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The lead of oil price rises on US equity market beliefs and preferences[*]

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Abstract

We find that oil price rises from a strengthening global economy have a state dependent lead on US equity market beliefs and preferences. When equity volatility is low, rising oil prices lead higher cashflow expectations, lower long run volatility and increased long run risk aversion. When volatility is high, markets focus on the contractionary effects of higher input costs, with rising oil prices leading decreased cashflow expectations, higher long run volatility and decreased long run risk aversion. Findings suggest important refinements for asset pricing, portfolio choice and models that link financial markets to the macro-economy.

JEL classification: G10; G13; C22; C30

Keywords: crude oil; risk neutral density; risk preferences; threshold regression

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1 Introduction

Decades of research have documented the significant effects of oil price shocks on the macro-economy (Hamilton, 1983; Hamilton and Herrera, 2004; Hamilton, 2011; Kilian, 2009; Kilian and Murphy, 2014). The macro-economic outlook has important implications for equity valuation, and so the relation between crude oil and equity markets has also been well studied. Kilian and Park (2009) for example, show that oil market fundamentals are an important determinant of US equity returns, with 22% of the long run variation in the US equity market driven by oil price shocks. Oil price shocks also have significantly different impacts across industries, which has important implications for risk management and asset allocation. See Maghyereh et al. (2016) for a review.

Despite this significant body of research, the direction of the relation between oil price shocks and equity returns is unclear. Evidence suggests it is either negative (Jones and Kaul, 1996; Sadorsky, 1999), insignificant (Chen et al., 1986; Huang et al., 1996), or state dependent, switching between a positive and negative relation over time (Aloui and Jammazi, 2009; Jammazi and Aloui, 2010; Conrad et al., 2014). The literature has focused on the relation between oil and *ex-post* equity returns and volatility. However the relation between oil and *forward looking* equity market beliefs (expected volatility, skewness and kurtosis) and preferences (or investor risk aversion) is not well understood.

We make a number of contributions to the literature. To our knowledge, we are the first paper to examine the lead of oil prices on forward looking equity market beliefs and preferences. This includes an examination into the lead of oil prices on expected higher order moments (skewness and kurtosis) as well as time varying risk preferences. Drawing on component models that link macro-economic fundamentals to long run volatility and correlation (Engle and Rangel, 2008; Engle et al., 2013; Conrad et al., 2014), we are also the first paper to consider the lead of oil prices on *long run* beliefs and preferences.

Previous oil research has paid little attention to skewness and kurtosis, despite their significance in asset pricing, portfolio choice, risk management and Value at Risk (Conrad et al., 2013; Jondeau and Rockinger, 2012; Chung et al., 2006). It has also failed to examine the relation between oil prices and preferences, which are a fundamental building block of choice theory. Preferences have important implications for many individual decisions as well as macroeconomic outcomes in the labour market, saving and investment, total factor productivity, and political outcomes. See Schildberg-Hörisch (2018) for further discussion.

There are many ways to extract preferences. Constant risk preferences have been estimated via expected utility and quadratic functions (Quiggin and Chambers, 2004), pricing kernels (Grith et al., 2013) or ambiguity and unawareness models (Karni and Vierø, 2013). Time varying risk preference estimation adds considerable complexity, with methods including consumption based asset pricing models (Campbell and Cochrane, 1999), time varying GARCH in mean models (Dias, 2017), or extraction via option prices (Aıt-Sahalia and Lo, 2000; Jackwerth, 2000; Rosenberg and Engle, 2002) and realised volatility (Bollerslev et al., 2011).

We examine the lead of oil prices on equity market beliefs and preferences using option prices for a number of reasons. Higher order moments based on a time series of the underlying asset are sensitive to outliers, noisy and more susceptible to estimation error. They are also usually poor estimates of expected future higher moments, as accurate estimates require a long time span. Option prices overcome these issues as a large number of prices are available cross-sectionally for a given underlying asset across strikes. This permits construction of expected density functions for the underlying asset that are less prone to estimation error. Multiple option maturity dates also means expected densities and preferences can be extracted for various horizons. This is not the case for many alternative methods based on underlying asset returns. Risk preference estimation via GARCH in mean models for example, typically employs daily data, and so little if anything can be inferred about forward looking preferences beyond the data frequency employed. Further, consumption based asset pricing models suffer from imprecise measurement of aggregate consumption, and time varying methods based on the underlying asset require selection of state variables where there is little consensus.

The risk neutral distribution (RND) extracted from option prices combines beliefs about the probability of future returns via the physical probability density function (PDF), with preferences towards those returns via the pricing kernel (PK) or stochastic discount factor. The PK explains most of the non-normality in the RND, and so much of the variation in higher order RN moments is due to changing preferences (Han, 2007).

To allow for time varying preferences, we employ the method of Rosenberg and Engle (2002). The physical PDF is obtained via simulation from a fitted model within the GARCH class of processes. The PK which is specified as a flexible generalised fourth order Chebyshev polynomial is then estimated using option prices. There is no need to select state variables and the PK provides a better fit than monotonically decreasing PKs which are rarely supported empirically (Cuesdeanu and Jackwerth, 2016).

Research examining the relation between oil and equity markets is confronted with many econometric issues. While exogeneity of oil prices is commonly assumed, more recent research recognises the same forces may be driving oil, equity markets and macroeconomic aggregates, with reverse causality also possible (Kilian, 2009; Kilian and Park, 2009). The source of oil price shocks can also be demand or supply side driven. An oil price rise from strengthening global demand for example, provides a very different signal to an oil price rise from a decrease in supply. Finally, the impact of oil price shocks on equity markets is state dependent (Aloui and Jammazi, 2009; Jammazi and Aloui, 2010; Conrad et al., 2014).

We are cognisant of these issues and employ multivariate systems of equations that allow for regime switching via threshold regression. A vector of *long run* (LR) RND moments (volatility, skewness, and kurtosis) is regressed against monthly lagged oil returns and macro-economic controls. A second threshold regression disentangles the

RND into its components, regressing a vector of LR physical PDF moments (volatility, skewness and kurtosis) and LR preferences (proxied by the slope of the PK) against the same variables. Lagged (as opposed to contemporaneous) oil returns and a wide variety of macro-economic controls, helps to alleviate endogeneity concerns. The reduced form means we can only examine if oil prices contain any additional *predictive* information beyond that contained in macro-economic aggregates. The paper therefore remains silent with respect to causality.

By focusing on long run predictability we overcome the need for a structural model that distinguishes between oil price changes from demand and supply side effects.[1] This is because production disruptions cause small and transitory oil price rises, while demand increases result in price rises that are large and sustained (Kilian, 2009). In fact, the global business cycle explained 87% of the long run variation in the oil price from 1973 to 2009, with speculative demand and flow supply shocks only accounting for 9% and 3% respectively (Kilian and Murphy, 2014).[2] The reduced form is therefore employed on the basis that oil prices over the long run are largely driven by the business cycle. This is consistent with Conrad et al. (2014), and avoids the need for a state-dependent structural model which would be heavily parameterised and possibly intractable.

Focusing on the long run also provides a number of other benefits. Only 1% of US stock return variation is driven by oil prices in the short run, but this increases to 22% in the LR (Kilian and Park, 2009). Macro-economic variables have been linked to LR ex-post equity volatility because this relates to unobservable expected cashflows and discount rates (Engle and Rangel, 2008; Engle et al., 2013) and option pricing models linking the LR component of risk neutral volatility to the macro-economy have signif-

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[1] This distinction is important in the short run though, where demand and supply side shocks impart different effects (Ready, 2018)

[2] They also show that the surge in oil prices from 2003 to 2008, was mainly caused by unexpected shifts in demand associated with the global business cycle. Commodity financialisation or diminishing oil supplies therefore do not explain the price surge. In their structural VAR, flow demand is driven by the global business cycle, flow supply by production, and speculative demand by anticipation of higher oil prices or precautionary motives.

icantly smaller pricing errors (Dorion, 2016). Filtering out short run dynamics should increase the power to detect whether oil returns lead beliefs and preferences, as short run dynamics are typically linked to liquidity and other short term factors. The removal of transitory components helps remove any noise in moment and PK estimates, and the lag between oil price shocks and the effects on the economy peaks after approximately 12 months, and remains significant for up to two to three years (Hamilton and Herrera, 2004; Kilian and Murphy, 2014).

This is the first paper to examine whether oil returns lead the second, third and fourth moments (herein moments) of the RND, as well as equity market beliefs and preferences. We consider whether oil prices contain predictive LR information over that embedded in macro-economic aggregates. We focus on oil because previous literature linking market beliefs and/or preferences to the macro-economy (Beber and Brandt, 2006; Äijö, 2008; Engle and Rangel, 2008; Engle et al., 2013) fails to consider oil despite its significant effect on the economy and financial markets. In a second step, we disentangle the LR predictive effect of oil on expectations into the expected cashflow and discount rate effects. This is done by regressing proxies for each variable against lagged oil returns and macro-economic controls.

Our simulated physical PDF moments are well calibrated. Higher order moment estimates are less noisy than realised moments, and they are more closely aligned with the unconditional distribution of S&P500 returns. Our PK estimates are consistent with the literature (Rosenberg and Engle, 2002; Aıt-Sahalia and Lo, 2000; Jackwerth, 2000) with relative risk aversion averaging approximately 9.5 over most of the sample period. In 1996 and the several years preceeding the GFC, PKs become steeply negatively sloped and average relative risk aversion doubled.[3] These changes in risk preferences are consistent with Bliss and Panigirtzoglou (2004), Brinkmann and Korn (2018) and Sichert (2020) as well as cumulative prospect theory (Tversky and Kahneman, 1992).

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[3] Aıt-Sahalia and Lo (2000) and the references within, report relative risk aversion estimates for the S&P500 ranging from 1 to 60 and so our estimates appear reasonable.

Our regressions consider: i) monthly oil returns; ii) separate effects for positive and negative monthly oil returns (via a slope dummy); and iii) positive oil returns only (with negative monthly oil returns set to zero). Results clearly demonstrate the superior model fit using positive monthly oil returns. This is consistent with the literature which finds that the economy and equity markets respond to oil price increases, with no significant effect from oil returns or oil price decreases. Hamilton (1988, 2003) attributes this asymmetry to adjustment costs arising from reallocation costs across and within sectors after an oil price change. Another explanation focuses on interest rate channels. Monetary policy responds to oil price increases but not decreases (Bernanke et al., 1999) and oil price rises increase the quality spread more than three times the size of the response to a negative shock (Balke et al., 2002).[4]

Findings reveal that oil price rises from a strengthening global economy have a state dependent lead on expected cashflows, but little effect on discount rates. When equity volatility is low (the good times), oil price rises lead increases in expected cashflows, lower levels of LR volatility (risk neutral and expected) and higher levels of LR risk aversion. When equity volatility is high (the bad times), the opposite occurs. Oil price rises lead decreases in expected cashflows, higher LR volatility and lower levels of LR risk aversion. This is due to markets placing more emphasis on the contractionary effects that rising input costs and inflation have on consumer income and aggregate demand.

The paper is structured as follows. Section 2 outlines the methodology used to examine the lead of oil returns on LR components of the RND, physical PDF and preferences. Section 3 applies the approach to the S&P500. Section 4 concludes with suggestions for further research.

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[4] See Jones and Kaul (1996) for a comprehensive review.

2 Methodology

Section 2.1 commences with the overall framework, which includes multivariate threshold regression. Section 2.2 then outlines estimation of the dependent variables, namely long run components of the RND and physical PDF moments as well as the pricing kernel.

