

Time-Varying Secondary Flows in Turbulent Boundary Layers over Surfaces with Spanwise Heterogeneity

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Abstract

The behaviour of turbulent boundary layers over surfaces composed of spanwise-alternating smooth and rough strips is investigated experimentally. The width of the strips S vary such that $0.32 \leq S/\bar{\delta} \leq 6.81$, where $\bar{\delta}$ is the boundary-layer thickness averaged over one spanwise wavelength of the heterogeneity. The experiments are configured to examine the influence of spanwise variation in wall shear stress over a large $S/\bar{\delta}$ range. Hot-wire anemometry (HWA) and particle image velocimetry (PIV) reveal that the half-wavelength $S/\bar{\delta}$ governs the diameter and strength of the resulting mean secondary flows. Three possible cases are observed: limiting cases where $S/\bar{\delta} \ll 1$ or $S/\bar{\delta} \gg 1$ and the secondary flows are either confined near the wall or near the roughness change, respectively, and intermediate cases ($S/\bar{\delta} \approx 1$), where the secondary flows fill the entire boundary layer and the outer layer similarity is destroyed. The size and strength of the time-averaged secondary flows are approximately capped by either the boundary-layer thickness $\bar{\delta}$ or the roughness patch width S . Instantaneously, however, these secondary flows appear very similar to naturally occurring large-scale structures that are spanwise-locked by the roughness transition with a residual meandering tendency about these locations. The efficacy of the roughness to lock the secondary flows in place and the meandering of the secondary flows are a function of $S/\bar{\delta}$, most prominent when $S/\bar{\delta} \approx 1$. Further analysis of the energy spectrograms and fluctuating flow fields obtained from PIV show that both secondary flows and the naturally occurring large-scale structures formed in turbulence over smooth walls meander in a similar manner and both coexist in the limits where $S/\bar{\delta} \ll 1$ and $S/\bar{\delta} \gg 1$.

Declaration of Authorship

This certifies that:

1. the thesis comprises only my original work towards the PhD,
2. due acknowledgement has been made in the text to all other material used,
3. the thesis is fewer than the 100,000 words in length, exclusive of tables, maps, bibliographies and appendices.

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Publications

Parts of the work presented in chapter [5](#), [6](#), [7](#), and [9](#) have been published in:

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List of Symbols

Roman Symbols

A	Intercept of viscous-scaled velocity profile for smooth wall (4.17)
A_{FR}	Intercept of viscous-scaled velocity profile for sand grain roughness (8.5)
a^+	Viscous-scaled variable a
a_i^n	POD coefficient of mode i of PIV snapshot n
a'_f	Spatially filtered fluctuating component a'
$\overline{a'b'}$	Reynolds stress of fluctuating components a' and b'
b	The width of roughness elements in ridge-type roughness
C	Constant related to the size of the time-averaged secondary flows
C_F	Front camera in SPIV measurements
C_f	Skin friction coefficient
C_n	Cameras in WPPIV measurements ($n = 1, 2, 3$)
C_p	Coefficient of pressure
C_R	Rear camera in SPIV measurements
$\mathcal{C}$	Advection of TKE term
c	Sutherland's law constant ($1.458 \times 10^{-6} \text{ kg m}^{-1} \text{ s}^{-1} \text{ K}^{-0.5}$)
$\tilde{\mathbf{c}}$	3×1 Camera position vector relative to physical coordinate system
$\mathcal{D}$	Viscous transport term
d	Streamwise misalignment between PIV calibration image plane and laser sheet
d_0	Measured initial distance between hot-wire and the surface
d_p	Diameter of seeding particle
E	Hot-wire output voltage
E_∞	Hot-wire output voltage in freestream
ΔE	Hot-wire output voltage drift from pre-calibration curve
F_O	Loss-of-correlation factor
f	Sampling frequency
$f/$	Lens aperture number
f_0	Focal length
H	Shape factor
$\mathcal{H}$	Pressure transport term
h	Height of roughness elements
$\Delta \bar{h}$	Average height difference between cardboard and sandpaper strip
$\mathbf{I}$	3×3 Identity matrix

$I_{\Lambda_{ci}}$	Time- and spatially averaged swirl strength
$I_{\Phi_{aa,x}}$	Integrated streamwise power spectral density of velocity component a
$I_{\Phi_{aa,y}}$	Integrated spanwise power spectral density of velocity component a
K	Turbulent kinetic energy
$\mathbf{K}$	3×3 Camera calibration matrix
k	Mean grain size of sandpaper roughness
k_{rms}	Root mean square of the deviation of grain size of sandpaper roughness from its mean
k_s	Equivalent sand grain roughness
k_x	Streamwise wavenumber
k_y	Spanwise wavenumber
L	Spacing between roughness elements in ridge-type roughness
l	Hot-wire spatial resolution
l_y	Spanwise extent of the time-averaged secondary flows
m_y	Number of pixels per unit distance in spanwise direction
m_z	Number of pixels per unit distance in wall-normal direction
$\mathbf{P}$	3×4 Camera calibration matrix
$\mathcal{P}$	Production term
P_{atm}	Atmospheric pressure
ΔP	Pressure difference
R	Perfect gas constant ($289.9 \text{ J kg}^{-1} \text{ K}^{-1}$)
$\mathbf{R}$	3×3 Camera orientation matrix
$R_{a'b'}$	Normalised two-point correlation coefficient of fluctuating components a' and b'
$\hat{R}_{a'b'}$	Two-point correlation coefficient of fluctuating components a' and b'
R_{cable}	Hot-wire probe cable resistance
R_{hw}	Hot-wire resistance
R_o	Hot-wire overheat resistance
Re_x	Reynolds number based on streamwise locations
Re_δ	Reynolds number based on boundary-layer thickness
Re_θ	Reynolds number based on momentum thickness
S	Width of spanwise heterogeneous roughness strip
S_r	Width of rough strip in strip-type roughness
S_s	Width of smooth strip in strip-type roughness
s	Skew parameter of the camera
s_{ij}	Rate of strain tensor
T	Air temperature
$\mathcal{T}$	Turbulent transport term
t	Time
Δt	Sampling interval
U	Time-averaged streamwise velocity
$\tilde{U}$	Time-averaged, spatial fluctuation of U
U_c	Convection velocity

U_1	Freestream velocity measured by spanwise traversing Pitot-static tube
U_∞	Freestream velocity
U_τ	Friction velocity
$\overline{U_\tau}$	Spanwise-averaged friction velocity
$\langle U \rangle_\Lambda$	Time and spatial average of U , averaged over an integer number of spanwise roughness wavelengths
ΔU^+	Hama roughness function
u	Total streamwise velocity
u'	Turbulent fluctuation of streamwise velocity
$\tilde{u}'$	The sum of u' and $\tilde{U}$
V	Time-averaged spanwise velocity
v	Total spanwise velocity
v'	Turbulent fluctuation of spanwise velocity
W	Time-averaged wall-normal velocity
$\mathcal{W}$	Modified wake function
w	Total wall-normal velocity
w'	Turbulent fluctuation of wall-normal velocity
X	Streamwise direction in image pixel coordinate system
X_f	Final streamwise grid location in image pixel coordinate system
X_i	Initial streamwise grid location in image pixel coordinate system
ΔX	Streamwise pixel displacement
x	Streamwise direction in physical coordinate system
x_f	Final streamwise grid location in physical coordinate system
x_i	Initial streamwise grid location in physical coordinate system
Δx	Streamwise spatial resolution
Y	Spanwise direction in image pixel coordinate system
Y_f	Final spanwise grid location in image pixel coordinate system
Y_i	Initial spanwise grid location in image pixel coordinate system
ΔY	Spanwise pixel displacement
y	Spanwise direction in physical coordinate system
$\hat{y}$	Distance between left- and right-leaning v'_f -conditioned correlation maps computed at $z/\bar{\delta} = 0.5$
y_c	Spanwise location of the center of the mean secondary flows
y_f	Final spanwise grid location in physical coordinate system
y_i	Initial spanwise grid location in physical coordinate system
y_{ref}	Reference point of two-point correlation in spanwise direction
y_u	Spanwise location of the common flow up
$\hat{y}_u$	Distance between left- and right-leaning v'_f -conditioned correlation maps computed at $z/\delta_u = 0.5$
Δy	Spanwise spatial resolution
Z	Wall-normal direction in image pixel coordinate system
ΔZ	Wall-normal pixel displacement
z	Wall-normal direction in physical coordinate system

z_r	Roughness sublayer height
z_{ref}	Reference point of two-point correlation in wall-normal direction
z_{sheet}	Wall-normal location of the laser sheet in wall-parallel PIV measurements
z_0	Initial wall-normal location of the hot-wire
Δz	Wall-normal spatial resolution

Greek Symbols

α	Angle in streamwise–spanwise plane between the camera viewing ray and lines parallel to the flow
β	Angle in streamwise–wall-normal plane between the camera viewing ray and lines parallel to the flow
$\bar{\delta}$	Spanwise-averaged 98% boundary-layer thickness
δ_r	Homogeneous rough wall 98% boundary-layer thickness
δ_s	Smooth wall 98% boundary-layer thickness
δ_u	98% Local boundary-layer thickness at the common flow up
δ_{99}	99% Boundary-layer thickness
ϵ	Error in hot-wire initial wall location measurement
ε	Dissipation term
Θ	Rotation of PIV calibration image plane about wall-normal axis
$\bar{\theta}$	Spanwise-averaged momentum thickness
θ_r	Homogeneous rough wall momentum thickness
θ_s	Smooth wall momentum thickness
κ	von Kármán constant (0.384)
Λ	Spanwise heterogeneous roughness wavelength
Λ_{ci}	Time-averaged swirl strength
λ_x	Streamwise wavelength
λ_y	Spanwise wavelength
ν	Kinematic viscosity of air
Π	Wake parameter
ρ	Density of air
ρ_p	Density of seeding particle
σ_a	Standard deviation of velocity component a
τ_p	Timescale of seeding particle
τ_r	Wall shear stress of the rough strip
τ_s	Wall shear stress of the smooth strip
τ_w	Wall shear stress
τ_η	Kolmogorov timescale of the flow
$\Phi_{a'b'}$	Power spectral density of the product of fluctuating components $a' \cdot b'$
$\phi_{a'}^i$	The i th POD mode of fluctuating component a'
Ψ	Rotation of PIV calibration image plane about spanwise axis
ψ	Angle between left- and right-leaning v'_f -conditioned correlation maps
Ω_x	Time-averaged streamwise vorticity

List of Abbreviations

BNC	Bayonet Neill-Concelman
C-D	Converging-Diverging
CCD	Charge-Coupled Device
CCW	Counter-Clockwise
CW	Clockwise
DAQ	Data Acquisition
DC	Direct Current
DNS	Direct Numerical Simulations
DSLR	Digital Single-Lens Reflex
FFT	Fast Fourier Transform
FOV	Field of View
HWA	Hot-Wire Anemometry
LES	Large Eddy Simulations
LHS	Left Hand Side
LSM	Large-Scale Motion
MDF	Medium-Density Fibreboard
MUCTA	Melbourne University Constant Temperature Anemometer
ND	Neutral Density
PIV	Particle Image Velocimetry
POD	Proper Orthogonal Decomposition
RHS	Right Hand Side
SPIV	Stereoscopic Particle Image Velocimetry
TKE	Turbulent Kinetic Energy
VLSM	Very Large-Scale Motion
WPPIV	Wall-Parallel Particle Image Velocimetry
ZPG	Zero Pressure Gradient

Chapter 1

Introduction

1.1 Background

The nature of wall-bounded turbulent flows over rough surfaces, whose roughness distribution is homogeneous, has been studied extensively and is relatively well-defined. To study such surfaces, engineers are equipped with tools such as the Moody chart (Moody, 1944) and the Hama roughness function (Hama, 1954). Most surfaces in nature and engineering applications, however, are heterogeneous and the heterogeneity can be arranged in an infinite number of ways. Examples include: rivets on aircraft, biofouling on ships, sedimentation on riverbeds, and forest and crop boundaries in the atmospheric surface layer. At present, there is no tool that can be used to accurately predict the skin friction drag of these heterogeneously rough surfaces.

This work considers a specific case of heterogeneous roughness where the roughness varies in the spanwise direction. Spanwise heterogeneity is imposed by alternating rough and smooth strips to form a test surface over which a turbulent boundary layer is developed (figure 1.1). This type of roughness heterogeneity induces secondary flows in the form of counter-rotating helical motion in the streamwise direction. In the existing studies, these secondary flows are almost always discussed in the time-averaged sense. Recent studies, however, have uncovered certain time-dependent features of the secondary flows which are masked by time averaging. A hint of these features is given by the 1-D energy spectrograms (Nugroho *et al.*, 2013; Awasthi & Anderson, 2018; Medjnoun *et al.*, 2018; Zampiron *et al.*, 2020), two-point correlation maps (Kevin *et al.*, 2017, 2019b), and POD (proper orthogonal decomposition) of the turbulent fluctuation fields (Vanderwel *et al.*, 2019). A streamwise periodic behaviour is implicit in many of these results, reminiscent of the meandering

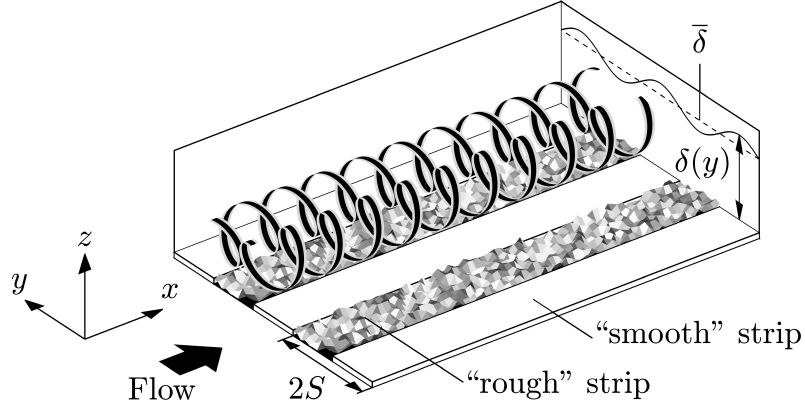


Figure 1.1: Illustration of turbulent boundary layer developing over a spanwise heterogeneous roughness surface, with a pair of counter-rotating roll modes embedded within the flow. Boundary-layer thickness $\delta(y)$ is marked by —, while the spanwise-averaged boundary-layer thickness $\bar{\delta}$ is marked by ---. S is the width of the roughness strips.

of very large-scale motions (VLSMs) that occur naturally in the log layer of wall-bounded turbulence (Hutchins & Marusic, 2007a). Meandering of the secondary flows induced by spanwise heterogeneity, akin to that observed for VLSMs, has been shown in Kevin *et al.* (2017, 2019b) and Vanderwel *et al.* (2019) for a surface whose spanwise wavelength is approximately equal to the boundary-layer thickness δ (close to the expected spanwise scale of the VLSM). This poses a question about the nature of the secondary flows imposed by surface roughness in comparison to the VLSMs and further, whether this observed unsteadiness is governed by the spanwise roughness wavelength. The meandering of the secondary flows has so far been observed in the case where $S \approx \delta$ (Kevin *et al.*, 2017, 2019b; Vanderwel *et al.*, 2019), where S is the width of the roughness strip. In Chung *et al.* (2018), the isovels of the mean streamwise velocity showed that in the scenarios where $S \gg \delta$ or $S \ll \delta$ the secondary flows are confined within certain parts of turbulent boundary layer while other regions approach locally homogeneous conditions. This leads to another question about the secondary flows and the large-scale structures: can they coexist with each other? If so, how would they affect one another?

In this study, we isolate a pure strip-type roughness (figure 1.1), similar to that of Hinze (1967, 1973), by minimising variations in surface elevation (ridges) between spanwise-alternating rough and smooth strips. In the absence of ridge-related behaviour, we aim to explore the effects of spanwise roughness wavelength $2S$ on the resulting secondary flows over a range of $S/\bar{\delta}$ ($0.32 \leq S/\bar{\delta} \leq 6.81$), where $\bar{\delta}$ is

the spanwise-averaged boundary layer thickness. This range is chosen to represent the intermediate cases, $S/\bar{\delta} \approx 1$, where meandering secondary flows have been observed (Kevin *et al.*, 2017, 2019b; Vanderwel *et al.*, 2019), and the two limits where $S/\bar{\delta} \gg 1$ and $S/\bar{\delta} \ll 1$, as discussed by Chung *et al.* (2018). To investigate the time-dependent behaviour of the secondary flows, we present the analysis of both time-averaged and the fluctuating velocity components, with focus on the fluctuating components. Hot-wire anemometry (HWA) and particle image velocimetry (PIV) measurements are conducted on turbulent boundary layers developing over surfaces composed of streamwise-aligned strips of sandpaper separated by smooth strips of cardboard laid down in a spanwise-alternating pattern. PIV measurements capture the flow field in two planes: stereoscopic PIV (SPIV) in the spanwise-wall-normal plane and wall-parallel PIV (WPPIV) in the streamwise-spanwise plane.

1.2 Objectives and thesis outline

The work presented in this thesis aims to:

- (a) Investigate the effects of spanwise roughness wavelength $2S$ on the resulting secondary flows over a range of $S/\bar{\delta}$.
- (b) Investigate the turbulent structures of the secondary flows imposed by surface roughness in comparison to the large-scale structures in wall-bounded turbulence and how the unsteadiness observed in both vary with the roughness wavelength.
- (c) Investigate the possibility of coexistence between the secondary flows and the large-scale structures and how these two might influence one another.

This thesis contains 10 chapters, with the outline of each chapter as follows: chapter 2 provides a review of previous studies in spanwise heterogeneous roughness and other relevant topics, such as surface roughness and coherent structures in wall-bounded turbulence. The experimental apparatus and techniques are presented in chapter 3 (test surfaces and HWA measurements) and chapter 4 (PIV measurements, calibration and velocity reconstruction techniques). Chapter 5 discusses the time-averaged statistics, recovery to outer layer similarity, and comparison to DNS of channel flows over similar spanwise heterogeneous roughness. Chapter 6 discusses the turbulence statistics (Reynolds stresses, turbulent kinetic energy, and the energy budget). In chapter 7, we present the 1-D streamwise and spanwise energy spectra,

and the 2-D energy spectra and co-spectra. The structures of the turbulence over the smooth wall are discussed in chapter [8](#), which are then compared to those over spanwise heterogeneous roughness in chapter [9](#). The summary of this work and possible future directions are presented in chapter [10](#).

Chapter 2

Literature review

2.1 Smooth and rough wall turbulent boundary layers

The velocity components of turbulent flows can be decomposed into the time-averaged value and the fluctuations,

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{U}(\mathbf{x}) + \mathbf{u}'(\mathbf{x}, t) \quad (2.1)$$

where $\mathbf{x} = (x, y, z)$ is the streamwise, spanwise, and wall-normal direction, respectively. $\mathbf{u} = (u, v, w)$ is the streamwise, spanwise, and wall-normal total (mean and fluctuating) velocity, $\mathbf{U} = (U, V, W)$ and $\mathbf{u}' = (u', v', w')$ are the mean and fluctuating components, respectively.

The structures of turbulent boundary layers over smooth walls are governed by two scales, depending on their vicinity to the wall (Townsend, 1976). Very close to the wall, ‘inner’ scaling, which is dominated by viscosity, is the relevant scaling. Through the dimensional analysis of Prandtl (1925), the mean velocity profile can be expressed as

$$\frac{U}{U_\tau} = f\left(\frac{zU_\tau}{\nu}\right) \quad (2.2)$$

where U_τ is the friction velocity, $U_\tau \equiv \sqrt{\tau_w/\rho}$ and where τ_w and ρ are the wall shear stress and the density of fluid, respectively. The variables normalised by viscous scaling are denoted by superscript ‘+’, e.g. $U^+ \equiv U/U_\tau$ and $z^+ = zU_\tau/\nu$. The viscous length scale is defined as ν/U_τ , where ν is the kinematic viscosity.

In the ‘outer’ layer of wall-bounded turbulence, the relevant scaling is the inertial scaling. Regardless of the condition near the wall, von Kármán (1930) showed that there is a universal form of velocity defect profile that scales on the boundary layer

thickness δ instead of ν/U_τ , such that

$$\frac{U_\infty - U}{U_\tau} = F\left(\frac{z}{\delta}\right) \quad (2.3)$$

where $(U_\infty - U)/U_\tau$ is the velocity defect profile and U_∞ is the freestream velocity.

In the inner layer of smooth wall turbulent boundary layers, the viscous sublayer ($z^+ < 5$) is modeled by

$$U^+ = z^+ \quad (2.4)$$

Further from the wall, there is an overlap between inner and outer layer, implying that the derivatives of equation (2.2) and (2.3) are equal in this overlapping region (Millikan, 1938).

$$z^+ \frac{df}{dz^+} = -\eta \frac{dF}{d\eta} = \frac{1}{\kappa} \quad (2.5)$$

where $\eta \equiv z/\delta$. Since the LHS term is solely a function of z^+ and the RHS term a function of η , then both must be equal to a constant $1/\kappa$, where κ is defined as the von Kármán constant. The integration gives a logarithmic mean velocity and velocity defect profiles

$$U^+ = \frac{1}{\kappa} \log z^+ + A \quad (2.6)$$

$$U_\infty^+ - U^+ = \frac{1}{\kappa} \log\left(\frac{z}{\delta}\right) + B \quad (2.7)$$

where κ and A are universal constants, while B depends on the flow geometry (and differs between channels, pipes and developing boundary layers). At $z/\delta \approx 0.15$ the mean velocity deviates from the logarithmic profile. Coles (1956) added a wake function to equation (2.6),

$$U^+ = \frac{1}{\kappa} \log z^+ + A + \frac{2\Pi}{\kappa} \mathcal{W}\left(\frac{z}{\delta}\right) \quad (2.8)$$

where Π is the wake parameter and $\mathcal{W}$ is the wake function. This log and wake profile shown in equation 2.8 is valid for $z^+ \gtrsim 50$ (Jiménez, 2004; Monkewitz *et al.*, 2007; Chauhan *et al.*, 2009).

For turbulent boundary layers developing over rough walls, Nikuradse (1933) showed that the logarithmic velocity profile is similar to that of the smooth wall boundary layer in equation (2.8). However, a ‘roughness’ scaling is introduced such that

$$U^+ = \frac{1}{\kappa} \log\left(\frac{z}{k_s}\right) + A_{FR} + \frac{2\Pi}{\kappa} \mathcal{W}\left(\frac{z}{\delta}\right) \quad (2.9)$$

where k_s is the equivalent sand grain roughness and $A_{FR} = 8.5$ is the fully rough intercept of the velocity profile of sand grain roughness (Nikuradse, 1933). The

definition of roughness scale may differ between communities. For example, geophysicists typically use roughness length z_0 instead of k_s ,

$$U^+ = \frac{1}{\kappa} \log \left(\frac{z}{z_0} \right) + \frac{2\Pi}{\kappa} \mathcal{W} \left(\frac{z}{\delta} \right) \quad (2.10)$$

where, from equation 2.9 and 2.10, $z_0 \equiv k_s \exp(-\kappa A_{FR}) = 0.033k_s$ (Raupach *et al.*, 1991; Jiménez, 2004). Alternatively, since roughness increases skin friction, equation (2.9) can be written as a vertical shift from the smooth wall velocity profile

$$U^+ = \frac{1}{\kappa} \log z^+ + A + \frac{2\Pi}{\kappa} \mathcal{W} \left(\frac{z}{\delta} \right) - \Delta U^+ \quad (2.11)$$

where ΔU^+ is the logarithmic shift, known as the Hama roughness function (Hama, 1954) and defined as

$$\Delta U^+ = \frac{1}{\kappa} \log k_s^+ + A - A_{FR} \quad (2.12)$$

where k_s is the equivalent sand grain roughness and $k_s^+ \equiv k_s U_\tau / \nu$.

The equivalent sand grain roughness k_s , as demonstrated above, is a more convenient way to define surface roughness instead of the roughness height k . Schlichting (1936) defined k_s as the size of uniform, closed-packed sand grains that make up a hypothetical surface. The drag of this hypothetical surface would be equal to that of a surface of interest if exposed to the same flow in the fully rough regime. By this definition, k_s is not a measure of physical roughness. Rather, k_s is a measure of roughness effect on the mean flow relative to the uniform sand grain roughness (Nikuradse, 1933) and can only be determined by measurements. Both k_s and ΔU^+ depend on the roughness geometry and configuration, such as: roughness density (Schlichting, 1936; Dirling, 1973; Sigal & Danberg, 1990), slope (Acharya *et al.*, 1986; Napoli *et al.*, 2008; Chan *et al.*, 2015), skewness (Schultz & Flack, 2009; Flack & Schultz, 2010, 2014), and height (Schultz & Flack, 2009; Chan *et al.*, 2015).

Nikuradse (1933) showed that for uniform sand grain surface, the departure from smooth wall drag occurs at $k_s^+ \approx 5$ ($k_s^+ < 5$ is a hydraulically smooth surface, where the wall shear stress is solely due to viscosity) and the surface becomes ‘fully’ rough at $k_s^+ \gtrsim 70$ (the wall shear stress is solely due to form drag on roughness elements). The region between $5 < k_s^+ < 70$ is the transitional region (wall shear stress arises from both viscous and form drag). It was shown in Jiménez (2004) and Flack & Schultz (2010) that ΔU^+ is solely a function of k_s^+ for various type of roughness when $k_s^+ \gtrsim 70$. Other studies, however, suggested that these limits may vary. For example, the transition starts at $k_s^+ = 3.5$ for honed pipe roughness (Shockling *et al.*, 2006) and as low as $k_s^+ = 1.4$ for a commercial steel pipe (Langelandsvik

et al., 2008); the transition region is reported to be $15 < k_s^+ < 55$ for packed spheres (Ligrani & Moffat, 1986) and $2.5 < k_s^+ < 25$ for a scratched surface (Flack *et al.*, 2007).

The presence of roughness alters the turbulent boundary layer from that of the canonical smooth wall. It replaces the inner layer with a roughness sublayer, where coherent roughness element-scaled structures instead of or in addition to the near-wall cycle are observed (Castro, 2007). There is a suggestion in the literature that the roughness sublayer height z_r (the wall-normal extent of the surface roughness effect on the turbulent boundary layer) is approximately 3 to 5 times the roughness physical height, $z_r \sim 3\text{--}5k$ (Raupach *et al.*, 1991; Flack *et al.*, 2007; Schultz & Flack, 2007; Wu & Christensen, 2007).

Similar to the inner-outer layer relationship in smooth wall boundary layers, the outer layer of a rough wall boundary layer ($z \gg z_r$) is independent of the roughness sublayer and the outer layer similarity in equation (2.3) holds. The validity of the outer layer similarity for rough wall boundary layers, however, depends on the roughness height relative to the flow depth (k/δ). In general, if $k/\delta \ll 1$, k is sufficiently small such that the effect of roughness is confined near the wall and the outer layer remains unaffected. The exact threshold of k/δ differs between studies, for example: $\delta/k_s \gtrsim 20$ (Schultz & Flack, 2005; Castro, 2007; Flack *et al.*, 2007), $\delta/k \gtrsim 40$ (Jiménez, 2004; Wu & Christensen, 2007), and $k/\delta < 0.15$ (Amir & Castro, 2011). Conversely, if k is comparable to δ , the roughness might affect the entire boundary layer and the outer layer similarity is invalid. It should also be noted that the outer layer similarity in rough wall assumes that the roughness is homogeneous (it is solely characterised by k or k_s). The existence of δ -scaled spatial heterogeneity in the roughness means that the roughness height k may not necessarily be the relevant roughness scale. For spanwise heterogeneous surfaces the spanwise wavelength of the surface could be large, relative to the boundary layer thickness, which can negate assumptions of outer layer similarity. Related to this, Chan *et al.* (2018) have recently suggested for a homogeneous roughness that the spanwise wavelength of the roughness Λ determines the height of the roughness sublayer, $z_r \approx 0.5\Lambda$. This is particularly relevant to the current study, where spanwise heterogeneous roughness will have spanwise wavelengths that in some cases are $> 2\delta$, meaning that the roughness sublayer (secondary flows generated by the heterogeneity) will occupy the entire turbulent boundary layer, affecting outer layer similarity.

2.2 Coherent structures in wall-bounded turbulent flows

In the context of wall-bounded turbulence, ‘coherent structure’ refers to a fluid motion that: (a) has spatial coherence and (b) lasts for a reasonable period of time (Panton, 2001). A conceptual drawing by Theodorsen (1952) suggested a horseshoe-shaped vortex as a model for this coherent structure. Without assumptions of the form for coherent structures, Townsend (1976) proposed the ‘attached eddy hypothesis’, where the coherent structures are regarded as eddies of various scales in turbulent flows. These eddies are attached to the wall and scaled proportionally to their distance from the wall. The flow visualisation of turbulent boundary layers in Head & Bandyopadhyay (1981), and later, direct numerical simulations (DNS) by Wu & Moin (2009*a,b*), showed a clear evidence of ‘a forest of hairpin vortices’ in the flow, similar to that suggested by Theodorsen (1952). Each hairpin vortex (figure 2.1b) is formed by two quasi-streamwise vortex rods (the ‘legs’) that extend downstream and are inclined away from the wall, forming a loop (the ‘head’). In Perry & Chong (1982), this vortex is referred to as Λ -shaped vortex and the relations between the mean flow, Reynolds stresses, and the energy spectra are derived based on this Λ -shaped model and Townsend’s attached eddy hypothesis.

When $\delta^+ \equiv \delta U_\tau / \nu \gtrsim 2000$, coherent structures in smooth wall turbulent boundary layers can be separated into two distinctive scales: the inner and outer scale (Hutchins & Marusic, 2007*a*). The viscous unit-scaled near-wall cycle, visualised as high- and low-momentum streaks (Kline *et al.*, 1967) associated with ejection (Q2 event, $u' < 0$ and $w' > 0$) and sweep (Q4 event, $u' > 0$ and $w' < 0$) due to quasi-streamwise vortices. Although the hairpin vortex has been a suggested representative eddy away from the wall, there is evidence that the near-wall coherent structure is more likely to be asymmetrical (‘one-legged’, ‘cane’-shaped or ‘arch’-like, see figure 2.1d) rather than symmetrical (hairpin-shaped). Zhou *et al.* (1999) observed near-wall quasi-streamwise vortices as a part of the hairpin vortex formation mechanism. It was suggested by Robinson (1991) that quasi-streamwise vortices are the dominant structures in the buffer layer, while symmetrical hairpin vortices are dominant in the wake region. A transient analysis in Schlatter *et al.* (2014) showed that hairpin vortices occupy the flow during transition, but quasi-streamwise and one-sided vortices are dominant when the flow is turbulent and increasingly so at higher Reynolds numbers. These quasi-streamwise vortices and accompanying streaks are a reminiscent of the ‘streak-vortex instability’ model (figure 2.1c) for

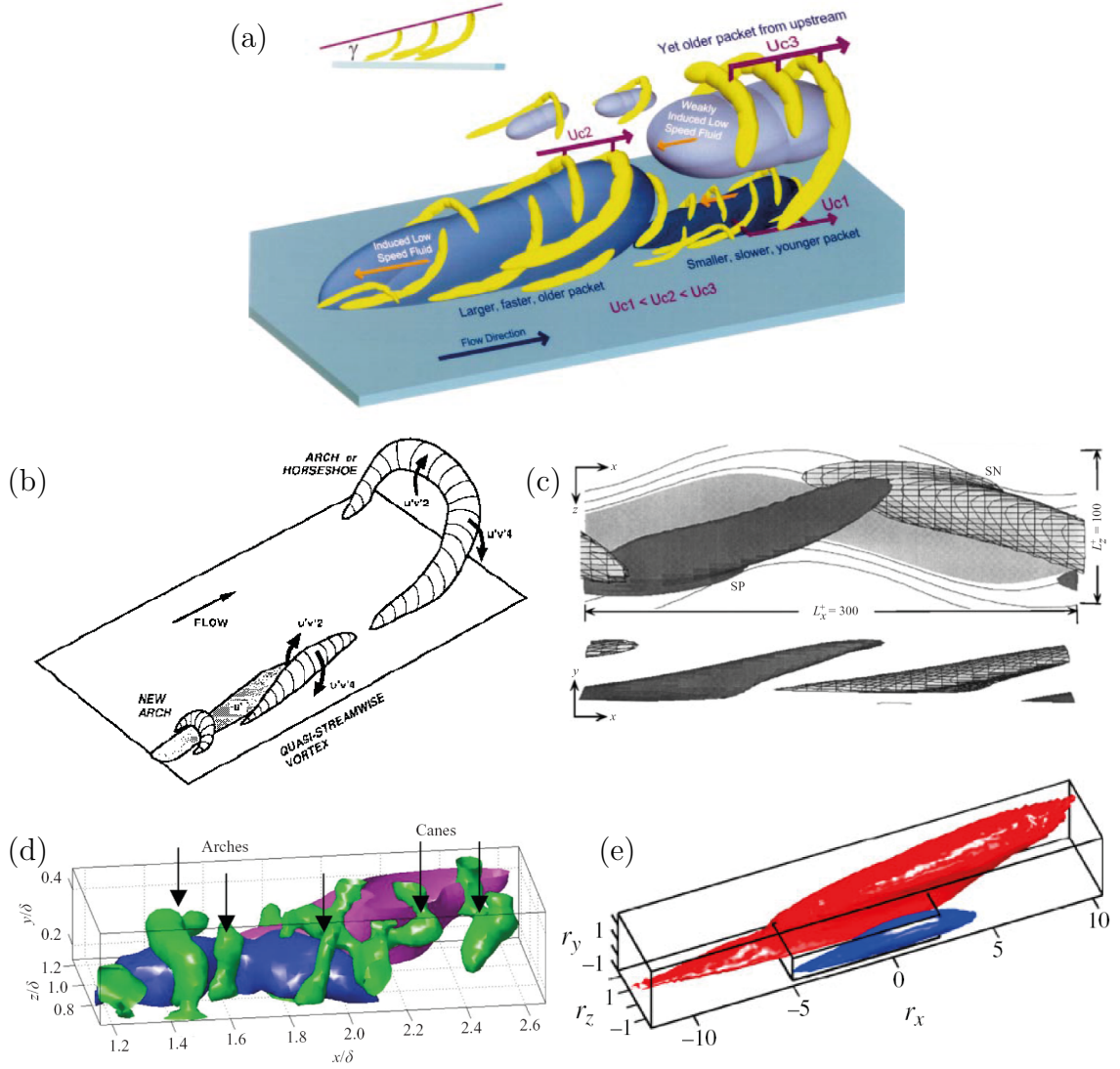


Figure 2.1: Coherent structures in wall-bounded flows: (a) illustration of the formation mechanism of LSM/VLSM from packets of hairpin vortices, adapted from Adrian *et al.* (2000). (b) Illustration of a hairpin vortex and a quasi-streamwise vortex, adapted from Johansson *et al.* (1991). (c) Vortex-instability model proposed for near-wall coherent structures, adapted from Schoppa & Hussain (2002). (d) ‘Arch’- and ‘cane’-like coherent structures, adapted from Elsinga *et al.* (2010). (e) Contours of conditionally averaged u' showing asymmetrical coherent structures, adapted from Lozano-Durán *et al.* (2012).

near-wall coherent structures of Jeong *et al.* (1997); Waleffe (2001) and Schoppa & Hussain (2002). Here, the coherent structures are regarded as alternating-sign, quasi-streamwise vortices that overlap to form the near-wall streaks (Jeong *et al.*, 1997).

In the outer layer, coherent structures exist at larger scales. This is consistent with the aforementioned attached eddy hypothesis where the characteristic scale of the eddies increases with increasing distance from the wall. The large-scale motion (LSM) was first observed near the edge of turbulent boundary layers as a coherent structure or interfacial bulge of $\sim 3\delta$ length (Kovasznay *et al.*, 1970). Later, these large-scale structures were also observed in both the logarithmic (overlap) and wake region of turbulent boundary layers (Adrian *et al.*, 2000; Ganapathisubramani *et al.*, 2003; del Álamo *et al.*, 2004), and also in pipe and channel flows (Liu *et al.*, 2001; Guala *et al.*, 2006; Monty *et al.*, 2007). These features have commonly been associated with hairpin packets (Adrian *et al.*, 2000; Tomkins & Adrian, 2003) and large-scale meandering high- and low-momentum streaks (Hutchins & Marusic, 2007*a,b*) with a streamwise wavelength of approximately 3δ (Guala *et al.*, 2006), where δ is either the boundary layer thickness, channel half-height, or pipe radius. In the logarithmic region, ‘superstructures’ or very-large-scale motions (VLSMs) of as long as $\sim 20\delta$ length are observed (Kim & Adrian, 1999; Guala *et al.*, 2006; Balakumar & Adrian, 2007; Hutchins & Marusic, 2007*a*; Dennis & Nickels, 2011*b*; Lee & Sung, 2011; Hutchins *et al.*, 2012; Wu *et al.*, 2012). In relation to the near-wall structures, the near-wall dynamics were regarded as a self-sustaining process, independent of the dynamics of the large-scale structures in the outer layer (Jiménez *et al.*, 2004). However, there are evidences suggesting that attached large scale structures affect this process through modulation of the near-wall cycle (Hutchins & Marusic, 2007*b*; Mathis *et al.*, 2009; Marusic *et al.*, 2010).

Adrian *et al.* (2000) suggested a ‘parent-offspring mechanism’ for the formation of large-scale coherent structures from hairpin vortices. A strong ejection (Q2) event occurs between the legs of a primary hairpin vortex, resulting in a low-momentum region in this area. As the primary hairpin vortex gains strength, the secondary hairpin vortex is formed downstream of the primary hairpin vortex, and so on. As more hairpins are formed, they can also reconnect and merge with each other (Tomkins & Adrian, 2003), until ‘a packet of hairpin vortices’ forms, aligned in the streamwise direction and convecting together, carrying an elongated large-scale low-momentum region between their legs (figure 2.1a). It has been suggested that cluster of hairpin packets can agglomerate or align themselves in streamwise direction, resulting in

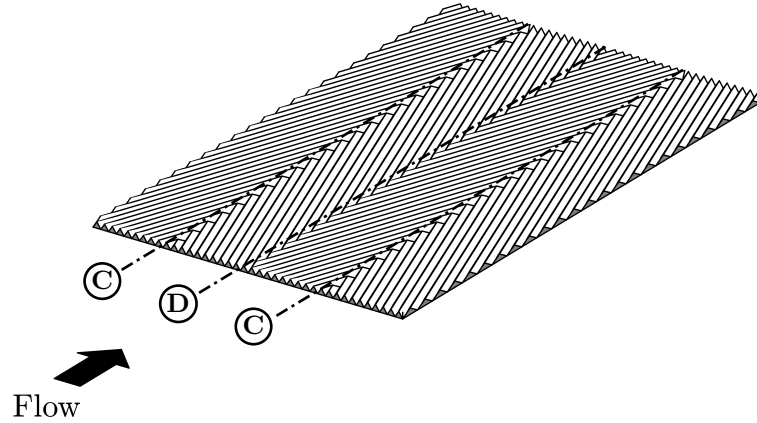


Figure 2.2: Illustration of converging-diverging (C-D) riblets (Kevin *et al.*, 2017, private communication). Dash-dot lines (---) labelled with $\textcircled{C}$ marks the converging region, $\textcircled{D}$ the diverging region.

VLSM (Kim & Adrian, 1999; Dennis & Nickels, 2011*b*). More recently, Lee *et al.* (2019) showed that the merging of the hairpin packets of LSM into VLSM involves stretching and shearing due to the wall-normal gradient of the mean velocity that convects the LSM.

The imprint of symmetrical hairpin vortex packets is evident in the resulting large-scale structures in wall-bounded turbulent flows. Two-point correlations of the fluctuating flow field exhibit an elongated low-momentum streak flanked by high-momentum streaks in the spanwise direction (Tomkins & Adrian, 2003; Ganapathisubramani *et al.*, 2003, 2005; Hutchins *et al.*, 2005; Hutchins & Marusic, 2007*a*; Marusic *et al.*, 2010; Dennis & Nickels, 2011*a*). Johansson *et al.* (1991), however, cautioned against this interpretation of the two-point correlation contours. Ensemble averaging and assumptions of spanwise homogeneity, as typically applied in the computation for smooth wall-bounded turbulence, enforce a plane of symmetry in the resulting conditional average. Asymmetrical large-scale structures (one-sided roll modes, see figure 2.1e) have also been observed in smooth walls (Lozano-Durán *et al.*, 2012; Kevin *et al.*, 2019*a*) and converging-diverging (C-D) riblets (Kevin *et al.*, 2017). In Elsinga *et al.* (2010), observation of the hairpin packets showed that they are likely to be made up of arch-like or cane-shaped structures (figure 2.1d). Similar to the near-wall cycle, an alternative model for the formation mechanism has been suggested for coherent structures in the outer layer. Large-scale elongated streaks with a sinuous instability and associated asymmetric staggered quasi-streamwise vortices have been observed in the logarithmic region and beyond (Flores & Jiménez,

2010; Cossu & Hwang, 2017; de Giovanetti *et al.*, 2017), which are similar to, but at a much larger scale than, structures associated with the streak-vortex instability, which was initially developed as a model for near-wall streak formation (Jeong *et al.*, 1997; Waleffe, 2001; Schoppa & Hussain, 2002). More recently, a similar asymmetric large-scale streak vortex structure has been elucidated in the log region from both PIV of C-D riblets and DNS of smooth wall channel flow (Kevin *et al.*, 2019a). This similarity hints at the possibility of similar formation mechanisms between surface-forced features (secondary flows) and the large-scale structures in canonical wall-bounded flow.

2.3 Prandtl's classification of the secondary flows

Prandtl (1952) classified secondary flows into two categories. The first kind is related to the skewing of mean flow due to heterogeneity in the streamwise direction, e.g. in a curved pipe or channel flow. It is an inviscid process, driven by pressure gradient instead of viscous stress. Therefore, it may occur in both laminar and turbulent flows (Bradshaw, 1987). The second kind is induced by heterogeneity and anisotropy of turbulence, hence the term ‘turbulence-induced’ or ‘stress-induced’ secondary flow. The secondary flow over a spanwise heterogeneous roughness belongs to the second kind (Hinze, 1967; Anderson *et al.*, 2015). Throughout this work, the term ‘secondary flow’ refers to the Prandtl’s secondary flow of the second kind.

The mechanism of turbulence-induced secondary flow generation is explained as follows. For steady and incompressible flow, assuming that there is very little to no skewness related to Prandtl’s secondary flow of the first kind, the mean vorticity equation (Einstein & Li, 1958; Perkins, 1970) in the cross section (yz -) plane is given by

$$\underbrace{V \frac{\partial \Omega_x}{\partial y} + W \frac{\partial \Omega_x}{\partial z}}_{\textcircled{A}} = \underbrace{\frac{\partial^2}{\partial y \partial z} (\overline{v'v'} - \overline{w'w'})}_{\textcircled{B}} + \underbrace{\left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \right) (-\overline{v'w'})}_{\textcircled{C}} + \underbrace{\nu \nabla^2 \Omega_x}_{\textcircled{D}} \quad (2.13)$$

where the mean streamwise vorticity $\Omega_x \equiv \partial W / \partial y - \partial V / \partial z$, $\textcircled{A}$ is the convection of vorticity, $\textcircled{B}$ is the production of vorticity, $\textcircled{C}$ is the Reynolds stress term, and $\textcircled{D}$ is the viscous term, which is negligible further from the wall (in the log region, $z^+ \gtrsim 100$ or $z/\delta \gtrsim 0.1$). From the measurements over a surface with spanwise heterogeneity, Nezu & Nakagawa (1984) observed that, due to the space-filling secondary flows, $\textcircled{B}$ is nearly balanced with $\textcircled{C}$ ($z/\delta \geq 0.2$) and the Reynolds stress term $\overline{v'w'}$ plays an important role in balancing the energy convected by the upward flow. Bradshaw

(1987) further distinguished secondary flows in the yz -plane into two types: cross flow, where term $\partial V/\partial z$ is dominant, and identifiable streamwise vortices, where $\partial W/\partial y$ and $\partial V/\partial z$ have the same order.

An alternative explanation on the formation of secondary flow is offered by Hinze (1967) from the derivation of the kinetic energy equation instead of mean vorticity (equation 2.13). The energy balance for steady, incompressible, and uniform flow in the streamwise direction is approximated by

$$\underbrace{V \frac{\partial K}{\partial y}}_{\textcircled{A}} + \underbrace{W \frac{\partial K}{\partial z}}_{\textcircled{B}} \approx \underbrace{-\overline{u'v'} \frac{\partial U}{\partial y} - \overline{u'w'} \frac{\partial U}{\partial z}}_{\textcircled{C}} - \varepsilon \quad (2.14)$$

where the turbulent kinetic energy $K \equiv (\overline{u'u'} + \overline{v'v'} + \overline{w'w'})/2$, $\textcircled{A}$ and $\textcircled{B}$ are the advection of K , $\textcircled{C}$ is the production term and ε is the viscous dissipation (the complete derivation of the energy equation and the analysis of each term are presented in chapter 6). Typically, $|\textcircled{A}| \ll |\textcircled{B}|$ and, away from the wall ($z^+ \gtrsim 100$), $\textcircled{B} < 0$. Hence, the sign of mean wall-normal velocity W depends on the interaction between the production and dissipation term in order to maintain energy balance. In the region where production is less than dissipation, there is a transport of highly turbulent flow to this region, which corresponds to upward W . Conversely, there is a transport of low turbulence fluid into a region where production exceeds dissipation, which is related to downward W . The example of this mechanism on spanwise heterogeneous roughness is discussed further in §2.5. It should be noted, however, that equation (2.14) explains the resulting large-scale secondary flows in the mean flow. In the instantaneous flow field, these secondary flows appear as large-scale streaks and counter-rotating roll modes, similar to the LSM/VLSM in the canonical spanwise homogeneous wall-bounded flows. It is possible that the two share similar formation mechanisms resulting from (instantaneous) spanwise variations in wall shear stress. For secondary flows these large-scale variations in the wall shear stress are persistent and spanwise periodic, while in a homogeneous flow they occur chaotically and convect downstream, as suggested by Townsend (1976) and Lee *et al.* (2019).

2.4 Classification of spanwise heterogeneous roughness

The existence of secondary flow was discovered in the 1940s as geologists observed sediment formation on riverbeds due to water circulation. Earlier studies by Vanoni

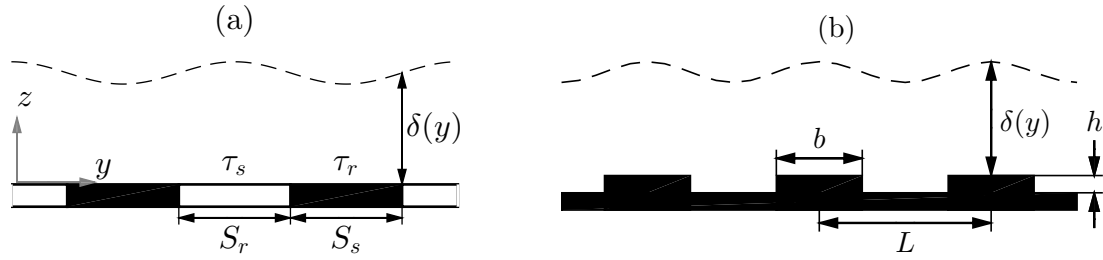


Figure 2.3: Type of spanwise heterogeneous roughness: (a) strip-type and (b) ridge-type. S_r and S_s are the width of ‘rough’ (high shear stress) and ‘smooth’ (low shear stress) strip, respectively, L is the spacing between roughness elements, b is the width of roughness elements, h is the height of roughness element, and δ is either boundary layer thickness, channel half-height, or flow depth, depending on the type of flow.

(1946), Kalinske & Hsia (1945), and aerial photographs by Kinoshita (1967) showed a spanwise-varying sediment concentration. It is believed that secondary flows along the river causes this particular pattern. Hinze (1967) and later Anderson *et al.* (2015) showed that these secondary flows are driven by the spanwise variation of Reynolds shear stress components. Colombini & Parker (1995) and Hwang & Lee (2018) further recognised two types of surfaces that may induce spanwise variation of Reynolds shear stress: strip-type (figure 2.3a), where spanwise variation of Reynolds shear stress is caused by roughness variations or directly imposed, and ridge-type (figure 2.3b), where spanwise variation of Reynolds shear stress is caused by variations in surface elevation.

In numerical simulations, strip roughness can be modeled as spanwise alternating regions of high and low wall shear stress (τ_r and τ_s in figure 2.3a) imposed directly as boundary conditions (Willingham *et al.*, 2014; Chung *et al.*, 2018). Superhydrophobic surfaces, with spanwise-alternating no-slip and free-slip boundary conditions, can also be considered as such (Jelly *et al.*, 2014; Türk *et al.*, 2014; Lee *et al.*, 2015; Stroh *et al.*, 2016). Experimentally, and also numerically, surfaces can be constructed from spanwise alternating roughness strips (Hinze, 1973; McLean, 1981; Nakagawa *et al.*, 1981; McLelland *et al.*, 1999; Wang & Cheng, 2005, 2006; Vermaas *et al.*, 2011; Bai *et al.*, 2018; Xie *et al.*, 2018; Stroh *et al.*, 2020). A selection of the studies involving strip-type roughness are summarised in table 2.1.

The recessed part of previously investigated ridge-type roughness is typically a flat surface, while the elevated part has many possible cross-sectional shapes: rectangular (Wang & Cheng, 2006; Hwang & Lee, 2018; Medjnoun *et al.*, 2018, 2020), trapezoidal (Nezu & Nakagawa, 1984), triangular (Goldstein & Tuan, 1998;

Zampiron *et al.*, 2020; Medjnoun *et al.*, 2020), semicircle (Medjnoun *et al.*, 2020), streamwise-aligned pyramids (Yang & Anderson, 2018), and smoother, sinusoidal-like spanwise elevation variations (Colombini, 1993; Wang & Cheng, 2006; Awasthi & Anderson, 2018). Studied surfaces often involve a combination of both ridge-type and strip-type roughness (Vanderwel & Ganapathisubramani, 2015; Vanderwel *et al.*, 2019; Stroh *et al.*, 2020). The studies involving ridge-type roughness are summarised in table 2.2.

A slightly different class of spanwise heterogeneity has been studied by Kevin *et al.* (2017) based on alternating arrangements of directional/anisotropic rough surfaces. Both strip- and ridge-type roughness as represented in figure 2.3(a,b) have locally isotropic surfaces, whereas converging-diverging (C-D) riblets (Nugroho *et al.*, 2013; Kevin *et al.*, 2017, 2019b) have spanwise varying anisotropy or directionality with no spanwise variation in roughness height or elevation. Converging-diverging riblets also generate secondary flows even in laminar boundary layers (Xu *et al.*, 2018), where no spanwise gradients of Reynolds shear stress are present. This possibly suggests a different generation mechanism to the Prandtl's secondary flows (of the second kind) typically attributed to spanwise heterogeneous roughness which are a turbulent phenomena. Surfaces with multiple spanwise roughness wavelengths (Barros & Christensen, 2014) have also been studied. Interestingly, although this surface has no clear dominant spanwise wavelength, δ -scaled secondary flows emerge in the time-averaged flow field. Possibly related, Nikora *et al.* (2019) has suggested that streamwise repeating features in tiled roughness can exacerbate these locked secondary flows.

2.5 The effect of spanwise heterogeneity on turbulent boundary layer

The effect of spanwise heterogeneity-induced secondary flows is observed as thickening and thinning of boundary-layer thickness in the cross-plane of the boundary layer (Nugroho *et al.*, 2013; Barros & Christensen, 2014; Vanderwel & Ganapathisubramani, 2015; Kevin *et al.*, 2017; Medjnoun *et al.*, 2018). This is associated with the formation of high momentum (HMPs) and low momentum pathways (LMPs) above the surfaces (Barros & Christensen, 2014; Willingham *et al.*, 2014; Anderson *et al.*, 2015).

Hinze (1967) hypothesised the occurrence of secondary flows as a consequence of the production-dissipation imbalance of turbulent kinetic energy over spanwise

Type 1: strip-type roughness

Case	Method	Flow	Spanwise heterogeneity	S_r/S_s	$\bar{S}/\delta$	k/δ	$Re_\delta (\delta^+)$
Hinze (1973)	experiments	channel	plastic grain & smooth strips	0.95	1.48	0.06	66000
Müller & Studerus (1979)	experiments	open channel	sand grain & PVC strips	0.60	1.00	0.03	44000
McLean (1981)	experiments	open channel	fine gravel & Eternit strips	1.00	1.44–2.88	0.04–0.08	(216–404)
Nakagawa <i>et al.</i> (1981)	experiments	channel	rough & smooth strips	1.00	0.44, 0.88		15170, 13500
Studerus (1982)	experiments	open channel	sand grain & smooth strips	0.60	≈ 1.00	≈ 0.03	42750–43760
McLelland <i>et al.</i> (1999)	experiments	open channel	coarse & fine sand grain strips	0.79	0.63	0.01	45300
Wang & Cheng (2005)	experiments	open channel	fine gravel & PVC strips	1.00	1.00	0.03	42000
Wang & Cheng (2006)	experiments	open channel	fine gravel & PVC strips	1.00, 2.00	1.00	0.03	38684–44206
Vermaas <i>et al.</i> (2011)	experiments	open channel	stones & wooden plate	1.00	4.51–12.4	0.03–0.09	18000–40000
Jelly <i>et al.</i> (2014)	DNS	channel	no-slip & free-slip boundary conditions	1.00	0.19	–	2800 (180)
Türk <i>et al.</i> (2014)	DNS	channel	no-slip & free-slip boundary conditions	0.33–3.00	0.02–1.56	–	(180)
Willingham <i>et al.</i> (2014)	LES	boundary layer	imposed high & low shear stress	0.06–0.47	1.57	–	(≈ 1000)
Lee <i>et al.</i> (2015)	DNS	channel	no-slip & free-slip boundary conditions	1.00	0.19	–	2800–10975 (180–590)
Bai <i>et al.</i> (2018)	experiments	boundary layer	40- & 240-grit sandpaper strips	0.50	1.36	0.08	54137 (1985)
Stroh <i>et al.</i> (2016)	DNS	channel	no-slip & free-slip boundary conditions	0.75–1.33	0.35–0.70	–	(120–360)
Chung <i>et al.</i> (2018)	DNS	channel	imposed high & low shear stress	1.00	0.39–6.28	–	(590)
Xie <i>et al.</i> (2018)	DNS	open channel	egg carton roughness & smooth strips	1.00	0.40–12.7	0.06	(590)
Present study	experiments	boundary layer	36-grit sandpaper & carboard strips	1.00	0.32–6.81	≈ 0.02	35800–74900 (≈ 2000)

Table 2.1: Summary of studies in spanwise heterogeneous roughness that belong to type 1: strip-type. The symbols follow the definition in figure 2.3(a), with $\bar{S} = (S_s + S_r)/2$ and k is the roughness height. $Re_\delta \equiv \delta U/\nu$, where δ is either the flow depth, channel half-height, or boundary layer thickness, and U is either freestream, bulk, maximum, or centerline velocity, depending on the case. Friction Reynolds number $\delta^+ \equiv \delta U_\tau/\nu$.

Type 2: ridge-type roughness							
Case	Method	Flow	Ridge's cross section	L/δ	b/δ	h/δ	Re_δ (δ^+)
Nezu & Nakagawa (1984)	experiment	channel	trapezoid	1.00, 2.00	0.50	0.13	13000
Colombini (1993)	experiment	open channel	sinusoid	1.00, 2.00	0.50	0.13	13000
Goldstein & Tuan (1998)	DNS	channel	triangle	0.18–3.00	0.18	0.07	2200 (125)
Wang & Cheng (2006)	experiment	open channel	sinusoid	1.92	0.96	0.15	48358
Wang & Cheng (2006)	experiment	open channel	rectangle	≈ 1.90	0.95, 1.25	0.06, 0.13	45149–49013
Vanderwel & Ganapathisubramani (2015) [†]	experiment	boundary layer	LEGO	0.30–1.76	0.15	0.09	(3800–4600)
Awasthi & Anderson (2018) [†]	LES	channel	sinusoid	3.14		0.05–0.10	($\sim O(10^6)$)
Hwang & Lee (2018)	DNS	boundary layer	rectangle	0.60–2.40	0.15–1.20	0.08	≈ 6000 (300)
Medjnoun <i>et al.</i> (2018)	experiment	boundary layer	rectangle	0.80–3.20	0.40	0.10	(1250–3340)
Yang & Anderson (2018)	LES	channel	streamwise-aligned pyramids	0.10–6.28	0.08	0.05, 0.07	(1×10^5)
Vanderwel <i>et al.</i> (2019) [†]	DNS	boundary layer	LEGO	1.00	0.15	0.10	7161 (495)
Medjnoun <i>et al.</i> (2020)	experiment	boundary layer	semicircle	1.17	0.17	0.09	(3125)
Medjnoun <i>et al.</i> (2020)	experiment	boundary layer	triangle	1.21	0.08	0.08	(3347)
Medjnoun <i>et al.</i> (2020)	experiment	boundary layer	rectangle	1.14–1.26	0.08–0.67	0.08–0.09	(2828–3239)
Stroh <i>et al.</i> (2020) [†]	DNS	open channel	smooth & rough strips, elevated smooth strips	1.00	0.50	0.04–0.09	5699–5756 (500)
Zampiron <i>et al.</i> (2020)	experiment	open channel	triangle	0.39–3.94	0.11	0.12	15500–17600

Table 2.2: Summary of studies in spanwise heterogeneous roughness that belong to type 2: ridge-type. The symbols follow the definition in figure 2.3(b) and table 2.1. [†]: combination of strip- and ridge-type roughness.

heterogeneity (see §2.3). For strip-type roughness (figure 2.3a), production exceeds the dissipation above the τ_r strips, transferring turbulent flows to the adjacent τ_s strips, and vice versa. This results in a secondary flow above the roughness interface between τ_r and τ_s , with the upwelling above τ_s strips and downwelling above τ_r . Locally, there will be low momentum pathways associated with upwelling ($W > 0$), and high momentum pathways associated with downwelling ($W < 0$). In ridge-type roughness, the occurring secondary flows have a less discernible pattern since they are deeply affected by the geometry and size of the ridges. In general, the secondary flow occurs somewhere near the junction between the raised and recessed sections of the surface, with the upwelling either above the junction (Wang & Cheng, 2006) or above the ridge (Nezu & Nakagawa, 1984; Colombini, 1993; Medjnoun *et al.*, 2018).

Recent studies showed that the size, spanwise location, and direction of the secondary flows may be affected by the specific configuration of spanwise heterogeneity (Goldstein & Tuan, 1998; Wang & Cheng, 2006; Türk *et al.*, 2014; Vanderwel & Ganapathisubramani, 2015; Stroh *et al.*, 2016; Awasthi & Anderson, 2018; Chung *et al.*, 2018; Hwang & Lee, 2018; Medjnoun *et al.*, 2018; Yang & Anderson, 2018; Zampiron *et al.*, 2020; Medjnoun *et al.*, 2020). For strip-type roughness, the spanwise roughness wavelength $\Lambda = S_s + S_r$ or $\Lambda = 2S$ when $S_s = S_r = S$ (figure 2.3a) seems to determine the behaviour of the secondary flows. The effect of the roughness wavelength over strip-type roughness is explored in Chung *et al.* (2018) and Xie *et al.* (2018), and over superhydrophobic surfaces by Türk *et al.* (2014) and Stroh *et al.* (2016). For ridge type roughness, the contributing factors are shown in figure 2.3(b): the spacing between ridges L (Goldstein & Tuan, 1998; Vanderwel & Ganapathisubramani, 2015; Medjnoun *et al.*, 2018; Yang & Anderson, 2018; Zampiron *et al.*, 2020), the width b (Hwang & Lee, 2018; Medjnoun *et al.*, 2020), and height h of the ridge (Wang & Cheng, 2006; Yang & Anderson, 2018; Awasthi & Anderson, 2018). As the geometry of the spanwise roughness becomes more complicated (Vanderwel & Ganapathisubramani, 2015; Vanderwel *et al.*, 2019; Stroh *et al.*, 2020), the resulting secondary flows are the combination of the aforementioned factors.

Despite the differences, existing studies have demonstrated the importance of spanwise spacing L or roughness width (half-wavelength) S in governing the behaviour of the secondary flows over both strip- and ridge-type roughness. Based on these studies, we categorize spanwise heterogeneous cases into the ‘limiting’ cases, where S is either very small or large compared to δ (figure 2.4a,c), and the ‘intermediate’ cases, where S is comparable to δ (figure 2.4b). The effects of S (relative to

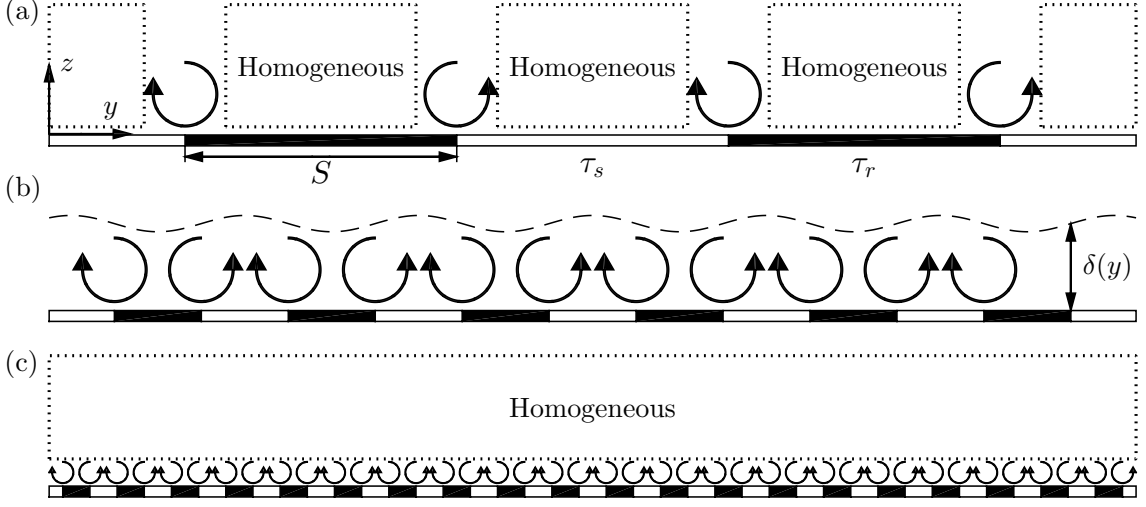


Figure 2.4: Illustration of limiting cases for surface with spanwise heterogeneous roughness, similar to that shown in Chung *et al.* (2018): (a) $S/\delta \gg 1$, (b) $S/\delta \approx 1$, and (c) $S/\delta \ll 1$. The boundary layer thickness is ---; areas marked by show the surface approaching spanwise homogeneity.

δ) on the turbulent flows developing over spanwise heterogeneous roughness surfaces are detailed as follows:

- (a) $S/\delta \gg 1$ (figure 2.4a): here, the spacing and roughness width is sufficiently large that the secondary flow is confined in some areas over the spanwise heterogeneity. Far removed from these areas, the flow behaves according to the local shear stress on the surface below it and the outer layer similarity is recovered. The limits documented in existing studies are: $L/(2\delta) > 0.88$ (Vanderwel & Ganapathisubramani, 2015), $S/\delta > 6$ (Chung *et al.*, 2018), $L/(2\delta) > 1.6$ (Medjnoun *et al.*, 2018), $L/(2\delta) \gtrsim 1$ (Yang & Anderson, 2018), $L/(2\delta) \gtrsim 2$ (Zampiron *et al.*, 2020). In ridge-type roughness, a smaller, weaker counter-rotating secondary flow pair (typically termed ‘tertiary’ flow) emerges between roughness elements (Goldstein & Tuan, 1998; Vanderwel & Ganapathisubramani, 2015; Zampiron *et al.*, 2020). This behaviour is also apparent with the increasing width of the ridges b (Hwang & Lee, 2018). A possibility of coexistence between the meandering secondary flows and LSM/VLSM was explored by Zampiron *et al.* (2020) in this limit ($L \approx 4\delta$).
- (b) $S/\delta \approx 1$ (figure 2.4b): the flow becomes truly ‘heterogeneous’ (Medjnoun *et al.*, 2018) since the secondary flow fills the entirety of space provided within the boundary layer/channel. The strength of the secondary flows is most

significant compared to other two cases (Medjnoun *et al.*, 2018; Yang & Anderson, 2018) and so is the skin friction drag (Chung *et al.*, 2018). As S/δ approaches 1, three phenomena are documented in the previous studies. First, in strip-type roughness, the flow slows down above the smooth strips instead of the rough strips, as opposed to the limit where $S/\delta \gg 1$ (Chung *et al.*, 2018; Xie *et al.*, 2018). Secondly, in ridge-type roughness, the direction of the secondary flow reverses (Yang & Anderson, 2018), which is thought to be result of competing effect between the δ -scaled secondary flows due to spanwise heterogeneity and the roughness-scaled secondary flows induced by the roughness elements (Goldstein & Tuan, 1998). However, Stroh *et al.* (2020) has shown that the same effect can also be achieved by increasing spanwise variation in virtual origin. Thirdly, regardless of the roughness type (ridge, strip, or C-D riblets), an additional outer layer peak emerges in the energy spectrograms of the turbulent fluctuations when $S/\delta \approx 1$, which hints at the potential time-varying behaviour of the secondary flows. The exact streamwise wavelength of the peak reported in these studies differs, approximately between $3 \lesssim \lambda_x/\delta \lesssim 8$ (Nugroho *et al.*, 2013; Awasthi & Anderson, 2018; Medjnoun *et al.*, 2018; Zampiron *et al.*, 2020). The streamwise periodicity extracted from conditional averaging reported in Kevin *et al.* (2019b) also falls within this range. The limits for $S/\delta \approx 1$ documented in existing studies are: $0.79 \leq S/\delta \leq 3.14$ (Chung *et al.*, 2018), $L/(2\delta) \approx 0.5$ (Medjnoun *et al.*, 2018), and $0.5 \lesssim L/(2\delta) \lesssim 1$ (Yang & Anderson, 2018; Anderson, 2019; Zampiron *et al.*, 2020).

- (c) $S/\delta \ll 1$ (figure 2.4c): the size of the secondary flow is limited by the small spacing or roughness width. The effect of the secondary flow on the boundary layer/channel is confined near the surface, where heterogeneity exists, but a large portion of the flow approaches spanwise homogeneous roughness surface. The limits documented in existing studies are: $L/(2\delta) \leq 0.23$ (Vanderwel & Ganapathisubramani, 2015), $S/\delta < 0.4$ (Chung *et al.*, 2018), and $L/(2\delta) \lesssim 0.5$ (Yang & Anderson, 2018; Anderson, 2019). Certain parallels with the study of homogeneous sinusoidal egg-carton type roughness by Chan *et al.* (2018) are also noted, which showed that spanwise homogeneity is attained when $z \gtrsim 0.5\Lambda$, where Λ is the spanwise wavelength of the sinusoids.

Chapter 3

Experimental setup

3.1 Facility

The measurements are performed in an open return boundary layer wind tunnel in the Walter Basset Aerodynamic Laboratory at the University of Melbourne. This facility has been used in several studies of turbulent boundary layers, such as Monty *et al.* (2011); Harun *et al.* (2013); Nugroho *et al.* (2013); Chandran *et al.* (2017); Bai *et al.* (2018); Kevin *et al.* (2019b), and Li *et al.* (2019). A centrifugal fan driven by 19 kW DC motor via a belt drive supplies the airflow into the tunnel. The flow passes through a diffuser, flow-straightening honeycomb, five mesh screens, and a three-dimensional contraction section with 8.9:1 ratio before entering the test section, which has the dimensions of $6.7 \times 0.94 \times 0.38$ m (length $\times$ width $\times$ height) and a freestream turbulence level of $\sqrt{u'u'}/U_\infty \approx 0.3\%$ for all test cases in this study, where $\sqrt{u'u'}$ is the streamwise turbulent intensity in the freestream and U_∞ is the freestream velocity. This is comparable to previous reported values from the same facility (Nugroho *et al.*, 2013) and also to the reported freestream intensities in other similar facilities (Medjnoun *et al.*, 2018). The axis system, illustrated in figure 3.1, is defined as $\mathbf{x} = (x, y, z)$, where x , y , and z refer to the streamwise, spanwise, and wall-normal directions, respectively. The test section is equipped with an adjustable perspex roof (**h** in figure 3.1) to enable zero-pressure gradient adjustment and detachable perspex windows on both sides. The roof has 10 sealable streamwise slots (**i** in figure 3.1), spaced by approximately 0.5 m in the streamwise direction x , which provide access for the two-dimensional (spanwise/wall-normal) traversing probe support. Each slot has the dimension of 250×15 mm, enabling measurements on spanwise/wall-normal (y - z) planes at 10 different streamwise locations. The boundary layer is tripped by a strip of P-40 sandpaper (length 115 mm, **a** in figure 3.1) prior to the inlet of the test section. The evolution of smooth wall



boundary layer with this tripping condition in a different facility but comparable unit Reynolds number ($U_\infty/\nu \approx 1.3 \times 10^6 \text{ m}^{-1}$, where ν is the kinematic viscosity of air) is documented in Marusic *et al.* (2015). The end of the trip is the inlet to the working section and the streamwise datum for the developing turbulent boundary layer ($x = 0$, figure 3.1).

The first 5.6 m-long part of the test section ($0 \leq x \leq 5.6 \text{ m}$) has an adjustable floor ((f) in figure 3.1) so that a surface can be easily slid in and installed, while the section from $x = 5.6 \text{ m}$ to $x = 6.7 \text{ m}$ is comprised of a smooth fixed surface ((g) in figure 3.1). For smooth wall boundary layer experiments, the surface is comprised of a $1600 \times 860 \times 8 \text{ mm}$ and two $2000 \times 860 \times 8 \text{ mm}$ melamine-coated MDF boards ((c), (d), and (e) in figure 3.1). These boards are laid on the floor of the test section, covering the adjustable floor from $0 \leq x \leq 5.6 \text{ m}$, and clamped on both sides using triangular fillets ((b) in figure 3.1) that are installed to prevent the formation of secondary flows at the corners of the working section. For surfaces with heterogeneous roughness (see §3.2 for details), strips of cardboard and sandpaper are aligned and glued to MDF boards of the same dimensions, while the reference rough wall test surface is made using sheets of P-36 sandpaper glued on top of the boards to obtain a homogeneous rough surface.

Some modifications are made in the tunnel to provide optical access for the PIV measurements. A vertical slot is made on one side of the tunnel at $x \approx 4 \text{ m}$ to provide access to the laser sheet ((m) in figure 3.1). On the other side of the tunnel and at roughly the same streamwise location, the perspex windows are replaced by glass ((n) and (o) in figure 3.1) to provide an improved optical access for the cameras (in forward scatter) for the stereoscopic PIV measurements.

For wall-parallel PIV measurements, the PIV cameras are mounted on top of the test section. Three sections of the wind tunnel roof, each sized $200 \times 85 \text{ mm}$, $170 \times 85 \text{ mm}$, and $200 \times 85 \text{ mm}$ ((j), (k), and (l) in figure 3.1), are replaced with glass. These access windows are located at $3.6 \leq x \leq 4 \text{ m}$ and are spaced 50 mm apart in x .

3.1.1 Air properties

Atmospheric pressure of the test facility, P_{atm} , is monitored using a digital barometer (SensorTechnics 144SC0811-BARO, 0.1% accuracy at full scale output). The ambient temperature T in $^\circ\text{C}$ is measured using an Omega DP25-TH Precision

Thermistor-meter (0.2°C accuracy, as provided by the manufacturer). For air density ρ and kinematic viscosity ν calculations, T in °C is converted to K using

$$T(\text{K}) = T(^{\circ}\text{C}) + 273.15 \quad (3.1)$$

A Pitot-static tube is positioned in the freestream at $x \approx 5.5$ m to measure pressure difference between the static and stagnation probe. The Pitot tube is connected to a 10-Torr MKS Baratron pressure transducer (0.12% accuracy) and MKS 670SC signal conditioner. Pressure difference ΔP is converted to freestream velocity U_{∞} using Bernoulli's equation

$$U_{\infty} = \sqrt{\frac{2\Delta P}{\rho}} \quad (3.2)$$

where air density ρ is calculated by assuming the perfect gas relation

$$P_{atm} = \rho RT \quad (3.3)$$

where T is air temperature in K and $R = 289.9 \text{ J kg}^{-1} \text{ K}^{-1}$. Kinematic air viscosity is calculated using Sutherland's law

$$\nu = c \frac{T^{1.5}}{\rho(T + 110.4)} \quad (3.4)$$

where $c = 1.458 \times 10^{-6} \text{ kg m}^{-1} \text{ s}^{-1} \text{ K}^{-0.5}$.

3.1.2 Streamwise pressure gradient

To ensure that the zero-pressure gradient (ZPG) condition in the wind tunnel is satisfied, the streamwise pressure gradient is measured for the reference smooth wall, reference rough wall, and all spanwise heterogeneous surfaces. This is assessed by measuring the pressure difference ΔP between static pressure taps located on the roof of the tunnel and a reference pressure tap at $x = 0$. Pressure difference is recorded at 11 points downstream through the wind tunnel's test section, spaced approximately by 0.5 m in x , using a 1-Torr MKS Baratron pressure transducer (0.12% accuracy) and MKS 670SC signal conditioner. The coefficient of pressure is defined as

$$C_p \equiv \frac{2\Delta P}{\rho U_{\infty}^2} \quad (3.5)$$

Figure 3.2(a) shows the variation of C_p in the streamwise direction x . For the reference smooth wall and all spanwise heterogeneous roughness cases, variation in C_p is within $\pm 1.3\%$ along the length of the test section. This range is close to the

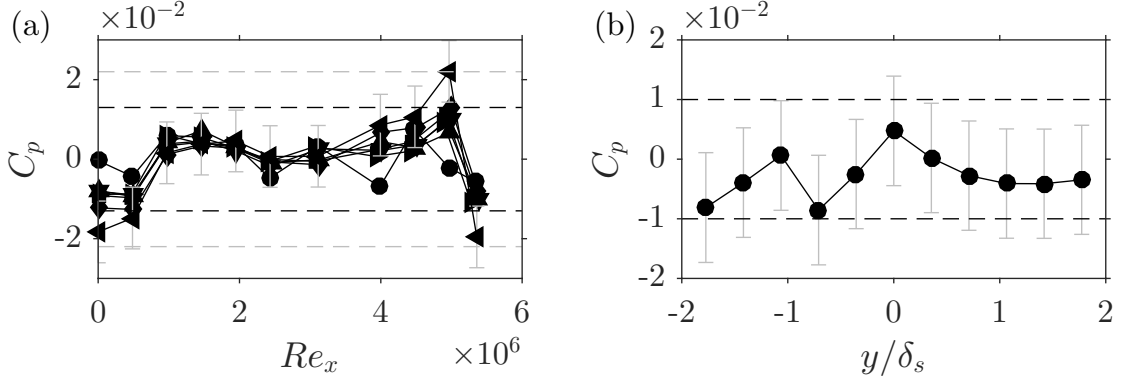


Figure 3.2: (a) Coefficient of pressure C_p along streamwise direction for surface (see table 3.1): reference smooth wall (SW, $\bullet$), SR25 ($\blacklozenge$), SR50 ($\blacksquare$), SR100 ($\blacktriangleleft$), SR160 ($\blacktriangledown$), SR250 ($\blacktriangleright$), and reference rough wall (RW, $\blacktriangleleft$); (b) C_p along spanwise direction for the reference smooth wall (SW) at 4 m downstream from the trip, $Re_x \equiv xU_\infty/\nu \approx 4 \times 10^6$. Dashed line denotes $\pm 1\%$ deviation from the mean value. Error bars (solid grey lines) denote the uncertainties introduced by the Omega DP25-TH Precision Thermistor-meter.

C_p variation ($C_p \pm 0.01$) observed for the zero-pressure-gradient smooth-wall cases conducted in the same facility previously by Harun *et al.* (2013), and also close to the recommended limits by Chauhan *et al.* (2009) for ZPG conditions. Without roof adjustment, the reference rough wall case has a maximum C_p variation of $\pm 2.2\%$. This is expected since the boundary layer grows more aggressively in the reference rough wall case. However, for the purposes of this study, since most measurements are made at $x = 4$ m ($Re_x \approx 4 \times 10^6$), where the variation in C_p for this case is within $\pm 0.78\%$, it is considered that the zero-pressure gradient is achieved.

Figure 3.2(a) also shows the uncertainties of the C_p measurements. Since the measurements are only conducted once for each surface, we cannot calculate the repeatability of the measurements. Thus, the uncertainties are calculated from the rated accuracy of the equipments utilised in the C_p measurements: the barometer (0.1% accuracy), the pressure transducer (0.12% accuracy), and the thermistor-meter (0.2°C accuracy). Of all these equipments, the largest source of uncertainties is introduced by the thermistor-meter (0.77% of the average measured temperature). The error bars (solid grey lines) in figure 3.2(a) show the uncertainties introduced by this thermistor-meter.

3.1.3 Spanwise pressure gradient

The current test facility does not allow direct pressure gradient measurement in the spanwise direction. However, it is possible to monitor the spanwise variation in the freestream velocity at a streamwise location located at $x = 4$ m, where most of the hot-wire measurements detailed in this study are made. Two Pitot-static tubes are employed: the first one is the stationary Pitot tube at $x = 5.5$ m and the second is a traversing Pitot tube positioned at $x = 4$ m, that can be traversed along the spanwise slot on the roof of the tunnel (❶ in figure 3.1). The traversing Pitot tube is positioned at the centreline of the tunnel. Freestream velocity is measured by the tube at 11 spanwise locations spanning approximately $\pm 2\delta_s$ from the tunnel centreline, as far as the length of the slot permits, where δ_s is the 98% smooth wall boundary-layer thickness at $x = 4$ m. The stationary Pitot tube, on the other hand, is off-centred (closer to the side wall). During the installation of both Pitot tubes on the facility, as the traversing (upstream) Pitot tube moves to both ends of the spanwise slot, the mean velocity measured by the stationary (downstream) Pitot tube is monitored and it is ensured that the mean velocity does not change. Thus, there is no wake effect from the traversing Pitot tube on the stationary Pitot tube downstream. Here, the coefficient of pressure C_p is defined as

$$C_p \equiv 1 - \frac{U_1^2}{U_\infty^2} \quad (3.6)$$

where U_1 is the freestream velocity measured by the traversing Pitot-static tube and U_∞ is measured by the stationary device. Figure 3.2(b) shows spanwise C_p variation and, by extension, freestream variation for the reference smooth wall at $x = 4$ m. The deviation is within $\pm 1\%$, which assures that the spanwise freestream variation is negligible. Similar to the streamwise pressure gradient in figure 3.2(a), the error bars of C_p in spanwise pressure gradient measurements are calculated from the uncertainties introduced by the thermistor-meter (0.92% of the average measured temperature).

3.2 Spanwise heterogeneity

Spanwise heterogeneous surfaces are constructed from strips of P-36 grit sandpaper (rough patch) and 2 mm-thick cardboard (smooth patch) of equal width S , as shown in figure 3.3(a). These strips are assembled in a spanwise-alternating pattern, covering the first 5.6 m of the test section after the trip ($0 \leq x \leq 5.6$ m), such that the

surfaces have spanwise periodic rough and smooth patches of wavelength $\Lambda = 2S$. The use of the 2 mm-thick cardboard strips for the smooth patches ensures, as far as possible, that the heterogeneity is imposed by the spanwise shear stress variation between the cardboard and sandpaper strips rather than any mismatch in virtual origin (strip- rather than ridge-type heterogeneity, figure 2.3). The thickness of the cardboard is selected to minimise variation in the virtual origin between the rough and smooth strips. A laser scan of a 60 mm $\times$ 20 mm sample of this surface (figure 3.3c) reveals that the cardboard surface is close to the maximum height of the sand grains, with the average height of the sandpaper grain 0.728 mm lower than the cardboard (see table 3.2 for the complete statistics of the sandpaper). The mean grain size of the sandpaper roughness can be estimated as $k \equiv 6k_{rms} = 1.053$ mm, which corresponds to $\delta_s/k \approx 54$ for the wind tunnel conditions used here, where δ_s is the smooth wall boundary-layer at $x = 4$ m. The definition of roughness height $k \equiv 6k_{rms}$ comes from the standard deviation of the Gaussian fit ($\pm 3\sigma$) of the sandpaper grain size, measured by laser profilometer, and follows the methodology of Squire *et al.* (2016). As a comparison, for the same definition of k and the same type of sandpaper, Squire *et al.* (2016) reported values of $k = 0.902$ and $\delta_s/k \approx 63$. It should be noted that while k of the sandpaper strips can be easily obtained from laser profilometry, the equivalent sand grain roughness k_s and the true virtual origin of the spanwise heterogeneous surfaces are unknown and, in fact, not well defined.

Table 3.1 describes all spanwise heterogeneous roughness (‘SR’) cases in this study. Five surfaces with varying S are constructed for this study to achieve a range of $S/\bar{\delta} = 0.32$ –6.81. Here, S is normalized by the spanwise-averaged boundary-layer thickness $\bar{\delta}$. The variation of boundary-layer thickness $\delta(y)$ across the span is under $\pm 20\%$ of its mean.

