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# Optimal schedule for monitoring a plant incursion when detection and treatment success vary over time

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# Abstract

Management of an invasive plant species can be viewed as two separate and successive processes. The first, *survey*, aims to find infested areas and remove individuals. The second, *monitoring*, consists of repeated visits to these areas in order to prevent possible re-emergence. As detection probability may vary over time, the timing and number of monitoring visits can dramatically impact monitoring efficacy. We explore the optimal timing and number of monitoring visits, by focusing on one infested site. Our decision-analysis framework defines an optimal monitoring schedule which accounts for a time-dependent probability of detection, based on the presence/absence of a flower. We use this framework to investigate the optimal monitoring schedule for *Hieracium aurantiacum*, an invasive species in the Australian Alps and many other countries. We also perform a sensitivity analysis to draw more general conclusions. For *Hieracium aurantiacum* eight monitoring visits (compared to 12 visits in the current program) are sufficient to obtain a 99% monitoring efficacy. When four or fewer visits to a site are allowed, it is optimal to visit during the high season, when the weed is likely to initiate flowering. Any extra visits should be scheduled in the early season, before the plants flower. The sensitivity analysis shows that increasing the detection probability early in the season has a greater impact than increasing it late in the season. An effective treatment method increases the value of site visits late in the season, when the detection probability is higher. Our decision-analysis framework can assist invasive species managers to reduce or reallocate management resources by determining the minimum number of monitoring visits required to satisfy an acceptable risk of re-emergence.

**Keywords:** Invasive Species, *Hieracium*, Weed Management, Monitoring, Scheduling, Imperfect detection.

# Introduction

1 Whether or not, or how, we can achieve eradication of an invasive species has become a  
2 contentious issue (Gardener et al., 2010). Management agencies are justifiably concerned  
3 about achieving the best outcome for their investment and need to ensure that their actions  
4 are as effective as possible. Eradication of an invasion requires delimitation of a population's  
5 extent, containment of the population to prevent further spread, and successful extirpation  
6 of all individuals and propagules (Panetta 2007). Imperfect detection during surveillance and  
7 monitoring can critically influence the optimal searching effort (Mehta et al., 2007; Hauser  
8 and McCarthy, 2009), the optimal management action (Regan et al., 2011; Emry et al., 2011;  
9 Moore et al., 2014), and the duration of an eradication program (Cacho et al., 2006; Rout  
10 et al., 2013). A common practice is to consider the probability of detection as a constant  
11 parameter, which only varies between species, for example because of species traits (Garrard  
12 et al., 2013) or appearance (O'Connell JR et al., 2006). But detection may also change in  
13 space and time, influenced by environmental factors such as habitat quality (e.g., Wintle  
14 et al., 2005), temperature (e.g., Kroll et al., 2008), rainfall (O'Donnell et al., 2015) or the  
15 changes in visibility between different stages of plant development.

16

17 The monitoring phase of an invasive plant eradication program aims to contain and ex-  
18 tirpate infestations, which may persist after their initial discovery and treatment due to: (1)  
19 individuals escaping detection during the original survey, (2) failure of the treatment method,  
20 (3) emergence of new plants from the seed bank, (4) reproduction of individuals at the site,  
21 or (5) secondary introduction from other populations (Panetta, 2007; Panetta et al., 2011).  
22 More than one visit to the infested area is generally needed to remove all the individuals  
23 and visit frequency should be set to prevent undetected individuals and new recruits from  
24 reproducing (Panetta, 2007; Moore et al., 2014). Given that a plant's detection probability  
25 may vary through time, monitoring visit frequency and timing are both crucial decisions for  
26 effective extirpation.

28 In this article we seek to determine the optimal timing of monitoring visits, in order to  
29 maximize the chance that

30 (i) every individual is removed and

31 (ii) no individual reproduces.

32 We define a monitoring schedule by the timing (position) and number of site visits, and  
33 propose a framework for evaluating schedules. We base the value of a monitoring schedule  
34 on the probability that every individual is removed and none reproduce, which is in turn  
35 defined as a function of the individual's probability of detection. We consider that control  
36 efficacy and detection are both triggered by time-dependent random factors, such as timing  
37 of flower development and seed release over spring and summer. What complicates moni-  
38 toring is that such factors cannot be known exactly before visiting. We propose a general  
39 structured decision-making approach, where the optimal monitoring schedule represents the  
40 average best compromise over all possible realisations of these random factors. This approach  
41 is particularly suitable for any monitored invasive species, plant or animal, which varies in  
42 detectability over time. In addition, this framework could also be used to optimise the sur-  
43 vey schedule of an endangered species, such that the probability of detecting the species is  
44 optimal.

45

46 A natural cause of temporal variation in its probability of detection is the presence or  
47 absence of flowers (Kéry and Gregg, 2003; Burrows, 2004). Flowers typically have a high  
48 colour contrast compared to stems and leaves and can lead to more rapid detection of weed  
49 species when present (Hauser and Moore, 2016). As a consequence the timing of a monitoring  
50 survey can highly impact its efficacy. For example, if sites are visited prior to flowering  
51 individuals may have a low probability of being detected; vegetative plants may be missed  
52 and may subsequently produce seeds. Conversely, later site visits should maximize the chance

53 of finding individuals, but are also risky because individuals may have already reproduced  
54 and dispersed seed. To complicate the trade-off further, factors affecting detection (such as  
55 flower presence) may not be precisely predictable. To the best of our knowledge, this problem  
56 has not received attention in the scientific literature. We illustrate this framework using an  
57 invasive flowering plant, *Hieracium aurantiacum*, which occurs in the USA, Canada, Japan,  
58 New Zealand and Australia where it is nationally prohibited (Williams and Holland, 2007).  
59 We compute an optimal monitoring schedule for this plant, and compare it to an ad-hoc  
60 strategy where the site is visited fortnightly. We then generalize the results by performing a  
61 sensitivity analysis.

## 62 **Materials and methods**

### 63 **A general monitoring model**

64 In this study, we are concerned with infestations that have already been discovered through  
65 general surveillance, and have now entered a *monitoring* phase. Our model describes only  
66 one site of infestation, as an equivalent monitoring schedule can be developed for every  
67 known infestation in the landscape. We assume that monitoring of the invasive population  
68 occurs periodically, every year. For species that cannot be detected all year, monitoring  
69 occurs during the species' active stage, where it is plausible to detect or remove individuals.  
70 For example, for a herbaceous perennial weed, the opportunity for monitoring corresponds  
71 to the period from re-emergence until it dies back after flowering. For an annual weed, it  
72 is the period between germination and death. For hibernating animals, it corresponds to  
73 the post-hibernation period or more generally for burrowing animals, it corresponds to their  
74 high surface activity period.

75 We consider that monitoring has two objectives: (i) remove every individual from the site  
76 and (ii) prevent reproduction. We devise a *monitoring schedule* of  $n_v$  visits planned during  
77 the management season according to a time schedule  $\sigma = \{t_1, \dots, t_{n_v}\}$ . The efficacy of a

78 monitoring schedule,  $U_\lambda(\sigma; \Theta^\sigma)$ , is based on the weighted probability that each objective is  
 79 fulfilled:

$$U_\lambda(\sigma; \Theta^\sigma) = \lambda \mathbb{P}(N^\sigma = 0 \mid \sigma, \Theta^\sigma) + (1 - \lambda) \mathbb{P}(Nb_o^\sigma = 0 \mid \sigma, \Theta^\sigma). \quad (1)$$

80 The expected outcomes depends on our schedule  $\sigma$ , i.e., the position of the visit days and  
 81 the number of visits  $n_v$ , and the set of factors that influence detection during monitoring  
 82  $\Theta^\sigma$ .

83  $N^\sigma$  is the number of individuals that are present in the site at the end of the monitoring  
 84 period and  $Nb_o^\sigma$  the number of offspring produced at the end of the monitoring period. Thus  
 85 monitoring is successful when  $N^\sigma = Nb_o^\sigma = 0$  and the efficacy  $U_\lambda(\sigma; \Theta^\sigma)$  is based on the  
 86 site's state at the end of the monitoring period. Weighting factor  $\lambda \in [0, 1]$  can be chosen to  
 87 determine the relative importance of the two monitoring objectives. For example, if the state  
 88 of the site during future years is not important, one only wants to remove every individual  
 89 before the end of the season, regardless of whether they have reproduced or not, and  $\lambda = 1$ .  
 90 Conversely, if the state of the site only matters during the following years, we can focus on  
 91 avoiding reproduction by setting  $\lambda = 0$ . When  $0 < \lambda < 1$ , we seek to address both objectives,  
 92 with  $U_\lambda = 1$  meaning that both objectives have been reached.

93 Note that setting  $\lambda = 1$  can also be used to optimize the survey schedule of an endangered  
 94 species. For example, the objective might be to mark all the individuals of a protected  
 95 population. In this case,  $N^\sigma$  could be defined instead as the number of unmarked animals  
 96 at the end of the survey and the survey is successful when  $N^\sigma = 0$ .