2.1 Overall framework

It is well understood that changes in the RND can occur because of changes in expected cashflows and discount rates and/or preferences. A call option price at time t with strike K and maturity T can be represented by

$$\begin{aligned} c_{t,K,T} &= e^{-r_f T} E_t^Q[(S_T - K)^+] \\ &= e^{-r_f T} \int_0^\infty (x - K)^+ f_Q(x) dx \\ &= \int_0^\infty (x - K)^+ m(x) f_P(x) dx \\ &= E_t^P[m(S_T)(S_T - K)^+] \end{aligned} \tag{1}$$

where $r_{f,T}$ is the risk free rate until maturity, $x^+ = max(0, x)$, $m(S_T)$ is the pricing kernel (PK) or stochastic discount factor, given by the risk-neutral probability density $f_Q(X)$ and the physical probability density $f_P(X)$, discounted at the risk free rate:

$$m(X) = e^{-r_f T} \frac{f_Q(X)}{f_P(X)} \tag{2}$$

and

$$E_t^P(S_T) = E_t^P \int_T^\infty D_t e^{-r_f(t-T)} dt \tag{3}$$

where D_t represents capital gains plus dividends at time t. Following Beber and Brandt (2006), the RND and physical PDF are summarised by their second, third and fourth moments, while preferences are proxied by the slope of the PK.

Oil is strongly linked to the macro-economy (Hamilton, 1983; Hamilton and Herrera, 2004; Hamilton, 2011; Kilian and Murphy, 2014) and equity markets (Jones and Kaul, 1996; Kilian and Park, 2009). Macroeconomic information also affects equity returns (Flannery and Protopapadakis, 2002), volatility (Engle and Rangel, 2008; Engle et al., 2013), skewness (Ghysels et al., 2011), and the pricing kernel (Rosenberg and Engle, 2002). Oil price shocks may therefore contain predictive information, leading the RND via the beliefs in the physical PDF and preferences embedded in the PK.

Given that oil price rises over the long run are largely driven by the global business cycle (Kilian and Murphy, 2014), they may contain long run predictive information (beyond that contained within macro-economic aggregates) with respect to: i) increased cashflows and possibly higher discount rates (from central banks combating demand pull inflation) with ii) a contractionary effect due to rising input costs and inflation reducing the disposable income of consumers and aggregate demand. This will decrease expected cashflows and possibly raise discount rates (due to central banks combating cost push inflation). The relative importance of these effects may be time varying and so any lead of oil price shocks on the RND, physical PDF and preferences may be state dependent.

Two systems of equations are estimated. The first regresses a 3×1 vector of LR components of S&P500 RN moments (volatility, skewness and kurtosis) against lagged crude oil returns and macroeconomic controls (US CPI, unemployment rate, industrial production, house starts, 10 year T bond rate, and macro variable volatility). The second regresses a vector of LR components of physical PDF moments (volatility, skewness and kurtosis) and LR preferences (proxied by the slope of the PK) against the same variables.

To specify the threshold regression, consider an $f \times 1$ dependent variable vector $\mu_t = (\mu_{1t}, \mu_{2t}, ..., \mu_{ft})'$. Let l denote the number of variables subject to switching, and a the number of variables not subject to switching. Define the $l \times 1$ vector of switching variables at time t as $v_t = (v_{1,t}, v_{2,t}, ...v_{l,t})'$ and the $a \times 1$ vector of non switching variables as $x_t = (x_{1,t}, ..., x_{a,t})'$. Let $V_{t-1} = (v'_{t-1}, ...v_{t-q'})'$ and $X_{t-1} = (1, x'_{t-1}, ..., x'_{t-p'})'$ where

$q^{'}$ and $p^{'}$ denote the number of lags. The state dependent lead is assumed to depend on the magnitude of an observable threshold variable h_t. Let $\gamma_1 < \gamma_2 < ... < \gamma_g$ denote strictly increasing threshold values where $\gamma_0 = -\infty$ and $\gamma_{g+1} = \infty$, with the system in regime c if $\gamma_c \leq h_t < \gamma_{c+1}$. For observations in regime $c = 0, 1, ..., g$

$$\mu_t^{'} = X_{t-1}^{'}\beta + V_{t-1}^{'}\delta_c + \epsilon_t^{'} \tag{4}$$

where β is the $(ap+1) \times f$ state-invariant parameter matrix, δ_c is the $(lq \times f)$ state-dependent parameter matrix and ϵ_t is the $f \times 1$ vector of residuals.

For each system of equations, three oil return measures are considered. We consider the lead of: i) monthly oil returns, equal to the average daily return over the month $r_{c,t}$; ii) separate effects for positive and negative monthly oil returns via a slope dummy (I_t), where $I_t = 1$ when $r_{c,t} > 0$ otherwise zero; and iii) positive monthly oil returns only, via $r_{c,t}^{+} = max(0, r_{c,t})$ as applied by Mork (1989).[5]

The model is estimated via least squares with Newey-West standard errors. Analytical estimation is important because we require many controls and a system of equations. Numerically estimated regime switching and smooth transition models are not suitable, as even parsimonious univariate models often suffer from convergence issues and multiple local maxima.

The Schwarz information criterion (SIC) is used to determine breakpoints, because the number of breaks that minimizes the SIC is a consistent estimator of the true number of breaks (Yao et al., 1988). In the interests of parsimony and intrepretability, oil is the only variable with state-dependent parameters $(l = 1)$. An increase in explanatory power from threshold effects, is therefore solely due to the state dependent lead of oil.

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[5] Contemporaneous regressors are not included as many macro variables are announced in the middle of the month. Their use would also require lagged dependent variables, i.e an ARDL model. This is not employed because the smooth dependent variables would mean that the auto-regressive lags would contain virtually all explanatory power, preventing any meaningful analysis of oil. As stated above, lagging variables also focuses more on the predictive nature of oil and helps minimise endogeneity concerns.

2.2 Dependent variables in the RND and beliefs and preferences regressions

To estimate dependent variables for the threshold regressions, we seek model free or flexible methods that place as few restrictions on the dynamics as possible. We also seek methods that do not require state variables to drive the evolution of the pricing kernel as there is little consensus on which variables to employ.

Many RND extraction methodologies exist, with more recent research generally applying the model free approach of Bakshi, Kapadia, and Madan (BKM) (Bakshi et al., 2003). To overcome the absence of a continuum of options with strikes from 0 to ∞, the indirect approach of Jiang and Tian (2007) is employed. A cubic spline is fit to implied Black-Scholes-Merton (BSM) volatilities against deltas, with an extrapolation beyond the end-points assuming constant endpoint volatility. A large number of equally spaced implied volatility (IV) observations are then back-transformed via the BSM formula to obtain call prices from which the RND is extracted. This approach is well understood and so we leave the details to the references.

Beliefs and preferences are estimated using the procedure of Rosenberg and Engle (2002). A fitted GARCH model is used to simulate the physical PDF and the associated option payoffs and probabilities for a given maturity. The simulated PDF and observed option prices are then used to fit the PK which is specified as a flexible generalised fourth order Chebyshev polynomial. Preferences are measured via the slope of the PK which represents a proxy for Arrow-Pratt relative risk aversion.

To model the physical PDF, Rosenberg and Engle (2002) employ an Asymmetric GARCH(1,1) process with conditionally Gaussian innovations. Their simulation draws innovations from the empirical residual density and so higher order moments are time invariant. We employ FIEGARCH(1,d,1) (Bollerslev and Mikkelsen, 1996) because it provides a much better fit, capturing asymmetries *and* long memory in volatility. This latter feature is particularly important over medium to long term horizons (Baillie and Morana, 2009) which is our focus. Given the emphasis on time varying higher order moments and risk preferences, the skewness and degrees of freedom parameters are time varying.[6] The model is specified as

$$r_{t-1,t} = \varrho_0 + \varrho_1 r_{t-2,t-1} + \epsilon_t \tag{5}$$

$$\epsilon_t = z_t \sigma_t \quad z_t \sim SkSt(0, 1, \varphi_t, \upsilon_t) \tag{6}$$

where $r_{t-1,t}$ is the continuously compounded return between $t-1$ and t, $SkSt(0, 1, \varphi_t, \upsilon_t)$ denotes the skewed student t distribution of Lambert and Laurent (2001), φ_t is the time varying skewness parameter, υ_t the time varying degrees of freedom parameter, and

$$\log(\sigma_t^2) = \omega + (1 - \beta_L)^{-1} (1 + \phi_L) (1 - L)^{-d} g(z_{t-1}) \tag{7}$$

where d is the order of fractional integration, $(1-L)^{-d} = 1 + dL + d(d+1)L^2/2! + d(d+1)(d+2)L^3/3! + ...$ and $g(z_t) = \chi(z_t) + \lambda [|z_t| - E(|z_t|)]$, where χ and λ capture the sign and magnitude effects respectively. Following Hansen (1994)

$$\varphi_t^* = f_{t-1}(.) \tag{8}$$

$$\upsilon_t^* = f_{t-1}(.) \tag{9}$$

where $f_{t-1}(.)$ denotes a function of the information set at time $t-1$, and the logistic transformation ensures φ_t and υ_t are within lower and upper bounds.[7]

To estimate the pricing kernel, let $P_{i,t}$ denote the option price for the i^{th} option on day t, $\hat{P}_{i,t}$ the fitted option price, L the number of option prices available on day t, ζ_t a time varying $B \times 1$ parameter vector and $r_{t,T}$ the continuously compounded return on

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[6] Models with Gaussian and skewed Student t innovations with constant skewness and kurtosis, provided similar volatility estimates but a worse fit according to the AIC, SIC and likelihood ratio tests.

[7] To illustrate, conditional degrees of freedom estimates are confined to be within lower(L) and upper(U) bounds via $\upsilon_t = L + (U-L)[1 + \exp(-\upsilon_t^*)]^{-1}$. Information criteria are used to determine the most appropriate conditional skewness and degrees of freedom specifications. See Appendix A for details.

the underlying between time t and T. The PK is the solution to

$$\text{Min}_{\zeta_t} \sum_{i=1}^{L} [P_{i,t} - \hat{P}_{i,t}(\zeta_t)]^2 \tag{10}$$

where

$$\hat{P}_{i,t}(\zeta_t) = \sum_{j=1}^{J} m(S_T;\zeta_t) g_i(S_{T,j}) pr(S_{T,j}) \tag{11}$$

and $m(S_T;\zeta_t)$ denotes the fitted pricing kernel, $g_i(S_{T,j})$ the option payoff for state j and $pr(S_{T,j})$ the physical probability of state j. The PK ($m(S_T;\zeta_t)$) follows a generalised fourth order Chebyshev polynomial defined over the domain $[-1,1]$ with terms $T_n(x) = \cos(n\cos^{-1}(x))$ where $x = (2r_{t,T} - a - b)/(b-a)$ where a and b are endpoints of the approximation interval. To ensure $m(S_T;\zeta_t)$ is strictly positive and setting $a = -20\%$ and $b = 20\%$

$$\begin{aligned} m(S_T;\zeta_t) &= [\zeta_{0,t}T_0(r_{t,T})]\exp[\zeta_{1,t}T_1(r_{t,T}) + \zeta_{2,t}T_2(r_{t,T}) + \zeta_{3,t}T_3(r_{t,T})] && (12) \\ &= \zeta_{0,t}\exp[\zeta_{1,t}r_{t,T} + \zeta_{2,t}\{\cos(2\cos^{-1}(\frac{r_{t,T}}{20}))\} + \zeta_{3,t}\{\cos(3\cos^{-1}(\frac{r_{t,T}}{20})\}] && (13) \end{aligned}$$

These procedures provide time series of moments (RN and Physical) and PK slopes for a given maturity. To extract their LR components through time, an Adaptive-ARFIMA process is employed (Baillie and Morana, 2012). The model is consistent with the class of models that decompose time series into transitory and LR components (Engle and Rangel, 2008; Engle et al., 2013; Conrad et al., 2014). The model is flexible, allowing for (but not imposing) long memory via fractional integration, with reversion to a LR component modelled via a flexible fourier form. The flexible fourier form is an excellent approximation to a wide range of nonlinear functions and has been used to model abrupt and smooth structural change (Baillie and Morana, 2009, 2012).

