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Money and Costly Credit*

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Abstract

We study an economy in which money and credit serve as a means of payment and the settlement of credit requires money. The model extends recent developments in microfounded monetary theory to address the choice of payment methods and the effects of inflation. Whether a buyer uses money or credit depends on the fixed cost of credit and the inflation rate. In particular, inflation not only makes money less valuable, but also makes credit more expensive because of the delayed settlement. The model predicts that either very low inflation or very high inflation hampers the use of credit.

JEL classification: E41, E50

Key words: money, costly credit, inflation

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1 INTRODUCTION

This paper develops a model of money and credit in order to study the choice of payment methods and the effects of inflation. Not too long ago, consumers typically paid for their purchases using either cash or checks. In recent decades, the payment instruments available to consumers have expanded to include debit and credit cards as well as other electronic payments. In this paper we focus on "consumer credit" which refers to credit card loans.[1] In a credit transaction, credit is usually offered by a third party, and this third party is willing to postpone debt settlement to the indefinite future – at high rates of interest. A prominent difference between credit and other means of payment is that acquisition of money in advance is not necessary to make a purchase using credit. In other words, credit transactions are settled "late".[2]

Technological advances in electronic record-keeping and financial deregulation promote the use of consumer credit. These technological advances allow credit to substitute for money and other means of payment in transactions. Based on U.S. household-level data, Duca and Whitesell (1995) estimate that for every 10% increase in the probability of owning a credit card, checking balances are reduced by 9%. Using the 2013 Survey of Consumer Payment Choice, Schuh and Stavins (2015) find that the share of consumers with a credit card is about 70% and credit card transactions account for 22.5% consumer payments in the U.S.. According to the Federal Reserve's Survey of Consumer Finances released in September 2017, there were 43.9% of households with a credit card balance.

If money and credit are purely substitutes, higher inflation would encourage more people to use credit. Nevertheless, evidence from countries that suffered high inflation episodes suggests that credit card usage actually increased after inflation was brought under control. For example, credit cards gained widespread popularity in Brazil following the successful reduction of inflation to sustainable levels, and the number of cards issued grew by 88% between 2000 and 2004. High inflation episodes also delayed the adoption and widespread use of credit cards in Turkey. Using the dataset developed by the European Credit Research Institute (ECRI), we compute the consumer credit to GDP ratio and plot it against inflation for each country in the sample for the year 2009-2010.[3] Figure 1 shows that in countries with relatively low inflation, the consumer credit to GDP ratio increases with inflation. Once inflation rises above a certain threshold (approximately 5.2% in our sample), the positive relationship between consumer credit and inflation becomes negative.[4]

[Insert Figure 1 here]

In this paper, we construct a model where money and credit are substitutes at low inflation but become complements at high inflation. The model is built on Lagos and Wright (2005). In monetary theory, frictions that render money essential make credit arrangements impossible. In order for credit to exist, we assume that there exist competitive financial intermediaries that can identify agents and have access to a record-keeping technology. There are two frictions associated with credit arrangements. First, arranging credit is costly.[5] In a bilateral trade, if a buyer wants to use credit, he must incur a fixed utility cost in order to make the seller and himself identified to a financial intermediary. Buyers use credit only when the benefit from using credit can cover the fixed cost. Second, the settlement of credit is available only at a particular time in each period, during which the financial intermediaries accept repayment of credit and settle debt. Due to the timing structure of the model, settlement is "delayed" and money becomes the only means of settlement.

These two features of the model allow some interesting interactions between money and credit. In the basic model, buyers use either money or credit as a means of payment. A credit equilibrium exists only when inflation is neither too low nor too high. We then generalize the basic model to allow for the coexistence of money and credit as a means of payment. In the generalized model, inflation tends to increase the fraction of buyers using credit at low inflation rates, but decrease the fraction of buyers using credit at high inflation rates. That is, the relationship between inflation and credit exhibits an inverse U-shape.

A major contribution of the paper is that the frictions associated with credit allow monetary policy to affect the interactions between money and credit. In contrast to existing models with costly credit, the delayed settlement makes credit transactions incur interest payments and subject to inflation distortion. Ferraris (2010) models the delayed settlement by letting agents for one period after production. Money is the only means to settle credit, and money and credit are complements. In fact, this idea can be traced back to Stockman (1981) who shows inflation reduces the capital stock if money and capital are complements. In reality, while convenience users of credit cards do not pay any interest, a large fraction of credit card users are revolvers. These revolvers do pay interest on their credit card debt. Fulford and Schuh (2015) find that around 65% of the credit card users between the ages of 25 and 50 were revolving some of their debt from month to month. Even at age 70, 45% of credit card users who used a credit card in the last month did not pay off their debt in full that month, and so will incur interest payments. According to NerdWallet's calculation, the average US household with revolving credit card debt carried a balance of $6,885 as of

June 2016 and paid $1,292 a year in interest, assuming an annual interest rate of 18.76%. In Australia, the Reserve Bank of Australia statistics on credit and charge card suggest that the amount of balances accruing interest as a fraction of total balances has remained above 60% since the statistics became available in 2002. Because of the close relationship between inflation and nominal interest rates, these observations suggest that credit transactions are not entirely free of inflation distortion.

Our model predicts a novel relationship between inflation and credit: an inverse U-shape relationship. This result strongly contrasts with previous models of money and credit that suggest a monotonic relationship. In particular, we show that both frictions associated with credit are needed to generate the non-monotonic relationship between inflation and credit. On one hand, inflation raises the cost of holding money, and therefore the benefit of using credit is more likely to cover its fixed cost. This fixed cost effect implies that buyers use more credit as inflation increases. On the other hand, the settlement of credit is delayed and credit transactions are subject to inflation distortion, which suggests that inflation worsens the credit transaction's terms of trade and lowers the benefit of using credit. The fixed cost effect and the delayed settlement effect have opposite implications on the choice of using credit as inflation fluctuates. Overall, the fixed cost effect dominates at low inflation rates and the delayed settlement effect dominates at high inflation rates. Consequently, very low or very high inflation hampers the use of credit. This implication is consistent with the empirical observations presented in Figure 1.[6]

This paper also contributes to the New Monetarist literature on money and credit.[7] Our model incorporates three features observed in an economy with money and credit: first, both money and credit serve as a means of payment; second, the choice of using money or credit is endogenous; and third, the settlement of credit requires money. In Berentsen, Camera and Waller (2007), banks can record financial history, but they cannot record goods transaction history. In this case, credit is available through bank loans in the form of money. Chiu and Meh (2011) extend Berentsen, Camera and Waller to study how credit/financial intermediation affects allocations and welfare in an economy where ideas (or projects) are traded among investors and heterogeneous entrepreneurs. Again, credit does not serve as a means of payment. Telyukova and Wright (2008) study the credit-card-debt puzzle by building a model in which agents can use money and credit. The market structure in their paper implies that agents do not use money and credit simultaneously. Monnet and Roberds (2008) examine the effect of monetary policy on credit card pricing. The choice of using credit is not endogenous in their model. The same is true in Sanches and Williamson (2010) who adopt the notion of limited participation in the sense that only an exogenous subset of agents can use credit. Lotz and Zhang (2016) consider a model with money and costly credit where sellers incur a cost to accept credit. They focus on how costly credit interacts with limited commitment in determining equilibrium outcomes. Gu, Mattesini and Wright (2016) study money and credit as substitutes in the payment process. They show that in a variety of environments, in equilibrium where money is valued, credit is inessential and changes in the debt limit are neutral. In such a situation, real balances adjust endogenously to changes in debt limit, keeping total liquidity the same.

The rest of the paper is organized as follows. Section 2 lays out the physical environment and considers the basic model in which buyers use either money or credit as a means of payment in equilibrium. To model the coexistence of money and credit as means of payment, Section 3 presents a generalized model by introducing match specific preference shocks. We conduct quantitative analysis of the generalized model in Section 4. Section 5 considers an extension of the basic model with secured credit. Finally, Section 6 concludes. All proofs are provided in the Appendix.

2 THE BASIC MODEL

Time is discrete and runs forever. In each period, there are three submarkets that open sequentially. The first submarket is characterized by bilateral trades and is labelled as market 1. The second submarket is characterized by a centrally located competitive spot market and is labelled as market 2. The third submarket, which is labelled as market 3, is a market for settlement. No trade occurs in market 3.

There are two permanent types of agents – buyers and sellers, each with measure 1. Buyers are those who want to consume in market 1 and sellers are those who produce in market 1. All agents are anonymous and lack commitment. There are two types of goods. Goods that are produced and consumed in market 1(2) are called good 1(2). All goods are nonstorable.

In market 1, buyers and sellers are matched randomly such that the probability that a buyer (seller) meets a seller (buyer) is σ with $0 < \sigma \leq 1$. Given that a buyer and a seller meet, the terms of trade are determined by the buyer's take-it-or-leave-it offer. After exiting market 1, all agents enter market 2. Buyers supply labor for production and consume good 2. Sellers purchase good 2 for consumption by using the revenue accumulated in market 1. The production technology in market 2 is assumed to be linear and 1 unit of labor can be converted into 1 unit of good 2.

The preference of a buyer is $u(q) + v(x) - h$, where $u(q)$ is the buyer's utility from consuming q units of good 1. The utility function $u(q)$ satisfies $u(0) = 0$, $u'(0) = \infty$ and $u''(q) < 0 < u'(q)$. In market 2, the buyer's utility from consuming x units of good 2 is $v(x)$, where $v'(0) = \infty$ and $v''(x) < 0 < v'(x)$. The buyer's disutility from working is h. The preference of a seller is $-q + y$, where q is the seller's disutility from producing q units of good 1. The seller has a linear utility in market 2, where y is the amount of consumption of good 2. All agents discount between market 3 and the next market 1. The discount rate is β.

