used and if other unavailable shapes are required, they need to be defined specifically for this use case or they just cannot be used.

The advantage of the sweep representation (SWP) is that the base form is drawn in 2D rather than directly in 3D, which makes it easier to use for most people. Another advantage is that it is possible to create complex objects with easy to use techniques (see Figure 4b). On the other hand, a sloped object is not easy to create as both of top and bottom surfaces must be either flat or sloped, not a mixture of flat and sloped (e.g. sloped top, flat bottom). In other words, objects created using sweeping are limited to objects with translational or rotational symmetry (Requicha, 1980). The creation paradigm does not give possibilities to change the models after creation. Furthermore, the newly generated object might not be described with mathematical formulae and would have self-intersections if it is not well-implemented by the software. Additionally, the use of SWP might result in objects that are not homogeneous and have lower geometric parts (e.g. "dangling faces", especially in general sweeping).

A strong aspect of using boundary representation (B-rep) models is that everything is definable to be "inside", "outside" or "on" the object, which facilitates internal and external checks. Furthermore, weight and volume calculations are possible using B-rep. Additionally, as every part of the object consists of a vertex with its adjacent edges and faces, a local modification can be done at every point and since triangulations can be implemented, it is even possible to do the modifications within faces (see Figure 6b). One drawback of B-rep solid models is that there may not be a guarantee of closedness (Jarroush and Even-Tzur, 2004). Additionally, it takes relatively long time to create a B-rep solid model because every face needs to be modelled separately.

Spatial partitioning representations (SPRs) produce unambiguous solid models and unique representations of objects. In addition, they are easy to validate. On the other hand, similar to 2D rasters, SPRs can only be an approximation and the size of the voxels or cells determines the accuracy of the resulting model, which might lead to very coarse representations. Therefore, SPRs may not be suitable for 3D cadastres as they must be highly accurate to not leave an empty space in legal partitioning of 3D space.

Advantages of constructive solid geometry (CSG) are similar to those of PI. Through the use of easy forms, this solid modelling paradigm is straightforward to use and does not require extended knowledge in the creation and drawing of 3D objects. It is relatively easy to create complex regular or irregular forms using just a few 3D primitives. Furthermore, the creation and combination of the 3D primitives ensures that the created 3D objects are "watertight" in most cases. On the other hand, due to unfortunate combinations of the Boolean operations, the derived CSG models may have dangling parts (points, edges or faces). Another disadvantage is that local modifications are not possible.

Table 2: Advantages and disadvantages of solid models

Solid Model	Ease of creation	Ease of validation/likelihood of erroneous geometry	Shape complexity	Local modifications	Other strengths (+)/limitations (-)
Primitive Instancing (PI)	High	High/Very low	Low	Not possible	- Elements can be predefined - only predefined primitives
Sweep Representation (SWP)	High	Low/Medium	Medium	Not possible	- limited to objects with translational or rotational symmetry - No mathematical description of generated object
Boundary Representation (B-rep)	Low	High/High	Very high	Highly modifiable	+ Everything is inside, outside or on the object + Weight and volume calculation possible - Mainly for graphical displays
Spatial-Partitioning Representations (SPRs)	High	High/Very low	Medium	Possible to a certain degree	+ Unambiguous and unique representations + Easy to validate - only approximation - validation difficult
Constructive Solid geometry (CSG)	High	High/Medium	High	Not possible	+ Conversion into other formats possible - Boolean operations might lead to dangling or disconnected parts

6 A proposed framework for using solid modelling approaches in 3D cadastres

The assessment presented in the previous section helped us to develop a framework for using solid models in the context of 3D cadastres. As presented in Table 2, each solid modelling approach has its own advantages and drawbacks. Thus, we propose using the easiest method possible and only use more complex ones if it is necessary. Choosing a suitable solid modelling approach depends on the geometric complexity of a 3D legal object. While B-rep is the most flexible approach, it is not easy to construct valid 3D geometries from scratch using this approach. Therefore, a more "fail-safe" and easy approach may be useful. In most cases, 3D legal objects to be created are flat polygons with a defined top and bottom surface. For these objects, it may be sufficient to model their shapes using SWP and simply extrude the polygons to create 3D legal objects. Therefore, we suggest that a solid modelling approach should be chosen based on the shape complexity of a 3D legal object. As presented in Section 2.1, typical forms of closed 3D legal objects are Polygonal Slice (PS), Single-Valued Stepped Slice (SVSS), Multi-Valued Stepped Slice (MVSS) and General 3D Parcels (G3D).