To define the model, let $Y_{i,t}$ denote the i^{th} variable at time t, and N the total number of observations. To ensure non-negativity of the second and fourth moments, $Y_{i,t}$ denotes the log transformation of the variable. The Ad-ARFIMA(p,d,q,k) model is

$$\theta_{i,1}(L)(1-L)^{d_i}(Y_{i,t}-\mu_{i,t}) = \theta_{i,2}(L)\varepsilon_{i,t} \tag{14}$$

$$\mu_{i,t} = \omega_i + \sum_{m=1}^{k_i}\left[\eta_{i,m}\sin\left(\frac{2mt\pi}{N}\right) + \varphi_{i,m}\cos\left(\frac{2mt\pi}{N}\right)\right] \tag{15}$$

where $(1-L)^{d_i} = 1 - d_i L - d_i(1-d_i)L^2/2! - d_i(1-d_i)(2-d_i)L^3/3! - ...$, all the p roots of $\theta_{i,1}(L)$ and q roots of $\theta_{i_2}(L)$ lie outside the unit circle, $\varepsilon_{i,t}$ is white noise and $\mu_{i,t}$ is the LR component. Hannan-Quinn information criterion determine k_i.[8]

Once extracted, LR components at time t are stacked to create the dependent variable vector μ_t. This consists of LR RN volatility $\mu_{1,t}$, skewness $\mu_{2,t}$ and kurtosis $\mu_{3,t}$ in the first system of equations. For the second system of equations μ_t consists of LR physical volatility, skewness and kurtosis as well as LR PK slopes.

3 Empirical

We examine the lead of oil returns on the 30 day ahead S&P500 RND. This maturity can be extracted using highly liquid options and is consistent with most studies as well as the CME VIX and SKEW indices. All regressions are estimated at a monthly frequency given our focus on the long run and the need for macro-economic controls. Section 3.1 estimates the RND and its LR components. Section 3.2 estimates LR components of beliefs and preferences. The empirical section closes with the regression analysis in Section 3.3.

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[8] Moving average or exponential smoothing techniques are not used to extract the LR component given their limitations. Moving averages equally weight past observations and can be very sensitive to the window size. Exponential smoothing imposes exponentially decreasing weights and requires selection of a smoothing parameter. In contrast, the number of fourier terms in the Ad-ARFIMA process is determined via information criteria, there are no choices regarding window size or smoothing parameters, and the flexible fourier form does not impose exponentially decaying weights.

3.1 Long run RND estimation

S&P 500 index options data from January 4, 1996 to April 29, 2011 is used to extract 30 day ahead RND moments for each day, as well as the PK in Section 3.2.[9] Table 1 reports descriptive statistics, Jarque-Bera tests and Q statistics for monthly time series created by averaging daily RN moments over the month. Results are in line with expectations, with volatility of 6.7% equivalent to 23% annualised, skewness -2.248 and kurtosis 15.9. All time series are non-normal with significant inter-temporal dependence. Correlations between moments are -0.89 (skewness and kurtosis), 0.37 (volatility and skewness) and -0.48 (volatility and kurtosis).

Table 1: RND Moment summary statistics

	Volatility	Skewness	Kurtosis
Mean	0.067	-2.248	15.930
Median	0.062	-2.215	14.185
Maximum	0.208	-0.952	55.250
Minimum	0.032	-4.541	3.716
Standard deviation	0.027	0.650	8.993
Skewness	2.158	-0.412	1.129
Kurtosis	10.342	2.989	4.456
Jarque-Bera	555.9^{***}	5.217^{*}	55.4^{***}
Q(10)	0.250^{***}	0.267^{***}	0.377^{***}

Moment estimates obtained using the indirect BKM extraction method (Jiang and Tian, 2007). This is applied to daily data from January 4, 1996 to April 29, 2011 and the average of the moment over the month calculated. The Jarque-Bera test has a null of normality, Q(k) denotes the Q statistic for the autocorrelation function for the kth lag, ***,**,* denotes significance at the 1%, 5% and 10% levels respectively (Jarque-Bera and Q stats only).

Figure 1 plots the average of the daily RN moments for the month, along with the average daily LR component.[10] All RNDs are fat tailed and negatively skewed. In

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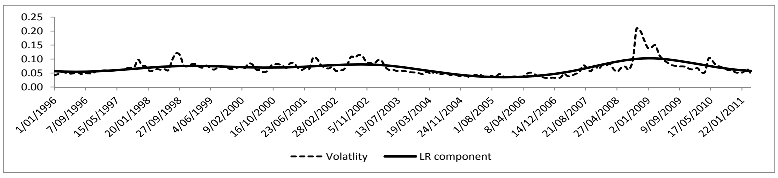
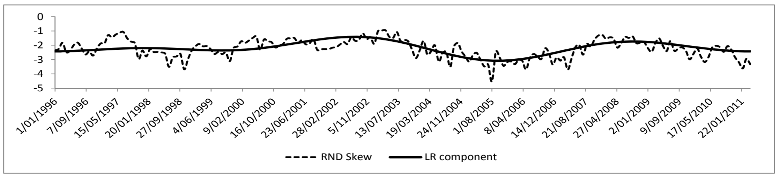
[9] Given that the reduced form relies on the findings that 87% of the variation in the oil price from 1973 to 2009 is explained by the global business cycle (Kilian and Murphy, 2014), significantly extending the analysis beyond April 2011 is beyond the scope of the paper. To avoid problems caused by far OTM options, large bid-ask spread and thin trading, all options with zero bid or ask price and zero volume are removed. Only options with the mid-point of the bid and ask greater than 0.50 and with the bid-ask spread not more than 30% of the average of bid and ask prices are employed. Options must also have a volume greater than 50 contracts over their life.

[10] See Table A.2 in Appendix A, for Ad-ARFIMA estimates fit to daily RN moments. All models have statistically significant estimates of the fractional differencing parameter, which is consistent with spectral density estimates and re-scaled range tests (Lo, 1991).

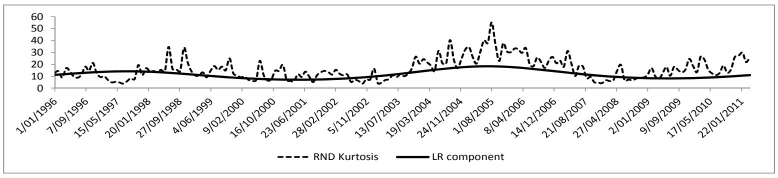
Figure 1: RND moment estimates and long run components



(a) Volatility



(b) Skewness



(c) Kurtosis

the several years preceeding the GFC, the RND had low volatility but high degrees of kurtosis and negative skew. During the GFC, RN volatility increased, but the RND became less negatively skewed and less fat tailed. These dynamics are similar to S&P500 RN moments extracted by Neumann and Skiadopoulos (2013) over a similar period.

3.2 Long run beliefs and preferences estimation

Following Rosenberg and Engle (2002), pricing kernel estimation uses out of the money call and put options with a 30 day maturity once per month.[11] On each of these dates, the AR(1)-FIEGARCH model (Equations 5 to 9) simulated a 30 day ahead return distribution conditional on the information set (see Table A.1 in the Appendix for details). 10,000 replications were employed and a Gaussian kernel used to discretise the densities into 200 equally spaced intervals.

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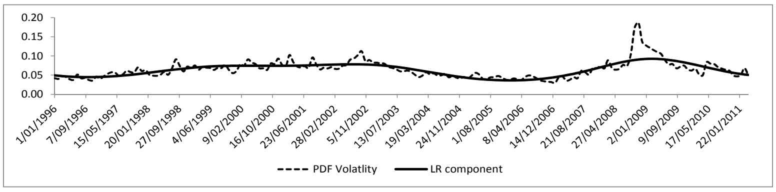
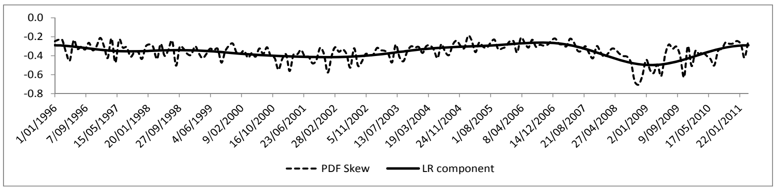
[11] On 11 out of the 184 months, there was no 30 day maturity so a 31 day maturity was used.

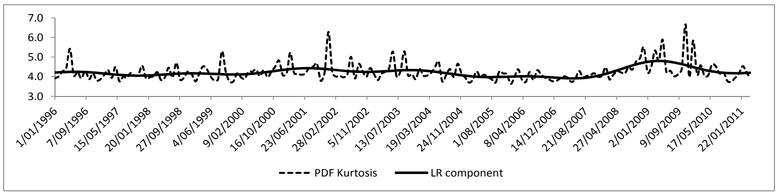
Figure 2: Physical PDF moment estimates and long run components



(a) Volatility



(b) Skewness



(c) Kurtosis

Table 2 reports descriptive statistics, Jarque-Bera tests and Q statistics for the physical PDF moments. For comparative purposes we report realised volatility (RV=$\sqrt{\sum_{i=1}^{N} r^2_{t-1,t,i}}$), skewness ($[\sqrt{N}\sum_{i=1}^{N} r^3_{t-1,t,i}]/RV^3$), and kurtosis ($[\sqrt{N}\sum_{i=1}^{N} r^4_{t-1,t,i}]/RV^4$) using daily returns where N is the number of returns in the month. Figure 2 plots simulated physical PDF moments along with LR components from unreported Ad-ARFIMA models.

Simulated physical PDF moments are similar to realised moments with two exceptions. Realised moments have considerably higher variation (or noise). This is particularly the case for skewness (standard deviation of 0.093 versus 0.808) and kurtosis (0.455 versus 1.310). It is also consistent with realised moment Q(10) statistics which fail to reject the null of no inter-temporal dependence.[12] The other difference is the average

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[12] Realised higher moment estimates using 5 minute returns were also considered, but they were no better than the reported measures.

Table 2: Physical PDF moment summary statistics

	Simulated physical PDF			Monthly realised estimates		
	Volatility	Skewness	Kurtosis	Volatility	Skewness	Kurtosis
Mean	0.064	-0.358	4.229	0.052	0.096	3.198
Median	0.063	-0.344	4.123	0.045	0.117	2.854
Minimum	0.030	-0.707	3.629	0.019	-3.275	1.829
Maximum	0.187	-0.191	6.664	0.239	2.566	13.617
Std	0.023	0.093	0.455	0.030	0.808	1.310
Skew	1.830	-1.038	2.332	2.723	-0.279	3.809
Kurtosis	9.148	4.456	10.284	14.249	4.018	25.712
Jarque Bera	392.5	49.3	573.5	1197.6	10.3	4399.9
Q(10)	0.318***	0.182***	0.114***	0.135***	-0.013	0.138

Physical PDF estimates are based on 30 day ahead Monte Carlo simulation of the AR(1)-FIEGARCH model (Equations 5 to 9) as reported in Appendix A - Table A.1. Thirty day ahead simulations are performed for the months of January 1996 to April 2011. Realised moment estimates are calculated using daily S&P500 returns. The Jarque-Bera test has a null of normality, Q(k) denotes the Q statistic for the autocorrelation function for the kth lag, *** denotes significance at the 1% level (J.B and Q stats only).

skewness of -0.358 for the physical PDF versus 0.096 for realised skewness.[13] The unconditional distribution of monthly S&P500 returns has a negative skew of -0.850 and kurtosis of 4.82. Simulated physical PDF moments are more in line with these estimates than the realised moments. Negative skewness is also consistent with the ubiquitous use of asymmetric GARCH models as well as leverage and volatility feedback effects. Our simulated physical PDF therefore appears well calibrated and provides a better fit than realised moments.