The spanwise heterogeneous surfaces are installed in the test section of the wind tunnel. For case SR25, SR50, and SR100, the centerline of the tunnel is aligned with the center of a smooth patch (figure 3.3d). Meanwhile, for the larger S (SR160 and SR250), the centerline of the tunnel is aligned with the roughness interface between a smooth and rough patch (figure 3.3d) to capture at least one-half of the secondary flows with minimal interference from the side walls of the tunnel.

All SR cases, except for SR250-1, are carried out at matched freestream velocity $U_\infty \approx 15 \text{ ms}^{-1}$ and at $x = 4$ m. Due to the limited test-section width (0.94 m), measurements are carried out at $x = 1.85$ m for SR250-1 to achieve $S/\bar{\delta} = 6.81$. As detailed in table 3.1, hot-wire measurements are conducted for all spanwise heterogeneous cases SR and all reference cases (SW and RW), while PIV data (crossplane

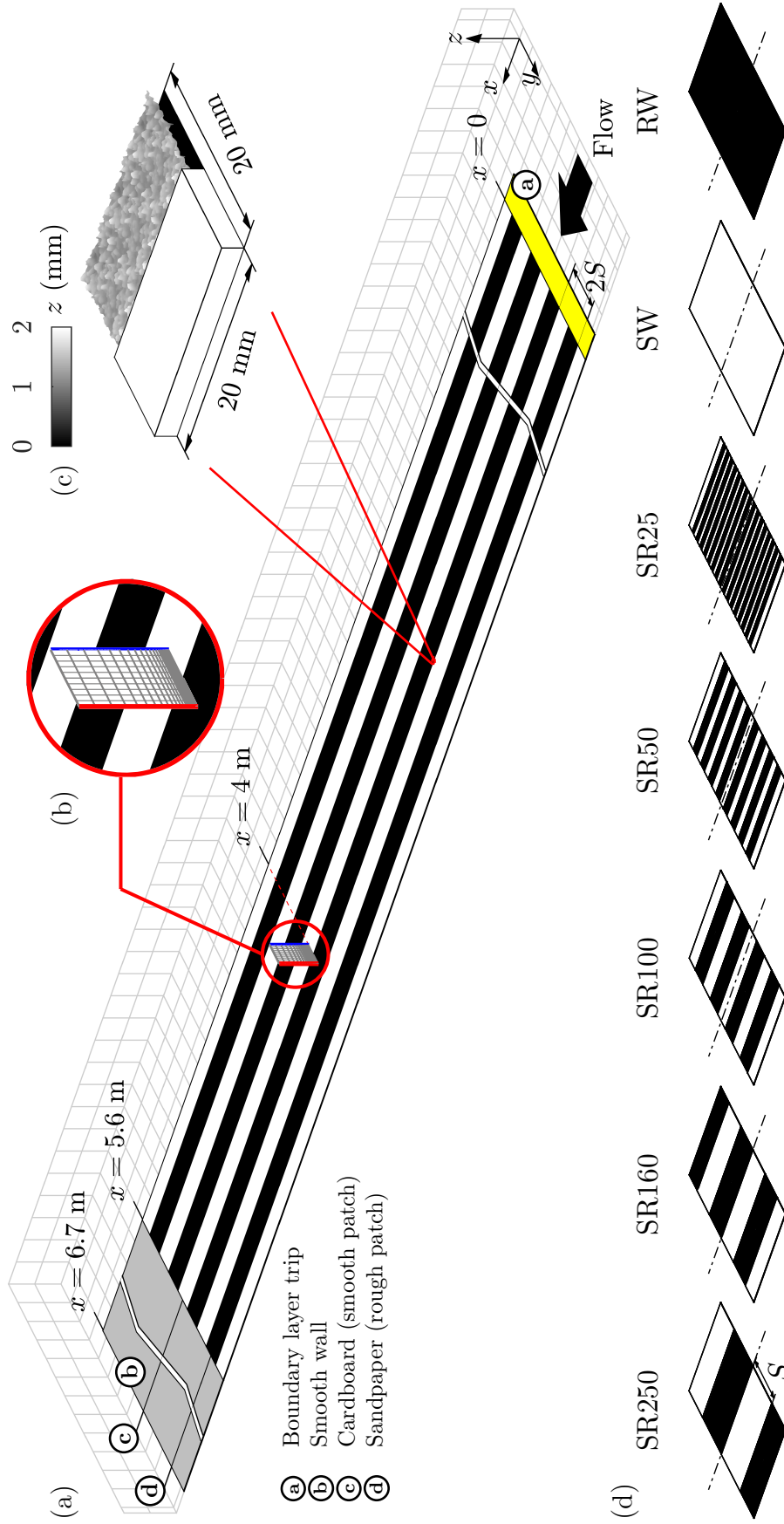


Figure 3.3: Illustration of spanwise heterogeneous roughness: (a) magnified view of $20 \text{ mm} \times 20 \text{ mm}$ sample of the surface. Rough patch profile is obtained from laser profilometry; (b) spanwise heterogeneous roughness surface and measurement plane at 4 m downstream of the trip. The white and black patches correspond to strips of cardboard and sandpaper, respectively; (c) hot-wire anemometry measurement grid. Measurements start at midpoint of the smooth patch (—) and end at midpoint of the adjacent rough patch (---). (d) Illustration of spanwise heterogeneous roughness surfaces, smooth wall and rough wall reference cases. Dash-dot line (---) is the tunnel centerline.

Spanwise heterogeneous roughness											
Case	S (mm)	$\bar{\delta}$ (mm)	$S/\bar{\delta}$	x (m)	U_∞ (ms^{-1})	Re_x ($\times 10^6$)	$Re_{\bar{\delta}}$ ($\times 10^4$)	$Re_{\bar{\theta}}$	HWA	SPIV	WPPIV
SR250-1	250	36.7	6.81	1.85	15.41	1.81	3.58	5070	✓	✗	✗
SR250-2	250	68.8	3.63	4.00	15.37	3.91	6.72	9090	✓	✓	✓
SR160	160	70.0	2.28	4.00	15.28	3.91	6.84	9200	✓	✓	✓
SR100	100	73.9	1.35	4.00	15.23	3.94	7.27	10150	✓	✓	✓
SR50	50	80.7	0.62	4.00	15.27	3.92	7.92	10220	✓	✓	✓
SR25	25	77.6	0.32	4.00	15.51	3.86	7.49	9990	✓	✓	✓
Reference smooth wall											
Case	S (mm)	δ_s (mm)	S/δ_s	x (m)	U_∞ (ms^{-1})	Re_x ($\times 10^6$)	Re_{δ_s} ($\times 10^4$)	Re_{θ_s}	HWA	SPIV	WPPIV
SW-1	—	30.6	—	1.85	15.08	1.81	2.98	3930	✓	✗	✗
SW-2	—	56.4	—	4.00	15.18	3.99	5.62	7320	✓	✓	✓
Reference rough wall											
Case	S (mm)	δ_r (mm)	S/δ_r	x (m)	U_∞ (ms^{-1})	Re_x ($\times 10^6$)	Re_{δ_r} ($\times 10^4$)	Re_{θ_r}	HWA	SPIV	WPPIV
RW-1	—	43.5	—	1.85	15.31	1.79	4.20	6240	✓	✗	✗
RW-2	—	80.0	—	4.00	15.36	3.86	7.72	11050	✓	✗	✗

Table 3.1: Summary of spanwise heterogeneous roughness cases and the reference smooth- and rough-wall cases. The statistics are obtained from hot-wire anemometry (HWA): $\bar{\delta}$ is the spanwise-averaged 98% boundary-layer thickness of the surface with spanwise heterogeneity, while δ_s and δ_r are the 98% boundary-layer thickness of the reference smooth- and rough-wall cases, respectively. $\bar{\theta}$ is the spanwise-averaged momentum thickness of the spanwise heterogeneous surfaces, while θ_s and θ_r are the momentum thickness of the reference smooth- and rough-wall cases, respectively. Reynolds number definitions: $Re_x \equiv xU_\infty/\nu$, $Re_\delta \equiv \delta U_\infty/\nu$, and $Re_\theta \equiv \theta U_\infty/\nu$. The last three columns show the availability of data from all measurement methods: HWA, stereoscopic PIV (SPIV), and wall-parallel PIV (WPPIV).

Parameter	Value	Units	Formula
$\Delta\bar{h}$	0.728	mm	–
k	1.053	mm	$6\sqrt{\overline{h'^2}}$
k_a	0.136	mm	$ \overline{h'} $
k_p	2.032	mm	$\max h' - \min h'$
k_{rms}	0.176	mm	$\sqrt{\overline{h'^2}}$
k_{sk}	0.743	–	$\overline{h'^3}/h_{rms}^3$
k_{ku}	4.116	–	$\overline{h'^4}/h_{rms}^4$
ES_x	0.538	–	$ \frac{dh'}{dx} $

Table 3.2: Roughness parameter of the sandpaper strips: $\Delta\bar{h}$ is the average height difference between the cardboard and the sandpaper and k is the sandpaper roughness size, where h' is the deviation about the mean height $h' = h - \bar{h}$.

SPIV and wall-parallel PIV) are available for SR cases at $x = 4$ m and SW-2. It should be noted that the single-normal hot-wire measurements only provide the time-resolved u velocity component, while the PIV in both crossplane and wall-parallel plane are non-time-resolved. All three velocity components u , v , and w are available from SPIV measurements, while the wall-parallel PIV measurements only provide u and v components.

3.3 Reference cases

Two reference cases for spanwise-homogeneous conditions are included in table 3.1 for comparison to SR cases. The first is the reference smooth-wall case (‘SW’) and the second is the reference rough-wall case (‘RW’), where the surface is constructed entirely from a sheet of P-36 grit sandpaper (the same sandpaper used to form the roughness strips for the SR cases). HWA measurements for these reference cases are taken at similar $Re_x \equiv xU_\infty/\nu$ to their counterparts, that is, SW-1 and RW-1 for case SR250-1 at $x = 1.85$ m, SW-2 and RW-2 for all other cases at $x = 4$ m.

For the SW cases, the wall friction velocity U_τ is determined by fitting the mean velocity profile from HWA measurements to the composite profile of Chauhan *et al.* (2009). The buffer region (between the viscous sublayer and the overlap region) is modeled using the Musker (1979) profile. In the outer region, the composite profile in equation (2.8) is used, with $z^+ \equiv (z - \epsilon)U_\tau/\nu$ and $\mathcal{W}$ is the modified wake function (Chauhan *et al.*, 2009). ϵ is the wall correction due to error in measuring the wire distance from the wall. Here, $\kappa = 0.384$ and $A = 4.17$, following Chauhan *et al.*

Case	U_τ (ms ⁻¹)	C_f ($\times 10^{-3}$)	δ_s^+ (δ_r^+)	l^+	ΔU^+	k_s (mm)	k_s^+	Symbol
SW-1	0.58	3.0	1150	38	—	—	—	○
SW-2	0.55	2.6	2030	36	—	—	—	●
RW-1	0.77	5.1	(2130)	49	7.6	2.01	98	△
RW-2	0.73	4.5	(3670)	46	7.4	1.99	91	▲

Table 3.3: Summary of reference smooth- and rough-wall cases in viscous-scaled units. δ_s and δ_r are the 98% boundary-layer thickness for the smooth- and rough-wall reference cases, respectively, and $\delta_s^+ \equiv \delta_s U_\tau / \nu$, $\delta_r^+ \equiv \delta_r U_\tau / \nu$. $l^+ \equiv l U_\tau / \nu$ is the viscous-scaled length of the etched portion of the hot wire. Coefficient of friction is defined as $C_f \equiv 2(U_\tau / U_\infty)^2$.

(2009). The viscous-scaled parameters for the reference cases are summarised in table 3.3.

For the RW cases, the profile is shifted from equation (2.8) according to the Hama roughness function (Hama, 1954) in equation (2.12), where A_{FR} is 8.5, following Nikuradse (1933). As shown in table 3.3, $k_s \approx 2$ mm for both RW-1 and RW-2, close to that obtained by Squire *et al.* (2016) for the same P-36 sandpaper surface. This value translates to 98 and 91 viscous units for RW-1 and RW-2, respectively.

HWA-measured boundary layer profiles for these reference cases are shown in figure 3.4. Figure 3.4(a) shows that U^+ for the smooth-wall reference cases SW-1 and SW-2 collapse to the log law $\kappa^{-1} \log z^+ + 4.17$, while U^+ for the rough-wall cases RW-1 and RW-2 are shifted by $\Delta U^+ \approx 7.5$ from this line. Viscous-scaled turbulence intensities $\overline{u'u'}^+$ are shown in figure 3.4(b). Reasonable collapse is observed for SW-1 and SW-2 to the DNS results of Sillero *et al.* (2014) at matched $Re_{\theta_s} \equiv \theta_s U_\infty / \nu$, where θ_s is the momentum thickness of the reference smooth wall cases. It should be noted that $\overline{u'u'}$ for the DNS results have been modified to account for the expected attenuation due to the hot-wire spatial resolution $l^+ \equiv l U_\tau / \nu$ following the method of Lee *et al.* (2016), with the missing energy, which is assumed to be function of l^+ , subtracted from the original DNS statistics. A measure of outer-layer similarity for these reference cases is given in figure 3.4(c), where velocity defect profiles $U_\infty^+ - U^+$ for all cases collapse to the line $2.3 - \kappa^{-1} \log(z/\delta_{99})$, consistent with Marusic *et al.* (2015). For both reference cases, skin friction coefficient $C_f \equiv 2(U_\tau / U_\infty)^2$ and shape factor $H \equiv \delta_s^* / \theta_s$ are shown in figure 3.4(d) as a function of Re_x . C_f for both cases collapses to the Schlichting's fit $C_f = (2 \log_{10} Re_x - 0.65)^{-7/3}$, while H follows the trend line $H = (1 - 7.1 \sqrt{2/C_f})^{-1}$, as given in Chauhan *et al.* (2009). Together these results validate that for the homogeneous surface conditions, the facility is behaving in a manner that is entirely consistent with the prior literature.

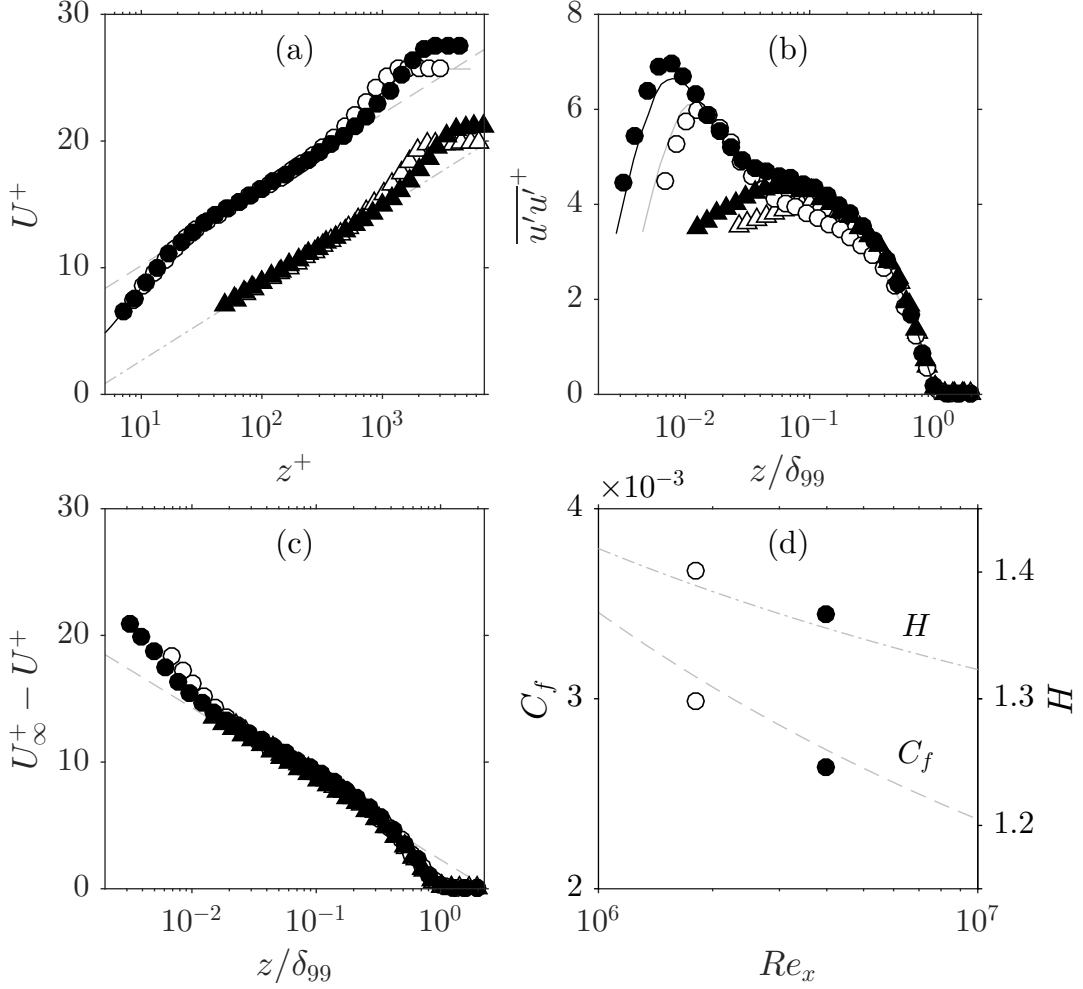


Figure 3.4: Symbols show boundary layer profiles from hot-wire measurements of reference smooth and rough wall cases, SW-1, SW-2, RW-1 and RW-2 (see table 3.3 for legend). (a) Mean streamwise velocity profile U^+ . DNS (Sillero *et al.*, 2014): SW-1 (—) and SW-2 (—). Dashed line (---) is $\kappa^{-1} \log z^+ + 4.17$, dash-dot line (— · —) is $\kappa^{-1} \log z^+ + 4.17 - \Delta U^+$. (b) Turbulence intensity $\overline{u'u'}^+$; DNS, filtered according to the spatial resolution l^+ for SW-1 (—) and SW-2 (—). (c) Velocity defect profile $U_\infty^+ - U^+$; ---, $2.3 - \kappa^{-1} \log(z/\delta_{99})$, where δ_{99} is the 99% boundary-layer thickness. (d) Skin friction coefficient $C_f \equiv 2(U_\tau/U_\infty)^2$ and shape factor $H \equiv \delta_s^*/\theta_s$ as a function of Re_x , where δ_s^* is the displacement thickness of the reference smooth wall boundary layer profile. Dashed line (---) is Schlichting's fit $C_f = (2 \log_{10} Re_x - 0.65)^{-7/3}$ and dash-dot line (— · —) is $H = (1 - 7.1\sqrt{2/C_f})^{-1}$ (Chauhan *et al.*, 2009). Data points are downsampled for clarity.

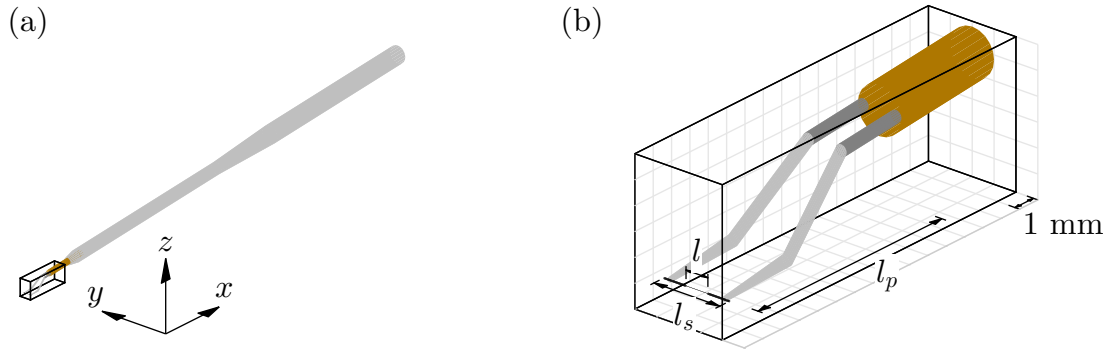


Figure 3.5: (a) Illustration of a modified Dantec 55P05 single boundary layer probe and probe support. Black box shows (b) an etched $5\text{ }\mu\text{m}$ hot-wire probe, where l is the length of the etched wire, l_s is the spacing between the prongs, and l_p is the length of the prongs. Figures are drawn to scale.

3.4 Hot-wire anemometry (HWA)

3.4.1 Hot-wire probe

Two sets of hot-wire measurements of streamwise velocity are performed for each case of spanwise heterogeneity in table 3.1. The first is a wall-normal traverse at 40 logarithmically spaced z locations over the centre of the smooth patch to obtain the near-wall profile. The second is a measurement over a spanwise-wall-normal grid (figure 3.3c), in which profiles are obtained at 11 (cases SR25–SR160) and 15 (cases SR250-1 and SR250-2) spanwise locations over one half-wavelength S . The grid spans from the midpoint of the smooth patch to the midpoint of the adjacent rough patch (figure 3.3c). At each spanwise location, the boundary layer is traversed at 30 logarithmically spaced wall-normal locations. For both smooth- and rough-wall reference cases (SW and RW), only the wall-normal traverse is conducted at the spanwise centre of the test section.

All hot-wire measurements are carried out using a modified Dantec 55P05 single-sensor boundary layer probe (figure 3.5a) attached to a Dantec 55H21 probe support. It should be noted that the single-sensor wire not only captures the streamwise velocity component u , but also parts of v and w . A discussion about hot-wire sensitivity to these velocity components is given in appendix A.1.

The hot-wire sensor is made by soldering a Wollaston wire with a $5\text{ }\mu\text{m}$ diameter platinum core to the tip of the modified Dantec 55P05 probe. A 1 mm-long portion of the wire is etched with nitric acid solution to remove the silver coating (figure 3.5b), yielding the recommended wire length-to-diameter ratio of 200 (Ligrani & Bradshaw, 1987). The viscous-scaled etched lengths l^+ for the reference cases are shown in

table 3.3. The total resistance of the etched hot-wire probe is $R_{hw} \approx 7\Omega$ (which includes the prong and lead resistance). The hot-wire overheat resistance R_o is calculated using

$$R_o = 1.8(R_{hw} - R_{cable}) + R_{cable} \quad (3.7)$$

where R_{cable} , the resistance of a 5 m-long BNC cable connecting the hot-wire probe to the anemometer, is measured using a shorting probe. Nominally, for the experiments reported here, $R_{cable} \approx 1\Omega$. A newly etched hot-wire probe is annealed overnight (> 8 hours) before the first measurement to minimize sensor drift.

3.4.2 Constant Temperature Anemometer

The anemometer used in this study is an in-house constant temperature anemometer (MUCTA) with a fixed overheat ratio of 1.8. Frequency response of the hot-wire is tuned using a square-wave test at one-third of the freestream velocity of the measurement ($\approx 5 \text{ ms}^{-1}$), so that the peak-to-peak frequency of the second order response is 8–9 kHz for the input square wave signal. A gain-offset amplifier is adjusted to match the output voltage from the MUCTA between -8 and 8 V, corresponding to zero tunnel velocity and $U_\infty \approx 15 \text{ ms}^{-1}$, respectively. The output signal from the MUCTA is sampled at 30 kHz and low-pass filtered at the cut-off frequency of $f = 15 \text{ kHz}$. This yields the viscous-scaled time $t^+ \equiv \Delta t U_\tau^2 / \nu = 1.34$ for the reference smooth wall case SW-2, where the sampling interval $\Delta t = 1/f$. The obtained t^+ is within the threshold $t^+ \leq 3$ set by Hutchins *et al.* (2009) to prevent an additional HWA-measured energy attenuation. Low-pass filtered data are digitised using a 16-bit Data Translation DT9836 data acquisition (DAQ) board which is controlled via MATLAB. Sampling time is set to maintain turnover $tU_\infty/\delta \geq 20000$ (Hutchins *et al.*, 2009) and thus allow the convergence of turbulence energy spectra.

3.4.3 Hot-wire calibration

Several possible causes of hot-wire calibration drift have been identified: changes in the ambient condition of the flow such as temperature (Comte-Bellot, 1976) and humidity (Durst *et al.*, 1996), wire degradation due to incomplete cleaning (Willmarth & Bogar, 1977), electro-migration (Willmarth & Sharma, 1984), contamination (dust particles) in the flow (Collis, 1952), and wire fouling (Wyatt, 1953). Of all these causes, temperature and humidity are easily measurable and thus they can be corrected for. To compensate for hot-wire drift due to temperature, the hot-wire probe is calibrated in the freestream prior to and after each measurement. The

relationship between output voltage E and freestream velocity U_∞ is obtained in each calibration. The calibration data are obtained at 15 different tunnel settings from $0 \leq U_\infty \lesssim 18 \text{ ms}^{-1}$. A third order polynomial is fitted to the calibration data,

$$U_\infty = c_1 E^3 + c_2 E^2 + c_3 E + c_4 \quad (3.8)$$

where c_n , $n = 1, \dots, 4$ are the calibration constants. For the single wall-normal traverse measurements, which typically take 1–2 hours, time-based interpolation between pre- and post-calibration curve is used to compensate for a slight sensor drift, so that the first measurement point falls on the pre-calibration curve and the last measurement point on the post-calibration curve. Figure 3.6(a) shows typical pre- and post-calibration curve for a wall-normal traverse. The sensor drift is $\approx 43 \text{ mV}$ ($\approx 0.6\%$ of the freestream voltage). Vertical and horizontal error bars in the inset of figure 3.6(a) show the uncertainties of E recorded by the hot-wire probe and U_∞ recorded by the Pitot tube during pre- and post-calibration (see appendix A.2 for details). For the wall-normal measurement shown in the inset of figure 3.6(a), the uncertainty of E at $U_\infty \approx 15 \text{ ms}^{-1}$ is $\pm 14 \text{ mV}$ ($\pm 0.2\%$ of U_∞). For the Pitot tube, the uncertainty is $\pm 0.034 \text{ ms}^{-1}$ ($\pm 0.2\%$ of U_∞).

For the spanwise wall-normal grid measurements, which take about 11–14 hours, sensor drift is accounted for using a modified version of the methodology suggested by Talluru *et al.* (2014). This method accounts for all possible causes of drift, but without identifying these causes individually. Here, the probe is traversed to the freestream after each wall-normal measurement (see the uppermost points of the measurement grid in figure 3.3c) and the hot-wire voltage at the freestream E_∞ is recorded. The black symbols in the inset of figure 3.6(a) shows E_∞ recorded throughout an example spanwise wall-normal measurement as a function of the freestream velocity U_∞ , measured by the Pitot tube at the same time. Sensor drift ΔE is measured as the difference between each E_∞ and the pre-calibration curve. At each wall-normal measurement point, ΔE is linearly interpolated and used to recalibrate the sensor, that is, the pre-calibration curve is shifted by the interpolated ΔE to give an intermediate calibration curve for each particular measurement point (see Talluru *et al.*, 2014 for details). Recorded ΔE from one of the longest measurements (approximately 14 hours) are shown in figure 3.6(b). The average drift in this measurement is $8 \pm 42 \text{ mV}$ ($\approx 0.1 \pm 0.5\%$ of the freestream voltage). Only the pre-calibration curve is used in this method for sensor correction. However, a post-calibration (figure 3.6a) is still performed to ensure that the overall sensor drift at the start and the end of the measurement remains small. In figure 3.6(b), red

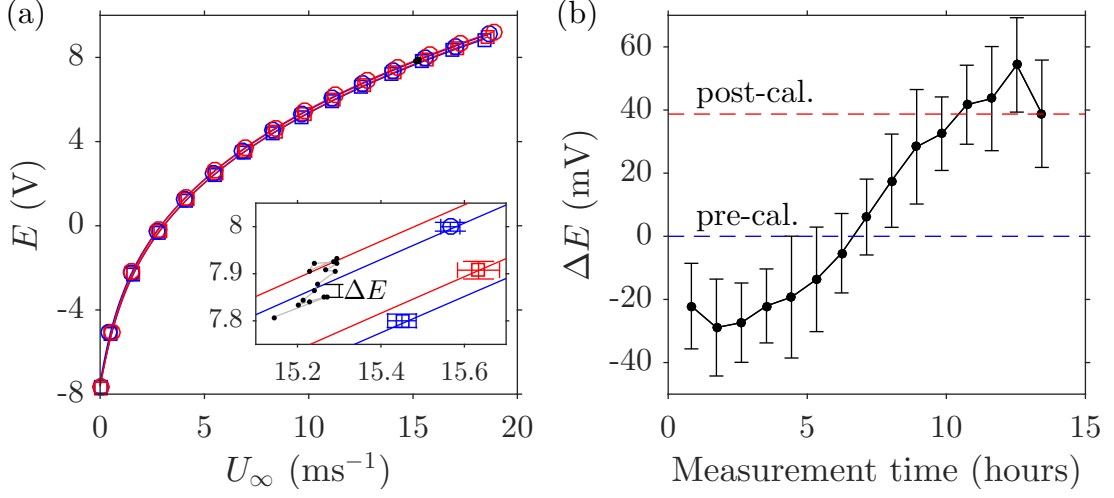


Figure 3.6: (a) Typical hot-wire calibration curves: wall-normal traverse measurement, pre- ($\square$) and post-calibration ($\square$), and spanwise wall-normal grid measurement, pre- ($\circ$) and post-calibration ($\circ$). Blue (—) and red solid lines (—) are the fitted 3rd-order polynomials. Inset: hot-wire voltage at the freestream E_∞ ($\bullet$). Grey solid line (—) is the time trajectory of E_∞ . (b) Voltage drift from the pre-calibration curve ΔE , recorded throughout the measurement. In (a), vertical and horizontal error bars show the uncertainties of hot-wire voltage output E and U_∞ measured by the Pitot tube due to freestream intensities, respectively, while in (b), the error bars show the uncertainties of E_∞ due to freestream intensities (see appendix A.2 for details).

dashed line (---) shows the sensor drifting by 39 mV ($\approx 0.5\%$ of the freestream voltage) from the pre-calibration curve at the end of the measurement. The error bars in figure 3.6(b) show the uncertainty of E_∞ (see appendix A.2 for details). When averaged throughout the measurement, the uncertainty is ± 15 mV ($\pm 0.2\%$ of U_∞).

3.4.4 Traversing system

A two-axis traversing system is employed in the facility to traverse the hot-wire probe in the spanwise and wall-normal direction. The system is built using two Vexta PKK266M-03A stepper motors and a Velmex VXM-2 controller. To ensure the precision of the probe's movement, the wall-normal traverse is equipped with a Renishaw RGH24 linear optical encoder (0.5 μm resolution), while the spanwise traverse is equipped with a Renishaw LM13 linear magnetic encoder (5 μm resolution). During hot-wire measurements, both motor controllers and encoders are controlled by a computer.

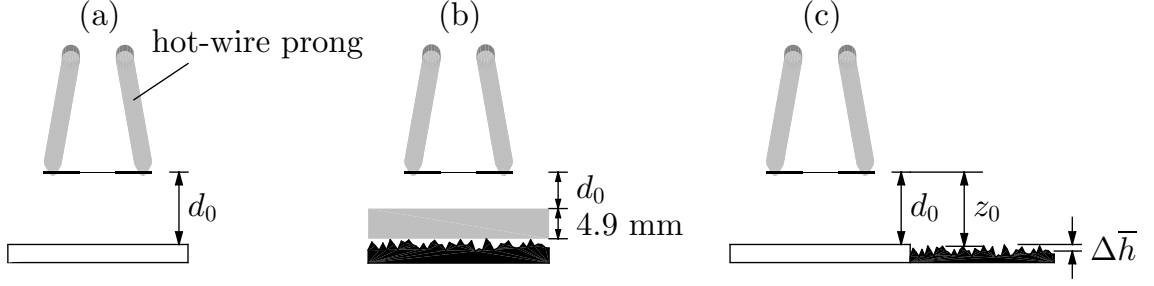


Figure 3.7: Illustration of the initial wall-normal location for: (a) reference smooth wall, (b) reference rough wall, and (c) spanwise heterogeneous roughness cases. In (c), $z_0 = d_0 + \Delta \bar{h} - 3k_{rms}$, where $\Delta \bar{h}$ is the average height difference between the smooth and rough strips and k is the sandpaper grain size. Figures are not drawn to scale.

The two-axis traverse sits on a carriage supported by 4 wheels, which runs on rails above the roof of the working section and can be positioned at any streamwise location on the test section. A cylindrical sting is attached to the wall-normal traverse which enters the working section via sealed spanwise slots in the ceiling (① in figure 3.1). The hot-wire probe and probe support are mounted on this traversing cylindrical sting.

3.4.5 Initial wall location

All measurements start in the near-wall region and end in the freestream as the hot-wire probe is traversed upward. To measure the initial wall-normal location z_0 , a digital z -axis vertical measuring displacement microscope (Titan Tool ZDM-1) is first focused on the wire and then shifted to focus on the surface. The measured displacement d_0 is the distance between the wire and the surface. For the reference smooth wall cases, the initial wall location of the wire is $z_0 = d_0$ (figure 3.7a), where $z_0 < 0.5$ mm. For reference rough wall cases, a plate with 4.9 mm thickness is placed between the wire and the sandpaper sheet (figure 3.7b). The initial wall location (in mm) is $z_0 = d_0 + 4.9$. The wire is then traversed downward so that $z_0 \approx 0.8$ mm above the surface.

For spanwise heterogeneity cases, the initial wall location is measured above the smooth patch. Since the virtual origin is unknown, the “true” initial wall location of the rough patch is estimated by d_0 measured above the smooth patch, the average height difference between the smooth and rough patch $\Delta \bar{h}$ (figure 3.7c), and the roughness size k measured by the laser scanner. Here, initial wall location above the rough strips is $z_0 = d_0 + \Delta \bar{h} - 3k_{rms}$.

Chapter 4

Particle Image Velocimetry (PIV)

4.1 Crossplane Stereoscopic PIV (SPIV)

4.1.1 Experimental setup

Non-time resolved cross-stream plane SPIV experiments are performed in the same zero pressure gradient, open return boundary layer wind tunnel in the Walter Basset Aerodynamic Laboratory at the University of Melbourne. The details of this facility is given in section 3.1. The cross-stream measurement plane is located at $x = 4$ m. The test surfaces are painted black with matte spray paint about the location of measurement plane to reduce background reflection in the PIV images. As shown in table 3.1, SPIV data are available for all spanwise heterogeneous cases and the reference smooth wall, except for cases located at $x = 1.85$ m (SR250-1 and SW-1) due to limited optical access. SPIV measurements are not conducted on the reference rough wall (RW-1 and RW-2).

The setup and parameters of the experiments are detailed in figure 4.1 and table 4.1. The measurement plane is illuminated by a laser sheet generated from an InnoLas SpitLight Compact PIV 400 dual pulse Nd:YAG laser and an optical configuration which consists of mirrors and cylindrical lenses (figure 4.1a). The laser sheet enters the test section through access window **(m)** in figure 3.1. For this setup, the in-plane components are y (spanwise) and z (wall-normal), and the out-of-plane component is x (streamwise), as shown in figure 4.1(a).

The flow is seeded with polyamide particles (Rosco fog fluid) generated by a Safex fog generator and the freestream velocity of the tunnel is maintained at $U_\infty \approx 15$ ms^{-1} for all cases. The particle has the diameter of $d_p = 1\text{--}2$ μm . The timescale of the particle τ_p is calculated using

$$\tau_p = \frac{d_p^2}{18} \frac{|\rho_p - \rho|}{\rho\nu} \quad (4.1)$$

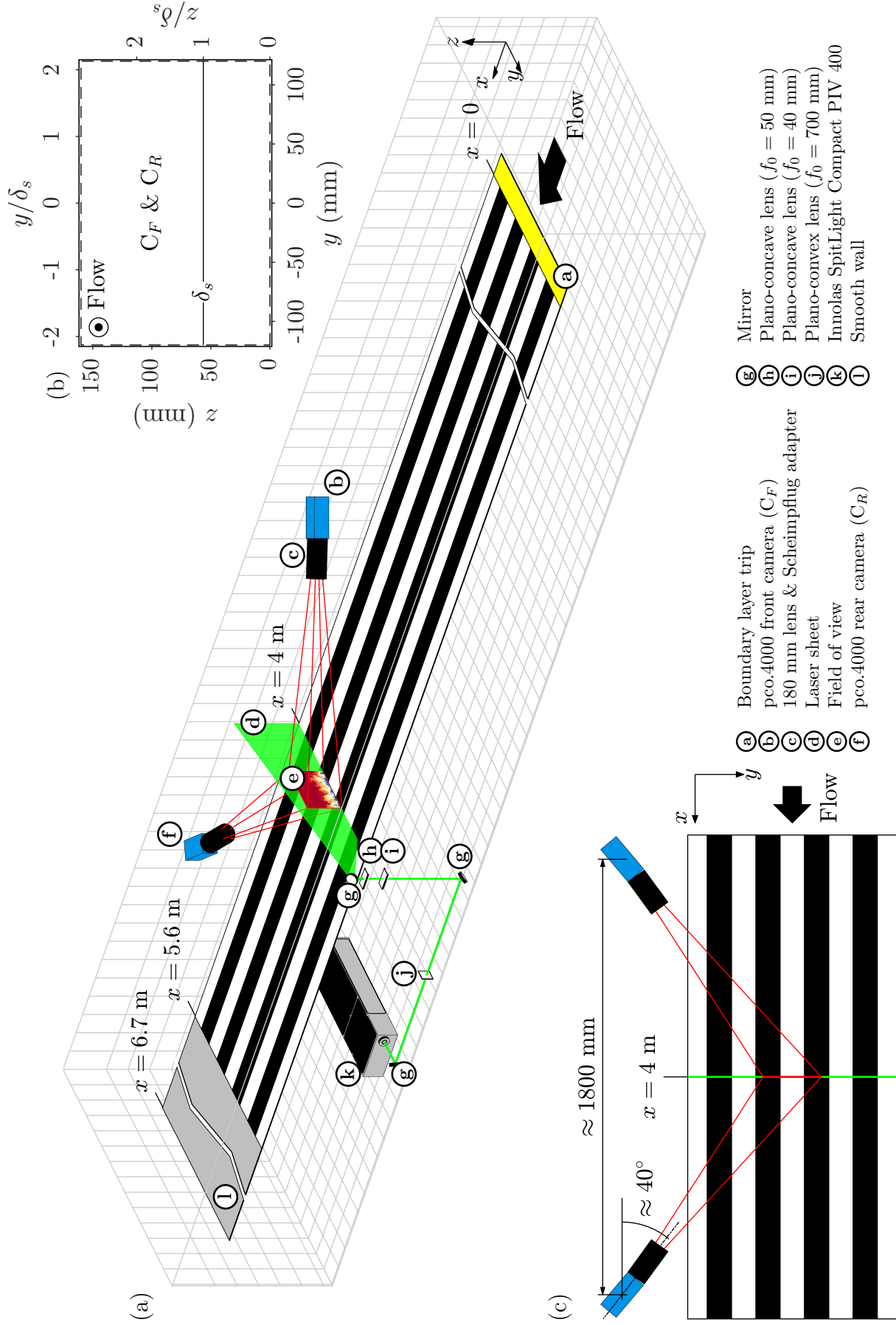


Figure 4.1: Illustration of cross-plane SPIV setup over a spanwise heterogeneous roughness surface: (a) isometric view, (b) the field of view, and (c) top view.

Time-resolved	No
Flow medium	air
Freestream velocity	$U_\infty \approx 15 \text{ ms}^{-1}$
Seeding	polyamide particles
Particle size	$\approx 1\text{--}2 \text{ }\mu\text{m}$
Laser sheet thickness	$\approx 4.5 \text{ mm}$
Sensor resolution	$4008 \times 2672 \text{ pixels}$
Aperture number	f/8
Camera angle	$\pm 40^\circ$
Image pixel size	$\approx 60 \text{ }\mu\text{m pixel}^{-1}$
Field of view	$240 \text{ mm} \times 160 \text{ mm} (\approx 4\delta_s \times 3\delta_s)$
Number of image pairs	4800 per case (1200 for SW-2)
Acquisition rate	1 Hz ($tU_\infty/\delta_s = 270$)
Time between images	$\Delta t = 85 \mu\text{s}$ ($t^+ = 1.7$) [†]
Interrogation window	$32 \times 32 \text{ pixels}$
Final resolution $\Delta x^+ \times \Delta y^+ \times \Delta z^+$	$165 \times 70 \times 70$ [†]

[†] for the reference smooth wall SW-2 ($\delta_s^+ \approx 2000$)

Table 4.1: Parameters of the SPIV experiments.

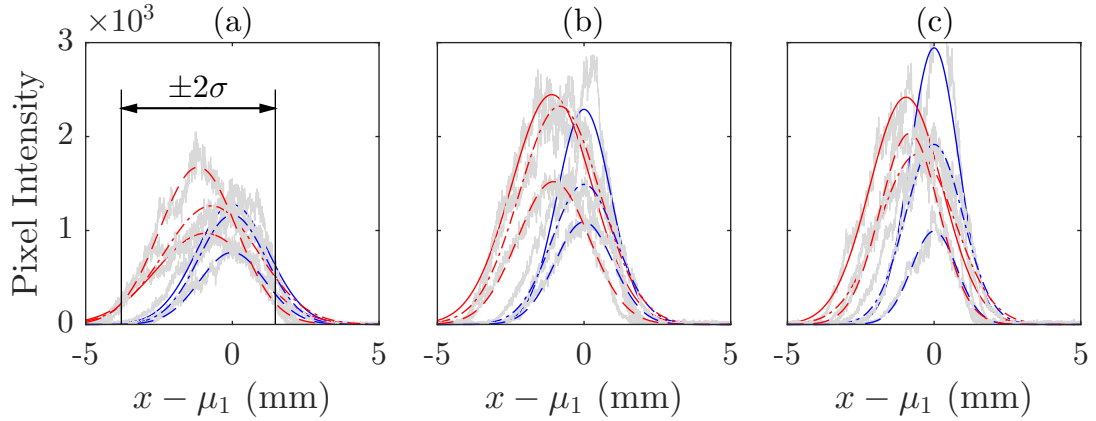


Figure 4.2: Crossplane SPIV laser sheet intensity profile (—) for Innolas SpitLight Compact PIV 4000. The overlap between the Gaussian fit of Head 1 (—) and Head 2 (—) is illustrated in the figures, where μ_1 is the mean streamwise location x of Head 1. Laser heads are profiled in three spanwise locations: (a) $y = -100 \text{ mm}$ (the camera side), (b) $y = 0$ (centerline of the tunnel), and (c) $y = 100 \text{ mm}$ (laser sheet side). At each spanwise location, profiles are taken at 3 wall-normal locations: $z = 40 \text{ mm}$ (— and —), $z = 80 \text{ mm}$ (--- and ---), and $z = 120 \text{ mm}$ (--- and ---).

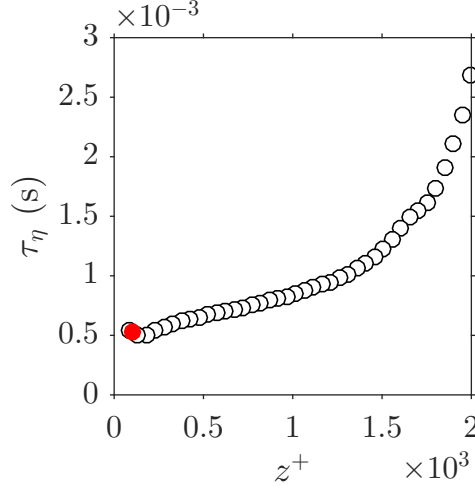


Figure 4.3: Estimated Kolmogorov timescale τ_η for the reference smooth-wall case SW-2 ($\delta_s^+ = 2030$) as a function of z^+ . Red dot ($\bullet$) shows the estimated τ_η at $z^+ = 100$. Data is obtained from SPIV measurements, downsampled for clarity.

where ρ_p is the density of the particle, ρ and ν are the density and kinematic viscosity of the fluid (air), respectively. The details of the measurements of air properties are given in §3, which yield $\rho = 1.2 \text{ kg m}^{-3}$ and $\nu = 1.5 \times 10^{-5} \text{ m}^2\text{s}^{-1}$. The density of the particle is approximated by the rated vapour density of Rosco fog fluid, 2.9 (relative to air). This gives $\tau_p = 7.04 \times 10^{-9} \text{ s}$ for $d_p = 1 \text{ }\mu\text{m}$. τ_p is then compared to the Kolmogorov timescale of the flow $\tau_\eta \equiv \sqrt{\nu/\varepsilon}$, where ε is the turbulent dissipation. Due to the spatial resolution of PIV measurements, the magnitude of ε cannot be accurately estimated. However, it is still possible to calculate a very rough estimation of ε for the comparison between particle and flow timescales (see §6 for more details about ε estimation). For channel flow at low Reynolds number $\delta^+ = 180$, the production-dissipation ratio in the log region ($z^+ = 98$) is $\mathcal{P}/\varepsilon = 0.91$ (Kim *et al.*, 1987; Pope, 2000). Hence, ε can be roughly estimated by the production term $\mathcal{P}$. Assuming that the mean velocity gradients are negligible except for $\partial(\cdot)/\partial z$,

$$\varepsilon_{est} \approx \frac{\partial U}{\partial z} \overline{u'w'} \quad (4.2)$$

Figure 4.3 shows the estimated Kolmogorov timescale τ_η of the reference smooth-wall case SW-2 ($\delta_s = 2030$). At $z^+ = 100$, $\tau_\eta = 5 \times 10^{-4} \text{ s}$. Since SPIV data are only available at $z^+ \gtrsim 100$ (see §4.3 for more details), the timescale of the seeding particle τ_p is still smaller than smallest possible τ_η by the $O(10^{-5})$.

The thickness of the laser sheet is approximated using laser sheet profiling method described in Grayson *et al.* (2018). A Nikon D5600 DSLR camera and

a series of ND filters (without lens) are placed inside the tunnel to capture the laser sheet. Profiles are taken for both laser heads, Head 1 (“Master”) and Head 2 (“Slave”), at three different spanwise locations: centerline of the tunnel ($y = 0$), 100 mm to the left from the tunnel centerline (camera side, $y = -100$ mm), and 100 mm to the right from the tunnel centerline (laser side, $y = 100$ mm). At each spanwise location, the profiles are taken at 3 wall-normal locations inside the FOV, $z = 40$, 80, and 120 mm above the wall. Mirror and crystal alignment inside the laser are adjusted beforehand to ensure an acceptable overlap between Head 1 and Head 2, such that the loss-of-correlation factor $F_O \geq 0.8$ (Scharnowski *et al.*, 2017; Grayson *et al.*, 2018). Laser sheet intensity profiles at all locations are shown in figure 4.2 (a–c). The pixel intensity profiles are fitted into a Gaussian curve and the thickness of the laser sheet is approximated by the standard deviation of the Gaussian fit, $\pm 2\sigma$ or ≈ 4.5 mm. In viscous unit this value scales to $\Delta x^+ \approx 165$ for case SW-2.

Two pco.4000 CCD cameras (14-bit, 4008×2672 pixels), each equipped with a Tamron macro lens (telephoto lens with 180 mm focal length, f/8 aperture), are positioned upstream and downstream of the measurement plane to capture the PIV images, as shown in figure 4.1. The lens distortion is accounted for in the calibration algorithm (see §4.1.2 for details). The cameras are arranged in forward scatter mode to maximise the particle light intensity, facing access windows (n) and (o) in figure 3.1, and are angled about $\pm 40^\circ$ with respect to the freestream. A Scheimpflug adapter is fitted between each camera and lens to compensate for the oblique imaging condition (Adrian & Westerweel, 2011). The current setup gives approximately $60 \mu\text{m pixel}^{-1}$ resolution, which corresponds to a field of view (FOV) of approximately $4\delta_s \times 3\delta_s$, where δ_s is the 98% boundary layer thickness of the reference smooth wall case SW-2 at the measurement plane. The FOV width is 240 mm or $\geq 2S$ for SR25, SR50, and SR100, but $< 2S$ for case SR160 and SR250-2. For these two cases, the FOV is limited to the region near to the rough-to-smooth interface and a complete pair of roll modes is not captured for these cases. For each heterogeneous roughness case, 4800 image pairs are acquired, while the reference case SW-2 has only 1200 realizations. The acquisition rate is 1 Hz, which corresponds to the boundary layer turnover rate of $tU_\infty/\delta_s = 270$ for case SW, meaning that each snapshot can be considered to be statistically independent. The timing between two images in a pair, $\Delta t = 85 \mu\text{s}$ ($t^+ \equiv \Delta t U_\tau^2 / \nu = 1.7$ for case SW-2).

4.1.2 Stereoscopic calibration

Stereoscopic calibration is performed by capturing images of a two-sided calibration target at 9 streamwise locations with 500 μm distance between each location prior to or after image acquisition. This calibration target is made from a $500 \times 255 \times 8$ mm anodized aluminium plate and comprises of 861 through holes in dotted pattern to ensure that every dot corresponds to the same location on the other side. Each dot has 3 mm diameter with 10 mm spacing in y and z between dots (see figure 4.5c). The through holes are filled with white craft clay for easier dot detection process.

An algorithm based on self-calibration technique (Wieneke, 2005) and developed in Gudla (2020) is employed to obtain the calibration matrix and correct slight misalignments between the image plane and the calibration target. It is assumed that the image coordinates $\mathbf{X} = [Y \ Z \ 1]^\top$ are related to physical coordinates $\mathbf{x} = [x \ y \ z \ 1]^\top$ by a camera matrix $\mathbf{P}$,

$$\mathbf{X} = \mathbf{P}\mathbf{x} \quad (4.3)$$

where a 3×4 camera matrix $\mathbf{P}$ is defined as

$$\mathbf{P} = \mathbf{K} \mathbf{R} \begin{bmatrix} 1 & 0 & 0 & -\tilde{c}_1 \\ 0 & 1 & 0 & -\tilde{c}_2 \\ 0 & 0 & 1 & -\tilde{c}_3 \end{bmatrix} \quad (4.4)$$

$\mathbf{K}$ is a 3×3 camera calibration matrix and $\mathbf{R}$ is a 3×3 matrix denoting the orientation of the camera. The matrix in equation (4.4) is a 3×4 matrix whose first three columns are the 3×3 identity matrix $\mathbf{I}$ and the fourth column is $-\tilde{\mathbf{c}}$, a 3×1 camera position vector relative to the physical coordinates. Assuming pin-hole, CCD sensor (possible non-square pixels), and finite projective camera model (Hartley & Zisserman, 2003), $\mathbf{K}$ is defined as

$$\mathbf{K} = \begin{bmatrix} m_y f_0 & s & Y_0 \\ 0 & m_z f_0 & Z_0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4.5)$$

where f_0 is the focal length, m_y and m_z are the number of pixels per unit distance in Y and Z , respectively, and s is the skew parameter of the camera, added for generality. Equation (4.3) can be rewritten as

$$\mathbf{X} \times \mathbf{P}\mathbf{x} = \mathbf{A}\mathbf{P} = 0 \quad (4.6)$$

where $\mathbf{A}$ is a skew-symmetric matrix from the cross product of $\mathbf{X}$ and $\mathbf{x}$, reduced by the last row (only the first and second row are independent),

$$\mathbf{A} = \begin{bmatrix} 0^\top & -\mathbf{x}^\top & Z\mathbf{x}^\top \\ \mathbf{x}^\top & 0^\top & -Y\mathbf{x}^\top \end{bmatrix} \quad (4.7)$$

Through single value decomposition, $\mathbf{A}$ can be decomposed into $\mathbf{A} = \mathbf{U}\mathbf{D}\mathbf{V}^\top$ and so we seek to minimize $\mathbf{U}\mathbf{D}\mathbf{V}^\top\mathbf{P}$. Here, $\mathbf{U}$ and $\mathbf{V}$ are orthogonal ($\mathbf{U}^\top\mathbf{U} = \mathbf{V}^\top\mathbf{V} = \mathbf{I}$) and $\mathbf{D}$ is a diagonal matrix with descending main diagonal entries,

$$\mathbf{D} = \text{diag}(d_1, d_2, \dots, d_{n-1}, d_n) \quad \text{where} \quad d_n < d_{n-1} < \dots < d_1 \quad (4.8)$$

Since d_n is the minimum entry of $\mathbf{D}$, $\mathbf{U}\mathbf{D}\mathbf{V}^\top\mathbf{P}$ is minimum if

$$\mathbf{P} = \mathbf{V} \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} \quad (4.9)$$

In other words, $\mathbf{P}$ is the last column of $\mathbf{V}$.

The flowchart of the self-calibration algorithm is shown in figure 4.4. For stereoscopic application in this study, which involves two cameras, subscript F denotes the “front” or upstream camera and R denotes the “rear” or downstream camera (see figure 4.1). The self-calibration algorithm starts with the centroid detection of calibration dots to obtain pixel locations (Y, Z) for each dot viewed by both front and rear camera, $C_F(Y_F, Z_F)$ and $C_R(Y_R, Z_R)$, respectively. An example of calibration image, taken by the rear camera C_R , and its axis system are shown in figure 4.5(a). In the physical coordinate system $\mathbf{x}$, the calibration plate is positioned at $x = 4$ m. Following the axis system in figure 4.5(a), $y = 0$ is the centerline of the calibration plate, which coincides with the centerline of the tunnel. The dots located at the bottom row are the closest to the wall. The procedure to determine the z location of this row is shown in figure 4.5(b). A ruler is positioned on top of the calibration plate and the z location of the dots at the bottom row is recorded.¹ Since both pixel and physical locations of each dots in the calibration images are known, the camera matrices of both cameras $\mathbf{P}_F$ and $\mathbf{P}_R$ can be obtained using the relation in equation 4.3.

The next step in the self-calibration algorithm is to calculate the disparity between images taken by the front and rear camera (figure 4.4). However, it should be noted that the images are distorted due to stereoscopic camera angle and lens distortion (see figure 4.5a for example). Thus, they are dewarped before the disparity is calculated (and later, before image cross-correlation to obtain displacement vectors). For image dewarping, the physical grid is determined first. Here, we use the full extent of the FOV, $4\delta_s \times 3\delta_s$ ($y \times z$, see table 4.1) and x is the location

¹This method also gives a rough estimation of the wall location. Later, after the velocity field is obtained, this location is further corrected. For the reference smooth-wall case SW-2, the velocity profile is fitted to the boundary-layer profile in equation 2.8, while for SR cases, the velocity profile is matched with the profile obtained from HWA measurements.

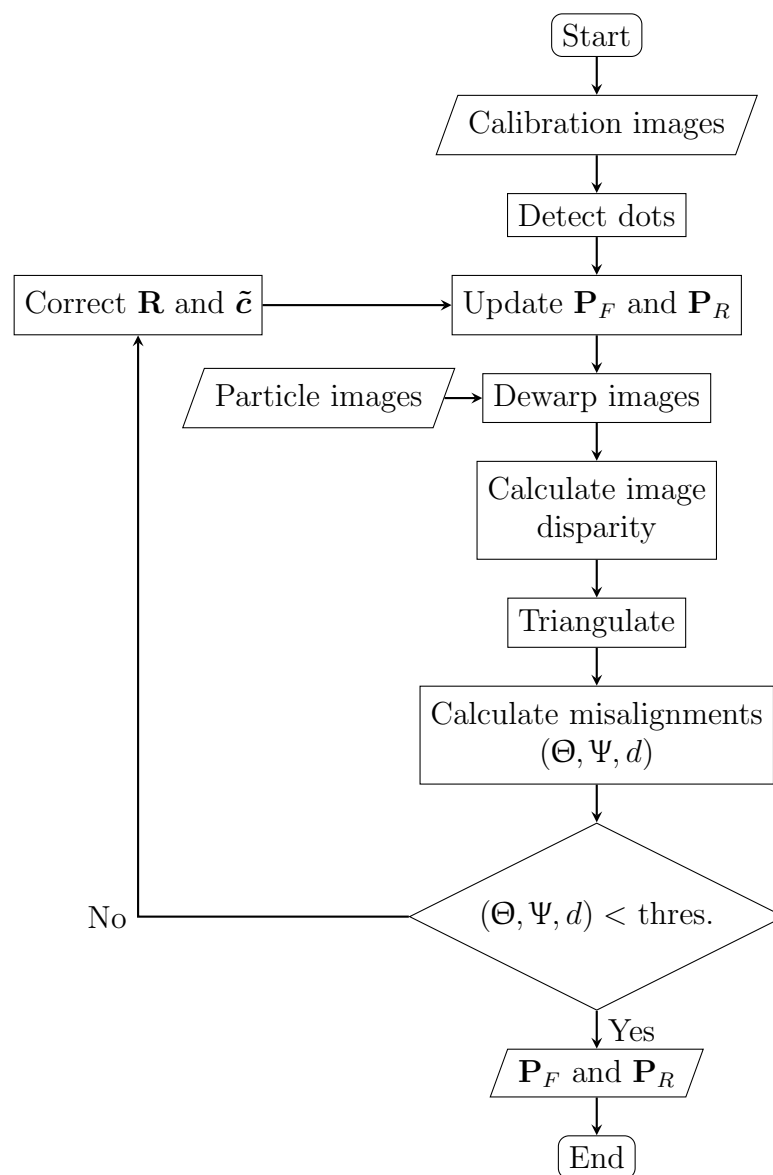


Figure 4.4: Flowchart of crossplane SPIV self-calibration algorithm.

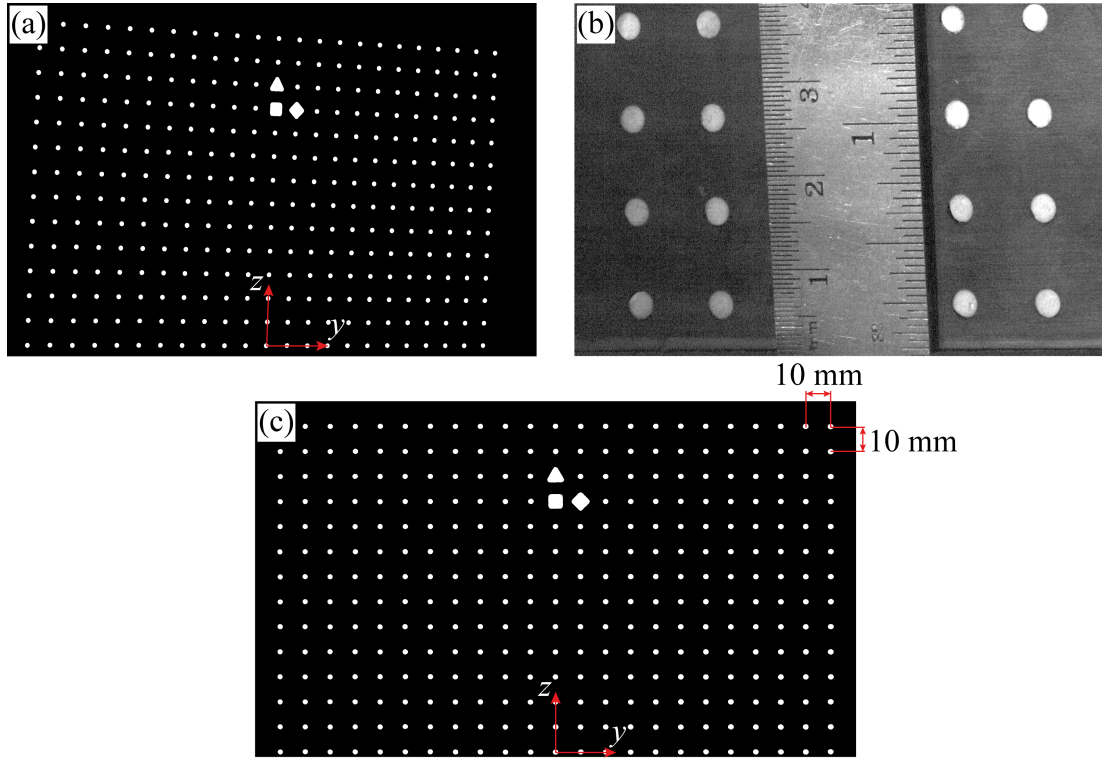


Figure 4.5: SPIV calibration images: (a) sample of warped calibration image, taken by the rear camera C_R at $x = 4$ m. The image is originally a grayscale image, which has been converted to binary image. (b) Magnified view of a calibration image taken by the front camera C_F with a ruler attached to measure the z location of the calibration dots at the bottom row. (c) Dewarped calibration image (a). Red arrows denote the axis system.

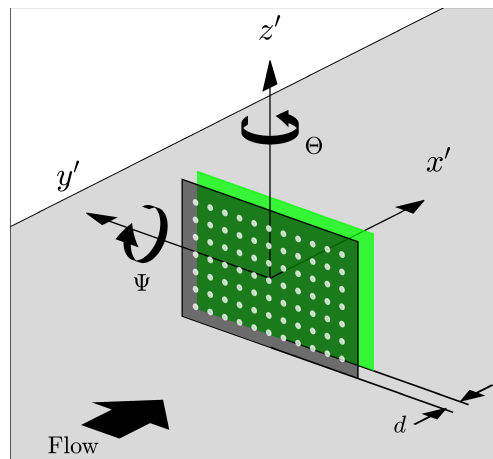


Figure 4.6: Misalignments between calibration image plane and laser sheet in SPIV setup: Θ , rotation about wall-normal axis z' ; Ψ , rotation about spanwise axis y' ; and offset between the two planes d . (x', y', z') is the Cartesian coordinate system at the center of image plane.

of the laser sheet, $x = 4$ m. As we would like to preserve the image resolution, the physical grid resolution is set to be equal to the image pixel size in $\mu\text{m pixel}^{-1}$ (table 4.1), which can be estimated from the centroid detection of the calibration dots. The pixel grid $\mathbf{X}$ (of the images) is related to this defined physical grid $\mathbf{x}$ by equation 4.3, where the camera matrix $\mathbf{P}$ is previously obtained from the calibration dots for both cameras. Finally, the dewarped image is obtained from a 2-D cubic interpolation between the defined pixel grid and every pixel in the warped image. Figure 4.5(a) shows an example of a warped image. The result of dewarping procedure of this image is shown in figure 4.5(c), which shows the image corrected for camera angle and lens distortion.

Five particle images are sampled to obtain the disparity between images taken by the front and rear camera. These images are dewarped (see example in figure 4.5c) and the disparity is calculated by cross-correlating the image pairs. A triangulation algorithm, explained in detail in Hartley & Zisserman (2003), is written to obtain physical coordinates from image coordinates of both cameras,

$$\mathbf{x} = \tau(\mathbf{P}_F, \mathbf{P}_R, \mathbf{X}_F, \mathbf{X}_R) \quad (4.10)$$

which is done by solving equation (4.6) for both cameras such that

$$\begin{bmatrix} \mathbf{X}_F \times \mathbf{P}_F \\ \mathbf{X}_R \times \mathbf{P}_R \end{bmatrix} \mathbf{x} = 0 \quad (4.11)$$

Since the calibration target is aligned by eye with the laser sheet when the laser is fired at low power, there is inevitably some slight misalignments between the image plane (captured in the calibration images) and the laser sheet (captured by the particle images). Three types of misalignments, shown in figure 4.6, are considered in this measurement. It is assumed that the calibration target is not parallel to the laser sheet; it has a small angular misalignment along the wall-parallel axis and the spanwise axis, Θ and Ψ , respectively. It also has a small offset d along streamwise axis with respect to the laser sheet. The misalignments are calculated using the resulting physical coordinates from triangulation. The obtained angular misalignment Θ and Ψ are used to correct camera orientation matrix $\mathbf{R}_F$ and $\mathbf{R}_R$, while the offset d is used to correct camera position $\tilde{\mathbf{c}}_F$ and $\tilde{\mathbf{c}}_R$. The camera matrix $\mathbf{P}_F$ and $\mathbf{P}_R$ are then updated for the next iteration. This process goes into a loop where the camera matrices are corrected iteratively until all three misalignments are smaller than a threshold value. For Θ and Ψ , the threshold is set to a small value of $< 0.02^\circ$ ($\approx 0^\circ$), while the image plane offset d is set to a threshold value of $< 50 \mu\text{m}$ or about 1% of the laser sheet thickness.

4.1.3 Image pre-processing

The pre-processing of the images is performed by subtracting the background image from the particle images. The background image for each camera is extracted by taking the average of particle images for every 1200 pairs. The top 2% pixel intensity of the particle images is saturated.

4.1.4 Image cross-correlation

The cross-correlation of image pairs and post-processing of the resulting vectors are performed using an in-house PIV package developed at University of Melbourne (de Silva *et al.*, 2014). The code employs multi-grid (Scarano & Riethmuller, 1999) and window deformation algorithm (Scarano, 2001). The images are cross-correlated with 64×64 pixels initial window size and 32×32 pixels final window size with 50% overlap. This window size and the estimated laser sheet thickness give the spatial resolution of $\Delta x^+ \times \Delta y^+ \times \Delta z^+ = 165 \times 70 \times 70$ for the smooth wall reference case SW-2 ($\delta_s^+ \approx 2000$). The choice of 50% window overlap is intended to increase vector density in the velocity field. It should be noted that the PIV spatial resolution, i.e the final size of the interrogation window, remains 32×32 pixels ($0.034\delta_s \times 0.034\delta_s$) and is not affected by overlap. It is also noted that adjacent PIV vectors (regardless of overlap) are not statistically independent relative to the length of the large-scale turbulent structures in the flow field ($\approx 6\delta_s$).

4.1.5 Velocity vector reconstruction

Three-dimensional displacement vectors $\mathbf{v}(\Delta x, \Delta y, \Delta z)$ are reconstructed from the pixel displacement vectors of front and rear camera, $C_F(\Delta Y_F, \Delta Z_F)$ and $C_R(\Delta Y_R, \Delta Z_R)$, respectively, using a method described in Brücker (1996) and Willert (1997).

The reconstruction process is illustrated in figure 4.7. Here, O is the origin, a point on the laser sheet captured by both cameras and P is a point on the laser sheet where the displacement vectors are to be determined. Figure 4.7(a) is the setup viewed from the top, where α_F and α_R are the angles in xy -plane between the viewing ray and the lines normal to the laser sheet (parallel to the flow). The magnified view of P is shown in figure 4.7(b), where it is decomposed into its physical displacements Δx and Δy . By assuming that the vector displacements and laser sheet thickness are much smaller than the distance between the cameras and the laser sheet, Δx and Δy can be determined geometrically from the spanwise pixel displacement vectors

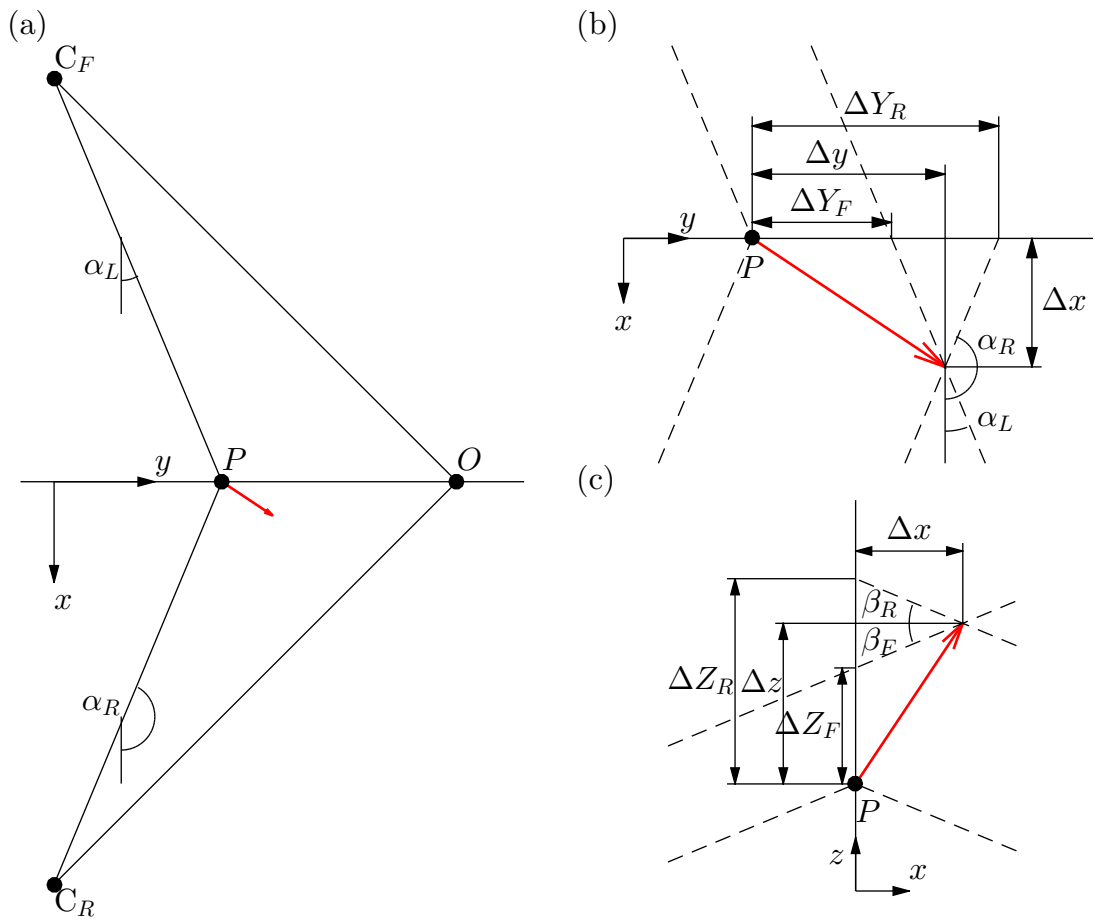


Figure 4.7: Illustration of 3-D physical displacement vectors from pixel displacements computed from the front (C_F) and rear (C_R) camera: (a) top view, magnified in (b), and (c) side view.

from both cameras, ΔY_F and ΔY_R

$$\Delta y = \begin{cases} \Delta Y_F + \Delta x \tan \alpha_F \\ \Delta Y_R + \Delta x \tan \alpha_R \end{cases} \quad (4.12)$$

By rearranging equation (4.12), the streamwise (out-of-plane) and spanwise (in-plane) physical displacements are obtained

$$\Delta x = \frac{\Delta Y_R - \Delta Y_F}{\tan \alpha_F - \tan \alpha_R} \quad \Delta y = \frac{\Delta Y_R \tan \alpha_F - \Delta Y_F \tan \alpha_R}{\tan \alpha_F - \tan \alpha_R} \quad (4.13)$$

The wall-normal displacement component is obtained geometrically from the side view (figure 4.7c). Here, β_F and β_R are the angles in xz -plane between the viewing ray and lines normal to the laser sheet. The wall-normal displacement is given by

$$\Delta z = \begin{cases} \Delta Z_F + \Delta x \tan \beta_F \\ \Delta Z_R + \Delta x \tan \beta_R \end{cases} \quad (4.14)$$

which can be rearranged into

$$\Delta z = \frac{\Delta Z_R \tan \beta_F - \Delta Z_F \tan \beta_R}{\tan \beta_F - \tan \beta_R} \quad (4.15)$$

In the current setup, $\beta_F \approx \beta_R \approx 0$, hence Δz is obtained by manipulating equation (4.13) and (4.15),

$$\Delta z = \frac{\Delta Z_F - \Delta Z_R}{2} + \frac{\Delta Y_F - \Delta Y_R}{2} \left(\frac{\tan \beta_F + \tan \beta_R}{\tan \alpha_F - \tan \alpha_R} \right) \quad (4.16)$$

The velocity vectors are obtained from the displacements,

$$u = \frac{\Delta x}{\Delta t} \quad v = \frac{\Delta y}{\Delta t} \quad w = \frac{\Delta z}{\Delta t} \quad (4.17)$$

where Δt is the delay between the Q-switch of Head 1 and Head 2 of the laser, $\Delta t = 85 \mu\text{s}$.

4.2 Wall-parallel PIV (WPPIV)

4.2.1 Experimental setup

Non-time resolved wall-parallel plane PIV experiments are also performed in the same facility. As shown in table 3.1, WPPIV data are available for all spanwise heterogeneous cases and the reference smooth wall, except for cases located at $x = 1.85 \text{ m}$ (SR250-1 and SW-1) due to limited optical access. WPPIV measurements are not conducted on the reference rough wall (RW-1 and RW-2).

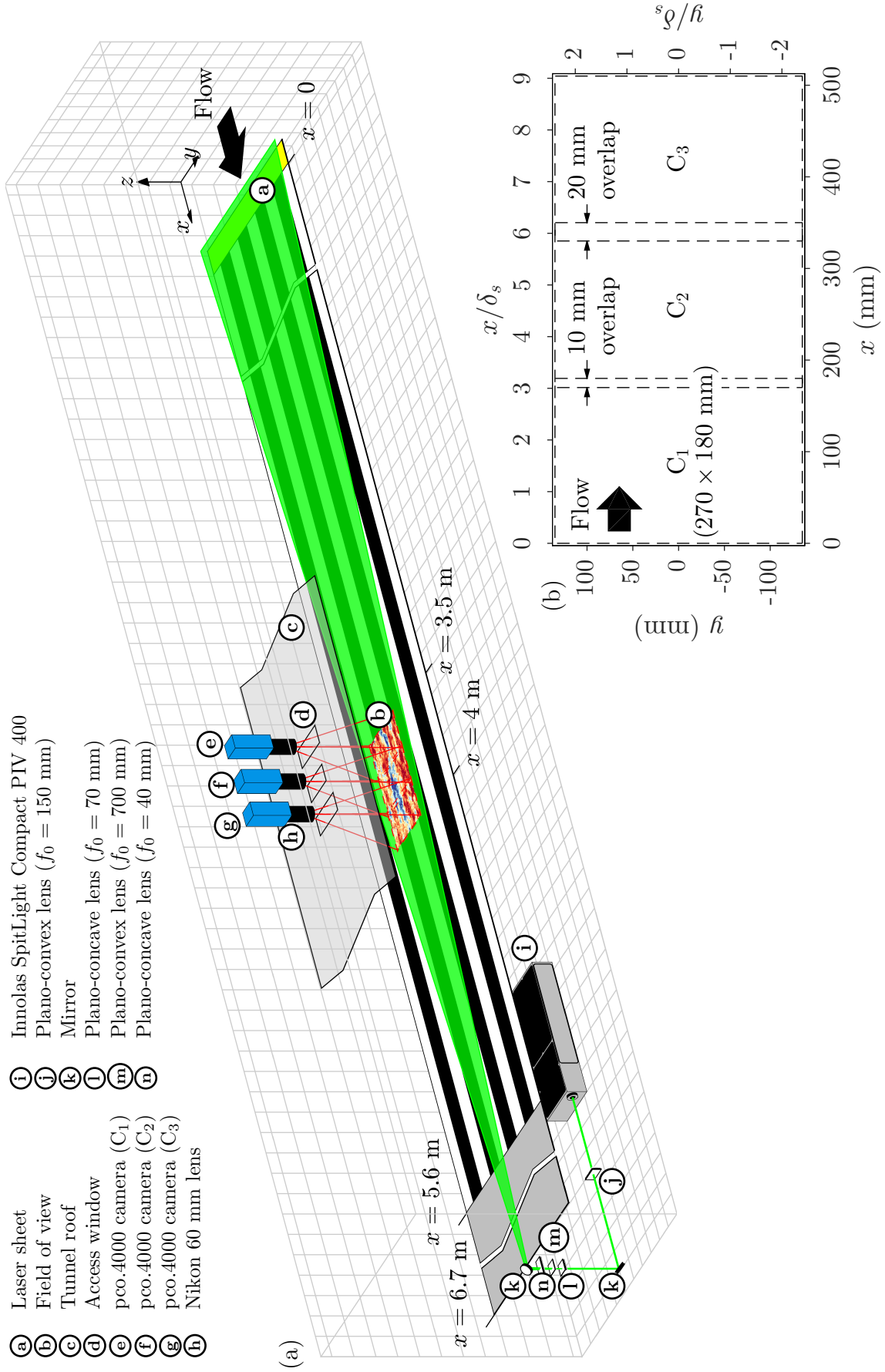


Figure 4.8: Illustration of wall-parallel PIV setup over a spanwise heterogeneous roughness surface: (a) isometric view and (b) the field of view.

Time-resolved	No
Flow medium	air
Freestream velocity	$U_\infty \approx 15 \text{ ms}^{-1}$
Seeding	polyamide particles
Particle size	$\approx 1\text{--}2 \text{ }\mu\text{m}$
Sensor resolution	$4008 \times 2672 \text{ pixels}$
Aperture number	f/2.8
Image pixel size	$\approx 70 \text{ }\mu\text{m pixel}^{-1}$
Field of view (each camera)	$270 \text{ mm} \times 180 \text{ mm} (\approx 5\delta_s \times 3\delta_s)$
Field of view (total)	$270 \text{ mm} \times 510 \text{ mm} (\approx 5\delta_s \times 9\delta_s)$
Number of image pairs	1200 per case (600 for SW-2)
Acquisition rate	1 Hz ($tU_\infty/\delta_s = 270$)
Time between images	$\Delta t = 60 \mu\text{s}$ ($t^+ = 1.2$) [†]
Interrogation window	$24 \times 24 \text{ pixels}$
Final resolution $\Delta x^+ \times \Delta y^+$	60×60 [†]

[†] for the reference smooth wall SW-2 ($\delta_s^+ \approx 2000$)

Table 4.2: Parameters of the WPPIV experiments.

The setup and parameters of the measurements are detailed in figure 4.8(a) and table 4.2. The measurements are conducted in the xy -plane, parallel to the surfaces. The plane is illuminated by a laser sheet generated from an InnoLas SpitLight Compact PIV 400 dual pulse Nd:YAG laser and an optical configuration, which consists of mirrors and cylindrical lenses as shown in figure 4.8(a). The light sheet is introduced into the tunnel working section by a 45° planar mirror ($\textcircled{\mathbf{k}}$ in figure 4.8a) that is located at $x = 6.7 \text{ m}$, approximately 2.7 m or $48\delta_s$ downstream of the measurement location. Hot-wire measurements have confirmed that the presence of the mirror in the downstream flow has no measurable impact on the boundary-layer characteristics at the measurement location. The flow is seeded with polyamide particles ($1\text{--}2 \text{ }\mu\text{m}$ diameter) using a Safex fog generator and the freestream velocity of the tunnel is maintained at $U_\infty \approx 15 \text{ ms}^{-1}$ for all cases. For this setup, the measured velocity components are the in-plane streamwise and spanwise components (u and v), as shown in figure 4.8(a). Unlike in the SPIV measurements, the thickness of the laser sheet is not measured in these measurements, but it is nominally 1 mm .

The total FOV covers the streamwise range $3.5 \lesssim x \lesssim 4.01 \text{ m}$ with a spanwise width of 0.27 m and is constructed by stitching three pco.4000 CCD cameras (14-bit, $4008 \times 2672 \text{ pixels}$), which are mounted above the wind tunnel roof, facing access windows ($\textcircled{\mathbf{d}}$), and positioned along the streamwise direction, as shown in figure 4.8. These cameras, labelled C_1 , C_2 , and C_3 from upstream to downstream, are equipped

with a Nikon lens (60 mm focal length, f/2.8 aperture). The current setup gives approximately $70 \mu\text{m pixel}^{-1}$ resolution, which for each camera corresponds to a FOV of approximately $5\delta_s \times 3\delta_s$ (spanwise $\times$ streamwise), where δ_s is the 98% boundary layer thickness of the reference smooth wall case SW-2 at the measurement plane. The FOV width is 270 mm or $\geq 2S$ for SR25, SR50, and SR100, but $< 2S$ for case SR160 and SR250-2. For these two cases, the FOV is limited to the region near to the rough-to-smooth interface and a complete pair of roll modes is not captured for these cases. To stitch the FOV, C_1 and C_2 are positioned such that a 10 mm-overlap exists between them, while 20 mm-overlap exists between C_2 and C_3 (figure 4.8b). The total FOV for the stitched images is 270×510 mm, or approximately $5\delta_s \times 9\delta_s$. For each heterogeneous roughness case, 1200 image pairs are acquired, while the reference case SW-2 has only 600 realizations. The acquisition rate is 1 Hz, which corresponds to the boundary layer turnover rate of $tU_\infty/\delta_s = 270$ for case SW-2 (each snapshot is statistically independent). The timing between two images in a pair, $\Delta t = 60\mu\text{s}$ ($t^+ = 1.2$ for case SW-2).

4.2.2 Measurement planes

A procedure for determining the wall-normal laser sheet location from the surfaces z_{sheet} is as follows: a calibration target of 9.7 mm thickness is placed above the surface, parallel to the laser sheet. Metal blocks of known (uniform) thickness are stacked under the calibration target until the desired z_{sheet} is achieved. Mirror $\textcircled{\text{k}}$ (figure 4.8a) is traversed until the laser sheet illuminates the calibration target.

For the spanwise heterogeneous cases, the laser sheet is set such that z_{sheet} is roughly at the center of the secondary flows due to spanwise heterogeneity, $z_{sheet}/\bar{\delta} = 0.5$ for $S/\bar{\delta} > 1$ or $z_{sheet}/S = 0.5$ for $S/\bar{\delta} < 1$. For the five spanwise heterogeneous cases in table 4.3, the measurements are conducted in three wall-normal locations of the laser sheet: $z_{sheet} = 33.7$ mm, 25.7 mm, and 13.8 mm above the surface, which correspond to $z_{sheet}/\bar{\delta} \approx 0.5$ for case SR250-2, SR160, and SR100 (figure 4.9a), and $z_{sheet}/S \approx 0.5$ for case SR50 and SR25 (figure 4.9b,c). It is noted that the cameras are not traversed in z for each light sheet location z_{sheet} . Instead, the cameras are refocused whenever z_{sheet} is adjusted. As a result, the image pixel size varies within the range of $69 \pm 1 \mu\text{m pixel}^{-1}$ across all cases. For the smallest spanwise roughness wavelength SR25 ($S/\bar{\delta} = 0.32$), an additional measurement is conducted approximately above the mean secondary flows (figure 4.9d) at $z_{sheet}/\bar{\delta} = 0.33$.