Our proposed framework comprises six steps for creating geometrically and topologically valid 3D legal objects using solid models: (1) identification, (2) instantiation, (3) combination, (4) modification, (5) visualization and (6) validation. Figure 8 shows a graphical of representation of the proposed framework.

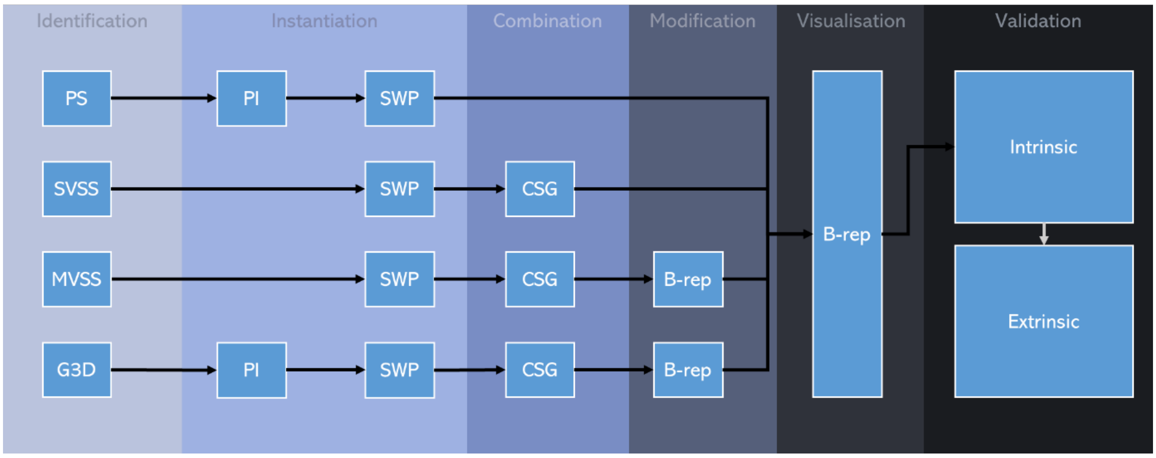

Figure 8: Suggested framework based on the evaluation results

Identification. In this first step, the geometry is assessed in a way to determine the complexity of a 3D legal object. For most 3D legal objects, this will be a polygonal slice (PS), while some of them may be of higher complexity, such as SVSS, MVSS and G3D which are prevalently defined in building strata subdivision. This is shown in Figure 9. On the left side, the shape complexity of the main 3D legal objects (property) is determined. The smaller object can be represented directly in form of a PS since it has no further subdivisions. However, the larger 3D legal object must be created in form of an MVSS due to further strata subdivisions which are represented in a form of SVSS. The place where the SVSS objects will be present, the MVSS cannot have any volume, it must have holes.

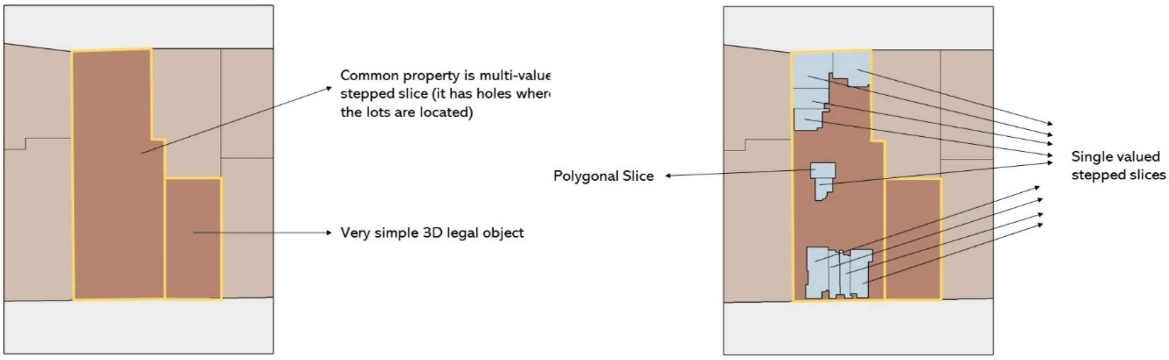

Figure 9: Identification of the shape complexity (left: for the primary legal objects, right: for subdivisions, such as building strata)