In this economy, money is essential because agents are anonymous and lack commitment. We assume that there is a monetary authority that controls the supply of money. Let M denote the aggregate money supply at any given date. It grows at a gross rate $\gamma > 0$, i.e., $\hat{M} = \gamma M$. Here "ˆ" denotes variables in the next period. We will consider $\gamma > \beta$ and $\gamma \to \beta$ from above. New money is injected (or withdrawn) via a lump-sum transfer (or tax) to each buyer at the beginning of market 2 in each period and the transfer is $\tau = (\gamma - 1)M$.

Besides the monetary authority, there exist competitive financial intermediaries. These financial intermediaries possess a record-keeping technology and can enforce the repayment of debt. The record-keeping technology allows the financial intermediaries to identify agents and keep track of goods market transaction history. The existence of financial intermediaries facilitate credit arrangements in this economy.[8] To sustain the essentiality of money, we assume two frictions associated with the record-keeping technology. The first friction is that the financial intermediaries do not participate in market 2. This restriction implies that agents may arrange credit transactions in market 1, but cannot settle their debt in market 2. As the financial intermediaries are available in market 3, buyers who have used credit in market 1 repay their debt and sellers who have extended credit receive repayment in market 3. Without such a restriction, agents would settle their debt in market 2. In some senses, the settlement of debt is delayed. Money is the only means of settlement because goods are nonstorable. The second friction associated with the record-keeping technology is that it is costly. Buyers in market 1 can incur a fixed utility cost k to use the record-keeping technology to arrange credit with sellers.[9] Without incurring the fixed cost, the buyer and the seller remain anonymous and cannot make credit arrangements. Figure 2 provides a timeline of events.[10]

[Insert Figure 2 here]

We restrict our attention to stationary allocation in what follows. Suppose a planner weights all agents equally and is subject to the random matching technology in market 1. The first-best allocation is given by (x^*, q^*) which solves $v'(x) = 1$ and $u'(q) = 1$.

2.1 Buyers

To facilitate the analysis, we begin with buyers' value functions in market 2. Suppose that in nominal terms, a buyer carries money balance m and debt ℓ at the beginning of market 2. Let $V_2^b(m, \ell)$ and $V_3^b(z, \ell)$ be the value functions for a buyer in markets 2 and 3, respectively. Notice that the buyer accumulates money balances in market 2 but must wait until financial intermediaries open in market 3 to pay off his debt. Let z denote the money balance that the buyer carries to market 3. The buyer's choice problem is

$$V_2^b(m,\ell) = \max_{x,h,z} \left\{ v(x) - h + V_3^b(z,\ell) \right\} \tag{1}$$

$$\text{st. } x + \phi z = \phi m + h + \phi \tau,$$

where ϕ is the inverse of the price level (or the value of money). Substituting h from the buyer's budget constraint into equation (1), the unconstrained problem is

$$V_2^b(m,\ell) = \phi m + \phi \tau + \max_{x,z} \{ v(x) - x - \phi z + V_3^b(z,\ell) \}.$$

The first order conditions for interior solutions are $v'(x) = 1$ and

$$\frac{\partial V_3^b(z,\ell)}{\partial z} = \phi. \tag{2}$$

As is standard, the choice of z does not depend on m; however, it depends on ℓ. If the buyer incurs more debt in market 1, he must accumulate more money in market 2 for repayment in market 3. The envelope conditions imply

$$\frac{\partial V_2^b(m,\ell)}{\partial m} = \phi \text{ and } \frac{\partial V_2^b(m,\ell)}{\partial \ell} = \frac{\partial V_3^b(z,\ell)}{\partial \ell}. \tag{3}$$

Note that $V_2^b(m,\ell)$ is linear in m.

For a buyer entering market 3, his value function is $V_3^b(z,\ell) = \beta V_1^b(z-\ell, 0)$ because the only activity for the buyer in market 3 is to repay debt. Without loss of generality, we assume that the buyer always repays debt within the same period. It will become clear later that the buyer should be indifferent between repaying the debt in the current market 3 and repaying in any future market 3. To simplify, let $\hat{m} = z - \ell$ be the buyer's money holding at the beginning of the next period and the associated value function is $V_1^b(\hat{m}, 0)$. The envelope conditions yield

$$\frac{\partial V_3^b(z,\ell)}{\partial z} = \beta \frac{\partial V_1^b(\hat{m},0)}{\partial \hat{m}} \text{ and } \frac{\partial V_3^b(z,\ell)}{\partial \ell} = -\beta \frac{\partial V_1^b(\hat{m},0)}{\partial \hat{m}}. \tag{4}$$

Combining equations (2), (3) and (4),

$$\beta \frac{\partial V_1^b(\hat{m},0)}{\partial \hat{m}} = -\frac{\partial V_2^b(m,\ell)}{\partial \ell} = \phi.$$

Therefore, $V_2^b(m,\ell)$ is linear in (m,ℓ) and $V_2^b(m,\ell) = \phi m - \phi \ell + V_2^b(0,0)$.

After exiting market 3, the buyer's value function in market 1 is

$$V_1^b(m,0) = \sigma[u(q) - k \cdot I(a) + V_2^b(m-d, a \cdot I(a))] + (1-\sigma)V_2^b(m,0),$$

where (q,d,a) are the terms of trade determined through the buyer's take-it-or-leave-it offer. With probability σ, the buyer spends d units of money and uses a units of credit in nominal terms in exchange for q units of good 1 from the seller. An indicator function $I(a)$ is such that $I(a) = 1$ if $a > 0$ and $I(a) = 0$ if $a = 0$. With probability $1-\sigma$, the buyer is not matched and carries his money to market 2.

2.2 *Sellers*

Let $V_2^s(m,\ell)$ and $V_3^s(z,\ell)$ be a seller's value functions in markets 2 and 3, respectively. Since the seller is the creditor, ℓ should be either 0 or negative. The seller's value function in market 2 is

$$V_2^s(m,\ell) = \max_{y,z}\left[y + V_3^s(z,\ell)\right] \text{ st. } y + \phi z = \phi m, \tag{5}$$

where z is the money balance that the seller carries to market 3. By substituting y from the constraint into the objective function in equation (5), the first order condition of the unconstrained problem is

$$\frac{\partial V_3^s(z,\ell)}{\partial z} \leq \phi, \text{ and } z = 0 \text{ if } \frac{\partial V_3^s(z,\ell)}{\partial z} < \phi. \tag{6}$$

The envelope conditions yield

$$\frac{\partial V_2^s(m,\ell)}{\partial m} = \phi \text{ and } \frac{\partial V_2^s(m,\ell)}{\partial \ell} = \frac{\partial V_3^s(z,\ell)}{\partial \ell}. \tag{7}$$

Again, $V_2^s(m,\ell)$ is linear in m.

For the seller in market 3, the value function is $V_3^s(z,\ell) = \beta V_1^s(z-\ell,0)$. If the seller has extended any credit in the previous market 1, the seller will receive repayment from the financial intermediary in market 3. Let $\hat{m} = z - \ell$ denote the seller's money holding at the beginning of the next market 1. The envelope conditions are

$$\frac{\partial V_3^s(z,\ell)}{\partial z} = \beta\frac{\partial V_1^s(\hat{m},0)}{\partial \hat{m}} \text{ and } \frac{\partial V_3^s(z,\ell)}{\partial \ell} = -\beta\frac{\partial V_1^s(\hat{m},0)}{\partial \hat{m}}. \tag{8}$$

The seller's value function in market 1 is

$$V_1^s(m,0) = \sigma[-q + V_2^s(m+d,-a)] + (1-\sigma)V_2^s(m,0). \tag{9}$$

If the seller meets a buyer, the seller sells q units of good 1, receives d units of money, and extends credit with the nominal value a if the buyer chooses to use credit.

2.3 Equilibrium

Now we proceed to define the equilibrium. First, we solve for the terms of trade in market 1. In a match, the buyer chooses whether to use credit (and incur the fixed cost k) or not. Consider a buyer who does not use credit. Recall that V_1^b and V_1^s are linear in m. The buyer's problem is

$$\max_{q,d}[u(q) - \phi d] \text{ st. } q = \phi d \text{ and } d \leq m,$$

where m is the buyer's money holding. The first constraint is the seller's incentive compatibility constraint resulting from the assumption that the buyer makes a take-it-or-leave-it offer. The second constraint is the buyer's liquidity constraint – the buyer cannot leave with a negative money balance. The solution to the buyer's problem is

$$\begin{cases} \text{if } d < m: & (q,d) \text{ are given by } u'(q) = 1 \text{ and } \phi d = q, \\ \text{if } d = m: & (q,d) \text{ are given by } d = m \text{ and } q = \phi d. \end{cases}$$

Consider a buyer who uses credit. Since V_2^b is linear in (m, ℓ), the buyer's problem is

$$\max_{q,d,a}[u(q) - k - \phi d - \phi a] \tag{10}$$

$$\text{st. } q = \phi d + V_2^s(0, -a) - V_2^s(0, 0) \text{ and } d \leq m.$$

It is obvious that the seller's money holding does not appear in the above problem. Therefore, the terms of trade (q, d, a) do not depend on the seller's money holding. In addition, recall that the terms of trade without credit do not depend on the seller's money holding either. Hence, $\partial V_1^s(\hat{m}, 0)/\partial \hat{m} = \hat{\phi}$ by virtue of equations (7) and (9). Two results are immediate. First, from equations (7) and (8), $\partial V_2^s(m, \ell)/\partial \ell = -\beta\hat{\phi}$ and $V_2^s(m, \ell) = \phi m - \beta\hat{\phi}\ell + V_2^s(0, 0)$. Second, equation (8) implies that $\partial V_3^s(z, \ell)/\partial z = \beta\hat{\phi}$. Since we focus on stationary allocations, the inflation rate is given by $\phi/\hat{\phi} = \gamma$. Together with equation (6), sellers do not carry money to market 3. That is, $z = 0$ for all sellers.

