# Optimal schedule for monitoring a plant incursion when detection and treatment success vary over time

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# Abstract

Management of an invasive plant species can be viewed as two separate and successive processes. The first, *survey*, aims to find infested areas and remove individuals. The second, *monitoring*, consists of repeated visits to these areas in order to prevent possible re-emergence. As detection probability may vary over time, the timing and number of monitoring visits can dramatically impact monitoring efficacy. We explore the optimal timing and number of monitoring visits, by focusing on one infested site. Our decision-analysis framework defines an optimal monitoring schedule which accounts for a time-dependent probability of detection, based on the presence/absence of a flower. We use this framework to investigate the optimal monitoring schedule for *Hieracium aurantiacum*, an invasive species in the Australian Alps and many other countries. We also perform a sensitivity analysis to draw more general conclusions. For *Hieracium aurantiacum* eight monitoring visits (compared to 12 visits in the current program) are sufficient to obtain a 99% monitoring efficacy. When four or fewer visits to a site are allowed, it is optimal to visit during the high season, when the weed is likely to initiate flowering. Any extra visits should be scheduled in the early season, before the plants flower. The sensitivity analysis shows that increasing the detection probability early in the season has a greater impact than increasing it late in the season. An effective treatment method increases the value of site visits late in the season, when the detection probability is higher. Our decision-analysis framework can assist invasive species managers to reduce or reallocate management resources by determining the minimum number of monitoring visits required to satisfy an acceptable risk of re-emergence.


**Keywords:** Invasive Species, *Hieracium*, Weed Management, Monitoring, Scheduling, Imperfect detection.

# Introduction

Whether or not, or how, we can achieve eradication of an invasive species has become a contentious issue (Gardener et al., 2010). Management agencies are justifiably concerned about achieving the best outcome for their investment and need to ensure that their actions are as effective as possible. Eradication of an invasion requires delimitation of a population's extent, containment of the population to prevent further spread, and successful extirpation of all individuals and propagules (Panetta 2007). Imperfect detection during surveillance and monitoring can critically influence the optimal searching effort (Mehta et al., 2007; Hauser and McCarthy, 2009), the optimal management action (Regan et al., 2011; Emry et al., 2011; Moore et al., 2014), and the duration of an eradication program (Cacho et al., 2006; Rout et al., 2013). A common practice is to consider the probability of detection as a constant parameter, which only varies between species, for example because of species traits (Garrard et al., 2013) or appearance (O'Connell JR et al., 2006). But detection may also change in space and time, influenced by environmental factors such as habitat quality (e.g., Wintle et al., 2005), temperature (e.g., Kroll et al., 2008), rainfall (O'Donnell et al., 2015) or the changes in visibility between different stages of plant development.

The monitoring phase of an invasive plant eradication program aims to contain and extirpate infestations, which may persist after their initial discovery and treatment due to: (1) individuals escaping detection during the original survey, (2) failure of the treatment method, (3) emergence of new plants from the seed bank, (4) reproduction of individuals at the site, or (5) secondary introduction from other populations (Panetta, 2007; Panetta et al., 2011). More than one visit to the infested area is generally needed to remove all the individuals and visit frequency should be set to prevent undetected individuals and new recruits from reproducing (Panetta, 2007; Moore et al., 2014). Given that a plant's detection probability may vary through time, monitoring visit frequency and timing are both crucial decisions for effective extirpation.

In this article we seek to determine the optimal timing of monitoring visits, in order to maximize the chance that

(i) every individual is removed and

(ii) no individual reproduces.

We define a monitoring schedule by the timing (position) and number of site visits, and propose a framework for evaluating schedules. We base the value of a monitoring schedule on the probability that every individual is removed and none reproduce, which is in turn defined as a function of the individual's probability of detection. We consider that control efficacy and detection are both triggered by time-dependent random factors, such as timing of flower development and seed release over spring and summer. What complicates monitoring is that such factors cannot be known exactly before visiting. We propose a general structured decision-making approach, where the optimal monitoring schedule represents the average best compromise over all possible realisations of these random factors. This approach is particularly suitable for any monitored invasive species, plant or animal, which varies in detectability over time. In addition, this framework could also be used to optimise the survey schedule of an endangered species, such that the probability of detecting the species is optimal.

A natural cause of temporal variation in its probability of detection is the presence or absence of flowers (Kéry and Gregg, 2003; Burrows, 2004). Flowers typically have a high colour contrast compared to stems and leaves and can lead to more rapid detection of weed species when present (Hauser and Moore, 2016). As a consequence the timing of a monitoring survey can highly impact its efficacy. For example, if sites are visited prior to flowering individuals may have a low probability of being detected; vegetative plants may be missed and may subsequently produce seeds. Conversely, later site visits should maximize the chance of finding individuals, but are also risky because individuals may have already reproduced and dispersed seed. To complicate the trade-off further, factors affecting detection (such as flower presence) may not be precisely predictable. To the best of our knowledge, this problem has not received attention in the scientific literature. We illustrate this framework using an invasive flowering plant, *Hieracium aurantiacum*, which occurs in the USA, Canada, Japan, New Zealand and Australia where it is nationally prohibited (Williams and Holland, 2007). We compute an optimal monitoring schedule for this plant, and compare it to an ad-hoc strategy where the site is visited fortnightly. We then generalize the results by performing a sensitivity analysis.

# Materials and methods

## A general monitoring model

In this study, we are concerned with infestations that have already been discovered through general surveillance, and have now entered a *monitoring* phase. Our model describes only one site of infestation, as an equivalent monitoring schedule can be developed for every known infestation in the landscape. We assume that monitoring of the invasive population occurs periodically, every year. For species that cannot be detected all year, monitoring occurs during the species' active stage, where it is plausible to detect or remove individuals. For example, for a herbaceous perennial weed, the opportunity for monitoring corresponds to the period from re-emergence until it dies back after flowering. For an annual weed, it is the period between germination and death. For hibernating animals, it corresponds to the post-hibernation period or more generally for burrowing animals, it corresponds to their high surface activity period.

We consider that monitoring has two objectives: (i) remove every individual from the site and (ii) prevent reproduction. We devise a *monitoring schedule* of $n_v$ visits planned during the management season according to a time schedule $\sigma = \{t_1, \ldots, t_{n_v}\}$. The efficacy of a monitoring schedule, $U_\lambda(\sigma;\Theta^\sigma)$, is based on the weighted probability that each objective is fulfilled:

$$U_\lambda(\sigma;\Theta^\sigma) = \lambda\mathbb{P}(N^\sigma = 0 \mid \sigma, \Theta^\sigma) + (1-\lambda)\mathbb{P}(Nb_o^\sigma = 0 \mid \sigma, \Theta^\sigma). \tag{1}$$

The expected outcomes depends on our schedule $\sigma$, i.e., the position of the visit days and the number of visits $n_v$, and the set of factors that influence detection during monitoring $\Theta^\sigma$.

$N^\sigma$ is the number of individuals that are present in the site at the end of the monitoring period and $Nb_o^\sigma$ the number of offspring produced at the end of the monitoring period. Thus monitoring is successful when $N^\sigma = Nb_o^\sigma = 0$ and the efficacy $U_\lambda(\sigma;\Theta^\sigma)$ is based on the site's state at the end of the monitoring period. Weighting factor $\lambda \in [0,1]$ can be chosen to determine the relative importance of the two monitoring objectives. For example, if the state of the site during future years is not important, one only wants to remove every individual before the end of the season, regardless of whether they have reproduced or not, and $\lambda = 1$. Conversely, if the state of the site only matters during the following years, we can focus on avoiding reproduction by setting $\lambda = 0$. When $0 < \lambda < 1$, we seek to address both objectives, with $U_\lambda = 1$ meaning that both objectives have been reached.

Note that setting $\lambda = 1$ can also be used to optimize the survey schedule of an endangered species. For example, the objective might be to mark all the individuals of a protected population. In this case, $N^\sigma$ could be defined instead as the number of unmarked animals at the end of the survey and the survey is successful when $N^\sigma = 0$.