We now compare the RN and physical moments in Tables 1 and 2. Physical and RN volatilities are similar with a correlation of 0.937 and averages of 0.064 and 0.067 respectively. In contrast, the physical PDF only captures some of the negative skewness and excess kurtosis of the RND. There are also significant differences in the evolution of the higher order RN and physical moments, with the negative correlation between physical and RN skewness (-0.326) and physical and RN kurtosis (-0.189) evident after 2004. Consistent with Han (2007) the PK therefore explains most of the non-normality

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[13] A t test fails to reject the null that realised skewness is zero.

in the RND.[14]

Figures 1 and 2 suggest that in the low volatility period leading up to the GFC, skewness and kurtosis expectations were relatively stable but risk aversion increased (given the decrease in RN skewness and increase in RN kurtosis). During the GFC, the opposite occurred and risk aversion appears to have decreased. This is because RN skewness increased and RN kurtosis decreased, despite their expectations going the other way (i.e expected skewness decreased and kurtosis increased).

To more robustly examine time varying risk aversion we consider the PK. Figure 3 plots two kernels. The first (Figure 3a) represents the median kernel for high volatility periods (Mar 97-Jan 04 and Sep 07-Mar 11 as identified by threshold regression in Table 7 below). The estimate is similar to PK estimates in Rosenberg and Engle (2002) and Aıt-Sahalia and Lo (2000). The overall negative slope supports declining marginal utility of returns. For large negative returns, PKs are steeply upward sloping and mildly downward sloping for large positive returns. There is a region of slightly increasing marginal utility in between both tails.

The other kernel (Figure 3b) represents the median kernel estimate over the remaining low volatility periods (Jan 96-Feb 97, Feb 04-Aug 07, Apr 11). Estimates show a strong preference towards large negative states reflecting a surge in demand for insurance due to higher risk aversion. The value and slope of the kernel beyond -16% is approximately zero. The significant increase in the PK slope is consistent with PK estimates over 2004 to 2007 in Sichert (2020).

The shape of the PK and its significant inter-temporal evolution, means that a single slope estimate for each kernel is an inadequate proxy for time varying risk preferences.

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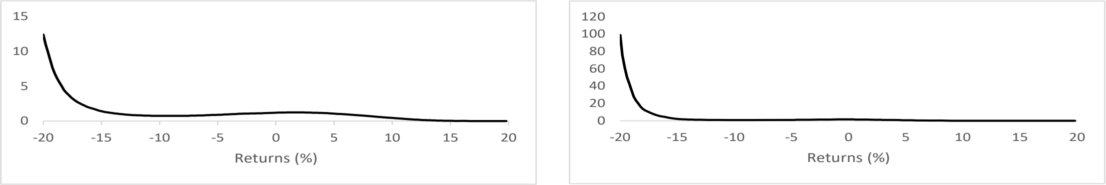
[14] This is evident on calculating RN moments *implied* by the Physical PDF and PK. Correlation between RN skewness(kurtosis) per BKM and *implied* RN skewness(kurtosis) is 0.720 (0.682). For the LR components correlations increase to 0.800 and 0.797 respectively. Slight differences between BKM and implied RN moments are attributable to the independent methodologies. BKM moments are the average over the month, while the implied RND measure is conditional on information in the middle of the month. Despite the differences, the impact of crude oil on RND moments (first threshold regression) and the physical PDF and PK (second threshold regression) are very similar. These alternative approaches therefore provide an additional level of robustness to the analysis.

Figure 3: Average pricing kernels



(a) High volatility (b) Low volatility

We therefore measure PK slopes over 20 equally spaced intervals over the PK domain $[-20\%, 20\%]$ through time. Table 3 provides summary statistics for the time series of PK slopes over each interval. Consistent with Rosenberg and Engle (2002), PK slopes over $[-20\%, -18\%]$ and $[-18\%, -16\%]$ have the largest magnitude and volatility, and so we focus on them.[15]

Table 3: Summary statistics: PK slopes

Return (%)	Mean	Median	Max	Min	Std dev
$[-20, -18]$	-35.746	-6.746	-0.213	-417.606	82.145
$[-18, -16]$	-6.711	-1.524	-0.062	-233.430	26.013
$[-16, -14]$	-1.054	-0.487	0.000	-16.589	2.301
$[-14, -12]$	-0.296	-0.166	0.000	-3.880	0.467
$[-12, -10]$	-0.099	-0.058	0.023	-1.364	0.155
$[-10, -8]$	-0.019	-0.002	0.060	-0.563	0.073
$[-8, -6]$	0.028	0.032	0.123	-0.260	0.053
$[-6, -4]$	0.063	0.059	0.213	-0.128	0.059
$[-4, -2]$	0.079	0.073	0.298	-0.077	0.064
$[-2, 0]$	0.057	0.059	0.194	-0.052	0.043
$[0, 2]$	-0.017	0.000	0.173	-0.267	0.081
$[2, 4]$	-0.092	-0.064	0.187	-0.421	0.122
$[4, 6]$	-0.122	-0.136	0.267	-0.295	0.098
$[6, 8]$	-0.113	-0.131	0.366	-0.247	0.083
$[8, 10]$	-0.089	-0.110	0.502	-0.252	0.087
$[10, 12]$	-0.063	-0.071	0.391	-0.308	0.076
$[12, 14]$	-0.042	-0.032	0.736	-0.576	0.084
$[14, 16]$	-0.027	-0.011	0.345	-0.381	0.064
$[16, 18]$	-0.014	-0.002	1.396	-0.654	0.124
$[18, 20]$	0.020	0.000	6.231	-0.746	0.464

The PK is estimated over the domain -20% to 20% once a month, using the method of Rosenberg and Engle (2002) from January 1996 to April 2011. Slopes over the return intervals through time were used to proxy time varying preferences.

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[15] This is even after winsorising the [-20%,-18%] slopes for the May 1996, November 2006, January 2007, April 2007 and May 2007 months, replacing them with the December 2006 estimate which represented the next largest [-20%,-18%] value. Further, the left tail of the January 2007 kernel from -20% to -12% had extreme outliers and so the entire kernel was replaced with the modified May 2007 kernel which represented the next most negatively sloped kernel.

Standard risk aversion measures are usually a function of the PK slope. This can be easily obtained for monotonically decreasing PKs like power utility, where relative risk aversion is the negative of the power parameter. The non-linearity in the estimated Chebyshev polynomial and its significant inter-temporal evolution complicates matters. To ensure the reasonableness of the PK slopes as relative risk aversion proxies, we approximate the relative risk aversion at the midpoint of the two left tail slopes $[-20\%, -18\%]$ and $[-18\%, -16\%]$. The slopes at these points are steeper than the rest of the kernel and so they represent an upper bound for relative risk aversion. A general expression for time varying risk aversion (γ_t) is

$$\gamma_t = \frac{-[r_{t,T} m_t'(r_{t,T})]}{m_t(r_{t,T})} \tag{16}$$

where $m_t'(r_{t,T})$ is the first derivative of the pricing kernel evaluated at $r_{t,T}$ and $m_t(r_{t,T})$ is the value of the kernel at that point. The relative risk aversion implied by Figure 3a over [-20%,-18%] is 9.6 and over [-18%,-16%] is 6.9.[16] Relative risk aversion in Figure 3b over the same intervals is 19.7 and 11.7. Most estimates of S&P500 risk aversion reported by Aıt-Sahalia and Lo (2000) are similar, and so our PKs appear reasonable.

Including LR components for each of the 20 PK slopes in the dependent variable vector (μ_t) is not feasible and so we focus on slopes over $[-20\%, -18\%]$ and $[-18\%, -16\%]$. Time variation in PK slopes over the remaining 18 intervals (from -16% to 20%) is captured by their first four principal components.[17] Long run preferences are therefore represented by LR components from Ad-ARFIMA models fit to six time series: PK slopes over $[-20\%, -18\%]$ and $[-18\%, -16\%]$ as well as the first four principal components of the remaining 18 slopes.

Figure 4 plots PK slopes and LR components over the $[-20\%, -18\%]$ and $[-18\%, -16\%]$

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[16] Where the returns used to scale $m_{t,T}'$ is the midpoint of the interval i.e -19% and -17% respectively.

[17] Principal components are extracted using the correlation matrix between PK slopes. The first four components explain 83.7% of the variation in the data.

intervals along with the S&P500 index and its returns. In the low volatility period leading up to GFC, strong gains in the equity market coincided with increasingly higher levels of risk aversion (as the left tail of the PK became more negatively sloped). Findings are inconsistent with risk aversion increasing when volatility rises (Bollerslev et al., 2011; Bekaert et al., forthcoming). If low volatility is associated with a strong economy, they are also at odds with counter cyclical risk aversion (Campbell and Cochrane, 1999; Rosenberg and Engle, 2002).[18]

Our findings (of equity market gains coinciding with low volatility and high risk aversion), are consistent with Bliss and Panigirtzoglou (2004), Brinkmann and Korn (2018) and Sichert (2020). Bliss and Panigirtzoglou (2004) for example show that risk aversion in low volatility periods exceeds high volatility periods by a factor of up to five. One explanation is that highly risk averse investors re-enter the market when volatility expectations are low, increasing the average risk aversion level. When volatility expectations are high, risk averse investors leave the market and the average level of risk aversion falls.

Another explanation draws on cumulative prospect theory (Tversky and Kahneman, 1992). One of the elements of prospect theory is diminishing sensitivity. This assumes a concave utility function above a reference point (commonly the purchase price or initial wealth) and convex utility below the reference point i.e an S shaped value function. Diminishing sensitivity therefore implies risk aversion after gains and risk seeking after losses. This effect has been used to explain more sales of stocks that earn profits i.e the disposition effect (Shefrin and Statman, 1985) and is consistent with our findings.

However, the main aspect of prospect theory is loss aversion which predicts the opposite effect i.e an increase in risk aversion after losses (or a positive relation between volatility and risk aversion). This is based on the premise that losses relative to the reference point hurt more than the satisfaction from gains of equal size. Barberis and Xiong

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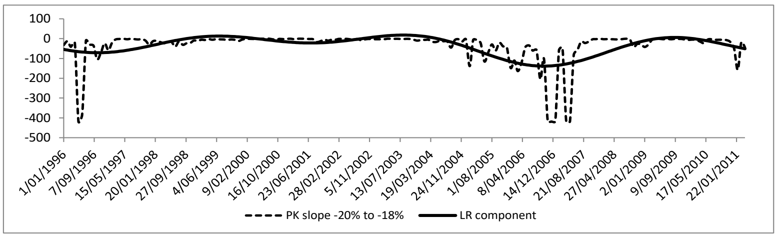
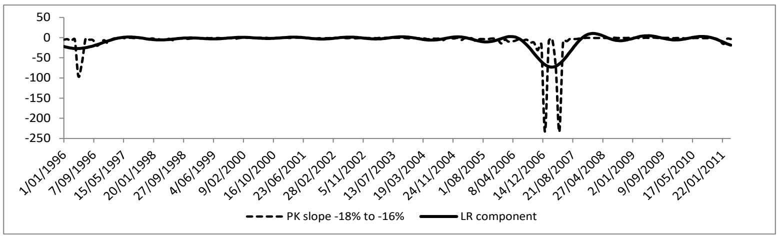
[18] We will see below that GDP growth and S&P500 volatility are only weakly correlated and so our results don't necessarily support pro-cyclical risk aversion either.