For the reference smooth wall case SW-2 (table 4.3), z_{sheet} is set at $z_{sheet}/\delta_s \approx 0.5$ and $z_{sheet}/\delta_s \approx 0.2$. The latter is related to the location of the center of large-scale

Spanwise heterogeneous roughness						
Case	S (mm)	$\bar{\delta}$ (mm)	$S/\bar{\delta}$	z_{sheet} (mm)	$z_{sheet}/\bar{\delta}$	z_{sheet}/S
SR250-2	250	68.8	3.63	33.7	0.49	0.13
SR160	160	70.0	2.28	33.7	0.48	0.21
SR100	100	73.9	1.35	33.7	0.46	0.34
SR50	50	80.7	0.62	25.7	0.32	0.51
SR25	25	77.6	0.32	13.8	0.18	0.55
SR25	25	77.6	0.32	25.7	0.33	1.03
Reference cases						
Case	S (mm)	δ_s (mm)	S/δ_s	z_{sheet} (mm)	z_{sheet}/δ_s	z_{sheet}/S
SW-2	—	56.4	—	25.7	0.46	—
SW-2	—	56.4	—	13.8	0.24	—

Table 4.3: Wall-normal locations of laser sheet in WPPIV measurements.

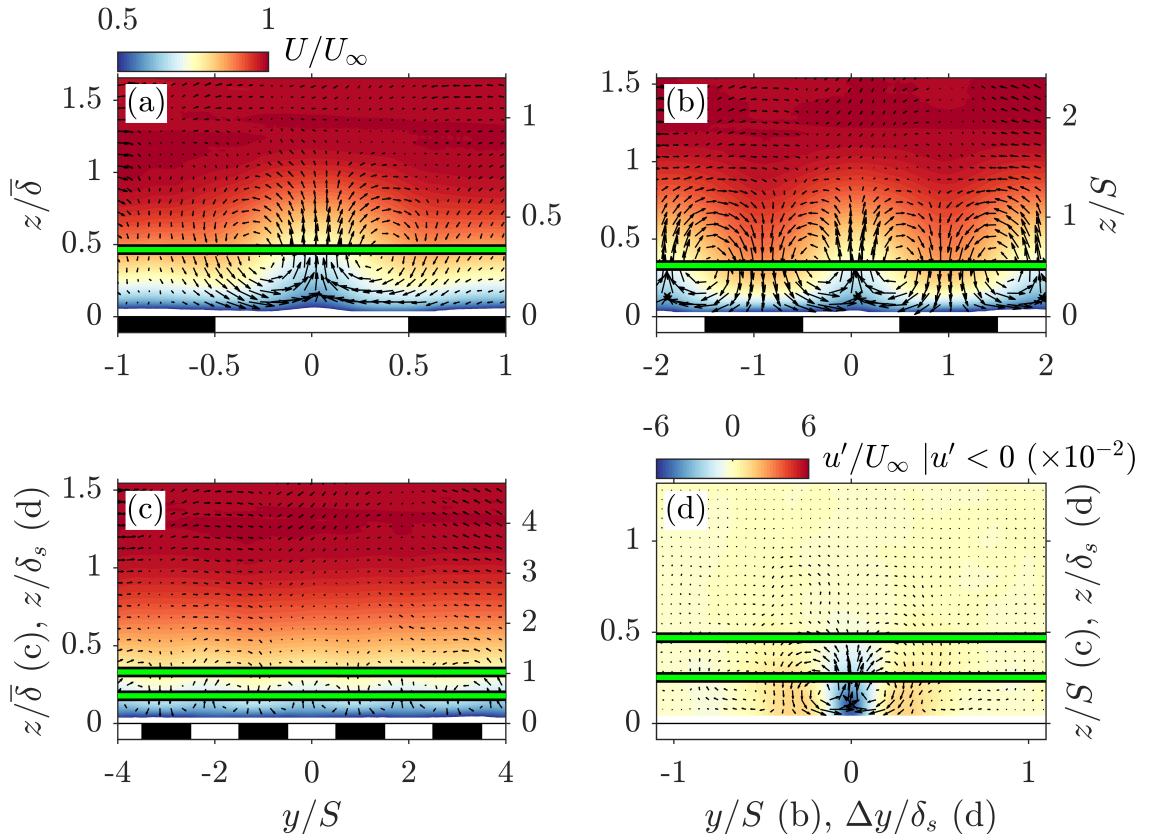


Figure 4.9: Wall-normal locations of laser sheet in WPPIV measurements (—) in case (a) SR100 ($S/\bar{\delta} = 1.35$), (b) SR50 ($S/\bar{\delta} = 0.62$), (c) SR25 ($S/\bar{\delta} = 0.32$), and (d) SW-2. The sheet is located approximately at the center of the secondary flows (vector fields in a-c), with an additional measurement plane in (c) above the mean secondary flows. In smooth wall reference case (d), the sheet is located at $z/\delta_s \approx 0.5$ and $z/\delta_s \approx 0.2$.

motions (LSM) that occur naturally in smooth wall turbulent boundary layers (this location is obtained from conditional averaging of streamwise velocity fluctuation u' conditioned on low speed ejection events, $u' < 0$).

4.2.3 Calibration

Wall-parallel PIV calibration is performed by taking an image of a calibration target with each camera prior to or after image acquisition. The calibration target is made from a $550 \times 250 \times 9.7$ mm anodized aluminium plate and comprises of 5500 white-coloured dots laser etched on one side of the target. Each dot has 1 mm diameter with 5 mm spacing in x and y between dots. The pixel location of each dot (X, Y) is detected by an algorithm similar to that used in stereoscopic calibration. Each pixel location (X, Y) of the dots in the calibration target corresponds to a known physical coordinate (x, y) .

The mapping between pixel and physical coordinates is obtained by fitting (X, Y) and (x, y) of each camera to two-dimensional third order polynomials. The mapping from pixel to physical coordinates is defined by

$$\begin{aligned} x &= A_1 + A_2X + A_3Y + A_4XY + A_5X^2 \\ &\quad + A_6Y^2 + A_7YX^2 + A_8XY^2 + A_9X^3 + A_{10}Y^3 \\ y &= B_1 + B_2X + B_3Y + B_4XY + B_5X^2 \\ &\quad + B_6Y^2 + B_7YX^2 + B_8XY^2 + B_9X^3 + B_{10}Y^3 \end{aligned} \quad (4.18)$$

The inverse of these polynomials is the mapping from physical to pixel coordinates

$$\begin{aligned} X &= a_1 + a_2x + a_3y + a_4xy + a_5x^2 \\ &\quad + a_6y^2 + a_7yx^2 + a_8xy^2 + a_9x^3 + a_{10}y^3 \\ Y &= b_1 + b_2x + b_3y + b_4xy + b_5x^2 \\ &\quad + b_6y^2 + b_7yx^2 + b_8xy^2 + b_9x^3 + b_{10}y^3 \end{aligned} \quad (4.19)$$

Although by definition the polynomials mapping pixel to physical coordinates (equation 4.18) are the inverse of the mapping from physical to pixel coordinates (equation 4.19), in practice these two sets of polynomials are obtained independently by fitting the calibration data from pixel to physical coordinates and vice versa.

4.2.4 Image pre-processing

The pre-processing of the images is performed by subtracting the background image from the particle images. The background image for each camera is extracted by taking the average of all particle images of each case. The top 2% pixel intensity of the particle images is saturated.

4.2.5 Image cross-correlation

The cross-correlation of image pairs and post-processing of the resulting vectors are performed using an in-house PIV package developed at University of Melbourne (de Silva *et al.*, 2014). The code employs multi-grid (Scarano & Riethmuller, 1999) and window deformation algorithm (Scarano, 2001). The images are cross-correlated with 48×48 pixels initial window size and 24×24 pixels final window size with 50% overlap. This window size gives the spatial resolution of $\Delta x^+ \times \Delta y^+ = 60 \times 60$ for smooth wall reference case SW-2 ($\delta_s^+ \approx 2000$).

4.2.6 Velocity vector reconstruction and stitching

Velocity vectors in the xy -plane (u, v) are reconstructed from pixel displacements ($\Delta X, \Delta Y$) for each camera with the following method. The cross-correlation is performed at the initial pixel grid locations (X_i, Y_i), with ($\Delta X, \Delta Y$) as the output. The final pixel grid location is defined as

$$\begin{aligned} X_f &= X_i + \Delta X \\ Y_f &= Y_i + \Delta Y \end{aligned} \quad (4.20)$$

In physical coordinates, (x_f, y_f) is mapped from (X_f, Y_f) using equation (4.18). Velocity vectors in the xy -plane are then simply obtained from

$$u = \frac{x_f - x_i}{\Delta t} \quad v = \frac{y_f - y_i}{\Delta t} \quad (4.21)$$

where (x_i, y_i) is the initial physical grid location, mapped from (X_i, Y_i) using equation (4.18).

Velocity vectors reconstruction is performed for C_1 , C_2 , and C_3 separately. To stitch the FOV, common grid points are defined first. These are the grid points that exist in C_1 , C_2 , and C_3 . As shown in figure 4.8(b), a 10 mm-overlap exists between C_1 and C_2 and 20 mm between C_2 and C_3 . The instantaneous velocity field in the FOV overlap between C_1 and C_2 , C_2 and C_3 are shown in figure 4.10 and 4.11, respectively. The rms error between each camera is less than 2% of the freestream velocity. To

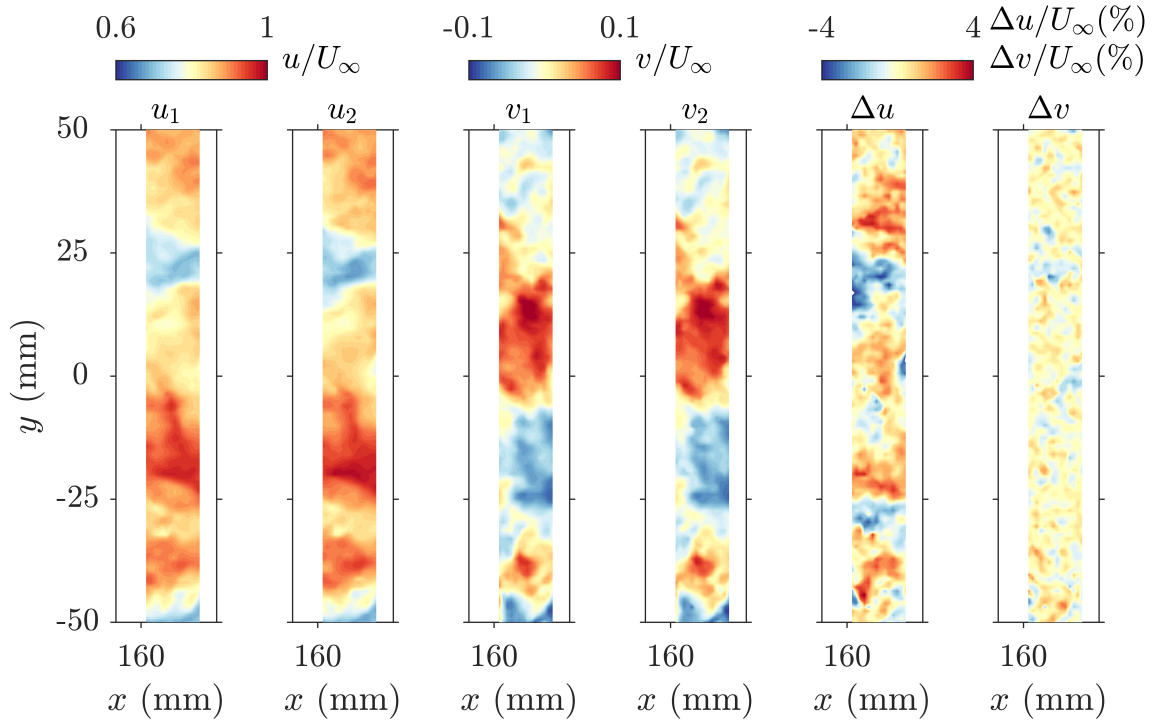


Figure 4.10: Instantaneous velocity field in the 10 mm-field of view overlap between $C_1(u_1, v_1)$ and $C_2(u_2, v_2)$. Δu and Δv are the differences between C_1 and C_2 in percentage of U_∞ .

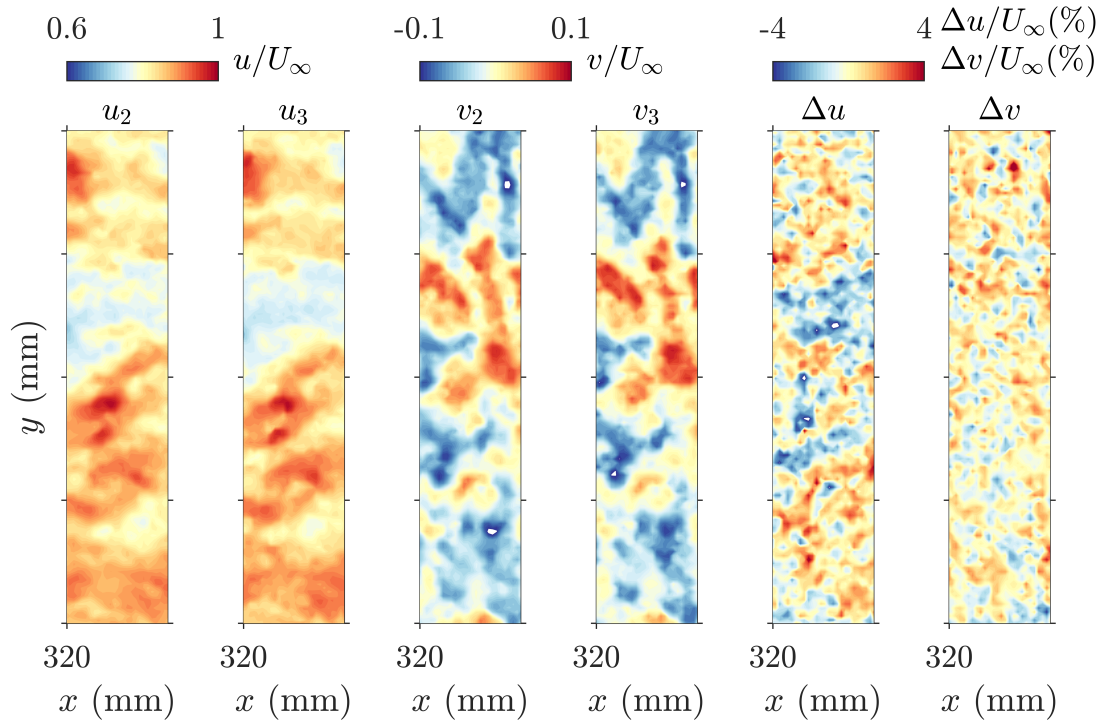


Figure 4.11: Instantaneous velocity field in the 20 mm-field of view overlap between $C_2(u_2, v_2)$ and $C_3(u_3, v_3)$. Δu and Δv are the differences between C_2 and C_3 in percentage of U_∞ .

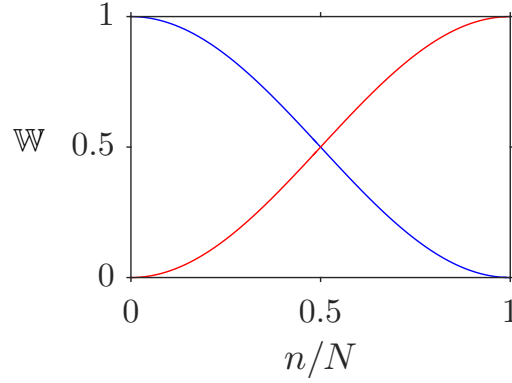


Figure 4.12: Weighting function $\mathbb{W}$ assigned in the C_1 - C_2 overlapping region of the WPPIV measurements: $\mathbb{W}_1$ (camera C_1 , —) and $\mathbb{W}_2$ (camera C_2 , —).

avoid any discontinuities in the stitched image, the final velocity components in the overlapping region between C_1 - C_2 are obtained by blending the velocity vectors from the downstream end of C_1 and the upstream end of C_2 (see figure 4.8b). Similarly, for C_2 - C_3 overlap, the velocity vectors are blended from those at the downstream end of C_2 and the upstream end of C_3 (see figure 4.8b). A weighting function in x is assigned to each vector in the overlapping region at the downstream end of C_1

$$\mathbb{W}_1(n) = 0.5 \left(1 + \cos \left(\pi \frac{n}{N} \right) \right), \quad 0 \leq n \leq N \quad (4.22)$$

At the upstream end of C_2 , the weighting function is given by

$$\mathbb{W}_2(n) = 0.5 \left(1 - \cos \left(\pi \frac{n}{N} \right) \right), \quad 0 \leq n \leq N \quad (4.23)$$

where $N+1$ is the number of overlapping grid points in x , $n=0$ is the upstream end of C_2 and $n=N$ is the downstream end of C_1 . A similar weight function was utilised by Kevin *et al.* (2019b) for a similar 2-D planar PIV setup in the same facility and with similar flow conditions (secondary flows over C-D riblets at closely matched Re). As shown in figure 4.12, at $n=0$, $\mathbb{W}_1=1$ and the information solely comes from C_1 . Meanwhile at $n=N$, $\mathbb{W}_2=1$ and the information solely comes from C_2 . In the middle of the overlap region ($n=0.5N$), the information is obtained equally from C_1 and C_2 . In this overlapping region between C_1 - C_2 , the resulting velocity vector is the sum of $C_1(u_1, v_1)$ and $C_2(u_2, v_2)$ multiplied by the weighting functions,

$$\begin{aligned} u &= \mathbb{W}_1 u_1 + \mathbb{W}_2 u_2 \\ v &= \mathbb{W}_1 v_1 + \mathbb{W}_2 v_2 \end{aligned} \quad (4.24)$$

The same weighting functions are also computed at the downstream end of C_2 and the upstream end of C_3 to obtain u and v in the overlap between C_2 - C_3 .

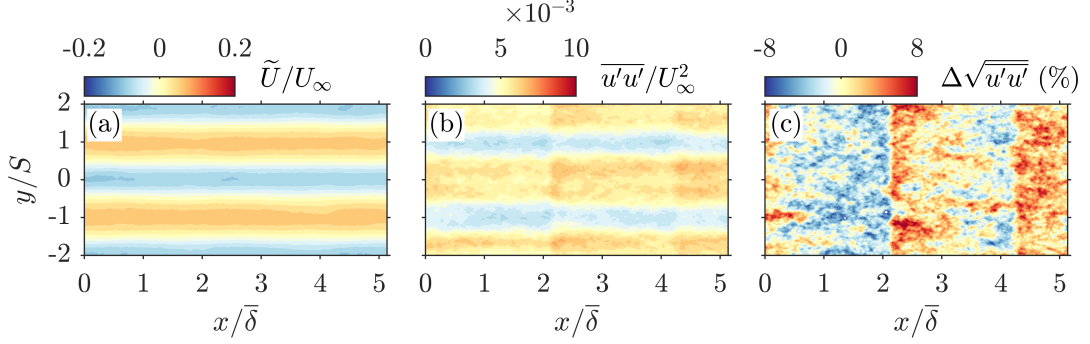


Figure 4.13: Velocity reconstruction error in the camera overlap of WPPIV measurements: (a) $\tilde{U}$ and (b) variance $\overline{u'u'}$ of case SR50 ($S/\bar{\delta} = 0.62$, $z/\bar{\delta} = 0.32$). Error between the standard deviation of u' , $\sqrt{\overline{u'u'}}$ and the same variable averaged along x .

Figures 4.13(a,b) show the mean statistics of streamwise velocity component for case SR50 ($S/\bar{\delta} = 0.62$ at $z/\bar{\delta} = 0.32$). Here, $\tilde{U}$ is the time-averaged, spanwise fluctuation of U (see §5.1 for velocity decomposition) and $\overline{u'u'}$ is the variance of u' . The contours of $\tilde{U}$ in figure 4.13(a) are continuous and uniform in x across the FOV, which means that in this case the blending functions chosen in equation (4.22) and (4.23) are able to reconstruct and stitch the FOV from all three cameras in the overlap region between C_1 - C_2 and C_2 - C_3 . Discontinuities, however, are apparent on $\overline{u'u'}$ (figure 4.13b) at $x/\bar{\delta} \approx 2.1$ (C_1 - C_2 overlap) and $x/\bar{\delta} \approx 4.3$ (C_2 - C_3 overlap). The error is examined in figure 4.13(c). The standard deviation of u' , $\sqrt{\overline{u'u'}}$ is compared to a reference value. Here, the reference value is the streamwise-averaged $\sqrt{\overline{u'u'}}$, $\langle \sqrt{\overline{u'u'}} \rangle_x$ such that the error is

$$\Delta \sqrt{\overline{u'u'}} = \frac{\sqrt{\overline{u'u'}} - \langle \sqrt{\overline{u'u'}} \rangle_x}{\langle \sqrt{\overline{u'u'}} \rangle_x} \quad (4.25)$$

The rms error is 3% of the reference value (0.2% of the freestream velocity). It is also noted that the contours of $\sqrt{\overline{u'u'}}$ error in figure 4.13(c) show a slight tendency of $\sqrt{\overline{u'u'}}$ to decrease from the upstream to downstream end of the camera FOV. This suggests that there are unmeasured out-of-plane components, probably due to a slight misalignment between the CCD plane of the camera and the laser sheet, and/or between the laser sheet and the freestream.

4.3 Reference case

SPIV and WPPIV data are validated for smooth wall reference case SW-2 against DNS data of boundary-layer flow from Sillero *et al.* (2014) at matched Re_{θ_s} . Fig-

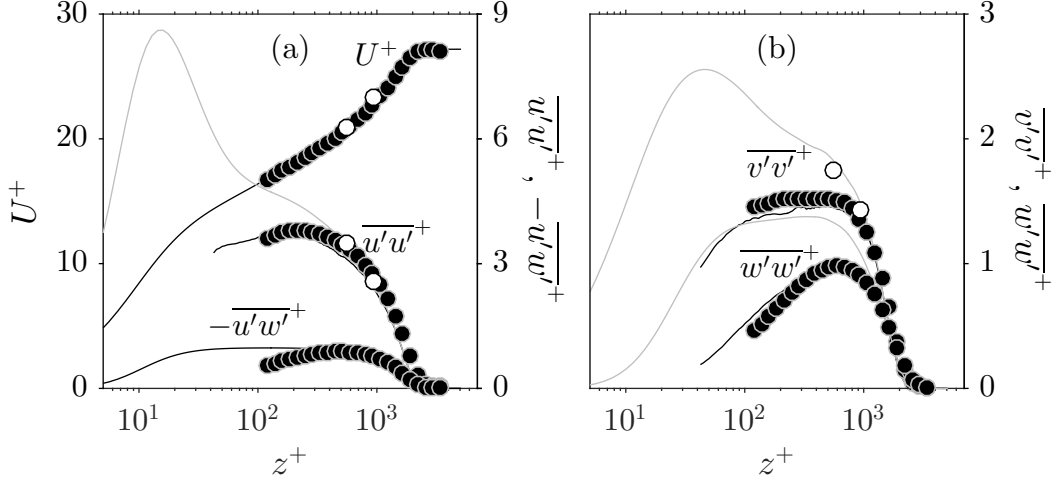


Figure 4.14: Boundary-layer profiles from SPIV (●) and WPPIV (○) measurements of reference smooth wall case SW-2. WPPIV measurements are conducted at $z/\delta_s = 0.46$ (table 4.3). (a) Mean streamwise velocity profile U^+ (left axis), $\overline{u'u'^+}$ and $-\overline{u'w'^+}$ (right axis). (b) $\overline{v'v'^+}$ and $\overline{w'w'^+}$ (right axis). For the second order statistics, solid lines are DNS data (Sillero *et al.*, 2014): fully resolved (—) and filtered according to the spatial resolution (—). Data points are downsampled for clarity.

Figure 4.14 shows the boundary-layer profiles of the reference smooth wall case SW-2, obtained from PIV measurements. Black circles (●) are the velocity profiles measured from SPIV measurements, with the WPPIV data at $z/\delta_s \approx 0.2$ and ≈ 0.5 shown in white circles (○). U_τ is shown in table 3.3, obtained from 2.8. The profile of U^+ collapses reasonably with the DNS data. The Reynolds stress for all three velocity components, $\overline{u'u'^+}$, $\overline{v'v'^+}$, $\overline{w'w'^+}$, and the Reynolds shear stress component $-\overline{u'w'^+}$ are also shown in the same figure. These second-order statistics are also validated with the DNS data (fully resolved profiles are shown in —). As with the HWA measurements data (figure 3.4), energy attenuation is expected, this time due to the PIV spatial resolution (table 4.2). The DNS data are filtered accordingly using a method proposed by Lee *et al.* (2016). The filtered DNS profiles are shown in solid black lines (—) in figure 4.14. Once the DNS data are corrected for the spatial attenuation from the PIV, we see very good agreement across all statistics, which validates the quality of the PIV measurements. It should be noted that the SPIV measurements can only obtain velocity profiles at $z/\delta_s \gtrsim 0.05$ ($z^+ \gtrsim 100$). Near-wall profiles are not captured due to wall reflections on the PIV images. However, the large-scale secondary flows due to spanwise heterogeneity, which is the focus of this study, are located in the outer layer and scale approximately between $0.3\bar{\delta}$ to $\bar{\delta}$ (figure 5.2). Hence, the obtained velocity profiles from PIV measurements

are sufficient to capture the dynamics of the large-scale secondary flows.

Chapter 5

Mean statistics

5.1 Velocity decomposition

The instantaneous velocity components are defined as $\mathbf{u} = (u, v, w)$, corresponding to the axis system $\mathbf{x} = (x, y, z)$. Due to spanwise periodicity, $\mathbf{u}$ can be decomposed into its temporal- and spatial-average, similar to the decomposition method utilized in Raupach & Shaw (1982); Finnigan (2000); Coceal *et al.* (2006). The total velocity $\mathbf{u}$ at a particular x location can be triply decomposed into

$$\mathbf{u}(y, z, t) = \mathbf{U}(y, z) + \mathbf{u}'(y, z, t) \quad (5.1)$$

$$= \langle \mathbf{U} \rangle_{\Lambda}(z) + \tilde{\mathbf{U}}(y, z) + \mathbf{u}'(y, z, t) \quad (5.2)$$

$$= \langle \mathbf{U} \rangle_{\Lambda}(z) + \tilde{\mathbf{u}}'(y, z, t) \quad (5.3)$$

where $\mathbf{U} = (U, V, W)$ is the Reynolds (temporal) average, or the local mean velocity, and $\mathbf{u}'$ is the turbulent fluctuation. $\mathbf{U}$ can be further decomposed into its yt -average $\langle \mathbf{U} \rangle_{\Lambda}$, or the global mean over the span, and the time-averaged spatial variations about this mean $\tilde{\mathbf{U}}$. Since $\langle V \rangle_{\Lambda} = \langle W \rangle_{\Lambda} = 0$, $\tilde{\mathbf{U}} = (\tilde{U}, V, W)$. We also introduce the quantity $\tilde{\mathbf{u}}' = (\tilde{u}', v, w)$, which is the sum of $\tilde{\mathbf{U}}$ and $\mathbf{u}'$. An example of this triple-decomposition for the streamwise velocity u is shown in figure 5.1. Here, $\tilde{U}$ shows the presence of low speed flows above the smooth strips and high speed flows above the rough strips, which coincide with the common flow up and down of the mean secondary flows, respectively (figure 5.2e). For the largest $S/\bar{\delta}$ cases, SR250-2 ($S/\bar{\delta} = 3.63$) and SR160 ($S/\bar{\delta} = 2.28$), the full spanwise wavelength is not captured in the FOV. For these two cases, the global mean $\langle \mathbf{U} \rangle_{\Lambda}$ and the time-averaged, spanwise variations $\tilde{\mathbf{U}}$ in (5.2) are obtained by reflecting the FOV about the centre of a smooth strip ($y = 0$).

It should be noted that there is a slight asymmetry of $\tilde{U}$ in figure 5.1. Two possible causes have been identified for this asymmetry in the flow. First, it is due

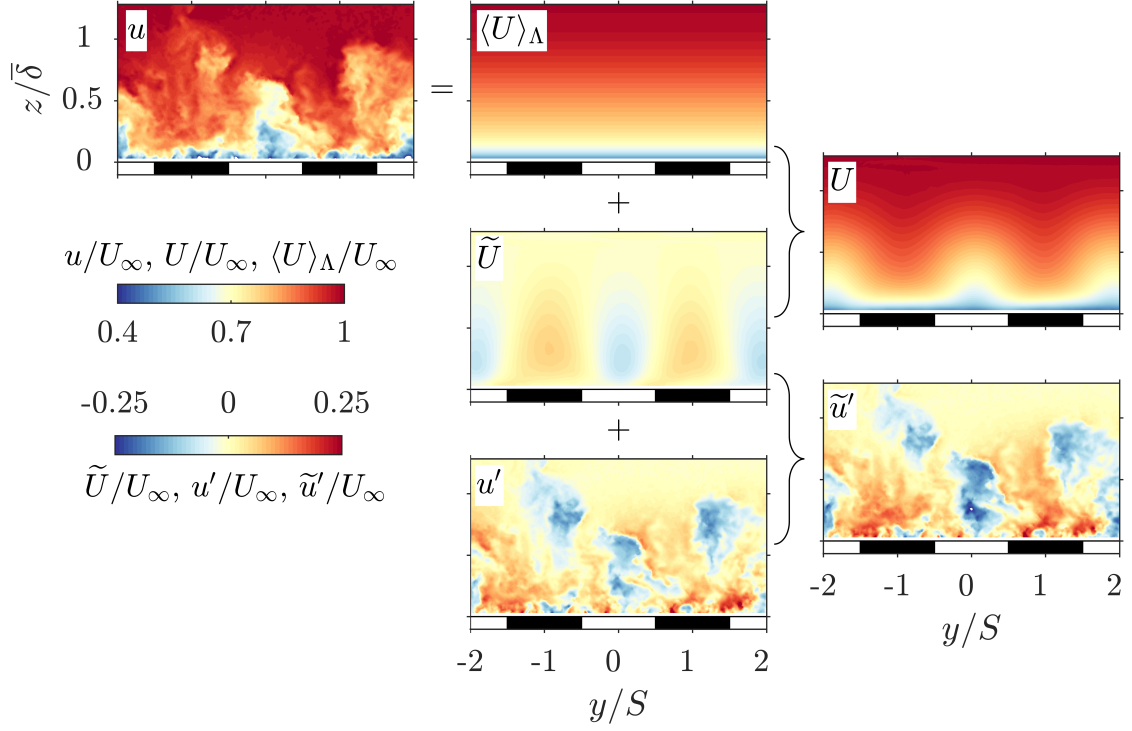


Figure 5.1: Triple-decomposition of streamwise velocity u for case SR50 ($S/\bar{\delta} = 0.62$).

to the lack of convergence in the rough-wall SPIV data, where we only have 4800 realisations at each (y, z) position for computing $\tilde{U}$. Secondly, the secondary flows are relatively weak compared to the turbulence. Although we go to a great length to carefully install the surfaces into the tunnel, ensuring that the roughness strips are parallel to the flow, there might be a slight asymmetry in the velocity field if the surfaces are slightly misaligned with respect to the flow.

5.2 Mean velocity

5.2.1 Mean streamwise velocity

Figure 5.2 shows the contours of mean streamwise velocity profiles U for the reference smooth wall case SW-2 (figure 5.2a) and all spanwise heterogeneous (SR) cases in table 3.1 (figure 5.2b–g). Contours are obtained from SPIV measurements, except for case SR250-1 (figure 5.2b) due to limited optical access at the $x = 1.85$ m location where the measurements are taken. For this case, the contours are constructed from the spanwise/wall-normal grid of HWA measurements. Since no attempt is made to measure U_τ for these cases (due to the heterogeneity), we present U normalised

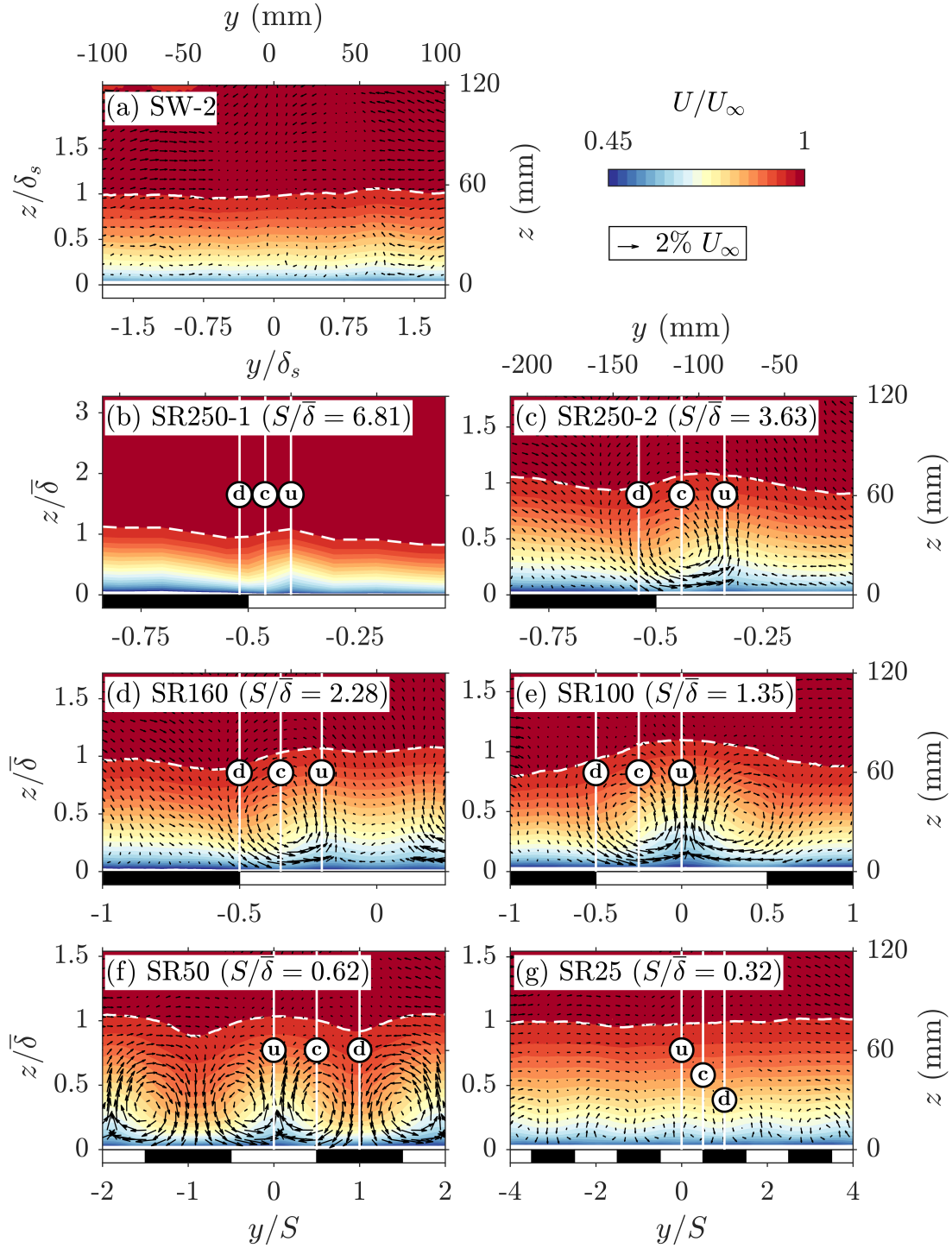


Figure 5.2: Contours of mean streamwise velocity U/U_∞ for reference smooth-wall case SW-2 (a) and spanwise heterogeneous (SR) cases (b-g), obtained from SPIV measurements except for (b). Contours in (b) are obtained from HWA measurements. The white and black patches correspond to smooth and rough strips, respectively. Vectors indicate V and W , downsampled for clarity. Solid white lines mark the three spanwise locations related to the mean secondary flows: common flow up ($\textcircled{u}$), center of mean secondary flows ($\textcircled{c}$), and common flow down ($\textcircled{d}$). White dashed lines are the 98% boundary-layer thickness δ as a function of y .

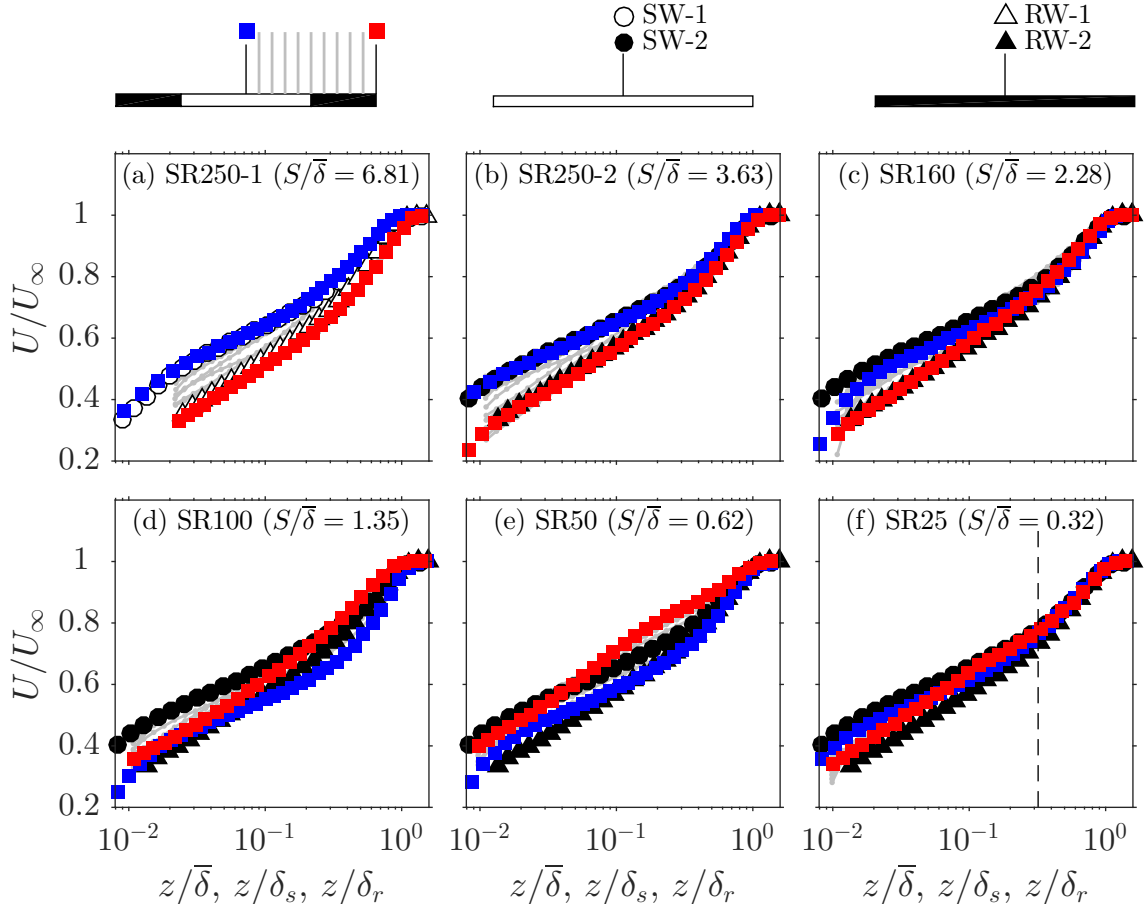


Figure 5.3: Mean streamwise velocity U/U_∞ for SR cases from HWA. Reference case SW and RW are plotted for comparison. Legends are on top of the page. In (f), dashed line (---) is $z/\bar{\delta} = 0.32$.

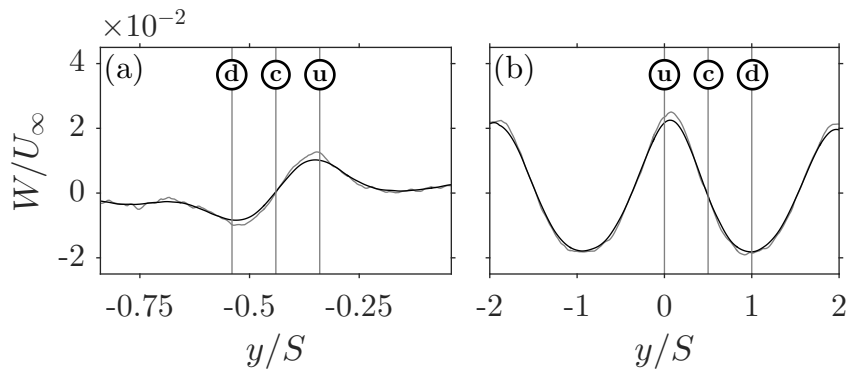


Figure 5.4: Mean wall-normal velocity W of case (a) SR250-2 ($S/\bar{\delta} = 3.63$) at $z/\bar{\delta} = 0.35$ and (b) SR50 ($S/\bar{\delta} = 0.62$) at $z/S = 0.5$ obtained from SPIV data (—), smoothed (---). The maxima of W is the common flow up (u), minima is the common flow down (d), and $W \approx 0$ is the centre of the secondary flow (c).

by the freestream velocity U_∞ as a function of outer-scaled wall location $z/\bar{\delta}$ and spanwise location y/S . The origin of z is at the smooth strips, whereas the roughness offset for the rough strips remains unknown. The contours are presented in the same spatial extent (see ordinate and abscissa on the right-hand sides and tops of panels in figure 5.2). It should be noted that for case SR250-1, SR250-2, and SR160 (figure 5.2b–d), the FOV is $< 2S$, hence only one half-pair of secondary flows is shown, whereas for case SR100, SR50, and SR25 (figure 5.2e–g), at least a complete pair of secondary flows is captured in the FOV.

The mean secondary flows are shown in figure 5.2 as vectors of V and W (mean spanwise and wall-normal velocity, respectively). As expected, large-scale mean secondary flows do not exist in the reference smooth wall case SW-2 (figure 5.2a). Where spanwise heterogeneity exists, however, large-scale secondary flows are apparent near the roughness transition between the smooth and rough patches. The direction of the secondary flows, for all SR cases, is consistent with that shown in Hinze (1967, 1973), where turbulent flows near the wall are transferred from the high shear-stress patch (rough strip) to the low shear-stress patch (smooth strip). The resulting mean secondary flows are characterised with regions of upwelling ($W > 0$) and downwelling ($W < 0$) over the smooth and rough strips, respectively. The spanwise locations of common flow up is identified as the maxima of W at a given wall-normal location (see for example figures 5.4a,b), while the common flow down is the minima. The maxima and minima of W are computed at $z/\bar{\delta} = 0.35$ for case SR250–SR100 (figure 5.2c–e) and $z/S = 0.5$ for case SR50 and SR25 (figure 5.2f,g) due to the decreasing size of the mean secondary flows. It should be noted that for case SR50 in figure 5.4b the common flow up is nominally located at the center of the smooth strip, i.e. at every harmonic $y/S = \{\dots, -4, -2, 0, 2, 4, \dots\}$. The slight asymmetry is due to the number of realisations and possible slight surface misalignment explained in §5.1. Between common flow up and down, the spanwise location where $W \approx 0$ is the centre of the secondary flows. The spanwise locations of common flow up, centre of the secondary flow, and common flow down in figure 5.2 are marked by symbol $\textcircled{u}$, $\textcircled{c}$, and $\textcircled{d}$, respectively. These regions are marked in figure 5.2 (for clarity only a single secondary flow cell is annotated in this manner). The local boundary-layer thickness $\delta(y)$ is shown by the white dashed line, where $\delta(y)$ is the 98% boundary layer thickness calculated at every location in y . It is observed in figure 5.2(c–g) that within the common flow up regions, positive W locally corresponds to a thickening of the boundary layers, whereas within the common flow down regions, negative W drives the flows towards the wall, corresponding to a

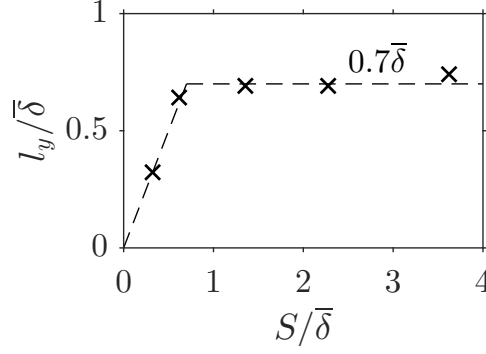


Figure 5.5: Mean secondary flows width l_y normalised by $\bar{\delta}$ as a function of $S/\bar{\delta}$. Equation (5.4) is shown by the dashed line (---).

local thinning of the boundary layers. Based on this observation of local thickening and thinning of the boundary layers, the locations of common flow up and down for case SR250-1 (figure 5.2b) can be estimated even though the vectors of V and W are not available for this case.

Figure 5.2 also shows how the size of the secondary flows vary with the roughness patch size. We define l_y as the width of the secondary flow, measured from the spanwise location of the common flow up to down ($\textcircled{\text{u}}$ and $\textcircled{\text{d}}$ in figure 5.2). l_y for various $S/\bar{\delta}$ can be approximated as

$$l_y \approx \begin{cases} C \bar{\delta}, & S/\bar{\delta} \geq C \\ S, & S/\bar{\delta} < C \end{cases} \quad (5.4)$$

The precise value of the constant C in (5.4) will vary according to the assessment method. An assessment of figure 5.2 and the extent of non-zero swirl strength in figure 5.9 suggests that $C \approx 0.7$ is a good approximation for the present data (see figure 5.5). Regardless of the assessment method, the overall trend shown by figure 5.5 holds; when $S/\bar{\delta}$ is small, the size of the secondary flow is a function of S , but when S becomes comparable to $\bar{\delta}$, the size of the secondary flow seems to asymptote to a constant. The effect of patch size (and consequently the size of the secondary flows) on U is discussed in the following.

The largest $S/\bar{\delta}$ cases correspond to SR250-1 ($S/\bar{\delta} = 6.81$) and SR250-2 ($S/\bar{\delta} = 3.63$). These cases (figure 5.2b,c) exhibit a thicker boundary layer above the rough patch than that above the smooth patch, with the exception of the area surrounding the secondary flows. For these largest $S/\bar{\delta}$ cases, away from the secondary flows, U is slower above the centre of a rough patch than above the center of a smooth patch at any $z/\bar{\delta}$. This is consistent with observations from either a homogeneous rough or smooth wall flow at a matched freestream velocity. Figure 5.3 shows the profiles

of U in various spanwise locations as a function of $z/\bar{\delta}$. For the largest spanwise wavelengths (SR250-1 and SR250-2, figure 5.3a,b, respectively), the profiles at the centres of the smooth and rough strips nearly collapse to the corresponding homogeneous smooth and rough reference cases (SW and RW, respectively), suggesting that for these largest $S/\bar{\delta}$, the centre of the rough and smooth strips are in near equilibrium with their local boundary conditions, as observed by Chung *et al.* (2018). Hinze (1967) suggests that secondary flows occur where the production–dissipation imbalance occurs, i.e. somewhere near the spanwise roughness transition. When S is larger than $\bar{\delta}$, the secondary flows have approximate diameter $\bar{\delta}$ and are confined near the transition between smooth–rough patch (figure 2.4a). Sufficiently far removed in the span from these secondary flows, the flow becomes locally homogeneous in accord with their local surface conditions and therefore the smooth and rough patches approach the reference SW and RW cases, respectively. In these regions, we would expect outer-layer similarity to be preserved. In figures 5.2 and 5.3, we observe locally homogeneous conditions for $S/\bar{\delta} \gtrsim 3$, which is comparable to the limit found in other studies, such as $S/\delta \gtrsim 6$ (Chung *et al.*, 2018) for strip-type roughness, and $L/(2\delta) > 1.6$ (Medjnoun *et al.*, 2018), $L/(2\delta) \gtrsim 1$ (Yang & Anderson, 2018), and $L/(2\delta) \gtrsim 2$ (Zampiron *et al.*, 2020) for ridge-type roughness, where L is the repeating spanwise wavelength of the heterogeneity. It should be noted that some of these studies featured substantial elevation change between the rough and the smooth strips (Medjnoun *et al.*, 2018; Yang & Anderson, 2018; Zampiron *et al.*, 2020).

The intermediate case of $S/\bar{\delta} \approx 1$ is approximately attained for cases SR100 and SR50, which correspond to $S/\bar{\delta} = 1.35$ and 0.62, respectively (figure 5.2e,f). For these cases, the boundary layer is thicker above the smooth patch than above the rough patch. The velocity profiles in figure 5.3(d,e) show that away from the immediate near wall region ($z/\bar{\delta} \gtrsim 0.05$), the flow slows down above the centres of the smooth strips and speeds up above the centres of the rough strips. This is the opposite behaviour to that observed for $S/\bar{\delta} \gg 1$ (figure 5.3a,b and figure 5.2b,c). We attribute the switch in this behaviour to the secondary flows at the limit of $S/\bar{\delta} \approx 1$. At this limit of $S/\bar{\delta} \approx 1$, the secondary flows fill the space inside the boundary layer such that in this case the centres of the smooth strips are characterised by common flow up, where the flow is pushed upward, reducing the velocity and thickening the boundary layers above the smooth strips (**u** in figure 5.2e,f). Common flow down is observed somewhere nearby the roughness transitions (**d** in figure 5.2e) or at the centres of the rough strips (**d** in figure 5.2f), leading to the thinning of the

boundary layers above the rough strips and higher local velocities. A similar switch in the isovels (a switch in whether low or high speed occurs above the rough or smooth strips) is also documented in other studies, with the range where behaviour switches estimated as: $S/\delta \approx 0.8\text{--}3$ (Chung *et al.*, 2018), $L/(2\delta) \approx 0.5$ (Medjnoun *et al.*, 2018), $0.5 \lesssim L/(2\delta) \lesssim 1$ (Yang & Anderson, 2018; Anderson, 2019; Zampiron *et al.*, 2020). Again it is reminded that some of these studies (Anderson, 2019; Medjnoun *et al.*, 2018; Yang & Anderson, 2018; Zampiron *et al.*, 2020) involved ridge-type arrangements. It should be noted that this behaviour is expected as $S/\bar{\delta}$ continues to decrease. As the strips become narrower, a secondary cell is restricted by its spanwise-adjacent such that the common flow up occurs above the centre of smooth strips and common flow down above the centre of rough strips.

The limiting case $S/\bar{\delta} \ll 1$ is represented by case SR25 ($S/\bar{\delta} = 0.32$) in figure 5.2(g). Since $S < \bar{\delta}$, the diameter of secondary flows is limited by S instead of $\bar{\delta}$ and hence these secondary flows are confined closer to the wall. The contour in figure 5.2(g) shows that for wall-normal distances beyond the influence of the secondary flows, the boundary layer approaches a spanwise homogeneous condition, as illustrated in figure 2.4(c), and the boundary-layer thickness is approximately constant across the spanwise extent. In figure 5.3(f), it is noted that U/U_∞ collapses at all spanwise positions for $z/\bar{\delta} \gtrsim 0.32 = S/\bar{\delta}$, suggesting spanwise homogeneity beyond the influence of the secondary flows. It is noted that this homogeneous outer velocity profile for $z/\bar{\delta} > 0.32$ falls somewhere between the reference SW and RW cases. This limiting case is documented in other studies as: $L/(2\delta) \leq 0.23$ (Vanderwel & Ganapathisubramani, 2015), $S/\delta < 0.4$ (Chung *et al.*, 2018), and $L/(2\delta) \lesssim 0.5$ (Yang & Anderson, 2018; Anderson, 2019). Certain parallels with the study of homogeneous sinusoidal egg-carton type roughness by Chan *et al.* (2018) are also noted, which showed that spanwise homogeneity is attained when $z \gtrsim 0.5\lambda$, where λ is the spanwise wavelength of the sinusoids.

5.2.2 Recovery to outer layer similarity

For a spanwise heterogeneous roughness model inside a channel flow, DNS results (Chung *et al.*, 2018) of the largest $S/\delta = 6.28$ (δ is the channel half-height) showed that the velocity defect in outer scale, when normalised with local friction velocity U_τ , collapses to a surface with spanwise uniform wall shear stress. At $S/\delta \gg 1$, the secondary flows are capped by δ and confined near the roughness transition, hence outer layer similarity recovers at spanwise locations sufficiently far removed from the secondary flows (i.e. the centres of smooth and smooth strips). Similarly, for

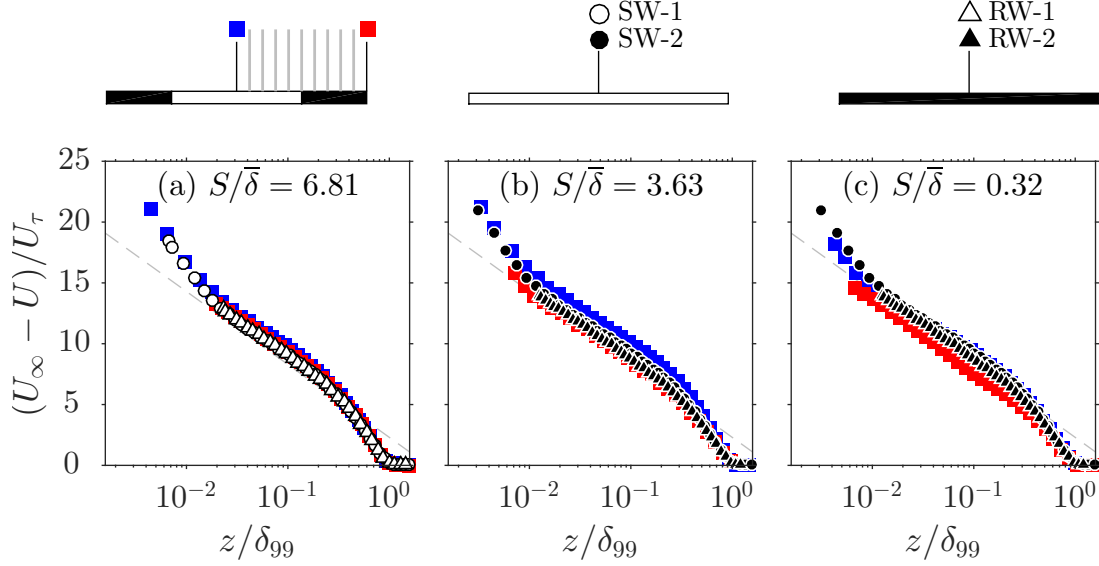


Figure 5.6: Velocity defect profiles for limiting case $S/\bar{\delta} \gg 1$ (a,b) and $S/\bar{\delta} \ll 1$ (c). The defect profiles are normalised by the local friction velocity U_τ . Dashed lines (---) are $2.3 - \kappa^{-1} \log(z/\delta_{99})$. Legends are on top of the page.

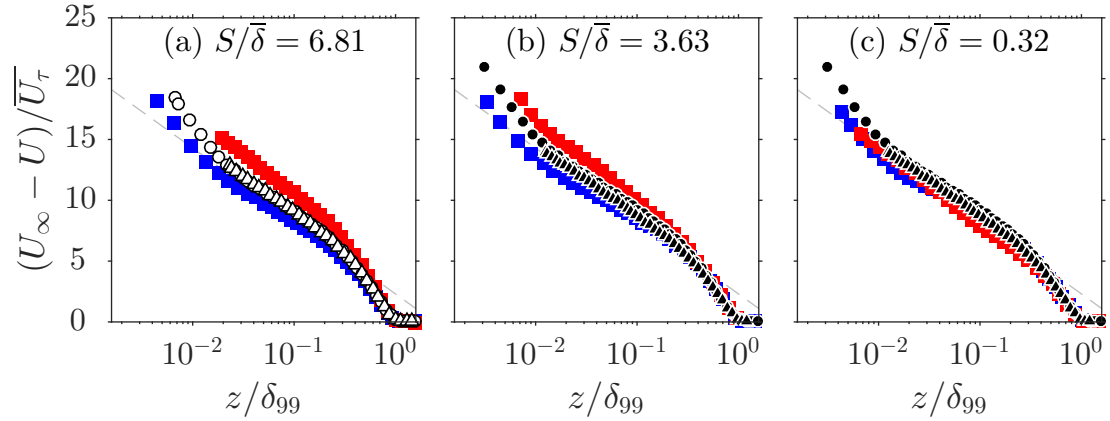


Figure 5.7: Velocity defect profiles for limiting case $S/\bar{\delta} \gg 1$ (a,b) and $S/\bar{\delta} \ll 1$ (c). The defect profiles are normalised by the average friction velocity $\bar{U}_\tau$. Dashed lines (---) are $2.3 - \kappa^{-1} \log(z/\delta_{99})$. Legends are on top of the page.

At the centre of a smooth strip ($y = 0$)						
Case	$S/\bar{\delta}$	U_τ (ms ⁻¹)	C_f ($\times 10^{-3}$)	ΔU^+	k_s (mm)	k_s^+
SR250-1	6.81	0.61	3.1	0.93	0.008	7.54
SR250-2	3.63	0.56	2.7	0.77	0.008	7.10
SR25	0.32	0.63	3.3	3.71	0.022	22.0
At the centre of a rough strip ($y/S = 1$)						
SR250-1	6.81	0.77	5.0	7.54	1.950	95.3
SR250-2	3.63	0.74	4.6	7.44	1.949	91.6
SR25	0.32	0.70	4.1	5.44	0.973	42.5

Table 5.1: Summary of $S/\bar{\delta} \gg 1$ and $S/\bar{\delta} \ll 1$ cases in viscous-scaled units. Profiles at the centre of a smooth strip and rough strip is fitted to the composite profile of Chauhan *et al.* (2009).

the present study, the profile at the centre of the smooth ($y = 0$) and rough strip ($y/S = 1$) are expected to collapse to the reference smooth and rough wall profile for cases where $S/\bar{\delta} \gg 1$: $S/\bar{\delta} = 6.81$ (SR250-1), and $S/\bar{\delta} = 3.63$ (SR250-2).

To obtain local U_τ , the measured profiles are fitted to the composite profile of Chauhan *et al.* (2009) given in equations (2.4), (2.8), and, when roughness is present, shifted according to equation (2.12). The defect profiles $U_\infty - U$ normalised by local U_τ (measured at $y = 0$ and $y/S = 1$) are shown in figure 5.6(a,b) for $S/\bar{\delta} \gg 1$ cases. The profiles for case SR250-1 (figure 5.6a) collapse to the reference smooth and rough wall case, showing recovery to outer layer similarity, with the exception of profiles above the roughness transition (where the secondary flow is). For case SR250-2 (figure 5.6), the profile at the centre of a smooth strip does not collapse to the reference smooth and rough wall case, but the profile at the centre of a rough strip does. This is related to the observation in the contours of U in figure 5.2, where the pair of secondary flows tends to move toward the centres of smooth instead of rough strips as $S/\bar{\delta}$ decreases. Table 5.1 shows the estimated U_τ from composite profile. The smooth strips are made of nominally smooth cardboard, but which the composite fit suggests corresponds to $k_s^+ \approx 7$, close to the criteria for hydraulically smooth surface of $k_s^+ \leq 5$ (Nikuradse, 1933), and $\Delta U^+ < 1$. The current estimation method has an expected U_τ error range of $\pm 5\%$ (Squire *et al.*, 2016). In figure 3.4(a), the reference smooth wall cases have $U_\infty^+ \equiv U_\infty/U_\tau \approx 27$, which corresponds to an error range of ± 1.35 for ΔU^+ , and hence $\Delta U^+ < 1$ estimated for the smooth strips is still within the expected error range for the current method. Figure 5.8 shows that the estimated ΔU^+ for the rough strips for both SR250-1 and SR250-2 collapse to

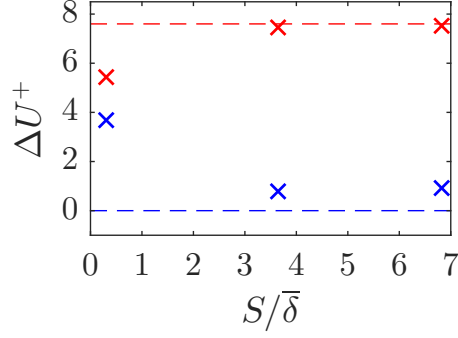


Figure 5.8: Estimated ΔU^+ for the reference cases and spanwise heterogeneous cases as a function of $S/\bar{\delta}$. Dashed lines are ΔU^+ of the reference smooth wall (---) and reference rough wall case (---). For the spanwise heterogeneous roughness cases: ΔU^+ at the centre of a smooth strip (x) and a rough strip (x).

the reference rough wall cases (red dashed line in figure 5.8), where the $\Delta U^+ \approx 7.5$ and the $k_s^+ \approx 90$. The equivalent sand grain roughness is estimated at $k_s \approx 2$ mm, similar to that obtained in Squire *et al.* (2016) for P-36 grit sandpaper. It is noted that the estimated ΔU^+ for the SR25 case ($S/\bar{\delta} \ll 1$) falls between the reference smooth and rough wall cases (figure 5.8).

Figure 5.6(c) shows the limiting case defect plots normalised by the local U_τ . The collapse to of the profile at the center of the smooth strip is quite good, but the collapse at the center of the rough shows a greater deviation. In some sense we would expect the collapse to be worse here, because for $z/\bar{\delta} > 0.32$, we have observed that the flow is spanwise homogeneous and so it is no longer in equilibrium with the local wall drag condition, but rather some aggregate value (presumably the spanwise average). To test this, figure 5.7 shows the same data, but here normalised by $\bar{U}_\tau$, the average of U_τ measured at the centre of a smooth ($y = 0$) and rough strip ($y/S = 1$). In this defect form the largest spanwise wavelengths (SR250-1 and SR250-2, figure 5.7a,b) do not exhibit collapse (as expected, since they are in approximate equilibrium with their *local* wall conditions), but the limiting case of $S/\delta = 0.32$ shows quite convincing collapse for $z/\bar{\delta} \gtrsim 0.32$ based on this spanwise averaged $\bar{U}_\tau$. It is noted that the estimated ΔU^+ falls somewhere between the reference smooth- and rough-wall case ($\Delta U^+ = 4.58$, averaged from ΔU^+ obtained at the center of a smooth and a rough strip). Taken together with $\bar{U}_\tau$ (U_τ for this surface is presumably of some aggregate value between the smooth and rough strips), these values correspond to $k_s^+ = 30.6$, which is within the transitional roughness region, $5 \leq k_s^+ \leq 70$ (Nikuradse, 1933).

5.2.3 Time-averaged strength of the secondary flows: mean swirl

The vectors of mean spanwise (V) and wall-normal (W) velocity component in figure 5.2 illustrate the secondary flows for all $S/\bar{\delta}$ cases. Figure 5.9 shows the swirl strength Λ_{ci} for all of the cases in table 3.1 normalized by freestream velocity and the boundary-layer thickness of the reference smooth-wall cases, U_∞/δ_s . The swirl is defined as the imaginary part of the complex eigenvalue of the mean velocity gradient tensor,

$$\frac{\partial U_i}{\partial x_j} = \begin{bmatrix} \frac{\partial V}{\partial y} & \frac{\partial V}{\partial z} \\ \frac{\partial W}{\partial y} & \frac{\partial W}{\partial z} \end{bmatrix} \quad (5.5)$$

The choice of using only the imaginary part of the complex eigenvalue enables the analysis of vortex flows without contamination from shear, unlike vorticity (Tomkins & Adrian, 2003). Physically, the axis along the eigenvectors of the velocity gradient tensor corresponding to the real eigenvalue represents either the stretching or compression of the flow, while the plane spanned by the complex pair of eigenvectors represents the swirling motion, of which the imaginary part represents the swirling strength (i.e., a scalar). The complete derivation and physical interpretation of the swirling strength are given in Chong *et al.* (1990) and Zhou *et al.* (1999). The direction of the vortex, either clockwise (CW) or counterclockwise (CCW), is obtained by multiplying the swirling strength by the sign of mean vorticity, $\Omega_x/|\Omega_x|$, $\Omega_x \equiv \partial W/\partial y - \partial V/\partial z$. CW secondary flows in figure 5.9(b–f) are shown by blue-filled contours and CCW is red. In this case, where there is very little variation in virtual origin between the rough and smooth patches, no reversal in the sign of the secondary flows, as suggested by Yang & Anderson (2018) and Stroh *et al.* (2020), is observed.

In figure 5.9(b–f), regions of non-zero swirl strength clearly show the counter-rotating large-scale mean secondary flows due to spanwise heterogeneity. The strength of the swirl appears to be dependent on $S/\bar{\delta}$, with the strongest swirl at case SR50 ($S/\bar{\delta} = 0.62$, figure 5.9e). The mean swirl then decays in strength and size as $S/\bar{\delta}$ decreases further to SR25 ($S/\bar{\delta} = 0.32$, figure 5.9f). To investigate this further, the strength of the mean secondary flows is quantified for every case by integrating $\Lambda_{ci}(\Omega_x/|\Omega_x|)$ across the spanwise location of the large-scale secondary flow, $y_c - l_y/2 \leq y \leq y_c + l_y/2$ (y_c is the spanwise location of the secondary flow, marked by © in figure 5.2) and the wall-normal location, $0 < z \leq l_y$, both obtained from

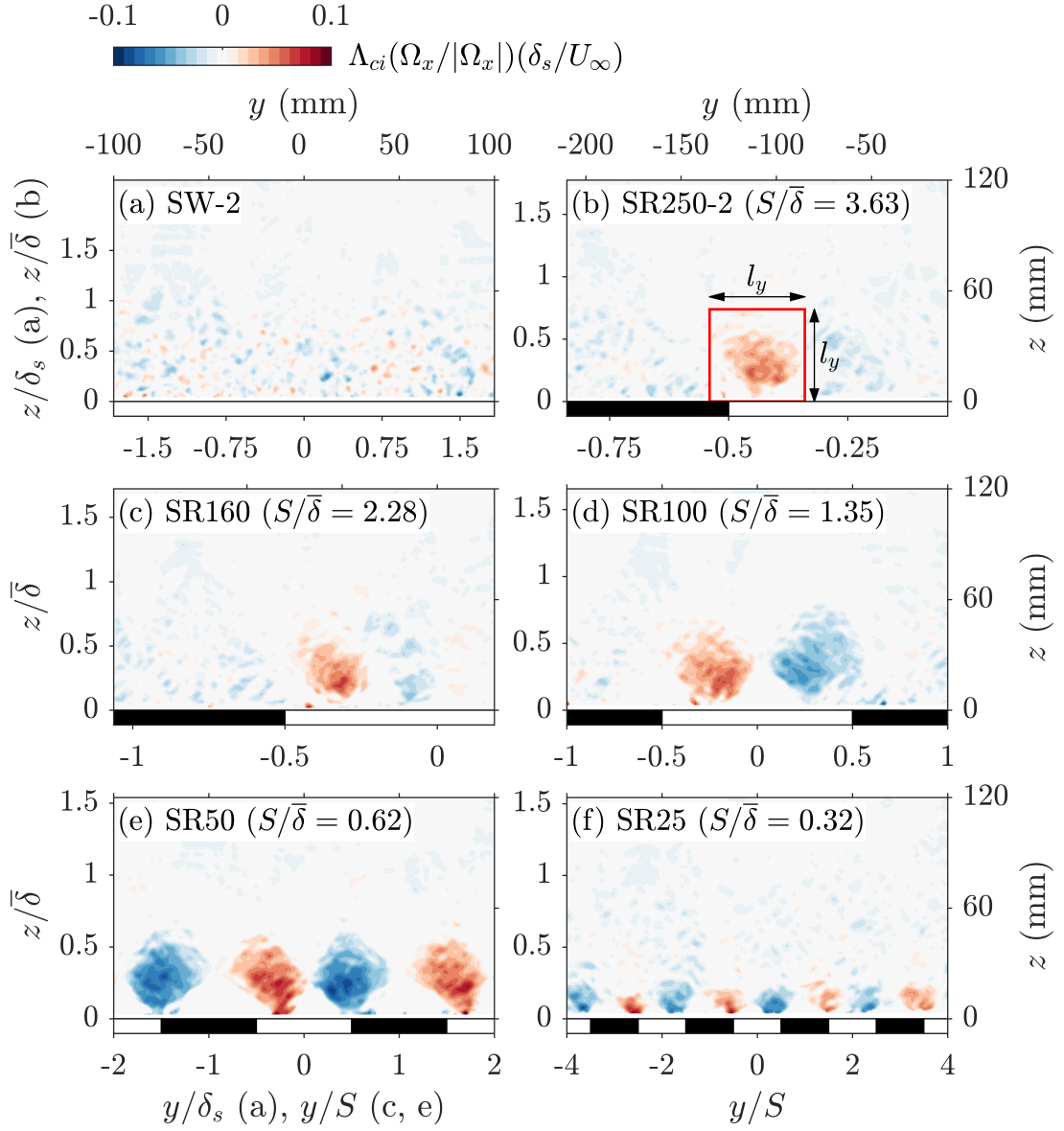


Figure 5.9: Large-scale secondary flows shown by the mean swirl strength multiplied by the sign of vorticity $\Lambda_{ci}(\Omega_x/|\Omega_x|)$ for cases (a) SW ($S/\delta_s = 0$), (b) SR250 ($S/\bar{\delta} = 3.63$), (c) SR160 ($S/\bar{\delta} = 2.28$), (d) SR100 ($S/\bar{\delta} = 1.35$), (e) SR50 ($S/\bar{\delta} = 0.62$), and (f) SR25 ($S/\bar{\delta} = 0.32$). Red indicates counterclockwise (CCW) roll modes, blue indicates clockwise (CW) roll modes. In (b), — illustrates the extent of the secondary flows.

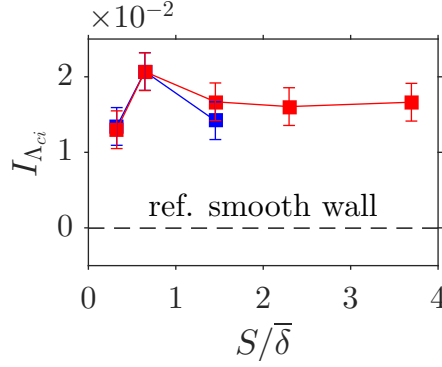


Figure 5.10: Plot of integrated mean swirl strength $I_{\Lambda_{ci}}$ as functions of $S/\bar{\delta}$. $I_{\Lambda_{ci}}$ of counterclockwise (■) and clockwise (■) roll modes inside the FOV (if any). Dashed line (---) is the reference smooth-wall case (SW-2). Error bars show the uncertainties of the measurement due to the strongest asymmetry between the CW and CCW swirl strength, which occurs here for case SR100 ($S/\bar{\delta} = 1.35$). See also appendix A.3 for error analysis of V and W components.

PIV results in figure 5.9,

$$I_{\Lambda_{ci}} = \frac{1}{N} \sum \frac{1}{l_y^2} \int_0^{l_y} \int_{y_c-l_y/2}^{y_c+l_y/2} \left| \Lambda_{ci} \frac{\Omega_x}{|\Omega_x|} \frac{\delta_s}{U_\infty} \right| dy dz \quad (5.6)$$

where l_y^2 is the area of integration given by equation (5.4) and marked by the red box in figure 5.9(b). For cases where a number (N) of roll modes are present in the FOV (cases SR25 and SR50), the integrated Λ_{ci} is averaged over N . For the reference smooth wall SW-2 case, Λ_{ci} is integrated over the entire FOV, $-2\delta_s \leq y \leq 2\delta_s$ and $0 \leq z \leq \delta_s$, yielding an integrated swirl strength $I_{\Lambda_{ci}}$ close to zero. Unfortunately, we are unable to compute the swirling strength of the reference rough-wall case RW-2 since we only have the U component from HWA data for this case (see table 3.1).

The integrated swirl strength $I_{\Lambda_{ci}}$ as a function of $S/\bar{\delta}$ in figure 5.10(a) confirms initial observations from figure 5.9. $I_{\Lambda_{ci}}$ is close to zero over a smooth wall (case SW-2), peaks when $S/\bar{\delta} \approx 1$, and decays as $S/\bar{\delta}$ increases further.

5.3 Comparison with DNS of channel flow at $\delta^+ = 590$

In this section, a comparison is made between the current experimental data and results from DNS of channel flow at $\delta^+ = 590$ by Chung *et al.* (2018) and Xie *et al.* (2018) over similar strip-type heterogeneous roughness, where δ is defined as the

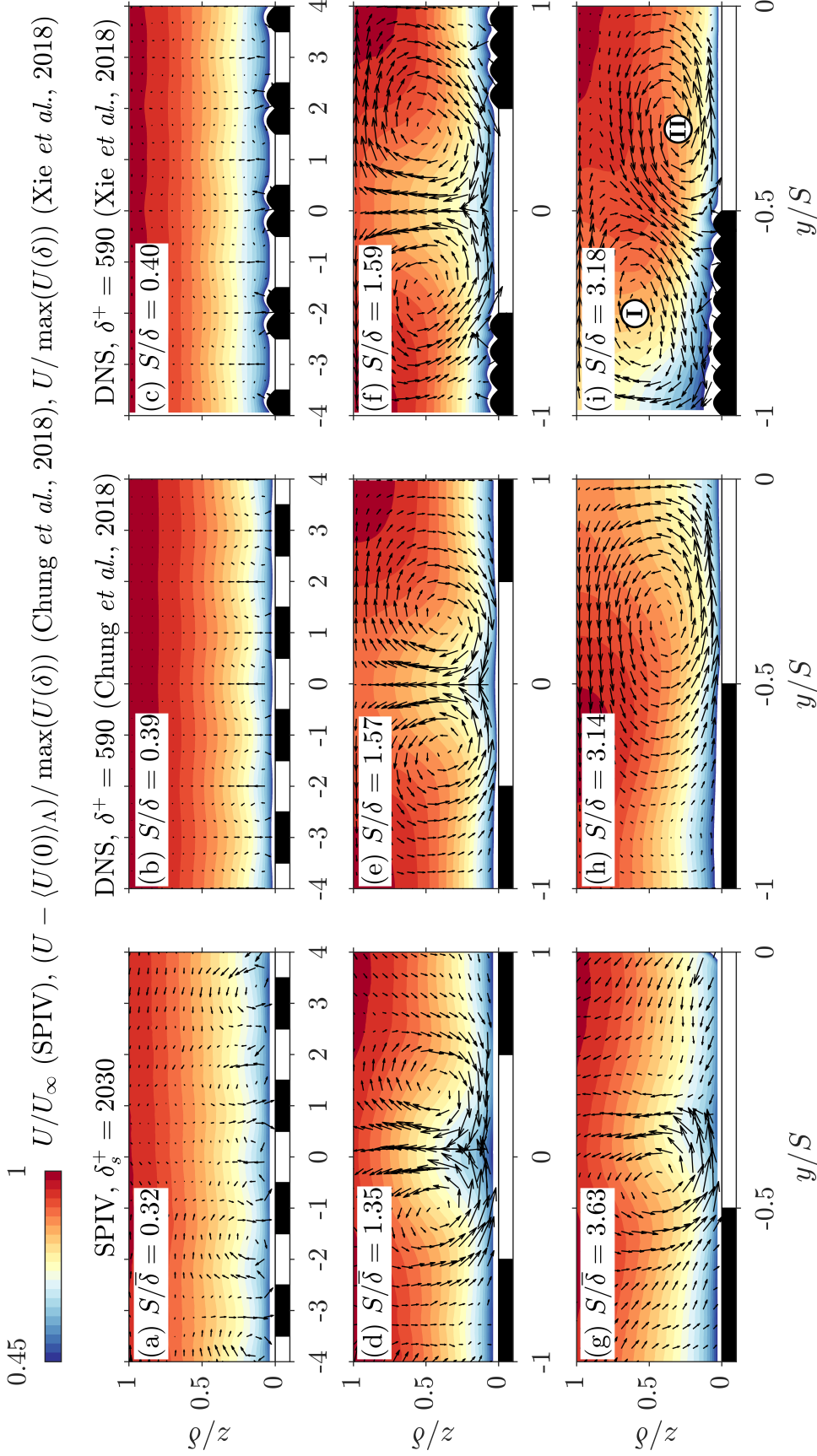


Figure 5.11: Contours of mean streamwise velocity U of spanwise heterogeneous roughness cases at various S/δ . Figures (a,d,g) are obtained from SPIV measurements of boundary layers at $\delta_s^+ = 2030$ for SW-2, (b,e,h) from DNS of channel flow at $\delta^+ = 590$ (Chung *et al.*, 2018), and (c,f,i) from DNS of open channel flow at $\delta^+ = 590$ (Xie *et al.*, 2018), where δ is defined as the channel half-height. White and black patch are the smooth (low wall shear stress) and rough (high wall shear stress) strips, respectively. Vectors indicate V and W , downsampled for clarity. Contours are normalised by U_∞ for SPIV data. For DNS data, contours show mean velocity relative to the wall, $U - \langle U(0) \rangle_\Lambda$, normalised by maximum U at channel half-height. Vectors of V and W are also relative to the wall, $V - \langle V(0) \rangle_\Lambda$ and $W - \langle W(0) \rangle_\Lambda$, respectively.

channel half-height. It is important to note here that the comparisons made in the following are between boundary-layer experiments (SPIV and HWA) and DNS of channel flow at lower δ^+ , and that both DNS are conducted over different types of spanwise heterogeneity. In Chung *et al.* (2018), the spanwise heterogeneity is imposed as a boundary condition in the DNS. The simulation domain comprises of strips of width S with spanwise-alternating uniform low and high shear stress magnitude, imposed via a Neumann boundary condition. In Xie *et al.* (2018), the spanwise variations of shear stress are imposed as roughness, with spanwise-alternating smooth strips and egg carton roughness corresponding to τ_s and τ_r , respectively. The egg carton roughness is the same surface investigated by Chan *et al.* (2015), and has the maximum height of $k/\delta = 0.056$ and wavelength of $7.1k$. In the present study, spanwise variation of shear stress are imposed as roughness, with spanwise-alternating strips of cardboard and sandpaper, where $k/\delta_s = 0.016$.

Figure 5.11 shows the side by side comparison of mean streamwise velocity U between boundary layer measurements (SPIV) and both available channel flow DNS. Three cases are selected from each dataset at the closest available $S/\bar{\delta}$ ratios to represent the limiting cases: $S/\bar{\delta} \ll 1$ (SPIV: $S/\bar{\delta} = 0.32$, DNS: $S/\delta = 0.39, 0.40$), $S/\bar{\delta} \approx 1$ (SPIV: $S/\bar{\delta} = 1.35$, DNS: $S/\delta = 1.57, 1.59$), and $S/\bar{\delta} \gg 1$ (SPIV: $S/\bar{\delta} = 3.63$, DNS: $S/\delta = 3.14, 3.18$). Mean secondary flows are shown on top of the contours as vectors of V and W . For the current SPIV data, U is normalised by U_∞ . For the DNS data of Xie *et al.* (2018), U , V , and W are normalised by maximum $U(\delta)$ (maximum U at channel half-height, $z = \delta$). For the DNS data of Chung *et al.* (2018), due to the slip boundary condition at the wall, U , V , and W are relative to the wall, $U(z) - \langle U(0) \rangle_\Lambda$, where $\langle U(0) \rangle_\Lambda$ is the spanwise-averaged mean velocity at wall ($z = 0$), and are normalised by the maximum $U(\delta)$. For both PIV and DNS data, the plots are shown within the same spanwise extent.

The cases for $S/\bar{\delta} \ll 1$ are shown in figure 5.11(a–c) for SPIV ($S/\bar{\delta} = 0.32$) and DNS ($S/\delta = 0.39, 0.40$), respectively. The contours of U and vectors of V and W show that the secondary flows behave similarly in both experiments and DNS. The direction of the counter-rotating pair of roll modes is the same (upwelling above “smooth” strips and downwelling above “rough” strips), and these roll modes are also limited by the size of S and are confined near the wall for both SPIV and DNS. A reasonable agreement between DNS and experiments is maintained when $S/\delta \approx 1$, $S/\bar{\delta} = 1.35$ (DNS: $S/\delta = 1.57, 1.59$) up to $z/\bar{\delta} \approx 0.8$ (figure 5.11d–f). However, we note that there are discrepancies between the experimental and DNS results in the outer region, especially closer to $z = \delta$. These discrepancies are expected

due to the geometry effects (both DNS are channel flows, each driven in a slightly different manner, while the current experiments are a developing ZPG turbulent boundary layer), and the manner in which the boundaries are imposed in the DNS. For turbulent boundary layers, the freestream velocity U_∞ must return to the same value over the rough and smooth patches, even when $S/\delta \gg 1$. However, for both channels, depending on how they are driven (and depending on the blockage due to the roughness k/δ) this is not the case. It should be noted that the DNS of channel flow in (Chung *et al.*, 2018) is driven by the wall, while in Xie *et al.* (2018), it is driven by a constant pressure gradient.