Typically, the value of the factors that influence detection $\Theta^\sigma$ are not known in advance. A common practice in decision analysis is to determine the decision that will provide the highest expected reward over all possible values of the environmental uncertainty (Huang et al., 2011). For our problem, the optimal monitoring schedule $\sigma^* = \{t_1^*, \ldots, t_{n_v}^*\}$ is thus defined as the one that has the highest expected efficacy $U_\lambda$, over all possible values of the factors $\Theta^\sigma$. Computing the optimal monitoring schedule for a fixed number of visits, $n_v$, consists of choosing the monitoring schedule $\sigma$ to maximise:

$$\mathbb{E}_{\Theta^\sigma}\left[U_\lambda\left(\sigma;\Theta^\sigma\right)\right]=\sum_{i=1}^{n_\Theta}\mathbb{P}\left(\Theta^\sigma=\Theta_i^\sigma\right)U_\lambda\left(\sigma;\Theta^\sigma\right), \tag{2}$$

where $\Theta_1^\sigma,\ldots,\Theta_{n_\Theta}^\sigma$ are all the possible values of the factors that influence detection, capturing the environmental uncertainty. The environmental uncertainty represented in $\Theta^\sigma$ can take different forms: (i) event timing uncertainty, (ii) model structural uncertainty or (iii) parameter uncertainty. Event timing uncertainty refers to the case where detection and reproduction are dependent on some particular event, $\Theta^t$ represents the realisation of the event on day $t$. For a flowering plant an important event is the first flowering day, as the plant becomes much easier to detect (see next section), but it is not possible to predict this event exactly. For amphibians, rain can greatly increase the availability of individuals (O'Donnell and Semlitsch, 2015) and consequently the probability of detection. In that case, previous rainfall records can be used to determine the probability of rain and model the probability of detection over time. Model structural uncertainty refers to the uncertainty around the model that describes the probability of detection or reproduction or removal. In this particular case, $\Theta$ is a random variable whose values tell the type of structure in the possible models. For example, adult lizards are generally easier to detect during the mating period as this corresponds to a period of high activity. But the functional form of the relationship between lizard activity over time and detection might be uncertain, such that several possible models might be considered. Finally, parameter uncertainty refers to the situation where the functional form of the probability of detection, reproduction or removal is assumed known but some parameters remain uncertain. In this case, $\Theta$ represents the possible values that these parameters can take. For example, birds are easier to detect when they are singing, so it is preferable to monitor a site when bird songs are frequent, such as at sunrise. But the value of the probability of detection when the song frequency is high might still be unknown.

Another example of parameter uncertainty is the initial number of individuals present. In all cases, the uncertainty on $\Theta$ must be described using a probability distribution, which can be either an empirical distribution or a functional form. It is the probability distribution of the event over time for the event timing uncertainty; the probability of each possible model representing the system for the model structural uncertainty and the probability distribution over all possible parameter values for the parameter uncertainty. When there is complete uncertainty, a uniform distribution over the set of possible values could be used. But when no information is available, the problem remains trivial as every monitoring schedule has the same estimated value, and there is no reason that a given monitoring schedule should be better than another one.

We illustrate this framework for herbaceous perennials and annual plants, where the probability of detection is influenced by the presence of flowers. In the following, we simplified the problem by considering that only one individual is present in the site.

# Monitoring a flowering plant

For a flowering plant we consider that the first flowering day of the plant is an event uncertainty, denoted by $\Theta = T_F$. We expect that the presence/absence of flowers greatly impacts the detection of the individuals, as it will be much easier to detect a plant when a colorful flower is present. In addition, the presence of flowers also provides useful information on the reproduction process, as seed production only starts sometime after the plant has had its first flower.

We parameterised this model for the management of orange hawkweed.

## Baseline case study: orange hawkweed

*Hieracium aurantiacum*(synonym *Pilosella aurantiaca*) is an invasive herbaceous perennial in the Asteraceae, spreading by both seeds and vegetatively by rhizomes and stolons. Each basal rosette of leaves can produce a single stem 15-40 cm tall that produces multiple conspicuous bright red-orange flowers. It is an invasive species in the USA, Canada, Japan, New Zealand and Australia where it is nationally prohibited (Williams and Holland, 2007). In Victoria (Australia), it is currently managed in the Victorian alpine region with the aim of complete eradication. Each site is currently visited as often as 12 times during the season. For this plant, the active season, where it is plausible to detect or treat the plant, is nearly 6 months and we set this to be $L_{Season} = 185$ days.

First, we fit a probability model for the first day of flowering. A negative binomial relationship models the number of trials required to obtain $r$ successes, with probability of success $q$ per trial. This is comparable to the concept of accruing sufficient degree-days before flowering can occur. Therefore we define the probability distribution of $T_F$, as a translated negative binomial distribution:

$$\mathbb{P}\big(T_F = t_f\big) = \begin{cases} \binom{n_{fd}-t_f+r}{n_{fd}-t_f+1} q^r (1-q)^{n_{fd}-t_f+1}, & \text{if } 1 \leq t_f \leq n_{fd}, \\ 0, & \text{if } n_{fd} < t_f \leq L_{Season}, \end{cases} \tag{3}$$

where $n_{fd}$ is the last day when the plant can have its first flower. After this day, it is assumed that the plant must have started flowering, i.e. $\mathbb{P}\big(T_F \leq n_{fd}\big) = 1$. Note that in this case, the equation (3) defines an approximate probability distribution since the probabilities do not necessarily sum to one. This is due to the fact that we forced the probability of having a first flower after $n_{fd}$ to be zero. However, the impact of this approximation is insignificant ($1 - \sum_{t_f} \mathbb{P}\big(T_F = t_f\big) \leq 0.001$) for the parameters we considered) and thus we still use $\mathbb{P}\big(T_F = t_f\big)$ as a probability distribution.

To use this probability distribution in practice, it is important to know the range of possible flowering days for the plant or the range of the most probable first flowering day. For orange hawkweed, monitoring program data show that flowers should be present after 92 days, i.e. $n_{fd} = 92$, and that the peak first flowering day is day 79. We tuned parameters $r$ and $q$ by hand, so as to visually find the values giving this range and most probable value. There may, however, be other ways to fit these parameters in different circumstances. For orange hawkweed, we selected $r = 4$ and $q = 0.17$. The probability distribution of the first flowering day is shown in Fig. (2).

We consider that the probability of detection can take two values, depending on whether a flower is present on the plant ($p_{High}$) or not ($p_{Low}$). We assume for simplicity that the high detection rate is maintained until the end of the season. The probability of detecting an individual hawkweed for each day $t$ is defined as follows:

$$p_D(t; T_F) = \begin{cases} p_{Low} & \text{if } 0 \leq t < T_F, \\ p_{High} & \text{if } T_F \leq t \leq L_{Season}. \end{cases}$$

We determined the values of these two detection probabilities from Hauser and Moore (2016) and Hauser et al. (2013): $p_{Low} = 0.47$ and $p_{High} = 0.99$. The value of the expected probability of detection is presented in Figure (2).

We suppose that, when detected, all the living parts of the plant are removed before the end of the season (Bear et al., 2012) and there is thus no risk of re-emergence.
Then, the removal success is only based on our ability to detect the plant and we have:

$$\mathbb{P}\left(Nb^\sigma = 0 \mid \sigma, \Theta^\sigma\right) = \sum_{t_d \in \sigma} \mathbb{P}\left(T_D = t_d \mid T_F\right).$$

Here $T_D$ is the random variable defining the first day of detection. When the plant is detected it is marked with a colored flag and its coordinates recorded via GPS. Thus, once detected, the plant will be detected during following visits. For a given monitoring schedule $\sigma$, the probability that the plant is first detected on day $t_d$ is:

$$\mathbb{P}(T_D = t_d \mid T_F) = \begin{cases} p_D(t_d; T_F) & \text{if } d = 1, \\ p_D(t_d; T_F) \prod_{i=1, d>1}^{d-1} \big(1 - p_D(t_i; T_F)\big) & \text{if } d > 1. \end{cases}$$

Similar to a geometric distribution, the left term of the second line of the equation is the probability that the plant is detected on day $t_d$ and the right term is the probability that the plant was not detected during the previous visits.

In this case, the efficacy of a monitoring schedule becomes:

$$U_\lambda(\sigma; T_F) = \sum_{t_d \in \sigma} \mathbb{P}(T_D = t_d \mid T_F)\left[\lambda + (1-\lambda)\left(\mathbb{P}(Nb_o^\sigma = 0 \mid T_D = t_d, T_F)\right)\right] \tag{4}$$

That is, successfully detecting a plant during the monitoring schedule $\sigma$ is sufficient to ensure $N^\sigma = 0$ at the end of the season. We assume that when found the plant is immediately treated with herbicide but, since the effect of herbicide is not instantaneous, reproduction might still occur even when the plant is detected.