97 Typically, the value of the factors that influence detection  $\Theta^\sigma$  are not known in advance.  
 98 A common practice in decision analysis is to determine the decision that will provide the  
 99 highest expected reward over all possible values of the environmental uncertainty (Huang  
 100 et al., 2011). For our problem, the optimal monitoring schedule  $\sigma^* = \{t_1^*, \dots, t_{n_v}^*\}$  is thus  
 101 defined as the one that has the highest expected efficacy  $U_\lambda$ , over all possible values of the

102 factors  $\Theta^\sigma$ . Computing the optimal monitoring schedule for a fixed number of visits,  $n_v$ ,  
 103 consists of choosing the monitoring schedule  $\sigma$  to maximise:

$$\mathbb{E}_{\Theta^\sigma} [U_\lambda(\sigma; \Theta^\sigma)] = \sum_{i=1}^{n_\Theta} \mathbb{P}(\Theta^\sigma = \Theta_i^\sigma) U_\lambda(\sigma; \Theta^\sigma), \quad (2)$$

104 where  $\Theta_1^\sigma, \dots, \Theta_{n_\Theta}^\sigma$  are all the possible values of the factors that influence detection, captur-  
 105 ing the environmental uncertainty. The environmental uncertainty represented in  $\Theta^\sigma$  can  
 106 take different forms: (i) event timing uncertainty, (ii) model structural uncertainty or (iii)  
 107 parameter uncertainty. Event timing uncertainty refers to the case where detection and re-  
 108 production are dependent on some particular event,  $\Theta^t$  represents the realisation of the event  
 109 on day  $t$ . For a flowering plant an important event is the first flowering day, as the plant  
 110 becomes much easier to detect (see next section), but it is not possible to predict this event  
 111 exactly. For amphibians, rain can greatly increase the availability of individuals (O'Donnell  
 112 and Semlitsch, 2015) and consequently the probability of detection. In that case, previous  
 113 rainfall records can be used to determine the probability of rain and model the probability  
 114 of detection over time. Model structural uncertainty refers to the uncertainty around the  
 115 model that describes the probability of detection or reproduction or removal. In this par-  
 116 ticular case,  $\Theta$  is a random variable whose values tell the type of structure in the possible  
 117 models. For example, adult lizards are generally easier to detect during the mating period  
 118 as this corresponds to a period of high activity. But the functional form of the relationship  
 119 between lizard activity over time and detection might be uncertain, such that several possible  
 120 models might be considered. Finally, parameter uncertainty refers to the situation where the  
 121 functional form of the probability of detection, reproduction or removal is assumed known  
 122 but some parameters remain uncertain. In this case,  $\Theta$  represents the possible values that  
 123 these parameters can take. For example, birds are easier to detect when they are singing, so  
 124 it is preferable to monitor a site when bird songs are frequent, such as at sunrise. But the  
 125 value of the probability of detection when the song frequency is high might still be unknown.

126 Another example of parameter uncertainty is the initial number of individuals present.  
127 In all cases, the uncertainty on  $\Theta$  must be described using a probability distribution, which  
128 can be either an empirical distribution or a functional form. It is the probability distribution  
129 of the event over time for the event timing uncertainty; the probability of each possible model  
130 representing the system for the model structural uncertainty and the probability distribution  
131 over all possible parameter values for the parameter uncertainty. When there is complete  
132 uncertainty, a uniform distribution over the set of possible values could be used. But when  
133 no information is available, the problem remains trivial as every monitoring schedule has  
134 the same estimated value, and there is no reason that a given monitoring schedule should  
135 be better than another one.

136 We illustrate this framework for herbaceous perennials and annual plants, where the  
137 probability of detection is influenced by the presence of flowers. In the following, we simplified  
138 the problem by considering that only one individual is present in the site.

## 139 **Monitoring a flowering plant**

140 For a flowering plant we consider that the first flowering day of the plant is an event uncer-  
141 tainty, denoted by  $\Theta = T_F$ . We expect that the presence/absence of flowers greatly impacts  
142 the detection of the individuals, as it will be much easier to detect a plant when a colorful  
143 flower is present. In addition, the presence of flowers also provides useful information on the  
144 reproduction process, as seed production only starts sometime after the plant has had its  
145 first flower.

146 We parameterised this model for the management of orange hawkweed.

## 147 **Baseline case study: orange hawkweed**

148 *Hieracium aurantiacum* (synonym *Pilosella aurantiaca*) is an invasive herbaceous perennial in  
149 the Asteraceae, spreading by both seeds and vegetatively by rhizomes and stolons. Each basal  
150 rosette of leaves can produce a single stem 15-40 cm tall that produces multiple conspicuous

151 bright red-orange flowers. It is an invasive species in the USA, Canada, Japan, New Zealand  
 152 and Australia where it is nationally prohibited (Williams and Holland, 2007). In Victoria  
 153 (Australia), it is currently managed in the Victorian alpine region with the aim of complete  
 154 eradication. Each site is currently visited as often as 12 times during the season. For this  
 155 plant, the active season, where it is plausible to detect or treat the plant, is nearly 6 months  
 156 and we set this to be  $L_{Season} = 185$  days.

157 First, we fit a probability model for the first day of flowering. A negative binomial  
 158 relationship models the number of trials required to obtain  $r$  successes, with probability of  
 159 success  $q$  per trial. This is comparable to the concept of accruing sufficient degree-days before  
 160 flowering can occur. Therefore we define the probability distribution of  $T_F$ , as a translated  
 161 negative binomial distribution:

$$\mathbb{P}(T_F = t_f) = \begin{cases} \binom{n_{fd}-t_f+r}{n_{fd}-t_f+1} q^r (1-q)^{n_{fd}-t_f+1}, & \text{if } 1 \leq t_f \leq n_{fd}, \\ 0, & \text{if } n_{fd} < t_f \leq L_{Season}, \end{cases} \quad (3)$$

162 where  $n_{fd}$  is the last day when the plant can have its first flower. After this day, it is assumed  
 163 that the plant must have started flowering, i.e.  $\mathbb{P}(T_F \leq n_{fd}) = 1$ . Note that in this case,  
 164 the equation (3) defines an approximate probability distribution since the probabilities do  
 165 not necessarily sum to one. This is due to the fact that we forced the probability of having  
 166 a first flower after  $n_{fd}$  to be zero. However, the impact of this approximation is insignificant  
 167 ( $1 - \sum_{t_f} \mathbb{P}(T_F = t_f) \leq 0.001$ ) for the parameters we considered) and thus we still use  
 168  $\mathbb{P}(T_F = t_f)$  as a probability distribution.

169 To use this probability distribution in practice, it is important to know the range of possible  
 170 flowering days for the plant or the range of the most probable first flowering day. For orange  
 171 hawkweed, monitoring program data show that flowers should be present after 92 days, i.e.  
 172  $n_{fd} = 92$ , and that the peak first flowering day is day 79. We tuned parameters  $r$  and  $q$   
 173 by hand, so as to visually find the values giving this range and most probable value. There  
 174 may, however, be other ways to fit these parameters in different circumstances. For orange

175 hawkweed, we selected  $r = 4$  and  $q = 0.17$ . The probability distribution of the first flowering  
 176 day is shown in Fig. (2).

177 We consider that the probability of detection can take two values, depending on whether  
 178 a flower is present on the plant ( $p_{High}$ ) or not ( $p_{Low}$ ). We assume for simplicity that the  
 179 high detection rate is maintained until the end of the season. The probability of detecting  
 180 an individual hawkweed for each day  $t$  is defined as follows:

$$p_D(t; T_F) = \begin{cases} p_{Low} & \text{if } 0 \leq t < T_F, \\ p_{High} & \text{if } T_F \leq t \leq L_{Season}. \end{cases}$$

181 We determined the values of these two detection probabilities from Hauser and Moore  
 182 (2016) and Hauser et al. (2013):  $p_{Low} = 0.47$  and  $p_{High} = 0.99$ . The value of the expected  
 183 probability of detection is presented in Figure (2).

184 We suppose that, when detected, all the living parts of the plant are removed before the  
 185 end of the season (Bear et al., 2012) and there is thus no risk of re-emergence.  
 186 Then, the removal success is only based on our ability to detect the plant and we have:

$$\mathbb{P}(Nb^\sigma = 0 \mid \sigma, \Theta^\sigma) = \sum_{t_d \in \sigma} \mathbb{P}(T_D = t_d \mid T_F).$$

187 Here  $T_D$  is the random variable defining the first day of detection. When the plant is detected  
 188 it is marked with a colored flag and its coordinates recorded via GPS. Thus, once detected,  
 189 the plant will be detected during following visits. For a given monitoring schedule  $\sigma$ , the  
 190 probability that the plant is first detected on day  $t_d$  is:

$$\mathbb{P}(T_D = t_d \mid T_F) = \begin{cases} p_D(t_d; T_F) & \text{if } d = 1, \\ p_D(t_d; T_F) \prod_{i=1, d>1}^{d-1} (1 - p_D(t_i; T_F)) & \text{if } d > 1. \end{cases}$$

191 Similar to a geometric distribution, the left term of the second line of the equation is the  
 192 probability that the plant is detected on day  $t_d$  and the right term is the probability that  
 193 the plant was not detected during the previous visits.