Back to the buyer's problem in equation (10), the first constraint is simplified to $q = \phi d + \beta\hat{\phi}a$. If the buyer brings any $m > 0$, the buyer should spend all of his cash holding whenever $\gamma > \beta$. The solutions of (q, a) are

$$u'(q) = \frac{\gamma}{\beta} = 1 + i, \tag{11}$$

$$\beta\hat{\phi}a = q - \phi m. \tag{12}$$

Here we define the nominal interest rate i from the Fisher equation. One important result from equation (11) is that q depends on γ. If a buyer uses credit in market 1, he will accumulate money for debt repayment in market 2. However, for the seller who extends the credit in the match, he will not be paid in the same market 2. Instead, the seller must wait to get settled in market 3. After receiving the money, the seller carries the money to the next market 1, but he cannot spend it since he does not want to consume. Hence, the seller actually spends the money one period after the buyer accumulates the money. There is an asymmetry between the time at which the buyer accumulates the money for repayment (production) and the time at which the seller can spend the money from repayment (consumption). The buyer must compensate the seller for the loss in the value of money. From equation (12), the buyer essentially borrows $\beta\hat{\phi}a/\phi$ and repays a in nominal terms. The nominal interest rate of credit is $1 + i = \phi/(\beta\hat{\phi}) = \gamma/\beta$. As γ gets higher, credit is more costly in nominal terms. Credit transactions are subject to inflation distortion.

As the structure of the model implies that money is the only means to settle credit, it is natural that inflation affects the terms of trade in a credit transaction. However, this is not necessarily true. The key feature that makes credit subject to inflation distortion is the time lag between buyer's production for repayment and seller's consumption from repayment.[11] Imagine an environment in which the settlement of credit requires money but financial intermediaries participate market 2. Sellers receive debt repayment in the form of money and can spend it in the same market 2 right away. It is clear that in this case the terms of trade in a credit transaction do not depend on inflation although money is imposed as the only means of settlement.

Having solved the terms of trade, we proceed to find the condition that determines whether a buyer uses credit or not. Notice that if the buyer uses credit, there is no need to carry any money for trade. Therefore, the buyer effectively chooses whether to use money or credit when choosing money holding in market 2. Let (q^c, q^m) denote quantities traded with credit and with money, respectively. If the buyer uses credit,

$$V_2^{b,c}(m,\ell) = \phi m + \phi\tau + v(x^*) - x^* + \beta\left\{\sigma[u(q^c) - (1+i)\,q^c - k] + V_2^b(0,0)\right\},$$

where q^c solves equation (11). If the buyer uses only money, the buyer's liquidity constraint always binds in market 1. It follows that

$$V_2^{b,m}(m,\ell) = \phi m + \phi\tau + v(x^*) - x^* + \max_z\left\{-\phi z + \beta\sigma\left[u\left(q^m\right) - \hat{\phi}z\right] + \beta\hat{\phi}z + \beta V_2^b(0,0)\right\},$$

where $\hat{\phi}z = q^m$. As $dq^m/dz = \hat{\phi}$, we can solve for z

$$u'\left(q^m\right) = 1 + \frac{i}{\sigma}. \tag{13}$$

Let $\Delta(i) = V_2^{b,c}(m,\ell) - V_2^{b,m}(m,\ell)$ be the net benefit of using credit for the buyer,

$$\Delta\left(i\right) = \beta\sigma\left[u\left(q^c\right) - \left(1+i\right)q^c - k\right] - \beta\left\{-iq^m + \sigma\left[u\left(q^m\right) - q^m\right]\right\}.$$

Notice that Δ depends on i and $\Delta'(i) = \beta\left[q^m\left(i\right) - \sigma q^c\left(i\right)\right]$.

LEMMA 1 *When $u\left(q\right)$ is isoelastic, $\lim_{i\to 0}\Delta(i) = \lim_{i\to\infty}\Delta(i) = -\beta\sigma k$ and there exists a unique interior solution to $\Delta'(i) = 0$ that maximizes $\Delta(i)$.*

All proofs are available in the appendix. Lemma 1 establishes that buyers use only money when the nominal interest rate approaches 0 or ∞. We label this type of equilibrium as a monetary equilibrium. Let $i^{\max}$ be the solution to $\Delta'(i) = 0$ and $i^{\max}$ depends only on σ and parameters in the utility function. We further define $k^{\max}$ as the value of k that satisfies $\Delta(i^{\max}) = 0$. Here $k^{\max}$ is a function of (β, σ) and parameters in the utility function. It follows that $\Delta(i^{\max}) > 0$ when $k < k^{\max}$ and $\Delta(i^{\max}) < 0$ when $k > k^{\max}$. Notice that an extreme case is when credit is not costly to use, i.e., $k = 0$. Buyers will not use money as a means of payment for any $i > 0$ because it is more costly to use money. However, money is still used by buyers to repay debt. We label this type of equilibrium as a credit equilibrium. When $k = 0$, only a credit equilibrium exists. When the cost of using credit is too big $(k > k^{\max})$, a credit equilibrium does not exist. When $k \in (0, k^{\max})$, $\Delta(i^{\max}) > 0$ and there are two solutions to $\Delta(i) = 0$. Let $(\underline{i}, \bar{\imath})$ denote these solutions where $\underline{i} < \bar{\imath}$. See Figure 3 for an illustration of $\Delta(i)$ for $k \in (0, k^{\max})$.

[Insert Figure 3 here]

When $k \in (0, k^{\max})$, a monetary equilibrium exists for very low or very high values of i. The benefit of using credit is that it partially avoids the inflation distortion and hence is associated with a higher q. To use money, a buyer has to accumulate the desired amount in advance even though the buyer may not find a seller in market 1. In contrast, the buyer uses credit when he is matched with a seller in market 1. There is no extra cost for the buyer when he is not matched with a seller. When $i > 0$, the benefit from using credit $\Delta(i)$ depends non-monotonically on i. Proposition 1 summarizes how monetary policy (inflation) affects the existence of a monetary equilibrium or a credit equilibrium.

PROPOSITION 1 *When $k \in (0, k^{\max})$, there are two thresholds of i, $\underline{i}$ and $\bar{\imath}$, such that a unique monetary equilibrium exists for $i \in [0, \underline{i}) \cup (\bar{\imath}, \infty)$ and a unique credit equilibrium* *exists for* $i \in (\underline{i}, \bar{\imath})$*. In the case that* $i = \underline{i}$ *or* $i = \bar{\imath}$*, buyers are indifferent between using money and credit.*

In the basic model, two frictions associated with using credit generate two channels through which inflation affects the choice of using credit. A higher inflation rate makes using money more costly and therefore the benefit from using credit is more likely to cover the fixed cost. This fixed cost effect implies that inflation raises the benefit from using credit. The other friction associated with using credit is the delayed settlement. As settlement of credit is delayed, credit transactions are also subject to inflation distortion. In particular, q^c is decreasing in inflation from equation (11). The delayed settlement effect suggests that inflation worsens the credit transaction's terms of trade and lowers the benefit from using credit. The fixed cost effect and the delayed settlement effect have opposite implications on the choice of using credit when the inflation rate changes.

Putting these two effects together, the basic model predicts that inflation induces buyers to use credit when the inflation rate is neither too low nor too high. That is, very low or very high inflation hampers the use of credit. This implication is consistent with the empirical observations discussed in the Introduction. The fixed cost effect dominates at low inflation rates and the delayed settlement effect dominates at high inflation rates.

In this model the Friedman rule ($\gamma \to \beta$) implements the first-best allocation.[12] Under the optimal monetary policy, all buyers use only money and no credit arrangement is needed.[13] Since buyers prefer to use credit at moderate inflation rates and credit transactions are subject to inflation distortion, it is worthwhile to check how monetary policy generally affects welfare in the basic model. Weighing all agents equally, welfare is defined as

$$\mathcal{W} = \frac{1}{1-\beta}\left\{v(x^*) - x^* + \sigma\left[u\left(q^m\right) - q^m\right]\right\}$$

if buyers use money and

$$\mathcal{W} = \frac{1}{1-\beta}\left\{v(x^*) - x^* + \sigma\left[u\left(q^c\right) - q^c - k - \frac{1-\beta}{\beta}q^c\right]\right\} \tag{14}$$

if buyers use credit.[14] In both cases, we can show that welfare is decreasing in inflation. Despite the fact that credit allows buyers to consume more, a higher inflation rate always reduces welfare.

3 THE GENERALIZED MODEL

In the basic model, buyers use either money or credit as a means of payment. We generalize the basic model by incorporating match specific preference shocks in order to have coexistence of money and credit as means of payment. We then use the generalized model to examine the interaction between money and credit, and how inflation affects credit through quantitative analysis. Let each buyer receive a match specific preference shock ε upon matching with a seller at the beginning of market 1 and the utility from consuming q units of good 1 is $\varepsilon u(q)$. Suppose that the preference shock follows a uniform distribution such that $\varepsilon \in [0,1]$ and is i.i.d. across buyers and time. The realization of these preference shocks is public information. It is straightforward that the first-best allocation $q^*(\varepsilon)$ satisfies $\varepsilon u'(q) = 1$. In the following, we describe the problems faced by buyers and sellers, and then solve for equilibrium.