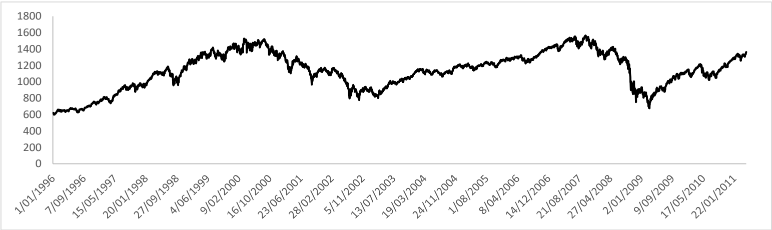
Figure 4: Pricing kernel slopes and the S&P500



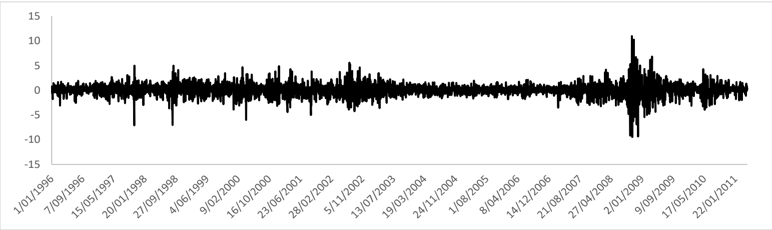
(a) Slope [-20%,-18%]



(b) Slope [-18%,-16%]



(c) S&P500



(d) S&P500 returns

(2009) argue that loss aversion dominates diminishing sensitivity. If so, prospect theory is inconsistent with our findings. More recent research however finds that loss aversion can predict a positive relation between returns and risk aversion, thereby reinforcing the effects of diminishing sensitivity. This is the case if lagged expected final wealth is the reference point (Meng and Weng, 2018) or dividends are negatively skewed (Li and Yang, 2013). S&P500 dividends are negatively skewed over the sample period (-1.13) and so our findings are consistent with prospect theory irrespective of the reference point.

3.3 The lead of oil returns on the RND, and beliefs and preferences

We now examine the state dependent lead of oil returns on the RND and beliefs and preferences via threshold regression (Equation 4). The first system of equations regresses a 3×1 vector of monthly LR RND moments (in Figure 1), against lagged oil returns and macro-economic controls. The second system employs the same regressors, with the 9×1 dependent variable vector consisting of three monthly LR physical PDF moments (in Figure 2) and six LR PK slopes (LR components of [-20%,-18%] and [-18%,-16%] in Figure 4, plus LR components of the first four principal components estimated across the 18 return intervals from -16% to 20%). Section 3.3.1 describes the independent variables and section 3.3.2 considers model selection. Section 3.3.3 presents the RND regression which is followed by the beliefs and preferences regression in Section 3.3.4. Section 3.3.5 examines possible economic mechanisms via expected cashflow and discount rate effects.

3.3.1 Independent variables

Table 4 summarises the eight independent variables (oil returns plus 7 macro-economic controls). All regressions include three lags of *all* variables. The table also reports data sources and supporting literature.[19] Growth rates using real time data are calculated

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[19] Much of this literature examines links between the macro-economy and equity returns, which is relevant given that relationships between returns and higher order moments also apply to RN measures (Conrad et al., 2013)

using the first information release.[20] The 10 year interest rate is the average daily change over the month. Oil returns $r_{c,t}$ are the average daily continuously compounded return for the month.

Table 4: Independent variables

Variable	Frequency	X_t/V_t	References
Real Time Data: Federal Reserve Bank Philadelphia			
CPI	Monthly	AGR(%)	Schotman and Schweitzer (2000)
Unemployment	Monthly	MGR(%)	McQueen and Roley (1993); Boyd et al. (2005)
Ind. production	Monthly	AGR(%)	Chen et al. (1986)
House starts	Monthly	MGR(%)	Estrella and Mishkin (1998)
Datastream			
CME oil futures	Daily	$\text{Avg}[\ln \frac{P_t}{P_{t-1}} = r_{c,t}]$	
US 10-yr T-bond	Daily	Avg Δ	Chen et al. (1986)
Constructed time series			
1st PC of vol	Monthly	-	Engle and Rangel (2008); Engle et al. (2013)
2nd PC of vol	Monthly	-	Gennotte and Marsh (1993)

All regressions include 3 lags of all of the above variables. AGR denotes the annual growth rate, MGR denotes the monthly growth rate. Continuously compounded returns are multiplied by 100. Avg denotes the average daily value over the month, 1st (2nd) PC of vol denotes the 1st (2nd) principal component of Riskmetrics volatility estimates. Oil futures link the nearby contracts and 10 year T bonds are at constant maturity.

Given that fluctuations in economic variables increase future cashflow uncertainty, risk premia (Gennotte and Marsh, 1993) and LR equity volatility (Engle and Rangel, 2008; Engle et al., 2013), two of the eight independent variables represent macro-variable volatility. Monthly volatility for each of the six regressors is initially calculated via Riskmetrics, where $\sigma_t^2 = 0.97\sigma_{t-1}^2 + 0.03\zeta_{t-1}^2$, and ζ_t denotes the monthly value in Table 4. Volatility correlations are all positive and average 0.49. In the interests of parsimony and tractability, macro-variable volatility is captured via the first two principal components of the volatility correlation matrix (which explain 82.5% of the variation in the data).

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[20] All releases have a one month lag and so an October growth rate for example is the November regressor at time t. CPI and industrial production growth rates are annualised, but house starts and unemployment growth rates are monthly. For the CPI the first release is employed, which commences in October 1998. Annual growth rates prior to this date are calculated manually using the same methodology $AGR_t = \left[(CPI_t/CPI_{t-1})^{12} - 1\right] \times 100$. Unemployment and housing start monthly growth rates are calculated via $MGR_t = [(X_t/X_{t-1}) - 1] \times 100$, where X_t denotes the 1st release. Monthly growth rates were used given that annualised rates were noisy. Further details can be found at https://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/.

3.3.2 Model selection

A large number of models are estimated to determine the most appropriate specification. Both systems of equations consider three alternative oil return measures: i) the average daily oil return for the month $r_{c,t}$; ii) separate effects for positive and negative oil returns via a slope dummy (I_t), where $I_t = 1$ when $r_{c,t} > 0$ otherwise zero (denoted r_c^+ and r_c^-); and iii) positive oil returns only via $r_{c,t}^+ = max(0, r_{c,t})$. Four threshold variables as well as their LR components are employed: i) the investor sentiment index of Baker and Wurgler (2006); ii) crude return volatility; iii) the CME Volatility index (VIX);[21] and iv) the annualised monthly growth rate in industrial production (IP). All models consider one or two breakpoints, threshold variable delays of one or two months and three lags ($q = p = 3$). Oil is the only variable subject to switching ($l = 1$).

The Schwarz information criterion (SIC) for all RND models is reported in Table 5, where the best fit of 686.7 is underlined. Similar results apply for the physical PDF and PK regressions. The following conclusions can be drawn. First, the best fit is obtained using the LR VIX as the threshold. This holds irrespective of the oil return specification, number of break points or the threshold delay. Industrial production is negatively correlated with the VIX (-0.5), and so high levels of volatility are only mildly correlated with low levels of economic activity. The state-dependent lead of oil is therefore more influenced by expected equity volatility, than the state of the economy, investor sentiment or crude volatility.

Second, models with oil price rises ($r_{c,t}^+$) provide a much better fit than otherwise equivalent models using returns ($r_{c,t}$) or returns with a slope dummy ($I_t = 1$ when $r_{c,t} > 0$ otherwise zero). This dominance applies across all threshold variables. The SIC for models using returns with a slope dummy is consistently the highest, providing clear evidence of model over-parameterisation. Most of the predictive power therefore comes

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[21] The VIX has a correlation of 0.98 with our RN volatility measure and so the model is similar to a self-exciting threshold regression.

Table 5: SIC for alternative RND threshold regressions

	LR threshold variable				Unfiltered threshold variable			
	Delay 1 month		Delay 2 months		Delay 1 month		Delay 2 months	
	1 break	2 breaks	1 break	2 breaks	1 break	2 breaks	1 break	2 breaks
r_c								
BW sentiment	765.3	780.8	765.2	791.4	772.5	805.1	775.9	805.4
Crude volatility	782.5	808.1	784.0	812.1	784.1	815.7	784.2	816.1
VIX	769.7	806.1	771.2	806.9	776.5	806.5	770.5	806.5
Industrial production	786.8	809.1	788.2	812.7	782.9	819.0	783.5	816.5
$r_c^+ and\ r_c^-$								
BW sentiment	807.9	861.9	802.6	855.3	833.9	898.4	838.0	904.0
Crude volatility	793.1	858.1	799.9	869.9	795.6	855.5	794.0	855.3
VIX	746.3	781.1	751.2	783.2	763.5	840.1	765.2	835.8
Industrial production	864.2	914.3	868.3	915.6	871.1	950.3	864.0	939.3
r_c^+								
BW sentiment	715.3	744.5	709.7	731.8	741.2	759.2	745.4	768.5
Crude volatility	717.1	743.8	720.1	752.7	721.6	736.2	716.0	736.9
VIX	686.7	702.0	690.5	706.4	698.7	735.5	698.9	724.8
Industrial production	776.7	788.7	780.0	799.0	779.1	817.5	774.9	808.1

Reports the SIC for each threshold regression, where the best result of 686.7 is underlined. BW sentiment is the investor sentiment index of Baker and Wurgler (2006), crude volatility the Risk Metrics measure calculated in Table 4, VIX the CBOE Volatility Index, and Industrial production the annualised monthly value. LR threshold variable results use long run components of the threshold from an Ad-ARFIMA process. Unfiltered threshold variable results use the original (raw) series as the threshold. All regressions employ three lags of all independent variables: CPI, industrial production, unemployment, house starts, change in 10 year T bonds, macro-variable volatility (proxied by the first two principle components of the volatility correlation matrix) and oil. The first panel regresses against lagged oil returns via $r_{c,t}$. The second panel (r_c^+ and r_c^-) regresses against lagged oil returns with a dummy $I_t = 1$ when $r_{c,t} > 0$ otherwise zero. The third panel regresses against lagged oil price rises via $r_{c,t}^+ = max(0, r_{c,t})$.

from oil price rises with falls having little effect. This is consistent with Hamilton (1988), Hamilton (2003), Bernanke et al. (1999), Balke et al. (2002) and Jones and Kaul (1996) as discussed above.

Third, a delay of one is usually slightly better than a delay of two, however the differences are small. This is particularly the case for thresholds based on the LR component due to their smooth nature.

Fourth, LR components of threshold variables have better explanatory power than their unfiltered counterparts. This suggests regimes are persistent and is consistent with Markov switching models typically inferring regimes via filtered or smoothed probabilities (as ex-ante probabilities are often noisy).