194 In this case, the efficacy of a monitoring schedule becomes:

$$U_\lambda(\sigma; T_F) = \sum_{t_d \in \sigma} \mathbb{P}(T_D = t_d | T_F) [\lambda + (1 - \lambda) (\mathbb{P}(Nb_o^\sigma = 0 | T_D = t_d, T_F))] \quad (4)$$

195 That is, successfully detecting a plant during the monitoring schedule  $\sigma$  is sufficient to en-  
 196 sure  $N^\sigma = 0$  at the end of the season. We assume that when found the plant is immediately  
 197 treated with herbicide but, since the effect of herbicide is not instantaneous, reproduction  
 198 might still occur even when the plant is detected.

199

200 In practice, production of viable seeds only starts after flowering of the plant and we  
 201 consider that no viable seeds are produced when the plant is detected and treated before  
 202 flowering day  $T_F$ , i.e.  $\mathbb{P}(Nb_o^\sigma = 0 | T_D, T_F) = 1$ , when  $T_D \leq T_F$ . We consider the simplified  
 203 situation where seeds are dispersed after a fixed time period of duration  $\delta_{spread}$  that follows  
 204 the flowering day. Thus, when the plant is detected after  $T_F + \delta_{spread}$ , i.e. after all viable  
 205 seeds have been released, we have  $\mathbb{P}(Nb_o^\sigma = 0 | T_D, T_F) = 0$ , when  $T_D > T_F + \delta_{spread}$ .  
 206 When the site is detected between flowering and the first day of seed release, we consider  
 207 that treatment can be either successful (i.e. no offspring are produced) with probability  $p_T$   
 208 or unsuccessful with probability  $1 - p_T$ . In this case, the probability that no offspring are  
 209 produced follows a geometric distribution:  $\mathbb{P}(Nb_o^\sigma = 0 | T_D, T_F) = \sum_{i=1}^{T_{days}} (1 - p_T)^{i-1} p_T$ , when  
 210  $T_F < T_D \leq T_F + \delta_{spread}$ .  $T_{days}$  is the number of treatments applied between flowering and  
 211 seed release:

$$T_{days} = \# \{i = 1 \dots n_v | T_F \leq T_D \leq t_i \leq T_F + \delta_{spread}\}.$$

212 In the definition of  $\mathbb{P}(Nb_o^\sigma = 0 | T_D, T_F)$ ,  $i$  is simply the index of the visit day where treatment  
 213 was successful.

214 Finally, we obtain the following definition of the probability that no offspring have been  
 215 produced:

$$\mathbb{P}(Nb_o^{t_{n_v}} = 0 \mid T_D, T_F) = \begin{cases} 1 & \text{if } T_D \leq T_F \\ \sum_{i=1}^{T_{days}} (1 - p_T)^{i-1} p_T & \text{if } T_F < T_D \leq T_F + \delta_{spread} \\ 0 & \text{if } T_D > T_F + \delta_{spread} \end{cases} \quad (5)$$

216 The probability of treatment success  $p_T = 0.51$  has been estimated from the current  
 217 management program.

218 Finally, the probability that the individual will not reproduce for a given monitoring  
 219 schedule is the expected probability that there is no offspring produced at the end of the  
 220 monitoring period, over all possible values of the day of first detection  $t_d$ :

$$\mathbb{P}(Nb_o^\sigma = 0 \mid \sigma, \Theta^\sigma) = \sum_{t_d \in \sigma} \mathbb{P}(T_D = t_d \mid T_F) \mathbb{P}(Nb_o^{t_{L_{season}}} = 0 \mid T_D = t_d, T_F) \quad (6)$$

221 A schematic representation of the entire process is available in Figure (1) and a summary  
 222 of the model parameters can be found in Table 1.

223 To allow visits to other sites, we set a minimum time interval  $\delta_{visits}$  between visits. We  
 224 considered here that a site can be visited at most once every 6 days, i.e.  $\delta_{visits} = 5$  days.

## 225 **Optimisation procedure**

226 We use three different values of the parameter  $\lambda = \{0, 1, 0.5\}$ , in order to discuss the effect  
 227 of each objective on the timing of the visit days.

228 We used a genetic algorithm to compute the optimal monitoring schedule for a fixed number  
 229 of visits  $n_v$ . More precisely, we use the function *ga* of Matlab, which converges relatively  
 230 easily to the optimal solution.

231

232 **Comparison with other strategies**

233 We compare the optimal monitoring schedule to a strategy which consists of visiting the site  
 234 once every two weeks; this has been commonly adopted in the orange hawkweed eradication  
 235 program. We suppose that the site is first visited during the first day of the management  
 236 season, and then 14 days later and so on until the site is visited 12 times. We compared  
 237 the fortnightly strategy to the optimal monitoring schedule computed for different numbers  
 238 of visits, from 1 to 12. Then we compute the gain, in percentage, of using the fortnightly  
 239 strategy over the optimal strategy:

$$Gain_{n_v} = \frac{100 * (U_{0.5}(\sigma_{Fortnightly}) - U_{0.5}(\sigma_{Opt}^{n_v}))}{U_{0.5}(\sigma_{Opt}^{n_v})}, \quad (7)$$

240 where  $\sigma_{Fortnightly}$  and  $U_{0.5}(\sigma_{Fortnightly})$  are the fortnightly schedule and its value (see  
 241 equ. (4)) and  $\sigma_{n_v}^*$  and  $U_{0.5}(\sigma_{n_v}^*)$  are the optimal monitoring schedule with  $n_v$  visits and  
 242 its value (see equ. (4)). Note that the number of visits of the fortnightly strategy is fixed  
 243 to 12 while the number of visits varies from 1 to 12 for the optimal monitoring schedule.  
 244 Then theoretically the value of the fortnightly strategy can be higher than the value of the  
 245 optimal monitoring schedule, computed for a smaller number of visits  $n_v < 12$ . The aim of  
 246 this comparison is to show that an optimal scheduling can perform as well or better than  
 247 this ad-hoc strategy, for a fewer number of visits.

Finally, we consider a random strategy, denoted  $\sigma_{Random}$ . The strategy selects randomly  
 12 monitoring days in the set  $\{1, \dots, L_{Season}\}$ , where each day can be selected at most one  
 with probability  $\frac{1}{L_{Season}}$  (random draw without replacement). There is not a unique random  
 strategy, so we propose to define  $U_{0.5}(\sigma_{Random})$  as an expected value instead. We first drew  
 5,000 possible random strategies  $(\sigma_{Random}^i)_{i=1}^{5000}$  and evaluated their values  $U_{0.5}(\sigma_{Random}^i)$  for  
 all  $i$ . We then define  $U_{0.5}(\sigma_{Random})$  as the average value of the 5,000 random strategies:

$$U_{0.5}(\sigma_{Random}) = \frac{\sum_{i=1}^{5000} U_{0.5}(\sigma_{Random}^i)}{5000}.$$

248 We chose to compare the optimal schedule with the random strategy, as it can be inter-  
249 preted as a lower bound in terms of efficacy.

## 250 Sensitivity analysis

251 Because the non-biological parameters are highly linked to orange hawkweed management  
252 practices, we now illustrate the model with varying management scenarios.

253 We first explore the impact of the treatment success, comparing a low  $p_T = 0.2$  and a high  
254  $p_T = 0.8$  value. Second, we explore the impact of the detection probability with  $p_{\text{Low}} = 0.1$   
255 or  $p_{\text{Low}} = 0.3$  and  $p_{\text{High}} = 0.6$  or  $p_{\text{High}} = 0.75$ . For clarity we reduce the possible number of  
256 visits  $n_v$  to 1, 3 or 6 and we only consider the case where both management objectives are  
257 accounted (i.e.  $\lambda = 0.5$ ).

258 We perform a *one-at-a-time* sensitivity analysis by first analysing the effect of the various  
259 detection probabilities and fix the treatment efficacy to the baseline scenarios (i.e.  $p_T =$   
260  $0.51$ ). We then analyse the effect of the treatment efficacy with various detection probabilities  
261 scenarios: (i) a low detection scenario with  $p_{\text{Low}} = 0.1$ ,  $p_{\text{High}} = 0.6$ , (ii) a middle detection  
262 scenario with  $p_{\text{Low}} = 0.3$ ,  $p_{\text{High}} = 0.75$  and (iii) a high detection scenario with the detection  
263 probabilities of the baseline scenario, i.e.  $p_{\text{Low}} = 0.47$ ,  $p_{\text{High}} = 0.99$ .

264 **Baseline parameters: orange hawkweed** The value of all the model parameters for  
265 *Hieracium aurantiacum* are summarized in Table 2. The probability distribution of the first  
266 flowering day is illustrated in Fig. 2, as well as the expected probability of detection and the  
267 value of all monitoring schedule with  $n_v = 1$  and  $\lambda = 0.5$ .

268 We can see that the expected probability of detection is equal to  $p_{\text{Low}}$  at the beginning of  
269 the season, where the probability of the plant having a flower is very low and, rosettes and  
270 seedlings are not easy to detect. The probability of having the first flower is higher than  
271  $1 \times 10^{-3}$  after the 39<sup>th</sup> day of the season and increases until day 79. The probability of  
272 having the first flower then quickly decreases and becomes zero after  $n_{fd} + 1 = 93$  days after

273 which the expected probability of detection is  $p_{High}$ .

