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Population dynamics modelling of child protection populations in Australia

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Abstract

Growing service demand is a perennial challenge for child protection systems in Australia, with the number of children involved in child protection (CP) investigations and admitted to out-of-home care (OOHC) increasing across Australian states and territories in the last decades. Since OOHC is costly and its effects on outcomes for children and youth are ambiguous, such a demand growth has attracted much scrutiny.

While factors influencing involvement in CP and placement in OOHC have been studied extensively, a review of existing research literature shows that no study has applied mathematical formulations of population dynamics to model CP population structures and trends. This study intends to fill the gap by asking the research question, “How can we use population dynamics modelling to study the macro-dynamical structures of CP populations across Australian states and territories?”

Two population dynamics models of a generic child protection system were constructed, one with four sub-populations and the other with three sub-populations. The models describe how the sub-populations evolve with time, simulate their interactions, and elucidate effects such as “churn” – repeated involvement with CP, both in terms of CP investigations and OOHC admissions. Analytical solutions of the system differential equations were obtained.

Applying the model solutions together with annual aggregate new and repeat client data from 2015-16 to 2017-18 to all Australian states and territories, this study obtained estimations of the investigation rates and OOHC admission rates for the sub-populations, trends of the sub-populations, as well as cross-sectional prevalence and lifetime prevalence of involvement in CP. The results show that both the investigation rates and OOHC admission rates for repeat clients were much higher than those for new clients across the states and territories. It was estimated that if the current rate scenarios persist, OOHC populations across the states and territories will approach asymptotic values significantly higher than the current values. How CP sub-populations response to changes in the CP process rates have also been investigated using hypothetical scenarios.

This study contributes to CP population research in three dimensions. Theoretically and methodologically, this is the first application of population dynamics models to study

the macro structure and trends of CP populations. Empirically, the models allow us to estimate CP prevalence and rates. Practically, the models allow us to estimate future trends of CP populations and to study how CP populations react to rate scenario changes which in turn, may result from practice or policy changes.

As in all modelling efforts, there are limitations and caveats. These include making the assumption that there is no system capacity limit in OOHC, thus making the models linear; not considering age structure; not considering socio-economic conditions and ethnic differences; and last but not least, having very limited data which are essential to such kind of modelling efforts. While future works have already been planned to address some of these limitations, any further progress will depend on the availability of high frequency data, from which conditions related to model limitations can be better understood and models can be better constrained.

Declaration

This is to certify that:

i. the thesis comprises only my original work toward the PhD;
ii. due acknowledgment has been made in the text to all other material used;
iii. the thesis is less than 100,000 words in length, exclusive of tables, maps, bibliographies and appendices.

Signed:

Wei Wu Tan

Acknowledgement

Some people say that doing a PhD is a lonely battle. I see it more like running a marathon. It is a race against oneself and the pace can vary from crawling like a snail to sprinting like a jaguar. It can be at once exasperating and exhilarating.

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Chapter 1: Introduction

Child maltreatment is a phenomenon that has been observed throughout human history across different societies. For example, early human literatures have shown that infanticide was a common practice for disposing of babies who were born with defects. In different cultures and countries in varying states of developments, children have been seen as the property of parents (Ten Bensel, Rheinberger, & Radbill, 1997). The concept of children being viewed as people in their own rights has only been developed in the recent past. The recognition of children's rights culminated in the adoption of the *United Nations Convention on the Rights of the Child* in 30 November 1989, which fully set out the safety, health, economic, social, cultural, civil and political rights of children (UN General Assembly, 1989).

The evolving conception of child rights and child protection went hand-in-hand with studies on the harms of child maltreatment. Research has firmly established that the repercussions of child maltreatment go beyond the immediate harms of abuse and neglect. In fact, child maltreatment could bring about long-lasting adverse outcomes for the victims, such as increased risks of behavioural, emotional, cognitive and social problems, the effects of which continue long into adulthood (Gilbert et al., 2009; Widom, 2014).

One of the means of addressing the problem of child maltreatment is to provide out-of-home care (OOHC) to children and young people who are at risk and deemed unsafe to live with their primary caregivers (Child Family Community Australia, 2018). As an important component of child protection (hereafter CP) systems in many western countries, OOHC is a state-mandated alternative not only to ensure the safety of children at risk, but ideally also to ensure their healthy development and wellbeing.

While the effect of OOHC to ensure the short-term safety of children at risk is self-evident, its causal effects on the wellbeing of children is hard to establish. This is mainly due to the difficulties of conducting studies which are free of selection bias, where subjects are truly comparable aside from the difference of OOHC and in-home care. Nevertheless, high quality existing research has shown that children placed into OOHC near the margin of placement threshold in terms of risk and safety assessments fared more poorly compared to those who were in similar situations but were not placed into OOHC (Doyle, 2007, 2013). Such outcomes have called into question the use of OOHC as an intervention for marginal cases.

In Australia, the per capita rate of children in OOHC has increased significantly over the last decades (Higgins, 2011; Steering Committee for the Review of Government Service Provision [SCRGSP], 2019), straining an already overwhelmed child welfare system. The growth in OOHC population has also rekindled the debate about the potentially detrimental effects of OOHC on children and their families (Maclean, Sims, O'Donnell, & Gilbert, 2016). This situation is particularly challenging for the Indigenous population, which is over-proportionately affected by these developments. The most recent data indicate that 37.3% of Indigenous children in Australia are in OOHC and that the relative risk of OOHC placement is 10.2 times higher for Indigenous children than for non-Indigenous children (Lewis et al., 2019).

While many studies have examined issues such as the effects, interventions, socio-economic factors of OOHC (Doyle, 2007; Forsman & Vinnerljung, 2012; Gilbert et al., 2009; Osborn, 2006), there have been few studies on the morphology and dynamics of CP sub-populations. To address this discrepancy, this work uses a population dynamics approach to study the macro-dynamical structure of OOHC in relation to other CP sub-populations.

This chapter provides an overview of the history, challenges, and practices of child protection in contemporary societies, focusing on the situations in Australia, to set the context for the research questions of the dissertation. It will also briefly describe a number of modelling approaches that are used to study population dynamics.

1.1 Child Protection in Contemporary Society

1.1.1 Brief history of child protection

Child maltreatment has existed throughout the ages and across civilizations. Infanticide was practiced in every ancient society with a recorded history. Throughout much of human civilisation, children have been regarded as the property of parents (Ten Bensel et al., 1997). For centuries, the use of severe physical punishment was commonly practiced, justified by the belief that such punishments were necessary to discipline and educate children, or even to expel evil spirits and appease deities (Radbill, 1974). In fact, what we understand today as essential practices of child welfare and protection should not be taken for granted. They have only come about after repeated critical reflections of human actions over the centuries.

Modernity and Conception of Child Rights

With the advent of modernity, the concept of human rights slowly developed and took root, eventually leading to the concept of child rights and legal protection of children. However, this process was long and has been evolving for hundreds of years. As early as the 1600s, there were already recorded cases of parents being prosecuted for "harming" their children. For examples, in 1642, there was a law in Massachusetts, USA that enabled the removal of children deemed not properly "trained up" by their parents (Myers, 2008); in 1672, a father in Connecticut, USA was executed after being found guilty of incestuous acts with his daughter (Tomison, 2001). However, in general, such examples did not reflect mature conception of child rights. At around the same time when there was the aforementioned provision for child removal in Massachusetts, the jurisdiction also adopted the Mosaic law which was based on biblical accounts and could impose the death penalty on unruly children (Tomison, 2001). In the Connecticut case, the child was blamed as a victim and was sentenced to a whipping, being perceived as having encouraged the abuse (Tomison, 2001).

It would take another two hundred years before the plight of maltreated children came to the fore of social conscious and child protection based on children's rights became a widespread concern to which significant societal and political efforts were devoted. Even in the early 1800s, it was still commonly accepted that parents, mainly fathers, had legitimate and almost total power over their children. Children had few or no legal rights, and were still viewed as the property of their parents (Stier, 1978). It was common for parents to treat their children as they wished. Corporal punishment was deemed appropriate universally, and child labour was rife.

First Wave of Child Rescue Movement

In the late 1800s, children began to be more widely seen not as the property of parents but as people in their own rights. The concept of children as a unique group of people who were vulnerable and need protection from parents, employers and others took shape. The state was then seen to have the responsibility to maintain the rights of children to protection and services (Alaimo, 2002).

1875 was often regarded as the year marking a new era of child protection (CP), often referred to as the “first wave of child rescue movement” (Liddell, 1993). This new era was heralded by a horrific case of abuse in New York City that aroused public outcry and widespread sympathy towards the victim, Mary Ellen.

In 1874, Mary Ellen was a ten-year old girl living with her adoptive parents in one of New York City’s notorious tenements. Concerned that she was being ill-treated, her neighbours contacted an organization which then sent volunteers to visit the family. They found that the girl had been badly neglected and abused. The parents refused to change, believing that they owned the girl and could do whatever they wished to her. There was no CP law at the time nor was there child protection services (hereafter CPS), and police declined to investigate. Eventually the case was brought to the courts under existing law against animal cruelty on the ground that a human being was a member of the animal kingdom. The court granted the removal of Ellen Mary from her adoptive parents (Tomison, 2001).

This case prompted advocates to establish the New York Society for the Prevention of Cruelty to Children (NYSPCC), the world’s first ever non-governmental charitable organization dedicated entirely to child protection, on December 15, 1874. Similar charitable organizations dedicated to child protection were to spread across the US continent, following the lead of the NYSPCC.

Legislation for Child Protection

Despite the USA being the ground that saw the first widespread movement for child protection, the first CP legislation was not enacted in the USA but in the UK. The formation of the NYSPCC had a direct influence across the Atlantic, inspiring British advocates who have observed how it worked to form similar organizations. In 1882, the first CP organization in the UK, the Liverpool Society for the Prevention of Cruelty to Children was established. In 1884, the London Society for the Prevention of Cruelty to Children was founded and was a few years later reconstituted as the National Society for the Prevention of Cruelty to Children (NSPCC) (Flegel, 2009).

In 1889, campaigns by NSPCC and others led to the UK parliament passing the “Prevention of Cruelty to, and Protection of, Children Act” which is commonly known as the “Children’s Charter.” While abuse of children was already prosecuted by the courts previously, the new legislation marked the first ever parliamentary act specially purposed for the protection of children from their caregivers, empowering the state to intervene in the cases of child maltreatment by their parents or custodians. Under the law, wilfully mistreating, neglecting or abandoning of a boy under 14 or a girl under 16 by his or her custodians was made an offence. If the carer was convicted upon investigation, a court could order the removal of the child and place him or her under the charge of another person (Fogarty, 2008).

Further progresses were to be made in the following decades, including refinements and reformations of existing legislations and establishments of juvenile courts. In 1899, the world's first juvenile court was established in Chicago, USA. Such juvenile justice legislations were designed to have jurisdiction on cases of abuse and neglect, in addition to their primary concern on handling juvenile delinquency (Myers, 2008). Similar juvenile courts were to spread in the US and established in Australia, Canada, and the UK subsequently.

Development in Australia

Developments of child protection awareness and practices in Australia were similar to those in the US and UK. In Australia, the New South Wales Society for the Prevention of Cruelty to Children (NSWSPCC) was established in 1890 and the Victorian Society for the Prevention of Cruelty to Children (VSPCC) was established in 1894, partly following the models in the US and the UK (Child Family Community Australia, 2015).

Despite the similarity in development, two features in the history of child protection in Australia were often singled out by researchers in the field – the so-called practice of "boarding out" and the state-based legislation and service system. These two aspects of early Australian child welfare practices were the precedents of similar patterns of preference for family-based care and state dictated CPS in contemporary CP practices in Australia.

Boarding out refers to the preference for keeping neglected children with other families instead of in institutional or residential facilities. In mid-1800s, institutionalization was the primary means of providing care to abandoned or abused children. However, the conditions of these institutional facilities were poor and children were deprived of the benefits of family life. These concerns led to the preference of "boarding out" to approved families over institutional facilities. While the preference of the system has oscillated between institutional care and family-based care subsequently, the latter has clearly become the service of favour in the last half-century or so (Tomison, 2001).

On the other hand, state dictated CPS can be traced to the lack of discussion and vision for a commonwealth system of CP in the convention debates preceding the establishment of the Commonwealth of Australia in 1901. While the Commonwealth did seek to legislate in child related area over the years, such efforts were mainly confined to the area of marriage and divorce, and CPS remained within the ambit of state legislation. In fact, it is clear from High Court rulings and current legislation that Family Court or Federal Magistrates Court orders regarding children under the Family Law Act are subservient to state CP orders. As a result, each state or territory continues to have its own CP system, although there are a lot of common practices between these systems (Tomison, 2001).

Second Wave of the Child Rescue Movement

The first wave of the child rescue movement was characterized by CP efforts being provided by non-governmental CP societies. Subsequent developments in the first decades of the twentieth century saw a shift towards governments getting more involved in CP both as service providers and as enforcer of standard of care based on legislations. This shift coincided with the increasing role of the governments in providing social services (Myers, 2008; Tomison, 2001).

However, it would take decades to herald a new era of CP practices – the "second wave" of child rescue movement, which was prompted and driven by research on child maltreatment. In 1962, a seminal paper titled "The Battered Child Syndrome" published by Dr Henry Kempe and colleagues in the USA proposed the use of radiological surveys of untreated bone injury as evidence of physical abuse (Kempe, Silverman, Steele, Droegemueller, & Silver, 1962). This was termed the "battered-child syndrome" and the researchers argued that for children under 3-year old, it was a significant cause of disability and death.

This study generated wide media coverage of stories on such syndrome, leading to further awareness of the problem of child abuse, kindling professional research interest and pressuring the government to take more responsibility. Changes in legislation identified CPS as part of public child welfare and mandated state-wide provision of child welfare services. Within a few years, mandatory reporting laws were also established (Myers, 2008).

Around the world, similar works were inspired by Dr Kempe's study. In Australia, Wurfel and Maxwell published a paper titled "The 'battered-child syndrome' in South Australia" in 1965 (Wurfel & Maxwell, 1965). Their study featured investigations on 26 children from 18 families at the Adelaide Children's Hospital, with half of the children aged six months or younger. More shockingly perhaps, was that eight of the children in their study had later on died of their injuries, showing the severity of the abuse.

Such research led to wide media coverage on child abuse in Australia, which was increasingly seen as a serious social issue that could only be adequately tackled with greater government response. The increasing public pressure has forced the state governments to take greater responsibility. Throughout the 1960s, with the exception of Victoria, Australian states and territories started to adopt a government-based CP approach, shifting towards professional practices, as opposed to voluntary practices. Except in Victoria, some level of mandatory reporting was also introduced (Fogarty, 2008).

In Victoria, the role of the state government was limited to the placement of children and CP investigations continued to be conducted by the police and the Children's Protection Society. This would last until 1985, when the state formally took over the provision of statutory CPS, with the Children's Protection Society no longer able to cope with increased demand for CPS (Scott & Swain, 2002).

Importantly, even with the transition to a professional- and government-based approach, CPS across Australia remain largely a "residual" system, with the state intervening only when families fail to meet their own needs (Bromfield, Arney, & Higgins, 2014).

1.1.2 Recognising Different Types of Child Maltreatment

In the 1970s, the definition of child maltreatment expanded. Early efforts of child protection had focused on severe physical abuse and extreme neglect. By the 1980s, it was common for CP systems in the western world to define four main types of child maltreatment – physical abuse, neglect, sexual abuse, and emotional abuse. Many systems also expanded the focus of abuse and neglect to include all children and young people up to 18 years of age (Lonne, Parton, Thomson, & Harries, 2009).

Among the four types of maltreatment, physical abuse and neglect were the first to catch public attention. In fact, the "rescue" of Mary Ellen which heralded the first wave of the child rescue movement was argued on the ground of cruelty, including physical abuse and neglect. Physical abuse was also the focus of the seminal work by Kempe and colleagues in 1962.

Physical Abuse

In contemporary CP practices, physical abuse is defined by the World Health Organization (WHO) as "the intentional use of physical force against a child that results in – or has a high likelihood of resulting in – harm for the child's health, survival, development or dignity. This includes hitting, beating, kicking, shaking, biting, strangling, scalding, burning, poisoning and suffocating. Much physical violence against children in the home is inflicted with the object of punishing." (Butchart, Harvey, Mian, & Fürniss, 2006, p. 10) Note that in contrast to earlier understanding of which injuries such as untreated fractured bones were the focus, injury is no longer a necessary factor. This reflects significant changes in our understanding of the harms that physical violence can cause, even if it doesn't result in an injury.

Neglect

In the beginning of the first wave of the child rescue movement, child neglect, as defined by the National Society for the Prevention of Cruelty to Children (NSPCC) in the UK, included issues such as "ill treatment or exposure of children; not provided children with food, nursing, clothing, medical aid or lodging; children taking part in a public exhibition or performance where life or limbs are endangered" (Swain, 2014, p. 7). Such a definition was largely adopted by Australian jurisdictions, leading to a significant shift from earlier legislation which was more concerned with the need of controlling destitute children than providing care to them (Swain, 2014, p. 6).

In contemporary practices, child neglect is defined by the WHO as "isolated incidents, as well as a pattern of failure over time on the part of a parent or other family member to provide for the development and well-being of the child – where the parent is in a position to do so – in one or more of the following areas: health, education, emotional development, nutrition, and shelter and safe living conditions." (Butchart et al., 2006, p.

10) In contrast to earlier definitions, impacts on development are now an essential factor, reflecting the contemporary understanding of the harms of neglect.

Sexual Abuse

In the first wave of the child rescue movement, intrafamilial child sexual abuse – incest, was identified as the most common form of child sexual abuse. For example, studies have shown that 10% of family violence cases on record in Boston child rescue organizations contained incest. Male brutality and lack of sexual control were seen as the causes of the problem. Such sympathy towards child victims was seen as a major achievement of the nineteenth century feminist movement (Gordon, 1988). Such an attitude was very different from that in earlier years, when the victims of incest were seen to play a part in the crime and thus punished (Tomison, 2001).

However, the view was to change significantly in the early twentieth century, with intra-familial abuse de-emphasized and extrafamilial abuse portrayed as the main problem. In the prevailing narrative, the streets replaced the home environment as the locus, perverted stranger replaced male family members as culprits, and sex delinquency of the victim replaced victim innocence. This shift in narrative was attributed to the fears of Bolshevism, sexual freedom, and feminism, which created a backlash against the feminism (Gordon, 1988).

It would take another wave of child advocacy and feminist movement to shift the narrative, leading eventually to contemporary definitions of child sexual abuse such as the following one provided by the WHO: "the involvement of a child in sexual activity that he or she does not fully comprehend, is unable to give informed consent to, or for which the child is not developmentally prepared, or else that violates the laws or social taboos of society." (Butchart et al., 2006, p. 10) Such a definition acknowledges the limited capacity of a child to give truly informed consent. It has also gone beyond incestuous acts to encompass all types of sexually abusive behaviours. Moreover, it is gender neutral, recognizing that the perpetrators of sexual abuse can be of any gender orientation.

Emotional Abuse

Among the four types of child maltreatment, emotional abuse was the latest to be "discovered". Emotional abuse, also known as psychological abuse, damages a child's mental wellbeing and social competence. It does not leave bodily injuries and is therefore, the most underestimated and elusive form of child maltreatment (Tomison & Tucci, 1997).

As a distinct form of abuse, emotional abuse such as verbal abuse, threat of abandonment, or exposure to domestic violence may lead to emotional trauma. However, emotional trauma may also occur as a consequence of other types of maltreatment. In fact, some researchers have argued that it is the emotional trauma resulting from physical and sexual abuse that is the most harmful for child development (O'Hagan, 1993).

Emotional abuse is defined by the WHO as "isolated incidents, as well as a pattern of failure over time on the part of a parent or caregiver to provide a developmentally appropriate and supportive environment. Acts in this category may have a high probability of damaging the child's physical or mental health, or its physical, mental, spiritual, moral or social development. Abuse of this type includes: the restriction of movement; patterns of belittling, blaming, threatening, frightening, discriminating against or ridiculing; and other non-physical forms of rejection or hostile treatment." (Butchart et al., 2006, p. 10)

1.2 Burdens of Child Maltreatment

Consequences of child maltreatment have long been the subject of extensive research since the seminal investigation of Elmer and Gregg in 1967 (Elmer & Gregg, 1967). Their work suggested that the deleterious consequences of child maltreatment were pervasive and severe. In fact, not only do maltreated children bear the consequences as individuals, society also bears a large cost.

However, it has also been recognized that the consequences of being exposed to similar maltreatment vary among children and young people, depending on the complex interactions of protective and risk factors (Widom, 2014). These factors are a range of positive or negative life experiences and family circumstances that influence maltreated children's resilience or vulnerability.

Protective factors in the face of child maltreatment include positive child attributes such as high self-esteem and independence, positive family environment such as good parenting quality and positive relationships with extended family, and extra-familial and community resources such as positive peer relationships and school environment (Haskett, Nears, Sabourin Ward, & McPherson, 2006). Risk factors in the face of child maltreatment include caregiver problems such as depression or alcohol and drug dependence, social isolation, socio-economic disadvantage, dangerous neighbourhoods, and child disability (Dubowitz & Bennett, 2007; Jaffee & Maikovich-Fong, 2011).

Protective and risk factors at different levels – individual, parents and family, neighbourhood and community – interact and produce complex behavioural responses to adverse experience (Widom, 2014). In general, for children whose protective factors were enough to foster resilience against risk factors and adverse experience, the outcomes of being maltreatment may be less adverse. However, in cases where risk factors are more dominant, the effects of maltreatment may be chronic and debilitating (Miller-Perrin & Perrin, 2013).

1.2.1 Individual Consequences

The most tragic consequence of child maltreatment is of course the death of a child. Other potential consequences can be examined in the physical, cognitive, social and behavioural, and psychological domains. The below gives a brief account of some of these potential consequences.

Consequences in the Physical Domain

Consequences of child maltreatment in the physical domain could either be a result of injuries sustained in maltreatment or impaired development due to deprivation. One of the most serious forms of injuries is brain injuries, which may result from being hit on the head or being vigorously shaken (Dykes, 1986).

On the other hand, it has been shown that children whose basic needs are not met suffer physical consequences including malnutrition, infections, failure to thrive, and impaired brain development (Block & Krebs, 2005). Research based on the United State Longitudinal Studies of Child Abuse and Neglect (LONGSCAN) has shown strong associations between maltreatment and significantly increased odds of poor physical health for children (Flaherty et al., 2006). It has also been found that all types of maltreatments were associated with major adolescent health risks (Hussey, Chang, & Kotch, 2006). Moreover, research on neurobiological development has shown that early childhood maltreatment could initiate a cascade of neurohumoral and physiological responses, leading to impaired brain development (Nemeroff, 2004) which would, in turn, increase the risk of psychopathology for the maltreated children, and when they turn into adults.

Consequences in the Cognitive Domain

Research has indicated that child maltreatment is associated with impaired cognitive and academic development (Mills et al., 2011). A meta-analysis of child maltreatment research showed that a majority of the studies examined indicated poor academic achievement and delays in language development in maltreated children (Veltman & Browne, 2001). Importantly, delayed intellectual development such as verbal intelligence has been shown to occur for maltreated children with no signs of neurological impairment (Augoustinos, 1987).

Consequences in the Social and Behavioural Domain

Consistent with the physical and cognitive problems associated with it, child maltreatment has been shown to associate with a wide range of behavioural, emotional, and social adjustment problems. Such problems include insecure attachment, interpersonal relationship problems, aggression, anti-social behaviours, violence, criminal activity, teen pregnancy, risky sexual behaviours, alcohol and drug abuse, mental health problems, youth suicide, and homelessness (Child Family Community Australia, 2014).

A series of meta-analyses have shown that infants and very young children exposed to maltreatment are more likely to experience deficiencies in the development of secure attachments to their primary caregiver (Cyr, Euser, Bakermans-Kranenburg, & Van Ijzendoorn, 2010). Secure attachment to primary caregiver plays an extremely important role in a child's emotional and social development. Deficient secure attachment has a cascading effect, negatively affecting a child’s communicating and interacting ability (Bacon & Richardson, 2001), subsequently leading to difficulties in peer and romantic relationships (Trickett, Negriff, Ji, & Peckins, 2011).

Research has also shown that exposure to maltreatment is associated with higher levels of withdrawal and aggression throughout childhood (Mills, 2004). Maltreated children were found to have lower levels of ego resiliency and more problems with the under-control of ego. They were more aggressive, more disruptive, and less cooperative

(Manly, Kim, Rogosch, & Cicchetti, 2001). Moreover, maltreated children could be much more likely to be arrested for criminal behaviour in adolescence (English, Widom, & Branford, 2001).

Research has shown that experiences of child sexual abuse may be associated with teenage pregnancy and risky sexual behaviours (Gilbert et al., 2009). Fergusson and colleagues found that young women exposed to child sexual abuse had significantly higher rates of sexual vulnerability, including teenage pregnancy, multiple sexual partners, and sexually transmitted disease (Fergusson, Horwood, & Lynskey, 1997). They also appeared to be more vulnerable to sexual assault after the age of 16. Meta-analyses of research in this area found strong associations between child sexual abuse and prostitution in adolescence or adulthood (Arriola, Louden, Doldren, & Fortenberry, 2005; Rind, Tromovitch, & Bauserman, 1998).

Consequences in the Psychological or Mental Health Domain

Child maltreatment has been found to be strongly associated with mental health problems such as depression, anxiety disorder, substance abuse, and post-traumatic stress disorder (PTSD) in adolescents (Gilbert et al., 2009). While earlier studies have focused on single disorder, more recent research has found that different disorders often co-occur, with each disorder capturing an aspect of the impairments to a child's complex self-regulatory and relational functioning (Cook et al., 2005).

Child maltreatment is also associated with increased risk of self-harm and suicide. In fact, studies have revealed a strong link between all types of child maltreatment and adolescent suicidal ideation and suicide attempts (Brown, Cohen, Johnson, & Smailes, 1999). However, some studies suggested that among the different types of maltreatment, sexual abuse and emotional abuse may have more explanatory relevance in explaining suicidal behaviour (Miller, Esposito-Smythers, Weismoore, & Renshaw, 2013).

All types of child maltreatment have been found to be associated with a significantly higher rate of tobacco, alcohol and illicit drug use in adolescents. Substance use is generally believed to be a coping strategy used by adolescents to reduce psychological pain resulting from a wide range of problems as a consequence of maltreatment exposure (Moran, Vuchinich, & Hall, 2004).

Consequences in Adulthood

Adverse outcomes of child maltreatment have been shown by extensive research, especially adverse childhood experiences (ACE) studies, to last into adulthood. A well-known ACE study jointly conducted by the Centers for Disease Control and Prevention (CDC) and the Department of Preventive Medicine at Kaiser Permanente (KP) San involved 17,421 adult KP Health Plan members (Felitti, 2002). It examined the relationships between current adult health status and adverse childhood experiences, including child maltreatment and home environment factors, and found a strong relationship between adverse experiences including maltreatment in childhood and many of the problems described in the previous sections. Such findings are consistent with reviews of research in the relationships of child maltreatment and associated consequences in adulthood (Gilbert et al., 2009).

Intergenerational Transmission of Child Maltreatment

The idea that past experience of being maltreated in childhood is associated with caregivers maltreating their own children can be found in early academic literature on child maltreatment. For example, in their seminal paper on battered child syndrome, Kempe and colleagues suggested that some parents who attacked their children were themselves the victims of similar maltreatment in childhood (Kempe et al., 1962). This phenomenon is now termed intergenerational transmission of child maltreatment (ITCM) and shows that the consequences of child maltreatment do not last merely into the adulthood of the victims, but potentially transmit across generations.

However, whether ITCM is real or is a result of detection bias remains debatable (Widom, Czaja, & DuMont, 2015). While there is now relative consensus that exposure to childhood maltreatment is a risk factor for victims to maltreat their own children, such transmission is far from inevitable and the rates obtained by researchers vary vastly (Berzenski, Yates, & Egeland, 2014).

1.2.2 Societal Costs

While the consequences of exposure to childhood maltreatment could be debilitating for the individuals, society also bears a huge cost. This is a result of individuals not being able to fully develop their potential as members of the society, and the resources that have to be deployed for the prevention and investigation of child maltreatment, as well as for intervention and treatment services provided to both offenders and victims.

A number of studies have been conducted to estimate the economic costs of child maltreatment. For example, taking into account healthcare costs in childhood and adulthood and costs borne by the government in CPS, child welfare, special education, and criminal justice, Fang and colleagues estimated that lifetime costs of nonfatal child maltreatment per victim was 210,012 USD (2010 dollars) in the US in 2008. The study also estimated that the total lifetime economic burden incurred by new cases of both fatal and nonfatal child maltreatment in 2008 amounted to 124 billion USD (Fang, Brown, Florence & Mercy, 2012).

In Australia, the recurrent expenditure on child protective services[1] was approximately 5.8 billion AUD in the 2017-18 financial year, amounting to 1055 AUD per child for every child 0 to 17 years of age in the Australian population (SCRGSP, 2019). A recent study by McCarthy and colleagues estimated that lifetime costs of new cases of child maltreatment were 9.3 billion AUD in 2012–13 (McCarthy et al., 2016). Such figures are generally comparable to those estimated for the US.

1.3 Child Protection Systems in Australia

As described previously, there are two important features with regards to CPS in Australia. The first is that child protection is largely a residual response to child maltreatment. The second is that due to historical reasons, child protection is mainly a state specific enterprise in Australia, with each state or territory having its own legislation, policies and practices, although there are a lot of common elements among them.

1.3.1 The National Framework

In recent years, there have been efforts to move from a “residual” approach, in which CP is merely a response to maltreatment, to an “institutional” approach, in which promoting the safety and wellbeing of children in general is the focus. Such a transition

[1] Including the four components of protective intervention services, out-of-home care, intensive family support services and family support services.

necessitates a public health model with a three-tier prevention and intervention system of services, as laid out in the National Framework for Protecting Australia's Children 2009–2020 (Council of Australian Governments, 2009). The priority – the primary tier – is universal supports or primary preventive services available to all families; the secondary tier consists of preventive interventions provided to families needing additional supports, with a focus on early intervention; the tertiary tier consists of intensive interventions for "at-risk" families and children.

In addition to providing guidance on the paradigm shift to a public health approach, the National Framework also seeks to coordinate reforms driven by the states and territories with Australian Government programs, policies and payments. It aims to deliver a more integrated response across the states and territories towards promoting the safety and wellbeing of children without chaining the separate responsibilities of the states and federal governments. Specifically, statutory CP will remain the responsibility of individual states and territories, while income support payments will remain the responsibility of the Australian Government (Council of Australian Governments, 2009).

In line with the national framework, preventive and early interventions services have been introduced into all states and territories. Child protection services across all states and territories now consist of four main components, as listed below (SCRGSP, 2019).

- Family support services, defined as "activities associated with the provision of lower level (that is, non-intensive) services to families in need, including identification and assessment of family needs, provision of support and diversionary services, some counselling and active linking and referrals to support networks." (p.16.37)
- Intensive family support services, defined as "specialist services that aim to prevent the imminent separation of children from their primary caregivers as a result of child protection concerns and to reunify families where separation has already occurred." (p.16.37)
- Protective intervention services, defined as "functions of government that receive and assess allegations of child abuse and neglect, and/or harm to children and young people, provide and refer clients to family support and other relevant services, and intervene to protect children." (p.16.39)

- Out-of-home care services, defined as "Overnight care, including placement with relatives (other than parents) where the government makes a financial payment. Includes care of children in legal and voluntary placements (that is, children on and not on a legal order) but excludes placements solely funded by disability services, psychiatric services, youth justice facilities and overnight child care services." (p.16-39)

Among these four components, family support services were added as a separate service category only in the 2011-12 financial years.

1.3.2 The Child Protection Process

While statutory CP is the responsibility of individual states and territories, with each state or territory having its own legislation, policies and practices, the general process of CP is the same. Once a report is lodged about a suspected case of child maltreatment, the ensuing process entails a few stages – notification, investigation, substantiation, and placement in OOHC, with decision making in each stage prescribed by state specific procedures and criteria.

Figure 1.1 shows the CPS and generic pathways of CP in Australia (adapted from Figure 16.1, SCRGSP, 2019, p.16.2). The rectangular boxes denote actions or options in the process of CP. The right-most column denotes the provision of prevention and support services provided to families at various levels of risk of child maltreatment. The three types of services correspond to the three-tier public health model of prevention and intervention. In this model, the first tier – universal prevention service, is not considered part of the CP system.

The figure also shows eight activity groups (AGs) along the pathways, each with its designated functions within the CP system.

- Activity group 1 (AG1) receives and assesses initial information about reports received by CPS.
- Activity group 2 (AG2) provides generic family support services.
- Activity group 3 (AG3) provides intensive family support services.
- Activity group 4 (AG4) gathers and assesses secondary information, i.e. investigation and decision on substantiation.

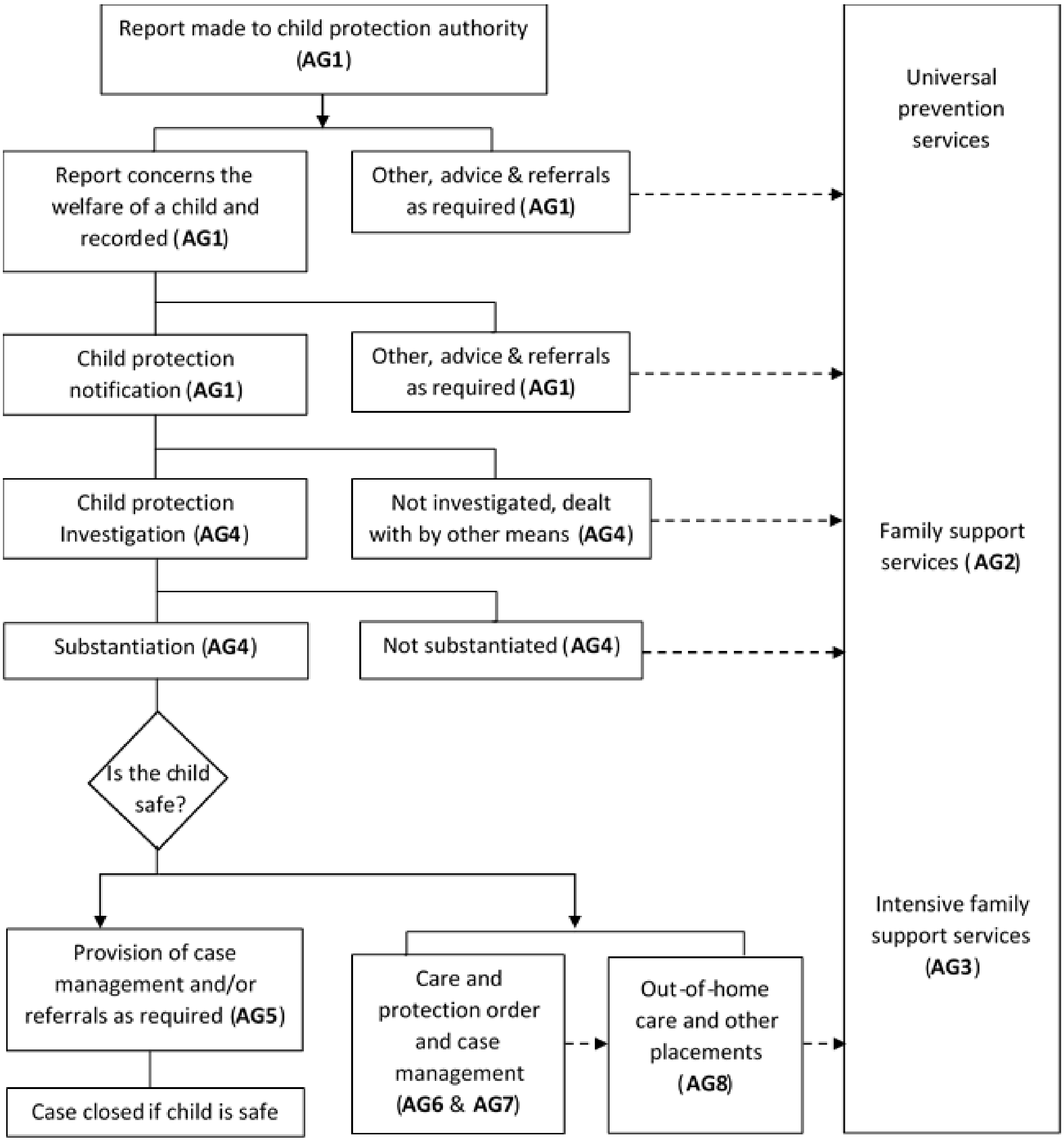


Figure 1.1 Pathways through the Child Protection System (Adapted from Figure 16.1, SCRGSP, 2019, p.16.2)

- Activity group 5 (AG5) provides short-term protective intervention and coordination services for children not on an order.
- Activity group 6 (AG6) deals with seeking a court order.
- Activity group 7 (AG7) provides long-term protective intervention, support and coordination services for children on an order.
- Activity group 8 (AG8) provides OOHC services

The CP process starts from a case being reported to State and Territories CPS, and potentially ends with a child being taken away from the caregiver and placed into OOHC. When a report is received by the system, an assessment is made, based on the information provided in the reports. If the report does not concern children and young people, it is diverted to universal primary services. A report concerning children and young people may also be referred to other services for support, e.g. school or other agencies.

If a report falls within the scope of CPS, it is "screened-in" and logged as an incident of notification in the system. Based on the information collected on notification, a case may still be referred to family support services instead of being investigated if assessment based on initial information deemed that to be the most appropriate action. Otherwise, an investigation will be initiated and a case file is opened. An investigation may consist of two stages, an office-based stage, and if necessary, a face-to-face stage where caseworkers meet with the clients for further assessment. If the case is substantiated in investigation, a decision on whether to take further action will depend on whether the child is deemed safe in the home environment. If safety is an immediate concern, a court order will be sought, and placement into OOHC or provision of alternative services will be arranged.

While the process described above is common to CPS in all states and territories, important differences abound across the states in how actions along the pathways are conducted. For example, all child maltreatment notifications are investigated in Queensland, while only a small percentage of notifications are investigated in other jurisdictions (Australian Institute of Health and Welfare [AIHW], 2019a). Each jurisdiction also has its own system of investigation protocol, including risk assessment and decision making.

An important aspect of the child protection system which is not included in the process diagram in Figure 1.1 is the mandatory reporting policy of child maltreatments (Child Family Community Australia, 2017). Such a policy, stemming from corresponding legislations, requires certain professionals who frequently deal with children in the course of their work, e.g. teachers, doctors, nurses, and police, to report cases of suspected child maltreatment and risk of child maltreatment to child protection agencies. The mandatory reporting policy varies across Australian states and territories in terms of the list of mandated reporters, grounds of reporting, and types of abuse. Since this study examines CP populations in terms of involvement in CP investigation, the effect of mandatory reporting is not considered. However, it is useful to keep in mind that differences in the mandatory reporting policy may contribute to the differences in the rates of notifications across the states and territories.

1.4 Contemporary Challenges in Australian Child Protection Systems

Challenges abound in child protective services across Australian states and territories. Some of these stem from the inherent complexity and uncertainty of CP practices. For example, risk assessment and decision making. Across all phases of the CP process shown in Figure 1.1, risk has to be assessed and decisions have to be made about the course of action, including the pan-ultimate decision of admitting a child to OOHC.

1.4.1 Risk Assessment and Decision Making

When investigating a CP notification, caseworkers have to make decision and devise a case plan based on a disperse but limited set of information from which risk factors have to be selected, organized, and weighted in a context of uncertainty (Shlonsky & Wagner, 2005). In addition to investigating circumstantial factors (case factors) of suspected maltreatment, caseworkers also have to take organizational and policy mandates into account, and make judgements using imperfect assessment tools (Gambrill & Shlonsky, 2000), with the process potentially compounded by their conscious and unconscious biases, as well as errors in thinking (Dawes, 1994).

To address some of these concerns, actuarial risk assessment instruments have increasingly been used. Researches on actuarial assessment of child maltreatment began in the early 1980s (Shlonsky & Wagner, 2005). Over the years, actuarial risk assessment instruments have been validated to a certain extent and studies have shown that they generally performed better than other approaches of child maltreatment risk assessment, such as consensus-based assessment and unaided clinical judgment in predicting risk (Baird & Wagner, 2000; Shlonsky & Wagner, 2005). Nevertheless, such instruments are generally atheoretical (Schwalbe, 2004) and are thus not able to illuminate the dynamics of decision making (Baumann, Law, Sheets, Reid, & Graham, 2005). Some studies have found that case workers did not use the instruments to the intended extent and there are worries that over-reliance on such instruments hamper professional development (Gillingham & Humphreys, 2009).

More recently, there have been efforts to use predictive modelling with big data in assessing future risk of maltreatment (Gillingham, 2016; Vaithianathan, Maloney, Putnam-Hornstein, & Jiang, 2013). Sustained efforts in the last decade have resulted in such systems being trialled in a number of jurisdictions in the USA (Packard, 2016). Nevertheless, ethical concerns and technical issues abound in such an approach (Gillingham & Graham, 2017; Keddell, 2014). Whether it will become a routine practice remains to be seen.

1.4.2 Unstainable Increase of Demand for Child Protection Services

In addition to the general problem with decision making, CP services in Australia have been struggling to grapple with the problem of ever-increasing and unmet demands. While the number of children involving in CPS have stabilised in the USA after decades of rising trends, the situation is different in Australia. In fact, across all Australian states and territories, there has been an increasing number and rate of children involved in CPS.

Taking the nation as a whole, over the last three decades, the population rates of child maltreatment notifications have risen enormously. In 1989-90, the number of notifications per 1000 children was 10.4; by 1999-2000, it has more than doubled to 22.5; by 2009-10, it has once again more than doubled to 56.2; by 2017-18, it has risen to 75.0 per 1000. Starkly, the rate in 2017-18 was more than 7 times the value in 1989-90 (Higgins, 2011; SCRGSP, 2019).

The per 1000 rate of children in OOHC at the end of the financial year (June 30) has also risen considerably since 1989. In June 30, 1989, there were 3.0 per 1000 children in OOHC; by 1999, the rate has risen by 20% to 3.6; by 2009, the rate has nearly doubled to 7.0; by 2018, the rate has risen to 8.2 (Higgins, 2011; SCRGSP, 2019). It was clear that the OOHC system saw a vast expansion between 1999 and 2009, and has continued to expand since 2009, at a slower but still significant rate.

Accompanying the vast increase in demand is the rise in the cost of service provision. The most recent figures (SCRGSP, 2019) show that the government’s real expenditure in OOHC services has nearly doubled from AUD 353 per child[2] in 2008-09 to AUD 617 per child in 2017-18; for protective investigation service, the figure has risen from AUD 183 per child in 2008-09 to AUD 257 per child in 2017-18.

Moreover, in 2017-18. protective investigation services accounted for 24.4% of the real expenditure; OOHC, a tertiary intervention, accounted for 58.5% of the real expenditure; family support services and intensive family support services accounting for only 9.0% and 8.1% of the real expenditure. This probably reflects the inconvenient truth that despite the rhetoric of the public health model, with its emphasis on three-tier prevention and intervention services, CP systems continue to be biased towards the most intensive form of tertiary intervention, OOHC.

The problems highlighted above have been examined in-depth by child welfare scholars (Bromfield et al., 2014) and the seemingly ever-increasing trend of OOHC population has prompted an independent reviewer of the New South Wales's OOHC system to label the system as unsustainable (Tune, 2016). A glance at state level figures on CP real expenditure shows that from 2008-09 to 2017-18, all states and territories have experienced a significant increase in OOHC real expenditure per child, ranging from 30.4% in Western Australia to 238.0% in South Australia (SCRGSP, 2019). The number of children involved in CPS has also risen across the states and territories. Therefore, one could argue that the scathing review on New South Wales's OOHC system (Tune, 2016) may well have indicated problems representative to all states and territories.

It should be noted that in New South Wales (NSW) and Australian Capital Territory (ACT), OOHC admission rates have been decreasing in the last few years. For example, in NSW, per 1000 rate of OOHC admission decreased by 43% from 2015-16 to 2017-18. However, across the same period, there was only a 4% reduction in OOHC population in NSW (Australian Institute of Health and Welfare, 2019). Such changes in OOHC admission rates and population should be viewed in light of other changes that may impact official counts of OOHC population. For example, the official definition of OOHC over the last few years has changed, with the exclusion of children on finalised

[2] Expenditure per child calculated as total real expenditure divided by general population of children below 18 years of age, in 2017-18 dollars.

third-party parenting order since 2015-16. Children on such an order, where parental responsibility has been transferred to a nominated person who may be a relative or government officer responsible for child protection, were previously counted as part of OOHC (Australian Institute of Health and Welfare, 2019a). While it is clear that such a change in definition directly reduced the official count of OOHC population, it is not clear how it affected OOHC admission rates. Regardless of the changes in OOHC admission rates and definitions, when consistent definitions are used, OOHC populations in most states and territories continue to exhibit a rising trend.

1.4.3 Racial Disparity

Related to the rise in the number of children involved in CPS is the issue of racial disparity. Specifically, Aboriginal and Torres Strait Islander children are vastly over-represented in CP involvement across all states and territories, at every phase of the CP process (Lewis et al., 2019). While this phenomenon of racial disparity is not new, it has aggravated over the years, primarily due the trend of CP involvement for Aboriginal and Torres Strait Islander children growing at a faster pace than that for non-Indigenous children.

In 2016-17, compared to non-Indigenous children, Aboriginal and Torres Strait Islander children were 5.5 times more likely to be involved in CP notifications, 6.4 times more likely to be involved in CP investigations, 6.8 times more likely to be involved in CP substantiations, 9.7 times more likely to be on care and protection order, and 10.0 times more likely to be in out-of-home care (Lewis et al., 2019).

Such severe and aggravating disparity has led to much soul searching and call for the federal government to “develop a national comprehensive Aboriginal and Torres Strait Islander children’s strategy that includes generational targets to eliminate over-representation and address the causes of Aboriginal and Torres Strait.” (Lewis et al., 2019, p. 91).

Notably, in September 2016, an independent review was commissioned by the NSW government to examine the high rates of Aboriginal children in OOHC and the implementation of the Aboriginal Child Placement Principle (ACPP) in the jurisdiction. The case files of 1,144 Aboriginal children entering OOHC in NSW system between 2015 and 2016 were reviewed. The final report, titled Family is Culture (Davis, 2019) was published in November 2019. It revealed that case workers often took the most intensive option of removing Aboriginal children, and some children who were not at risk of harm were placed into OOHC. The conclusions were damning, highlighting issues such as the lack of transparency, the lack of an effective regulator, and the lack of genuine consultation with the Aboriginal community. Altogether, 125 recommendations to the NSW Government were made to improve the child protection system.

1.5. The Future of Child Protection Services in Australia

With the stipulated period of the National Framework for Protecting Australia's Children 2009–2020 drawing near to an end, the aggravated state of CPS in terms of expanded and unmet demand is a clear indication that past reform efforts need to be "reformed".

In analysing the reform efforts associated to the National Framework, Bromfield et al. (2014) observed that the quest to improve the mechanism for accessing "the right services at the right time" has been the primary assumption underpinning contemporary CP reforms. In Australia, this assumption has led to two inter-related reform efforts: shifting towards prevention-oriented pathways, primarily family support services; and increasing investment in intensive family support services for highly vulnerable families.

As we have shown above, since 2009, demand for CPS has continued to rise and racial disparity has worsened. Moreover, real expenditure has continued to skew towards OOHC – the most intensive form of tertiary intervention. In this light, it can be said that the National Framework has failed to achieve the objective of reducing the demand on CPS.

In their analysis, Bromfield et al. (2014) suggested that one of the issues that could hamper the reforms as set out in the National Framework was the tendency to view a public health approach to CP only within the scope of CP practices and investment, by reforming statutory intake and referral pathways, and increasing provision of prevention and intervention services for vulnerable families. The truth is, while these reforms are key components of a public health approach, they are not the core components of the public health model, which includes addressing social determinants of child maltreatment.

Moreover, Bromfield et al. (2014, p. 127) pointed out that "Australia continues to lack national data on the incidence or prevalence of child abuse and neglect in the community—a core component of a public health approach required to determine whether reforms are creating meaningful population change."

Indeed, the dynamics of CP population in Australia, including issues of prevalence and interactions among various sub-populations, remains poorly understood. This study aims to make a small but hopefully, meaningful contribution to fill this gap. Specifically, it aims to provide a mathematic-based approach to understand the population dynamics of CP populations in Australia, in the hope of leading to a better understanding of the trends and prevalence of CP sub-populations.

Before stating the research question, a brief descriptive introduction to mathematic-based approach to population dynamics is provided to facilitate a better understanding of the subject matter. The exact mathematical formulation is provided in the methodology chapter (Chapter 3).

1.6 Mathematical Models of Population Dynamics

Generally speaking, population dynamics is "a branch of knowledge concerned with the sizes of populations and the factors involved in their maintenance, decline, or expansion." ("Population Dynamics," 2019a) In particular, changes in the sizes of populations of humans, animals, and disease infections have been a focus of scientific inquiries for centuries. In these inquiries, mathematics, especially the use of differential equations, has played an essential role.

The first recognised use of mathematics to analyse population dynamics was Fibonacci's description of a growing population of rabbits with the famous Fibonacci sequence, shown in a book he finished writing in 1202 (Bacaër, 2011). However, the first mathematical formulation of population dynamics, in terms of the use of differential equation to describe the change of a population with time, was Daniel Bernoulli's study of smallpox inoculation in 1760 (Bacaër, 2011). In the early 18th century, it was known that future smallpox infections could be prevented by introducing a small amount of pustule material from a mild case of smallpox into the body. However, inoculation could sometimes be deadly and the challenge was to compare its long-term benefit with the immediate risk of dying. To this end, Bernoulli first formulated differential equations for the susceptible population and survival population, based on parameters representing the probabilities of infection and death due to infection. Using observed data and life table, he estimated the risk of death from smallpox infections for increasing age. The effect of inoculation in eradicating smallpox and risk of dying due to inoculation were then taken into account, and the comparison of long-term benefits and short-term risk could be made.

Bernoulli's study described above was an application of differential equations to study the population dynamics of infectious disease. The first application of differential equations to describe the growth of a general population was proposed by Pierre-Francois Verhulst in 1838 (Bacaër, 2011), who used a version of the logistic equation to counter Thomas Malthus' concept of exponential population growth. In Malthus' conceptual model, which was not formalised with mathematical statements, birth rate was larger than death rate. Without imputing mechanisms other than death to arrest the net rate of growth, this would lead to unimpeded exponential growth. Verhulst formulated a differential equation for a population which included an ad hoc stabilising factor in the population growth rate. This stabilising factor would later on be associated with the concept of carrying capacity of an ecological system. The key concept is that when a population approaches the carrying capacity of the system, per capita resource will become more and more scarce. This will result in the growth rate becoming smaller and smaller, thus stabilising the population growth to the maximum level – the carrying capacity.

In 1920, Alfred Lotka (Bacaër, 2011) published an article in which he formulated the differential equations for a two-species ecological system – the predator-prey system of herbivores and plants, showing that in a predator-prey system, the populations could oscillate permanently. A few years later, the same model was formulated independently by Vito Volterra to study a different predator-prey system, in which cartilaginous fish were predators and smaller fish were preys. This the predator-prey model was later on known as the Lotka-Volterra model, becoming one of the most commonly cited in the study of ecological system. The model and its variants continue to be used to these days.

A decade before Lotka's seminal work, in 1911, Ronald Ross (Bacaër, 2011) formulated a differential equation to study Malaria infection, thus pioneering the concept of basic reproduction number (commonly labelled as R_0) – the expected number of secondary infections due to one primary infection. When R_0 is smaller than 1, the infection will eventually die out; if R_0 larger than 1, the infection will eventually spread through the population.

These early seminal works in the application of differential equations to study the population dynamics of ecological system and infectious diseases have shown the power and values of such an approach to unravel the structures of multiple populations and their interactions. In this regard, Ross' plead in favour of mathematical modelling in epidemiology (Bacaër, 2011, p. 69) is still relevant today:

> As a matter of fact all epidemiology, concerned as it is with the variation of disease from time to time or from place to place, must be considered mathematically, however many variables are implicated, if it is to be considered scientifically at all. To say that a disease depends upon certain factors is not to say much, until we can also form an estimate as to how largely each factor influences the whole result. And the mathematical method of treatment is really nothing but the application of careful reasoning to the problems at issue.

In fact, mathematical modelling of population dynamics is a consolidation of the knowledge gained through qualitative studies and other quantitative methods. It has the potential to uncover insights which would not be readily available in its absence, and the results obtained would inevitably lead to further studies using other methodologies, both qualitative and quantitative, contributing to an iterative cycle of knowledge accumulation.

1.7 Objectives and Research Questions

Despite the common use of mathematical modelling in studying the population dynamics of ecological system and infectious disease, to the best of our knowledge, there has been no study in existing child protection research literature that used such an approach to formulate differential equations which specifically describe CP populations, based on the process and knowledge of contemporary child protection practices.

We intend to take the first step to filling this gap by conceptualizing population dynamics models of up to four sub-populations, deriving the corresponding differential equations, making assumptions to simplify the equations, solving the equations, and using data to illuminate how such an approach may help us deepen our understanding of the interactions of CP sub-populations.

Specifically, this study is guided by the following research question:

> How can we use population dynamics modelling to study the macro-dynamical structures of CP populations across Australian states and territories?

The rest of the dissertation will be organised as follows:

- Chapter 2 will provide a literature review of methodologies used in the study of CP populations.
- Chapter 3 will formulate conceptual models of CP populations, set up the differential equations corresponding to the conceptual models, simplify the equations based on CP knowledge base, and derive solutions to these equations. It will also describe how the solutions can be used in conjunction with CP data across Australian states and territories to obtain insights into the trends and prevalence of CP populations.
- Chapter 4 will provide a descriptive analysis of CP data across Australian states and territories. Such a descriptive analysis serves both the purposes of portraying CP populations as reported by CP agencies and preparing data that are needed in subsequent modelling studies.
- Chapter 5 will report the results obtained using a three-population model of CP populations.
- Chapter 6 will report the results obtained using a four-population model of CP populations, illuminating the issue of OOHC re-entry after reunification.
- Chapter 7 will provide a discussion of the results, including limitations of the present study and recommendations for future works.

Chapter 2: Literature Review

In this chapter, we review the methodologies that have been used to study population dynamics of CP populations. To obtain a better coverage of existing literature, a systematic search similar to that used in a scoping review was conducted, albeit the procedure of study selection and presentation were simplified. The advantage of this approach is that materials selected for review are less ad hoc compared to traditional narrative review. More importantly, a systematic search provides a panoramic view of the evolution of the research area, making it easier to identify the research gap. We will refer to this approach as *narrative review with a systematic search* in this study.

In the following sections, we define the key concepts, describe the systematic search and selection of articles in detail, classify the articles according to the methodology used, provide a chronological narrative based on methodological classification, and identify the gap in research methodology in relation to our study.

2.1 Systematic Search

The first step of the review is to conduct a systematic search based on the standard procedure of a scoping review (Peters et al., 2015). This process starts with defining the *population*, *concept*, and *context* (hereafter "PCC elements") of concern.

2.1.1 Definitions of Key Terms

Recall that this study is guided by the following research question:

> How can we use population dynamics modelling to study the macro-dynamical structures of CP populations across Australian states and territories?

The PCC elements of the systematic search are in turn informed by the above research question. However, as preliminary searches revealed that there had been no existing study that explicitly formulated population dynamics models of CP populations, the scope of the systematic search and the subsequent review will be broader than the specific scope of the research question, as described below.

Population/Participants

The population of concern are CP populations, in other words, children who are involved with CPS. As described in Chapter 1, CPS refer to governmental provisions of supports and interventions that promote child and family wellbeing, and protect children and young people below 18 years of age at risk of maltreatment by their carers, or whose carers are not able to provide care and protection (SCRGSP, 2019). Such services include handling reports of alleged child maltreatment, investigating child maltreatment reports, removing at-risk children and admitting them to OOHC, and providing preventive and support services to families at-risk of involving or having been involved in child maltreatment (SCRGSP, 2019).

In this study, CP population is defined as the population of children who have been involved in at least one CP investigation. As defined in detail in Chapter 3, the general population of children below 18 years of age is divided into the following sub-populations: 1. Children who have never been involved in any CP investigation; 2. Children who were the subject of at least one investigation but never admitted to OOHC; 3. Children currently in OOHC; and 4. Children who exited OOHC. CP population in this study refers to the latter three sub-populations. However, to broaden the scope for the literature review, studies that examine the population of children who are involved with in CPS, including those who are reported for alleged maltreatment but not involved in CP investigations, will be included.

Concept

There are two concepts in the research question, namely population dynamics and macro-dynamical structures.

Population dynamics is “a branch of knowledge concerned with the sizes of populations and the factors involved in their maintenance, decline, or expansion.” (“Population Dynamics,” 2019a) To be more specific, it is “the study of how and why populations change in size and structure over time.” (“Population Dynamics, ” 2019b) When applied to the context of children involved in CP, population dynamics refers to the study of the sizes of these CP populations and factors associated with the maintenance and changes of these populations.

Wulczyn (1996) referred to this topic as caseload dynamics and made a distinction between dynamics at the individual level and dynamics at the aggregate level. The former are studies on how outcomes of the individuals are associated with or affected by different factors, with individuals as the unit of analysis; the latter are studies on changes of populations as a group, with groups of populations or variables associated with groups, e.g. process rates and count, as the unit of analysis.

In fact, what we mean by macro-dynamical structure in the research question of this study is none other than dynamics at the aggregate level. By focusing on macro-dynamical structures of CP population, this study is only concerned with the exchange of members among the various sub-populations described in the section above and their resulting changes in size. It doesn’t aim to investigate the trajectories of individuals through the CP system or factors related to the outcomes for individuals in these sub-populations.

In accordance with the aim of the study, the literature review will focus on studies which analyse the sizes and changes in CP populations. While factors of child maltreatment, especially ecological factors, do shed light on CP populations and their maintenance, decline, or expansion, they are not the focus of this study. Moreover, preliminary searches of the literature have revealed that there is a large body of literature on factors of child maltreatment both at the individual level and the aggregate level. In particular, since the ground-breaking study on the community context of child maltreatment by Garbarino & Crouter (1978), which used multiple-regression analysis to examine the relationship of child maltreatment with socioeconomic, demographic, and attitudinal factors, using neighbourhood areas and census tracts as units of analysis, researchers have increasingly placed their attention on ecological factors of child maltreatment. In fact, there is a significant number of review papers on the topics of child maltreatment factors (e.g. Coulton, Crampton, Irwin, Spilsbury, & Korbin, 2007; Freisthler, Merritt, & LaScala, 2006; Hindley, Ramchandani, & Jones, 2006; Stith et al., 2009; White, Hindley, & Jones, 2014; Zielinski & Bradshaw, 2006). We will thus only include risk factor studies which aim to explain temporal changes in foster care population or rates. Risk factor studies which do not include a temporal dimension will be excluded from our review and we refer interested readers to the review articles listed above for a broad overview of related literature.

Context

While the focus of this study is Australian states and territories, the literature review will include studies conducted on the CP systems in any country or jurisdiction, as long as they are within the scope of the other two PCC elements described above.

2.1.2 Eligibility criteria

To reduce bias in the articles selected for review, we define a set of eligibility criteria. Table 2.1 shows the inclusion and exclusion criteria that will guide the selection of articles obtained from the systematic searches, with pairs of corresponding inclusion and exclusion criteria positioned side by side in the respective column.

Focusing on “Concept” among the PCC elements, we first note that there are separate criteria for studies with regression-based methodologies and non-regression-based methodologies. This is due to the decision to not review regression-based risk factors studies which do not include a temporal dimension, as explained in the *Concept* section above.

Specifically, non-regression-based studies are included if they analyse and explain CP population structures or their trends. In contrast, studies which merely report incidences, or population rates of children involved with CP will not be selected for review, including those comparing incidences or rates between different groups. In other words, an analytical element is required, whether it is done with a modelling or simulation method or a descriptive analysis (e.g. using in-depth analysis of entry and exit rates to unravel changes in OOHC populations). Note that population rates (e.g. number per 1000) are sometimes referred to as prevalence by some researchers. In this study, two types of prevalence are defined – cross sectional prevalence and lifetime prevalence. The former refers to the fraction of children who have been involved in at least one CP investigation among the general population at any point in time; the latter refers to the cumulative risk or probability of being involved with at least one CP investigation at a certain age level. Some researchers use the term cumulative risk to denote cumulative exposure to risk factors. That is not the definition used in this study.

For studies using regression-based methodology, they are included only if the regression model used in the studies includes a temporal dimension. In other words, risk factor studies will only be included if they attempt to analyse changes in population. Risk factor studies which do not include a temporal dimension will be excluded.

In contrast to mere reporting of incidences or annual prevalence, studies which estimate lifetime prevalence will be included. As we will show in Chapter 3, lifetime prevalence is linked to cross sectional prevalence and is an important indicator of population dynamics.

Table 2.1. Inclusion and Exclusion Criteria

Inclusion criteria	**Exclusion criteria**
Population	
• Studies focusing on children and young people involved in CPS	• Studies not focusing on children and young people involved in CPS • Studies focusing on outcomes for carers, service providers, and other professionals involving in CP works
Concept	
• Studies analysing and explaining the CP population structure and/or trends of CP populations (non-regression-based methodologies) • Studies on factors associated with child maltreatment or CP involvement at the aggregate level, aiming to explain temporal changes (regression-based methodologies) • Studies on lifetime prevalence or cumulative risk of involvement with CPS • Studies including all types of abuses	• Studies merely reporting incidences or prevalence (population rates of children) of CP involvement, including trends (non-regression-based methodologies) • Studies merely calculating and comparing transition rates between different groups • Studies on factors associated to child maltreatment or CP involvement either at the individual level or the aggregate level, without aiming to explain temporal changes (regression-based methodologies) • Studies focusing only on individual types of abuse • Studies focusing on interventions and their impacts on CP outcomes. • Qualitative studies
General	

• Peer reviewed journal papers
• Studies published in English
	• Grey literature
	    ○ Book chapters
	    ○ Reports
	    ○ Dissertations and theses
	• Studies not published in English

We also focus on studies that examine child maltreatment in general, instead of those focusing only on specific types of maltreatment, such as on physical abuse only or on sexual abuse only. Preliminary searches have revealed that studies inclusive of maltreatment types and studies focusing on specific maltreatment type often use similar methodologies. Including only studies which include all maltreatment types is a sensible criterion for a narrative review.

Lastly, we focus only on peer-reviewed journal articles. We understand that by excluding grey literature such as dissertations and research reports, we may miss some literature of relevance. However, it is our viewpoint that new methodologies used in dissertation studies and research reports will eventually make it into the sphere of peer-reviewed publications. This is indeed the case for several selected studies which were based on dissertation works.

In a scoping review, at least two reviewers are required to screen the search results in two rounds using the eligibility criteria – first by title and abstract, and then by full text. Since this is a narrative literature review informed by systematic search, the final selection of articles is conducted solely by the author of the dissertation, with no independent verification.

2.1.3 Information Sources

Four academic databases which are known to host a wide range of publications on child protection research are selected as the information sources, as listed below.

i. SocIndex (Sociology) on EBSCO
ii. CINAHL (Nursing and allied health) on EBSCO
iii. Medline (Life sciences and biomedical sciences) on OVID
iv. PsycINFO (Psychology) on OVID

By selecting databases focusing predominantly on four different fields in which social work scholars publish their research, the chance of locating relevant studies is enhanced. Preliminary searches confirmed that there are significant overlaps and unique variations among these databases.

2.1.4 Search Queries and Results

To effectively locate relevant articles for review, we conducted a series of preliminary searches, experimenting with various search queries. The search queries used in the final search is shown in Table 2.2, for the CINAHL database on the EBSCO platform. The queries are based on two key themes – child protection and population dynamics and the final search was based on the following steps:

1. For the "child protection" key theme, three subgroups of terms were used and the results were combined with the OR operator.
2. For the "population dynamics" key theme, three subgroups of terms were used for "population". The search results for these subgroups were combined with the OR operator. This was, in turn, combined with the search results with terms representing "dynamics" using the AND operator. Another group of specialized terms associated with the "population dynamics" key theme was used separately.
3. Results from the two steps above were combined with the AND operator to obtain the final outcome.
4. To further reduce the number of irrelevant articles, we assembled a set of terms related to medical studies or gerontology studies and filtered out articles including these terms in their titles.

Final searches on other databases used the same search queries, with necessary changes of syntax in accordance with the platform used (EBSCO or OVID). Searches from all databases were compiled and imported into the Endnote reference management software. Automatic de-duplication was performed, resulting in 7205 articles eligible for screening. Search results were screened first by titles and abstracts, and then by full text, resulting in 34 articles selected for review. Three additional articles were identified from the references in the originally selected article, bring the total of studies reviewed to 37. A screening flow chart is not provided as in a scoping review since the screening was not done following standard scoping review protocol. It should also be noted that among the articles selected are 2 studies which did not fully meet the eligibility criterion for study population, with OOHC children being only parts of the study population. However, since these studies used unique methodologies not seen in other selected studies, they are included for review. This will be highlighted in the narrative.

