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1     **MUSCLE CONTRIBUTIONS TO TAKE-OFF VELOCITY IN THE LONG JUMP**

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22 **ABSTRACT**

23 **Purpose:** A key determinant of long jump performance is the ability to increase the vertical  
24 velocity of the center of mass (COM) while minimizing the loss in forward velocity (running  
25 speed) during the take-off phase, but exactly how this occurs is not fully understood. We  
26 combined a three-dimensional musculoskeletal model of the body with dynamic optimization  
27 theory to simulate the biomechanics of the long jump take-off and determine the  
28 contributions of the individual leg muscles to jump performance. **Methods:** The body was  
29 modelled as a 29-degree-of-freedom skeleton actuated by a combination of muscles and net  
30 joint torques. A dynamic optimization problem was solved to reproduce full-body motion and  
31 ground-force data recorded from experienced sub-elite jumpers. The optimization solution  
32 then was analyzed to determine each muscle's contribution to the ground-force impulse and  
33 hence the change in velocity of the COM during the take-off phase. **Results:** The hip, knee  
34 and ankle extensors dominated the change in velocity of the COM during take-off. VAS  
35 generated the highest support impulse and contributed one-third (33%) of the increase in  
36 vertical COM velocity generated by all the muscles. SOL and GMAX also developed  
37 substantial support impulses and contributed 24% and 16% of the increase in vertical COM  
38 velocity, respectively. VAS also generated the highest braking impulse and contributed  
39 approximately one-half (55%) of the loss in forward COM velocity generated by all the  
40 muscles, while SOL and GMAX made much smaller contributions (12% and 7%,  
41 respectively). **Conclusions:** VAS, SOL, and GMAX contributed nearly three-quarters (73%)  
42 of the increase in vertical COM velocity at take-off, suggesting that these muscles ought to be  
43 prioritized in strength training programs aimed at improving long jump performance.

44 **Keywords:** sprinting, impulse, muscle training, quadriceps, gluteal, ankle plantarflexors

45

**INTRODUCTION**

46

47 Elite long jumpers run up at fast speeds, lower their centers of mass in the last few strides, and  
48 plant their legs at relatively steep angles to the ground to propel their bodies as far as possible  
49 (1, 2). There are four distinct phases of the long jump – approach, take-off, flight, and landing  
50 – but the take-off phase, defined as the period from touchdown to take-off, is thought to be the  
51 most critical (3-5). A key determinant of jump performance is the ability to increase the vertical  
52 velocity of the center of mass (COM) while minimizing the loss in forward velocity during the  
53 take-off phase, but exactly how this happens is not fully understood, primarily because there is  
54 no information on how the leg muscles generate the required ground force impulse and change  
55 in velocity of the COM.

56 Many investigators have measured the kinematic and kinetic patterns used by elite and sub-  
57 elite athletes to produce a successful long jump (1, 3-6). Hay (1) reported that elite male long  
58 jumpers reach speeds slightly higher than 10 m/s four strides before take-off. During the last  
59 four strides they also lower their COMs by approximately 10 cm relative to the approach height  
60 to lengthen the vertical path over which the body is accelerated, thus increasing the vertical  
61 velocity of the COM at take-off (1). From touchdown to take-off, vertical velocity is increased  
62 by 3-4 m/s while forward velocity is reduced by 1-2 m/s (3). The corresponding force  
63 transmitted to the leg during take-off can be remarkably high: during the initial impact the peak  
64 vertical ground reaction force is around 11 times body weight (BW) whereas the peak fore-aft  
65 braking force reaches ~4.5 BW (7).

66 Lees and co-workers (4-6) proposed a mechanism which they called the ‘pivot’ to explain the  
67 observed increase in vertical velocity of the COM during take-off. They reasoned that an  
68 upward velocity is generated at the expense of forward velocity because the body pivots about  
69 a stationary foot after touchdown in much the same way as an inverted pendulum pivots about

70 a fixed support. Dapena and Chung (8) used a kinematic model of the take-off phase of the  
71 high jump that may also be relevant to the long jump. They explained that radial motion of the  
72 COM toward the stationary foot after touchdown allows the jumper to increase the vertical  
73 velocity of the COM by generating a higher vertical ground reaction force and therefore a  
74 higher ground force impulse. Alexander (2) and later Seyfarth et al. (9, 10) developed simple  
75 muscle-actuated computational models to simulate the long jump take-off. Alexander (2) found  
76 that the longest jump is achieved at the highest run-up speed with the leg set down at an angle  
77 of  $70^\circ$  relative to the ground, while Seyfarth et al. (10) showed that the necessary increase in  
78 vertical velocity could be generated by rapid stretching (eccentric contractions) of the knee  
79 extensor muscles during take-off. An important limitation of these models is that they were  
80 unable to reproduce the first peak in the vertical ground reaction force generated during the  
81 initial impact. Taking an entirely different approach, Graham-Smith and Lees (5) used  
82 multivariate regression analyses to show that the vertical COM velocity during take-off  
83 increases with a more extended knee and a lower peak knee flexion velocity at touch-down,  
84 whereas the loss in forward COM velocity is minimized by reducing hip adduction and  
85 increasing hip extension.

86 Few studies have recorded muscle electromyographic (EMG) activity during the long jump  
87 take-off. Kakihana and Suzuki (11) used kinematic, kinetic, and muscle EMG data to explain  
88 the different techniques used by two elite jumpers in terms of the coordinated actions of their  
89 leg muscles. They found that higher levels of activity in the hip and knee extensors during the  
90 first half of the take-off phase correlated with an increase in the vertical velocity of the COM  
91 and the distance jumped. No study to our knowledge has described how the individual leg  
92 muscles generate the observed changes in vertical and forward velocities of the COM during  
93 the take-off phase.

94 The main aim of the present study was to determine the contributions of the individual lower-  
95 limb muscles to performance of the long jump (jump distance). We combined a three-  
96 dimensional musculoskeletal model of the body with dynamic optimization theory to determine  
97 the contributions of the individual leg muscles to the ground force impulse and change in  
98 velocity of the COM during the take-off phase. Our specific aims were to (1) simulate the  
99 biomechanics of the long jump take-off by accurately reproducing full-body motion and ground  
100 reaction force data recorded from experienced sub-elite jumpers; and (2) identify those muscles  
101 responsible for generating the observed increase in vertical velocity of the COM and the  
102 concomitant loss in forward velocity during the take-off phase. A comprehensive  
103 understanding of the functional roles of the individual leg muscles during the long jump take-  
104 off would help to improve training methods aimed at maximizing jump performance.

## 105 **METHODS**

106 Nine experienced long jumpers (4 males, 5 females; age  $19.1 \pm 1.5$  years; mass  $57.4 \pm 6.1$  kg;  
107 height  $171 \pm 8$  cm) volunteered to participate in this study. The study was approved by the  
108 Institutional Review Board of Fu Jen Catholic University and each participant gave written  
109 informed consent prior to data collection. All jumps were performed in an indoor track-and-  
110 field facility and were unaided by wind. A maximum run-up distance of 12 m was available  
111 prior to take-off. After landing on the take-off board during the final stride, the participant  
112 jumped as far as possible into a sandpit. Each participant performed three trials, and the longest  
113 jump was selected for further analysis. Approximately 4-5 minutes of rest time was provided  
114 between each trial to minimize fatigue.

115 Forty-seven reflective markers were placed on prescribed bony landmarks located on 14 body  
116 segments, including the head, trunk, upper arm, forearm, thigh, shank, and foot segments (12).  
117 An infrared motion capture system sampling at 250 Hz (Motion Analysis, Rohnert Park, CA,

118 USA) was used to measure the three-dimensional positions of the markers for the duration of  
119 the take-off phase, while the corresponding ground reaction force (GRF) was measured  
120 simultaneously using a force plate sampling at 2,000 Hz (AMTI, Watertown, WA, USA). The  
121 force plate was placed at the take-off location with the front edge of the plate aligned with a  
122 foul line marked on the ground. In the present study, a fair jump was determined by the take-  
123 off foot landing behind the foul line, and jump performance was taken as the distance measured  
124 from the toes of the take-off foot to the rear-most point of the body at landing. The force-plate  
125 data were filtered using a third-order Butterworth IIR low-pass filter with a cut-off frequency  
126 of 50 Hz. Joint angles were estimated from the measured marker positions by implementing an  
127 inverse kinematics algorithm available in OpenSim 3.3, and these data were then filtered using  
128 a third-order IIR low-pass filter with a cut-off frequency of 15 Hz. Muscle electromyographic  
129 (EMG) data were recorded from a subset of three subjects using a wireless EMG system  
130 (DELSYS Inc., Natick, MA, USA) sampling at 1,962 Hz. EMG data were recorded from six  
131 muscles of the take-off leg: gluteus medius, soleus, medial gastrocnemius, vastus medialis,  
132 rectus femoris, and semitendinosus. The muscle EMG data were filtered using a fourth-order  
133 Butterworth high-pass filter with a cut-off frequency of 20 Hz, and subsequently full-wave  
134 rectified and filtered using a fourth-order Butterworth low-pass filter with a cut-off frequency  
135 of 8 Hz. A high-pass filter was used to remove noise from the raw EMG signal caused by sensor  
136 motion artifact (13). Full-wave rectification and low-pass filtering were then used to produce  
137 a linear envelope of the EMG signal (14). The processed EMG data were normalized by the  
138 peak EMG activity recorded for one complete trial of the long jump encompassing the run-up,  
139 approach, take-off and flight phases. All experimental data were extracted to cover the entire  
140 take-off phase as well as a pre-contact period equivalent to 30% of the take-off phase. The  
141 early, middle, late, and terminal portions of the take-off phase were defined as 0-20%, 20-60%,  
142 60-80%, and 80-100% of ground contact time, respectively.