274 The early season is defined between the first day of the season and day 39, when it is very  
275 unlikely that the plant already has its first flower. The high season is then defined from day  
276 40 to day  $n_{fd} + 1 = 93$ , when it is likely that a plant initiates an inflorescence ; the late  
277 season is defined from day 94 to  $L_{season} = 185$ , when it is likely that the plant will already  
278 have its first flower.

279 Naturally, the value of a monitoring schedule  $U_{0.5}(\{t\})$  is approximately equal to  $p_{Low}$  in the  
280 early season, as long as there is a low probability of flowering. Thus, if the plant is found it  
281 is automatically removed before it can spread seeds. Then,  $U_{0.5}(\{t\})$  increases because the  
282 expected probability of detection increases with the probability of having the first flower.  
283 If the site is visited late in the high season, there is an increasing chance that the plant  
284 can disperse seeds even if it is discovered and treated. As a consequence, the value of a  
285 monitoring schedule  $\{t\}$  decreases from day  $79 + \delta_{spread}$  to day  $93 + \delta_{spread}$ . After day 93  
286  $+ \delta_{spread}$ , the schedule value is equal to  $0.5 \times p_{High}$  because the plant has already spread  
287 seeds and thus  $\mathbb{E}_{T_F} [p_{SR}(t; T_F)] = 0$ . Thus, when the site is visited only once, it is optimal  
288 to visit it during the first day where the expected probability of detection is maximal.

## 289 Results

290 **Timing of the visit days** The optimal timing of the visit days are presented in Fig. 3.  
291 When  $\lambda = 1$ , the only objective is to minimise the chance that the plant will be present next  
292 season. Then, it is sufficient to maximise the chance of detection, as long as we consider no  
293 failure in the removal method. The optimal placement of the visit day is naturally in the late  
294 season, when the plant is easy to detect. In our case study,  $p_{High}$  is close to one and there is  
295 no clear advantage of visiting the site more than one time, since  $U_1(\{t_1^*\}) = p_{High} \simeq 1$ . As  
296 illustrated in Fig. 4, the value of the monitoring schedule is close to 1, whatever the number  
297 of visits. For example,  $U_1(\{t_1^*\})$  and  $U_1(\{t_1^*, t_2^*\})$  are within  $10^{-5}$  of each other. For  $n_v \geq 9$ ,

298 some visit days are placed during the early season since the value of the monitoring schedule  
 299 is the same with a precision of  $10^{-12}$ , the precision we used with the genetic algorithm.  
 300 When  $\lambda = 0$ , the only objective is to minimise the chance that the plant disperses seeds.  
 301 In this case, repeated visits to the site are justified to increase the treatment success and  
 302 decrease the chance that the plant will produce seeds. When the number of visits is low,  
 303  $n_v \leq 4$ , it is optimal to visit the site during the high season, maximising the chance of plant  
 304 detection, even if there is a risk that the treatment will not entirely prevent seed production.  
 305 When the number of visits increases,  $n_v > 4$ , it is optimal to also schedule visit days early  
 306 in the season, when treatment is most effective and no seeds can be released after the plant  
 307 is found. But visits during the high season are still needed. For  $n_v \geq 8$ , there is more than  
 308 a 99% chance that the plant will not release seed, and there is then no real advantage, in  
 309 terms of monitoring value, of increasing the number of visits.  
 310 When both monitoring objectives are considered with  $\lambda = 0.5$ , the optimal dynamic of the  
 311 monitoring schedule is close to the case with  $\lambda = 0$ . Generally, the optimal visit days are  
 312 postponed compared to the case where  $\lambda = 0$  in order to maximise the probability of detec-  
 313 tion and then remove the plant. Even if we apply the  $\lambda = 0$  optimal monitoring schedule  
 314 and assess it using  $\lambda = 1$  utility, the probability of removal of the plant becomes close to one  
 315 when  $n_v \geq 6$ . Then, the objective of plant removal no longer influences the timing of visits  
 316 and their position becomes nearly the same as when  $\lambda = 0$ .  
 317 Finally, we can see in Fig. (4) that the value of the optimal monitoring schedule is ap-  
 318 proximately constant for the higher numbers of visits. For 4 visits or more, whatever the  
 319 management objective, the value of the optimal monitoring schedule is greater than 0.9 (i.e.  
 320 0.923 for  $\lambda = 0$ , 0.999 for  $\lambda = 1$  and 0.954 for  $\lambda = 0.5$ ). When 8 visits are allowed, the  
 321 values start to be all greater than 0.99 (i.e. 0.993 for  $\lambda = 0$ , 0.999 for  $\lambda = 1$  and 0.996  
 322 for  $\lambda = 0.5$ ).

323 **Comparison with other strategies** The gain of the 12-visits fortnightly strategy over  
324 the optimal monitoring schedule with a varying number of visits is presented in Fig. (5).  
325 The difference between the value of the 12-visits fortnightly strategy and the 6-visits optimal  
326 monitoring schedule is lower than 1%. Optimal monitoring schedules with  $n_v \geq 8$  visits have  
327 slightly higher values (less than 1%) compared to the 12-visits fortnightly strategy.  
328 Thus, when the timing of the visits is optimal, only 8 visits are needed in order to have a  
329 higher value than a simple common fortnightly strategy, which uses 12 visits.

330  
331 Furthermore, the random 12-visits monitoring schedule  $\sigma_{Random}$  only performed better  
332 than the 1-visit optimal schedule and even there is only a 1% improvement.

333 **Sensitivity analysis** We first analyse the sensitivity of the optimal schedule to the prob-  
334 abilities of detection. The optimal timing of the visit days is described by Fig. 6.  
335 For most of the optimal schedules, when the number of visits is low, i.e.  $n_v \leq 3$ , it is optimal  
336 to visit the site during the high season . But when the number of visits is high (i.e.  $n_v = 6$ ),  
337 extra visits are timed during the early season.

338 The placement of the visit days are remarkably similar when  $p_{Low} = 0.1$  and  $p_{Low} = 0.3$ ,  
339 even if the site is visited slightly earlier when  $p_{Low} = 0.3$ . When  $p_{Low}$  is sufficiently high and  
340  $p_{High}$  is low (here  $p_{Low} = 0.47$  and  $p_{High} = 0.6$ , see Fig. 6 (b) and (c)), it is optimal to visit  
341 the site only during the early season. But increasing  $p_{High}$  encourages visits during the high  
342 season. For  $p_{High} = 0.75$ , 1 and 4 visits are during the early season, for  $n_v = 3$  and  $n_v = 6$ .  
343 Only the highest value of  $p_{High}$  leads to site visits mostly during the high season.

344 For the different treatment models, we first discuss their efficacy. The expected prob-  
345 ability that no seeds will be released (i.e.  $\mathbb{E}[p_{AR}(t, T_F)]$ ) when only one visit is allowed is  
346 presented in Fig. 7. Obviously, when it is unlikely that the plant has flowered (i.e.  $t \leq 40$   
347 days) or it is likely that seeds are already released (i.e.  $t \geq n_{fd} + \delta_{spread} + 1 = 114$  days),  
348 all models agree: there is either an expected probability of  $p_{Low}$  (i.e.  $t \leq 40$  days) or 0 that

349 no seeds are released ( $t \geq 114$  days). But between these situations, each model acts differ-  
350 ently and the baseline scenario model gives intermediate efficacy. For the highest treatment  
351 success  $p_T = 0.8$ , the expected probability that no seeds will be released increased after day  
352  $n_{fd} + 1 = 93$ , due to an increase in detection probability (see Fig. 2). For the baseline  
353 scenario  $p_T = 0.51$ , the effect of this increase in detection probability is much less and there  
354 is no effect when  $p_T = 0.2$ . The treatment success is too low and the expected probability  
355 that no seeds will be released decreased until day  $n_{fd} + \delta_{spread} + 1 = 114$  and becomes 0  
356 thereafter.

357 The optimal timing of the visits for the low, middle and high detection scenarios and the  
358 different treatment success are shown in Fig. 8. In the low detection scenario, the timing of  
359 most visits is during the high season, which increases the expected probability of detection  
360 and gives some chance to visit the site before seeds have been released. For the highest  
361 treatment success  $p_T = 0.51$  or  $p_T = 0.8$ , even one visit is allowed at the beginning of the  
362 late season when  $n_v = 6$ .

363 For the highest treatment efficacy (i.e.  $p_T = 0.8$ ), all the visits are timed during the high  
364 season, whatever the detection scenario and the number of visits. The treatment method  
365 is sufficiently successful in killing plants to delay the visits during the season and thus  
366 increase the probability of detection. In this case, increasing the probability of detection  
367 allocates early site visits. For the baseline scenario  $p_T = 0.51$ , optimal visits are during the  
368 high season and increasing the probability of detection allows earlier visits. For the lowest  
369 treatment success  $p_T = 0.2$ , the optimal schedule is more complicated. When detection is  
370 low, visits during the early season are highly unlikely to detect plants and visits are timed  
371 for late in the high season to increase the chance of detection. In the intermediate detection  
372 scenario, some visits are located during the early season to help increase the probability of  
373 avoiding reproduction, as long as the treatment success is low. Finally, in the high detection  
374 scenario, the monitoring schedule becomes similar to the one in the low detection scenario,  
375 to take advantage of the high detection probability after flowering.