Table 2.2. Search Queries for CINAHL

CINAHL (EBSCO)	
[1] Child protection, OR Combination of three subgroups below	
Search TI, SU, KW and AB Peer-reviewed journals	"child protect*" OR "child welfare"
	(infan* OR child* OR minor* OR toddler* OR baby OR babies OR adolescent* OR teen* OR "young person" OR youth OR "young people") N1 (maltreat* OR neglect* OR abuse*)
	"foster care" OR "group home" OR "group care" OR "residential care" OR "congregate care" OR "kinship care" OR "relative care" OR "customary care" OR "shelter care" OR "temporary care" OR "looked after child*" OR "child place*" OR "place* in care" OR "out-of-home care" OR "out of home care" OR OOHC OR "foster child*" OR "foster youth" OR "kinship guardian*"
[2] Population dynamics, OR Combination of [2a] with [2b]	
[2a.1] Population	population* N2 (infan* OR child* OR minor* OR toddler* OR baby OR babies OR adolescent* OR juvenile OR teen* OR “young person” OR youth OR “young people”)
	number N2 (infan* OR child* OR minor* OR toddler* OR baby OR babies OR adolescent* OR juvenile OR teen* OR “young person” OR youth OR “young people”)
	percent* N2 (infan* OR child* OR minor* OR toddler* OR baby OR babies OR adolescent* OR juvenile OR teen* OR “young person” OR youth OR “young people”)
	OR Combination of three subgroups above
[2a.2] Dynamics	grow* OR trend* OR chang* OR evol* OR distribution* OR movement
[2a], AND combination of [2a.1] with [2a.2]	
[2b] Specialized terms	demograph* OR epidemiolog* OR dynamic* OR space-time OR spatio-temporal OR "stock and flow" OR caseload* OR "cumulative prevalence" OR “cumulative risk” OR "lifetime prevalence" OR “life table” OR "placement stability" OR "placement breakdown" OR re-entry OR reentry OR re-enter* OR reenter* OR rereturn OR re-return OR reuni* OR re-uni* OR rerefer* OR re-refer* OR recurren* OR "ag* out" OR "carrying capacity" OR "system* capacity" OR discontinuity OR transition* OR forecast* OR predict* OR pathway* OR trajector*
Final result: [1] AND [2]	

Methodologies used in these studies include:

1. Diagnostic analysis
2. Regression analyses
3. Markov models of transition rates

4. Simulation study on the effects of entry cohorts
5. Bayesian spatio-temporal models
6. Latent growth curve analyses
7. Lifetime prevalence or cumulative risk analyses
8. Agent-based model
9. Risk terrain model
10. Convergent cross-mapping of admission and discharge rates

2.3 Review of Methodologies

In this section, a narrative review of individual studies in each of the methodology groups above is provided in chronological order. Note that the term "foster care" and OOHC are used interchangeably as some authors used the former to denote all types of OOHC.

2.3.1 Diagnostic analyses

Most of the studies in this category seek to analyse and explain changes in foster care population, i.e. foster care caseloads, which has been a persistent concern for child welfare agencies. These studies mainly examined time series of admission and discharge rates or descriptive statistics for epidemiological analysis.

Prompted by the increase in the number of children needing care by local authority since 1970 in the UK, Boldy and Howell (1980) devised a simple model to forecast the number of children in care, according to type, in the South West region for use in service planning. The model consists of two components: (1) forecasting the number of children remaining in care at the end of forecast period, starting from a base number at initial time; (2) forecasting the number of children entering and remaining in care within the forecasting time frame. These two components were estimated using past rates of admission and discharge for different categories of children (sex, legal status, and age), taking into account the projected population growth. Professional advice was sought with regard to the assumed rates of admission and discharge, with possible adjustments made accordingly.

As in the UK, a surge in child maltreatment reports beginning in the late 1970s USA prompted similar efforts of estimating and forecasting foster care population. Between 1978 and 1984, the number of child maltreatment reports increased by 86% (Ards, 1989). Policy response and resource allocation were hampered by the lack of data at the local or county level. There was thus a need to estimate the rate of abuse at the local level, using national level data such as those obtained in the National Study of the Incidence and Severity of Child Abuse and Neglect conducted in 1979. Ards (1989) deliberated on three methods to estimate local rates of child abuse. The first method simply assumes that local abuse rates is proportionate to national abuse rate, in accordance with the proportion of local population within the national population. The second method is based on the arguments that proportionality of local abuse rates to national abuse rates can only be applied to subgroups in the populations. Thus, one must first decompose the population into subgroups, calculate the abuse rates proportionately within the subgroups, and then aggregate them to obtain the local abuse rates. The third method is based on the observations that child abuse depends on social and demographic factors. Therefore, estimations should take into account social and demographic variations across local populations. This approach makes use of regression methods, with selections of predictors based on existing knowledge on structural factors in child abuse.

In the early 1990s, the need to examine the influences of Adoption Assistance and Child Welfare Act of 1980 on foster care populations came to the fore of the field. Passed by the federal government in 1980, the Act formally adopted the principle of attaining permanency, mainly by moving children out of temporary substitute care as soon as possible and reunifying them with their natural family (Wulczyn, 1991). This practice paradigm may increase foster care discharge, which in turns may increase the risk of recurrence of maltreatment and thus re-try into care, creating a positive or reinforcing feedback loop. If the capacity of foster care system is fixed (e.g. limited bed space in residential care), the rise of population in care will in turn hasten the discharge of children in care, leading to a vicious cycle of increasing population. These phenomena were examined by Wulczyn (1991), who used New York State data to study three issues: (1) the relative sizes of repeat and new clients entering care; (2) risk of re-entry using proportional hazard models; and (3) the size of an entry cohort in relation to brief placements. The study provided evidence that re-entry had contributed to the observed rise of population in care. It also established the importance of understanding the subtle interplay between admissions and discharges in the studies of foster care population dynamics.

In a similar vein, Wulczyn & George (1992) examined admission and discharge rates in-depth to understand the rapid rise of foster care population in New York and Illinois in late 1980s. They were able to establish that the rise was related to an increase in foster care admission of young children, especially those under the age of 10. While this study did not use formal statistical models to study the population of concern, it showed that descriptive analysis of time series could go a long way in explaining observed trends. In fact, such an approach is fundamentally no difference from using a mathematical model of population dynamics, although the latter is more complex and complete, and is thus able to provide more insights into how the component groups of population interact.

In an attempt to explain the significant rise of children in care between 1993 and 1998 in Ontario, Trocmé, Fallon, MacLaurin, & Neves (2005)conducted an extensive analysis of various variables, including substantiation levels, maltreatment types, child characteristics, case characteristics, investigation outcomes, and source of referrals. Using cross-tabulation of descriptive statistics, they deduced that the increase could not be explained by the lowering of intervention threshold. Rather, there was a distinct shift in maltreatment types and their rates. This study again illustrated the usefulness of judicial application of descriptive statistics.

In mid-2000s, point-in-time data in the USA showed that there had been a sharp decline of the number of children in care, especially those in kinship care. This prompted the important question of whether kinship care usage has decreased or increased. Using four years of data from the Adoption and Foster Care Analysis and Reporting System (AFCARS), Vericker, Macomber, & Geen (2008) demonstrated that while the point-in-time number of children in kinship care indeed declined, the number of children entering care, and thus the use of kinship care across the vast majority of states actually increased. While this study only made use of descriptive statistics, it was able to unravel important insights in how we interpret foster care data. In particular, it showed that using point-in-time data alone to assess foster care population trends can obscure important population dynamics.

2.3.2 Regression analyses

Tucker and Hurl (1992) studied the populations of foster homes in the regional municipality of Hamilton-Wentworth, Ontario from 1968 to 1990. They took foster home market entry as an arrival process and assumed that the rate of arrival could be modelled by an exponential function of environmental factors, including average foster care entry age, legislative change, labor participation rate of women, economic incentives; and intrapopulation factors such as density of foster homes and service demand. Time lag for each of the independent variables was estimated using cross-correlation between time-series of foster home entries and that of the variables. This resulted in a compound Poisson regression model. Maximum likelihood estimation was then performed to obtain the coefficients of the independent variables. The study established the importance of dynamic change processes in getting a better understanding of populations of foster homes, with the eventual aim to facilitate planning and improve the wellbeing of children in care. This study is, strictly speaking, not a study of CP population. It is nevertheless included in the literature review as the number of foster home population is intimately related to CP population and such studies are very rare.

Prompted by the tripling of children reported for child maltreatment between 1976 and 1992 in the USA, Albert & Barth (1996) analysed the relationship among the number of maltreatment reports and a number of demographic, social and economic factors of child maltreatment reports in 18 urban, suburban and rural counties of California, using a fixed effect regression approach with monthly time series. Differences in predictors were found among the different county types although each fixed-effects model was able to explain about 98% of the variance in monthly count of child maltreatment reports. The model was then used to predict the number of maltreatment reports from January 1985 to December 1991. The predicted series was able to replicate the actual series in each of the counties to a large extent, although the results for rural counties were poorer, perhaps due to the lower and less accurate counts of maltreatment reports.

Using a similar model, Albert & King (1996) examined the effect of funding mechanism on the number of maltreatment reports in 7 counties in California, using data from January 1985 to December 1992. The model differed from that described in the previous paragraph with the number of emergency child maltreatment responses used as the dependent variable, and funding mechanism (caseload-based or population-based) added as an explanatory variable. The results suggested that a caseload-based funding mechanism could unintendedly drive up CP caseloads.

In an attempt to explain the doubling of foster care caseloads from 1985 to 2000 in the USA, Swann & Sylvester (2006) used an ordinary least square regression model with state fixed effects and year effects to analyse the relationship between foster care rate and state-specific factors and child-family specific factors. They found that the growth in foster care caseloads from 1985 to 2000 could be explained by increase female incarcerations and reduced cash welfare benefits.

Parental substance abuse has long been established as a factor associated with child maltreatment (e.g. Wolock & Magura, 1996). To examine the issue of alcohol use and child maltreatment from an ecological perspective, Freisthler, Gruenewald, Remer, Lery, & Needell (2007) studied the relationship between alcohol outlet density and CP referrals, substantiations, and foster care entries in 579 zip codes in California, using panel data spanning 1998 to 2003. Statistical analyses using spatial random effects panel models with generalized least squares estimation showed that zip codes with density of off-premise alcohol outlets (e.g., convenience or liquor stores) had higher maltreatment rates. The study also estimated the impact of the decrease in off-premise outlet per zip code on the reductions in CP referrals, substantiations, and foster care entries.

Hiilamo (2009) used regression models to analyse the large increase in OOHC placements in Finland from 1991 to 2006 despite growth in GDP. In the study, sub-region was used as the unit of analysis, the number of children in care was the dependent variables, and indicators on risk factors such as family structure, welfare, parental mental health, etc. were explanatory variables. Single year and time series regressions were conducted using two different sets of factors, and each analysis was repeated with ordinary linear regression and generalised linear model. The results suggested that OOHC placement in Finland was associated with long-term economic hardships, with the rate of change over the years associated with alcohol and substance abuse.

Since the late 1990s, the private sector has played an increasingly prominent role in foster care provision in the USA (e.g. Westat & Chapin Hall Center for Children, 2002). Research has shown that in Florida, where the private sector has assumed core responsibilities of service provision in the foster care system, with the role of public agency increasingly reduced to that of contract manager, the percentages of re-abuse and re-entry into care have risen (Office of Program Policy Analysis and Government Accountability, 2006). This prompted Steen & Duran (2013) to examine the impact of foster care privatisation on multiple placements in Florida. Hierarchical linear analysis models were used with a data set consisting of 47 of Florida's 67 counties and annual time points from 2002 to 2007. The dependent variable was county-level percent of foster children with fewer than three placements and the explanatory variables were privatisation policy indicators and rate of children in care. The results showed that post-privatisation years had significant lower percentages of children with fewer than three foster care placements although the effect varied across counties.

While studies on changes in CP populations based on the ecological perspective have been quite common in the USA, such studies were relatively rare in non-Western and non-Christian countries, especially those that examined temporal changes. Partially to fill this gap, Ben-Arieh (2015) used two-points-of time fixed effects regression analysis to explore changes in the rates of reported child maltreatment and socio-economic factors and availability of social services. The data set consisted of 169 Israeli localities of various sizes. Their results supported hypotheses that ecological factors and availability of social services were correlated with reported child maltreatment rates.

2.3.3 Markov models of transition rates

In a study which was closely related to population dynamic modelling, Pandiani, Maynard, & Schacht (1994) used a first order Markov transition model to predict movement of children with severe emotional disturbance among five types of living situations – living in their own family, living in foster care, living in residential mental health treatment, living in detention centre, and living in other arrangements. In such a model, each transition between various types of living situations is an event that is independent of past history. Using residential placement data collected at 6-month intervals by interagency teams in Vermont from 1990 to 1992, transition probabilities between different types of living situations were calculated. When starting from an initial condition where all children lived with their own families, the model quickly progressed to a state in which different types of living situations assume stable proportions. The model was used to predict how the populations of various living situations react to hypothetical changes in the transition probabilities, which were, in turn, due to policy or practice changes. This study is, strictly speaking, not a study of population dynamics of CP population, as its scope of was Vermont's system of mental health residential care, with foster care being only one of the five types of living situations. It is nevertheless included in the literature review as it is partially related to foster care and the methodology can easily be applied to CP populations.

2.3.4 Simulation study on the effects of entry cohorts

Simulation methods have long been a staple tool in population study (e.g. Barrett, 1969; Dyke & MacCluer, 1974). However, such methods have not been widely used in the study of CP populations. In a study on the use of outcomes such as length of stay in foster care to evaluate child welfare system performance, Wulczyn, Kogan, & Dilts (2001) employed the simulations of annual entry cohorts comparable to a medium size foster care system in the USA to examine how sizes of entry cohorts, average length of stay for the cohorts, and case mix (mixture of multiple distributions of length of stay) affect the observed length of stay. Through the simulations and analysis of empirical data, the authors illustrated how observed length of stay is affected by basic population dynamics and sampling strategy. Based on the analysis, the authors suggested that the use of exit cohort data was problematic, and recommended that for federal reporting system, longitudinal data based on entry cohorts should be the norm. This study serves as a good example of how simulation methods can be used to unravel effects of basic population dynamics which are hard to explain based on observed data alone.

2.3.5 Lifetime prevalence or cumulative risk analyses

Lifetime prevalence or cumulative risk is an important indicator of child maltreatment and an important aspect of CP population dynamics. Interpretation of the risk of child maltreatment based on annual incidences do not capture the full extent and severity of such risk. Moreover, cumulative risk allows the risk of maltreatment over children's developmental span to be estimated. To estimate cumulative risk, two approaches are often used – longitudinal study with one birth cohort or cross-sectional study with synthetic cohorts. The latter has the advantage of using data from a fixed time period, normally one year, to estimate age-specific cumulative risk, instead of having to follow a birth cohort from birth to the age of interest. In cases where maltreatment risks change gradually from year to year, such an estimation is not likely to differ significantly from that obtained using birth cohorts.

Life table is a common method in epidemiology and demography (e.g. Preston, Heuveline, & Guillot, 2001). Sabol, Coulton, & Polousky (2004) were one of the first to estimate cumulative risk of maltreatment using the life table approach. Using records of child maltreatment reports in Cuyahoga County in Ohio between 1999 and 2001, they constructed synthetic cohort (or period) life tables to estimate the probability or cumulative risk of child maltreatment up to 10 years of age. It was found that 31.0% of children were involved in at least one child maltreatment investigation by age 10, with 19.5% had at least one substantiated maltreatment. Cumulative risk was also estimated based on race and urbanicity.

While Sabol et al. (2004) studied the age-specific cumulative risk of child maltreatment in one county, Wilderman et al. (2014) studied the cumulative risk of foster care placement across the USA using data from the Adoption and Foster Care Analysis and Reporting System (AFCARS). Synthetic cohort life tables were constructed annually from 2001 to 2011, up to age 18. It was found that in 2005, the cumulative risk of foster care placement was 5.91% up to age 18. Moreover, the risks for Native American children (15.44%) and Black children (11.54%) were much higher compared to the national risk and to other major ethnic groups. Overall, cumulative risk of foster care placement decreased from 2001 to 2011.

Using a similar approach, Wildeman & Emmanuel (2014) constructed synthetic cohort life tables with the National Child Abuse and Neglect Data System (NCANDS) data spanning 2004 to 2011 to estimate cumulative prevalence of substantiated maltreatment by 18 years of age. They found that in 2011, the cumulative risk of substantiated maltreatment was 12.5% for US children by 18 years of age. Moreover, minorities such as Black, Native American, and Hispanic children had higher cumulative risk compared to White children. On the other hand, Asian/Pacific Islander children had the lowest risk. The results indicated that the risk of child maltreatment is drastically understated if it is interpreted based on annual rates of substantiated maltreatment, which was around 10%.

Fallesen, Emanuel, & Wildeman (2014) replicated the study of Wilderman & Emanuel (2014) to study cumulative risk of foster care placement for Danish children, using Danish registry data from 1998 to 2010. Synthetic cohort life tables constructed from the data showed that in 1998, the cumulative risks of foster care placement were similar for Danish children and American children. However, the cumulative risk for Danish children showed larger decline in subsequent years, reaching only half the value for American children in 2010. In contrast to the US, variations by ethnicity was much smaller than those among Danish children. Moreover, age-specific risk profile exhibited higher risks for older Danish children, in contrast to the profile for American children, which exhibited higher risk for children in the first few years of life.

Using a birth cohort approach instead of a synthetic cohort approach, Ubbesen, Gilbert, & Thoburn (2015) examined the cumulative incidences of OOHC entry of children before their 16th birthday in Demark and 8 localities in England. Data from 1992 to 2008 were grouped into 6 cohorts to avoid low counts and to simplify the analysis. Life tables were constructed for each cohort. The results revealed that the cumulative OOHC placement risk has risen in the local authorities in England over the years, while the opposite was true in Denmark. It was also found that cumulative risks of placements were similar in England and Denmark by 3 years of age. However, the risk increased for older children in Denmark compared to that in England.

Also using the birth cohort approach, Mc Grath-Lone, Dearden, Nasim, Harron, & Gilbert (2016) examined the cumulative proportion of OOHC placement from 1992 to 2011 for children in England. The data were grouped into 7 cohorts, changes in cumulative risk were examined over time, and the determinants of the changes were explored using decomposition methods. They found that Black children had much higher cumulative risk of OOHC placement than White children did, and over time, the cumulative risk of placement had increased. Decomposition analysis showed that the overall increase was driven mainly by the increase in cumulative risk of placement among White children.

In a similar vein, O'Donnell et al. (2016) studied the cumulative incidence of OOHC placement in the Australian state of Western Australia (1994 to 2005) and the Canadian province of Manitoba (1998 to 2008). Children were grouped into 3 cohorts and tracked over the time span of the respective data set. Multivariate Cox regression analysis was used to examine the relationships between OOHC placement and risk factors at the individual and socio-economic levels. The results showed that the risk of placement was much higher in Manitoba than Western Australia by age 12. Moreover, in both jurisdictions, children were placed in OOHC at a younger age over time.

Kim, Wildeman, Jonson-Reid, & Drake (2017) extended the study of Wildeman et al. (2014) using child maltreatment data from the National Child Abuse and Neglect Data System Child Files and population census data from 2003 to 2014 to estimate the lifetime prevalence or cumulative risk of involvement in child maltreatment investigations in the US. Specifically, their study examined all maltreatment investigations instead of just substantiated maltreatment. They also provided estimations for different maltreatment types. The results, obtained using synthetic cohort life tables, showed that the cumulative risk of involvement with child maltreatment investigations was 37.4% by18 years of age.

Rouland & Vaithianathan (2018) studied the 1998 birth cohort in New Zealand longitudinally until 2015 to calculate the cumulative prevalence of involvement with child protective services, including child maltreatment notifications, substantiated maltreatment, and OOHC placements. Unlike most other studies, they were able to take migration into account. They found that the lifetime risk of CPS involvement, at 17 years of age, was 23.5% for notification, 9.7% for substantiation, and 3.1% for OOHC placement, with the risk being higher for boys.

Using synthetic cohort life tables, Fong (2019) estimated the cumulative risk of involvement in CPS investigations and substantiations for children living in different types of neighbourhoods in Connecticut, USA from 1997 to 2015. The results revealed substantial variations of involvement in CPS notifications by neighbourhoods, with the cumulative risk of children in high-poverty neighbourhoods being three times higher than that of those in low-poverty neighbourhoods.

Kim and Drake (2019) used national CPS records and Census data in the USA in 2015 to estimate the cumulative risk of onset and recurrences of child maltreatment reports by rank order – 1st occurrence to the 6th recurrence – from birth to age 11. In the study, rank order was identified using a longitudinal dataset constructed from the National Child Abuse and Neglect Data System Child Files from 2003 to 2015. The results showed that by age 12, the cumulative risk of being involved in maltreatment reports was 32.41% for at least 1 report, 13.71% for at least 2 reports, 7.57% for at least 3 reports, 4.50% for at least 4 reports, 2.80% for at least 5 reports, and 1.79% for at least 6 reports. Moreover, the risk of recurring involvement increased with the number of prior reports. The study also found that onset and recurring rates were much lower for

Asian/Pacific Islanders than for other ethnic groups while onset rates were higher for Blacks, Native Americans, and Hispanics than for Whites. However, the rates of recurrence were slightly higher for Whites than for non-Whites after the first involvement.

2.3.6 Bayesian spatio-temporal models

Bayesian spatio-temporal mapping is another common methodology in public health and epidemiology research. However, it has not been used in the field of child maltreatment research until recent years. While this approach can also be classified under the rubric of regression analyses, it is treated as a separate category due to its use as a unique methodology in disease studies.

One of the earliest studies using Bayesian space-time method was conducted by Freisthler & Weiss (2008). Using annual panel data of 58 California counties over 4 years, they employed conditionally autoregressive Bayesian models to examine the spatio-temporal relationships between child maltreatment referrals and a number of substance use and welfare factors. The results showed that maltreatment referrals in Californian counties were affected by both spatial and temporal processes, with significant correlations between referrals and some of the explanatory factors.

To study the relationship between changes in drug market activity and changes in child maltreatment rates, Freisthler, Kepple, & Holmes (2012) employed Bayesian conditionally autoregressive space–time models using a data set covering 95 census tracts in California over a 7-year time period (2002 to 2008). The results showed a pattern of spatial and temporal lagged correlations between drug sale and child maltreatment referrals, substantiations, and foster care entries.

Gracia, Lopez-Quilez, Marco, & Lila (2017) conducted a study in the city of Valencia, Spain, using data of 552 census block groups spanning a period of 12 years (2004 to 2015). The aim was to analyse the relationships between neighbourhood characteristics and spatio-temporal patterns of child maltreatment risk, using Bayesian spatio-temporal models and disease mapping methods. In addition to illustrating linkage between unequal distribution of neighbourhood risk factors and unequal distribution of substantiated child maltreatment in urban areas, the study also showed the importance of including temporal changes in analysing variations in child maltreatment risk.

Without taking the temporal dimension into account, trends of changes in risk would not be detected.

In a similar vein, Barboza (2019) studied the spatio-temporal risk of child maltreatment in Los Angeles. Bayesian hierarchical spatial models with integrated nested Laplace approximations were used to analyse counts of substantiated maltreatment across 1678 census tracts spanning four years (2006 to 2009). Areas of increased risk and the relationships between structural disadvantage and substantiated child maltreatment were identified.

2.3.7 Latent growth curve models

Kim, Chenot, & Ji (2011) analysed disparity index (DI) scores in 48 counties in California from 2005 to 2008 to study disparity levels and long-term disparity changes among different ethnic groups in the child welfare system, and to investigate how their relationships changed over time with county-level factors such as child poverty rates, unemployment rates, and rurality. The study used latent growth curve to estimate the trajectories of DI scores, obtaining the average DI score at baseline and the average rate of change in DI scores. Based on the variations in the average DI scores at baseline and average rate of change in DI scores, contextual factors which might explain DI scores across counties were explored.

2.3.8 Agent-based model

To simulate the dynamics of child maltreatment and the impact of child maltreatment prevention, Hu & Keller (2015) developed an agent-based model which incorporated multi-level social ecological factors of child maltreatment. Agents in the model are families and behaviours of individual agents were informed by functions representing child need, family resources, social network and community openness, parental efficacy, and stress level, based on literature associated with theory of planned behaviour, self-efficacy theory, and models of parenting stress. These functions specify the behavioural and cognitive modelling of the agents, accounting for the behavioural and cognitive process and underlying mechanisms of child maltreatment. The impacts of different factors were tested in model simulations. By adding functions representing intervention and prevention, the impacts of child maltreatment prevention and intervention strategies, such as those aiming to reduce community stress and build social connections, were also simulated.

2.3.9 Risk terrain model

Risk terrain modelling (RTM) is a type of geospatial analysis which uses mapping techniques to predict potential hotspots associated with a particular crime outcome (Caplan, Kennedy, & Miller, 2011). RTM starts with identifying all risk factors in the literature of a particular outcome and then assigning a value to each of the risk factors across a given geography. This produces a coverage or raster map for each of the risk factors which are combined in a Geographic Information System (GIS). The resulting product is a risk terrain map on which every unit on the map is assigned a composite risk value associated with the particular outcome, thus enabling the prediction of potential hotspots for the outcome. In other words, instead of using past statistics associated with the outcome, it uses current values of established factors associated with the outcome to predict it (Caplan et al., 2011).

Daley et al. (2016) employed RTM which incorporated environmental risk factors of child maltreatment to predict child maltreatment substantiations in Fort Worth, Texas. Address-level risk values were calculated using data of substantiated child maltreatment, poverty, domestic violence, aggravated assaults, runaways, murders, and drug crimes, obtained from various sources in 2013. Model prediction was validated against actual child maltreatment substantiation data in 2014. Fifty-two percent of all cases in 2014 were predicted in the 10% of highest risk cells; almost all incidents of substantiated maltreatment were located in cells with elevated risk prediction.

2.3.10 Convergent cross-mapping of admission and discharge rates

Prompted by the perceived lack of research using the systems science perspective to tackle the issue of improving foster care systems, Wulczyn and Halloran (2017) turned to the field of population biology to examine whether resource restraints in the form of congregate care bed space led to a coupling of admission and discharge. Using administrative records in a mid-sized state in the USA, the daily counts of children under 18 years of age admitted to and discharged from foster care were calculated over a period of 15 years, resulting in two time series for analysis. The salient method used in this study is convergent cross-mapping (CCM), which was developed in the field of population biology to test for causal relationships in time series from the same system. Specifically, the method produces CCM coefficient values which are high for time series coupled by some underlying structure or mechanism in the system. It is thus a tool to distinguish between random and non-random time series. Two CCM analyses were performed, one using exits as predictor and entries as outcome (exits-favor-entries) and the other using entries as predictor and exits as outcome (entries-favor-exits). The results showed evidence that foster care admission and discharge were coupled by some underlying structure, which was hypothesised to be bed space constraints in congregate care.

2.4 Summary

The methodologies reviewed above can also be classified according to their usage, at different levels of complexity: 1. Analysing and explaining population structure and its changes; 2. Quantifying association of factors with population structure and its changes; 3. Forecasting population structure and its changes; 4. Modelling population structure and its changes.

2.4.1 Analysing and explaining population structure and its changes

Many of the studies listed above were driven by the need to analyse and explain population structure and its changes. All 10 methodology groups can serve this purpose.

2.4.2 Quantifying association of factors with population structure and its changes

In addition to providing explanation, there is often a need to quantify the association of factors with population structure and its changes. Methodological groups such as regression analyses, Bayesian spatio-temporal models and latent growth curve models have been used for this purpose.

2.4.3 Forecasting population structure and its changes

Among the methodologies reviewed in this section, some regression models, Markov models of transition rates, and risk terrain models have explicitly been used for forecasting or predicting purposes. It should be noted that in general, the suitability of a regression-based model for forecasting purposes hinges on its ability to explain a high percentage of the variance in the dependent variable. While the Bayesian spatio-temporal models and agent-based model were not used for such a purpose in the studies reviewed, they have been used for forecasting purposes by researchers in other contexts.

2.4.4 Modelling population structure and its changes

Modelling can either be done to obtain insights into population structure and its changes, or to simulate how population structure evolves with different scenarios. Markov models of transition rates and agent-based model were explicitly used for this purpose.

2.5 Conclusion

The narrative review informed by systematic search shows that since early 1990s, macro- or aggregate level dynamics of CP populations has been a topic of considerable concern over the years, partly due to the need of policy response and resource planning. Diagnostic analyses, regression analyses, and lifetime prevalence (cumulative risk) analyses were some of the most commonly used methods. In recent years, researchers have increasingly turned to complex methods developed in other fields, including econometrics, geoinformatics, and population biology. However, to the best of our knowledge, there has been no study that formally constructs a mathematically based population dynamics model of CP populations. We intend to take the first step to filling this gap by conceptualizing population dynamics models of up to four sub-populations, deriving the corresponding differential equations, making assumptions to simplify the equations, solving the equations, and using data to illuminate how such an approach may help us deepen our understanding of the interactions of CP sub-populations. We attempt to show that such an approach can provide us with a valuable tool to study both the cross-sectional and lifetime prevalence of CP involvement, in addition to simulate the trend of CP sub-populations in response to changes in transition rates.

Chapter 3: Methodology

This study employs a population dynamics model to investigate the macro-dynamical structure of populations of children involved with CPS. In particular, this study seeks to better understand how observed flows of children in child maltreatment investigation, and entry into and exit from OOHC influence the dynamic evolution of sub-populations including OOHC population, children previously in OOHC, and children involved in CPS but not in OOHC. These flows are necessarily governed by factors such as policy mandates, decision making practices and system capacity of OOHC. However, in terms of macro-dynamics of the populations, the observables are the stocks and flows of children across the various components of the CP system, namely, counts of children in notification investigation, substantiation, OOHC entry, OOHC, and OOHC exit. It is based on these stocks and flows that population dynamic models can be constructed and constrained to better understand the CP system.

In this chapter, the population dynamics model is conceptualised, the differential equations corresponding to the conceptual model are derived, and simplified versions of the equations are analytically solved. While the analytical solutions do not describe aspects of the system neglected in the process of simplification, they do provide important insights to the morphology of the system.

The population dynamics models set up in this chapter will be used to study the following topics:

1. Obtaining insights into the morphology of the populations and the relevant flow rates across CPS, including prevalence of involvement with CPS.
2. Studying how CP populations respond to external shocks such as changes in investigation and placement rates.
3. Modelling the current levels of OOHC populations using data from Australia.

3.1 A General Four-population Model of CP Systems

3.1.1 Conceptualizing a Population Dynamics Model

The processes of CP potentially ending with a child being placed into state care is described in detail in Chapter 1 and delineated with a schematic flow chart shown in Figure 1.1. From a simplistic perspective, the process of statutory CP is initiated by a case of suspected maltreatment being reported to the CPS. If the case is deemed to be related to children and young people (CYP), it is formally logged and investigations into the case will be carried out. Such investigations may consist of more than one stages, which may include face-to-face interviews with the children and the carers. If a case of alleged maltreatment is substantiated, the children involved may be removed and enter into OOHC.

Being involved in each phase of the process described above carries its corresponding consequences. For example, CP data in Australia have shown that repeat clients[3] account for most of the cases in various types of CPS, despite them being a smaller population group than children who have never been involved with CPS. This indicates that risk of further CPS involvement is higher on average for children who have previous involvement with CPS than those who have never been involved with CPS. Part of this may have to do with repeat clients continue to live in households with adult members who were subject to allegation of child maltreatment.

This is perhaps best illustrated by the use of history of maltreatment or alleged maltreatment in the decision-making process of child maltreatment investigation. For example, actuarial risk assessments such as those in structured decision making (SDM) (e.g. Department of Child Safety, Youth and Women, 2013) are point-based instruments with thresholds. Children in a household with adult members who were involved in past incidences of CP investigations add to the point total of the instruments and make it more easily to reach the substantiation threshold of the risk of harm. In a similar vein, children who were in OOHC previously but have exited care may be more likely to be substantiated if they are involved in another investigation.

Clearly, when constructing a population dynamics model for OOHC systems, one can classify children under 18 years of age in the general population into four distinct categories: 1. Children who have never been involved with CPS; 2. Children who have been investigated but have never been in OOHC; 3. Children in OOHC; 4. Children

[3] "Repeat clients are children or young people who have previously been the subject of an investigation, or who were discharged (according to national specifications) from any type of care and protection order or funded out-of-home care placement (excluding respite placements lasting less than 7 days), or whose earliest order and/or placement in the current reporting period is part of a preceding continuous episode of care." (AIHW, 2019a).

who have exited OOHC. The morphology of OOHC population cannot be fully understood unless the dynamics among these four populations is understood. As the first step, each population is assumed to be homogenous and no demographic distributions within the populations are considered. The homogeneity assumptions can be easily relaxed to accommodate more demographic categories.

To construct the population dynamics model used in this study, one can begin by delineating the relationships between the four populations with a diagram that depicts inflows and outflows linking the populations, as shown in Figure 3.1. In the figure, P_0, P_1, P_2, and P_3, enclosed in large circles, denote populations of children eligible for CP but have never been involved with CPS, children who have been involved in child maltreatment investigation but have never been in OOHC, children in OOHC, and children who have exited OOHC, respectively. The sum of these populations is the total number of children under 18 years of age in the general population. The arrows pointing towards the circles denote inflows while arrows pointing away from the circles denote outflows. Symbols alongside the arrows denote the flow rates which will be defined in the next section.

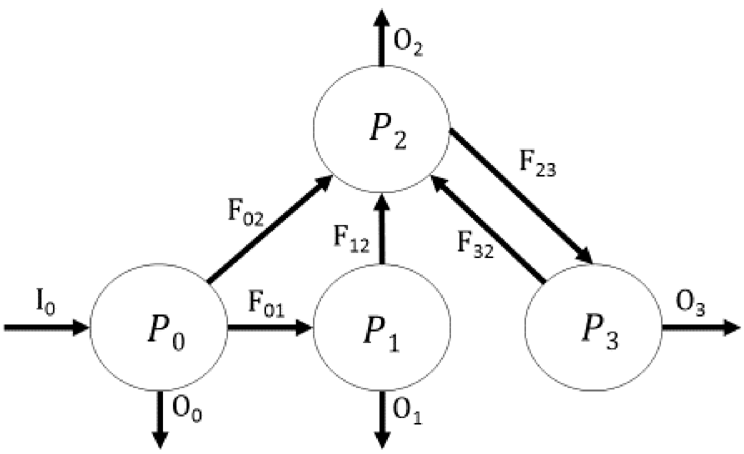


Figure 3.1 Four-population Model of CP Population

For P_0, population of children eligible for CP but have never been involved with CPS, there is only one inflow (I_0), which is the increase in children younger than 18 years of age due to birth and migration. Outflows include children growing beyond 18 years of age and thus are no longer eligible for CP (O_0). Since all populations exhibit such an outflow, for the sake of simplicity, the term “aged-out” will be used to denote the phenomenon of children flowing out of the populations having grown beyond the statutory age limit of 18 years old. The other two outflows denote children involving in child maltreatment investigation for the first time without going into OOHC (F_{01}) and children entering OOHC as the result of the first involvement with child maltreatment investigation (F_{02}). It is important to note that in our formulation, all inflows and outflows are defined as number of children per unit time.

For P_1, population of children who have been investigated but have never been in OOHC, the inflow is the children who have just been involved in child maltreatment for the first time without being sent into OOHC (F_{01}). There are two outflows, children aging-out (O_1) and children getting into OOHC as the result of subsequent involvement in a child maltreatment investigation (F_{12}). Note that children who have been involved in more than one child maltreatment investigation will stay in P_1 as long as they do not get into OOHC.

For P_2, population of children in OOHC, there are three inflows, children coming into OOHC from P_0 as a result of the first involvement with child maltreatment investigation (F_{02}), children coming into OOHC from P_1 having been involved in more than one unique child maltreatment investigations (F_{12}), and children coming into OOHC having exited OOHC previously (F_{32}). There are two-outflows, children aging-out (O_2) and children exiting OOHC for reasons other than aging out (F_{23}).

For P_3, population of children who have previously been in OOHC and still under the statutory age limit of 18, the inflow (F_{23}) is children leaving OOHC for reasons other than aging out, for example, reunifications. The outflows are children aging-out (O_3) and children re-entering OOHC (F_{32}) as the result of another involvement with child maltreatment investigation.

In the context of CP process described earlier, there are five set of factors that facilitate the flow rates and thus changes of the number of children in the populations:

1. *Growth* rate of children in the general population;
2. *Notification, investigation, substantiation, OOHC placement, and OOHC exit* rates of child maltreatment allegations for children who have never been involved with CPS;
3. *Notification, investigation, substantiation, OOHC placement, and OOHC exit* rates of child maltreatment allegations for children who have been involved with CPS but have never been in OOHC;

4. *Notification, investigation, substantiation, OOHC placement, and OOHC exit* rates of child maltreatment allegations for children who have been in OOHC but have exited care;
5. *Aging* of children beyond the statutory limit of 18-year-old.

As the decision-making ecology framework has recognized, the values of the CP related factors depend on a complex web of socio-economic conditions, decision making practices, CP policies, and CP resources, among others (Graham, Dettlaff, Baumann, & Fluke, 2015). Since the main objective of this study is to explore the macro-dynamical features of the OOHC populations, it will focus on using the observed flow rates – investigation rates, OOHC admission rates, and OOHC discharged rates, without explicitly referring to these conditions. However, the results of the macro-dynamical model may be discussed in the context of these conditions.

Having laid out the structure of populations related to OOHC, a set of differential equations of the population dynamic model can be derived for quantitative studies. This will be done by establishing functional forms of the flow rates, laying out the system equations, and making assumptions to derive models with different levels of complexity. Each of these models will facilitate our understanding of OOHC population dynamics at different levels of complexity.

3.1.2 Mathematical Formulation of the Flow Rates

Derivation of the differential equations begins with the following postulations: 1) there is a growth in the number of children eligible for CPS; 2) the populations decrease due to children aging out; and 3) the flow or movement of children from one population to another is proportional to the source population. The larger the source population, the larger it increases or decreases, and the larger the flow of children moving to another population will be. As such, the flows of children (apart from I_0 which will be dealt with separately), as indicated by the arrows pointing towards the circles in Figure 3.1, take the following forms:

$$O_0 = \eta_0 P_0$$

$$O_1 = \eta_1 P_1$$

$$O_2 = \eta_2 P_2$$

$$O_3 = \eta_3 P_3$$

$$F_{01} = \lambda_{01} P_0$$

$$F_{02} = \lambda_{02} P_0$$

$$F_{12} = \lambda_{12} P_1$$

$$F_{32} = \lambda_{32} P_3$$

$$F_{23} = \lambda_{23} P_2$$

Equivalently, the above equations can be re-written using i and j to denote the indices:

$$\eta_i = \frac{O_i}{P_i}$$

$$\lambda_{ij} = \frac{F_{ij}}{P_i}$$

In these expressions, η_i and λ_{ij} are time averaged variables representing the fraction of the growth, decrease, and flows per unit time in proportion to the source population, respectively. If η_i and λ_{ij} are time varying variables, the formulation of the flow being proportional to the source population will always be viable. The question is, of course, whether there are some time averaged values of η_i and λ_{ij} that can give good approximations to observed structures and trends in model simulations. This will have to be borne out using CP data.

The term I_0 is now considered. I_0 denotes the growth of P_0 due to growth in the general population. As noted previously, the sum of P_0, P_1, P_2 and P_3 is the total number of children under 18 years of age in the general population, which is designated as P.

$$P = P_0 + P_1 + P_2 + P_3$$

It is clear that P_0, P_1, P_2 and P_3 are none other than the partitioning of the general population of children under 18 years of age into four distinct categories.

It is assumed that number of children under 18 years of age in the general population grows with rate α. It is also assumed that the addition of children due to the growth of P leads to the growth of P_0 but does not result in a direct increase of the populations of children already involved with CPS. In other words, involvement with CPS has to originate from P_0.

With these assumptions, one obtains

$$I_0 = \alpha P$$

Now the λ_{ij}s can be further elaborated by observing that P_2, population of children in OOHC, is fundamentally different from the other populations. The capacity of OOHC is limited as it is constrained by resources allocated to the service, and also by the availability of qualified carers. As such, flows into P_2 will be limited by some system capacity or carrying capacity.

In classical population dynamics models, system capacity acts to limit the natural growth rate of a population and is often modelled by a logistic limitation term. In the case of OOHC population considered here, system capacity acts to limit the growth of OOHC population by modifying the flow rates from source populations. Despite this difference, it is still possible to borrow from classical population dynamics and impose a growth limiting term to flows into P_2.

In the presence of system capacity limit C_2, it is assumed that there is a flow adjustment term, Δ_{i2}, the form of which is to be specified, and the rates are rewritten as follows[4].

$$F_{i2} = \lambda_{i2} P_i + \Delta_{i2} \qquad \text{(Equation E.1)}$$

In the case of a logistic-liked limiting term,

$$F_{i2} = \lambda_{i2} P_i \left(1 - \frac{P_2}{C_2}\right) \qquad \text{(Equation E.2)}$$

Equivalently,

$$\Delta_{i2} = -\lambda_{i2} \frac{P_i P_2}{C_2}$$

This is akin to postulating a natural rate of removal into care, λ_{i2}, that is contingent on societal, economical, and political conditions, among others, and which is limited by how much the system can carry. It is imaginable that the effect of system capacity has a much steeper feature compared to the functional form assumed above. For example, system capacity limit may not come into play until the system is very near capacity, producing a step-function like effect with abrupt cut-off of inflows. In reality, how

[4] Only equations that are explicitly referred to in subsequent text are labelled.

system capacity affects the inflows have to be examined with actual observations of how placement decisions are made by case workers. Nevertheless, for the purpose of exploring dynamical properties, the above functional form is a reasonable first step.

More generally, the effect of system capacity can be modelled in the following manner.

$$F_{i2} = \lambda_{i2} P_i f_{i2}$$

In this formulation, f_{i2} is a multiplicative capacity factor that modifies the effect of the "natural" rate.

As defined in our formulation, there is no system capacity limit for the sub-populations P_1 and P_3, keeping in mind that these two sub-populations are not institutionalized populations. They are informal and are defined based on the different pathways of children through CPS. They can also be seen as labels used to establish group identifies based on the differing experience of children in CPS. As such, flows into P_1 and P_3 are not constrained by capacity inherent to the respective target populations. On the other hand, it is possible that flows into P_1 and P_3 may be influenced by the system capacity of P_2. For examples, when P_2 is near capacity, some children who may otherwise enter P_2 may end up in P_1, and outflow from P_2 to P_3 may be quickened to free up space for children who are perceived to need OOHC service more than some who are already in service.

It should be clarified that there are two types of capacity issues. The first is related to process bottleneck due to resource constraint while the second is the capacity of system to accommodate or carry a population. In the conceptual model shown in Figure 3.1, the first type of capacity constrains the flow rates while the second type of capacity constrains the level of a sub-population. For example, CP investigations may be limited by staffing issues, leading to many cases not fully investigated, and in turn affecting the flow rates into OOHC. However, as far as level of population is concerned, P_1 and P_3 are not capped by a potential upper limit due to resource constraint. It is essential to keep in mind that the concept of system capacity in this study refers only to the second type of capacity issues, not the first type.

3.1.3 Differential Equations of the System

With the flow terms formulated, the differential equations that govern the dynamic evolution of the system can be specified. Referring to Figure 3.1, it is observed that change of a population per unit time equals to the balance of inflows and outflows for the population. This can be described by the following equations.

$$\frac{dP_0}{dt} = I_0 - F_{01} - F_{02} - O_0$$

$$\frac{dP_1}{dt} = F_{01} - F_{12} - O_1$$

$$\frac{dP_2}{dt} = F_{02} + F_{12} + F_{32} - F_{23} - O_2$$

$$\frac{dP_3}{dt} = F_{23} - F_{32} - O_3$$

In the next section, the growth and decrease in a population, and the flows between populations are formulated as being proportional to the source population. Together with the formulation of the effect of system capacity in multiplicative form, the set of differential equations become:

$$\begin{aligned}\frac{dP_0}{dt} &= \alpha P - \lambda_{01}P_0f_{01} - \lambda_{02}P_0f_{02} - \eta_0P_0\\ \frac{dP_1}{dt} &= \lambda_{01}P_0f_{01} - \lambda_{12}P_1f_{12} - \eta_1P_1\\ \frac{dP_2}{dt} &= \lambda_{02}P_0f_{02} + \lambda_{12}P_1f_{12} + \lambda_{32}P_3f_{32} - \lambda_{23}P_2f_{23} - \eta_2P_2\\ \frac{dP_3}{dt} &= \lambda_{23}P_2f_{23} - \lambda_{32}P_3f_{32} - \eta_3P_3\end{aligned} \qquad \text{(Equations S.1)}$$

Although it was argued in the previous section that there is no system capacity limit for P_1 and P_3, the effects of system capacity are retained in the above questions for now and the effects of system capacity are explored in more detail.

Note that since $P = P_0 + P_1 + P_2 + P_3$, summing the above equations gives

$$\frac{dP}{dt} = \alpha P - \sum_{i=0}^{3}\eta_iP_i \qquad \text{(Equation E.3)}$$

This is the net growth rate in the number of children below 18 years of age in the general population. The second term on the right-hand side simply means that loss of general populations is the sum of the population loss of the four sub-populations. Since the number of children "aging out" from P is an observed quantity, it is a constraint in model simulations.

System Capacity

The terms involving f_{01} and f_{02} are now examined. It is noted again that P_1 designates the population of children who have received at least one investigation but have never been placed in OOHC while P_2 designates the population of children in OOHC. In actual practice, any children not transferred from P_0 to P_2, due to the latter reaching its system capacity C_2, will automatically be diverted into P_1 since they would have to be investigated. Therefore, any reduction in the flow from P_0 into P_2 will lead to a corresponding increase in the flow from P_0 into P_1. This is clear if the transfer terms are re-written in additive form. Specifically, by using *Equation E.1*, the two transfer terms in the equation for P_0 in *Equations S.1* can be re-written as

$$\lambda_{01} P_0 f_{01} = \lambda_{01} P_0 + \Delta_{02}$$

and

$$\lambda_{02} P_0 f_{02} = \lambda_{02} P_0 - \Delta_{02}$$

where Δ_{02} denotes the adjustment of flow into P_2 due to the effect of system capacity C_2. This leads to

$$\lambda_{01} P_0 f_{01} + \lambda_{02} P_0 f_{02} = \lambda_{01} P_0 + \lambda_{02} P_0$$

Therefore, the net outflow from P_0 is not affected by the adjustment due to system capacity C_2.

It is further assumed that the effect of system capacity C_2 is the same for all flows into P_2, i.e.

$$f_{02} = f_{12} = f_{32} = f_2$$

where f_2 indicates that the subscript for the source populations can be ignored. In practice, this condition means that decision making for placement into OOHC under the effect of system capacity does not depend on whether a child is a new or repeat client to CPS, or whether he or she has been in OOHC previously.

Remember that there are no system capacity limits for P_1 and P_3, i.e. $f_{i1} = f_{i3} = 0$. Further assume that when a system approaches the system capacity limit C_2, it does not quicken the release of children from OOHC, i.e., $f_{23} = 1$. The differential equations of the system, i.e. *Equations S.1*, become:

$$
\begin{aligned}
\frac{dP_0}{dt} &= \alpha P - \lambda_{01}P_0 - \lambda_{02}P_0 - \eta_0 P_0 \\
\frac{dP_1}{dt} &= \lambda_{01}P_0 + \lambda_{02}P_0 - \lambda_{02}P_0 f_2 - \lambda_{12}P_1 f_2 - \eta_1 P_1 \\
\frac{dP_2}{dt} &= \lambda_{02}P_0 f_2 + \lambda_{12}P_1 f_2 + \lambda_{32}P_3 f_2 - \lambda_{23}P_2 - \eta_2 P_2 \\
\frac{dP_3}{dt} &= \lambda_{23}P_2 - \lambda_{32}P_3 f_2 - \eta_3 P_3
\end{aligned}
\qquad \text{(Equations S.2)}
$$

Equations S.2 form the basis for simulating populations in OOHC systems.

Fractional Populations

Equations S.2 can be simplified, with the variable P eliminated by defining fractional population Q_i for $i = 0, 1, 2$ and 3, where

$$Q_i = \frac{P_i}{P}$$

And

$$\sum_{i=0}^{3} Q_i = 1 \qquad \text{(Equation E.4)}$$

The fractional system capacity is defined as

$$B_2 = \frac{C_2}{P}$$

Recall the logistic-liked multiplicative term described in *Equation E.2* in the previous section.

$$f_2 = 1 - \frac{P_2}{C_2} = 1 - \frac{\frac{P_2}{P}}{\frac{C_2}{P}} = 1 - \frac{Q_2}{B_2}$$

The final equality is obtained by substituting the expressions for Q_2 and B_2. This shows that as long as the effect of system capacity f_2 comes into play as some function of the proportion of current population to the system capacity limit, the multiplicative term f_2

can also be written as a function of the proportion of fractional population and fractional system capacity limit. This is equivalent to making the assumption that system capacity grows proportionately with population growth.

One can designate

$$H = \frac{1}{P}\sum_{i=0}^{3}\eta_i P_i = \sum_{i=0}^{3}\eta_i Q_i \qquad \text{(Equation E.5)}$$

By the chain rule of differentiation,

$$\frac{1}{P}\frac{dP_i}{dt} = \frac{d}{dt}\left(\frac{P_i}{P}\right) + \frac{P_i}{P}\left(\frac{1}{P}\frac{dP}{dt}\right)$$

Using *Equation E.3* and then *Equation E.5*, one obtains

$$\frac{1}{P}\frac{dP_i}{dt} = \frac{dQ_i}{dt} + Q_i\left(\alpha - \sum_{i=0}^{3}\eta_i Q_i\right) = \frac{dQ_i}{dt} + Q_i(\alpha - H)$$

Substituting the above expression into *Equations S.2*, one obtains

$$\begin{aligned}
\frac{dQ_0}{dt} &= \alpha - (\alpha + \lambda_{01} + \lambda_{02} + \eta_0 - H)Q_0 \\
\frac{dQ_1}{dt} &= (\lambda_{01} + \lambda_{02} - \lambda_{02}f_2)Q_0 - (\alpha + \lambda_{12}f_2 + \eta_1 - H)Q_1 \\
\frac{dQ_2}{dt} &= \lambda_{02}f_2Q_0 + \lambda_{12}f_2Q_1 - (\alpha + \lambda_{23} + \eta_2 - H)Q_2 + \lambda_{32}f_2Q_3 \\
\frac{dQ_3}{dt} &= \lambda_{23}Q_2 - (\alpha + \lambda_{32}f_2 + \eta_3 - H)Q_3
\end{aligned} \qquad \text{(Equations S.3)}$$

This is the set of differential equations for the fractional populations.

3.1.4 System with Uniformed Aged-out Rates and No Capacity Limit

Having set up the system equations, one can gain some insight into the morphology of the populations by examining a system with uniformed aged out rates across the populations and where there is no system capacity limit. While such a system inevitably involves simplifications of some terms in the full equations, it allows us to get an intuitive grasp of the interactions among the flow rates in influencing the populations.

With the condition of uniformed aged out rates, all η_i can be set to η and the term H defined in *Equation E.5* becomes

$$H = \sum_{i=0}^{3} \eta Q_i = \eta \sum_{i=0}^{3} Q_i = \eta \qquad \text{(Equation E.6)}$$

Further setting f_2 to 1 and rearrange the terms, *Equations S.3* become:

$$\frac{dQ_0}{dt} = \alpha - (\alpha + \lambda_{01} + \lambda_{02})Q_0$$
$$\frac{dQ_1}{dt} = \lambda_{01}Q_0 - (\alpha + \lambda_{12})Q_1$$
$$\frac{dQ_2}{dt} = \lambda_{02}Q_0 + \lambda_{12}Q_1 - (\alpha + \lambda_{23})Q_2 + \lambda_{32}Q_3 \qquad \text{(Equations S.4)}$$
$$\frac{dQ_3}{dt} = \lambda_{23}Q_2 - (\alpha + \lambda_{32})Q_3$$

This is a set of non-homogeneous first order differential equations which can be written in vector form

$$\frac{d\boldsymbol{y}}{dt} = A\boldsymbol{y} + \boldsymbol{b}$$

with solution (e.g. Braun, 1983)

$$\boldsymbol{y} = U\boldsymbol{y_0} + U \int U^{-1}\, \boldsymbol{b}dt \qquad \text{(Equation E.7)}$$

where

$$U = e^{\int Adt}$$

Here **y** represents the vector of fractional populations {Q_0, Q_1, Q_2, Q_3}, and $\mathbf{y_0}$ represents the initial conditions of **y**. For the sake of simplicity, time dependence of the term *U*, an integrating factor which can be chosen as the exponential of an antiderivative of the coefficient matrix, is not explicitly indicated.

The method of integrating factor outlined above can now be used to solve for Q_0, Q_1 and Q_2, one by one, and Q_3 can then be obtained as 1 – ($Q_0 + Q_1 + Q_2$).

To simplify the equation for Q_0 in *Equations S.4*, define

$$h_0 = \alpha + \lambda_{01} + \lambda_{02}$$

and choose the integrating factor

$$U = e^{-h_0 t}$$

Using *Equation E.6*, the solution can be written as

$$Q_0 = c_0 e^{-h_0 t} + e^{-h_0 t} \int \alpha e^{h_0 t}\, dt$$

Carrying out the integration, one obtains

$$Q_0 = \frac{\alpha}{h_0} + c_0 e^{-h_0 t} \qquad \text{(Equation E.8)}$$

where c_0 is an integration constant to be determined by initial conditions.

For Q_1, define

$$h_1 = \alpha + \lambda_{12}$$

and choose the integrating factor

$$U = e^{-h_1 t}$$

Using *Equation E.6*, the solution can be written as

$$Q_1 = c_1 e^{-h_1 t} + e^{-h_1 t} \int \lambda_{01} Q_0 e^{h_1 t}\, dt$$

Carrying out the integration and rearranging terms, one obtains

$$Q_1 = \frac{\alpha\lambda_{01}}{h_0 h_1} - \frac{c_0 \lambda_{01}}{h_0 - h_1} e^{-h_0 t} + \left[c_1 + \lambda_{01}\left(\frac{c_0}{h_0 - h_1} - \frac{\alpha}{h_0 h_1}\right)\right] e^{-h_1 t} \qquad \text{(Equation E.9)}$$

where c_1 is an integration constant to be determined by initial conditions. Note that for the above expression, the assumption that $h_0 \neq h_1$ has been made. This will be explained below. Also note that while one can redefine the integration constant and simplify the coefficient of the last exponential term above, the more complex form is kept since in this form, c_1 is none other than the initial value of Q_1 when t = 0.

For Q_2, substitute $Q_3 = 1 - (Q_0 + Q_1 + Q_2)$ into the differential equation for Q_2 in *Equations S.4* to obtain

$$\frac{dQ_2}{dt} = \lambda_{32} + (\lambda_{02} - \lambda_{32})Q_0 + (\lambda_{12} - \lambda_{32})Q_1 - h_2 Q_2$$

Define

$$h_2 = \alpha + \lambda_{23} + \lambda_{32}$$

and choose the integrating factor

$$U = e^{-h_2 t}$$

Using *Equation E.6*, the solution can be written as

$$Q_2 = c_2 e^{-h_2 t} + e^{-h_2 t} \int [\lambda_{32} + (\lambda_{02} - \lambda_{32})Q_0 + (\lambda_{12} - \lambda_{32})Q_1] e^{h_2 t}\, dt$$

Substituting Q_0 and Q_1, and then carry out the integration, one obtains

$$\begin{aligned} Q_2 = &\frac{(\alpha + \lambda_{32})(\alpha\lambda_{02} + \lambda_{01}\lambda_{12} + \lambda_{02}\lambda_{12})}{h_0 h_1 h_2} - \\ &\frac{c_0(\lambda_{02} - \lambda_{12})(\lambda_{01} + \lambda_{02} - \lambda_{32})}{(h_0 - h_1)(h_0 - h_2)} e^{-h_0 t} - \\ &\frac{\lambda_{12} - \lambda_{32}}{h_1 - h_2}\left[c_1 + \lambda_{01}\left(\frac{c_0}{h_0 - h_1} - \frac{\alpha}{h_0 h_1}\right)\right] e^{-h_1 t} + \\ &\left\{c_2 - \frac{(\alpha + \lambda_{32})(\alpha\lambda_{02} + \lambda_{01}\lambda_{12} + \lambda_{02}\lambda_{12})}{h_0 h_1 h_2}\right. \\ &+ \frac{c_0(\lambda_{02} - \lambda_{12})(\lambda_{01} + \lambda_{02} - \lambda_{32})}{(h_0 - h_1)(h_0 - h_2)} \\ &\left. + \frac{\lambda_{12} - \lambda_{32}}{h_1 - h_2}\left[c_1 + \lambda_{01}\left(\frac{c_0}{h_0 - h_1} - \frac{\alpha}{h_0 h_1}\right)\right]\right\} e^{-h_2 t} \end{aligned} \qquad \text{(Equation E.10)}$$

where c2 is an integration constant to be determined by initial conditions. Once again, instead of redefining c_2 to simplify the equation, the more complex functional form above is kept since in this form, c_2 is none other than the initial value of Q_2 at t = 0. Also note that in the expression above, it has been assumed that $h_0 \neq h_2$ and $h_1 \neq h_2$, in addition to the assumption that $h_0 \neq h_1$. This will be explained below.

With the above solutions, the equation $Q_3 = 1 - (Q_0 + Q_1 + Q_2)$ can be used to obtain Q_3.

$$\begin{aligned} Q_3 = &\frac{\lambda_{23}(\alpha\lambda_{02} + \lambda_{01}\lambda_{12} + \lambda_{02}\lambda_{12})}{h_0 h_1 h_2} + \\ &\frac{c_0 \lambda_{23}(\lambda_{02} - \lambda_{12})}{(h_0 - h_1)(h_0 - h_2)} e^{-h_0 t} + \\ &\frac{\lambda_{23}}{h_1 - h_2}\left[c_1 + \lambda_{01}\left(\frac{c_0}{h_0 - h_1} - \frac{\alpha}{h_0 h_1}\right)\right] e^{-h_1 t} - \end{aligned} \qquad \text{(Equation E.11)}$$

$$\left\{c_2 - \frac{(\alpha+\lambda_{32})(\alpha\lambda_{02}+\lambda_{01}\lambda_{12}+\lambda_{02}\lambda_{12})}{h_0 h_1 h_2} + \frac{c_0(\lambda_{02}-\lambda_{12})(\lambda_{01}+\lambda_{02}-\lambda_{32})}{(h_0-h_1)(h_0-h_2)} + \frac{\lambda_{12}-\lambda_{32}}{h_1-h_2}\left[c_1+\lambda_{01}\left(\frac{c_0}{h_0-h_1}-\frac{\alpha}{h_0 h_1}\right)\right]\right\} e^{-h_2 t}$$

Note that as time (t) increases, the exponential terms decrease and approach 0. As a result, the fractional populations become

$$Q_0 = \frac{\alpha}{\alpha+\lambda_{01}+\lambda_{02}}$$

$$Q_1 = \frac{\alpha\lambda_{01}}{(\alpha+\lambda_{12})(\alpha+\lambda_{01}+\lambda_{02})}$$

$$Q_2 = \frac{(\alpha+\lambda_{32})(\alpha\lambda_{02}+\lambda_{02}\lambda_{12}+\lambda_{01}\lambda_{12})}{(\alpha+\lambda_{01}+\lambda_{02})(\alpha+\lambda_{12})(\alpha+\lambda_{23}+\lambda_{32})}$$

$$Q_3 = \frac{\lambda_{23}(\alpha\lambda_{02}+\lambda_{02}\lambda_{12}+\lambda_{01}\lambda_{12})}{(\alpha+\lambda_{01}+\lambda_{02})(\alpha+\lambda_{12})(\alpha+\lambda_{23}+\lambda_{32})}$$

What happens is that as time increases, inflows and outflows are balanced or are in equilibrium, and the fractional populations Q_0, Q_1, Q_2 and Q_3 become stabilized. The term asymptotic solutions will be used when referring to this condition.

The time varying analytical solutions will form the basis of our analyses in Chapter 6.

One important thing to note is that the above solutions are derived based on the conditions that $h_0 \neq h_1$, $h_0 \neq h_2$ and $h_1 \neq h_2$. In general, h_2, related to OOHC discharge rate, is much larger than h_0 and h_1. Therefore, the conditions $h_0 \neq h_2$ and $h_1 \neq h_2$ will hold. However, the values of h_0 and h_1 could have the same value if OOHC entry rate for repeat clients equals to the sum of investigation rate and OOHC entry rate for new clients (i.e. $\lambda_{12} = \lambda_{01} + \lambda_{02}$) and thus one cannot rule out the situation where $h_0 = h_1$.

For $h_0 = h_1$, the solution for Q_0 is not affected. For the other sub-populations, repeating the method of integrating factor above, one obtains

$$Q_1 = \frac{\alpha\lambda_{01}}{h_0 h_1} + \left(c_1 - \frac{\alpha\lambda_{01}}{h_0 h_1} + c_0\lambda_{01} t\right) e^{-h_1 t}$$

$$Q_2 = \frac{(\alpha+\lambda_{32})(\alpha\lambda_{02}+\lambda_{01}\lambda_{12}+\lambda_{02}\lambda_{12})}{h_0h_1h_2} - \frac{c_0(\lambda_{02}-\lambda_{32})}{h_0-h_2}e^{-h_0t} +$$
$$\frac{\lambda_{12}-\lambda_{32}}{h_1-h_2}\left(c_1 - \frac{\alpha\lambda_{01}}{h_0h_1} - \frac{c_0\lambda_{01}}{h_1-h_2}\right)e^{-h_1t} +$$
$$\left[c_2 - \frac{(\alpha+\lambda_{32})(\alpha\lambda_{02}+\lambda_{01}\lambda_{12}+\lambda_{02}\lambda_{12})}{h_0h_1h_2} + \frac{c_0(\lambda_{02}-\lambda_{32})}{h_0-h_2}\right.$$
$$\left. - \frac{\lambda_{12}-\lambda_{32}}{h_1-h_2}\left(c_1 - \frac{\alpha\lambda_{01}}{h_0h_1} - \frac{c_0\lambda_{01}}{h_1-h_2}\right)\right]e^{-h_2t} - \frac{c_0\lambda_{01}(\lambda_{12}-\lambda_{32})}{h_1-h_2}te^{-h_1t}$$

$$Q_3 = \frac{\lambda_{23}(\alpha\lambda_{02}+\lambda_{01}\lambda_{12}+\lambda_{02}\lambda_{12})}{h_0h_1h_2} + \frac{c_0(\lambda_{23}-\lambda_{01})}{h_0-h_2}e^{-h_0t} -$$
$$\left[\left(c_1 - \frac{\alpha\lambda_{01}}{h_0h_1}\right)\left(\frac{2\lambda_{12}-\lambda_{23}-2\lambda_{32}}{h_1-h_2}\right) - \frac{c_0\lambda_{01}(\lambda_{12}-\lambda_{32})}{(h_1-h_2)^2}\right]e^{-h_1t} -$$
$$\left[c_2 - \frac{(\alpha+\lambda_{32})(\alpha\lambda_{02}+\lambda_{01}\lambda_{12}+\lambda_{02}\lambda_{12})}{h_0h_1h_2} + \frac{c_0(\lambda_{02}-\lambda_{32})}{h_0-h_2}\right.$$
$$\left. - \frac{\lambda_{12}-\lambda_{32}}{h_1-h_2}\left(c_1 - \frac{\alpha\lambda_{01}}{h_0h_1} - \frac{c_0\lambda_{01}}{h_1-h_2}\right)\right]e^{-h_2t} + \frac{c_0\lambda_{01}\lambda_{23}}{h_1-h_2}te^{-h_1t}$$

For the above equations (with $h_0 = h_1$), at $t = 0$,

$$Q_0 = \frac{\alpha}{h_0} + c_0$$

$$Q_1 = c_1$$

$$Q_2 = c_2$$

$$Q_3 = 1 - (Q_0 + Q_1 + Q_2) = 1 - \frac{\alpha}{h_0} - c_0 - c_1 - c_2$$

Moreover, the asymptotic solutions are exactly the same as the case for $h_0 \neq h_1$.