As S increases further to the limit of $S/\bar{\delta} \gg 1$, vectors of V and W from SPIV (figure 5.11g) show that the secondary flow remains confined near the roughness transition ($y = 0.5$) and the diameter is approximately capped by $\bar{\delta}$ in both spanwise and wall-normal extent. In the contours obtained from DNS in figure 5.11(h,i), however, the secondary flow height is capped by channel half-height but stretched in the spanwise direction such that common flow up is at the center of the smooth strip. In Xie *et al.* (2018) (figure 5.11i), the secondary flow is flanked by counter-signed rotation above the egg carton roughness (see label ① and ② in figure 5.11i). Chung *et al.* (2018) note that their DNS results (figure 5.11b,e,f) exhibit two possible types of secondary flows based on Bradshaw (1987), depending on S/δ : ‘type (a)’ or ‘identifiable streamwise vortices’ when $S/\delta \lesssim 1.57$ ($\partial V/\partial z$ has the same order as $\partial W/\partial y$) and ‘type (b)’ or ‘cross-flow’ when $S/\delta \gtrsim 3.14$ ($\partial V/\partial z \gg \partial W/\partial y$). No such distinction is observed in the boundary-layer measurements, where the secondary flows belong to type (a) in all $S/\bar{\delta}$ cases. It should be noted, however, that this is not a one-to-one comparison. DNS results presented here are of channel flow instead of boundary layer, which will lead to differences as $z \rightarrow \delta$, and the Re is roughly 4 times smaller for the DNS than the boundary-layer experiments. How the simulations treat surface roughness may also lead to this discrepancy. The roughness in Chung *et al.* (2018) is imposed as Neumann boundary condition instead of a ‘real’ roughness and the no-slip boundary condition is not satisfied on the wall. This leads to a difference in the freestream over the high stress (rough) or the low stress strips (smooth) for $S/\delta \gg 1$ (figure 5.11h). For the SPIV data, measured in an evolving turbulent boundary layer, the freestream velocity becomes invariant in y for $z/\delta > 1$ and so these two have very different upper boundary conditions. In Xie *et al.* (2018), the size of roughness element k/δ is approximately five times larger than that of the current study. Because the flow is driven by a constant pressure gradient (instead of an evolving ZPG boundary layer), in the limit of $S/\delta \gg 1$ the wall shear stress

across the smooth and rough strips approach spanwise homogeneity while a strong variation in the centerline velocity above the smooth and the rough strips is observed at the channel half-height. The opposite occurs in the current study: the smooth and rough strips approaches their respective spanwise homogeneous roughness condition (reference smooth and rough wall, respectively) as shown in §5.2.2, but $U = U_\infty$ in the freestream, which may explain the noticeable differences in secondary flows between the current study and the study of Xie *et al.* (2018).

Other comparisons between boundary-layer experiments and boundary-layer DNS are available based on previous studies. Boundary-layer measurements over ridge-type roughness in Medjnoun *et al.* (2018) ($1000 \lesssim \delta^+ \lesssim 3000$, $0.4 \leq L/(2\delta) \leq 1.6$) showed similar flow topology to some cases in boundary-layer DNS of Hwang & Lee (2018) ($\delta^+ \sim O(10^2)$, $0.3 \leq L/(2\delta) \leq 1.2$), where L is the spacing between ridges. Similar conclusion has also been drawn in Vanderwel *et al.* (2019), when comparing boundary-layer SPIV measurements ($\delta^+ = 4700$) to boundary-layer DNS ($\delta^+ \approx 500$) at $L/(2\delta) = 0.44$. For these cases the outer geometry of the flows are matched and k/δ is comparable, even though the Re numbers are very different. Together this implies that it is difference in geometry and the manner that the roughness is imposed that lead to the differences observed in figure 5.11.

Chapter 6

Turbulence statistics

6.1 Reynolds stresses and turbulent kinetic energy

6.1.1 Normal stresses

The contours of time-averaged normal stresses in the cross-plane for all three velocity components, $\overline{u'u'}$, $\overline{v'v'}$, and $\overline{w'w'}$, normalised by U_∞^2 are shown in figure 6.2, 6.3, and 6.4, respectively. The contours are obtained from SPIV measurements for the reference smooth wall case SW-2 and all SR cases at $x = 4$ m. The profiles of $\overline{u'u'}$ as a function of $z/\bar{\delta}$ are also available from HWA measurements in figure 6.1. As with the profiles of U in figure 5.3, the limiting and intermediate cases are also observed through $\overline{u'u'}$ profiles. The limiting case $S/\bar{\delta} \gg 1$, represented by figure 6.1(a,b), shows less energetic turbulence above the smooth patch compared to the rough patch, as expected for the homogeneous smooth and rough reference cases (SW and RW). The profiles at the centre of the smooth and rough strips (far removed from the secondary flow) nearly collapse to the reference cases. It should be noted that the discrepancy in the profiles and the inner peak of $\overline{u'u'}$ of case SR250-1 and SR250-2 (figure 6.1a,b) is maybe partially due to error while measuring the wall-normal location of the hot-wire probe in the near-wall region and also error during hot-wire etching with nitric solution to remove the silver coating. This latter issue can result in ambiguity in terms of the final attained spanwise length of the wire and the length-to-diameter ratio, both of which can effect the measured intensities (Ligrani & Bradshaw, 1987). At the other limit where $S/\bar{\delta} \ll 1$ (figure 6.1f), $\overline{u'u'}$ is approximately spanwise homogeneous for $z/\bar{\delta} \gtrsim 0.5$. Comparing the turbulence intensity and the mean velocity profiles for the $S/\bar{\delta} \ll 1$ case (figure 6.1f and 5.3f) reveals that the spanwise homogeneity for this smallest

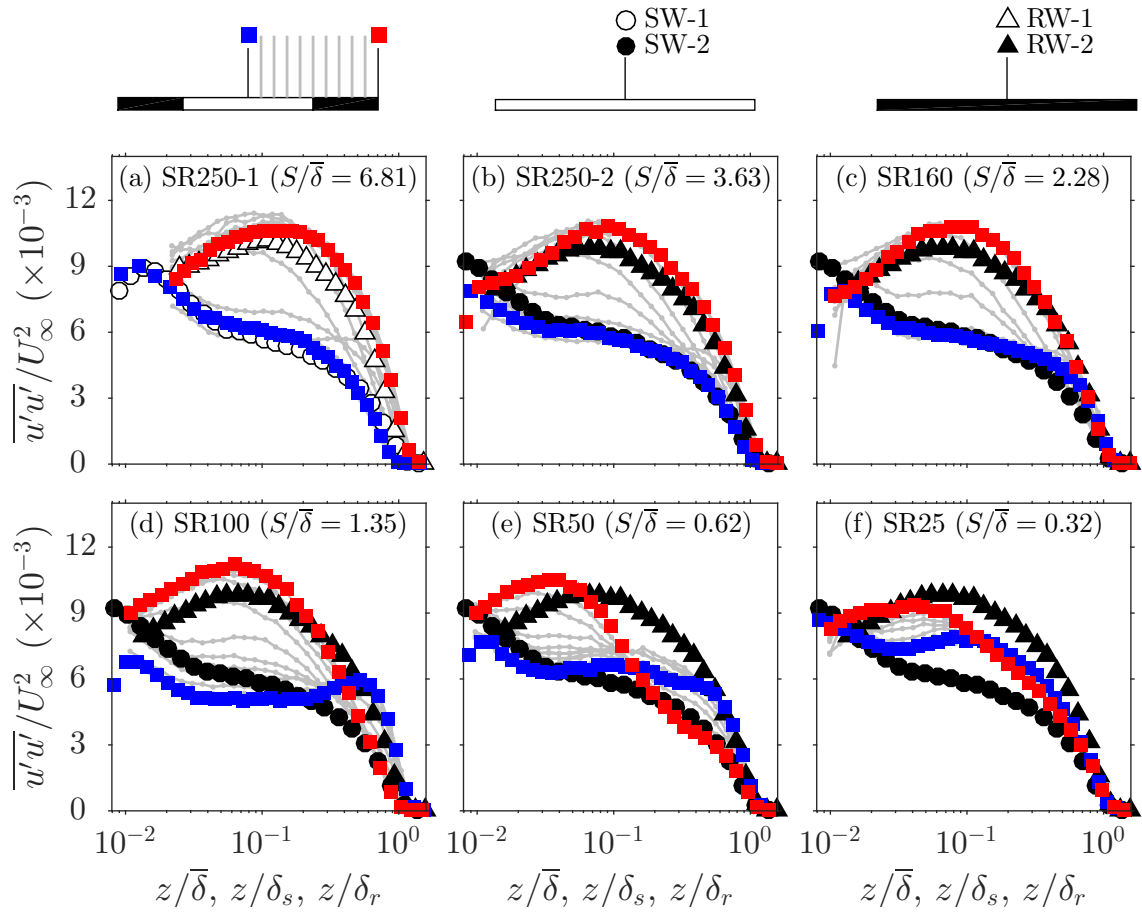


Figure 6.1: Reynolds stress $\overline{u'u'}/U_\infty^2$ for SR cases. Reference case SW and RW are plotted for comparison. Legends are on top of the page.

wavelength case occurs further from the wall for the turbulence intensity ($z/\bar{\delta} \gtrsim 0.5$) than for the mean velocity ($z/\bar{\delta} \gtrsim 0.32$). This difference is also seen in Chan *et al.* (2018). For both limiting cases ($S/\bar{\delta} \gg 1$ and surprisingly for $S/\bar{\delta} \ll 1$), the near-wall profiles ($z/\bar{\delta} \lesssim 0.05$) over the center of the rough and smooth patches conform closely to the reference cases.

For the intermediate cases where $S/\bar{\delta} \approx 1$ (figure 6.1d,e), we see departures in the near-wall region from the rough and smooth reference cases over the centres of the patches. In this regime, the near-wall intensity over the smooth strip is reduced as compared to SW, and the near-wall intensity over the rough strip is elevated as compared to RW. As with the mean velocity profiles (figure 5.3d,e), we see an apparent reversal in behaviour away from the wall for $\overline{u'u'}$. For these spanwise wavelengths, the outer region of the boundary layer ($z/\bar{\delta} > 0.1$) exhibits increased turbulence intensity above the smooth patch ($z/\bar{\delta} \gtrsim 0.5$ for SR100 and $z/\bar{\delta} \gtrsim 0.1$ for SR50), which approximately corresponds to the wall-normal location of the center of secondary flows.

Unlike the profiles of $\overline{u'u'}$ in figure 6.1, the near-wall profiles are not captured by the contours of normal stresses in figure 6.2, 6.3, and 6.4. However, a clearer picture on how the normal stresses are affected by large-scale secondary flows can still be shown by these contours. In general, $\overline{u'u'}$, $\overline{v'v'}$, $\overline{w'w'}$, and the turbulent kinetic energy (TKE) $K = (\overline{u'u'} + \overline{v'v'} + \overline{w'w'})/2$ (figure 6.5) show similar behaviour. The contours of SW-2 (figure 6.2a, 6.3a–6.5a) are spanwise homogeneous, as expected for the reference smooth wall case. All contours for spanwise heterogeneous roughness cases show higher normal stresses and TKE over the rough strips close to the wall, which confirms the observation from $\overline{u'u'}$ profiles (figure 6.1) that the flow conforms to surface conditions near the wall. However, away from the wall, the wall-normal extent of high normal stresses above the rough strips deteriorates with decreasing $S/\bar{\delta}$. For the cases where $S/\bar{\delta} \gg 1$ (figure 6.2b–d, 6.3b,c–6.5b,c), the rough strips are characterised by high normal stresses up to $z/\bar{\delta} \lesssim 0.5$. Near the roughness transition, the wall-normal extent of high normal stresses above the smooth strips decreases as a peak emerges at the edge of adjacent smooth strips. The contours show that this coincides with the thinning and thickening of the boundary-layer thickness, which correspond to the common flow down and up, respectively (see the vectors of V and W in figure 6.2–6.5). As $S/\bar{\delta}$ approaches 1 (figure 6.2e,f, 6.3d,e–6.5d,e), the peak in common flow up regions is stronger (this manifests as the outer layer peak in the $\overline{u'u'}$ profiles, figure 6.1d,e) while the wall-normal extent of high normal stresses decreases further across the spanwise extent of the rough strips, until a reversal is

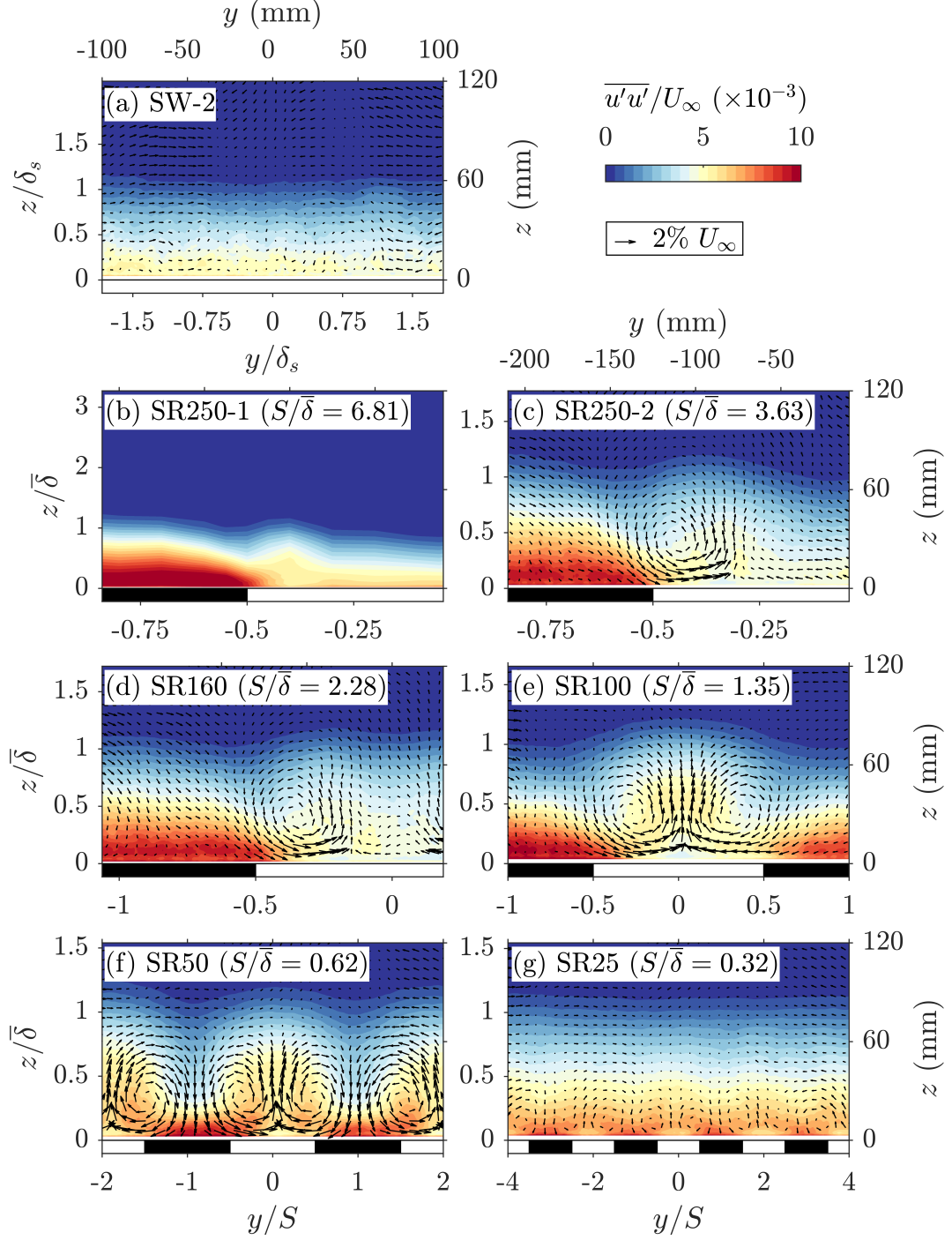


Figure 6.2: Contours of Reynolds stress $\overline{u'u'}$ for SR cases and reference case SW-2. Contours in (a, c–g) are plotted from SPIV results, while (b) is from HWA. Vectors indicate V and W , downsampled for clarity.

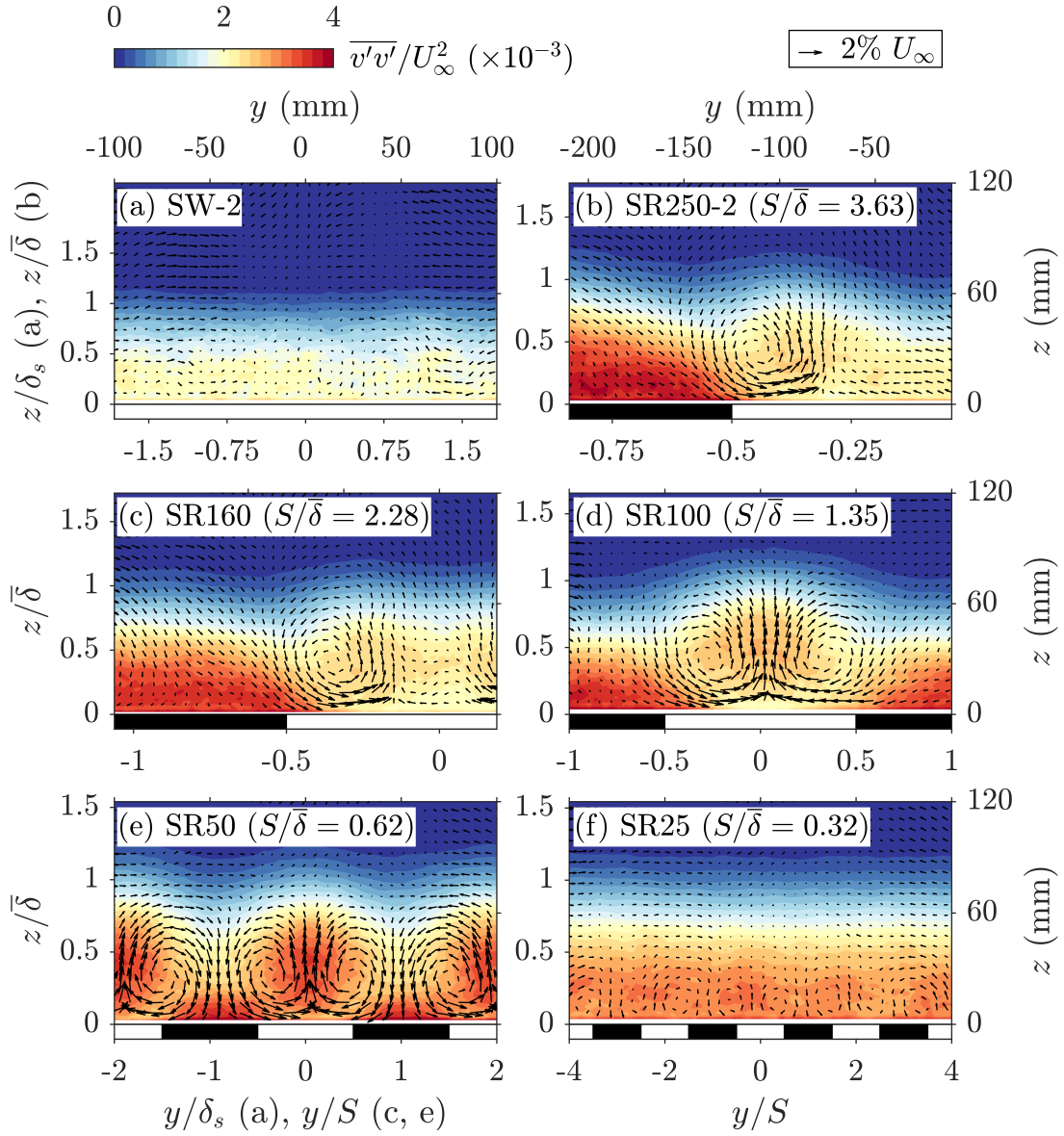


Figure 6.3: Contours of Reynolds stress $\overline{v'v'}$ for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Contours are plotted from SPIV results. Vectors indicate V and W , downsampled for clarity.

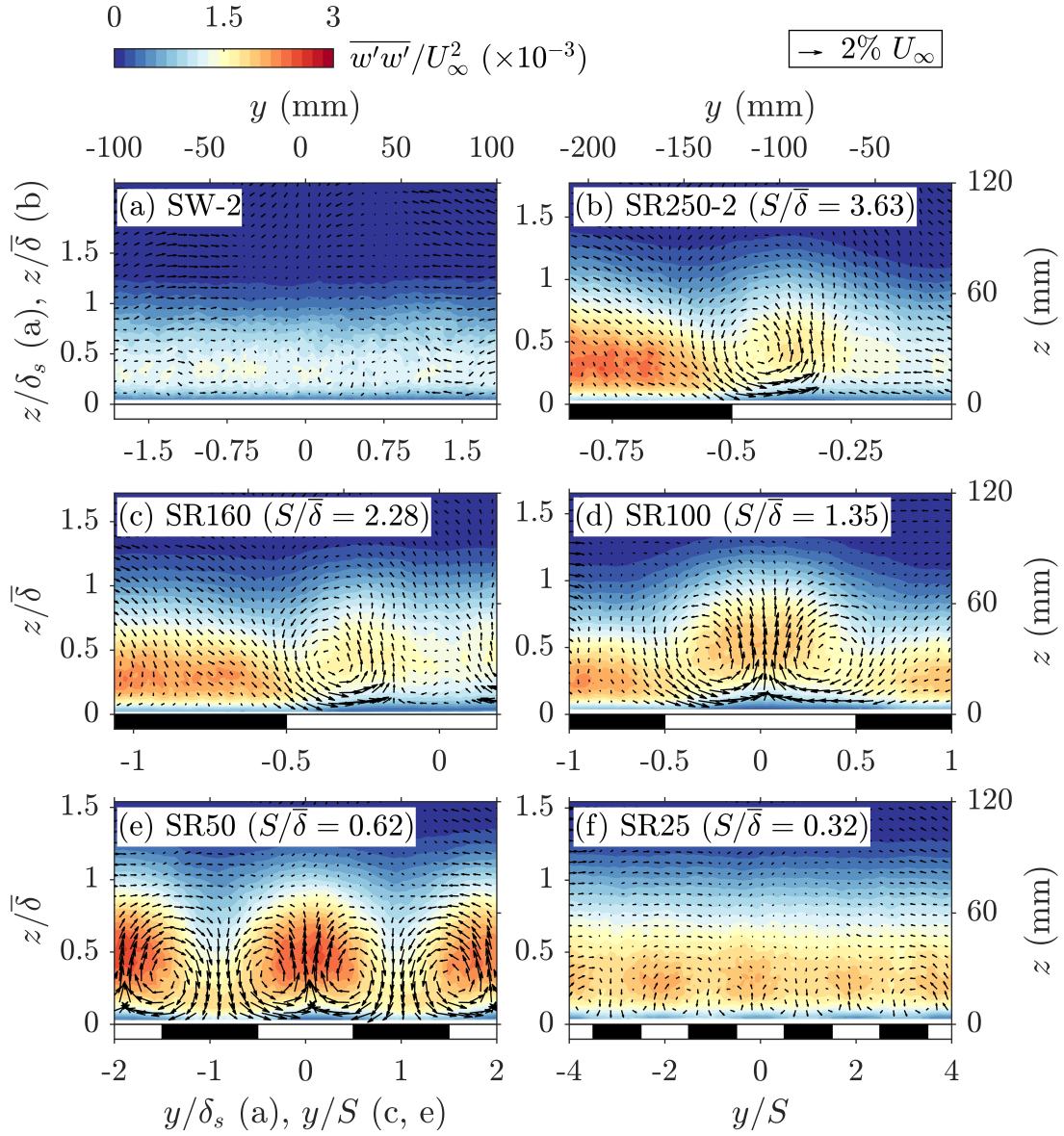


Figure 6.4: Contours of Reynolds stress $\overline{w'w'}$ for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Contours are plotted from SPIV results. Vectors indicate V and W , downsampled for clarity.

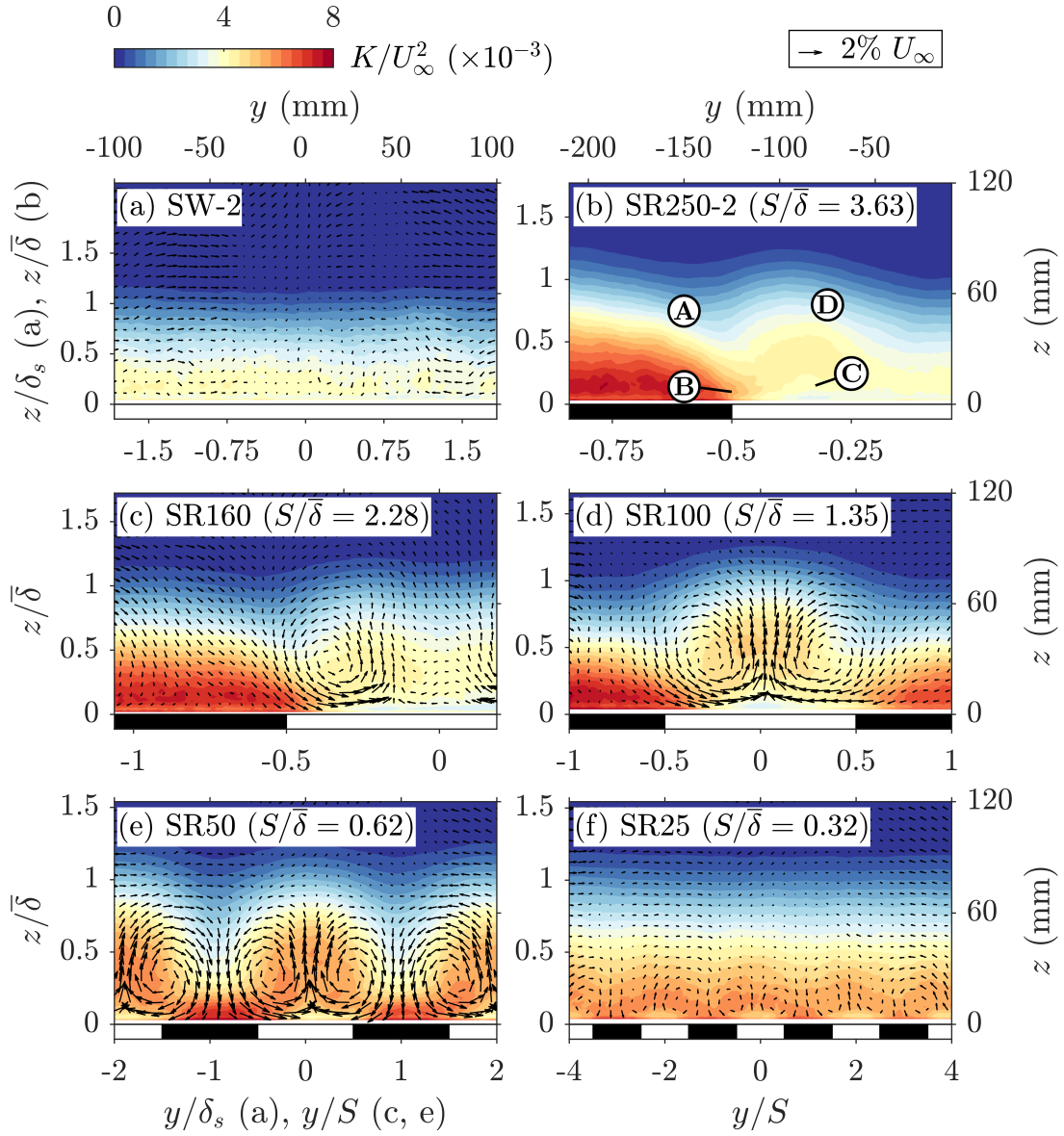


Figure 6.5: Contours of turbulent kinetic energy tke for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Contours are plotted from SPIV results. Vectors indicate V and W , downsampled for clarity.

observed above the surface (high stresses above the smooth strips). Anderson *et al.* (2015) attributed this pattern to the redistribution of TKE due to the presence of large-scale secondary flows. Finally, a nearly spanwise homogeneous behaviour is observed when $S/\bar{\delta} \ll 1$ (figure 6.2g, 6.3f–6.5f), although near the wall, the flow still follows the surface condition.

6.1.2 Shear stresses

The contours of time-averaged Reynolds shear stresses in the cross-plane for all three velocity components, $-\overline{u'w'}$, $\overline{u'v'}$, and $\overline{v'w'}$, normalised by U_∞^2 are shown in figure 6.6, 6.7, and 6.8, respectively. The contours are obtained from SPIV measurements for the reference smooth wall case SW-2 and all SR cases at $x = 4$ m. The contours of $-\overline{u'w'}$ (figure 6.6) show a similar pattern to the normal stresses, with typically high shear stress above the rough strips. As $S/\bar{\delta}$ approaches 1, the wall-normal extent of high shear stress diminishes above the rough strips and a peak above the smooth strips, corresponding to the common flow up ($W > 0$, see the vectors in figure 6.6), strengthens in the outer layer ($z/\bar{\delta} \approx 0.5$ in figure 6.6d,e). Figure 6.8 shows the magnitude of $\overline{v'w'}$ is the least compared to $\overline{u'w'}$ and $\overline{u'v'}$ (a comparison of the magnitude of each Reynold stress component for one spanwise heterogeneous roughness case is shown in figure 6.9). In Anderson *et al.* (2015), it was estimated that the order of magnitude of $\overline{v'w'}$ is about a decade lower than $-\overline{u'w'}$ and $\overline{u'v'}$. Similar patterns of $-\overline{u'w'}$ and $\overline{v'w'}$ were also observed for other types of spanwise heterogeneous roughness: strip-type roughness with imposed boundary condition (Anderson *et al.*, 2015), ridge-type roughness (Hwang & Lee, 2018), a combination of ridge- and strip-type (Vanderwel & Ganapathisubramani, 2015), and C-D riblets (Kevin *et al.*, 2017).

Figure 6.7 shows the contours of $\overline{u'v'}$, which in some sense approximately resemble the pattern of time-averaged large-scale secondary flows, shown as the mean swirl strength in figure 5.9. These contours are close to zero for the reference smooth wall case SW-2 (figure 6.7a) but marked by spanwise periodicity for the spanwise heterogeneous cases (figure 6.7b–f). The red-filled contours coincide with the occurrence of CCW roll modes, while the blue-filled with the CW roll modes. Willingham *et al.* (2014) suggested that $\overline{u'v'}$ is responsible for sustaining the spanwise momentum transfer near roughness transition, which results in the mean large-scale secondary flows. It is also noted that the magnitude and size of $\overline{u'v'}$ are a function $S/\bar{\delta}$, similar to those observed in the mean swirl strength figure 5.9) and the trendline in

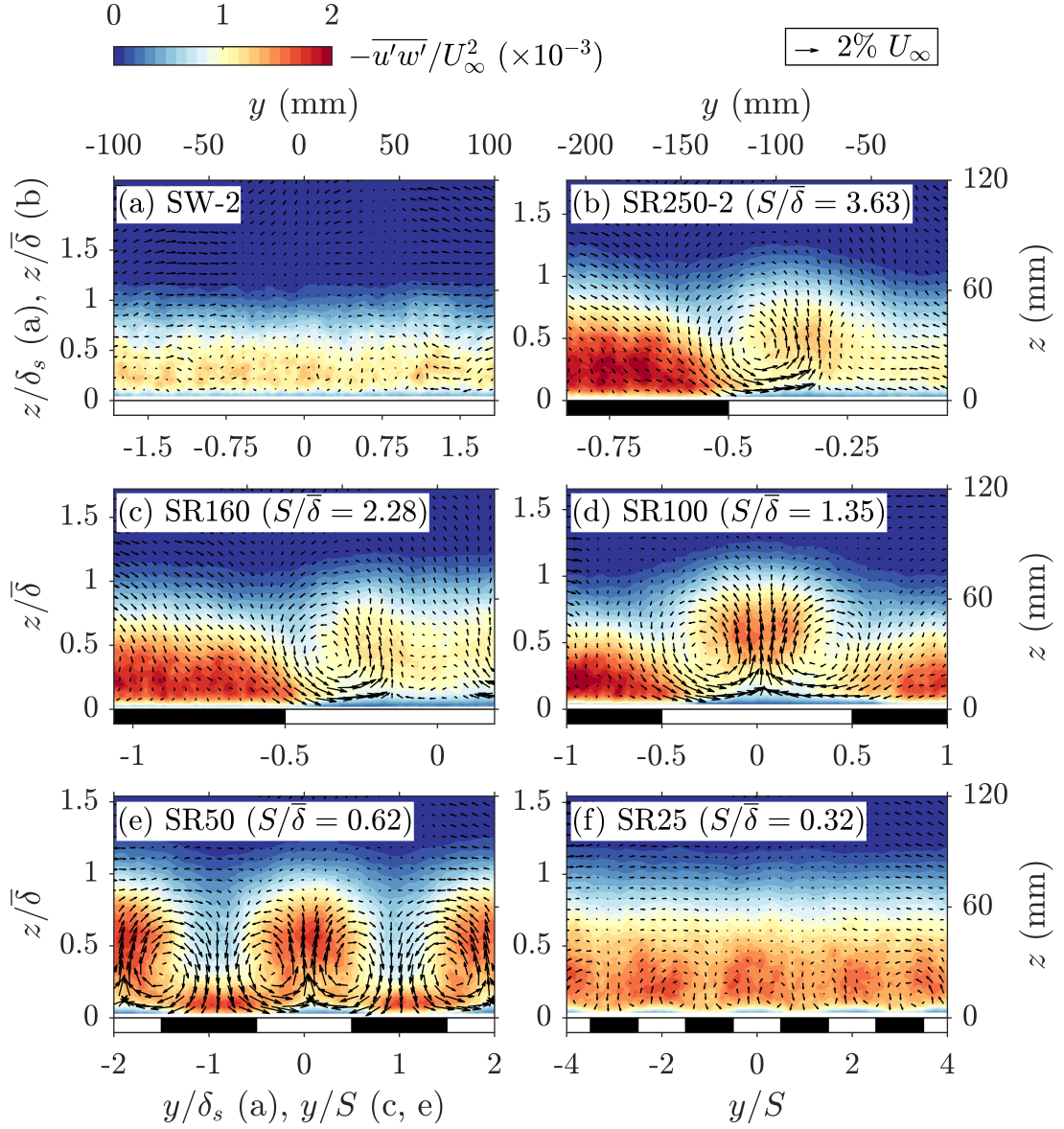


Figure 6.6: Contours of Reynolds shear stress $-\overline{u'w'}$ for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Contours are plotted from SPIV results. Vectors indicate V and W , downsampled for clarity.

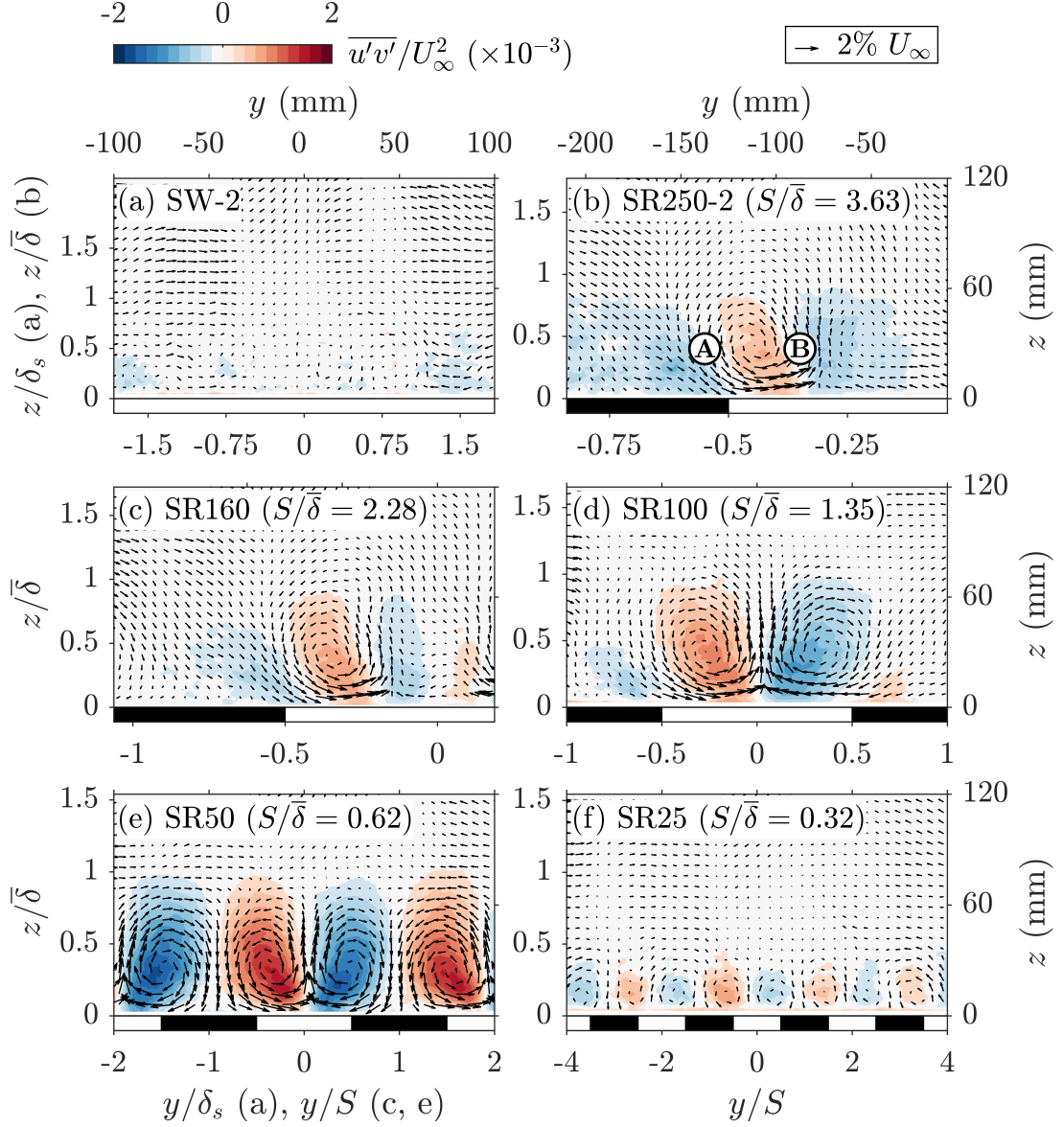


Figure 6.7: Contours of Reynolds shear stress $\overline{u'v'}$ for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Contours are plotted from SPIV results. Vectors indicate V and W , downsampled for clarity.

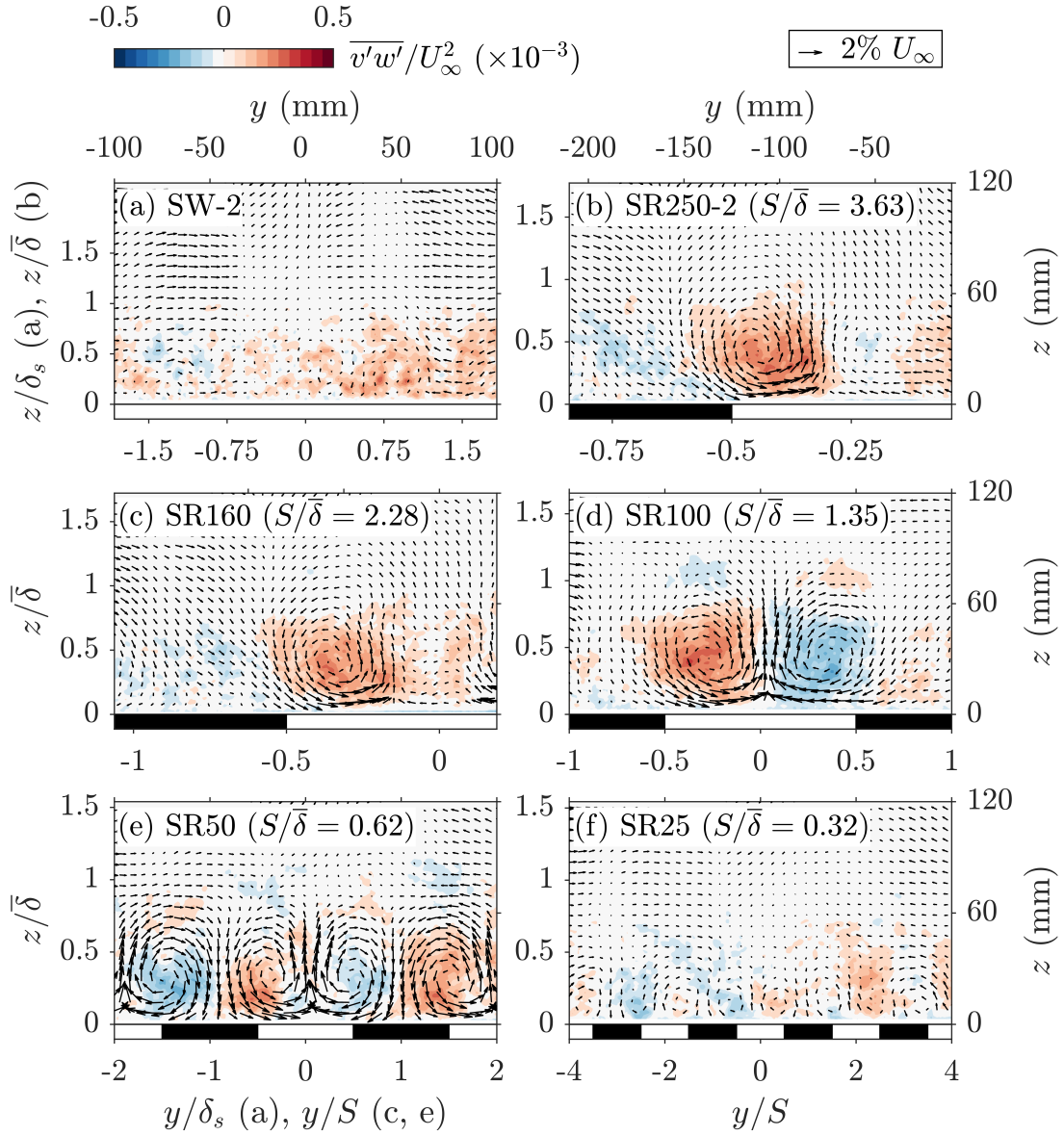


Figure 6.8: Contours of Reynolds shear stress $\overline{v'w'}$ for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Contours are plotted from SPIV results. Vectors indicate V and W , downsampled for clarity.

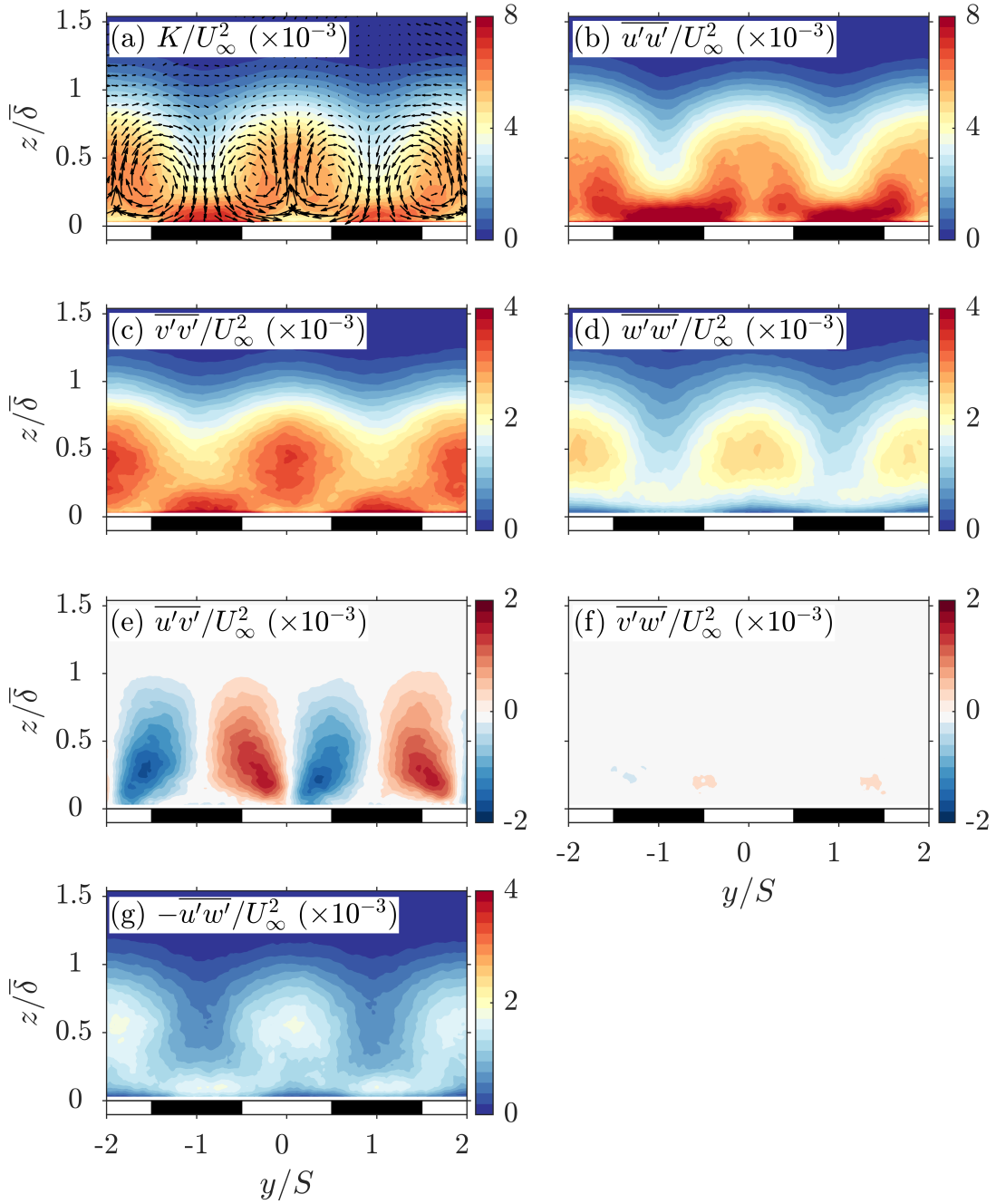


Figure 6.9: Contours of TKE and Reynolds stresses for SR50 case: (a) TKE, (b) $\overline{u'u'}$, (c) $\overline{v'v'}$, (d) $\overline{w'w'}$, (e) $\overline{u'v'}$, (f) $\overline{v'w'}$, and (g) $-\overline{u'w'}$. Contours are plotted from SPIV results. In (a), vectors indicate V and W , downsampled for clarity.

figure 5.10. The magnitude of $\overline{u'v'}$ is the strongest at SR50 case ($S/\bar{\delta} = 0.62$, figure 6.7e), while the strength and size decrease as $S/\bar{\delta}$ decreases further (case SR25, figure 6.7).

A possible interpretation of the contours of $\overline{u'v'}$ is that the spanwise periodic pattern of $\overline{u'v'}$ is caused by the time-dependent behaviour of the secondary flows. While the time-averaged secondary flows appear as a counter-rotating streamwise vortices (figure 5.9), they have been shown to emulate the instantaneous behaviour of the large-scales structures that occur naturally in the wall-bounded turbulence (Kevin *et al.*, 2019b; Vanderwel *et al.*, 2019; Zampiron *et al.*, 2020). These large-scales structures meander along x and, when viewed through yz -plane, lean sideways in y -direction. A hint of this behaviour is given in §7 and explored further in §9. Figure 6.7(b) shows the mean secondary flows (plotted as V and W vectors) on top of the contours of $\overline{u'v'}$. The location of common flow down and up is marked by **(A)** and **(B)**, respectively. Assuming that instantaneously, the secondary flows are spanwise locked and meander (lean sideways) about this location, the occurrence of a high-speed streak ($u' > 0$) in **(A)** coincides with $w' < 0$ (downward flow, as shown in the positive contours of $-\overline{u'w'}$ in figure 6.6). This event can be accompanied by either $v' > 0$ (structure leans to the right, resulting in positive $u'v'$ on the right hand side of **(A)**) or $v' < 0$ (structure leans to the left, negative $u'v'$ on the left of **(A)**). Similarly, the occurrence of a low-speed streak ($u' < 0$) in **(B)** coincides with $w' > 0$ (upward flow) and is accompanied by either $v' > 0$ (negative $u'v'$ on the right of **(B)**) or $v' < 0$ (positive $u'v'$ on the left of **(B)**). It is noted that the size and magnitude of the red and blue filled contours of $\overline{u'v'}$ are approximately similar within each cases, with the intermediate cases ($S/\bar{\delta}$) showing the strongest magnitude among all cases. However, in figure 6.7(d) a strong pair of red and blue filled contours (which coincide with CCW and CW roll mode, respectively) is flanked by a much weaker and smaller one. It is possible that as $S \approx \bar{\delta}$ and the secondary flows are stronger, the occurrence of downward flow ($w' < 0$ and $u' > 0$) at $y/S = -0.5$ (figure 6.7d) is strongly accompanied by $v' > 0$ closer to the wall rather than $v' < 0$. On its counter-rotating pair, the occurrence of downward flow at $y/S = 0.5$ is strongly accompanied by $v' < 0$ closer to the wall. Another possible interpretation of this weaker pattern is the emergence of smaller and weaker counter-rotating pairs of secondary flows (typically called ‘tertiary’ flow in the previous studies) between the ‘main’ counter-rotating secondary flows, as shown in Goldstein & Tuan (1998); Vanderwel & Ganapathisubramani (2015); Zampiron *et al.* (2020). However, it is noted that the weaker mean secondary flows have only been observed in ridge-type

roughness and are not visible in the mean swirl strength plots in figure 5.9 for strip-type roughness.

6.2 Energy balance

The complete Reynolds-averaged TKE transport equation is given as

$$\frac{\partial K}{\partial t} + U_j \frac{\partial K}{\partial x_j} = - \underbrace{\frac{\partial}{\partial x_j} \frac{\overline{p u'_j}}{\rho}}_{\mathcal{H}} - \underbrace{\frac{\partial}{\partial x_j} \frac{\overline{u'_i u'_i u'_j}}{2}}_{\mathcal{T}} + 2\nu \underbrace{\frac{\partial \overline{s_{ij} u'_i}}{\partial x_j}}_{\mathcal{D}} - \underbrace{\overline{u'_i u'_j} \frac{\partial U_i}{\partial x_j}}_{\mathcal{P}} - \varepsilon \quad (6.1)$$

where the second term on the left hand side is the advection of TKE by the mean flow, $\mathcal{H}$, $\mathcal{T}$, and $\mathcal{D}$ are the transport of TKE by pressure fluctuations, turbulent fluctuations, and the viscous stresses, respectively, $\mathcal{P}$ is the production term, and ε is the dissipation term, $\varepsilon \equiv 2\nu \overline{s_{ij} s_{ij}}$, s_{ij} is the rate-of-strain tensor (symmetric components of the gradient of u'_i), $s_{ij} = (\partial u'_i / \partial x_j + \partial u'_j / \partial x_i) / 2$. The term $\mathcal{D}$ is only significant very close to the wall (Hinze, 1967) and the pressure and turbulent transport term, $\mathcal{H}$ and $\mathcal{T}$, are only significant at the center of a pipe (Laufer, 1954). Furthermore, $\mathcal{T}$ is less significant in strip-type roughness (as opposed to ridge-type roughness) due to the lack of change in spanwise elevation (Hwang & Lee, 2018). Assuming steady and streamwise uniform flow ($\partial(\cdot)/\partial t = \partial(\cdot)/\partial x = 0$), Anderson *et al.* (2015) reduced equation (6.1) to

$$\underbrace{V \frac{\partial K}{\partial y}}_{\mathcal{C}_y} + \underbrace{W \frac{\partial K}{\partial z}}_{\mathcal{C}_z} \approx - \underbrace{\overline{u' v'} \frac{\partial U}{\partial y}}_{\mathcal{P}_{u'v'}} - \underbrace{\overline{u' w'} \frac{\partial U}{\partial z}}_{\mathcal{P}_{u'w'}} - \underbrace{\overline{v' w'} \left(\frac{\partial V}{\partial y} + \frac{\partial W}{\partial y} \right)}_{\mathcal{P}_{v'w'}} - \underbrace{\overline{v' v'} \frac{\partial V}{\partial y}}_{\mathcal{P}_{v'v'}} - \underbrace{\overline{w' w'} \frac{\partial W}{\partial z}}_{\mathcal{P}_{w'w'}} - \varepsilon \quad (6.2)$$

The total production $\mathcal{P}$ is the sum of all the production terms

$$\mathcal{P} = \mathcal{P}_{u'v'} + \mathcal{P}_{u'w'} + \mathcal{P}_{v'w'} + \mathcal{P}_{v'v'} + \mathcal{P}_{w'w'} \quad (6.3)$$

Hinze (1967) further reduced equation (6.2) based on the most dominant production terms to

$$\mathcal{C}_y + \mathcal{C}_z \approx \mathcal{P}_{u'v'} + \mathcal{P}_{u'w'} - \varepsilon \quad (6.4)$$

For $S/\delta \approx 1$, $V \approx 0$ at the center of smooth and rough strips, and $\partial K / \partial z < 0$. Hence, the sign of W depends on whether $\mathcal{P}$ exceeds ε or not so that the energy balance is maintained. In the region where $\mathcal{P} < \varepsilon$ (the smooth strips), there is

a transport of highly turbulent flow to this region, which corresponds to $W > 0$. Conversely, there is a transport of low turbulence fluid into a region where $\mathcal{P} > \varepsilon$ (the rough strips), which is related to $W < 0$.

All production terms in 6.3 are shown in figure 6.10–6.14 and the sum of all production terms $\mathcal{P}$ is shown in figure 6.15. As shown in Anderson *et al.* (2015) for strip-type roughness, the magnitude of production term $\mathcal{P}_{u'w'} = -\overline{u'w'} \partial U / \partial z$ (figure 6.10) is the most significant compared to other terms and hence it dominates the total production $\mathcal{P}$, as shown in the contours in figure 6.15, while $\mathcal{P}_{v'w'}$ (figure 6.12) is the least significant. A comparison of figures 6.15 and 6.10 reveal that the contours of total production $\mathcal{P}$ are very similar in shape and magnitude to those of $\mathcal{P}_{u'w'}$. All cases in figure 6.10(b–f), regardless of $S/\bar{\delta}$, show high $\mathcal{P}_{u'w'}$ above the rough strips, close to the wall. Away from the wall ($z/\bar{\delta} \approx 0.5$), a peak emerges above the smooth strips. The spanwise location of this peak approximately corresponds to the common flow up (thickening of the boundary layer). The magnitude of the rest of the production terms, $\mathcal{P}_{u'v'}$, $\mathcal{P}_{v'v'}$, and $\mathcal{P}_{w'w'}$ (figure 6.11, 6.13, 6.14), are roughly 5 times less than $\mathcal{P}_{u'w'}$ (a comparison of the magnitude of each term in equation (6.2) for one spanwise heterogeneous roughness case is shown in figure 6.16). The contours of $\mathcal{P}_{u'v'}$ (figure 6.11) are similar to those of $\overline{u'v'}$ in figure 6.7 (except for the sign). $\mathcal{P}_{u'v'}$ also exhibits a similar $S/\bar{\delta}$ trend as the mean swirl in figure 5.9, which represents the strength of the mean secondary flows, with $\mathcal{P}_{u'v'}$ becoming more prominent at $S/\bar{\delta}$ approaches 1.

With the contributions from each production term considered, the total production $\mathcal{P}$ can be approximated by its most significant contributor, $\mathcal{P} \approx \mathcal{P}_{u'w'}$, and equation (6.4) can be reduced further to

$$\mathcal{C}_y + \mathcal{C}_z \approx \mathcal{P}_{u'w'} - \varepsilon \quad (6.5)$$

The approximation in equation (6.5) has been utilized by Anderson *et al.* (2015) to explain the mechanism of secondary flow generation through production–dissipation imbalance. All terms in equation (6.5) are presented here, but it should be noted that the magnitude of dissipation ε is severely underestimated due to the spatial resolution of the PIV (table 4.1). In figure 6.17, ε is calculated from the rate-of-strain tensor of the turbulence measured by SPIV, $\varepsilon \equiv 2\nu \overline{s_{ij}s_{ij}}$. The magnitude of ε here is less than the production term $\mathcal{P}$ (figure 6.15) by a factor of approximately 10. Another alternative is to utilise the HWA measurement data (u component) to estimate ε . Assuming homogeneous isotropic turbulence, ε can be calculated from

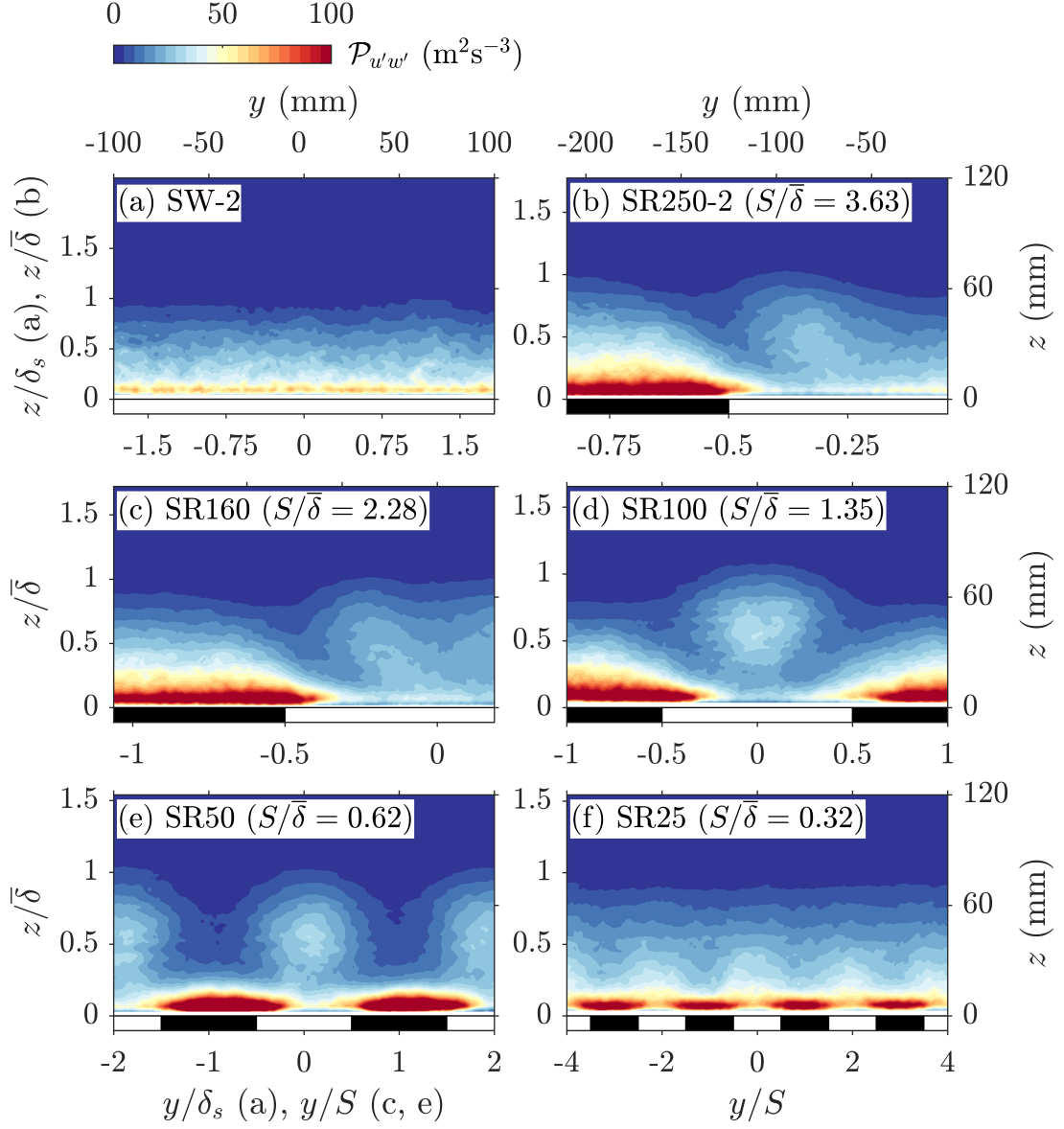


Figure 6.10: Contours of production term $\mathcal{P}_{u'w'} = -\overline{u'w'} \partial U / \partial z$ for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Contours are plotted from SPIV results.

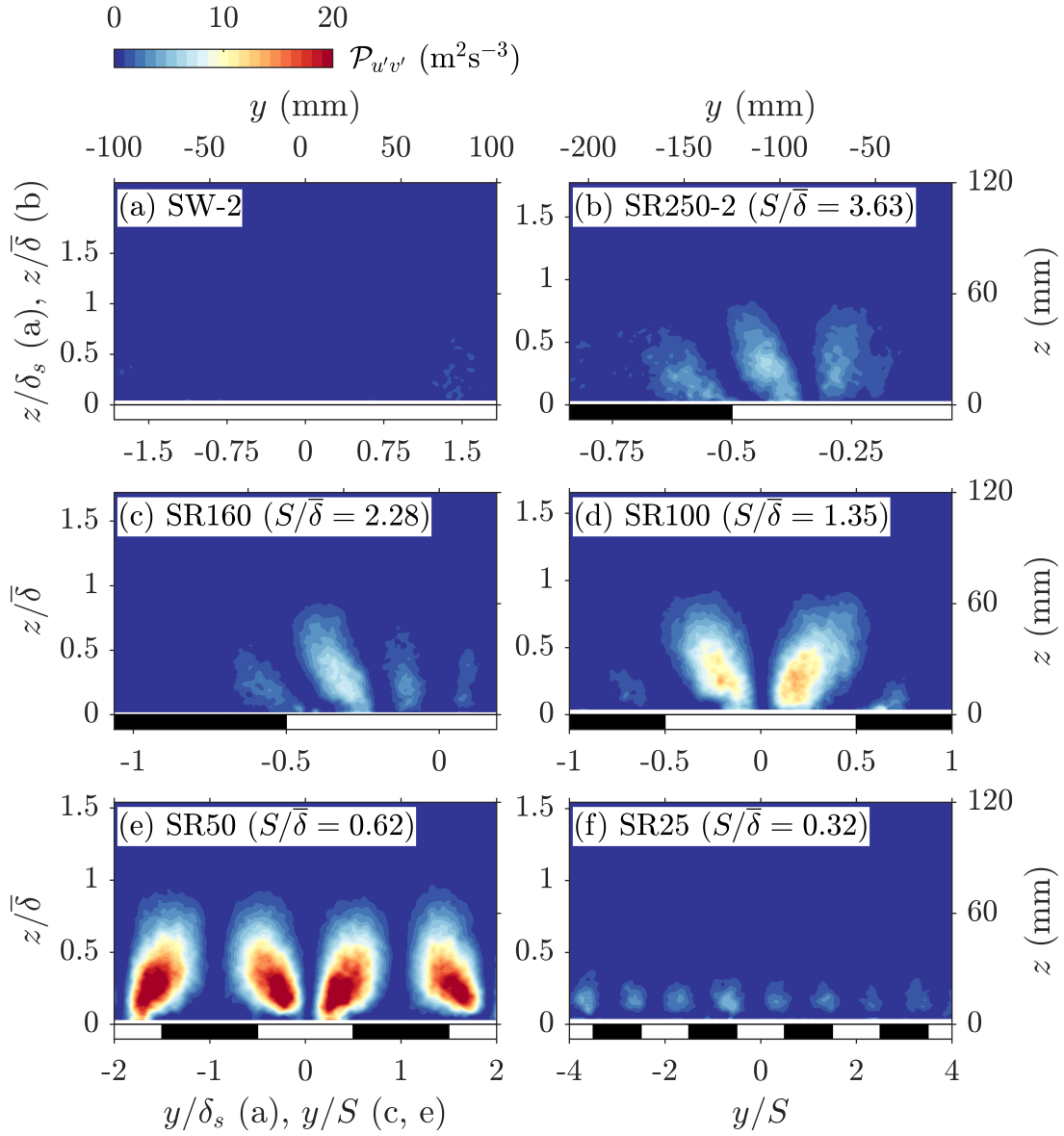


Figure 6.11: Contours of production term $\mathcal{P}_{u'v'} = -\overline{u'v'} \partial U / \partial y$ for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Contours are plotted from SPIV results.

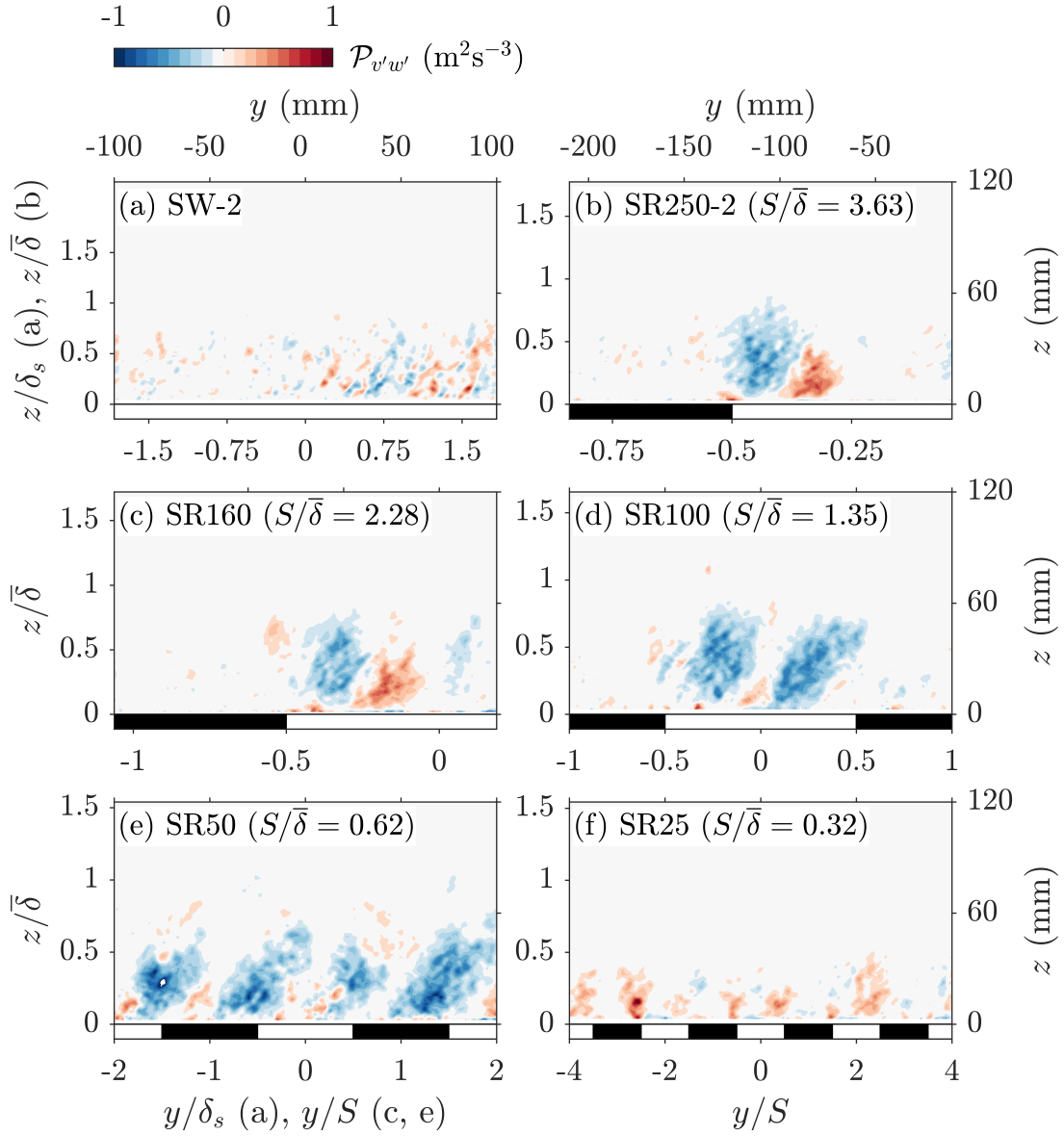


Figure 6.12: Contours of production term $\mathcal{P}_{v'w'} = -\overline{v'w'} (\partial V/\partial y + \partial W/\partial y)$ for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Contours are plotted from SPIV results.

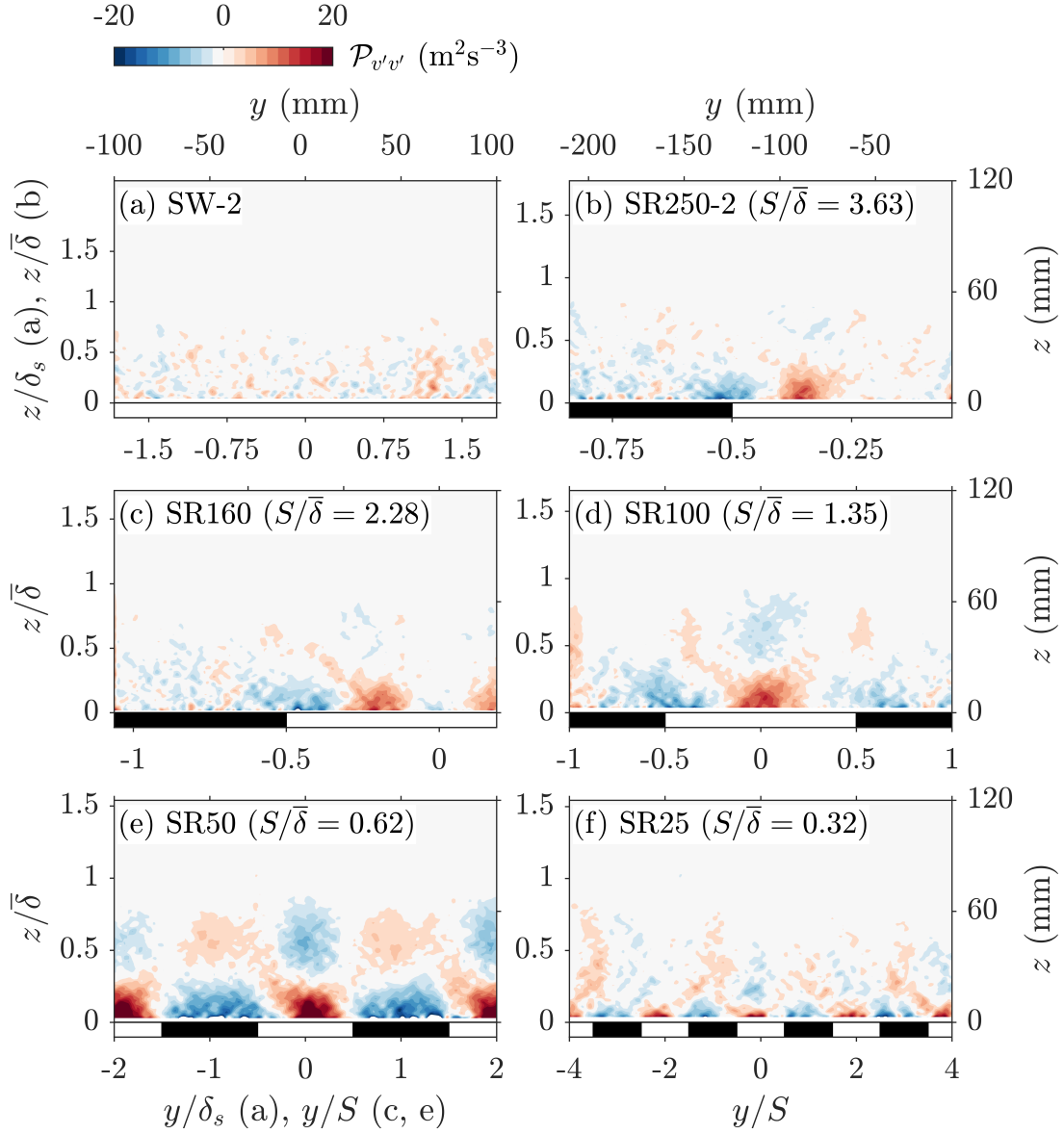


Figure 6.13: Contours of production term $\mathcal{P}_{v'v'} = -\overline{v'v'} \partial V / \partial y$ for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Contours are plotted from SPIV results.

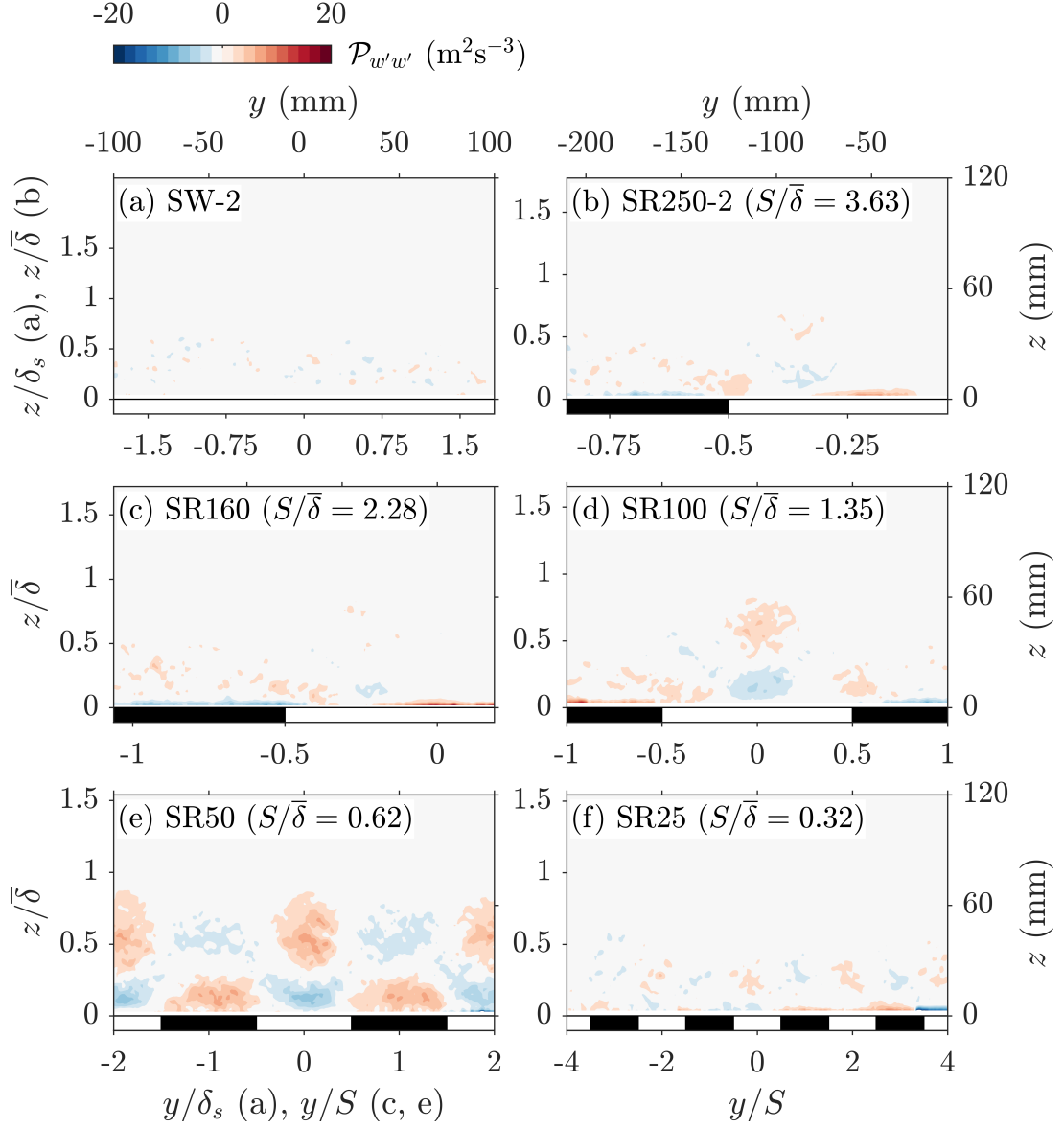


Figure 6.14: Contours of production term $\mathcal{P}_{w'w'} = -\overline{w'w'} \partial W / \partial z$ for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Contours are plotted from SPIV results.

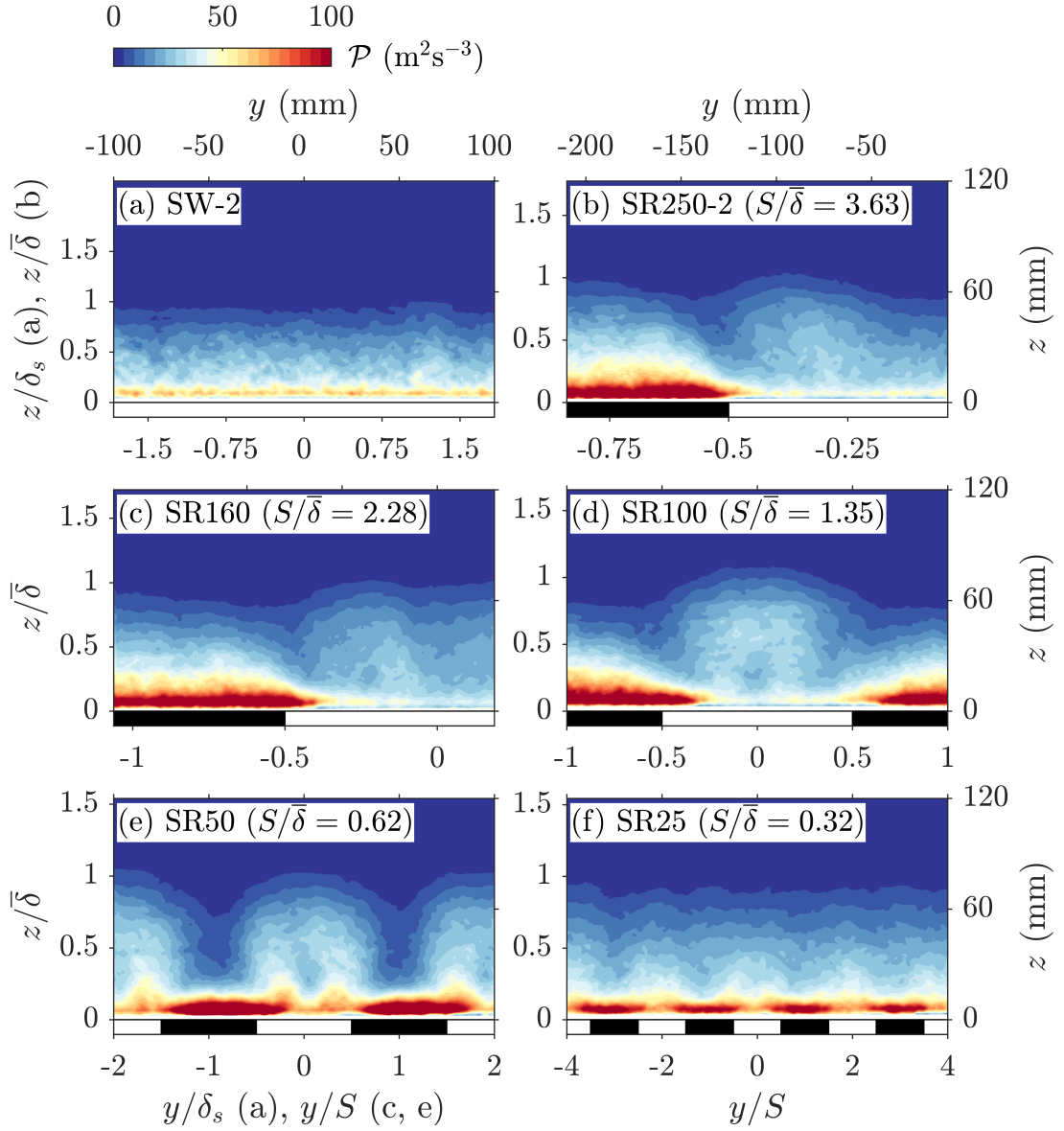


Figure 6.15: Contours of total production $\mathcal{P} = \mathcal{P}_{u'v'} + \mathcal{P}_{u'w'} + \mathcal{P}_{v'w'} + \mathcal{P}_{v'v'} + \mathcal{P}_{w'w'}$ for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Contours are plotted from SPIV results.