3.1 Buyers

A buyer's value function in market 2 $V_2^b(m, \ell)$ remains the same as in the basic model. For the buyer entering market 3, the value function is

$$V_3^b(z, \ell) = \beta \int V_1^b(z - \ell, 0; \varepsilon) d\varepsilon.$$

Again, let $\hat{m} = z - \ell$ be the buyer's money holding at the beginning of the next period. If the buyer receives a preference shock ε at the beginning of the next period, let $V_1^b(\hat{m}, 0; \varepsilon)$ be his value function. As $V_1^b(\hat{m}, 0; \varepsilon)$ depends on ε, we take the expected value for the buyer in the next market 1 and discount it by β. The envelope conditions yield

$$\frac{\partial V_3^b(z, \ell)}{\partial z} = \beta \int \frac{\partial V_1^b(\hat{m}, 0; \varepsilon)}{\partial \hat{m}} d\varepsilon \text{ and } \frac{\partial V_3^b(z, \ell)}{\partial \ell} = -\beta \int \frac{\partial V_1^b(\hat{m}, 0; \varepsilon)}{\partial \hat{m}} d\varepsilon. \quad (15)$$

Following similar steps as in the basic model, we find that $V_2^b(m, \ell) = \phi m - \phi \ell + V_2^b(0, 0)$. In market 1, for the buyer with ε, the value function is

$$V_1^b(m, 0; \varepsilon) = \sigma[\varepsilon u(q) - k \cdot I(a) + V_2^b(m - d, a \cdot I(a))] + (1 - \sigma) V_2^b(m, 0).$$

3.2 Sellers

A seller's value function in market 2 $V_2^s(m,\ell)$ remains the same as in the basic model. For the seller entering market 3, the value function is

$$V_3^s(z,\ell) = \beta \int V_1^s(z-\ell, 0; \varepsilon) d\varepsilon.$$

Similarly, we take the expected value function of the seller because the seller anticipates that a potential buyer he will meet in the next market 1 may have a preference shock ε drawn from $[0,1]$. Let $\hat{m} = z - \ell$ denote the seller's money holding at the beginning of the next market 1. The envelope conditions are

$$\frac{\partial V_3^s(z,\ell)}{\partial z} = \beta \int \frac{\partial V_1^s(\hat{m},0;\varepsilon)}{\partial \hat{m}} d\varepsilon \text{ and } \frac{\partial V_3^s(z,\ell)}{\partial \ell} = -\beta \int \frac{\partial V_1^s(\hat{m},0;\varepsilon)}{\partial \hat{m}} d\varepsilon. \tag{16}$$

For the seller who potentially meets a buyer with ε, the value function in market 1 is

$$V_1^s(m,0;\varepsilon) = \sigma[-c(q) + V_2^s(m+d, -a)] + (1-\sigma)V_2^s(m,0).$$

3.3 Equilibrium

There are two types of trades in market 1, depending on whether the buyer in a match uses credit or not. Suppose that a buyer with ε uses only money. The buyer's problem is

$$\max_{q,d} [\varepsilon u(q) - \phi d] \text{ st. } q = \phi d \text{ and } d \leq m,$$

The solution is given by

$$\begin{cases} if\ \ d < m: \ \ (q, d) \text{ are given by } \varepsilon u'(q) = 1 \text{ and } \phi d = q, \\ if\ \ d = m: \ \ (q, d) \text{ are given by } d = m \text{ and } q = \phi d. \end{cases}$$

Suppose that the buyer with ε uses credit. We use similar steps as in the basic model to find $V_2^s(m, \ell) = \phi m - \beta\hat{\phi}\ell + V_2^s(0, 0)$. Therefore, the buyer solves

$$\max_{q,d,a}[\varepsilon u(q) - k - \phi d - \phi a] \text{ st. } q = \phi d + \beta\hat{\phi}a \text{ and } d \leq m.$$

One can verify that $d = m$ always holds. The solutions for (q, a) are

$$\varepsilon u'(q) = \frac{\gamma}{\beta} = 1 + i, \tag{17}$$

and equation (12). Terms of trade with credit still depend on the inflation rate or the nominal interest rate as before.

With preference shocks, whether a buyer uses credit or not also depends on the preference shock ε. For a buyer with ε in market 1, if he uses only money,

$$V_1^b(m, 0; \varepsilon) = \sigma[\varepsilon u(q) - q] + \phi m + V_2^b(0, 0).$$

If the same buyer uses credit,

$$V_1^b(m, 0; \varepsilon) = \sigma[\varepsilon u(q) - (1 + i)\, q + i\phi m - k] + \phi m + V_2^b(0, 0).$$

Let $T(\varepsilon)$ be the net benefit of using credit, where

$$T(\varepsilon) = \sigma[\varepsilon u(q^c) - (1+i)\, q^c + i\phi m - k] - \sigma[\varepsilon u(q^m) - q^m]. \tag{18}$$

We again use (q^c, q^m) to denote the quantities traded with credit and without credit.

LEMMA 2 *For any given i, there exist two threshold values of ε, ε_0 and ε_1 such that*

$$\begin{cases} 0 \leq \varepsilon \leq \varepsilon_0, \text{ the buyer spends } d < m,\ a = 0 \text{ and consumes } q^* \text{ where } \varepsilon u'(q^*) = 1, \\ \varepsilon_0 \leq \varepsilon \leq \varepsilon_1,\ \text{the buyer spends } d = m,\ a = 0 \text{ and consumes } q \text{ where } q = \phi m, \\ \varepsilon_1 \leq \varepsilon \leq 1,\ \text{the buyer spends } d = m,\ a > 0 \text{ and consumes } q^c \text{ where } \varepsilon u'(q^c) = 1 + i. \end{cases}$$

Lemma 2 is intuitive. If a buyer receives a very low value of ε, he has enough money to afford q^*, which is the optimal consumption for him. Here ε_0 is the threshold that determines whether a buyer is liquidity constrained. For a buyer who receives an intermediate value of ε, the money may not be enough to afford his q^*. The buyer is liquidity constrained. Using credit can relax the buyer's liquidity constraint, but this is costly. Therefore, buyers with intermediate values of ε find it optimal not to use credit, because the benefit from using credit is not enough to cover the fixed cost. For those buyers who have large values of ε, paying the fixed cost to relax their liquidity constraints becomes optimal. The threshold ε_1 determines whether a buyer uses credit.

The decision to use credit is related to the size of purchase in this environment. Buyers use credit for large purchases. This result is in accordance with the evidence on consumers' choices of payment methods. Empirically, the mean value of cash purchases is smaller than the mean value of credit purchases. In English (1999), the mean values of credit card purchases and cash purchases are \$54 and \$11, respectively. Klee (2008) documents that these respective mean values are \$30.85 and \$14.2.

With different groups of buyers in terms of their choices of payment methods, we can now characterize the equilibrium. We define (q_0, q_1) such that

$$\varepsilon_0 u'(q_0) = 1, \tag{19}$$

$$\varepsilon_1 u'(q_1) = 1 + i. \tag{20}$$

Notice that $q_0 = \phi m$ represents the transaction demand for money. In market 3, the expected marginal benefit of 1 unit money is

$$\beta \int \frac{\partial V_1^b(\hat{m}, 0; \varepsilon)}{\partial \hat{m}} d\varepsilon = \beta \hat{\phi} \left\{ \int_0^{\varepsilon_0} d\varepsilon + \int_{\varepsilon_0}^{\varepsilon_1} [\sigma \varepsilon u'(q_0) + (1-\sigma)] d\varepsilon + \int_{\varepsilon_1}^{1} (1 + \sigma i)\, d\varepsilon \right\}.$$

The marginal cost of 1 unit money is ϕ. The optimal q_0 is determined by

$$\varepsilon_0 + \frac{1}{2} u'(q_0)(\varepsilon_1^2 - \varepsilon_0^2) + (1+i)(1-\varepsilon_1) = 1 + \frac{i}{\sigma}. \tag{21}$$

The last condition that completes the characterization of the equilibrium is derived from $T(\varepsilon) = 0$,

$$\varepsilon_1 u(q_1) - (1+i)\, q_1 - k = \varepsilon_1 u(q_0) - (1+i)\, q_0. \tag{22}$$

It is possible that no buyer uses credit and $\varepsilon_1 = 1$. When i is close to 0, the rate of return of money is high enough so that there is no need to use credit. When i approaches ∞, the gain from using credit cannot cover the fixed utility cost k. In equation (18), $T(\varepsilon)$ is negative and buyers use only money. Depending on the values of (i, k, σ), there are two types of monetary equilibrium.

DEFINITION 1 *A monetary equilibrium with credit is characterized by* $(\varepsilon_0, \varepsilon_1, q_0, q_1)$ *satisfying equations (19), (20), (21) and (22). A monetary equilibrium without credit is characterized by* $\varepsilon_1 = 1$ *and* (ε_0, q_0) *satisfying equations (19) and (21).*

PROPOSITION 2 *For any* $i > 0$, *a monetary equilibrium exists. The optimal monetary policy is the Friedman rule* ($i \to 0$ *or* $\gamma \to \beta$).

In general, a monetary equilibrium without credit is unique. The uniqueness of monetary equilibrium with credit is less clear. However, we have not found multiple equilibria in the numeral analysis presented in the next section. As in the basic model, the Friedman rule is the optimal monetary policy. At the Friedman rule, buyers can hold enough money to afford $q^* (\varepsilon)$. Credit is driven out as a means of payment. Given that both money and credit can function as the means of payment in market 1, they substitute each other. More interestingly, money and credit are also complements because the settlement of credit requires money.

The values of parameters (i, k, σ) are important in determining whether credit is valued in a monetary equilibrium. The fixed cost k of using credit affects a buyer's choice of payment methods. A lower k can be viewed as an improvement in credit transaction technology, which is likely to promote the use of credit. The trading probability σ reflects frictions in the goods market. A higher σ implies fewer trading frictions. The comparative statics results show that $(\varepsilon_0, \varepsilon_1, q_0, q_1)$ are all increasing in k and σ in a monetary equilibrium with credit, .

Analytically, it is less clear how i affects the equilibrium allocation. Inflation is a tax on money, so it generally lowers buyers' incentives to hold money. A higher i leads to a lower q_0 and a lower ε_0. The fixed cost effect and the delayed settlement effect through which i affects ε_1 still exist. The fixed cost effect implies that inflation makes more buyers use credit. This is because high inflation makes more buyers liquidity constrained so that more buyers may find using credit beneficial enough to cover the fixed cost. Through the fixed cost channel, i decreases ε_1. The delayed settlement effect on the other hand lowers incentives for buyers to use credit because of a deterioration in the terms of trade. Through the delayed settlement channel, i increases ε_1. Given that ε_1 hits the boundary 1 when either $i \to 0$ or $i \to \infty$, the total effect of i on ε_1 should be non-monotonic. We will rely on numerical results in the next section to verify these conjectures.