3.1.5 Quadratic System

The case where system capacity limit cannot be ignored is now explored as an example. To facilitate a tractable exploration of the effects of system capacity, it is assumed that f_2 takes the logistic form.

$$f_2 = 1 - \frac{Q_2}{B_2}$$

With the above assumption and for the case of uniformed aged out rates (see *Equation E.5* and *Equation E.6*), *Equations S.3* become

$$\frac{dQ_0}{dt} = \alpha - (\alpha + \lambda_{01} + \lambda_{02})Q_0$$

$$\frac{dQ_1}{dt} = \left(\lambda_{01} + \lambda_{02}\frac{Q_2}{B_2}\right)Q_0 - \left(\alpha + \lambda_{12} - \lambda_{12}\frac{Q_2}{B_2}\right)Q_1$$

$$\frac{dQ_2}{dt} = \left(\lambda_{02} - \lambda_{02}\frac{Q_2}{B_2}\right)Q_0 + \left(\lambda_{12} - \lambda_{12}\frac{Q_2}{B_2}\right)Q_1 - (\alpha + \lambda_{23})Q_2 + \left(\lambda_{32} - \lambda_{32}\frac{Q_2}{B_2}\right)Q_3$$

$$\frac{dQ_3}{dt} = \lambda_{23}Q_2 - \left(\alpha + \lambda_{32} - \lambda_{32}\frac{Q_2}{B_2}\right)Q_3$$

Noting that $Q_0 = 1 - (Q_1 + Q_2 + Q_3)$ and rearranging terms, one obtains

$$\frac{dQ_0}{dt} = \alpha - (\alpha + \lambda_{01} + \lambda_{02})Q_0$$

$$\frac{dQ_1}{dt} = \lambda_{01}(1 - Q_1 - Q_2 - Q_3) - (\alpha + \lambda_{12})Q_1 + \frac{\lambda_{02}}{B_2}(1 - Q_1 - Q_2 - Q_3)Q_2 + \frac{\lambda_{12}}{B_2}Q_1Q_2$$

$$\frac{dQ_2}{dt} = \lambda_{02}(1 - Q_1 - Q_2 - Q_3) + \lambda_{12}Q_1 - (\alpha + \lambda_{23})Q_2 + \lambda_{32}Q_3 - \frac{\lambda_{02}}{B_2}(1 - Q_1 - Q_2 - Q_3)Q_2 - \frac{\lambda_{12}}{B_2}Q_1Q_2 - \frac{\lambda_{32}}{B_2}Q_2Q_3$$

$$\frac{dQ_3}{dt} = \lambda_{23}Q_2 - (\alpha + \lambda_{32})Q_3 + \frac{\lambda_{32}}{B_2}Q_2Q_3$$

Further rearrangement gives

$$\frac{dQ_0}{dt} = \alpha - (\alpha + \lambda_{01} + \lambda_{02})Q_0$$

$$\frac{dQ_1}{dt} = \lambda_{01} - (\alpha + \lambda_{01} + \lambda_{12})Q_1 - \left(\lambda_{01} - \frac{\lambda_{02}}{B_2}\right)Q_2 - \lambda_{01}Q_3 + \left(\frac{\lambda_{12}}{B_2} - \frac{\lambda_{02}}{B_2}\right)Q_1Q_2 - \frac{\lambda_{02}}{B_2}Q_2^2 - \frac{\lambda_{02}}{B_2}Q_2Q_3$$

$$\frac{dQ_2}{dt} = \lambda_{02} + (\lambda_{12} - \lambda_{02})Q_1 - \left(\alpha + \lambda_{02} + \lambda_{23} + \frac{\lambda_{02}}{B_2}\right)Q_2 + (\lambda_{32} - \lambda_{02})Q_3 - \left(\frac{\lambda_{12}}{B_2} - \frac{\lambda_{02}}{B_2}\right)Q_1Q_2 + \frac{\lambda_{02}}{B_2}Q_2^2 + (\frac{\lambda_{02}}{B_2} - \frac{\lambda_{32}}{B_2})Q_2Q_3$$

$$\frac{dQ_3}{dt} = \lambda_{23}Q_2 - (\alpha + \lambda_{32})Q_3 + \frac{\lambda_{32}}{B_2}Q_2Q_3$$

3.2 Three-Population Model of CP Systems

One can also consider the case where no distinction is made between children who are involved in in CPS but have never been in OOHC and those who have exited OOHC, in terms of their rates of notification, investigation, substantiation, placement into OOHC, exit from OOHC, and aging out. In this case, children in CPS consist of two populations, those in OOHC and those not in OOHC. This is equivalent to combining P_1 and P_3 in the formulation corresponding to the conceptual model in Figure 3.1 and setting the following:

$$\lambda_{32} = \lambda_{12}$$

$$\lambda_{23} = \lambda_{21}$$

$$\eta_1 = \eta_3$$

At this point, it is again noted that relevant CP data for Australian states and territories which are needed for this type of modelling effort are limited. In particular, there are no publicly available data on how children who have exited OOHC fare compared to children who have never been involved in CP or children who are involved but have never been admitted to OOHC. As a result, the reduced, three-population system is more useful in our effort to model the structures of Australian CP populations. Nevertheless, the four-population model is still necessary for us to explore and gain some insight into the issue of OOHC “churn” – repeated involvement with OOHC.

The three-population system is shown in Figure 3.2, where P_1 is the combination of P_1 and P_3 in the original system.

The procedure for deriving the differential equations and solving them is the same as that described in detail for the four-population mode. Therefore, only an abbreviated description is provided below.

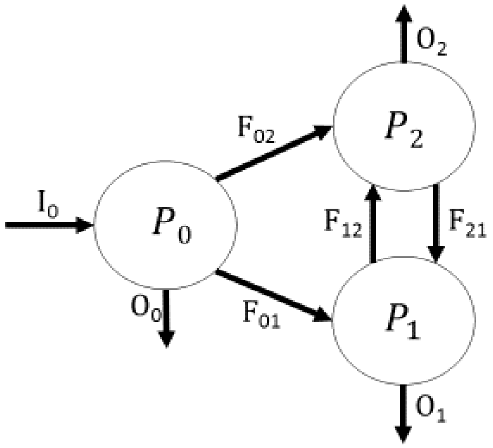


Figure 3.2 Three-population Model of CP Population

As in the case for the four-population model, it is assumed that there is no system capacity limit for P_1. The corresponding equations for the fractional populations are

$$\frac{dQ_0}{dt} = \alpha - (\alpha + \lambda_{01} + \lambda_{02} + \eta_0 - H)Q_0$$

$$\frac{dQ_1}{dt} = (\lambda_{01} + \lambda_{02} - \lambda_{02}f_2)Q_0 - (\alpha + \lambda_{12}f_2 + \eta_1 - H)Q_1 + \lambda_{21}Q_2$$

$$\frac{dQ_2}{dt} = \lambda_{02}f_2Q_0 + \lambda_{12}f_2Q_1 - (\alpha + \lambda_{21} + \eta_2 - H)Q_2$$

where

$$H = \frac{1}{P}\sum_{i=0}^{2}\eta_iP_i = \sum_{i=0}^{2}\eta_iQ_i$$

Quadratic System

To facilitate a tractable exploration of the effects of system capacity, it is assumed that f_2 takes the logistic form.

$$f_2 = 1 - \frac{Q_2}{B_2}$$

For the case of uniformed aged out rates, the equations become

$$\frac{dQ_0}{dt} = \alpha - (\alpha + \lambda_{01} + \lambda_{02})Q_0$$

$$\frac{dQ_1}{dt} = \left(\lambda_{01} + \lambda_{02}\frac{Q_2}{B_2}\right)Q_0 - \left(\alpha + \lambda_{12} - \lambda_{12}\frac{Q_2}{B_2}\right)Q_1 + \lambda_{21}Q_2$$

$$\frac{dQ_2}{dt} = \left(\lambda_{02} - \lambda_{02}\frac{Q_2}{B_2}\right)Q_0 + \left(\lambda_{12} - \lambda_{12}\frac{Q_2}{B_2}\right)Q_1 - (\alpha + \lambda_{21})Q_2$$

Noting that $Q_0 = 1 - (Q_1 + Q_2)$ and rearranging terms, one obtains

$$\frac{dQ_0}{dt} = \alpha - (\alpha + \lambda_{01} + \lambda_{02})Q_0$$

$$\frac{dQ_1}{dt} = \lambda_{01}(1 - Q_1 - Q_2) - (\alpha + \lambda_{12})Q_1 + \lambda_{21}Q_2 + \frac{\lambda_{02}}{B_2}(1 - Q_1 - Q_2)Q_2 + \frac{\lambda_{12}}{B_2}Q_1Q_2$$

$$\frac{dQ_2}{dt} = \lambda_{02}(1 - Q_1 - Q_2) + \lambda_{12}Q_1 - (\alpha + \lambda_{21})Q_2 - \frac{\lambda_{02}}{B_2}(1 - Q_1 - Q_2)Q_2 - \frac{\lambda_{12}}{B_2}Q_1Q_2$$

Further rearrangement gives

$$\frac{dQ_0}{dt} = \alpha - (\alpha + \lambda_{01} + \lambda_{02})Q_0$$

$$\frac{dQ_1}{dt} = \lambda_{01} - (\alpha + \lambda_{01} + \lambda_{12})Q_1 + \left(\lambda_{21} + \frac{\lambda_{02}}{B_2} - \lambda_{01}\right)Q_2 + \left(\frac{\lambda_{12}}{B_2} - \frac{\lambda_{02}}{B_2}\right)Q_1Q_2 - \frac{\lambda_{02}}{B_2}Q_2^2$$

$$\frac{dQ_2}{dt} = \lambda_{02} + (\lambda_{12} - \lambda_{02})Q_1 - \left(\alpha + \lambda_{02} + \lambda_{21} + \frac{\lambda_{02}}{B_2}\right)Q_2 + \left(\frac{\lambda_{02}}{B_2} - \frac{\lambda_{12}}{B_2}\right)Q_1Q_2 + \frac{\lambda_{02}}{B_2}Q_2^2$$

Linear System

For the case of uniformed aged out rates and where there is no system capacity limit, the equations are reduced to

$$\frac{dQ_0}{dt} = \alpha - (\alpha + \lambda_{01} + \lambda_{02})Q_0$$

$$\frac{dQ_1}{dt} = \lambda_{01}Q_0 - (\alpha + \lambda_{12})Q_1 + \lambda_{21}Q_2$$

$$\frac{dQ_2}{dt} = \lambda_{02}Q_0 + \lambda_{12}Q_1 - (\alpha + \lambda_{21})Q_2$$

Define

$$h_0 = \alpha + \lambda_{01} + \lambda_{02}$$

$$h_1 = \alpha + \lambda_{12} + \lambda_{21}$$

and note that

$$Q_2 = 1 - (Q_0 + Q_1)$$

Carrying out the method of integrating factor described in Section 3.1.4, the following solutions are obtained.

$$\begin{aligned}
Q_0 &= \frac{\alpha}{h_0} + c_0 e^{-h_0 t} \\
Q_1 &= \frac{\alpha\lambda_{01} + \lambda_{01}\lambda_{21} + \lambda_{02}\lambda_{21}}{h_0 h_1} - \frac{c_0(\lambda_{01} - \lambda_{21})}{h_0 - h_1} e^{-h_0 t} + \\
&\left[c_1 - \frac{\alpha\lambda_{01} + \lambda_{01}\lambda_{21} + \lambda_{02}\lambda_{21}}{h_0 h_1} + \frac{c_0(\lambda_{01} - \lambda_{21})}{(h_0 - h_1)}\right] e^{-h_1 t} \\
Q_2 &= \frac{\alpha\lambda_{02} + \lambda_{01}\lambda_{12} + \lambda_{02}\lambda_{12}}{h_0 h_1} - \frac{c_0(\lambda_{02} - \lambda_{12})}{h_0 - h_1} e^{-h_0 t} - \\
&\left[c_1 - \frac{\alpha\lambda_{01} + \lambda_{01}\lambda_{21} + \lambda_{02}\lambda_{21}}{h_0 h_1} + \frac{c_0(\lambda_{01} - \lambda_{21})}{(h_0 - h_1)}\right] e^{-h_1 t}
\end{aligned} \qquad \text{(Equations S.5)}$$

where c_0, c_1 and c_2 are integration constants to be determined by initial conditions.

Once again, as time increases, the exponential terms approach 0 and one obtains

$$\begin{aligned}
Q_0 &= \frac{\alpha}{\alpha + \lambda_{01} + \lambda_{02}} \\
Q_1 &= \frac{\alpha\lambda_{01} + \lambda_{01}\lambda_{21} + \lambda_{02}\lambda_{21}}{h_0 h_1} \\
Q_2 &= \frac{\alpha\lambda_{02} + \lambda_{01}\lambda_{12} + \lambda_{02}\lambda_{12}}{h_0 h_1}
\end{aligned} \qquad \text{(Equations S.6)}$$

As in the four-population system, the above solutions are derived based on the conditions that $h_0 \neq h_1$. In this system, h_1 is related to OOHC discharge rate, and is much larger than h_0. Therefore, the conditions $h_0 \neq h_1$ will generally hold.

Equations S.5 and *Equations S.6* will form the basis of our analyses in Chapter 5.

3.3 CP Data for Australian States and Territories

Our main objectives in developing the population dynamics models, as stated in Chapter 1, are: 1. Using the models to better understand how various parameters influence the structure of populations of children involved with CP; 2. Using the models to study the populations of children involved with CP across the Australian States and Territories. The second objective involves both estimating the fraction of children involved in CP and analysing how the various sub-populations evolve. These objectives can only be achieved when the models used are adequately constrained and are thus consistent with the observed data.

Ideally, one would analyse high frequency unit-record data from various CP systems in relation to the parameters specified in the generic CP population system described above. However, permissions to use such data for this study could not be obtained. Therefore, this study will focus on using publicly available annual aggregate data as a first step. As such, our study should be viewed as a proof of concept attempt which will be strengthened with better data and by taking into account issues such as socio-economic factors, demographic structures (age) and ethnicity.

In terms of publicly available CP in Australia, our main source is *Child Protection Australia* (AIHW, 2019a), an annual report published by the Australian Institute of Health and Welfare (AIHW). Based on the nationally agreed standard (definitions and technical specifications), AIHW works with state and territory CP departments to collate, analyse and publish CP data. The data published in the annual report are drawn mainly from the Child Protection National Minimum Data Set (CP NMDS), which consists of unit record or child-level data provided by the states and territories, from their CP administrative data sets.

The following list highlights a number of important features of the data published in the annual reports:

- Annual aggregate figures such as number of children in various stages of CPS: notification, investigation, substantiation, protection orders, admission to OOHC, in OOHC, and discharge from OOHC.
- Classifications in terms of jurisdictions, age group, indigenous status, care types, length of stay in OOHC, etc.
- Beginning from the 2012-2013 financial year report, breakdowns of the number of children involved with CP in terms of new and repeat clients for some jurisdictions were provided. However, up until 2014-15, only aggregate data across a few states were provided.

- In the 2015-16 financial year report, AIHW began to provide breakdowns into new and repeat clients for all states and territories.

Note that critical information such as classification of notifications in terms of new and repeat clients is not available. Also, there are no data on children who have experienced repeated admission OOHC, commonly known as the issue of "churn". Nevertheless, the available data do provide constraints that are useful to modelling efforts.

Recall that in the four-population model, among children who are involved with CP but not currently in OOHC, a distinction is made between those who have previously been in OOHC and those who have never been in OOHC. As such, data on "churn" is needed to fully leverage the model to better utilise the model. With such data not available, this study will focus on using the three-population model to analyse CP population across Australian states and territories. However, the four-population model will still be used to extend the analyses.

Table 3.1 exhibits the matching of the parameters needed in the three-population model described in section 3.2 to the variables provided by annual aggregate AIHW data. The first and second columns show the list of parameters in the model and descriptions of these parameters. The third and fourth columns show the list of corresponding data from AIHW and its description. The last column provides comments to the matching and data estimation.

Note that in general, population growth rates and age-out rates can be estimated from demographic structures, both of the general population of children and children in OOHC. However, even though the number of children in investigation and the number of children admitted to OOHC are reported for repeat clients, the population of children who are involved in CP but not currently in OOHC (P_1 and thus Q_1) is not known. Likewise, the population of children who have never been involved in CP is also not known. As a result, the actual process rates, defined as proportional to the relevant populations, cannot be directly calculated from the number of new and repeat clients in investigations and OOHC admission. An iterative process will be used to obtain a set of rates for new and repeat clients based on the model solutions. Since Q_0 and Q_1 are not observed, in obtaining these rates, the condition imposed in the iterative process is that the resulting number of children in investigation and OOHC admission given by these rates should be consistent with the reported number of children in investigation and OOHC admission. This process will be described in the following section.

Table 3.1 System Parameters and Available Data for the Three-Population Model

Parameters		Available Data		Comments
α	Gross growth rate of child population (P) due to population increase.	αP	Sum of net growth in general child population (below 18 years of age) and children "aging out".	α – H = net growth in general child population (below 18 years of age), where H designates those who "age out".
η_0	Rate of children aging out from the eligible population (P_0)	$\eta_0 P_0$	Children between 16 and 17 years of age	Rough approximation
η_1	Rate of children aging out from non-OOHC CPS (P_1)	N/A	N/A	P_1 is not known. Age-out rate for general population will be used.
η_2	Rate of children aging out from OOHC (P_2)	η_2	Children between 16 and 17 years of age in OOHC	Can be estimated from OOHC demographic structure
λ_{01}	Rate of children involved with CP for the first time	$\lambda_{01} P_0$	Number of new clients in investigation minus number of new clients entering OOHC	Range can be estimated
λ_{02}	Rate of children entering OOHC (P_2) in first-time involvement in CP	$\lambda_{02} P_0$	Number of OOHC entry for new clients	Range can be estimated
λ_{12}	Rate of children entering OOHC (P_2) in non-first-time involvement in CP	$\lambda_{12}\ P_1$	Number of OOHC entry for repeat clients	Range can be estimated
λ_{21}	Rate of children leaving OOHC (P_2) not as a result of aging out	λ_{21}	OOHC discharge	Range can be estimated

While not all necessary data are available in view of the model parameters, and only coarse estimations could be obtained for much of the data which are available, they are sufficient to provide an opportunity to examine and analyse CP populations across the Australian states and territories. As the results described in Chapter 5 and Chapter 6 show, it is indeed possible to obtain a more complete picture of the structures of CP populations with this approach.

In Chapter 4, a descriptive analysis of the annual aggregate data will be provided. Such analyses are not only useful and informative in and of themselves, but they will also provide observed trends and ranges of the parameters to be used in the models.

3.4 Modelling CP Populations Using the Analytical Solutions with Data Constraint

In this section, the process of using the analytical solutions derived in the previous sections to model CP populations with the constraints of reported data will be described. While the analytical solutions allow us to explore how the population structure changes with different regimes of flow rates, applying it to model actual CP populations is not a straightforward task. This is mainly due to the limited available data and the ensuing difficulties in estimating relevant parameters that drive the population change, as mentioned in the previous section.

As described in Chapter 1, this is a two-fold task. The first is model construction and the use of such models to better understand CP population structure; the second is parameter estimations, which may involve the use of complex statistical methods of estimations. Due to the limited amount of available data for this study, no attempt will be made to construct a sophisticated method of estimation which involves a good description of error statistics. Instead, a simple heuristic approach is used to demonstrate the use of model solutions constrained by limited data, specific to this particular study.

To this end, an iterative process is devised to obtain a set of rates which are based on the model solutions and are consistent with the reported number of children in investigation and OOHC admission for new and repeat clients. This iterative process consists of 6 steps. It is shown schematically in Figure 3.3 and described below.

Step 1

It is first noted that only up to 3 years of annual aggregate new and repeat client data are available. In ACT, only 1 year of data are available (2015-16); in NSW, only 2 years of data are available (2015-16 and 2016-17). Also, due to changes in counting, the 2017-18 OOHC data for VIC will be excluded. These will be described in Chapter 4.

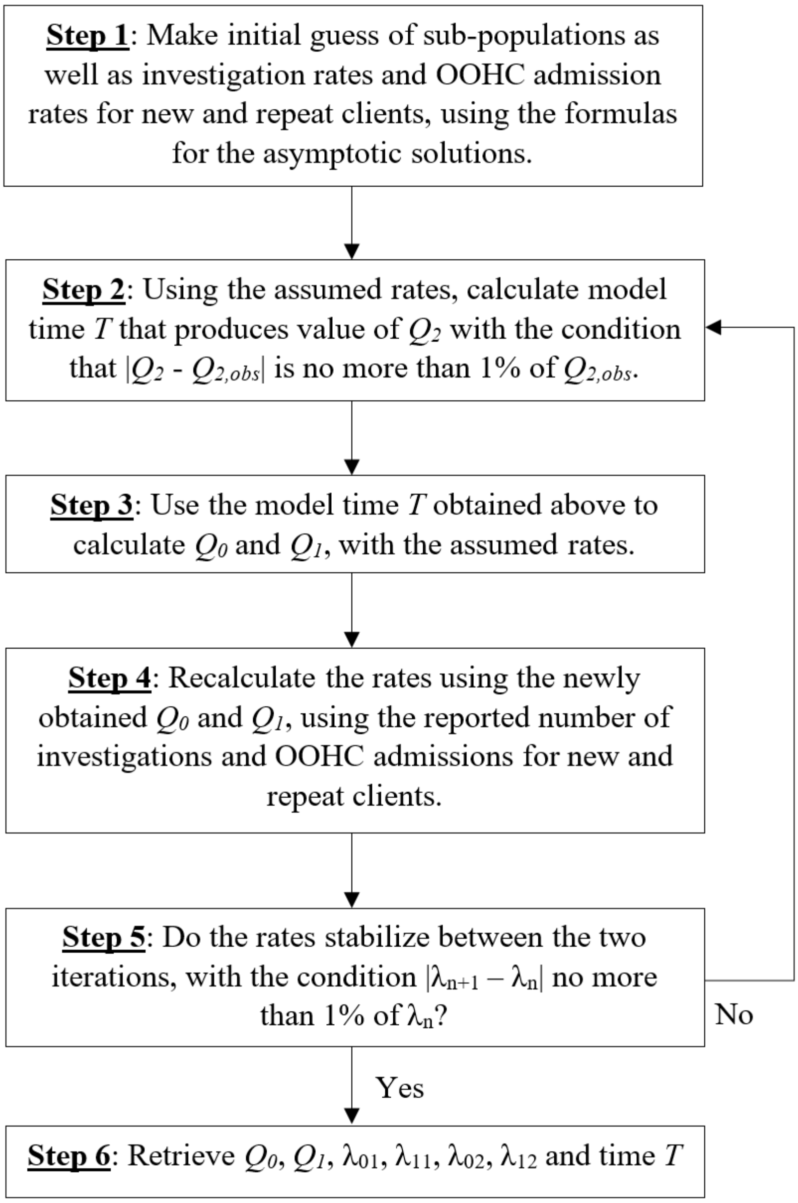


Figure 3.3 Schematic for Iterative Process

These limited data are combined to produce the average number of children in investigations, OOHC admission, and in OOHC, over the 1-year to 3-year spans where data are available. The task will thus be of one of single point estimation.

One first makes initial guess of the sub-populations as well as investigation rates and OOHC admission rates for new and repeat clients based on the average data. Note that while the population growth rates, age-out rates, and OOHC discharge rates can be estimated directly from available data, the investigation rates and OOHC admission rates for new and repeat clients cannot be directly estimated, due to Q_0 and Q_1 being unknown. An initial range of values for Q_0 from 0 and 1 are assumed, where a value of 1 means that no child is involved with CP, and a value of 0 means that every child is involved in CP. With Q_2 fixed by the observed or reported value, designated $Q_{2,obs}$, Q_1 can be calculated as $1 - Q_0 - Q_{2,obs}$. For each of these sets of assumed values for Q_0 and Q_1, one can calculate the corresponding values of λ_{01}, λ_{02} and λ_{12}, based on the reported number of children in investigation and OOHC admission for new and repeat clients.

These set of rates calculated based on assumed values of Q_0 and Q_1 are then used to re-calculate Q_0, Q_1 and Q_2, using the formulas for the asymptotic solutions (*Equations S.6*). Note that by using the formulas for the asymptotic solutions, one will most likely overestimate Q_0 and underestimate Q_1 and Q_2. This is because reported values of Q_2 in all Australian states and territories exhibit a rising trend. The analytical solutions of the three-population model (*Equations S.5*) shown in the preceding section indicate that in such a situation, the value of Q_1 and Q_2 will continue to rise before reaching the asymptotic values.

This is demonstrated using data from NSW, as shown in Figure 3.4. Details of the data used in this calculation will be described in Chapter 5. Figure 3.4 shows the results of "initial guess" for NSW. In the figure, the horizonal axis is the assumed fraction of children involved in CP, which is the sum of Q_1 and Q_2. The primary vertical axis, on the left, shows the values for Q_0, Q_1 and $Q_1 + Q_2$ while the secondary vertical axis, on the right, shows the values for Q_2. The long-dashed-dotted curve is Q_0; the solid curve is Q_1; the short-dashed curve is Q_2; the long-dashed curve is the sum of Q_1 and Q_2; the short-dashed horizontal line is $Q_{2,obs}$; the solid sloped line is the line with the slope of 1; the vertical dotted line indicates the position where the calculated value for the sum of Q_1 and Q_2 equals to the assumed value (intersection of slope-1 line and long-dashed curve).

In Figure 3.4, along the range of assumed values of the fraction of population involved in CP, there is only one set of values for Q_0 and Q_1 that matches the calculated set of values. The values of λ_{01}, λ_{02} and λ_{12} corresponding to this matched set form the "initial guess" of the investigation rates and OOHC admission rates for new and repeat clients. These initial values will be used in Step 2.

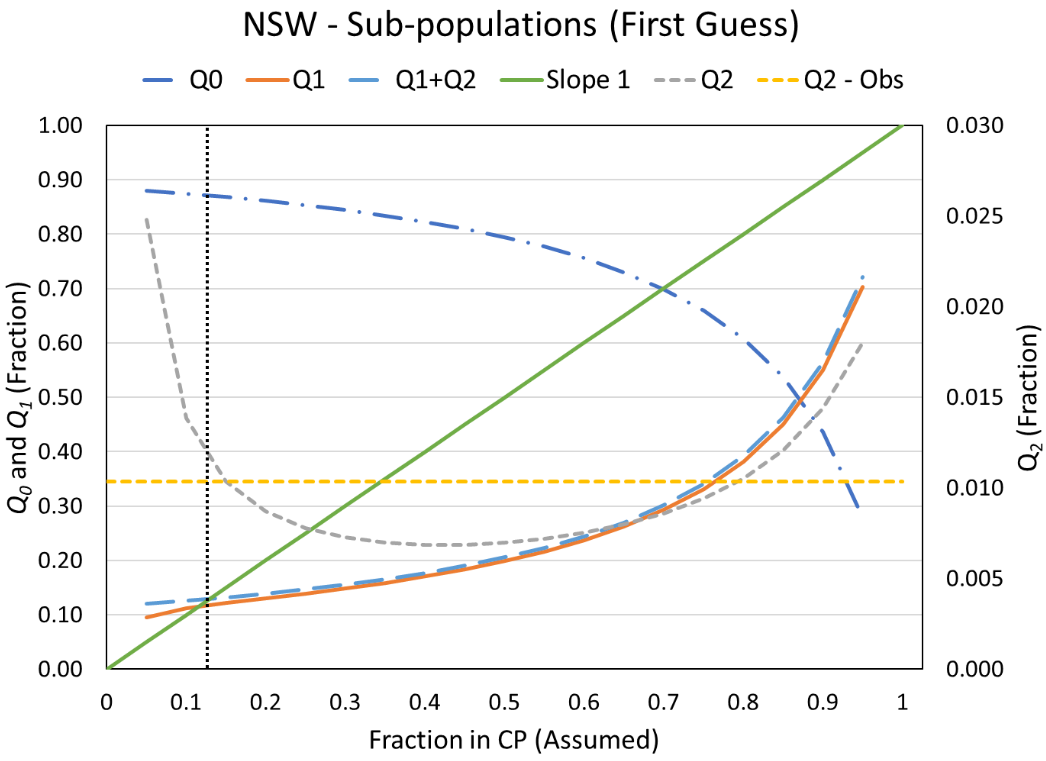


Figure 3.4 Initial Guess of Sub-populations Using Formulas for Asymptotic Solutions

Step 2

Step 2 involves using the initial values of λ_{01}, λ_{02} and λ_{12} obtained in Step 1 in conjunction with the time-varying analytical solution of Q_2. To do so, it is assumed that the sub-populations evolve from the following initial conditions:

$$c_0 \ = 1 - \frac{\alpha}{h_0}$$

$$c_1 \ = 0$$

Doing so is equivalent to using model solutions which start out with $Q_0 = 1$, $Q_1 = 0$ and $Q_2 = 0$. In fact, different sets of initial conditions can be used. This will be demonstrated in Chapter 5.

With the initial conditions and the initial guess of the rates, one obtains the model time T from the time-varying analytical solution for Q_2, to within 1% of the reported or observed value of Q_2, designated $Q_{2,obs}$. Note that the 1% error threshold is arbitrarily chosen although it is not an unreasonable one giving the fact that parameters such as population growth rates and age-out rates are all coarse estimations. While the error threshold can be more generally formulated, the 1% value will be used consistently for the analyses in this study.

Step 3

With the model time T obtained in Step 2, the corresponding values of Q_0 and Q_1 can be calculated using their time-varying analytical solutions.

Step 4

The values of Q_0 and Q_1 obtained in Step 3 are then used to recalculate the relevant rates, with the formulas shown below:

$$\lambda_{01} = \frac{N_{inv,0}}{Q_0 P}$$

$$\lambda_{02} = \frac{N_{adm,0}}{Q_0 P}$$

$$\lambda_{11} = \frac{N_{inv,1}}{Q_1 P}$$

$$\lambda_{12} = \frac{N_{adm,1}}{Q_1 P}$$

In the formulas above, $N_{inv,0}$, $N_{inv,1}$, $N_{adm,0}$ and $N_{adm,1}$ are the number of children in investigation for new clients, the number of children in investigation for repeat clients, the number of children admitted to OOHC for new clients, and the number of children admitted to OOHC for repeat clients, respectively. All these are reported values. The variable P is the general population of children below 18 years of age. Recalling that Q_0 is the fraction of children among the general population who are not involved in CP, Q_0P gives the total number of children in the general population not involved in CP. Likewise, Q_1P gives the total number of children in the general population involved in CP but not in OOHC.

Step 5

The new set of rates calculated in Step 4 is compared to the set of initial guesses. If the differences between the two set of rates are larger than the error threshold of 1%, the iterative process is continued by using the new set of rates in Step 2. If the differences are equal to or smaller than 1%, the iterative process is stopped and the rates are retrieved (Step 6). The rates thus obtained are the rates that are consistent with the observed data and fit with the time-varying solutions. The time-varying solutions thus produced will be used to explore sub-populations in CP.

This iterative process will be used in Chapter 5 for each of the Australian states and territories.

3.5 Prevalence of Child Protection Involvement

In this section, the relationship between the models described above and the prevalence of CP involvement is elucidated. As mentioned in the first chapter, prevalence of involvement in CPS among children below 18 years of age in the general population is an important variable in our attempt to understand the scale of child maltreatment and its cost. Most existing efforts to study this issue use the life table method (Preston et al., 2001) to estimate lifetime cumulated risk. Two approaches can be taken with the life table method – the birth cohort approach and the synthetic cohort approach. The former entails following a birth cohort through the years, while the latter entails constructing cohorts of various age-groups (thus the term synthetic cohorts) from cross-sectional data. With either of the approaches, age specific conditional probabilities of being involved in a CP investigation are estimated from first-time investigation rates. With these age-specific risks, one can obtain the cumulative probability for a child to have a child maltreatment investigation throughout the age span of CP eligibility. This probability can be expressed as the percentage of a cohort of children who are the subjects of investigated child maltreatment reports before their 18th birthday.

In the population dynamics models delineated in the previous sections, the criterion for children entering the CP system is defined as having been involved in at least one investigated child maltreatment report. For the four-population model, it is noted that the sum of P_1, P_2 and P_3 constitutes the total population of children under 18 years of age in the CP system. Equivalently, the sum of Q_1, Q_2 and Q_3 is the fraction of children under 18 years of age in the current general population who are the subject of at least one investigated child maltreatment report. This is a measure of the prevalence of involvement in CPS among children below 18 years of age in the general population at any time.

Likewise, in the three-population model, the sum of P_1 and P_2 constitutes the total population of children under 18 years of age in the CP system. Equivalently, the sum of Q_1 and Q_2 is the fraction of children under 18 years of age in the current general population who are the subject of at least one investigated child maltreatment report.

Therefore, whether in the four-population model or in the three-population model, the prevalence of child maltreatment at any time can be obtained just by summing the relevant fractional populations. This will be referred to this as cross-sectional prevalence, in contrast to lifetime prevalence.

Both the lifetime prevalence of maltreatment investigation of a birth cohort and the cross-sectional prevalence in the CP system provide important information about the prevalence of CP involvement. The relationship of the two measures is now explored.

In the life table method, lifetime cumulative risk is given by the formula (Kim et al., 2017)

$$L_x = 1 - \prod_{i=0}^{x} p_{n,i}$$

where n is the length of the age interval (normally taken to be 1 year), x is age in years, and $p_{n,x}$ is the age-specific conditional probability of not receiving a maltreatment investigation by the end of the age bracket. Life table can be used in two approaches, the birth cohort approach and the synthetic cohort approach. In the birth cohort approach, the age-specific probability, $p_{n,x}$, is calculated for a birth cohort as it progresses in age; in the synthetic cohort or period approach, $p_{n,x}$ is calculated using first time maltreatment investigation for all age brackets using data in a single time period (Kim et al., 2017). Due to the difficulties of tracking a birth cohort longitudinally, synthetic cohort is often used to approximate lifetime prevalence or cumulative risk. This approximation is justifiable when age-specific risks exhibit little change over time.

The four-population model described in the previous section does not consider age-structure. Nevertheless, the fraction of children within an age bracket in the population involved in CP (the sum of Q_1, Q_2 and Q_3) is none other than the lifetime cumulative risks of the age cohort, accumulated over the course of model integration. For example, the fraction of children in the 17-year old age bracket in the CP system is none other than the actual lifetime prevalence of the corresponding cohort of children born 17 years ago.

In a population model which takes into account age structure, lifetime prevalence for any age bracket at any moment in time is readily given by the age bracket in the sum of Q_1, Q_2 and Q_3. In fact, the relation between population prevalence of maltreatment and lifetime maltreatment prevalence of the age cohorts constituting the population is as follows.

$$P_{cps} = P_1 + P_2 + P_3 = \sum_{x=0}^{17} P_{cps,x} = \sum_{x=0}^{17} P_{-x,0}\, L_x + \varepsilon$$

where $P_{cps,x}$ denotes the current number of children in the CP system in the x age bracket, $P_{-x,0}$ denotes the number of children in the age 0 cohort, x number of years prior to the year of concern. For example, $P_{0,0}$ denotes the number of children in the age 0 cohort in the current year; $P_{-17,0}$ denotes the number of children in the age 0 cohort, 17 years prior to the current year. This is none other than the recognition that the population of children in the current year is made up of the birth cohorts of children born in previous years[5]. The term ε accounts for the effects of death censoring and migration. Dividing the above equation by the total number of children in the current year P, one obtains:

$$Q_{cps} = Q_1 + Q_2 + Q_3 = \sum_{x=0}^{17} Q_{cps,x} = \sum_{x=0}^{17} W_x\, L_x + \varepsilon'$$

Where $Q_{cps,x}$ denotes the fraction of children in the x age bracket in the current CP population and ε' denotes the effect of migration and death censoring. Moreover, by definition,

$$W_x = \frac{P_{-x,0}}{P}$$

Up to an unspecified error term, the fraction of children in the x bracket is given by

[5] Note that death rates and immigration rates are not taken into account. Censoring due to death or migration out of the country has to be carefully considered for a more rigorous formulation. The relation shows here is an estimation without taking these effects into account.

$$Q_{cps,x} = W_x L_x$$

Note that in general, the sum of children age 0 in the 18 birth cohorts ($\Sigma P_{-x,0}$, x ranging from 0 to 17) does not equal to P, the current population of children below 18 years of age. In fact, the balance between P and $\Sigma P_{-x,0}$ gives an indication of ε, the effect of migration and death censoring.

From the equations above, one can see that theoretically, the cross-sectional prevalence or the fraction of children currently involved in CP is just the sum of lifetime prevalence for each of the age brackets, weighted by the fractional contribution of the age bracket to the total population, modified by the effects of migration and death censoring. Population data across the Australian states and territories will be used to discuss the issue of CP prevalence in the result chapters.

In this chapter, the methods modelling and estimating CP sub-populations have been set up in detail. The relation between cross-sectional prevalence of CP involvement and lifetime prevalence of CP involvement has also been discussed. The next chapters present the results of this study in three aspects: 1) Analysis of publicly available CP population data in Australia; 2) Analytical solutions of the three-population model and their applications; 3) Analytical solutions of the four-population model and their applications.

Chapter 4: Analysis of Publicly Available CP Population Data

This chapter presents descriptive analyses of CP population data across Australian states and territories. The analyses serve two purposes:

1. Illuminating systems behaviours in the states and territories, across the stages of protective intervention services, i.e. notifications, investigations, substantiations, OOHC entries, and OOHC exits. Not only do such analyses reveal patterns of systems behaviours which can inform practitioners and policy makers, but they also form the basis of comparative works using analytical solutions and model simulations.
2. Estimating ranges of parameters needed for the modelling and simulation of CP subpopulations.

4.1 CP Process Data across Australian States and Territories

There are two main publicly available sources of CP process data across Australian states and territories – *Child Protection Australia* (AIHW, 2019a) published by Australia Institute of Health and Welfare (AIHW) and *Report on Government Services* (SCRGSP, 2019) published under the direction of Steering Committee for the *Review of Government Service Provision* (SCRGSP). Both of these publish annual aggregate CP data across Australian states and territories. There are significant data overlap between the two publications, with AIHW being the primary providers in certain aspects of CP data.

These annual aggregate data cover a wide range of variables, including CP process data such as the number of children in notification, investigation, substantiation, OOHC, as well as types of abuse, demographic information of children involved in CPS, CP expenditures, among others. In line with the objectives of this dissertation, the focus will be on CP process data related to the populations of children involved in CPS.

The trends of CP process data across the states and territories are examined first. Prior to the 2009-10 financial year[6], the number of children involved in investigation related data were reported for children between 0 to 16 years of age. From 2009-10 onward, the

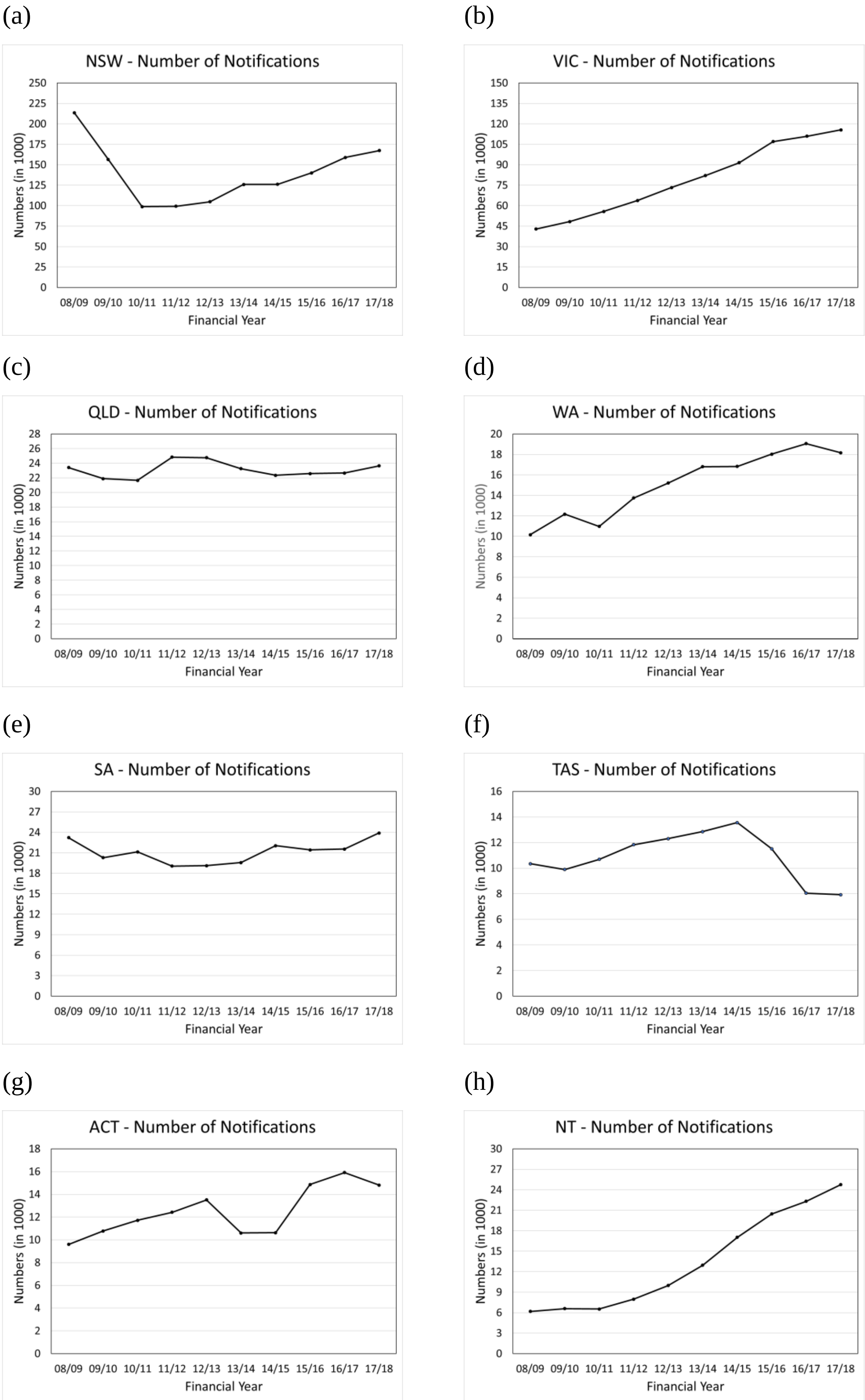
[6] 1 July 2009 to 30 June 2010.

data were provided for children between 0 to 17 years of age. For consistency reason, only data from 2009-10 to 2017-18 are included unless specifically noted.

It should be noted that the focus here will be on describing changes in the trends of the variable in each jurisdiction and not on directly comparing the number of notifications across the jurisdictions. This is due to the fact that legislation and policies regarding CP notifications vary across the jurisdictions. For example, there are two different ways of logging a CP report as notifications, upon initial contact with CP authority – caller-defined and agency-defined (AIHW, 2019a). In the caller-defined approach, caller reports are recorded as notifications without assessment by the CP agency. This approach is used in Victoria (VIC) and the Australian Capital Territory (ACT). In the agency-defined approach, an initial assessment based on criteria defined by the jurisdiction is made to determine if a report should be recorded as a notification. This approach is used in New South Wales (NSW), Queensland (QLD), Western Australia (WA), South Australia (SA), Tasmania (TAS), and the Northern Territory (NT). Focusing on the trends in each jurisdiction accentuates changes in the jurisdiction without having to reconcile the different practices across the jurisdictions.

4.1.1 Number of Notifications

Figure 4.1 shows the trends of the number of notifications for all states and territories. In each of the plots, the horizontal axis indicates financial years while the vertical axis indicates number in 1000. For this variable, the state level data are available from 2008-09 to 2017-18 and the data for the longest possible period are plotted so that large systemic changes in some jurisdictions can be shown. Trends differ across the jurisdictions. While the numbers of notifications have been relatively stable in QLD and SA over the years, those in other jurisdictions exhibited significant changes. In VIC and NT, the numbers of notifications increased monotonically. WA also largely registered an increasing trend. The number of notifications in NSW decreased greatly in 2009-10, reaching a low in 2010-11, but then slowly grew to the level in 2009-10. The precipitous drop started in 2009-10 in NSW was due to legislation change which resulted in the adoption of a higher threshold for reporting of concerns (NSW Ombudsman, 2011). In TAS, the number of notifications grew until 2014-15, before taking a downward turn in 2015-16, partly due to the change from the caller-defined



Raw data source: SCRGSP (2017); AIHW (2018, 2019a)

Figure 4.1. Trends of the Number of Notifications

approach to the agency-defined approach. Finally, in ACT, the trend displayed a trough from 2013-14 to 2014-15, before growing again and staying stable.

Table 4.1 shows the differences in the number of notifications between 2009-10 and 2017-18 across the jurisdictions. Note that the differences were calculated using 2009-10 as the base year to facilitate comparisons with other variables for which consistent data are available only starting 2009-10. It can be seen that VIC and NT registered the largest increase, with 139% and 276% from the 2009-10 level, respectively. Differences in other jurisdictions reflect the description given in the paragraph above. For NSW, using 2010-11 instead of 2009-10 as the base year, the number of notifications increased by 69% (69000 more notifications).

Table 4.1 Differences in Number of Notifications between 2009-10 and 2017-18

2009-10 to 2017-18	NSW	VIC	QLD	WA	SA	TAS	ACT	NT
Difference (in 1000)	11	67	2	6	4	-2	4	18
% Differences	7%	139%	8%	49%	18%	-20%	37%	276%
Raw data source: SCRGSP (2017); AIHW (2018, 2019a)								

Trends in the number of notifications are informative and important, as they indicate societal responses to suspect maltreatments. Moreover, the more notifications there are, the more resources will be spent dealing with them. However, the number of notifications does not reflect the population of children involved in CP notifications, since an incident that triggers a CP involvement may have multiple notifications. Moreover, a child may be involved in distinct incidents of CP notifications within the year.

4.1.2 Children in Notifications

To get a better picture of the population of children involved in the intake phase of CP process, Figure 4.2 shows the number of children in notifications from 2009-10 to 2017-18. In each of the plots, the horizontal axis indicates financial years while the vertical axis indicates number in 1000. It can be seen that the trends are similar to the trends of the number of notifications. VIC and NT registered the largest changes; the numbers in QLD and SA were relatively stable; WA, and ACT registered relatively modest growth; TAS was the only jurisdiction with an overall negative trend, due to the change in notification definition in 2015-16; NSW exhibited a sharp drop, reaching a low in 2010-11 before rising monotonously to the level before the drop.

As shown in Table 4.2, percentage differences in number of children in notifications between 2009-10 and 2017-18 were smaller than those in number of notifications. For NSW, using 2010-11 instead of 2009-10 as the base year, the number increased by 51%.

Table 4.2 Differences in Number of Children in Notifications between 2009-10 and 2017-18

2009-10 to 2017-18	NSW	VIC	QLD	WA	SA	TAS	ACT	NT
Difference (in 1000)	3.4	39.8	1.3	4.6	1.0	-0.9	1.7	7.2
% Difference	4%	105%	6%	44%	7%	-13%	31%	153%
Raw data source: SCRGSP (2019)								

The differences between percentage changes in number of children in notifications and the number of notifications are reflections of changes in the number of notifications per child in notifications. Table 4.3 shows the ratios of the number of notifications to the number of children in notifications, which serves as a proxy of number of notifications per child in notifications. It can be seen that NT exhibited the largest ranges across the year, and the largest change from 2009-10 to 2017-18, followed by VIC and SA. Once again, TAS was the only jurisdiction that exhibited a drop. For NSW, using 2010-11 instead of 2009-10 as the base year, the increase would be 13% instead of 3%.

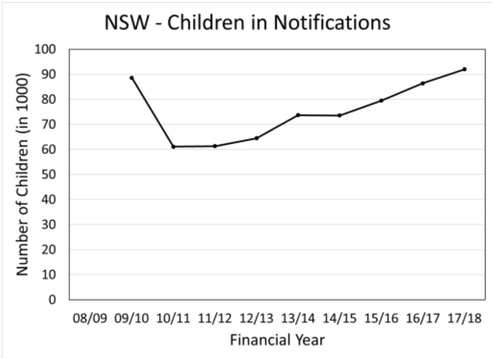
Table 4.3 Ratio of Number of Notifications and Children in Notification

2009-10 to 2017-18	NSW	VIC	QLD	WA	SA	TAS	ACT	NT
Mean	1.72	1.42	1.41	1.18	1.56	1.45	2.04	1.66
Range	1.62 to 1.84	1.28 to 1.52	1.11 to 1.17	1.13 to 1.23	1.50 to 1.64	1.31 to 1.55	1.92 to 2.14	1.35 to 2.07
% Difference	3%	16%	2%	4%	10%	-8%	4%	49%
Raw data source: SCRGSP (2019)								

4.1.3 Per 1000 Rates of Children in Notifications, Investigations and Substantiations

The number of children in notifications does not take into account of changes of the general populations. In order for CP process data to be compared across the years, taking into account changes in general populations of children, one can examine these data in terms of per capita or per 1000 rates of the number of children across the process (e.g. notifications, investigations, substantiations, etc). Figure 4.3 shows the per 1000 rates of children in notifications, finalised investigations, and substantiations of

(a)



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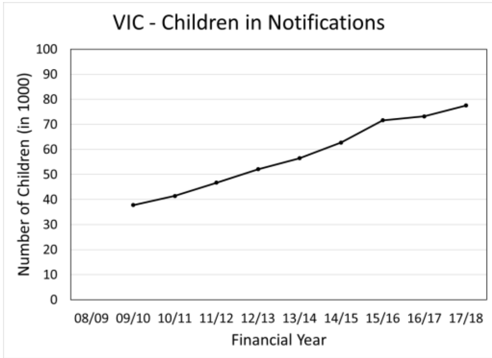

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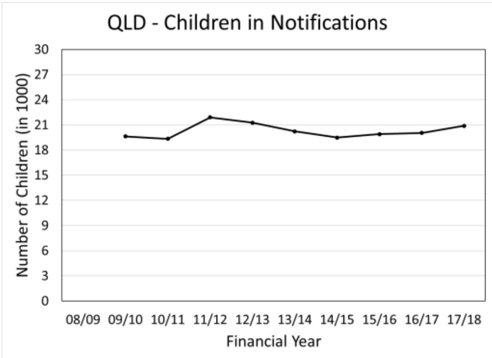

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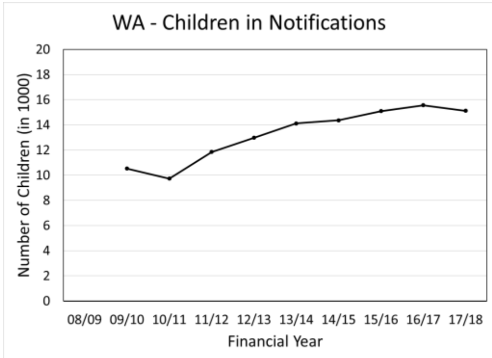

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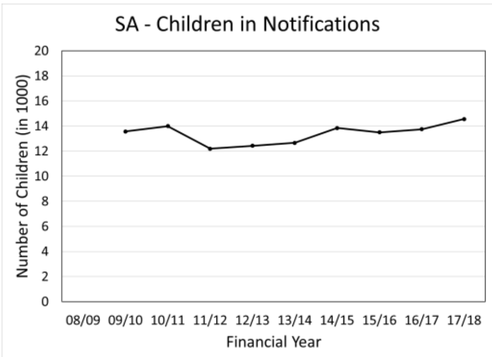

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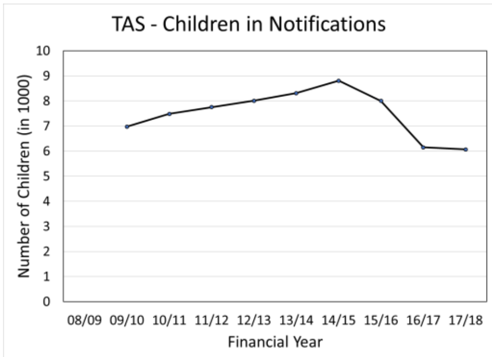

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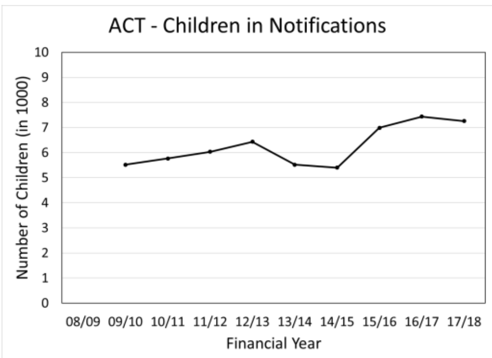

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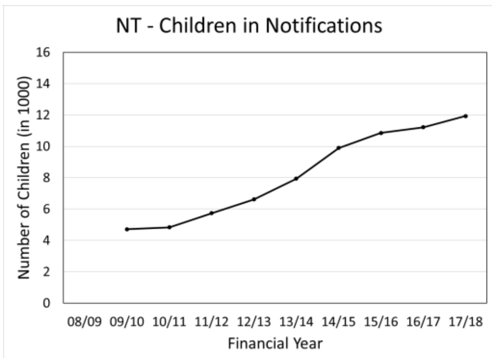


Raw data source: SCRGSP (2019)

Figure 4.2. Trends of Children in Notifications (in Thousands of Children)

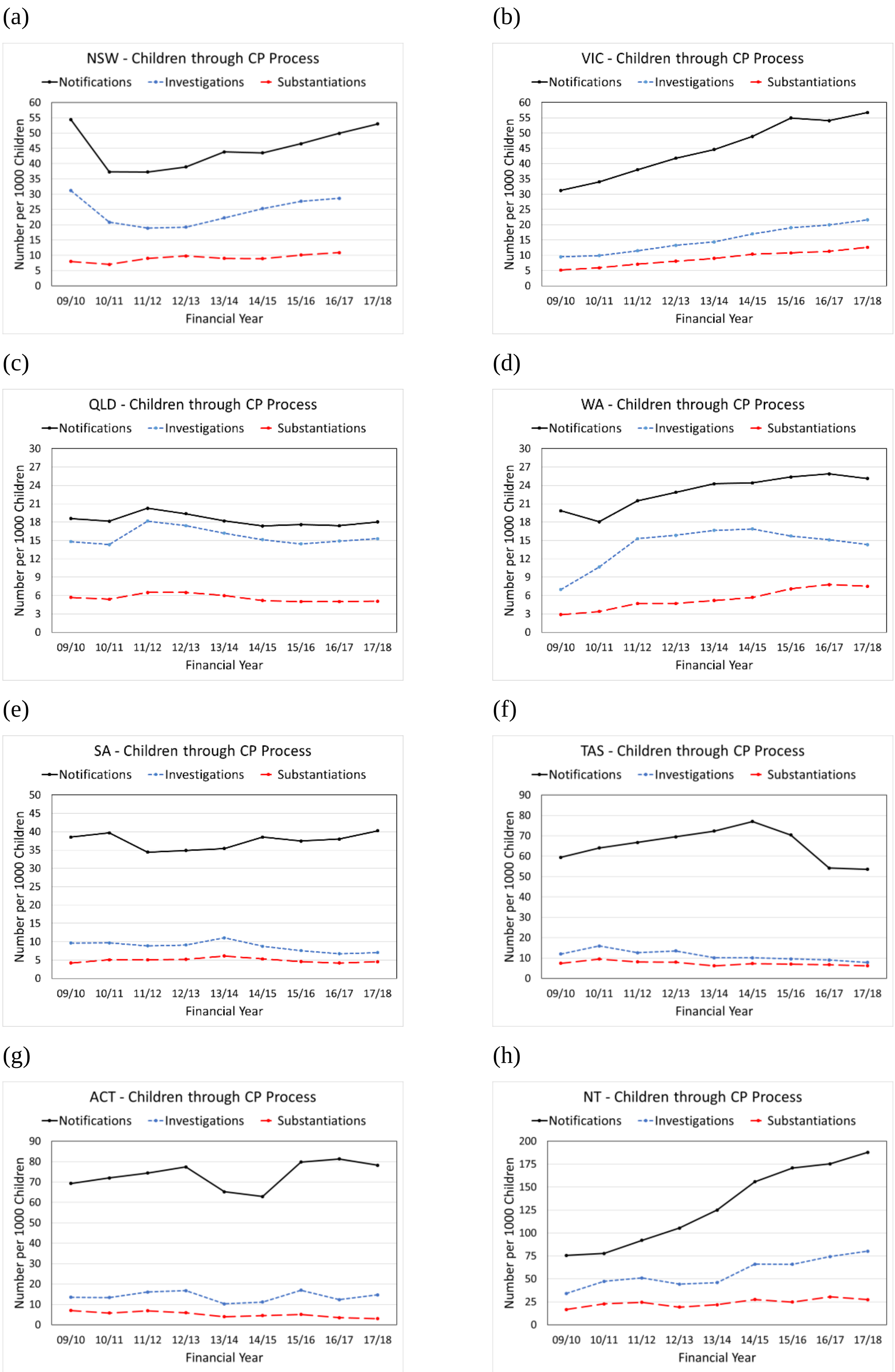
investigations across the states and jurisdictions. In each of the plots, the horizontal axis indicates financial years while the vertical axis indicates number per 1000. Note that data for children in finalised investigations and substantiations were not available for NSW in 2017-18 due to ongoing changes in its CP data system.

Among all the jurisdictions, QLD is unique in that the per 1000 rate of children in finalised investigation tracked very closely that of children in notifications, both in trend and magnitude. This is because all notifications are assessed as investigations in QLD. The discrepancy in magnitude between the rate of children in notifications and the rate of children in investigations most likely stemmed from the fact that not all investigations for notifications received within the year were finalised, and that some investigations were closed without reaching an outcome. Finalised investigations have been used in Figure 4.3 since this is the variable consistently provided in the published reports.

In NSW, the trend of children in investigations resembles that of children in notifications, with a large drop from 2009-10 to 2010-11. It stabilizes between 2010-11 and 2012-13, before slowly rising again. In VIC and NT, per 1000 rate of children in investigations exhibited a rising trend from 2009-10 to 2017-18, resembling the trends of children in notifications. In SA and TAS, per 1000 rate of children in investigations decreased from 2009-10 to 2017-18. Per 1000 rates of children in investigations remained stable although there were fluctuations across the years.

As shown in Table 4.4, VIC and NT exhibited the largest increases in per 1000 rates of children in notifications between 2009-10 and 2017-18, at 82% and 149%, respectively. WA exhibited a sizable increase of 27% while ACT registered an increase of 13%. TAS exhibited a decrease of 10%. The rates for QLD and SA were stable. NSW registered a decrease of 3%. However, using 2010-11 instead of 2009-10 as the base year, the rate in NSW increased by 42%, as reflected in the trend shown in Figure 4.3 (a).

The per 1000 rate of children in investigations is particularly important for the objectives of this study, in which involvement with the CP system is defined as having been involved in at least one CP investigation (see Chapter 3, definition of Q_1). Three jurisdictions – VIC, WA, and NT – exhibited increases larger than 100% from 2009-10 to 2017-18. ACT registered an increase of 8%. The rate for QLD was relatively stable. For SA, per 1000 rate of children in investigations decreased by 27%, despite the rate of



Raw data source: SCRGSP (2019)

Figure 4.3. Per 1000 Rates of Children across CP Process

children in notifications staying relatively stable. The rate for TAS decreased by 35%. NSW registered a decrease of 8%. However, using 2010-11 instead of 2009-10 as the base year, the rate in NSW increased by 37%.

For children in substantiations, VIC and WA exhibited the largest increase, at 143% and 159%, respectively. While NT exhibited increases larger than 100% in the other two variables, the increase in children in substantiations was only 65%. SA registered a relatively small increase of 9%. ACT exhibited a large decrease of 57%. QLD and TAS exhibited decreases of 11% and 16% respectively. Interestingly, NSW exhibited an increase of 36% even when using 2009-10 as the base year. As can be seen in Figure 4.3 (a), while per 1000 rates of children in notifications and investigations exhibited precipitous drops in 2010-11 due to the change in legislation, per 1000 rate of children in substantiations did not exhibit a similar drop.

Table 4.4 Changes in Per 1000 Rates of Children through CP Process

2009-10 to 2017-18	NSW	VIC	QLD	WA	SA	TAS	ACT	NT
Children in Notifications (number per 1000)								
Difference	-1.4	25.5	-0.6	5.3	1.7	-5.8	9.0	112.3
% Difference	-3%	82%	-3%	27%	4%	-10%	13%	149%
Children in Investigations (number per 1000)								
Difference	-2.6	12.1	0.5	7.3	-2.6	-4.2	1.1	46.1
% Difference	-8%	128%	3%	105%	-27%	-35%	8%	135%
Children in Substantiations (number per 1000)								
Difference	2.9	7.4	-0.6	4.6	0.4	-1.2	-4.0	10.8
% Difference	36%	143%	-11%	159%	9%	-16%	-57%	65%
Raw data source: SCRGSP (2019)								

4.1.4 Children in OOHC

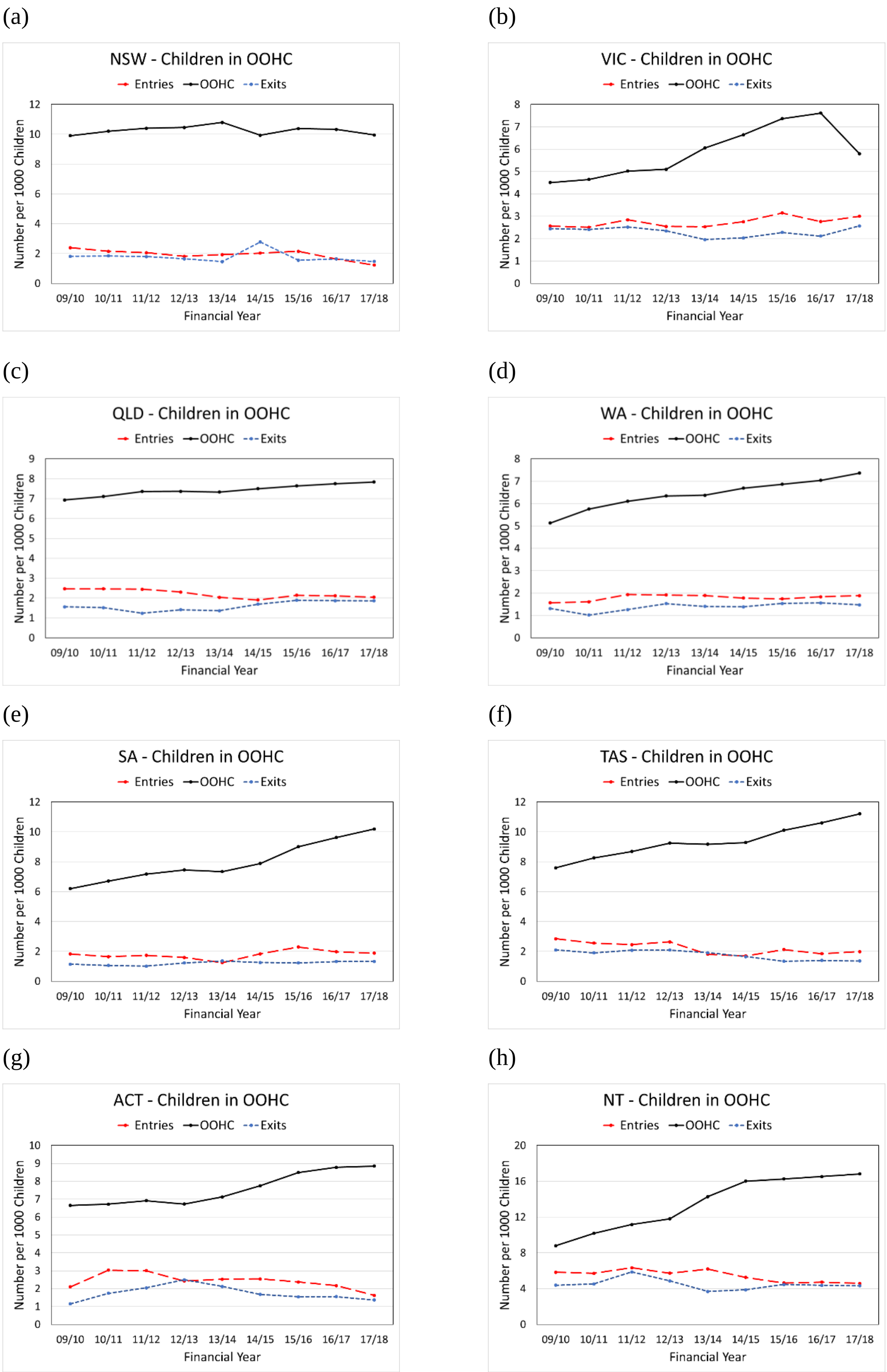
Having shown the trends of CP process variables for the intake and investigation phases, the analysis turns to OOHC data, including children entering OOHC, exiting OOHC, and in OOHC. For entries and exits, the data were annual aggregate data, with children counted only once even if they had more than one OOHC admissions. For children in OOHC, the data were point-in-time data on June 30, i.e. the number of children in OOHC at the end of the financial years.

When viewing the data, one should also keep in mind changes in counting rules, of which one of the most significant is the inclusion or exclusion of children on third-party parental responsibility orders (TPRO) from OOHC reporting. Children on such orders have historically been included in OOHC counting. However, in 2014-15, NSW started to exclude them from OOHC reporting. WA did the same starting 2015-16, and VIC starting 2017-18 (AIHW, 2019a).

Figure 4.4 shows the per 1000 rates of children in OOHC, OOHC entries and OOHC exits. In each of the plots, the horizontal axis indicates financial years while the vertical axis indicates number per 1000; the solid curve represents OOHC population; the long-dashed curve represents OOHC entries; the short-dashed curve represents OOHC exits. It can be seen that all states except NSW exhibited a rising trend in the rate of children in OOHC. VIC had a rising trend until 2017-18, when a new counting rule took effect, as described above. As a result of excluding children on TPRO in OOHC, a sharp drop was registered in 2017-18. In NSW, there was a rising trend until 2014-15, when children on TPRO was excluded from OOHC. The exclusion resulted in a spike in children exiting from OOHC, together with a corresponding drop in the OOHC trend. While WA excluded children on TPRO from OOHC in 2015-16, there was no sharp drop in the OOHC trend, as the proportion of children on TPRO in WA was relatively small.

In each of the jurisdictions, the trends in OOHC largely reflected the trends of the differences between entries and exits. For examples, in SA and TAS, the trends in OOHC exhibited a period of flatline around 2013-14 wedged between two stretches of relatively steeper growth, as indicated by the gaps between entries and exits. Also, the drop in the NSW OOHC trend in 2014-15 was reflected by the spike in OOHC exits in the same year, due to the exclusion of children on TPRO. In VIC, steeper slopes along OOHC trend reflected larger gaps between entries and exits, except for 2017-18, which could be due to the exclusion of children on TPRO not being included in exits. Excluding NSW and VIC, QLD exhibited the smallest growth, while all other jurisdictions exhibited rather significant growth.

The net changes in per 1000 rates of children in OOHC from 2009-10 to 2017-18 are shown in Table 4.5. With the exclusion of children on TPRO, the rates in NSW stayed essentially unchanged. As mentioned above, QLD registered a modest growth of 13%, while VIC exhibited a growth of 28.5%. Note that if 2016-17 were used to calculate the change in VIC, there would be a growth of 68.8%. NT registered the largest growth, with per 1000 rate of children in OOHC almost double between 2009-10 to 2017-18.



Raw data source: SCRGSP (2019); AIHW (2011, 2016, 2019a)

Figure 4.4. Per 1000 Rates of Children in OOHC

Per 1000 rate of children in OOHC, labelled as Q_2 in Chapter 3, is one of the variables of focus in our population dynamics model. The figures above will be used in the sections that explore model behaviours.