In practice, production of viable seeds only starts after flowering of the plant and we consider that no viable seeds are produced when the plant is detected and treated before flowering day $T_F$, i.e. $\mathbb{P}(Nb_o^\sigma = 0 \mid T_D, T_F) = 1$, when $T_D \leq T_F$. We consider the simplified situation where seeds are dispersed after a fixed time period of duration $\delta_{spread}$ that follows the flowering day. Thus, when the plant is detected after $T_F + \delta_{spread}$, i.e. after all viable seeds have been released, we have $\mathbb{P}(Nb_o^\sigma = 0 \mid T_D, T_F) = 0$, when $T_D > T_F + \delta_{spread}$. When the site is detected between flowering and the first day of seed release, we consider that treatment can be either successful (i.e. no offspring are produced) with probability $p_T$ or unsuccessful with probability $1 - p_T$. In this case, the probability that no offspring are produced follows a geometric distribution: $\mathbb{P}(Nb_o^\sigma = 0 \mid T_D, T_F) = \sum_{i=1}^{T_{days}} (1-p_T)^{i-1} p_T$, when $T_F < T_D \leq T_F + \delta_{spread}$. $T_{days}$ is the number of treatments applied between flowering and seed release:

$$T_{days} = \#\left\{i = 1 \ldots n_v \mid T_F \leq T_D \leq t_i \leq T_F + \delta_{spread}\right\}.$$

In the definition of $\mathbb{P}(Nb_o^\sigma = 0 \mid T_D, T_F)$, $i$ is simply the index of the visit day where treatment was successful.

Finally, we obtain the following definition of the probability that no offspring have been produced:

$$\mathbb{P}(Nb_o^{t_{n_v}} = 0 \mid T_D, T_F) = \begin{cases} 1 & \text{if } T_D \leq T_F \\ \sum_{i=1}^{T_{days}} (1-p_T)^{i-1} p_T & \text{if } T_F < T_D \leq T_F + \delta_{spread}. \\ 0 & \text{if } T_D > T_F + \delta_{spread} \end{cases} \tag{5}$$

The probability of treatment success $p_T = 0.51$ has been estimated from the current management program.

Finally, the probability that the individual will not reproduce for a given monitoring schedule is the expected probability that there is no offspring produced at the end of the monitoring period, over all possible values of the day of first detection $t_d$:

$$\mathbb{P}\left(Nb_o^{\sigma} = 0 \mid \sigma, \Theta^{\sigma}\right) = \sum_{t_d \in \sigma} \mathbb{P}\left(T_D = t_d \mid T_F\right) \mathbb{P}\left(Nb_o^{t_{L_{\text{season}}}} = 0 \mid T_D = t_d, T_F\right) \tag{6}$$

A schematic representation of the entire process is available in Figure (1) and a summary of the model parameters can be found in Table 1.

To allow visits to other sites, we set a minimum time interval $\delta_{visits}$ between visits. We considered here that a site can be visited at most once every 6 days, i.e. $\delta_{visits} = 5$ days.

## Optimisation procedure

We use three different values of the parameter $\lambda = \{0,\ 1,\ 0.5\}$, in order to discuss the effect of each objective on the timing of the visit days.

We used a genetic algorithm to compute the optimal monitoring schedule for a fixed number of visits $n_v$. More precisely, we use the function *ga* of Matlab, which converges relatively easily to the optimal solution.

## Comparison with other strategies

We compare the optimal monitoring schedule to a strategy which consists of visiting the site once every two weeks; this has been commonly adopted in the orange hawkweed eradication program. We suppose that the site is first visited during the first day of the management season, and then 14 days later and so on until the site is visited 12 times. We compared the fortnightly strategy to the optimal monitoring schedule computed for different numbers of visits, from 1 to 12. Then we compute the gain, in percentage, of using the fortnightly strategy over the optimal strategy:

$$Gain_{n_v} = \frac{100 * \left(U_{0.5}\left(\sigma_{Fortnightly}\right) - U_{0.5}\left(\sigma_{Opt}^{n_v}\right)\right)}{U_{0.5}\left(\sigma_{Opt}^{n_v}\right)}, \tag{7}$$

where $\sigma_{Fortnightly}$ and $U_{0.5}\left(\sigma_{Fortnightly}\right)$ are the fortnightly schedule and its value (see equ. (4)) and $\sigma^*_{n_v}$ and $U_{0.5}\left(\sigma^*_{n_v}\right)$ are the optimal monitoring schedule with $n_v$ visits and its value (see equ. (4)). Note that the number of visits of the fortnightly strategy is fixed to 12 while the number of visits varies from 1 to 12 for the optimal monitoring schedule. Then theoretically the value of the fortnightly strategy can be higher than the value of the optimal monitoring schedule, computed for a smaller number of visits $n_v < 12$. The aim of this comparison is to show that an optimal scheduling can perform as well or better than this ad-hoc strategy, for a fewer number of visits.

Finally, we consider a random strategy, denoted $\sigma_{Random}$. The strategy selects randomly 12 monitoring days in the set $\{1, \ldots, L_{\text{Season}}\}$, where each day can be selected at most one with probability $\frac{1}{L_{\text{Season}}}$ (random draw without replacement). There is not a unique random strategy, so we propose to define $U_{0.5}\left(\sigma_{Random}\right)$ as an expected value instead. We first drew 5,000 possible random strategies $(\sigma^i_{Random})_{i=1}^{5000}$ and evaluated their values $U_{0.5}\left(\sigma^i_{Random}\right)$ for all $i$. We then define $U_{0.5}\left(\sigma_{Random}\right)$ as the average value of the 5,000 random strategies:

$$U_{0.5}\left(\sigma_{Random}\right) = \frac{\sum_{i=1}^{5000} U_{0.5}\left(\sigma^i_{Random}\right)}{5000}.$$

We chose to compare the optimal schedule with the random strategy, as it can be interpreted as a lower bound in terms of efficacy.

## Sensitivity analysis

Because the non-biological parameters are highly linked to orange hawkweed management practices, we now illustrate the model with varying management scenarios.

We first explore the impact of the treatment success, comparing a low $p_T = 0.2$ and a high $p_T = 0.8$ value. Second, we explore the impact of the detection probability with $p_{\text{Low}} = 0.1$ or $p_{\text{Low}} = 0.3$ and $p_{\text{High}} = 0.6$ or $p_{\text{High}} = 0.75$. For clarity we reduce the possible number of visits $n_v$ to 1, 3 or 6 and we only consider the case where both management objectives are accounted (i.e. $\lambda = 0.5$).

We perform a *one-at-a-time* sensitivity analysis by first analysing the effect of the various detection probabilities and fix the treatment efficacy to the baseline scenarios (i.e. $p_T = 0.51$). We then analyse the effect of the treatment efficacy with various detection probabilities scenarios: (i) a low detection scenario with $p_{\text{Low}} = 0.1$, $p_{\text{High}} = 0.6$, (ii) a middle detection scenario with $p_{\text{Low}} = 0.3$, $p_{\text{High}} = 0.75$ and (iii) a high detection scenario with the detection probabilities of the baseline scenario, i.e. $p_{\text{Low}} = 0.47$, $p_{\text{High}} = 0.99$.

**Baseline parameters: orange hawkweed** The value of all the model parameters for *Hieracium aurantiacum* are summarized in Table 2. The probability distribution of the first flowering day is illustrated in Fig. 2, as well as the expected probability of detection and the value of all monitoring schedule with $n_v = 1$ and $\lambda = 0.5$.

We can see that the expected probability of detection is equal to $p_{Low}$ at the beginning of the season, where the probability of the plant having a flower is very low and, rosettes and seedlings are not easy to detect. The probability of having the first flower is higher than $1 \times 10^{-3}$ after the 39$^{\text{th}}$ day of the season and increases until day 79. The probability of having the first flower then quickly decreases and becomes zero after $n_{fd} + 1 = 93$ days after which the expected probability of detection is $p_{High}$.

The early season is defined between the first day of the season and day 39, when it is very unlikely that the plant already has its first flower. The high season is then defined from day 40 to day $n_{fd} + 1 = 93$, when it is likely that a plant initiates an inflorescence ; the late season is defined from day 94 to $L_{season} = 185$, when it is likely that the plant will already have its first flower.

Naturally, the value of a monitoring schedule $U_{0.5}(\{t\})$ is approximately equal to $p_{Low}$ in the early season, as long as there is a low probability of flowering. Thus, if the plant is found it is automatically removed before it can spread seeds. Then, $U_{0.5}(\{t\})$ increases because the expected probability of detection increases with the probability of having the first flower. If the site is visited late in the high season, there is an increasing chance that the plant can disperse seeds even if it is discovered and treated. As a consequence, the value of a monitoring schedule $\{t\}$ decreases from day 79 + $\delta_{spread}$ to day 93 + $\delta_{spread}$. After day 93 + $\delta_{spread}$, the schedule value is equal to $0.5 \times p_{High}$ because the plant has already spread seeds and thus $\mathbb{E}_{T_F}\left[p_{SR}(t; T_F)\right] = 0$. Thus, when the site is visited only once, it is optimal to visit it during the first day where the expected probability of detection is maximal.