Instantiation. Depending on the result of the identification, the 3D legal object geometry can then be instantiated. For example, PS may be created using only primitive instancing (creating a cuboid) or using the footprint with extrusion techniques. Other complex forms such as SVSS may use several instances of SWP-cuboids or a combination of PI and SWP.
Since the instantiation is based on the identification, 3D legal objects can be created as shown in Figure 10. The basic properties might be created using PI and SWP (left side) while building strata can be created using SWP. For instantiation, only PI and SWP should be used as these methods only require the object form and its dimensions (e.g. cuboid with specified height, width and length) or the object form (flat polygon) and a height for the SWP method. PI can only be used for very easy object shapes such as the small 3D legal object in Fig. 10 (left). While the more complex object on the left as well as all shapes on the right have been created using SWP.

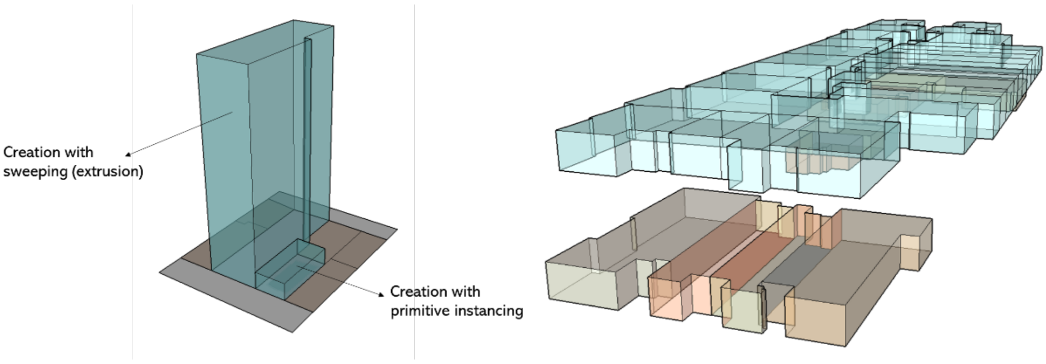

Figure 10: Instantiation of the 3D legal object geometries using primitive instancing and/or sweeping

Combination. The created basic geometry might then be aggregated/combined using CSG-techniques. Using PI and/or SWP for the basic shape creation allows for easy and valid shape creation that can then be combined or subtracted using CSG-operators to “glue them together” or to subtract parts.
One example for subtraction can be found in Figure 10. The 3D legal objects on the right are strata subsets of the same property as the large object on the left. The larger object occupies the space around the strata objects but cannot be at the same space. Therefore, to create the common property surrounding the strata objects, the strata must be subtracted from the common property to create non-overlapping 3D legal objects. An example for the combination

of objects is shown in Figure 11. In the top part, the cadastral information (in 2D) is instantiated with SWP technique. However, the 3D legal object on the right (turquoise) is in fact one connected legal object but has been instantiated as two non-connected objects. Therefore, the two objects must be combined using CSG. It is easier to create two simple objects and combine them than to directly create a complex object. The operations that can be done in this step are union, difference and intersection of instantiated geometries to create more complex 3D legal objects from simple instantiations.