Finally, models with one breakpoint dominate models with two. For a given threshold variable, models with one or two breakpoints identify a similar γ_1. The additional regime in the two breakpoint models (where $\gamma_1 \leq h_{t-1} < \gamma_2$) often had no significant relation to oil, was short in duration and varied considerably between models. These findings along with the SIC, suggest that γ_1 is well identified, and models with two breakpoints are over-parameterised.[22] The remaining analysis therefore uses one break point, oil price rises ($r^+_{c,t}$), and the LR VIX as the threshold variable delayed one period.

3.3.3 RND moment regression

We now examine the lead of monthly oil returns on LR RN moments. Given the focus on oil, it is the only variable subject to switching ($l = 1$) and only oil coefficients and model diagnostics are presented. Results in Table 6 are in four panels. The first and second panels are benchmark models without threshold effects for oil price rises ($r^+_{c,t}$) and returns ($r_{c,t}$). The third and fourth panels report threshold regressions using oil price rises ($r^+_{c,t}$) and returns ($r_{c,t}$). Models in panels 3 and 4 have an SIC of 686.7 and 769.7 respectively (as reported in Table 5). Regressions against returns with a slope

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[22] Nonetheless, regime 2 results with one break also mirror results for regime 3 with two breaks.

dummy are not reported as they generally have the highest SIC.

Without threshold effects, oil contains no predictive information. None of the oil coefficients in panels 1 or 2 are statistically significant, and partial R^2 results for oil are low, ranging from 0.3% to 2%.

On using price rises ($r^{+}_{c,t}$), the Tsay (1998) statistic in the third panel strongly rejects the null of no threshold effects. This is supported by the threshold regression: i) on allowing for a breakpoint at $\gamma_1 = 18.1$, the adjusted R^2 increased from 42.1% to 70.0% for volatility, 37.7% to 60.0% for skewness and 30.6% to 65.6% for kurtosis; ii) the partial R^2 for oil increased from 2.0% to 41.4% for volatility, 0.3% to 37.2% for skewness and 1.7% to 43.9% for kurtosis; and iii) likelihood ratio tests for each equation reject the null of common oil coefficients for each lag across regimes.

The fourth panel using oil returns reports similar though weaker findings. Coefficient magnitudes are smaller and less significant, and adjusted and partial R^2 statistics are much lower. The threshold parameter is similar ($\gamma_1 = 19.0$ versus $\gamma_1 = 18.1$) and likelihood ratio and Tsay (1998) statistics support a structural break. These weaker results provide further evidence of oil price rises providing more significant predictive information than returns.

Conditions in each regime are depicted in Figure 5. Regime 1 is highlighted against plots of the S&P500 index (Fig 5a), S&P500 returns (Fig 5b) and US real GDP growth (Fig 5c). Regime 1 occurs over most of 1996 as well as January 2004 to September 2007. This latter period experienced steadily rising crude oil prices driven by unexpected increases in world oil consumption (Kilian and Murphy, 2014).[23] Regime 1 therefore generally experienced steadily rising equity and oil prices, low equity market volatility and solid economic growth. Regime 2 in contrast, has higher levels of equity volatility, rising and falling oil and equity prices, and considerable variation in economic activity.

Oil price rises have opposite effects in each regime, which is why models without break

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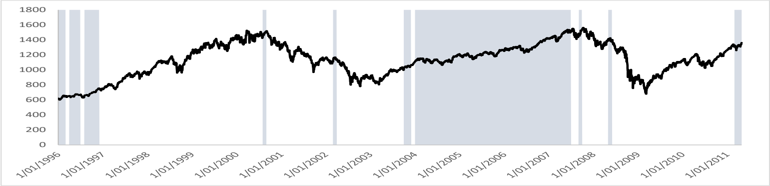
[23] Crude futures at January 5, 2004 were 33.78USD and reached 81.66USD by 28 September, 2007.

Table 6: RND regression

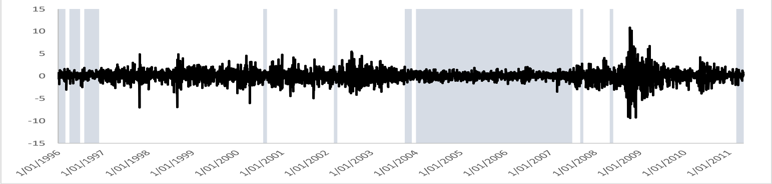
	Volatility		Skewness		Kurtosis	
	coeff	std err	coeff	std err	coeff	std err
No threshold: Price rises						
$r^+_{c,t-1}$	0.0057	0.0038	0.0521	0.0956	-1.1121	0.6974
$r^+_{c,t-2}$	0.0034	0.0037	-0.0072	0.0996	-0.7074	0.8101
$r^+_{c,t-3}$	0.0019	0.0039	-0.0662	0.0942	-0.2941	0.8257
Adj R^2	42.1%		37.7%		30.6%	
Partial R^2	2.0%		0.3%		1.7%	
No threshold: Returns						
$r_{c,t-1}$	0.0014	0.0024	0.0322	0.0561	-0.4261	0.47437
$r_{c,t-2}$	0.0022	0.0020	0.0443	0.0553	-0.4075	0.48445
$r_{c,t-3}$	0.0020	0.0022	0.0205	0.0529	-0.2028	0.54935
Adj R^2	41.6%		37.8%		30.1%	
Partial R^2	1.1%		0.6%		1.0%	
Threshold: Price rises						
Regime 1: (low volatility: $\gamma_1 < 18.1$)						
$r^+_{c,t-1}$	-0.0191^{***}	0.0070	-0.5222^{***}	0.1820	4.2513^{***}	1.2114
$r^+_{c,t-2}$	-0.0218^{***}	0.0043	-0.6576^{***}	0.1040	5.0592^{***}	1.0287
$r^+_{c,t-3}$	-0.0225^{***}	0.0067	-0.6903^{***}	0.1577	5.2315^{***}	1.1031
Regime 2 (high volatility: $\gamma_1 \geq 18.1$)						
$r^+_{c,t-1}$	0.0080^{**}	0.0034	0.1016	0.0874	-1.5959^{**}	0.6862
$r^+_{c,t-2}$	0.0071^{**}	0.0029	0.0873	0.0824	-1.5420^{**}	0.6635
$r^+_{c,t-3}$	0.0071^{**}	0.0030	0.0639	0.0890	-1.4655^{**}	0.6300
Adj R^2	70.0%		60.0%		65.6%	
Partial R^2	41.4%		37.2%		43.9%	
-Regime 1	54.0%		52.0%		66.2%	
-Regime 2	32.9%		24.4%		27.8%	
LR	94.6^{***}		85.1^{***}		103.3^{***}	
Tsay	284.7^{***}					
Threshold: Returns						
Regime 1: (low volatility: $\gamma_1 < 19.0$)						
$r_{c,t-1}$	-0.0066	0.0059	-0.1295	0.1319	1.7098^{*}	0.9571
$r_{c,t-2}$	-0.0056	0.0064	-0.1312	0.1547	1.5040	1.0057
$r_{c,t-3}$	-0.0113^{**}	0.0043	-0.2932^{**}	0.1203	3.2021^{***}	1.0959
Regime 2 (high volatility: $\gamma_1 \geq 19.0$)						
$r_{c,t-1}$	0.0033	0.0028	0.0700	0.0608	-0.9393^{*}	0.5070
$r_{c,t-2}$	0.0026	0.0022	0.0542	0.0575	-0.5022	0.5134
$r_{c,t-3}$	0.0041^{*}	0.0022	0.0718	0.0567	-0.7614	0.5518
Adj R^2	45.1%		40.5%		36.6%	
Partial R^2	8.8%		6.6%		11.9%	
-Regime 1	6.1%		5.4%		10.8%	
-Regime 2	11.2%		8.0%		12.8%	
LR	14.9^{***}		11.5^{***}		21.46^{***}	
Tsay	235.9^{***}					

Results are for the regression of a 3×1 vector of the LR components of the RND moments against returns ($r_{c,t}$) or price rises ($r^+_{c,t} = max(0, r_{c,t})$), where $r_{c,t}$ is the average daily crude return for the month. Regressions include three lags of the following macroeconomic controls: CPI, industrial production, unemployment, house starts, change in 10 year T bonds and macro-variable volatility. The partial R^2 is for oil. LR is an equation by equation likelihood ratio test for the equality of the crude oil coefficients for each lag across the two regimes. Tsay is the test statistic of Tsay (1998), which has a null of no structural break. The identified regimes using $r^+_{c,t}$ are depicted in Figure 5. ***,**,* denotes significance at the 1%, 5% and 10% levels respectively.

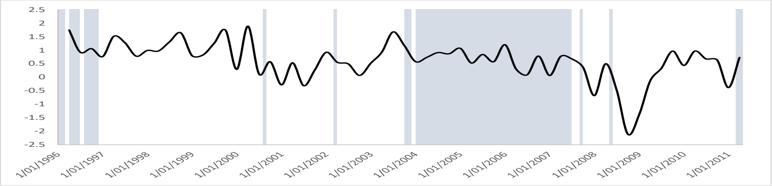
Figure 5: Regimes identified by RND regression



(a) S&P500



(b) S&P500 returns



(c) Real US GDP growth (%)

points found no relation between oil and the RND. In the low volatility regime, higher oil prices lead lower LR RN volatility, more negative LR skewness and higher LR RN kurtosis. Crude oil price rises had the opposite effect in the high volatility regime, leading higher LR RN volatility and lower LR RN kurtosis. Oil also had higher explanatory power in the low volatility regime, with larger coefficient estimates (in absolute terms) and partial R^2 values.

Given the PK explains most of the variation in RN skewness and kurtosis, higher oil prices leading more negative LR skewness and higher LR kurtosis, suggests an increase in LR risk aversion when volatility is low. The opposite occurred in the high volatility regime, suggesting higher oil prices lead decreased LR risk aversion. A proper investigation requires disentangling the RND into beliefs and preferences, and so this is left for section 3.3.4 below.

Robustness checks found that oil coefficients and standard errors were stable in the absence of macro-economic controls, or when dropping them sequentially via alternative model reduction paths. Unfiltered RN moments as dependent variables provided similar (though weaker) relations in the low volatility regime, but there was no lead of oil price rises in the high volatility regime. This is consistent with the higher predictive power of oil in the low volatility regime, and emphasises the importance of filtering out SR dynamics.

3.3.4 Beliefs and preferences regression

This section examines the lead of oil price rises on long run beliefs and preferences. Equations mirror the RND regression, with positive oil returns ($r^+_{c,t}$) the only variable subject to switching ($l = 1$), the same controls (US CPI, unemployment rate, industrial production, house starts, 10 year T bond rate, and macro variable volatility), lag lengths ($q = p = 3$), number of breakpoints (one) and delay parameter (one). Given most variation in the PK occurs from -16% to -20%, only results for PK slopes over [-20%,-18%] and [-18%,-16%] are reported.

Results in Table 7 are consistent with the RND. Without threshold effects there is little evidence supporting the lead of oil price rises on LR physical PDF moments or preferences. Only 4 out of 27 oil coefficients are statistically significant (3 only at 10%), and the partial R^2 for oil ranges from 0.2% (kurtosis) to 5.1% (volatility).

The Tsay (1998) statistic rejects the null of no breakpoints. This is supported by the threshold regression: i) the breakpoint ($\gamma_1 = 19.3$) is similar to the RND regression ($\gamma_1 = 18.1$); ii) with the exception of kurtosis (where all coefficients are not significant), likelihood ratio tests of common oil coefficients for each lag across regimes are rejected; and iii) threshold effects increase the average partial R^2 for oil (across the 9 equations) from 2.4% to 56.9%.