143 Participant-specific computer simulations of the take-off phase of the long jump were  
144 generated based on a generic model of the body (15). The skeleton was represented as a 29  
145 degree-of-freedom (DOF) articulated linkage, with the pelvis represented as a 6-dof rigid body.  
146 The shoulder and hip joints were each modeled as a ball-and-socket joint whereas the elbow,  
147 knee, ankle, and metatarsophalangeal (MTP) joints were each represented as a hinge. The  
148 ipsilateral take-off leg (i.e., the leg in contact with the ground during the take-off phase) was  
149 actuated by 40 muscle-tendon units while the MTP joint (16), contralateral swing leg, and arms  
150 were driven by ideal torques. Each musculotendon unit was modeled as a Hill-type muscle in  
151 series with an elastic tendon (17). Peak isometric muscle force,  $F_0^M$ , was calculated using the  
152 muscle volume data reported by Miller et al. (18) and assuming a muscle specific tension (i.e.,  
153 muscle force divided by muscle physiological cross-sectional area) of 60 N/cm<sup>2</sup> (19). Muscle  
154 activation dynamics was assumed to be first-order (20) with activation and deactivation time  
155 constants of 0.01 and 0.04, respectively (21). Six Hunt-Crossley contact spheres were used to  
156 model the interaction of the take-off leg with the ground (22): four spheres were placed under  
157 the hindfoot segment and two spheres under the toes segment. The static and dynamic frictional  
158 coefficients were assumed to be 0.9 and 0.3 respectively, for all contact spheres.

159 Participant-specific musculoskeletal models were created using a built-in scaling algorithm  
160 available in OpenSim 3.3. Segment-specific scaling factors were found by comparing the  
161 distance between virtual markers defined in the generic model and corresponding marker  
162 positions measured on each participant. These scaling factors were used to scale the segment  
163 lengths, segment inertial properties, and muscle attachment sites for each participant. An  
164 inverse kinematics analysis then was performed to calculate the joint angles by minimizing the  
165 differences between the measured marker positions and the positions of the virtual markers  
166 assigned to the model. Participant-specific simulations were generated by solving a dynamic  
167 optimization problem to track the joint angles and GRFs measured during the take-off phase.

168 The dynamic optimization problem was solved by discretizing each state (i.e., joint angles,  
169 joint angular velocities, muscle forces, and muscle activations) into 100 evenly-spaced nodes  
170 (23, 24). Values of the states at each node were found by minimizing the sum of all muscle  
171 activations as well as the differences between the model-predicted and measured joint angles  
172 and GRFs. The dynamic optimization solution was computed using a modified version of the  
173 direct collocation algorithm developed by Lin and Pandy (23) (see Supplementary Digital  
174 Content 1, which presents details relating to the dynamic optimization problem for the long  
175 jump take-off and the initial guess used to compute the optimization solution). All simulations  
176 were performed in OpenSim (v 3.3) and MATLAB (R2020a).

177 A pseudo-inverse force decomposition method (25) was used to calculate the contribution of  
178 each muscle force to the GRF generated during the take-off phase. At each time step of the  
179 simulation, individual muscle contributions to the GRF were found by solving an optimization  
180 problem that minimized the induced accelerations of the ground contact spheres subject to the  
181 dynamical equations of motion (26). Individual muscle contributions to the vertical and fore-  
182 aft GRF impulses were found by integrating each muscle's contribution to the GRF over the  
183 duration of the take-off phase. These results were then normalized by body mass to obtain each  
184 muscle's contribution to the change in velocity of the COM during take-off.

## 185 **RESULTS**

186 Descriptive data for the best long jump recorded for each participant are given in Table 1.  
187 Model-predicted joint angles, motion of the whole-body COM, and GRFs were in good  
188 agreement with corresponding measurements obtained from experiment (Figs 1 and 2) (see  
189 also Supplementary Digital Content 2, which presents a comparison of model-predicted and  
190 experimental results for joint angles). Root mean square (RMS) errors for pelvic translations  
191 and rotations were  $\leq 2$  cm and  $\leq 1^\circ$ , respectively. Measured hip, knee, and ankle joint angles

192 were also accurately tracked with RMS errors of  $\leq 3.5^\circ$ . RMS errors in the model-predicted  
193 GRFs were 0.12 BW, 0.14 BW, and 0.11 BW in the fore-aft, vertical, and mediolateral  
194 directions, respectively. The simulated positions and velocities of the COM were also closely  
195 similar to corresponding experimental data.

196 The sequence and timing of model-predicted muscle activations were consistent with measured  
197 EMG data, albeit with some differences evident (Fig. 3). Perhaps most notably, model-  
198 predicted activations for the ankle plantarflexors, GAS and SOL, were higher than the levels  
199 of EMG activity recorded for these muscles during terminal swing, whereas model-predicted  
200 activations for the quadriceps (VAS and RF) and hamstrings (HAMS) were lower than  
201 measured EMG at this time.

202 VAS developed the highest peak force of all the muscles, followed by SOL and GMAX (Fig.  
203 4). VAS developed a peak force of 12.5 BW ( $\sim 8000$  N) at 40% of ground contact time, as the  
204 knee approached maximum flexion (Fig. 4, VAS). The peak force in SOL was 6.0 BW, which  
205 occurred at 60% of ground contact time when the ankle reached maximum dorsiflexion (Fig.  
206 4, SOL). GMAX, a uniarticular hip extensor, was activated shortly after touchdown and  
207 developed a peak force of 5.5 BW during the initial foot-ground impact (Fig. 4). ILPSO, a  
208 uniarticular hip flexor, was activated much later and developed a peak force of 4.7 BW just  
209 before the body left the ground. GAS developed a peak force of 3.0 BW, which was much  
210 lower than the peak force in SOL and occurred closer to take-off. The force in HAMS remained  
211 relatively constant throughout the take-off phase and varied between 2.0 BW and 3.0 BW.

212 Model-predicted sagittal-plane net moments developed about the hip, knee and ankle joints  
213 were consistent with corresponding measurements obtained from inverse dynamics (Fig. 5A).  
214 The mean net hip-extensor moment peaked at 5.2 Nm/kg whereas the mean net knee-extensor  
215 and ankle-plantarflexor moments peaked at slightly lower magnitudes of 4.5 Nm/kg and 3.6

216 Nm/kg, respectively. GMAX generated the bulk of the net hip extensor moment throughout  
217 the take-off phase, with HAMS contributing substantially (Fig. 5B). ILPSO generated a flexor  
218 moment at the hip that increased prior to take-off in preparation for the subsequent flight phase  
219 (see Supplementary Digital Content 2, which presents the time history of hip flexion during  
220 take-off). VAS generated most of the net extensor moment at the knee whereas HAMS  
221 generated a flexor moment that slowed knee extension and prevented the knee from  
222 hyperextending at take-off. SOL generated a large fraction of the net ankle-plantarflexor  
223 moment, but GAS also contributed appreciably, especially near take-off where its contribution  
224 grew to exceed that of SOL (Fig. 5B).

225 The contributions of the individual muscles of the ipsilateral (take-off) leg to the vertical and  
226 fore-aft GRFs revealed a proximal-to-distal sequence of muscle coordination during take-off  
227 (Fig. 6). The hip extensor (GMAX), with assistance from the hip abductors (GMED and GMIN)  
228 and hip adductor (ADD), were the major contributors during early stance (0-20% ground  
229 contact time); the knee extensors (VAS) dominated in midstance (40-60%); the ankle  
230 plantarflexors (SOL and GAS) contributed most significantly during late stance (75-90%); and  
231 the metatarsal joint was dominant immediately before take-off (90-100%).

232 GMAX, VAS and SOL were the major contributors to the vertical (support) GRF (Fig. 6A and  
233 B, top). GMAX and VAS generated peak support forces of 2.15 BW and 2.60 BW during early  
234 and mid-stance, respectively, while SOL generated a peak support force of 2.19 BW in late  
235 stance. GAS also contributed appreciably to support and generated a peak force of 0.95 BW  
236 just prior to lift-off.

237 GMAX, VAS and SOL also contributed most significantly to the fore-aft GRF (Fig. 6A and B,  
238 bottom). VAS induced the highest braking force, with a peak of 1.43 BW compared to peak  
239 braking forces of 0.6 BW and 0.5 BW generated by GMAX and SOL, respectively. The hip

240 extensors, GMAX and HAMS, also contributed significantly to propulsion, with HAMS  
241 generating a propulsive force for the entire take-off phase (Fig. 6A, bottom).