4 QUANTITATIVE ANALYSIS

In this section, we derive more implications from the generalized model by solving it numerically. For the numerical exercise, we adopt some specific functional forms for $\varepsilon u(q)$ and $v(x)$ that have been used in the monetary search literature. Let $\varepsilon u(q) = \varepsilon q^\rho/\rho$ and $v(x) = B \log x$ where $0 < \rho < 1$. In market 1, the matching technology is the urn-ball matching function, where $\sigma = 1 - e^{-1}$. There are four parameters (β, B, ρ, k) to be determined. The period length in this model is set to be 1 year mainly to facilitate comparisons with past work on the welfare cost of inflation.

The time preference parameter β is set $\beta^{-1} = 1.04$, so the implied annual real interest rate is 0.04. For the other parameters, we follow Lucas (2000) and Lagos and Wright (2005), and fit the model's money demand to the US money demand data by nonlinear least squares. The data covers the annual nominal interest rate and the "money demand" (or the inverse of the velocity of money) for the period $1900-2000$.[15] The "money demand" predicted by the model is

$$L(i)=\frac{M}{PY}=\frac{q_0+\frac{\sigma}{\beta}\int_{\varepsilon_1}^{1}[q^c(\varepsilon)-q_0]d\varepsilon}{Y_c+\sigma[\int_0^{\varepsilon_0}q^*(\varepsilon)d\varepsilon+\int_{\varepsilon_0}^{\varepsilon_1}q_0 d\varepsilon+\int_{\varepsilon_1}^{1}q^c(\varepsilon)d\varepsilon]}.$$

where

$$\begin{aligned}Y_c &= x+\sigma[\int_0^{\varepsilon_0}q^*(\varepsilon)d\varepsilon+\int_{\varepsilon_0}^{\varepsilon_1}q_0 d\varepsilon+\int_{\varepsilon_1}^{1}q^c(\varepsilon)d\varepsilon] \\ &\quad +\frac{\sigma(1-\beta)}{\beta}\int_{\varepsilon_1}^{1}[q^c(\varepsilon)-q_0]d\varepsilon.\end{aligned}$$

The parameters from the best fit are in Table 1. The values of (ρ, B) are in the ballpark of existing studies in monetary search literature. We use a consumption equivalence measure to evaluate the plausibility of the value of k. The utility cost $k=0.0739$ is worth 1% of consumption for buyers.

[Insert Table 1 here]

Based on these parameter values, we numerically solve the model and show the results in Figure 4. The upper-left and upper-right panels are the effects of inflation on the threshold ε_1 and the credit to GDP ratio, respectively.[16] The lower panels are the comparisons with a no-credit economy. The lower-left panel presents the total demand for real money balances in the credit economy and the no-credit economy.[17] The lower-right panel shows the welfare improvement of having credit based on a consumption equivalence measure. That is, the number on the vertical axis is the fraction of consumption that a buyer is willing to give up to live in a credit economy instead of a no-credit economy. Welfare $\mathcal{W}$ in this economy is measured by

$$(1-\beta)\mathcal{W} = \sigma\Psi(\varepsilon_0, \varepsilon_1, q_0) + [v(x^*) - x^*] - \sigma(\frac{1-\beta}{\beta})\int_{\varepsilon_1}^{1}[q^c(\varepsilon) - q_0]d\varepsilon, \tag{23}$$

where

$$\Psi(\varepsilon_0, \varepsilon_1, q_0) = \int_0^{\varepsilon_0}[\varepsilon u(q^*(\varepsilon)) - q^*(\varepsilon)]d\varepsilon + \int_{\varepsilon_0}^{\varepsilon_1}[\varepsilon u(q_0) - q_0]d\varepsilon + \int_{\varepsilon_1}^{1}[\varepsilon u(q^c(\varepsilon)) - q^c(\varepsilon) - k]d\varepsilon.$$

Again the third term in equation (23) represents the production distortion associated with credit transactions. Note that welfare is specifically buyers' welfare since sellers in this economy earn 0 surplus from trades and their welfare is 0.

[Insert Figure 4 here]

There are several interesting findings from Figure 4. Inflation induces more buyers to use credit at low inflation rates and fewer buyers to use credit at high inflation rates. This is consistent with the analytical finding from the basic model. Moreover, the credit to GDP ratio predicted by the model has an inverse U-shape against inflation. The numerical results suggest that the fixed cost effect dominates the delayed settlement effect at low inflation rates, but the delayed settlement effect dominates the fixed cost effect at high inflation rates. High inflation makes using credit involve high repayments and hence implies unfavorable terms. This exactly describes the consumer credit market in Brazil during the late 1980s.[18]

Compared to a no-credit economy, credit lowers demand for real money balances at low to moderate inflation rates, but slightly increases demand for real money balances at high inflation rates. As the repayment of credit also requires money, the repayment demand may increase as the inflation rate increases. Overall, credit and money are substitutes at low to moderate inflation rates, but are complements at high inflation rates. The first half of the result can be supported by the empirical work using US data, since the inflation rates in the US have been low to moderate in recent decades. See Duca and Whitesell (1995) for an example.

The lower-right panel reveals that having credit does not always benefit the society in terms of welfare. Knowing that individuals optimally choose to use money versus credit, it seems puzzling that credit can hurt the economy. From equation (23), credit improves welfare by relaxing the liquidity constraint for some buyers, but may hurt welfare because of the production distortion and the fixed cost. Besides these direct effects, credit affects welfare through the general equilibrium effect. As analyzed above, credit may lower the demand for real money balances and thus the value of money, which will generate a negative externality on agents who use money. On the other hand, credit may increase the demand for real money balances and the value of money, which will generate a positive externality on agents who use money. The general equilibrium effect implies that credit may hurt welfare at low to moderate inflation rates, but improve welfare at high inflation rates. Similar results appear in Chiu and Meh (2011). Our numerical results suggest credit improves welfare when the inflation rate exceeds a threshold.

As for the effect of monetary policy, the model predicts that welfare and aggregate output are decreasing in the inflation rate. This is not surprising although the model does introduce a channel through which inflation may potentially increase output at low inflation rates by encouraging more buyers to use credit. However, the effect from this channel does not appear to be strong.

In Figure 4, the threshold value for credit to improve welfare is an inflation rate of around 20%, which is fairly high. The potential problem is that fitting (ρ, B, k) together implicitly assumes that these parameters do not change over one hundred years. However, it is hard to believe that the cost of credit transactions stays constant over time. Nevertheless, since there is no direct data that measures how k evolves over time and the focus of the paper is not to match any moment in the data, we take a simple approach to evaluate the model's predictions by varying k and fixing (ρ, B). To highlight the effect of changing k, we show in Figure 5 the credit to GDP ratio and the welfare improvement when $k = 0.01$, 0.05 and 0.1.

[**Insert Figure 5 here**]

In Figure 5, a lower cost of credit promotes the use of credit. Using the average inflation rate 7.387 from $1969 - 2000$, the predicted credit to GDP ratio is 0.26% and the predicted "money demand" is 0.395 when $k = 0.1$. For $k = 0.01$, the predicted credit to GDP ratio is 10.02% and the predicted "money demand" is 0.201. The low cost credit regime is featured by more credit and less "money demand". It has been noted that there is a trend decline in "money demand" in the recent decade, which has been viewed as a shift of the money demand curve. Clearly, improvements in the credit transaction technology contribute to the trend decline in "money demand".

In terms of welfare, more costly credit makes credit less beneficial to the society. The threshold for credit to be welfare-improving is higher when k is higher. In practice, if sellers receive repayment in the form of money, they may deposit the money to avoid any inflation distortion, which makes credit more beneficial. This type of argument can be built into the model by allowing a fraction of agents to settle in market 2 and the rest to settle in market 3. While this is a nice extension, the current model still serves as a benchmark for analyzing the effect of inflation on credit in a world in which credit is not entirely free of inflation distortion.

As a robustness check, we choose values of (ρ, B) by varying the sample period and evaluate the model's predictions. In these experiments, the value of ρ varies from 0.387 to 0.589, but the value of B which is around 1.4 does not change much. The model's predictions emerging from Figure 4 are robust.[19]

To study how introducing credit affects the welfare cost of inflation, we compute the welfare cost of 10% inflation based on the parameter values in Table 1. The measure of the welfare cost follows the recent literature by using the consumption equivalence measure. The numbers reported in Table 2 are the fraction of consumption a buyer is willing to give up to have 0% inflation rather than 10% inflation. As a benchmark, we compute the welfare cost for a no-credit economy, i.e., $k = \infty$ and hence $\varepsilon_1 = 1$. The welfare cost of 10% inflation is 1.12% in the benchmark economy, which is relatively small because the take-it-or-leave-it offer by buyers avoids the holdup problem.[20]

[Insert Table 2 here]

We then compute the welfare cost of inflation for different values of k. The introduction of credit can raise the welfare cost when credit is costly enough. For low values of k, the cost for buyers to substitute credit for money is relatively low. Therefore, inflation does not generate a large welfare loss. On the contrary, if the cost for buyers to switch from money to credit is high, inflation can result in a higher welfare loss compared to the benchmark economy. Note that if k is too big, no buyer uses credit and the economy is essentially the benchmark economy. Dotsey and Ireland (1996) and Lacker and Shreft (1996) both emphasize that credit costs are quantitatively important as a component of the welfare cost of inflation. The results in Table 2 further confirm their results.

5 EXTENSION: SECURED CREDIT

So far the credit we have considered is unsecured. We now extend the basic model (where buyers use either money or credit) to include a real asset to study the case of secured credit.[21] Suppose a real asset as the standard tree in Lucas (1978) exists and its supply is normalized to 1. In market 2, agents can buy its shares at price ψ and the asset pays dividend $R > 0$, both measured in units of good 2. Following Kiyotaki and Moore (1997) and Gu, Mattesini and Wright (2016), buyers can use this asset in market 1 as collateral to secure loans from financial intermediaries. The size of the loan is limited by the value of the collateral $\theta(\hat{\psi} + R)\mu$, where μ is the amount of real assets that a buyer possesses and $\theta \leq 1$ is the loan-to-value (LTV) ratio. For instance, $\theta = 0.8$ indicates that financial intermediaries are willing to lend up to 80% of the total value of the collateral. Again, we assume that financial intermediaries can enforce the repayment of loans and settlement is delayed: in market 3, buyers repay loans with money. No other types of loans are available to buyers.