In the low volatility state, oil price increases lead expectations of lower LR volatility but more positive LR skewness. The lead of oil price rises on LR volatility expectations

Table 7: Beliefs and preferences regression

	Beliefs (Physical PDF)						Preferences (Pricing Kernel)			
	Volatility		Skewness		Kurtosis		[-20%,-18%]		[-18%,-16%]	
	coeff	std err	coeff	std err	coeff	std err	coeff	std err	coeff	std err
No threshold: Price rises										
$r^+_{c,t-1}$	0.0065^{**}	0.0033	-0.0165	0.0130	0.0118	0.0339	13.7297^{*}	8.2448	4.5196	3.5801
$r^+_{c,t-2}$	0.0064^{*}	0.0034	-0.0168	0.0119	-0.0095	0.0300	7.8126	8.3465	6.0002^{*}	3.4217
$r^+_{c,t-3}$	0.0051	0.0037	-0.0136	0.0115	-0.0160	0.0277	5.3619	8.6212	5.7739	3.6093
Adj R^2	44.3%		53.1%		69.6%		60.7%		20.6%	
Partial R^2	5.1%		2.9%		0.2%		2.5%		3.2%	
Threshold: Price rises										
Regime 1: (low volatility: $\gamma_1 < 19.3$)										
$r^+_{c,t-1}$	-0.0106^{**}	0.0043	0.0314^{**}	0.0155	-0.0345	0.0473	-19.0239	12.7606	-6.5300	4.4502
$r^+_{c,t-2}$	-0.0108^{***}	0.0039	0.0219	0.0172	-0.0226	0.0425	-24.3678^{**}	9.4771	-0.2149	4.4303
$r^+_{c,t-3}$	-0.0166^{***}	0.0046	0.0426^{***}	0.0148	-0.0841	0.0494	-35.4291^{**}	15.9332	1.1510	5.7449
Regime 2: (high volatility: $\gamma_1 \geq 19.3$)										
$r^+_{c,t-1}$	0.0122^{***}	0.0032	-0.0329^{**}	0.0150	0.0285	0.0403	24.5457^{***}	8.0158	8.5784^{*}	4.4857
$r^+_{c,t-2}$	0.0102^{***}	0.0033	-0.0243^{*}	0.0124	-0.0132	0.0349	14.9589^{*}	9.1415	7.5310^{**}	3.8019
$r^+_{c,t-3}$	0.0090^{***}	0.0027	-0.0236^{**}	0.0103	-0.0008	0.0329	12.5958^{*}	7.8124	5.7041^{*}	3.3701
Adj R^2	68.4%		63.9%		70.0%		71.1%		24.0%	
Partial R^2	47.2%		26.7%		3.1%		29.6%		9.1%	
-Regime 1	50.9%		30.6%		4.4%		25.7%		5.8%	
-Regime 2	44.5%		23.9%		2.1%		32.3%		16.8%	
LR	117.6^{***}		57.2^{***}		5.7		64.5^{***}		17.6^{***}	
Tsay	772.8^{***}									

The regressand is a 9×1 vector consisting of three LR physical PDF moments (volatility, skewness and kurtosis) and LR preferences proxied by six PK slopes over [-20%,-18%] and [-18%,-16%] plus the first four principal components of the remaining intervals from [-16%,20%] (which are unreported). Regressors include three lags of $r^+_{c,t} = max(0, r_{c,t})$ plus three lags of the following macroeconomic controls: CPI, industrial production, unemployment, house starts, change in 10 year T bonds and macro-variable volatility. The partial R^2 is for $r^+_{c,t}$. LR is an equation by equation likelihood ratio test for the equality of oil coefficients across the two regimes. Tsay is the test statistic of Tsay (1998), which has a null of no structural break. The low volatility period is from January 1996 to February 1997, February 2004 to August 2007, and April 2011 a total of 58 monthly observations. The high volatility period is from March 1997 to January 2004 and September 2007 to March 2011, the remaining 126 observations. ***,**,* denotes significance at the 1%, 5% and 10% levels respectively.

is consistent with RN volatility in Table 6, which is unsurprising given their similarities. However, consistent with the macro-announcement effects in Beber and Brandt (2006), oil coefficients for expected and RN skewness have opposite signs. This can be explained by the LR pricing kernel. Oil price rises lead more negative slopes in the left tail of the PK, consistent with higher LR risk aversion. This effect completely dominates increases in expected LR skewness, and so the net effect in Table 6 is a decrease in LR RN skewness.

In the high volatility state, oil price rises had the opposite effect, leading higher expected LR volatility and more negative expected LR skewness. Results for RN and expected volatility are consistent with Aloui and Jammazi (2009) and Jammazi and Aloui (2010), who find oil price rises increase *ex-post* equity volatility in high volatility states, but decrease *ex-post* equity volatility in low volatility states. Results also find that higher oil prices lead more positive slopes in the left tail of the PK reflecting lower LR risk aversion. This effect is offset by the more negative expected LR skewness, and explains why there was no lead of oil on LR RN skewness in Table 6.[24]

Finally, robustness checks find oil coefficients and standard errors stable in the absence of controls, or when dropping them sequentially via alternative model reduction paths. Unfiltered dependent variables also provided similar (though weaker) findings.[25]

3.3.5 Interpretation

Findings have similarities with McQueen and Roley (1993) and Boyd et al. (2005), who show the same news can have different effects on equities depending on the state of the economy. In a strong economy the increase in discount rates from good news, dominates

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[24] For the rest of the PK, only oil coefficients on the 2nd principle component were significant. This component was highly correlated with the PK from [-16%,-10%]. The significant negative slopes for the 2nd principle component in regime 1 and positive slopes in regime 2, further support oil price rises leading increases (decreases) in LR risk aversion in the low (high) volatility state.

[25] Additional robustness checks also represented preferences via the first four principal components of the correlation matrix across *all* PK slopes. Findings remain unchanged and so the paper focuses on slopes over [-20%,-18%] and [-18%,-16%] for ease of interpretation.

the increase in expected cashflows due to capacity constraints, and stock prices decrease. The opposite occurs in a weak economy, where stock prices rise in response to good news, because the expected cashflow increase dominates the discount rate increase.

The state dependent relation in this paper is a function of equity market volatility, which is only *weakly* related to GDP (see section 3.3.3), and so capacity constraints seem unlikely. One possibility is over the LR, oil price rises from a strengthening global economy contain predictive information beyond that contained in macro-economic aggregates, bundling the effects of: i) increased cashflows and possibly higher discount rates (from central banks combating demand pull inflation) with ii) a contractionary effect due to rising input costs reducing the disposable income of consumers. This will decrease expected cashflows and possibly raise discount rates (due to central banks combating cost push inflation). If the relative importance of these effects is state dependent, the lead of oil price rises will also be state dependent.

To test this conjecture, cashflow and discount rate proxies are regressed against lagged oil price rises. The S&P500 log dividend growth rate is the main cashflow proxy, because it is a managed value tied to expectations regarding long-run profitability (Hecht and Vuolteenaho, 2006). Regressions also consider the LR log dividend growth rate from an Ad-ARFIMA(1,d,1) process.[26] Following McQueen and Roley (1993) and Boyd et al. (2005), the change in the 10 year US T bond rate, change in the 3 month US T bill rate, and default spread between Baa and Aaa corporate bonds are the discount rate proxies.

To allow for state dependence we estimate single equation threshold regression. Given that the LR VIX with a delay of one provided the best fit in sections 3.3.3 and 3.3.4, it is used as the threshold variable h_t. For observations in regime $c = 0, 1, ..., g$

$$\mu_{1,t} = X_{t-1}^{'}\beta + V_{t-1}^{'}\delta_c + \epsilon_{1,t} \tag{17}$$

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[26] Growth in industrial production is also considered and provides similar, though slightly weaker findings.The 12 month inflation adjusted real dividend per share is sourced from www.multpl.com.

where $\mu_{1,t}$ and $\epsilon_{1,t}$ are scalars and the remaining variables and lags are the same as sections 3.3.3 and 3.3.4 (i.e 3 lags of all the variables in Table 4). For the cashflow regressions, the same variables are then used as regressors against cashflow proxies for one (t+1), two (t+2) and three months ahead (t+3).

Table 8: Cashflow proxy regression results

	Same month		1 month ahead		2 months ahead		3 months ahead	
	coeff	std err	coeff	std err	coeff	std err	coeff	std err
No threshold								
LR log dividend growth rate								
$r^{+}_{c,t-1}$	-0.0016	0.0015	-0.0019	0.0015	-0.0022	0.0015	-0.0022	0.0016
$r^{+}_{c,t-2}$	-0.0021	0.0015	-0.0022	0.0015	-0.0022	0.0016	-0.0022	0.0016
$r^{+}_{c,t-3}$	-0.0022	0.0017	-0.0020	0.0017	-0.0021	0.0018	-0.0021	0.0018
Adjusted R^2	49%		47%		46%		44%	
Threshold								
LR log dividend growth rate								
Regime 1 (low volatility)								
$r^{+}_{c,t-1}$	0.0043^{**}	0.0018	0.0045^{**}	0.0018	0.0045^{**}	0.0019	0.0049^{**}	0.0021
$r^{+}_{c,t-2}$	0.0034^{*}	0.0020	0.0033	0.0021	0.0037^{*}	0.0022	0.0037^{*}	0.0021
$r^{+}_{c,t-3}$	0.0033^{**}	0.0015	0.0038^{**}	0.0016	0.0039^{**}	0.0016	0.0046^{**}	0.0019
Regime 2 (high volatility)								
$r^{+}_{c,t-1}$	-0.0030^{**}	0.0015	-0.0041^{***}	0.0017	-0.0038^{**}	0.0016	-0.0035^{**}	0.0016
$r^{+}_{c,t-2}$	-0.0040^{**}	0.0017	-0.0033^{**}	0.0016	-0.0030^{*}	0.0017	-0.0031^{*}	0.0017
$r^{+}_{c,t-3}$	-0.0043^{***}	0.0015	-0.0037^{**}	0.0016	-0.0043^{***}	0.0016	-0.0045^{***}	0.0017
Adjusted R^2	62%		62%		61%		61%	
γ_1	23.23		23.25		23.23		23.24	
Log dividend growth rate								
Regime 1 (low volatility)								
$r^{+}_{c,t-1}$	-0.0005	0.0021	0.0025	0.0021	0.0020	0.0020	0.0056^{**}	0.0025
$r^{+}_{c,t-2}$	0.0012	0.0021	0.0022	0.0020	0.0038^{*}	0.0023	0.0051^{*}	0.0028
$r^{+}_{c,t-3}$	0.0025	0.0020	0.0040^{*}	0.0023	0.0062^{**}	0.0027	0.0043^{*}	0.0024
Regime 2 (high volatility)								
$r^{+}_{c,t-1}$	-0.0097^{***}	0.0020	-0.0028	0.0019	-0.0028	0.0018	-0.0024	0.0017
$r^{+}_{c,t-2}$	-0.0043^{**}	0.0020	-0.0053^{**}	0.0023	-0.0025	0.0017	-0.0048^{**}	0.0021
$r^{+}_{c,t-3}$	-0.0051^{**}	0.0023	-0.0054^{**}	0.0023	-0.0064^{**}	0.0025	-0.0063^{***}	0.0019
Adjusted R^2	53%		51%		55%		58%	
γ_1	23.23		23.23		23.25		23.23	

Results report regressions of cashflow proxies (log dividend growth rate and LR log dividend growth rate) against lagged $r^{+}_{c,t}$ plus three lags of all the macroeconomic controls in Table 4. See Equation 17 for the model specification. Results with no threshold effects using the log dividend growth rate are similar to the reported LR log dividend growth rate. ***,**,* denotes significance at the 1%, 5% and 10% levels respectively.