242 The torques actuating the joints of the contralateral (swing) leg and the arms contributed  
243 relatively little to the vertical and fore-aft components of the GRF (Fig. 6C, Swing Leg and  
244 Arms; see also Supplementary Digital Content 3, which presents the time histories of the  
245 torques applied at the joints of the swing leg, back, and arms). The contribution from all  
246 gravitational forces was also comparatively small (Fig. 6C, Gravity). The inertial forces  
247 (Coriolis and centrifugal forces) contributed significantly during the initial foot-ground impact  
248 whereas the MTP joint contributed markedly just before lift-off (Fig. 6C, Inertia and MTP).

249 Most muscles delivered a positive vertical (support) impulse that increased the vertical velocity  
250 of the COM during take-off (Fig. 7A and B). VAS generated the largest support impulse and  
251 increased the vertical COM velocity by 1.8 m/s, which accounted for one-third (33%) of the  
252 increase in vertical COM velocity generated by all the muscles. SOL and GMAX generated  
253 smaller support impulses and increased the vertical COM velocity by 1.3 m/s and 0.8 m/s,  
254 respectively, accounting for 24% and 16% of the increase in vertical COM velocity generated  
255 by all the muscles. HAMS and ILPSO (not shown) generated small downward impulses that  
256 reduced the vertical COM velocity during take-off.

257 The muscles of the take-off leg also delivered positive fore-aft (propulsive) and negative fore-  
258 aft (braking) impulses that either increased or reduced the forward velocity of the COM (Fig.  
259 7A and B). VAS generated the highest braking impulse and contributed most significantly to a  
260 loss in forward COM velocity (Fig. 7A, right panel). VAS reduced the forward COM velocity  
261 by 0.92 m/s, which accounted for approximately one-half (55%) of the loss in forward COM  
262 velocity generated by all the muscles. The hip and ankle muscles delivered much smaller  
263 braking impulses. The ankle plantarflexors, SOL and GAS, generated braking impulses during

264 the first half of the take-off phase and reduced the forward COM velocity by 0.20 m/s and 0.06  
265 m/s, respectively (Fig. 7A, right panel). SOL contributed 12% while GAS contributed just 4%  
266 of the loss in forward COM velocity generated by all the muscles (Fig. 7B). GMAX generated  
267 a braking impulse immediately after touchdown and reduced the forward COM velocity by 0.1  
268 m/s, which represented 7% of the loss in forward COM velocity generated by all the muscles.

269 HAMS delivered the largest propulsive impulse and increased the forward COM velocity by  
270 0.17 m/s, which accounted for 30% of the increase in forward COM velocity generated by all  
271 the muscles (Figs 7A and B). HAMS was the only muscle that delivered a propulsive impulse  
272 throughout the take-off phase, with no braking effect. The ankle plantarflexors also delivered  
273 propulsive impulses during the second half of the take-off phase: SOL and GAS increased the  
274 forward COM velocity by 0.09 m/s and 0.12 m/s and accounted for 15% and 20% of the  
275 increase in forward COM velocity generated by all the muscles, respectively.

276 Overall, the net increase in vertical COM velocity generated by all the forces acting on the  
277 body during the take-off phase, including gravity and inertia, was 3.3 m/s, whereas the net loss  
278 in forward COM velocity was 1.1 m/s (Fig. 7C). The knee extensors (VAS and RF) generated  
279 the largest support impulse and increased the vertical COM velocity by 1.9 m/s, while the ankle  
280 plantarflexors (SOL and GAS) and hip extensors (GMAX, GMED, ADD and HAMS)  
281 increased the vertical COM velocity by 1.6 m/s and 1.2 m/s, respectively. Gravity and inertia  
282 generated substantial downward impulses, which reduced the vertical COM velocity by 1.1  
283 m/s and 0.5 m/s, respectively. The knee extensors, primarily VAS, generated the largest  
284 braking impulse and contributed most of the loss in forward COM velocity (1.0 m/s), while the  
285 ankle plantarflexors and hip extensors made much smaller contributions to slowing the forward  
286 velocity of the COM (0.25 m/s and 0.1 m/s, respectively). The hip extensors and ankle  
287 plantarflexors also delivered the largest propulsive impulses and increased the forward COM  
288 velocity by 0.25 m/s and 0.20 m/s, respectively.

289

**DISCUSSION**

290 A key determinant of long jump performance is the ability to increase the vertical velocity of  
291 the COM while minimizing the loss in forward velocity (running speed) during the take-off  
292 phase. We found that the uniarticular hip, knee and ankle extensors dominated the change in  
293 COM velocity during take-off. VAS, SOL and GMAX combined to increase the vertical COM  
294 velocity by 3.9 m/s, which accounted for 73% of the increase in vertical COM velocity  
295 generated by all the muscles (Figs 7A and B). VAS generated the highest support impulse and  
296 contributed one-third (33%) of the increase in vertical COM velocity generated by all the  
297 muscles. SOL and GMAX also generated substantial support impulses and contributed 24%  
298 and 16%, respectively, of the increase in vertical COM velocity. The net increase in the vertical  
299 COM velocity was 3.3 m/s, less than the increase in vertical COM velocity due to VAS, SOL  
300 and GMAX combined (Fig. 7C). The difference is explained by the actions of gravity and  
301 inertia, which generated negative support impulses, accelerated the COM downwards, and  
302 reduced the vertical COM velocity at take-off.

303 While increasing the vertical COM velocity, VAS, SOL, and to a lesser extent, GMAX, also  
304 reduced the forward velocity of the COM during take-off (Figs 7A and B). VAS generated the  
305 highest braking impulse and contributed approximately one-half (55%) of the loss in forward  
306 COM velocity generated by all the muscles. SOL delivered a substantial braking impulse  
307 during the first half of the take-off phase and contributed 12% of the loss in forward COM  
308 velocity, whereas GMAX generated a braking impulse immediately after touchdown and  
309 contributed 7% of the loss in forward COM velocity. Altogether, the lower-limb muscles  
310 reduced forward COM velocity by 1.5 m/s, which was greater than the net loss of 1.1 m/s in  
311 forward COM velocity (Fig. 7C). The difference is explained by the fact that most muscles also  
312 generated a propulsive impulse that increased the forward COM velocity during certain periods  
313 of the take-off phase. For example, HAMS delivered a propulsive impulse throughout the take-

314 off phase which accounted for 30% of the increase in forward COM velocity generated by all  
315 the muscles (Figs 7A and B).

316 Why is an increase in vertical COM velocity accompanied by a loss in forward COM velocity  
317 during take-off? Upward velocity of the COM is generated at the expense of forward velocity  
318 because the body pivots about a stationary foot as the stance leg compresses during the first  
319 half of the take-off phase (4, 6, 8). Dapena and Chung (8) used a simple kinematic model of  
320 the take-off phase of the high jump to show that vertical COM velocity increases after  
321 touchdown even though the COM moves radially toward the fixed foot due to flexion of the  
322 stance-leg joints. They explained that radial motion of the COM increases the vertical velocity  
323 of the COM because the leg muscles experience faster eccentric contractions, allowing them  
324 to develop higher forces and generate a higher peak vertical GRF (8). Similar to the behavior  
325 described by Dapena and Chung (8) for the high jump, in the long jump take-off vertical  
326 velocity of the COM increases shortly after touchdown while the COM moves radially toward  
327 the fixed foot (see Supplementary Digital Content 4, which presents the trajectories of the  
328 position and velocity of the COM in the vertical and radial directions). A negative radial  
329 velocity of the COM (directed toward the foot) is converted to an upward vertical velocity by  
330 eccentric (lengthening) contractions of the extensor muscles as they are stretched to resist  
331 compression of the stance leg during the first half of the take-off. Our model simulation results  
332 showed that the uniarticular hip extensor, GMAX, lengthened immediately after touchdown,  
333 developed a peak force of 5.5 BW (Fig. 4, GMAX), and contributed significantly to the first  
334 peak in the vertical GRF (Fig. 6A). As the knee and ankle joints then flexed for the remainder  
335 of the first half of the take-off phase, VAS and SOL lengthened appreciably, developed peak  
336 forces of 12.5 BW and 5.5 BW respectively (Fig. 4, VAS and SOL), and contributed  
337 significantly to the second peak in the vertical GRF (Fig. 6B) (see Supplementary Digital  
338 Content 4, which presents the lengths and velocities of the muscle fibers computed for the

339 uniarticular hip, knee and ankle extensors during take-off). The vertical velocity of the COM  
340 continued to increase upward during the second half of the take-off phase as GMAX, VAS and  
341 SOL all underwent concentric contractions (shortening) until lift-off.