376 For all these scenarios, the value of the optimal monitoring schedule increases with the  
377 detection probability, number of visits and treatment efficacy.

## 378 Discussion

379 We have proposed a general framework to develop an efficient and effective monitoring  
380 schedule for an invasive flowering plant. Unlike other frameworks, it accounts for a changing  
381 probability of detection. This innovation is an highly realistic situation for invasive species  
382 management but also for biological surveys in general and to the best of our knowledge,  
383 ours is the first study which explicitly incorporates a systematically changing probability of  
384 detection. Indeed, previous optimal monitoring research has focused on the optimal number  
385 of visits (e.g. Garrard et al., 2008; Wintle et al., 2012), based on a constant probability that  
386 the species will be detected during a visit. But in many cases, the probability of detection is  
387 likely to change over time with for example, the emergence of flowers or an increasing activity  
388 of the species during mating. This makes the timing of the visits particularly important to  
389 detect the species, and not only the number of visits.

390 We illustrate our framework with the orange hawkweed control program in Victoria, where  
391 we showed that 8 optimally-scheduled visits perform nearly as well as 12 visits scheduled  
392 every two weeks. In this case, we also showed that when the monitoring budget is low (i.e.,  
393 at most 4 visits), it is optimal to visit the site during high season. As the monitoring bud-  
394 get increases, additional observations should be placed in the early season. The sensitivity  
395 analysis, seems to confirm this as a general result: visit in the high season as a first priority  
396 and then, if possible, during the early season. We also show that the maximum number of  
397 visits (i.e.  $n_v = 12$ ) is not needed to obtain satisfactory optimal monitoring schedule values.  
398 Indeed, the optimal monitoring schedule with 4 visits (respectively 8 visits) is already highly  
399 efficient, with monitoring objectives within 10% (respectively 0.1%) of perfect performance.  
400 This translates to a substantial saving of labour for the orange hawkweed program, where

401 the number of newly discovered sites requiring monitoring has grown every year.

402 More generally, this work can be classified as a structured decision-making approach, par-  
403 ticularly adapted when decisions have to be made while facing uncertainty. This approach  
404 consists first in defining the objectives (e.g. remove all plants and avoid reproduction) and  
405 defining a value function able to quantify the value of different alternatives (here  $U_\lambda(\sigma; \Theta^\sigma)$ ).

406 An important point is to account for uncertainty when defining the value function. It is quite  
407 obvious that probability of detection is influenced by the presence of flowers. But at first,  
408 it might seem ambitious to account for the presence of a flower because the first flowering  
409 day is unknown at the beginning of monitoring. Structured decision making makes a bridge  
410 between using an exact value of the first flowering day and ignoring the fact that flowers  
411 will emerge during the season. This bridge is made possible by considering a probability  
412 distribution of the emergence of first flower and then computing a decision that is best on  
413 average. This approach is better than ignoring the environmental uncertainty and of course,  
414 worse than knowing the exact value of all the environmental variables.

415 The model could be improved in several ways. We used a simple step function to model the  
416 probability of detection. A more flexible could use a logit function which can give different  
417 probabilities of detection for the different plant growth stages between rosette and flowering.

418 This might change the timing of the visit days, because a higher probability of detection  
419 would be expected early in the season. Nevertheless, a step function remains a plausible  
420 model and is easier to define. In addition, one can consider more than one growing stage  
421 (i.e. instead of just flowering) which affects the probability of detection. We studied here  
422 the simple case where only one site has to be monitored. In practice, hundreds of sites may  
423 have to be monitored with limited resources. In addition, the probability of detection can  
424 also vary across space. The probability that the plant will be present in the site can also  
425 be incorporated into the model and a different number of visits to each site can be allowed.

426 Then each presence probability can be updated from year to year or week to week and the  
427 optimal schedule can be computed dynamically, at the beginning of the season or every week.

428 These additions will increase the complexity of the model and the expertise required from  
429 management agencies. In this case, the optimal schedules have to be computed from more  
430 complex approximate resolution methods.

431 Another model simplification was to consider that only one individual was present in the  
432 site. The model can easily be extended to the case where more than one individual is present  
433 in the site, but then information on the number of individuals and the relation between the  
434 number of individuals in the site and the probability of detection will be required. If we  
435 consider that the individuals are independently detected at the site, then the optimal moni-  
436 toring schedule will not change from the one computed here for one individual, only its value  
437 will differ. If we consider that individuals in the site can be viewed as one unique entity,  
438 which might be the case for sufficiently small site, then the conclusions of this article remain  
439 the same.

440 In spite of these simplifying assumptions, this model provides a promising framework for  
441 managers to determine the optimal monitoring schedule by specifying a fixed number of  
442 visits, or alternatively, determining the number of necessary visits to achieve an accept-  
443 able level of performance. Our relatively simple model of changing detection and stochastic  
444 flowering demonstrates that successful monitoring can be achieved with a small number of  
445 well-scheduled repeat visits. Such prudent scheduling is likely to save considerable resources  
446 when many sites are monitored across an entire weed population.

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# Figures

## ENVIRONMENTAL UNCERTAINTY

$\theta = T_F$ , the plant's first flowering day

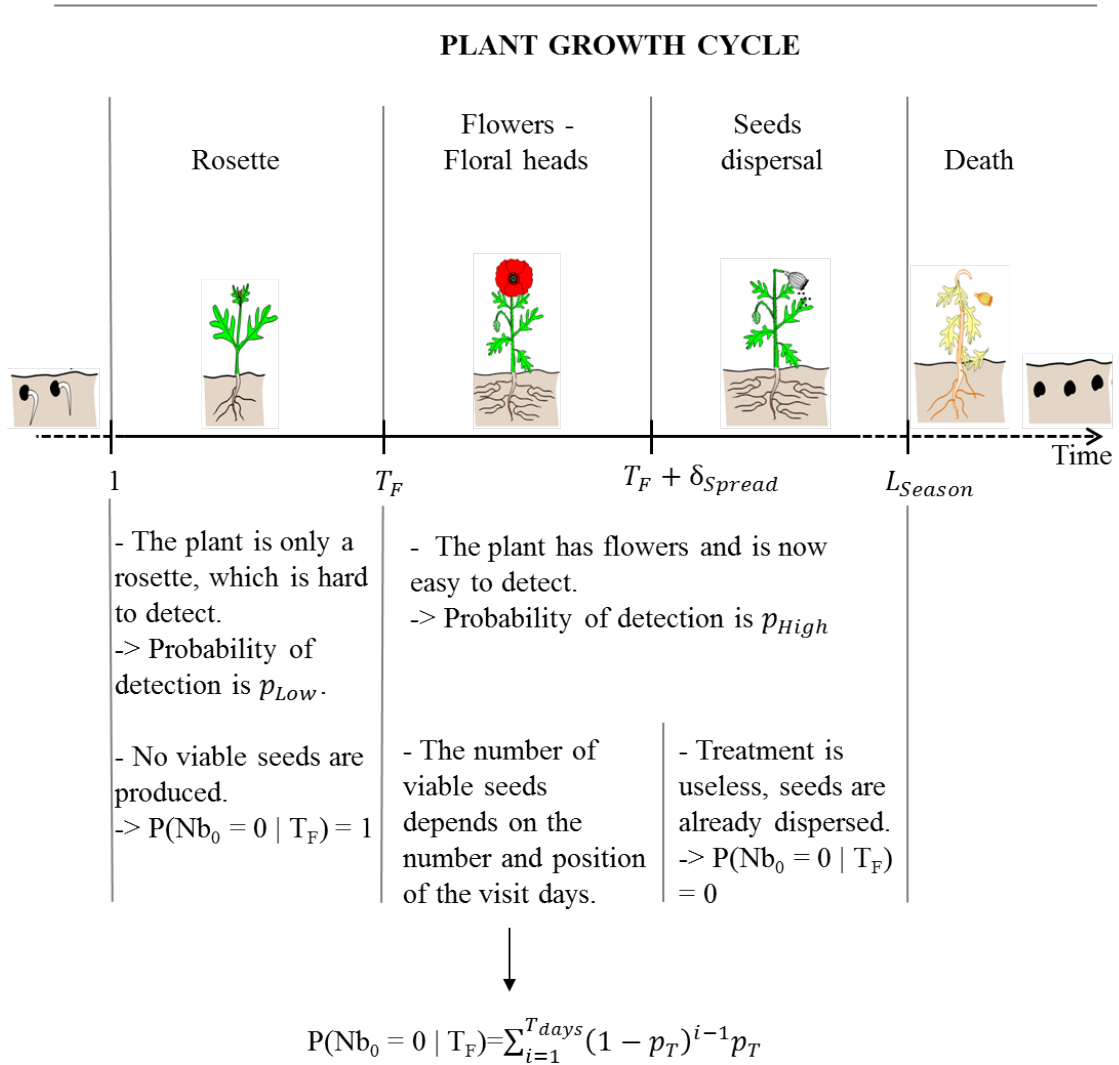


Figure 1: .