# Results

**Timing of the visit days** The optimal timing of the visit days are presented in Fig. 3. When $\lambda = 1$, the only objective is to minimise the chance that the plant will be present next season. Then, it is sufficient to maximise the chance of detection, as long as we consider no failure in the removal method. The optimal placement of the visit day is naturally in the late season, when the plant is easy to detect. In our case study, $p_{High}$ is close to one and there is no clear advantage of visiting the site more than one time, since $U_1(\{t_1^*\}) = p_{High} \simeq 1$. As illustrated in Fig. 4, the value of the monitoring schedule is close to 1, whatever the number of visits. For example, $U_1(\{t_1^*\})$ and $U_1(\{t_1^*, t_2^*\})$ are within $10^{-5}$ of each other. For $n_v \geq 9$, some visit days are placed during the early season since the value of the monitoring schedule is the same with a precision of $10^{-12}$, the precision we used with the genetic algorithm.

When $\lambda = 0$, the only objective is to minimise the chance that the plant disperses seeds. In this case, repeated visits to the site are justified to increase the treatment success and decrease the chance that the plant will produce seeds. When the number of visits is low, $n_v \leq 4$, it is optimal to visit the site during the high season, maximising the chance of plant detection, even if there is a risk that the treatment will not entirely prevent seed production. When the number of visits increases, $n_v > 4$, it is optimal to also schedule visit days early in the season, when treatment is most effective and no seeds can be released after the plant is found. But visits during the high season are still needed. For $n_v \geq 8$, there is more than a 99% chance that the plant will not release seed, and there is then no real advantage, in terms of monitoring value, of increasing the number of visits.

When both monitoring objectives are considered with $\lambda = 0.5$, the optimal dynamic of the monitoring schedule is close to the case with $\lambda = 0$. Generally, the optimal visit days are postponed compared to the case where $\lambda = 0$ in order to maximise the probability of detection and then remove the plant. Even if we apply the $\lambda = 0$ optimal monitoring schedule and assess it using $\lambda = 1$ utility, the probability of removal of the plant becomes close to one when $n_v \geq 6$. Then, the objective of plant removal no longer influences the timing of visits and their position becomes nearly the same as when $\lambda = 0$.

Finally, we can see in Fig. (4) that the value of the optimal monitoring schedule is approximately constant for the higher numbers of visits. For 4 visits or more, whatever the management objective, the value of the optimal monitoring schedule is greater than 0.9 (i.e. 0.923 for $\lambda = 0$, 0.999 for $\lambda = 1$ and 0.954 for $\lambda = 0.5$). When 8 visits are allowed, the values start to be all greater than 0.99 (i.e. 0.993 for for $\lambda = 0$, 0.999 for $\lambda = 1$ and 0.996 for $\lambda = 0.5$).

**Comparison with other strategies** The gain of the 12-visits fortnightly strategy over the optimal monitoring schedule with a varying number of visits is presented in Fig. (5). The difference between the value of the 12-visits fortnightly strategy and the 6-visits optimal monitoring schedule is lower than 1%. Optimal monitoring schedules with $n_v \geq 8$ visits have slightly higher values (less than 1%) compared to the 12-visits fortnightly strategy. Thus, when the timing of the visits is optimal, only 8 visits are needed in order to have a higher value than a simple common fortnightly strategy, which uses 12 visits.

Furthermore, the random 12-visits monitoring schedule $\sigma_{Random}$ only performed better than the 1-visit optimal schedule and even there is only a 1% improvement.

**Sensitivity analysis** We first analyse the sensitivity of the optimal schedule to the probabilities of detection. The optimal timing of the visit days is described by Fig. 6. For most of the optimal schedules, when the number of visits is low, i.e. $n_v \leq 3$, it is optimal to visit the site during the high season . But when the number of visits is high (i.e. $n_v = 6$), extra visits are timed during the early season. The placement of the visit days are remarkably similar when $p_{\text{Low}} = 0.1$ and $p_{\text{Low}} = 0.3$, even if the site is visited slightly earlier when $p_{\text{Low}} = 0.3$. When $p_{\text{Low}}$ is sufficiently high and $p_{\text{High}}$ is low (here $p_{\text{Low}} = 0.47$ and $p_{\text{High}} = 0.6$, see Fig. 6 (b) and (c)), it is optimal to visit the site only during the early season. But increasing $p_{\text{High}}$ encourages visits during the high season. For $p_{\text{High}} = 0.75$, 1 and 4 visits are during the early season, for $n_v = 3$ and $n_v = 6$. Only the highest value of $p_{\text{High}}$ leads to site visits mostly during the high season.

For the different treatment models, we first discuss their efficacy. The expected probability that no seeds will be released (i.e. $\mathbb{E}\left[p_{AR}\left(t, T_F\right)\right]$) when only one visit is allowed is presented in Fig. 7. Obviously, when it is unlikely that the plant has flowered (i.e. $t \leq 40$ days) or it is likely that seeds are already released (i.e. $t \geq n_{fd} + \delta_{spread} + 1 = 114$ days), all models agree: there is either an expected probability of $p_{\text{Low}}$ (i.e. $t \leq 40$ days) or 0 that no seeds are released ($t \geq 114$ days). But between these situations, each model acts differently and the baseline scenario model gives intermediate efficacy. For the highest treatment success $p_T = 0.8$, the expected probability that no seeds will be released increased after day $n_{fd} + 1 = 93$, due to an increase in detection probability (see Fig. 2). For the baseline scenario $p_T = 0.51$, the effect of this increase in detection probability is much less and there is no effect when $p_T = 0.2$. The treatment success is too low and the expected probability that no seeds will be released decreased until day $n_{fd} + \delta_{spread} + 1 = 114$ and becomes 0 thereafter.

The optimal timing of the visits for the low, middle and high detection scenarios and the different treatment success are shown in Fig. 8. In the low detection scenario, the timing of most visits is during the high season, which increases the expected probability of detection and gives some chance to visit the site before seeds have been released. For the highest treatment success $p_T = 0.51$ or $p_T = 0.8$, even one visit is allowed at the beginning of the late season when $n_v = 6$.

For the highest treatment efficacy (i.e. $p_T = 0.8$), all the visits are timed during the high season, whatever the detection scenario and the number of visits. The treatment method is sufficiently successful in killing plants to delay the visits during the season and thus increase the probability of detection. In this case, increasing the probability of detection allocates early site visits. For the baseline scenario $p_T = 0.51$, optimal visits are during the high season and increasing the probability of detection allows earlier visits. For the lowest treatment success $p_T = 0.2$, the optimal schedule is more complicated. When detection is low, visits during the early season are highly unlikely to detect plants and visits are timed for late in the high season to increase the chance of detection. In the intermediate detection scenario, some visits are located during the early season to help increase the probability of avoiding reproduction, as long as the treatment success is low. Finally, in the high detection scenario, the monitoring schedule becomes similar to the one in the low detection scenario, to take advantage of the high detection probability after flowering.

For all these scenarios, the value of the optimal monitoring schedule increases with the detection probability, number of visits and treatment efficacy.

# Discussion

We have proposed a general framework to develope an efficient and effective monitoring schedule for an invasive flowering plant. Unlike other frameworks, it accounts for a changing probability of detection. This innovation is an highly realistic situation for invasive species management but also for biological surveys in general and to the best of our knowledge, ours is the first study which explicitly incorporates a systematically changing probability of detection. Indeed, previous optimal monitoring research has focused on the optimal number of visits (e.g. Garrard et al., 2008; Wintle et al., 2012), based on a constant probability that the species will be detected during a visit. But in many cases, the probability of detection is likely to change over time with for example, the emergence of flowers or an increasing activity of the species during mating. This makes the timing of the visits particularly important to detect the species, and not only the number of visits.

We illustrate our framework with the orange hawkweed control program in Victoria, where we showed that 8 optimally-scheduled visits perform nearly as well as 12 visits scheduled every two weeks. In this case, we also showed that when the monitoring budget is low (i.e., at most 4 visits), it is optimal to visit the site during high season. As the monitoring budget increases, additional observations should be placed in the early season. The sensitivity analysis, seems to confirm this as a general result: visit in the high season as a first priority and then, if possible, during the early season. We also show that the maximum number of visits (i.e. $n_v = 12$) is not needed to obtain satisfactory optimal monitoring schedule values. Indeed, the optimal monitoring schedule with 4 visits (respectively 8 visits) is already highly efficient, with monitoring objectives within 10% (respectively 0.1%) of perfect performance. This translates to a substantial saving of labour for the orange hawkweed program, where the number of newly discovered sites requiring monitoring has grown every year.