Table 4.5 Changes in Per 1000 Rates of Children in OOHC

2009-10 to 2017-18	NSW	VIC	QLD	WA	SA	TAS	ACT	NT
Difference	0.05	1.29	0.9	2.23	3.99	3.62	2.19	8.01
% Difference	0.5%	28.5%	13.0%	43.5%	64.3%	47.6%	32.9%	91.0%

4.2 Coupling of CP Process

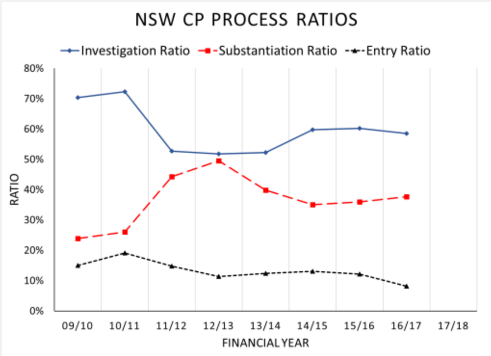
In a study on the structure and performance of child abuse reporting systems in the USA, Van Voorhis & Gilbert (1998) found a negative correlation between the volume of CP reports and substantiation percentage. They attributed this to the decreased capacity of the CP investigative agencies to fully investigate cases or the increased inclusion of cases which did not meet the criteria of maltreatments. Both these situations have a strong impact on the populations of children involved in the CP systems.

In this section, CP process variables and their bivariate relations were analysed as a means to uncover systemic coupling which may indicate a sign of capacity issues. Specifically, the following variables were constructed:

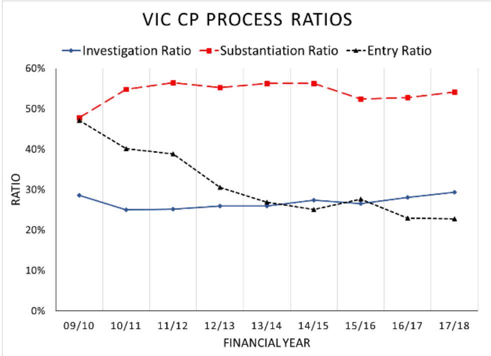
- Investigation ratio, which is the ratio of the number of investigations and the number of notifications. This indicates the fraction of notifications investigated within the financial years.
- Substantiation ratio, which is the ratio of the number of substantiations and the number of investigations. This indicates the fraction of investigations that led to substantiations.
- Entry ratio, which is the ratio of the number of OOHC entries and the number of substantiations. This indicates the fraction of substantiations that led to OOHC entries.

Figure 4.5 shows the above three ratios from 2009-10 to 2017-18. In each of the plots, the horizontal axis indicates financial years while the vertical axis indicates the ratio in percent; the solid curve indicates investigation ratios; the long-dashed curve indicates

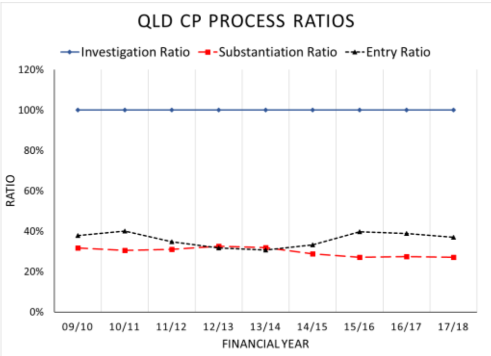
(a)



(b)



(c)



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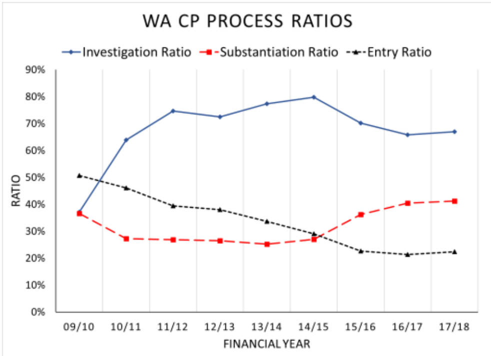
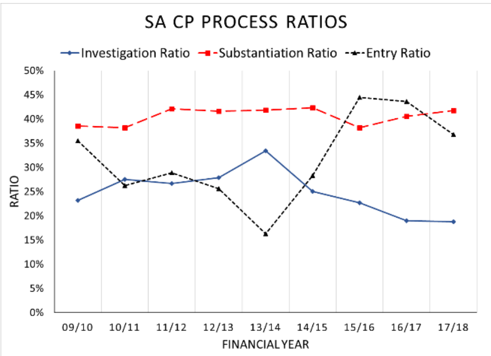
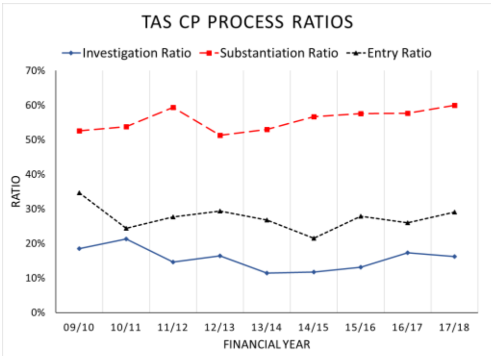


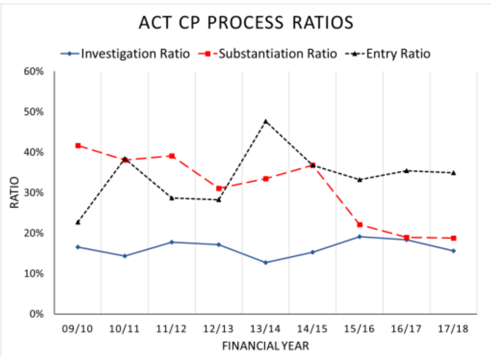
(e)



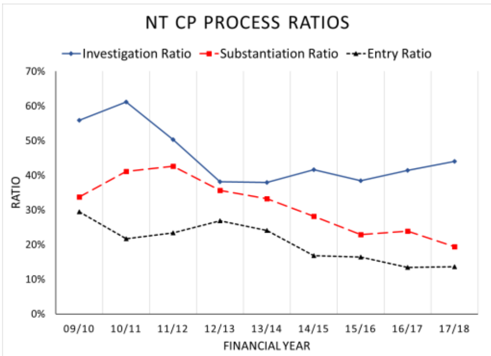
(f)



(g)



(h)



Raw data source: SCRGSP, 2017, 2019; AIHW, 2011, 2016, 2019

Figure 4.5. Ratios of CP Process Variables

substantiation ratios; the dotted curve indicates entry ratios. For NSW, data were only plotted up to 2016-17 due to changes in counting rules for CP investigations in 2017-18. Four jurisdictions, VIC, SA, TAS and ACT, had relatively low investigation ratios, which were below 30% on average. QLD had a 100% investigation ratio as all notifications had to be investigated. The ratios for NSW, WA and NT were relatively high, ranging from 46% to 68% on average.

Substantiation ratios for the four jurisdictions with low investigations ratios were relatively high, except for ACT in recent years. VIC, SA and TAS registered the highest substantiation ratios on average, ranging from 40% to 55%. Substantiation ratios were relatively stable through the years in VIC, QLD, SA and TAS. They had decreased over the years in ACT and NT.

Entry ratios ranged from 13.9% in NSW to 37.1% in WA. Four jurisdictions, NSW, VIC, WA and NT exhibited a downward trend. In QLD and TAS, the values were relatively stable across the years. In SA and ACT, there were large fluctuations. It should be noted that in VIC and WA, while the per 1000 rate of children entering OOHC has been rising or flat (see Figure 4.4), the entry ratio has been decreasing. This may either be due to OOHC system capacity issues or the increasing use of alternative means to serve children with substantiated maltreatments.

Pearson correlation for each of the pairs of the constructs were calculated across the states and territories. The results are shown in Figure 4.6, where the first bar (solid) in each jurisdiction shows the correlation between investigation ratio and substantiation ratio; the second bar (horizonal stripes) shows the correlation between investigation ratio and entry ratio; the third bar (slanted stripes) shows the correlation between investigation ratio and exit ratio). For correlations with a p value smaller than 0.05, the values are shown.

One can see that in NSW, the investigation ratio and the substantiation ratio had a statistically significant negative correlation. In SA, the investigation ratio and the entry ratio had a statistically significant negative correlation. In NT, substantiation ratio had a statistically significant positive correlation with entry ratio. In VIC, QLD, WA, TAS, and ACT, none of the variables are significantly correlated. In the following sections, the statistically significantly correlated variables in each jurisdiction are examined.

NSW

From Figure 4.5 (a), the strong negative correlation between the investigation ratio and the substantiation ratio is apparent. Figure 4.7 (a) shows another way of visualizing the linear relationship, with the x-axis being the investigation ratio and the y-axis being the

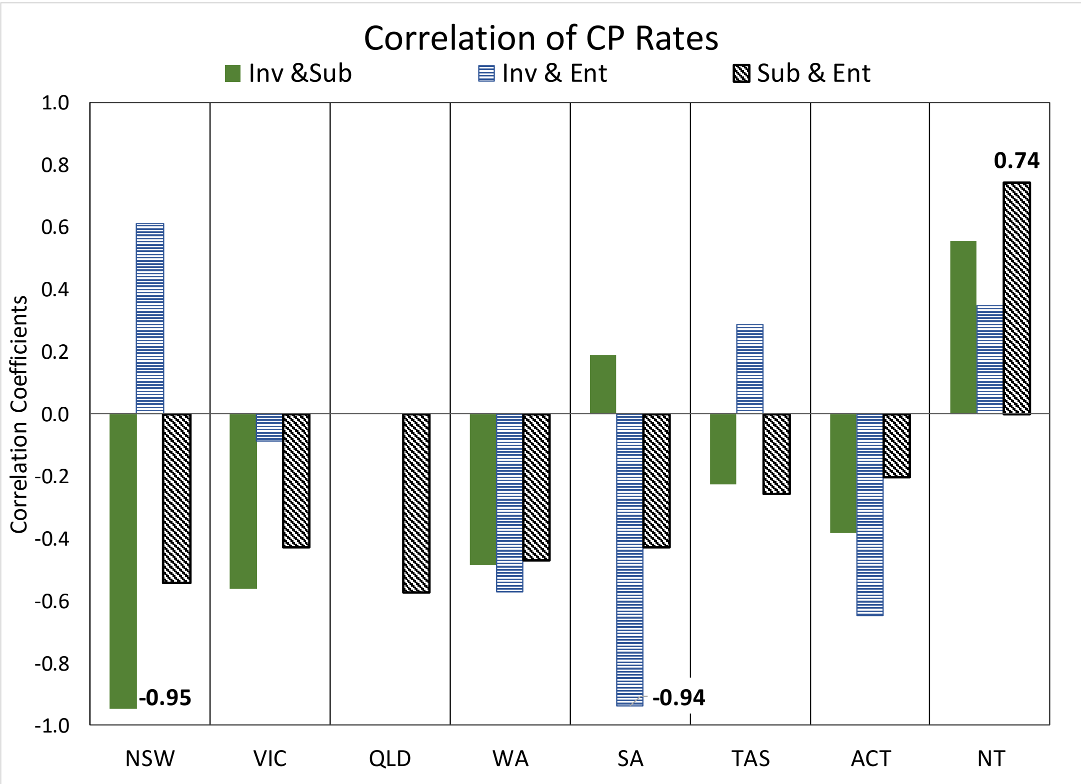


Figure 4.6 Correlations of CP Process Variables

substantiation ratio. R^2 of the linear regression line, which is just the square of the correlation coefficient, shows that about 90% of the variance in the substantiation ratio can be explained by that in the investigation ratios. This indicates a potential similarity to the situation pointed by Van Voorhis & Gilbert (1998). In other words, a large investigation ratio may indicate either a decreased capacity of the CP investigative agencies to fully investigate cases or the increased inclusion of cases which did not meet the criteria of substantiation.

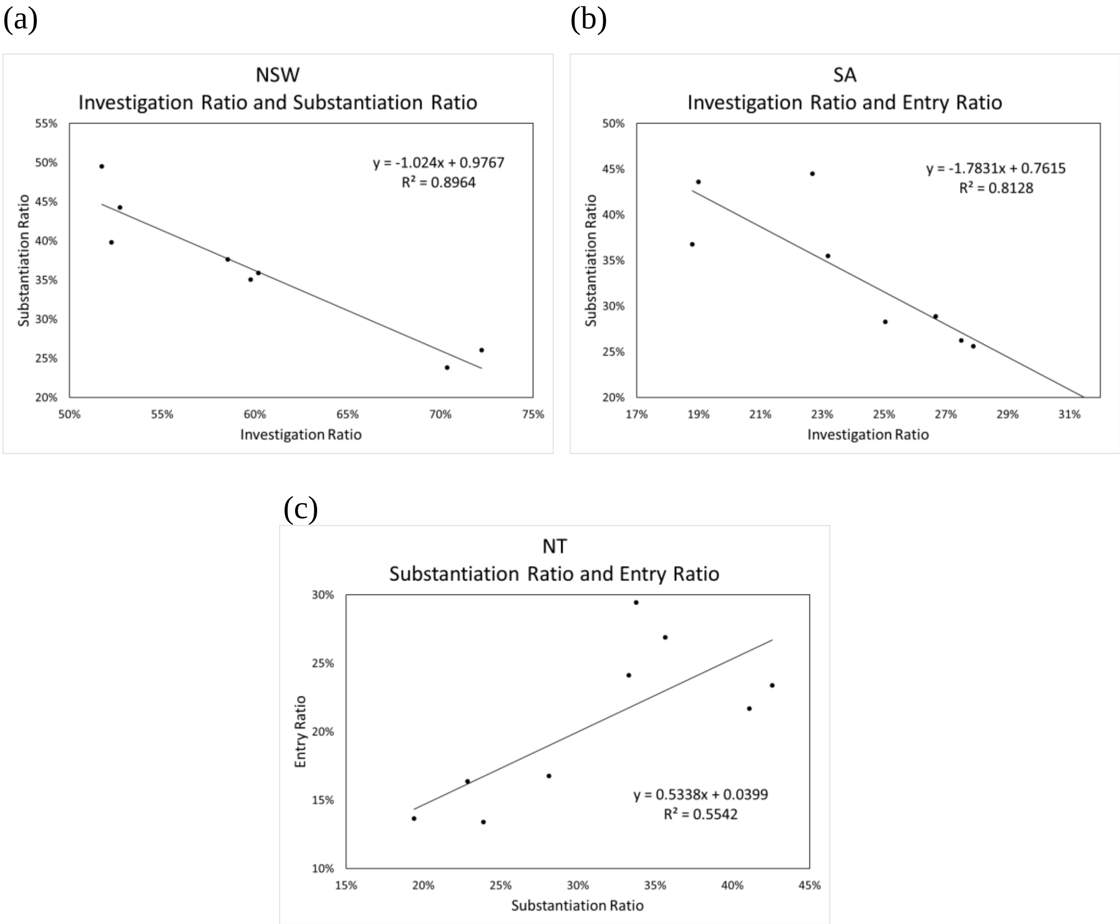


Figure 4.7 Ratio Coupling in NSW, SA and NT

SA

In Figure 4.5 (e) the strong negative correlation between the investigation ratio and the entry ratio is apparent, especially before 2015-16. Figure 4.7 (b) shows another way of visualizing the linear relationship, with the x-axis being the investigation ratio and the y-axis being the entry ratio. R^2 of the linear regression line, which is just the square of the correlation coefficient, shows that about 81% of the variance in the entry ratio can be explained by that in the investigation ratios. Further analysis of higher frequency data is needed to better understand the relationship identified in these graphs.

NT

In Figure 4.5 (h) one can see the correlation between the substantiation ratio and the entry ratio. Figure 4.7 (c) shows another way of visualizing the linear relationship, with the x-axis being the investigation ratio and the y-axis being the entry ratio. R^2 of the linear regression line, which is just the square of the correlation coefficient, shows that about 55% of the variance in the entry ratio can be explained by that in the investigation ratios. The correlation is not as statistically significant as those shown in NSW and SA.

The above analyses are simplistic and are based on annual aggregate data. However, they could serve as useful signposts to examine the issue of CP process capacity, which are not only highly relevant to the modelling study, but are important in and of themselves.

4.3 New and Repeat Clients

The analysis now turns to CP data for new and repeat clients. Since 2014, AIHW began to publish data that provide breakdowns of the number of children receiving CPS, the number of children in investigations, the number of children on care and protection orders, and the number of children in OOHC, in terms of new and repeat clients, for some jurisdictions. In subsequent years, data on new and repeat clients provided by AIHW have undergone some variations, as highlighted below.

- 2012-13 financial year: Aggregate data including SA, TAS and NT.
- 2013-14 financial year: Aggregate data excluding QLD and ACT.
- 2014-15 financial year: Aggregate data excluding ACT.
- 2015-16 financial year: States and Territories breakdown for all jurisdictions.
- 2016-17 financial year: States and Territories breakdown for jurisdictions excluding ACT.
- 2017-18 financial year: States and Territories breakdown for jurisdictions excluding NSW and ACT.

Since data on new and repeat clients are key to our objectives of modelling the sub-populations of children involved in CPS, the available data are compiled and analysed in detail.

Table 4.6 shows the breakdown in terms of new and repeat clients for children in investigations and children in OOHC, aggregating SA, TAS and NT. Data in 2012-13 were originally reported as the aggregate of the three jurisdictions (AIHW, 2014); the data for the other three years were aggregated from individual jurisdiction data. This aggregated data series spans the longest time since such data were published. The most significant feature is the increase in the proportion of repeat clients. For children in investigations, new clients accounted for 47.0% of the total number of clients in 2012-

13 but only 28.1% by 2017-18. For children in OOHC, new clients accounted for 12.4% of the total number of clients in 2012-13 but only 5.5% in 2017-18. The proportion of new clients in investigations decreased by 40.1% while the proportion of new clients in OOHC decreased by 55.5%. Note that there was a large increase from 2012-13 to 2015-16, after which the proportions fluctuated within a small range. This phenomenon will be interpreted with model simulations later in this chapter.

Table 4.6 Aggregate New and Repeat Client Data for SA, TAS & NT

	Children in Investigations		Children in OOHC	
	New Clients %	Repeat Clients %	New Clients %	Repeat Clients %
2012-13	47.0%	53.0%	12.4%	87.6%
2015-16	30.6%	69.4%	5.9%	94.1%
2016-17	27.8%	72.2%	6.0%	94.0%
2017-18	28.1%	71.9%	5.5%	94.5%

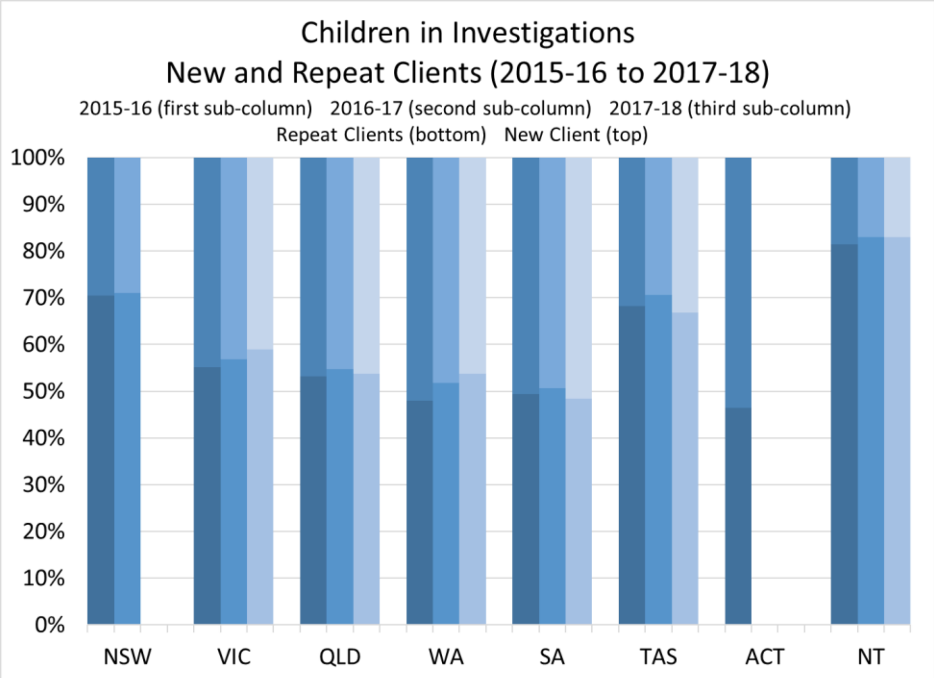
Table 4.7 shows the new- and repeat-client breakdown for children in investigations and children in OOHC, aggregating NSW, VIC, WA, SA, TAS and NT. Except for 2013-14, the data for the other two years were aggregated from individual jurisdiction data. Unlike the large changes between 2012-13 and subsequent years shown in Table 4.6, the proportions of new clients and repeat clients were stable across the three years, especially for children in OOHC. For children in investigations, new clients accounted for 38.5% of the total number of clients in 2013-14, but only 35.0% in 2016-17, decreasing by 9.1%.

Table 4.7 Aggregate New and Repeat Client Data for NSW, VIC, WA, SA, TAS & NT

	Children in Investigations		Children in OOHC	
	New Clients %	Repeat Clients %	New Clients %	Repeat Clients %
2013-14	38.5%	61.5%	5.0%	95.0%
2015-16	36.4%	63.6%	5.4%	94.6%
2016-17	35.0%	65.0%	4.6%	95.4%

Figure 4.8 (a) shows the breakdown of new clients and repeat clients for children in investigations, from 2015-16 to 2017-18. In the data for each jurisdiction (horizontal axis), the three sub-columns from left to right show the data from 2015-16 to 2017-18, respectively. Data were not available for NSW in 2017-18, and for ACT in 2016-17 and 2017-18. From the figure, it can be seen that ACT had the lowest proportion of repeat clients while NT had the highest proportions. For all jurisdictions, the proportions of

(a)



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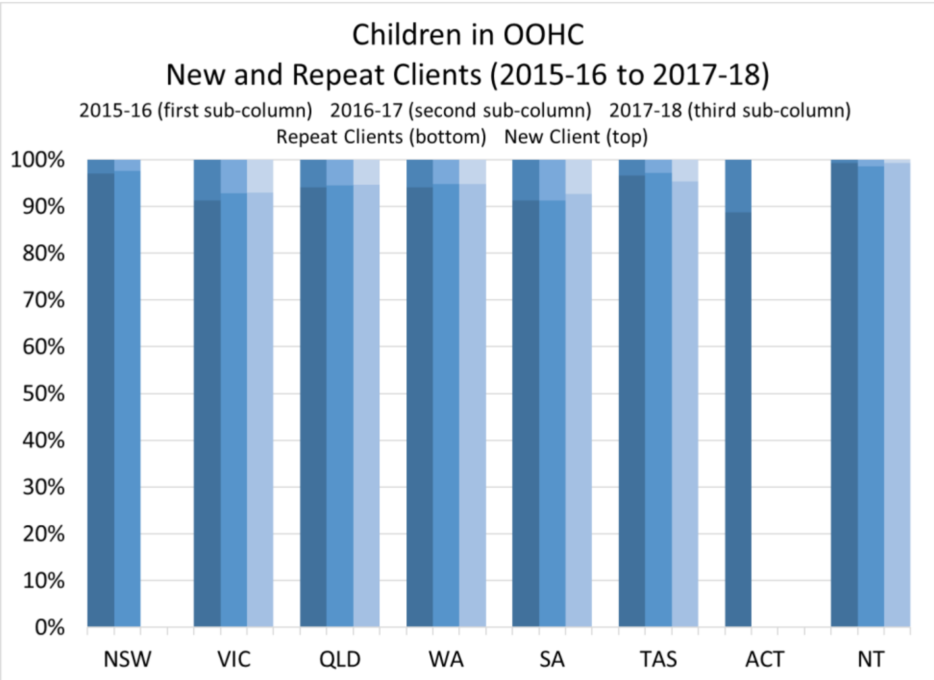


Figure 4.8 New and Repeat Clients for Children in Investigations and OOHC

repeat clients were rather stable, except for VIC and WA, in which the proportions of repeat clients rose by 6.9% and 12.1%, respectively.

Figure 4.8 (b) shows the breakdown of new clients and repeat clients for children in OOHC, from 2015-16 to 2017-18, with the same layout as in Figure 4.8 (b). Once again, ACT had the lowest proportion of repeat clients (88.7%) while NT had the highest proportions (99.0% on average). Overall, repeat clients accounted for the absolute majority of children in OOHC.

4.3.1 Investigation Rates and OOHC Entries Rates for New and Repeat Clients

The data provided by AIHW give the proportions of new and repeat clients for children in investigations and children in OOHC. However, such breakdowns are not provided for children in notifications and children entering OOHC. Some information on these two variables is important for our population dynamics modelling efforts. Moreover, information on these two variables is important in and of themselves as it provides a more complete picture of the phenomenon of "churn". Information about these variables can be deduced using supplementary information on children in notifications and children entering OOHC. The third row in Table 4.8 shows the number of children in notification in 2015-16, as provided in Table 16A.1 of the 2019 Reports on Government Services (SCRGSP, 2019). While a breakdown in terms of new and repeat clients were not given, the lower and upper bounds of such information can be deduced by noticing that the minimum number of new clients for children in notifications should be the number of new clients for children in investigations. On the other hand, the maximum number of new clients for children in notifications should be the total number of children in notifications minus the number of repeat clients in investigations. The above relations can be expressed with the following equations.

$$Min(N_{new|notifications}) = N_{new|investigations}$$

$$Max\left(N_{new|notifications}\right) = N_{notifications} - N_{repeat|investigations}$$

Likewise, the lower and upper bounds of repeat clients in notifications can be expressed with the following equations.

$$Min(N_{repeat|notifications}) = N_{repeat|investigations}$$

$$Max\left(N_{repeat|notifications}\right) = N_{notifications} - N_{new|investigations}$$

The lower bounds (LB) and upper bounds (UB) of new and repeat clients in notifications can thus be obtained, as shown in the fourth to seventh rows in Table 4.8. The eighth to eleventh rows in the table show the lower bounds and upper bounds of the proportions of new and repeat clients. Subsequent blocks in the table show the data for 2016-17 and 2017-18.

Table 4.8 Uncertainties for New and Repeat Client Breakdown for Children in Notifications

Notifications	NSW	VIC	QLD	WA	SA	TAS	ACT	NT
2015-16								
All clients	79,487	71,663	19,905	15,090	13,496	8,004	6,991	10,851
New LB	14,219	11,559	8,563	5,696	1,631	425	918	1,014
New UB	45,470	57,410	10,165	9,850	11,901	7,091	6,195	6,406
Repeat LB	34,017	14,253	9,740	5,240	1,595	913	796	4,445
Repeat UB	65,268	60,104	11,342	9,394	11,865	7,579	6,073	9,837
New LB %	17.9%	16.1%	43.0%	37.7%	12.1%	5.3%	13.1%	9.3%
New UP %	57.2%	80.1%	51.1%	65.3%	88.2%	88.6%	88.6%	59.0%
Repeat LB %	42.8%	19.9%	48.9%	34.7%	11.8%	11.4%	11.4%	41.0%
Repeat UB %	82.1%	83.9%	57.0%	62.3%	87.9%	94.7%	86.9%	90.7%
2016-17								
All clients	86,426	73,203	20,051	15,558	13,750	6,149	7,436	11,222
New LB	14,629	12,086	8,509	5,191	1,408	372	N/A	1,000
New UB	50,657	57,310	9,785	10,005	12,304	5,255	N/A	6,346
Repeat LB	35,769	15,893	10,266	5,553	1,446	894	N/A	4,876
Repeat UB	71,797	61,117	11,542	10,367	12,342	5,777	N/A	10,222
New LB %	16.9%	16.5%	42.4%	33.4%	10.2%	6.0%	N/A	8.9%
New UP %	58.6%	78.3%	48.8%	64.3%	89.5%	85.5%	N/A	56.5%
Repeat LB %	41.4%	21.7%	51.2%	35.7%	10.5%	14.5%	N/A	43.5%
Repeat UB %	83.1%	83.5%	57.6%	66.6%	89.8%	94.0%	N/A	91.1%
2017-18								
All clients	92,007	77,526	20,888	15,129	14,566	6,072	7,257	11,937
New LB	N/A	12,436	9,031	4,795	1,530	356	N/A	1,148
New UB	N/A	59,626	10,393	9,577	13,129	5,357	N/A	6,345
Repeat LB	N/A	17,900	10,495	5,552	1,437	715	N/A	5,592
Repeat UB	N/A	65,090	11,857	10,334	13,036	5,716	N/A	10,789
New LB %	N/A	16.0%	43.2%	31.7%	10.5%	5.9%	N/A	9.6%
New UP %	N/A	76.9%	49.8%	63.3%	90.1%	88.2%	N/A	53.2%
Repeat LB %	N/A	23.1%	50.2%	36.7%	9.9%	11.8%	N/A	46.8%
Repeat UB %	N/A	84.0%	56.8%	68.3%	89.5%	94.1%	N/A	90.4%

In most jurisdictions, the lower bounds for the proportions of new clients for children in notifications were stable across the three years, except for WA and SA. In WA, the lower bound for the proportion of new clients decreased from 37.7% to 31.7% from 2015-16 to 2017-18, by 15.9%. In SA, the lower bound for the proportion of new clients decreased from 12.1% to 10.5%, by 13.2%. The upper bounds for the proportion of new clients were quite stable (changes < 5%) in all jurisdictions, except in NT where it decreased by 10.0%. For the proportion of repeat clients, the lower bounds increased by

16.1% in VIC and by 14.4% in NT, and it decreased by 16.5% in SA. In other jurisdictions, there were increases of smaller than 6%. The upper bounds increased by 9.7% in WA and were stable (<2%) in other jurisdictions.

Figure 4.9 shows the uncertainties (ranges) in the proportions of new and repeat clients for children in notification across the jurisdictions, in 2015-16. Only ranges in 2015-16 are shown since changes across the years were not large, as described in the last paragraph. In the figure, the solid lines are the ranges of the proportions of new clients while the dashed lines are the ranges of the proportion of repeat clients. For VIC, SA, TAS, and ACT, the uncertainties for new and repeat clients very much overlap, indicating that without further information, it is not possible to meaningfully distinguish between the two. Moreover, the uncertainties in the proportions of new and repeat clients for VIC, SA, TAS, and ACT were larger than 60%. Note that large uncertainties are due to low investigation ratios and large overlaps between the uncertainties for new and repeat clients are due to the relatively small difference in new and repeat clients. Where investigation ratios were large, the uncertainties for the breakdown of children in notifications into new and repeat clients were also smaller. For example, QLD has the

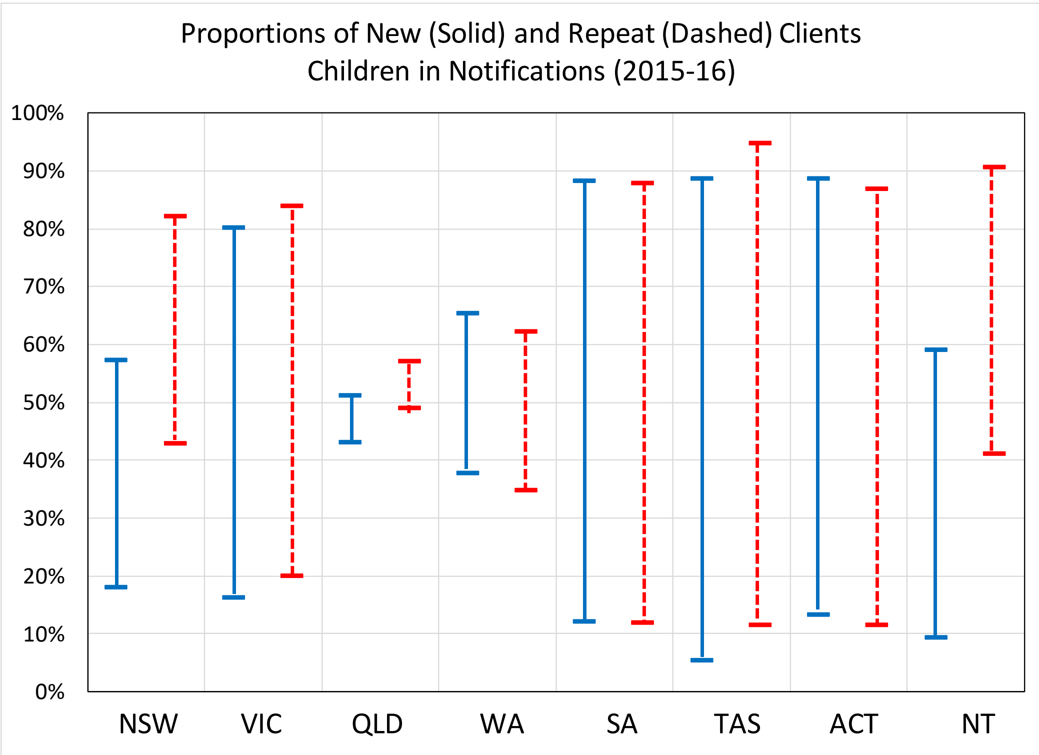


Figure 4.9 Proportions of New and Repeat Clients for Children in Notifications

highest investigation ratios and the certainties for the breakdown in notifications are therefore smallest among the jurisdictions. The relatively small shift between the uncertainties for new and repeat clients is due to the small difference between new and repeat clients in investigations. For NSW and NT, the differences between new and repeat clients in investigations were large; as a result, although the uncertainties were large, the shifts between them were also large.

With the upper bounds of new and repeat clients for children in notifications established, the corresponding lower bounds of the investigation ratios, defined here as children in investigations divided by the upper bounds of children in notifications, can also be calculated. The results for 2015-16 are shown in Table 4.9. The investigation ratio is a measure of the proportion of children in notifications being the subject of investigations. The investigation ratios of new and repeat clients partially inform the parameters λ_{01} and λ_{12} in Chapter 3.Different investigation ratios for new and repeat clients will lead to differences in sub-population structures, and to differences in the composition of new and repeat clients in the OOHC, which will be shown later in model simulations. For the time being, it is noted that among the jurisdictions, QLD once again stands out for having the highest investigation ratios, which were essentially the same for both new and repeat clients. The lower bounds in other jurisdictions largely reflect that shown in the lower bounds for children in notifications, as shown in Figure 4.9.

Table 4.9 Lower Bounds of Investigation Ratios for New and Repeat Clients (2015-16)

Investigation ratios	**NSW**	**VIC**	**QLD**	**WA**	**SA**	**TAS**	**ACT**	**NT**
New Clients LB	31.3%	20.1%	84.2%	57.8%	13.7%	6.0%	14.8%	15.8%
Repeat Clients LB	52.1%	23.7%	85.9%	55.8%	13.4%	12.0%	13.1%	45.2%

So far, only the breakdown into new and repeat clients for children in notifications, children in investigations, and children in OOHC have been examined. While the breakdown into new and repeat clients for children entering OOHC was not provided in the published reports, it can be deduced. To do so, one can simply note that, by definition, new clients in OOHC are children who enter OOHC directly upon being involved in CP investigation for the first time. This is none other than F_{02} in our population dynamics formulation (see Figure 3.1).

Based on the number of children entering OOHC and the number of new clients in OOHC for 2015-16, Table 4.10 shows the breakdown of children entering OOHC into new and repeat clients. Except for ACT, in which new and repeat clients contributed almost equally to children entering OOHC, all other jurisdictions exhibited very uneven distributions, with repeat clients accounted for at least 61.4% (SA) of the children. The jurisdiction with the most lopsided distribution was NT, where 96.6% of the clients were repeat clients.

More important than just the proportion of the numbers of new and repeat clients entering OOHC is the percentages of new and repeat clients admitted to OOHC among the children in investigations. This is shown in the last three rows in Table 4.10, for 2015-16. It can be seen that the ratios ranged from 1.3 in ACT to 6.5 in NT. For NT, this means among the children in investigations, repeat clients were 6.5 times more likely than new clients to be admitted to OOHC.

Table 4.10 New and Repeat Clients for Children Entering OOHC (2015-16)

Client Type	**NSW**	**VIC**	**QLD**	**WA**	**SA**	**TAS**	**ACT**	**NT**
New	605	1,097	635	294	319	44	99	10
Repeat	3,076	3,018	1,793	741	507	198	109	284
New %	16.4%	26.7%	26.2%	28.4%	38.6%	18.2%	47.6%	3.4%
Repeat %	83.6%	73.3%	73.8%	71.6%	61.4%	81.8%	52.4%	96.6%
	Percentage placed from investigations							
New	4.3%	9.5%	7.4%	5.2%	19.6%	10.4%	10.8%	1.0%
Repeat	9.0%	21.2%	18.4%	14.1%	31.8%	21.7%	13.7%	6.4%
Ratio (R/N)	2.1	2.2	2.5	2.7	1.6	2.1	1.3	6.5

Figure 4.10 shows the ratios of repeat and new clients, for percentages of children in investigations admitted to OOHC, from 2015-16 to 2017-18 (bars in solid fill, bars in horizontal stripes, and bars in slanted stripes, respectively). Note that data were not provided for NSW in 2017-18, and for ACT in 2016-17 and 2017-18. TAS and SA exhibited large decreases of 43.5% and 17.3%, respectively. NT exhibited rather large fluctuations (from 6.5 to 3.2, and back to 6.4). The ratios in other jurisdictions were quite stable.

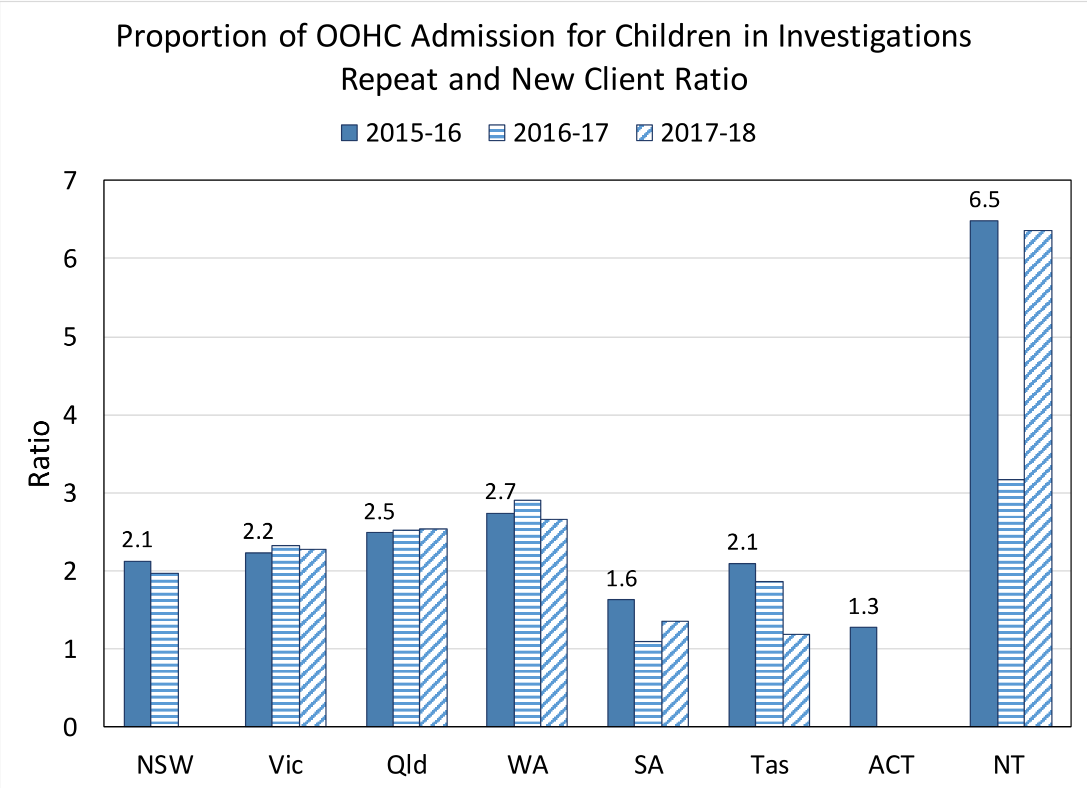


Figure 4.10 Proportion of OOHC Admission for Children in Investigations

4.4 Parameters for the Population Dynamics Model

In this section, data from 2009-10 to 2017-18 are used to estimate the parameters needed for the three-population model (see Chapter 3), shown in Table 4.11. Note that our aim is not to obtain statistically robust estimations as in most cases, as that cannot be done due to the limited information provided in the published reports. Our aim is to logically obtain ranges of parameters that will inform the modelling efforts.

In the following sections, the term "age-out" will be used to denote both children leaving OOHC or moving out of the scope of any stages of CP process because they have grown beyond the statutory CP age limit, which is currently below 18 years of age. It is also used as a short-hand for children in the general population moving out of the 0 to 17 age bracket.

Table 4.11 System Parameters and Available Data for the Three-Population Model

Parameters		Available Data	
α	Gross growth rate of child population (P) due to population increase.	αP	Sum of net growth in general child population (0 to 17 years of age) and the "age out".
η_0	Rate of children aging out from the eligible population (P_0)	$\eta_0 P_0$	Proportion of 17-year olds among children in the 0-17 age bracket
η_1	Rate of children aging out from non-OOHC CPS (P_1')	N/A	N/A
η_2	Rate of children aging out from OOHC (P_2)	η_2	From CP data
λ_{01}	Rate of children entering non-OOHC CPS (P_1') for the first time	$\lambda_{01} P_0$	Number of investigations minus number of OOHC entries for new clients
λ_{02}	Rate of children entering OOHC (P_2) in first-time involvement with CPS	$\lambda_{02} P_0$	Number of OOHC entries for new clients
λ_{12}	Rate of children entering OOHC (P_2) in non-first-time involvement in CPS	$\lambda_{12} P_1$	Number of OOHC entries for repeat clients
λ_{21}	Rate of children leaving OOHC (P_2) not as a result of aging out	λ_{21}	From CP data

Growth Rates of Children in the 0-17 Age Bracket

Figure 4.11 shows the annual growth rate of children in the 0-17 age bracket, calculated using estimated June 30 population data from *Child Protection Australia* (AIHW, 2015, 2019a) and *Australian Demographic Statistics* (Australian Bureau of Statistics, 2018)[7]. The triangles and the associated number show the average annual growth rates (AAGR) from 2010 to 2018 in each jurisdiction. The rate for each year was calculated as the fractional change between the population of that year and the population in the preceding year. It can be seen that population in TAS has shrunk over the years, with an average annual rate of -0.51%. Population in NT has grown but then started to shrink in recent years, resulting in a net growth rate of 0.08%. ACT registered the largest AAGR (1.88%), followed by VIC (1.72%). In NSW, VIC and ACT, there have been an accelerated rate of growth in general. In QLD, the growth rate was relatively stable across the year. In WA, the rate has dropped in recent years. Note that these are the net growth rates, which is the balance of gross population growth and age-out.

[7] 2009 data

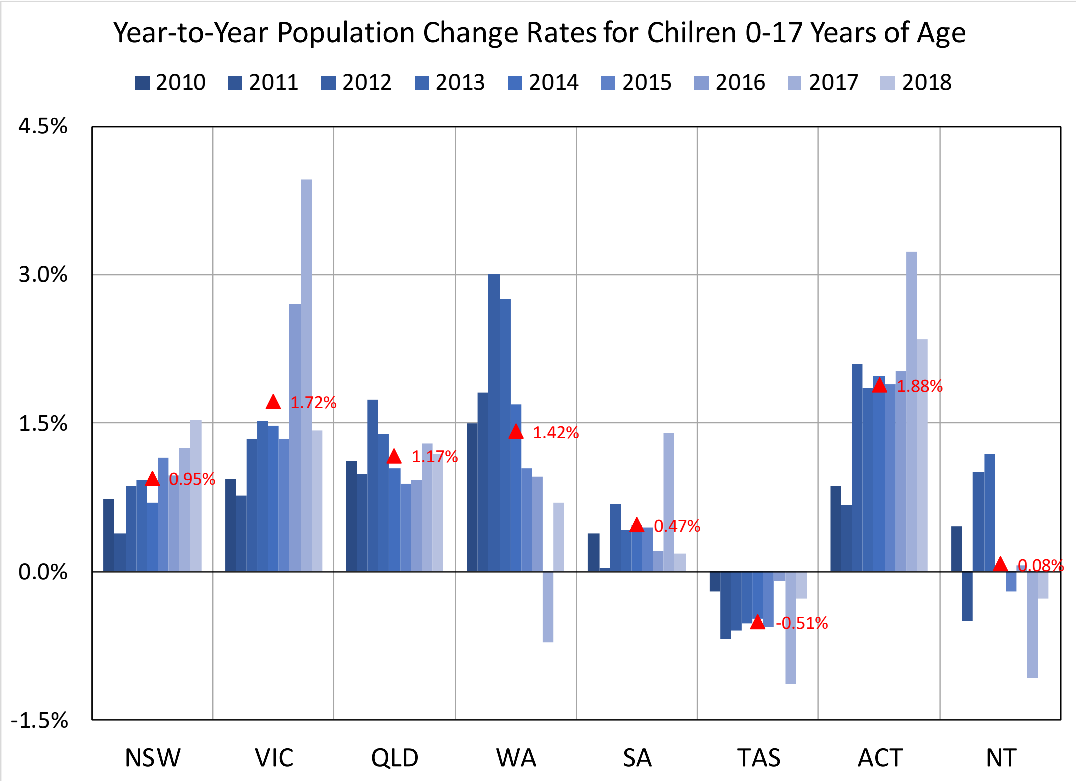


Figure 4.11 Annual Growth Rates of Children in the 0-17 Age Bracket

Children in the General Population Aging Out of the 0 to 17 Age Bracket

To estimate the number of children aging out of the 0 to 17 age bracket in the general population, jurisdiction data from Australian Demographic Statistics (Australian Bureau of Statistics, 2019b) are used. Estimation of the range of age-out rates from the general population of 0 to 17 year olds is done by calculating the proportion of 17 year olds within children in the 0-17 age bracket using annual population figures from 2009 to 2018. Figure 4.12 shows the trends of this variable for all jurisdictions. Table 4.12 shows the means for 2009 to 2018, percentage changes from 2009 to 2018, and standard error of the means for 2009 to 2018.

Table 4.12 Arithmetic Means and Standard Errors of Age-out Rates (2009 to 2018)

	NSW	VIC	QLD	WA	SA	TAS	ACT	NT
Mean	5.47%	5.60%	5.50%	5.40%	5.83%	5.77%	5.63%	4.90%
% Diff	-7.22%	-10.14%	-4.99%	-10.37%	-8.02%	-1.89%	-21.34%	-3.86%
Std Err	0.05%	0.07%	0.04%	0.07%	0.05%	0.04%	0.14%	0.08%

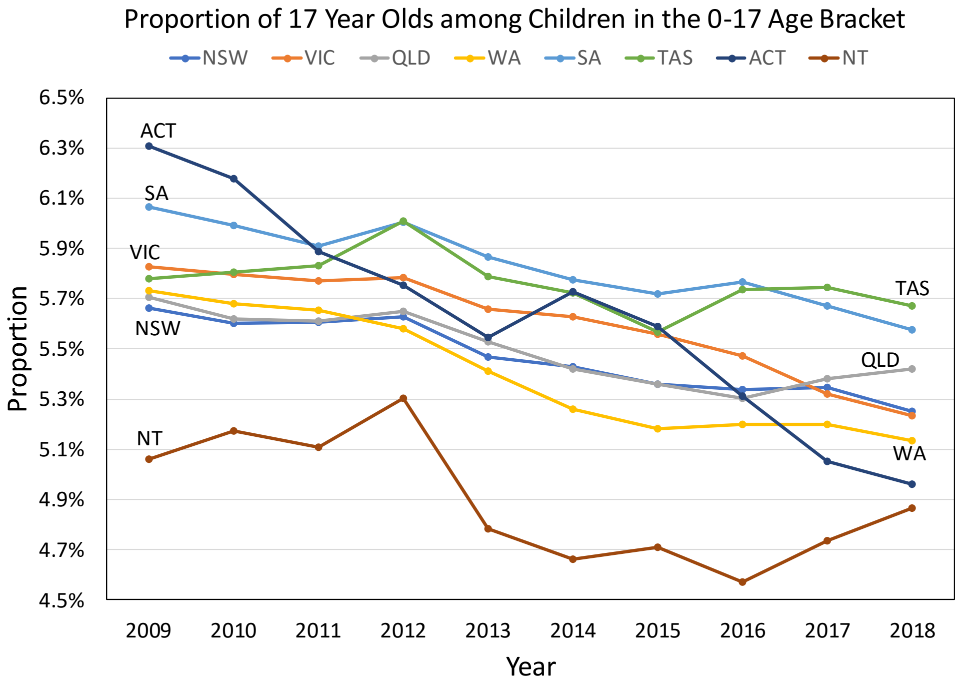


Figure 4.12 Proportion of 17-Year Olds (2009 to 2018)

It can be seen that in general, proportions of 17 year olds had been in decline across all jurisdictions. ACT had the largest decline (21.34%), followed by WA (10.37%) and VIC (10.14%). TAS exhibited the smallest decline (1.89%). QLD and NT exhibited a relatively large decline before stabilizing and rising again, resulting in a net decline of 3.86% and 4.99% across the years. All other jurisdictions exhibited essentially monotonous declines across the years.

Rates of Children Aging Out from OOHC

Publicly available data on OOHC provide breakdowns of children admitted to OOHC, discharged from OOHC, and in OOHC for different age groups – < 1, 1 to 4, 5 to 9, 10 to 14, and 15 to 17. Since data for discrete age are not available, one can only obtain the upper limits of OOHC age-out rates, which are calculated as the number of children in the 15 to 17 age group discharged from OOHC divided by the total number of children in OOHC. Figure 4.13 shows the results across the jurisdictions. Comparing Figure 4.13 to Figure 4.12, it is noticed that while the age-out rates in the general population exhibit downward trends in general, there is no clear trend in the upper limits of OOHC age-out rates.

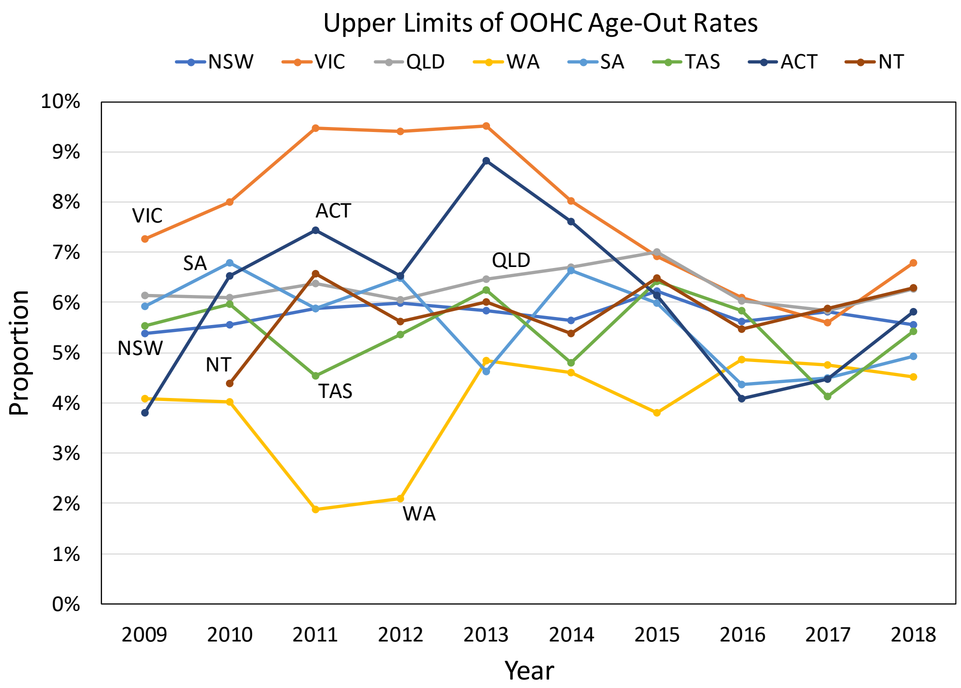


Figure 4.13 Upper Limits of OOHC Age-Out Rates (2009 to 2018)

Assuming that the discharged rates of the 15- and 16-year old are similar to that of the 10 to 14 age group, one may attempt to obtain indicative estimates for OOHC age-out rates involving only the 17-year old in OOHC. To do this, one can calculate the average number of discharges per age year in the 10 to 14 age group and subtract the corresponding values for the 15- and 16-year old from the upper limits shown in Figure 4.13. Table 4.13 shows the means of age-out rates in the general population, the upper limits of OOHC age-out rates, and the indicative estimates calculated using the approach outlined above.

Table 4.13 Means of Age-out Rates for the General Population and OOHC (Upper Limits)

	NSW	VIC	QLD	WA	SA	TAS	ACT	NT
Means (2009 to 2018)								
General Population	5.47%	5.60%	5.50%	5.40%	5.83%	5.77%	5.63%	4.90%
OOHC Upper Limits	5.75%	7.71%	6.30%	3.95%	5.61%	5.43%	6.13%	5.79%
OOHC Indicative	5.05%	6.41%	5.40%	3.08%	5.10%	4.67%	5.31%	4.44%
Means (2014 to 2018)								
General Population	5.34%	5.44%	5.38%	5.20%	5.70%	5.69%	5.33%	4.71%
OOHC Upper Limits	5.77%	6.69%	6.37%	4.51%	5.29%	5.33%	5.63%	5.90%
OOHC Indicative	5.10%	5.64%	5.44%	3.60%	4.81%	4.79%	4.86%	4.87%

The estimates shown in Table 4.13 have to be interpreted in caution. Here it is merely noted that comparing age-out rates between OOHC (upper limits) and the general population, there are a number of regimes:

- OOHC age-out rates much lower than that for the general population – WA;
- OOHC age-out rates generally lower than that for the general population – SA and TAS;
- OOHC age-out rates within the range of that for the general population – NSW, QLD, ACT and NT;
- OOHC age-out rates much higher than that for the general population – VIC.

These regimes will inform our simulation efforts.

Rates of Children Aging Out from Non-OOHC CP Involvement

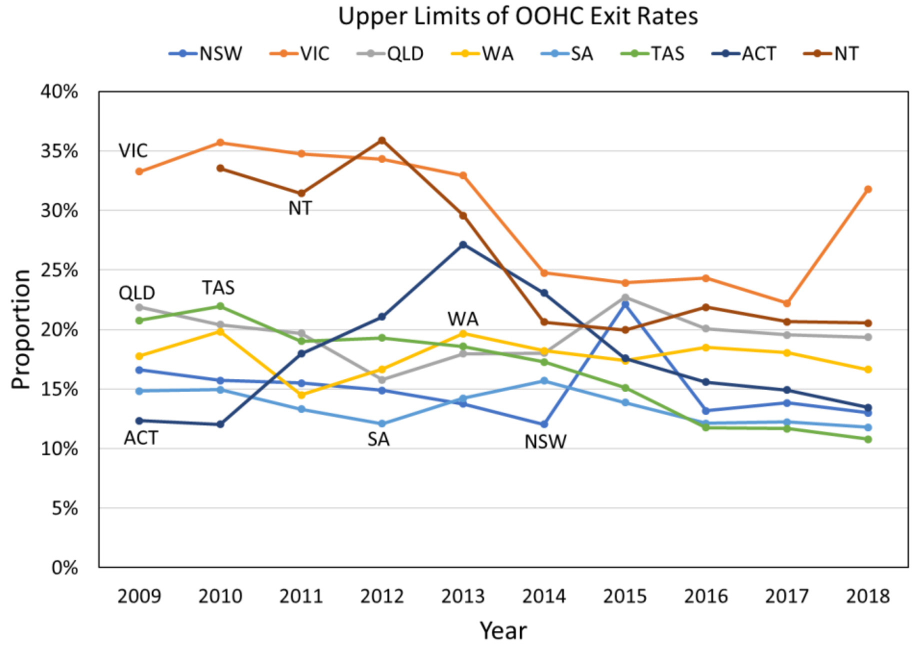
The next parameter of concern is the age-out rate for children who are involved in CP but have never been admitted to OOHC. There are no relevant data for this parameter. For the purpose of simulations, a range of values informed by age-out rates in OOHC and the general population will be used.

Rates of Children Exiting OOHC Not as a Result of Aging Out

Rates of children exiting OOHC (not including aging-out) can be estimated using the publicly available data on OOHC discharge for different age groups. Since 17-year old children are included in the 15 to 17 age group, one can only obtain the lower limits and upper limits of the exit rates. The upper limits of the exit rates are none other than the discharged rates for children of all ages in OOHC; the lower limits of the exit rates are the exit rates of all children from 0 to 14 years of age since this age group doesn't contain any children old enough to age-out. These lower and upper limits are plotted in Figure 4.14 (a) and (b) for all jurisdictions. Figures 4.15 shows the limits for each jurisdiction, providing a clearer picture of the value ranges.

From Figure 4.14 and Figure 4.15, it can be noticed that there is a large uptick in the NSW data in 2015 and VIC data in 2018. These were likely to be due to the change in counting rules where children on finalised third-party parental responsibility orders were no longer counted as being in OOHC, and thus "discharged" or "exited" from the system. Overall, exit rates were relatively stable for NSW, QLD, WA and SA. The rates in VIC and NT exhibited large drops and then stabilized (except for the large uptick associated with the change of counting rules in VIC in 2017-18). The rate in TAS has decreased over the years while the rate in ACT increased to a peak in 2013, before declining again to the level in 2009.

(a)



(b)

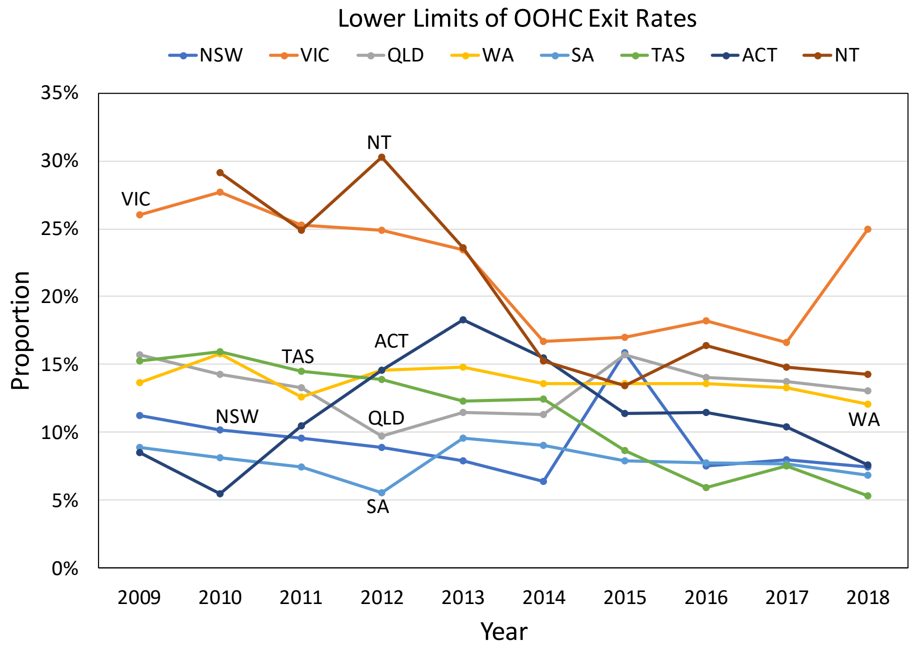


Figure 4.14 Lower and Upper Limits of OOHC Exit Rates (2009 to 2018)

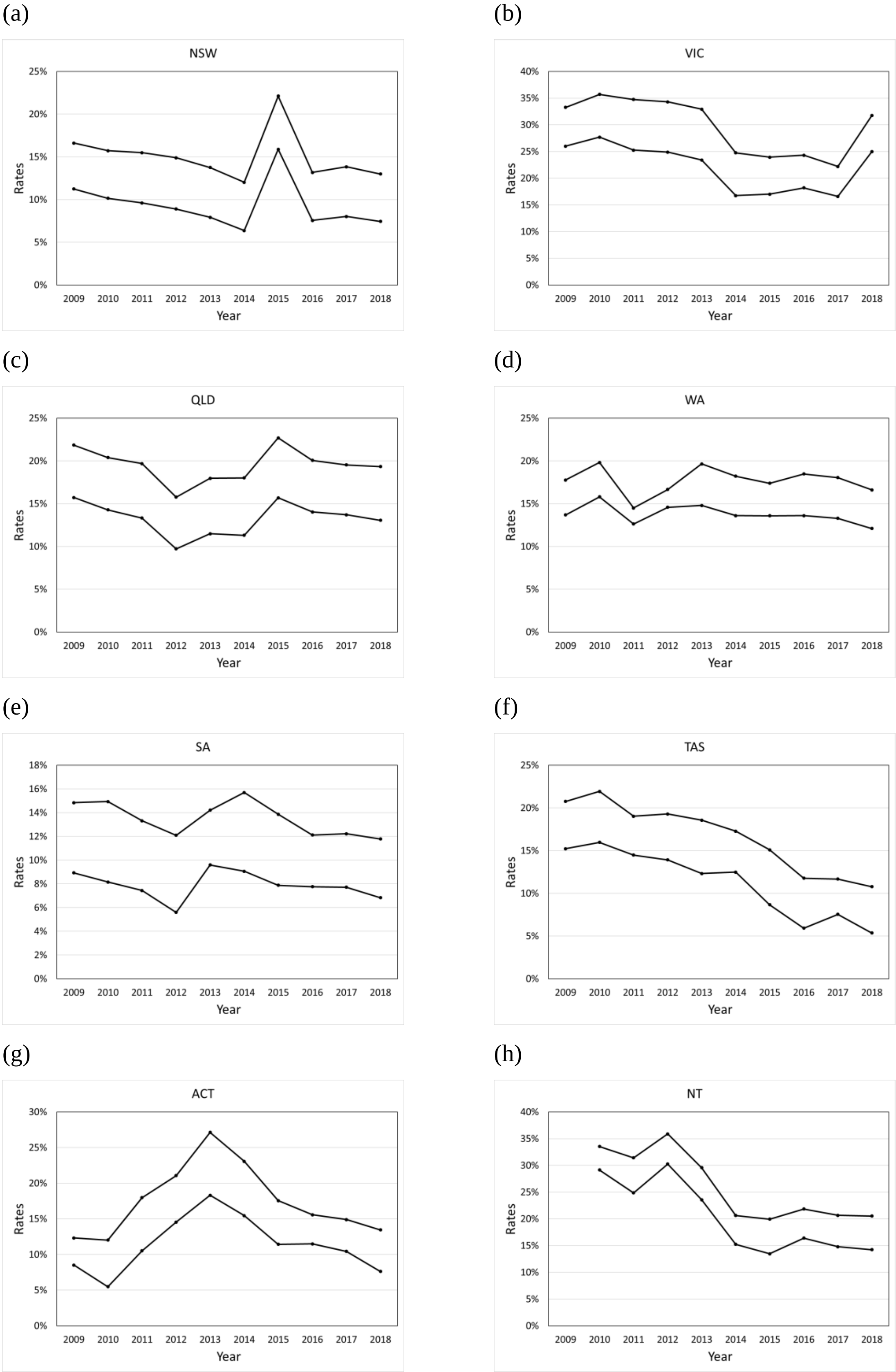


Figure 4.15. Range of OOHC Exit Rates (Excluding Age-Out)

Rates of Children Entering non-OOHC CPS for the First Time

In Section 4.1.3, the breakdown of children in investigations and children entering OOHC into new and repeat clients have been examined. However, new clients who were involved in investigations but not in OOHC have not been examined. This parameter is important in our population dynamics models. Note that upon being investigated, children may or may not be admitted to OOHC. Those who are not admitted to OOHC will be in the sub-population P_1 (having been involved in CPS but not in OOHC) in our model formulation; those who are admitted to OOHC upon first involvement in an investigation will be in the sub-population P_2. To obtain estimations of this parameter, one can simply subtract the number of new clients admitted to OOHC from the number of new clients in investigations.

With the upper bounds of new and repeat clients for children in notifications established previously, one can also calculate the corresponding lower bound of the ratios of children involved only in investigation, defined here as children in investigations only divided by the upper bound of children in notifications. The results for 2015-16 are shown in Table 4.14. The results are very similar to those shown in Table 4.9, except that the values are generally lower. Note that the results for new clients will inform the rates of children being involved in CPS for the first time without being admitted to OOHC.

Table 4.14 Lower Bounds of Investigation-Only Ratios (2015-16)

Investigation ratios	NSW	VIC	QLD	WA	SA	TAS	ACT	NT
New Clients	29.9%	18.2%	78.0%	54.8%	11.0%	5.4%	13.2%	15.7%
Repeat Clients	47.4%	18.7%	70.1%	47.9%	9.2%	9.4%	11.3%	42.3%

Rates of Children Admitted to OOHC in First-time and Repeat Involvement with CPS

The rates of children admitted to OOHC in first-time and repeat involvement in CPS have been examined in Section 4.1.3, with the relevant results shown in Table 4.10.

4.5 Conclusion

In this chapter, descriptive analyses of CP population data across Australian states and territories have been presented. These include trends from 2008-09 to 2017-18 for variables across the stages of protective intervention services, i.e. notifications, investigations, substantiations, OOHC entries, and OOHC exits.

For per 1000 rates of children in investigation, 3 jurisdictions – VIC, WA, and NT – exhibited increases larger than 100% from 2009-10 to 2017-18. ACT registered an increase of 8%. The rate for QLD was relatively stable. For SA, per 1000 rate of children in investigations decreased by 27%. The rate for TAS decreased by 35%. NSW registered a decrease of 8%. However, using 2010-11 instead of 2009-10 as the base year, the rate in NSW increased by 37%.

For per 1000 rates of children in OOHC, from 2009-10 to 2017-18, the rates in NSW stayed essentially unchanged, with children on third-party parental responsibility orders were excluded. QLD registered a modest growth of 13%, while VIC exhibited a growth of 28.5%. However, if 2016-17 were used as the end year to calculate the change in VIC, there would be a growth of 68.8%. NT registered the largest growth, with the per 1000 rate of children in OOHC almost double between 2009-10 to 2017-18.

Correlation analyses show that in NSW, the investigation ratio and substantiation ratio had a statistically significant negative correlation. In SA, the investigation ratio and the entry ratio had a statistically significant negative correlation. In NT, the substantiation ratio had a statistically significant positive correlation with the entry ratio. While the correlation analyses shown in this chapter are simplistic and are based on annual aggregate data, coupling of the process, as indicated by statistically significant correlations, could serve as signposts to further investigate potential issues of CP process capacity.