523 **Caption of Figure (1):** Schematic representation of our model for an invasive flowering  
524 plant. The major source of uncertainty is the first flowering day of the plant, i.e.  $\Theta = T_F$ .  
525 The presence of a flower improves detection as it is much easier to detect a flower than a  
526 rosette. Thus, before the appearance of the flower the probability of detection  $p_D$  is low  
527 ( $p_{Low}$ ) and much higher after the plant has its first flower ( $p_{high}$ ). The appearance of the  
528 first flower indicates that viable seeds are not yet present on the plant. Before the first  
529 flowering day, i.e.  $t \leq T_F$ , there is no seeds so the plant did not reproduce. Between the first  
530 flowering day and the day of seed release, i.e.  $T_F \leq t \leq T_F + \delta_{Spread}$ , treatment is successful  
531 with probability  $p_T$ . Finally, it is useless to visit the site only after the seeds release day, as  
532 seeds are already dispersed and reproduction cannot be avoided. Weeds illustrations can be  
533 found online at [svt.ac-dijon.fr](http://svt.ac-dijon.fr)

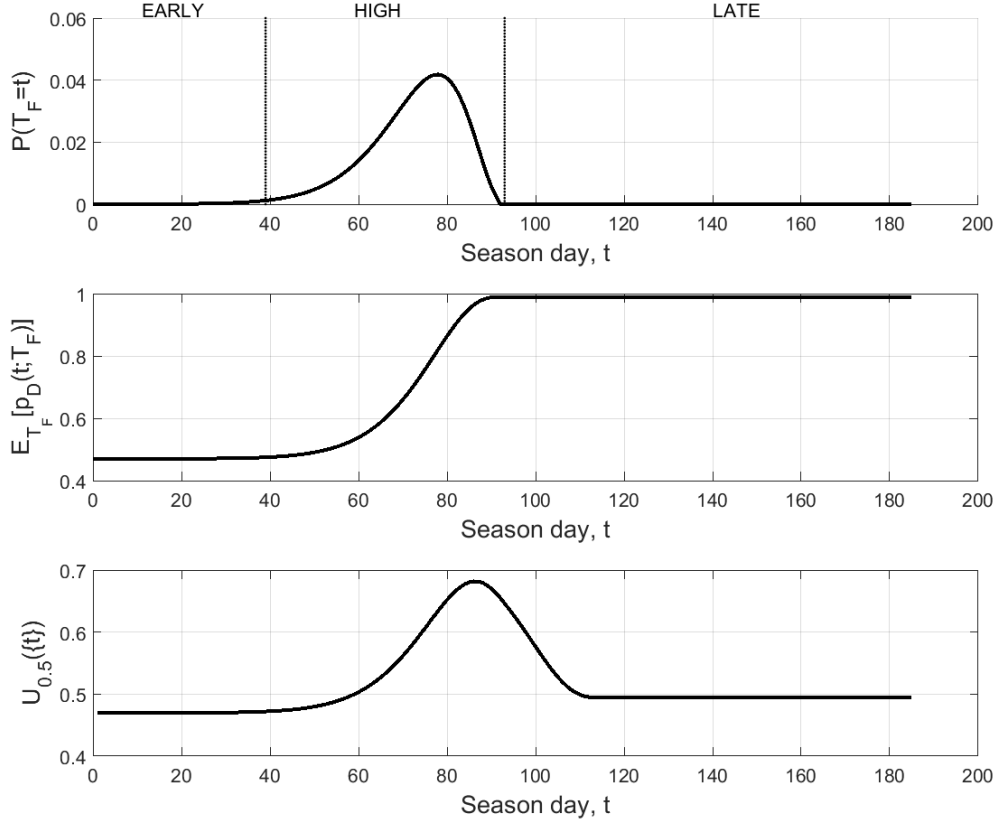


Figure 2: The likely timing and value of flowering throughout a perennial or annual plant's season. (a) The probability distribution of the first flowering day  $P(T_F = t)$ . The early season is the period where the plant is unlikely to have flowered. The high season is the period where the plant is likely to have its first flower. Finally, the late season is the period where the plant should already have flowered. (b) The influence of flowering day on the probability of detection  $E_{T_F} [P_D(t; T_F)]$ . The expected probability of detection is approximately  $p_{low}$  during the early season as the plant should not have flowered and  $p_{high}$  during late season as the plant should then have flowered. During the high season, the probability of detection smoothly increases with the probability that the plant has already flowered. (c) The variation in the probability of detection affects the value of a monitoring visit, displayed for  $\lambda = 0.5$ , meaning that both removing plants and avoiding reproduction matter. One can see here that it is optimal to visit the site during the first day where the probability of detection is maximal.

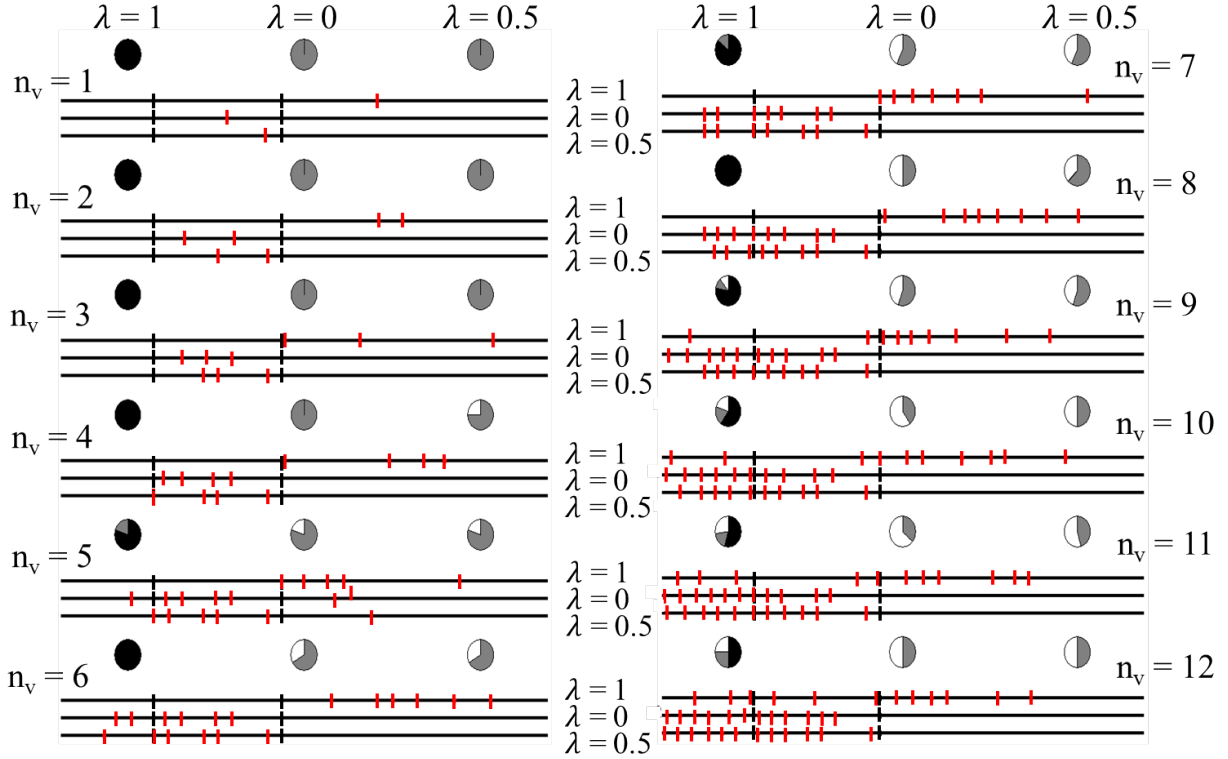


Figure 3: Optimal schedule of monitoring visits when the objectives is to remove the plant ( $\lambda = 1$ ), avoid seed release ( $\lambda = 0$ ), both removing the plant and avoiding seed release ( $\lambda = 0.5$ ). The exact positions of the visit days are displayed with red points, while the black points are boundaries of the season period, i.e. early-high-late. The pie charts give the proportions of visit days within each season period and for each management objective. The lines represent the timeline of season days, from 1 to  $L_{Season} = 185$ . Results are presented for different number of visits, from  $n_v = 1$  to  $n_v = 12$ . Each combination of three arrows and three pie charts give the optimal monitoring schedule for a given number of visits  $n_v$ . For example, the top right corner of the figure provides the results when 7 visits are allowed to the site. The first, second and third pie charts are the proportion of the visit days within each season period when  $\lambda = 1$ ,  $\lambda = 0$  and  $\lambda = 0.5$ . And the red points on the first, second and third arrow provide the optimal position of the monitoring visit when  $\lambda = 1$ ,  $\lambda = 0$  and  $\lambda = 0.5$ .