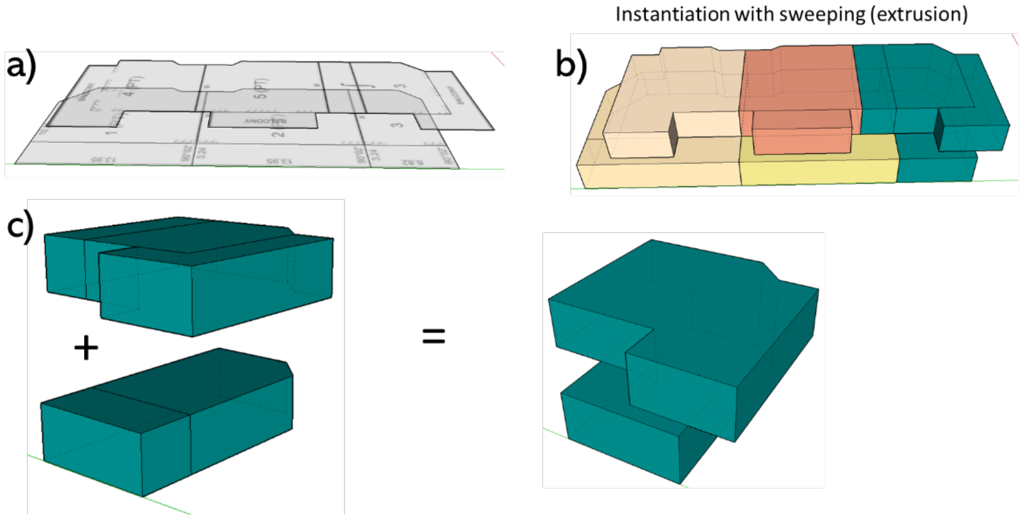

Figure 11: a) Geometry in 2D for two floors which will be b) instantiated using SWP. c) The final shape of legal object is created using union operation in CSG.

Modification. The steps above already provide the methods for the creation of rather complex objects. However, sometimes, even the combination step may not be able to create the required level of complexity of object shapes. Thus, it is necessary to provide the means for further modifications in terms of the creation of complex G3D objects or local modifications. Since B-rep is the only method that allows for local modifications and at the same time, it is straightforward to convert objects into a B-rep model, it can be used to allow for local modifications. This is shown in Figure 12, where the object created in Figure 11 gets a local modification at one side.

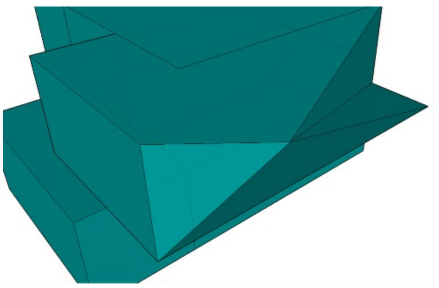

Figure 12: Local modifications of objects

Visualization. After the creation of the geometry, everything should be finally transformed into B-rep for visualization purposes, most 3D visualization applications typically use B-rep. Most solid models can be transformed in a lossless manner into the B-rep representation. During the creation steps, it should already be decided whether the final model uses flat or curved surfaces as most standards only implement and manipulate flat surfaces. This needs to be considered during the conversion phase. Therefore, in this step, all solid geometries are converted into B-rep, regardless of their creation method or shape complexity.

Validation. The validation can be done for both 3D digital data that exists in the 3D database and at data entry into 3D database. The 3D digital data before entry to database should be validated itself. However, after inserting the new 3D digital data into the database, the 3D digital data that already exists in the database needs to be validated against the new 3D digital data to make sure that the entire 3D digital datasets are valid. In this case, it will be done for 3D digital data that already exists in the database.

Intrinsic Validation. As shown in Figure 13, the validation rules as defined in Section 2.2 need to be checked (closedness, connectedness, orientable, homogeneity, no intersections (edges, faces and interior) and no duplications). Furthermore, it needs to be decided whether self-touching is allowed or not and to perform the corresponding checks.