Analyses of new and repeat clients from 2015-16 to 2017-18 show that across the states and territories, repeat clients accounted for the absolute majority of children in OOHC. Among the jurisdictions, ACT had the lowest proportion of repeat clients (88.7% in 2015-16, missing data for the other 2 years) and NT having the highest proportions (99.0%, 3-yr average). Repeat clients accounted for more than half of the children in investigations in all states and territories, except ACT and SA. ACT had the lowest proportion of repeat clients (46.4% in 2015-16) while NT had the highest proportion (> 82.5%, 3-yr average).

Finally, parameters for the three-population model of CP populations have been estimated. This will facilitate the use of model solutions to analyse the population structure in-depth in the following chapter.

Chapter 5: The Three-Population Model – Analytical Solutions and Applications

In Chapter 3, the differential equations for a three-population model and a four-population model of children involved in CP are set up. The equations are simplified for the case with no systems capacity limits and with uniform age-out rates across the sub-populations, resulting in sets of linear differential equations that could be solved analytically. In the following section, the analytical solutions of the three-population model are explored and used to model the populations of children involved in CP.

5.1 Analytical Solutions for Three-Population Model

For the three-population model, the analytical solutions for the case with no system capacity limit and with uniform age-out rates across the sub-populations are (Equations S.5 in Chapter 3):

$$Q_0 = \frac{\alpha}{h_0} + c_0 e^{-h_0 t}$$

$$Q_1 = \frac{\alpha\lambda_{01} + \lambda_{01}\lambda_{21} + \lambda_{02}\lambda_{21}}{h_0 h_1} - \frac{c_0(\lambda_{01} - \lambda_{21})}{h_0 - h_1} e^{-h_0 t} + \left[c_1 - \frac{\alpha\lambda_{01} + \lambda_{01}\lambda_{21} + \lambda_{02}\lambda_{21}}{h_0 h_1} + \frac{c_0(\lambda_{01} - \lambda_{21})}{(h_0 - h_1)}\right] e^{-h_1 t}$$

$$Q_2 = \frac{\alpha\lambda_{02} + \lambda_{01}\lambda_{12} + \lambda_{02}\lambda_{12}}{h_0 h_1} - \frac{c_0(\lambda_{02} - \lambda_{12})}{h_0 - h_1} e^{-h_0 t} - \left[c_1 - \frac{\alpha\lambda_{01} + \lambda_{01}\lambda_{21} + \lambda_{02}\lambda_{21}}{h_0 h_1} + \frac{c_0(\lambda_{01} - \lambda_{21})}{(h_0 - h_1)}\right] e^{-h_1 t}$$

At t = 0,

$$Q_0 = \frac{\alpha}{h_0} + c_0$$

$$Q_1 = c_1$$

$$Q_2 = 1 - (Q_0 + Q_1) = 1 - c_0 - c_1 - \frac{\alpha}{h_0}$$

As time (t) increases, the solutions approach the following asymptotic values (Equations S.6 in Chapter 3):

$$Q_0 = \frac{\alpha}{h_0}$$

$$Q_1 = \frac{\alpha\lambda_{01} + \lambda_{01}\lambda_{21} + \lambda_{02}\lambda_{21}}{h_0 h_1}$$

$$Q_2 = \frac{\alpha\lambda_{02} + \lambda_{01}\lambda_{12} + \lambda_{02}\lambda_{12}}{h_0 h_1}$$

As explained in Chapter 3, this corresponds to the situation in which the number of children in the subpopulations P_0, P_1 and P_2 grows in tandem with time, leading to stable fractional subpopulations Q_0, Q_1 and Q_2. This will simply be referred to as the steady state solutions.

Note that in the solutions above,

$$h_0 = \alpha + \lambda_{01} + \lambda_{02}$$

$$h_1 = \alpha + \lambda_{12} + \lambda_{21}$$

where the parameters are listed in Table 4.12.

Asymptotically, Q_0 and correspondingly, the fraction of children involved in CP (sum of Q_1 and Q_2), are fully determined by the population growth rate, and the investigation rate and OOHC admission rate for new clients (λ_{01} and λ_{02}).

The effects of the parameters on the asymptotic solutions are first explored. Taking the partial derivatives of the steady solutions of Q_0, Q_1 and Q_2 with respect to each of the parameters, one obtains:

1. $\frac{\partial Q_0}{\partial \alpha} > 0$. This means with fixed investigation rate and OOHC admission rate for new clients, a larger population growth rate leads to a larger fraction of children not involved in CP.
2. $\frac{\partial Q_0}{\partial \lambda 01} < 0$. This means with fixed population growth rate and OOHC admission rate for new clients, a larger investigation rate leads to a smaller fraction of children not involved in CP.
3. $\frac{\partial Q_0}{\partial \lambda 02} < 0$. This means with fixed population growth rate and investigation rate for new clients, a larger OOHC admission rate leads to a smaller fraction of children not involved in CP.

4. $\frac{\partial Q_1}{\partial \alpha} < 0$ for the range of parameters generally observed. This is just a corollary of the first point above, i.e. all else equal, larger population growth leads to a larger fraction of children not involved in CP.
5. $\frac{\partial Q_1}{\partial \lambda 01} > 0$. This means with all other parameters held fixed, a larger investigation rate for new clients leads to a larger fraction of children involved in CP but not in OOHC.
6. $\frac{\partial Q_1}{\partial \lambda 02} > 0$ for the range of parameters generally observed. With all other parameters held fixed, a larger OOHC admission rate for new clients who are not currently in OOHC will lead to a larger fraction of children involved in CP. This is the case since in general, OOHC discharge rates are larger than investigation rates and OOHC admission rates for new clients. Higher rates of new clients being admitted to OOHC also mean that a larger number will be discharged, leading to growth for children involved in CP but not in OOHC. In contrast, if OOHC admission rates for new clients are larger than OOHC discharge rates, the opposite sign will hold.
7. $\frac{\partial Q_1}{\partial \lambda 12} < 0$. This means with all other parameters held fixed, a larger OOHC admission rate for repeat clients leads to a smaller fraction of children involved in CP but not in OOHC.
8. $\frac{\partial Q_1}{\partial \lambda 21} > 0$. This means with all other parameters held fixed, a larger OOHC discharge rate leads to a larger fraction of children involved in CP but not in OOHC.
9. $\frac{\partial Q_2}{\partial \alpha} < 0$ for the range of parameters generally observed. This is just a corollary of the first point above, i.e. larger population growth leads to a larger fraction of children not involved in CP.
10. $\frac{\partial Q_2}{\partial \lambda 01} > 0$ for the range of parameters generally observed. This condition is true when the OOHC admission rate for repeat client is larger than that for new clients, as when the investigation rate increases, Q_1 increases, eventually leading to a larger fraction of children in OOHC.
11. $\frac{\partial Q_2}{\partial \lambda 02} > 0$. This means with all other parameters held fixed, larger OOHC admission for new clients leads to a larger fraction of children in OOHC.

12. $\frac{\partial Q_2}{\partial \lambda 12} > 0$. This means with all other parameters held fixed, a larger OOHC admission rate for repeat clients leads to a larger fraction of children in OOHC.
13. $\frac{\partial Q_2}{\partial \lambda 21} < 0$. This means with all other parameters held fixed, a larger OOHC discharge rate leads to a smaller fraction of children in OOHC.

5.2 Time Dependence of the Solutions

Time variations of the solutions are dependent on the two exponential terms, with the constants h_0 and h_1 determining the pace of change. To illustrate this, the case for a set of typical parameters similar to that observed in NSW is examined, as shown below.

$$\alpha = 0.06$$

$$\lambda_{01} = 0.01$$

$$\lambda_{02} = 0.0004$$

$$\lambda_{12} = 0.012$$

$$\lambda_{21} = 0.11$$

In the scenario above, the gross population growth rate is 6% per annum; the investigation rate for new clients not ending up in OOHC is 1% per annum; the OOHC admission rate for new clients is 0.04% per annum; the OOHC admission rate for repeat clients is 1.2% per annum; the OOHC discharge rate is 11% per annum. Note that the rate ratio between repeat and new clients for OOHC admission is 30. In the subsection on asymptotic solutions below, how the parameters are obtained will be shown.

For this set of parameters,

$$h_0 = 0.0704$$

$$h_1 = 0.182$$

Since the h_0 and h_1 differ quite substantially, the solutions contain a more slowly varying component and a more quickly varying component. The more slowly moving component is mainly associated with flows related to new clients while the more quickly moving component is mainly associated with flows related to repeat clients and OOHC discharge. Note that Q_0 only contains the more slowly varying exponential term while Q_1 and Q_2 contain both exponential terms, i.e. there is a more quickly varying component modulated by the more slowly moving part. Whether the more quickly varying component or the more slowly moving component dominates depends on the initial conditions.

Figure 5.1 shows the time varying plots of Q_0 (solid curve), Q_1 (dot-dash curve) and Q_2 (long-dash curve) calculated using the set of parameters specified above, with the following initial conditions of Q_0 =0.568, Q_1 = 0.346 and Q_2 =0.086. The x-axis is year after the initial time and the y-axes are populations normalized to 1000. Q_0 and Q_1 are plotted on the primary vertical axis, to the left; Q_2 is plotted on the secondary vertical axis, to the right.

In this example, the initial population has 568 per 1000 children not involved in CP, 346 per 1000 children involved in CP but not in OOHC, and 86 per 1000 children in OOHC. In this case, all three sub-populations are initially far from the asymptotic values of 852.2 per 1000 children, 136.2 per 1000 children, and 11.6 per 1000 children, respectively. As time progresses, Q_0 increases towards its asymptotic value, while Q_1 and Q_2 decrease towards their respective asymptotic values. The choice of initial conditions leads to Q_2 being dominated by the more quickly varying term in the beginning, and thus decreasing much more quickly than the other two curves.

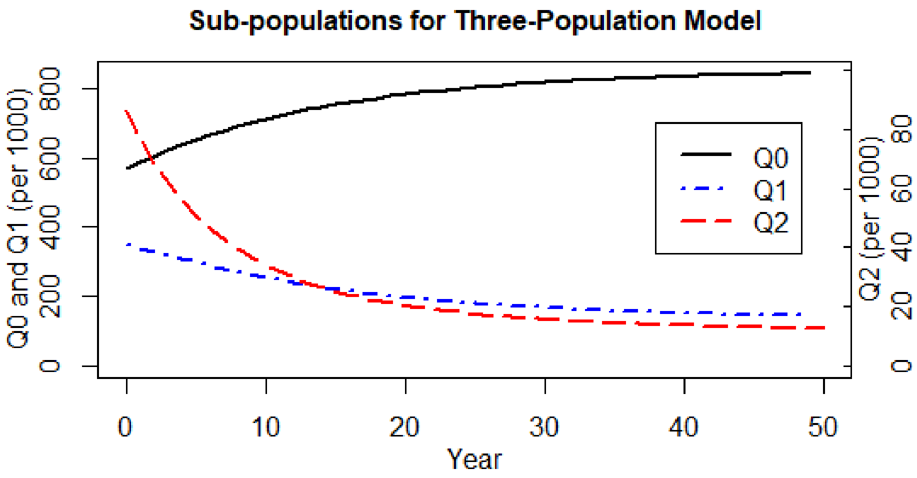


Figure 5.1 Example of Time Varying Solutions

5.3 Model Solutions with Various Scenarios

Having examined analytically how the various sub-populations vary with the parameters and illustrated the time dependence of the solutions, how the solutions vary with time when the parameters change will be demonstrated. This is done for 5 scenarios, with the variations of parameters shown in Table 5.1, with each scenario described below:

- In Scenario 1, it is assumed that the gross population growth rate is 5% per annum; the investigation rate for new clients not ending up in OOHC is 2.5% per annum; the OOHC admission rate for new clients is 0.03% per annum; the OOHC admission rate for repeat clients is 1.2% per annum; and the OOHC discharge rate is 18% per annum. Note that this scenario is not unlike that in NT. This will be shown in the subsection on asymptotic solutions below.
- In Scenario 2, it is assumed that the gross population growth rate is only half that of Scenario 1 (note that the net population growth rate is the difference between the gross population growth rate and the age-out rate, as defined in Table 3.1). All other parameters are kept the same.
- In Scenario 3, the population growth rate is kept the same as in Scenario 1. The investigation rate for new clients is doubled while the OOHC admission rate for new clients is quadrupled. This corresponds to the situation with a vast increase in alleged maltreatment (investigation) and maltreatment resulting in OOHC admission for children not previously involved in CP.

- In Scenario 4, the parameters are the same as those in Scenario 3, except that the OOHC admission rate for repeat clients is also quadrupled. This corresponds to either a change of OOHC admission criteria or an increase in cases where OOHC is deemed the best option.
- In Scenario 5, the parameters are the same as those in Scenario 4, except that the OOHC discharged rate is halved. This corresponds to the situation where children are staying longer in OOHC.

The calculations are carried out for two different initial conditions:

Table 5.1 Typical observed values of the parameters

Parameters	α	λ_{01}	λ_{02}	λ_{12}	λ_{21}
Scenario 1	0.050	0.025	0.00030	0.012	0.18
Scenario 2	0.025	0.025	0.00030	0.012	0.18
Scenario 3	0.050	0.050	0.00120	0.012	0.18
Scenario 4	0.050	0.050	0.00120	0.048	0.18
Scenario 5	0.050	0.050	0.00120	0.048	0.09

1. The system starts out with the initial conditions that no child is involved in CP. In this case $Q_0 = 1$, $Q_1 = 0$, and $Q_2 = 0$ at t = 0. Also, $c_1 = 0$ and
$$c_0 = 1 - \frac{\alpha}{h_0}$$
2. The system starts out with the initial conditions defined by
$$c_0 = -\frac{\alpha}{3h_0}$$
$$c_1 = 0.8\left(1 - \frac{2\alpha}{3h_0}\right)$$

A system with the five scenarios, each with the above two initial conditions, will be examined.

Figure 5.2 shows the results of the 5 scenarios with the first initial conditions, where the system starts out with no children involved in CP. As in Figure 5.1, the solid curve represents Q_0, the dot-dash curve, represents Q_1, and the long-dash curve represents Q_2. In each scenario, the solutions start out with no child involved in CP, i.e. $Q_0 =1$, $Q_1 = 0$ and $Q_2 = 0$. The x-axis is year after the initial time and the y-axes are populations normalized to 1000. Q_0 and Q_1 are plotted on the primary vertical axis, to the left; Q_2 is plotted on the secondary vertical axis, to the right.

In order to facilitate easier comparisons between the scenarios, the asymptotic values of the sub-populations, together with the exponents of the time varying terms, are placed in Table 5.2.

Table 5.2 Exponents of Time Varying Terms and Asymptotic Values of Sub-populations

	Q_0	Q_1	Q_2	h_0	h_1
Scenario 1	664.2	318.3	17.5	0.0753	0.242
Scenario 2	500.5	471.3	28.2	0.0503	0.217
Scenario 3	494.1	478.4	27.5	0.1012	0.242
Scenario 4	494.1	416.4	89.5	0.1012	0.278
Scenario 5	494.1	373.6	132.3	0.1012	0.188

From Figure 5.2 and Table 5.2, one sees that in Scenario 1, the sub-populations (normalised to 1000) converge to Q_0 =664.2, Q_1 = 318.3 and Q_2 = 17.5. In Scenario 2, the gross population growth rate was halved. In accordance with conditions 1, 4 and 9 in the previous sub-section where the effects of the parameters on the sub-populations are

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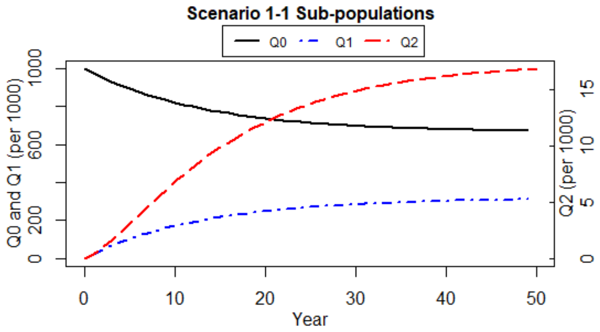

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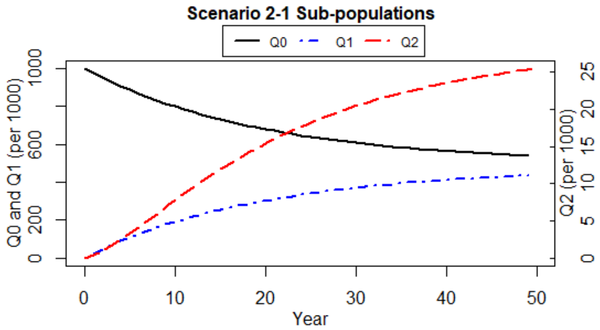

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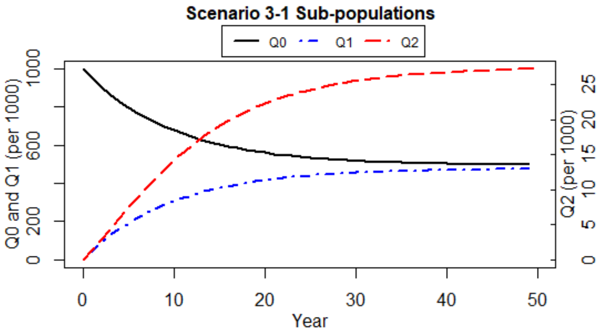

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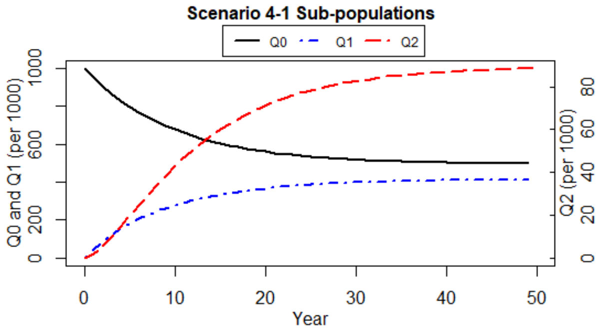

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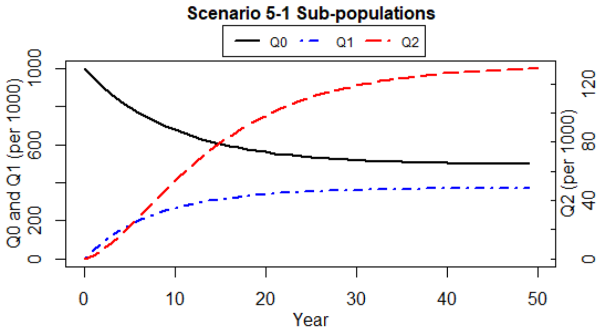


Figure 5.2 Initial Conditions with no Child Involved in CP

examined, Q_0 is lower than that in Scenario 1, while Q_1 and Q_2 are higher. In fact, halving the gross population leads to the sub-populations (normalised to 1000) converging to Q_0 =500.5, Q_1 = 471.3 and Q_2 = 28.2. With a smaller gross population growth rate, one ends up having nearly half of all children involved in CP.

In Scenario 3, the sub-populations converge to Q_0 =494.1, Q_1 = 478.4 and Q_2 = 27.5. Scenario 3 differs from Scenario 1 with the doubling of the investigation rate and quadrupling of the OOHC admission rate for new clients. The changes in the sub-populations are in accordance with conditions 2, 3, 5, 6, 10 and 11 in the previous subsection. Note that the asymptotic values of the sub-populations are very similar to those in Scenario 2. However, the curves converge to the asymptotic values more quickly, due to the larger exponents in the time varying terms. This shows that scenarios with different parameters may lead to a similar rate of children in OOHC.

In Scenario 4, the gross population growth rate, the investigation rate for new clients not ending in OOHC, and the OOHC admission rate are the same with those in Scenario 3. As a result, the fraction of children not involved in CP is also the same as that in Scenario 3. In fact, the curve of Q_0 is identical in both scenarios. The sub-populations converge to Q_0 =494.1, Q_1 = 416.4 and Q_2 = 89.5. Note that in Scenario 4, while all other parameters are the same as those in Scenario 3, the OOHC admission rate for repeat clients quadruples. In accordance with conditions 7 and 12 in the previous subsection, Q_1 decreases and Q_2 increases compared to those in Scenario 3. In comparison to Scenario 3, this scenario illustrates the potential effect of “churning”. However, since the three-population model does not treat children having left OOHC as a separate sub-population, the effect of “churning” cannot be fully investigated.

Scenario 5 differs from Scenario 4 in OOHC discharge rate being halved in the latter. In this case, the curve of Q_0 is identical to that in Scenario 3 and Scenario 4. However, the sub-population of children involved in CP but not in OOHC is lower while the sub-population of children in OOHC is higher, in accordance with conditions 7 and 12 in the previous subsection. The sub-populations converge to Q_0 =494.1, Q_1 = 373.6 and Q_2 = 132.3. Note that in Scenario 4, while all other parameters are the same as those in Scenario 3, the OOHC admission rate for repeat clients quadruples. In comparison to Scenario 4, this scenario illustrates the effect of providing stable placements, where children stay longer in OOHC, leading to a smaller discharge rate.

Table 5.3 shows the second batch of initial conditions. These initial conditions are specified with α, λ_{01} and λ_{02}, and thus vary for some of the scenarios. In all these initial conditions, as opposed to the previous calculations, one starts out with more children involved in CP than those not involved in CP. For Scenario 1, Q_0 and Q_1 start out roughly equal, each accounting for about 44% of the total population, while Q_2 accounts for about 11% of the total population. For Scenario 2, Q_1 is the dominant sub-population, accounting for about 54% of the total population. Q_0 only accounts for about 33% and Q_2 about 13%. The last three scenarios have the same initial conditions, which are very similar to that in Scenario 2. All these initial conditions correspond to situations where the system has a very large number of children involved in CP, mainly having been involved in at least one investigation. While the levels of children in OOHC are unrealistic, in terms of the observed number of children in OOHC, these scenarios provide a good means of examining how a system initially populated with children involved in CP evolve. Such initial conditions make it easier to see the changes.

Table 5.3 Initial Conditions for the Scenarios with Children Involved in CP

Initial Sub-populations	Q_0	Q_1	Q_2
Scenario 1	442.7	445.9	111.4
Scenario 2	331.4	534.9	133.7
Scenario 3	329.4	536.5	134.1
Scenario 4	329.4	536.5	134.1
Scenario 5	329.4	536.5	134.1

Figure 5.3 shows the results of the 5 scenarios. As in Figure 5.2, , the solid curve represents Q_0, the dot-dash curve, represents Q_1, and the long-dash curve represents Q_2. The x-axis is year after the initial time and the y-axes are populations normalized to 1000. Q_0 and Q_1 are plotted on the primary vertical axis, to the left; Q_2 is plotted on the secondary vertical axis, to the right. Once again, Table 4.16 will be used when referring to the asymptotic values of the sub-populations and the exponents of the time varying terms.

In all the calculations with the initial conditions listed in Table 5.3, the sub-populations evolve towards the same asymptotic values shown in Table 5.2. This is because the asymptotic values are fully determined by the parameters or rates used in the calculations, and are thus independent of initial conditions. As in the scenarios shown in

Figure 5.2, Q_2 is dominated by the more quickly varying term in the beginning, thus decreasing much more quickly than the other two curves, except for Scenario 4.

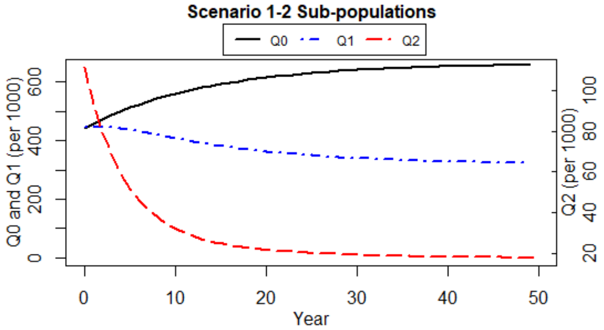
Figure 5.3 (a) shows the results for Scenario 1, from which it is observed that Q_0 and Q_1 are roughly equal in the beginning. Q_1 starts out near the peak value, stays flat and then starts to decline slowly. This is partly due to the more quickly varying exponential term having a very small magnitude with the choice of parameters and initial conditions. By the 10th year, Q_0 increases 53% towards the asymptotic value; Q_1 drops to 29% towards the asymptotic value; Q_2 drops to 85% towards the asymptotic value. At the 20th year point, the value of Q_2 drops from the initial value of 111.4 per 1000 to 21.5 per 1000, which is already in the range of observed value in NT. This shows that large deviations from the asymptotic value, which happens essentially when the flows are unbalanced, tend to be "corrected" quickly.

In Scenario 2, Q_1 starts out with a higher value than Q_0. It rises to the peak value quickly, before it slowly declines and eventually reaches the asymptotic value which is lower than that of Q_0. Q_1 crosses Q_0 between the 55th year and the 56th year. All three sub-populations drop more slowly towards the asymptotic values, due to the smaller values of the exponents in the time varying terms. Note that the asymptotic value of Q_2 in this scenario is 28.2 per 1000, which is higher than the observed value in NT but much lower than the initial value.

In Scenario 3, the exponent of the more slowly varying term is larger than that in Scenario 2. As a result, all three components vary more quickly with time. Q_1 crosses Q_0 between the 30th year and the 31st year. All three sub-populations approach the asymptotic values much more quickly compared to Scenario 2. In this scenario, the asymptotic value of Q_2 in this scenario is 27.5 per 1000, which is also higher than the observed value in NT but much lower than the initial value.

In Scenario 4, the exponents in the time varying terms are the largest among the scenarios. However, the more quickly varying of the two exponential terms is more than one order of magnitude smaller than the more slowly varying term. As a result, Q_2 is dominated by the more slowly moving term, and it approaches the asymptotic value much more slowly compared to the previous scenarios. By the 10th year, Q_2 drops to only 39% of the asymptotic value, which is 89.5 per 1000.

(a)



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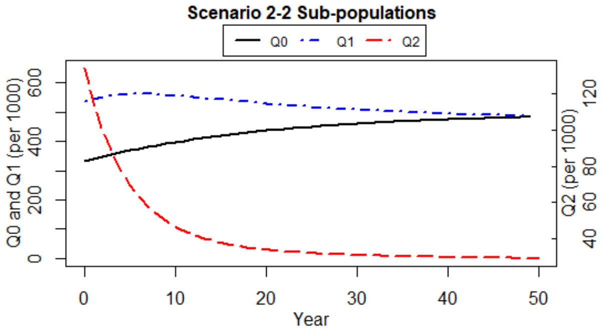

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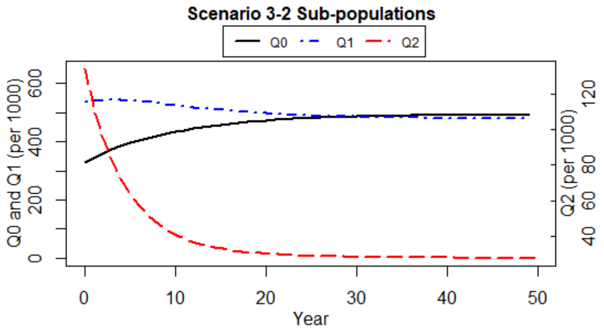

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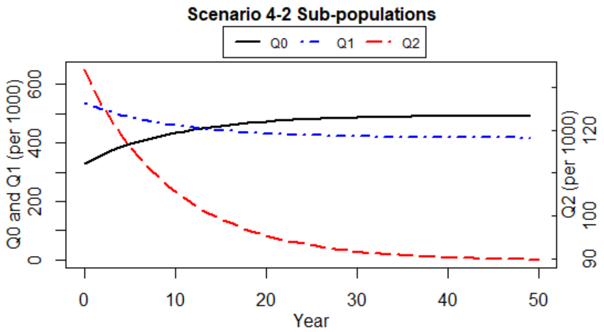

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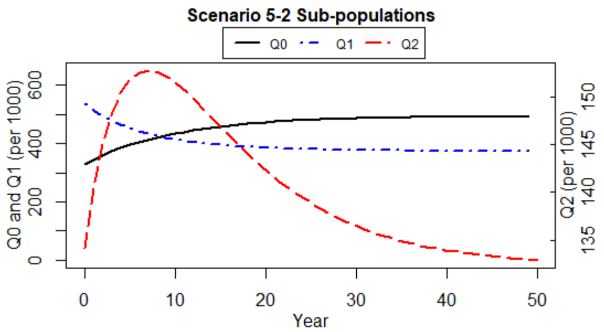


Figure 5.3 Initial Conditions with Children Involved in CP

Scenario 5 is the most complex among all the scenarios. It is first observed that Q_0 in this scenario is exactly the same as that in Scenario 4. This is due to the two scenarios starting from the same initial conditions and using the same parameters that determine Q_0, i.e. the gross population growth rate, the investigation rate for new clients, and the OOHC admission rate for new clients are the same in the two scenarios. The only thing that differs between the two scenarios is OOHC discharge rate, the value of which in Scenario 5 is halved of that in Scenario 4. Halving the discharge rate leads to Q_2 growing sharply from 134 per 1000 to 153 per 1000 within the span of six years. This is due to the value of the more quickly varying term being negative, as opposed to being positive in Scenario 4. As the magnitude of this term approaches 0 very rapidly with time, Q_2 comes to be dominated by the more slowly varying exponential term after reaching the peak.

Results from the various scenarios in Figure 5.2 and Figure 5.3 show that even with a linearized three-population model, the system behaviours can be very rich and complex. This is due to the interplay of the various parameters and the initial conditions. The diverse system behaviours show that without using such models, it is not easy to analyse the system based on the observed data alone.

5.4 Initial Rates Calculations

In the previous sections, various scenarios of time-varying sub-populations using two basic sets of parameters are demonstrating, noting that the set of parameters used in Figure 5.1 is not unlike that deduced based on NSW data, and the set of parameters used in Figure 5.2 and Figure 5.3 is not unlike that based on NT data. These parameters or rates are calculated using limited observed data. In this section, the calculations will be demonstrated and the asymptotic solutions for all jurisdictions will be further analysed.

In this section, the iterative process described in Section 3.4 (Figure 3.3) is first used to obtain a set of values for α, λ_{01}, λ_{02}, λ_{12} and λ_{21} which are consistent with the observed data. Available data which would allow us to get a sense of these parameters has been examined in the previous section. The major piece of missing data that will allow us to fully calculate these parameters is related to notifications. First of all, breakdowns into new and repeat clients for children involved in CP notifications but not in other stages of the process are not known. Second, the rates of children in notifications were calculated based on the number of children aged 0 to 17 in the general population, not based on the number of children in the respective sub-populations as defined in our model (P_0 and P_1).

Note that the published rate of children in investigations (designated as R_I) is a weighted average of the per capita rates of the subpopulations ($R_{I,0}$, $R_{I,1}$ and $R_{I,2}$ for the sub-populations P_0, P_1 and P_2, respectively). Since Q_0, Q_1 and Q_2 are the fractional sub-populations (see Chapter 3), one gets:

$$R_I = Q_0 R_{I,0} + Q_1 R_{I,1} + Q_2 R_{I,2}$$

with

$$Q_0 + Q_1 + Q_2 = 1$$

In our model formulation, there is a tacit assumption that $R_{I,2} = 0$, i.e., number of CP investigations for children currently in OOHC is negligible. This gives:

$$R_I = Q_0 R_{I,0} + Q_1 R_{I,1}$$

For Step 1 of the iterative procedure described in Section 3.4 (Figure 3.3), one proceeds as follows:

1. Calculating per capita investigation rates for new clients not admitted to OOHC, for Q_0 ranging from 0 to 1, with Q_1 taken as $1 – Q_0 – Q_{2,obs}$, where $Q_{2,obs}$ is the observed per capita rate of children in OOHC. This step gives λ_{01}.
2. Calculating per capita OOHC admission rates for new and repeat clients, for Q_0 ranging from 0 to 1, with Q_1 taken as $1 – Q_0 – Q_{2,obs}$. This gives λ_{02}, and λ_{12}.
3. Obtaining per capita OOHC discharge or exit rates for children in OOHC. This gives λ_{21}.
4. Using the above information, calculating Q_0, Q_1 and Q_2 for the range of assumed P_0 fractions, discarding assumed values which are not matched with calculated values.
5. Using the values of Q_0, Q_1 and Q_2 obtained in the last step, repeat the calculation for the relevant rates and continue with the iterative process until changes between iteration are negligible.

Since the breakdown into new and repeat clients is necessary to obtain the parameters, the above calculation is performed only for 2015-16, 2016-17 and 2017-18, using three-year averages for all jurisdictions where data are available for all three years. For NSW, data for 2017-18 are not available, and two years means are used; for ACT, data are available only for 2015-16, and single year data are used; for VIC and TAS, only 2015-16 and 2016-17 are used, due to data consistency issues for other variables needed in applying the model.

Note that for the following calculations in this chapter, $Q_{2,obs}$ refers to the total number of children in OOHC on June 30 of the financial year. Children on third-party parental responsibility orders (TPRO) are not included, as presented in Figure 4.4 and Table 4.5, even though doing so will ensure that the number of children OOHC used in calculations are largely consistent despite changes in counting rule. This is because breakdowns into new and repeat clients are not provided for children on TPRO, making it impossible to infer the corresponding rates for new and repeat clients.

5.4.1 Obtaining Per Capita Per Annum OOHC Discharge Rates for Children in OOHC

The per capita per annum OOHC discharge rates from 2008-09 to 2017-18 are shown in Figure 4.16. In order to ensure that the rates are consistent with other data used in the calculations in this chapter, the following adjustments are made in the calculations:

- Instead of using the total number of unique children in OOHC within the year as the base population in calculating the discharge rates, the number of children in OOHC on June 30 is used. This is because there is only one data point each year, and the solutions of the model are point-in-time sub-populations. The OOHC data used in the iterative process in the calculations are point-in-time data. If the discharge rates are calculated using the total number of unique children in OOHC within the year as the base number, the model will not be able to produce the constraint of the number of children who are discharged.
- Since children aging-out from care are included in the discharge data provided by AIHW (2019a), and the data are provided in age-groups instead of single age-level, it is very difficult to disentangle age-out from discharge. To reduce the effect of double counting, the number of children aging-out is obtained from the estimated fraction of children in the 17-year-old bracket. This value is then deducted from the total discharge rate of children aged 0-17 to obtain the discharge rate.

- Relating to the condition above, there are two ways to obtain the age-out rate from the fraction of children in the 17-year-old bracket. The first uses the age structure in the OOHC population. The other uses the age structure in the general population of children between 0-17 years of age. While the first approach is a more accurate reflection of the actual OOHC age-out rate, it doesn't reflect the model assumption. Recall that when deriving the analytical solutions for the three-population model in Section 3.1.4, the assumption of uniform age-out rates across all sub-populations was made. Therefore, in order to use the model solutions in our calculations, age-out rates corresponding to those calculated from the general population will be used to obtain the discharge rates. Table 5.4 shows the discharge rates calculated using the two approaches.
- For NSW, VIC and TAS, data for 2015-16 and 2016-17 were used to calculate the averages, due to counting rule change in VIC and data problem in NSW and TAS for 2016-17. For ACT, only 2015-16 data were used as that was the only year when new and repeat client data were available.

Table 5.4 shows the discharge rates calculated using the two approaches described above. Rates calculated using OOHC age-out rates are labelled Scenario 1 while rates calculated using general population age-out rates are labelled Scenario 2. The last row shows the percentage differences of the discharge rates between the two scenarios. It is observed that for VIC and NT, the discharge rates for Scenario 2 are larger than those in Scenario 1 but the magnitudes of the differences are smaller than 4%. In all other jurisdictions, the discharge rates in Scenario 2 are smaller than those in Scenario 1. In NSW, QLD and ACT, the magnitudes of the differences are smaller than 6.1%. The magnitudes of the differences are 9.30% in WA, 12.53% in TAS, and 12.87% in SA. The discharge rates in Scenario 2 will be used as they are consistent with the assumption made in the analytical solutions.

Table 5.4 Per Capita Per Annum OOHC Discharge Rates for Children in OOHC

Scenario 1. OOHC Discharge Rates (2015-16 to 2017-18)							
NSW	VIC	QLD	WA	SA	TAS	ACT	NT
10.28%	23.95%	19.28%	17.78%	9.11%	8.54%	13.85%	20.77%
Scenario 2. Discharge Rates Calculated Using Age-out Rates for General Population							
9.91%	24.22%	18.78%	16.13%	7.93%	7.47%	13.00%	21.54%
% Difference (Scenario 2 – Scenario 1)/ Scenario 1							
-3.58%	1.13%	-2.59%	-9.30%	-12.87%	-12.53%	-6.10%	3.72%

5.4.2 Calculating Per Capita Per Annum Investigation Rates for New Clients Not Admitted to OOHC

Table 5.5 shows the number of children in investigations obtained from the new and repeat client data described in Section 4.3, together with the population of children between 0 and 17 years of age. These are average values from 2015-16 to 2017-18. However, for NSW, VIC and TAS, only data for 2015-16 and 2016-17 were used so that they are consistent with the averages for other variables, where there was a change in counting rule or a data problem. For ACT, only 2015-16 data were used as that was the only year in which new and repeat client data were available. Also, it should be noted that the number of children admitted to OOHC has been subtracted from the raw data for the number of children in investigations to make the data consistent with our model formulation.

Table 5.5 Average Number of Children in Investigations (2015-16 to 2017-18)

	NSW	VIC	QLD	WA	SA	TAS	ACT	NT
New	13876	10807	8089	4949	1201	358	819	1042
Repeat	32177	12142	8373	4646	1068	720	687	4691
General Population	1729296	1355132	1152513	596702	364502	113296	88668	63114

Using these values, one can calculate the per capita investigation rates for new and repeat clients, for Q_0 ranging from 0 to 1, constrained by $Q_{2,obs}$, the observed values of children in OOHC. This is the first part of Step 1 in the iterative process described in Section 3.4 (see Figure 3.3 and associated text). The results are shown in Figure 5.4, where the horizontal axis is the fraction of children involved in CP (sum of Q_1 and Q_2). The vertical axis is the number of children per 1000. The solid curve represents the rates for repeat client and the dash curve represents the rates for new clients. It should be noted that per capita rates are presented as per 1000 rates.

In the graphs shown in Figure 5.4, the further to the right, the larger it is the proportion of children involved in CP. In all jurisdictions, the two curves cross each other above the 0.45 value on the horizontal axis. NSW, TAS and NT exhibited the most lop-sided situations where the two curves do not cross each other until above 0.65. In NT, the two curves cross above 0.8. The implication is that for all jurisdictions, unless the fraction of children involved in CP was larger than 45%, investigation rates for repeat clients were higher or much higher than those of repeat clients. These per capita rates of new clients are the potential values for λ_{01}.

5.4.3 Calculating Per Capita OOHC Admission Rates for New and Repeat Clients

Table 5.6 shows the number of children in investigations obtained from the new and repeat client data described in Section 4.3, together with the population of children between 0 and 17 years of age. These are average values from 2015-16 to 2017-18. As in the case of children in investigations, only data for 2015-16 and 2016-17 were used for NSW, VIC and TAS. For ACT, only 2015-16 data were used as that was the only year in which new and repeat client data were available.

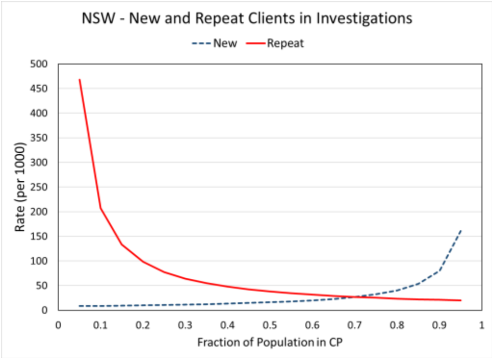
Using these values, one can calculate the per capita rates of OOHC admission for new and repeat clients, for Q_0 ranging from 0 to 1, with Q_1 taken as $1 - Q_0 - Q_{2,obs}$. Again, this is the first part of Step 1 in the iterative process described in Section 3.4 (see Figure 3.3 and associated text).The results are shown in Figure 5.5, where the horizontal axis is the fractions of children involved in CP (sum of Q_1 and Q_2). The vertical axis indicates the number of children per 1000. The solid curve represents the rates for repeat client and the dash curve represents the rates for new clients. Note that per capita rates are presented as per 1000 rates.

Table 5.6 Average Number of Children Admitted to OOHC (2015-16 to 2017-18)

	NSW	VIC	QLD	WA	SA	TAS	ACT	NT
New	549	1016	612	278	322	41	99	12
Repeat	2716	2932	1794	803	424	184	109	280
General Population	1729296	1355132	1152513	596702	364502	113296	88668	63114

In Figure 5.5, as in the case in Figure 5.4, the further to the right, the larger it is the proportion of children involved in CP. In all jurisdictions, the two curves cross each other only above the 0.5 value on the horizontal axis. Except for SA and ACT, in which the two curves cross each other between 0.5 and 0.65, in all other jurisdictions, they cross each other above 0.7. For NT, the rates for repeat clients are larger than those of new clients across the whole horizonal axis. The per capita rates of new and repeat clients are the potential values for λ_{02} and λ_{12}, respectively.

(a)



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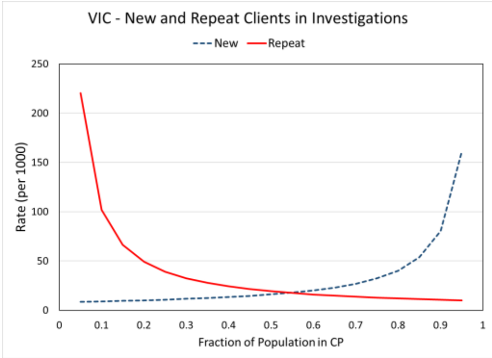

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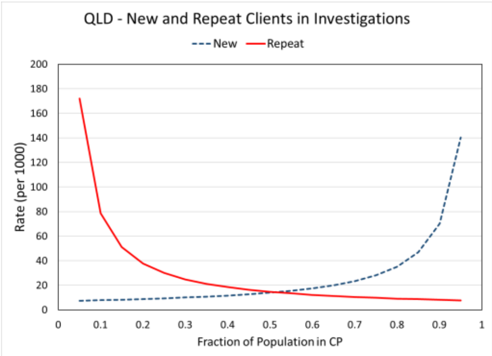

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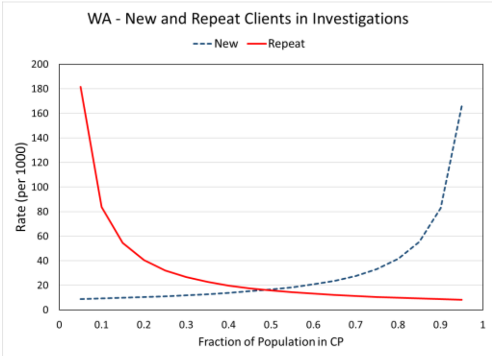

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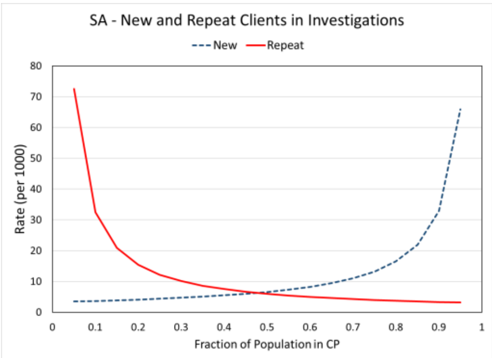
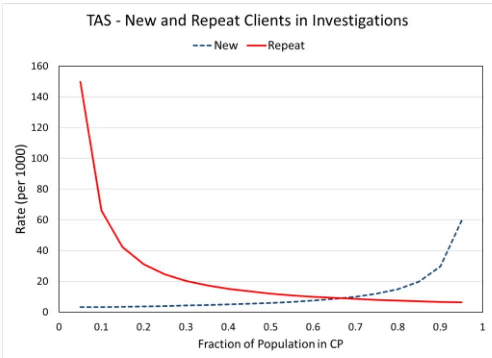
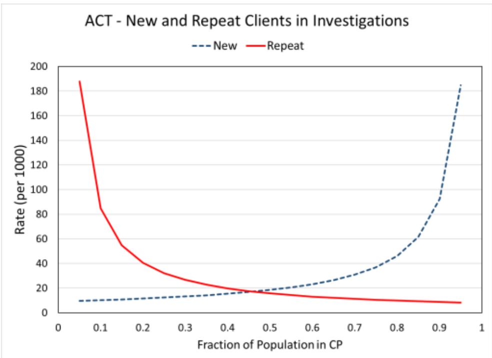


(f)



(g)



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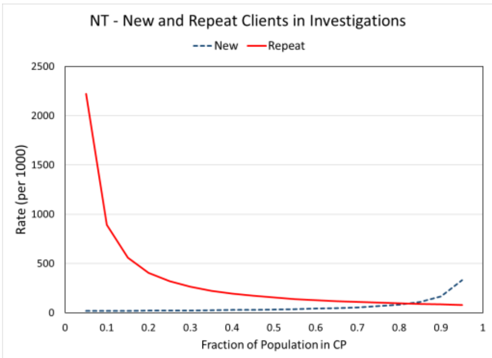


Figure 5.4. Average Per Capita Investigation Rates for New and Repeat Clients

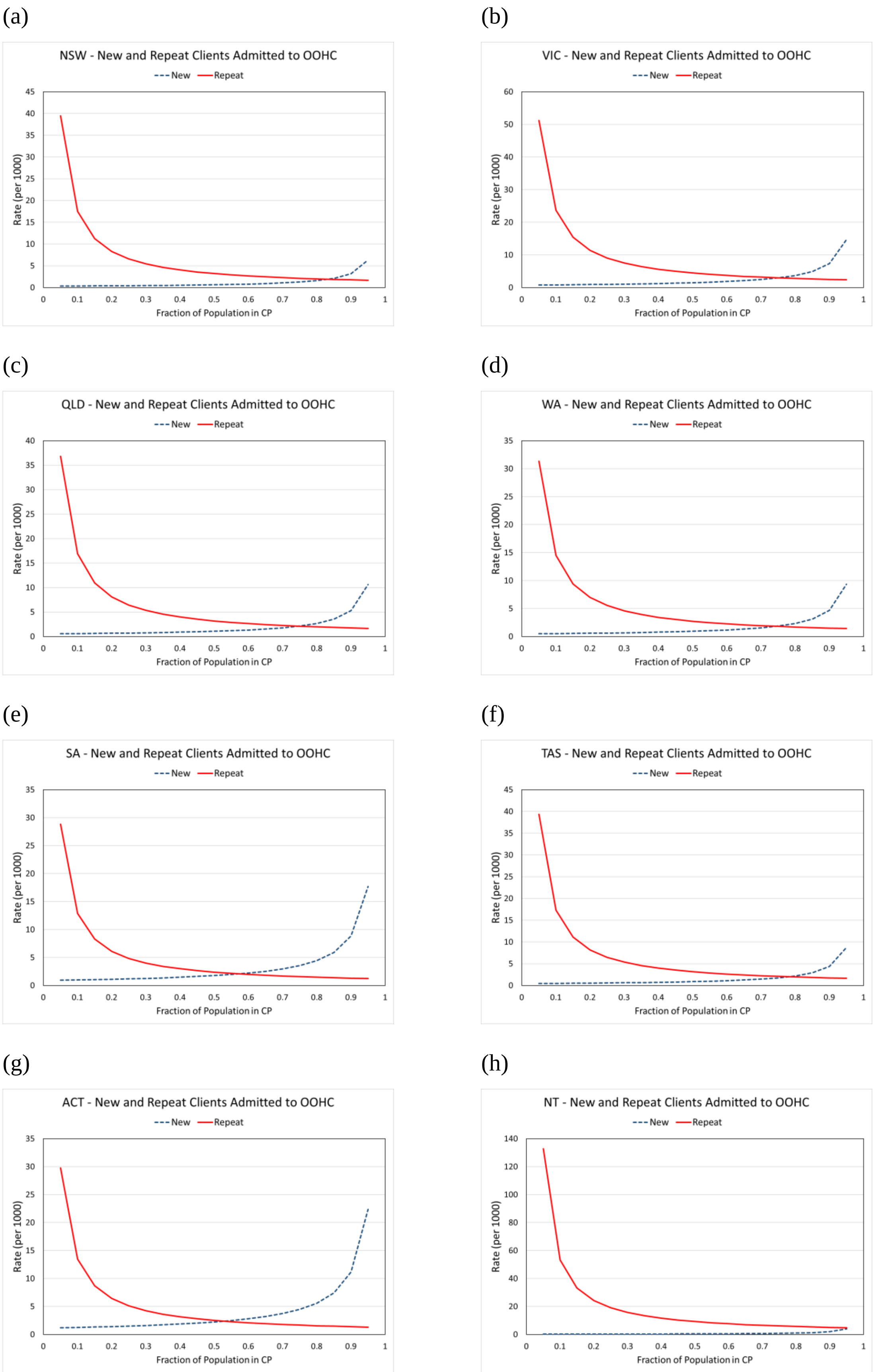


Figure 5.5. Average Per Capita OOHC Admission Rates for New and Repeat Clients

5.5 Iterative Calculations of Q_0, Q_1, Q_2, Investigation Rates and OOHC Admission Rates

Based on the population growth rates, per capita investigation rates of new clients not ending up in OOHC, per capita rates of OOHC admission for new clients and repeat clients, and per capita discharge rates from OOHC (Scenario 2 rates, see Section 5.4.1), the second part of Step 1 in the iterative process described in Section 3.4 (see Figure 3.3 and associated text) is carried out further. As described in Section 3.4, the formulas for the asymptotic solutions are used to obtain the "initial guess" of Q_0 and Q_1 and thus a set of rates consistent with them.

Completing Step 1 of the Iterative Process

Using the range of values of λ_{01}, λ_{02} and λ_{12}, as shown in Figure 5.4 and 5.5, Q_0, Q_1 and Q_2 are calculated using the asymptotic formulas. Figure 5.6 (a) shows the results for NSW. In the figure, the horizonal axis is the assumed fraction of children involved in CP, which is the sum of Q_1 and Q_2. The primary vertical axis, on the left, shows the values for Q_0, Q_1 and $Q_1 + Q_2$ while the secondary vertical axis, on the right, shows the values for Q_2. The dot-dashed curve is Q_0; the solid curve is Q_1; the short-dash curve is Q_2; the long-dash curve is the sum of Q_1 and Q_2; the short-dash horizontal line is $Q_{2,obs}$; the solid sloped line is the line with the slope of 1; and the vertical dotted line indicates the position where the calculated value for the sum of Q_1 and Q_2 equals to the assumed value (intersection of the slope-1 line and the long-dash curve).

In Figure 5.6 (a), along the range of assumed values of the fraction of population involved in CP, there is only one value that matches the calculated value. The asymptotic values of Q_0, Q_1 and Q_2 can be read from the intersection of the vertical dotted line with the lines indicating the three subpopulations. There is a discrepancy between $Q_{2,obs}$, the assumed value of Q_2 used in the calculation and the calculated value of Q_2, as shown by the gap between the short-dash curve and the short-dash horizontal line, along the vertical dotted line.

Using the values of Q_0, Q_1 and Q_2 determined above, the corresponding values of λ_{01}, λ_{02} and λ_{12} are obtained. These form the "initial guesses" that will be used in Step 2 of the iterative process.

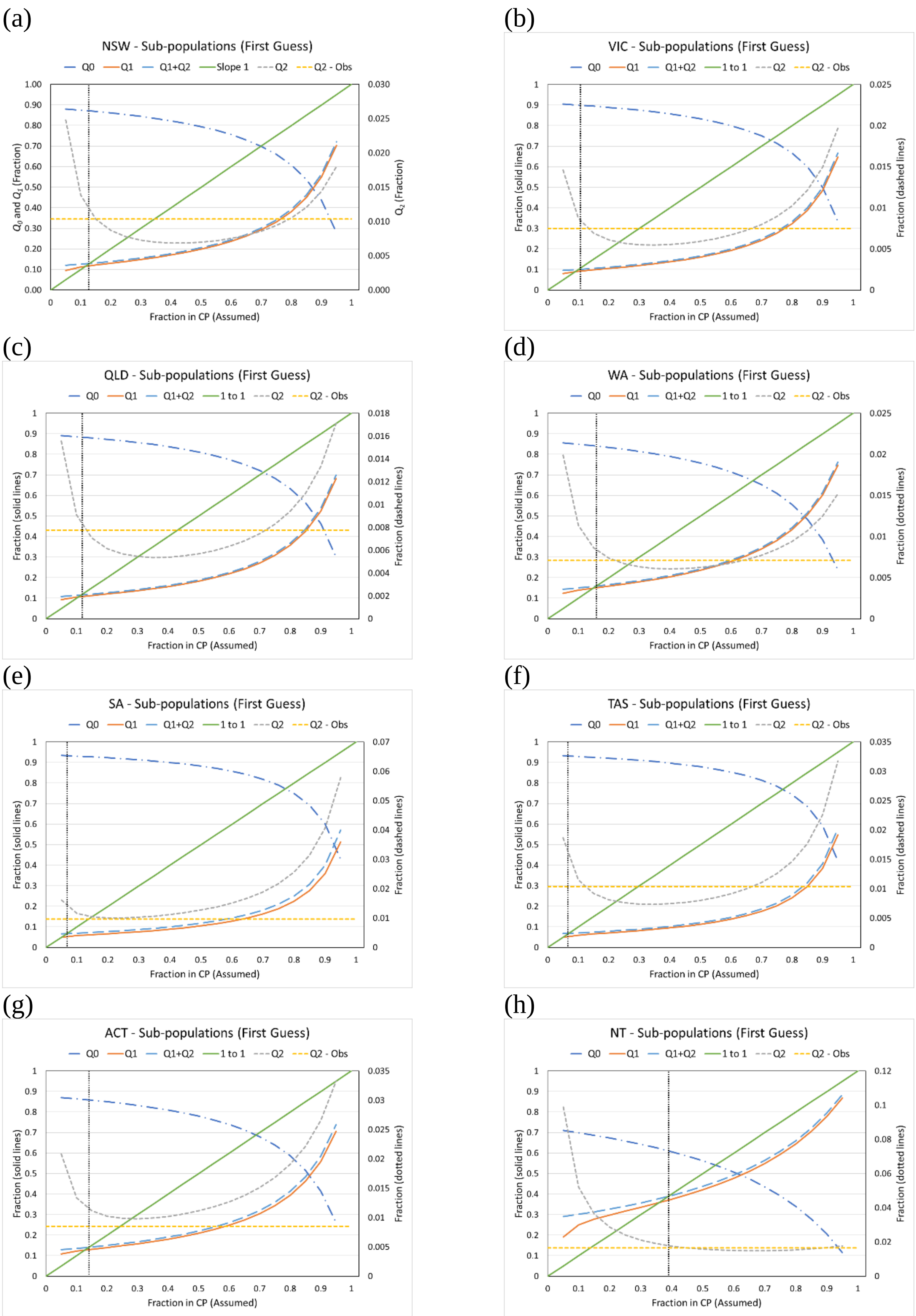


Figure 5.6. First Guesses of Sub-populations across the States and Territories

Calculations are carried out for all states and territories, with the results shown in the rest of Figure 5.6. Table 5.7 shows a compilation of the values of Q_0, Q_1 and Q_2 corresponding to the dashed vertical lines in Figure 5.6. The "initial guesses" of λ_{01}, λ_{02},

λ_{12} are also shown, together with the investigation rates for repeat clients (designated λ_{01}) and the reported values of children in OOHC, $Q_{2,obs}$.

It is noted that the values of Q_0 range from 610 per 1000 in NT to 935 per 1000 in TAS. Correspondingly, the values of Q_1 range from 47.5 per 1000 in TAS to 367.8 per 1000 in NT. The variations in these sub-populations across the jurisdictions are driven by the investigation rates for new clients (λ_{01}), with NT having the highest value, at 27.1 per 1000 per annum, and TAS having the lowest value, at 3.4 per 1000 per annum. OOHC admission rates also vary greatly across the jurisdictions. At this point, a complete description of the results shown in Figure 5.6 and Table 5.7 is not given, as these are just "initial guesses" which will be used in subsequent steps of the iterative process. A complete description of the results obtained at Step 6 of the iterative process will be given below.

Table 5.7 Initial Guesses of Q_0, Q_1, Q_2 (per 1000), λ_{01}, λ_{02}, λ_{11} and λ_{12} (per 1000 per annum)

	NSW	VIC	QLD	WA	SA	TAS	ACT	NT
Q_0	873.0	890.0	884.0	840.0	934.0	935.0	863.0	610.0
Q_1	114.3	98.8	106.0	150.0	49.9	47.5	123.1	367.8
Q_2	12.7	11.2	10.0	10.0	16.1	17.5	13.9	22.2
$Q_{2,obs}$	10.2	7.5	7.7	7.1	9.6	10.4	8.7	16.5
λ_{01}	9.2	9.0	7.9	9.9	3.5	3.4	10.7	27.1
λ_{11}	162.8	90.7	68.5	51.9	58.7	133.8	62.9	202.1
λ_{02}	0.36	0.36	0.36	0.38	0.34	0.34	0.37	0.52
λ_{12}	13.7	21.9	14.7	9.0	23.3	34.2	10.0	12.1

Step 2 to Step 5 of the Iterative Process

In Step 2, armed with the "initial guesses" calculated in the previous step, the time-dependent solution of Q_2 is used to obtain the model time which will produce a value of Q_2 consistent with the reported value, $Q_{2,obs}$.

Recall that the time-dependent solution of Q_2 is

$$Q_2 = \frac{\alpha\lambda_{02} + \lambda_{01}\lambda_{12} + \lambda_{02}\lambda_{12}}{h_0 h_1} - \frac{c_0(\lambda_{02} - \lambda_{12})}{h_0 - h_1} e^{-h_0 t} - \left[c_1 - \frac{\alpha\lambda_{01} + \lambda_{01}\lambda_{21} + \lambda_{02}\lambda_{21}}{h_0 h_1} + \frac{c_0(\lambda_{01} - \lambda_{21})}{(h_0 - h_1)}\right] e^{-h_1 t}$$

where

$$h_0 = \alpha + \lambda_{01} + \lambda_{02}$$

$$h_1 = \alpha + \lambda_{12} + \lambda_{21}$$

For our calculations, the initial conditions are set with the following values:

$$c_0 = 1 - \frac{\alpha}{h_0}$$

$$c_1 = 0$$

This is equivalent to assuming a solution curve that has evolved from an initial state with $Q_0 = 1$, $Q_1 = 0$ and $Q_2 = 0$. In addition to this set of initial conditions, calculations with other initial conditions which start out with Q_1 and Q_2 not equal to 0 are also explored. However, the larger the initial value Q_1 is assigned, the smaller the model time T is at which $Q_2 = Q_{2,obs}$ and the model curves become too steep when compared with the reported trend of children in OOHC in preceding years. As a result, the initial state with $Q_0 = 1$, $Q_1 = 0$ and $Q_2 = 0$ is the one that produces the best results and will be used in the calculations.

It should be noted that the reported values of children in OOHC used to constrain the model solutions are averaged over two to three years (only one year for ACT), depending on data availability. Model values averaging over the same number of years are used in the iterative calculations. Using averaged values of children in OOHC produce better results because it constrains both the number of children and the rates of change in the number of children over the years where data are available.

Table 5.8 shows the initial guesses and final results of the model time T, relevant parameters (per 1000 per annum), and the sub-populations (per 1000) at the model time T, retrieved at the end of the iterative calculations (Step 6 in Figure 3.3). Note that in step 2, for the first iteration, the model values of Q_2 and the reported values ($Q_{2,obs}$) are allowed to differ by as much as 15% in ACT and by 10% in VIC and QLD. This is because for these jurisdictions, model value of Q_2 obtained using the "initial guess" is smaller than the reported value by larger than 15%. As a result, the 1% condition could not be achieved in the first iteration, as the model values of Q_2 are smaller than the reported values by an amount larger than 1% at all model time. Instead of fixing the initial guesses, the condition for the differences between the calculated values and the reported value were simply relaxed for the first iteration. From the second iteration, the 1% was imposed with no problem.

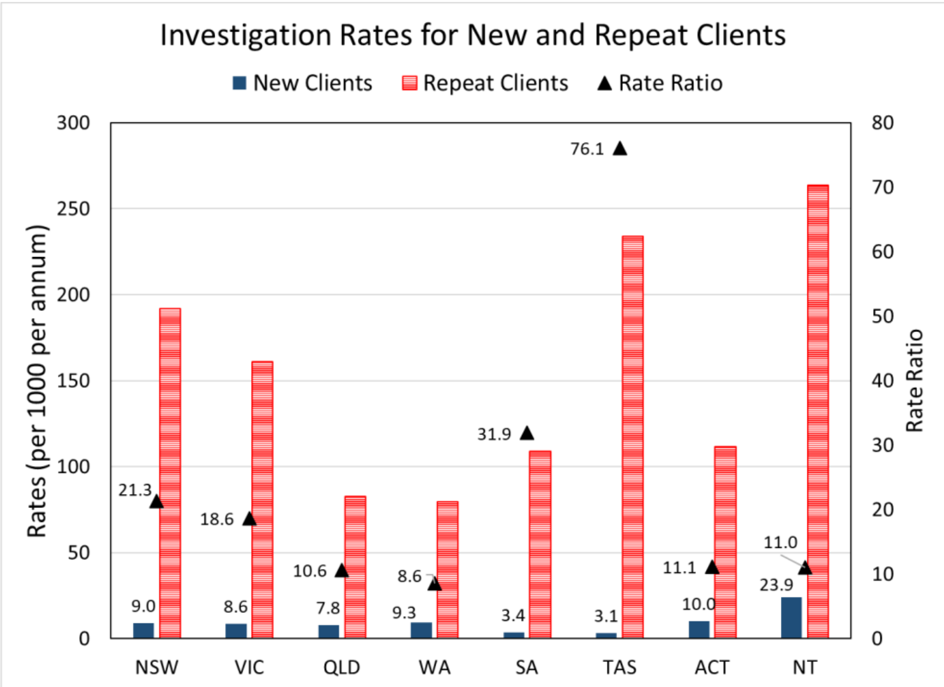
Table 5.8 Model Time and Parameters Obtained through the Iterative Process

		NSW	VIC	QLD	WA	SA	TAS	ACT	NT
T (yr)	Initial	31.5	39.7	41.4	40.1	47.4	23.0	43.9	33.8
	Final	24.8	11.9	25.1	18.3	12.7	15.1	9.9	27.0
λ_{01}	Initial	9.2	9.0	7.9	9.9	3.5	3.4	10.7	27.1
	Final	9.0	8.5	7.8	9.3	3.4	3.3	10.0	23.5
λ_{02}	Initial	0.36	0.36	0.36	0.38	0.34	0.34	0.37	0.52
	Final	0.36	0.80	0.59	0.52	0.92	0.38	1.21	0.27
λ_{11}	Initial	162.8	90.7	68.5	51.9	58.7	133.8	62.9	202.1
	Final	194.0	153.8	82.6	79.7	108.9	235.3	117.0	264.1
λ_{12}	Initial	13.7	21.9	14.7	9.0	23.3	34.2	10.0	12.1
	Final	16.4	37.1	17.7	13.8	43.2	60.1	18.6	15.8
Q_0	Initial	1000	1000	1000	1000	1000	1000	1000	1000
	Final	892.2	932.0	903.2	893.3	962.9	963.7	921.2	691.5
Q_1	Initial	0	0	0	0	0	0	0	0
	Final	97.1	58.9	88.0	97.8	26.9	25.2	69.5	282.0
Q_2	Initial	0	0	0	0	0	0	0	0
	Final	10.1	7.4	7.7	7.0	9.5	10.3	8.6	16.4

For all parameters, across all jurisdictions, stable values (defined as differences in magnitudes between iteration smaller than 1%, see Figure 3.3) were reached after fewer than 9 iterations of calculations. Among the parameters, λ_{01} and λ_{02} converged the most quickly (at or before the 5th iteration), as they were better constrained, with the condition $Q_2 = Q_{2,obs}$ imposed in the calculations. The other two parameters, λ_{11} and λ_{12}, converged more slowly (at or before the 8th iteration), as they were more poorly constrained. It is emphasised that since Q_0 and Q_1 are not directly observed, the parameters should be taken only as values consistent with the model caveats, as well as the reported number of children in OOHC and process data (investigation and OOHC admission rates).

Figure 5.7 (a) plots the values of investigate rates for new clients (λ_{01}) and those for repeat clients (λ_{11}), as shown in Table 5.8, across all jurisdictions. The vertical bars with solid shade show the investigation rates for new clients and the vertical bars with horizontal stripes show the investigation rates for repeat clients. They are plotted against the primary vertical axis, at the left, with the unit of per 1000 per annum. The triangles are the repeat client to new client rate ratios, plotted against the secondary vertical axis, at the right.