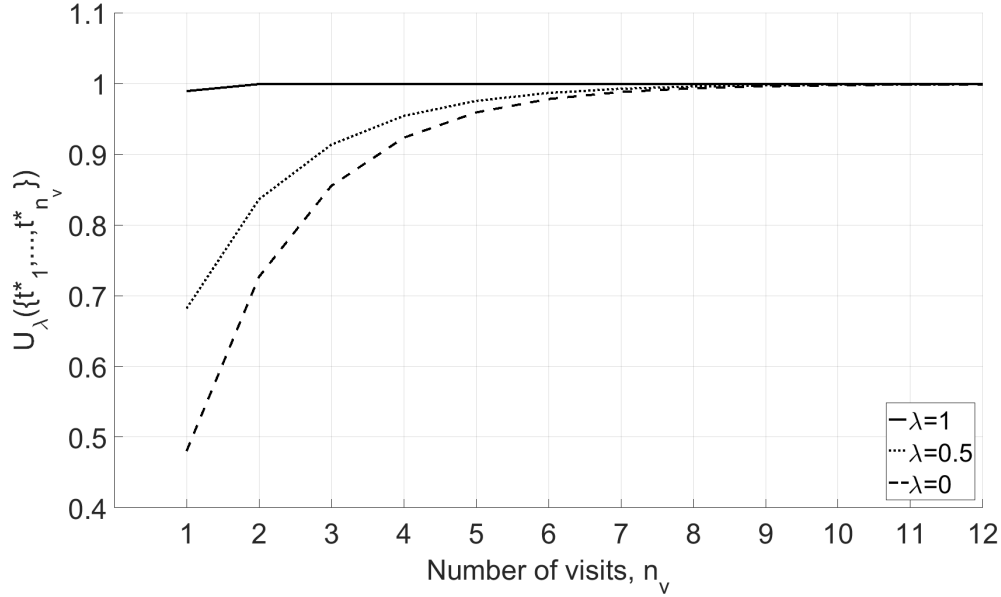


Figure 4: Value of the optimal monitoring schedule,  $U_{\lambda}(\{t_1^*, \dots, t_{n_v}^*\})$ , for various  $\lambda$  and number of visits  $n_v$ . When  $\lambda = 0$  the objective is to avoid reproduction and  $U_0(\{t_1^*, \dots, t_{n_v}^*\})$  is then the expected probability that the individual did not reproduce when the optimal monitoring schedule is used. At least 6 visits are needed in order to have a risk of reproduction close to 0. When  $\lambda = 1$  the objective of monitoring is only to optimize the detection of the individual and  $U_1(\{t_1^*, \dots, t_{n_v}^*\})$  is then the expected probability of detection when the optimal monitoring schedule is used. Here it is easy to have a probability of detection close to one when the site is visited during the late season. Finally, when  $\lambda = 0.5$  the monitoring objective is to optimize the detection of the plant and to avoid reproduction. The value of the optimal monitoring schedule,  $U_{0.5}(\{t_1^*, \dots, t_{n_v}^*\})$  is mostly influenced by the second objective of avoiding reproduction. In this case, 6 visits are also needed in order to have an objective value above 99%.

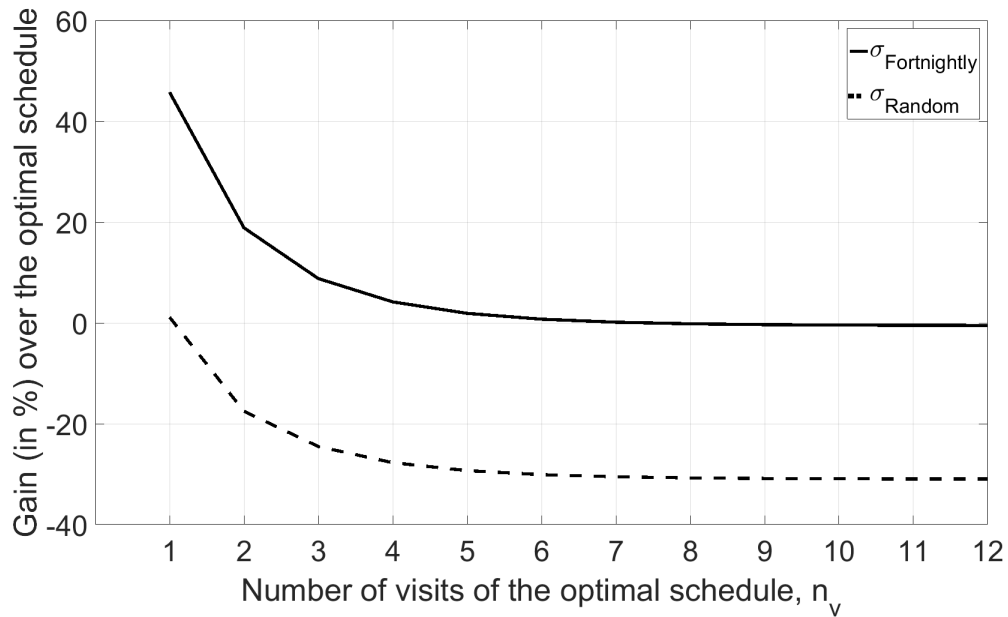


Figure 5: Percentage gain (see equ. (7)), in monitoring schedule value, for the 12-visit fortnightly strategy (straight line) and the random 12-visit strategy (dashed line) over the optimal monitoring schedule, computed for different numbers of visits. A negative (positive) gain implies that the optimal schedule has a better (worse) efficacy. For example, efficacy of the 12-visits random strategy is about 20% worse than the efficacy of the 2-visit optimal monitoring schedule. Or in other words, it is possible to increase monitoring efficacy by 20% with two visits scheduled optimally, compared to 12 visits scheduled randomly.

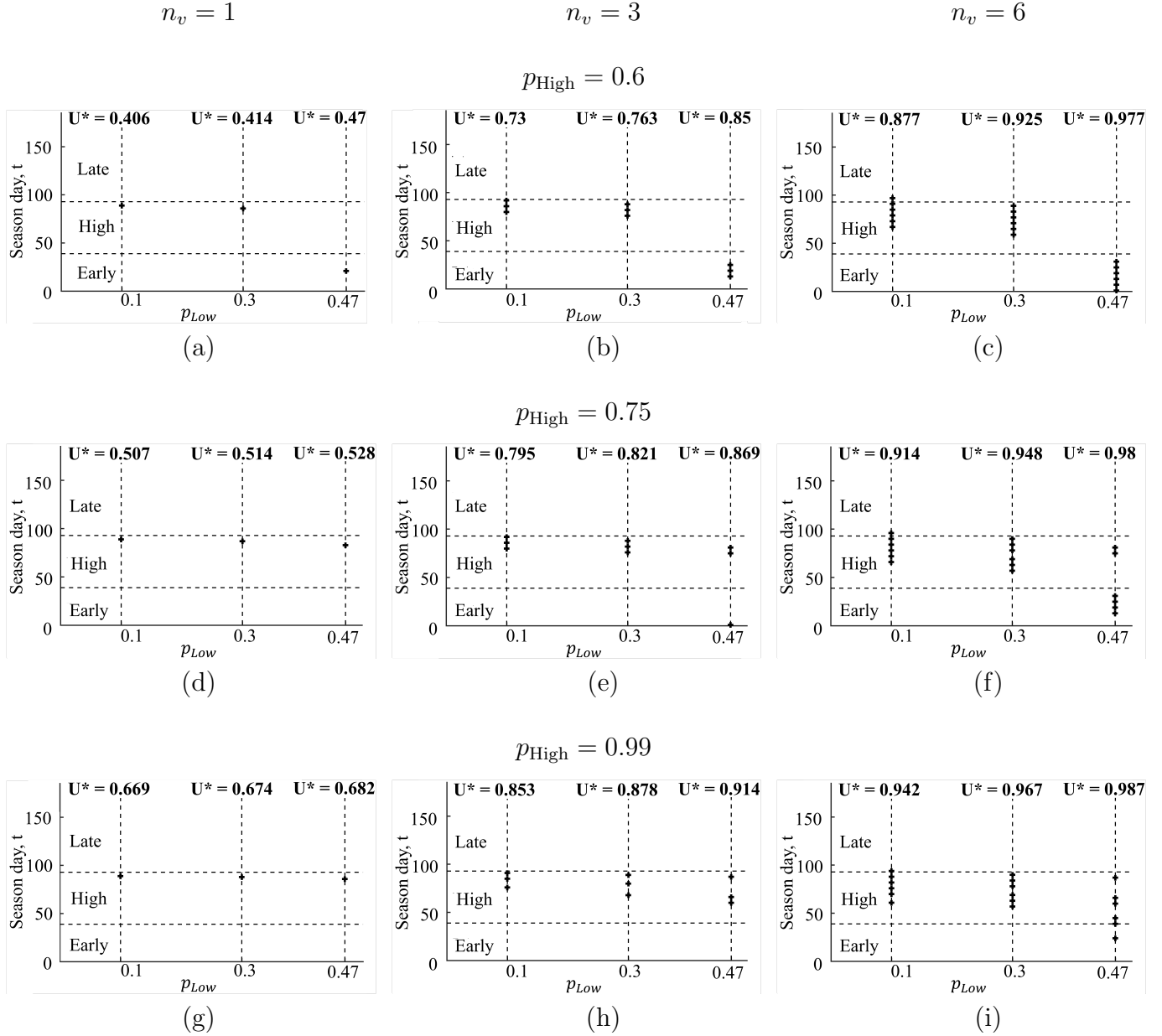


Figure 6: Optimal monitoring schedules given different values of  $p_{\text{Low}}$  and  $p_{\text{High}}$  and all other parameters equal to the baseline scenario. The management objective is both removing plant and avoiding seed release (i.e.  $\lambda = 0.5$ ). Each row gives the optimal schedule for a fixed value of  $p_{\text{High}}$  and each plot within a row gives the optimal schedule for different values of  $p_{\text{Low}}$ . On top of the optimal schedule is the schedule's value (i.e.  $U^*$ ). Each column displays results for a different number of visits. For example, the top right graphic gives the optimal monitoring schedule when 6 visits are allowed and with  $p_{\text{High}} = 0.6$ ,  $p_{\text{Low}} = 0.1$ , 0.3 and 0.47. The value of the optimal monitoring schedule for  $p_{\text{Low}} = 0.1$  is 0.884.