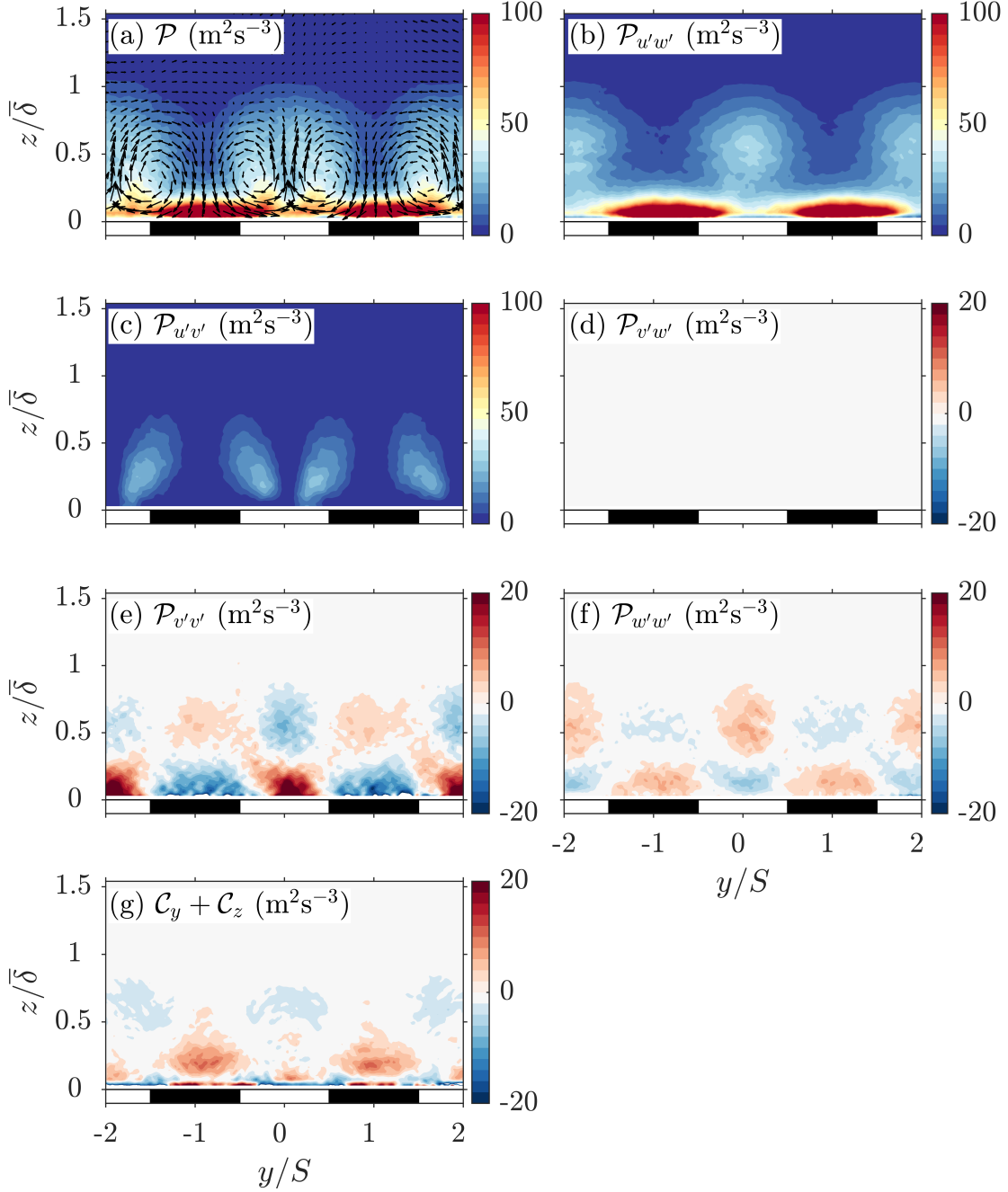


Figure 6.16: Contours of TKE transport equation terms for case SR50: (a) total production $\mathcal{P}$, (b) $\mathcal{P}_{u'w'} = -\overline{u'w'}\partial U/\partial z$, (c) $\mathcal{P}_{u'v'} = -\overline{u'v'}\partial U/\partial y$, (d) $\mathcal{P}_{v'w'} = -\overline{v'w'}(\partial V/\partial y + \partial W/\partial y)$, (e) $\mathcal{P}_{v'v'} = -\overline{v'v'}\partial V/\partial y$, (f) $\mathcal{P}_{w'w'} = -\overline{w'w'}\partial W/\partial z$, and (g) TKE advection terms (LHS terms in equation 6.2). Contours are plotted from SPIV results. In (a), vectors indicate V and W , downsampled for clarity.

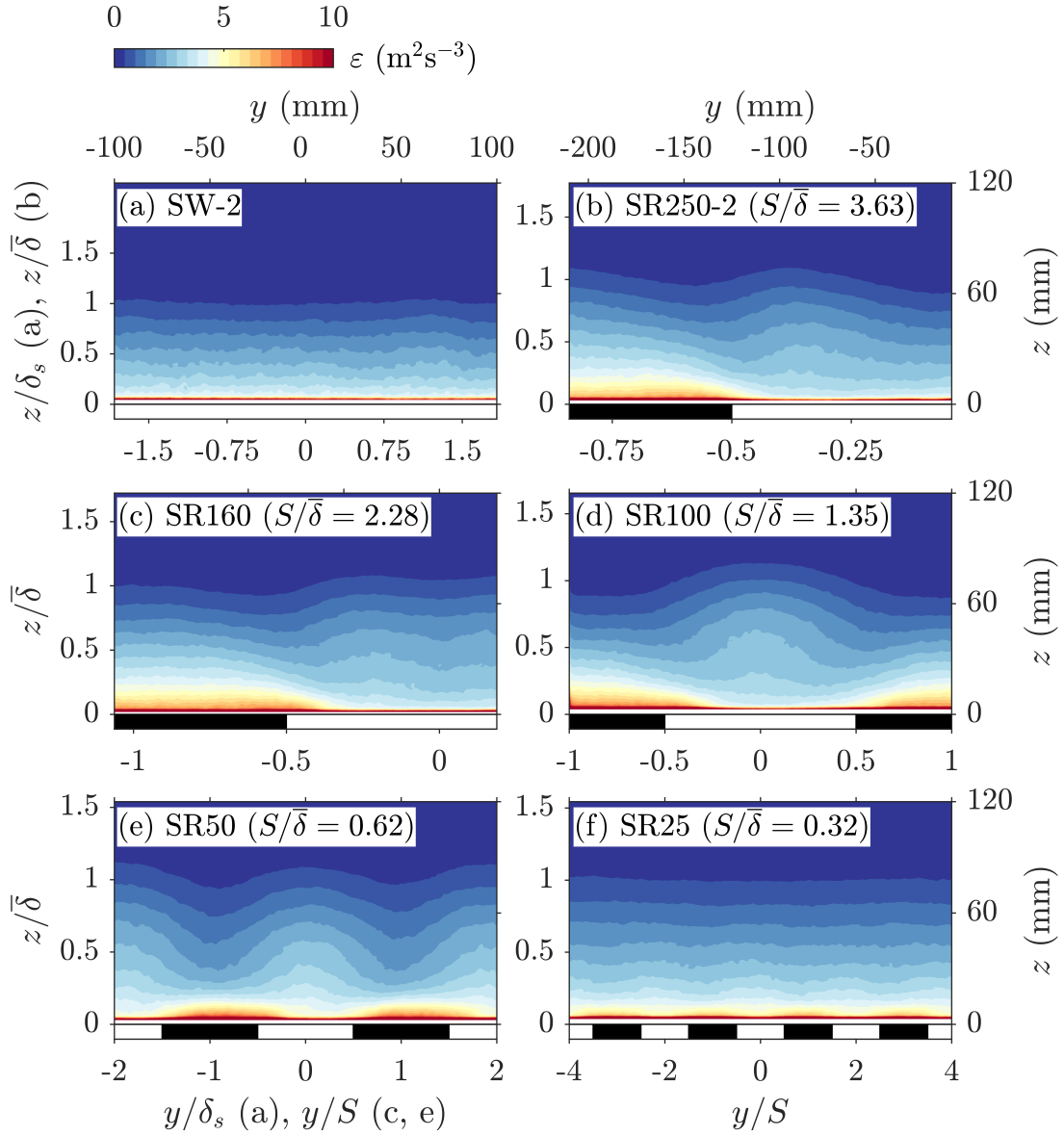


Figure 6.17: Contours of dissipation $\varepsilon \equiv 2\nu\overline{s_{ij}s_{ij}}$ for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Contours are plotted from SPIV results.

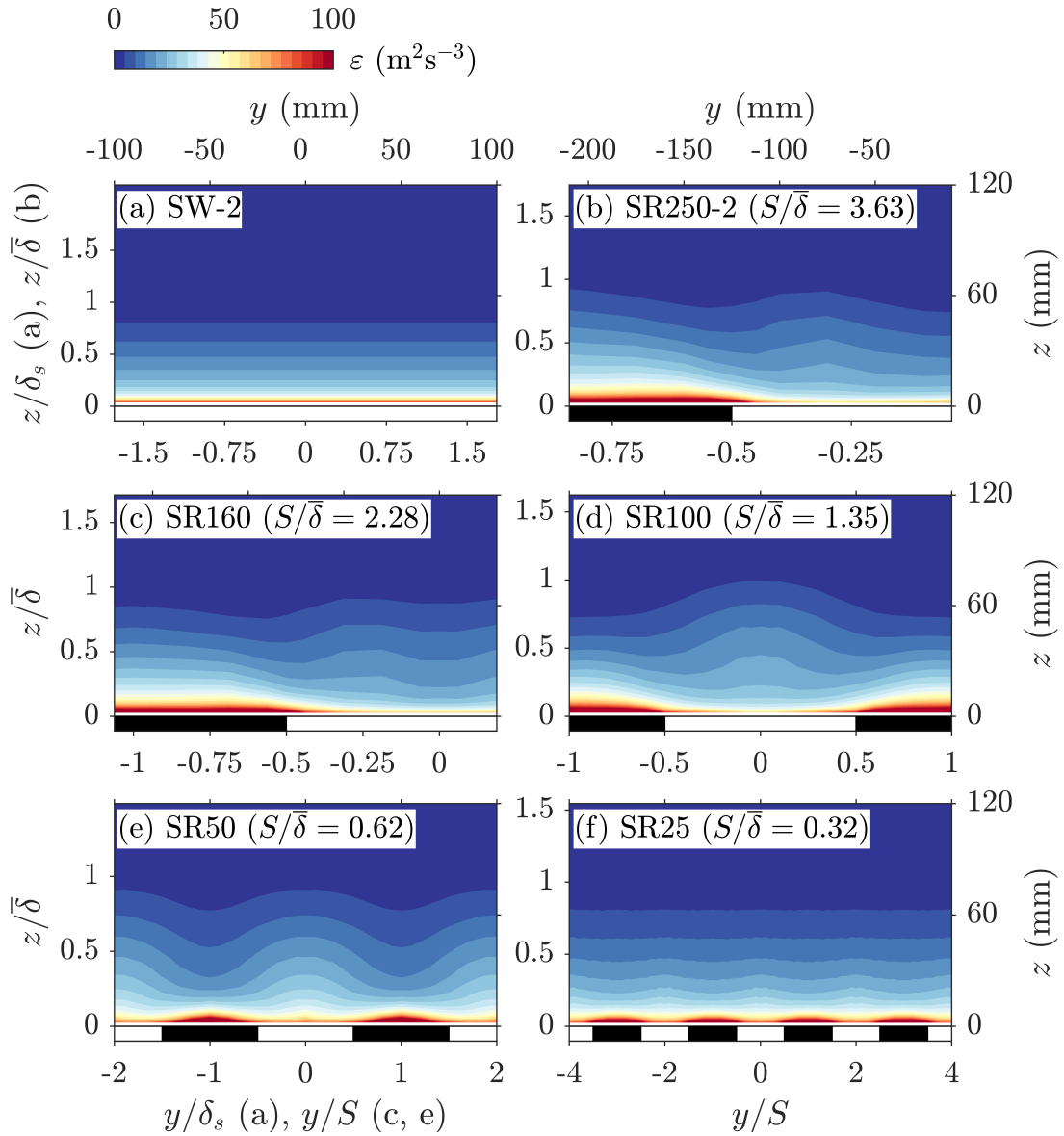


Figure 6.18: Contours of estimated dissipation $\varepsilon_{est} = 15\nu/U^2(\overline{\partial u'/\partial t})^2$ for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Contours are plotted from HWA results, assuming homogeneous isotropic turbulence. For (a), measurements are conducted over one y location and contours are repeated across y . For (b–f), the measurement domains extend half of the spanwise roughness wavelengths in y , the rest of the contours are reflected and repeated owing to presumed periodicity.

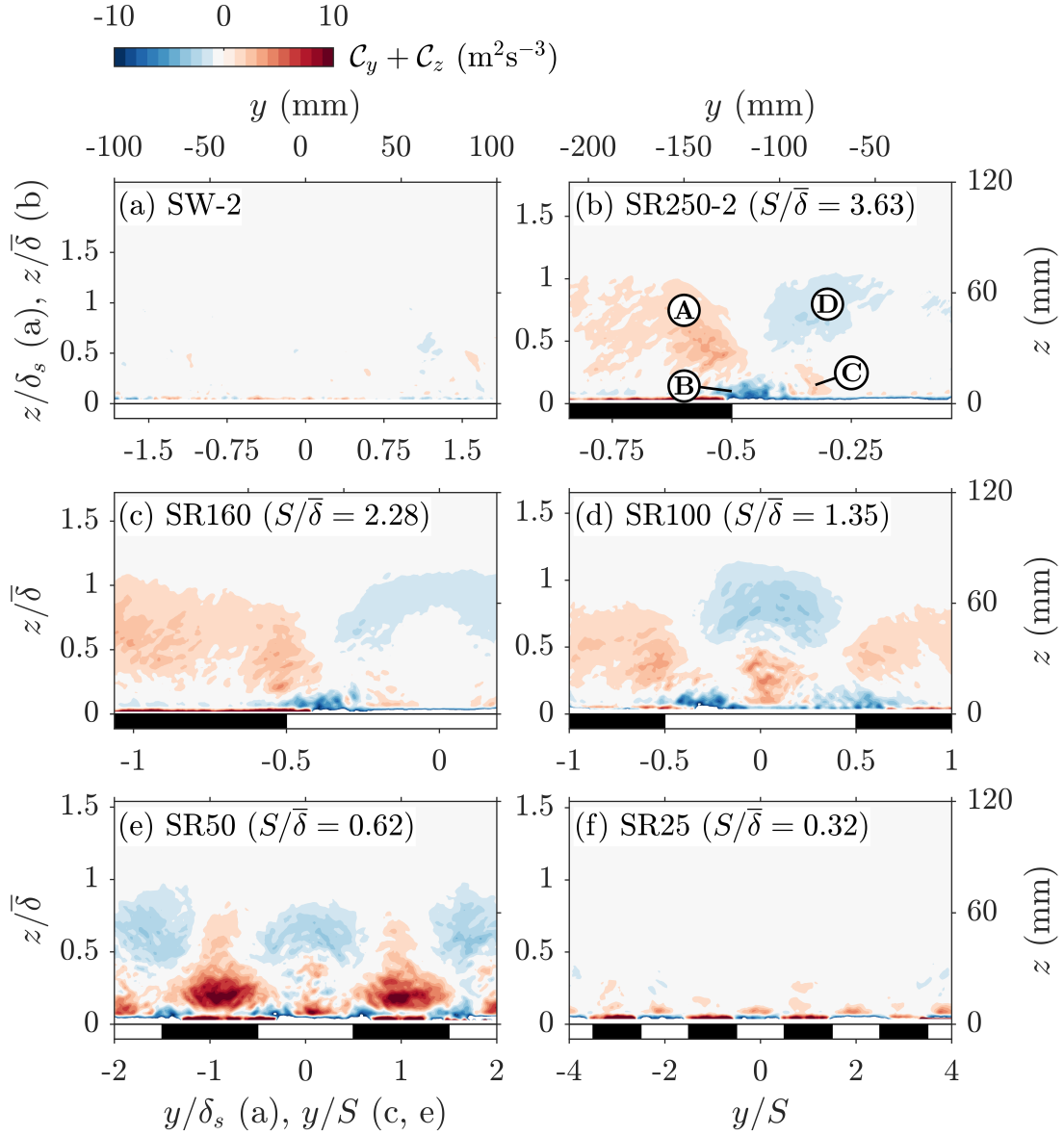


Figure 6.19: Contours of turbulent kinetic energy advection term $C_y + C_z$ (LHS terms in 6.2) for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Contours are plotted from SPIV results.

the velocity gradient (Wang *et al.*, 2021)

$$\varepsilon_i = 15\nu \overline{\left(\frac{\partial u'_i}{\partial x_i}\right)^2} \quad (6.6)$$

Assuming Taylor's frozen flow hypothesis (1938) holds, spatial information can be converted to time-series data such that

$$\varepsilon_{est} = \frac{15\nu}{U^2} \overline{\left(\frac{\partial u'}{\partial t}\right)^2} \quad (6.7)$$

Here, $\partial u'/\partial t$ is calculated from the time-series data provided by HWA measurements. Figure 6.18 show the estimated ε for the same cases as those in figure 6.17. Although equation (6.7) gives a more accurate estimation of ε (it now has the same magnitude as $\mathcal{P}$ in figure 6.15), it should be noted that the equation only holds for homogeneous isotropic turbulence, which is not what the flow investigated in this study is. Therefore, production–dissipation imbalance is inferred from TKE advection terms (LHS terms in equation 6.5) instead. The sum of these TKE advection terms is shown in figure 6.19. Positive $\mathcal{C}_y + \mathcal{C}_z$ ($\mathcal{P} > \varepsilon$) is observed above the rough strips (see the region marked by (A) in figure 6.19b). Near the roughness transition, where common flow down occurs, $V \approx 0$, and hence $\mathcal{C}_z = W\partial K/\partial z > 0$. Since $\partial K/\partial z < 0$ ((A) in figure 6.5b), then $W < 0$ (common flow down). Adjacent to the region with $\mathcal{C}_y + \mathcal{C}_z > 0$ above the rough strip, negative value is observed closer to the wall ((B) in figure 6.19b). Assuming $W \approx 0$ in this area, the remaining term becomes $\mathcal{C}_y = V\partial K/\partial y < 0$. The spanwise gradient $\partial K/\partial y$ is negative since TKE near wall is higher above rough rather than smooth strips ((B) figure 6.5b) and so $V > 0$. The downwelling above rough strips towards the wall is followed by spanwise transfer of the flow from rough to smooth strip. Adjacent to this region, a weak positive $\mathcal{C}_y + \mathcal{C}_z$, which becomes more prominent as $S/\bar{\delta}$ approaches 1, is observed near the wall ((C) in figure 6.19b). The spanwise location of this region is associated with the common flow up ($V \approx 0$), which is also marked by an emerging peak in TKE at $z/\bar{\delta} \approx 0.5$ (figure 6.5). Closer to the wall ($z/\bar{\delta} \lesssim 0.5$), $\partial K/\partial z > 0$ ((C) in figure 6.5b), and thus $W > 0$ above the smooth strip. Above this region ($z/\bar{\delta} \gtrsim 0.5$), $\mathcal{C}_y + \mathcal{C}_z < 0$ ((D) in figure 6.19b) as the wall-normal gradient of TKE switches sign above the emerging peak to $\partial K/\partial z < 0$ ((D) in figure 6.5b) and the direction of W is maintained, $W > 0$.

Chapter 7

Energy spectra

7.1 1-D Streamwise energy spectra (from HWA)

To further explore the nature of the turbulence under the imposed spanwise heterogeneities, the energy spectrograms of all SR cases are presented and discussed in the following. The 1-D energy spectra of the streamwise turbulent velocity fluctuations $\Phi_{u'u'}$ are calculated from HWA signals for each case as a function of wall-normal and spanwise location. The relationship between streamwise velocity fluctuation u' and $\Phi_{u'u'}$ is given by

$$\overline{u'u'} = \int_0^\infty \Phi_{u'u'} d(k_x) \quad (7.1)$$

where $k_x \equiv 2\pi/\lambda_x$ is the streamwise wavenumber and λ_x is the streamwise wavelength. We assume Taylor's frozen flow hypothesis (1938) to project time-series data spatially using $\Delta x = -U_c \Delta t$, with $U(y, z)$, the mean streamwise velocity at each spanwise-wall-normal grid datapoint, as the assumed convection velocity U_c . The streamwise wavelength is defined as $\lambda_x = U(y, z)/f$, where f is the frequency of a given Fourier mode. This study is focused on the energetic signatures that are in the range of $\lambda_x/\bar{\delta} = 3$ –6. The limitation of Taylor's hypothesis over large projected streamwise distance was investigated by Dennis & Nickels (2008). They confirmed the accuracy of Taylor's approximation for projection distances up to 6δ , which covers the range of events investigated here. Furthermore, it is shown later in §7.3 and §9.2 that the results from energy spectra are supported by true spatial measurements from WPPIV. The magnitude of the premultiplied energy spectrograms are normalized by freestream velocity U_∞^2 , since a reliable estimate for U_τ is not available for the spanwise heterogeneous cases.

Figure 7.1(a) shows the energy spectrograms for the homogeneous smooth and rough wall reference cases (SW-2 and RW-2). The premultiplied energy spectro-

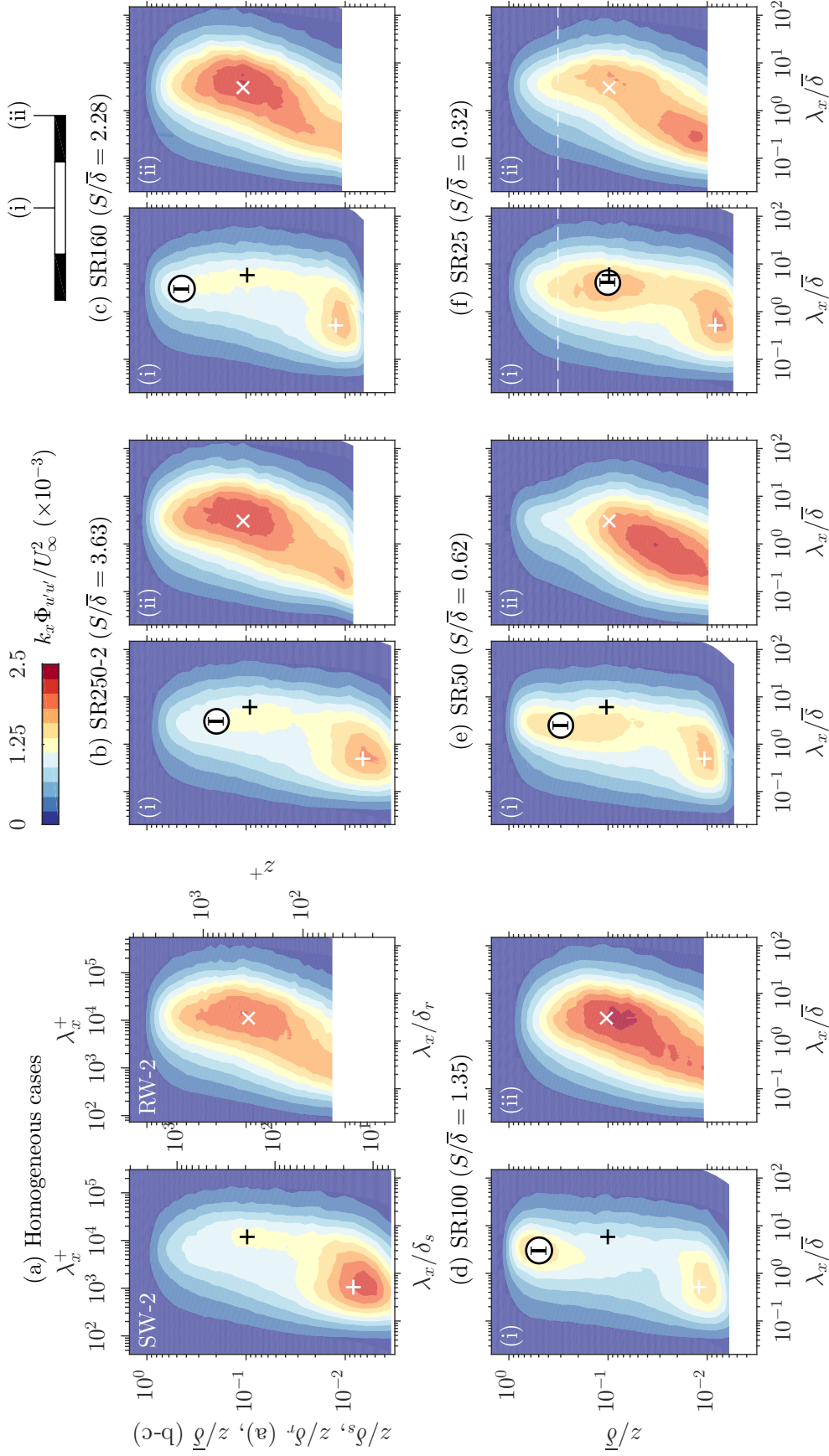


Figure 7.1: Premultiplied 1-D streamwise energy spectra $k_x \Phi_{u'u'}$ as a function of streamwise wavelength λ_x and wall location z of (a) reference homogeneous cases (SW-2 and RW-2) and (b-f) SR cases. (i) is the spectrogram at the center of the smooth patch and (ii) is at the center of the rough patch (legends are at the top of the page). Label $\textcircled{I}$ indicates the energetic peak associated with the secondary flow. Inner-scaled axes are available for (a) at the right and top, where U_τ is obtained by composite fit described in §3.3. Dashed line in (f) is $z/\delta = 0.32$.

grams are shown as a function of λ_x and z , normalized by δ_s (SW-2) or δ_r (RW-2) at the left-hand side and bottom axis. The inner-scaled λ_x and z (λ_x^+ and z^+) are shown in the right-hand side and top axis for the reference cases only. The inner peak representing the viscous near-wall cycle (Hutchins & Marusic, 2007b) of SW-2 ($\delta_s^+ = 2030$), marked by the white ‘+’ in figure 7.1(a), is captured at $z/\delta_s \approx 0.01$ and $\lambda_x/\delta_s \approx 0.5$, which correspond to $z^+ \approx 15$ and $\lambda_x^+ \approx 1000$. A smaller outer peak (marked by a black ‘+’) emerges at $z/\delta_s \approx 0.1$ with wavelength $\lambda_x/\delta_s \approx 6$, consistent with that of long, meandering large-scale structures observed in the logarithmic region by Hutchins & Marusic (2007a,b); Mathis *et al.* (2009) and others. The reference rough wall case RW-2 in figure 7.1(a) has overall higher energy than that of SW-2. This is expected under this scaling, since under an assumption of Townsend’s (Townsend, 1976) outer layer similarity, we would expect u'^2/U_τ^2 to be matched at a given z/δ . Since U_τ/U_∞ for the rough wall reference is larger than that for the smooth reference, u'^2/U_∞^2 must be greater for the rough surface. A peak (marked by a white ‘×’) is detected within the logarithmic region at $z/\delta_r \approx 0.1$, $\lambda_x/\delta_r \approx 3$.

The energy spectrograms of the spanwise heterogeneous cases are shown for the cases measured at $x = 4$ m: SR250-2 ($S/\bar{\delta} = 3.63$, figure 7.1b), SR160 ($S/\bar{\delta} = 2.28$, figure 7.1c), SR100 ($S/\bar{\delta} = 1.35$, figure 7.1d), SR50 ($S/\bar{\delta} = 0.62$, figure 7.1e), and SR25 ($S/\bar{\delta} = 0.32$, figure 7.1f), plotted as a function of $\lambda_x/\bar{\delta}$ and $z/\bar{\delta}$. For each case, the spectrogram is shown at two spanwise locations: the centre of a smooth strip (marked by (i) in figure 7.1) and the centre of a rough strip (marked by (ii) in figure 7.1).

Comparisons are drawn between the spectrograms of the reference smooth wall case SW-2 and the spanwise heterogeneous roughness cases. In terms of the viscous near-wall cycle, the reference smooth wall case SW-2 and all of the spanwise heterogeneous cases over the centre of a smooth strip show a nearly constant location of the inner peak (marked by a white ‘+’) at $z/\bar{\delta} \approx 0.01$, $\lambda_x/\bar{\delta} \approx 0.5$ (figure 7.1b(i)–f(i)). For the nominal friction Reynolds numbers of these cases ($\delta^+ \approx 2000$) this is close to the identified viscous scaled location for this peak found in Hutchins & Marusic (2007b). Later, we show that the increasing strength of secondary flows and increasing meandering are observed for $\Lambda/\bar{\delta} \approx 2$ ($\Lambda = 2S$), which roughly corresponds to $\Lambda^+ \approx 4000$ wall units or 40 times the Kline spacing ($\approx 100U_\tau/\nu$) of the near-wall streaks (Kline *et al.*, 1967). This supports the observation in Stroh *et al.* (2016) and Vanderwel *et al.* (2019), that outer scaling is more relevant than inner scaling in the formation of secondary flows. The spectrograms in the outer

layer, however, differ between the reference smooth wall and spanwise heterogeneous cases. In case SR250-2 (the limit where $S/\bar{\delta} \gg 1$, figure 7.1b), we observe that the energy spectrograms over the centre of the smooth and the rough strips best resemble the homogeneous reference cases (figure 7.1a) as compared to other cases. This observation from the spectra is in line with that from the mean velocity profiles of figures 5.3 and 5.2 for $S/\bar{\delta} \gg 1$, and consistent with the notion that the flow over the rough and smooth patches approaches the homogeneous limit for very large $S/\bar{\delta}$ (see figure 2.4a). The outer peak for SW-2 (figure 7.1a) occurs at $\lambda_x/\delta_s \approx 6$, $z/\delta_s \approx 0.1$ and is associated with the large-/very large-scale motions (LSM/VLSM) in the log region. For SR250-2, this peak (marked by a white ‘+’ in figure 7.1b(i)) is still present over the centre of the smooth strip but is now supplemented with additional energy at $\lambda_x/\bar{\delta} \approx 3$, $z/\bar{\delta} \approx 0.2$ (labelled **Ⓘ** in figure 7.1). The additional energetic sites labeled **Ⓘ** in these figures are presumed to be related to the secondary flows formed over these surfaces. As $S/\bar{\delta}$ decreases to 2.28 (figure 7.1c(i)), this additional outer bump of energy at $\lambda_x/\bar{\delta} \approx 3$ is pushed outwards towards $z/\bar{\delta} \approx 0.5$, which approximately coincides with the wall-normal location of the centre of the secondary flow (figure 5.2d). The relation between the peak associated with LSM/VLSM and secondary flows due to heterogeneity is discussed further in §9.4. Meanwhile, the energy spectra above the rough strips (figure 7.1b(ii),c(ii)) still resembles those above the reference rough wall RW-2 (figure 7.1a), which is expected since the counter-rotating pairs of secondary flows tend to move towards the centres of the smooth strips (instead of rough strips) as $S/\bar{\delta}$ decreases (figure 5.2).

The spectrograms for cases where $S/\bar{\delta} \approx 1$ (figure 7.1d,e) offer a new insight. In this case, the spectrogram above the smooth strip exhibits a strong additional energetic site at a wavelength of $\lambda_x/\bar{\delta} \approx 3$ and a wall-normal location of $z/\bar{\delta} \approx 0.5$ for SR100 (figure 7.1d(i)) and $z/\bar{\delta} \approx 0.3$ for SR50 (figure 7.1e(i)), roughly corresponding to the wall-normal location of the center of the secondary flows for these configurations (figure 5.2e,f). As this additional outer peak above the centre of a smooth strip becomes stronger in case SR50 ($S/\bar{\delta} = 0.62$, figure 7.1e(i)), the energy in the outer layer of the rough strip becomes less prominent (figure 7.1e(ii)).

As $S/\bar{\delta}$ decreases further to the small limit where $S/\bar{\delta} \ll 1$ (SR25, figure 7.1f), the energy levels over the smooth and rough strips appear more equal, and no longer resemble the reference cases. We note that the outer peak over the centre of a smooth and rough strip is now located at $z/\bar{\delta} \approx 0.1$ (close to the location in the reference cases). For $z/\bar{\delta} > 0.32$ (corresponding to $z > S$, marked by the dashed line in figure 7.1f), the energy spectrograms over the rough and smooth strips are almost

the same, indicating a return to spanwise homogeneity in the outer region for this case (see figure 2.4c).

In figure 7.2, the 3-D isosurfaces of streamwise energy spectrograms are plotted for all SR cases to show the complete spanwise variation in energy. The isosurfaces are plotted as a function of the streamwise wavelength $\lambda_x/\bar{\delta}$, spanwise location y/S , and wall-normal location $z/\bar{\delta}$. The isosurfaces span one half-wavelength S , where $y/S = 0$ is the centre of a smooth strip, $y/S = 0.5$ is the roughness transition, and $y/S = 1$ is the centre of the adjacent rough strip. For clarity, only three levels are shown. Note that the aspect ratio of spanwise to wall-normal axes is not preserved between plots in figure 7.2. For example, figure 7.1(a) covers a much wider spanwise domain ($6.81\bar{\delta}$) than figure 7.2(f) ($0.32\bar{\delta}$).

At the limit where $S/\bar{\delta} \gg 1$, the energy spectrograms of SR250-1 ($S/\bar{\delta} = 6.81$) in figure 7.2(a) exhibit the expected behaviour. The rough patch has higher energy than the smooth patch and the behaviour is relatively homogeneous across the rough and smooth strips. A small bulge of energy in the outer layer is noticeable near the roughness transition, at $y/S \approx 0.4$ (visible in the blue isosurface and labeled ①). As $S/\bar{\delta}$ reduces through figures 7.2(b,c), the degree of spanwise uniformity over either the rough or smooth patches diminishes. The bulge remains as $S/\bar{\delta}$ decreases and extends towards the centre of the smooth patch. This bulge is centered at $y/S \approx 0.3$ in case SR250-2 (figure 7.2b) and nearly throughout the spanwise extent of the smooth patch for case SR160 (figure 7.2c), appearing to be related to the location of the centre of the secondary flows (figure 5.2b–d, the spanwise location of the centre of the secondary flows are shown by dashed white lines).

As $S/\bar{\delta}$ decreases to the intermediate case where $S/\bar{\delta} \approx 1$ in figure 7.2(d) (SR100, $S/\bar{\delta} = 1.35$), the counter-rotating secondary flows fill the entire spanwise and wall-normal extent of the boundary layer and common flow up occurs over the centre of smooth strips. The extra bulge of energy now becomes stronger still (yellow isosurface) and is approximately centered at $z/\bar{\delta} \approx 0.5$ and with a characteristic lengthscale of $\lambda_x/\bar{\delta} \approx 3$. The bulge is even stronger (red isosurface) as $S/\bar{\delta}$ decreases to 0.62 (SR50, figure 7.2e), appearing over the roughness transition ($y/S = 0.5$) and spanning the wall-normal location $0.1 \lesssim z/\bar{\delta} \lesssim 0.5$. The streamwise wavelength of the bulge is roughly unchanged from the previous case at $\lambda_x/\bar{\delta} \approx 3$.

In the limit where $S/\bar{\delta} \ll 1$, figure 7.2(f) shows that the energy spectrograms of the smooth and rough strips approach spanwise homogeneity. The two roughness strips have similar energy spectrograms in the outer layer. Closer to the surface, energy spectrograms are governed by their respective wall conditions, notably with

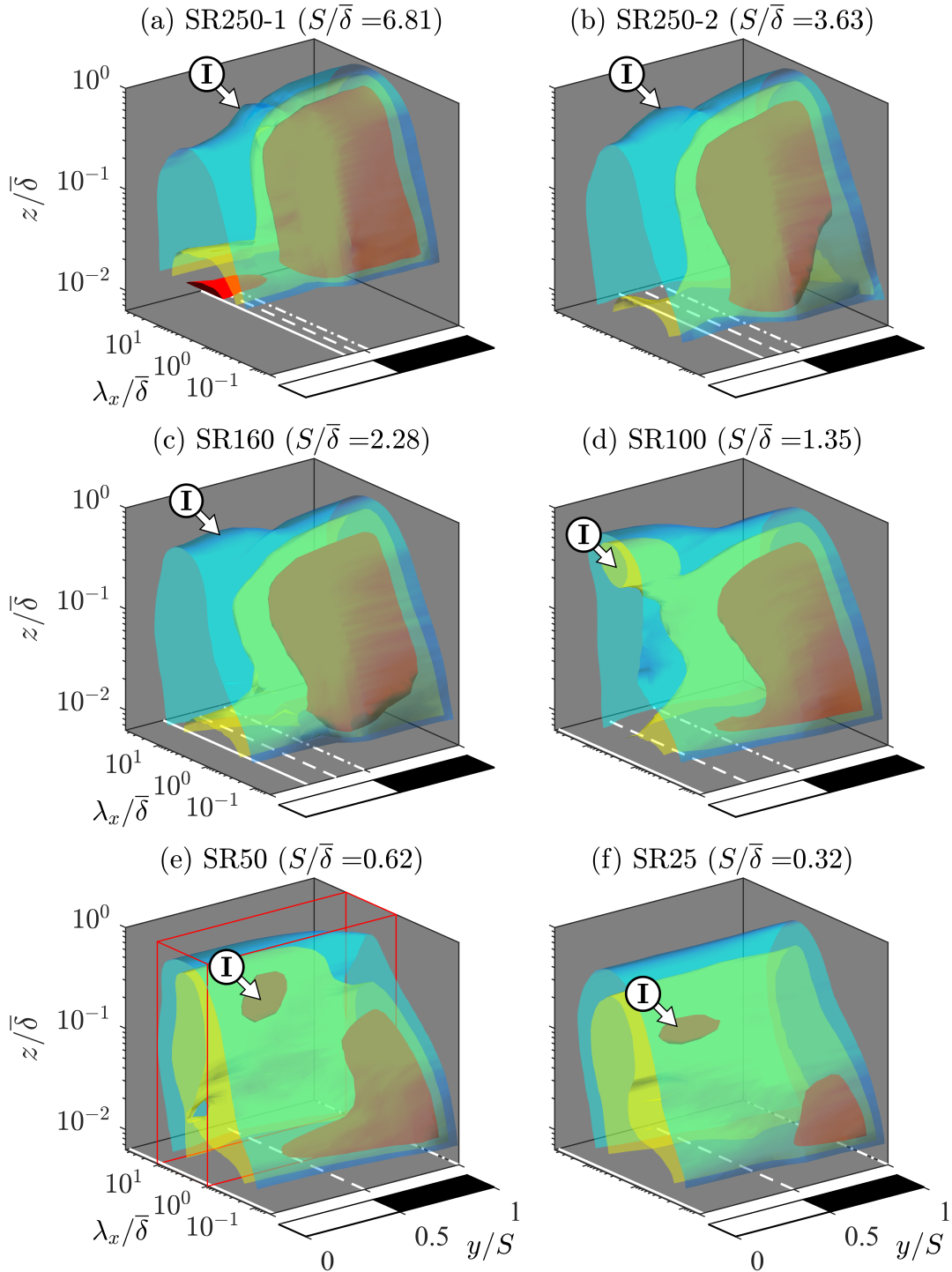


Figure 7.2: Isosurface of 1-D streamwise energy spectra $k_x \Phi_{u'u'}$ across half-wavelength S for all spanwise heterogeneous roughness cases. Colour level: blue ($k_x \Phi_{u'u'}/U_\infty^2 = 0.001$), yellow (0.0015), and red (0.002). White solid lines (—) are the spanwise location of common flow up (Ⓐ in figure 5.2), white dashed lines (---) are the center of the secondary flows (Ⓑ in figure 5.2), and white dash-dot lines (-.-) are the common flow down (Ⓒ in figure 5.2). Label Ⓐ denotes the peak associated with the secondary flows exist. The red box in (e) is the integration domain in (7.2).

high energy above the rough patch. An energy bulge is detected above the roughness transition ($y/S = 0.5$) at $\lambda_x/\bar{\delta} \approx 3$ and $z/\bar{\delta} \approx 0.1$. However, the main observation from figure 7.2(f) is that beyond the expected secondary flows, i.e. $z/\bar{\delta} \gtrsim 0.32$, the energy across the rough and smooth strips is homogeneous, marking the return to spanwise homogeneity as suggested by Chung *et al.* (2018) and Medjnoun *et al.* (2018), and as illustrated in figure 2.4(c).

The additional information provided by the energy spectra in figures 7.1 and 7.2 is the presence of an additional outer energetic peak approximately above the roughness transition, where secondary flows occur. We quantify the strength of this emergent peak in the energy spectra by integrating $k_x \Phi_{u'u'}$ within the volume in figure 7.2, in which this emergent peak resides. The integration volume (figure 7.2e) comprises a range of streamwise wavelengths encompassing a region in the spectrograms where the energy peak is observed ($1 \leq \lambda_x/\bar{\delta} \leq 10$), the wall-normal extent up to the boundary-layer height ($0 < z \leq \bar{\delta}$), and a span $\pm 0.5l_y$ about the spanwise location of the center of the secondary flows y_c (© in figure 5.2), l_y is the width of the mean secondary flows defined in (5.4). The integrated energy is given by

$$I_{\Phi_{u'u'},x} = \frac{1}{\bar{\delta}l_y} \int_0^{\bar{\delta}} \int_{y_c-l_y/2}^{y_c+l_y/2} \int_{2\pi/\bar{\delta}}^{2\pi/(10\bar{\delta})} \frac{\Phi_{u'u'}}{U_\infty^2} dk_x dy dz \quad (7.2)$$

For comparison, the integrated energy is also calculated for the reference smooth and rough wall cases

$$I_{\Phi_{u'u'},x} = \frac{1}{\delta} \int_0^\delta \int_{2\pi/\delta}^{2\pi/(10\delta)} \frac{\Phi_{u'u'}}{U_\infty^2} dk_x dz \quad (7.3)$$

where δ is either δ_s for the reference smooth wall cases (SW-1 and SW-2) or δ_r for the reference rough wall cases (RW-1 and RW-2).

The magnitude of $I_{\Phi_{u'u'},x}$ as a function of $S/\bar{\delta}$ is shown in figure 7.3, along with the integrated energy from both smooth and rough reference cases. In the limit where $S/\bar{\delta} \gg 1$, the integrated energy of case SR250-1 ($S/\bar{\delta} = 6.81$) approaches its reference smooth wall counterpart (SW-1, dashed line in figure 7.3). In the other limit, as $S/\bar{\delta} \ll 1$, we would expect the energy within the integral bounds to asymptote to some value between the reference SW and RW cases. This case is represented by SR25 ($S/\bar{\delta} = 0.32$), whose integrated energy falls below its reference rough wall counterpart (RW-2, dash-dot line in figure 7.3). Clearly, figure 7.3 indicates that there is an $S/\bar{\delta}$ dependency in the energy within the integral limits, with a clear

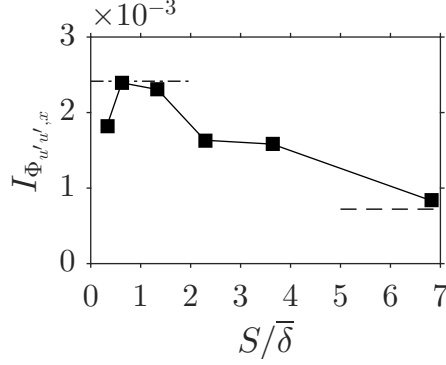


Figure 7.3: Integrated energy spectra $I_{\Phi_{u'u',x}}$ as a function of $S/\bar{\delta}$ for SR cases ($\blacksquare$). Dashed line (---) is the integrated energy for the reference smooth wall SW-1 and the dash-dot line (-.-.) is the reference rough wall RW-2. The uncertainty of $\overline{u'u'}$ measured by HWA is approximately $\pm 2\%$ (twice larger than that of U , see Jørgensen, 1996), which is smaller than the symbols. See also appendix A.4 for the convergence analysis of energy spectra.

peak occurring at $S/\bar{\delta} \approx 1$ (this trend is insensitive to the y limits of the integration in equation 7.2). The peak in the energy indicates an additional periodicity that becomes prominent at cases SR50 and SR100 ($S/\bar{\delta} \approx 1$) and seems to be related to the location of the secondary flows. This suggests that there is some time-varying dependence or instability of the secondary flows that emerges at these spanwise wavelengths. We also note that the trend shown in figure 7.3 is similar to that of the mean secondary flow strength in figure 5.10. At this point, with only hot-wire time-series data, we lack the ability to reconstruct a view of the flow structure that leads to these features in the energy spectra. The flow structure, however, can be viewed through the instantaneous velocity fields in the xy -plane taken from the WPPIV measurements, which will be discussed further in §9.1.

7.2 1-D Spanwise energy spectra and co-spectra (from SPIV)

The current hot-wire measurements provide time-resolved turbulent fluctuation u' signals, which enable $k_x \Phi_{u'u'}$ computation, but not other velocity components. The energy spectra of v' and w' are computed instead from the non-time resolved PIV data. The spanwise energy spectra and co-spectra, which are related to spanwise wavenumber k_y and wavelength λ_y , are computed from the fluctuating velocity field obtained from SPIV measurements. At any wall-normal location z in the FOV, the non-normalised two-point correlation of the fluctuating velocity component a' and

b' is given by

$$\hat{R}_{a'b'}(\Delta y, z) = \overline{a'(y, z)b'(y + \Delta y, z)} \quad (7.4)$$

where the overbar denotes ensemble average at all y and for all SPIV realizations and Δy is the spanwise shift. The Fourier transformation of $\hat{R}_{a'b'}$ yields the energy spectra of the fluctuating velocity component

$$\Phi_{a'b'} = \int_{-\infty}^{\infty} \hat{R}_{a'b'} e^{-j2\pi k_y \Delta y} d(\Delta y) \quad (7.5)$$

where k_y is the spanwise wavenumber and $\lambda_y = 2\pi/k_y$ is the spanwise wavelength. The relationship between the Reynolds stress component $\overline{a'b'}$ and $\Phi_{a'b'}$ is given by

$$\overline{a'b'} = \int_0^{\infty} \Phi_{a'b'} d(k_y) \quad (7.6)$$

Due to the limited spanwise extent of the FOV, the velocity field is padded with zeros in y to increase the length of the signal to three times the total width of FOV (from $4\delta_s$ to about $12\delta_s$) and thus increase the resolution in k_y and λ_y .

Figure 7.4 shows the validation of 1-D spanwise energy spectra computation for the reference smooth wall case SW-2. The method, detailed in (7.4)–(7.6), is validated for Reynolds stress components: $\overline{u'u'}$, $-\overline{u'w'}$ (figure 7.4a) and $\overline{v'v'}$, $\overline{w'w'}$ (figure 7.4b). The figures show a collapse between Reynolds stresses obtained from SPIV measurements (solid lines, —) and integrated energy spectra across all resolved k_y (●). This method for 1-D spanwise energy spectra computation, however, must be applied with caution for the spanwise heterogeneous roughness cases. Figure 7.5(a) shows the instantaneous turbulent fluctuation component u' for case SR50 ($S/\bar{\delta} = 0.62$), whose mean (for all snapshots) across the spanwise extent of the FOV is zero. Its variance $\overline{u'u'}$, however, is heterogeneous in y (figure 7.5b). Using this method, integration of the energy spectra across k_y in (7.6) results in the *average* of $\overline{u'u'}$ in y , and yields no information regarding the spanwise heterogeneity of the flow as shown in figure 7.5(b).

Figure 7.6 shows the premultiplied energy spectra $k_y \Phi_{u'u'}$ for the reference smooth wall case SW-2 (figure 7.6a) and SR cases (figure 7.6b–f). In general, all SR cases have higher energy than the reference smooth wall case SW-2, which is expected since the presence of surface roughness results in higher magnitude of energy (when normalised by U_∞). Further, all cases (SR and SW-2) also show an outer peak ('+') at $z/\bar{\delta} \approx 0.2$ and $\lambda_y/\bar{\delta} \approx 1$ ($z/\delta_s \approx 0.2$ and $\lambda_y/\delta_s \approx 1$ for SW-2) which is associated with the width and spacing between large-scale low- and high-speed streaks for the large-scale and very large-scale motions (figure 4.9d). It is important to note

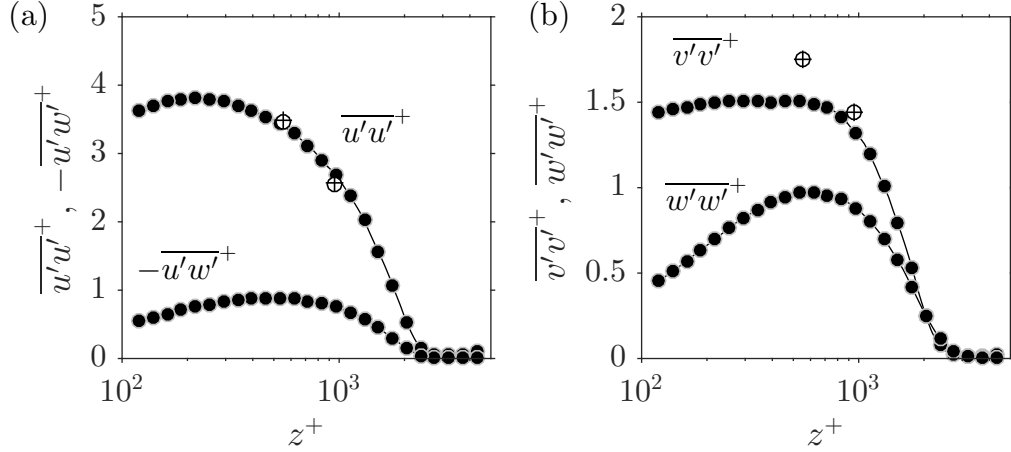


Figure 7.4: Reynolds stress components for reference smooth wall case SW-2: (a) $\overline{u'u'}^+$, $-\overline{u'w'}^+$ and (b) $\overline{v'v'}^+$, $\overline{w'w'}^+$, obtained from SPIV measurements (—) and integrating energy spectra across all resolved k_y (●), as shown in (7.6). Data are downsampled from clarity. Data from WPPIV measurements at two wall-parallel slices are shown in '+', while data obtained by integrating energy spectra across all resolved k_x and k_y are shown in '○', as shown in (7.12).

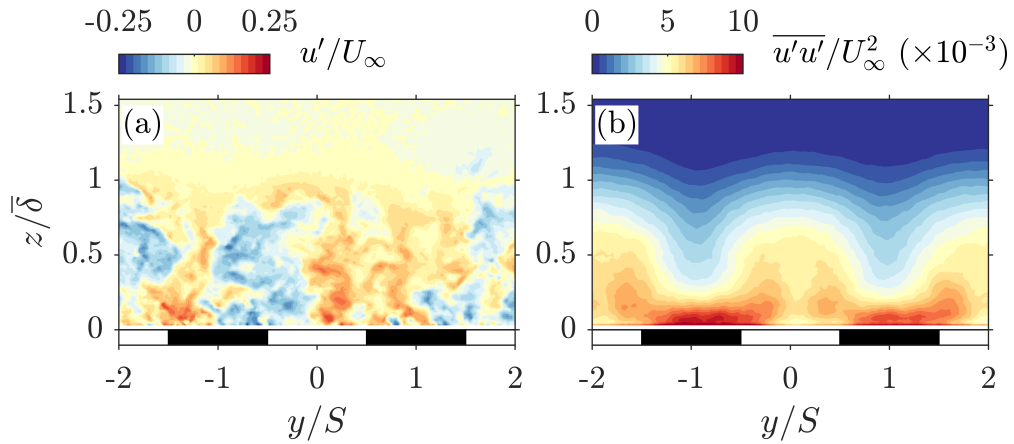


Figure 7.5: (a) Instantaneous turbulent fluctuation u' and (b) turbulence intensity $\overline{u'u'}$ for case SR50.

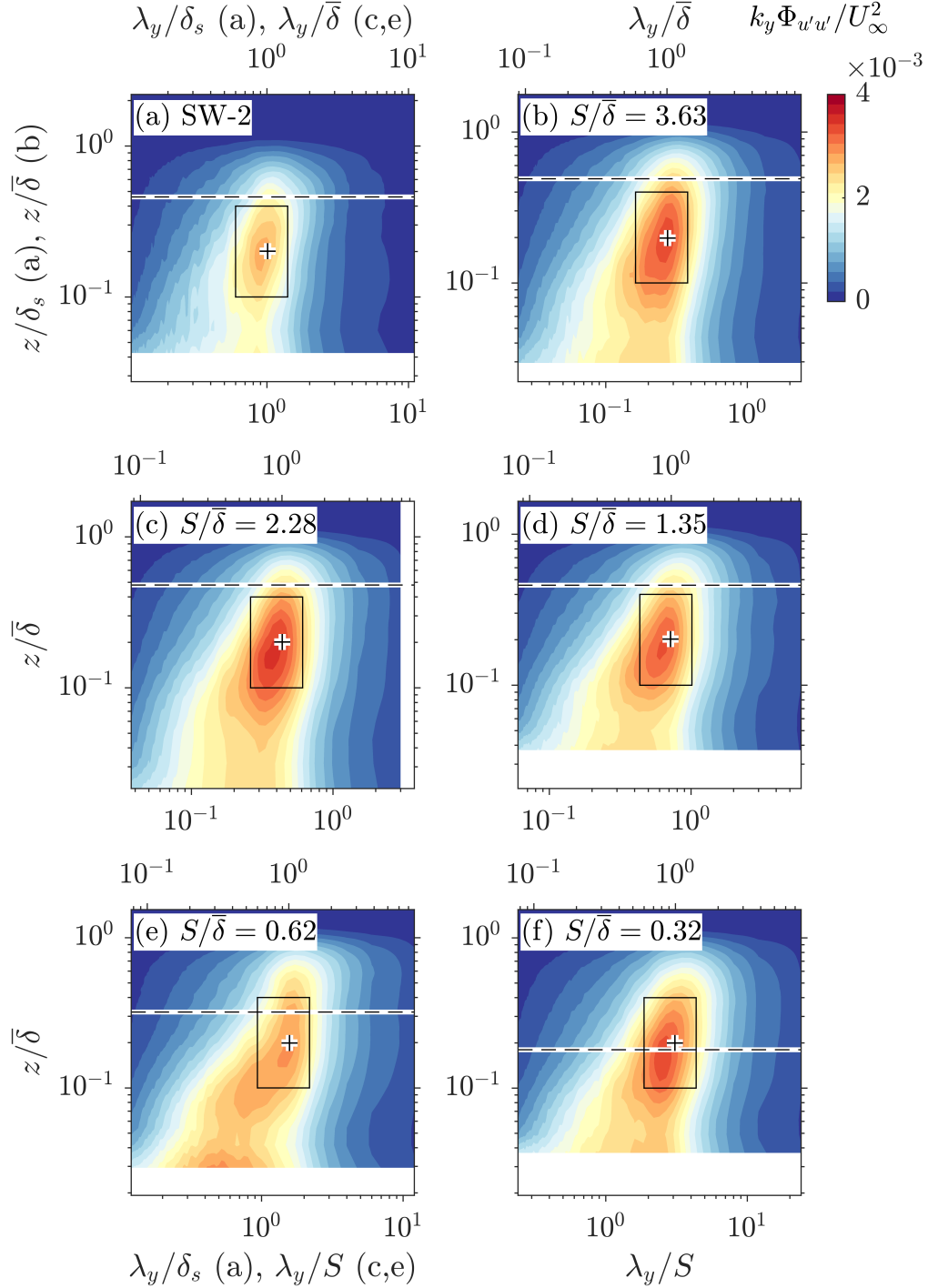


Figure 7.6: Contours of premultiplied energy spectra $k_y \Phi_{u'u'}$ as a function of λ_y and z for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Data are obtained from SPIV measurements. '+' marks the peak of energy spectra in the outer layer at $z/\bar{\delta} = 0.2$ and $\lambda_y/\bar{\delta} = 1$ (b-f), $z/\delta_s = 0.2$ and $\lambda_y/\delta_s = 1$ in (a). Dashed lines are the wall-normal locations of xy -planes in figure 7.11. Black boxes are the area of integration in equation (7.7).

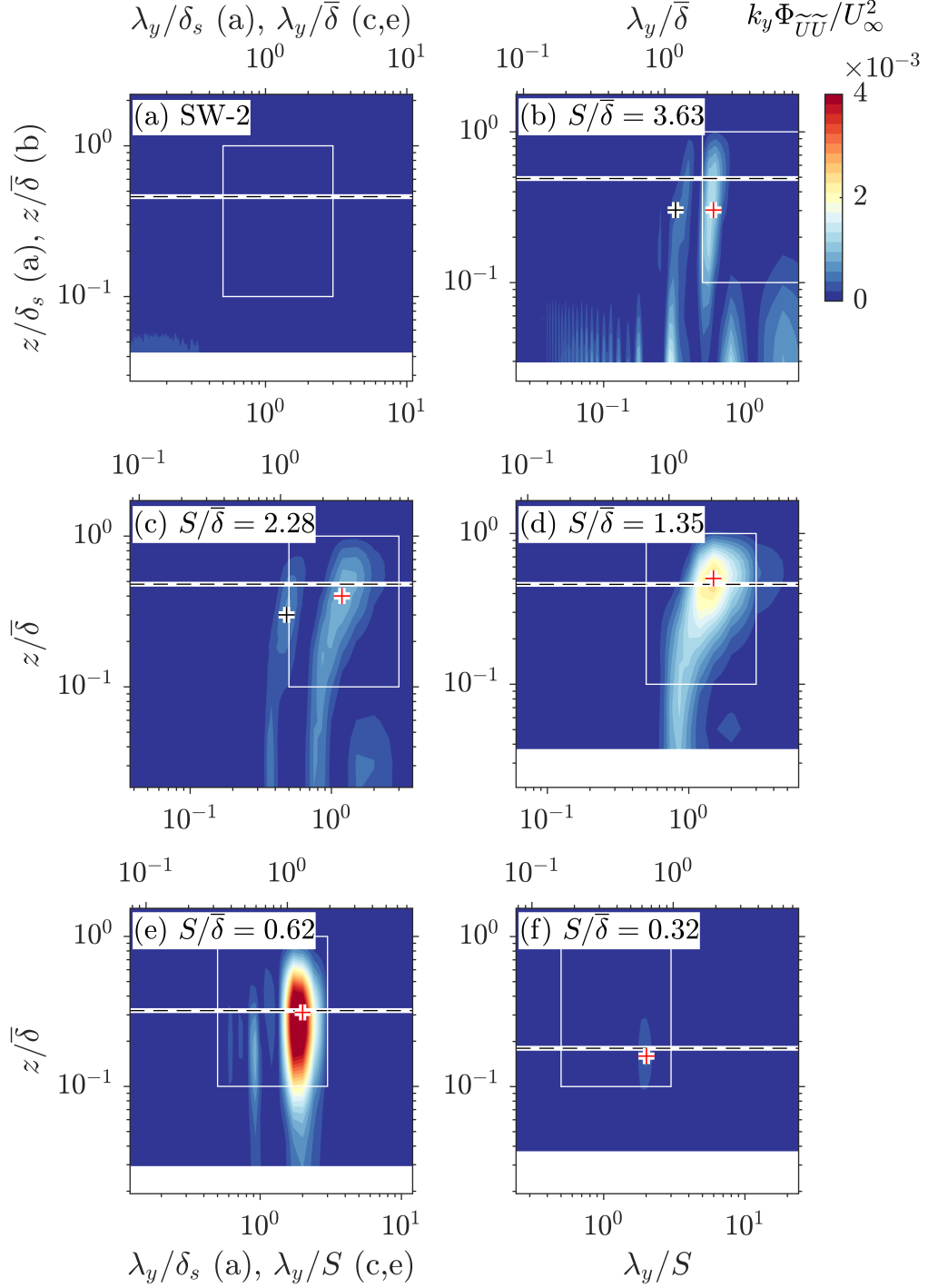


Figure 7.7: Contours of premultiplied energy spectra $k_y \Phi_{\tilde{U}\tilde{U}}$ as a function of λ_y and z for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Data are obtained from SPIV measurements. ‘+’ and ‘+’ mark the peaks of energy spectra in the outer layer related to the spanwise wavelengths shown in black and red arrows in figure 7.8. Dashed lines are the wall-normal locations of xy -planes in figure 7.12. White boxes are the area of integration in equation (7.8) and (7.9).

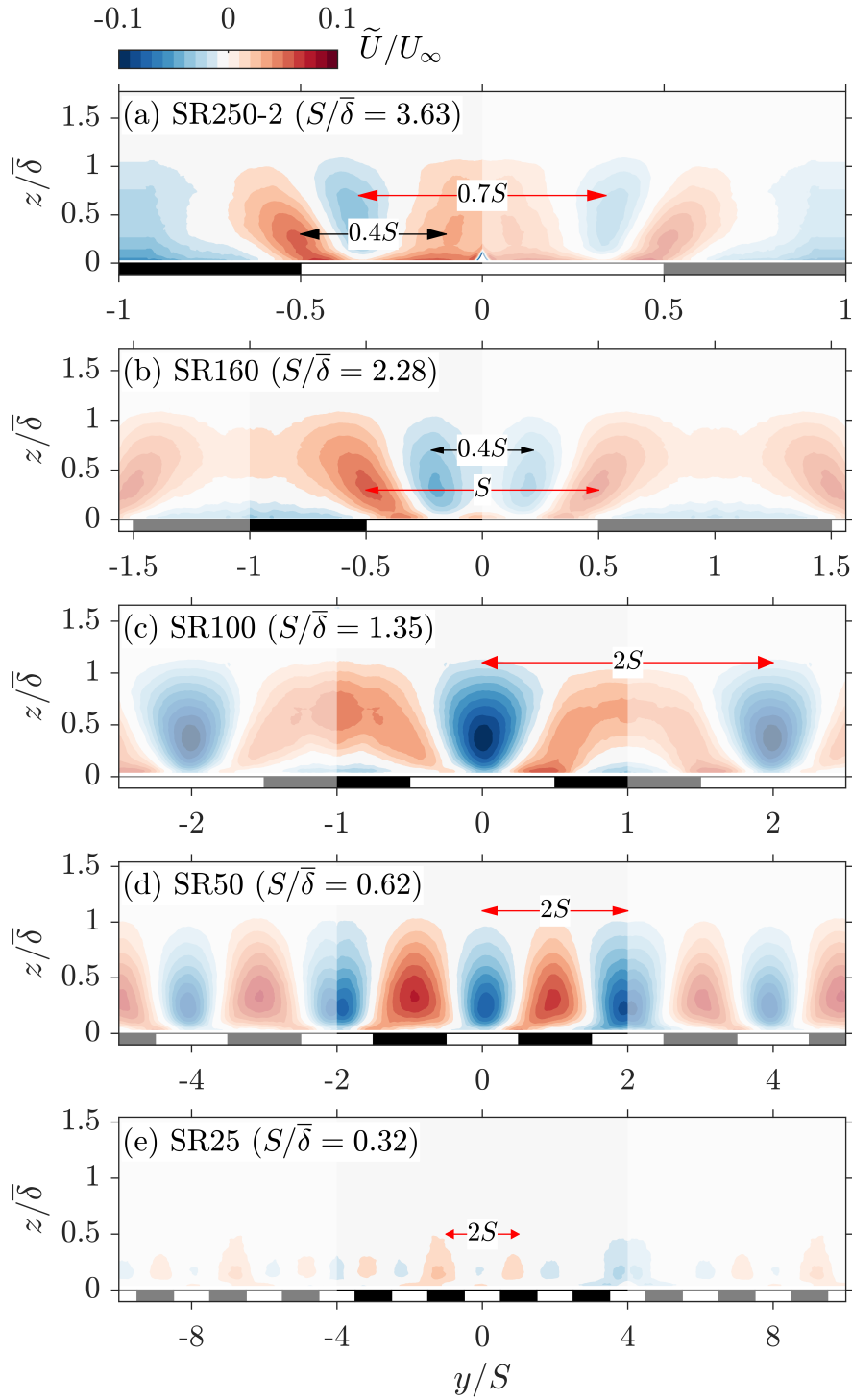


Figure 7.8: Contours of time-averaged, spanwise fluctuation of u , $\tilde{U}$, for SR cases: (a) SR250-2, (b) SR160, (c) SR100, (d) SR50, and (e) SR25. Lightly coloured contours are reflected and repeated from the FOV captured by SPIV cameras. Black arrows show the spanwise wavelength λ_y marked by '+' in figure 7.7, red arrows are λ_y marked by '+' in figure 7.7.

that $k_y \Phi_{u'u'}$ does not contain information regarding the stationary secondary flows (which are contained in $\tilde{U}$ – see figure 7.7). Figure 7.6 contains information about non-stationary turbulent fluctuations, and these seem largely similar between the smooth and heterogeneously rough surfaces. The contours of spanwise energy spectra for SR cases calculated from the SPIV data also do not contain a clear imprint of the time-dependent (meandering) secondary flows, unlike the streamwise energy spectra $k_x \Phi_{u'u'}$ in figure 7.1 and 7.2, which is expected since this assumed meandering behaviour is related primarily to the phase of the spanwise Fourier modes which are not analysed in the amplitude spectra (although increased meandering might affect the magnitude of the resolved $k_y \Phi_{u'u'}$ energy). For completeness, the contours of 1-D spanwise energy spectra for other Reynolds stress components ($k_y \Phi_{v'v'}$, $k_y \Phi_{w'w'}$, and $k_y \Phi_{u'w'}$) are presented in Appendix B.

It is also worth noting that compared to other SR cases, the magnitude of $k_y \Phi_{u'u'}$ is the lowest for case SR50 (figure 7.6e) – the case which shows the strongest outer peak in $k_x \Phi_{u'u'}$ and the most prominent meandering (figure 7.2e). To understand this phenomenon, another attempt is made to examine the characteristics of the secondary flows that appear in the spanwise energy spectra. Instead of the turbulent fluctuations, the energy spectrograms are computed for the time-averaged spatial velocity fluctuations $\tilde{U}$ (figure 5.1), V , and W , which carry the footprint of the time-averaged secondary flows. The method for computation is similar to that of turbulent fluctuations in (7.4)–(7.6). To ensure zero mean across y , the FOV is reflected for the largest S cases (SR250-2 and SR160) so that it captures one spanwise roughness wavelength $\Lambda = 2S$. For the cases where $S/\bar{\delta} \lesssim 1$: SR100, SR50, and SR25, the FOV is clipped so that it captures an integer numbers of spanwise wavelengths, Λ , 2Λ , and 4Λ , respectively.

Figure 7.7 shows the premultiplied energy spectra of $\tilde{U}$ (for completeness, the contours of premultiplied energy spectra of V and W are presented in Appendix B). As expected, the magnitude of $k_y \Phi_{\tilde{U}\tilde{U}}$ is zero for the reference smooth wall case SW-2 in figure 7.7(a). Clear, prominent modes in the energy spectra are observed in all SR cases in figure 7.7(b–e). These peaks are related to the mean secondary flows and are expected to occur at $\lambda_y = 2S$, i.e. the spanwise roughness wavelength. However, multiple modes are observed for the cases where $S/\bar{\delta} \gg 1$ (figure 7.7b,c). For case SR250-2 ($S/\bar{\delta} = 3.63$, figure 7.7b), harmonics are observed at $\lambda_y/\bar{\delta} \approx 1$ ($\lambda_y/S \approx 0.4$, marked by ‘+’ in figure 7.7b) and $\lambda_y/S \approx 0.7$ (marked by ‘+’ in figure 7.7b). Weaker harmonics are also observed closer to the wall, at $\lambda_y/S \approx 1$ and $\lambda_y/S \approx 2$. Figure 7.8(a) shows the contours of $\tilde{U}$ for case SR250-2 (the FOV is

reflected to capture Λ). Here, the mode $\lambda_y/S \approx 0.7$ is associated with the mean secondary flows due to spanwise heterogeneity near the roughness interface. However, the spectrogram also picks up other modes that are not necessarily related to the secondary flows. The mode $\lambda_y/S \approx 0.4$ appears because the flow approaches local homogeneity far removed from the wall (the center of a smooth strip has higher velocity than a rough strip, figure 7.8a). This demonstrates how the choice of velocity decomposition in (5.1)–(5.3), especially when $S/\bar{\delta} \gg 1$, can be misleading when the secondary flows are viewed through spanwise energy spectrograms. Multiple modes are also observed in case SR160 ($S/\bar{\delta} = 2.28$, figure 7.7b). Here, both prominent modes at $\lambda_y/S \approx 0.4$ ($\lambda_y/\bar{\delta} \approx 1$) and $\lambda_y/S \approx 1$ are related to a pair of counter-rotating roll modes separated by large S (figure 7.8b). As S decreases, the secondary flow becomes space-filling and the prominent mode in $k_y \Phi_{\tilde{U}\tilde{U}}$ approaches $2S$. Figure 7.7(d) shows a prominent mode for case SR100 ($S/\bar{\delta} = 1.35$) at $\lambda_y/S \approx 2$ away from the wall, shifting to weaker $\lambda_y/S \approx 1$ tendency in the near-wall region. The contours of $\tilde{U}$ in figure 7.8(c) show that mode $\lambda_y/S \approx 1$ emerges near the wall as the flow reverts to its expected local surface condition with low speed regions located at the centres of the rough strips (note that figures 7.8c–e show a slight asymmetry in the flow field due to lack of convergence and surface misalignment discussed in §5.1). The prominent mode is locked at $\lambda_y = 2S$ for cases where $S/\bar{\delta} \leq 1$. Contours of $k_y \Phi_{\tilde{U}\tilde{U}}$ show a very strong mode at $\lambda_y/S = 2$ for case SR50 ($S/\bar{\delta} = 0.62$, figure 7.7e) and a much weaker one for case SR25 ($S/\bar{\delta} = 0.32$, figure 7.7f). Similar to the results observed in the strength of mean secondary flows (figure 5.9) and the 1-D streamwise energy spectrograms (figure 7.2), the strongest magnitude of energy in $\tilde{U}$ occurs at SR50 ($S/\bar{\delta} = 0.62$) and it decreases as S approaches $S/\bar{\delta} \ll 1$.

The premultiplied energy spectra of u' and $\tilde{U}$ seem to show an opposing phenomenon: when compared to the rest of the SR cases, the magnitude of $k_y \Phi_{u'u'}$ is minimum but $k_y \Phi_{\tilde{U}\tilde{U}}$ is maximum for case SR50 (see figure 7.6e and 7.7e). To further examine this, $k_y \Phi_{u'u'}$ is integrated across the area enclosing the outer peak (white boxes in figure 7.6): $0.6 \leq \lambda_y/\bar{\delta} \leq 1.4$ and $0.1 \leq z/\bar{\delta} \leq 0.4$,

$$I_{\Phi_{u'u',y}} = \frac{1}{0.3\bar{\delta}} \int_{0.1\bar{\delta}}^{0.4\bar{\delta}} \int_{10\pi/3\bar{\delta}}^{10\pi/7\bar{\delta}} \frac{\Phi_{u'u'}}{U_\infty^2} dk_y dz \quad (7.7)$$

For the reference smooth wall cases SW-2, δ_s is used instead of $\bar{\delta}$. $k_y \Phi_{\tilde{U}\tilde{U}}$ for SR cases is integrated across the area enclosing the prominent $\tilde{U}$ modes (white boxes

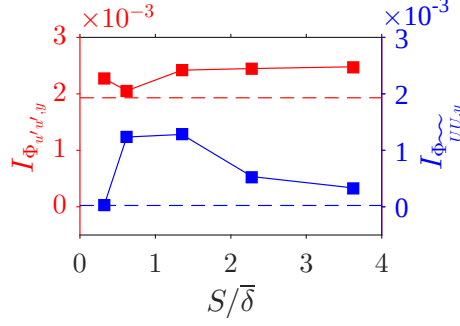


Figure 7.9: Integrated energy spectra of u' ($I_{\Phi_{u'u',y}}$, $\blacksquare$) and $\tilde{U}$ ($I_{\Phi_{\tilde{U}\tilde{U},y}}$, $\blacksquare$) as a function of $S/\bar{\delta}$ for SR cases. Red and blue dashed lines (--- and ---) are $I_{\Phi_{u'u',y}}$ and $I_{\Phi_{\tilde{U}\tilde{U},y}}$ for the reference smooth wall SW-2, respectively. The uncertainties of $\tilde{U}^2$ and $\overline{u'u'}$ are approximately $\pm 4\%$ (see appendix A.3 for the error analysis of U), which are smaller than the symbols. See also appendix A.4 for the convergence analysis of energy spectra.

in figure 7.7b–f): $0.5 \leq \lambda_y/S \leq 3$ and $0.1 \leq z/\bar{\delta} \leq 1$,

$$I_{\Phi_{\tilde{U}\tilde{U},y}} = \frac{1}{0.9\bar{\delta}} \int_{0.1\bar{\delta}}^{\bar{\delta}} \int_{4\pi/S}^{2\pi/3S} \frac{\Phi_{\tilde{U}\tilde{U}}}{U_\infty^2} dk_y dz \quad (7.8)$$

For the reference smooth wall case in figure 7.7(a), the integration area encompasses $0.5 \leq \lambda_y/\delta_s \leq 3$ and $0.1 \leq z/\delta_s \leq 1$,

$$I_{\Phi_{\tilde{U}\tilde{U},y}} = \frac{1}{0.9\delta_s} \int_{0.1\delta_s}^{\delta_s} \int_{4\pi/\delta_s}^{2\pi/3\delta_s} \frac{\Phi_{\tilde{U}\tilde{U}}}{U_\infty^2} dk_y dz \quad (7.9)$$

Integrated energy spectra of u' and $\tilde{U}$ are shown in figure 7.9 as functions of $S/\bar{\delta}$ (dashed lines show the reference smooth wall case SW-2). The lines in figure 7.9 suggest a relationship between u' and $\tilde{U}$: as $S/\bar{\delta}$ approaches 1, $\tilde{U}$ and u' reach a maxima and minima, respectively. As $S/\bar{\delta} \gg 1$, the rise of u' is accompanied by a decrease in $\tilde{U}$. In some sense this fits with the conceptual picture that the secondary flows are spanwise-locked large-scale structures. Under this assumption, SR50 ($S/\bar{\delta} = 0.62$) presumably is the case where the locking is most complete, since the secondary flows are the strongest (figure 5.10) and fill the entire wall-normal and spanwise extent of the boundary layer (this is later shown in the instantaneous velocity fields in figure 9.2). If the secondary flows can really be viewed as locked turbulent structures, it seems reasonable to assume that for heterogeneous surfaces that generate secondary flows some of the large-scale structure energy will be transferred from

turbulent fluctuations u' to the dispersive component $\tilde{U}$. This interplay between turbulent fluctuations and the dispersive components (as one goes up, the other one goes down and vice versa) is strongly reminiscent of similar observations by Nikora *et al.* (2019) of secondary flows in rough-bed open channel flows and by Modesti *et al.* (2018) of secondary flows in a square duct. In the latter case, Modesti *et al.* (2018) also observed that when the secondary flows are artificially suppressed, the turbulent fluctuations rise to compensate the loss of dispersive component. Taken together, a possible interpretation of these results would be that the $\tilde{U}$ component may be an indicator of the efficacy of the surface at locking turbulent structures in place. For example, in the smooth surface, $\tilde{U}$ is zero (blue dashed line in figure 7.9 and figure 7.7a), and the turbulent structures are randomly occurring, so all energy ends up in the u' component. It is noted that the case where $S/\bar{\delta} \ll 1$ in figure 7.9 (SR25, $S/\bar{\delta} = 0.32$) approaches this condition, which the integrated energy in $\tilde{U}$ approaching the smooth wall reference case. To summarize, a possible conclusion from the above analysis is that for $S/\bar{\delta} \approx 1$, the turbulent structures are optimally locked or trapped by the surface, and hence energy from u' is surrendered into $\tilde{U}$ (which respectively reach a minima and a maxima at these $S/\bar{\delta}$).

7.3 2-D Energy spectra and co-spectra

The 2-D energy spectra and co-spectra as a function of streamwise and spanwise wavelength, λ_x and λ_y respectively, are computed from the fluctuating velocity field obtained from WPPIV measurements at a certain wall-normal height. Like the computation of 1-D spanwise energy spectra, no assumption is necessary for the conversion of time-signal to spatial information. The non-normalised two-point correlation of the fluctuating velocity component a' and b' is given by

$$\hat{R}_{a'b'} = \overline{a'(x, y)b'(x + \Delta x, y + \Delta y)} \quad (7.10)$$

where the overbar denotes ensemble averaging of all WPPIV realizations and at all x and y , Δx is the streamwise shift. The 2-D Fourier transformation of $\hat{R}_{a'b'}$ yields the 2-D energy spectra of the fluctuating velocity component $\Phi_{a'b'}$

$$\Phi_{a'b'} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{R}_{a'b'} e^{-j2\pi(k_x \Delta x + k_y \Delta y)} d(\Delta x) d(\Delta y) \quad (7.11)$$

The relationship between the Reynolds stress component $\overline{a'b'}$ and $\Phi_{a'b'}$ is given by

$$\overline{a'b'} = \int_0^{\infty} \int_0^{\infty} \Phi_{a'b'} d(k_x) d(k_y) \quad (7.12)$$

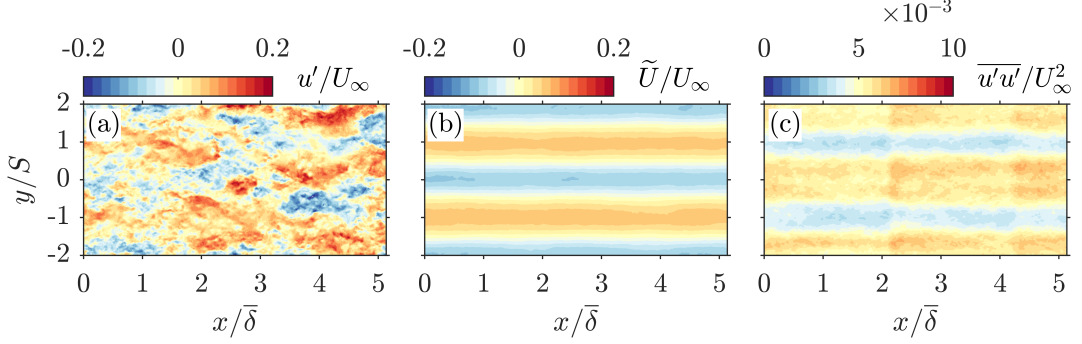


Figure 7.10: (a) Instantaneous turbulent fluctuations u' , (b) time-averaged spatial fluctuations of streamwise velocity component $\tilde{U}$, and (c) variance $\overline{u'u'}$ for case SR50 (see §4.2.6 for the error analysis in $\overline{u'u'}$).

Due to the limited spanwise extent of the FOV, the velocity field is padded with zeros in y to increase the length of the signal to three times the original FOV width (from approximately $5\delta_s$ to $15\delta_s$), thus increasing the resolution of k_y and λ_y . The velocity field is not padded with zeros in the streamwise direction, giving the total length of signal of $9\delta_s$ in x . For the reference smooth wall case SW-2, the 2-D spectrograms are computed at $z/\delta_s = 0.46$, while the spectrograms are computed approximately at the center of mean secondary flows for the SR cases (table 4.3)

Figure 7.4 shows the validation of 2-D spanwise energy spectra and co-spectra computation for the reference smooth wall case SW-2 at $z/\delta_s \approx 0.2$ and $z/\delta_s \approx 0.5$. The method, detailed in (7.10)–(7.12), is validated for Reynolds stresses components: $\overline{u'u'}$, $-\overline{u'w'}$ (figure 7.4a) and $\overline{v'v'}$, $\overline{w'w'}$ (figure 7.4b). The figures show that these Reynolds stresses components obtained from WPPIV measurements (‘+’) collapse with those computed from integration of energy spectra across all resolved k_x and k_y (‘o’) at both wall-normal slices. As with the method used to compute 1-D spanwise energy spectra, the method for 2-D energy spectra computation must also be applied with caution for the spanwise heterogeneous roughness cases. Figure 7.10(a) shows the instantaneous turbulent fluctuation component u' for case SR50 ($S/\bar{\delta} = 0.62$), whose mean (for all snapshots) across the spanwise extent of the FOV is zero. The variance, however, is heterogeneous across y (figure 7.10c) and the integration of energy spectra across k_x and k_y in (7.12) results in the xy -average of a Reynolds stress component.

Figure 7.11 shows the premultiplied 2D spectrograms $k_x k_y \Phi_{u'u'}$ (for completeness, the contours of $k_x k_y \Phi_{v'v'}$ and $k_x k_y \Phi_{u'v'}$ are shown in Appendix C). Similar to the 1-D spanwise spectra in §7.2, the shape of 2D spectrograms is similar between the reference smooth wall case SW-2 (figure 7.11a) and SR cases (figure 7.11b–f). It

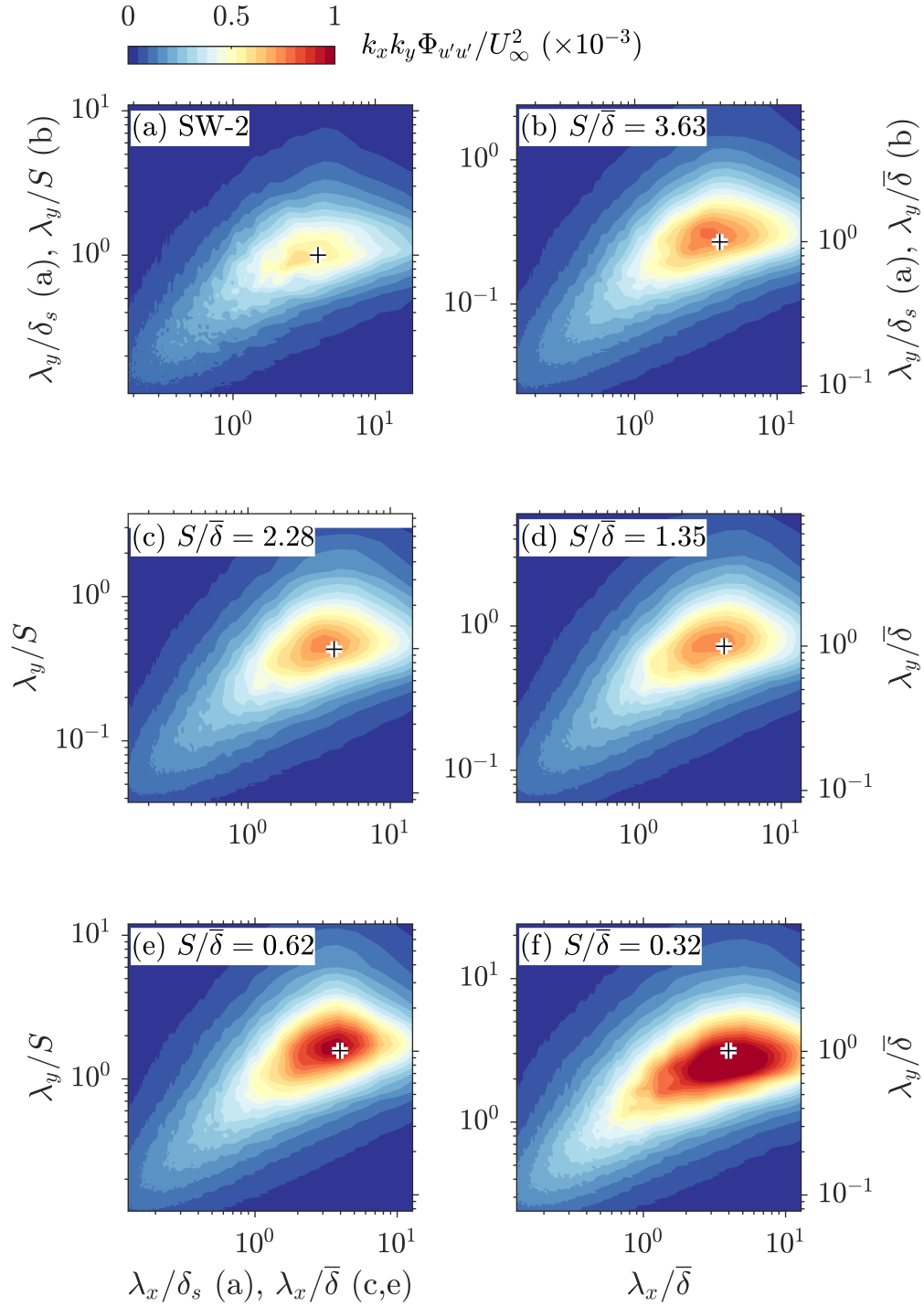


Figure 7.11: Contours of premultiplied energy spectra $k_x k_y \Phi_{u'u'}$ as a function of λ_x and λ_y for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Data are obtained from WPPIV measurements. ‘+’ marks the peak of energy spectra in the outer layer. The wall-normal locations of WPPIV planes are shown by the dashed lines in figure 7.6.