(a)



(b)

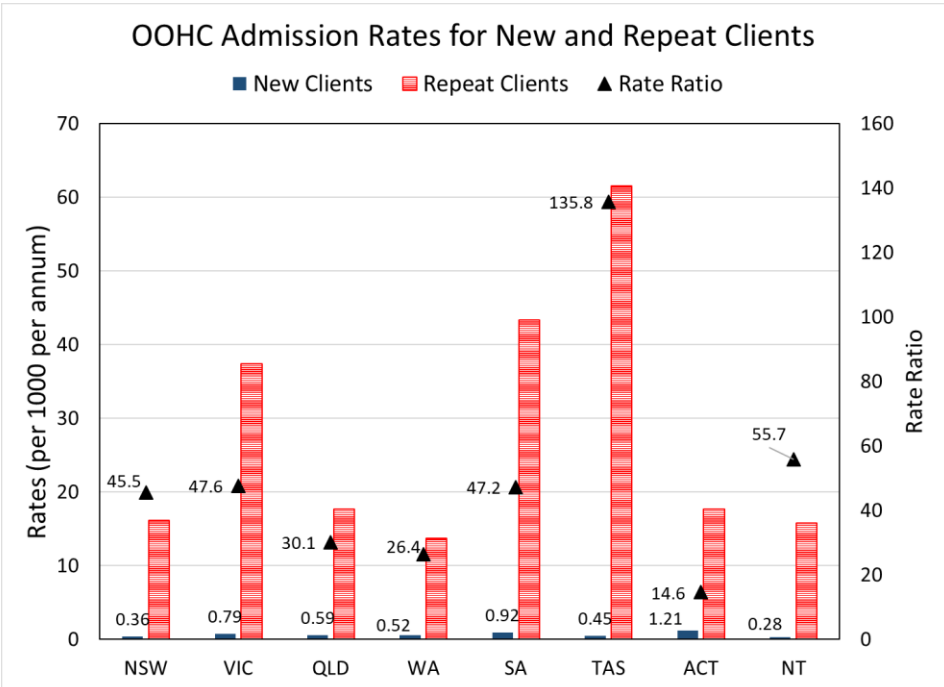


Figure 5.7 Investigation Rates and OOHC Admission Rates

It can be seen that NT has the highest investigation rates for both new clients and repeat clients, at 23.5 and 264.1 per 1000, respectively. The investigation rate for new clients in NT is 2.53 times the value in ACT, which has the next highest rate (10.7 per 1000).

On the other hand, the investigation rate for repeat clients in NT is 1.12 times the value in TAS, which has the next highest rate (235.3 per 1000). TAS and SA have the lowest investigation rates for new clients (3.3 and 3.4 per 1000, respectively), leading to them having the lowest fractions of children involved in CP across all jurisdictions (sum of Q_1 and Q_1 in Table 5.8). TAS also exhibits the highest rate ratio between repeat and new clients, with repeat clients 71.7 times more likely to be involved in CP investigations. WA has the lowest rate ratio between repeat and new clients, with repeat clients 8.6 times more likely to be involved in CP investigations.

Figure 5.7 (b) plots the values of OOHC admission rates for new clients (λ_{02}) and repeat clients (λ_{12}), as shown in Table 5.8, across all jurisdictions. The vertical bars with solid shade show the OOHC admission rates for new clients and the vertical bars with horizontal stripes show the OOHC admission rates for repeat clients. They are plotted against the primary vertical axis, at the left, with the unit of per 1000 per annum. The black triangles are the repeat client to new client rate ratios, plotted against the secondary vertical axis, at the right.

The pictures for OOHC admission rates are more complex. Overall, it is relatively rare across all jurisdictions for children who have never been involved in CP to be admitted to OOHC upon being involved in CP for the first time. Even for ACT, which

has the highest OOHC admission rate for new clients, the value is only 1.21 per 1000 per annum. In comparison, the investigation rates for new clients in ACT is 10.7 per 1000 per annum, 8.8 times as large. NT has the lowest OOHC admission rate for new clients (0.27 per 1000 per annum). In comparison, the investigation rates for new clients in NT is 27.0 per 1000 per annum, 100 times as large.

It is also observed that although TAS and SA have the lowest fractions of children involved in CP (sum of Q_1 and Q_2 in Table 5.8), they have the highest OOHC admission rates for repeat clients (60.1 and 43.2 per 1000 per annum, respectively). Given that TAS also has the second highest investigation rate for repeat clients, this may indicate a particularly serious problem with “churn” in TAS. In other words, even though the fractions of children involved in CP and OOHC are small, this small group of children have repeated involvement with CP and OOHC admission.

As in the case with investigation rate, TAS also exhibits the highest rate ratio between repeat and new clients admitted to OOHC, with the rate for repeat clients 159.9 times that for new clients. Once again, this is due to the relatively small fraction of children who are involved in CP, i.e. the base population for calculating the rates are particularly small. ACT has the smallest rate ratio between repeat and new clients (15.4). In all other jurisdictions, the rate ratios are larger than 25. NT has the second highest rate ratio, 58.2.

5.6 Trends of OOHC Populations across the States and Territories

With the rates shown in Table 5.8 and the time varying equations shown in the beginning of this chapter, the trend of OOHC population in each of the jurisdictions can be obtained. The results are shown in Figure 5.8. In the figure, the solid curves represent model solutions of the number of children in OOHC; diamonds represent the reported number of children in OOHC; crosses in the graphs for NSW, VIC and WA represent the number of children in OOHC with those on TPRO added, as shown in Figure 4.5. The vertical axis indicates per 1000 population while the horizontal axis shows financial years (e.g. 2010 represents the 2009-10 financial year).

Figure 5.8 (a) shows the results for NSW. Note that only data for 2015-16 to 2016-17 were used for NSW. The model curve is within the range of the last few data points, including 2018. However, the model trend does not reflect that of the reported data. Also, when children on TPRO were added to OOHC, the reported values became much higher, as expected. The first 5 reported data points (2010 to 2014) were all located much higher than the model curve, indicating that they might have been driven by a different rate regime. This will be further elucidated in the following section. Note that there was no attempt to use the TPRO corrected values of OOHC in the calculations because the OOHC admission rates reported are for values of children in OOHC without counting children on TPRO.

Figure 5.8 (b) shows the results for VIC. Note that only data for 2015-16 to 2016-17 were used for VIC. While there is an apparent kink in the reported data in 2013, the model curve is largely in accordance with the reported data points, except for 2018, when the counting rule was changed in VIC, with children on TPRO no longer included in the count of children in OOHC. When children on TPRO were added to the count of children in OOHC, the last data point moved to be in accordance with the model curve. The model curve also crosses the first two reported data points (2010 and 2011). This indicates that while there may have been a different rate regime in earlier years, the current rate regime may still be used to explain the average behaviour of OOHC population going back many years.

Figure 5.8 (c) shows the results for QLD. The reported data points from 2014 to 2018 all match the model curve. However, the reported data points from 2010 to 2013 are all located much higher than the model curve. This once again indicates the possibility that the earlier reported data points were driven by a different rate regime.

Figure 5.8 (d) shows the results for WA. All reported data points except those from 2011 to 2013 match the model curve. However, when children on TPRO were added to the count of children in OOHC, the last 3 reported data points (2016 to 2018) shifted upward from the curve.

Figure 5.8 (e) shows the results for SA. The reported data have an apparent kink in 2014. Overall, the model curve matches the reported data from 2016 to 2018 very well. However, for the first segment of the time series (2010 to 2013), the reported data points are all larger than the model curve, and the slope is smaller than that of the model curve.

Figure 5.8 (f) shows the results for TAS. Note that only data for 2015-16 to 2016-17 were used for TAS. All reported data points from 2015 to 2018 match the model curve very well. There is a kink between 2014 and 2015. Similar to SA, for the first segment of the time series (2010 to 2014), the reported data points are all larger than the model curve, and the slope is smaller than that of the model curve.

Figure 5.8 (g) shows the results for ACT. Note that only data for 2015-16 were used for ACT. The segment from 2013 to 2016 matches the model curve but the model curve does not capture the last two data points. Also, reported data from 2010 to 2012 had a very flat slope, compared to the much steeper slope of the model curve. Their values were also much higher than those of the model curve. This again indicates a different rate regime driving the earlier reported data points.

Figure 5.8 (h) shows the results for NT. The last 4 reported data points (2015 to 2018) match the model curve very well. However, the reported data exhibit a kink in 2014, with the data from 2010 to 2013 being much lower in value compared to the model curve. There appears to be a different rate regime for the earlier years.

(a) (b) (c) (d)

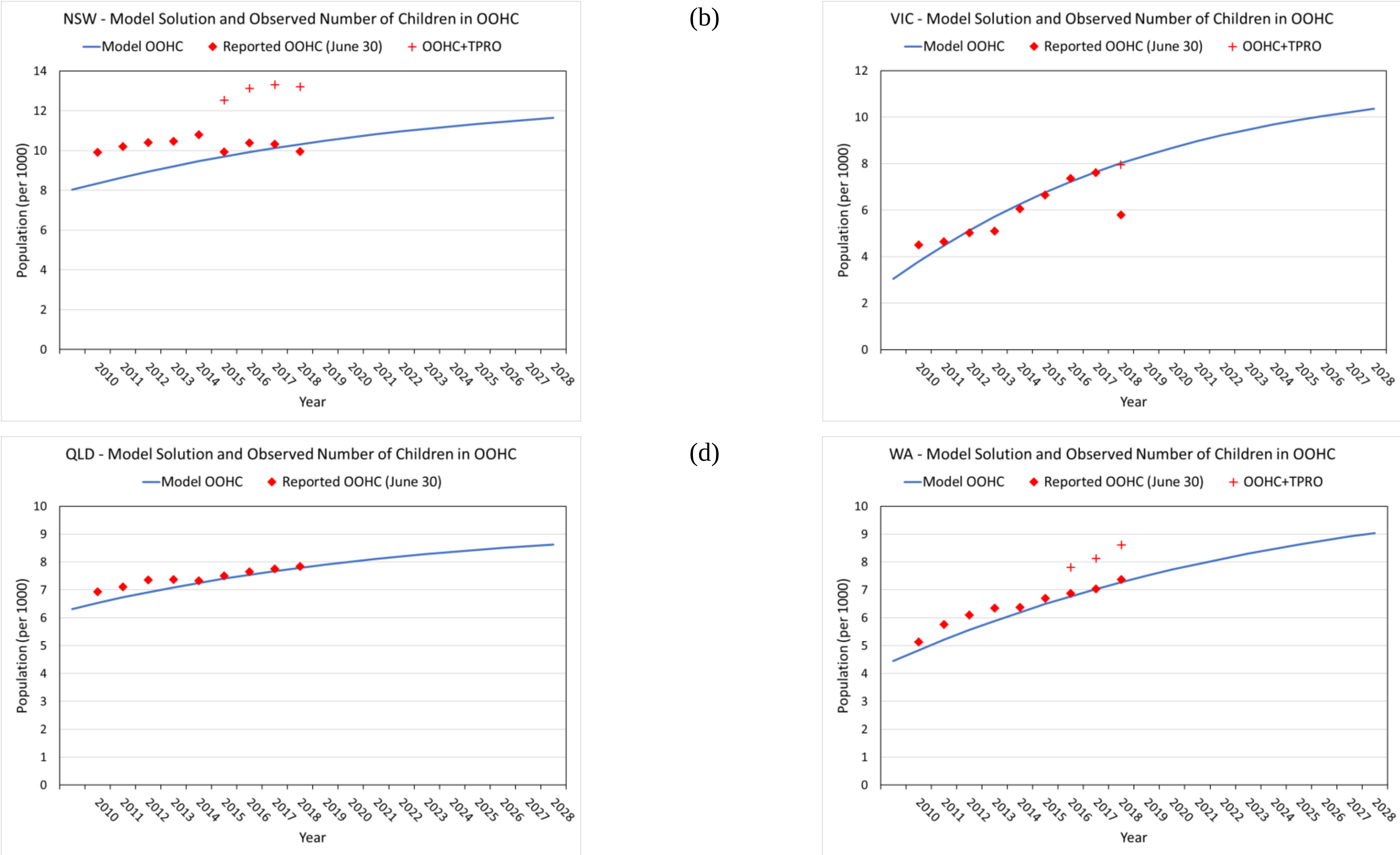


Figure 5.8 Number of Children in OOHC – Model Calculations and Reported Values (Continue on next page)

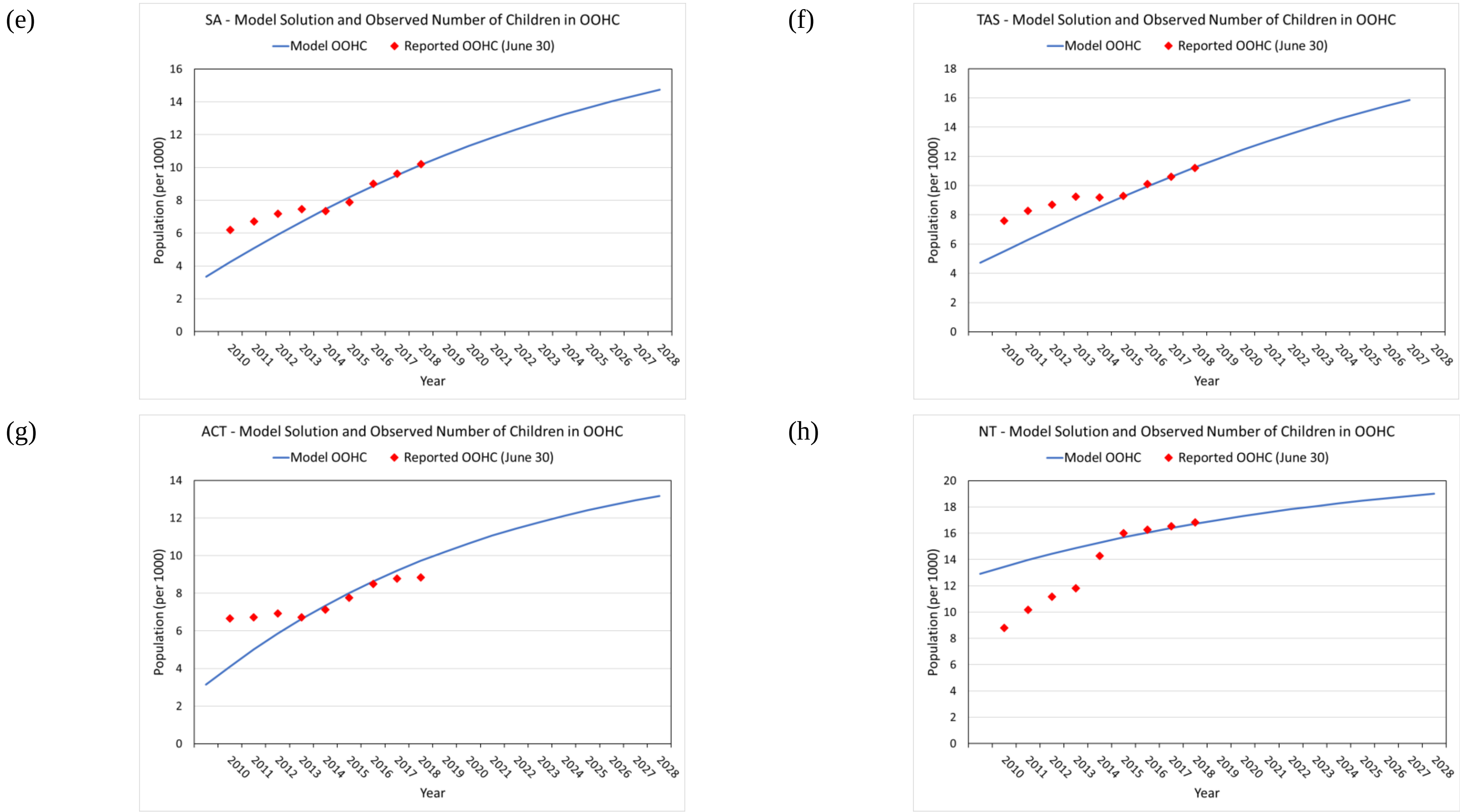


Figure 5.8 Number of Children in OOHC – Model Calculations and Reported Values (Continue from last page)

5.7 Asymptotic Values of the Sub-populations

Figure 5.9 shows the asymptotic values of the three sub-populations across the jurisdictions. Due to Q_2 being at least one order of magnitude smaller than Q_1, it is plotted on the secondary vertical axis, to the right, represented by the vertical bars with slanted stripes. Q_0 (vertical bars with solid shade) and Q_1 (vertical bars with horizontal stripes) are plotted on the primary vertical axis, to the left.

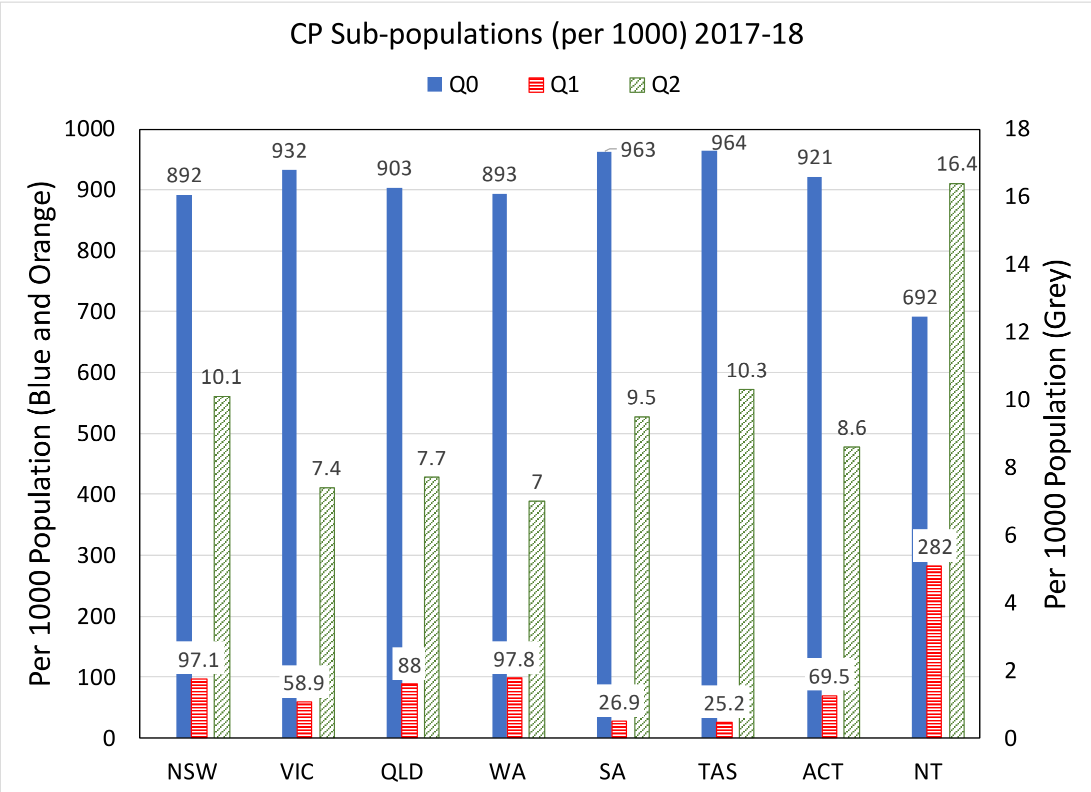


Figure 5.9 Asymptotic Solutions across the Jurisdictions

To better describe the results in Figure 5.9, the jurisdictions are ordered in terms of the values of each of the three sub-populations, from the lowest values to the highest values. The results are shown in Table 5.9. It is worthwhile to recall that the asymptotic solutions are the values where the sub-populations will eventually approach, if the rates used in the calculations do not change.

From Table 5.9, it can be observed that asymptotically, NT has the lowest population of children not involved in CP (644.5 per 1000) and the highest population of children involved in CP but not in OOHC (334.2 per 1000). It also has the second highest population of children in OOHC (21.3 per 1000). SA and TAS are similar in that they have the highest populations of children not involved in CP (935.3 and 933.3 per 1000, respectively); they also have the lowest populations of children involved in CP but not in OOHC (45.0 and 43.2 per 1000, respectively). On the other hand, they have the highest and third highest population of children in OOHC (23.5 per 1000 in TAS and 19.7 per 1000 in SA). This again shows a complex picture of CP involvement where per 1000 of OOHC alone is far from being able to provide a complete situation of CP involvement. QLD has the lowest population of children in OOHC, at 9.4 per 1000. All other jurisdictions exhibit similar asymptotic values, with the populations of children not involved in CP ranging from 848.8 to 903.4 per 1000.

Table 5.9 Ordering of the Jurisdictions in Terms of the Three Asymptotic Sub-populations

Q_0 ranking	NT	WA	ACT	NSW	QLD	VIC	TAS	SA
Q_0	644.5	848.8	867.6	873.6	886.3	903.4	933.3	935.3
Q_1	334.2	140.2	116.6	113.1	104.4	84.8	43.2	45.0
Q_2	21.3	11.0	15.8	13.2	9.4	11.8	23.5	19.7
Q_1 ranking	TAS	SA	VIC	QLD	NSW	ACT	WA	NT
Q_1	43.2	45.0	84.8	104.4	113.1	116.6	140.2	334.2
Q_2	23.5	19.7	11.8	9.4	13.2	15.8	11.0	21.3
Q_0	933.3	935.3	903.4	886.3	873.6	867.6	848.8	644.5
Q_2 ranking	QLD	WA	VIC	NSW	ACT	SA	NT	TAS
Q_2	9.4	11.0	11.8	13.2	15.8	19.7	21.3	23.5
Q_1	104.4	140.2	84.8	113.1	116.6	45.0	334.2	43.2
Q_0	886.3	848.8	903.4	873.6	867.6	935.3	644.5	933.3
Children in CP	SA	TAS	VIC	QLD	NSW	ACT	WA	NT
$Q_1 + Q_2$	64.7	66.7	96.6	113.7	126.4	132.4	151.2	355.5
Q_2 percentage	30.5%	35.2%	12.2%	8.2%	10.5%	11.9%	7.3%	6.0%

The second to the last row in Table 5.9 shows the total of asymptotic populations of children involved in CP (sum of Q_1 and Q_2) in each of the jurisdictions, ranked from the lowest to the highest. Once again, SA and TAS have the lowest populations of children involved in CP – 64.7 and 66.7 per 1000, respectively. However, in these two jurisdictions, the percentage of children in OOHC among all children involved in CP are the highest – 30.5% and 35.2% respectively. On the other hand, while NT has the highest population of children in OOHC, children in OOHC only accounts for 6% of all children involved in CP. Apart from NT, the total of the asymptotic populations of children involved in CP in all jurisdictions is below 152 per 1000, i.e. less than 15.2% of all number of children. In all jurisdictions other than TAS and SA, children in OOHC account for less than 12.2% of all children involved in CP. Note that the relatively high asymptotic populations of children in OOHC in TAS and SA are most likely driven by the small discharge rates of children in these jurisdictions (as shown in Table 5.4), in conjunction with the relatively high OOHC admission rates, as shown in Figure 5.7 (b).

5.8 Piecewise-Constant Parameter Modelling for SA, TAS and NT

In the previous section, it was seen that OOHC data in most jurisdictions exhibited two apparent rate regimes. The transition periods of these apparently different regimes were around 2013 and 2014. In this section, this issue is further analysed. This task is difficult due to limited data on new and repeat clients. As mentioned in Chapter 4, it was only in 2012-13 that breakdowns into new and repeat clients were first provided, and only aggregating SA, TAS and NT. In the previous section, our calculations were based on jurisdiction specific data from 2015-16 to 2017-18. CP data across these three years show little variation in the magnitudes of various process rates for new and repeat clients.

However, by aggregating the data in SA, TAS, and NT 2015-16 to 2017-18 and comparing them to the aggregate data provided by AIHW in 2012-13, it is evident that breakdowns into new and repeat clients for children in investigation and admission to OOHC underwent significant changes between 2012-13 and subsequent years, as shown in Figure 5.10. In the figure, percentages of new and repeat clients among children in investigations are represented by the solid and hollow diamonds, respectively; percentages of new and repeat clients among children admitted to OOHC are represented by the solid and hollow rectangles, respectively. It is seen that in 2012-13, repeat clients (53%) accounted for a slightly higher percentage than new clients (47%) for children in investigations. However, by 2015-16, the percentage of repeat clients in investigation had risen to 69.4%, while the percentage of new clients in investigations had dropped to 30.6%. In 2016-17 and 2017-18, while there were fluctuations, the new levels were generally maintained.

The situation for children admitted to OOHC was similar. In 2012-13, the percentage of repeat clients (45.7%) was actually lower than that of new clients (54.3%). By 2015-16, the percentage of repeat clients in investigation had risen to 72.6%, while the percentage of new clients in investigations had dropped to 27.4%. In 2016-17 and 2017-18, the percentage of new clients rebounded slightly, but the percentage of repeat clients remained much higher than that of new clients compared to 2012-13 level. Comparable data for the intervening years were not available. Therefore, it is not possible to know exactly whether the changes between 2012-13 and 2015-16 happened gradually or abruptly. Regardless, such changes should have had profound implications on the prevalence of children involved in CP. Changes in the relative fractions of new and repeat clients in investigations and OOHC admission reflect the changes in investigation rates and OOHC admission rates of new and repeat clients.

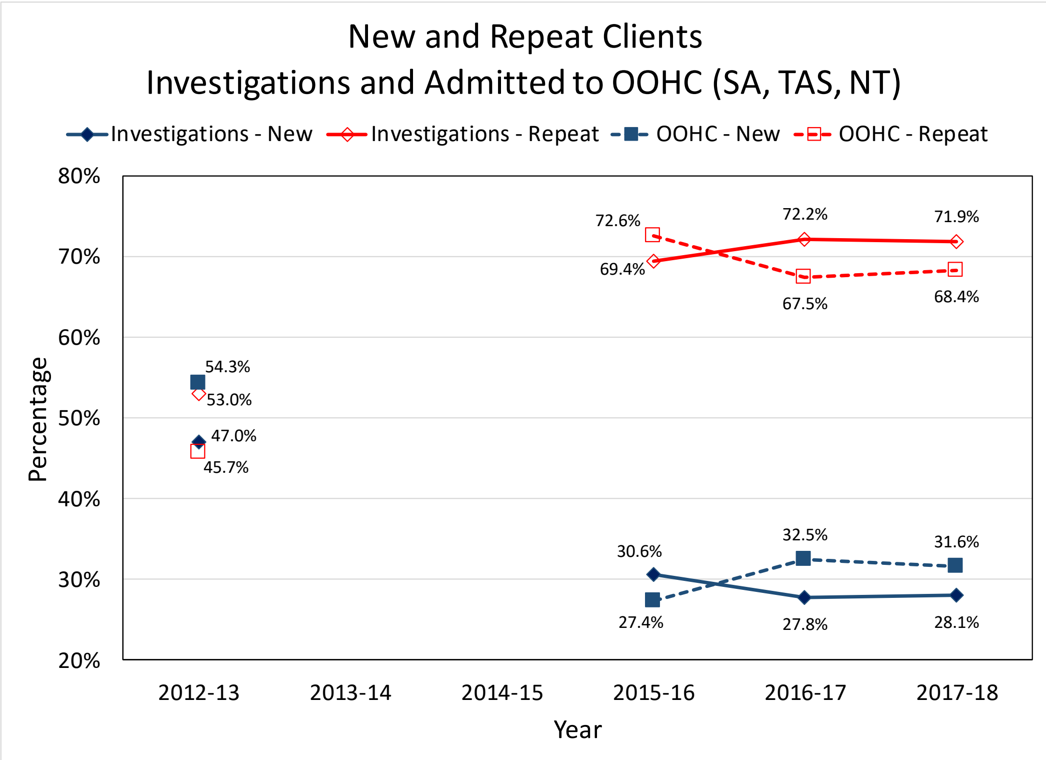


Figure 5.10 New and Repeat Clients Data for SA, TAS and NT (aggregate)

As shown in previous sections, substantial changes in these rates would lead to profound changes in the structure of the sub-populations of children involved in CP. In fact, taking the changes shown in Figure 5.10 to represent two typical rate regimes which exhibit significant differences, such changes can be modelled with the three-population model, in a piecewise continuous manner.

Due to the different CP practices and counting rules in the three jurisdictions of concern, the three jurisdictions will not be treated as if they belonged to the same CP system. Rather, it is assumed that the two rate regimes shown in Figure 5.10 are applicable to each of these jurisdictions. This is equivalent to making the assumption that in each of these jurisdictions, there are two different rate regimes with the transition point being around 2012-13. In each of the jurisdictions, it is assumed that in 2012-13 and before, the investigation rates as well as OOHC admission rates for new and repeat clients were similar to the earlier regime shown in Figure 5.10. On the other hand, after 2012-13, the rates were taken to be those used in the calculations shown in the previous sections.

Using data from 2009-10 and 2012-13, the mean population of children aged 0-17, number of children in investigations, number of children admitted to OOHC, and number of children discharged from OOHC for SA and NSW were calculated. For NT, only data from 2011-12 to 2012-13 were used. The reason for using data averaging over a number of years is that doing so better constrains Q_2 over time. However, when using four years of data for NT, the outcome was poor, with the model values matching poorly with the first three data points, and the trend was much steeper than that of the reported data.

Population growth rates were also calculated and age-out rates were estimated. These numbers were then used to obtain relevant parameters for new and repeat clients, using Step 1 of the iterative procedure described in Section 3.4 (Figure 3.3). With these estimated rates (labelled Regime 1), time series for the sub-populations (Q_0, Q_1 and Q_2) were calculated using the solutions of the three-population model, following Step 2 to Step 6 of the iterative process. These represent the time varying sub-populations preceding and up to 2012-13.

Table 5.10 shows the data used in the calculations. Note that as described above, it was assumed in the calculations that in each of the jurisdictions, the breakdown into new and repeat clients was the same as the aggregate data for 2012-13. Also, the number of children in investigations was adjusted by subtracting the number of children admitted to OOHC, for both the new and repeat clients. This allowed us to obtain the number of children in investigations only. By doing so, the fraction of new and repeat clients in investigations deviated slightly from the values shown in Figure 5.10. However, this was necessary to make the data consistent with the model formulation.

The data shown in Table 5.10 were first used in Step 1 of the iterative process described in Section 3.4 (Figure 3.3) to obtain “initial guesses” of the rates and sub-populations. Table 5.11 shows the comparison of these “initial guesses” for the data described above

(Regime 1) with those used in the previous sections (Regime 2). Instead of giving an extensive description of the differences, it is simply noted that the differences between the two rate regimes are large. A more thorough description of the differences will be given when the results obtained from the complete iterative process are examined.

Table 5.10 Mean CP Data for Time Varying Calculations for Regime 1

	SA	TAS	NT
Children in investigations only	3158	1518	3136
New	1484	713	1474
Repeat	1674	804	1662
Children admitted to OOHC	604	305	382
New (54.3%)	328	166	207
Repeat (45.7%)	276	139	175
General population	354545	116254	63338
OOHC discharge rate	10.91%	17.15%	27.69%
Age-out rate	5.97%	5.84%	5.04%
Net population growth rate	0.38%	-0.50%	1.10%
Gross population growth rate	6.35%	5.34%	5.63%

Table 5.11 Comparisons of Initial Parameters

per 1000 per annum		**SA**	**TAS**	**NT**
λ_{01}	Regime 1	5.4	5.3	41.1
	Regime 2	3.5	3.4	27.1
λ_{02}	Regime 1	1.52	1.61	5.77
	Regime 2	0.34	0.34	0.52
λ_{11}	Regime 1	10.7	56.1	63.1
	Regime 2	58.7	133.8	202.1
λ_{12}	Regime 1	2.1	11.7	6.6
	Regime 2	23.3	34.2	12.1

With the initial guesses of parameters shown in Table 5.11, the rest of the iterative process was carried out and a consistent set of investigation rates and OOHC admission rates, as well as CP sub-populations were obtained. Table 5.12 shows a comparison of the model results between the two rate regimes. The units are per 1000 per annum for the rates and per 1000 for the sub-populations.

The last row in Table 5.12 shows the model values of per 1000 OOHC populations (Q_2) as well as the reported values ($Q_{2,obs}$). It can be verified that the differences between the reported values and the model values are smaller than 1% (using values with one more significant figure instead of the round off values shown in the table). It is observed that for SA and TAS , the investigation rates for new clients (λ_{01}) were much higher in Regime 1 than those in Regime 2; in NT, the rate was only slightly higher in Regime 1. On the other hand, the investigation rate for repeat clients (λ_{11}) in Regime 1 was higher in SA, much lower in TAS, and slightly lower in NT, compared to Regime 2. The OOHC admission rate for new clients (λ_{02}) was much higher in Regime 1 compared to Regime 2 for TAS and NT, and only slightly higher in Regime 1 for SA. On the other hand, the OOHC admission rate for repeat clients (λ_{12}) in Regime 1 was much lower for SA and TAS, and much higher for NT, compared to Regime 2. Lastly, the OOHC discharge rate (λ_{21}) was slightly lower in Regime 1 for SA, almost twice as high for TAS, and higher for NT, compared to Regime 2.

Table 5.12 Final Model Parameters Obtained through the Iterative Process

		SA	**TAS**	**NT**
λ_{01}	Regime 1	4.4	6.5	26.3
	Regime 2	3.4	3.3	23.5
λ_{02}	Regime 1	0.96	1.51	3.70
	Regime 2	0.92	0.38	0.27
λ_{11}	Regime 1	155.4	153.5	254.1
	Regime 2	108.9	235.3	264.1
λ_{12}	Regime 1	25.6	26.5	26.8
	Regime 2	43.2	60.1	15.8
λ_{21}	Regime 1	77.0	138.5	276.9
	Regime 2	79.3	74.7	215.4
Q_0	Regime 1	961.8	945.3	884.4
	Regime 2	962.9	963.7	691.5
Q_1	Regime 1	30.4	45.1	103.3
	Regime 2	26.9	25.2	282.0
Q_2	Regime 1	6.8	8.4	11.4
	Regime 2	9.5	10.3	16.4
$Q_1 + Q_2$	Regime 1	37.2	53.5	114.7
	Regime 2	36.4	35.5	298.4
$Q_{2,obs}$	Regime 1	6.9	8.5	11.5
	Regime 2	9.6	10.4	16.5

For the ease of comparison, the ratios of the rates and sub-populations are plotted in Figure 5.11, in which the ratios are displayed along the vertical axis and the states across the horizontal axis. It is clear from the figure that apart from the investigation rate for repeat clients (λ_{11}) in TAS, the OOHC admission rate for repeat clients (λ_{12}) in

SA and TAS, and the total population of children involved in CP (Q_1 + Q_2), the rates were either similar or have mostly decreased in Regime 2. Recall that for the model used in this analysis, the time rate of change of Q_2 is given by the following equation:

$$\frac{dQ_2}{dt} = \lambda_{02}Q_0 + \lambda_{12}Q_1 - (\alpha + \lambda_{21})Q_2$$

With λ_{12} vastly larger in SA and TAS, and vastly smaller in NT for Regime 2, the trends in OOHC population became much steeper in the later years (2016 to 2018) compared to those in the earlier years (2010 to 2013) for SA and TAS. The reverse is true for NT. This can be easily verified from Figure 5.8 (e), (f) and (h).

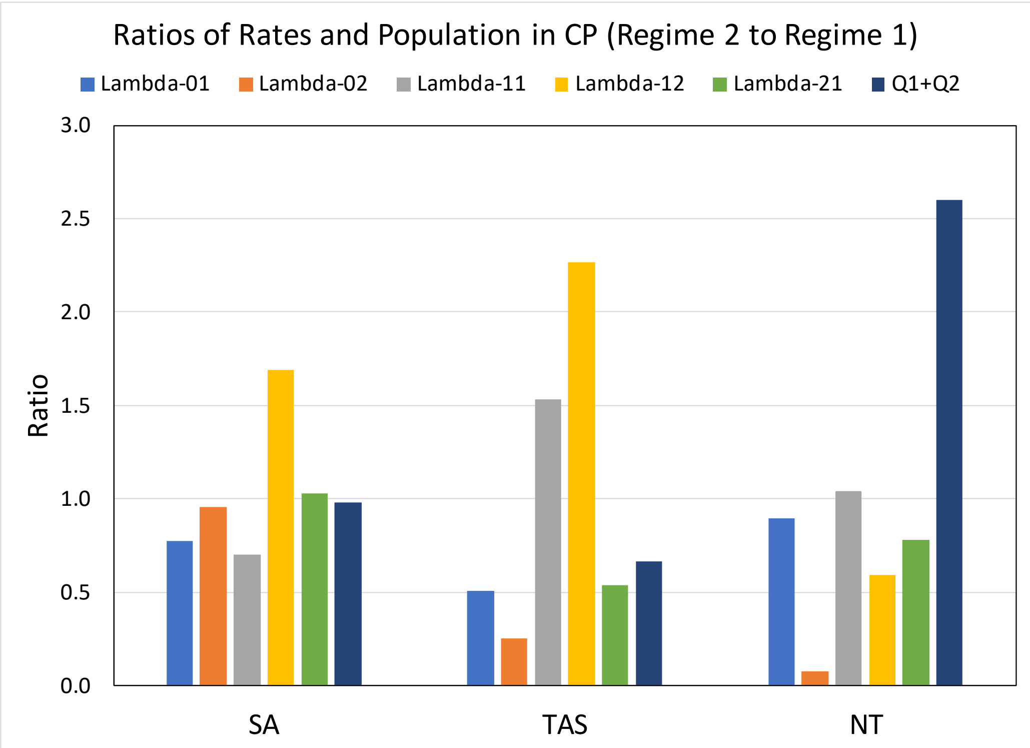


Figure 5.11 Ratios of Rates and CP Population (2012-13 Values)

Table 5.13 compares the asymptotic sub-populations for the two regimes. It can be seen that for SA and TAS, the asymptotic values for the population of children involved in CP but not in OOHC (Q_1) have decreased substantially in Regime 2, compared to Regime 1; the opposite is true for NT. Interestingly, the scenario is reverse for OOHC population (Q_2), where for SA and TAS, the asymptotic values have increased in Regime 2, compared to Regime 1; the opposite is true for NT. The implication is that in the later years (2016 to 2018), CP practice and policy changes have led to increased focus on children with repeatedly CP involvement, instead of new clients.

The differences between the two regimes are the starkest in TAS. In TAS, as shown in Figure 5.11, the investigations rate and OOHC admission rate for new clients (λ_{01} and λ_{02}) have decreased substantially in Regime 2 (2016 to 2018), while those for repeat clients (λ_{11} and λ_{12}) have increased substantially. This has led to a dramatic decrease in the asymptotic value of Q_1 (from 101.1 per 1000 to 43.2 per 1000). A vastly lower discharge rate in Regime 2 also contributed to the decrease in Q_1. Note that the magnitude of the overall population involved in CP is driven mainly by the investigation rates on new clients, due to the large base population of new clients (Q_0).

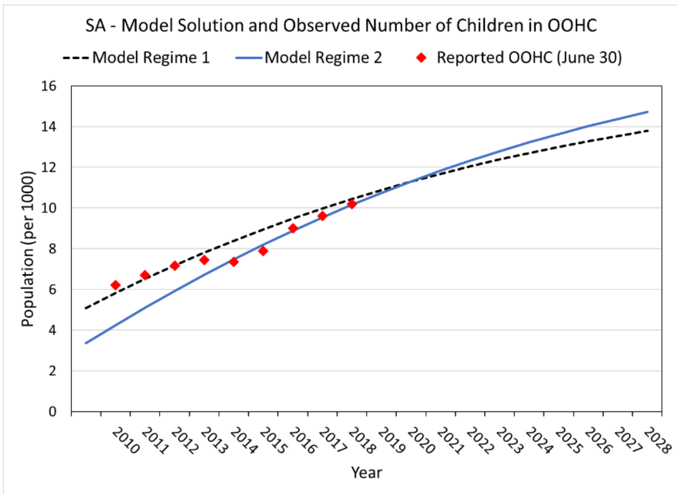
At this point, it is worthwhile to emphasise that the comparison of the two regimes presented above are predicated on the caveat that the proportion of new and repeat clients in investigations and OOHC admission obtained from aggregate data are applicable to each of the jurisdictions discussed. The extent to which this is true is questionable, as the analysis below shows.

Table 5.13 Asymptotic Values of Sub-populations for SA, TAS and NT

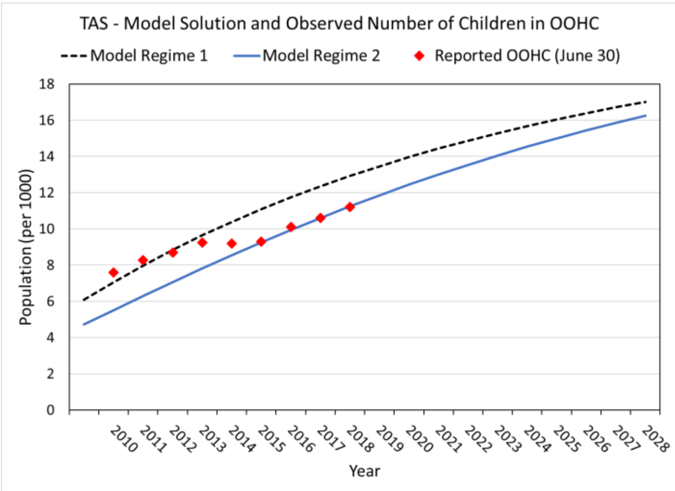
	SA	TAS	NT
Regime 1 (2010 to 2013)			
Q_0	922.5	870.1	671.7
Q_1	60.2	108.1	297.5
Q_2	17.3	21.8	30.9
$Q_1 + Q_2$	77.5	129.9	328.4
Regime 2 (2016 to 2018)			
Q_0	935.3	933.3	644.5
Q_1	45.0	43.2	334.2
Q_2	19.7	23.5	21.3
$Q_1 + Q_2$	64.7	56.7	355.5

Figure 5.12 plots the time varying trends of OOHC populations for both rate regimes. The vertical axis indicates per 1000 population while the horizontal axis indicates financial years. The dashed curves represent model solutions for Regime 1; the solid curves represent model solutions calculated using Regime 2; the diamonds represent the reported numbers of children in OOHC.

(a)



(b)



(c)

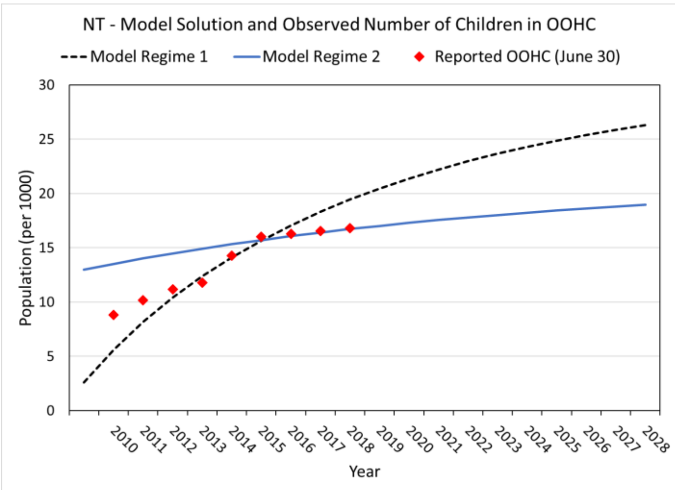


Figure 5.12 Piecewise Time Varying Population Trends for SA, TAS and NT

In Figure 5.12 (a), it can be seen that using two different rate regimes capture most of the reported data points of OOHC quite well, keeping in mind the caveat that aggregate new and repeat client data were projected to each of the jurisdictions. The dashed back curve shows an over-estimation of the trend in the earlier years (Regime 1, 2010 to

2013) and an under-estimation of the trend in the later years (Regime 2, 2016 to 2018). The kink around 2014 could not be captured by either curve. It is clear from the two curves that the blue curve (Regime 2) has a higher asymptotic value compared to the dashed black curve (Regime 1), as shown in Table 5.13.

Figure 5.12 (b) shows the results for TAS. The dashed black curve captures the reported data points in the earlier years although the trend was over-estimated. The two curves are almost parallel for the segment shown in the graph, but converge in the latter segment, leading to the blue curve eventually taking over the dashed black curve before tapering off to the asymptotic value. As in the case for SA, the kink around 2014 could not be captured.

Figure 5.12 (c) shows the results for NT. The dashed black curve somehow captures the reported data points from 2014 to 2016 but not for earlier years. The trend was also vastly over-estimated, leading to a far larger asymptotic value of OOHC population in Regime 1.

The fact that the model results could not capture the reported data points in Regime 1 for NT may not be surprising, given that SA accounted for 66% of the aggregate population of the 3 jurisdictions, TAS for 22% and NT for only 12%. It is possible that the rate regime shown in Figure 5.10 may not reflect the situation in NT. This indicates the importance for new and repeat client data to be provided consistently in order to build a better understanding of the structure of CP sub-population.

5.8 Comparisons with Projections Based on Past Growth Rates

In this section, OOHC population projections using the model solutions shown in the previous sections are compared to those using past OOHC growth rates. One way of obtaining future projection of a population is to use past growth rates of the population to calculate future population. However, without understanding the underlying mechanism that gives rise to the growth rates, this simple projection runs the risk of vastly misrepresenting the population. As seen in Figure 5.1 to Figure 5.3, a set of constant rate regimes representing a fixed process of population movements may give rise to vastly different growth rates at different stages in the evolution of a population.

Table 5.14 shows the annual population growth rates (APGR) of OOHC from 2010 to 2018 across the Australian states and territories. Due to changes in counting rule, the data used in three jurisdictions are limited as follows: 2016 to 2018 for NSW, 2010 to 2017 for VIC, and 2017 to 2018 for WA. Based on the mean growth rates shown in Table 5.14, the projection of the OOHC populations across the jurisdictions are calculated and compared to the model results shown in Figure 5.8.

Table 5.14 Annual OOHC Population Growth Rates from 2010 to 2018

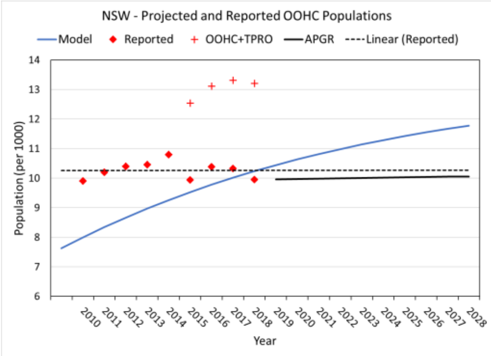
	NSW	VIC	QLD	WA	SA	TAS	ACT	NT
Mean	0.11%	7.24%	1.63%	3.56%	6.67%	5.66%	4.02%	9.18%
Std. Deviation	3.36%	5.46%	1.20%	1.06%	3.91%	3.46%	3.80%	6.61%

Figure 5.13 shows the projected OOHC populations based on the model solutions and the AGPR projection. In addition, a linear trendline based on the reported data is also shown. In each of the graphs, the vertical axis displays the per 1000 population and the horizontal axis displays the financial years (e.g. 2015 refers to the 2014-15 financial year). The diamonds show the reported values of OOHC population. The crosses in NSW, VIC and WA show the values of OOHC population with children on TPRO added. The solid curve shows the model solutions based on data from 2016 to 2018 (see Section 5.5 for details). The solid curve in the second half shows the projection calculated using the mean APGR in Table 5.14. The dashed curve shows the linear trend line of the reported data. For NT, an extra projection using mean APGR from 2016 to 2018 is also shown (dashed line in the second half).

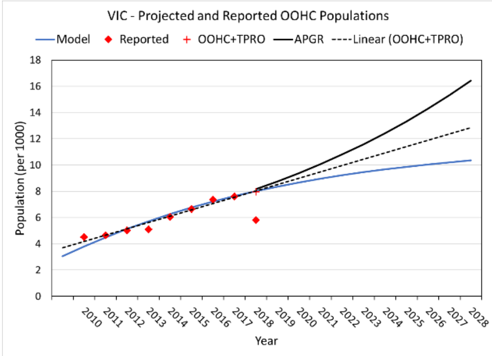
As noted in Section 5.6, the model solution calculated using 2016 and 2017 data does not capture the reported data points for NSW. Likewise, neither does the APGR projection nor the linear trendline capture the complex structure of the reported data points, as shown in Figure 5.13 (a). In fact, the reported data appear to exhibit two distinct regimes, with or without children on TPRO added to the OOHC population (red diamonds and red crosses).

Figure 5.13 (b) shows the projections for VIC. As mentioned in Section 5.6, the model solution for VIC was calculated using only data from 2016 and 2017, due to a change in counting rule in which children on TPRO were excluded from the count of children in OOHC. Despite this, the model solution correctly captures the data point with TPRO added (red cross) in 2018. The APGR projection for 2018 is almost 3% higher than the TPRO data point while both the model solution and the linear trendline are within the

(a)



(b)



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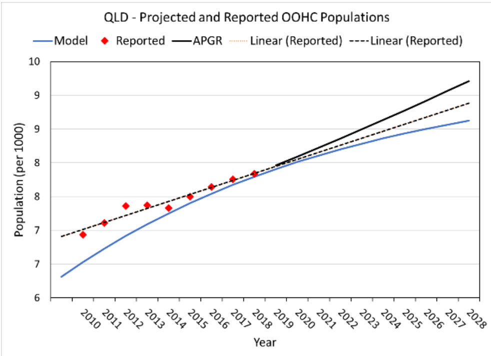
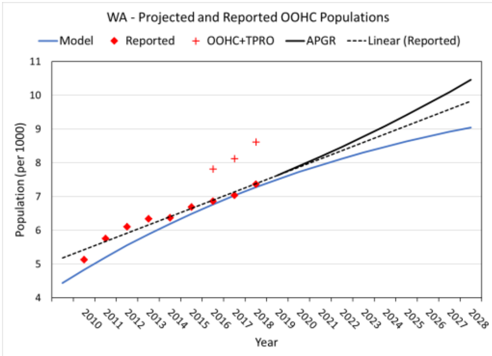
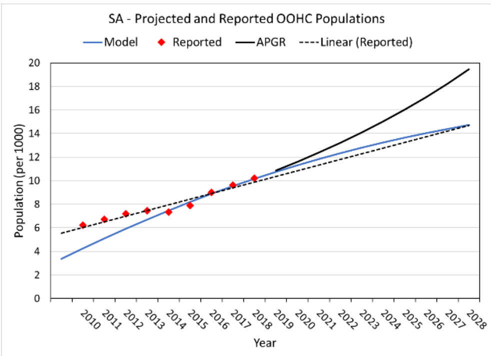


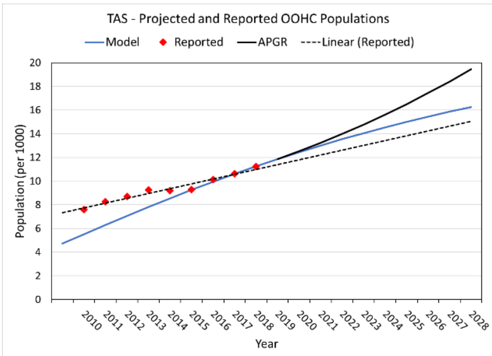
(d)



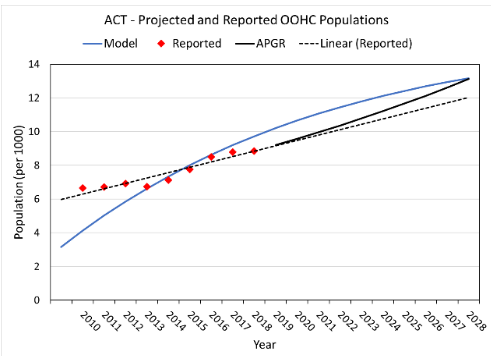
(e)



(f)



(g)



(h)

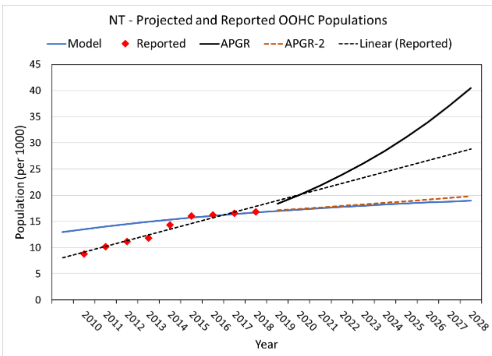


Figure 5.13. OOHC Population Projections

1% range of the data point. Both the APGR projection and the linear trendline continue to diverge from the model solution, resulting in much larger OOHC populations in 2028. The average annual population growth rate for the model projection from 2018 to 2028 is 2.9%, compared to the rate of 7.2% used in the APGR projection.

Figure 5.13 (c) shows the projections for QLD. In this case, the linear trendline best captures the last four reported data points while the model solution underestimates their values by an average of 1.9%. As in VIC, both the AGPR projection and the linear trendline continue to diverge from the model solution, resulting in larger OOHC populations in 2028, although not to the extent in VIC. The average annual population growth rate for the model projection from 2018 to 2028 is 1.1%, compared to the rate of 1.6% used in the APGR projection.

Figure 5.13 (d) shows the projections for WA. In this case, both the linear trendline and the model solution capture the last four reported data points well. The model solution underestimates their values by an average of 1.4%. As in the cases in VIC and QLD, the model solution has the lowest value compared to the APGR projection and the linear trendline in 2018. The average annual population growth rate for the model projection from 2018 to 2028 is 2.4%, compared to the rate of 3.56% used in the APGR projection.

Figure 5.13 (e) shows the projections for SA. In this case, the model solution best captures the last four reported data points, including the trend. The linear trendline exhibits the lowest values among the projections until 2018, in which it catches up with the model solution. The average annual population growth rate for the model projection from 2018 to 2028 is 4.5%, compared to the rate of 6.7% used in the APGR projection.

Figure 5.13 (f) shows the projections for TAS. As for SA, the model solution best captures the last four reported data points. The linear trendline exhibits the lowest values among the projections until 2018 and does not catch up with the model solution until much further ahead. The average annual population growth rate for the model projection from 2018 to 2028 is 4.5%, compared to the rate of 5.7% used in the APGR projection.

Figure 5.13 (g) shows the projections for ACT. ACT is another jurisdiction for which the model solution does not do well. The most likely factor in the poor performance is that only the 2016 of data were available when calculating the model solution. As a result, the solution cannot capture the trend of the reported data points and it overestimates OOHC populations for 2017 and 2018. The linear trendline exhibits the best fit with the reported data overall and has the lowest projected value in 2028. The APGR projection also has lower values compared to the model solution but it eventual catches up with the model solution in 2028.

Figure 5.13 (h) shows the projections for NT. In this case, the model solution best captures the last four reported data points, underestimating their values by an average of only 1.1%. The linear trendline overestimates the reported data point in 2018 and continues to diverge from the model solution afterwards. The average annual population growth rate for the model projection from 2018 to 2028 is 1.3%, compared to the rate of 9.2% used in the APGR projection. However, if the APGR is calculated using only reported data from 2016 to 2028, the value drops to 1.6% and the projection, shown as the brown dashed curve, is much closer to the model solution.

Overall, it can be seen that using APGR to project the OOHC population produces the largest increase in population, except for NSW and ACT, where data issues plagued the applications of the model solutions. In VIC, QLD, WA and NT, the model projected the smallest increase in OOHC population compared to both the APGR projection and the linear trendline based on the reported data. In SA and TAS, the model projections produced higher values than the linear trendline, although the latter eventually caught up.

5.9 Cross-sectional Prevalence and Lifetime Prevalence of CP Involvement

In this section, population data are used to examine the relationship between cross-sectional prevalence and lifetime prevalence of CP involvement. Recall that in Section 3.5, the relation between the cross-sectional prevalence of children in CP and lifetime prevalence of CP involvement is examined. It is shown that these two types of prevalence are linked by the following equation:

$$Q_{cps} = Q_1 + Q_2 = \sum_{x=0}^{17} Q_{cps,x} = \sum_{x=0}^{17} W_x \, L_x + \varepsilon'$$

where Q_{cps} is the fraction of children currently involved in CP, ε' denotes the effect of migration and death censoring, L_x is the lifetime cumulative risk of CP involvement for children in the "x" age bracket, and W_x is a weighting factor, each defined below.

$$L_x = 1 - \prod_{i=0}^{x} p_i$$

$$W_x = \frac{P_{-x,0}}{P}$$

In the formulas above, p_i is the age-specific probability of not receiving a maltreatment investigation. $P_{-x,0}$ denotes the number of children in the age 0 cohort, x number of years prior to the current year of concern. P is the general population of children 0 to 17 years of age. The relation between cross sectional prevalence of children in CP and lifetime prevalence of CP involvement is based on the recognition that the population of children in the current year is made up of the birth cohorts of children born in previous years.

The three-population model formulated in this study does not explicitly take age-structure into account. However, for demonstration purposes, some assumptions can still be made to provide a crude estimation of the lifetime cumulative risk or lifetime prevalence of CP involvement. To do so, it is assumed that there exists a mean value of p_i, the age-specific probability of not receiving a maltreatment investigation, which will produce the correct value of Q_{cps}. This is a strong assumption that may not hold in reality. However, for the purpose of demonstration here, this is assumed to be the case. Doing so, the equations for L_x (Equation A) and Q_{cps} (Equation B) can be rewritten as:

$$L'_x = 1 - p^{x+1}$$

$$Q_{cps} = \frac{1}{P} \sum_{x=0}^{17} P_{-x,0} \left(1 - p^{x+1}\right)$$

where p is the mean value of p_i. With p being one of the solutions of Equation B, it is not likely that using this value will reproduce the actual value of L_x. For this reason, the original L_x in Equation A differs from L'_x above by an error term.

A published study of lifetime prevalence estimation, done with the life-table approach, will be first used to examine the procedure described above, especially the error term in L_{17} – lifetime prevalence of CP involvement up to 17 years of age (Kim et al., 2017). In their study, lifetime prevalence was estimated for the overall population of children aged 0 to 17 in the US, as well as for various sub-populations, in 2014.

In this study the focus is on the estimation for the overall population. Following the procedure in Kim et al. (2017), the populations of children in the age 0 bracket from 1997 to 2014 in the USA is first obtained, using the CDC WONDER online database[8], excluding a number of states as described in the supplementary materials accompanying the paper. The total population of children aged between 0 to 17, as well as age-specific probability of having a maltreatment investigation, are then obtained from Table A of the supplementary materials. Carrying out the procedure described above, the following is found:

- The sum of the number of children in the age 0 bracket from 1997 to 2014 was 64743509, while the general population of children aged 0 to 17 in 2014 was 66769330, giving a percentage difference of negative 3.1%.
- Using Equation B, with the error term stemming from migration and death censoring neglected, a value of 0.971 for p (a mean of p_i, the age-specific probability of not receiving a maltreatment investigation) was obtained. The corresponding value of L'_{17} is 0.414. The actual value of lifetime prevalence calculated in the cited study was 0.374, giving a difference of 10.7%. In other words, the crude estimation done using the procedure described above overestimated the lifetime prevalence by around 10%.

The above analysis using data from published study provides a perspective of the magnitude of error one can expect in the crude estimation. With that in mind, one can proceed to perform the estimation for the Australian states and territories. Using the model calculations in Section 5.5 and Section 5.6, Q_{cps} or $Q_1 + Q_2$ across the Australian states and territories for 2018 are compiled and shown below in Table 5.15. Note that for Q_2 (OOHC population), the model solution instead of reported data was used so that Q_1 and Q_2 are both consistent with the model.

Table 5.15 Model Results of per 1000 Population of Children Involved in CP (2018)

	NSW	VIC	QLD	WA	SA	TAS	ACT	NT
Q1	95.9	60.5	87.7	97.4	26.9	27.9	74.1	277.9
Q2	10.2	8.0	7.8	7.3	10.2	11.2	9.7	16.7
Q1 + Q2	106.1	68.5	95.4	104.6	37.1	39.1	83.8	294.7

[8] https://wonder.cdc.gov/

Table 5.16 shows the number of children in the age 0 bracket from 2001 to 2018. They correspond to $P_{-x,0}$ in the equation above, with $P_{0,0}$ denoting the number of children age 0 in 2018 and $P_{-17,0}$ denoting the number of children age 0 in 2001. The last three rows show the sum of the 0 age cohorts from 2001 to 2018, the general population of children below 18 years of age in 2018, and percentage differences between these two variables. The differences are calculated using the sum of 0 age cohorts as the base number. Therefore, positive values indicate net in-flow and negative values indicate net out-flow.

Table 5.16 Number of Children in the Age 0 Bracket (2001 to 2018)

	NSW	VIC	QLD	WA	SA	TAS	ACT	NT
2001	86686	59543	48603	24867	17553	6206	4070	3746
2002	84726	60237	48426	24383	17567	5806	3994	3647
2003	85563	60166	48302	24096	17346	5730	4024	3700
2004	85285	61358	49815	24550	17395	5710	4140	3535
2005	86480	61910	52204	25425	17558	5977	4169	3470
2006	89501	64220	54045	26810	17834	6361	4446	3635
2007	94650	68232	57939	29130	18905	6668	4584	3722
2008	95642	69595	61021	30315	19538	6635	4590	3849
2009	97157	70310	62173	30544	19719	6665	4823	3847
2010	97771	71569	61792	31256	19669	6384	5123	3850
2011	94818	70073	60155	31245	19312	6176	4946	3650
2012	98177	74786	63003	32951	20231	6242	5295	3994
2013	99227	76791	63466	34443	20424	6076	5491	3986
2014	96398	76705	62990	34588	20203	5960	5546	3904
2015	98112	77846	62125	34874	20071	5786	5652	3946
2016	101604	82400	62587	35730	20671	6034	5696	4083
2017	98195	78229	61150	34749	19335	5615	5600	3899
2018	105254	78645	61865	33875	18925	5514	5652	3798
Total	1695246	1262615	1041661	543831	342256	109545	87841	68261
GP-18	1766635	1401120	1166543	598005	366636	112340	93681	62771
Diff	4.2%	11.0%	12.0%	10.0%	7.1%	2.6%	6.6%	-8.0%

Among the jurisdictions, NT was the only jurisdiction with a net outflow, with a percentage difference of negative 8%; QLD, VIC and WA exhibited the largest percentage differences between the two variables, at positive 12%, 11% and 10% respectively; SA and ACT were similar, with percentage differences of positive 7.1% and 6.6%, respectively; NSW and TAS had the smallest percentage differences between the two variables, at positive 4.2% and 2.6% respectively. It should be noted that only

NSW and TAS had an error term which is comparable to the published study in the USA described above. These results give a hint of error term (*ε'*) in the equation for Q_{cps} above. It is once again emphasised that the estimation of lifetime prevalence in this section has to be viewed with these caveats in mind.

Using the data in Table 5.15 and Table 5.16, and Equation B above, one solves for *p* – the mean probability of children in a one-year age bracket for not being involved in a CP investigation. One then calculates L'_{17}, the lifetime cumulative risk for a person below 18 years of age to receive a CP investigation, which by definition is the lifetime prevalence of CP involvement. The probabilities of no CP involvement are shown in Table 5.17 while the lifetime prevalence of CP involvement is shown in Figure 5.14.

Table 5.17 Probability of No CP Involvement for One-Year Age Bracket (2018)

NSW	VIC	QLD	WA	SA	TAS	ACT	NT
98.7%	99.1%	98.7%	98.6%	99.6%	99.6%	98.9%	96.5%

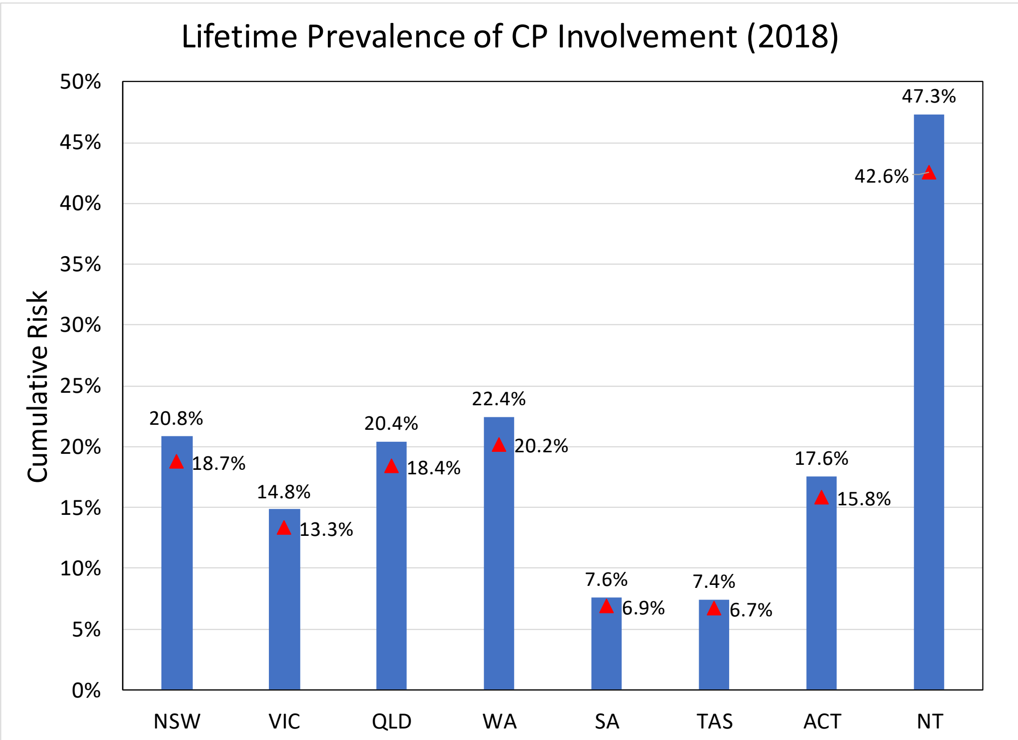


Figure 5.14 Lifetime Prevalence of CP Involvement

In Figure 5.14, the vertical axis shows the cumulative risk of CP involvement up to 17 years in age. This is the lifetime prevalence of CP involvement. The shaded bars and the corresponding numbers show the estimations of lifetime prevalence of CP involvement calculated as described above. The triangles and the corresponding numbers show the values which are 10% lower than the estimated values represented by the shaded bars.

It can be seen that SA had the lowest lifetime prevalence, with a 7.6% probability of being involved in a CP investigation over the 18-year span of childhood. TAS has the second lowest lifetime prevalence, with a 7.4% probability. On the other hand, NT has the highest lifetime prevalence, with a probability of 47.3%. WA has the second highest lifetime prevalence, with a probability of 22.4%. The lifetime prevalence in all other jurisdictions range from 14.8% in VIC to 20.8% in NSW. Even by effecting a 10% reduction as an indicative value of over-estimation, the lifetime prevalence in NT was as high as 42.6 %. This value is indeed very high but is perhaps not implausible, given that with a much meticulous procedure using the life table approach, a value of 37.4% for the USA in 2014 was obtained in the published study cited above.