342 Maximizing forward COM velocity (running speed) during the approach phase is an important  
343 feature of the long jump (1, 2). Not unexpectedly then, there are similarities in how the lower-  
344 limb muscles are used to maximize performance in steady-speed sprinting and the long jump  
345 take-off. In both of these activities the combined actions of VAS and the ankle plantarflexors,  
346 SOL and GAS, contribute a large fraction of the support impulse generated at each step (27).  
347 Further, in both steady-state sprinting and the long jump take-off, VAS contributes practically  
348 all the braking impulse generated on the ground (27). However, one major difference between  
349 the two activities is that the net fore-aft impulse during steady-speed sprinting is nearly zero  
350 whereas during the long jump take-off the net braking impulse is substantial with a  
351 magnitude  $>1.0$  N.s/kg. The reason the net fore-aft impulse remains close to zero in steady-  
352 speed sprinting is because the ankle plantarflexors, SOL and GAS, generate large propulsive  
353 impulses which overcome the braking action of VAS (27). The ankle plantarflexors also  
354 generate propulsive impulses in the second half of the long jump take-off, however their total  
355 contribution to the net fore-aft impulse during take-off is much smaller than that in sprinting,  
356 which explains why the net fore-aft impulse during the long jump take-off is dominated by the  
357 braking action of VAS. Another key difference between steady-speed sprinting and the long  
358 jump take-off is the contribution made by the hip muscles to the support impulse. In steady-  
359 state sprinting, the hip muscles, GMAX, GMED and ADD, contribute relatively little to the  
360 support impulse generated at each step (27), whereas these muscles each make a notable  
361 contribution to the support impulse generated during the long jump take-off (Figs 6A and 7A).  
362 The ability to generate vertical velocity during take-off is a critical determinant of long jump  
363 performance. We found that VAS contributes most significantly to the vertical velocity of the

364 COM at take-off (Fig. 7), implying that coaches and athletic trainers ought to focus foremost  
365 on strengthening the quadriceps muscles if performance of the long jump is to be improved.  
366 Our simulation results also show that the hip and ankle extensors play a major role in generating  
367 an upward velocity of the COM during take-off (Fig. 6). GMAX produced most of the support  
368 impulse generated during impact immediately after touchdown, whereas SOL contributed a  
369 large fraction of the support impulse generated during the second half of the take-off phase  
370 (Figs 6 and 7). Thus, GMAX and SOL also should be targeted in strength training programs  
371 aimed at maximizing jump performance. It should be noted that increasing the volumes of these  
372 muscles would also alter the inertial properties of the limbs (specifically the masses and  
373 locations of the centers of mass of the thigh and shank segments) and may negatively impact  
374 jump performance by increasing body mass and/or altering muscle coordination during the run-  
375 up. Addressing this issue quantitatively requires a predictive modelling approach, like the  
376 dynamic optimization technique used recently by Lin and Pandy (28) who investigated the  
377 effects of changes in muscle strength and other muscle-tendon properties on sprint performance.  
378 Perhaps the most significant limitation of the present study was the limited run-up distance  
379 available to our jumpers, which most likely explains the shorter jump distances recorded by  
380 our athletes compared to data reported by others (3-6). Run-up distances used in official  
381 competitions are typically 40 m, allowing the athlete to complete around 14 strides before  
382 reaching the board (29). By comparison, the maximum run-up distance available in the present  
383 study was just 12 m, which would have limited approach velocity and hence jump distance  
384 appreciably. Lees and colleagues measured long jump performance in elite female (6) and male  
385 (4) collegiate athletes. They reported mean jump distances of 6.51 m for women and 7.67 m  
386 for men. At touchdown, the mean forward and vertical velocities were reported to be 8.75 m/s  
387 and -0.03 m/s, respectively, for women, compared to mean forward and vertical velocities of  
388 9.88 m/s and -0.15 m/s for men. At take-off, the mean forward and vertical velocities were 7.55

389 m/s and 3.05 m/s, respectively, for women, compared to mean forward and vertical velocities  
390 of 8.75 m/s and 3.01 m/s for men. The mean jump distance for our cohort of male and female  
391 Taiwanese athletes was much lower at 5.18 m, with corresponding mean forward and vertical  
392 velocities at touchdown of 7.65 m/s and -0.58 m/s, respectively, and mean forward and vertical  
393 velocities at take-off of 6.65 m/s and 2.67 m/s, respectively. Nonetheless, the change in position  
394 and velocity of the COM from touchdown to take-off measured for our athletes closely  
395 resembled the data published by Lees and colleagues (4, 6). For both men and women, these  
396 authors found the vertical position of the COM to dip slightly just after touchdown and then to  
397 increase sharply after maximum knee flexion until take-off. We also found a slight dip in the  
398 vertical position of the COM immediately after touchdown as the velocity of the COM was  
399 directed downwards (Fig. 1). The vertical velocity of the COM then increased sharply before  
400 declining just before take-off, consistent with the trend noted by Lees et al. (4, 6). Another  
401 limitation of the present study concerns the structure of the model used to simulate the long  
402 jump take-off. Whereas the joints of the ipsilateral (take-off) leg were actuated by muscles, the  
403 joints of the contralateral (swing) leg, arms and torso were driven by ideal torques. We  
404 simplified the musculoskeletal model in this way because the mechanics of the take-off leg is  
405 known to contribute more substantially to the vertical velocity of the COM generated during  
406 take-off than either the swing leg or the arms (4). Indeed, we found that the torques actuating  
407 the joints of the contralateral leg and arms contributed little to the vertical and fore-aft  
408 components of the GRF (Fig. 6C). However, this does not mean that the arms and contralateral  
409 leg are not important contributors to performance of the long jump. Swinging the contralateral  
410 leg and arms during the take-off phase serves to transfer angular momentum to the whole body  
411 prior to lift-off, which promotes an efficient landing (30). Future studies should examine how  
412 muscle actuation of the swing leg and arms affect take-off biomechanics and jump distance.  
413 Finally, we simulated only the take-off phase of the long jump and ignored the preceding strides

414 where the forward approach velocity is known to peak (3). Future work should consider  
415 expanding the analysis presented here to include the last 2 to 3 strides in addition to the take-  
416 off phase to better understand how the leg muscles maximize long jump performance.

417 In summary, the uniarticular hip, knee and ankle extensors dominated the change in COM  
418 velocity during the long-jump take-off. VAS, SOL, and GMAX contributed nearly three-  
419 quarters (73%) of the increase in the vertical velocity of the COM at take-off, with VAS and  
420 SOL contributing more than one-half (57%) of the total increase in vertical velocity. VAS also  
421 delivered the highest braking impulse and contributed more than one-half (55%) of the  
422 reduction in forward velocity of the COM during take-off. These results suggest that the hip,  
423 knee and ankle extensors ought to be prioritized in strength training programs aimed at  
424 improving long jump performance.

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429 Lin for his advice related to model development.

#### 430 **CONFLICT OF INTEREST STATEMENT**

431 The authors declare no competing or financial interests. The results of the present study do not  
432 constitute endorsement by the American College of Sports Medicine and are presented clearly,  
433 honestly, and without fabrication, falsification, or inappropriate data manipulation.

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## REFERENCES

- 436 1. Hay JG. The biomechanics of the long jump. *Exerc Sport Sci Rev.* 1986;14(1):401-  
437 46.
- 438 2. Alexander RM. Optimum take-off techniques for high and long jumps. *Philos Trans R*  
439 *Soc Lond B Biol Sci.* 1990;329(1252):3-10.
- 440 3. Hay JG, Miller JA, Canterna RW. The techniques of elite male long jumpers. *J*  
441 *Biomech.* 1986;19(10):855-66.
- 442 4. Lees A, Graham-Smith P, Fowler N. A biomechanical analysis of the last stride,  
443 touchdown, and takeoff characteristics of the men's long jump. *J Appl Biomech.*  
444 1994;10(1):61-78.
- 445 5. Graham-Smith P, Lees A. A three-dimensional kinematic analysis of the long jump  
446 take-off. *J Sports Sci.* 2005;23(9):891-903.
- 447 6. Lees A, Fowler N, Derby D. A biomechanical analysis of the last stride, touch-down  
448 and take-off characteristics of the women's long jump. *J Sports Sci.* 1993;11(4):303-14.
- 449 7. Luhtanen P, Komi PV. Mechanical power and segmental contribution to force  
450 impulses in long jump take-off. *Eur J Appl Physiol Occup Physiol.* 1979;41(4):267-74.
- 451 8. Dapena J, Chung CJM. Vertical and radial motions of the body during the take-off  
452 phase of high jumping. *Med Sci Sports Exerc.* 1988;20(3):290-302.
- 453 9. Seyfarth A, Friedrichs A, Wank V, Blickhan R. Dynamics of the long jump. *J*  
454 *Biomech.* 1999;32(12):1259-67.
- 455 10. Seyfarth A, Blickhan R, Van Leeuwen JL. Optimum take-off techniques and muscle  
456 design for long jump. *J Exp Biol.* 2000;203(4):741-50.
- 457 11. Kakihana W, Suzuki S. The EMG activity and mechanics of the running jump as a  
458 function of takeoff angle. *J Electromyogr Kinesiol.* 2001;11(5):365-72.