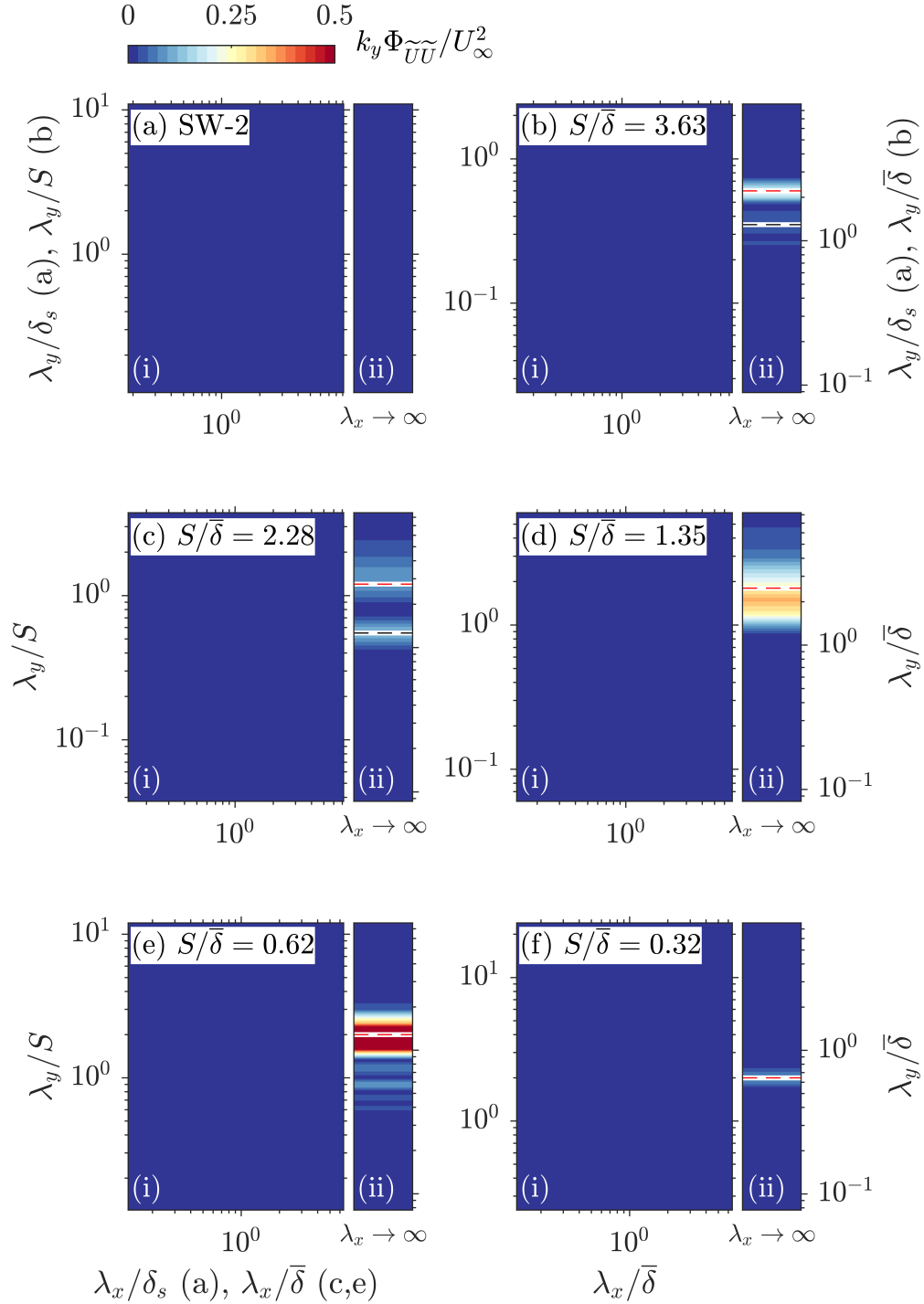


Figure 7.12: (i) Contours of premultiplied energy spectra $k_y \Phi_{\tilde{u}\tilde{u}}$ as a function of λ_x and λ_y for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Data are obtained from WPPIV measurements. (ii) Contours of non-premultiplied energy spectra at the zeroth mode of $k_y \Phi_{\tilde{u}\tilde{u}}$, $\lambda_x \rightarrow \infty$ ($k_x = 0$). The wall-normal locations of WPPIV planes are shown by the dashed lines in figure 7.7.

should be noted that the contours of case SW-2 in figure 7.11(a) are less converged than those of the SR cases because it has only 600 instead of 1200 realisations per case. All peaks of the energy spectra are observed at $\lambda_y/\bar{\delta} \approx 1$ ($\lambda_y/\delta_s \approx 1$ for SW-2), with no clear distinction between SR cases. This is in line with the observation in the 1-D spanwise spectra $k_y\Phi_{u'u'}$ (figure 7.6), suggesting that the energy spectra of u' captures the scale and magnitude of the turbulent (non-stationary) structures, and that the characteristic scale of these structures is largely unaffected by the surface heterogeneity. Certainly, in u' there is no obvious sign of energy at $\lambda_y/S = 2$. However, contrary to $k_y\Phi_{u'u'}$ (figure 7.6), the contours of $k_xk_y\Phi_{u'u'}$ in figure 7.11 show a stronger peak in case SR50 and SR25 (figure 7.11e and f, respectively). This is an artefact of the chosen wall-normal position at which these wall-parallel PIV planes are acquired. For all SR cases, the 2-D spectrograms are computed approximately at a wall-parallel plane that roughly slices the center of the secondary flows. For cases SR250-2, SR160, and SR100, the plane is located at $z/\bar{\delta} \approx 0.5$, while for SR50 and SR25 it is located at $z/\bar{\delta} = 0.32$ and $z/\bar{\delta} = 0.18$, respectively. The wall-normal locations of the WPPIV are shown by the dashed lines in figure 7.6. For SR50 and SR25, the planes are closer to the outer peak of $k_y\Phi_{u'u'}$ (figure 7.6) at $z/\bar{\delta} \approx 0.2$, which is related to the naturally occurring large-scale turbulent structures, and hence there are magnitude contours of $k_xk_y\Phi_{u'u'}$ in figure 7.11(e) and (f).

As with the 1-D spanwise spectra in §7.2, an attempt is made to examine the secondary flows through the spectrograms of time-averaged spatial velocity fluctuations instead of the turbulent fluctuations. Figure 7.10(b) shows the contours of $\tilde{U}$ for case SR50 ($S/\bar{\delta} = 0.62$). Strips of high- and low-speed flows can be observed clearly in the contours, which results in at least one dominant mode in y at the zero wavenumber mode of x (infinite wavelength, $\lambda_x \rightarrow \infty$). Figure 7.12 shows the premultiplied energy spectra $k_y\Phi_{\tilde{U}\tilde{U}}$ (instead of $k_xk_y\Phi_{\tilde{U}\tilde{U}}$) for the reference smooth wall case SW-2 and SR cases. Axes (i) show a two-dimensional energy spectra for all resolved modes in x and y (k_y and $k_x > 0$). The energy spectra approaches zero for both SW-2 and all SR cases. The strong spanwise periodicity in SR cases illustrated in figure 7.10(b) is concentrated at mode $k_x = 0$ ($\lambda_x \rightarrow \infty$), shown in axes (ii) in figure 7.12(b–f). The prominent modes, marked by dashed lines (--- and ---), correspond to the modes marked by ‘+’ and ‘+’ in $k_y\Phi_{\tilde{U}\tilde{U}}$ (figure 7.7b–f) and the spanwise periodicity marked by black and red arrows in the contours of $\tilde{U}$ (figure 7.8). All three figures show multiple prominent modes for cases where $S/\bar{\delta} \gg 1$

due to the choice of velocity decomposition, which merge into a single prominent mode $\lambda_y/S \approx 2$ when $S/\bar{\delta} \approx 1$, as the secondary flows become space filling.

To summarise, the energy spectra computed from both HWA and PIV results give an insight on the instantaneous behaviour of the secondary flows. Based on the velocity decomposition in figure 5.1, $\tilde{U}$ shows secondary flows as stationary (locked) structures and u' is the (supposedly random) turbulent fluctuations. The 1-D spanwise energy spectrograms from SPIV suggest that there is an interplay between u' and $\tilde{U}$, with the possibility of u' energy leeching into $\tilde{U}$. Further, the results also suggest that the efficacy of the surfaces in locking turbulent structures (and how much of the u' leeches to $\tilde{U}$) are a function of $S/\bar{\delta}$. When $S/\bar{\delta} \approx 1$, the locking is most complete and the energy is mostly deposited to $\tilde{U}$. The streamwise energy spectra ($k_x \Phi_{u'u'}$) from HWA further suggest that there is an unsteadiness of the secondary flows that manifests as an extra site of u' energy at the location where secondary flows are locked and with a preferred streamwise wavelength of $\lambda_x/\bar{\delta} = 3$ –4. We should, however, exercise caution when interpreting energy spectrograms from both SPIV and WPPIV measurements for heterogeneous surfaces, where there is no homogeneity in the spanwise direction. The computed spanwise energy spectra will yield the spanwise average of the variance when integrated. This variance is homogeneous across the span for the reference cases, but is a function of $\Delta y/S$ for the spanwise heterogeneous surfaces. With this in mind, the next step is to analyse the secondary flows further through the instantaneous velocity field in §9.

Chapter 8

The structure of spanwise homogeneous wall-bounded turbulence

To further examine the time-varying behaviour of the secondary flows shown by the HWA-measured energy spectrograms, the instantaneous velocity fields of turbulent boundary layers developing above the reference smooth wall cases are examined in this chapter. Two-point correlations of the turbulent fluctuation fields, u' , v' , and w' , obtained from SPIV and WPPIV measurements are computed to examine the spatial coherence.

8.1 Two-point correlations

The correlations are computed for all turbulent fluctuation components u' , v' , and w' (see equation 2.1 and illustrations in figure 5.1). In this study, we limit the observations to the large-scale structures, which are approximately the scale of the secondary flows induced by spanwise heterogeneity, and filter out smaller scales. Thus, the velocity fields are filtered with a box filter of $0.1\delta_s \times 0.1\delta_s$ size. The filtered streamwise turbulent fluctuations are denoted by subscript ' f ', giving u'_f , v'_f , and w'_f for streamwise, spanwise, and wall-normal component, respectively. Regarding the choice of velocity filter type and size, it should be noted that the resulting conditionally averaged structures are, to an extent, insensitive to these parameters. Figure 8.1 shows a comparison between different filter type and size in the resulting contours of normalised two-point correlation coefficient of u'_f , $R_{u'_f u'_f}$ for the reference smooth-wall case SW-2. The contours are taken from two wall slices, $z/\delta_s = 0.24$ (figures 8.1a,c,e) and $z/\delta_s = 0.46$ (figures 8.1b,d,f). Filter types and sizes are systematically varied and applied in the instantaneous velocity fields: figures 8.1(a,b) use

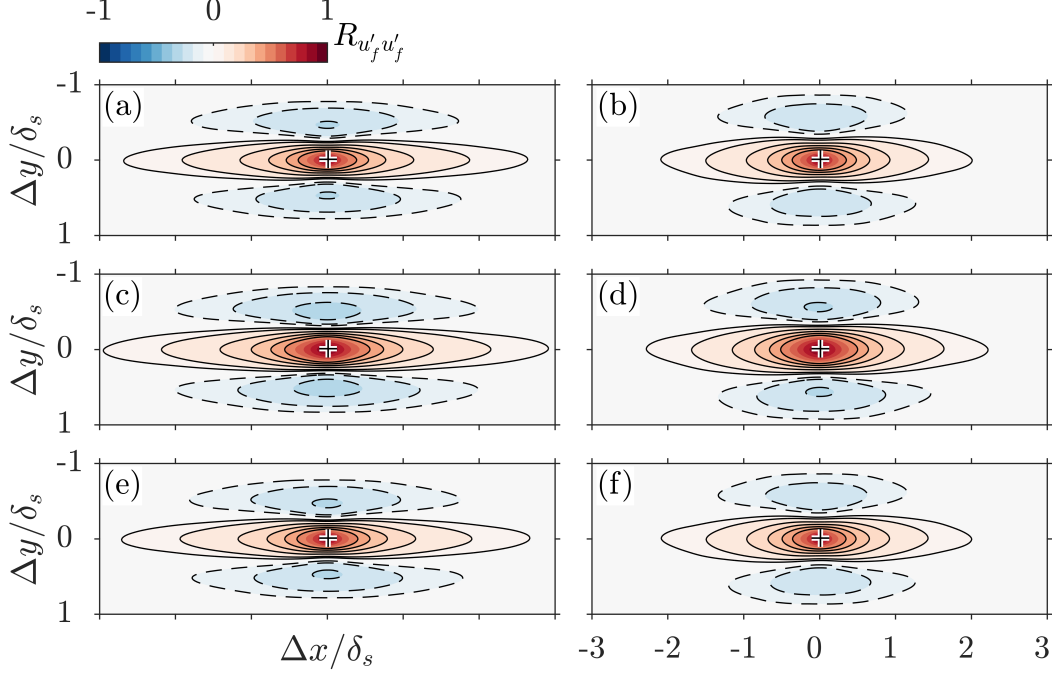


Figure 8.1: Velocity field filter applied on the reference smooth-wall case SW-2 at (a,c,e) $z/\delta_s = 0.24$ and (b,d,f) $z/\delta_s = 0.46$. Contours show the normalised two-point correlation coefficient $R_{u'_f u'_f}$ calculated by equation (8.2). In (a,b), the instantaneous velocity fields are filtered with a box filter of $0.1\delta_s \times 0.1\delta_s$ size; in (c,d), box filter of $0.25\delta_s \times 0.25\delta_s$ size; (e,f) uses a 2-D Gaussian filter ($\sigma = 2$) of $0.25\delta_s \times 0.25\delta_s$ size.

a box filter of $0.1\delta \times 0.1\delta_s$ size, figures 8.1(c,d) use a box filter of $0.25\delta \times 0.25\delta_s$ size, and figures 8.1(e,f) use a 2-D Gaussian filter ($\sigma = 2$) of $0.25\delta_s \times 0.25\delta_s$. The resulting $R_{u'_f u'_f}$ contours have identical shapes in all cases compared here. Figures 8.1(a,b) are identical to figures 8.1(e,f). When the same filter is used with a larger filter size ($0.25\delta \times 0.25\delta_s$), a slightly longer structures are observed (figures 8.1c,d). However, this does not alter the conclusion of the study.

Spatial homogeneity is assumed in the computation of two-point correlations. For the reference smooth wall case SW-2 in the yz -plane (SPIV plane), homogeneity is assumed in the spanwise direction. The normalised two-point correlation coefficient between fluctuating signal a' and b' is defined as

$$R_{a'b'}(z_{ref}) = \frac{\overline{a'(y, z_{ref})b'(y + \Delta y, z)}}{\sigma_a(z_{ref})\sigma_b(z)} \quad (8.1)$$

where z_{ref} is the wall-normal reference point where the correlations are taken, Δy is the spanwise shift between datapoints in a' and b' , and σ_a and σ_b are the standard deviation of signal a and b , respectively. For the reference smooth wall case SW-2

in the xy -plane (WPPIV plane), it is assumed that the flow is homogeneous in both streamwise and spanwise (x - and y -directions). The normalised two-point correlation coefficient between fluctuating signals is defined as

$$R_{a'b'} = \frac{\overline{a'(x, y)b'(x + \Delta x, y + \Delta y)}}{\sigma_a \sigma_b} \quad (8.2)$$

where Δx is the streamwise shift between datapoints in a' and b' .

The normalised two-point autocorrelation contours of filtered streamwise turbulent fluctuations u'_f , $R_{u'_f u'_f}$, is shown in figure 8.2 for $z_{ref}/\delta_s = 0.24$ and figure 8.3 for $z_{ref}/\delta_s = 0.46$. The correlations are taken at the wall-normal locations of the WPPIV planes (table 4.3) and contours are shown for both the xy - and the yz -planes. The regions of positive $R_{u'_f u'_f}$ are shown by solid contours, while dashed contours show negative $R_{u'_f u'_f}$. These figures show a streamwise elongated region of positive correlation flanked in the span by regions of negative correlation. This pattern is a known representation of long, low- and high-speed structures in turbulent boundary layers flanked by opposite signed events (Tomkins & Adrian, 2003; Ganapathisubramani *et al.*, 2005; Hutchins & Marusic, 2007a; Sillero *et al.*, 2014). Figure 8.4(f) and 8.5(f) show the autocorrelation coefficient $R_{u'_f u'_f}$ at z_{ref} as a function of y , with the maximum negative value of $R_{u'_f u'_f} \approx -0.2$ on either side of the positive correlation at $\Delta y/\delta_s < 0.5$ at $z_{ref}/\delta_s = 0.24$ and $\Delta y/\delta_s > 0.5$ at $z_{ref}/\delta_s = 0.46$. In xy -plane, the streamwise coherence length is reduced as the reference point z_{ref} moves further from the wall due to the inclination of the head of the hairpin vortices (Ganapathisubramani *et al.*, 2005). The streamwise extent of the $R_{u'_f u'_f} = 0.05$ contour is approximately $6\delta_s$ when $z_{ref}/\delta_s = 0.24$ (figure 8.2), which is reduced to $4\delta_s$ at $z_{ref}/\delta_s = 0.46$ (figure 8.3). It is noted that the $R_{u'_f u'_f} > 0$ correlation map has a slight hourglass shape (**(A)** in figure 8.4(a)ii and 8.5(a)ii) flanked by triangular $R_{u'_f u'_f} < 0$ contours on each side (**(B)** in figure 8.4(a)ii and 8.5(a)ii), which has previously been interpreted as an evidence of meandering of LSM/VLSM in turbulent boundary layers (Hutchins & Marusic, 2007a). It is also suggested in Sillero *et al.* (2014) that the hourglass shape is the result of superposition of two opposite events (events yawed in the positive or negative y direction).

Superposition is also likely the reason behind the square-ish shape of the autocorrelation of spanwise fluctuations $R_{v'_f v'_f}$ (figure 8.4b and 8.5b). The shape is the result of diagonal alignment of v' in the instantaneous velocity fields leaning either to the positive or negative y direction, depending on whether $v' > 0$ or $v' < 0$ (Sillero *et al.*, 2014). The shape of the autocorrelation of wall-normal fluctuations $R_{w'_f w'_f}$ (figure 8.4d and 8.5d) is similar to $R_{u'_f u'_f}$, except that it is taller and narrower

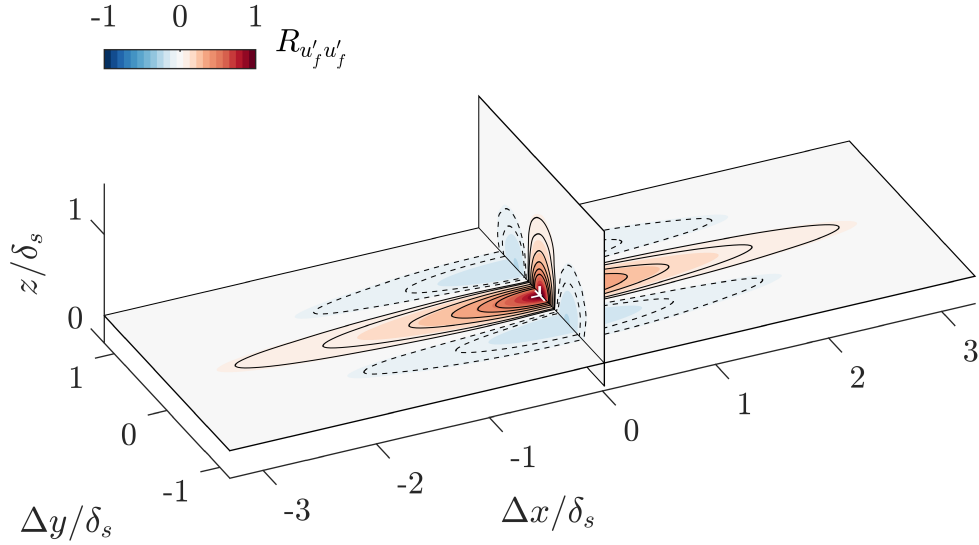


Figure 8.2: Contour of normalized two-point correlation coefficient of filtered streamwise turbulent fluctuation $R_{u'_f u'_f}$ for the reference smooth wall case SW-2, constructed from spanwise-wall-normal and streamwise-spanwise planes. ‘+’ symbol indicates the reference wall-height point at $z_{ref}/\delta_s = 0.24$. Contour levels: $R_{u'_f u'_f} = 0.05, 0.1, 0.2, \dots, 0.5$ (—) and $R_{u'_f u'_f} = -0.05, -0.1$ (---).

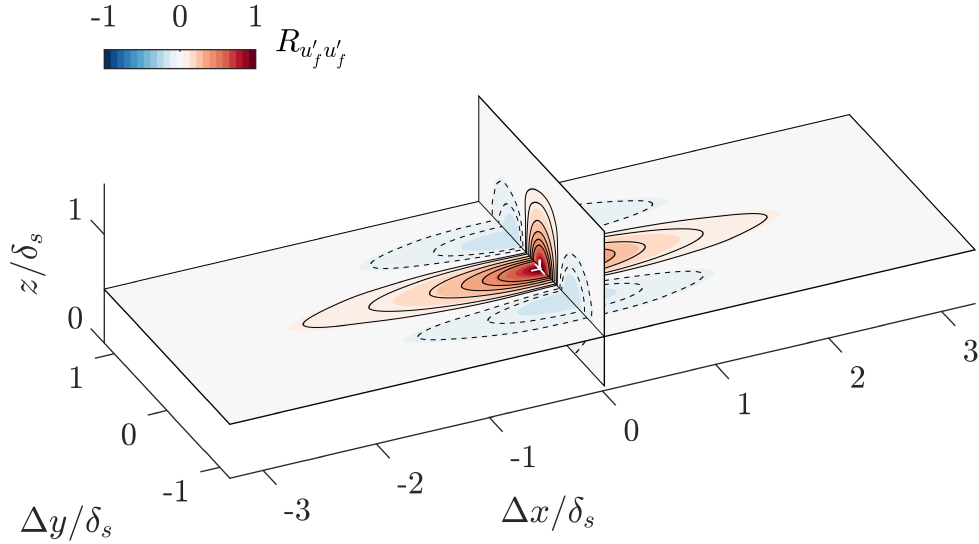


Figure 8.3: Contour of normalized two-point correlation coefficient of filtered streamwise turbulent fluctuation $R_{u'_f u'_f}$ for the reference smooth wall case SW-2, constructed from spanwise-wall-normal and streamwise-spanwise planes. ‘+’ symbol indicates the reference wall-height point at $z_{ref}/\delta_s = 0.46$. Contour levels: $R_{u'_f u'_f} = 0.05, 0.1, 0.2, \dots, 0.5$ (—) and $R_{u'_f u'_f} = -0.05, -0.1$ (---).

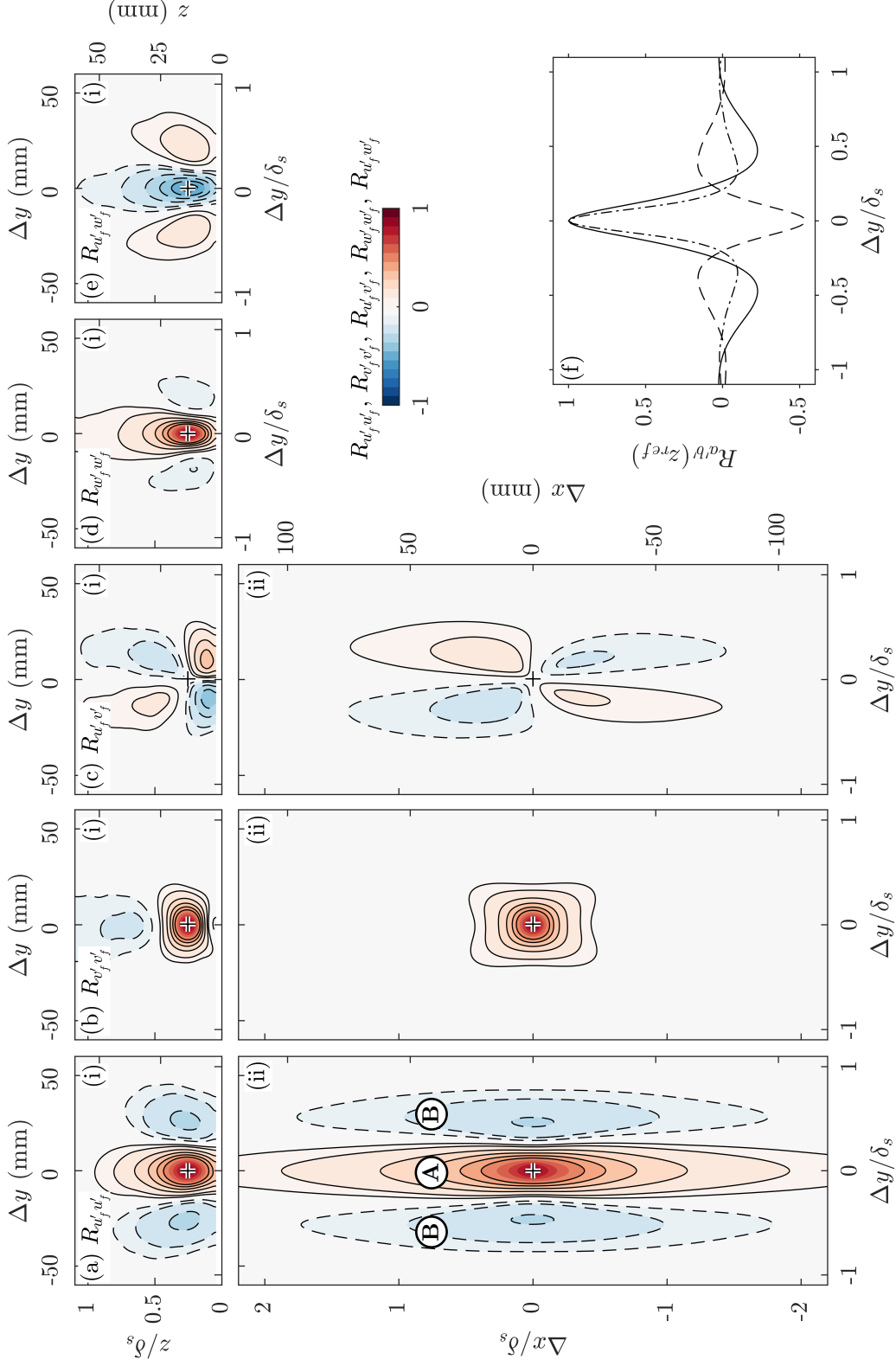


Figure 8.4: Contour of normalized two-point correlation coefficient of filtered turbulent fluctuation components: (a) $R_{u'_f u'_f}$, (b) $R_{u'_f v'_f}$, (c) $R_{u'_f w'_f}$, (d) $R_{v'_f v'_f}$, (e) $R_{v'_f w'_f}$, and (f) $R_{w'_f w'_f}$ for the reference smooth wall case SW-2. '+' symbol indicates the reference wall-height point at $z_{ref}/\delta_s = 0.24$. Contours are shown in (i) spanwise-wall-normal and (ii) streamwise-spanwise plane (when the data are available). Contour levels: $R_{u'v'} = 0.05, 0.1, 0.2, \dots, 0.5$ (—) and $R_{u'v'} = -0.2, -0.1, -0.05$ (---). (f) $R_{u'_f u'_f}$ (—), $R_{u'_f v'_f}$ (---), and $R_{u'_f w'_f}$ (---) at the reference wall height z_{ref} .

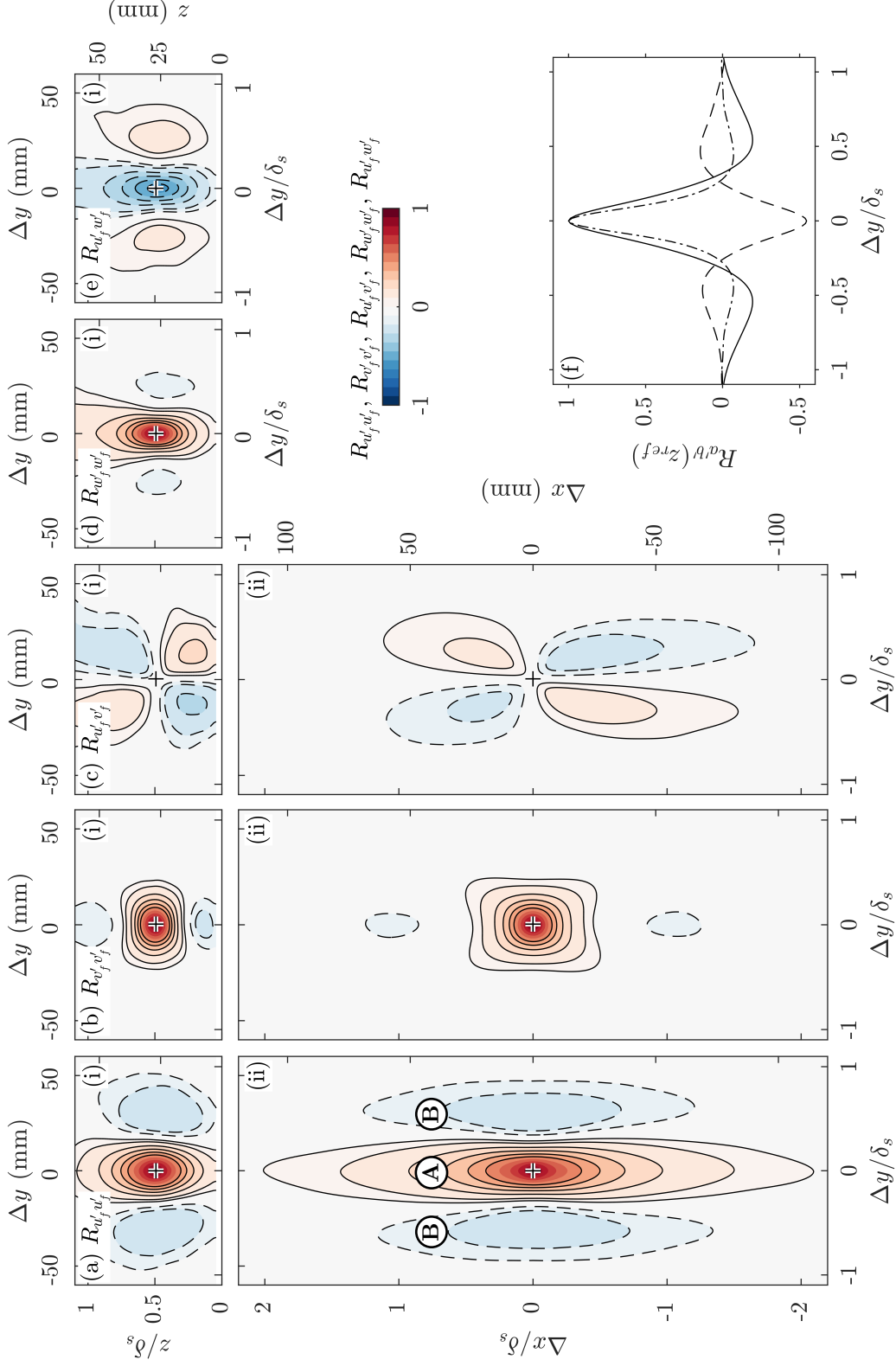


Figure 8.5: Contour of normalized two-point correlation coefficient of filtered turbulent fluctuation components: (a) $R_{u'_f u'_f}$, (b) $R_{u'_f v'_f}$, (c) $R_{u'_f w'_f}$, (d) $R_{v'_f v'_f}$, (e) $R_{v'_f w'_f}$, and (f) $R_{w'_f w'_f}$. Contours are shown in (i) spanwise-wall-normal and (ii) streamwise-spanwise plane (when the data are available). Contour levels: $R_{u'_f u'_f} = 0.05, 0.1, 0.2, \dots, 0.5$ (—) and $R_{u'_f v'_f} = -0.2, -0.1, -0.05$ (---). (f) $R_{u'_f u'_f}$ (—), $R_{u'_f v'_f}$ (---), and $R_{u'_f w'_f}$ (---) at the reference wall height z_{ref} .

(figure 8.4f and 8.5f). The contours of $R_{u'_f w'_f}$ show a negative correlation flanked by positive contours on each side (figure 8.4e and 8.5e), with the maximum negative value of $R_{u'_f w'_f} \approx -0.5$. Together with the long, low-speed structures represented in the contours of $R_{u'_f u'_f}$, the negative correlation of $R_{u'_f w'_f}$ implies that the occurrence of low-speed structures ($u' < 0$) coincides with upwelling $w' > 0$. This is related to the quadrant analysis of Reynolds shear stress, where the events of $u'w' < 0$ (Q2 and Q4) are more dominant and less random than $u'w' > 0$ (Willmarth & Lu, 1972; Lu & Willmarth, 1973). It is also observed in figure 8.4(f) and 8.5(f) that the change of sign of $R_{u'_f u'_f}$ and $R_{u'_f w'_f}$ from positive to negative and vice versa occurs at a similar spanwise extent ($\Delta y/\delta_s \approx 0.2$ for $z_{ref}/\delta_s = 0.24$ and $\Delta y/\delta_s \approx 0.3$ for $z_{ref}/\delta_s = 0.46$), which suggests that the Reynolds shear stress $u'w'$ and streamwise velocity have a similar spanwise scale (Ganapathisubramani *et al.*, 2005). The shape of $R_{u'_f v'_f}$ is shown in figure 8.4(c) and 8.5(c). As discussed in Volino *et al.* (2007); Hutchins *et al.* (2012); Mejia-Alvarez & Christensen (2013), the shape represents the opposing sign of quasi-streamwise roll-modes (occasionally attributed to legs of hairpin vortices) that flank a low-speed structure. It should be noted, however, that symmetry in y about the reference point is enforced in these two-point correlations, and we should remain open-minded to a more asymmetric model of instantaneous structure.

8.2 Instantaneous velocity fields

The instantaneous velocity fields for the reference smooth wall case SW-2 at $z_{ref}/\delta_s = 0.24$ and 0.46 are given in figure 8.6 and 8.8, respectively. As shown in the contours of two-point correlation coefficient $R_{u'_f u'_f}$ in figure 8.2 and 8.3, the large-scale structures has longer coherence closer to the wall at $z_{ref}/\delta_s = 0.24$ (figure 8.6d) than 0.46 (figure 8.8d). It should be noted that in the instantaneous SPIV snapshots shown in figure 8.6 and 8.8, (a–c) are not taken simultaneously with the WPPIV snapshots shown in plots (d,e). The anti-correlated pattern of $R_{u'_f w'_f}$, shown previously in figure 8.4e and 8.5e, is clearly evident in the instantaneous velocity field of u' and w' in figure 8.6(a,c) and 8.8(a,c). The occurrence of low-speed streaks ($u' < 0$, blue-filled contours, solid black contour lines in plot a) coincides with $w' > 0$ events (red filled contours in plot c) and vice versa. The pattern between u' and v' events (figure 8.6b,e and 8.8b,e) is less clear. However, figure 8.7(a,b) and 8.9(a,b) show that the square shape of $R_{v'_f v'_f}$ originates from a tendency for instantaneous spanwise velocity fluctuations v' to be arranged at either of two characteristic angles, 135° or

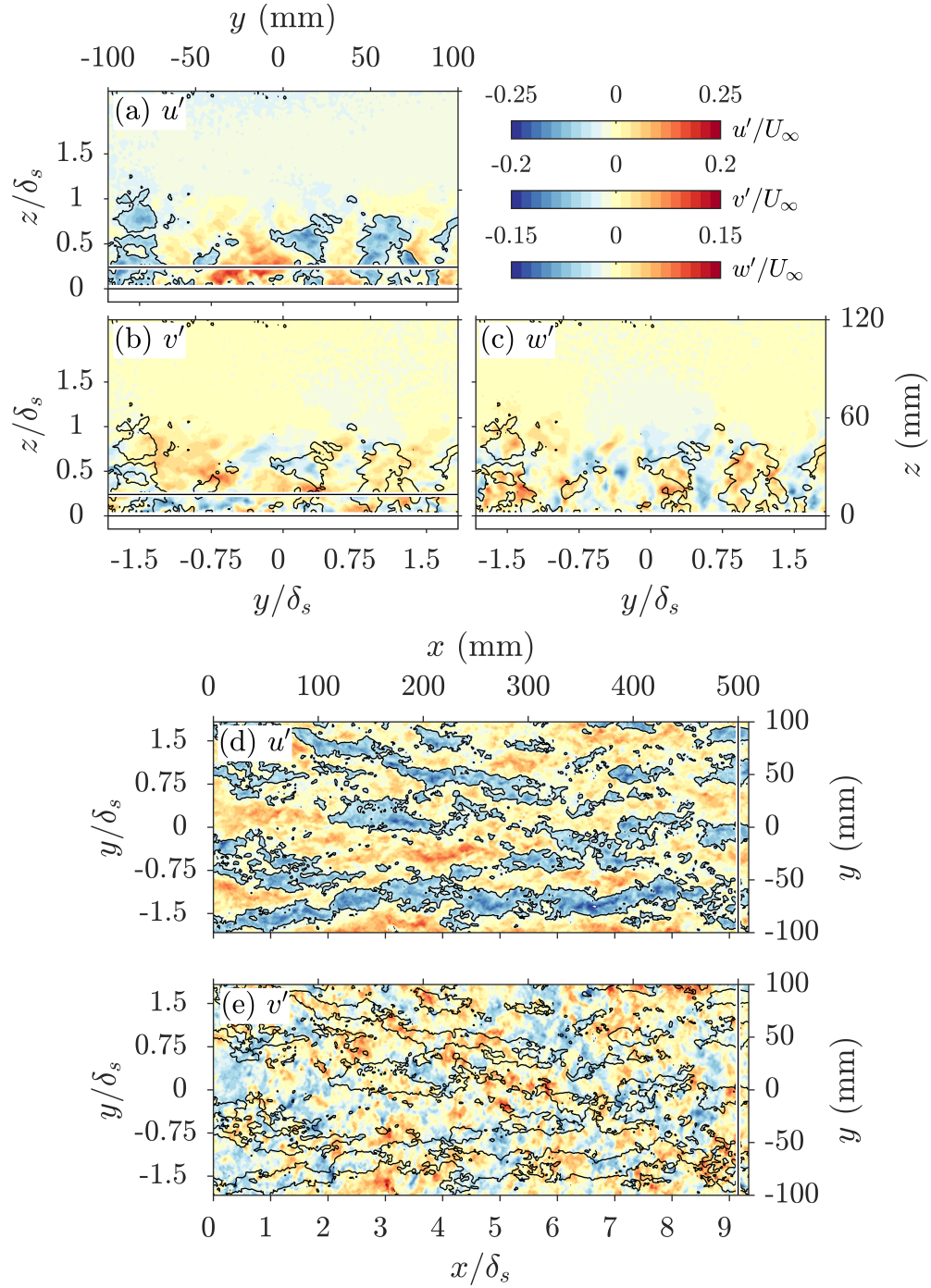


Figure 8.6: Instantaneous velocity field of case SW-2 in yz -plane: (a) u' , (b) v' , (c) w' , and in xy -plane at $z_{ref}/\delta_s = 0.24$: (d) u' , (e) v' . Solid line (—) in (a-c) is the z location of the WPPIV plane, Solid line (—) in (d,e) is the x location of the SPIV plane. Solid contour lines in (a-e) are $u'/U_\infty = -0.05$.

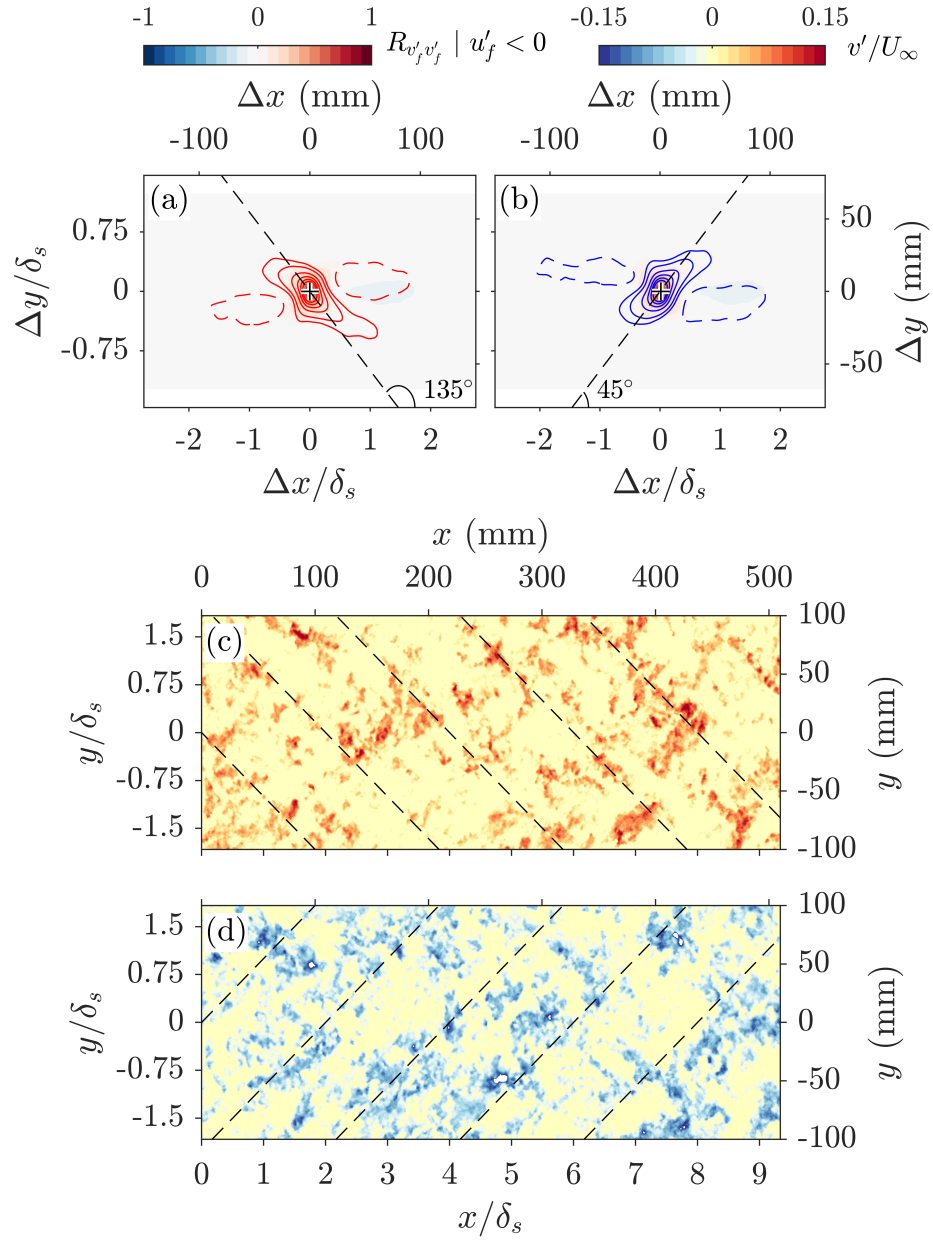


Figure 8.7: Contours of normalized autocorrelation coefficient $R_{v'_f v'_f}$ for reference smooth wall case SW-2 at $z_{ref}/\delta_s = 0.24$, conditioned at $u' < 0$. Contour lines show the additional condition: (a) $v' > 0$ (—), (b) $v' < 0$ (---). Contour levels: 0.1, 0.2, ..., 0.5 (solid lines) and -0.05, -0.1 (dashed lines). '+' symbol is the reference point where correlations are taken. Instantaneous velocity field of v' : (c) $v' > 0$ and (d) $v' < 0$. In (a-d), dashed lines (---) shows the angle of v' alignment at $\pm 45^\circ$.

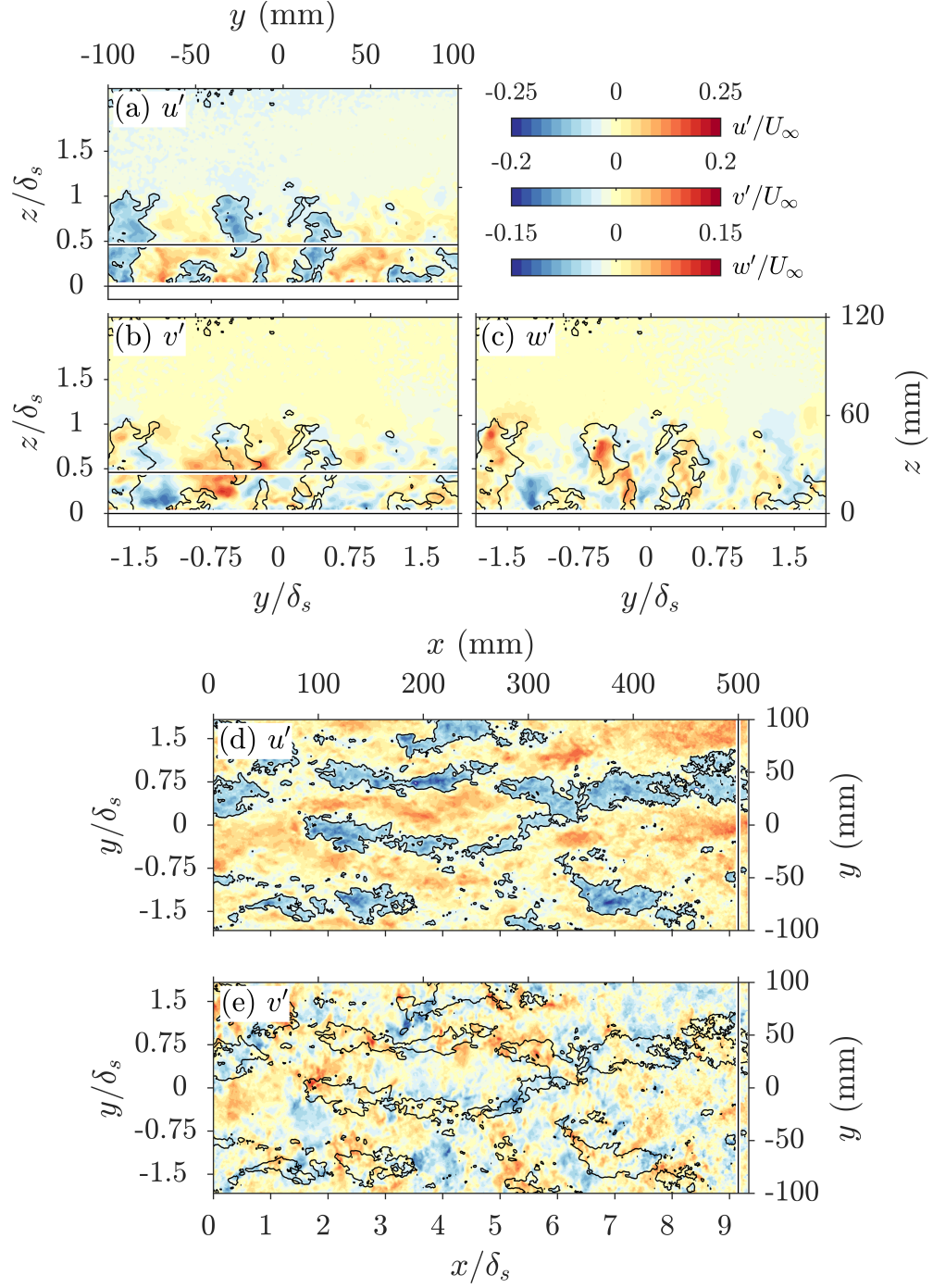


Figure 8.8: Instantaneous velocity field of case SW-2 in yz -plane: (a) u' , (b) v' , (c) w' , and in xy -plane at $z_{ref}/\delta_s = 0.46$: (d) $\tilde{u}'$, (e) v' . Solid line (—) in (a-c) is the z location of the WPPIV plane, Solid line (—) in (d,e) is the x location of the SPIV plane. Solid contour lines in (a-e) are $u'/U_\infty = -0.05$.

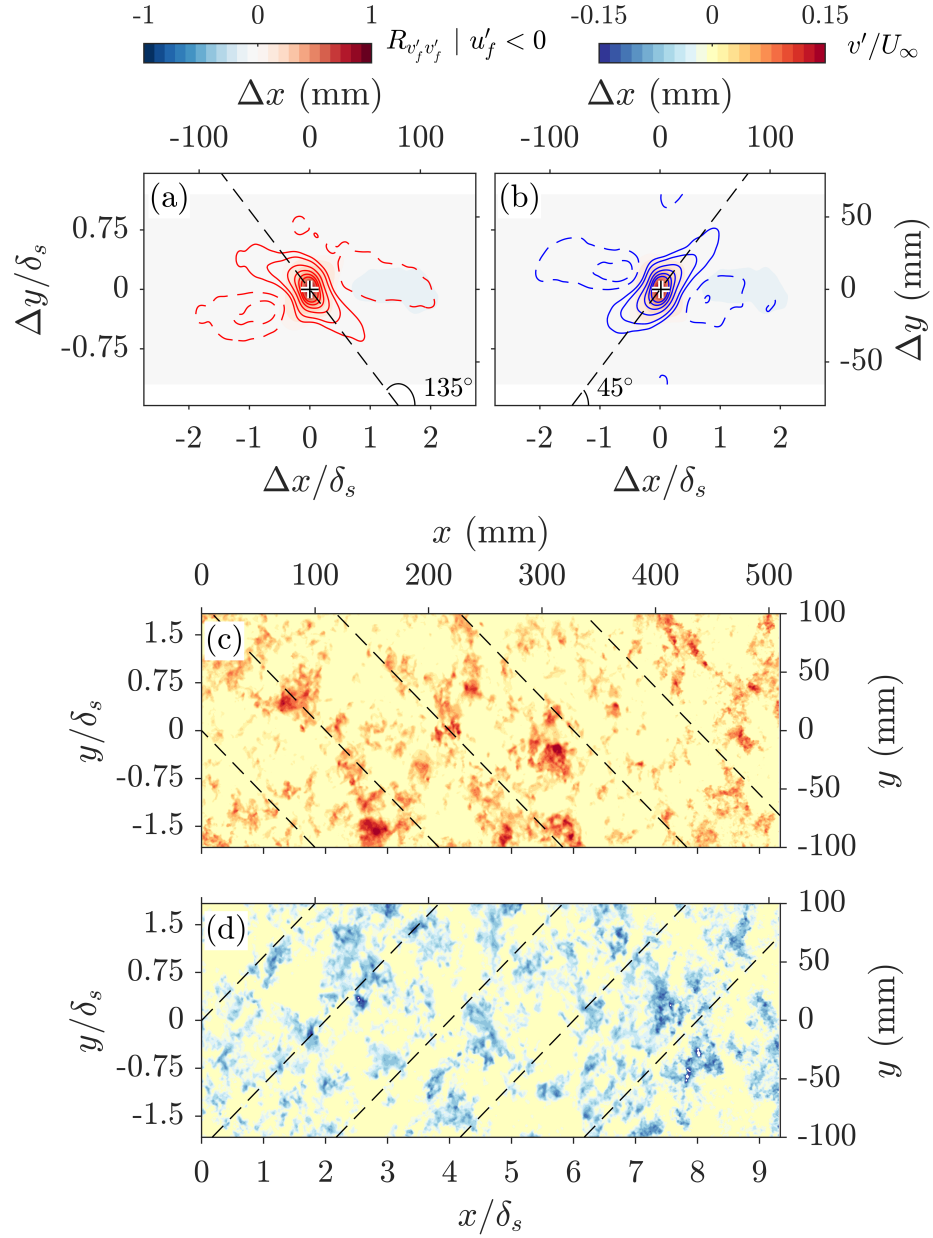


Figure 8.9: Contours of normalized autocorrelation coefficient $R_{v'_f v'_f}$ for reference smooth wall case SW-2 at $z_{ref}/\delta_s = 0.46$, conditioned at $u' < 0$. Contour lines show the additional condition: (a) $v' > 0$ (—), (b) $v' < 0$ (—). Contour levels: 0.1, 0.2, ..., 0.5 (solid lines) and -0.05, -0.1 (dashed lines). '+' symbol is the reference point where correlations are taken. Instantaneous velocity field of v' : (c) $v' > 0$ and (d) $v' < 0$. In (a-d), dashed lines (---) shows the angle of v' alignment at $\pm 45^\circ$.

45°, which emerges when $R'_{v'_f v_f}$ is conditioned on a low speed event $u' < 0$ & $v' > 0$ and $u' < 0$ & $v' < 0$, respectively. This diagonal tendency in v' was first shown by Sillero *et al.* (2014) and later by de Silva *et al.* (2018). Although not as clear, this diagonal alignment is also apparent in the instantaneous v' field in figure 8.7(c,d) and 8.9(c,d). Here, some features of v' adhere to the 135°/45° alignment, others do not. It has been observed in de Silva *et al.* (2018) that the adherence of instantaneous v' features to the 135°/45° alignment increases further away from the wall. Having compiled the statistical behaviour of the instantaneous large-scale structures in the homogeneous smooth wall reference in this chapter, the next stage (chapter 9) is to compare this to the cases with introduced spanwise heterogeneity.

Chapter 9

The structure of spanwise heterogeneous wall-bounded turbulence

To further examine the time-varying behaviour of the secondary flows shown by the HWA-measured energy spectrograms, the instantaneous velocity fields of turbulent boundary layers developing above spanwise heterogeneous roughness are examined in this chapter. Two-point correlations and conditional averaging of the turbulent fluctuation fields, u' , v' , and w' , obtained from SPIV and WPPIV measurements are computed to examine the spatial coherence. The resulting correlation maps and conditionally averaged velocity fields are compared to the smooth wall reference and further examined to reveal whether the behaviour of the turbulent structures is dependent on $S/\bar{\delta}$.

9.1 Instantaneous velocity fields

Figure 9.1 and 9.2 show the instantaneous velocity fields of the intermediate cases where $S/\bar{\delta} \approx 1$, where a strong outer peak in the energy spectra is observed (SR100 and SR50, figure 7.2d and e, respectively). It should be noted that the HWA-measured energy spectrograms are constructed from the turbulent fluctuations u' , while the velocity fields in figure 9.1 and 9.2 are shown as $\tilde{u}'$ (these two quantities are equal in the reference smooth wall case).

Similar to the reference smooth wall case (figure 8.6 and 8.8), the flow fields in the cross-stream plane (a,b,c) are characterised by ejection ($u' < 0$, $w' > 0$) and sweep ($u' > 0$, $w' < 0$) events and large-scale streaks are observed in the wall-parallel plane (d,e). However, it is apparent in figure 9.1 and 9.2 that these events are phase-locked. Low speed events ($u' < 0$, blue-filled contour in figure 9.1a and 9.2a)

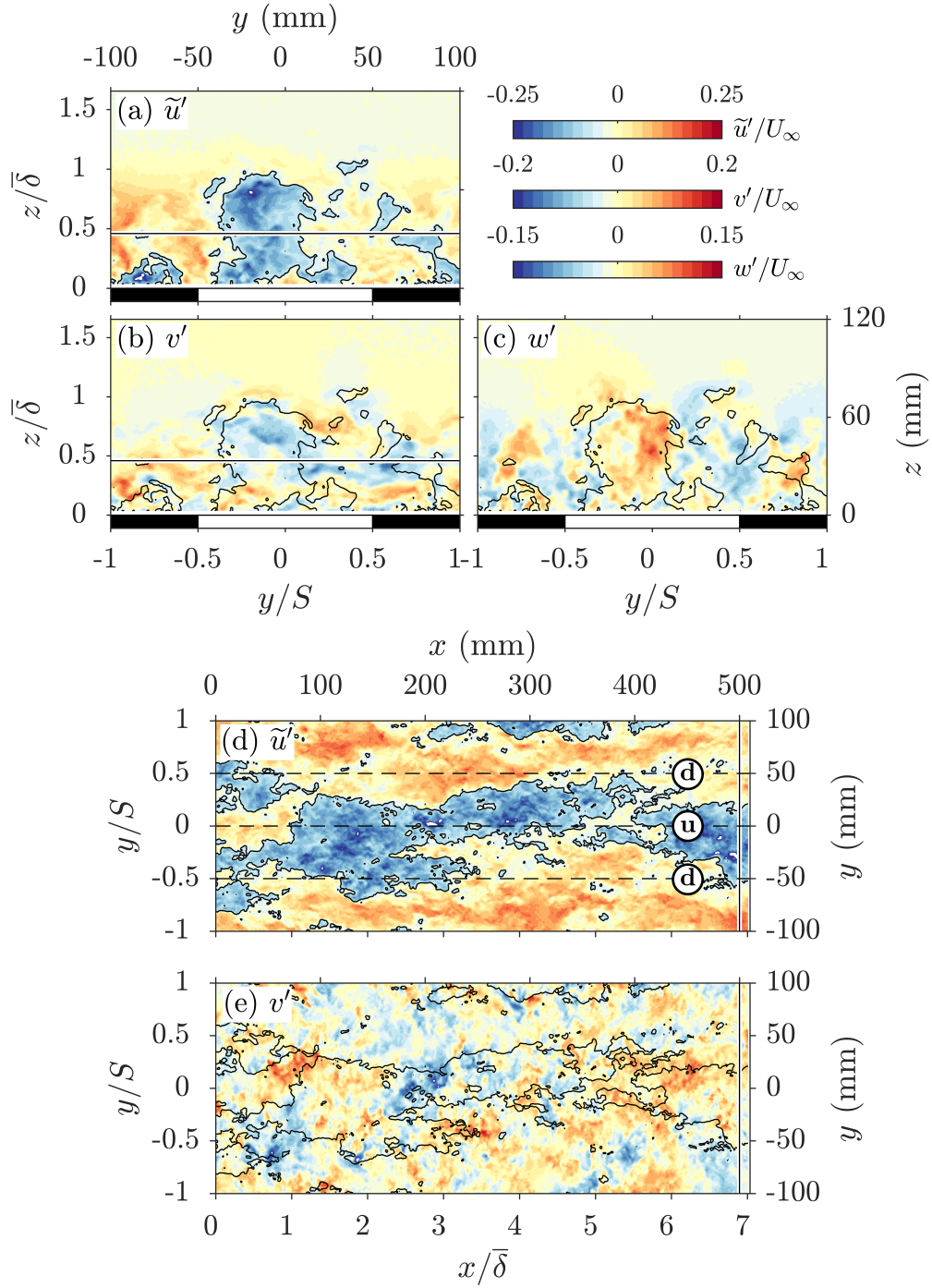


Figure 9.1: Instantaneous velocity field of case SR100 ($S/\bar{\delta} = 1.35$) in yz -plane: (a) $\tilde{u}'$, (b) v' , (c) w' , and in xy -plane at $z_{ref}/\bar{\delta} = 0.46$: (d) $\tilde{u}'$, (e) v' . Solid line (—) in (a–c) is the z location of the WPPIV plane, Solid line (—) in (d,e) is the x location of the SPIV plane. Solid contour lines in (a–e) are $\tilde{u}'/U_\infty = -0.05$. Dashed lines (---) are the spanwise locations related to the common flow up (u) and common flow down (d) of the mean secondary flows.

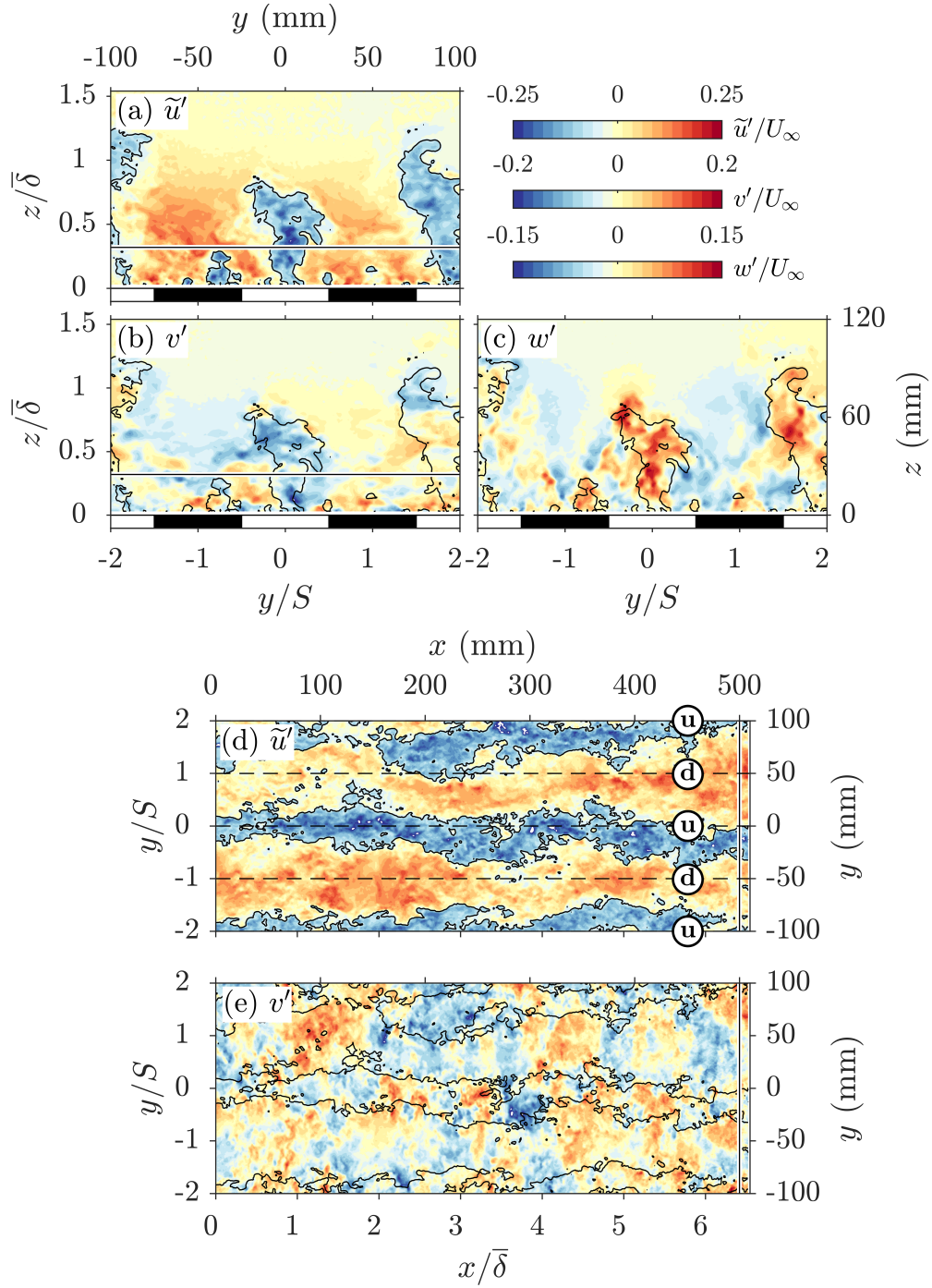


Figure 9.2: Instantaneous velocity field of case SR50 ($S/\bar{\delta} = 0.62$) in yz -plane: (a) $\tilde{u}'$, (b) v' , (c) w' , and in xy -plane at $z_{ref}/\bar{\delta} = 0.32$ ($z_{ref}/S \approx 0.5$): (d) $\tilde{u}'$, (e) v' . Solid line (—) in (a–c) is the z location of the WPPIV plane, Solid line (—) in (d,e) is the x location of the SPIV plane. Solid contour lines in (a–e) are $\tilde{u}'/U_\infty = -0.05$. Dashed lines (---) are the spanwise locations related to the common flow up ($\textcircled{u}$) and common flow down ($\textcircled{d}$) of the mean secondary flows.

coincide with ejections ($w' > 0$, red-filled contour in figure 9.1c and 9.2c), but these events are locked at the center of the smooth patch. When averaged in time, this location corresponds to the common flow up ($W > 0$).

These WPPIV planes approximately slice the center of the mean secondary flows for the spanwise heterogeneous cases, hence we believe that we capture the instantaneous flow fields of the secondary flows. In figure 9.1(d) and 9.2(d), the low-speed structures span the entire streamwise extent of the FOV, longer than those observed in the reference smooth wall case (figure 8.6d and 8.8d). These structures, though nominally fixed at the center of the smooth wall strip, meander about the location of the common flow up (labeled $\textcircled{\text{u}}$), while the high-speed structures meander about the mean location of the common flow down (labeled $\textcircled{\text{d}}$). This suggest that the secondary flows are not a time-invariant feature, and instead exhibit a prominent spatial meandering.

9.2 Streamwise turbulent fluctuations

For the spanwise heterogeneous roughness case, the velocity fields are filtered with a box filter of $0.1\bar{\delta} \times 0.1\bar{\delta}$ size (filtered velocity is denoted with subscript f , see §8.1 for discussion about the choice of filter). For the spanwise heterogeneous roughness cases in the yz -plane (SPIV plane), it is assumed that the flow is heterogeneous in both spanwise and wall-normal (y - and z -direction). The normalised two-point correlation coefficient between fluctuating signal a' and b' is defined as

$$R_{a'b'}(y_{ref}, z_{ref}) = \frac{\overline{a'(y_{ref}, z_{ref})b'(y_{ref} + \Delta y, z)}}{\sigma_a(y_{ref}, z_{ref})\sigma_b(y_{ref} + \Delta y, z)} \quad (9.1)$$

where y_{ref} is the spanwise reference point where the correlations are taken.

The filled contours in figure 9.3 show the normalised autocorrelation coefficient of filtered u' , $R_{u'_f u'_f}$, in the cross-plane computed at a reference wall height $z_{ref}/\delta_s = 0.1$ for case SW-2 and $z_{ref}/\bar{\delta} = 0.1$ for SR cases (marked by '+'), as close to the wall as the FOV permits. The correlations are conditioned on low speed events, $u'_f < 0$, which is approximately 50% of available vectors for all cases. For SR cases in figure 9.3(b,e,h,k,n), the correlations are taken at the spanwise location of the common flow up ($\textcircled{\text{u}}$), in figure 9.3(c,f,i,l,o) at the center of the secondary flows ($\textcircled{\text{c}}$), and in figure 9.3(d,g,j,m,p) at the location of common flow down ($\textcircled{\text{d}}$).

For the common flow up regions, the positive correlation maps $R_{u'_f u'_f} > 0$ (red filled contours) do not show a marked difference between the smooth wall (figure 9.3a) and heterogeneous case (figure 9.3b,e,h,k,n), except for the wall-normal

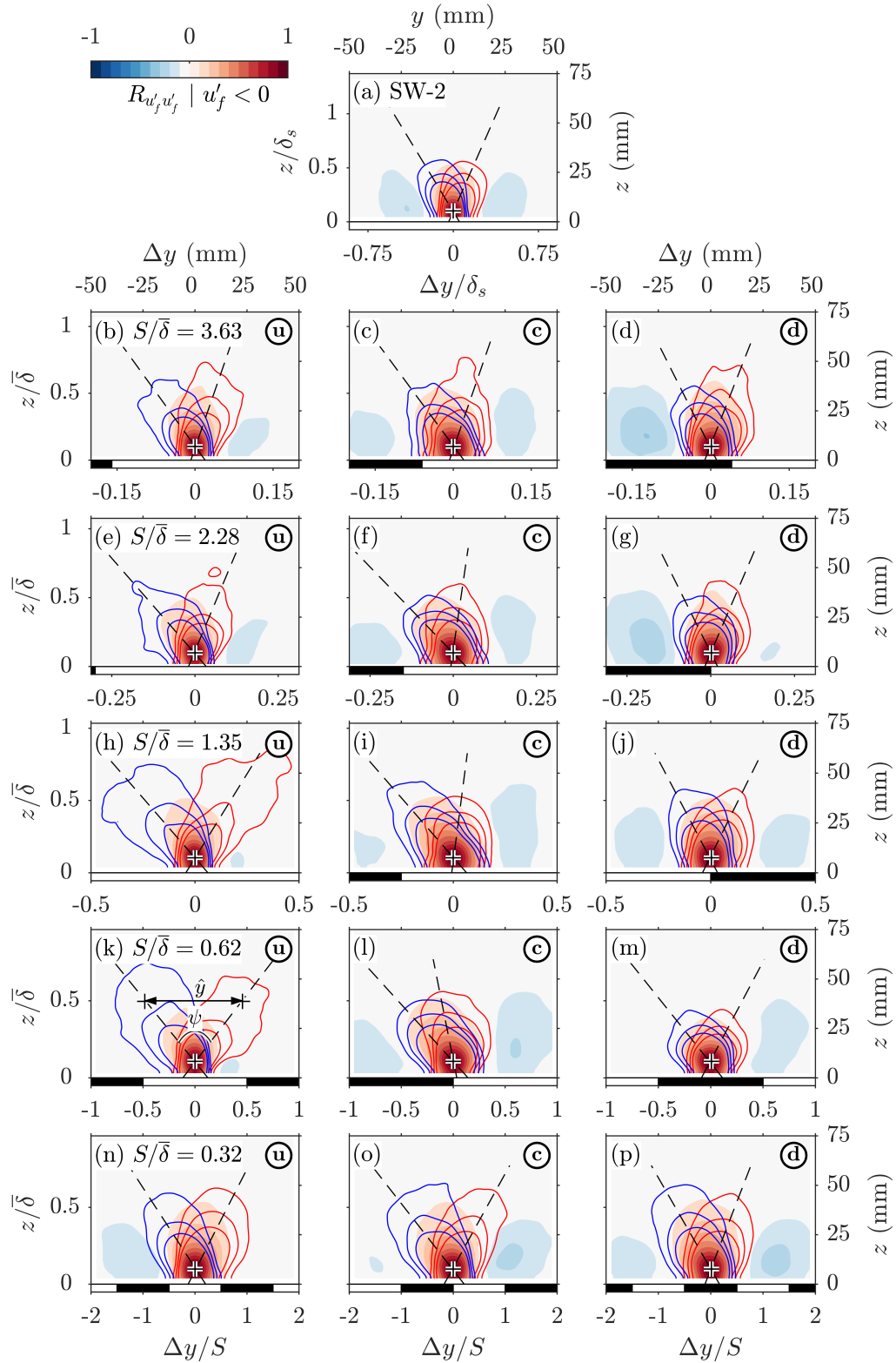


Figure 9.3: Contours of the normalized one-sided two-point correlation coefficient $R_{u'_f u'_f}$ computed for low-speed events only ($u'_f < 0$) for (a) reference smooth-wall case SW-2, (b–d) SR250-2, (e–g) SR160, (h–j) SR100, (k–m) SR50, and (n–p) SR25. Red and blue solid contour lines are further conditioned based on $v'_f > 0$ (—) and $v'_f < 0$ (—).

extent of the correlation maps. The wall-normal height in mm of the filled $R_{u'_f u'_f} > 0$ contours in all SR cases is taller than that in the SW case. This is expected since the structures scale with the local boundary layer thickness, which is larger at the common flow up region due to the presence of large-scale secondary flows ($\textcircled{\mathbf{u}}$ in figure 5.2). It should be noted that all plots in figure 9.3 are shown with the same spatial extent (see ordinate and abscissa on the right hand side and top).

The maps of $R_{u'_f u'_f} > 0$ over the center of a roll mode are shown by the filled contours in figure 9.3(c,f,i,l,o). Conditional averaging is taken over the spanwise location of the center of a counterclockwise (CCW) roll mode ($\textcircled{\mathbf{c}}$ in figure 5.2). Corresponding conditional averages for clockwise (CW) roll modes are not shown since they mirror the CCW calculations. Here, the regions of $R_{u'_f u'_f} > 0$ (red filled contours) are typically inclined to the left. The inclination to the left hints at how a CCW secondary flow affects the streamwise alignment of large-scale structures. We observe a dependency in the inclination with the spanwise wavelength of heterogeneity. The correlation map leans strongly for case SR50 (figure 9.3l, $S/\bar{\delta} \approx 1$) and less so for case SR250 (figure 9.3c, $S/\bar{\delta} \gg 1$) and SR25 (figure 9.3o, $S/\bar{\delta} \ll 1$).

The maps of $R_{u'_f u'_f} > 0$ over the common flow down are shown by the filled contours in figure 9.3(d,g,j,m,p). The correlation map in SR50 (figure 9.3m) is shorter and wider than those above the common flow up (figure 9.3k). This is a result of the common flow down tendency in this region, giving way to spanwise diverging flows as the wall is approached and the local thinning of boundary layers at this spanwise location ($\textcircled{\mathbf{d}}$ in figure 5.2). A less stark difference is observed in the correlation map of case SR25 in terms of leaning tendency (figure 9.3o) and correlation map height (figure 9.3p), which perhaps is an evidence of the decaying size (and influence) of the secondary flows as the surface approaches spanwise homogeneity for this $S/\bar{\delta}$ range.

It was shown in Sillero *et al.* (2014) for smooth wall boundary layer and channel flow and Kevin *et al.* (2017) for C-D riblets that the shape of ensemble-averaged correlation maps might result from the superposition of two opposing events. To investigate this, the events in the velocity fields are conditioned further based on the sign of the filtered spanwise turbulent fluctuations v'_f at the reference point. Note that this additional condition shows the two-point correlation coefficients conditioned on negative u'_f fluctuations, with a further added condition on the sign of v'_f . Hence, each case (either $u'_f < 0, v'_f < 0$ or $u'_f < 0, v'_f > 0$) will occur for approximately 25% of snapshots at a given reference location. The correlation maps for this additional $v'_f > 0$ and $v'_f < 0$ condition are shown by the solid red and blue

contour lines in figure 9.3, respectively. Only contour lines for $R_{u'_f u'_f} > 0$ are shown for clarity. The contours depict a large-scale structure leaning sideways to the left or right (leaning in the negative or positive y direction) depending on the sign of v'_f . The standard two-point correlation of negative u_f events without the additional v'_f condition, as shown by the filled red contours in figure 9.3, is merely the superposition of these left and right leaning events. We can relate this spanwise-leaning motion to the time-varying behaviour of the secondary flows inferred from the energy spectrograms of figures 7.1 and 7.2 and also from the instantaneous WPPIV shown in figure 9.1(d) and 9.2(d). The red and blue contours in figure 9.3 show that an ejection ($u' < 0$) event, which is associated with the low-speed streak, has a possibility of either leaning left ($v' < 0$) or right ($v' > 0$) depending on the sign of the local spanwise fluctuation. When projected into the wall-parallel plane, this left- and right-leaning tendency will give rise to long, meandering low-speed sinusoidal-like structures (as evident from blue-coloured contours in the instantaneous WPPIV shown in figure 9.1d and 9.2d).

In general from figure 9.3, it is observed that the spanwise leaning tendency is most pronounced in the common flow up region ($\textcircled{\mathbf{u}}$) shown in the first column (figure 9.3b,e,h,k,n). In this region figures 9.3(h,k) demonstrate that the spanwise-leaning tendency becomes somewhat more prominent in the intermediate cases ($S/\bar{\delta} \approx 1$) compared to other SR cases (figure 9.3b,e,n) and the reference smooth wall case (figure 9.3a). We attempt to measure the magnitude of spanwise-leaning tendency by fitting a major axis of the v'_f conditioned correlation contours in the common-flow-up location, drawn by taking the linear trend of maximum $R_{u'_f u'_f}$ at every wall-normal location (dashed lines in figure 9.3). Spanwise distance between two major axes of v'_f -conditioned correlation maps $\hat{y}$ is measured at $z/\delta_s = 0.5$ for SW and $z/\bar{\delta} = 0.5$ for spanwise heterogeneous cases. Another method is to measure the angle between positive- and negative-leaning major axis, ψ . Figure 9.3(k) shows a graphical representation of $\hat{y}$ and ψ extraction from the conditional correlation maps.

Figure 9.4 shows the amplitude of the extracted spanwise leaning in the common flow up region, normalised by δ_s or $\bar{\delta}$ for each case, as a function of $S/\bar{\delta}$. This trend is maximum when $S/\bar{\delta} \approx 1$ and decays as $S/\bar{\delta}$ increases and when $S/\bar{\delta} \ll 1$, similar to that obtained from integrated energy spectrograms in figure 7.3 and the integrated mean swirl strength in figure 5.10. So far, these results (from HWA and SPIV) suggest that the secondary flows meander, and the behaviour of the meandering is dependent of the roughness patch width $S/\bar{\delta}$. Both results also show that the

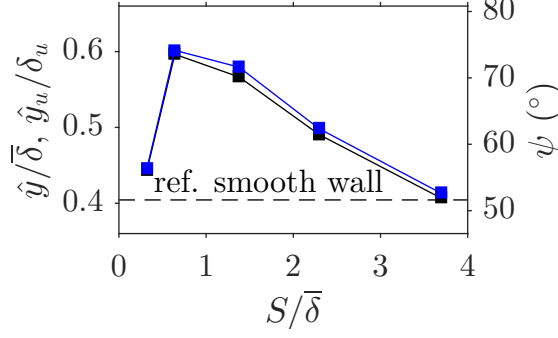


Figure 9.4: Spanwise distance between v'_f -conditioned correlation maps in figure 9.3(b,e,h,k,n). $\blacksquare$: $\hat{y}$, computed at $z/\bar{\delta} = 0.5$, $\blacksquare$: $\hat{y}_u$, computed at $z/\delta_u = 0.5$ where δ_u is the local boundary-layer thickness at common flow up, ---: reference smooth wall case SW-2. Right hand side ordinate is ψ , the angle between two major axis (figure 9.3k).

meandering is most prominent when $S/\bar{\delta} \approx 1$. A similar trend is also obtained when ψ is used as a definition of spanwise leaning magnitude since $\psi \approx \tan^{-1}(\hat{y}/\bar{\delta})$ and when $\hat{y}$ is normalised by the local boundary layer thickness at the common flow up region ($\textcircled{\text{u}}$) δ_u rather than the spanwise average value (see figure 5.2 for an example of the spanwise variation of the boundary layer thickness). The HWA-measured spectra shown in figure 7.2 are converted from time-series to space with the assumption that Taylor's hypothesis holds. In addition, the meandering of large-scale secondary flows can only be inferred (from the left and right leaning tendency) in the instantaneous cross-plane snapshots obtained from the SPIV. Hence, to complete the analysis on the meandering of the secondary flows, we compute the two-point correlations of the velocity fluctuation $R_{u'_f u'_f}$ in the xy -plane using the WPPIV data.