5.10 Conclusion

In this chapter, the analytical solutions of the three-population model have been used to simulate OOHC population in each of the Australian states and territories, and examine the total number of children involved in CP. Overall, the model is able to capture the observed trend of OOHC population in most states and territories, except for NSW and ACT. When projected into the future, the trends obtained from the model calculations exhibit significant differences from projections using historical annual population growth rates. Moreover, it has been demonstrated that the model solutions can be used to estimate lifetime prevalence of CP involvement with annual aggregate data.

The results in this chapter demonstrate that with adequate information on new and repeat clients, the three-population model, even with the linearization caveat, is able to capture the trend of OOHC population, except where there appear to be a break in population structure. In such cases, it has been demonstrated that a piecewise-constant parameter approach, with separate sets of rate regimes for apparently different structural segment in the OOHC population time series, may help us provide a good picture across the apparently different regimes. Various projections of OOHC populations have been compared and the issue of lifetime prevalence of CP involvement has been elucidated. In Chapter 6, the structures of CP population will be further investigated using the four-population model.

Chapter 6: The Four-Population Model – Analytical Solutions and Applications

In the previous chapter, the analytical solutions of the three-population model were explored and used to analyse the populations of children involved in CP across Australian states and territories. In this chapter, the analytical solutions of the four-population model are examined and numerical simulations are conducted to further explore the dynamics of the CP populations, especially the issue of "churn" – repeated involvement in the CP system.

6.1 Analytical Solutions for Four-Population Model

As shown in sub-section 3.1.4, for the four-population model, the analytical solutions for the case with no system capacity limit and with uniform age-out rates across the sub-populations are (Equations E.8 to E.11 in Chapter 3):

$$Q_0 = \frac{\alpha}{h_0} + c_0 e^{-h_0 t}$$

$$Q_1 = \frac{\alpha\lambda_{01}}{h_0 h_1} - \frac{c_0\lambda_{01}}{h_0 - h_1} e^{-h_0 t} + \left[c_1 + \lambda_{01}\left(\frac{c_0}{h_0 - h_1} - \frac{\alpha}{h_0 h_1}\right)\right] e^{-h_1 t}$$

$$\begin{aligned} Q_2 = {} & \frac{(\alpha + \lambda_{32})(\alpha\lambda_{02} + \lambda_{01}\lambda_{12} + \lambda_{02}\lambda_{12})}{h_0 h_1 h_2} - \\ & \frac{c_0(\lambda_{02} - \lambda_{12})(\lambda_{01} + \lambda_{02} - \lambda_{32})}{(h_0 - h_1)(h_0 - h_2)} e^{-h_0 t} - \\ & \frac{\lambda_{12} - \lambda_{32}}{h_1 - h_2}\left[c_1 + \lambda_{01}\left(\frac{c_0}{h_0 - h_1} - \frac{\alpha}{h_0 h_1}\right)\right] e^{-h_1 t} + \\ & \left\{c_2 - \frac{1}{h_0 h_1 h_2}(\alpha + \lambda_{32})(\alpha\lambda_{02} + \lambda_{01}\lambda_{12} + \lambda_{02}\lambda_{12}) + \frac{c_0(\lambda_{02} - \lambda_{12})(\lambda_{01} + \lambda_{02} - \lambda_{32})}{(h_0 - h_1)(h_0 - h_2)}\right. \\ & \left. + \frac{\lambda_{12} - \lambda_{32}}{h_1 - h_2}\left[c_1 + \lambda_{01}\left(\frac{c_0}{h_0 - h_1} - \frac{\alpha}{h_0 h_1}\right)\right]\right\} e^{-h_2 t} \end{aligned}$$

$$\begin{aligned} Q_3 = {} & \frac{\lambda_{23}(\alpha\lambda_{02} + \lambda_{01}\lambda_{12} + \lambda_{02}\lambda_{12})}{h_0 h_1 h_2} + \\ & \frac{c_0\lambda_{23}(\lambda_{02} - \lambda_{12})}{(h_0 - h_1)(h_0 - h_2)} e^{-h_0 t} + \\ & \frac{\lambda_{23}}{h_1 - h_2}\left[c_1 + \lambda_{01}\left(\frac{c_0}{h_0 - h_1} - \frac{\alpha}{h_0 h_1}\right)\right] e^{-h_1 t} - \end{aligned}$$

$$\left\{c_2 - \frac{1}{h_0 h_1 h_2}(\alpha + \lambda_{32})(\alpha\lambda_{02} + \lambda_{01}\lambda_{12} + \lambda_{02}\lambda_{12}) + \frac{c_0(\lambda_{02} - \lambda_{12})(\lambda_{01} + \lambda_{02} - \lambda_{32})}{(h_0 - h_1)(h_0 - h_2)} + \frac{\lambda_{12} - \lambda_{32}}{h_1 - h_2}\left[c_1 + \lambda_{01}\left(\frac{c_0}{h_0 - h_1} - \frac{\alpha}{h_0 h_1}\right)\right]\right\} e^{-h_2 t}$$

In the equations above, c_0, c_1 and c_2 are integration constants to be determined by initial conditions, while h_0, h_1 and h_2 are as defined below.

$$h_0 = \alpha + \lambda_{01} + \lambda_{02}$$

$$h_1 = \alpha + \lambda_{12}$$

$$h_2 = \alpha + \lambda_{23} + \lambda_{32}$$

The rest of the parameters and data from which they may be obtained are shown in Table 6.1. Note that in the formulation of the four-population model, the discharge rate from OOHC in the three-population model, λ_{21}, is substituted by λ_{23}, as children having exited OOHC are represented as Q_3, a sub-population which is distinct from those who have never been admitted to OOHC. With Q_3 added, there is a separate OOHC admission rate corresponding to it. This is labelled as λ_{32}. Note that h_1 is different from the one used in the three-population model. This is due to the addition of Q_3, resulting in children having exited OOHC no longer being lumped into Q_1 which now designates only children having been involved in CP investigation but not being admitted to OOHC.

Table 6.1 System Parameters and Available Data for the Four-Population Model

Parameters		**Available Data**
α	Gross growth rate of child population (P) due to population increase.	Sum of net growth in general child population (0 to 17 years of age) and the "aged out".
λ_{01}	Rate of children entering non-OOHC CPS (P_1') for the first time	Number of investigations for new clients minus number of OOHC entries for new clients
λ_{02}	Rate of children entering OOHC (P_2) in first-time involvement with CPS	Number of new clients admitted to OOHC
λ_{12}	Rate of children entering OOHC (P_2) after previous involvement with CP but never been admitted to OOHC	Part of the number of repeat clients admitted to OOHC
λ_{32}	Rate of children re-admitted to OOHC after previously exiting from it	Part of the number of repeat clients admitted to OOHC
λ_{23}	Rate of children leaving OOHC (P_2) not as a result of aging out	Child discharged from OOHC minus those aging out

In the above equations, as time (t) increases, the exponential terms decrease and eventually drop out. The following are obtained.

$$Q_0 = \frac{\alpha}{\alpha + \lambda_{01} + \lambda_{02}}$$

$$Q_1 = \frac{\alpha\lambda_{01}}{(\alpha + \lambda_{12})(\alpha + \lambda_{01} + \lambda_{02})}$$

$$Q_2 = \frac{(\alpha + \lambda_{32})(\alpha\lambda_{02} + \lambda_{02}\lambda_{12} + \lambda_{01}\lambda_{12})}{(\alpha + \lambda_{01} + \lambda_{02})(\alpha + \lambda_{12})(\alpha + \lambda_{23} + \lambda_{32})}$$

$$Q_3 = \frac{\lambda_{23}(\alpha\lambda_{02} + \lambda_{02}\lambda_{12} + \lambda_{01}\lambda_{12})}{(\alpha + \lambda_{01} + \lambda_{02})(\alpha + \lambda_{12})(\alpha + \lambda_{23} + \lambda_{32})}$$

As in the three-population mode, asymptotically, Q_0 and correspondingly, the fraction of children involved in CP (sum of Q_1 and Q_2), are fully determined by the population growth rate as well as the investigation rate and OOHC admission rate for new clients (λ_{01} and λ_{02}).

The effects of the parameters on the asymptotic solutions are first explored. Taking the partial derivatives of the steady solutions of Q_0, Q_1 and Q_2 with respect to each of the parameters, the following are obtained.

1. $\frac{\partial Q_0}{\partial \alpha} > 0$, as in the three-population model.
2. $\frac{\partial Q_0}{\partial \lambda 01} < 0$, as in the three-population model.
3. $\frac{\partial Q_0}{\partial \lambda 02} < 0$, as in the three-population model.
4. $\frac{\partial Q_1}{\partial \alpha} < 0$, as in the three-population model.
5. $\frac{\partial Q_1}{\partial \lambda 01} > 0$, as in the three-population model.
6. $\frac{\partial Q_1}{\partial \lambda 02} < 0$, opposite to the three-population model. This is because in the four-population formulation, children exiting OOHC are classified into Q_3, a different subpopulation as opposed to re-classifying into Q_1, as in the three-population formulation. As a result, with a larger rate of new clients being directly admitted to OOHC, there will be a smaller fraction of clients being involved in CP but not having been admitted to OOHC.
7. $\frac{\partial Q_1}{\partial \lambda 12} < 0$, as in the three-population model.

8. $\frac{\partial Q_2}{\partial \alpha} < 0$ for the range of parameters generally observed, as in the three-population model.

9. $\frac{\partial Q_2}{\partial \lambda 01} > 0$ for the range of parameters generally observed, as in the three-population model.

10. $\frac{\partial Q_2}{\partial \lambda 02} > 0$, as in the three-population model.

11. $\frac{\partial Q_2}{\partial \lambda 12} > 0$ for the range of parameters generally observed, as in the three-population model.

12. $\frac{\partial Q_2}{\partial \lambda 23} < 0$. This means with all other parameters held fixed, a higher OOHC discharge rate leads to a smaller fraction of children in OOHC.

13. $\frac{\partial Q_2}{\partial \lambda 32} > 0$. This means with all other parameters held fixed, a higher OOHC re-admission rate leads to a larger fraction of children in OOHC.

14. $\frac{\partial Q_3}{\partial \alpha} < 0$. This is just a corollary of the first point above, i.e. larger population growth leads to a smaller fraction of children involved in CP, and thus smaller fraction of children who have exited OOHC.

15. $\frac{\partial Q_3}{\partial \lambda 01} > 0$ if $\lambda_{12} > \lambda_{02}$; $\frac{\partial Q_3}{\partial \lambda 01} < 0$ if $\lambda_{12} < \lambda_{02}$. This means an increase in the investigation rate (without leading to OOHC admission) will eventually lead to a larger fraction of children who have exited OOHC, if repeat clients are admitted to OOHC at a higher rate than new clients. In contrast, if new clients are admitted to OOHC at a higher rate than repeat clients, the increasing investigation rate will lead to a smaller fraction of children who have exited OOHC.

16. $\frac{\partial Q_3}{\partial \lambda 02} > 0$. This means with all other parameters held fixed, a higher OOHC admission rate for new clients leads to a larger fraction of children who have exited OOHC.

17. $\frac{\partial Q_3}{\partial \lambda 12} > 0$. This means with all other parameters held fixed, a higher OOHC admission rate for repeat clients leads to a larger fraction of children who have exited OOHC.

18. $\frac{\partial Q_3}{\partial \lambda 23} > 0$. This means with all other parameters held fixed, a higher exit rate from OOHC leads to a larger fraction of children who have exited OOHC.

19. $\frac{\partial Q_3}{\partial \lambda 32} < 0$. This means with all other parameters held fixed, a higher OOHC re-admission rate leads to a smaller fraction of children who have exited OOHC.

6.2 Time Dependence of the Solutions

Time variations of the solutions are dependent on the three exponential terms, with the constants h_0, h_1 and h_2 determining the pace of change. It should be noted that Q_0 only depends on the exponential term containing h_0; Q_1 only depends on the exponential terms containing h_0 and h_1. The exponential term containing h_2 plays a role only in the time variation of Q_2 and Q_3. This is in accordance with the logical facts that the population of children not involved in CP is fully determined by the population growth rate, the investigation rate for new clients, and OOHC admission rate for new clients; the population of children involved in CP but not having been admitted to OOHC is fully determined by the population growth rate, the investigation rates for new clients, and OOHC admission rates for new and repeat clients. They are not influenced by the OOHC discharge rate and readmission rate of children who have exited OOHC. The main opportunity with the addition of Q_3 is to examine and simulate the phenomenon of “churn” – repeated involvement in OOHC.

To illustrate the time dependence of the four-population model, the following demonstration uses the exact same case as that used to illustrate the three-population model in Section 5.2, with a set of typical parameters similar to that observed in NSW, as shown below.

$$\alpha = 0.06$$

$$\lambda_{01} = 0.01$$

$$\lambda_{02} = 0.0004$$

$$\lambda_{23} = 0.11$$

It should be noted that for the corresponding calculation in the three-population model, λ_{12} is set to a value of 0.012 per capita per annum. In this case, the overall rate of repeat clients, including both with and without prior OOHC admission experience, should be 0.012 for the case to be comparable. This means the weighted average of the OOHC admission rates for Q_1 and Q_3 should be 0.012, i.e.

$$\frac{\lambda_{12}Q_1 + \lambda_{32}Q_3}{Q_1 + Q_3} = 0.012$$

However, since there is no information on λ_{32}, readmission rate of children who have exited OOHC has to be hypothesised. For the case where λ_{32} equals λ_{12}, i.e. when OOHC admission rate is homogenous for all repeat clients, regardless of whether they have previously been admitted to OOHC and exited, the results are identical to that in the three-population model. To demonstrate this, the value of 0.012 is assigned to both λ_{12}, and λ_{32}. For this full set of parameters, the gross population growth rate is 6% per annum; the investigation rate for new clients not ending up in OOHC is 1% per annum; the OOHC admission rate for new clients is 0.04% per annum; the OOHC admission rate for repeat clients, both with and without experience in OOHC, is 1.2% per annum; the OOHC discharge rate is 11% per annum. Moreover, the exponents of the time varying terms are:

$$h_0 = 0.0704$$

$$h_1 = 0.072$$

$$h_2 = 0.182$$

It should be noted that h_0 and h_1 are similar in magnitude. However, h_2 is much larger in magnitude. As the case in the three-population model, the solutions contain a more slowly varying component and a more quickly varying component. The more slowly moving component is mainly associated with flows related to new clients while the more quickly moving component is mainly associated with flows related to OOHC discharge. Since in the four-population formulation, OOHC discharge is now directed towards Q_3, only Q_2 and Q_3 contain the more quickly moving components. As a result, Q_0 and Q_1 only contain the more slowly varying exponential terms while Q_2 and Q_3 contain both the more slowly and quickly varying exponential terms.

Figure 6.1 shows the time varying plots of Q_0 (solid curve), Q_1 (dash curve), Q_2 (two-dash curve) and Q_3 (long-dash curve) for the scenario specified above. The dot-dash curve shows the sum of all children involved in CP but not currently in OOHC, i.e. Q_1 and Q_3. The x-axis is year after the initial time and the y-axes indicate populations normalised to 1000. Q_0 and Q_1 are plotted on the primary vertical axis, to the left; Q_2 and Q_3 are plotted on the secondary vertical axis, to the right.

(a)

(b)

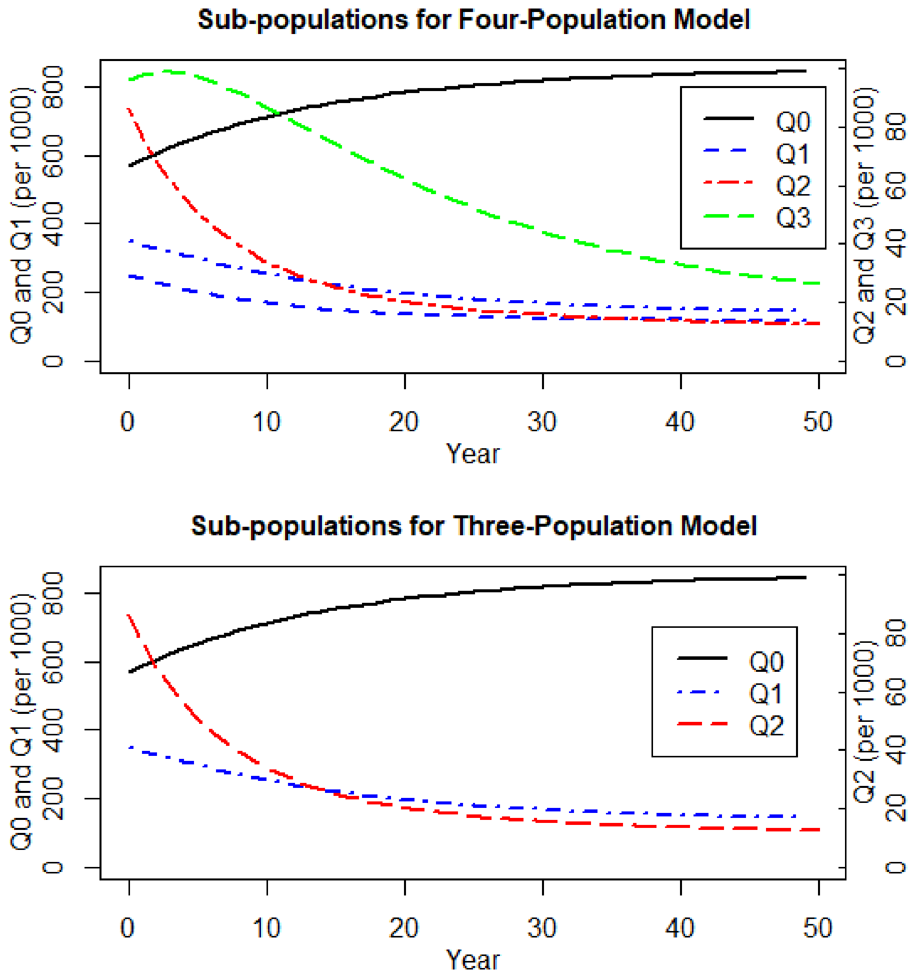


Figure 6.1 Example of Time Varying Solutions

In Figure 6.1, the initial population has 568 per 1000 children not involved in CP (Q_0), 250 per 1000 children involved in CP but not having been in OOHC (Q_1), 86 per 1000 children in OOHC (Q_2), and 96 per 1000 children who have exited OOHC (Q_3). Note that the sum of Q_1 and Q_3 is 346 per 1000, which is exactly the same as the number of children involved in CP but not currently in OOHC, the corresponding example in Section 5.2, the results of which is reproduced in Figure 6.1 (b) for reference. However, it should be noted once again that in the three-population model, Q_1 includes both children who are only involved in CP investigation but have never been admitted to OOHC and children who have exited OOHC. In the four-population model illustrated here, Q_1 only includes children who are involved in CP investigations but have never been admitted to OOHC. Children who have been admitted to OOHC and exited are now represented by Q_3.

In this case, all four sub-populations are initially far from the asymptotic values of 852.2 per 1000 children, 118.4 per 1000 children, 11.6 per 1000 children, and 17.8 per 1000 children, respectively. As time progresses, Q_0 increases towards its asymptotic value, while Q_1 and Q_2 decrease towards their respective asymptotic values. Q_3 increases for a few years before decreasing towards its asymptotic value. It is observed that the blue dashed curve in Figure 6.1 (a) is identical to the blue curve in Figure 6.1 (b). Likewise, the curves for Q_0 and Q_2 are also identical in the two models. As indicated above, the only difference between the two models in this case is that children who have previously in OOHC but are not currently in care are lumped into Q_1 in the three-population model. In the four-population model, they are separated out as a distinct sub-population. In fact, when λ_{32} and λ_{12} are identical, the four-population model is reduced to the three-population model.

Two scenarios are now explored: one in which λ_{32} is larger than λ_{12}, and the other in which λ_{32} is smaller than λ_{12}. In both scenarios, λ_{12} and λ_{32} are subject to the constraint of the combined OOHC admission rate above, i.e. the combined OOHC admission rate of repeat client is 0.012. For these two scenarios, all parameters are the same as those used in Figure 6.1, except for λ_{12} and λ_{32}. In Scenario 1, λ_{12} is set to a fixed value of 0.014, while λ_{32} is calculated using the constraint equation above in every time step, with the combined rate being 0.012. Note that with λ_{12} fixed, λ_{32} will vary with time, in order for the combined rate to be 0.012. In Scenario 2, λ_{32} is assigned the fixed value of 0.04, while λ_{12} varies with time, subjecting to the constraint of the combined OOHC admission rate being 0.012.

Since the solutions were derived from a set of differential equations with constant rate parameters (coefficients), allowing λ_{32} to vary with time essentially means that a piecewise continuous approach is taken in using the solutions, with the values of the variables at each time interval serving as the initial condition for the next time interval.

Figure 6.2 shows the time varying sub-populations for the two scenarios described above. Q_0 is plotted as the solid curve, Q_1 as the dash curve, Q_2 as the two-dash curve, and Q_3 as the long-dash curve. The x-axis is year after the initial time and the y-axes indicate populations normalised to 1000. Q_0 and Q_1 are plotted on the primary vertical axis, to the left; Q_2 and Q_3 are plotted on the secondary vertical axis, to the right.

Q_0 is identical in the two scenarios as it does not depend on λ_{12} and λ_{32}. In addition, Q_2 approaches the value of 11.6 per 1000 asymptotically in both scenarios, as in the case in Figure 6.1 (a). In Scenario 1, the value of λ_{12} is fixed at 0.014 while the values of λ_{32} range from 0.0010 to 0.0083, with a mean of 0.0034. λ_{32} approaches 0.0010 asymptotically. In Scenario 2, the value of λ_{32} is fixed at 0.04, while the values of λ_{12} range from 0.00055 to 0.0091, with a mean of 0.0074. λ_{12} approaches 0.0091 asymptotically. This means that the OOHC admission rate for children who have previously been admitted to OOHC is much larger in Scenario 2 compared to that in Scenario 1. This is reflected in the much steeper decline of Q_3 in Scenario 2. For the same reason, the asymptotic value of Q_3 is 21.0 per 1000 in Scenario 1 and 12.8 per 1000 in Scenario 2, much lower for the latter. The sum of Q_1 and Q_3 also approaches the same asymptotic value of 136.1 per 1000, with the time varying values differing from each other by no more than 1% at all time.

(a)

(b)

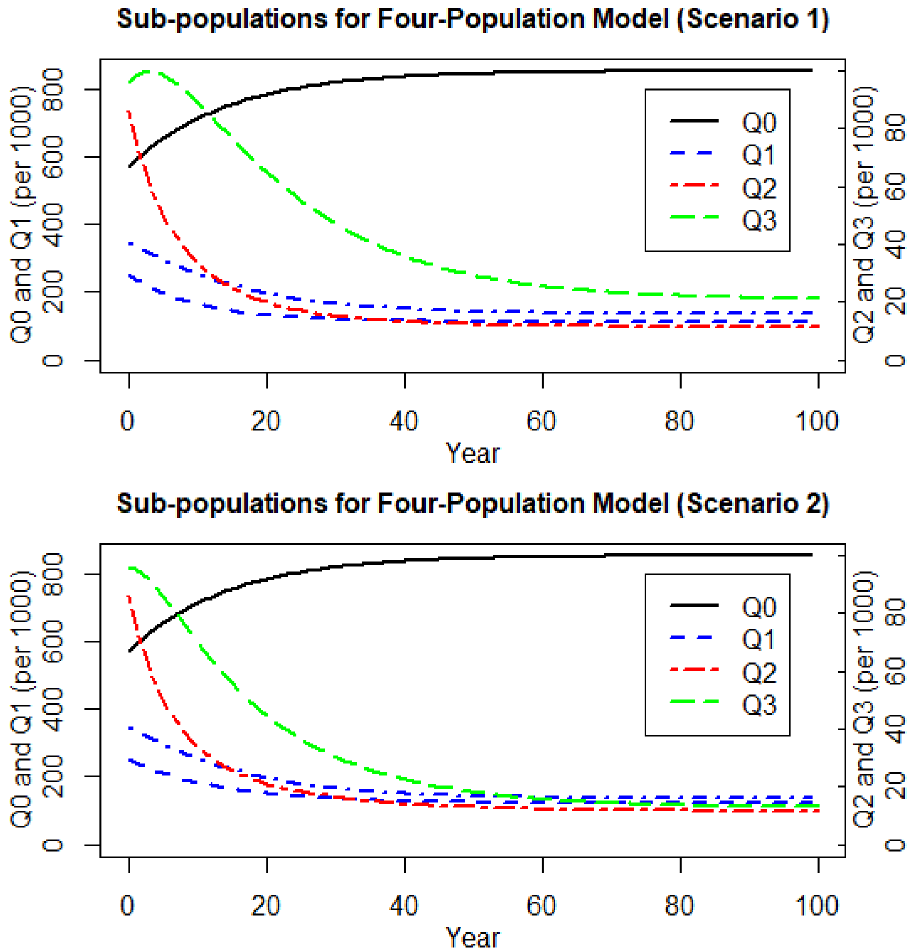


Figure 6.2 Time Varying Solutions for Two Sets of OOHC Admission Rates

The results in Figure 6.1 and Figure 6.2 indicate that designating children who have exited OOHC as a distinct sub-population has little effect on the dynamics of OOHC, as long as the overall OOHC admission rate is preserved. However, if the number of children re-entering care led to an increase in the overall OOHC admission rate, OOHC population would rise as a result. It should be cautioned that the calculations in Figure 6.2 are purely for the purpose of scenario testing and no attempt was made to offer an interpretation of time varying λ_{32}, which have been applied in a piecewise-constant manner in the calculations, assuming that it is constant from year to year.

6.3 Modelling CP Populations across Australian States and Territories

In this section, the four-population model is used to further examine the issue of repeated involvement in OOHC or “churn”. The calculations are based on the results obtained in Section 5.6 and the analysis in this section can be considered an extension of the corresponding analysis using the three-population model.

In the following calculations, the rate parameters α, λ_{01}, λ_{02}, and λ_{23} are fixed, with values taken from the corresponding calculations in Section 5.6. As of λ_{12} and λ_{32}, three scenarios are considered. In Scenario 1, λ_{12} and λ_{32} are assumed to be equal. This will reproduce the results as in Section 5.6, but with the sub-population of repeat clients not currently in OOHC split into two distinct sub-populations. In this scenario, the issue of “churn” is serious, as children who have exited care are as likely as other repeated clients to re-enter care. In Scenario 2, λ_{32} is set to the value of λ_{02}, which is much smaller than λ_{12}. This can be interpreted as most reunifications being successful and very few children who exited care re-enter the OOHC system, i.e. the issue of “churn” is negligible. In Scenario 3, λ_{32} is set to twice the value of λ_{12}. This means children who have exited OOHC are significantly more likely than other repeat clients to enter OOHC, i.e. the issue of “churn” is even more serious than that in Scenario 1.

Calculations using the iterative process described in Section 3.4 (see Figure 3.3 and associated text) were carried out. Data needed for Step 1 in the iterative proves are retrieved from the final results in the corresponding calculations in Section 5.6. As in Section 5.6, all sub-populations and parameters stabilized after a few iterations. The results for Scenario 1 are first presented and then a summary comparing the three scenarios is provided.

For Scenario 1, it was first verified that a number of parameters and variables were practically identical to those obtained in Section 5.6. These include the model time (T), population of children not involved in CP (Q_0) both at model time T and asymptotically, population of children in OOHC (Q_2) both at model time T and asymptotically, the investigation rates for new and repeat clients (λ_{01} and λ_{11}), and the OOHC admission rates for new and repeat clients (λ_{02} and λ_{12}). In NT, the magnitudes of the differences for all these variables and parameters, between Scenario 1 in this section and those in Section 5.6 were smaller than 0.8%. For all other jurisdictions, the magnitudes of the differences were smaller than 0.04%.

Figure 6.3 shows the time varying plots of the sum of Q_1 and Q_3 (solid curve), Q_1 (dash curve), Q_2 (two-dash curve) and Q_3 (long-dash curve) for Scenario 1. The x-axis indicates financial years (e.g. 2010 indicates the 2009-10 financial year) and the y-axes indicate populations normalised to 1000. Q_1 and the sum of Q_1 and Q_3 are plotted on the primary vertical axis, to the left; Q_2 and Q_3 are plotted on the secondary vertical axis, to the right.

In all jurisdictions, the solid curve represents the total population of children involved in CP but not currently in OOHC. This is exactly the same as the result for Q_1 in Section 5.6. As a reminder, it is noted that in the four-population model, children who have exited care are designated as a separate sub-population, represented by the long-dash curve. The results represent the breakdown of the population of children involved in CP but not currently in OOHC into two sub-populations, provided that they cannot be distinguished in terms of the rates of investigation and OOHC admission.

In NSW, VIC, WA and ACT, Q_3 starts out smaller than Q_2 but eventually outgrows Q_2, doing so more quickly in some jurisdictions than others. In QLD and NT, Q_3 starts out larger than Q_2 and remains so into the future. In SA and TAS, Q_3 starts out smaller than Q_2 and remains smaller into the future. These differences are driven mainly by the small OOHC exit rates in SA and TAS, leading to very small populations of children who have exited OOHC in these two jurisdictions.

Through the course of the data shown in Figure 6.3, the value of Q_3 increases by a much larger magnitude compared to the value of Q_2, across all jurisdictions. Moreover, the value of Q_3 plateaus off much later than that of Q_2, across all jurisdictions. This is due mainly to the fact that OOHC exit rates are much larger than OOHC admission for any of the three sub-populations. Even as changes in the population of children in OOHC (Q_2) start to slowdown as in-flows and out-flows balance, the population of children who have exited OOHC (Q_3) will continue to rise, as the balancing of in-flows and out-flows will not happen until Q_3 becomes large enough, wherein out-flows from Q_3 start to approach the values of in-flows from OOHC.

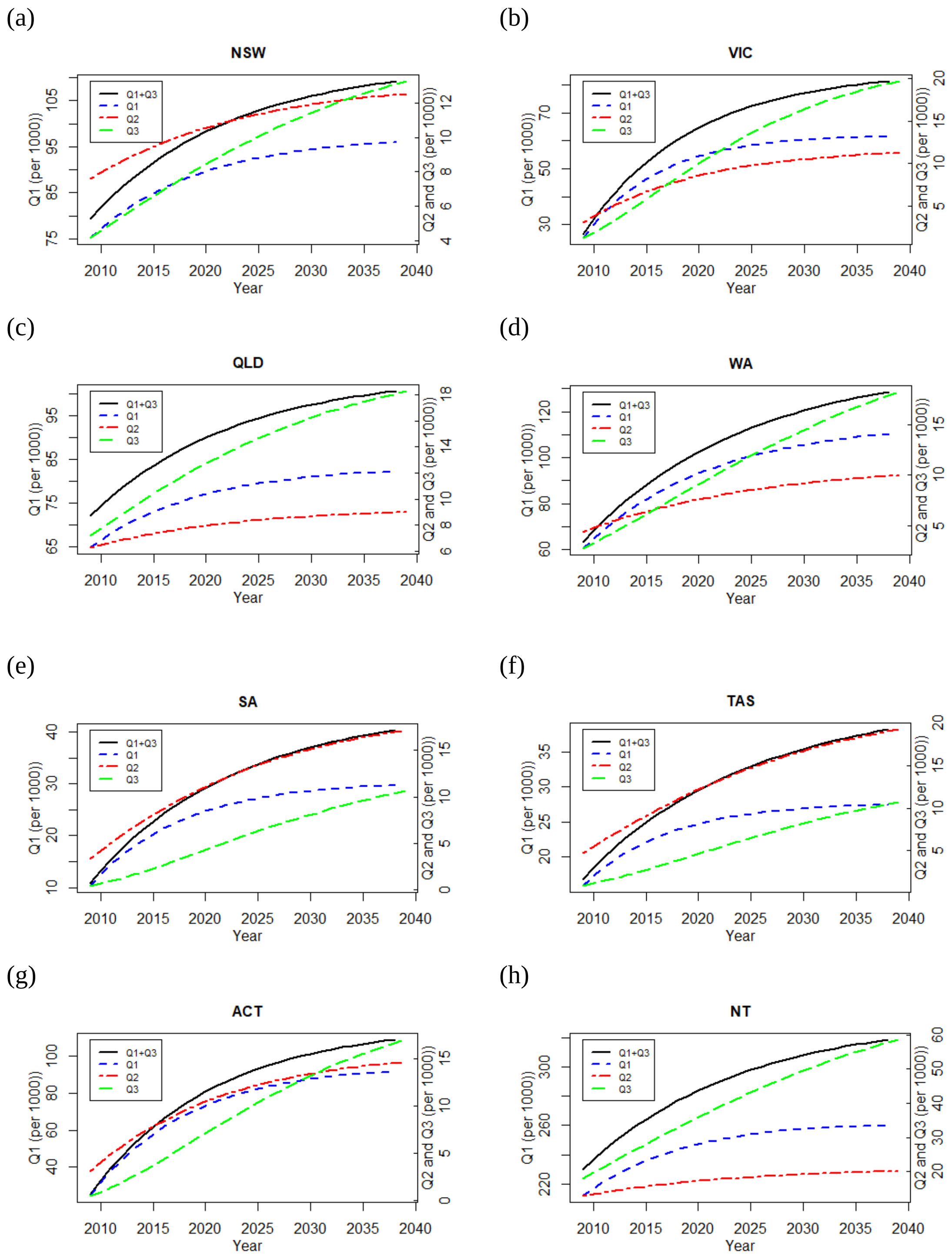


Figure 6.3. CP Sub-populations for Four-Population Model (Scenario 1)

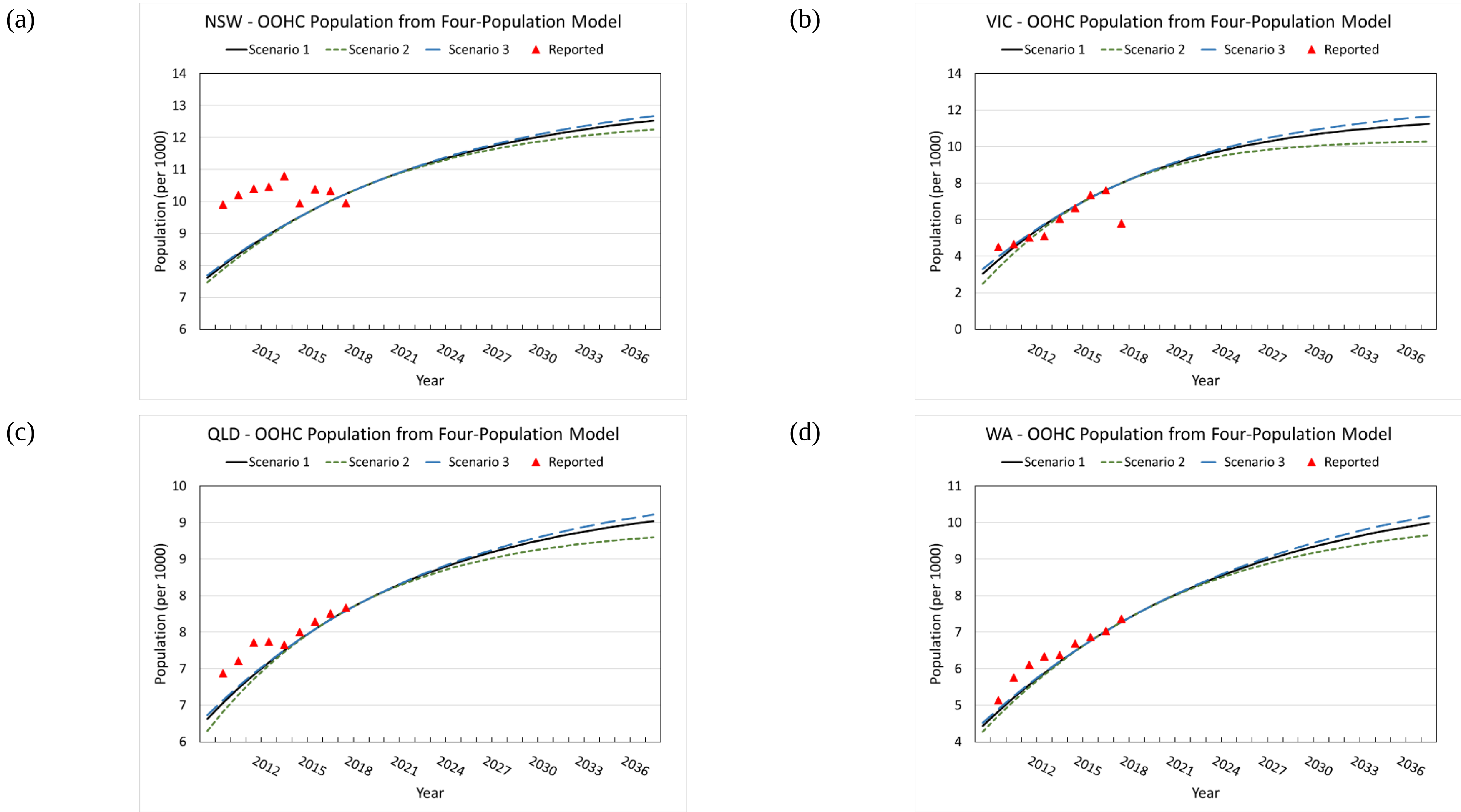
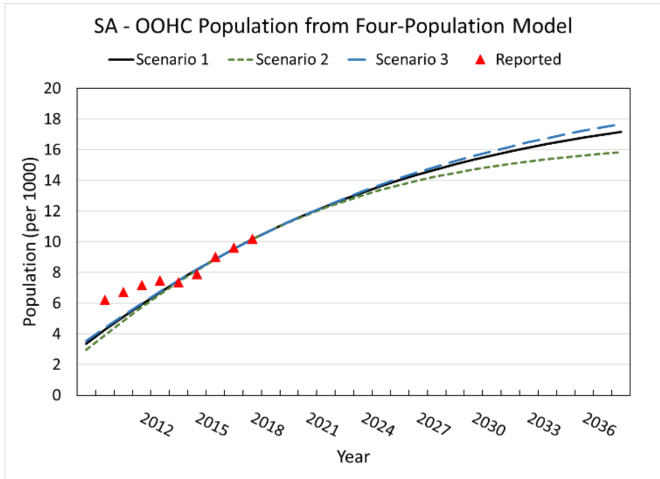
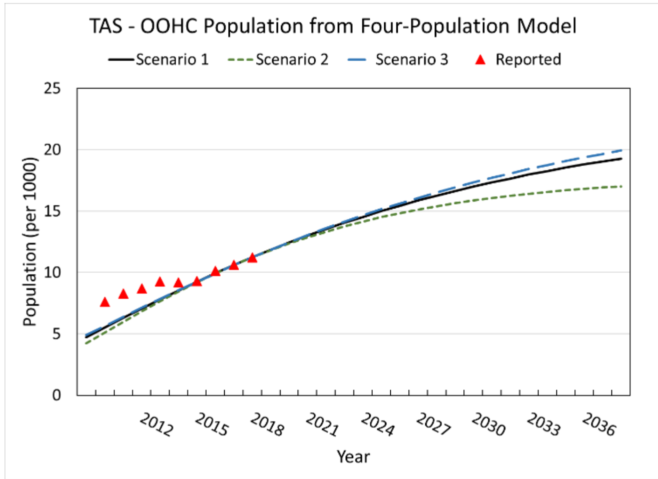


Figure 6.4 Population of Children in OOHC for Various Scenarios (Continue on next page)

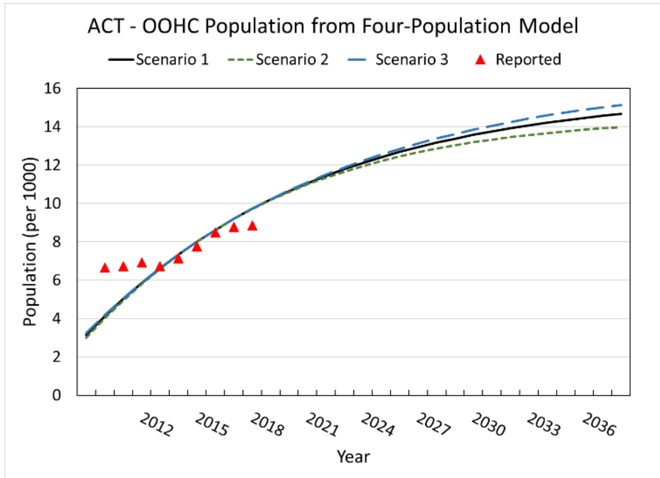
(e)



(f)



(g)



(h)

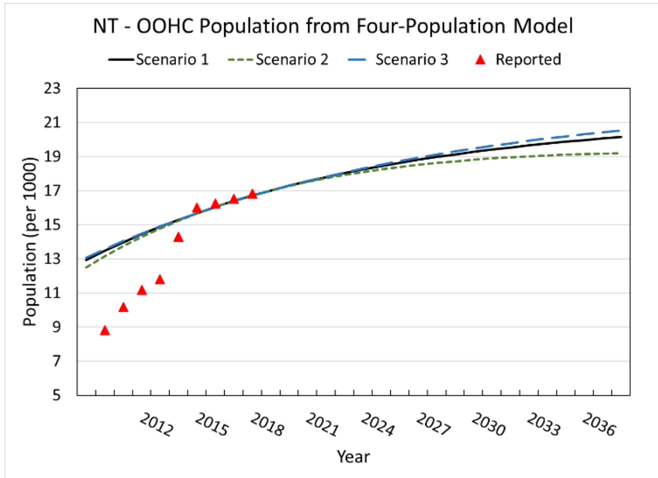


Figure 6.4 Population of Children in OOHC for Various Scenarios (Continue from last page)

Figure 6.4 shows the results of Q_2 for the three scenarios, as well as the reported number of children in OOHC. In the figure, the solid curves represent Q_2 for Scenario 1; the dash curves represent Q_2 for Scenario 2; the long-dash curves represent Q_2 for Scenario 3; diamonds represent the reported numbers of children in OOHC. The vertical axis indicates populations normalised to 1000 while the horizonal axis shows financial years (e.g. 2012 represents the 2011-12 financial year). This figure is based on Figure 5.8, with adjustments in the ranges of the vertical axes for different jurisdictions. A thorough comparison of the reported values and model values (Scenario 1) is provided in the text accompanying Figure 5.8.

It is noted that across all jurisdictions, Q_2 for the three scenarios are almost identical from around 2014 to 2021. Further to the right of the plots, the curves for the three scenarios start to diverge, with Scenario 2 having the lowest values and Scenario 3 having the highest values. Such differences are driven by the fact that in Scenario 3, OOHC admission rates for children who have exited care (λ_{32}) are the largest among the three scenarios.

Table 6.2 shows the values of Q_2 at the end of the time series shown in the plots (year 2038). The three scenarios are presented in the order from smaller values to larger values, with the last row in the table showing divergence as defined by the percentage difference of the lowest value and the highest value among the three scenarios. It is noted that TAS, VIC and SA exhibit the largest divergences, with values of 17.2%, 13.4% and 11.7%, respectively. On the other hand, NSW, QLD and WA exhibit the smallest divergences, with values of 3.4%, 3.5% and 5.4% respectively. This is not surprising as TAS, SA and VIC had the highest OOHC admission rates for children who have exited care (Q_3), while NSW, QLD and WA were among the four jurisdictions with the lowest OOHC admission rates for the same sub-population.

Table 6.2 Values of Q_2 at 2038 for the Three Scenarios

	NSW	VIC	QLD	WA	SA	TAS	ACT	NT
Scenario 2	12.3	10.3	8.8	9.7	15.8	17.0	14.0	19.2
Scenario 1	12.5	11.2	9.0	10.0	17.2	19.3	14.7	20.1
Scenario 3	12.7	11.7	9.1	10.2	17.7	19.9	15.1	20.5
Divergence	3.4%	13.4%	3.5%	5.4%	11.7%	17.2%	8.3%	6.8%

One important point to note is that while the weighted sum of investigation rates for repeat clients (sum of λ_{11} and λ_{33} weighted using Q_1 and Q_3) and that of OOHC admission rates for repeat clients (sum of λ_{12} and λ_{32} weighted using Q_1 and Q_3) in Scenario 2 and Scenario 3 are matched with the investigation rates and OOHC admission rates for repeat clients in Section 5.6 in the years where data were available, their values are not constant in time, as Q_1 and Q_3 vary with time. In Scenario 1, the weighted sum of investigation rates and OOHC admission rates are constant in time as the component values are equal. As a result, for Scenario 2 and Scenario 3, the combined investigation rates and OOHC admission rates for repeat clients are not equal to those in Scenario 1 and the corresponding cases in Section 5.6 throughout the time series. Moving beyond the years where data were available to constrain the model solutions, the combined investigation rates and OOHC admission rates in Scenario 2 become lower than those in Scenario 1; the rates in Scenario 3 become higher than those in Scenario 1. This also contributes to the divergence of the three curves shown in Figure 6.4, albeit it is not the major factor.

6.3.1 Behaviours of Q_3

For Q_3, instead of showing the results in all jurisdictions, the results in WA are used to further illustrate its behaviours for the three scenarios. Figure 6.5 shows Q_3 in WA, as well as Q_2 for the convenience of reference. Once again, the black curves represent Scenario 1; green dashed curves represent Scenario 2; and blue curves represent Scenario 3. The vertical axis indicates populations normalised to 1000 while the horizontal axis shows financial years.

It can be seen that for Q_3, the segment where the three curves are almost identical is much shorter than that for Q_2. In fact, for the years where data were available to constrain the model solutions, namely 2016 to 2018, the three curves have visibly diverged. In contrast to Q_2, Q_3 has the highest values in Scenario 2, following by that in Scenario 1, and then Scenario 3. By the end of the time series shown in the Figure 6.5 (b), the divergence ranges from 22.3% to 91.2%, which is much higher than that for Q_2. Similar to that in Q_2, TAS, SA and VIC exhibit the largest divergences, with values of 91.2%, 59.4% and 41.1%, respectively. For all other jurisdictions, the divergences range from 22.3% in WA to 30.8% in QLD. Once again, it is noted that for each jurisdiction, Scenario 3 has the highest OOHC admission rates for children who have exited care

(λ_{32}). This explains why the curve for Q_3 in Scenario 3 has the smallest values among the three scenarios, as out-flow from Q_3 is the largest in Scenario 3. On the other hand, the relatively larger divergence between the curves in TAS, SA and VIC compared to other jurisdictions are mainly due to these states having the largest OOHC admission rates for children who have exited care (Q_3) among the jurisdictions.

(a)

(b)

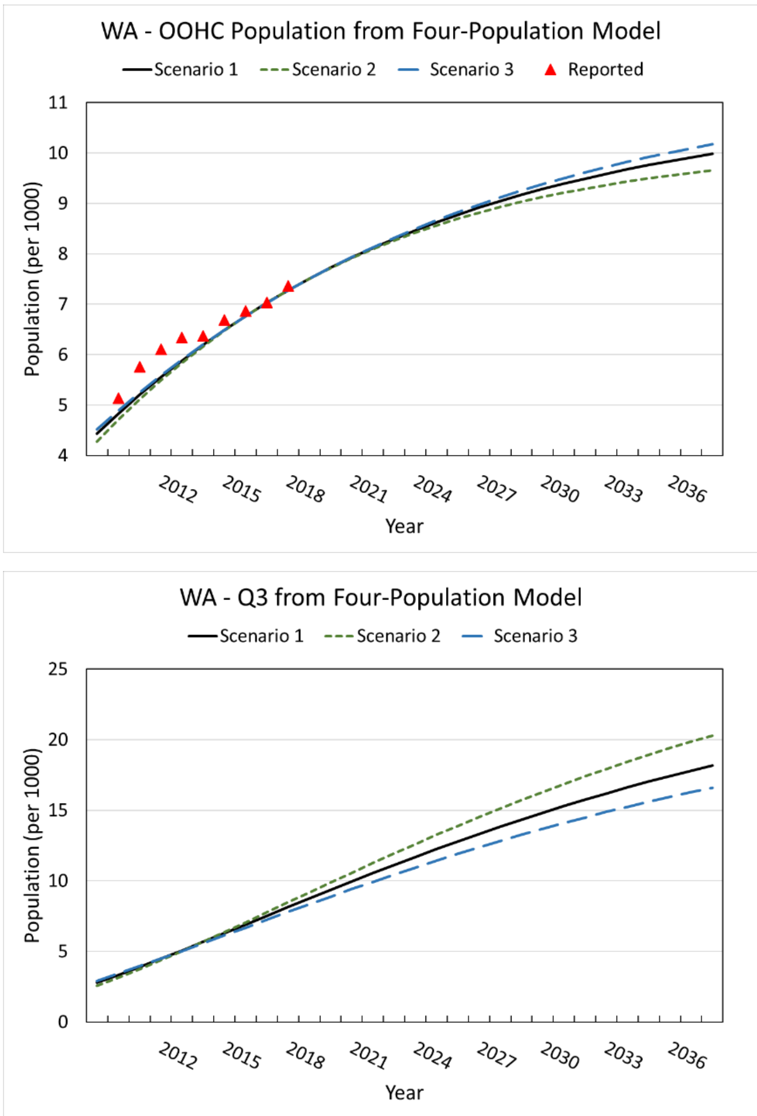


Figure 6.5 OOHC Related Populations for WA

6.3.2 Comparison of Asymptotic Solutions

Having briefly described the time series of Q_3, the asymptotic values of CP subpopulations across the jurisdictions are now discussed. Figure 6.6 shows the asymptotic values of CP sub-populations, namely Q_1, Q_2 and Q_3 across the Australian states and territories, for the three scenarios. The asymptotic values of the population of children not involved in CP (Q_0) is not shown as it is just the complement of the sum of the other three sub-populations.

For ease of understanding, fractional populations are presented with the total population normalised to 1000. In Figure 6.6, the vertical axis populations normalised to 1000 while the horizontal axis indicates the states and territories. For each jurisdiction, there are three stacked columns, representing CP sub-populations of Q_1, Q_2 and Q_3, ordered from top to bottom. For examples, the bottom cell of the first stacked column for NSW represents Q_3 in Scenario 1; the middle cell of the second stacked column for VIC represents Q_2 in Scenario 2; the top cell of the third stacked column for NT represents Q_1 in Scenario 3. The numbers amidst the stacked columns for NT show the values of Q_1 for NT for the three scenarios. It is opted that the values are shown instead of extending the vertical axis to cover the whole range of values since doing so will vastly reduce the resolution of the plots in other jurisdictions.

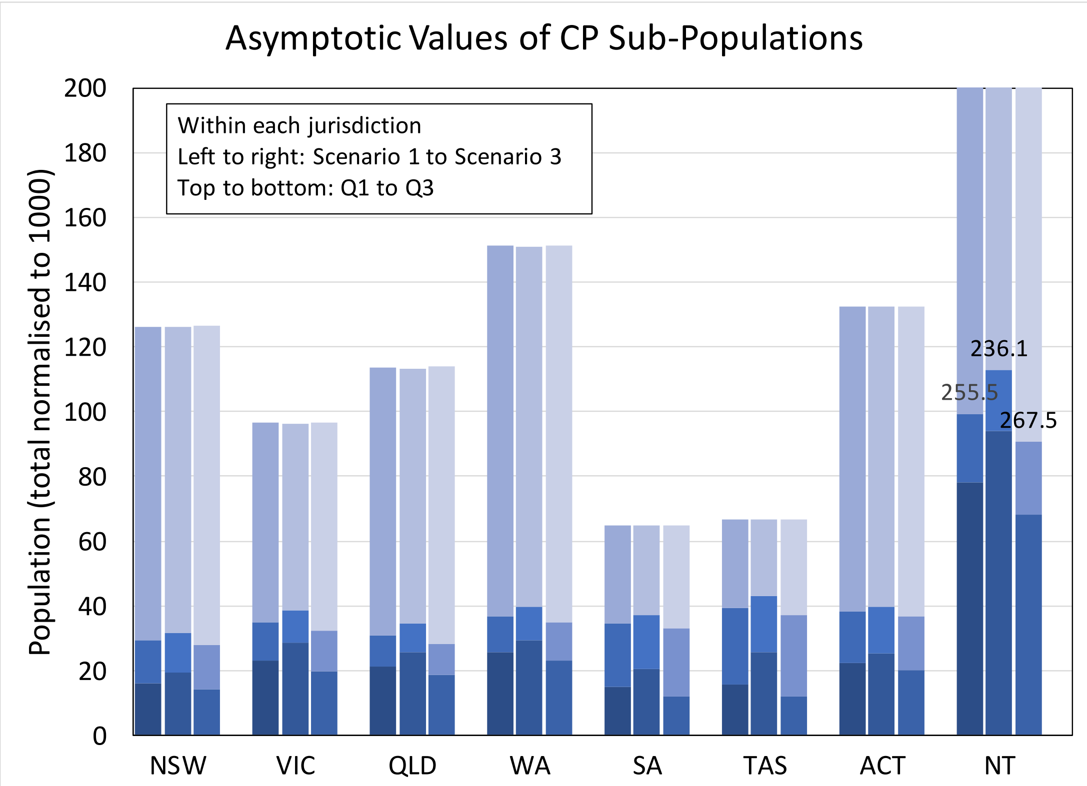


Figure 6.6 Asymptotic Values of CP Sub-populations

It is first noted that the sums of the three CP sub-populations for each jurisdiction, the values of which can be read from the tip of each stacked column, are very much constant across the three scenarios. This is not surprising since Q_0 and thus its complement, which is just the sum total of the three CP sub-populations, are fully determined by the gross population growth rate (α), and the investigation rate and OOHC admission rate for new clients (λ_{01} and λ_{02}). While α is constant in each of the scenarios, there are slight variations in λ_{01} and λ_{02} across the scenarios when reported data are fitted to the model values during the iteration process. Table 6.3 shows the values of λ_{01} and λ_{02}, as well as the sum of all CP sub-populations for the three scenarios, from which it is verified that the variations are indeed small. The last row shows the variations of the total CP sub-populations, defined as the percentage differences between the largest and smallest values among the three scenarios. The largest variation occurs in NT, with a variation of 2.7%. In all other jurisdictions, the variations are smaller than 1%. The small variations in the sum of CP sub-populations provide some assurance that the iterative process used in this study is robust, for when the investigation rate and OOHC admission rate for new clients are allowed to vary in the calculations, constrained only by the data, the variations of the results related to these parameters should be small, which is indeed the case.

Table 6.3 Variations of λ_{01}, λ_{02}, and Sum of CP Sub-populations for the Three Scenarios

	NSW	VIC	QLD	WA	SA	TAS	ACT	NT
λ_{01} (per 1000 per annum)								
Scenario 1	8.98	8.54	7.76	9.26	3.42	3.28	9.98	23.36
Scenario 2	8.95	8.51	7.73	9.23	3.42	3.28	9.97	22.76
Scenario 3	8.99	8.55	7.78	9.28	3.42	3.29	9.99	23.72
λ_{02} (per 1000 per annum)								
Scenario 1	0.35	0.80	0.58	0.52	0.92	0.38	1.21	0.26
Scenario 2	0.36	0.80	0.59	0.52	0.92	0.38	1.21	0.27
Scenario 3	0.36	0.80	0.59	0.52	0.92	0.38	1.21	0.27
$Q_1 + Q_2 + Q_3$ (per 1000)								
Scenario 1	126.4	96.6	113.7	151.2	64.7	66.7	132.4	354.8
Scenario 2	126.1	96.3	113.3	150.8	64.7	66.5	132.3	348.9
Scenario 3	126.5	96.8	114.0	151.5	64.8	66.8	132.5	358.2
% Variation	0.4%	0.5%	0.6%	0.5%	0.2%	0.3%	0.1%	2.7%

Returning to Figure 6.6, the following observations are noted:

1. In Scenario 2, with λ_{32} set to the value of λ_{02}, which is much smaller than λ_{12}, the asymptotic values of Q_3 are larger than the corresponding values in Scenario 1

(in which λ_{32} is equal to λ_{12}). This is in accordance with Condition 19 in Section 6.1.

2. In Scenario 3, with λ_{32} set to twice the value of λ_{12}, which is much larger than λ_{02}, the asymptotic values of Q_3 are smaller than the corresponding values in Scenario 1 (in which λ_{32} is equal to λ_{12}). This is again in accordance with Condition 19 in Section 6.1.
3. In Scenario 2, the asymptotic values of Q_2 are smaller than the corresponding values in Scenario 1, having started from the same values in both scenarios. This is due to the value of λ_{32} being smaller than that in Scenario 1. This is in accordance with Condition 13 in Section 6.1.
4. In Scenario 3, the asymptotic values of Q_2 are larger than the corresponding values in Scenario 1, having started from the same values in both scenarios. This is due to the value of λ_{32} being larger than that in Scenario 1. This is in accordance with Condition 13 in Section 6.1.
5. In Scenario 2, the asymptotic values of Q_1 are smaller than the corresponding values in Scenario 1, having started from the same values in both scenarios. This is due to the value of λ_{12} being larger than that in Scenario 1. It should be noted that with the value of λ_{32} being much smaller than λ_{12} in Scenario 2, λ_{12} has to be larger than that in Scenario 1 so that the number of OOHC admission for repeat clients can still match the reported values. This is in accordance with Condition 7 in Section 6.1.
6. In Scenario 3, the asymptotic values of Q_1 are larger than the corresponding values in Scenario 1, having started from the same values in both scenarios. This is due to the value of λ_{12} being smaller than that in Scenario 1. It should be noted that with the value of λ_{32} being twice as large as λ_{12} in Scenario 2, λ_{12} has to be smaller than that in Scenario 1 so that the number of OOHC admission can still match the reported values. This is again in accordance with Condition 7 in Section 6.1.

In addition to the above, it is observed that in all scenarios across all jurisdictions, the asymptotic values of Q_3 are larger than that of Q_2, with the exception of Scenario 1 and Scenario 3 in SA and TAS. Table 6.4 shows Q_2 and Q_3 for SA and TAS across the three scenarios. For ease of understanding, fractional populations shown in the table are normalised, with 1000 being the total population. As noted, Q_3 is larger than Q_2 only in

Scenario 2. Recall that for Scenario 1 and Scenario 3, the OOHC admission rates for children who have exited care (λ_{32}), i.e. OOHC re-admission rates, are equal to or higher than the OOHC admission rates for children involved in CP but have never been in OOHC (λ_{12}), and much higher than OOHC admission rates for new clients (λ_{02}). In these scenarios, the issue of "churn" can be considered particularly serious. With λ_{32} being very high, many children who have exited care are being repeatedly returned to OOHC. It should be noted that without actual data on OOHC readmission, the scenarios shown here are merely for the purpose of demonstrating the potential effects of "churn" and should not be taken as what is actually happening.

Table 6.4 Q_2 and Q_3 for SA and TAS (with total population normalised to 1000)

	SA		TAS	
	Q_2	Q_3	Q_2	Q_3
Scenario 1	19.7	14.8	23.5	15.7
Scenario 2	16.5	20.6	17.6	25.5
Scenario 3	20.9	12.0	25.1	12.0

6.4 A Hypothetical Future with Substantially Reduced Investigation and OOHC Admission Rates

In this section, a scenario where the investigation rates and OOHC admission rates for repeat clients are reduced substantially is explored. For this purpose, the results of Scenario 1 in Section 6.3 in 2018 are taken and used as the initial conditions to project the sub-populations into the future. Again, three scenarios are examined. These are labelled as Scenario A, Scenario B, and Scenario C to prevent confusion with the scenarios in Section 6.3. In Scenario A, the investigation rates for new clients are reduced by half compared to the rates obtained in Section 6.3. In Scenario B, the OOHC admission rates for repeat clients, λ_{12} and λ_{32}, are reduced by half. In Scenario C, the investigation rates for new clients and OOHC admission for repeat clients are all reduced by half.

6.4.1 Scenario A

Figure 6.7 shows the time varying plots of the sum of Q_1 and Q_3 (solid curve), Q_1 (dash curve), Q_2 (two-dash curve) and Q_3 (long-dash curve) for Scenario A. The x-axis indicates financial years and the y-axes indicate populations normalised to 1000. Q_1 and

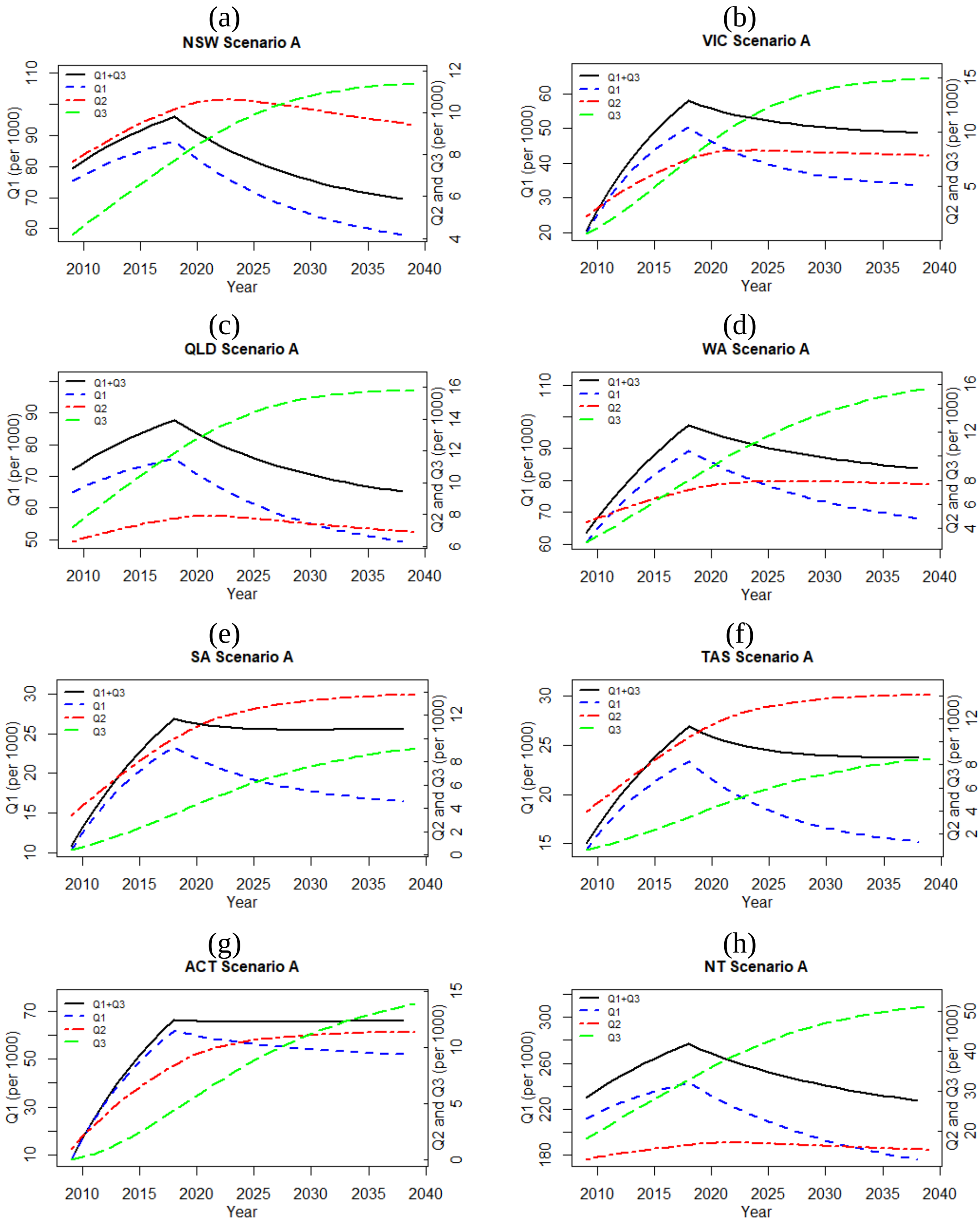


Figure 6.7 CP Sub-Populations for Scenario A

the sum of Q_1 and Q_3 are plotted on the primary vertical axis, to the left; Q_2 and Q_3 are plotted on the secondary vertical axis, to the right. In each of the plots, the kinks in the solid and dash curves are located in year 2018, the position where two segments of time series calculated using the different sets of parameters join. With investigation rates for new clients reduced by half, the population of children involved in CP but never in OOHC (Q_1) starts to drop after 2018, tending towards new asymptotic values which are much smaller.

The curves for Q_2 and Q_3 do not exhibit a kink since the time derivatives of these two sub-populations do not directly depend on the investigation rate for new clients, unlike the time derivative of Q_2. In NSW, VIC, QLD, WA and NT, the values of Q_2 continue to rise after 2018 before reaching a peak and then start to decline towards the new asymptotic values, which are much lower than the original values. In SA, TAS and ACT, the values of Q_2 continue to rise monotonically towards the new asymptotic values, which are much lower than the original values. In all jurisdictions, the values of Q_3 continue to rise monotonically towards the new asymptotic values which are also much lower than the original values. The asymptotic values in the three scenarios will be compared to the original values after describing the time series.

6.4.2 Scenario B

Figure 6.8 shows the time varying plots of the sum of Q_1 and Q_3 (solid curve), Q_1 (dash curve), Q_2 (two-dash curve) and Q_3 (long-dash curve) for Scenario B. The x-axis indicates financial years and the y-axes indicate populations normalised to 1000. Q_1 and the sum of Q_1 and Q_3 are plotted on the primary vertical axis, to the left; Q_2 and Q_3 are plotted on the secondary vertical axis, to the right. It is recalled that in Scenario B, the OOHC admission rates for repeat clients, λ_{12} and λ_{32}, are reduced by half after year 2018. Unlike Scenario A where only the time derivative of Q_1 is affected by the abrupt change in the investigation for new clients, the time derivatives of all curves in Scenario B are affected by the changes in OOHC admission rates for repeat clients. As a result, all curves exhibit a kink in year 2018, where the two segments of time series calculated with different sets of parameters join. The kink is the most obvious in Q_2 (the two-dash curve), where the term containing λ_{12} (OOHC admission for repeat clients) dominates the various terms effecting changes in Q_2. The kink is the least obvious in Q_3 (the long-dash curve), as it is dominated by the term associated with OOHC exit.