- 459 12. Cappozzo A, Cappello A, Croce UD, Pensalfini F. Surface-marker cluster design  
460 criteria for 3-D bone movement reconstruction. *IEEE Trans Biomed Eng.* 1997;44(12):1165-  
461 74.
- 462 13. De Luca CJ. The use of surface electromyography in biomechanics. *J Appl Biomech.*  
463 1997;13(2):135-63.
- 464 14. Lai A, Schache AG, Brown NA, Pandy MG. Human ankle plantar flexor muscle–  
465 tendon mechanics and energetics during maximum acceleration sprinting. *J R Soc Interface.*  
466 2016;13(121).
- 467 15. Lai AKM, Arnold AS, Wakeling JM. Why are antagonist muscles co-activated in my  
468 simulation? A musculoskeletal model for analysing human locomotor tasks. *Ann Biomed*  
469 *Eng.* 2017;45(12):2762-74.
- 470 16. Chen W-M, Shim VP-W, Park S-B, Lee T. An instrumented tissue tester for  
471 measuring soft tissue property under the metatarsal heads in relation to metatarsophalangeal  
472 joint angle. *J Biomech.* 2011;44(9):1801-4.
- 473 17. De Groot F, Kinney AL, Rao AV, Fregly BJ. Evaluation of direct collocation  
474 optimal control problem formulations for solving the muscle redundancy problem. *Ann*  
475 *Biomed Eng.* 2016;44(10):2922-36.
- 476 18. Miller R, Balshaw TG, Massey GJ, Maeo S, Lanza MB, Johnston M, et al. The  
477 muscle morphology of elite sprint running. *Med Sci Sports Exerc.* 2020;53(4):804-15.
- 478 19. Rajagopal A, Dembia CL, DeMers MS, Delp DD, Hicks JL, Delp SL. Full-body  
479 musculoskeletal model for muscle-driven simulation of human gait. *IEEE Trans Biomed Eng.*  
480 2016;63(10):2068-79.
- 481 20. Zajac FE. Muscle and tendon: properties, models, scaling, and application to  
482 biomechanics and motor control. *Crit Rev Biomed Eng.* 1989;17(4):359-411.

- 483 21. Winters JM, Stark L. Estimated mechanical properties of synergistic muscles involved  
484 in movements of a variety of human joints. *J Biomech.* 1988;21(12):1027-41.
- 485 22. Sherman MA, Seth A, Delp SL. Simbody: multibody dynamics for biomedical  
486 research. *Procedia Iutam.* 2011;2(1):241-61.
- 487 23. Lin Y-C, Pandy MG. Three-dimensional data-tracking dynamic optimization  
488 simulations of human locomotion generated by direct collocation. *J Biomech.* 2017;59(1):1-  
489 8.
- 490 24. Lin Y-C, Walter JP, Pandy MG. Predictive simulations of neuromuscular  
491 coordination and joint-contact loading in human gait. *Ann Biomed Eng.* 2018;46(8):1216-27.
- 492 25. Lin YC, Kim HJ, Pandy MG. A computationally efficient method for assessing  
493 muscle function during human locomotion. *Int J Numer Method Biomed Eng.*  
494 2011;27(3):436-49.
- 495 26. Anderson FC, Pandy MG. Individual muscle contributions to support in normal  
496 walking. *Gait Posture.* 2003;17(2):159-69.
- 497 27. Pandy MG, Lai AKM, Schache AG, Lin YC. How muscles maximize performance in  
498 accelerated sprinting. *Scand J Med Sci Sports.* 2021;31(10):1882-96.
- 499 28. Lin Y-C, Pandy MG. Predictive Simulations of Human Sprinting: Effects of Muscle-  
500 Tendon Properties on Sprint Performance. *Med Sci Sports Exerc.* 2022.
- 501 29. Bridgett LA, Linthorne NP. Changes in long jump take-off technique with increasing  
502 run-up speed. *J Sports Sci.* 2006;24(8):889-97.
- 503 30. Bouchouras G, Moscha D, Papaiakovou G, Nikodelis T, Kollias I. Angular  
504 momentum and landing efficiency in the long jump. *Eur J Sport Sci.* 2009;9(1):53-9.  
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506

**FIGURE CAPTIONS**

507 Fig. 1: Time histories of model-predicted vertical and fore-aft displacements and velocities  
508 (red lines) of the center of mass (COM) of the body during the take-off phase of the  
509 long jump. The blue lines are the mean of the experimental data recorded for the  
510 participants. The shaded regions represent  $\pm 1$  standard deviation from the mean for  
511 both the model and experimental data. 0% and 100% of ground contact time represent  
512 touchdown and take-off, respectively. The origin (0,0) of the coordinate system used  
513 to measure COM position was located at the upper left corner of the force plate  
514 embedded in the ground.

515 Fig. 2: Time histories of model-predicted vertical, fore-aft, and mediolateral components of  
516 the ground reaction force (GRF) (red lines) generated during the take-off phase of the  
517 long jump. The blue lines are the mean of the experimental data recorded for the  
518 participants, The shaded regions represent  $\pm 1$  standard deviation from the mean for  
519 both the model and experimental data. 0% and 100% of ground contact time represent  
520 touchdown and take-off, respectively. BW, body weight.

521 Fig. 3: Comparison of model-predicted activations (red) and measured EMG activity (blue)  
522 for selected muscles crossing the hip, knee and ankle joints. Shaded regions depict  $\pm$   
523 1 standard deviation from the mean. EMG data were measured from 3 of the 9  
524 participants and then averaged across participants. EMG data for each muscle were  
525 normalized by the peak value recorded during the take-off phase. Muscle symbols are:  
526 GASMED: medial gastrocnemius; GMED: anterior portion of gluteus medius; RF:  
527 rectus femoris; SEMITEN: semitendinosus; SOL: soleus; VASMED: vastus medialis.  
528 0% and 100% of ground contact time represent touchdown and take-off, respectively.

529 Fig. 4: Time histories of the forces calculated in representative muscles during the take-off  
530 phase of the long jump. The solid lines are mean muscle forces calculated for all

531 participants and the shaded regions represent  $\pm 1$  standard deviation from the mean.  
 532 Muscle forces were normalized by each participant's body weight (BW). Muscle  
 533 symbols are: GMAX, superior, middle, inferior portions of gluteus maximus  
 534 combined; RF, rectus femoris; GMED, anterior and posterior portions of gluteus  
 535 medius combined; HAMS, semimembranosus, semitendinosus, and biceps femoris  
 536 long head and short head combined; ILPSO, iliacus and psoas combined; VAS, vastus  
 537 medialis, intermedius, and lateralis combined; SOL, soleus; GAS, medial and lateral  
 538 gastrocnemius combined. 0% and 100% of ground contact time represent touchdown  
 539 and take-off, respectively.

540 Fig. 5: (A) Time histories of the net moments developed by all the muscles spanning the hip,  
 541 knee, and ankle joints during the take-off phase of the long jump. Moments were  
 542 normalized by each participant's body mass and averaged across all participants. The  
 543 shaded regions represent  $\pm 1$  standard deviation from the mean. Hip extension, knee  
 544 extension and ankle plantarflexion are positive. 0% and 100% of ground contact time  
 545 represent touchdown and take-off, respectively.

546 (B) Contributions of individual muscles to the net joint moments exerted about the  
 547 hip, knee, and ankle during the take-off phase. Muscle moments were normalized by  
 548 each participant's body mass and averaged across all participants. Hip extension, knee  
 549 extension and ankle plantarflexion are positive. Muscle symbols are defined in Fig. 4.  
 550 0% and 100% of ground contact time represent touchdown and take-off, respectively.

551 Fig. 6: Mean contributions of the hip-spanning muscles (A), knee- and ankle-spanning  
 552 muscles (B), and gravity, inertia, swing leg, arms, and metatarsal joint (C) to the  
 553 vertical and fore-aft ground reaction forces generated during the take-off phase of the  
 554 long jump. The light black dashed lines represent the total GRF induced by the  
 555 ipsilateral (take-off) leg. The blue shaded regions are the measured vertical and fore-

556 aft GRFs averaged across all participants. 0% and 100% of ground contact time  
 557 represent touchdown and take-off, respectively. Muscle symbols are: GMAX,  
 558 superior, middle, inferior portions of gluteus maximus combined; GMED, anterior  
 559 and posterior portions of gluteus medius combined; GMIN, anterior and posterior  
 560 portion of the gluteus minimus; RF, rectus femoris; ADD, adductor brevis, adductor  
 561 longus and adductor magnus combined. HAMS, semimembranosus, semitendinosus,  
 562 and biceps femoris long head and short head combined; VAS, vastus medialis,  
 563 intermedius, and lateralis combined; SOL: soleus; GAS, medial and lateral  
 564 gastrocnemius combined; OTHER, summed contributions from all other muscles  
 565 included in the model. Other symbols appearing in the diagram are: MTP,  
 566 metatarsophalangeal joint torque applied to the take-off leg; Swing leg, combined  
 567 joint torques applied to the contralateral (swing) leg; Arms, combined joint torques  
 568 applied to the left and right arms; Gravity, sum of all gravitational forces acting on the  
 569 body; Inertial, sum of all velocity-related forces including Coriolis and centrifugal  
 570 forces, applied to the body.