For the spanwise heterogeneous cases in the xy -plane (WPPIV plane), it is assumed that the flow is homogeneous in the streamwise (x -direction) but heterogeneous in spanwise (y -direction). The normalised two-point correlation coefficient between fluctuating signal a' and b' is defined as

$$R_{a'b'}(y_{ref}) = \frac{\overline{a'(x, y_{ref})b'(x + \Delta x, y_{ref} + \Delta y)}}{\sigma_a(y_{ref})\sigma_b(y)} \quad (9.2)$$

Figure 9.5 shows the contours of $R_{u'_f u'_f}$ in the xy -plane for the reference smooth wall case (a) and SR cases (b-f). $R_{u'_f u'_f}$ is computed without conditions in both u'_f and v'_f . The contours in figure 9.5 are presented in the same spatial extent (see ordinate and abscissa on the right-hand side and top of panels). For the SR cases, the two-point correlations are computed at the spanwise location that corresponds to the center of the secondary flows ($\textcircled{\text{c}}$ in figure 5.2), which is approximately where the

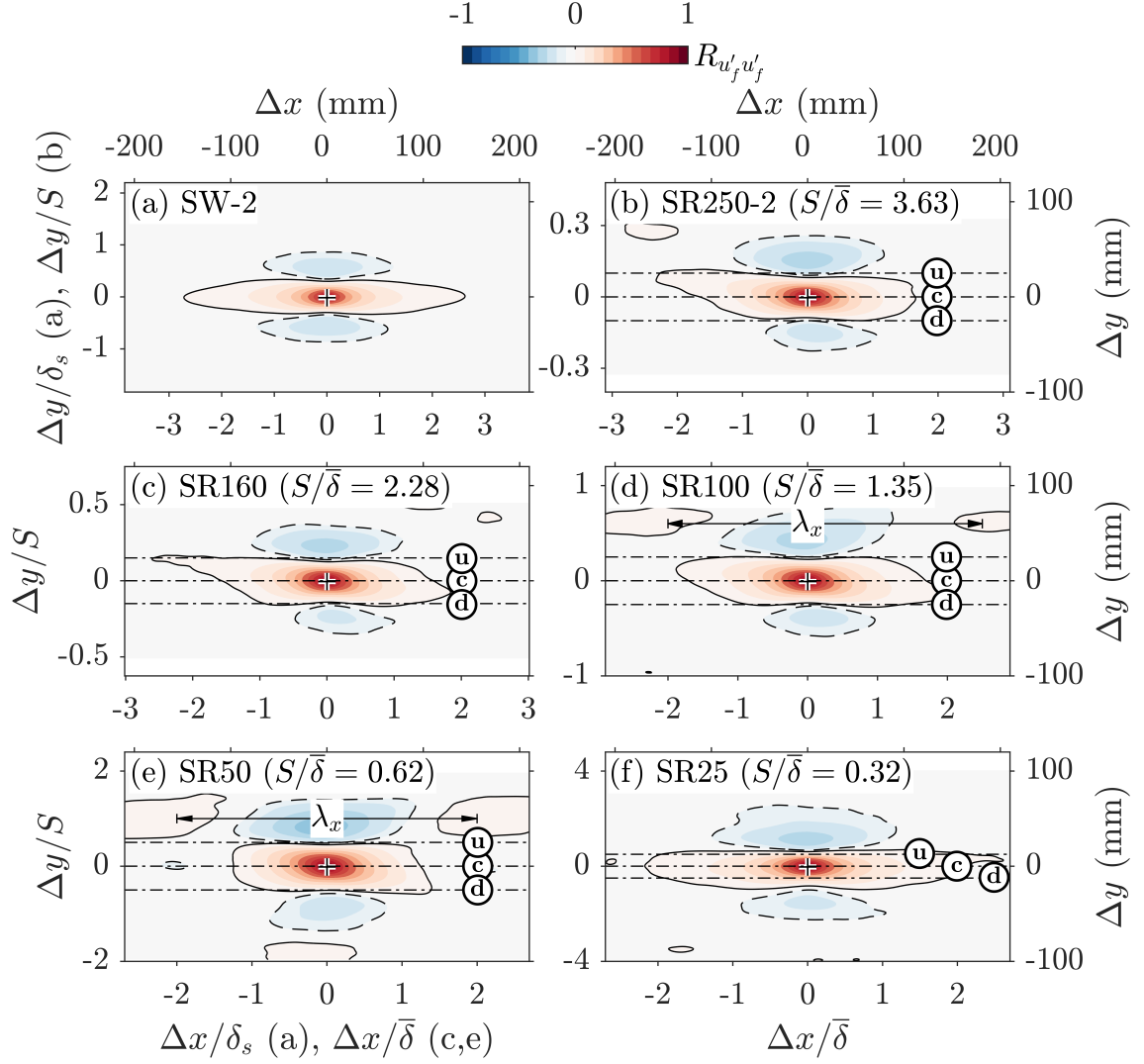


Figure 9.5: Contours of normalized two-point correlation coefficient $R_{u'_f u'_f}$ in xy -planes from the WPPIV data. Correlations are taken at the center of secondary flows (Ⓢ). Symbol '+' indicates the reference point. Contour levels: $R_{u'_f u'_f} = 0.05$ (—) and $R_{u'_f u'_f} = -0.025$ (---). Dash-dot lines (---) are the spanwise locations related to the mean secondary flows, common flow up (u), center of the secondary flow (c), and common flow down (d).

energetic peak is observed in the intermediate cases (figure 7.2d,e). This spanwise location is approximately midway between the high- and low-speed streak formed by the secondary flow (see for example figure 9.2d). Contours of $R_{u'_f u'_f} > 0$ and $R_{u'_f u'_f} < 0$ are shown by the solid and dashed black lines, respectively. Dash-dot lines in figure 9.5(b–f) show the three spanwise locations of the upwelling, downwelling and centre of the secondary flows (symbols are defined in figure 5.2). The contours of $R_{u'_f u'_f}$ for the reference smooth wall case SW-2 in xy - and yz -planes (figure 9.5a and figure 9.3a, respectively) depict the high and low speed large-scale structures in the canonical spanwise homogeneous turbulent boundary layers, where a streamwise elongated region of positive correlation is flanked by two negative (anticorrelated) lobes. A largely similar pattern is observed in the cases where $S/\bar{\delta} \gtrsim 1$ (SR250-2 and SR160, figure 9.5b,c) and $S/\bar{\delta} \ll 1$ (SR25, figure 9.5f). It is also observed that the contours of positive correlation $R_{u'_f u'_f} > 0$ (red-filled contours) for SR250-2 ($S/\bar{\delta} = 3.63$, figure 9.5b) and SR160 ($S/\bar{\delta} = 2.28$, figure 9.5c), and also the intermediate cases (SR100 and SR50, figure 9.5d,e), are tilted towards the regions of common flow up ($\odot$ in figure 9.5). This is thought to be the result of near-wall tail of low-speed structures being driven towards the common flow up regions while the head, which is further from the wall, migrates towards the common flow down regions due to the oppositely signed spanwise velocity v that occurs at these different distances from the wall, see figure 10 in Kevin *et al.* (2019b).

However, a distinctly different pattern in $R_{u'_f u'_f}$ is observed for the intermediate cases (SR100 and SR50, figure 9.5d and e, respectively), but not in the cases where $S/\bar{\delta} \gg 1$ or $S/\bar{\delta} \ll 1$. Here, a pronounced anticorrelated pattern emerges in x (a streamwise alternating pattern of positive and negative correlation, with antiphase behaviour to the span). It is noted that a similar anticorrelated pattern was also observed above the yawed region of C-D riblets in Kevin *et al.* (2019b), where $S/\bar{\delta} \approx 1$. This pattern suggests a strong periodicity in the flow field for the turbulent fluctuations u' . The distance between two $R_{u'_f u'_f} > 0$ regions in x can be used to estimate the streamwise wavelength of this flow periodicity (in figure 9.5e this can also be estimated from the separation between consecutive $R_{u'_f u'_f} < 0$ regions). The streamwise wavelength is estimated at $\lambda_x/\bar{\delta} \approx 4$. The time-series hot-wire data, when converted to space using Taylor's hypothesis, also show an additional energetic peak at $\lambda_x/\bar{\delta} \approx 3$ –4 in the premultiplied energy spectrograms of u' for the same cases (SR100 and SR50, figure 7.2d and e, respectively), at spanwise and wallnormal locations that approximately correspond to the center of the secondary flows. Taken together, the extra bump of energy evident in the pre-multiplied energy spectrograms, along with

the side-to-side leaning apparent in the cross-plane SPIV, and now the anticorrelated behaviour observed in the wall parallel plane, together strongly suggest that the secondary flows meander in the streamwise direction. Such meandering is also evident in the instantaneous WPPIV snapshots shown in figure 9.1(d) and 9.2(d). This meandering is masked by the tendency to consider secondary flows in only a time-averaged sense. Further, it is observed that the degree of meandering is strongly dependent on the size of the roughness patches ($S/\bar{\delta}$), with the most prominent meandering modes occurring when $S/\bar{\delta} \approx 1$. This also corresponds to the spanwise heterogeneous wavelength at which the strongest secondary flows are observed for a given heterogeneity condition, and where the secondary flows are approximately space-filling within the turbulent boundary layer. As a final observation from figure 9.5, it is noted that this anticorrelated behaviour is only seen when the two-point correlations are computed at the center of the secondary flows, which is approximately where the spanwise left and right leaning shown in figure 9.3 will manifest most strongly. In other spanwise locations the two-point correlations, even for the intermediate SR cases, exhibit correlation contours that look very similar to the smooth reference.

9.3 A note on meandering of the large-scale structures

It is not clear, at this point, what causes the secondary flows over spanwise heterogeneous roughness to meander and why this meandering should be the strongest when $S/\bar{\delta} \approx 1$. We note, however, that there are similarities between the secondary flows (when viewed instantaneously) and the large-scale structures that occur naturally in the log layer of canonical spanwise homogeneous wall-bounded turbulence (Kim & Adrian, 1999; Ganapathisubramani *et al.*, 2003; Tomkins & Adrian, 2003; Hutchins & Marusic, 2007*a,b*). Both of these structures are characterised by elongated high- and low-momentum streaks and, in a mean sense, by large-scale counter-rotating roll modes. Certainly these large-scale features in canonical flows have also been observed to meander. However, unlike the large-scale structures, the secondary flows are time-persistent, spanwise-locked, do not diminish with Reynolds averaging, and, in this study, occupy the wake region instead of residing predominantly in the log layer (the centre of the secondary flows is at $0.1 \lesssim z/\bar{\delta} \lesssim 0.5$).

Recent studies have shown that the secondary flows meander in a similar manner to the large-scale structures in canonical flows. When analysing the instantaneous

structure of the secondary flows formed over converging-diverging riblets, Kevin *et al.* (2017) showed that instantaneously, the secondary flows exhibited counter-rotating behaviour in only approximately 30% of snapshots, with highly asymmetric one-sided roll-modes (and associated flapping from side to side of the low speed regions) in a further 30% of snapshots. In a subsequent extension of this study, Kevin *et al.* (2019a) demonstrated that the roll-modes that flank the naturally occurring superstructures in canonical turbulent boundary layers exhibit almost identical behaviour. A structural model was sketched by Kevin *et al.* (2019a) to explain this behaviour, which consisted of asymmetric roll-modes arranged about a meandering low-speed streak, similar to the self-sustaining streak-vortex model that has been proposed for the near-wall structure (Jeong *et al.*, 1997; Waleffe, 2001; Schoppa & Hussain, 2002), and that has recently gained traction as a model for the structure in the logarithmic region (Flores & Jiménez, 2010; Cossu & Hwang, 2017; de Giovanetti *et al.*, 2017). The new information presented in this study is the suggestion that this meandering behaviour of the secondary flows is a strong function of the spanwise wavelength of heterogeneity, with the most pronounced meandering occurring in cases where the spanwise repeating wavelength is approximately in the range of $\bar{\delta}$ – $2\bar{\delta}$. Curiously, this range corresponds quite closely to the observed spanwise wavelength of the naturally occurring very large-scale motions.

The WPPIV measurements data for the reference smooth wall case and spanwise heterogeneous roughness at various roughness wavelengths enable further examination on how similar (or perhaps how different) the secondary flows are to the naturally occurring (unforced by surface geometry) LSM/VLSM in homogeneous canonical flows. An attempt is made to reconstruct the meandering of large-scale structures and secondary flows through conditional averaging of the fluctuating velocity field. Figure 9.6 shows the low-speed structures at an instantaneous velocity field taken from WPPIV measurements for the reference smooth wall case SW-2 (figure 9.6a) at $z/\delta_s = 0.46$ and SR cases (figure 9.6b–f) approximately at the center of the mean secondary flows. Gray coloured contours are negative fluctuations of $\tilde{u}'$ (total velocity with the global yt -average subtracted), $\tilde{u}'/U_\infty < -0.03$. To limit the analysis to long, large-scale structures, the velocity field is filtered with a box filter of the size $0.1\delta_s \times 0.1\delta_s$ for SW-2 and $0.1\bar{\delta} \times 0.1\bar{\delta}$ for SR cases and only structures with length $\geq 3\bar{\delta}$ ($\geq 3\delta_s$ for SW-2) are retained for the analysis. The ‘spine’ of each detected low-speed structure (gray solid lines, — in figure 9.6) is constructed from the midpoint of the structure in y across every coordinate point in x . This ‘spine’ is further smoothed with a 1-D low-pass filter whose length is $\bar{\delta}$

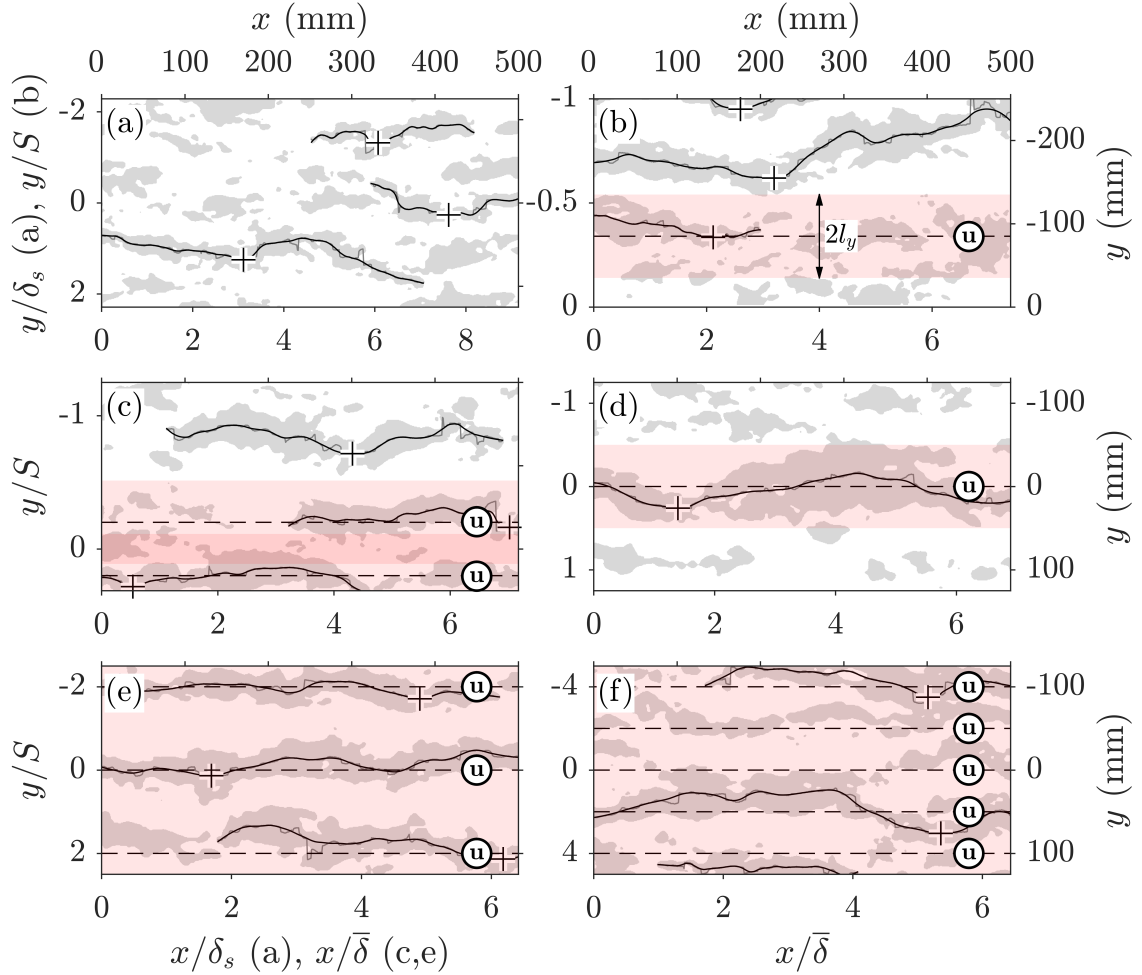


Figure 9.6: Detected low-speed structures for (a) reference case SW-2 at $z/\delta_s = 0.46$ and SR cases: (b) SR250-2 ($z/\bar{\delta} = 0.49$), (c) SR160 ($z/\bar{\delta} = 0.48$), (d) SR100 ($z/\bar{\delta} = 0.46$), (e) SR50 ($z/\bar{\delta} = 0.32$), (f) SR25 ($z/\bar{\delta} = 0.18$), about the center of the secondary flows. Grey coloured contours are the low-speed structures, $\tilde{u}'/U_\infty < -0.03$. '+' marks the minima of a low-speed structure. The spines of the detected low-speed structures are shown in solid lines (from PIV data: —, smoothed: —). Dashed lines (---) are the spanwise locations of the common flow up of the secondary flows (marked by $\textcircled{u}$, see also figure 5.2 these locations in the yz -plane). The low-speed structures related to the secondary flows due to spanwise heterogeneity are assumed to occur inside the red-shaded area, spanning $2l_y$ about the location of common flow up, as shown in (b).

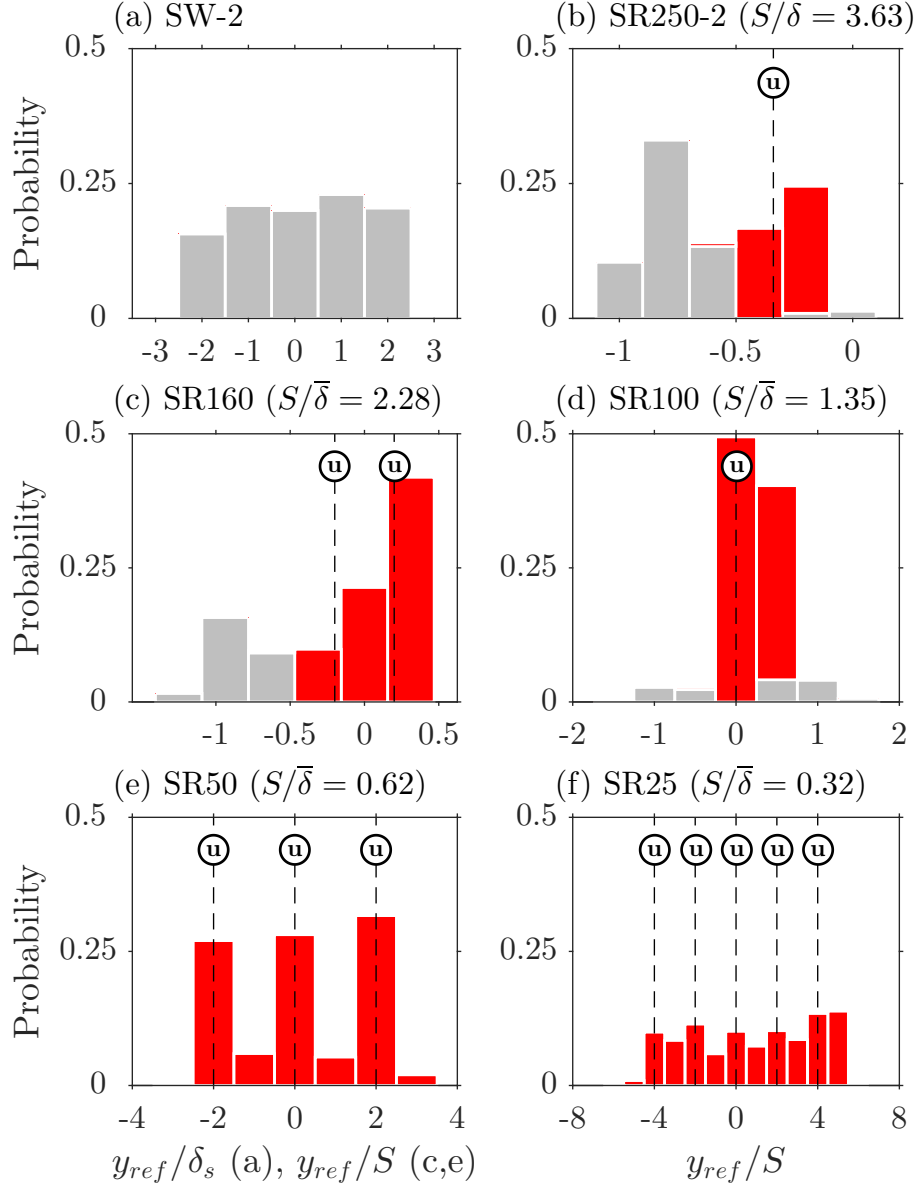


Figure 9.7: Histogram of the spanwise location of the detected minima of low-speed structures y_{ref} for (a) reference case SW-2 at $z/\delta_s = 0.46$ and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25 about the center of the secondary flows. Red bars (■) indicate y_{ref} detected inside the red-shaded area in figure 9.6, while gray bars (■) are detected at the rest of the FOV. Dashed lines (---) are the spanwise locations of the common flow up of the mean secondary flows (marked by Ⓢ).

(δ_s for SW-2), shown in black solid lines (—) in figure 9.6. Conditional averaging of the turbulent fluctuation u' (instead of $\tilde{u}'$) is computed at the minima (in y) of the detected low-speed structures marked by ‘+’ in figure 9.6.

Figure 9.6(d,e) also shows how the meandering of low-speed structures is clearly ‘phase-locked’ about the spanwise location of the common flow up of the mean secondary flows (marked by dashed lines, --- and $\textcircled{\mathbf{u}}$). This is expected since $\tilde{u}' = \tilde{U} + u'$ and $\tilde{U}$ is phase-locked. Ensemble averaging of the total velocity field, as shown in the contours of U (figure 5.2) and swirl strength Λ_{ci} (figure 5.9), reveal time-averaged large-scale secondary flows even in the case when $S/\bar{\delta} \gg 1$ and $S/\bar{\delta} \ll 1$. Following this pattern, it can be assumed that the secondary flows in the instantaneous velocity fields meander about a certain spanwise location for all $S/\bar{\delta}$ cases. Here, it is assumed that the secondary flows due to spanwise heterogeneity cause low-speed streaks to meander about the spanwise location of common flow up ($\textcircled{\mathbf{u}}$ in figure 9.6), spanning the area shaded by red colour in the same figure. This area spans $y_u \pm l_y$ (figure 9.6b), where y_u is the spanwise location of common flow up and l_y is the width of the mean secondary flow roll-mode, as illustrated in figure 5.9. Only low-speed structures detected in these areas are included in the conditional averaging. With this assumption, all detected low-speed structures might belong to either secondary flows due to spanwise heterogeneity (red shaded area) or (possibly) LSM/VLSM over homogeneous roughness in the cases where $S/\bar{\delta} > 1$ (figure 9.6b–d), while in the cases where $S/\bar{\delta} < 1$ (figure 9.6e,f) all detected low speed structures will be categorized as belonging to the secondary flows, and hence this categorisation is somewhat flawed. It should be noted that although the secondary flows fill the entire spanwise extent of the turbulent boundary layers in these cases, larger structures whose scale is $\bar{\delta}$ also coexist with the secondary flows. However, because the currently available WPPIV snapshots at xy -plane slice the center of the roll modes and the method used to separate secondary flows and LSM/VLSM is based only on the spanwise location of the secondary flows, these two cannot be distinguished in the cases where $S/\bar{\delta} \leq 1$.

Figure 9.7 shows the histogram of the detected low speed streaks (identified in the manner depicted in figure 9.6) for all SR cases, split into those classified as secondary flows (occurring within the red regions of figure 9.6) and LSM/VLSM (occurring beyond the red regions). The histogram counts the observed y_{ref} minima (denoted by the black + sign in figure 9.6) in these two regions. The probability of the occurrence of long, low-speed structures are distributed equally across the span of the FOV in the reference smooth wall case SW-2 (figure 9.7a). For the case

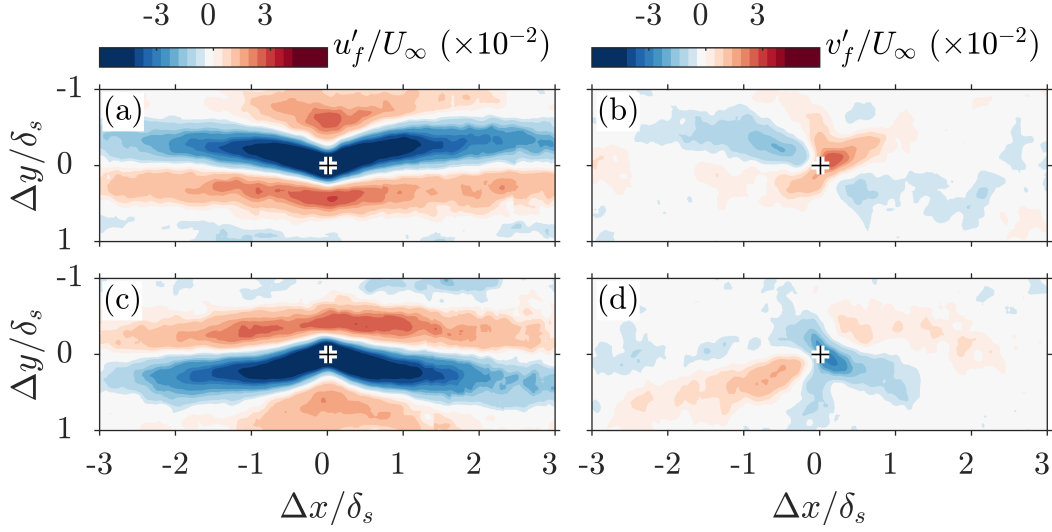


Figure 9.8: Contours of turbulent fluctuation (a,c) u' and (b,d) v' conditionally averaged at (a,b) the minima and (c,d) maxima of the detected low-speed structures illustrated in figure 9.6 for the reference case SW-2 at $z/\delta_s = 0.46$.

SR250-2 ($S/\bar{\delta} = 3.63$, red bars in figure 9.7b) and SR160 ($S/\bar{\delta} = 2.28$, figure 9.7c), the low-speed structures related to the secondary flows comprise 41% and 74% of all structures detected across the FOV, respectively (as a reference the red shade regions associated with the secondary flows consist of 38% and 53% respectively of the available total area in these cases). The secondary flows dominate as $S/\bar{\delta}$ approaches 1 (86% for case SR100, figure 9.7d), and they are also clearly ‘phase-locked’ to the location of common flow up (case SR50, figure 9.7e). A finer histogram bin width is used in the smallest $S/\bar{\delta}$ case (case SR25, figure 9.7f), showing a residual spanwise locking of the structures.

Figure 9.8 shows the conditional average of filtered turbulent fluctuations u'_f (figure 9.8a,c) and v'_f (figure 9.8b,d) for the reference smooth wall case SW-2 at $z/\bar{\delta}_s = 0.46$. The velocity fields are filtered with a box filter of $0.1\delta_s \times 0.1\delta_s$ size ($0.1\bar{\delta} \times 0.1\bar{\delta}$ for SR cases). Figure 9.8(a,b) shows u'_f conditionally averaged at the minima of the detected low-speed structures as illustrated in figure 9.6(a), while figure 9.8(c,d) is conditionally averaged at the maxima of the detected low-speed structures (not pictured). The contours of u'_f show a low-speed structure flanked by two high-speed structures leaning to the left and right depending on the reference location (minima or maxima) where the conditional average is computed. The leaning tendency is also shown in v'_f , where the conditional average at the minima of low-speed structures largely correspond to $v'_f > 0$ (figure 9.6b) and the maxima to $v'_f < 0$ (figure 9.6d). Diagonal alignment of v' , similar to that observed in figure 8.9,

is also apparent in these contours.

To obtain a complete picture of how the secondary flow meanders, similar conditional averaging is also computed in the yz -plane from SPIV measurements. Since the minima and maxima of a low-speed structure cannot be observed in the yz -plane, the conditional average is computed at the common flow up ($\textcircled{\mathbf{u}}$ in figure 5.2) in y and at $z/\bar{\delta} = 0.1$, as close to the wall as the FOV permits. For the reference case SW-2, the average is computed at any point in y along $z/\delta_s = 0.1$ where the conditions are satisfied. The conditions are: $\tilde{u}' < 0, v' > 0$ (25% of all realisations), and $\tilde{u}' < 0, v' < 0$ (25% of all realisations). The velocity fields in the yz -plane are filtered with a box filter of $0.1\delta_s \times 0.1\delta_s$ size ($0.1\bar{\delta} \times 0.1\bar{\delta}$ for SR cases). It should be noted, however, that these condition vectors for the ensemble averaging differ between the xy - and yz -plane and are only intended to show the representation of the high- and low-speed structures above the reference smooth wall case and spanwise heterogeneous roughness cases. As a direct comparison, figure 9.9 and 9.10 show the conditional average in yz - and xy - plane at the exact same conditions for the reference smooth wall case SW-2 and SR100 ($S/\bar{\delta} = 1.35$), respectively. For the reference smooth wall case (figure 9.9), the conditional average is computed at $z_{ref}/\delta_s = 0.46$ and any point in x and y that satisfies the following conditions: $\tilde{u}' < 0, v' > 0$ (a,b) and $\tilde{u}' < 0, v' < 0$ (c,d). The same conditions also apply for case SR100 in figure 9.10. However, due to spanwise heterogeneity, the conditional average is computed at $z_{ref}/\bar{\delta} = 0.46$ and at the spanwise location of common flow up ($y_{ref} = 0$, along the dashed line marked by $\textcircled{\mathbf{u}}$ in figure 9.1d). Contours of conditionally averaged u' for both cases show a good agreement between SPIV and WPPIV data (yz - and xy -plane, respectively). However, due to the choice of conditional point, which is limited to quantities that can be computed in both planes and leads to increased phase jitter in the ensemble, the shape of the meandering large-scale structures is not as clear as that computed at the minima/maxima of a low-speed structure (figure 9.8a,c). In figures 9.9 and 9.10, the structures yaw to the left or right, depending on v' , rather than exhibiting the clear meandering demonstrated in figure 9.8.

Conditionally averaged u'_f for $S/\bar{\delta} \approx 1$ (SR100 and SR50) and the reference smooth wall case SW-2 are shown in figure 9.11. The plots show a low-speed structures flanked by two high-speed structures in yz - (i and iii) and xy -plane (ii and iv). Subplot (i) show the tendency of the structures to lean sideways to the right ($v'_f > 0$), corresponding to the minima of y_{ref} in (ii), while subplot (iii) show the left-leaning tendency ($v'_f < 0$) of the structures, corresponding to the maxima of

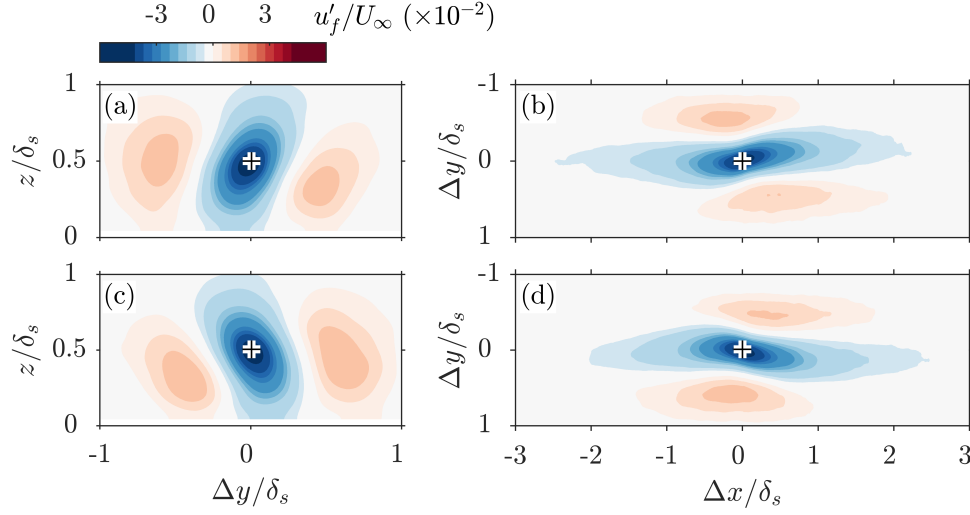


Figure 9.9: Contours of conditionally averaged u' in (a,c) yz -plane and (b,d) xy -plane for the reference case SW-2 at $z/\delta_s = 0.46$. The conditions are: (a,b) $\tilde{u}' < 0$, $v' > 0$ and (c,d) $\tilde{u}' < 0$, $v' < 0$, computed at $z_{ref}/\delta_s = 0.46$ for (a,c). Homogeneous condition is assumed in x - and y -direction.

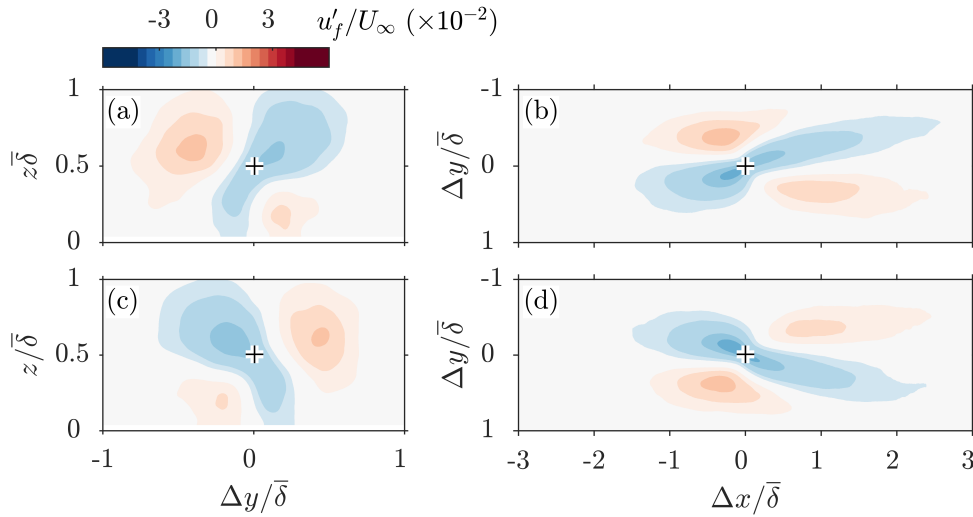


Figure 9.10: Contours of conditionally averaged u' in (a,c) yz -plane and (b,d) xy -plane for case SR100 ($S/\bar{\delta} = 1.35$) at $z/\bar{\delta} = 0.46$. The conditions are: (a,b) $\tilde{u}' < 0$, $v' > 0$ and (c,d) $\tilde{u}' < 0$, $v' < 0$, computed at $z_{ref}/\bar{\delta} = 0.46$ for (a,c) and the spanwise location of common flow up ($y_{ref} = 0$).

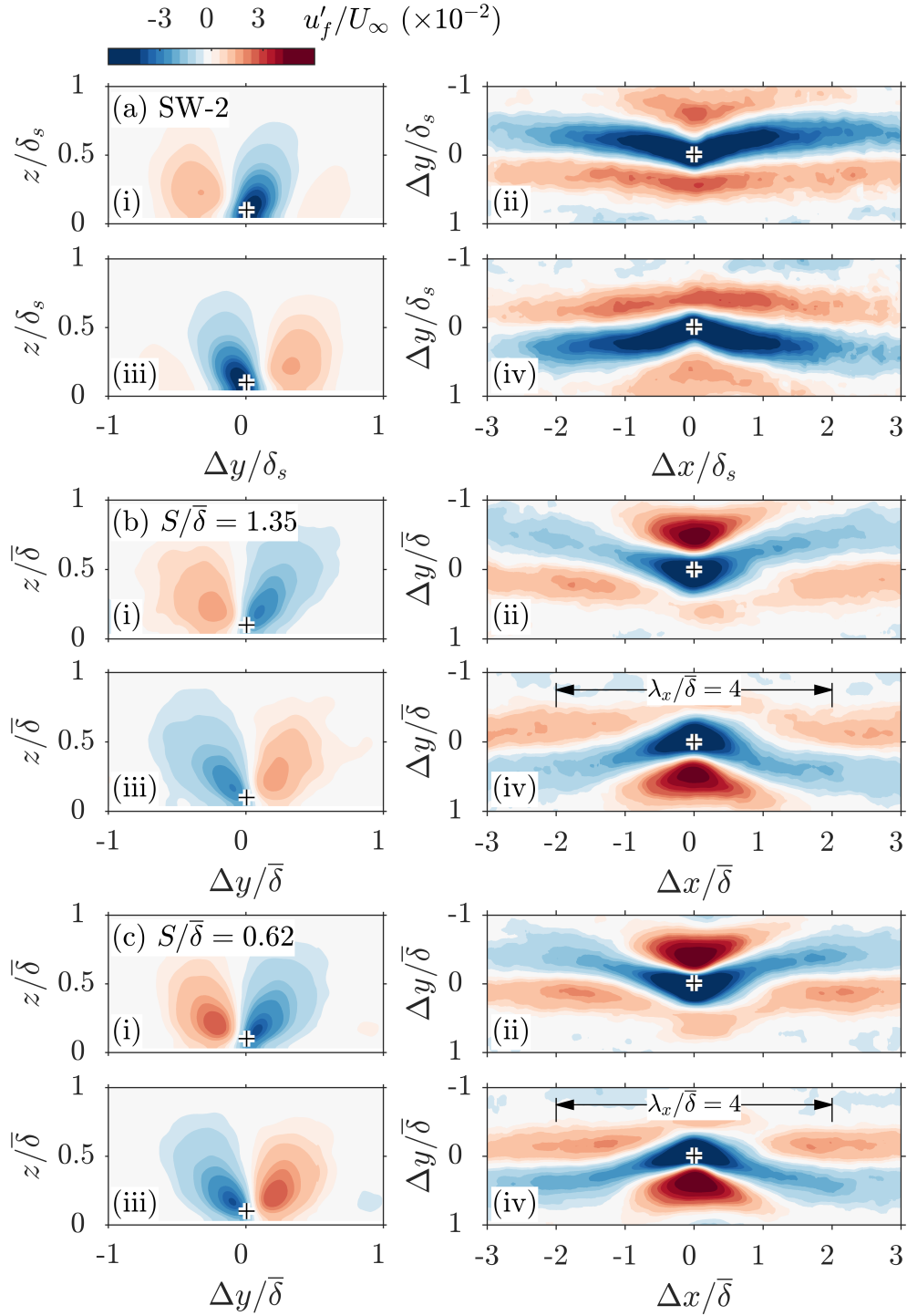


Figure 9.11: Contours of conditionally averaged turbulent fluctuation u' for (a) reference case SW-2 at $z/\delta_s = 0.46$ and SR cases where $S/\bar{\delta} \approx 1$: (b) SR100 and (c) SR50. Conditions: (i) $\tilde{u}' < 0$ and $v' > 0$, (ii) $\tilde{u}' < 0$ and $v' < 0$, (iii) minima and (iv) maxima of detected low-speed structures ($\tilde{u}'/U_\infty < -0.03$) illustrated in figure 9.6.

y_{ref} in (iv). The leaning is strongly one-sided for the cases where $S/\bar{\delta} \approx 1$ in figure 9.11(b,c), subplot (i) and (iii), where the low-speed structures lean to one side and are flanked by asymmetric high-speed structures which are highly one-sided, while for the reference smooth wall case SW-2 in figure 9.11(a), the sideways leaning of the low-speed event is less prominent with a more symmetric arrangement of flanking high-speed events. The conditionally averaged velocity fields in xy -plane (ii and iv) further confirm the differences between smooth wall case and $S/\bar{\delta} \approx 1$ cases. The structures strongly lean to one side (dark red coloured contours) in figure 9.11(b,c), subplot (ii) and (iv). The averaged velocity fields also show sinusoidal, strongly meandering structures, compared to the reference smooth wall case in figure 9.11(a). The shape of these structures are akin to that observed in the contours of normalised two-point correlation of u' in figure 9.5(e,f), where a clear streamwise anti-phase pattern is shown by $S/\bar{\delta} \approx 1$ cases. The streamwise wavelength observed in figure 9.11 is approximately $\lambda_x/\bar{\delta} \approx 4$ (see figure 9.11b and c, subplot iv), similar to that observed in the contours of two-point correlation coefficients (figure 9.5e,f) and spectrograms from HWA measurements (figure 7.2e,f).

Figure 9.12 show the comparison between conditionally averaged u' for the reference smooth wall case (figure 9.12a) and the two limiting cases: $S/\bar{\delta} \gg 1$ (figure 9.12b) and $S/\bar{\delta} \ll 1$ (figure 9.12c). For the SR cases the conditional average is computed about the spanwise location of the secondary flows. In the crossplane (yz -plane) the structures in both limiting cases still show a somewhat stronger one-sided leaning compared to those in SW-2, as shown in figure 9.12(b,c), subplot (i) and (iii). When viewed in the xy -plane (subplot ii and iv), however, these structures do not show a marked difference to those observed in SW-2. The shape is similar, and so is the size when normalised by their respective boundary-layer thickness (δ_s for SW-2 and $\bar{\delta}$ for SR cases). It should be noted that longer and thinner streaks observed in SR25 ($S/\bar{\delta} = 0.32$, figure 9.12c, subplots ii and iv) is due to the wall-normal location of the WPPIV laser sheet in this case which, at $z/\bar{\delta} = 0.18$, is closer to the wall compared to other SR cases and the smooth wall reference case ($z/\delta_s = 0.46$) due to the diminishing size of the secondary flows.

These similarities and differences between secondary flows and the large-scale motions raise many important questions. Firstly, why do the large-scale structures strongly meander only when $S/\bar{\delta} \approx 1$, but not at the limiting cases where either $S/\bar{\delta} \gg 1$ or $S/\bar{\delta} \ll 1$? At these two limits the secondary flows are very similar to the large-scale motions observed in the reference smooth wall case SW-2. This

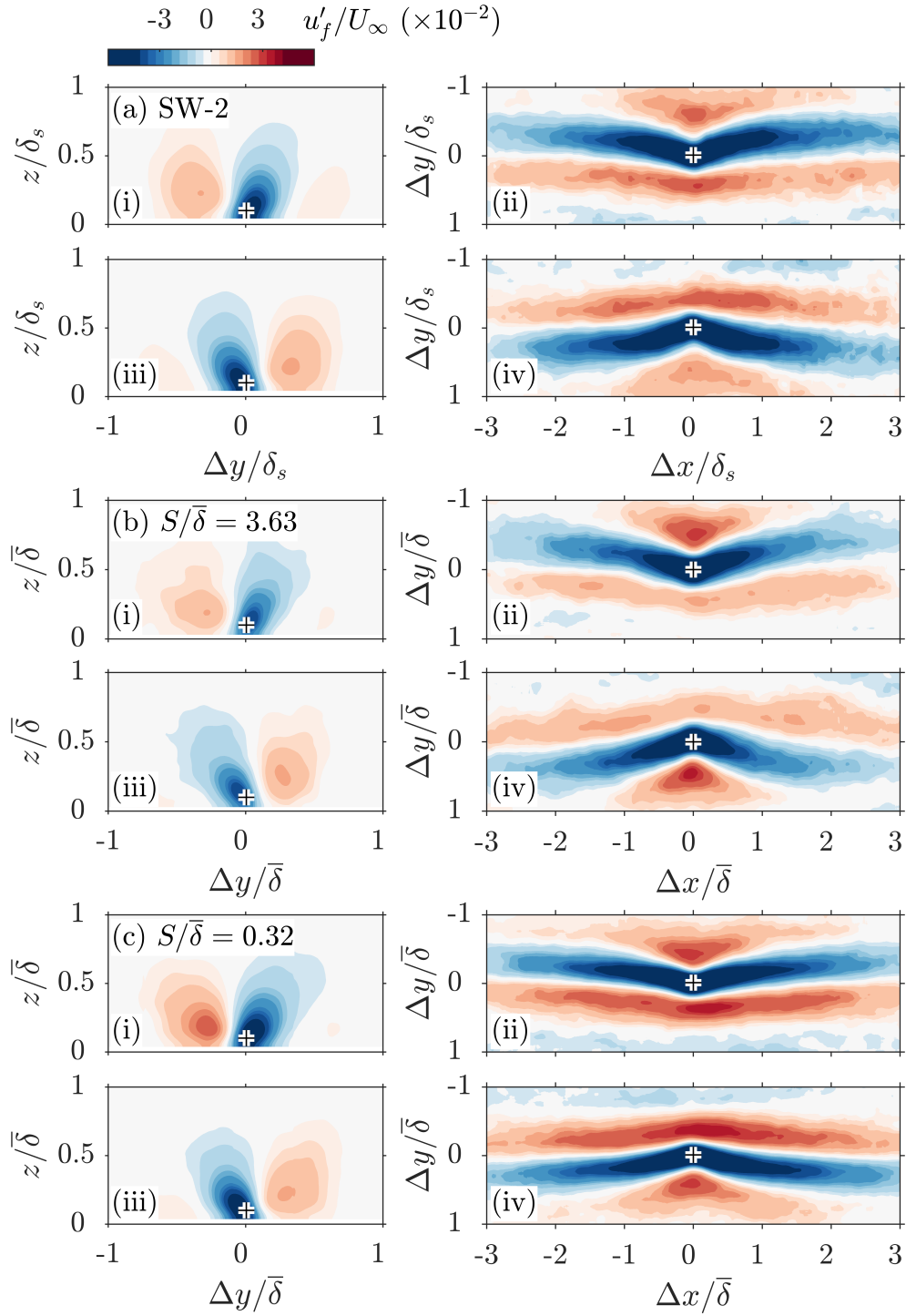


Figure 9.12: Contours of conditionally averaged turbulent fluctuation u' for (a) reference case SW-2 at $z/\delta_s = 0.46$ and SR cases where (b) $S/\bar{\delta} \gg 1$ (SR250-2) and (c) $S/\bar{\delta} \ll 1$ (SR50). Conditions: (i) $\tilde{u}' < 0$ and $v' > 0$, (ii) $\tilde{u}' < 0$ and $v' < 0$, (iii) minima and (iv) maxima of detected low-speed structures ($\tilde{u}'/U_\infty < -0.03$) illustrated in figure 9.6.

leads to the second question: is it possible that secondary flows due to spanwise heterogeneity and large-scale motions share similar formation mechanisms? Lee *et al.* (2019) suggested parallels between the formation of large-scale motions in homogeneous turbulent pipe flows due to instantaneous lateral variations in the wall shear stress and the secondary flows that form over spanwise heterogeneous roughness, where the lateral variations in wall shear stress are imposed on the surface. It has also been shown by Townsend (1976) that transient (impulsive imposed) lateral variations in wall shear stress with δ -scaled spanwise wavelength can excite persistent large-scale secondary flows. Instantaneously, spanwise heterogeneous surfaces of the type investigated here will generate large-scale streaks and roll-modes. These may likely undergo a similar streak-vortex instability to that proposed for the naturally occurring large-scale structures (Jeong *et al.*, 1997; Waleffe, 2001; Schoppa & Hussain, 2002; Flores & Jiménez, 2010; Cossu & Hwang, 2017; de Giovanetti *et al.*, 2017) and will give rise to the observed characteristics of instantaneous asymmetry and meandering. Figure 9.13 and figure 9.14 show the 3-D reconstruction of the large-scale structures from xy - and yz -plane for case SW-2 and SR100 ($S/\bar{\delta} = 1.35$), respectively¹. Although the meandering is more prominent in case SR100 (secondary flows rather than LSM/VLSM), both structures show similarities to the streak-vortex models proposed in the previous studies. Philosophical and circular questions can often arise. For example, are secondary flows naturally occurring VLSMs that are locked at a certain spanwise location? Or, following the suggestion of Lee *et al.* (2019) and Townsend (1976), are VLSMs instantaneously formed and transient secondary flows? Either argument is difficult to unpick. As $S/\bar{\delta}$ approaches 1, however, previous studies show that, regardless of the roughness type (ridge, strip, or C-D riblets), in the intermediate cases secondary flows thoroughly modify the flow such that an additional outer layer peak emerges in the energy spectrograms of the turbulent fluctuations. The exact streamwise wavelength of the peak reported in these studies differs, approximately between $3 \lesssim \lambda_x/\delta \lesssim 8$ (Nugroho *et al.*, 2013; Awasthi & Anderson, 2018; Medjnoun *et al.*, 2018; Zampiron *et al.*, 2020). The streamwise periodicity extracted from conditional averaging reported in Kevin *et al.* (2019b) also falls within this range. It is noted that this is also the order of the large-scale structures in smooth wall-bounded turbulence; $\lambda_x/\bar{\delta} = 3\text{--}4$ is associated with the meandering secondary flows in this study, while the outer peak associated with meandering large-scale structures occurs at $\lambda_x/\delta_s = 6$ (SW-2, figure 7.1a). One

¹It should be noted that the condition vectors for the ensemble averaging differ between the xy - and yz -plane and these figures are only intended to show the representation of the high- and low-speed structures above the reference smooth wall case and an intermediate case.

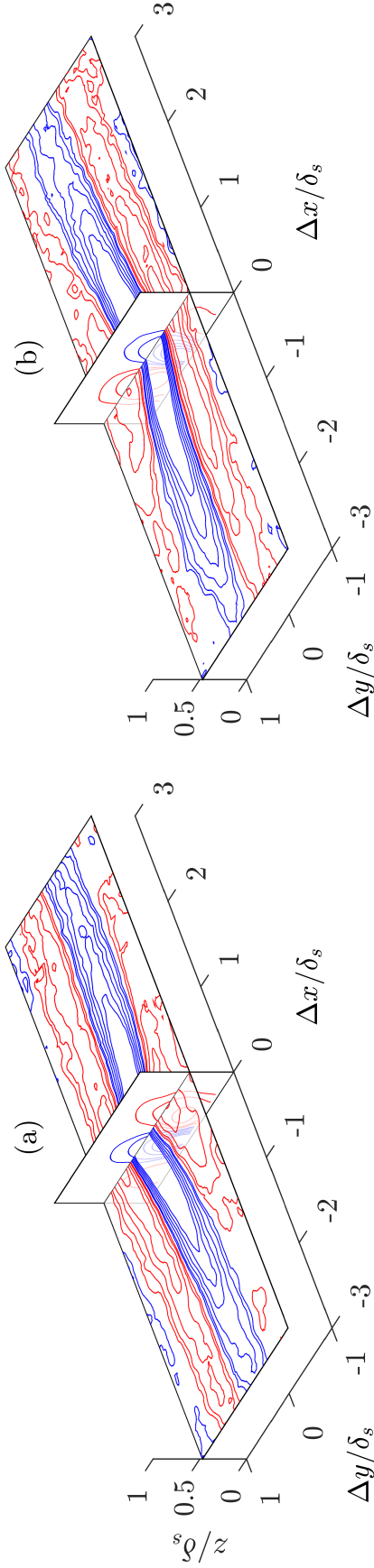


Figure 9.13: Illustrations of meandering low-speed structures flanked by two high-speed structures constructed by conditionally averaged u' from SPIV and WPPIV plane at the (a) minima and (b) maxima of detected low-speed structures illustrated in figure 9.6 for reference case SW-2 at $z/\delta_s = 0.46$.

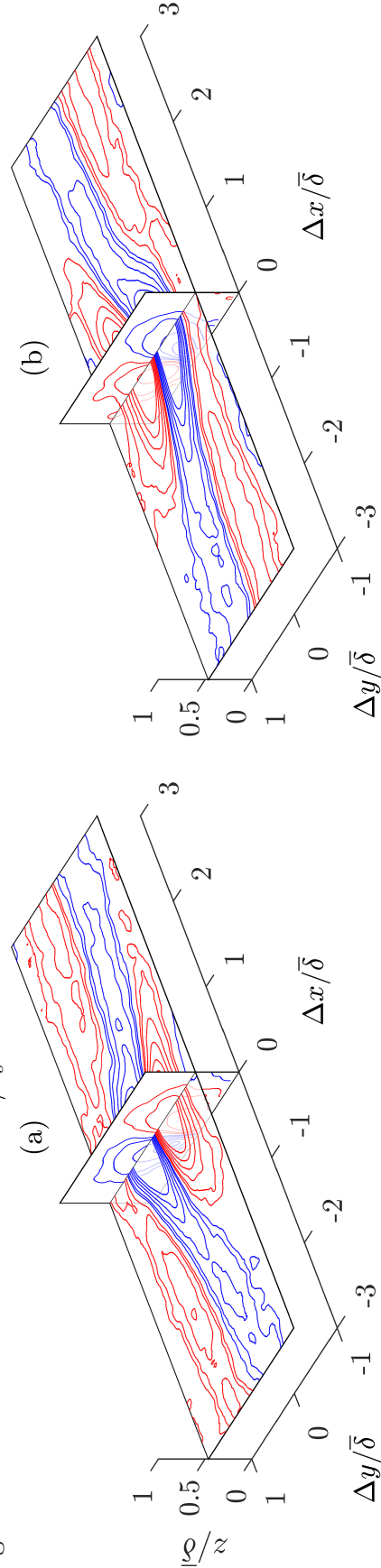


Figure 9.14: Illustrations of meandering low-speed structures flanked by two high-speed structures constructed by conditionally averaged u' from SPIV and WPPIV plane at the (a) minima and (b) maxima of detected low-speed structures illustrated in figure 9.6 for reference case SR100.

element that is missing from the present analysis is any consideration of the convection velocity of the meandering features, or their temporal development. Ultimately, time-resolved data may be able to better address these questions.

9.4 Coexistence between secondary flows and large-scale structures

Another question that arises from the analysis of the fluctuating velocity fields is whether the secondary flows and the large-scale structures coexist with each other. Coexistence between the VLSMs and secondary flows was hinted at Zampiron *et al.* (2020) at larger spanwise wavelengths, where these two features exist at a very different scale in the energy spectrograms. It should be noted, however, that the degree to which we can separate naturally occurring large-scale structures and secondary flows is very much swayed by the choice of decomposition, and this becomes increasingly difficult when the spanwise scales of these two features become roughly equal ($S/\bar{\delta} \approx 1$). Meanwhile, at both limits when $S/\bar{\delta} \gg 1$ and when $S/\bar{\delta} \ll 1$, we might expect the VLSM to exist in a relatively unaltered form, especially when we are far from the secondary flows (far from the roughness transition for the case with $S \gg \bar{\delta}$, or $z > S$ for the case where $S \ll \bar{\delta}$). In this section, an attempt is made to separate the secondary flows due to spanwise heterogeneity and (presumably) unaltered, naturally occurring LSM/VLSM in the limiting cases: $S/\bar{\delta} \gg 1$ (§9.4.1) and $S/\bar{\delta} \ll 1$ (§9.4.2). Conditional averaging and POD (Proper Orthogonal Decomposition) analysis are conducted for the two largest wavelength: SR250-2 ($S/\bar{\delta} = 3.63$) and SR160 ($S/\bar{\delta} = 2.28$), and the smallest wavelength, SR25 ($S/\bar{\delta} = 0.32$).

9.4.1 Limiting case: $S/\bar{\delta} \gg 1$

Figure 9.15 show the low-speed structures at an instantaneous velocity field of these two cases taken from WPPIV measurements approximately at the center of the mean secondary flows ($z/\bar{\delta} = 0.46$). Gray coloured contours are the field of $\tilde{u}'$ (total velocity with yt -average subtracted), $\tilde{u}'/U_\infty < -0.03$. The velocity field is filtered with a box filter of the size $0.1\bar{\delta} \times 0.1\bar{\delta}$. The ‘spine’ of each detected low-speed structure (gray solid lines, — in figure 9.15) is constructed from the midpoint of the structure in y across every coordinate point in x . This ‘spine’ is further smoothed with a 1-D low-pass filter whose length is $\bar{\delta}$, shown in black solid lines (—) in figure 9.15. Only structures with length $\geq 3\bar{\delta}$ are kept for the analysis. Conditional averaging is computed at the minima (in y) of the detected low-speed

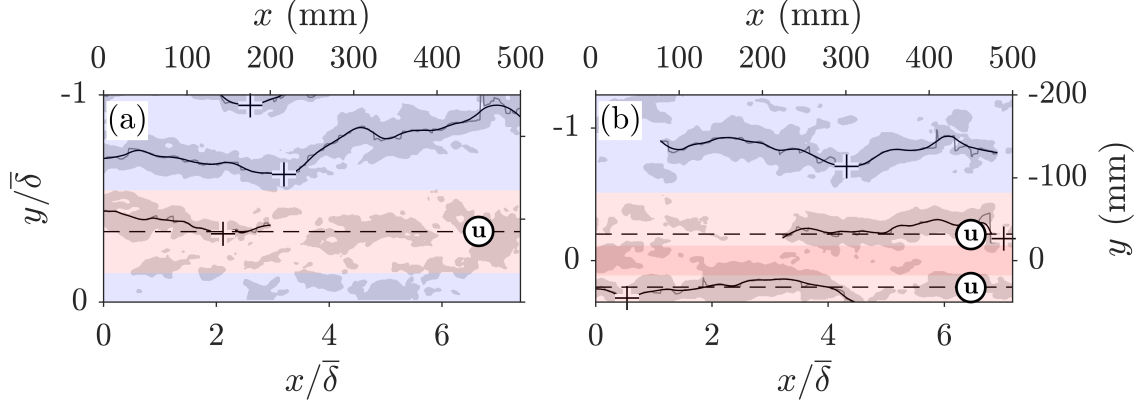


Figure 9.15: Detected low-speed structures for cases where $S/\bar{\delta} \gg 1$: (a) SR250-2 and (b) SR160. Grey coloured contours are the low-speed structures, $\tilde{u}'/U_\infty < -0.03$. '+' marks the minima of a low-speed structure. The spines of the detected low-speed structures are shown in solid lines (from PIV data: —, smoothed: —). Dashed lines (---) are the spanwise locations of the common flow up of the secondary flows (marked by $\oplus$). The low-speed structures related to the secondary flows due to spanwise heterogeneity are assumed to occur inside the red-shaded area, while those related to the LSM/VLSM naturally occurring on homogeneous roughness are inside the blue-shaded area.

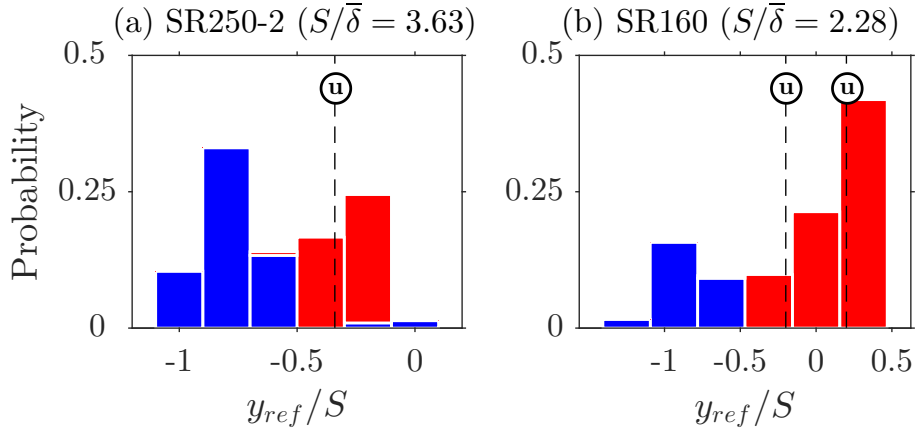


Figure 9.16: Histogram of the spanwise location of the detected minima of low-speed structures y_{ref} for cases where $S/\bar{\delta} \gg 1$: (a) SR250-2 and (b) SR160. Red bars (■) indicate y_{ref} detected inside the red-shaded area in figure 9.6, while blue bars (■) inside the blue-shaded area in figure 9.6. Dashed lines (---) are the spanwise locations of the common flow up of the mean secondary flows (marked by $\oplus$).

structures marked by the ‘+’ symbol in figure 9.15. Instantaneously, the large-scale structures induced by spanwise heterogeneity meander about a certain spanwise location, which results in δ -scaled secondary flows in the time-averaged velocity field near the interface between rough and smooth strips (contours of U in figure 5.2, swirl strength in figure 5.9). Here, it is assumed that the low-speed structures related to the secondary flows (upwelling motions) meander about the spanwise location of common flow up $y_u \pm l_y$, where l_y is the spanwise extent of the mean secondary flows. This area is shaded red in figure 9.15. Far removed from the secondary flows, the flow becomes locally homogeneous (either smooth wall or homogeneous rough wall). In figure 9.15, areas shaded in blue (closer to the centerline of smooth and rough strips) represent this condition. It is assumed that secondary flows occur in the red-shaded area, while LSM/VLSM occur in the blue-shaded area (these two terms are used here to distinguish between naturally occurring turbulent structures and those that are imposed or locked by the roughness heterogeneity). Histograms in figure 9.16 show that, secondary flows (red bars) comprise 41% and 74% of all low-speed structures detected across the FOV for case SR250-2 and SR160, respectively, and thus LSM/VLSM (blue bars) comprise the rest (59% and 26%).

Similar to the analysis conducted in §9.2, the conditional average is also calculated at the maxima of the detected low-speed structures and in the yz -plane from SPIV measurements, using a different but related condition vector. For the yz -plane, the average is computed at $z_{ref}/\bar{\delta} = 0.1$, based on the condition of $\tilde{u}'$ and v' (see §9.3). Figure 9.17 shows the conditional average of case SR250-2 (figure 9.17b,c) compared to the reference smooth wall case SW-2 (figure 9.17a). Conditional average for the secondary flows (‘SF’) due to spanwise heterogeneity in (b) is computed at the spanwise location of common flow up ($\textcircled{u}$ in figure 5.2) in subplot (i) and (iii). It is computed at the minima and maxima of the detected low-speed structures inside the area shaded by red (figure 9.15a) in subplot (ii) and (iv). The conditional average for the LSM/VLSM is computed at $y_{ref}/S = -0.72$, close to the centerline of a rough strip ($y/S = -1$). A one-sided leaning tendency is observed in the conditionally averaged low-speed structures related to the secondary flows (figure 9.17b). However, both secondary flows and LSM/VLSM (figure 9.17c) have similar shape and size (when scaled by $\bar{\delta}$) to the LSM/VLSM in the reference smooth wall case. But what about the strength? A cursory inspection of figure 9.17 suggests that the LSM/VLSM are stronger (higher in magnitude) than the secondary flows, which is expected since the conditionally averaged u' is normalised by U_∞ instead of U_τ and the conditionally averaged LSM/VLSM in figure 9.17 are obtained above a rough

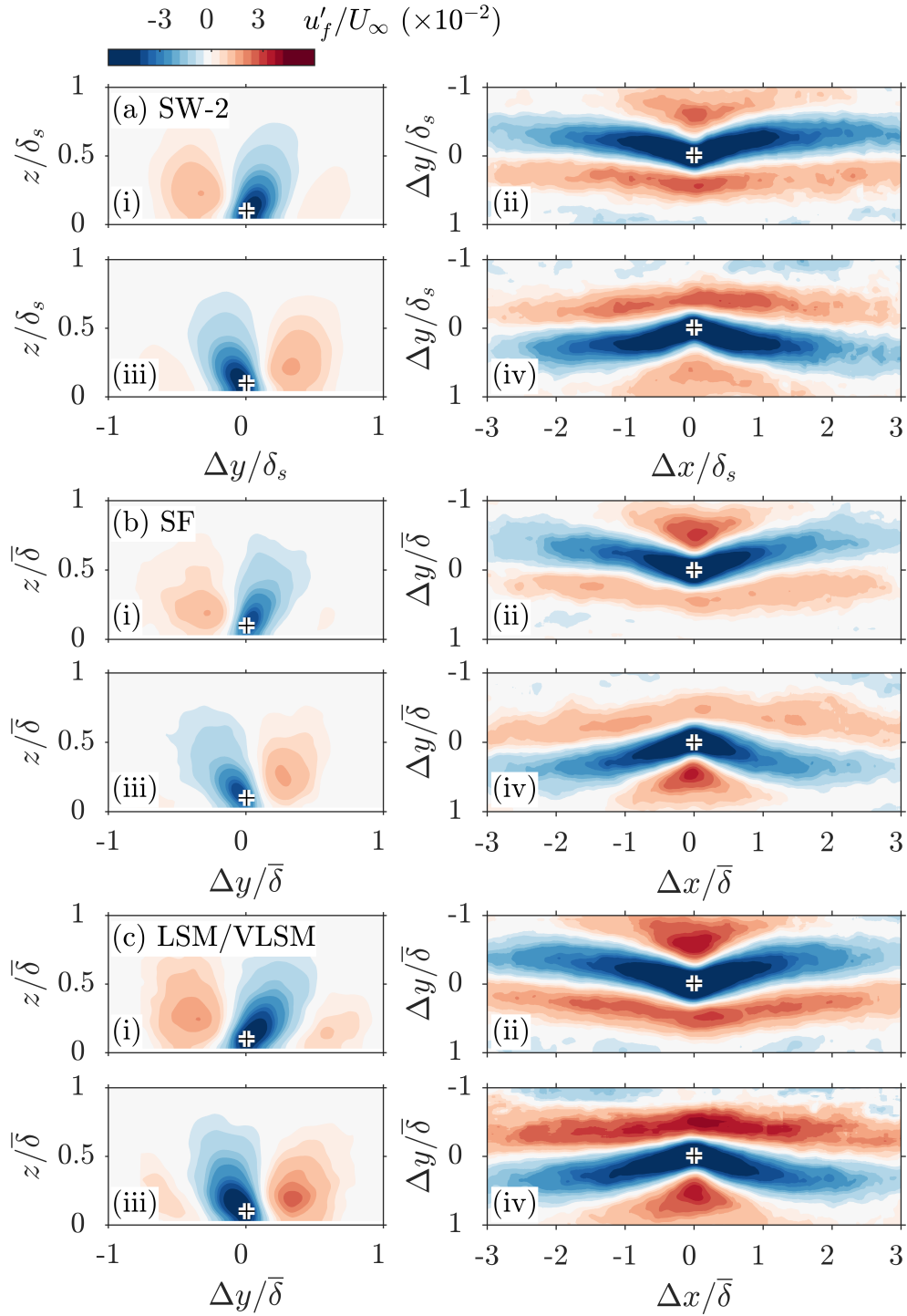


Figure 9.17: Contours of conditionally averaged turbulent fluctuation u' for (a) reference case SW-2 at $z/\delta_s = 0.46$ and SR250-2: conditioned at low-speed structures associated with (b) secondary flows (SF) and (c) LSM/VLSM. Conditions: (i) $\tilde{u}' < 0$ and $v' > 0$, (ii) $\tilde{u}' < 0$ and $v' < 0$, (iii) minima and (iv) maxima of detected low-speed structures ($\tilde{u}'/U_\infty < -0.03$) illustrated in figure 9.15(a).

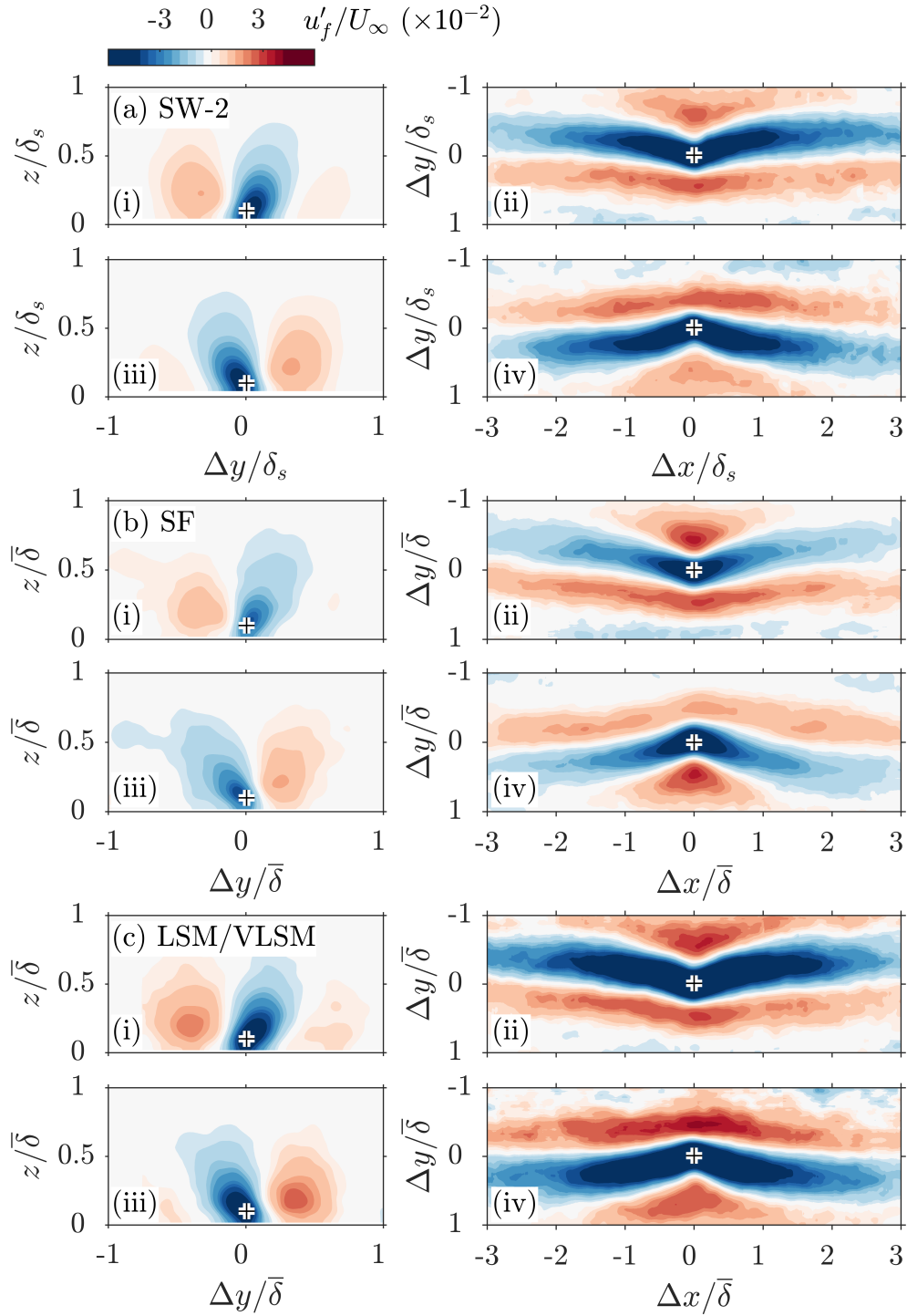


Figure 9.18: Contours of conditionally averaged turbulent fluctuation u' for (a) reference case SW-2 at $z/\delta_s = 0.46$ and SR160: conditioned at low-speed structures associated with (b) secondary flows (SF) and (c) LSM/VLSM. Conditions: (i) $\tilde{u}' < 0$ and $v' > 0$, (ii) $\tilde{u}' < 0$ and $v' < 0$, (iii) minima and (iv) maxima of detected low-speed structures ($\tilde{u}'/U_\infty < -0.03$) illustrated in figure 9.15(b).

strip. From Townsend's 1976 outer layer similarity hypothesis, we might expect at fixed z/δ that u'/U_τ at the center of the rough strip would be the same as that measured on the smooth wall. However, since the roughness increases C_f , we know that U_∞/U_τ for the rough strip is greater than the smooth, and hence we might expect u'/U_∞ to be higher over the center of the rough strip than over the smooth surface (with higher magnitude large scale fluctuations). This observation also holds for SR160 ($S/\bar{\delta} = 2.28$) in figure 9.18. Here, the low-speed structures related to naturally occurring LSM/VLSM are conditionally averaged at $y_{ref}/S = -0.75$ (centerline of a rough strip is at $y/S = -1$) for subplot (i) and (iii).

9.4.2 Limiting case: $S/\bar{\delta} \ll 1$

For the smallest $S/\bar{\delta}$ case, SR25 ($S/\bar{\delta} = 0.32$), the WPPIV measurements plane is set as close as possible to the center of the secondary flows at $z/\bar{\delta} = 0.18$, while the diameter of the secondary flows is approximately $0.32\bar{\delta}$. As shown in figure 5.9(f), the secondary flows fill the entire spanwise extent at this wall height (see also red shaded area in figure 9.6f). Similar to the cases where $S/\bar{\delta} \gg 1$, conditional average is also calculated for the minima and maxima of the detected low-speed structures in the xy -plane. In the yz -plane, the conditional average is computed at the spanwise location of the common flow up ($\odot$ in figure 5.2) and $z_{ref}/\bar{\delta} = 0.1$, based on the condition of $\tilde{u}'$ and v' (see §9.3).

Figure 9.19 show the comparison of conditionally averaged u' between the reference smooth wall case SW-2 (figure 9.19a) and SR25 (figure 9.19b). In subplot (ii) and (iv), the xy -plane is located at $z/\delta_s = 0.24$ for SW-2 and $z/\bar{\delta} = 0.18$ for SR25. Although the secondary flows are space-filling in case SR25 for $z/\bar{\delta} \lesssim 0.32$, the conditionally averaged low-speed structures do not seem to show a marked difference to the LSM/VLSM in the reference smooth wall case; the size (when normalised by $\bar{\delta}$) and shape are very similar. Another comparison is also shown at a different wall height in figure 9.20. Here, the xy -plane is located at $z/\delta_s = 0.46$ for SW-2 and $z/\bar{\delta} = 0.33$ for SR25, approximately at the interface between space-filling secondary flows and the flow reaching spanwise homogeneity. As expected, the conditionally averaged low-speed structures are indistinguishable between SW-2 and SR25.

These observations for the $S/\bar{\delta} \ll 1$ case reveal the problem of using spanwise locations to separate secondary flows and LSM/VLSM. This strategy works for $S/\bar{\delta} \gg 1$ cases because the both secondary flows and LSM/VLSM are δ -scaled and the secondary flows are confined about the roughness interface, while the rest of the flow approaches local homogeneity (smooth wall or homogeneous rough wall).

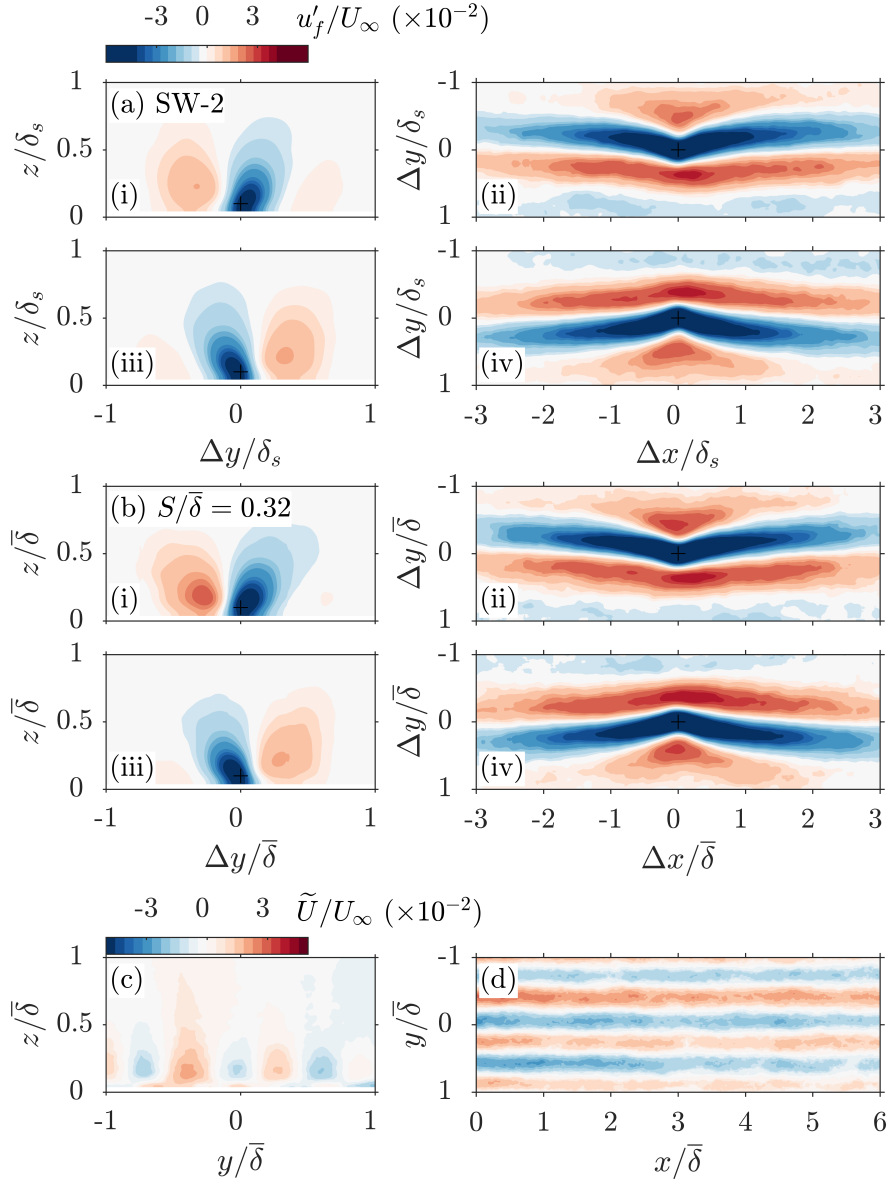


Figure 9.19: Contours of conditionally averaged turbulent fluctuation u' for (a) reference case SW-2 at $z/\delta_s = 0.24$ and (b) SR25 at $z/\bar{\delta} = 0.18$. Conditions: (i) $\tilde{u}' < 0$ and $v' > 0$, (ii) $\tilde{u}' < 0$ and $v' < 0$, (iii) minima and (iv) maxima of detected low-speed structures ($\tilde{u}'/U_\infty < -0.03$) illustrated in figure 9.6(f). Mean spatial fluctuation $\tilde{U}$ of SR25 case in: (c) yz -plane and (d) xy -plane at $z/\bar{\delta} = 0.18$.

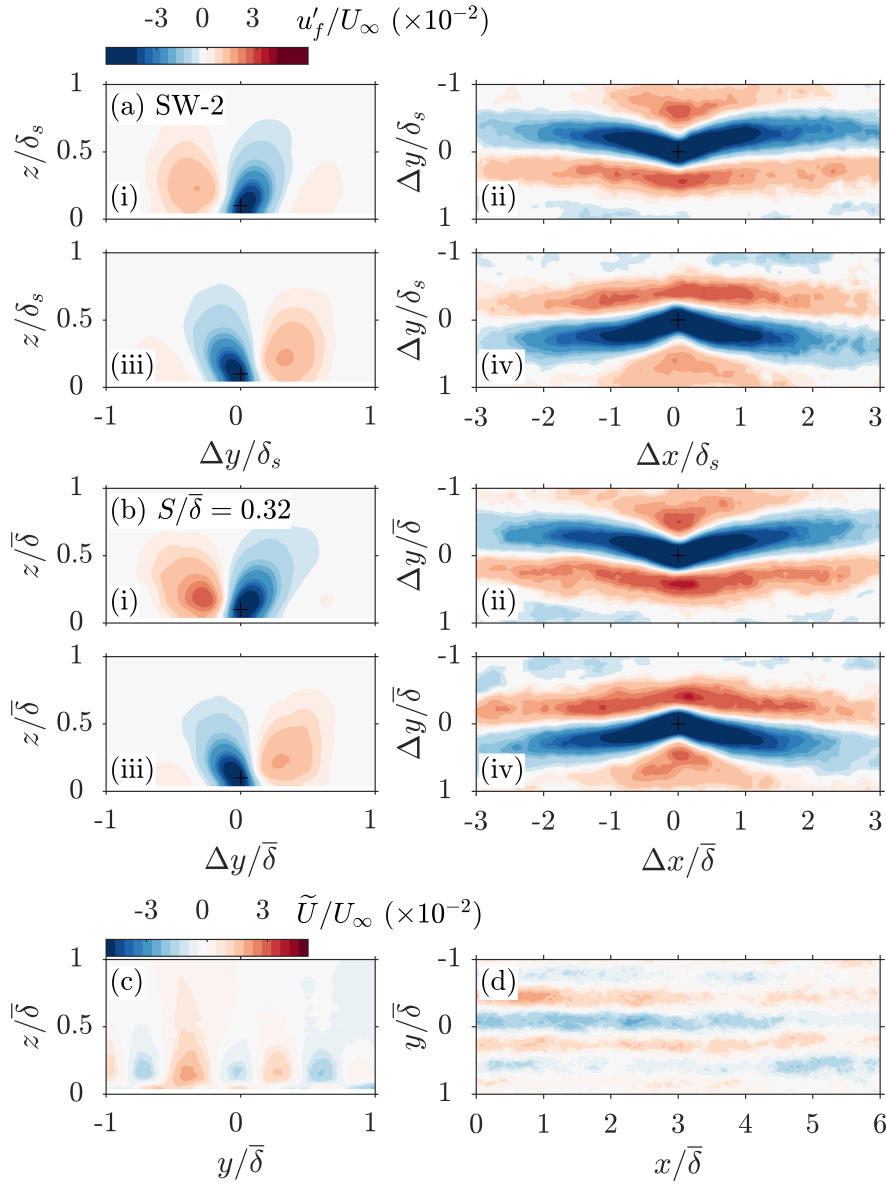


Figure 9.20: Contours of conditionally averaged turbulent fluctuation u' for (a) reference case SW-2 at $z/\delta_s = 0.46$ and (b) SR25 at $z/\bar{\delta} = 0.33$. Conditions: (i) $\tilde{u}' < 0$ and $v' > 0$, (ii) $\tilde{u}' < 0$ and $v' < 0$, (iii) minima and (iv) maxima of detected low-speed structures ($\tilde{u}'/U_\infty < -0.03$) illustrated in figure 9.6(f). Mean spatial fluctuation $\tilde{U}$ of SR25 case in: (c) yz -plane and (d) xy -plane at $z/\bar{\delta} = 0.33$.

However, for the other limit where $S/\bar{\delta} \ll 1$, the secondary flows and LSM/VLSM coexist in different scales. LSM/VLSM are δ -scaled, while the secondary flows are scaled by S ($S \ll \delta$) and aligned according to the roughness strips (see figure 9.19c,d and 9.20c,d). For case SR25, specifically, the scale of the secondary flows is approximately $0.32\bar{\delta} \approx S$. Hence, to study the coexistence between LSM/VLSM and secondary flows in the case where $S/\bar{\delta} \ll 1$, another method is necessary to separate the two.