With OOHC admission rates for repeat clients reduced by half, OOHC populations in all jurisdictions start to drop after 2018, except in SA. Changes to OOHC populations induced by the reduction in OOHC admission rates are much more complex compared to those in Scenario A. In SA, as shown in Figure 6.8 (e), the reduction in OOHC admission rate is not large enough to cause the OOHC population to drop but merely to reduce the rate of growth. With the changes in OOHC admission rates, the OOHC population in SA continues to grow, with smaller growth rates, towards a new asymptotic value which is much smaller than the original asymptotic value. In all other jurisdictions, OOHC populations start to drop after 2018, with some more visibly and for a longer period of time than others. Eventually, they all reach a minimum and start to rise again towards the new asymptotic values, which are much smaller than the original values.

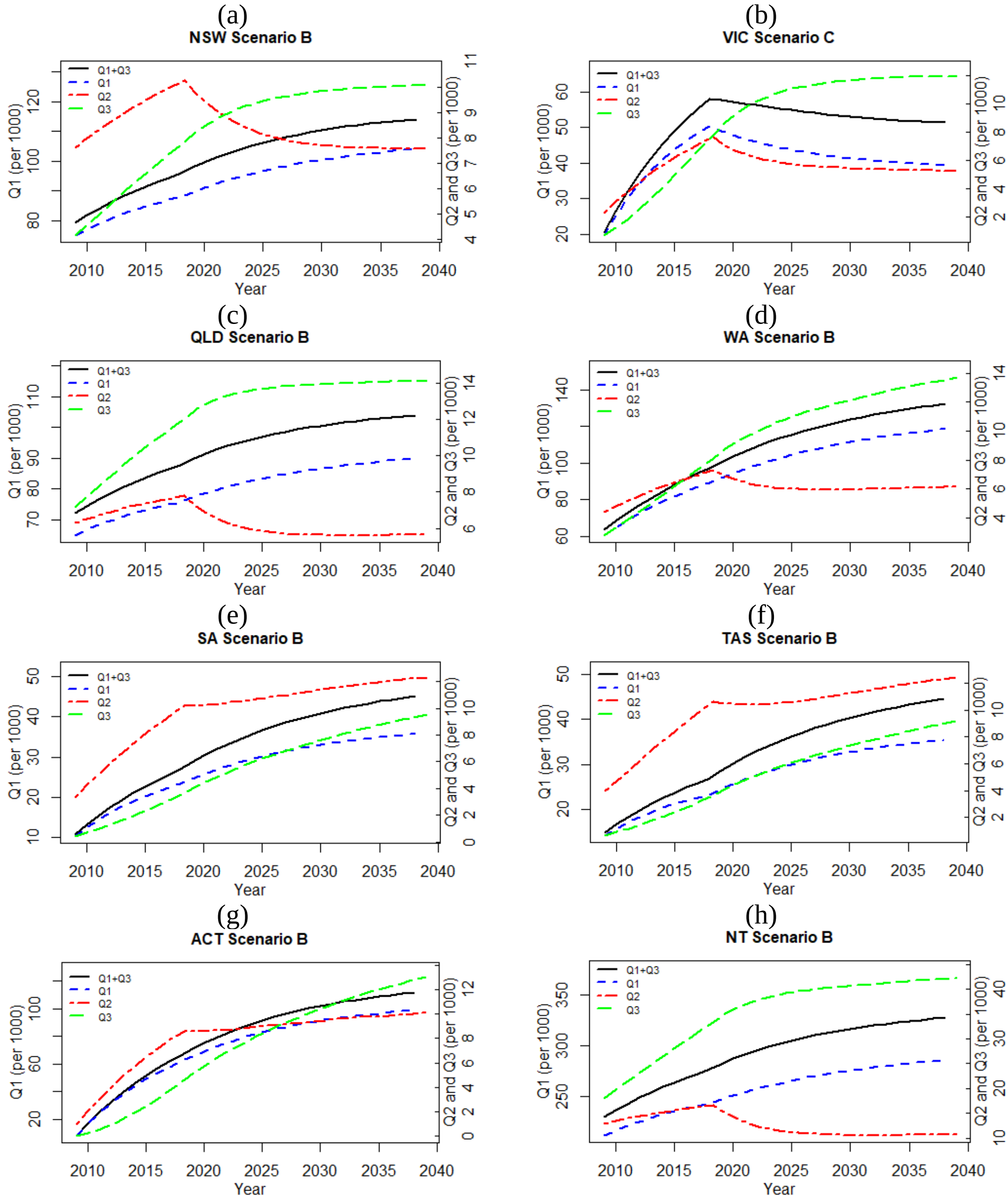


Figure 6.8 CP Sub-Populations for Scenario B

On the other hand, Q_1, the populations of children involved in CP but never admitted to OOHC continue to rise after year 2018, approaching the new asymptotic values which are larger than the original values.

In all jurisdictions, the reduction in OOHC admission rates for repeat clients lead to Q_3 growing at much smaller growth rates compared to the originals. Nevertheless, the values of Q_3 continue to rise monotonically towards the new asymptotic values, which are much lower than the original values.

Comparing Q_2 and Q_3, it is simply noted that they diverge much more quickly compared to Scenario A and the original scenario, in most jurisdictions. It is also noted that the values of Q_2 remain larger than Q_3, in SA and TAS, albeit the differences appear to have been reduced. This will be explored more thoroughly when the asymptotic values are examined.

6.4.3 Scenario C

Figure 6.9 shows the time varying plots of the sum of Q_1 and Q_3 (solid curve), Q_1 (dash curve), Q_2 (two-dash curve) and Q_3 (long-dash curve) for Scenario B. The x-axis indicates financial years and the y-axes indicate populations normalised to 1000. Q_1 and the sum of Q_1 and Q_3 are plotted on the primary vertical axis, to the left; Q_2 and Q_3 are plotted on the secondary vertical axis, to the right. It is recalled that in Scenario C, both the investigation rates for new clients and the OOHC admission rates for repeat clients are reduced by half compared to those in the original scenario. In Scenario C, the time derivatives of all curves are affected by the large reductions of the rates, with changes in Q_1 and Q_2 being the most abrupt, exhibiting very sharp kinks. As in Scenario B, the kink is the least obvious in Q_3 (the long-dash curve), as it is dominated by the term associated with OOHC exit.

In Scenario C, due to the double effects of reduction in the population of repeat clients who have never been admitted to OOHC and reduction in their rate of being admitted to OOHC, the drop of value for Q_2 is the largest among the three scenarios in all jurisdictions, except in SA, TAS and ACT, for which the values in 2018 are already close to the new asymptotic values.

The value of Q_2 decreases monotonically in all jurisdictions except in SA and ACT. In SA, as shown in Figure 6.9 (e), Q_2 actually increases monotonically with a low growth rate towards the new asymptotic value. In ACT, Q_2 decreases after 2018 and then increases again towards the new asymptotic value.

For Q_3, its value in all jurisdictions continues to increase after 2018. In SA, TAS and ACT, the increase is monotonic, tending towards the new asymptotic values which are lower than the original values. In the other jurisdictions, its values reach a peak and then start to decrease towards the new asymptotic values which again are lower than the original values.

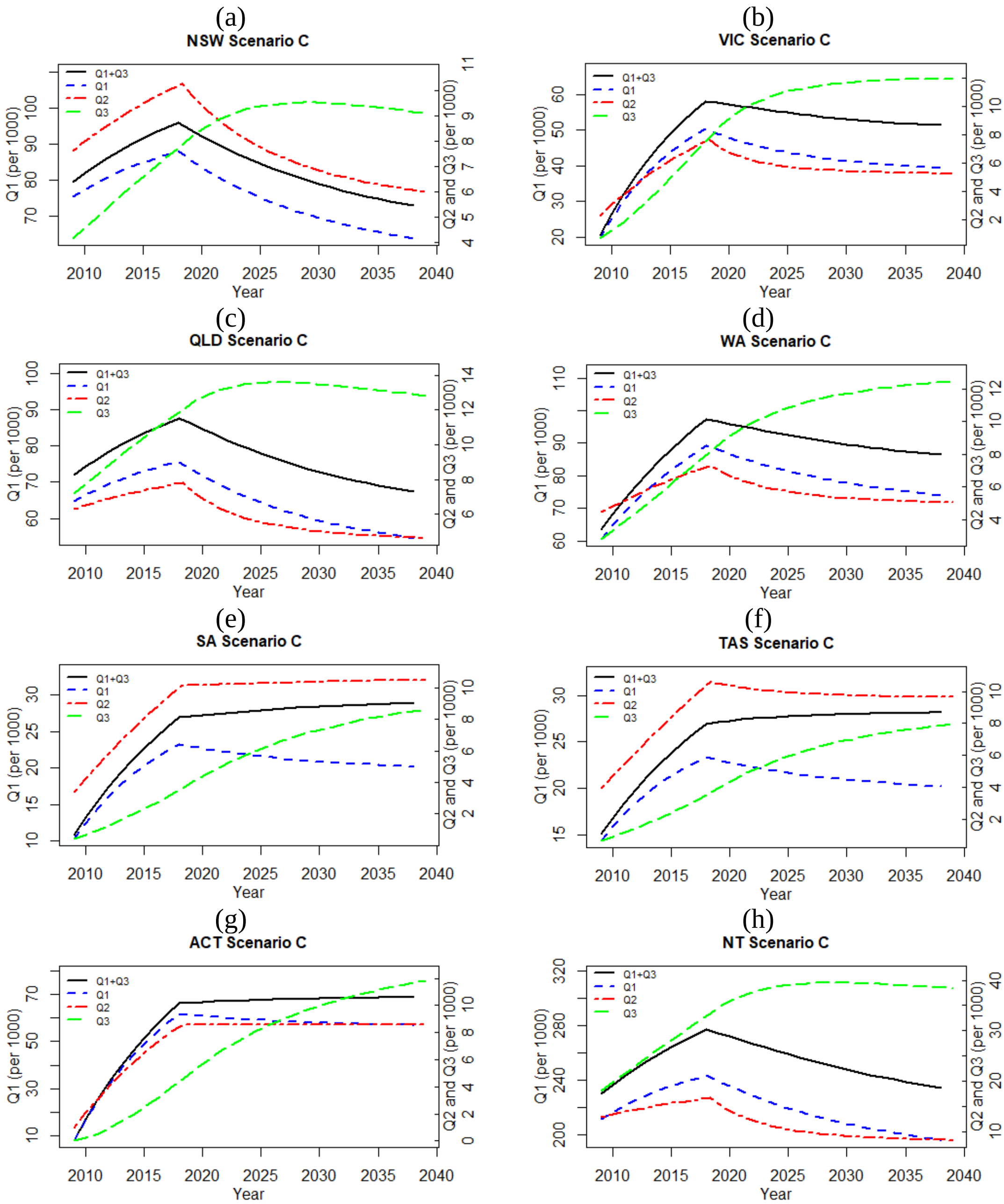


Figure 6.9 CP Sub-Populations for Scenario C

6.4.4 Comparisons of Asymptotic Values

A comparison of asymptotic values of the sub-populations will now be made. Table 6.5 shows asymptotic values (per 1000) of Q_2, Q_3 and the total CP population (sum of Q_1, Q_2 and Q_3) for the three scenarios and Scenario 1 in Section 6.3. It is also recalled that the latter is the extension of the results shown in Section 5.6, with repeat clients split into two sub-populations.

It is first noticed that for the total CP population, the asymptotic values in Scenario B are exactly the same as those in the original scenario (Scenario 1 in Section 6.3); and the asymptotic values in Scenario A are exactly the same as those in Scenario C. This is not surprising as the total CP population is just the complement of the population of children not involved in CP, as $Q_1 + Q_2 + Q_3 = 1 - Q_0$. From the analytical solution of Q_0 in Section 6.1, it can be seen that its asymptotic value depends only on the gross population growth rate (α), and the investigation rate and OOHC admission rate for new clients (λ_{01} and λ_{02}). For Scenario B, these rates are the same as those in the original scenario. Likewise, these rates are the same for Scenario A and Scenario C. Overall, halving the investigation rates for new clients leads to the total CP population being reduced by 37.8% (SA) to 44.7% (NSW). On the other hand, halving the OOHC admission rates for repeat clients has no impact on the total CP population. This is because the repeat clients are already part of the CP population.

Examining the changes in Q_2, it is observed that halving the investigation rates for new clients leads to Q_2 being reduced by 27.2% (ACT) to 39.6% (TAS). In contrast, halving the OOHC admission rates for repeat clients leads to Q_2 being reduced by 29.9% (SA) to 47.2% (NT). Halving both investigation rates for new clients and OOHC admission rates for repeat clients leads to Q_2 being reduced by 45.6% (ACT) to 65.9% (NT). In all jurisdictions except TAS, halving the OOHC admission rates for repeat clients appear to have a larger impact on the reduction of OOHC population (Q_2) compared to halving the investigation rate for new clients. However, how Q_2 reacts to the changes in these CP process rates depends on a combination of factors, including the relative sizes of gross population growth rate (α) and the CP process rates (λs).

Examining the changes in Q_3, it is observed that halving the investigation rates for new clients leads to Q_3 being reduced by 27.2% (ACT) to 39.5% (TAS). In contrast, halving the OOHC admission rates for repeat clients leads to Q_2 being reduced by 11.5% (SA) to 38.6% (NT). The impact on SA and TAS is much smaller than that in other jurisdictions due to these two jurisdictions having the smallest investigation rates for new clients in the original scenario. The investigation rates for new clients in SA and TAS in the original scenario were 3.4 and 3.1 per 1000 per annum, respectively. In other jurisdictions, the values were between 7.8 (QLD) and 23.9 (NT) per 1000 per annum. The small values mean that their contribution to the asymptotic values of Q_3 are proportionally small. As a result, when their magnitudes are reduced, the impact is smaller compared to that on other jurisdictions. Similar reasoning can be applied to explain the large reduction in the asymptotic value of Q_3 in NT, which has the highest value of investigation rate for new clients, and thus the largest reduction in the asymptotic value of Q_3 when the investigation rate is reduced.

When both investigation rates for new clients and OOHC admission rates for repeat clients are halved, Q_3 is reduced by 31.8% (SA) to 60.3% (NT). It should be noted that in all jurisdictions except NT, halving the investigation rates for new clients appears to have a larger impact on the reduction of the population of children who have exited OOHC (Q_3) compared to halving the OOHC admission rates for new clients. However, as in Q_2, how Q_3 reacts to the changes in these CP process rates depends on a combination of factors, including the relative sizes of gross population growth rate (α) and the CP process rates (λs).

Table 6.5 Asymptotic Values of CP Sub-populations across the Scenarios

	NSW	VIC	QLD	WA	SA	TAS	ACT	NT
Q_2 (per 1000)								
Original	13.2	11.8	9.4	11	19.7	23.5	15.8	21.4
Scenario A	8.2	7.6	6.2	7.2	14.1	14.2	11.5	13.5
Scenario B	7.8	7.3	5.8	6.7	13.8	15.1	10.7	11.3
Scenario C	5.3	5.1	4.3	4.8	10.7	9.6	8.6	7.3
Q_3 (per 1000)								
Original	16.2	22.9	21.3	25.8	14.8	15.7	22.4	78
Scenario A	10	14.9	14.2	17	10.6	9.5	16.3	49.2
Scenario B	10.7	16.6	14.9	17.4	13	13.9	16.9	47.9
Scenario C	7.2	11.7	10.9	12.5	10.1	8.8	13.6	31
$Q_1 + Q_2 + Q_3$ (per 1000)								
Original	126.4	96.6	113.7	151.2	64.7	66.7	132.4	354.8
Scenario A	69.8	54.9	64.3	85.8	40.2	37.9	78.0	217.6
Scenario B	126.4	96.6	113.7	151.2	64.7	66.7	132.4	354.8
Scenario C	69.8	54.9	64.3	85.8	40.2	37.9	78.0	217.6

6.5 Conclusion

In this chapter, the analyses in Chapter 5 have been extended using the solutions of the four-population model. The four-population model is an extension of the three-population model by further distinguishing children who have been to OOHC only once from those who have been to OOHC more than once. This facilitates the study of the issue of "churn" in OOHC.

Since data for repeated admission to OOHC were not available, this chapter focuses on testing hypothetical scenarios with different CP process rates across the states and territories. Three scenarios were tested, using the rates calculated in Chapter 5 as the basis for comparison. In the first scenario, the investigation rates for new clients were reduced by half. In the second scenario, the OOHC admission rates for repeat clients were reduced by half. In the third scenario, both the investigation rates for new clients and OOHC admission for repeat clients were reduced by half. The results show that in all jurisdictions except for TAS, halving the OOHC admission rates for repeat clients appears to have a larger impact on the reduction of OOHC population, as compared to halving the investigation rates for new clients.

In the following chapter, how the insights gained from these analyses may illuminate the issue of "churn" will be discussed. The analyses presented in Chapter 5 and Chapter 6 will also be discussed in the context of CP policies and practices.

Chapter 7: Discussions and Conclusions

In the previous chapters, the results of this study based on the objectives and research questions stated in Chapter 1 have been presented. Specifically, these results are structured using the following themes:

- In Chapter 4, thorough descriptive analyses of publicly available CP data across the Australian states and territories are provided, focusing on issues which have not been explored by other researchers. These include further examining the breakdown of children into new and repeat clients in the investigation stage and OOHC admission stage of the CP process; and exploring structural relationship between various CP stages, for example, correlations between investigation ratios and substantiation ratios.
- In Chapter 5, the analytical solutions of the linear three-population model are explored and applied to model CP population structure and its time varying trends across the Australian states and territories, using the data analysed in Chapter 4 to constrain these model solutions. A crude but useful estimation of lifetime prevalence of CP involvement based on the model solutions is also provided.
- In Chapter 6, the analysis and modelling in Chapter 5 are extended using the linear four-population model. This extension explores the sub-population of children who have exited OOHC and elucidates the issue of "churn" – repeated OOHC admission and exit.

This chapter contains an in-depth discussion on data used in this study, the model formulation, and the results presented in Chapter 5 and Chapter 6.

7.1 Publicly Available CP Data in Australia

An important aspect of this study is the analysis of CP data across Australian states and territories and using them to constrain the population dynamics model. Optimally, the models discussed in the previous chapters could be applied with high frequency data from respective state authorities. However, despite best efforts to procure such datasets, permission to use them could not be obtained in time to complete this dissertation study within the required time frame. The analyses were therefore restricted to publicly available annual aggregate data. While using high frequency data will enable one to conduct a more thorough analysis, publicly available aggregate data are still very useful. In fact, much insights can be gained from an in-depth analysis of such data across Australian states and territories. In addition, they are adequate to drive the model analysis, albeit only at the yearly time level.

As mentioned in Section 3.3, the main source of data used in this study is the annual series published by Australian Institute of Health and Welfare (AIHW) titled *Child Protection Australia* (AIHW, 2019a). This is the staple source for CP data in Australia and it has been used in secondary analyses and reports. For example, these data form the basis of analyses in the annual *Family Matters Report: Measuring Trends to Turn the Tide on the Over-representation of Aboriginal and Torres Strait Islander Children in Out-of-home Care in Australia*, published by the Secretariat of National Aboriginal and Islander Child Care (e.g. Lewis et al., 2019). The data have also been used to drive discussions on CP issues in Australia (e.g. Higgins, 2011).

While there are concerns in using statutory CP data for child maltreatment research in Australia, mainly due to issues regarding reporting accuracy on the incidence of maltreatment (Bromfield & Higgins, 2004), statutory CP data suit our purpose since the focus of our models is population of children involved in statutory CP, and not actual child maltreatment.

Descriptive data analyses presented in Chapter 4 describe the trends of CP incidences (e.g. CP notifications, investigations, substantiations, OOHC entries and exits, and OOHC population) over the last decade. The focus is on two issues not often studied in existing literature: 1. Examining potential coupling between various stages of the CP process; and 2. Analysing new and repeat client data to establish bounds of breakdown into new and repeat clients beyond the numbers provided by AIHW. These analyses not only serve to inform the modelling efforts in this study, but they also reveal aspects of CP process, which was not the focus of previous studies.

7.1.1 Coupling of CP process

The analysis of the potential coupling reveals that there are statistically significant correlations between CP related rates or ratios in some jurisdictions. For example, investigation ratios and substantiation ratios are negatively correlated in NSW, with the Pearson correlation coefficient as high as negative 0.95. Investigation ratios and OOHC entry ratios are negatively correlated in SA, with a correlation coefficient of negative

0.94. On the other hand, substantiation ratios and OOHC entry ratios are positively correlated in NT, with a correlation coefficient of 0.74.

Such correlations have to be interpreted with caution. It is recalled that investigation ratio is defined as the ratio of the number of investigations and the number of notifications; the substantiation ratio is defined as the ratio of the number of substantiations and the number of investigations; the entry ratio is defined as the ratio of the number of OOHC admission and the number of substantiations. A negative correlation between investigation ratios and substantiation ratios may arise if the numbers of investigations fluctuate but the numbers of notifications and substantiations stay constant, as the ratios will be inversely related to each other. However, a check of these numbers in NSW revealed that the situation was more complex. Moreover, the fact that only a limited number of jurisdictions exhibited such couplings but not others indicate that the situations were not so simple.

These couplings point towards potential structural constraints in the CP process. For example, a previous study in the US by Van Voorhis & Gilbert (1998) revealed a similar pattern between investigation ratios and substantiation ratios. They postulated that the negative correlation between these two process rates might indicate either a decreased capacity of the CP investigative agencies to fully investigate cases or the increased inclusion of cases which did not meet the criteria of substantiation. The implication of either causes are significant and such correlations point towards the need of a thorough investigation into the phenomena with a more complete administrative dataset for the relevant jurisdictions. Moreover, with only a limited number of jurisdictions exhibiting such couplings, there may be systemic differences either in resource allocation or in practice conventions across the jurisdictions which warrant an in-depth investigation. However, due to the lack of access to high frequency administrative datasets, the issues could not be further analysed in this study.

7.1.2 New and Repeat Clients

As described in Section 3.3, breakdowns into new and repeat clients for the number of children involved in CP investigation and in OOHC, aggregating over a number of jurisdictions, were provided for the first time in 2012-13[9]. In 2015-16, AIHW started to provide new and repeat client data for each state or territory. Unfortunately, in 2016-17, data for ACT were withheld due to quality reasons, and in 2017-18, data for NSW and ACT were not provided.

Much of the ongoing discourse on CP in Australia has focused on the issue of OOHC population (e.g. Australian Law Reform Commission, 2017; Community Affairs Committee Secretariat, 2015; Lewis et al., 2019; Tune, 2016). The implications of investigation rates have seldom come to the fore of discussion. The focus on OOHC is understandable and necessary as the OOHC system is perceived to be in need of serious reforms. However, one can argue that it is equally important for policy makers and CP practitioners to fully engage in an in-depth discussion on the larger pattern and problems of CP involvement, not just on OOHC. It should be noted that OOHC is just the most intensive form and often the last step of CP interventions. Without fully understanding previous stages in the CP process, it is not likely that we could fully tackle problems in OOHC. This is because unchanged practices in investigation routine and services provided to families involved in CP investigation will likely ensure that there will be a large number of children deemed necessary to enter care.

In this regard, this study shows that in order to fully understand the extent of CP involvement and to be able to use population dynamics to model CP populations, new and repeat client data are essential. Consistent data for the breakdowns into new and repeat clients will show us whether there is an increasingly larger proportion of children who are involved in CP, or CP involvement is mainly driven by repeated involvement of a segment of the population. The latter is normally referred to as the phenomenon of "churn". Whether the former or the latter is the dominating driver will call for policy and practice responses which can be very different. For example, if a particular segment of the population is repeatedly involved in CP, we have to question the effectiveness of interventions and availability of services provided to help these families, and whether more focus has to be placed on larger socio-economic patterns that may drive such repeated involvement (e.g. Bromfield et al., 2014).

[9] Financial year, e.g. the 2012-13 financial year spanned 1 July 2012 to 30 June 2013.

The analysis of aggregate data for SA, TAS and NT from 2012-13 to 2017-18 (Table 4.6 and Figure 5.10) indicates that there has been a large change in the proportion of new and repeat clients who are involved in CP investigation and OOHC admission. The data indicate that this major change occurred between 2013-14 and 2014-15. However, the data available are very limited in time span and jurisdictional details, making it impossible to conduct an in-depth analysis of the issue. It would be emphasised that the importance of making more detailed new and repeat client data available to researchers cannot be under-estimated.

Another important issue is that no new and repeat client information has been provided in terms of Indigenous status. The overrepresentation of Indigenous children in OOHC population (Lewis et al., 2019) is one of the most serious problems of social justice in Australia. When narrating the issue of overrepresentation in CP, most studies focus on rate ratios between Indigenous and Non-Indigenous populations in OOHC and in other stages of CP process (e.g. children in notification, investigation, substantiation) (Lewis et al., 2019). However, this study has shown that in some jurisdictions, the overall population of children involved in CP but not in OOHC could be a much larger issue, which could be as critical as the growing OOHC population, and perhaps it holds the key to understanding more deeply about the growth in OOHC population. An attempt had been made to apply the models to study the disparity of CP involvement in terms of Indigenous status. Unfortunately, due to the lack of information on new and repeat clients for Indigenous status, the analysis could not be carried out very far.

Last but not least, while breakdowns into new and repeat clients have been provided for children receiving CPS, children in investigation, children on care and protection order, and children in OOHC, there are no new and repeat client breakdowns for children in substantiation. As a result, one cannot establish substantiation rates for new and repeat clients, which is an important indicator for analysing false negatives of maltreatment referrals.

7.1.3 Other Data Limitations

In addition to the limited coverage of new and repeat client data, there are several other data issues, many are due to changes in counting rules.

Changes in Age Aggregation

The first among these is that before 2009-10, CP investigation related data were reported for children between 0 to 16 years of age. From 2009-10 onward, the data were provided for children between 0 to 17 years of age. As a result, although data are available for a much longer period of time, only those from 2009-10 are used in this study.

Children on Third-party Parental Responsibility Orders

Prior to 2014-15, children on third-party parental responsibility orders (TPRO) were included in the count of children in OOHC. In 2014-15, NSW started to exclude children on such orders from OOHC counts; in 2015-16 and 2017-18, WA and VIC started to do the same, respectively. With AIHW working on improving the comparability of OOHC reporting, similar changes may happen in other jurisdictions (AIHW, 2019a). Such changes result in inconsistency of data before and after the year of change. So far, AIHW has not provided a reconstruction of data using the new rules. One has to be very careful working on such data as changes in counting will inevitably lead to artificial changes in the number of children in the relevant category. For example, the exclusion of children on TPRO in 2017-18 in VIC led to a large drop in OOHC population.

Limited Data on Age Structure

Age structure is available for children in substantiation, children on care and protection orders, children in OOHC, children admitted to and discharged from care and protection orders, children admitted to and discharged from OOHC, and children receiving CPS (all services). However, age structure is not available for children in investigations. For this model, in order to consider age structure, having such data for children in investigations is crucial. It can be argued that independent of the modelling need, having information on age structure for children in investigations is important in providing us with a good understanding of the extent of involvement in CP. This is especially important in view of that a large proportion of annual CP related resources are spent on CP investigation related services. This issue will be addressed in a later section. Lastly, it is noted that there is no age structure for data on new and repeat clients.

Data Frequency

Currently, only annual data (either in the form of annual aggregates or cross-sectional snapshots) are available. For example, data on the number of children in OOHC are provided both for June 30, at the end of the financial year, and for the annual aggregate of the unique number of children. Inevitably, with such a low data frequency, characteristics of shorter time scales could not be captured. For example, about 15% of all OOHC episodes had durations shorter than one year. As a result, annual snapshots are bound to miss a large portion of these short episodes. This is reflected in the snapshot values of OOHC population being lower than the annual aggregate values. For a more thorough understanding of OOHC stays of varying lengths, higher frequency data are necessary.

In view of the limitations described above, this study has been conducted by limiting the years of analysis and by re-constructing the time series so that they are consistent, or not including the missing variables in the model (e.g. age and ethnicity). It should also be kept in mind that the overriding objective of this study is on the overall structure of CP populations, in accordance with different types of involvement in CP. Age structure and ethnicity have not been the foci from the outset. However, for future works which aim to provide more comprehensive analyses of CP populations by taking into account all of the variables heretofore unexplored, it is of utmost importance to overcome these limitations, either with AIHW providing better data coverage, or with access to high frequency data sets, ideally unit records data.

7.2 Model Formulation

The review of existing literature shows that no attempt has previously been made to study CP populations based on the mathematical formulation of population dynamic models. This study is the first application of such an approach. While it has been shown in Chapter 5 and Chapter 6 that the models are able to produce results that are in accordance with the observed CP process data, it is important to recognize that there is currently a lack of relevant data for a more complex formulation that takes into account a comprehensive list of factors which may be used to categorize children involved in CP. As a result, the study is only able to examine CP sub-populations in the broadest sense, with only up to four sub-populations used in the model formulation.

On the other hand, since the models are formulated based on the general structure of CP populations – children who have never been involved in CP, children who have been involved in a CP investigation but have never been admitted into OOHC, children in OOHC, and children who have exited OOHC (in the case of the four-population model) , they are generic models that can be applied to study CP population in any jurisdiction with similar CP practices, not just those in Australia.

7.2.1 Expanding Model Sub-populations

While the particular population dynamics models used in this study consist of only up to four sub-populations, as mentioned above, it is easy to expand them by adding mutually exclusive categories if necessary, and if relevant data are available. For example, by considering gender, the sub-populations will be doubled, since there is essentially no exchange of population between the genders. The model equations will stay the same, but with two separate sets of sub-populations exchanging members among themselves.

One can also easily take ethnicity into account as ethnic labels can be considered mutually exclusive as far as CP systems are concerned, e.g. Indigenous population and Non-Indigenous population in Australia. With each additional ethnic group considered, there will be an additional set of sub-populations exchanging members among themselves through CP involvement. In fact, an attempt had been made to apply the models to study differences of CP involvement between the Indigenous population and the Non-Indigenous population across the Australian states and territories. However, due to the absence of breakdown into new and repeat clients for each of these ethnic grouping, no meaningful results can be obtained.

7.2.2 Model Assumptions and Limitations

Age Structure

While adding mutually exclusive group is a simple task, as mentioned above, there are other model assumptions and limitations which are more complicated. The first is demographic or age structure. Age structure in population models is normally realized using discrete age groups, often with an age range of one year for each discrete group (e.g. Brauer & Castillo-Chavez, 2012; Keeling & Rohani, 2008). The age groups are then linked, with group members moving from lower age groups to the next higher age groups. Figure 7.1 shows a schematic of a model with age structure. In this diagram, items in black represent the sub-populations and CP process for children in the 17-year old age bracket, while those in red and blue represent the sub-populations and process for the 16-year-old and 15-year-old age brackets, respectively. The dashed vertical arrows in red and blue, on the right-hand side of the diagram, indicates the uni-directional flow from a lower age bracket to a higher age bracket for P_3, as children grow in age. In fact, these out-flows from lower age brackets are none other than the out-flows due to "aging-out" from these age brackets. Flows between the age brackets are not shown as doing so will make the diagram incomprehensible. Instead of using a single year age brackets, it is also possible to use a broader age range, depending on the objectives of the study.

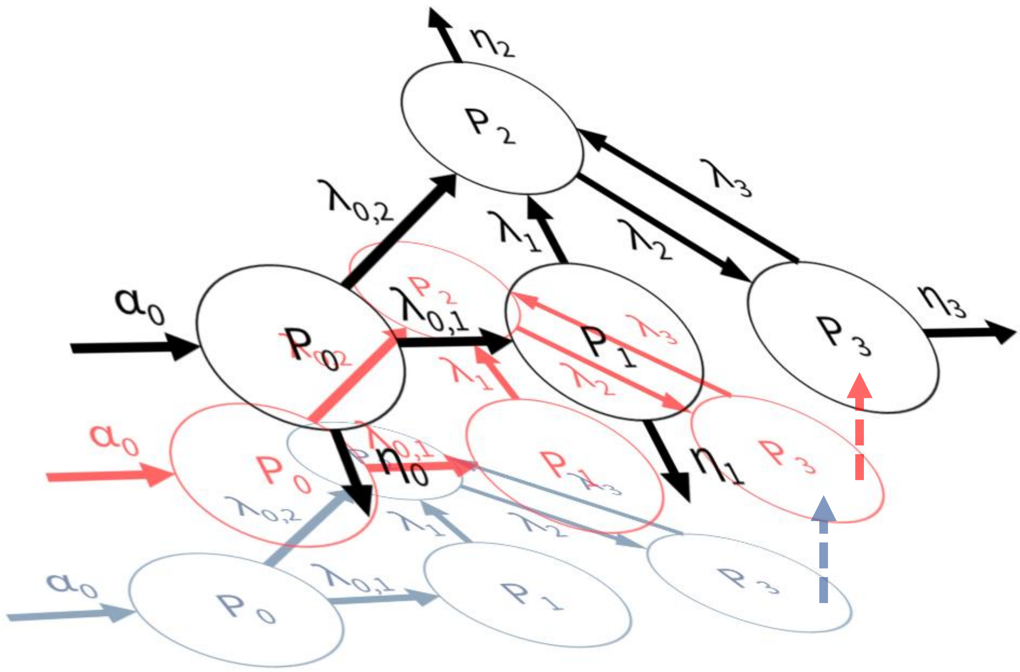


Figure 7.1 Model with Age Structure

There are methods that have been developed in dealing with age structure in population dynamic models (e.g. Brauer & Castillo-Chavez, 2012; Keeling & Rohani, 2008). Here it is merely noted that with age-brackets being coupled, solving the model differential equations becomes much more difficult, and numerical integration becomes necessary. For this study, the first and foremost purpose is to understand the broad structure of CP population, age structure is thus a secondary concern. Moreover, due to the absence of age structure for new and repeat client breakdown in investigation rates and OOHC admissions, which are the most important data for the model, taking age into account is not feasible. As such, age structure has not been introduced into the models.

Age-out Rate

In solving the differential equations for both the three-population model and the four-population model, the assumption that age-out rates for all the sub-populations are homogeneous has been made. This was partially because it was impossible to obtain the age-out rate for each of the sub-populations based on the data used in the study. Recall that among the sub-populations conceptualised in the models, data are available for only Q_2 (OOHC population). While Q_1 (population of children involved in CP but not in OOHC) can in principle be inferred from reported number of new clients involved in CP investigations, this is not possible with the available data. Moreover, to calculate age-out rates, detailed age structure is needed. This is only available in coarse age brackets for OOHC population and not available for children in investigations, as noted in the discussion in Section 7.1.3.

The assumption of homogeneous age-out rate is not expected to have a serious impact on the sub-population of children not involved in CP (Q_0) as this sub-population accounts for the large majority of the general population of children below 18 years of age. Recall that the total CP sub-population (i.e. $Q_1 + Q_2 + Q_3$) is the complement (i.e. $1 - Q_0$) of the sub-population of children not involved in CP. Therefore, by extension, the assumption of homogeneous age-out will also not have a serious impact on the total CP sub-populations. However, further work is needed to examine how this assumption impacts on the division within the total sub-populations, i.e. children involved in CP but not in OOHC and children in OOHC.

Flows between Sub-populations

Another important aspect of the model used in Chapter 5 and Chapter 6 is that the flows between the sub-populations are formulated as linearly proportional to the source populations. This is a common practice in population dynamics models when non-linear effects such as system capacity or carrying capacity are not considered. Another assumption is that the rates (coefficients of the proportion) are constant. In reality, the rates fluctuate with time. Constant rates throughout a certain time period means that the model is deterministic, as knowing the states of the system at the beginning of the time period (in this case the initial sub-populations) will fully determine the future sub-populations. The choice of constant rates can be viewed as essentially an assumption that there is a mean rate regime which drives the dynamics of the sub-populations within the time period of concern. Results in Chapter 5 show that in jurisdictions with adequate and consistent data, the model solutions obtained with this assumption (together with other assumptions to be discussed below) can indeed capture the time varying trend of OOHC population (Q_2 in the model). It has also been shown in Chapter 5 that in some jurisdictions, apparent structural disruptions in the OOHC population can be modelled using a piecewise continuous approach.

The assumption of constant rates can be relaxed by making the rate coefficients time dependent, if the form of such dependency can be deduced from data analysis; or by adding a stochastic term, if effects of the deviations from the constant rates are deemed to be random and following a certain distribution. Adding a stochastic term results in what is termed stochastic differential equation (SDE). In such an approach, stochasticity, or the non-deterministic and random part is often modelled as white noise.

Techniques have been developed to deal with the addition of stochastic terms (e.g. Tuma & Hannan, 1984). However, this will generally make the set of differential equations not directly integrable. While adding stochasticity does make the equations more realistic, there is a trade-off between using deterministic differential equations which enable us to understand the dominant or mean behaviours of the system and obtaining model simulations with outcomes which are more realistic in terms of their fluctuations in time. In the case of CP systems, which are very much tightly controlled systems constrained by reporting, risk and safety assessment and practice guidelines, it seems reasonable to assume that the dominant behaviours of the systems are deterministic within periods in which socio-economic conditions and social policies are stable. In such a situation, the random parts of the equations will probably not lead to abrupt changes in the trend described by the deterministic parts. When there are significant disruptions in socio-economic conditions or changes in social policies, beyond the transition periods, the populations can be modelled with a piecewise approach.

Perhaps the most critical issue is that due to the difficulty in obtaining high frequency data sets (as mentioned in Section 3.3), there is no means to characterize the distributions of the rate parameters used in this study, especially the investigation rates and OOHC rates in terms of new and repeat clients. In fact, to carry this line of inquiry forward, having access to high frequency data – optimally unit-record data, will be necessary.

System Capacity and Non-Linearity

In formulating the population dynamics model used in this study, the issue of system capacity (often termed carrying capacity in population biology) and how it may be handled (Section 3.1.2) have been addressed. There are two types of capacity issues. The first is related to process bottleneck due to resource constraint while the second is the capacity of a system to accommodate or carry a population. The first limits the flow rates while the second limits the level of a sub-population. In Section 3.1.2, it is argued that system capacity is the mechanism that will give rise to non-linearity in CP population modelling. It has also been demonstrated that one way to represent the effect of system capacity is to assume a logistic functional form in the relevant terms.

Figure 7.2 shows an illustration of the effect of using a logistic function to model system capacity for OOHC population. The horizontal axis indicates time while the y-axis indicates per 1000 population. The dashed horizontal line represents the population level of 20 per 1000. The solid curve represents a solution not unlike those obtained from the linear models used in this study. In the case here, the solid curve tends towards its asymptotic value, and it is set to 20 per 1000, as in-flows and out-flows of the population are in balance. It should be emphasized that in a linear system without system capacity limit, this balance is essentially the result of increasing outflows due to the growth of OOHC population and decreasing inflows due to the decline of population eligible for admission into OOHC.

The long-dash curve represents an illustrative solution for the scenario with a system capacity limit of 20 per 1000. It has a S-shape representative of a logistic function. Unlike the solid curve, the population limit in this case is externally imposed. Moreover, the effect of imposing a system capacity limit reduces the flow rates into the target population (OOHC), all else being equal (see Section 3.1.2).

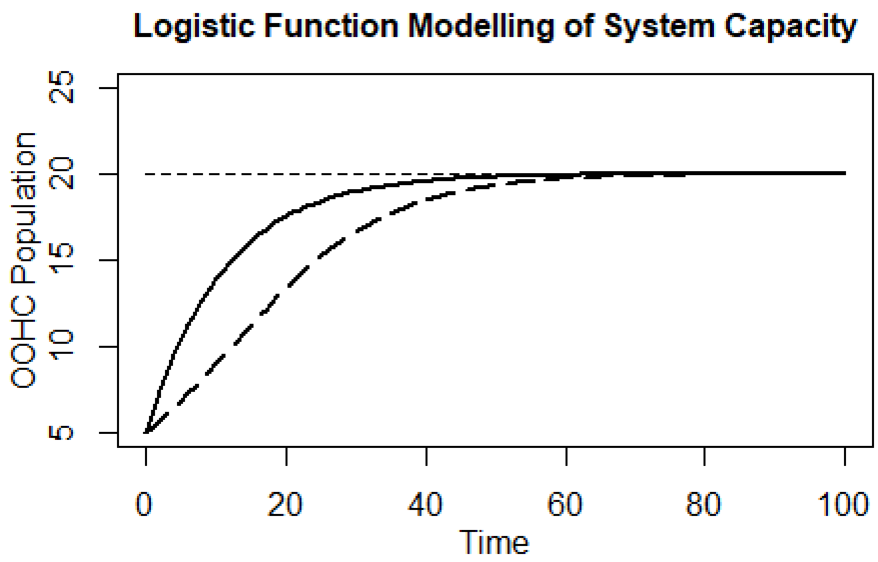


Figure 7.2 Illustration of Logistic Function Modelling of System Capacity

The most important thing to note for the purpose of this study is that modelling system capacity limit with a logistic function or its variants necessitates the effect of system capacity to kick in from the very beginning, when the population level is far from its system-imposed limit. While this is reasonable in ecological studies where the population of a species is constrained by limited resources, as even a slight increase in population leads to a reduction in per capita resource which slows down the growth of the population, the same may not be true for CP practice. In situations where there is an abundance of carers to accept children placed in OOHC, it is hardly plausible that children deemed in-need of care will be denied admission into care. A more likely scenario is the effect of system capacity coming into play when the limit is close, with the pool of carers stretched to the limit and placement becomes increasingly difficult.

In fact, in order to properly model the effects of system capacity, it is necessary to study how such an effect comes into play in practice. Without empirical information on the phenomenon, attempts to model it using an arbitrary functional form may not lead to an improvement of the model.

Socio-economic Conditions

Another important assumption in the formulation model is the homogeneity of the model populations in terms of socio-economic conditions. Studies show that risks of child maltreatment are related to socio-economic conditions (e.g. Hussey et al., 2006; Raissian & Bullinger, 2017). In cases where investigation rates and OOHC admission rates result in asymptotic values of CP sub-populations which are relatively small, this may not be a serious problem. However, when the rates are high, the asymptotic values of the total of all CP sub-populations may get near 1000 per 1000, meaning that 100% of all children are involved in CP. This is of course, unrealistic.

Although the highest asymptotic value of the total of all CP sub-populations obtained in this study is 298.4 per 1000 (in the Northern Territory), the effects of assuming homogeneity in terms of socio-economic conditions should be further evaluated in future works. As in the case for age-structure, socio-economic classes can be modelled by further stratifying the populations into socio-economic classes, with movements between the classes permitted.

Death and Migration

Last but not least, the models used in this study do not take death and migration of CP sub-populations into account. It is noted that child death rates have been dropping and are very low in Australia. For example, the mortality rate of children in the 0 to 4 age-bracket was 77 deaths per 100,000 population (AIHW, 2019b). Therefore, it can be expected that not taking death rates into account will not affect the model results.

As for migrations, the net effect of migrations is captured in the population growth rates for children not involved in CP. In 2018, the magnitudes of net interstate migration across the states and territories were all below 0.5% of the population, except for NT, which was -1.54% of the population (Australian Bureau of Statistics, 2018, 2019a). If one assumes that the migration rates for CP sub-populations are the same as that of the general populations, a correction will have to be made for CP.

The effect of interstate migration can be roughly estimated by adjusting the age-out rates in each of the jurisdiction. When this was done, it was found that migration had very small impacts on jurisdictions except NT. This is not surprising given that the magnitudes of the migration rates were lower than 0.5% in these jurisdictions. In NT, due to the relatively high migration rate, the impact was larger. However, it should be cautioned that adjusting age-out rate is a very rough means of gauging the effect of interstate migration. It merely points to whether such an effect should be further examined and the estimation cannot be taken literally.

7.3 Model Solutions and Their Applications

In Chapter 5 and Chapter 6, the use of the three-population model and the four-population model to obtain a complete picture of CP sub-populations (as defined in the models), including their time-varying trends, has been demonstrated. While data needed for the modelling efforts were very limited, with new and repeat client data available only from 2015-16 to 2017-18 (fewer years for some jurisdictions), the results were encouraging.

Per 1000 rates of children involved in CP investigation (Q_1) and children in OOHC (Q_1) for each of the states and territories have been estimated. In conjunction with these sub-populations, the investigation rates and OOHC admission rates for new and repeat clients have also been obtained, providing a much-needed picture on the issue of "churn" – repeated involvement in CP both in investigations and OOHC admission. A number of important issues from these results are highlighted and discussed below.

7.3.1 Model Applications with Iterative Process

In applying the model solutions to model CP sub-populations across Australian States and Territories, a heuristic iterative process (Section 3.4) has been used, due to the extremely limited number of data points obtained from CP annual reports. This iterative process starts with making an initial guess of the rate parameters and sub-populations, and then calculating the model time points at which the model OOHC values (Q_2) match the reported values within an error threshold. The model time points are used to re-calculate the other two sub-populations (Q_1 and Q_1), and then re-calculate the investigation rates and OOHC admission rates for new and repeat clients. This ensures that the final rates and sub-populations obtained through this iterative process produce the reported values of OOHC population, as well as the number of investigations and OOHC admission for new and repeat clients. This is essentially a simple curve fitting process that is pragmatic. With the number of data points spanning from one to three and with the uncertainties in many of the process rates (e.g. population growth, age-out, discharge), this heuristic approach serves the purpose of this study well. With more data points, a formal curve fitting approach, such as ordinary least square (OLS) or maximum likelihood estimation, will be developed.

7.3.2 CP Prevalence

Out-of-Home Care has been the dominant focus of current discourse on CP practices in Australia, especially the issue of disparity in terms of Indigenous status. This is essentially important. In contrast, little attention has been given to the overall prevalence of involvement in CP, defined in this study as the proportion of children in the general population who have been involved in at least a CP investigation. Being able to estimate this variable is important if we are to take a public health approach in formulating CP policies. However, neither cross sectional prevalence nor lifetime prevalence has been routinely assessed in Australia. Annual CP reports provide the number of children in investigations and other phases of the CP process, but these values have not been translated into prevalence of CP involvement.

This modelling approach provides a means to estimate the prevalence of CP involvement, with the results shown in Chapter 5 and Chapter 6. The strength of the population dynamics approach is that it is not merely an empirical statistical estimation which ignores the inherent relations among the sub-populations. In fact, the sub-populations are related to one another through the flows of population among them. The formulation of these flow-relationships in the models is merely the expression of the conservation of overall population flows. These relationships are not arbitrary and are true in any system. This results in a set of analytically solutions which can then be used in conjunction with reported data, through a choice of curve fitting procedure. While several assumptions have to be made in reducing the models to solvable forms, it is believed that these assumptions are reasonable, as discussed in Section 7.2.

The results of this study show that across the Australian states and territories, there are large differences in cross sectional CP prevalence (the sum of Q_1 and Q_2 in the three-population model shown in Chapter 5). To recap, the 2017-18 values of CP prevalence across the states and territories given by the three-population model are listed below, arranged from the lowest value to the highest value, with the asymptotic values shown in parentheses:

- TAS, 35.5 per 1000 (Asymptote 66.7 per 1000)
- SA, 36.4 per 1000 (Asymptote 64.7 per 1000)
- VIC, 66.3 per 1000 (Asymptote 96.6 per 1000)
- ACT, 78.1 per 1000 (Asymptote 132.4 per 1000)

- QLD, 95.7 per 1000 (Asymptote 113.7 per 1000)
- WA, 104.8 per 1000 (Asymptote 151.2 per 1000)
- NSW, 107.2 per 1000 (Asymptote 126.4 per 1000)
- NT, 298.4 per 1000 (Asymptote 355.5 per 1000)

The larger the value is, the more widespread CP involvement is. There is no question that these numbers have to be interpreted with caution, as they were inferred based on reported number of CP investigation. The definitions of CP investigation could be different across the jurisdictions and could be even changed within a jurisdiction. For example, NSW changed the definition of CP investigation in 2017-18 (AIHW, 2019a).

In NSW, CP investigation consists of two phases. An initial phase and a follow-up involving face-to-face contact with clients. A large fraction of CP investigation does not get to the second phase. Before 2017-18, all cases of investigations, whether they have gone through the second phase, are counted among the number of investigations. Beginning in 2017-18, only cases that go through the second phase are officially counted among the number of investigations.

If the investigation rates based on the new counting rule were used, CP prevalence in NSW would exhibit large drop. However, in terms of the modelling effort in this study, such a change presents two difficulties. The first is the issue of data consistency. A change in counting rule means that a consistent time series of the relevant data needs to be carefully reconstructed if they are to be used for long-term study. The second is the actual effect of being involved in CP investigation. It is not clear how children who were involved only in the first phase of investigation fared compared to those who were involved in both phases of investigation. Are they more or less likely to be re-reported and re-involved in another investigation or OOHC admission? If so, this will warrant the creation of another sub-population in the models.

From the list above, it can be seen that TAS and SA were estimated to have the lowest CP prevalence while the CP prevalence for NT far out stripped the rest. However, in terms of the asymptotes, the current values (2017-18) in NSW, QLD and NT are the closest to their respective asymptotic values, meaning that the room for the total CP sub-population to grow is small (about 18% in NSW to 19% in QLD and NT). On the other hand, the growth potential for the total CP sub-population is the largest in TAS and SA (88% and 77%, respectively), although the asymptotic values in these jurisdictions are still far lower than those in other jurisdictions.

Any comparison between the states and territories has to be viewed in caveat of the differences in counting rules, data limitations, and limitations in model formulations, as discussed in the preceding sections. However, they are useful at least as an indication that in-depth research with better data is needed to better understand the differences and that collaborative efforts among CP agencies across the jurisdictions in order to understand respective practices that might have led to the differences.

7.3.3 CP Involvement and OOHC admission for New and Repeat Clients

In Chapter 4, new and repeat client data for CP investigations and OOHC admission published by AIHW (2019a) are described. An attempt has also been made to infer the range of the breakdown of children in notifications into new and repeat clients (e.g. Figure 4.9). In Chapter 5, the analytical solutions of the three-population model are applied to obtain the investigation rates and OOHC admission rates for new and repeat clients.

As mentioned in Section 3.3, data reported by AIHW contain the number of new and repeat clients in investigations, on care and protection orders, and in OOHC. While one can calculate the respective fractions of new and repeat clients for each of these categories of variables, these fractions do not reveal the per capita rate of investigation and OOHC admission without information on the populations of new and repeat clients. By using the three-population model, each of the sub-populations in the model could be estimated, together with the investigation rates and OOHC admission rates associated with these sub-populations.

The results (e.g. Figure 5.7) show that for the investigation rates, the rate ratios of repeat clients over new clients ranged from 8.6 in WA to 71.7 in TAS; for OOHC admission rates, the rate ratios of repeat clients over new clients ranged from 15.4 in ACT to 159.9 in TAS. For all jurisdictions, these ratios are vastly higher than the ratios of the percentages of repeat and new clients obtained from the data. The implication is that in all jurisdictions, children with previous CP involvement are much more likely than those with no previous involvement to be involved in another investigation and to be admitted to OOHC. In other words, the issue of "churn", in terms of repeated involvement in CP investigations, is serious. Without being able to estimate the respective sub-populations of new client and repeat clients (Q_0 and Q_1), it would not be possible to obtain their respective rates. Nevertheless, further validations of these results are needed by using higher frequency data.

Aggregate data for SA, TAS and NT show evidence of a structural change in new and repeat breakdowns between 2012-13 to 2015-16. In the earlier period, the proportions of new and repeat clients are nearly equal, with the proportion of repeat clients being slightly higher for the number of children in investigations, and the proportion of new clients being slightly higher for the number of children admitted to OOHC. The situation changed to one in which the proportion of repeat clients is decidedly higher than that of new clients in 2015-16. It is not clear whether all of these jurisdictions exhibiting this change as the aggregate of the three sub-systems was dominated by SA, the most populous among the jurisdictions. However, this is definitely a phenomenon which warrants an in-depth analysis using higher frequency data from each of the jurisdictions. Such analysis will reveal much about how policy or practice change affects the population dynamics of children involved in CP.

7.3.4 OOHC Re-admission and Recurring CP Investigations

In Chapter 6, the results of the four-population model are shown. A number of hypothetical scenarios of investigation and OOHC admission rates for children who have exited OOHC (Q_3) were made to explore the issue of OOHC re-admission or "churn". This issue has been the focus of much research as OOHC "churn" or repeated admission to OOHC could be a driver for the increase of OOHC population (e.g. Courtney, 1995; Shaw, 2006; Wells & Guo, 1999; Wulczyn, 1991).

The analysis shown in Chapter 6 used three hypothetical scenarios in which OOHC re-admission rate for children who have exited OOHC were: 1) The same as the rate for children who have not been involved with CP; 2) the same as the rate for children who are involved with CP but have never been admitted to OOHC; and 3) twice the rate for children who are involved with CP but have never been admitted to OOHC. The first scenario corresponds to the situation where reunification is successful, with children who have been reunified with their original guardians having OOHC admission rate which is no difference from those who have never been involved in CP. The second scenario corresponds to the situation where reunification is not successful, with children re-admitted to OOHC with a rate as high as those who have been involved in CP but have never been in OOHC. The third scenario corresponds to the situation where reunification frequently fails, with the rate of re-admission of children who have been reunited with the original guardian being much higher than the rate for those who have been involved in CP but have never been in OOHC.

When constrained with the reported data, neither of the two scenarios with higher re-admission rates was shown to have driven OOHC population much higher when projected up to 2027, for each of the jurisdictions. However, for some jurisdictions, the asymptotic values of OOHC populations may vary by large magnitudes among the three scenarios. For example, in TAS, the asymptotic values for OOHC population between the scenarios with the lowest and highest re-admission rates differing by more than 40%. On the other hand, the difference is only 8% in NSW. When all else are equal, there is no question that a larger OOHC re-admission rate will lead to an increase of OOHC population, as shown in condition 13 in Section 6.1. However, the varying results shown in the hypothetical studies above indicate that the issue of “churn” or re-admission as a driver of OOHC population is not straightforward. When data are drawn only from cross-sectional snapshots with limited temporal coverage, one should be careful in making generalizations from the data.

Modelling studies with other hypothetical scenarios have also been conducted. In these scenarios, the OOHC admission rates and investigation rates for repeat clients were reduced by half, separately and in combination. These hypothetical scenarios were initialized with model sub-populations for 2017-18 which were obtained using the analytical solutions of the four-population model with the constraints of reported data. The results show that reducing either the investigation rates for repeat clients or OOHC admission rates for repeat clients will lead to a reduction in OOHC population. However, the effect of reduction in investigation rates for repeat clients manifests much more slowly.

In reality, when investigation rates are reduced, it may not necessarily lead to a reduction in OOHC population, especially in jurisdictions where the process rates are resource constrained. For example, it is shown in Figure 4.2 that for NSW, investigation rates and substantiation rates appeared coupled. Reduced investigation rates may lead to a rise in substantiation rates. In such cases, the pool of children in substantiation remain large despite a drop in the investigation rate. As long as there is a large enough pool of children who are deemed at risk and thus eligible for consideration of OOHC placements, OOHC population may not drop despite reduced investigation rates. This phenomenon is not reflected in the model as it does not take such a coupling into account.

7.3.5 Future Projections of OOHC Populations

For policy formulations and advocacy for social changes, future projections of CP populations, in particular OOHC population, are of significant interest. The population dynamics model formulated in this study can be used for projecting future populations. In Section 5.8, the projections based on the model are compared to projections based on linear regression of past populations, as well as those based on mean annual growth rates obtained from past-year data.

In most jurisdictions, projections done with the model solutions exhibit the most modest growth, especially in the long run. In particular, the population curves based on the model solutions taper off towards some asymptotic values. This is a well-known property of linear models which exhibits so-called linear partial-adjustment. In the models shown here, the growth of CP population eventually slows down when the in-flows and out-flows balance, leading to the CP sub-populations approaching a balanced state (asymptotic state).

It is contended that projections based on population dynamics models such as those presented in this study are more robust in view of the necessary dynamics structure among the sub-populations. In fact, as shown in the results presented through Chapter 5 and Chapter 6, the derivatives of different segments of the solution curves could be quite different. With enough data to correctly locate the segment of the curves that represent the current sub-populations, the future rates of change are also more accurately reflected. This is especially relevant when there is an abrupt change in rate regimes. When such abrupt changes occur, future trends could no longer be predicted using past-year growth rates. The ability of the model to handle such abrupt changes are demonstrated with the hypothetical cases discussed in Section 7.3.4. When the investigation rates and the OOHC admission rates associated with the new rate regimes are known, they can be immediately applied to estimate the new population curves, and this can be done with very few data points. In fact, the results shown in Chapter 5 and Chapter 6 were obtained with limited data. In contrast, empirical curve fittings are based on past-year population data, not on the internal process variables which actually dictate the trends of the populations. As such, when an abrupt change of process rate regimes has not effected a corresponding change in the population, curve fitting based on past-year growth rates could be of limited use.

7.3.6 Cross-sectional Prevalence and Lifetime Prevalence of CP Involvement

In Section 5.9, the relationship between cross-sectional prevalence and lifetime prevalence of CP involvement is elaborated. In conjunction with the relationship, a procedure has been devised to obtain a crude estimation of lifetime prevalence, based on model solution of cross-sectional prevalence. The crude estimation of lifetime prevalence using this procedure ranges from 7.4% in TAS to 47.3% in NT.

While such a crude estimation comes with many caveats, an analysis using data from a published study of lifetime prevalence of CP involvement in the USA shows that the results of the crude estimations are not implausible. By analysing the data from the published study, it was deduced that the procedure for the crude estimation over-estimated the lifetime prevalence by around 10% compared to that in the published study. Even when taking this level of over-estimation into account, the estimated values of lifetime prevalence in three jurisdictions (NSW, QLD and WA) are nearly 20%, i.e. there is a one in five chance for a child in these jurisdictions to be involved in CP investigations. The value in NT, around 40%, was the most alarming, meaning that on average, there is nearly a one in two chance of a child to be involved in CP investigations.

The crude estimation obtained in this study can also be compared with the results of a recent study by Segal et al. (2019) which used linked administrative data in SA from 1986–2017 to estimate lifetime risk of CP involvement for Aboriginal and non-Aboriginal children. In this study, the estimated values of cumulative lifetime prevalence for involvement in CP investigation for the 1998-2003 cohort was 7.1% for non-Aboriginal children and nearly 35% for Aboriginal children. The study did not estimate the prevalence for all children. Using the proportions of Aboriginal and non-Aboriginal children to calculate the weighted mean of the above values, one can deduce that the cumulative lifetime prevalence for all children should range from 7.1% to 8.4%. The value estimated in Chapter 5 is 7.6% which is within this range.

It should be cautioned again that the results need to be viewed in perspective of the caveats, especially for NT. In particular, it is mentioned in a preceding section that the model formulation in this study does not take socio-economic class into account. Not doing so may lead to CP involvement becoming more widespread than it really is. However, sorting out the effects of using homogenous rates instead of taking into account socio-economic heterogeneity requires an in-depth study using higher frequency CP data.

7.4 Implications for Policy and Practice

In this section, insights gained from this study, which may have policy and practice implications, are summarised. These are organised into two broad areas. The first discusses expenditure and investment in terms of the issue of repeated CP involvement. The second discusses how the population dynamics approach may be used as a tool for policy making purposes.

7.4.1 Expenditure and Investment in Preventative and Intensive Interventions

As shown by both the published data on new and repeat clients (e.g. Figure 4.8) as well as this study’s estimations of investigation rates and OOHC admission rates for new and repeat clients (e.g. Figure 5.7), CP systems across Australian states and territories are heavily dominated by repeat clients. This may be a deliberate choice of practice in accordance with the perspective of three-tier prevention and intervention as depicted in official CP publications in Australia (e.g. AIHW, 2019a, p. 2, Figure 1.1). In such a three-tier model, new clients who come in touch with CP would preferably be directed towards preventive services and support services, except for cases which needed more intensive interventions. This is in fact among the stated objectives of CPS as delineated in the Report on Government Services, in which it is stated among others that:

> … governments seek to provide child protection services that:
>
> - are targeted to children and young people who are at greatest risk
> - support and strengthen families so that children can live in a safe and stable family environment
>
> (SCRGSP, 2019, Chapter 16, Box 16.1, p. 16.5)

With only about 10% (2016-17) of the children who were involved in CP investigation admitted to OOHC, one would assume that many would receive support through either Family Support Services (FSS) or Intensive Family Support Service (IFSS). However, FSS and IFSS only accounted for 17.3% of the national CP expenditure in 2016-17 (SCRGSP, 2019), with protective intervention services accounting for 23.1% of the expenditure and OOHC accounting for 59.5% of the expenditure. This pattern of expenditure has been consistent since 2011-12.

Recall that protective intervention services are "Functions of government that receive and assess allegations of child abuse and neglect, and/or harm to children and young people, provide and refer clients to family support and other relevant services, and intervene to protect children." (SCRGSP, 2019, p. 16.39). Taken at face value, the figures on expenditure quoted above would mean that more money has been spent on CP investigation related activities than on providing actual support services to those involved in CP situations. This is a cause for concern, especially in view of the findings from this study that there is a high cross-sectional prevalence of children who have been involved in CP investigations, and correspondingly, the lifetime prevalence of involvement in CP is alarmingly high in some jurisdictions.

7.4.2 Using Population Dynamics Models for Policy Making Purposes

It has been repeatedly cautioned throughout this dissertation that both the three-population model and the four-population model were derived without taking potential system capacity limit of OOHC population into account, and that the process rates were assumed to be constant in solving the differential equations. However, it has also been stressed that when there are changes in the rates across different periods, the models can still be used if these rates are relatively stable within these time periods. In Chapter 5, the three-population model was used to demonstrate how this could be used to describe the apparent structural changes in OOHC populations in SA and TAS. Moreover, Chapter 6 demonstrates the use of the four-population model for scenario testing purposes. Three scenarios were conducted to test how OOHC population changes in reaction to reductions in investigation rates and the rate of OOHC "churn".

The above applications of the models, together with the results for future projections of OOHC reported in Section 5.8, show that when used skilfully, the population dynamics approach has the potential to be used for the following policy making purposes:

- The models can be used to project future service demands, both for the population of children involved in CP investigation, thus needing family support services, and the population of children needing OOHC services. In fact, results in Section 5.8 show that when compared with projections estimated using historical growth rates, projections using the model exhibit more moderate trends of growth. This is because the model projections implicitly account for internal structures governing how the sub-population may change in relation to each other. This is not possible when using only historical growth rates.
- Similar to projecting future service demands but in a different strain, the models can be used to test the impacts of different policy scenarios on CP populations. This will involve first deducing how the policy scenarios change the relevant CP process rates and then using the new rates in the model to obtain how the target populations react to the changes. Cost-benefit analysis can then be performed, based on CP population changes associated with each of the policy scenario.

7.5 Future Works

There are ongoing plans to continue developing the model by addressing some of the limitations described in Section 7.2. However, further development will depend on whether high frequency data across Australian states and territories, preferably unit record data, are available. Alternatively, data in other countries, e.g. the USA, may be used to facilitate further development. In particular, the issues of system capacity and non-linearity have to be studied first using case management records. More complexly formulated models will take into account age structure, ethnicity and socio-economic conditions.

Perhaps the most important work along the line of inquiry proposed in this study is to apply the current model formulation in conjunction with high frequency data. Only with high frequency data would it be possible to examine the various insights or outcomes gained from this study.

Model projections of OOHC will also continue to be tested with newly published annual aggregate data. Moreover, there is an ongoing project which applies the model solutions to other jurisdictions, such as California, USA. By doing comparative studies in different countries using the same model, one may be able to discover patterns of differences which may inform practitioners and policy makers alike in these countries.

7.6 Conclusion

In this study, an in-depth descriptive analysis of publicly available CP data across the Australian states and territories has been provided. Most importantly, population dynamics models of CP sub-populations have been successfully formulated and the model solutions have been applied to study CP populations across the Australian states and territories. A review of published literature in CP indicates that this is the first time that such models have been formulated and applied to studying the dynamics of CP populations.

By making a number of reasonable assumptions, a set of analytical solutions for the sub-populations included in the model have been derived. These sub-populations include children who have never been involved in CP, children who have been involved in CP but have never been in OOHC, children who are currently in OOHC, and children who have exited OOHC. When used with CP data obtained from publicly available sources, the investigation rates for new and repeat clients, as well as cross-sectional prevalence of CP involvement across the jurisdictions were successfully estimated. A review of the literature shows that this is the first time that such estimations have been made.

The results of this study provide a more complete picture of the extent of CP involvement, which is of great interest if child protection is to be viewed as a public health concern. The models formulated in this study can also be used as a tool to explore how changes in CP practices and policies (e.g. CP investigations and OOHC admission for new and repeat clients) may lead to changes in the sub-populations. Such a modelling approach is useful as without a structural model, it is difficult to get a more complete understanding of population structures and trends just by studying the data empirically.

A corollary outcome of the study is that it accentuates the importance of making more extensive data on new and repeat clients available to researchers. Even without high frequency data, breakdowns into new and repeat clients for more data categories, e.g. indigenous status and children in substantiations, will go a long way in facilitating a better understanding of CP populations across Australian states and territories, especially if such data are used in conjunction with population dynamics models such as those formulated in this study.

In conclusion, this study contributes to the field of CP population research in three dimensions. Theoretically and methodologically, the review of existing literature shows that this is the first application of population dynamics models to study the macro structure and trends of CP populations. Empirically, the models allow us to estimate all CP sub-populations included, with the requirements of consistency with observed data. Practically, the models allow us to estimate future trends of CP populations and to study how CP populations react to rate scenario changes which in turn, may result from practice or policy changes.

As in all modelling efforts, there are limitations and caveats associated with the proposed models. These include making the assumption that there is no system capacity limit in OOHC, thus making the models linear; not taking into account age structure; not considering socio-economic conditions and ethnic differences in CP involvements; and last but not least, having very limited data which are essential to such kind of modelling efforts. While future works have already been planned to address some of these limitations, it should be emphasized that any further progress of this approach will depend on the availability of high frequency data, from which conditions related to model limitations can be better understood and models can be better constrained.

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