More generally, this work can be classified as a structured decision-making approach, particularly adapted when decisions have to be made while facing uncertainty. This approach consists first in defining the objectives (e.g. remove all plants and avoid reproduction) and defining a value function able to quantify the value of different alternatives (here $U_\lambda\left(\sigma;\Theta^\sigma\right)$). An important point is to account for uncertainty when defining the value function. It is quite obvious that probability of detection is influenced by the presence of flowers. But at first, it might seem ambitious to account for the presence of a flower because the first flowering day is unknown at the beginning of monitoring. Structured decision making makes a bridge between using an exact value of the first flowering day and ignoring the fact that flowers will emerge during the season. This bridge is made possible by considering a probability distribution of the emergence of first flower and then computing a decision that is best on average. This approach is better than ignoring the environmental uncertainty and of course, worse than knowing the exact value of all the environmental variables.

The model could be improved in several ways. We used a simple step function to model the probability of detection. A more flexible could use a logit function which can give different probabilities of detection for the different plant growth stages between rosette and flowering. This might change the timing of the visit days, because a higher probability of detection would be expected early in the season. Nevertheless, a step function remains a plausible model and is easier to define. In addition, one can consider more than one growing stage (i.e. instead of just flowering) which affects the probability of detection. We studied here the simple case where only one site has to be monitored. In practice, hundreds of sites may have to be monitored with limited resources. In addition, the probability of detection can also vary across space. The probability that the plant will be present in the site can also be incorporated into the model and a different number of visits to each site can be allowed. Then each presence probability can be updated from year to year or week to week and the optimal schedule can be computed dynamically, at the beginning of the season or every week.

These additions will increase the complexity of the model and the expertise required from management agencies. In this case, the optimal schedules have to be computed from more complex approximate resolution methods.

Another model simplification was to consider that only one individual was present in the site. The model can easily be extended to the case where more than one individual is present in the site, but then information on the number of individuals and the relation between the number of individuals in the site and the probability of detection will be required. If we consider that the individuals are independently detected at the site, then the optimal monitoring schedule will not change from the one computed here for one individual, only its value will differ. If we consider that individuals in the site can be viewed as one unique entity, which might be the case for sufficiently small site, then the conclusions of this article remain the same.

In spite of these simplifying assumptions, this model provides a promising framework for managers to determine the optimal monitoring schedule by specifying a fixed number of visits, or alternatively, determining the number of necessary visits to achieve an acceptable level of performance. Our relatively simple model of changing detection and stochastic flowering demonstrates that successful monitoring can be achieved with a small number of well-scheduled repeat visits. Such prudent scheduling is likely to save considerable resources when many sites are monitored across an entire weed population.

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## Figures

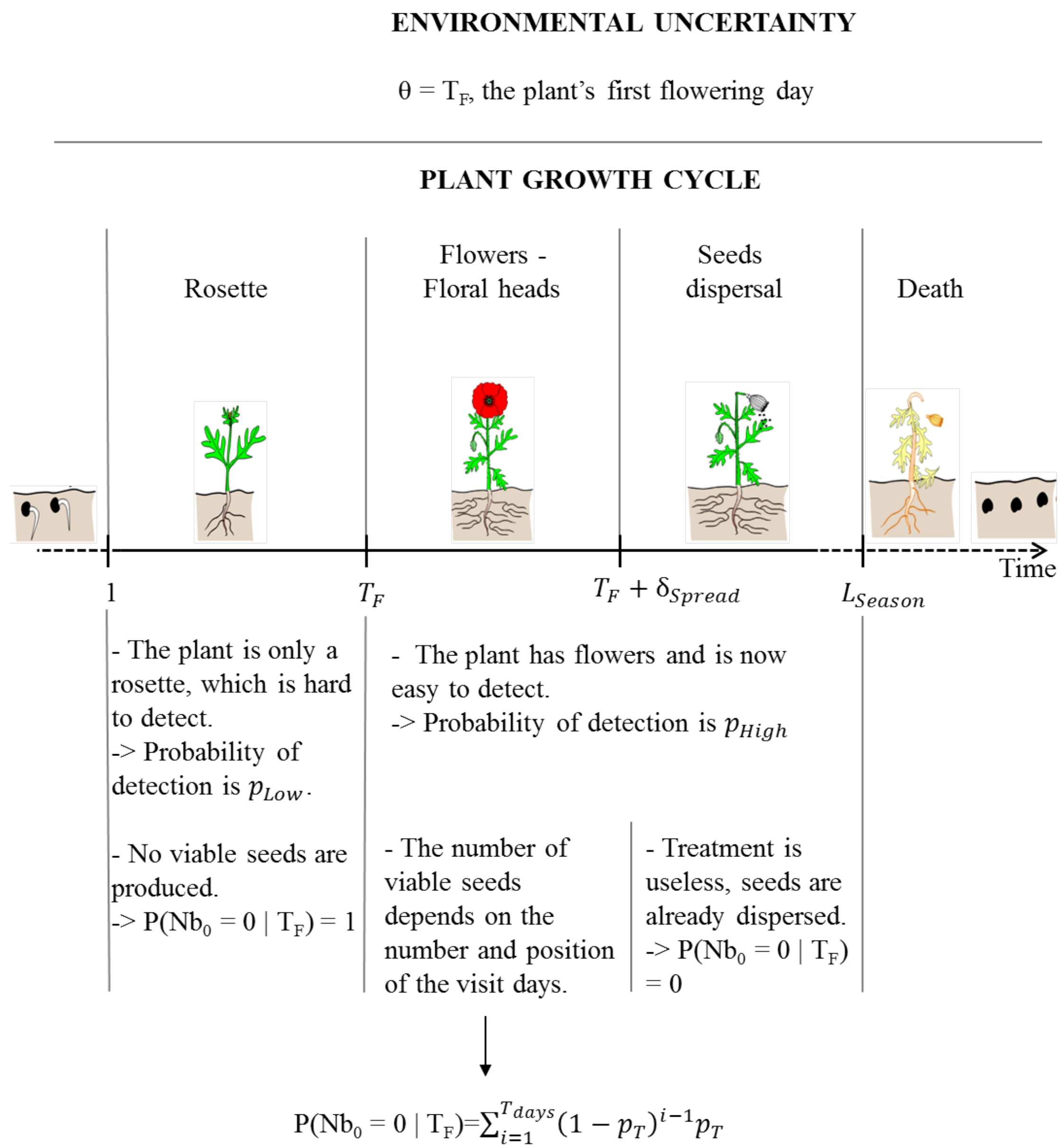


Figure 1: .

**Caption of Figure (1)**: Schematic representation of our model for an invasive flowering plant. The major source of uncertainty is the first flowering day of the plant, i.e. $\Theta = T_F$. The presence of a flower improves detection as it is much easier to detect a flower than a rosette. Thus, before the appearance of the flower the probability of detection $p_D$ is low ($p_{Low}$) and much higher after the plant has its first flower ($p_{high}$). The appearance of the first flower indicates that viable seeds are not yet present on the plant. Before the first flowering day, i.e. $t \leq T_F$, there is no seeds so the plant did not reproduce. Between the first flowering day and the day of seed release, i.e. $T_F \leq t \leq T_F + \delta_{Spread}$, treatment is successful with probability $p_T$. Finally, it is useless to visit the site only after the seeds release day, as seeds are already dispersed and reproduction cannot be avoided. Weeds illustrations can be found online at svt.ac-dijon.fr