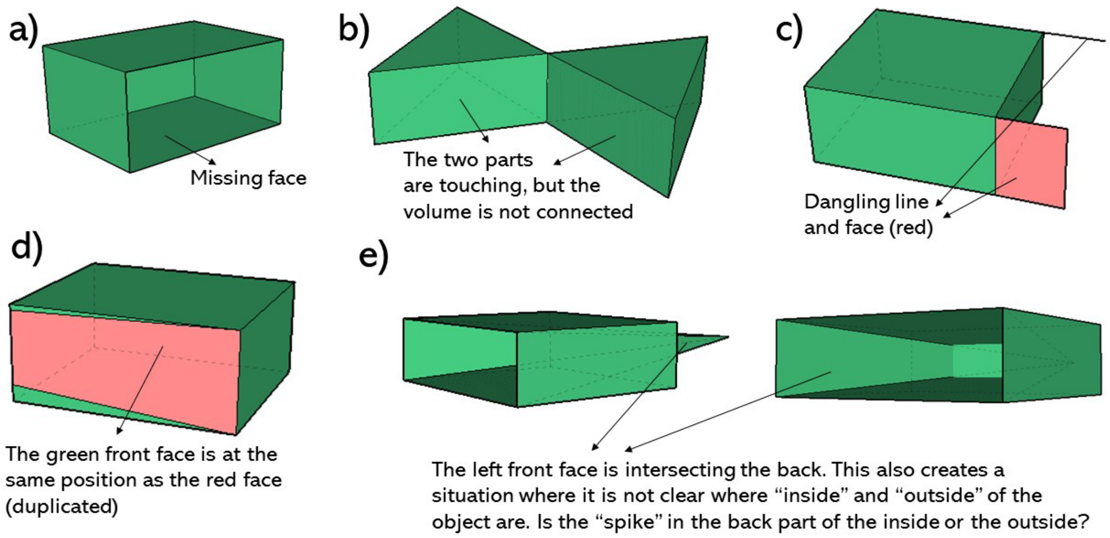

Figure 13: Intrinsic validation checking for the creation of the validity of each single 3D legal object (a-e showing examples for intrinsic topology errors)

Extrinsic Validation. If a model passes the intrinsic validation, the extrinsic validation can be done. In the extrinsic validation, it is checked if no parcels are overlapping, if parcels are touching the neighbours (definition of a distance that distinguishes "touch" and "disjoint"). Furthermore, it needs to be checked whether sub parcels are fully contained within primary parcels (see Figure 14).

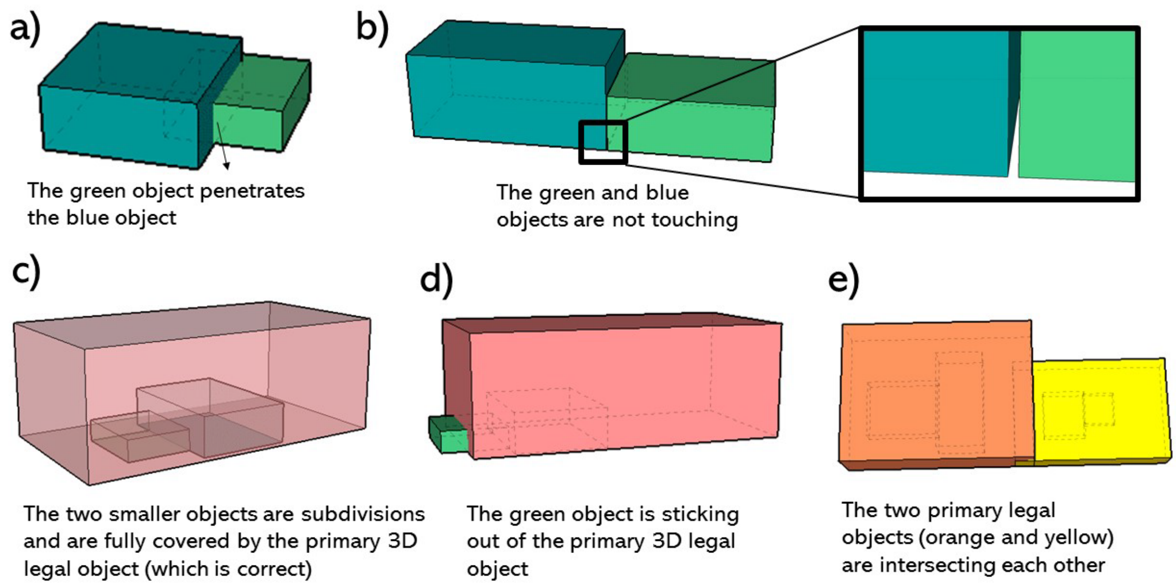

Figure 14: Extrinsic validation between 3D legal objects

7 Discussion

Traditional cadastral systems rely on 2D plans to demarcate the legal ownership of land and properties. From the policy angle, legislative frameworks and organizational settings in most jurisdictions are highly entrenched in using 2D methods for managing land and property ownership. For instance, the subdivision regulations in Victoria State of Australia use the term "plan" as the medium to communicate subdivision of ownership rights (Victorian Government, 2011). In addition, building subdivision guidelines published by the Victorian Government rely on the use of 2D diagrams to represent examples of legal spaces. These regulations and guidelines dictate the use of 2D methods to subdivide 3D reality of multi-storey structures (Land Use Victoria, 2015). 3D cadastres aim to shift this traditional paradigm into 3D digital information environments. From a technical point of view, 3D solid modelling methods enable us to define the 3D geometry and topology of legal spaces. However, current subdivision guidelines and regulations need to be modified in order to support the use of solid models in 3D digital subdivision of land and properties in complex aboveground and underground areas.