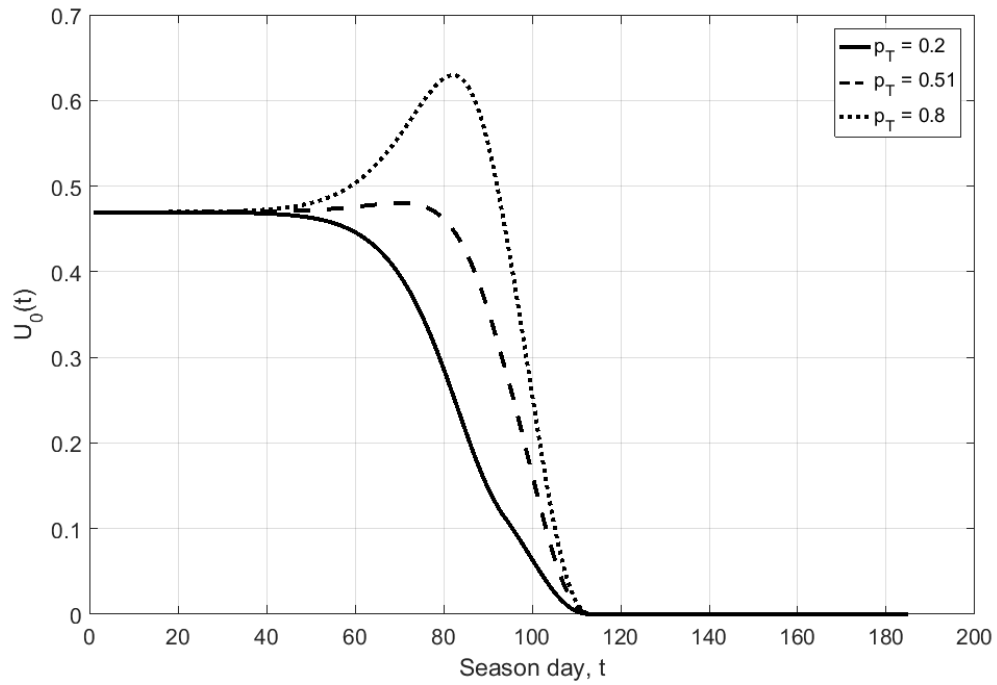


Figure 7: Expected probability that no plant reproduces for probabilities of treatment success  $p_T = 0.2$  (solid line),  $p_T = 0.51$  (dashed line) and  $p_T = 0.8$  (dot-dashed line). All the other parameters are from the baseline scenario.

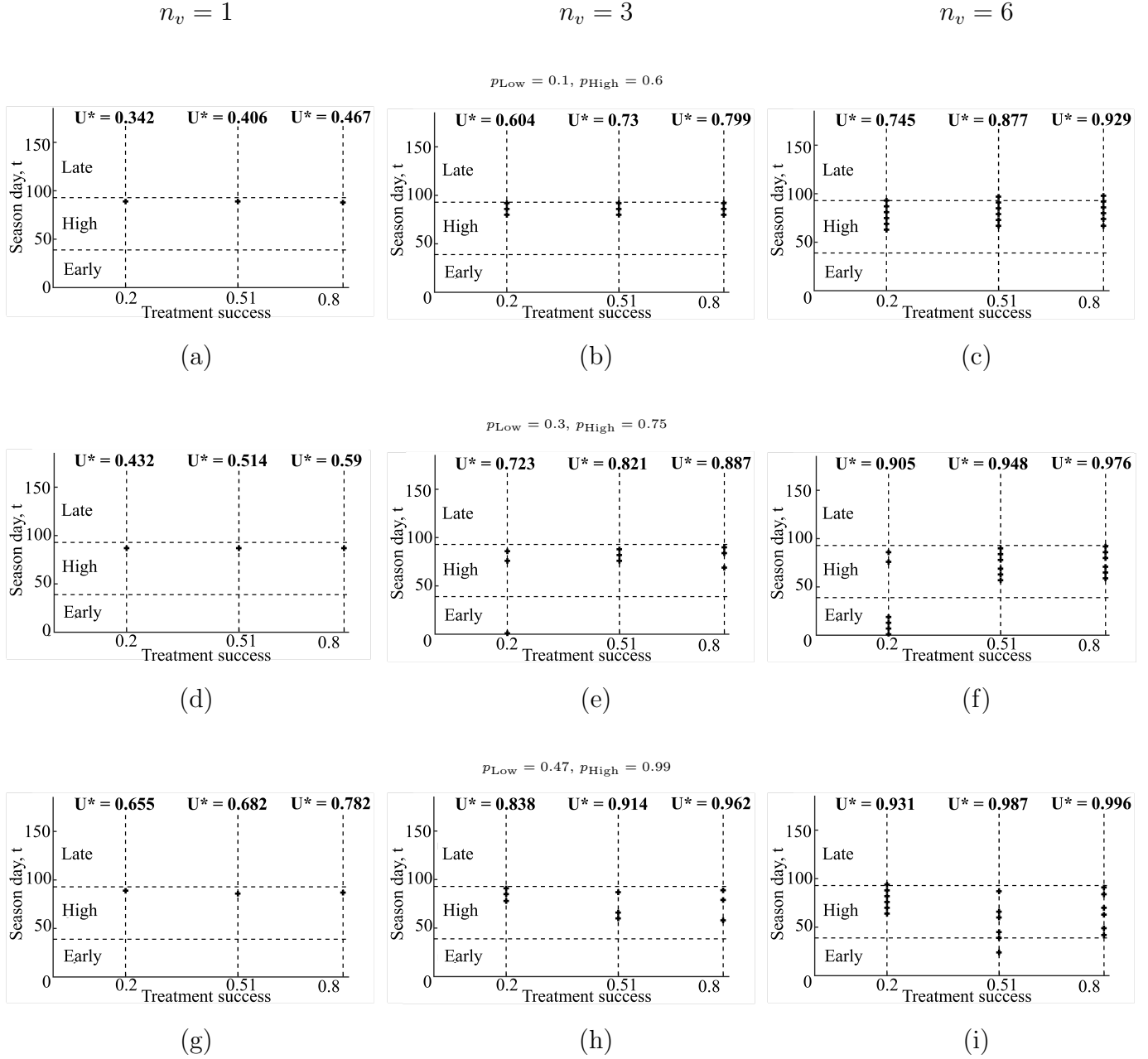


Figure 8: Position of the visit days for different probability of treatment success. The Figure is organized as in Figure (6) but here the rows are used for fixed values of  $p_{\text{High}}$  and  $p_{\text{Low}}$ .

Notation	Meaning
$L_{Season}$	The length of the monitoring season.
$\Theta$	The set of factors defining the environmental uncertainty.
$\sigma$	A given monitoring schedule of visit days.
$\lambda$	The relative importance of the two monitoring objectives.
$n_v$	The number of monitoring visits of a given schedule.
$N^\sigma$	The number of individuals still present at the end of the monitoring period.
$Nb_o^\sigma$	The number of offspring produced during the season.
$\sigma^*$	The optimal monitoring schedule.
$\delta_{visits}$	The minimum time between two visits to the site.
$T_F$	The random variable defining the first flowering day.
$(n_{fd}, r, q)$	Parameters used to define the probability distribution of $T_F$ .
$p_D(t; T_F)$	The detection probability on day $t$ if the first flower appears on day $T_F$ .
$p_{Low}$ ( $p_{High}$ )	Probability of detection before (after) the flowering day.
$T_D$	The random variable defining the first day of detection.
$\delta_{Spread}$	Time between flowering day and seed release day.
$p_T$	Probability of treatment success when a plant is treated after first flowering, but before seed release.

Table 1: Summary of the notations used.

Name, value	Meaning	Sources
<b>General parameters</b>		
$L_{Season} = 185$ days	Length of the management season.	Program.
$(n_{fd}, r, q) = (92, 4, 0.17)$	Parameters of the probability distribution of $T_F$ .	Program.
$\delta_{Spread} = 21$ days	Time lag between flowering and seed release.	Program.
$\delta_{visits} = 5$ days	Time lag between two visits.	Program.
<b>Probability of detection</b>		
$p_{Low} = 0.47$	Probability of detection of a rosette.	Hauser et al. (2012), Hauser and Moore (2016).
$p_{High} = 0.99$	Probability of detection of a flower.	Hauser et al. (2012), Hauser and Moore (2016).

Table 2: Summary of the parameters used for the baseline scenario. *Program* means that the parameter is directly estimated from the eradication program data.