With secured loans, a buyer's choice problem is market 2 becomes

$$V_2^b(m, \mu, \ell) = \max_{x,h,z,\hat{\mu}} \left\{ v(x) - h + V_3^b(z, \hat{\mu}, \ell) \right\}$$

$$\text{st. } x + \phi z + \psi\hat{\mu} = \phi m + h + \phi\tau + (\psi + R)\mu,$$

where $\hat{\mu}$ is the units of the real asset that the buyer brings into market 3. The buyer's value functions in market 3 and market 1 remain almost identical to those in the basic model except for the additional state variable μ to be included: $V_3^b(z, \hat{\mu}, \ell) = \beta V_1^b(z - \ell, \hat{\mu}, 0)$ and $V_1^b(m, \mu, 0) = \sigma[u(q) - k \cdot I(a) + V_2^b(m - d, \mu, a \cdot I(a))] + (1 - \sigma)V_2^b(m, \mu, 0)$.

In market 1, the buyer can use money or secured credit to purchase good 1. Terms of trade in cash transactions remain the same as in the basic model. For credit transactions, we need to amend the buyer's problem to include an additional constraint - a borrowing constraint which is irrelevant in the basic model:

$$\max_{q,d,a}[u(q) - k - \phi d - \phi a]$$

$$\begin{aligned} \text{st. } q &= \phi d + V_2^s(0, \mu, -a) - V_2^s(0, \mu, 0) \\ d &\leq m, \text{ and} \\ \phi \hat{a} &\leq \theta(\hat{\psi} + R)\mu. \end{aligned}$$

The borrowing constraint $\phi \hat{a} \leq \theta(\hat{\psi} + R)\mu$ implies that the size of the debt never exceeds the value of the collateral. Note that the first constraint can be simplified to $q = \phi d + \hat{\beta}\phi a$ due to the linearity of seller's value function and the fact that $z = 0$ for all sellers.

Following the same procedure we can derive the two conditions characterizing optimal choice of z and $\hat{\mu}$ as equation (13) and

$$u'\left(q^c\right) = 1 + i + \frac{\frac{\psi}{\beta(\psi+R)} - 1}{\sigma\theta\beta}. \tag{24}$$

Interestingly, replacing unsecured credit by secured credit does not affect the amount of goods traded with money. However, comparing equations (24) with (11) reveals that amounts of goods traded in credit transactions q^c across the two different types of credit (unsecured credit versus secured credit) coincide if and only if $\psi = \beta(\psi + R)$. This result is intuitive: the term $\psi - \beta(\psi + R)$ measures the cost of carrying an additional unit of the asset across periods. In any credit equilibrium, $\psi \geq \beta(\psi + R)$ because otherwise agents' demand for the asset will be infinite. When $\psi = \beta(\psi + R)$, the cost of transferring assets across periods is zero and agents are indifferent between holding assets or not. In this case, the borrowing constraint is not binding. The model with secured credit achieves the same q^c as in the basic model. If $\psi > \beta(\psi + R)$, the cost of carrying assets across periods becomes strictly positive, which reduces buyers' demand for real assets as well as q^c. In this case, the borrowing constraint must be binding and we can solve for the equilibrium value of ψ from the asset market clearing condition $\mu = 1$. It is worth remarking that there are two types of inefficiencies associated with using secured credit: one due to the delayed settlement and the other due to the cost of carrying assets. The delayed settlement implies that increases in i reduce q^c as in the basic model, whereas the last term in equation (24) captures the cost of holding the asset. Hence, $q^c = q^*$ if and only if $i = 0$ and $\psi = \beta(\psi + R)$.

To facility the comparison between secured and unsecured credit, we focus on the special case $\psi = \beta(\psi + R)$ in what follows.[22] Only in this special case, the economy with secured credit can replicate the allocation achieved with unsecured credit. The existence of credit equilibrium requires the asset market clearing condition to hold, which implies $1 \geq \mu \geq q^c/[\theta\beta(\psi + R)]$. The first inequality requires that buyers' demand for assets μ cannot be greater than its supply in equilibrium. The second inequality follows from the buyer's borrowing constraint, where the term $q^c/[\theta\beta(\psi + R)]$ measures the amount of asset used as collateral by buyers to purchase q^c. The asset market clearing condition generates restrictions on exogenous parameters θ and R. Assuming the same isoelastic utility function $u(q) = q^\rho/\rho$, the sufficient condition to guarantee $1 \geq q^c/[\theta\beta(\psi + R)]$ is $\theta R\beta/(1-\beta) \geq 1$.[23] Note that the term $R\beta/(1-\beta)$ is the present discounted value of dividends. In other words, the values of the dividend R and/or the LTV ratio θ must be high enough such that buyers are willing to hold a sufficient amount of asset as collateral to purchase q^c.

To summarize, when credit takes the form of collateralized lending secured by real assets, the equilibrium allocation replicates the allocation with unsecured credit if and only if the asset price satisfies $\psi = \beta(\psi + R)$. In this case, all main results in the basic model hold. The sufficient condition that guarantees the existence of a credit equilibrium is $\theta R\beta/(1-\beta) \geq 1$.

6 CONCLUSION

Both money and credit are widely used as means of payment. It is important to understand how the use of credit affects money demand and hence the effects of monetary policy. We construct a model in which money and credit serve as a means of payment and money is the means of settlement.[24] There are two frictions associated with using credit – a fixed utility cost and the delayed settlement. The models show that inflation hampers the use of credit at very low and very high inflation rates. Our numerical results predict an inverse U-shape relationship between inflation and the credit to GDP ratio. Costly credit does not always improve social welfare. Depending on the fixed utility cost of credit, allowing credit as a means of payment may raise the welfare cost of inflation.

The optimal monetary policy in our models is always the Friedman rule, at which credit is driven out of existence. As our main results on the interaction between money and credit hinge on monetary policy being away from the Friedman rule, it will be useful to modify the model to allow credit to have a role even at the socially optimal monetary policy. Moreover, both the Bank of Canada and the Federal Reserve Bank of Boston have recently started to collect data on consumer payment choice. It will be of interest to use these data to conduct more serious calibrations of our model or similar models with endogenous choice of payment methods. In this way, we can learn quantitatively how advances in payment technologies and monetary policy affect consumer payment choice and social welfare. These are left for future research.

A APPENDIX

A.1 Proof of Lemma 1

PROOF. When i approaches 0, $q^m = q^c = q^*$. So we have $\lim_{i\to 0}\Delta(i) = -\beta\sigma k$ and $\lim_{i\to 0}\Delta'(i) = \beta(1-\sigma)q^*$. When i approaches ∞, both q^m and q^c approaches 0 so that $\lim_{i\to\infty}\Delta(i) = -\beta\sigma k$ and $\lim_{i\to\infty}\Delta'(i) = 0$. It implies that $i \to \infty$ is a solution to $\Delta'(i) = 0$. We know that $q^c(i) = q^m(i) = 0$ if and only if $i \to \infty$. When $i < \infty$ so that $q^c(i) > 0$ and $q^m(i) > 0$, solving $\Delta'(i) = 0$ is equivalent to solving $q^m(i)/q^c(i) = \sigma$. Define $\Psi(i) = q^m(i)/q^c(i)$. We know that $\Psi(i \to 0) = 1$ and

$$\Psi'(i) = \frac{q^c(i)\frac{dq^m(i)}{di} - q^m(i)\frac{dq^c(i)}{di}}{\left[q^c(i)\right]^2}.$$

From equations (11) and (13),

$$\frac{dq^c(i)}{di} = \frac{1}{u''(q^c)} \text{ and } \frac{dq^m(i)}{di} = \frac{1}{\sigma u''(q^m)}.$$

It follows that the sign of $\Psi'(i)$ depends on the sign of

$$\frac{1}{\sigma q^m(i)\, u''(q^m)} - \frac{1}{q^c(i)\, u''(q^c)}. \tag{A1}$$

Notice that equations (11) and (13) also imply

$$\frac{u'(q^c) - 1}{u'(q^m) - 1} = \sigma \text{ for } i > 0.$$

The sign of $\Psi'(i)$ then depends on the sign of

$$\frac{u'(q^m) - 1}{q^m u''(q^m)} - \frac{u'(q^c) - 1}{q^c u''(q^c)}.$$

Proof. Suppose the isoelastic utility function is $u(q) = q^\rho/\rho \;\forall\; 0 < \rho < 1$. We have

$$\frac{u'(q) - 1}{qu''(q)} = \frac{1 - q^{\rho-1}}{(1-\rho)\, q^{\rho-1}},$$

which is increasing in q. Since $q^m < q^c$, we know the sign of equation (A1) is negative. It implies that $\Psi(i)$ is decreasing in i for $i > 0$. Furthermore, $\Psi(i \to \infty) = \sigma^{1/(1-\rho)} < \sigma$. Therefore, there is a unique solution to $\Psi(i) = \sigma$. It implies that there is a unique interior solution to $\Delta'(i) = 0$. Let $i^{\max}$ denote the solution. To check the solution is a maximum, we take the second derivative of $\Delta(i)$,

$$\Delta''(i) = \beta\left[\frac{dq^m(i)}{di} - \sigma\frac{dq^c(i)}{di}\right] = \beta\left[\frac{1}{\sigma u''(q^m)} - \frac{\sigma}{u''(q^c)}\right].$$

Evaluating the above expression at $i^{\max}$, we can substitute σ by q^m/q^c and

$$\Delta''(i^{\max}) = \beta q^c q^m\left[\frac{1}{q^{m2}u''(q^m)} - \frac{1}{q^{c2}u''(q^c)}\right].$$

One can show that $\Delta''(i^{\max}) < 0$ given the isoelastic utility function. It follows that $i^{\max}$ maximizes $\Delta(i)$. ■

A.2 Proof of Proposition 1

PROOF. In Lemma 1, we know that there is a unique interior solution to $\Delta(i) = 0$ and the solution gives to a local maximum. As long as $\Delta(i^{\max}) > 0$, a credit equilibrium exists for some i. Now recall that $\lim_{i\to 0}\Delta(i) = \lim_{i\to\infty}\Delta(i) = -\beta\sigma k$. When $\Delta(i^{\max}) > 0$, it means that $\Delta(i) = 0$ has two solutions. That is, $\Delta(i)$ is first increasing in i until it reaches $\Delta(i^{\max})$ and then is decreasing in i. These solutions are labelled as $\underline{i}$ and $\bar{\imath}$ where $\underline{i} < \bar{\imath}$. Therefore, $\Delta(i) > 0$ only when $\underline{i} < i < \bar{\imath}$. A credit equilibrium exists only when $\underline{i} < i < \bar{\imath}$. For $i < \underline{i}$ or $i > \bar{\imath}$, $\Delta(i) < 0$ and only a monetary equilibrium exists. It is easy to check from the second order condition that whenever a monetary equilibrium or a credit equilibrium exists, it must be unique.