Our proposed framework provides jurisdiction independent guidelines for subdividing 3D legal objects with various spatial complexities. However, each jurisdiction has its unique guidelines for subdividing legal objects in multi-storey structures. As an example, building subdivision guidelines in Victoria provide different examples of 2D floor plan and cross section diagrams to describe the horizonal and vertical boundaries of 3D legal objects. In addition, the guidelines provide examples of using textual notations in complex ownership situations. These guidelines also refer to The Subdivision (Registrars Requirements) Regulations (Victorian Government, 2011)for defining legal boundaries. 3D cadastre cannot be realised without any change in these guidelines and legalisations. Therefore, there is a need to provide examples of defining 3D digital objects in building subdivision guidelines to show

how the land surveyors should draft and draw them in a 3D digital environment. To achieve this, the guidelines should show how 3D legal objects from simple to complex shapes can be drawn and represented using solid models as we proposed in our framework. For example, PI solid model can be suggested in the guidelines for 3D mapping of a private property with a simple rectangular footprint, while a common property area requires a combined use of SWP and CSG techniques.

In addition, solid models need to be considered in examining and registering 3D legal objects. The land registry examiners typically use two sets of validation rules for examining spatial and legal integrity: semantic and geometric/topologic. The geometric/topologic validation rules in 3D space can be significantly different to the rules in 2D space. The examination guidelines therefore should include 3D geometric/topologic rules for validating 3D legal objects. Ensuring internal and external validation of solid models in the examination phase can enable land registries to perform the land and property registration in a 3D digital environment.

Since the LADM standard is a widely used international standard for land administration, the final discussion point is to comment on this standard which also describes 3D geometry and whether it can be used in modelling 3D solids. In the latest version of LADM, "createVolume()" and volumeClosed()" methods are used to create volumetric spatial units and ensure the closure of the constructed volume, respectively (Rajabifard et al., 2019). Although the "createVolume()" method is based on "GM_MultiSolid", there is no entity in the "Surveying and Representation" sub package to define solid models and use them within this method. In addition, "volumeClosed()" only defines Boolean values. However, Eular-Poincaré formula should be considered as a constraint in this method to ensure closure of volumetric spatial units. Therefore, LADM may not explicitly allow for solid models to construct the geometric aspect of 3D legal objects. In fact, LADM LADM adopts a multi-surface geometric representation, which is not as strong as solid models for modelling 3D legal objects (Atazadeh et al., 2017; Pouliot et al., 2013).

8 Conclusion

In this study, we investigated a broad range of solid modelling approaches that are widely used in the general domain of 3D object modelling. The major contribution was a proposed framework for manipulating 3D legal objects with different spatial complexities. The framework suggests that 3D legal objects with basic geometry, such as polygonal slice, can be constructed using sweeping and primitive instancing methods, while more complex 3D legal objects, such as multi-valued stepped slice, should be created using the CSG approach. To visualise and validate 3D legal objects, B-rep is considered as a suitable solid modelling technique in our suggested framework. However, using B-rep does not guarantee spatial validity of 3D legal objects. Therefore, we identified two sets of principles for validating 3D legal objects on both internal and external topological levels. The internal topology principles provide a mechanism to ensure that an individual 3D legal object is valid from a 3D cadastre point of view, while the external topology principles checks adjacency relationships between two or more 3D legal objects and ensures that gaps and overlaps are avoided.

Despite our suggestions for adopting solid models for 3D cadastre, there are still non-technical challenges that need to be addressed in the future research. This will help us to develop appropriate legal and institutional frameworks to support the use of these models in current cadastral systems. In line with legal and institutional requirements, the existing methods of 3D data capture and visualisation needs to be considered to facilitate adoption of solid models by authorities. For technical future research, we plan to implement a prototype system to test the proposed solid modelling framework in current cadastral practices to facilitate 3D digital subdivision of ownership arrangements in complex urban areas. We will also investigate how 3D standards in building information modelling, such as Industry Foundation Classes, and geospatial domain, such as CityGML, are able to accommodate valid 3D legal objects in line with geometric and topological rules required in 3D cadastral systems. In addition, appropriate database schemas will be developed to support storage of valid 3D legal objects in a 3D spatial database.

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