571 Fig. 7: (A) Mean contributions of selected muscles to the change in COM velocity during the  
 572 take-off phase of the long jump. Positive values represent upward- and forward-  
 573 directed velocities while negative values represent downward- and backward-directed  
 574 velocities. (A) Contributions of individual muscles to the change in vertical COM  
 575 velocity (left panel) and forward COM velocity (right panel). (B) Percentage  
 576 contributions of individual muscles to the change in the vertical and forward velocities  
 577 of the COM. 100% represents the contribution made by all muscles in the model to  
 578 the change in the vertical and forward velocity of the COM. (C) Mean contributions  
 579 of the hip extensors (GMAX, GMED, ADD and HAMS), knee extensors (VAS and  
 580 RF), ankle plantarflexors (SOL and GAS), metatarsophalangeal (MTP) joint,



## Muscle Contributions to Take-off Velocity in the Long Jump SUPPLEMENTARY DIGITAL CONTENT 1

### Dynamic Optimization Problem for the Long Jump Take-off

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#### Dynamic optimization problem

The data-tracking optimal control problem was converted into a nonlinear optimization problem by discretizing the model states into 100 evenly spaced nodes (1, 2). Values of the joint angles ( $q$ ), joint angular velocities ( $u$ ), muscle-tendon forces ( $F$ ), muscle activations ( $a$ ), and ideal torques ( $\tau$ ) applied at the torque-driven joints at each node, along with the values of a subset of the time-invariant Hunt-Crossley contact parameters (specifically, height of the contact plane, dissipation constant and viscous friction coefficient) were found by minimizing the performance criterion  $J$ :

$$J = J_q + J_{GRF} + J_{GRM} + J_{ID} + J_a + J_\tau \quad (1)$$

Here  $J_q$  minimizes the integral sum of the differences between model-predicted and measured joint angles with a weighting factor of  $10^4$ ;  $J_{GRF}$  and  $J_{GRM}$  respectively minimize the integral sum of the differences between model-predicted and measured ground reaction forces and moments with

weighting factors of  $10^5$  and  $10^3$ ;  $J_{ID}$  minimizes the integral sum of the differences between model-predicted and measured joint torques for the take-off leg (specifically, hip extension, adduction, knee extension, and ankle plantarflexion torque) with a weighting factor of  $10^3$ ; and  $J_a$  and  $J_\tau$  minimize respectively the integral sum of muscle activations squared and joint torques squared with weighting factors of 1 and  $10^2$ .

The data-tracking dynamic optimization problem was solved subject to a trapezoidal state defect constraint (1) at each node, thus:

where  $x$  is the vector of model states comprised of the generalized coordinates, generalized speeds, muscle activations, and muscle fiber lengths ( $l_m$ );  $n$  is the node number; and  $t$  is time.

### **Initial guess for dynamic optimization solution**

To obtain better tracking performance, an initial guess for the optimization solution was generated using a two-step procedure. In Step 1, an optimization-based inverse dynamics problem was solved using ideal joint torques to track the measured joint coordinates. The simulated time history of the joint motion closely tracked the measured data while maintaining dynamical consistency (3). The 6-dimensional pelvis residual forces and torques were minimized as pelvic motion was not explicitly generated by muscles.

Step 1 involved calculating the variables ( $q, u, \tau$ ) needed to minimize the objective function:

$$w_q(q - q_{EXP})^T(q - q_{EXP}) + \frac{1}{2}\tau^T H \tau \quad (3)$$

subject to the constraints

$$u_{i+1} = u_i + \frac{1}{2} \Delta t (q_i + q_{i+1}) \quad (4)$$

$$a_{i+1} = a_i + \frac{1}{2} \Delta t (u_i + u_{i+1}) \quad (5)$$

where  $w_q = 100$  is the weighting factor applied to the coordinate tracking term in the objective function, and  $H$  is a square diagonal weighting matrix with dimension equal to the number of model DOFs. For the pelvis residual torque, a higher weighting factor of 10 was used to minimize the pelvic residuals while a lower weighting factor of 0.1 was used for the remaining joint torques to ensure the model tracked the movement path with minimal effort. In equation (5) above,  $a$  is a vector of joint accelerations and was computed using explicit dynamics based on the dynamical equations of motion:

$$a = M(q)^{-1}(F_{inertial}(q, u) + F_{GRF} + F_{gravity} + \tau) \quad (6)$$

Step 1 converged to a solution within 10-20 mins with a cluster of 6 CPUs in parallel.

In the Step 2, the joint torques  $\tau$  obtained in the Step 1 were decomposed into individual muscle forces by solving a static optimization problem at each time step along the movement trajectory. Specifically, Step 2 involved calculating the muscle activations  $a$  and the ideal torques  $\tau_q$  needed to minimize the quadratic objective function:

$$a^T a + w \tau_q^T \tau_q \quad (7)$$

subject to the linear constraints

$$\tau = \tau_q + MA(FV \cdot FL \cdot \cos(\alpha) a) \quad (8)$$

with bounds

$$a \in [0,1] \quad (9)$$

where  $w$  is a weighting factor whose value was taken to be 1000. A large weighting factor was used to penalize the residual torques in the muscle-actuated joints. Also,  $\alpha$  was a vector of muscle pennation angles, and  $FV$  and  $FL$  were vectors containing multipliers used to specify each muscle's force-length and force-velocity properties.  $FV$ ,  $FL$  and  $\alpha$  were derived assuming a rigid tendon for each musculotendon unit included in the model. Step 2 solved very efficiently in less than 5 seconds because the optimization problem was posed as a quadratic programming problem.

The time histories of the joint angles ( $q$ ) and joint angular velocities ( $u$ ) obtained in Step 1 were amalgamated with the time histories of the muscle activations ( $a$ ) and muscle forces ( $F$ ) obtained in Step 2 to form the initial guess for the dynamic optimization problem for the long jump take-off.

### **Optimizer Settings**

The optimization was solved using the MATLAB built-in solver “fmincon” (MathWorks, Natick, MA, USA). The optimization algorithm used was ‘interior-point’ and the ‘SubproblemAlgorithm’ selected was ‘cg’, which is the abbreviation for the conjugate gradient algorithm. The optimization algorithm was previously developed and verified by Byrd et al. (4).

The Jacobian of the constraint function and the gradient of the objective function were provided to the solver. The multibody dynamics state derivatives with respect to the skeletal states and generalized forces were estimated using finite differences while the derivatives of the muscle states with respect to the muscle states and muscle length were derived analytically using symbolic operation. Since the muscle length and muscle moment arm with respect to the joint angles were trained using smooth polynomials, the derivatives of the constraint functions with respect to all optimization design variables could be derived using the derivative chain rule. The Hessian matrix was estimated using the built-in option 'bfgs'.

Finally, the termination step length tolerance (`StepTolerance`) was  $1e-6$ ; the constraint tolerance (`ConstraintTolerance`) was  $1e-2$ ; and the first-order optimality tolerance (`OptimalityTolerance`) was  $1e-5$ . Each optimization program took between 1.5 hours and 4 hours to converge on a 6-core, 32GB RAM desktop with an Intel i9 processor.

## REFERENCES

1. Lin Y-C, Pandy MG. Three-dimensional data-tracking dynamic optimization simulations of human locomotion generated by direct collocation. *J Biomech.* 2017;59(1):1-8.
2. Lin Y-C, Walter JP, Pandy MG. Predictive simulations of neuromuscular coordination and joint-contact loading in human gait. *Ann Biomed Eng.* 2018;46(8):1216-27.
3. Runge C, Zajac F, Allum J, Risher D, Bryson A, Honegger F. Estimating net joint torques from kinesiological data using optimal linear system theory. *IEEE Trans Biomed Eng.* 1995;42(12):1158-64.
4. Byrd RH, Hribar ME, Nocedal J. An interior point algorithm for large-scale nonlinear programming. *SIAM J Optim.* 1999;9(4):877-900.