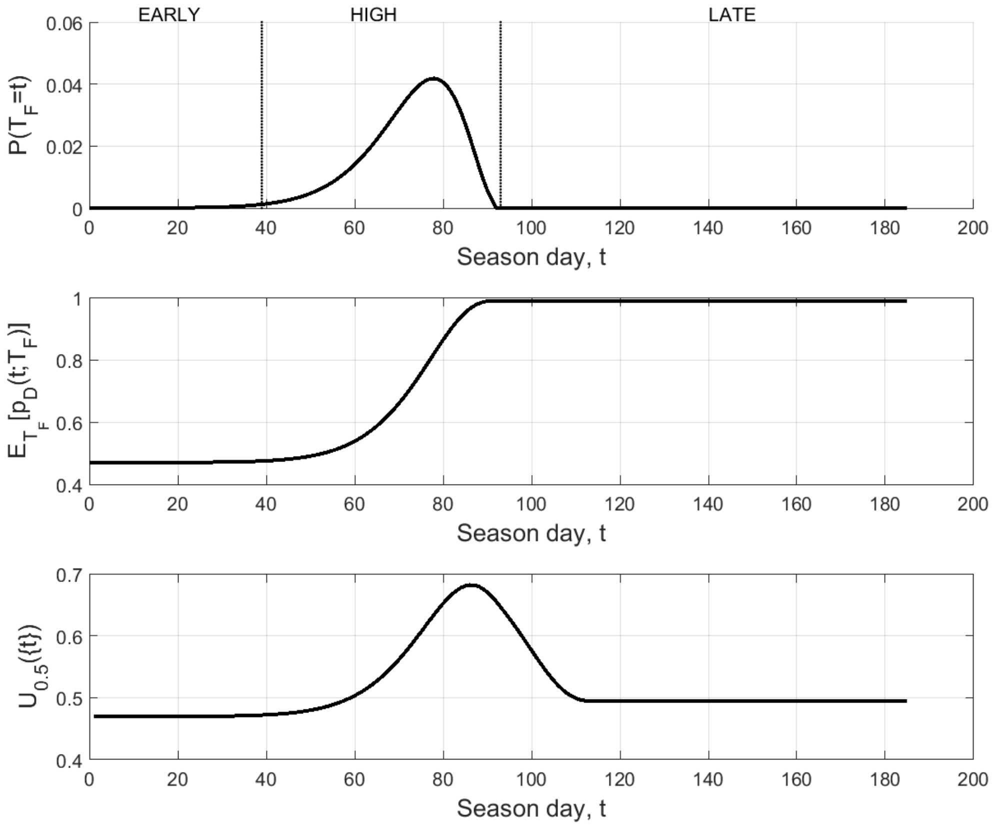


Figure 2: The likely timing and value of flowering throughout a perennial or annual plant's season. (a) The probability distribution of the first flowering day $P(T_F = t)$. The early season is the period where the plant is unlikely to have flowered. The high season is the period where the plant is likely to have its first flower. Finally, the late season is the period where the plant should already have flowered. (b) The influence of flowering day on the probability of detection $E_{T_F}[P_D(t; T_F)]$. The expected probability of detection is approximately $p_{low}$ during the early season as the plant should not have flowered and $p_{high}$ during late season as the plant should then have flowered. During the high season, the probability of detection smoothly increases with the probability that the plant has already flowered. (c) The variation in the probability of detection affects the value of a monitoring visit, displayed for $\lambda = 0.5$, meaning that both removing plants and avoiding reproduction matter. One can see here that it is optimal to visit the site during the first day where the probability of detection is maximal.

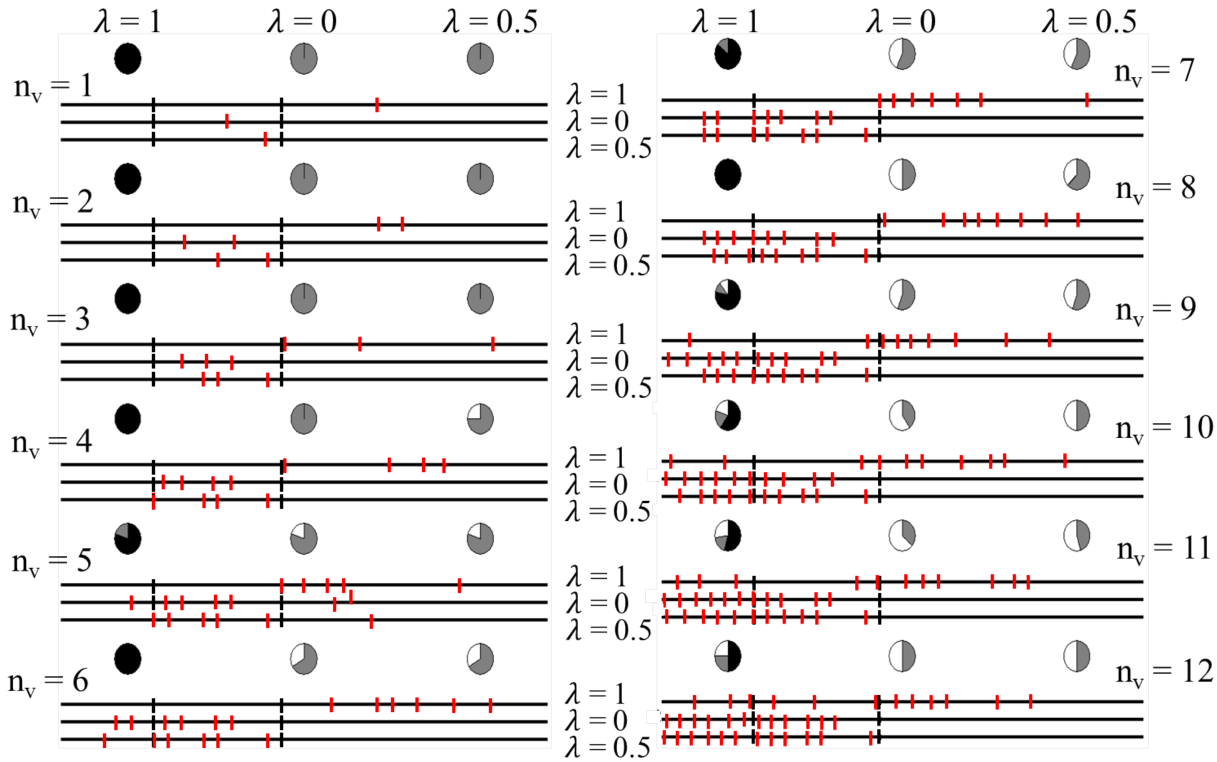


Figure 3: Optimal schedule of monitoring visits when the objectives is to remove the plant ($\lambda = 1$), avoid seed release ($\lambda = 0$), both removing the plant and avoiding seed release ($\lambda = 0.5$). The exact positions of the visit days are displayed with red points, while the black points are boundaries of the season period, i.e. early-high-late. The pie charts give the proportions of visit days within each season period and for each management objective. The lines represent the timeline of season days, from 1 to $L_{Season} = 185$. Results are presented for different number of visits, from $n_v = 1$ to $n_v = 12$. Each combination of three arrows and three pie charts give the optimal monitoring schedule for a given number of visits $n_v$. For example, the top right corner of the figure provides the results when 7 visits are allowed to the site. The first, second and third pie charts are the proportion of the visit days within each season period when $\lambda = 1$, $\lambda = 0$ and $\lambda = 0.5$. And the red points on the first, second and third arrow provide the optimal position of the monitoring visit when $\lambda = 1$, $\lambda = 0$ and $\lambda = 0.5$.

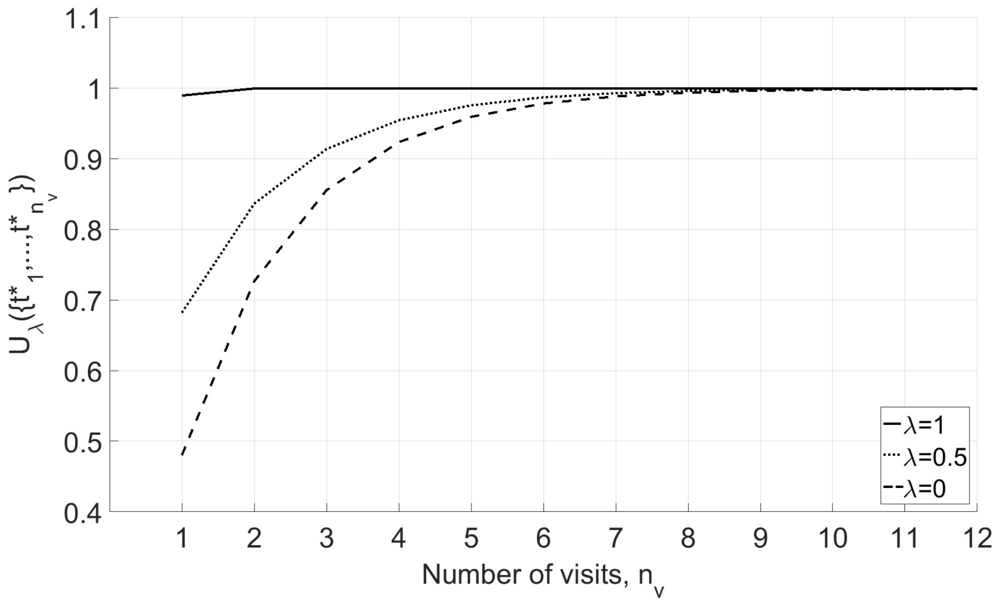


Figure 4: Value of the optimal monitoring schedule, $U_\lambda(\{t_1^*, \ldots, t_{n_v}^*\})$, for various $\lambda$ and number of visits $n_v$. When $\lambda = 0$ the objective is to avoid reproduction and $U_0(\{t_1^*, \ldots, t_{n_v}^*\})$ is then the expected probability that the individual did not reproduce when the optimal monitoring schedule is used. At least 6 visits are needed in order to have a risk of reproduction close to 0. When $\lambda = 1$ the objective of monitoring is only to optimize the detection of the individual and $U_1(\{t_1^*, \ldots, t_{n_v}^*\})$ is then the expected probability of detection when the optimal monitoring schedule is used. Here it is easy to have a probability of detection close to one when the site is visited during the late season. Finally, when $\lambda = 0.5$ the monitoring objective is to optimize the detection of the plant and to avoid reproduction. The value of the optimal monitoring schedule, $U_{0.5}(\{t_1^*, \ldots, t_{n_v}^*\})$ is mostly influenced by the second objective of avoiding reproduction. In this case, 6 visits are also needed in order to have an objective value above 99%.