A.3 Proof of Lemma 2

PROOF. As discussed in the paper, if a buyer uses credit, he must spend all his money. Define $\bar{q}$ as the solution to $u'(q) = c'(q)$. One can show that $\phi m < c(\bar{q})$ as long as $\gamma > \beta$. It implies that there exists a threshold ε_0 such that $\varepsilon_0 u'(q_0) = 1$ where q_0 is from $q_0 = \phi m$. For buyers with $\varepsilon > \varepsilon_0$,

$$T'(\varepsilon) = \sigma[\varepsilon u'(q^c) - (1+i)]\frac{dq^c}{d\varepsilon} - \sigma[\varepsilon u'(q^m) - 1]\frac{dq^m}{d\varepsilon} + \sigma[u(q^c) - u(q^m)].$$

Notice that $\varepsilon u'(q^c) = 1 + i$ from equation (11) and $dq^m/d\varepsilon = 0$. Because $q^c > q^m$, it follows that $T'(\varepsilon) > 0$ for $\varepsilon > \varepsilon_0$. Define ε_1 such that $T(\varepsilon_1) = 0$. If an interior ε_1 exists, then [1] buyers with $\varepsilon_0 < \varepsilon < \varepsilon_1$ are liquidity constrained and do not use credit; and [2] buyers with $\varepsilon > \varepsilon_1$ are liquidity constrained and use credit.

A.4 Proof of Proposition 2

PROOF. Notice that equation (21) determines q_0, where ε_0 and ε_1 are functions of q_0. From equation (19), one can solve for $\varepsilon_0(q_0)$ and $d\varepsilon_0/dq_0 > 0$. To solve for ε_1, both equations (20) and (22) are used. The choice problem of q_0 can be rewritten as

$$\frac{1}{2}\frac{1}{u'(q_0)} + \frac{1}{2}u'(q_0)\varepsilon_1^2(q_0) - (1+i)\,\varepsilon_1(q_0) = i\left(\frac{1}{\sigma} - 1\right).$$

As we focus on $q_0 \in [0, q^*]$ where q^* is given by $u'(q) = 1$, a solution of q_0 must exist by the Theorem of Maximum. For any $i > 0$, $q_0 > 0$ for any monetary equilibrium with or without credit. It follows that monetary equilibrium must exist for $i > 0$.

Proof. When the monetary authority implements $i \to 0$, $\varepsilon_0 = \varepsilon_1 = 1$ and $q(\varepsilon) = q^*(\varepsilon)$ for all ε. The Friedman rule achieves the efficient allocation. ■

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Footnotes

1. We focus on credit card loans for two reasons: First, credit card loans are the most common form of consumer credit. Consumer credit is the credit extended to individuals for household, family, and other personal expenditures, excluding loans secured by real estate. Credit card loans account for roughly half of all unsecured debt in the United States (Federal Reserve, 2005). Second, it is natural to model consumer credit in the New Monetarist literature as the models explicitly describe the decentralized trading environment including the choices of payment methods between buyers and sellers.

2. A recent paper by Schuh and Stavins (2014) also emphasizes "credit cards authorize payments to be settled much later and hence are based on medium-term debt (up to 30 days) rather than money". Checks can be viewed as a short-term form credit. However, since the time until settlement is typically very short for checks, checks are not considered as "settled later" in this paper.

3. Data source: ECRI Lending to Households in Europe (1995-2011). Our sample includes 24 countries: Australia, Belgium, Bulgaria, Canada, Germany, Greece, Spain, Finland, France, Hungary, Italy, Lithuania, Malta, Netherlands, Poland, Portugal, Romania, Sweden, Slovenia, Slovakia, Switzerland, Turkey, UK and US. Consumer credit corresponds to the outstanding amounts (stocks) of loans at the end of the year granted by the resident monetary financial institution sector to resident households and non-profit institutions serving households for consumption purposes. Consumer credit includes loans related to credit cards as well as overdrafts.

4. Evidence based on the total private credit to GDP ratio, a broader measure of credit, suggests that inflation tends to have a negative impact on credit at high rates, but not at low rates. Using a sample of 97 countries, Boyd, Levine and Smith (2001) conclude that inflation has a negative impact on credit. See also Boyd and Champ (2003). Later, Khan, Senhadji and Smith (2006) use a large cross-country sample, but they find that there is a threshold effect of inflation on credit. Inflation has a negative impact on credit when it exceeds a threshold.

5. The fixed cost of using credit has been assumed in some cash-in-advance models and OLG models. For cash-in-advance models, see Lacker and Shreft (1996), Aiyagari, Braun and Eckstein (1998), and English (1999) for examples. For the OLG framework, see Freeman and Huffman (1991) for an example.

6. Azariadis and Smith (1996) obtain a similar result. However, the key friction driving their result is asymmetric information associated with credit.

7. The literature on money and credit is vast and thus we do not intend to review all these papers. Rather, we only mention those articles that are directly related to this paper. For recent attempts to rationalize the coexistence of inside and outside money, see Cavalcanti and Wallace (1998), Kocherlakota and Wallace (1998), Mills (2007), and Sun (2011). Lester, Postlewaite and Wright (2012) study the coexistence of multiple assets that differ in their return and liquidity.

8. We model financial intermediaries as a third-party that possesses the record-keeping technology. This is a convenient way to model the settlement of credit as a separate stage, which is crucial for our main results. Alternatively one could allow a fraction of sellers to have access to the record-keeping technology and thus can make loans directly to buyers, such

as in Gu, Mattesini and Wright (2016), Lotz and Zhang (2016), and Araujo and Hu (2018). To generate a role for both money and credit, these models consider two frictions: limited commitment and imperfect monitoring. The assumption of limited commitment generates endogenous debt limits that satisfy the incentive compatibility constraints for repayment. With imperfect monitoring (i.e. only a fraction of sellers can access past records), credit coexists with money. Such an environment is more complicated as the issues of repayment and enforcement naturally arise. Understanding how these frictions affect coessentiality of money and credit could be of interest in its own, however, it is not the intended focus of this paper.

9. One may argue that in reality sellers actually pay the cost of using credit. The model can be modified to allow the seller to pay the fixed cost in a match. All the main results hold. See Nosal and Rocheteau (2011) and Lotz and Zhang (2016) for examples of models where sellers incur the fixed cost to use credit.

10. Note that the IOU in Figure 2 refers to the record kept by the financial intermediaries. Financial intermediaries do not issue physical IOUs. We also rule out the existence of private claims on credit.

11. If sellers can deposit the money from repayment in a bank and earn interest as in Berentsen, Camera and Waller (2007), then they will be fully compensated and the terms of trade will not be affected by inflation. However, a full general equilibrium analysis that incorporates bank deposits involves more careful consideration. For example, since financial intermediaries are not available in market 2, sellers cannot withdraw their deposits (if there are any) in market 2, which impedes the use of deposits. It is also worth mentioning that the only role played by financial intermediaries in our model is to offer credit (record goods

transactions in market 1 and settle repayment in market 3). In practice financial intermediaries have many other roles, such as taking deposits and issuing interest-bearing liabilities. This paper abstracts from these other roles as we focus on the relationship between money and credit.

12. The result that the Friedman rule is the optimal monetary policy is robust across many other models of money and credit with unconstrained monetary policy, such as Berentsen, Camera and Waller (2007), Gu, Mattesini and Wright (2016), Lotz and Zhang (2016), and Telyukova and Wright (2008).

13. The literature have made a few attempts to generate coexistence of money and credit at the optimal policy. One approach is to let the optimal monetary policy deviate from the Friedman rule. For example, in Monnet and Roberds (2008) some agents are always liquidity constrained, even at the Friedman rule. The optimal monetary policy deviates from the Friedman rule and credit has is essential to support consumption by the liquidity constrained agents at the optimal policy. Another example is Sanches and Williamson (2010) in which money is subject to theft. Some inflation is useful to prevent theft and credit coexists with money under the optimal monetary policy. In both cases, monetary policy is unconstrained and the Friedman rule is implementable. The other approach to generate the essentiality of credit is to restrict to feasible policies that respect underlying constraints and technology. For example, Gomis-Porqueras and Sanches (2013) find that a positive inflation rate is optimal because it improves the credit system. Araujo and Hu (2018) show that money and credit are essential under the optimal policy in a model with limited commitment and limited monitoring. In both papers, some inflation hurts monetary trades but benefit credit trades by relaxing the credit constraints.

14. A novel term in equation (14) is $(1-\beta)\, q^c/\beta$, which captures production distortion from using credit. Consider a seller who extends credit in market 1. He receives payment from the financial intermediary in market 3 and must wait till the next market 2 to spend the money. As discussed earlier, the buyer in this transaction should pay nominal interest to compensate the seller. The cost is reflected by the term $(1-\beta)q^c/\beta$.