Table 8 reports results without threshold effects using the LR dividend growth rate.

Dividend growth rates without threshold effects are similar, revealing no lead of oil price rises on expected cashflows. Threshold regression however identifies $\gamma_1 = 23.2$ across all months and cashflow proxies. Oil coefficients are consistently positive in the low volatility state and negative in the high volatility state. Higher oil prices therefore lead increased cashflow expectations when volatility was low, and lower expected cashflows when volatility was high.

The same regressions were performed using discount rate proxies and their LR components. Interest rates can adjust instantly and so only regressions for the same month are performed. No threshold effects were identified using unfiltered discount rates or the LR component of the 10 year T-bond. Results for each of these threshold regressions are identical to results in Table 9 without threshold effects. When using unfiltered discount rate proxies, oil price increases have very little effect, with all but one coefficient statistically insignificant. The evidence supporting a positive response of discount rates to oil price increases is stronger using LR components of the 3 month T-bill rate and default spreads. Both threshold regressions identify a breakpoint, with a positive response to oil price rises in low volatility states. Overall though, results only provide weak evidence of a positive response of discount rates to oil price increases.

Findings therefore reveal that the same news (oil price rises from a strengthening global economy) have a state dependent lead on expected cashflows, but little to no effect on discount rates.[27] Oil price rises in the low volatility state, saw markets emphasise the strong economic outlook and expected cashflows increased. Equities increased because the cashflow effect dominated any increase in discount rates. This saw a decrease in LR volatility expectations given that good news (oil price rises signalling a stronger economy), in good times (or low volatility states), decreases uncertainty (Veronesi, 1999). The increase in equities and decrease in volatility, is also consistent with leverage and

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[27] This is consistent with McQueen and Roley (1993) who show that state dependence is due to changes in expected cashflows (with discount rates having little effect) and Jones and Kaul (1996) who show that US stock price movements can be accounted for by the impact of oil shocks on cashflows.

Table 9: Discount rate proxy regression results

	10 year T-bond		3 month T-bill		Default spread	
	coeff	std err	coeff	std err	coeff	std err
No threshold						
LR Component						
$r^+_{c,t-1}$	0.0010	0.0010	0.0055	0.0296	0.0831	0.0660
$r^+_{c,t-2}$	0.0002	0.0010	0.0304	0.0301	0.1395^{**}	0.0670
$r^+_{c,t-4}$	0.0003	0.0011	0.0596^{*}	0.0340	0.1115	0.0757
Adjusted R^2	21%		43%		57%	
Unfiltered						
$r^+_{c,t-1}$	0.0031	0.0042	-0.0297	0.0544	0.0382	0.0778
$r^+_{c,t-2}$	0.0071^{**}	0.0034	0.0584	0.0553	0.0532	0.0790
$r^+_{c,t-3}$	-0.0048	0.0040	0.0548	0.0625	0.1240	0.0893
Adjusted R^2	2%		26%		70%	
Threshold						
LR Component						
Regime 1 (low volatility)						
$r^+_{c,t-1}$	0.0010	0.0010	0.1011^{*}	0.0604	0.1942^{**}	0.0761
$r^+_{c,t-2}$	0.0002	0.0010	0.2128^{***}	0.0564	0.2357^{***}	0.0760
$r^+_{c,t-3}$	0.0003	0.0011	0.2516^{***}	0.0623	0.1463^{*}	0.0822
Regime 2 (high volatility)						
$r^+_{c,t-1}$	–	–	-0.0054	0.0273	-0.0780	0.1026
$r^+_{c,t-2}$	–	–	0.0035	0.0280	-0.0085	0.1036
$r^+_{c,t-3}$	–	–	0.0174	0.0304	0.0291	0.1111
Adjusted R^2	21%		57%		60%	
γ_1	-		16.10		25.95	

Results report regressions of discount rate proxies (change in the 10 year US T bond rate, change in the 3 month US T bill rate, and default spread between Baa and Aaa corporate bonds) against lagged $r^+_{c,t}$ plus 3 lags of all the macroeconomic controls in Table 4. See Equation 17 for the model specification. When using the unfiltered regressands and the LR component of the 10 year T-bond, no threshold effects were identified. Threshold regression results are therefore identical to the no threshold results. ***,**,* denotes significance at the 1%, 5% and 10% levels respectively.

volatility feedback effects.

In the high volatility state, the effects of rising input costs and a decrease in consumer disposable income dominated the market's response. Despite the strengthening global economy, higher oil prices lead decreases in expected cashflows (from the decrease in consumer income) and there was little if any discount rate effect. Consistent with Veronesi (1999), good news (rising oil prices from a strengthening economy) in bad times (or high volatility states) increased uncertainty.

4 Conclusion

This paper documents a state-dependent lead of oil price increases on long run US equity market beliefs and preferences. When S&P500 volatility was low, higher oil prices from a strengthening global economy, preceded a rise in expected cashflows and equity prices, a fall in LR expected volatility and an increase in LR risk aversion. The opposite occurred in the high volatility state, where oil price rises saw markets place more emphasis on the inflationary pressures from higher input costs and the reduction in the disposable income of consumers. Higher oil prices preceded a decrease in expected cashflows and LR risk aversion and higher levels of LR expected volatility. Given opposite reactions in each volatility state, the predictive information contained in oil price rises was masked in the absence of state-dependent parameters.

Findings suggest many avenues for further research. A state-dependent structural model linking economic activity, crude oil prices, and equity and option markets may provide further insights. Extending the model to include oil supply and speculative oil demand would enable an examination into the short and long run effects of crude oil prices. Finally, given that dynamic expectations of higher order moments improve investment outcomes (Jondeau and Rockinger, 2012), the implications of oil price rises on long run asset allocation in a state-dependent framework may be interesting.

Data Availability Statement

The data that support the findings of this study are available from Datastream and OptionMetrics. Restrictions apply to the availability of these data, which were used under license for this study. Data are available at https://melbourne.figshare.com/articles/dataset/Data_xls/14552562 with the permission from both data providers.

A Appendix A: Physical PDF and Ad-ARFIMA estimation

The AR(1)-FIEGARCH (1,d,1) process provides a good fit for the conditional mean and volatility and so this part of the Physical PDF model was held constant. The following conditional skewness and kurtosis specifications were considered

- Model 1: $\varphi_t^* = \varphi_0 + \varphi_1 \epsilon_{t-1} + \varphi_2 \varphi_{t-1}^*$, $\upsilon_t^* = \upsilon_0 + \upsilon_1 \epsilon_{t-1}^4 + \upsilon_2 \upsilon_{t-1}^*$
- Model 2: $\varphi_t^* = \varphi_0 + \varphi_1 \epsilon_{t-1}^3 + \varphi_2 \varphi_{t-1}^*$, $\upsilon_t^* = \upsilon_0 + \upsilon_1 \epsilon_{t-1}^4 + \upsilon_2 \upsilon_{t-1}^*$
- Model 3: $\varphi_t^* = \varphi_0 + \varphi_1 z_{t-1} + \varphi_2 \varphi_{t-1}^*$, $\upsilon_t^* = \upsilon_0 + \upsilon_1 z_{t-1}^4 + \upsilon_2 \upsilon_{t-1}^*$
- Model 4: $\varphi_t^* = \varphi_0 + \varphi_1 z_{t-1}^3 + \varphi_2 \varphi_{t-1}^*$, $\upsilon_t^* = \upsilon_0 + \upsilon_1 z_{t-1}^4 + \upsilon_2 \upsilon_{t-1}^*$.

Parameter estimates for the selected model (Model 1) are reported in Table A.1. Estimates are consistent with previous studies that document autoregressive return dynamics and asymmetric fractionally integrated volatility (Bollerslev and Mikkelsen, 1996). Diagnostic tests are reasonable and the evidence supporting time varying skewness is strong with the φ_1 and φ_2 coefficients statistically significant. The evidence supporting time varying degrees of freedom is mixed with the t statistics for υ_1 and υ_2 insignificant, however restricting $\upsilon_1 = \upsilon_2 = 0$ provided no improvement in the Akaike Information Criterion (AIC). The physical PDF is simulated using the reported model, with similar results obtained using a model with constant degrees of freedom ($\upsilon_1 = \upsilon_2 = 0$).[28]

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[28] Unreported models with Gaussian and Skewed Student innovations with $\varphi_1 = \varphi_2 = \upsilon_1 = \upsilon_2 = 0$ (i.e constant skewness and degrees of freedom), did not provide as good a fit as assessed via the Akaike information criterion (AIC), Schwarz information criterion (SIC) and likelihood ratio tests.

Table A.1: AR(1)-FIEGARCH(1,d,1)-$SkSt(0,1,\varphi_t,\upsilon_t)$ estimates

	coeff	std err
ϱ_0	0.047^{***}	0.011
ϱ_1	0.063^{***}	0.002
ω	0.622	0.640
d	0.485^{***}	0.109
ϕ	-0.676^{***}	0.089
β	0.902^{***}	0.042
χ	-0.036^{***}	0.009
λ	0.146^{***}	0.009
φ_0	-0.408^{***}	0.059
φ_1	0.137^{***}	0.026
φ_2	0.362^{***}	0.086
υ_0	-1.039	1.022
υ_1	$2.9\times10^{-}5$	$2.3\times10^{-}5$
υ_2	0.482	0.504
Diagnostics		
LL	-5449.2	
Q(10)	7.175	
$Q^2(10)$	14.201	

Results from AR(1)-FIEGARCH(1,d,1)-$SkSt(0,1,\varphi_t,\upsilon_t)$ model fit to daily S&P500 returns from February 1, 1978 to December 29, 1995. LL is the log likelihood, $Q(10)$ and $Q^2(10)$ denotes Box-Pierce statistics for the 10th lag for z_t, and z_t^2 respectively, ***,**,* denotes significance at the 1%, 5% and 10% levels. The monthly regressor for the month was calculated as the average daily LR component.

Table A.2: Ad-ARFIMA(1,d,1,k) estimates: RND moments

	Volatility		Skewness		Kurtosis	
	coeff	std err	coeff	std err	coeff	std err
θ_1	0.989^{***}	0.001	0.385^{***}	0.158	0.992^{***}	0.004
θ_2	-1.003^{***}	0.000	-0.516^{***}	0.166	-0.979^{***}	0.006
d	0.762^{***}	0.019	0.371^{***}	0.046	0.312^{***}	0.020
σ_ε	0.082^{***}	0.002	0.876^{***}	0.050	0.559^{***}	0.010
ω	-1.483^{***}	0.052	-2.196^{***}	0.195	2.404^{***}	0.267
η_1	0.102	0.069	0.152	0.212	-0.140	0.260
η_2	-0.282^{***}	0.047	-0.439^{***}	0.134	0.371^{*}	0.195
η_3	0.013	0.042	0.239^{**}	0.106	-0.051	0.217
φ_1	0.106^{*}	0.059	-0.043	0.136	0.047	0.169
φ_2	-0.023	0.040	0.076	0.128		
φ_3	-0.214^{***}	0.045	-0.267^{**}	0.117		
Diagnostics						
LL	4196.1		-4964.0		-3230.2	
HQ	-2.163		2.585		1.684	
Q(10)	8.489		5.262		2.704	

Reports Ad-ARFIMA(1,d,1,k) models fit to daily 30 day ahead RN moments extracted via (Jiang and Tian, 2007). LL is the log likelihood, HQ is the Hannan-Quinn information criteria (used to determine k_i), and $Q(10)$ is Box-Pierce statistic for the 10th lag. ***,**,* denotes significance at 1%, 5% and 10% levels.

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