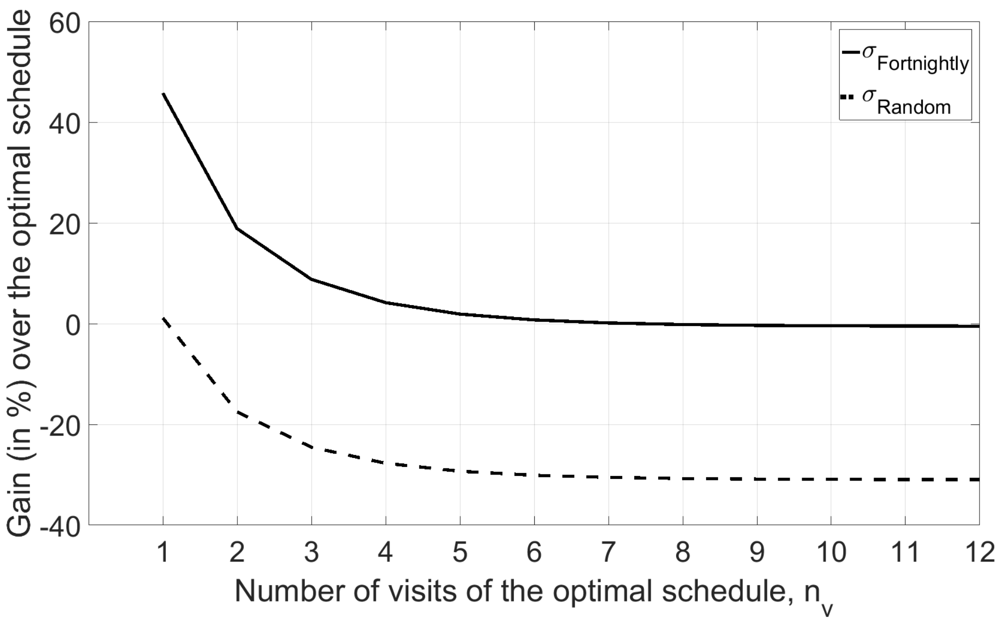


Figure 5: Percentage gain (see equ. (7)), in monitoring schedule value, for the 12-visit fortnightly strategy (straight line) and the random 12-visit strategy (dashed line) over the optimal monitoring schedule, computed for different numbers of visits. A negative (positive) gain implies that the optimal schedule has a better (worse) efficacy. For example, efficacy of the 12-visits random strategy is about 20% worse than the efficacy of the 2-visit optimal monitoring schedule. Or in other words, it is possible to increase monitoring efficacy by 20% with two visits scheduled optimally, compared to 12 visits scheduled randomly.

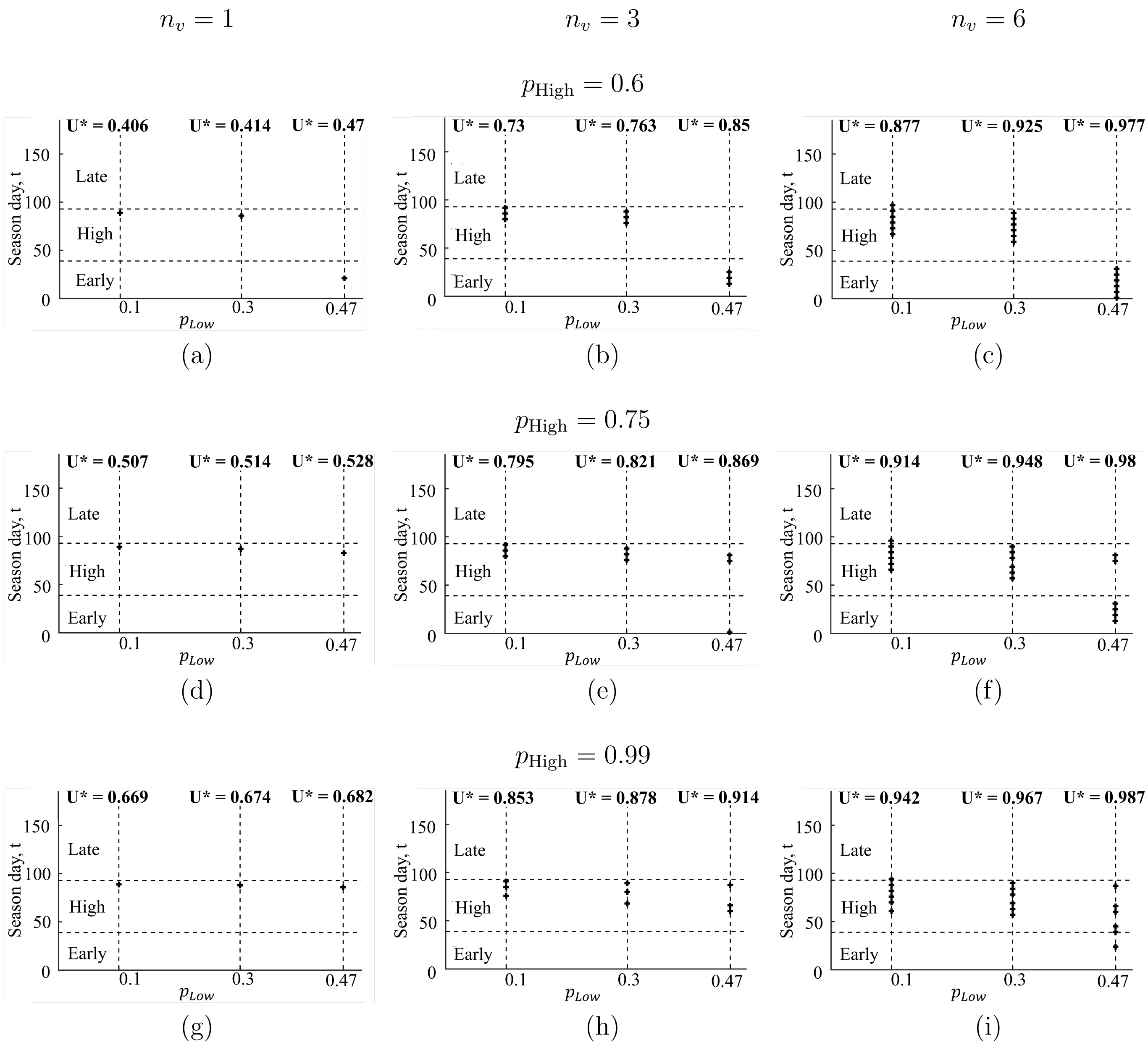


Figure 6: Optimal monitoring schedules given different values of $p_{\text{Low}}$ and $p_{\text{High}}$ and all other parameters equal to the baseline scenario. The management objective is both removing plant and avoiding seed release (i.e. $\lambda = 0.5$. Each row gives the optimal schedule for a fixed value of $p_{\text{High}}$ and each plot within a row gives the optimal schedule for different values of $p_{\text{Low}}$. On top of the optimal schedule is the schedule's value (i.e. $U^*$). Each column displays results for a different number of visits. For example, the top right graphic gives the optimal monitoring schedule when 6 visits are allowed and with $p_{\text{High}} = 0.6$, $p_{\text{Low}} = 0.1,\ 0.3$ and $0.47$. The value of the optimal monitoring schedule for $p_{\text{Low}} = 0.1$ is 0.884.