15. The original data are from Craig and Rocheteau (2008).

16. The predicted credit to GDP ratio from the model is

$$\frac{\sigma \int_{\varepsilon_1}^{1} [q^c(\varepsilon) - q_0] d\varepsilon}{Y_c + \sigma[\int_0^{\varepsilon_0} q^*(\varepsilon) d\varepsilon + \int_{\varepsilon_0}^{\varepsilon_1} q_0 d\varepsilon + \int_{\varepsilon_1}^{1} q^c(\varepsilon) d\varepsilon]}.$$

17. The total demand for real money balance is

$$\phi M = q_0 + \frac{\sigma}{\beta} \int_{\varepsilon_1}^{1} [q^c(\varepsilon) - q_0] d\varepsilon,$$

where q_0 reflects the transaction demand for real money balance.

18. Due to the long time delay in credit card charges clearing through the banking system, vendors have been documented to normally add on a 20 to 30 percent surcharge to the price of the purchased item. In this way, vendors can protect themselves from the depreciation of money during the time the vendors are waiting to be paid by the credit card companies.

19. Taking one year as the period length in the model implies that the settlement is delayed for one year, which seems to be too long. To address this issue, we can take one month or one quarter as the model's period length and compute the model's annual "money

demand" by aggregating from monthly or quarterly "money demand". We can also take one quarter as the period length and use quarterly data on treasury bill rate per annum, M1 and GDP from IFS. The quarterly "money demand" data is computed by converting the treasury bill rate and GDP into quarterly measures. All these methods do not change the qualitative predictions of the model including the inverse U-shape relationship between inflation and credit. However, the quantitative predictions depend on the specific method and the initial values assigned to (ρ, B, k).

20. In Lagos and Wright (2005), the welfare cost of 10% inflation is 1.4% when buyers have all the bargaining power. Our result does not deviate from their estimate.

21. This extension was inspired by an anonymous referee, who made excellent suggestions concerning ways to incorporate real assets into our model.

22. In credit equilibrium where $\psi > \beta\left(\psi + R\right)$, assets are costly to carry across periods. Buyers hold enough assets to use as collateral to purchase q^c and sellers do not hold any assets. We can solve for equilibrium q^c and ψ from equation (24) and the asset market clearing condition $\mu = 1$. Then we can solve the buyer's decision on using credit or not following sane steps as in the basic model.

23. To see why this is the case, note that $q^c = q^* = 1$ at the Friedman rule when $\psi = \beta(\psi + R)$. Thus, $q^c/[\theta\beta(\psi + R)]$ can be simplified into $(1 - \beta)/(\theta R \beta)$. Recall that q^c decreases with γ; therefore, $1 \geq q^c/[\theta\beta(\psi+R)]$ is always satisfied as long as $\theta R \beta/(1-\beta) \geq 1$.

24. We also consider a modified environment where enforcement of credit repayment is imperfect following Berentsen, Camera and Waller (2007). In this case, a buyer who defaults will be excluded from using credit in all future periods. An incentive compatibility constraint that ensures repayment of credit generates an endogenous credit limit. From the

numerical examples, this credit limit tends to increase at low inflation rates, but decrease at high inflation rates. However, credit limits only bind at very low inflation rates. Interestingly, imperfect enforcement may improve social welfare because it avoids socially inefficient borrowing.

Table and Figures

Tables

Parameters	ρ	B	k
Values	0.4732	1.4436	0.0739

Table 1: Parameter Values

k	benchmark	0.01	0.05	0.0739	0.1
Welfare Cost	1.12%	0.22%	0.85%	1.32%	1.40%

Table 2: Welfare Cost of 10% Inflation

Figures

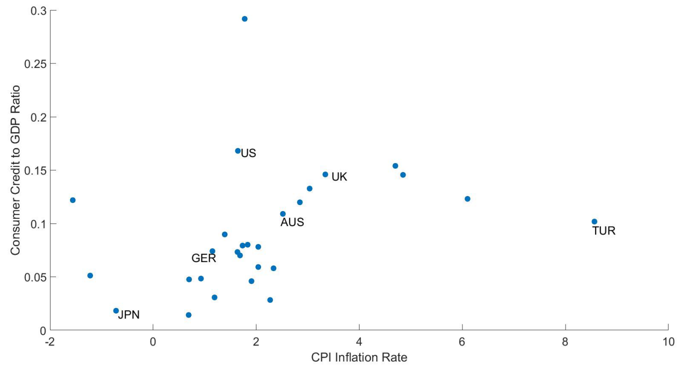

Figure 1: Inflation and Consumer Credit

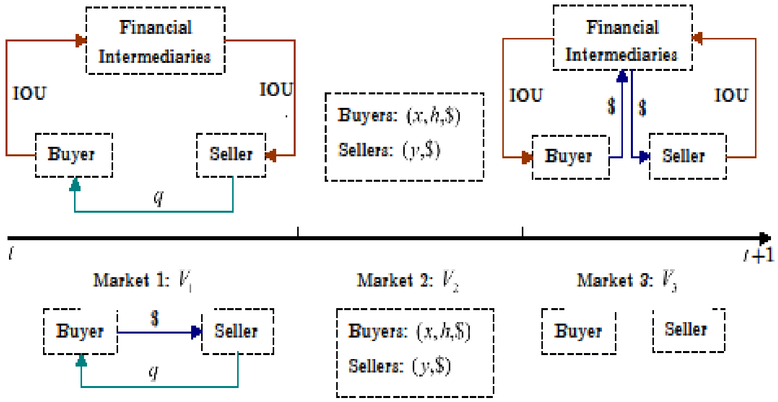

Figure 2: Timeline of Events

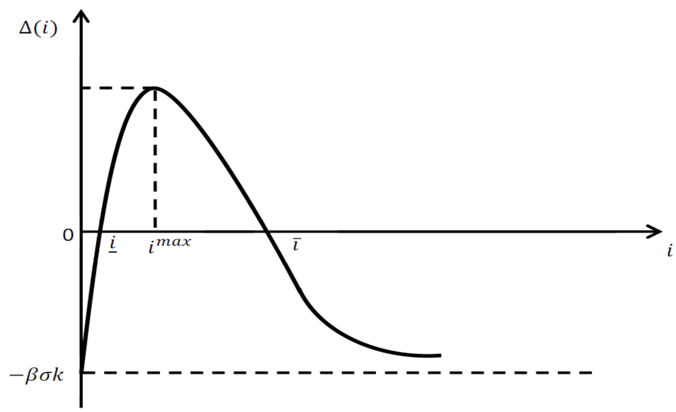

Figure 3: Net Benefit of Using Credit

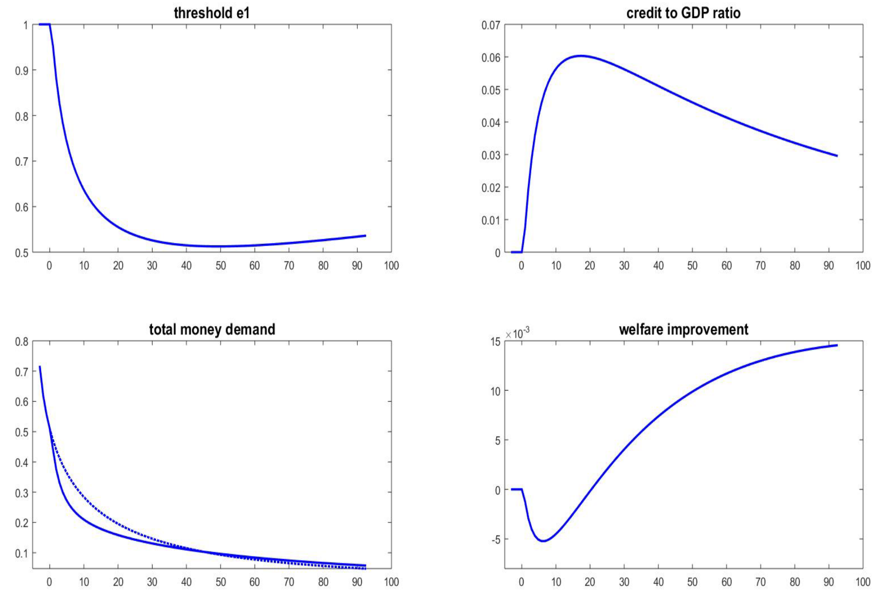

Figure 4: Effects of Inflation

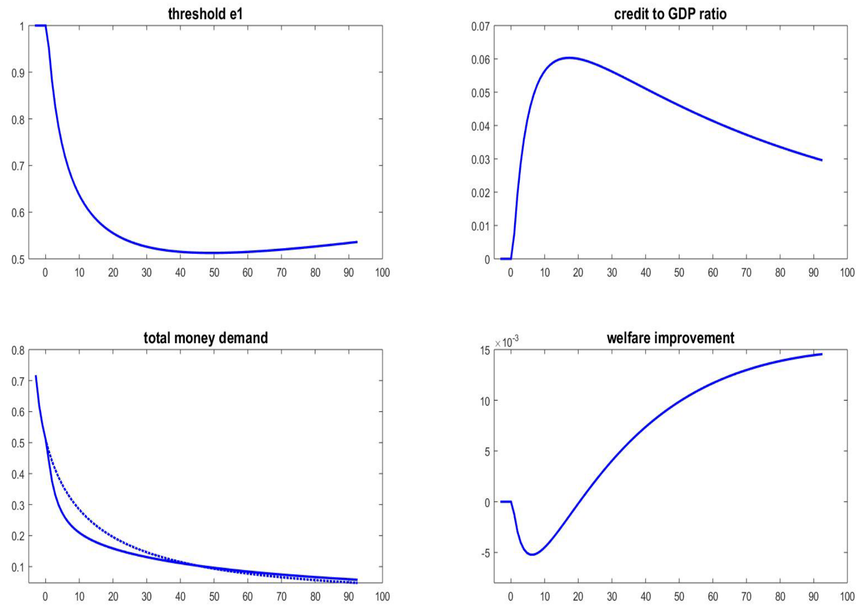

Figure 5: Comparative Statics – Varying k