**Muscle Contributions to Take-off Velocity in the Long Jump**  
**SUPPLEMENTARY DIGITAL CONTENT 3**

**Model-computed Joint Torques During the Long-jump Take-off**

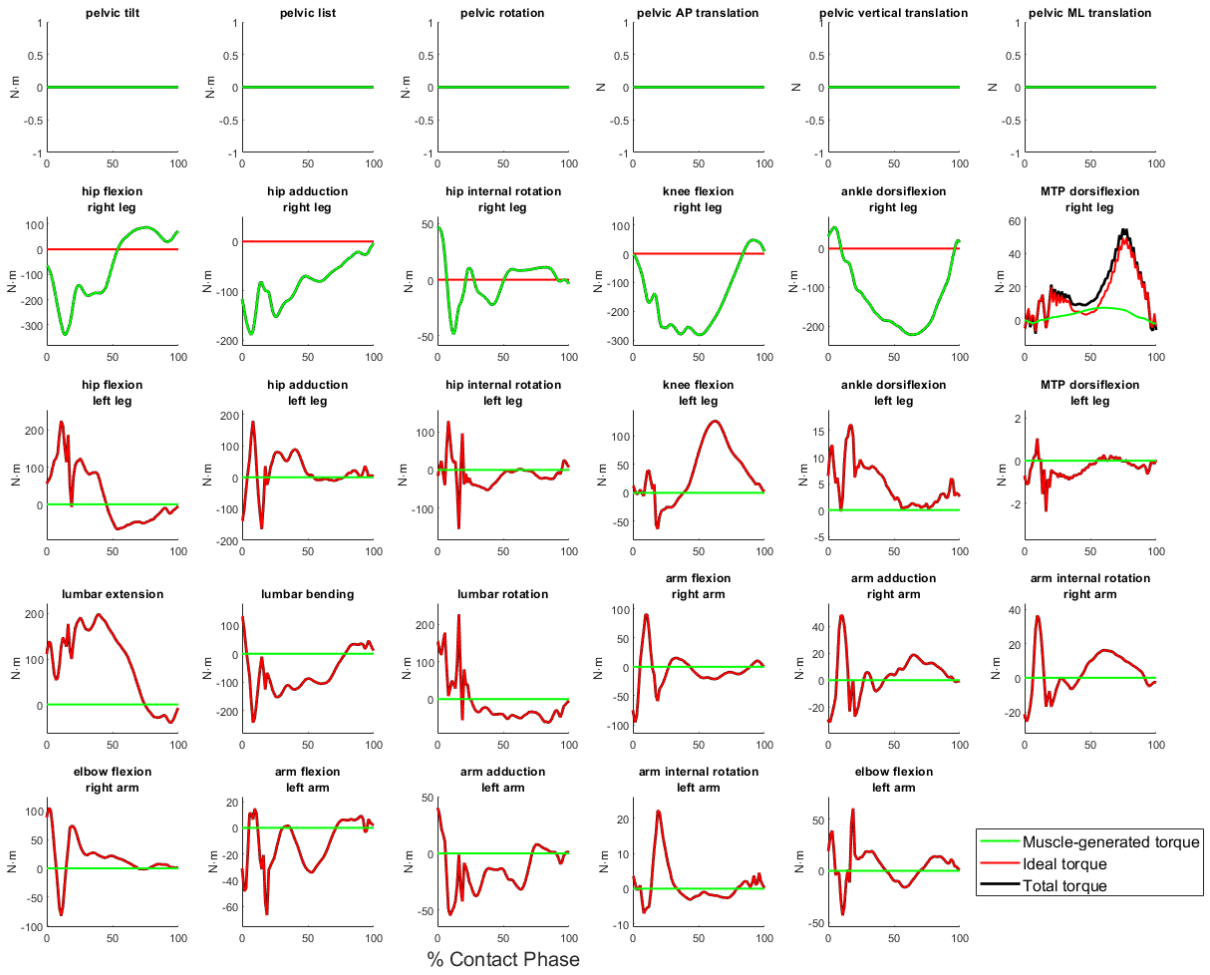
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Below we present plots of the histories of model-predicted joint torques (black lines), ideal joint torques (red lines), and muscle-generated joint torques (green lines) calculated in the model for participant #1 during the take-off phase of the long jump. Participant #1 used her right leg as the takeoff leg. 0% and 100% of ground contact time represent touchdown and take-off, respectively.



## **Muscle Contributions to Take-off Velocity in the Long Jump**

### **SUPPLEMENTARY DIGITAL CONTENT 2**

#### **Model-predicted Joint Motion**

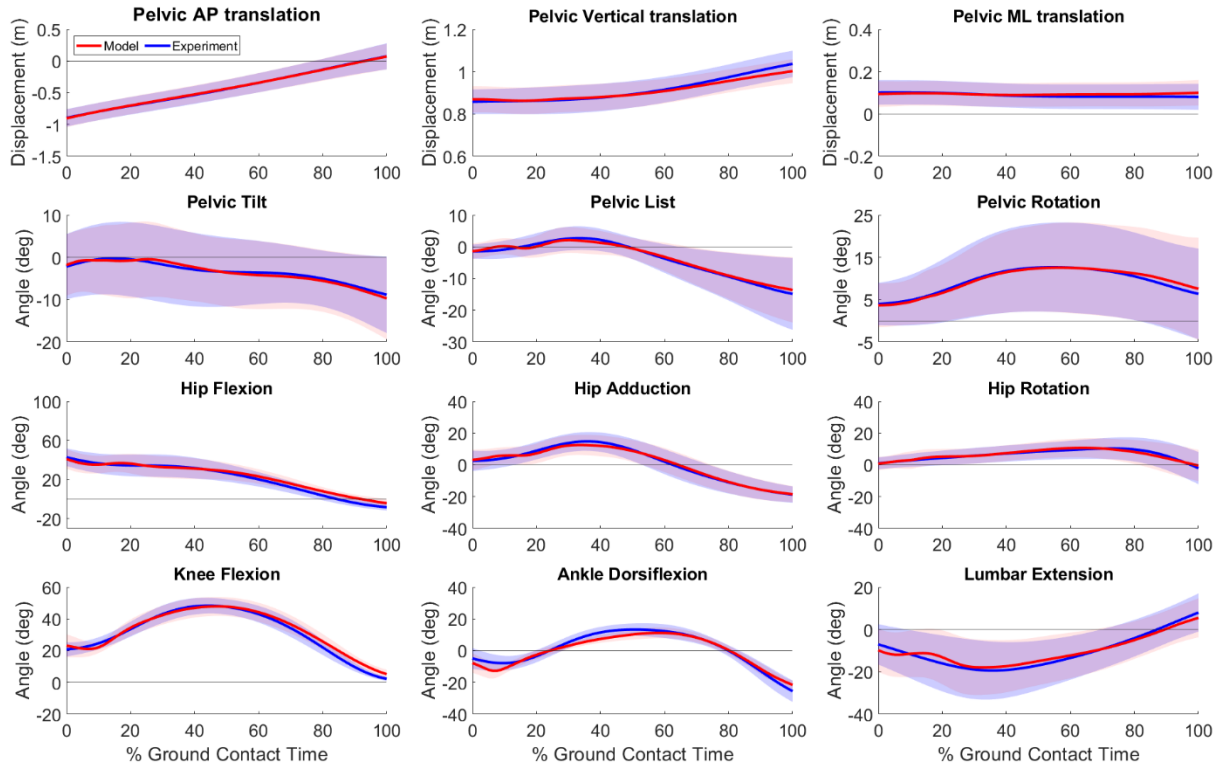
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Below we present plots of the time histories of model-predicted joint motion (red lines) during the take-off phase of the long jump. Data for the hip, knee, and ankle joint angles are for the take-off leg. The blue lines are the mean of the experimental data recorded for the participants while the shaded regions represent  $\pm 1$  standard deviation from the mean. 0% and 100% of ground contact time represent touchdown and take-off, respectively.



**Muscle Contributions to Take-off Velocity in the Long Jump**  
**SUPPLEMENTARY DIGITAL CONTENT 4**

**Kinematics of the Mass Center and Lower-limb Joints and Muscles**

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Below we present plots of the position and velocity of the center of mass of the whole body during the take-off phase of the long jump (Figure S4.1) followed by plots of the time histories of muscle-fiber lengths and velocities computed during the take-off phase (Figure S4.2). Also shown in Figure S4.2 are the angular displacements and velocities of the hip, knee, and ankle joints calculated in the model during the take-off phase.