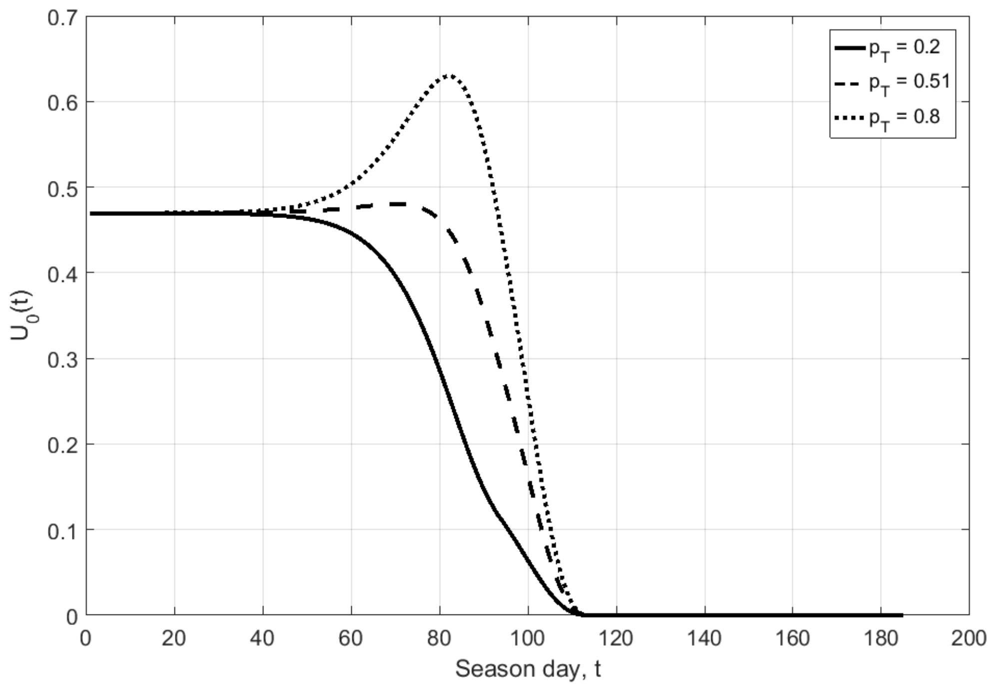


Figure 7: Expected probability that no plant reproduces for probabilities of treatment success $p_T = 0.2$ (solid line), $p_T = 0.51$ (dashed line) and $p_T = 0.8$ (dot-dashed line). All the other parameters are from the baseline scenario.

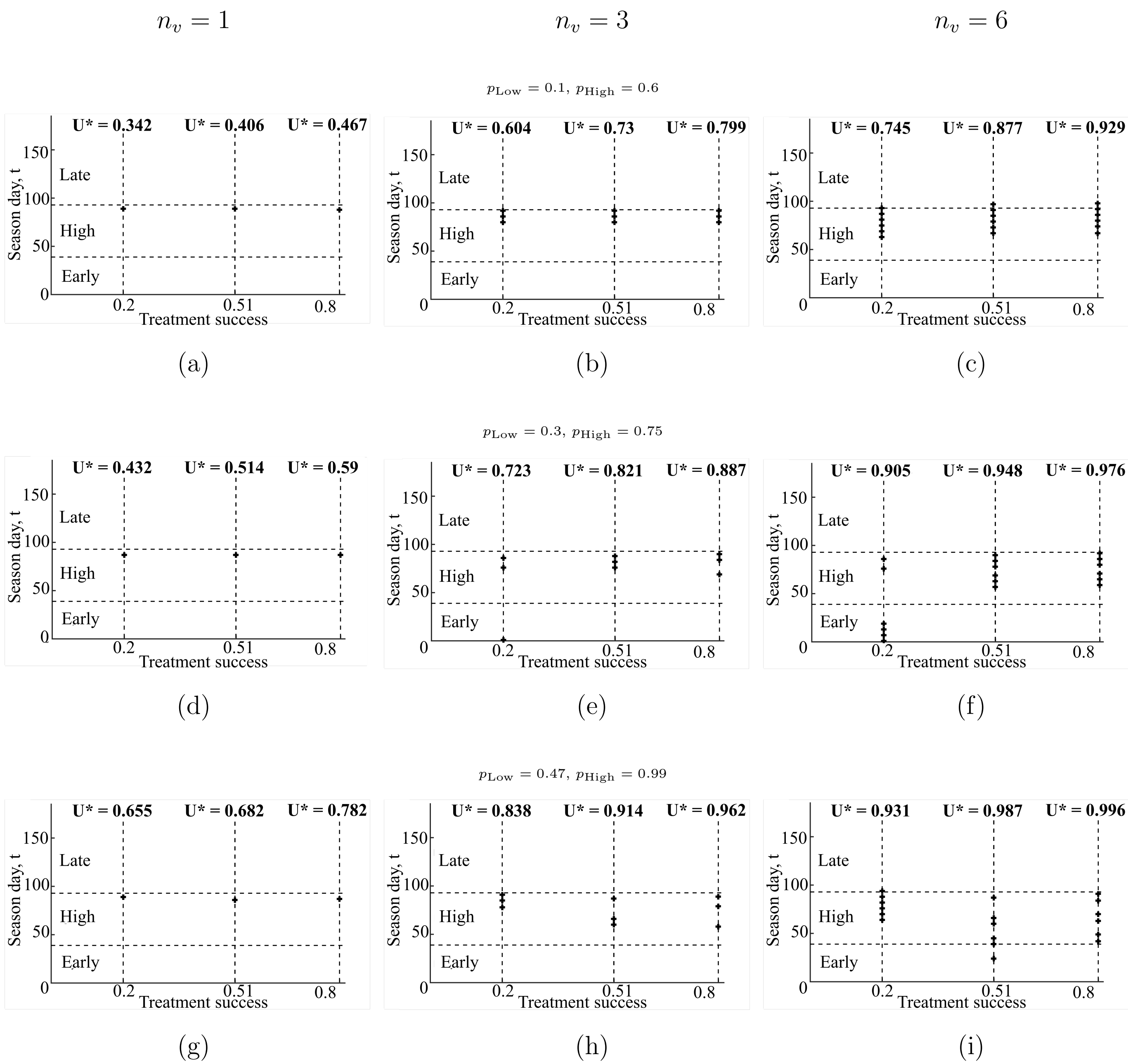


Figure 8: Position of the visit days for different probability of treatment success. The Figure is organized as in Figure (6) but here the rows are used for fixed values of $p_{\text{High}}$ and $p_{\text{Low}}$.

## Tables

| Notation | Meaning |
|---|---|
| $L_{Season}$ | The length of the monitoring season. |
| $\Theta$ | The set of factors defining the environmental uncertainty. |
| $\sigma$ | A given monitoring schedule of visit days. |
| $\lambda$ | The relative importance of the two monitoring objectives. |
| $n_v$ | The number of monitoring visits of a given schedule. |
| $N^\sigma$ | The number of individuals still present at the end of the monitoring period. |
| $Nb_o^\sigma$ | The number of offspring produced during the season. |
| $\sigma^*$ | The optimal monitoring schedule. |
| $\delta_{visits}$ | The minimum time between two visits to the site. |
| $T_F$ | The random variable defining the first flowering day. |
| $(n_{fd}, r, q)$ | Parameters used to define the probability distribution of $T_F$. |
| $p_D(t; T_F)$ | The detection probability on day $t$ if the first flower appears on day $T_F$. |
| $p_{low}$ ($p_{High}$) | Probability of detection before (after) the flowering day. |
| $T_D$ | The random variable defining the first day of detection. |
| $\delta_{Spread}$ | Time between flowering day and seed release day. |
| $p_T$ | Probability of treatment success when a plant is treated after first flowering, but before seed release. |

Table 1: Summary of the notations used.

| Name, value | Meaning | Sources |
|---|---|---|
| | **General parameters** | |
| $L_{Season} = 185$ days | Length of the management season. | Progam. |
| $(n_{fd}, r, q) = (92, 4, 0.17)$ | Parameters of the probability distribution of $T_F$. | Program. |
| $\delta_{Spread} = 21$ days | Time lag between flowering and seed release. | Program. |
| $\delta_{visits} = 5$ days | Time lag between two visits. | Program. |
| | **Probability of detection** | |
| $p_{Low} = 0.47$ | Probability of detection of a rosette. | Hauser et al. (2012), Hauser and Moore (2016). |
| $p_{High} = 0.99$ | Probability of detection of a flower. | Hauser et al. (2012), Hauser and Moore (2016). |

Table 2: Summary of the parameters used for the baseline scenario. *Program* means that the parameter is directly estimated from the eradication program data.