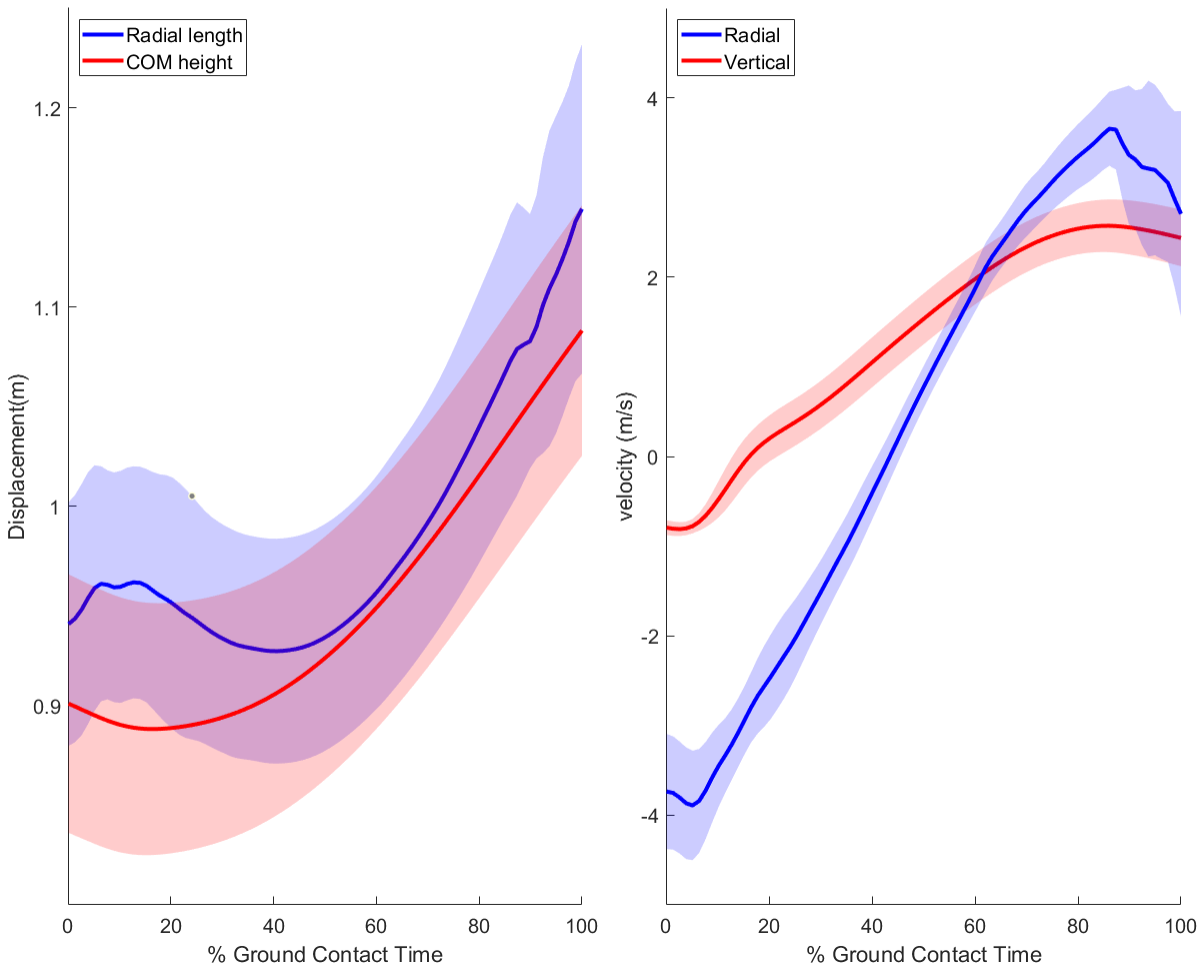


Figure S4.1: Left Panel: Radial (blue line) and vertical (red line) displacement of the center of mass over the contact phase.

Right Panel: Radial (blue line) and vertical (red line) velocity of the center of mass over the contact phase. The solid lines are the mean of the model-predicted results for the participants, and the shaded regions represent  $\pm 1$  standard deviation from the mean. 0% and 100% of ground contact time represent touchdown and take-off, respectively.

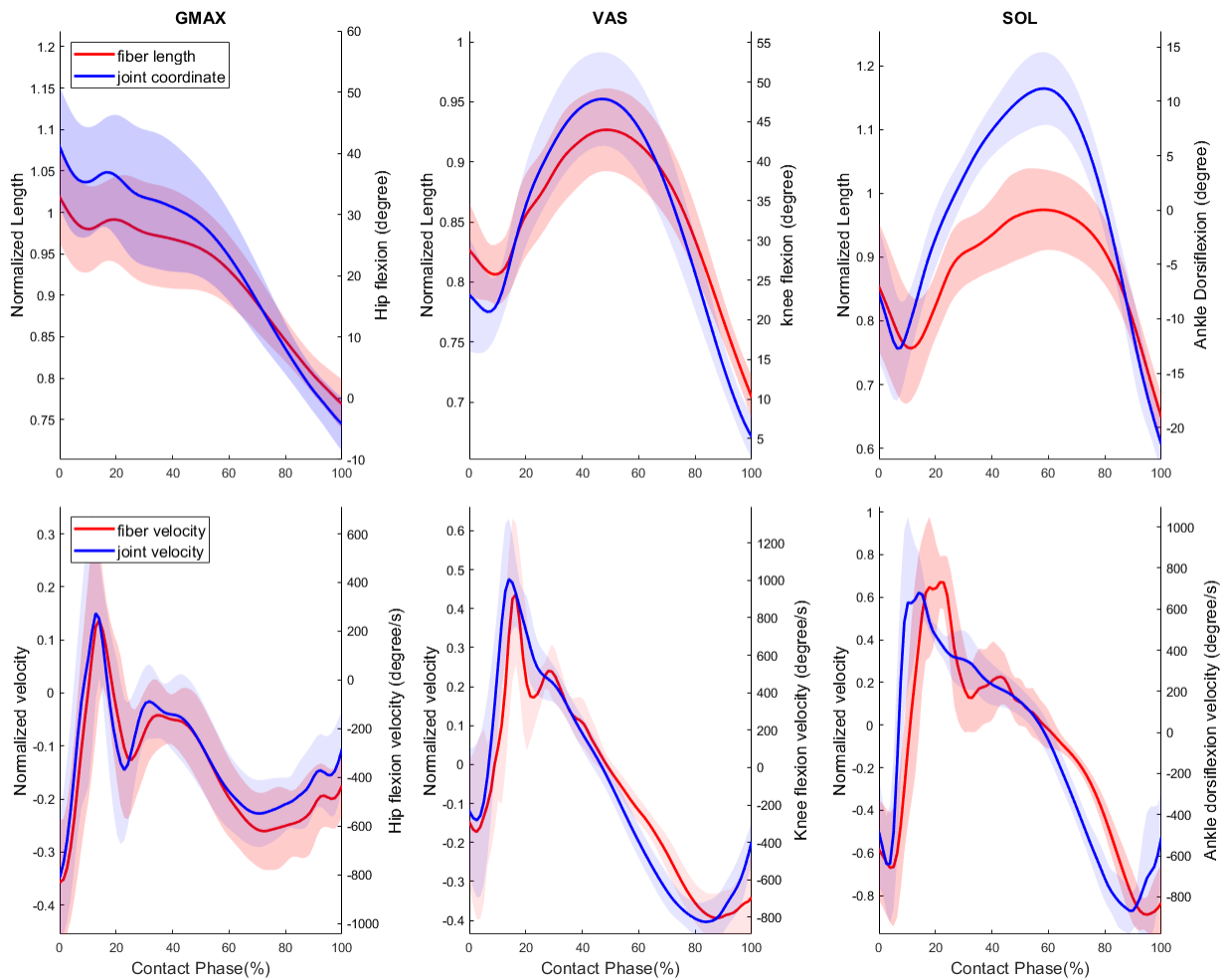


Figure S4.2: GMAX, VAS, and SOL fiber kinematics. First row: Fiber lengths computed for GMAX, VAS, and SOL normalized by their corresponding optimal muscle-fiber lengths (red lines). Also plotted on the same set of axes are the angular displacements of the hip, knee and ankle joints (blue lines). The solid lines are the mean of the model-predicted joint angles and joint angular velocities calculated across all participants, and the shaded regions represent  $\pm 1$  standard deviation from the mean. 0% and 100% of ground contact time represent touchdown and take-off, respectively.

Second row: Velocities of the muscle fibers computed for GMAX, VAS, and SOL normalized by their corresponding maximum shortening velocities (red lines). Also plotted on the same set of axes are the angular velocities of the hip, knee and ankle joints (blue lines). The solid lines are the mean of the model-predicted muscle-fiber lengths, muscle-fiber velocities, and joint kinematics calculated across all participants, and the shaded regions represent  $\pm 1$  standard deviation from the mean. 0% and 100% of ground contact time represent touchdown and take-off, respectively. Muscle symbols are: GMAX, superior, middle, inferior portions of gluteus maximus combined; VAS, vastus medialis, intermedius, and lateralis combined; SOL, soleus.

Table 1: Physical characteristics of study participants and spatiotemporal data recorded for each participant's longest jump. Year-best Performance: Longest jump recorded for each participant in 2020; Touchdown Leg Angle: Angle measured in the sagittal plane between the ground and the line connecting the COM of the body and the center of pressure acting under the take-off foot at the instant of touchdown (i.e., 0% of the stance phase); Takeoff Angle: Angle measured in the sagittal plane between the resultant velocity of the COM of the body and the ground. Jump Distance was defined as the distance measured from the last point of contact of the foot immediately before take-off to the rearmost mark in the sandpit upon landing. s.d., standard deviation.

Participant	Mass (kg)	Gender	Height (cm)	Age (yrs)	Take-off Leg	Year-Best Performance (m)	Approach Velocity		Takeoff Velocity		Touchdown Leg Angle (deg)	Takeoff Angle (deg)	Ground Contact Time(sec)	Jump Distance (m)
							Fore-aft (m/s)	Vertical (m/s)	Fore-aft (m/s)	Vertical (m/s)				
1	62	F	170	22	R	6.06	7.366	-0.729	5.913	2.906	59.07	26.17	0.145	5.34
2	60	M	180	20	R	6.50	8.046	-0.459	7.357	2.492	60.90	18.71	0.152	6.23
3	54	F	169	19	L	5.92	7.570	-0.602	6.738	2.408	62.01	19.67	0.143	5.36
4	60	M	180	20	R	6.50	8.111	-0.630	7.171	2.726	62.50	20.81	0.151	6.04
5	44	F	159	20	R	5.91	6.867	-0.576	5.779	2.924	61.31	26.84	0.136	4.92
6	59	F	172	18	R	5.20	6.637	-0.612	6.168	2.130	67.97	19.05	0.138	4.62
7	54	F	158	18	R	5.32	6.817	-0.638	5.680	2.591	60.75	24.52	0.136	4.53
8	59	M	173	17	L	6.69	8.333	-0.561	7.386	2.773	60.72	20.58	0.120	6.05
9	65	M	181	18	L	7.18	9.111	-0.442	7.684	3.147	58.17	22.27	0.123	6.40
Mean±s.d.	57.44±6.13		171.3±8.5	19.1±1.5		6.14±0.64	7.65±0.82	-0.58±0.09	6.65±0.78	2.67±0.31	61.49±2.78	22.07±3.07	0.138±0.011	5.50±0.71