‘Snapshot’ proper orthogonal decomposition (POD) is the chosen method in the present study to separate LSM/VLSM and secondary flows in the SR25 case. Snapshot POD was first introduced by Sirovich (1987) and the algorithm used in the present study was explained in detail in Meyer *et al.* (2007). In the context of PIV data, each instantaneous velocity field, obtained from cross-correlating a consecutive image pair, is considered as a ‘snapshot’ and the fluctuating velocity components of snapshot n , $\mathbf{u}'^n = (u'^n, v'^n, w'^n)$, can be written as a linear expansion

$$\mathbf{u}'^n = \sum_{i=1}^N \mathbf{a}_i^n \phi^i \quad (9.3)$$

where ϕ^i is the i th POD mode, $\mathbf{a}_i^n$ is the POD coefficient of mode i of snapshot n , and N is the total number of snapshots. In snapshot POD, the number of resolved POD modes is equal to the number of snapshots, which corresponds to 4800 for the spanwise heterogeneous cases in the yz -plane (1200 for the reference smooth wall case SW-2) and 1200 snapshots in the xy -plane (600 for SW-2). It is noted that the POD modes ϕ^i are ordered according to their contribution to the total turbulent kinetic energy such that the first mode ϕ^1 has the largest fraction of total energy, followed by the second mode ϕ^2 , and so forth.

Figure 9.21 shows POD modes of streamwise velocity fluctuation ϕ_u^i for case SR25 in the yz -plane. Three modes are selected: mode $i = 1, 5$, and 18, which contribute to 3.3%, 2.2%, and 0.9% of the total kinetic energy, respectively. All three selected modes show a spanwise periodic pattern of low- and high-speed streaks (blue and red filled contours, respectively). However, it is assumed that these streaks (either LSM/VLSM and secondary flows) coexist in different scales due to the spanwise roughness wavelength. To measure the ‘scale’ of turbulence in each mode, the Fast Fourier Transform (FFT) is computed for each ϕ_u^i at a reference wall-normal location $z/\bar{\delta} = 0.1$ (as close to the wall as the FOV permits, see solid black lines in figure 9.21a–c). The 1-D power spectral density of ϕ_u^i at $z/\bar{\delta} = 0.1$ is defined as

$$\Phi_{\phi\phi} = \frac{2c_n c_n^*}{\Delta k_y} \quad (9.4)$$

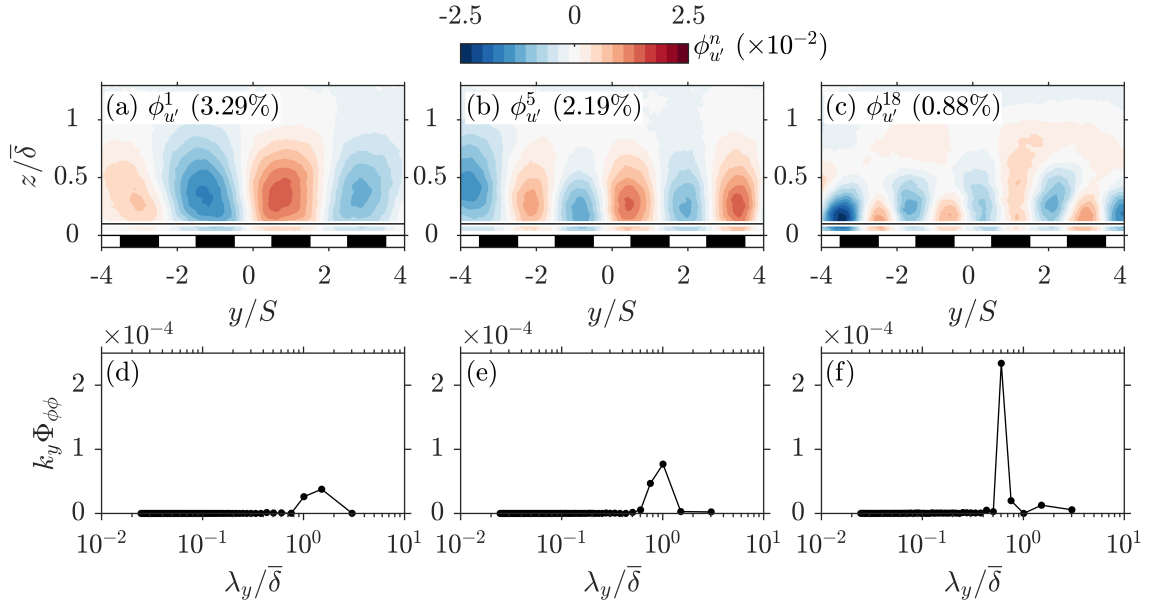


Figure 9.21: POD modes of u' for SR25 case in yz -plane: (a,d) mode 1, (b,e) mode 5, and (c,f) mode 18. Brackets on top left of the figures show the fraction of energy of the POD modes. (d,e,f) are 1-D premultiplied energy spectra of the modes $k_y \Phi_{\phi\phi}$ as a function of λ_y , computed at $z/\bar{\delta} = 0.1$ (solid black lines in a,b,c). The dominant modes are: (a) $\lambda_y/\bar{\delta} \approx 1.5$, (b) $\lambda_y/\bar{\delta} \approx 1$, and (c) $\lambda_y/\bar{\delta} \approx 0.64$ ($\lambda_y/S \approx 2$).

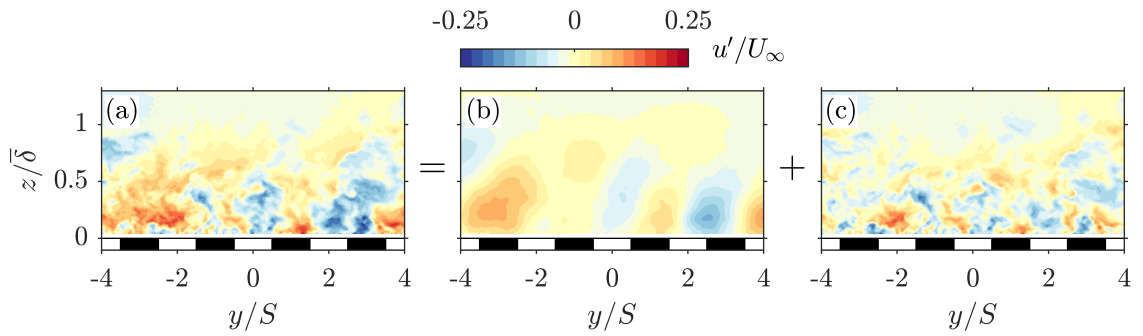


Figure 9.22: Turbulent velocity fluctuations u' in yz -plane: (a) all resolved scales, (b) large scales (constructed from u' POD modes with dominant $\lambda_y/\bar{\delta} > 0.64$), (c) small scales (constructed from u' POD modes with dominant $\lambda_y/\bar{\delta} \leq 0.64$).

where c_n is the n th Fourier coefficient, c_n^* is its complex conjugate, and k_y is the spanwise wavenumber. Figures 9.21(d–f) show the 1-D energy spectra premultiplied by k_y of mode 1, 5, and 18, respectively. These figures show that the spanwise periodic pattern observed in figure 9.21(a–c) corresponds to a dominant spanwise wavelength λ_y , i.e. maximum $k_y \Phi_{\phi\phi}$. Mode 1, for example, has the dominant $\lambda_y/\bar{\delta} \approx 1.5$, while the dominant wavelength for mode 5 and 18 are $\lambda_y/\bar{\delta} \approx 1$ and ≈ 0.64 , respectively. Dominant spanwise wavelengths are then computed for all 4800 resolved POD modes and separated into two groups: ‘large’ scales (dominant $\lambda_y/\bar{\delta} > 0.64$, presumably related to LSM/VLSM) and ‘small’ scales (dominant $\lambda_y/\bar{\delta} \leq 0.64 = 2S$ for case SR25, presumably related to the secondary flows due to spanwise heterogeneity). With this threshold for ‘large’/‘small’ scale separation, the POD modes associated with ‘large’ and ‘small’ scales make up 0.6% and 99.4% of all resolved modes, but contribute to 33.7% and 66.3% of the total kinetic energy, respectively. The fluctuating velocity field are then reconstructed from both ‘large’ and ‘small’ scale POD modes in equation 9.3. Figure 9.22 shows an instantaneous fluctuating velocity field u' of case SR25 ($S/\bar{\delta} = 0.32$) in the yz -plane, with figure 9.22(a) showing the broadband PIV measurement result (all resolved scales), figure 9.22(b) is the instantaneous u' reconstructed only from POD modes whose dominant $\lambda_y/\bar{\delta} > 0.64$ (‘large’ scales), and figure 9.22(c) is reconstructed only from ‘small scales’ ($\lambda_y/\bar{\delta} \leq 0.64$).

A similar strategy to separate turbulent scales related to LSM/VLSM and the secondary flows is also applied for the fluctuating velocity fields in the xy -plane (WPPIV data). Figure 9.23(a,c,e) show the first, ninth, and thirty-second mode of u' ($\phi_{u'}^1$, $\phi_{u'}^9$, and $\phi_{u'}^{32}$, respectively) of case SR25 ($S/\bar{\delta} = 0.32$) in xy -plane at $z/\bar{\delta} = 0.18$. This wall-normal location of the xy -plane is close to the centre of the secondary flows, which is estimated at $z/\bar{\delta} \approx 0.16$, $z/S \approx 0.5$. Similar to the POD modes in yz -plane, the modes in the xy -plane show spanwise-alternating, elongated low- and high-speed streaks (blue and red filled contours, respectively).

To obtain the ‘scales’ of turbulent flows, FFT is once again computed for each $\phi_{u'}$ in both the x - and y -directions. The 2-D power spectral density of $\phi_{u'}$ in the xy -plane is defined as

$$\Phi_{\phi\phi} = \frac{4c_n c_n^*}{\Delta k_x \Delta k_y} \quad (9.5)$$

where k_x is the streamwise wavenumber. Figure 9.23(b,d,f) show the contours of premultiplied 2-D energy spectra of POD modes, $k_x k_y \Phi_{\phi\phi}$, for each POD mode in figure 9.23(a,c,e). Symbol ‘+’ marks the maxima of $k_x k_y \Phi_{\phi\phi}$ contours, which corresponds to the dominant streamwise and spanwise wavelength (λ_x and λ_y) of each POD mode. It should be noted that the accuracy of this method to detect

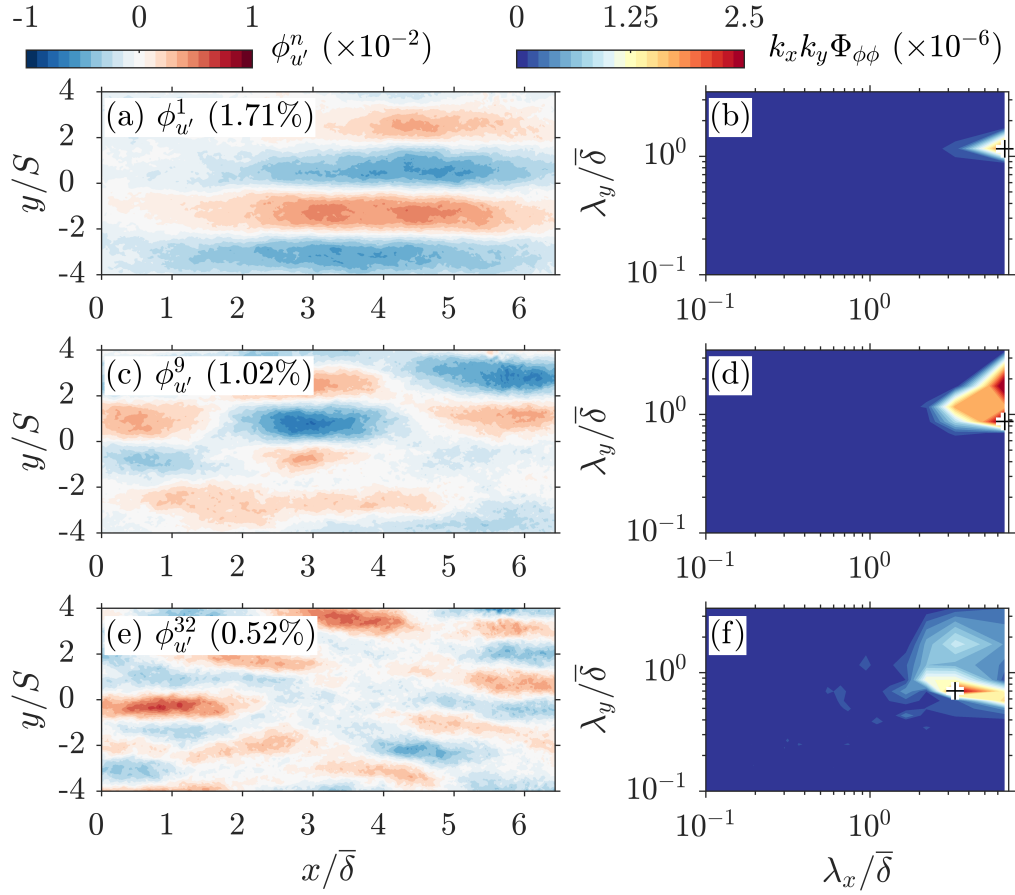


Figure 9.23: POD modes of u' for SR25 case in xy -plane ($z/\bar{\delta} = 0.18$): (a,b) mode 1, (c,d) mode 9, and (e,f) mode 32. Brackets on top left of the figures show the fraction of energy of the POD modes. (b,d,f) are the contours of premultiplied 2-D energy spectra of the modes $k_x k_y \Phi_{\phi\phi}$ as a function of λ_x and λ_y . The dominant modes (marked by '+' symbols) are: (a) $\lambda_x/\bar{\delta} \approx 6.6$, $\lambda_y/\bar{\delta} \approx 1.2$, (b) $\lambda_x/\bar{\delta} \approx 6.6$, $\lambda_y/\bar{\delta} \approx 0.9$, and (c) $\lambda_x/\bar{\delta} \approx 3.3$, $\lambda_y/\bar{\delta} \approx 0.64$ ($\lambda_y/S \approx 2$).

dominant λ_x and λ_y is limited by the streamwise and spanwise extent of the FOV. In this case, mode 1 and 9 (figure 9.23b,d) have the dominant wavelength of $\lambda_x/\bar{\delta} \approx 6.6$, which is the maximum streamwise extent of the FOV. It is possible that the actual dominant λ_x for these modes is longer than $6.6/\bar{\delta}$. It is also noted that mode 32 (figure 9.23f) might be related to the secondary flows due to spanwise heterogeneity: it has the dominant $\lambda_y/\bar{\delta} \approx 0.64 = 2S$ (the spanwise wavelength of the heterogeneous roughness strips) and dominant $\lambda_x/\bar{\delta} \approx 3.3$, which corresponds to the additional peak in the energy spectra of turbulent fluctuations u' , $k_x \Phi_{u'u'}$ (label ① in figure 7.2f). The distribution of dominant λ_x and λ_y across all resolved POD modes is shown in figure 9.24(a). The distribution of this dominant wavelengths is such that the small scales are bounded by linear relationship $\lambda_y \sim \lambda_x$ and the large scales by $\lambda_y/\bar{\delta} \sim (\lambda_x/\bar{\delta})^{0.5}$ (red solid line and dashed lines in figure 9.24a). The same linear relationships are also observed in the premultiplied 2-D energy spectra of u' ($k_x k_y \Phi_{u'u'}$) in figure 9.24(b) for case SR25 ($S/\bar{\delta} = 0.32$, $z/\bar{\delta} = 0.18$), a smooth wall boundary layer at $\delta_s^+ \approx 2000$ (Chandran *et al.*, 2017), and channel flow at similar δ_s^+ (del Álamo *et al.*, 2004).

Similar to the POD modes of u' in the yz -plane, $\phi_{u'}$ in the xy -plane are also separated into two groups based on the dominant λ_y : ‘large’ scales (dominant $\lambda_y/\bar{\delta} > 0.64$, presumably related to LSM/VLSM) and ‘small’ scales (dominant $\lambda_y/\bar{\delta} \leq 0.64 = 2S$, presumably related to the secondary flows), as shown by the dash-dot line in figure 9.24(a). No such threshold for scale separation is applied for the dominant λ_x . With this threshold for ‘large’/‘small’ scale separation, the POD modes associated with ‘large’ and ‘small’ scales make up 6.3% and 93.7% of all resolved 1200 modes, but contribute to 32.8% and 67.2% of the total kinetic energy, respectively. The fluctuating velocity field are then reconstructed from both ‘large’ and ‘small’ scale POD modes in equation 9.3. Figure 9.25 shows an instantaneous fluctuating velocity field u' of case SR25 in the xy -plane. Figure 9.25(a) is the broadband PIV result (all resolved scales), figure 9.25(b) is the instantaneous u' reconstructed from POD mode whose dominant $\lambda_y/\bar{\delta} > 0.64$ (‘large’ scales), and figure 9.25(c) is reconstructed only from ‘small scales’ ($\lambda_y/\bar{\delta} \leq 0.64$).

The conditional average of turbulent fluctuation u' is computed for case SR25 at $z/\bar{\delta} = 0.18$. In the xy -plane, the condition points are at the minima and maxima of the low-speed structures. The same algorithm to detect ‘spines’ of the structures and the minima/maxima in §9.4.1 is also applied for SR25. However, for cases where $S/\bar{\delta} \gg 1$ in §9.4.1 the low-speed structures are separated into LSM/VLSM and secondary flows based on their spanwise locations. Here, where $S/\bar{\delta} \ll 1$,

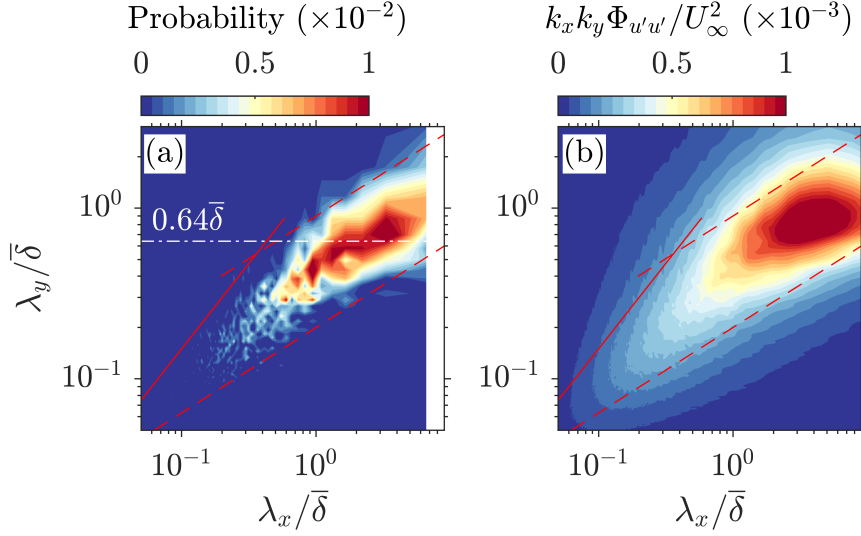


Figure 9.24: (a) Distribution of dominant wavelength λ_x and λ_y obtained from the maxima of premultiplied 2-D energy spectra of u' POD modes, $k_x k_y \Phi_{\phi\phi}$ ($\circ$) for case SR25. (b) Contours of premultiplied 2-D energy spectra of u' , $k_x k_y \Phi_{u'u'}$, for case SR25 at $z/\bar{\delta} = 0.18$. Dash-dot line (---) in (a) is the threshold for large/small scales separation, $\lambda_y/\bar{\delta} = 0.64$. Solid red lines (—) denote the relationship $\lambda_y/\bar{\delta} \sim \lambda_x/\bar{\delta}$, while dashed red lines (---) denote $\lambda_y/\bar{\delta} \sim (\lambda_x/\bar{\delta})^{0.5}$.

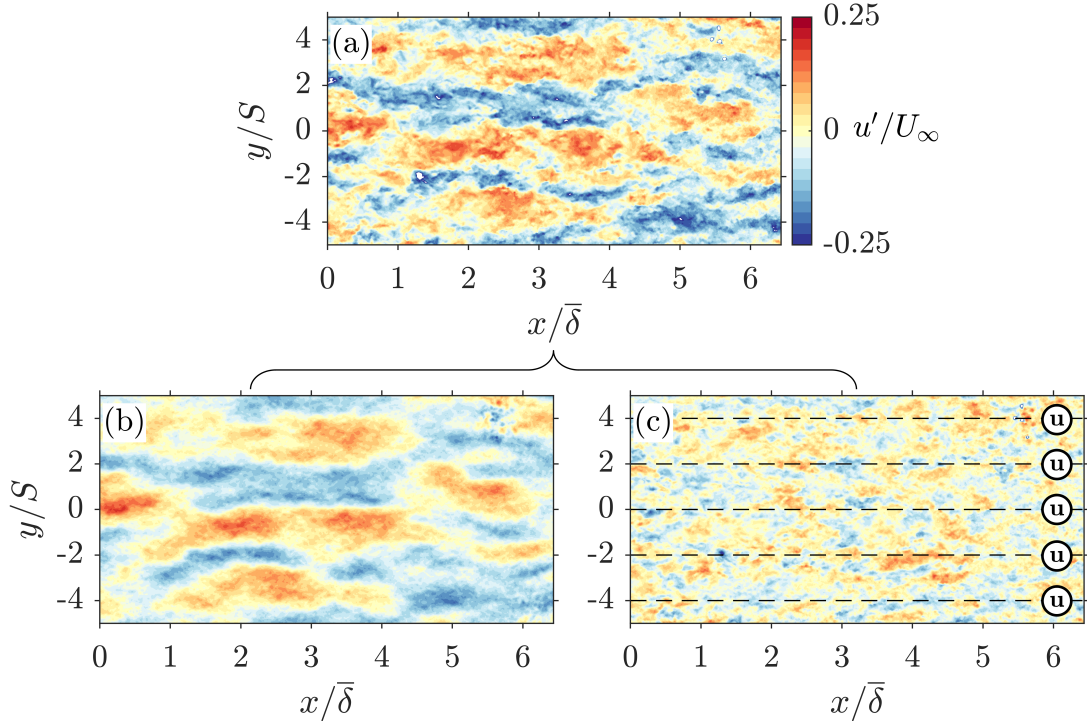


Figure 9.25: Turbulent velocity fluctuations u' in xy -plane: (a) all resolved scales, (b) large scales (constructed from u' POD modes with dominant $\lambda_y/\bar{\delta} > 0.64$), (c) small scales (constructed from u' POD modes with dominant $\lambda_y/\bar{\delta} \leq 0.64$). Dashed lines (---) are the spanwise locations of the common flow up of the secondary flows (marked by $\textcircled{u}$).

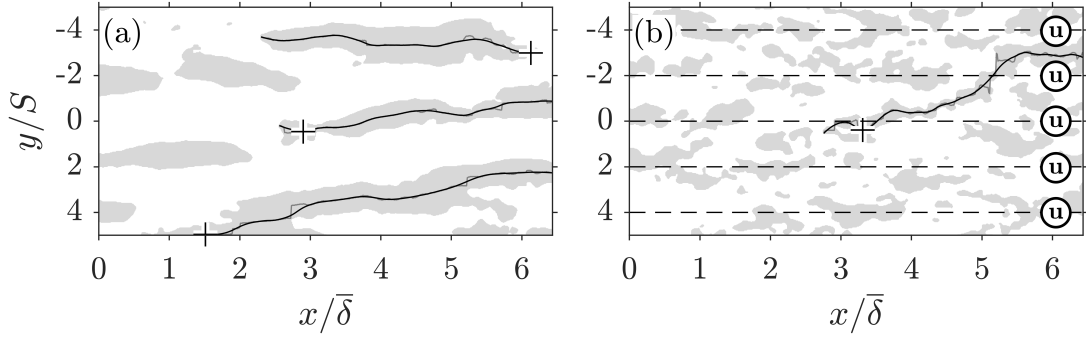


Figure 9.26: Detected low-speed structures for case SR25 ($z/\delta = 0.18$): (a) large scales (constructed from u' POD modes with dominant $\lambda_y/\delta > 0.64$), (b) small scales (constructed from u' POD modes with dominant $\lambda_y/\delta \leq 0.64$). Grey coloured contours are the low-speed structures, $\tilde{u}'/U_\infty < -0.03$. '+' marks the minima of a low-speed structure. The spines of the detected low-speed structures are shown in solid lines (from PIV data: —, smoothed: —). Dashed lines (---) are the spanwise locations of the common flow up of the secondary flows (marked by $\textcircled{u}$).

each instantaneous turbulent fluctuation field is separated into the ‘large’ scales (presumably LSM/VLSM) and ‘small’ scales (presumably including the secondary flows) based on the dominant λ_y of the POD modes. An example of detected low-speed structures for case SR25 ($z/\delta = 0.18$) are shown in figure 9.26(a) for the ‘large’ scales and in figure 9.26(b) for the ‘small’ scales. Gray filled contours are $\tilde{u}'/U_\infty < -0.03$ and only structures whose length is $\geq 3\bar{\delta}$ are included in the computation (these criteria for low-speed structure detection is the same as those in §9.4.1).

Conditional averages of u' are computed in the yz - and xy -planes for both ‘large’ and ‘small’ scales. As the scale of the secondary flows is considerably smaller than the LSM/VLSM when $S/\bar{\delta} \ll 1$, the velocity field is unfiltered so that all scales are accounted for, unlike the analysis of $S/\bar{\delta} \gg 1$ cases in §9.4.1. In the yz -plane, the condition point of u' (‘+’ in figure 9.27i and iii) is at $z/\bar{\delta} = 0.1$ (as close to the wall as the FOV permits) and at the spanwise locations of common flow up (centre of the smooth strips, see $\textcircled{u}$ in figure 5.2g) for consistency with the previous analysis in §9.3 and §9.4.1 (for ‘larger’ scales, the resulting conditionally averaged structures are insensitive to the spanwise location of the condition vector). The conditions are as follows: $\tilde{u}' < 0$ (low-speed streaks) and $v' > 0$, $\tilde{u}' < 0$ (low-speed streaks) and $v' < 0$. In the xy -plane, the condition point is at the minima and maxima of the low-speed streaks, which corresponds to $v' > 0$ and $v' < 0$ (see figure 9.8 for an example). It is assumed that the minima and maxima of the low-speed streaks occur

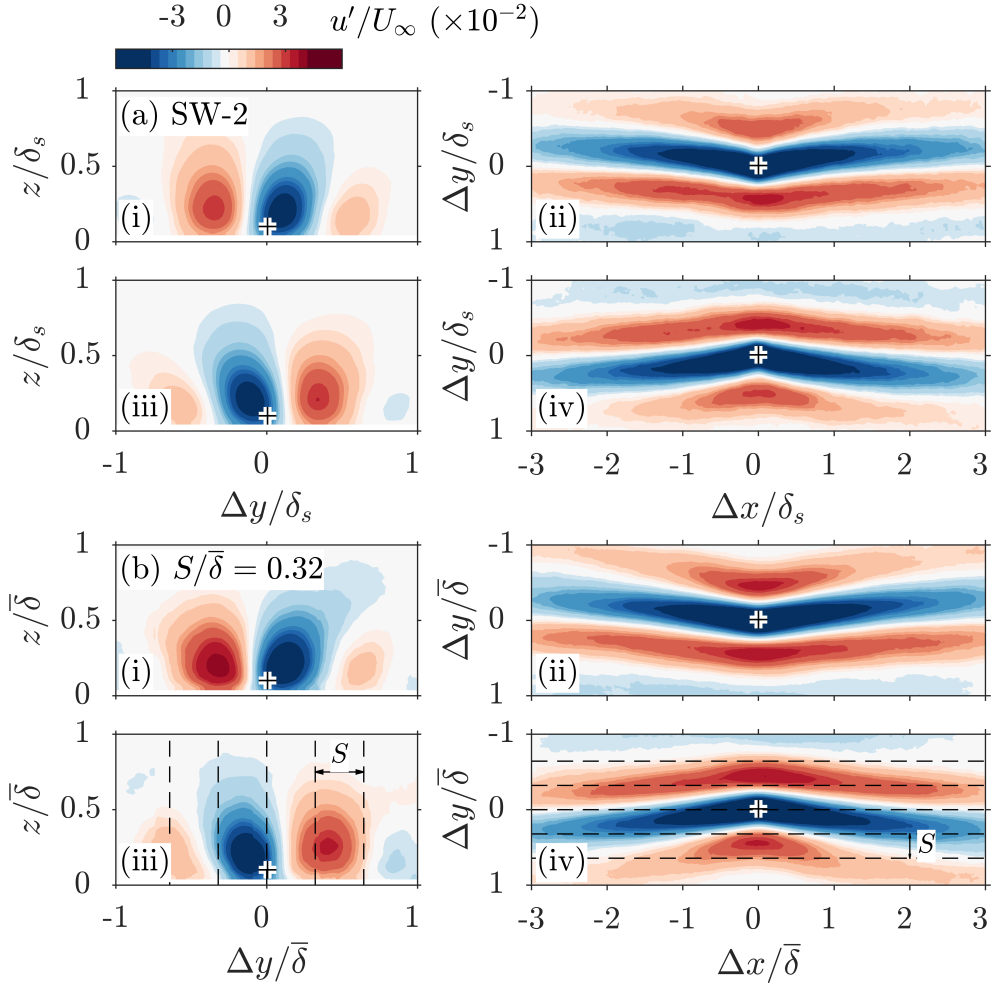


Figure 9.27: Contours of conditionally averaged large-scale turbulent fluctuation u' (constructed from u' POD modes with dominant $\lambda_y/\bar{\delta} > 0.64$ or $\lambda_y/\delta_s > 0.64$) for (a) reference case SW-2 at $z/\delta_s = 0.24$ and (b) SR25 at $z/\bar{\delta} = 0.18$. Conditions: (i) $\tilde{u}' < 0$ and $v' > 0$, (ii) $\tilde{u}' < 0$ and $v' < 0$ at '+', (iii) minima and (iv) maxima of detected low-speed structures (at '+', $\tilde{u}'/U_\infty < -0.03$) illustrated in figure 9.26(a) (refer to §9.3 for the method used to extract the 'spines' of low-speed structures). In (b)(iii) and (iv), dashed lines (---) show the width of the roughness strips S .

about the spanwise location of the common flow up, spanning a spanwise extent of $\pm l_y$ (see figure 9.6f). As a comparison to the $S/\bar{\delta} \ll 1$ case, the same POD and conditions are also applied to the reference smooth wall case SW-2 with appropriate scaling (δ_s instead of $\bar{\delta}$). The velocity field is separated into ‘large’ (POD modes with dominant $\lambda_y/\delta_s > 0.64$) and ‘small’ scales ($\lambda_y/\delta_s \leq 0.64$). In the yz -plane, the condition point of u' is at $z/\delta_s = 0.1$ and spanwise homogeneity is assumed. In the xy -plane, only structures with $\geq 3\delta_s$ length are included and spanwise homogeneity is also assumed.

Figure 9.27(a) show the contours of conditionally averaged ‘large’-scale u' for the reference smooth wall case SW-2 at and (b) SR25 case ($S/\bar{\delta} = 0.32$). Conditionally averaged u' in the yz -plane is shown in subplot (i) and (iii). The xy -plane (subplot ii and iv) is located at $z/\delta_s = 0.24$ for SW-2 and $z/\bar{\delta} = 0.18$ for SR25 (close to the centre of the secondary flows). The contours show the structures leaning to the left and right, similar to those shown in §9.3 and §9.4.1. It is interesting to note, however, that when only ‘large’ scales (i.e. scales that are larger than the spanwise heterogeneity) are considered, the structures of SW-2 and SR25 are identical. This further suggests that in the case of spanwise heterogeneity where the roughness $S/\bar{\delta} \ll 1$, scales that are $\gtrsim S$ can be considered as the naturally occurring turbulent structures and they exist in a relatively unchanged form compared to those observed in the reference smooth wall.

Figure 9.28(a) show the contours of conditionally averaged ‘small’-scale u' for the reference smooth wall case SW-2 at and (b) SR25 case ($S/\bar{\delta} = 0.32$). It is noted that the conditionally averaged ‘small’-scale u' in the xy -plane (subplot ii and iv) is less converged than the ‘large’ scales (figure 9.27). As shown in figure 9.26(b), the ‘small’-scale low-speed streaks are significantly shorter than the ‘large’-scale streaks in (a), which means that the total number of detected low-speed streaks in the ‘small’-scale turbulent velocity field will be less than that in the ‘large’-scale velocity field. For comparison, only 21.3% of all detected low-speed streaks belong to the ‘small’ scales for case SR25. For the smooth wall reference SW-2, only 2.4% of detected streaks are ‘small’. Despite the issues with convergence, the differences in the ‘small’ scales between SW-2 and SR25, however, are still discernible. In the yz -plane, the structures for both SW-2 and SR25 are capped in the wall-normal direction at $z/\bar{\delta} \lesssim 0.32$ ($z/\delta_s \lesssim 0.32$). However, a clear spanwise periodic pattern is observed in the SR25 case (figure 9.28b, subplot i and iii), which is largely absent from the smooth wall reference (figure 9.28a, subplot i and iii). The wavelength of

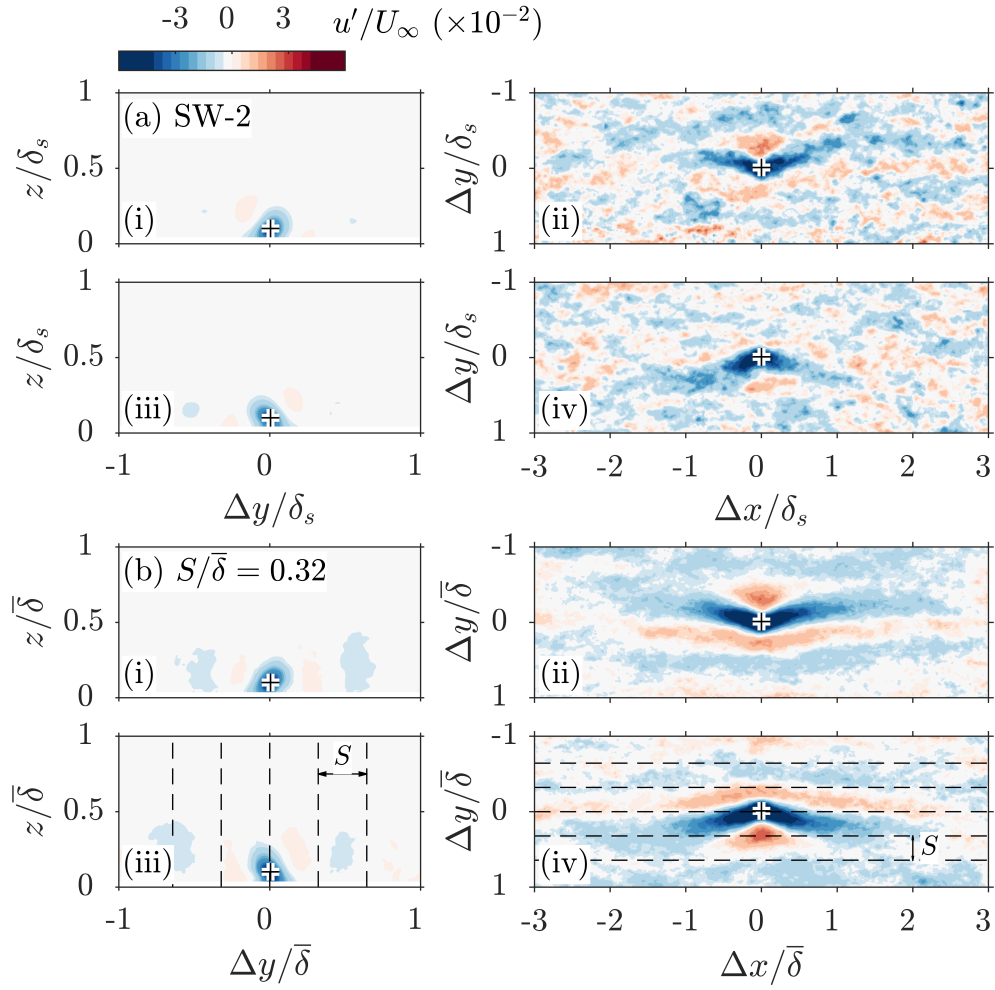


Figure 9.28: Contours of conditionally averaged small-scale turbulent fluctuation u' (constructed from u' POD modes with dominant $\lambda_y/\bar{\delta} \leq 0.64$ or $\lambda_y/\delta_s \leq 0.64$) for (a) reference case SW-2 at $z/\delta_s = 0.24$ and (b) SR25 at $z/\bar{\delta} = 0.18$. Conditions: (i) $\tilde{u}' < 0$ and $v' > 0$, (ii) $\tilde{u}' < 0$ and $v' < 0$ at '+', (iii) minima and (iv) maxima of detected low-speed structures (at '+', $\tilde{u}'/U_\infty < -0.03$) illustrated in figure 9.26(b) (refer to §9.3 for the method used to extract the 'spines' of low-speed structures). In (b)(iii) and (iv), dashed lines (---) show the width of the roughness strips S .

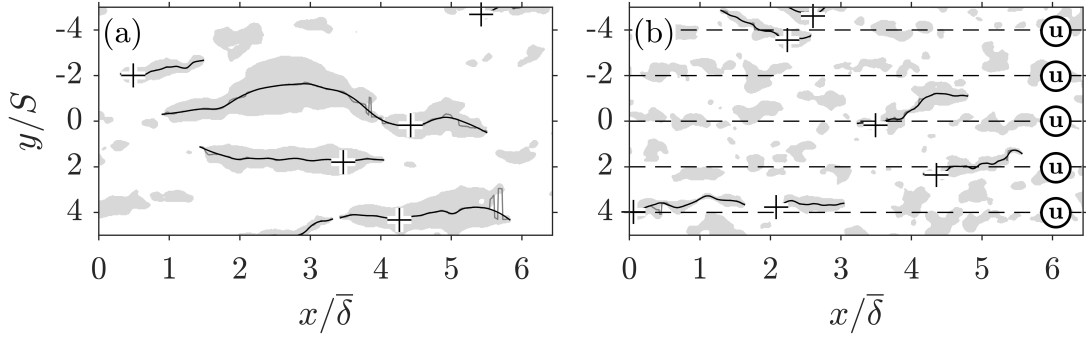


Figure 9.29: Detected low-speed structures for case SR25 ($z/\bar{\delta} = 0.33$): (a) large scales (constructed from u' POD modes with dominant $\lambda_y/\bar{\delta} > 0.64$), (b) small scales (constructed from u' POD modes with dominant $\lambda_y/\bar{\delta} \leq 0.64$). Grey coloured contours are the low-speed structures, $\tilde{u}'/U_\infty < -0.03$. '+' marks the minima of a low-speed structure. The spines of the detected low-speed structures are shown in solid lines (from PIV data: —, smoothed: —). Dashed lines (---) are the spanwise locations of the common flow up of the secondary flows (marked by $\textcircled{u}$).

this spanwise periodic pattern is $2S$ (which is also the spanwise roughness wavelength Λ), as shown by the dashed lines in figure 9.28(b)(iii). A slight misalignment is attributed to the condition of v' , such that structures lean slightly to the left and right. In the xy -plane, a similar pattern is also observed. The pronounced spanwise periodicity for the SR25 case is largely absent for the smooth wall reference. Furthermore, the streaks for case SR25 appear to be elongated in x (approximately $3-4\bar{\delta}$ long, figure 9.28(b)(ii) and (iv)) to a greater extent than those observed for SW-2 ($\sim 2\delta_s$). This length is similar to the streamwise wavelength associated with the additional energy spectra peak in figure 7.2f), which has been previously associated with meandering of the secondary flows. A possible explanation for this elongated streaks is due to a phase-locked, imposed heterogeneity (i.e. interface between rough and smooth strips) on the surface along the streamwise direction. This further ensures that in the case of spanwise heterogeneity where the roughness $S/\bar{\delta} \ll 1$, the secondary flows due to spanwise heterogeneity exist instantaneously as S -scaled, phase-locked structures along with naturally occurring $\bar{\delta}$ -scaled LSM/VLSM.

Another question worth considering is how far above the surface the S -scaled structures exist. For figure 9.27 and 9.28, the xy -plane is close to the center of the mean secondary flows). However, figure 5.9(f) shows that the secondary flows are capped and the flow approaches spanwise homogeneity above the secondary flows (illustrated in figure 2.4c), which suggests that S -scaled, phase-locked structures due to spanwise heterogeneity should diminish at $z/\bar{\delta} \gtrsim 0.32$. Similar analysis is

repeated for case SR25, for a different wall-parallel slice ($z/\bar{\delta} = 0.33$, approximately at the edge of the secondary flows due to spanwise heterogeneity) and the reference smooth wall case at $z/\delta_s = 0.46$. Figure 9.29(a) show the detected ‘spines’ of ‘large’-scale low-speed streaks (POD modes have the dominant $\lambda_y/\bar{\delta} > 0.64$), while (b) show the ‘small’-scale low-speed streaks (dominant $\lambda_y/\bar{\delta} \leq 0.64$). It should be noted that the xy -plane here is further away from the wall, which means that the streaks are shorter than those detected in $z/\bar{\delta} = 0.18$ wall-parallel slice. With the previous length threshold for low-speed streaks detection ($\geq 3\bar{\delta}$), no detected low-speed streak satisfies this condition. Therefore, for the wall-parallel slice at $z/\bar{\delta} = 0.33$, the threshold is lowered: only structures whose length is $\geq \bar{\delta}$ are included in the computation. The same length threshold is also applied for the reference smooth wall case SW-2 at $z/\delta_s = 0.46$, with adjusted scaling (δ_s instead of $\bar{\delta}$).

Figure 9.30 and 9.31 show the conditionally averaged u' for ‘large’- and ‘small’-scale turbulent fluctuation fields, respectively. Similar to what is observed in figure 9.27, the ‘large’-scales are very similar between the reference smooth wall case SW-2 (figure 9.27a) and case SR25 (figure 9.27b), with slightly higher magnitude of conditionally averaged u' for SR25 since it is normalised by U_∞ instead of U_τ . The conditions and condition points in the yz -plane (subplot i and iii) are exactly the same as those in figure 9.27 and 9.28. The xy -planes, however, are further from the wall ($z/\bar{\delta} = 0.33$ for SR25, $z/\delta_s = 0.46$ for SW-2), which explains the shorter (in x) streaks in figure 9.30. For the ‘small’ scales shown in figure 9.31, a spanwise periodic behaviour of $2S$ wavelength, similar to those in figure 9.28(b), is again observed adjacent to the detected events (figure 9.31b). The structures are once again shorter due to the wall-normal location of the xy -plane and the length threshold for low-speed streak detection ($\bar{\delta}$ instead of $3\bar{\delta}$). The observed spanwise periodic pattern in SR25 case, however, suggests that the structures associated with S -sized secondary flows due to spanwise heterogeneity do not immediately diminish at $z \approx S$, even though the mean velocity field shows that the flow approaches spanwise homogeneity when $z \gtrsim S$ (from which we may have inferred that only LSM/VLSM occur here).

In summary, the coexistence between LSM/VLSM and secondary flows due to spanwise heterogeneity are observed in the two limits where $S/\bar{\delta} \gg 1$ and $S/\bar{\delta} \ll 1$. Two different methods are utilised to separate LSM/VLSM and secondary flows for these cases. In the case where $S/\bar{\delta} \gg 1$, the secondary flows are confined above the roughness interfaces while LSM/VLSM occur above either smooth or rough strips far removed from the interfaces, where the flow approaches (local) homogeneity

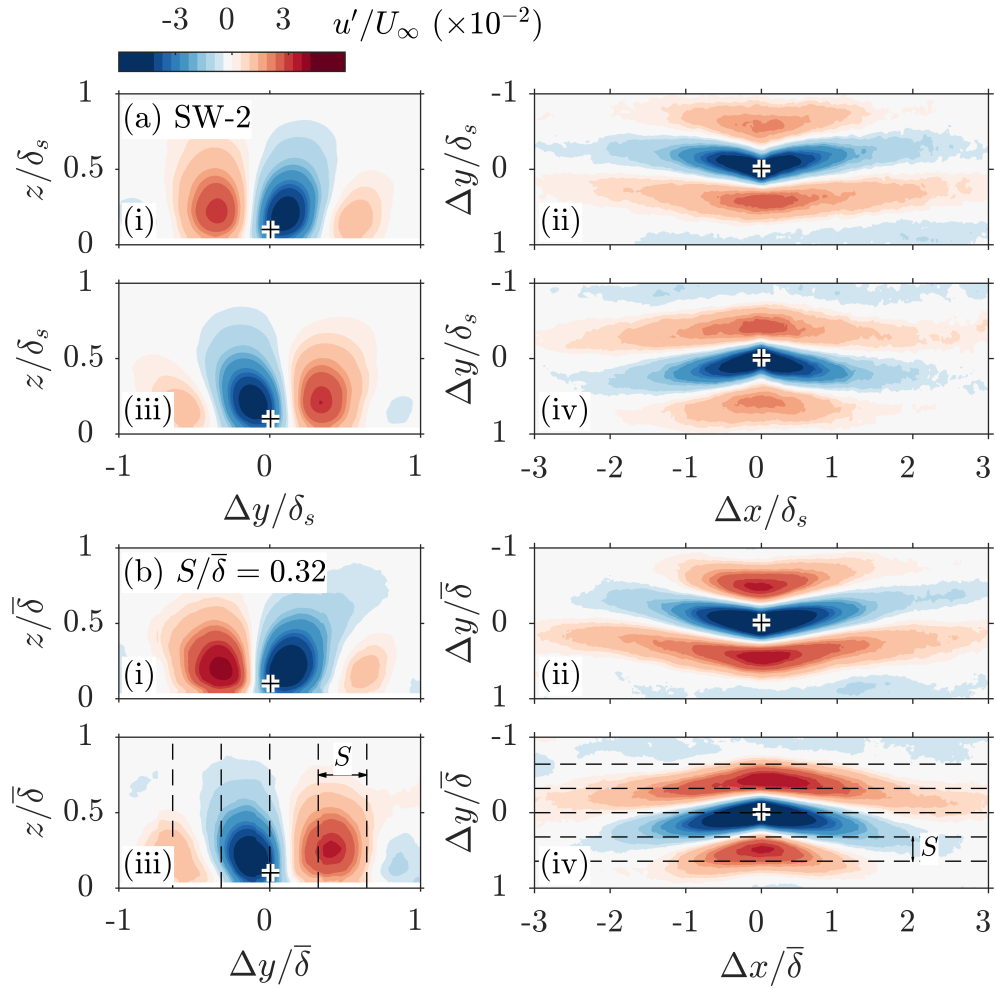


Figure 9.30: Contours of conditionally averaged large-scale turbulent fluctuation u' (constructed from u' POD modes with dominant $\lambda_y/\bar{\delta} > 0.64$ or $\lambda_y/\delta_s > 0.64$) for (a) reference case SW-2 at $z/\delta_s = 0.46$ and (b) SR25 at $z/\bar{\delta} = 0.33$. Conditions: (i) $\tilde{u}' < 0$ and $v' > 0$, (ii) $\tilde{u}' < 0$ and $v' < 0$ at '+', (iii) minima and (iv) maxima of detected low-speed structures (at '+', $\tilde{u}'/U_\infty < -0.03$) illustrated in figure 9.29(a). In (b)(iii) and (iv), dashed lines (---) show the width of the roughness strips S .

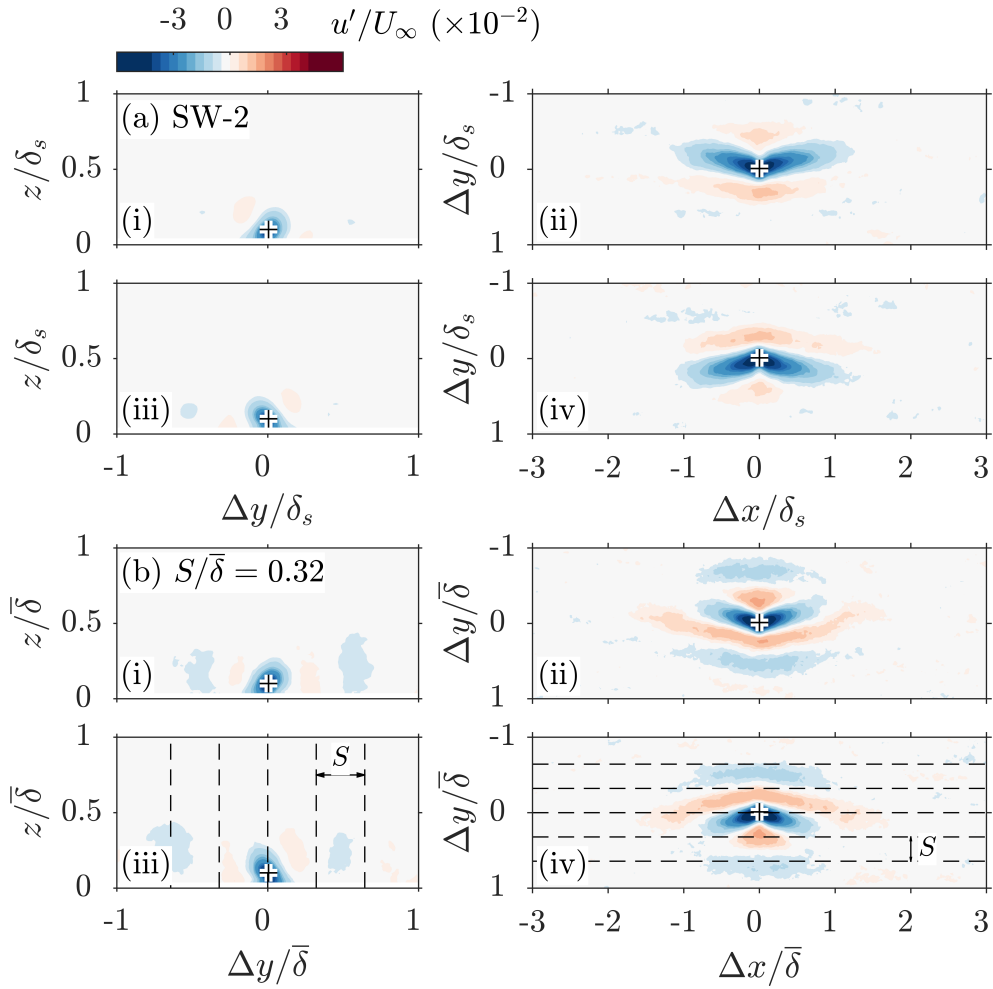


Figure 9.31: Contours of conditionally averaged small-scale turbulent fluctuation u' (constructed from u' POD modes with dominant $\lambda_y/\bar{\delta} \leq 0.64$ or $\lambda_y/\delta_s \leq 0.64$) for (a) reference case SW-2 at $z/\delta_s = 0.46$ and (b) SR25 at $z/\bar{\delta} = 0.33$. Conditions: (i) $\tilde{u}' < 0$ and $v' > 0$, (ii) $\tilde{u}' < 0$ and $v' < 0$ at '+', (iii) minima and (iv) maxima of detected low-speed structures (at '+', $\tilde{u}'/U_\infty < -0.03$) illustrated in figure 9.29(b). In (b)(iii) and (iv), dashed lines (---) show the width of the roughness strips S .

(figure 2.4a). Hence, it is assumed that the low-speed streaks occurring about the roughness interfaces are caused by the spanwise heterogeneity, while those occurring elsewhere above the surface are assumed to be LSM/VLSM. With this separation method, conditional averages of the turbulent fluctuation u' reveal that both secondary flows and LSM/VLSM have a similar shape and scale ($\bar{\delta}$ -scaled), with stronger spanwise-leaning tendency for the secondary flows (but not as prominent as the intermediate case where $S/\bar{\delta} \approx 1$). Further, these structures are identical to the LSM/VLSM observed in the reference smooth wall case SW-2. In the case where $S/\bar{\delta} \ll 1$, the secondary flows are confined to $z \lesssim S$ while LSM/VLSM occur above this wall-normal extent, where the flow approaches spanwise homogeneity (figure 2.4c). Snapshot POD is utilised to separate an instantaneous velocity field into two scales: ‘large’ scales ($> 2S$, related to LSM/VLSM) and ‘small’ scales ($\leq 2S$, related to the secondary flows). Conditional average of the turbulent fluctuation u' shows that the LSM/VLSM in $S/\bar{\delta} \ll 1$ case and the reference smooth wall are identical. Phase-locked secondary flows coexist with LSM/VLSM in smaller scales (S instead of $\bar{\delta}$) in the heterogeneous roughness case, but not in the reference smooth wall case. Since this scale is considerably smaller than LSM/VLSM, conditionally averaging u' without separating the scales causes the smaller scales (including those associated with the secondary flows) to diminish with averaging, as shown in figure 9.19 and 9.20. Moreover, conditional averages of u' at another wall-parallel slice further away from the wall show that the structures associated with secondary flows still coexist with LSM/VLSM even at the interface between space-filling secondary flows and spanwise homogeneity ($z \approx S$).

Chapter 10

Conclusions and future work

10.1 Conclusions

From the analysis of secondary flows and perturbed turbulent boundary layers resulting from spanwise varying surface conditions within a range of spanwise half-wavelength $0.32 \leq S/\bar{\delta} \leq 6.81$, it can be concluded that:

- (a) The behaviour of the turbulent boundary layer in the presence of spanwise heterogeneous surface roughness can be categorised into two limiting cases and one intermediate case: $S/\bar{\delta} \gg 1$, $S/\bar{\delta} \ll 1$, and $S/\bar{\delta} \approx 1$, respectively. When $S/\bar{\delta} \gg 1$, the diameter of the secondary roll modes are capped by $\bar{\delta}$ and confined in the span to the vicinity of the roughness transition. Farther from the transition, the flow becomes locally homogeneous above each spanwise-alternating roughness strip with outer-layer similarity approximately recovered based on the local wall condition. In this case, higher-speed flow occurs above the smooth strip and lower-speed flow occurs above the rough strip, as would be expected in the homogeneous rough or smooth cases. At the other limit when $S/\bar{\delta} \ll 1$, the diameter of the secondary flow, which is limited by S , is confined near the wall and the region above this secondary flow approaches spanwise homogeneity (and global outer-layer similarity based on the spanwise averaged wall-shear stress). Outer-layer similarity is destroyed when $S/\bar{\delta} \approx 1$, where the secondary flow fills the entire boundary layer. In this regime, we also observe a reversal in the isovels as observed in DNS by Chung *et al.* (2018), with high-speed regions occurring above the rough surface and low-speed regions occurring above the smooth patch. These high- and low-speed regions are driven by the local common flow down and up due to the arrangement of secondary flows. Similar regions also exist in the limiting cases $S/\bar{\delta} \gg 1$

and $S/\bar{\delta} \ll 1$, but these regions are localised within the secondary flows. Further from the secondary flows, the flow approaches the spanwise homogeneous condition, as discussed above.

- (b) The size and strength of time-averaged secondary flows are functions of $S/\bar{\delta}$. The size is approximately capped by either the boundary-layer thickness or the roughness patch width (figure 5.5b), while the strength is maximum when $S/\bar{\delta} \approx 1$ (figure 5.10). The latter leads to two possible scenarios for $S/\bar{\delta} \approx 1$ cases: either (i) that these cases induce the strongest streamwise vortices or (ii) the secondary flows are more effectively locked in place for these cases.
- (c) Following the hypothesis in (b), we might consider the secondary flows (instantaneously) as spanwise-locked large-scale turbulent structures. The 1-D spanwise energy spectra demonstrates that there is an interplay between the turbulent fluctuations and the stationary secondary flows, where energy of the first might leech into the latter. The amount of energy being exchanged in this manner is also a function of $S/\bar{\delta}$. In the intermediate cases ($S/\bar{\delta} \approx 1$) where the secondary flows are space filling, the energy deposited into the stationary component and turbulent fluctuations reach maximum and minimum, respectively. This further demonstrate that $S/\bar{\delta}$ is also a measure of the efficacy of roughness wavelengths in locking the secondary flows/turbulent structures in place. The large-scale structures are most effectively locked in place when $S/\bar{\delta} \approx 1$, but not as much in the limits where $S/\bar{\delta} \gg 1$ or $S/\bar{\delta} \ll 1$.
- (d) In addition to being spanwise locked, the secondary flows also exhibit a streamwise unsteadiness. This time-dependent feature is quantifiable and also governed by $S/\bar{\delta}$. It manifests as an outer layer peak in the 1-D streamwise energy spectrograms. The strength of this emergent peak is a function of $S/\bar{\delta}$, and becomes prominent around $S/\bar{\delta} \approx 1$ (figure 7.3), suggesting that the secondary flows meander. Further analysis of the instantaneous turbulent fluctuations fields from PIV also show the tendency of these large-scale structures to lean sideways, most prominently when $S/\bar{\delta} \approx 1$ (figure 9.4). Both 1-D streamwise energy spectra (figure 7.2) and two-point correlations of u' (figure 9.5) suggest an additional periodicity in the flow with a preferred streamwise wavelength of $\lambda_x \approx 3-4\bar{\delta}$ when $S/\bar{\delta} \approx 1$.
- (e) Although the meandering is most prominent when $S/\bar{\delta} \approx 1$, conditional averaging of u' for these intermediate cases and the reference smooth wall produce a

structure (figure 9.13 and 9.14) that are both similar and strongly reminiscent of the streak-vortex instability model proposed in Jeong *et al.* (1997); Waleffe (2001); Flores & Jiménez (2010); Cossu & Hwang (2017) and de Giovanetti *et al.* (2017).

- (f) In the limits where $S/\bar{\delta} \gg 1$ and $S/\bar{\delta} \ll 1$, it is possible to examine the coexistence of secondary flows and the naturally occurring large-scale structures. Following (a), in the cases where $S/\bar{\delta} \gg 1$ both the secondary flows and large-scale structures are δ -scaled. The first are locked about the roughness transition and still carry the imprint of meandering secondary flows, although not as strong as the intermediate cases, while the latter occur farther from the transition and are identical to those observed in the reference smooth wall case (figure 9.17). In the other limit where $S/\bar{\delta} \ll 1$, secondary flows and large-scale structures scale on S and δ , respectively. POD modes are computed and each velocity field snapshot is separated by scales $> S$ or $\leq S$. The conditional average of u' of the larger scales show that the structures of turbulence in both limiting case and the reference smooth wall are identical (figure 9.27). The smaller scales, however, exhibit the imprint of secondary flows leeching into u' , characterised as long meandering spanwise alternating streaks that are locked and aligned with the roughness strips.

10.2 Future work

Secondary flows induced by surface roughness have recently gained traction due to their possible applications in flow control, heat transfer, sediment transport, flood mitigation, and so on. In Chung *et al.* (2018), for example, certain S/δ configurations were shown to increase drag. The present study demonstrates that this $S/\bar{\delta}$ condition corresponds with the case where the secondary flows are strongest and where the meandering of the secondary flows is most pronounced. As a predictive tool, if engineers wish to avoid this drag penalty, or wish to avoid the effects of strong and meandering secondary flows, roughness heterogeneities with these spanwise wavelengths ($S/\bar{\delta} \approx 1$) should be avoided. In other scenarios, where one is looking to locally promote mixing, transpiration or heat transfer, one might wish to promote these secondary flows by deliberately assigning heterogeneities with this wavelength. In addition to that, the present study demonstrates similarities between the secondary flows and the naturally occurring large-scale structures in wall-bounded turbulence. It suggests these secondary flows may in fact be locked

large-scale structures, which opens a possibility of using the secondary flows, artificially generated by surface roughness, as a useful test case to develop and test models for coherent structure formation in wall-bounded flows, and also to conveniently test different control strategies that target large-scale structures. In terms of engineering design, the predictive power of the current study is limited, as only the parameter $S/\bar{\delta}$ has been investigated in detail. In the future, design parameters such as roughness geometry (e.g. k) and Re should be examined.

It is also noted that some questions remain unanswered by the results and analysis presented in this work. First, it is still unclear what causes the secondary flows to meander and, more importantly, why these secondary flows strongly meander at the intermediate case ($S/\bar{\delta} \approx 1$) but not at the limits ($S/\bar{\delta} \gg 1$ and $S/\bar{\delta} \ll 1$). Secondly, the analysis of streamwise unsteadiness of the secondary flows in heterogeneous roughness cases and the naturally occurring turbulent structures in both heterogeneous roughness and the reference smooth wall in chapter 9 is based on the spanwise fluctuation v' as the condition vector (i.e. $v' > 0$ and $v' < 0$ correspond to the sideways leaning of the large-scale structures). This warrants a further inspection of v' component in both heterogeneous roughness and the smooth wall. There is also a possibility conducting the analysis of secondary flows using vorticity fluxes, which recently have been established to reveal many key mechanisms of these flows embedded in turbulence. Finally, it has been demonstrated that these secondary flows share similarities with the naturally occurring large-scale structures. Kevin *et al.* (2019a) suggested that, given the appropriate scaling, both are identical. This opens a possibility of the secondary flows and large-scale structures sharing the same formation mechanism (the first being locked and the latter transient), as suggested by Lee *et al.* (2019). With the currently available data, we are unable to examine these questions further. One element that is missing from the present analysis is any consideration of the convection velocity of the meandering features, or their temporal development. Ultimately, time-resolved data may be able to better address these questions.

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Appendix A

Error analysis

A.1 Hot-wire probe sensitivity

There is a consensus that a hot-wire probe responds differently to all three velocity components. Thus, the effective velocity u_e measured by a wire is not equal to the resultant velocity u_r (the sum of u , v' , and w' vectors). Jørgensen (1971) postulated that the effective velocity u_e is determined by the three velocity components and the wire's sensitivity to these components

$$u_e^2 = u^2 + a_1^2 v'^2 + a_2^2 w'^2 \quad (\text{A.1})$$

For a single-sensor hot-wire probe, the u and w' velocity components are normal to the wire (see figure A.1), while v' is tangential to the wire. This type of wire is almost completely insensitive to the tangential component v' , and thus the effect of this component is negligible. For a standard plated hot-wire, the typical values of the yaw and pitch coefficients, a_1 and a_2 , are 0.2 and 1.05, respectively (Bruun,

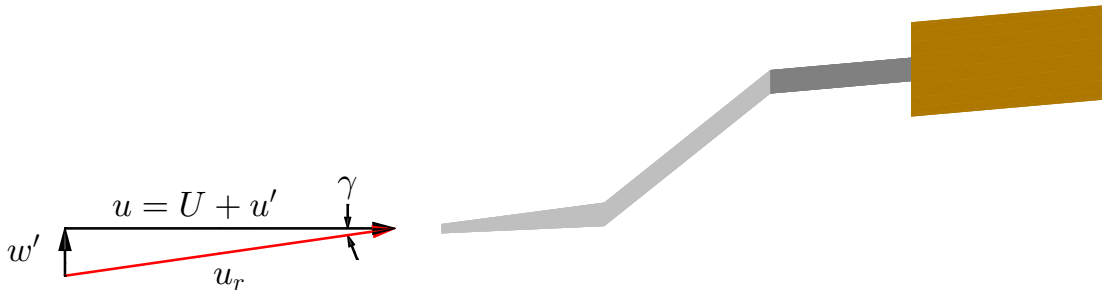


Figure A.1: Illustration of single-wire boundary layer type probe and the corresponding velocity components, where γ is the pitch angle and u_r is the resultant velocity vector.

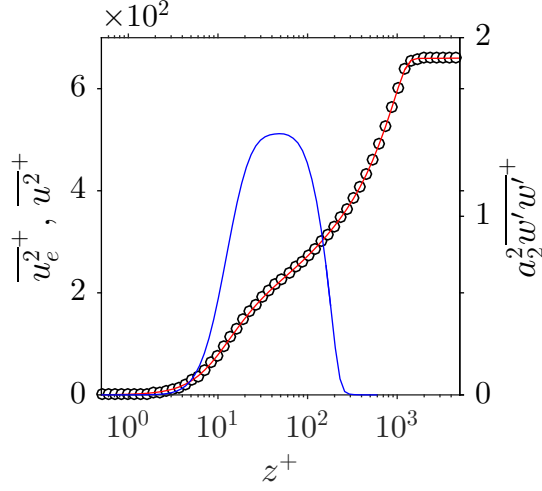


Figure A.2: Smooth-wall boundary layer profiles at $Re_{\theta_s} = 6500$ from the DNS of Sillero *et al.* (2014). The left hand side ordinate: time-averaged effective velocity $\overline{u_e^2}^+$ ($\circ$, data are downsampled for clarity) and streamwise velocity $\overline{u^2}^+$ (—). The right hand side ordinate: variance of w' multiplied by the pitch coefficient, $a_2^2 \overline{w'w'}^+$ (—).

1995), and equation (A.1) is reduced to

$$u_e^2 = u^2 + a_2^2 w'^2 \quad (\text{A.2})$$

$$= \|u_r\|^2 (\cos \gamma + a_2^2 \sin \gamma) \quad (\text{A.3})$$

where γ is the pitch angle (figure A.1) and $\|u_r\|$ is the magnitude of resultant velocity. For turbulent boundary layers, γ decreases with the increasing distance from the wall and the magnitude of w' is relatively small compared to u . Taking the time average of equation (A.2)

$$\overline{u_e^2} = \overline{u^2} + a_2^2 \overline{w'w'} \quad (\text{A.4})$$

$$= U^2 + \overline{u'u'} + a_2^2 \overline{w'w'} \quad (\text{A.5})$$

Figure A.2 shows the smooth-wall boundary layer profiles from the DNS of Sillero *et al.* (2014) at $Re_{\theta_s} = 6500$, comparable to that of the reference smooth wall case SW-2 in this study (see table 3.1). All terms in equation (A.4) are shown: $\overline{u_e^2}^+$, $\overline{u^2}^+$ (see ordinate on the left hand side), and $a_2^2 \overline{w'w'}^+$ (ordinate on the right hand side). The velocity profiles illustrate how the w' component measured by the wire ($a_2^2 \overline{w'w'}^+$) is smaller than $\overline{u_e^2}^+$ and $\overline{u^2}^+$ by the $O(10^{-2})$, and also that $\overline{u_e^2}^+$ collapses to the profile of $\overline{u^2}^+$.

In addition to the scale of u compared to w' , it has also been observed in Dahm & Rasmussen (1969) that the measured velocity is relatively unaffected by γ if the two

prongs of the wire probe are spaced by $l_s \geq 2$ mm and the length of the prongs $l_p \geq 8$ mm (see figure 3.5b). The Dantec 55P05 probe used in this study fits this criteria since $l_s = 3$ mm and $l_p = 8$ mm. Studies (see Bruun, 1995) have also observed that the variation in a_2 with γ is small and a_2 further decreases as the velocity increases (it should be noted the measurements in this study are conducted at relatively high Re , $\delta_s^+ = 2030$).

A.2 Hot-wire calibration

Due to the number of test cases (table 3.1) and the length of each hot-wire measurement for these cases, all measurements in this study are conducted only once. As such, we cannot check for the repeatability of these measurements. Uncertainties of the hot-wire measurements, however, can be estimated from the time series of hot-wire output voltage E and the freestream velocity U_∞ measured by the Pitot tube. Figures A.3(a,b) show an example of these time series. Here, the fluctuation of E and U_∞ about the time-average $\overline{E}$ and $\overline{U_\infty}$ are plotted as a function of measurement time t . The time series is sampled every 0.5 s from a wall-normal measurement, during pre-calibration at $U_\infty \approx 15$ ms⁻¹. The fluctuation of the time series data in figures A.3(a,b) is $\pm 0.2\%$ about its time average for both E and U_∞ . For the pre- and post-calibration curves shown in figure 3.6(a), the uncertainty of each data point is calculated from its time-series of E and U_∞ . It should be noted, however, that the time series of E and U_∞ shown here are not measurement errors, they are a measure of freestream intensity in the tunnel.

Figures A.4(a,b) shows an example of U_∞ and freestream voltage E_∞ time series, sampled every 0.5 s from a spanwise-wall-normal measurement. The fluctuation of the time series data in figures A.4(a,b) is $\pm 0.2\%$ about its time average for both E and U_∞ . For the trajectory of E_∞ shown in figure A.4(c), recorded throughout the measurement, the uncertainty of each data point is calculated from its time-series of E_∞ , as in (b). For the hot-wire voltage drift in the freestream ΔE shown in figure 3.6(b), the uncertainty of each data point is calculated from its time-series of E_∞ . The time-averaged uncertainty of E_∞ in figure A.4(c) is ± 15 mV ($\pm 0.2\%$ of U_∞). It should be noted, however, that the time series of E_∞ and U_∞ shown here are not measurement errors, they are a measure of freestream intensity in the tunnel.

Figure A.5 shows the convergence of HWA time-series signal of the reference rough-wall case RW-2. This case has the longest sampling time (120 s) over all

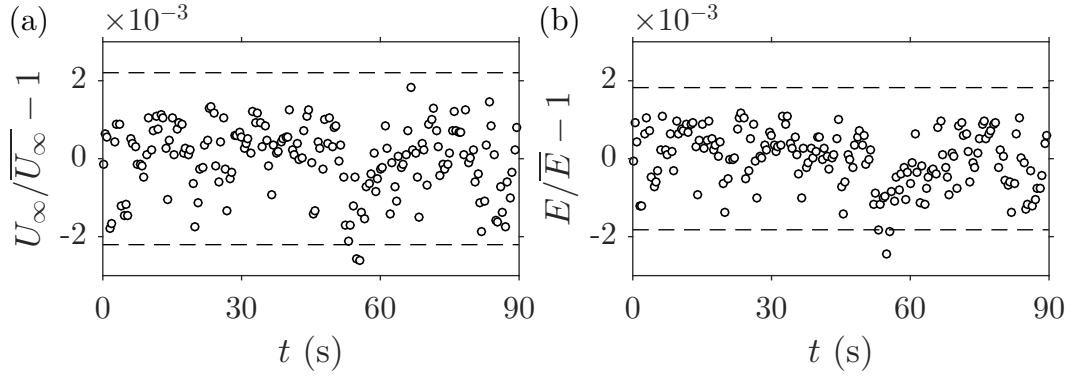


Figure A.3: Time-series of (a) U_∞ and (b) hot-wire output voltage E , fluctuation about its time average $\overline{E}$ from a wall-normal measurement, pre-calibration at $U_\infty \approx 15 \text{ ms}^{-1}$. U_∞ is measured by the Pitot tube. Datapoints ($\circ$) are the time-series signal averaged every 0.5 s. Dashed lines (---) show the uncertainties of the measurement due to freestream intensities of the tunnel (quoted as 0.3%).

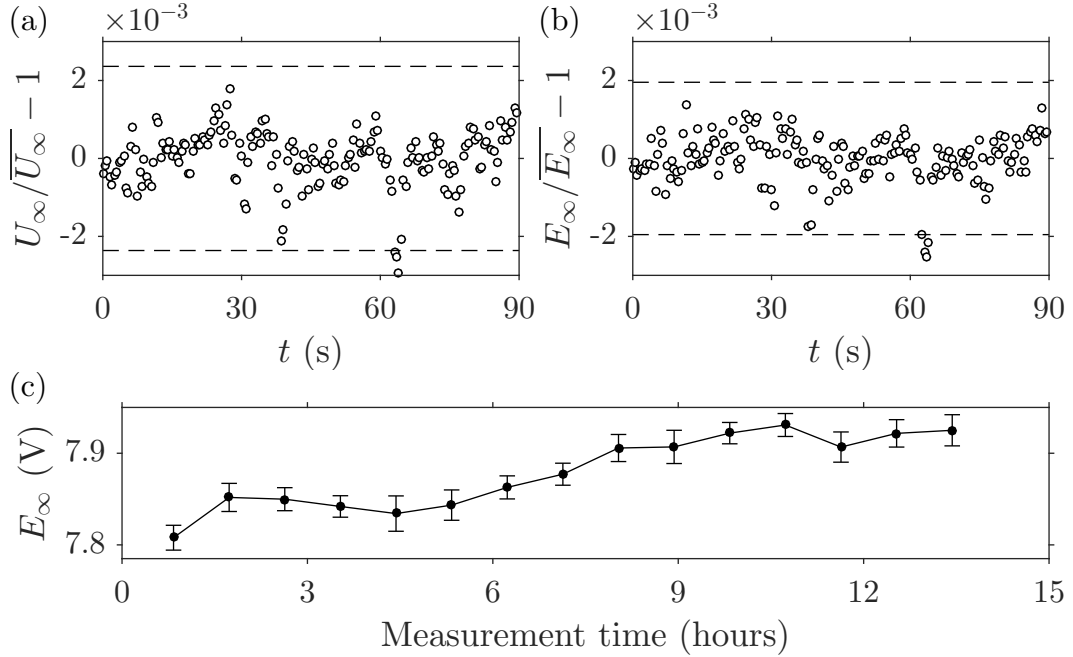


Figure A.4: Time-series of (a) U_∞ and (b) hot-wire freestream voltage E_∞ , fluctuation about its time average $\overline{E_\infty}$ from spanwise-wall-normal measurement. E_∞ is sampled throughout the measurement, U_∞ is measured by the Pitot tube. Datapoints ($\circ$) are the time-series signal averaged every 0.5 s. Dashed lines (---) show the uncertainties of the measurement due to freestream intensities. (c) Trajectory of freestream voltage E_∞ , recorded throughout a spanwise-wall-normal grid measurement. Error bars show the uncertainties of the measurement due to freestream intensities of the tunnel (quoted as 0.3%), as in (b).

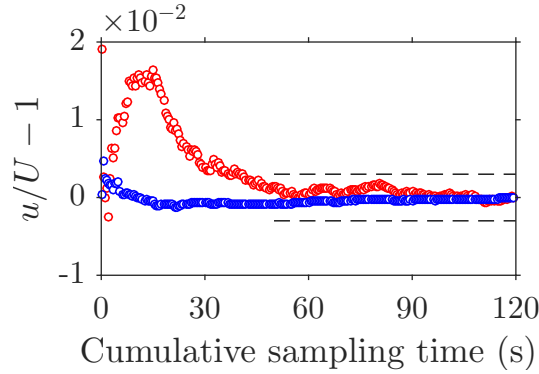


Figure A.5: Time-series of u of the reference rough-wall case RW-2, $\circ$: u when $\overline{u'u'}$ is maximum ($z^+ \approx 250$) and $\circ$: u at the freestream, $U \approx U_\infty$ as a function of cumulative sampling time. Dashed lines (---) are the quoted freestream intensity of the tunnel, 0.3%.

other cases, both reference smooth wall and heterogeneous roughness cases, and the thickest boundary layer (see table 3.1). Time-series of u (normalised by its average U) is shown for two wall-normal locations: the most turbulent (maximum $\overline{u'u'}$, $\circ$ at $z^+ \approx 250$) and at the freestream ($\circ$). The datapoints in figure A.5 are averaged for every 0.5, 1, 1.5 s, and so on, shown as a function of the cumulative sampling time in x -axis. As expected, the signal converges much faster in the freestream, but for the most turbulent scenario, it takes approximately 50 s for the signal to converge (the dashed lines in figure A.5 show the quoted freestream intensity of the tunnel, 0.3%). For all HWA measurements in this study (reference rough and smooth walls, spanwise heterogeneous surfaces), the sampling time is $t > 50$ s.

A.3 SPIV measurements

Due to the number of test cases (table 3.1) and the time required to conduct SPIV measurements and image processing, all SPIV measurements in this study are conducted only once. As such, we cannot check for the repeatability of these measurements. Uncertainties of the SPIV measurements, however, can be estimated from the obtained velocity field. For all cases in table 3.1, it is expected that the time-averaged velocity component $U = U_\infty$ and $V = W = 0$ in the freestream. Uncertainties of SPIV measurements are calculated based on the deviation from these values in the freestream across the span.

Figure A.6 show the contours of mean velocities of the reference smooth-wall case SW-2. The contours are normalised by U_∞ measured by the Pitot tube, simultaneously with the SPIV measurements. Measurement errors are visible in these figures,

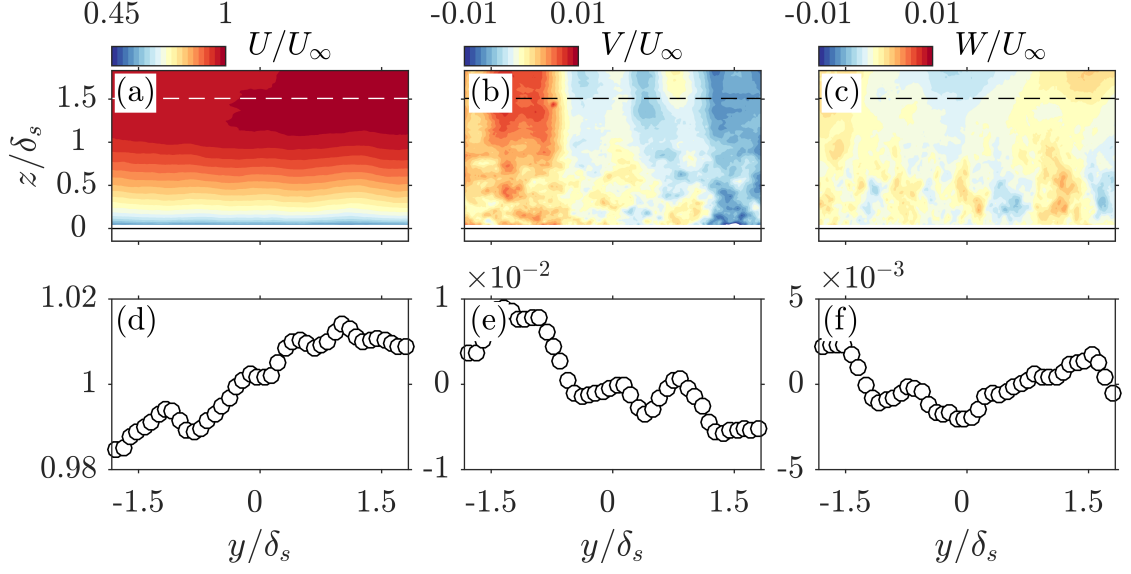


Figure A.6: Mean velocities of the reference smooth-wall case SW-2: (a,d) U , (b,e) V , and (c,f) W . Contours are obtained from SPIV measurements. U_∞ is measured by the Pitot tube, simultaneously with SPIV measurements. Figures (d–f) show the mean velocities measured at the freestream wall-normal location, $z/\bar{\delta} = 1.5$ (see the dashed lines, ---, in a–c).

especially in figure A.6(a), where there is a non-zero spanwise velocity gradient in the freestream of the U field, and in figure A.6(b,c), where non-zero V and W are observed. Uncertainties are quantified at a freestream wall-normal location, here defined as $z/\delta_s = 1.5$. Figure A.6(d–f) show the mean velocities of the reference smooth-wall case SW-2 at this wall-normal location ($z/\delta_s = 1.5$) as a function of y . The uncertainties of each mean velocity component are as follows: U ($\pm 2\%U_\infty$), V ($\pm 1\%U_\infty$), W (less than $\pm 0.5\%U_\infty$). For the subsequent analysis of the velocity field obtained from SPIV measurements in this study, it is assumed that the uncertainties originate from the uncertainty of the mean velocity at a freestream wall-normal location ($z/\delta_s = 1.5$ or $z/\bar{\delta} = 1.5$) for each test case in table 3.1 (when SPIV data are available). When necessary, propagation of errors is assumed: that the uncertainty of a variable is twice the uncertainty of its square root. It should also be noted that due to the discrepancies in U_∞ measured by SPIV and the Pitot tube (see figure A.6d), U_∞ for SPIV data is defined as the freestream velocity obtained from PIV cross-correlation instead of the Pitot tube.

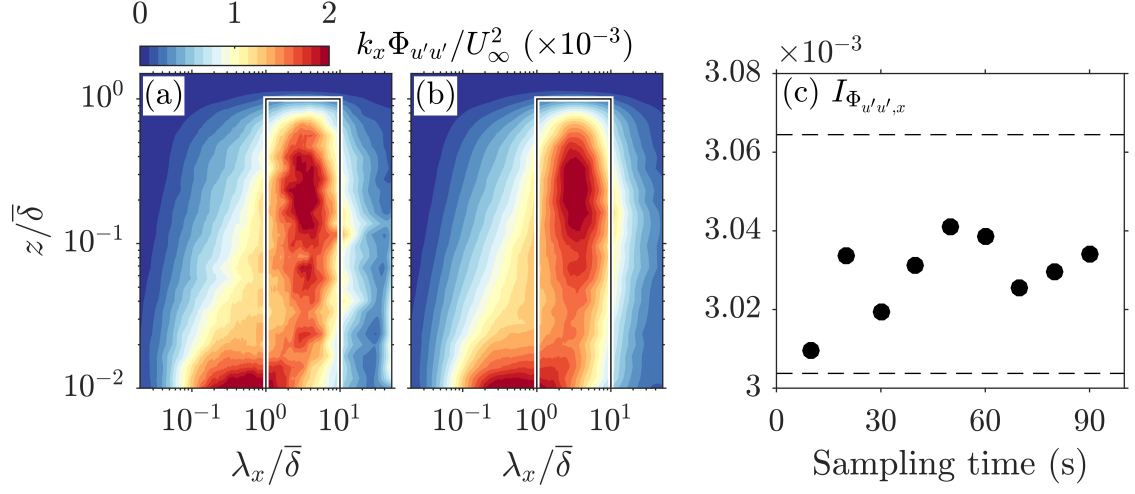


Figure A.7: Contours of premultiplied energy spectra $k_x \Phi_{u'u'}$ of case SR50 ($S/\bar{\delta} = 0.62$) at the centre of the mean secondary flows, $y/S = 0.5$. Contours are obtained from HWA measurements, shown as a function of λ_x and z . In (a), only the first 10 s of the HWA time series are used to compute the energy spectra, while in (b), 90 s. (c) Integrated energy spectra $I_{\Phi_{u'u'},x}$ (see equation 7.3) over the area marked by the black boxes in (a,b), as a function of the sampling time in s. Dashed lines (---) denote the $\pm 1\%$ deviation from the $I_{\Phi_{u'u'},x}$ calculated with the full sampling time.

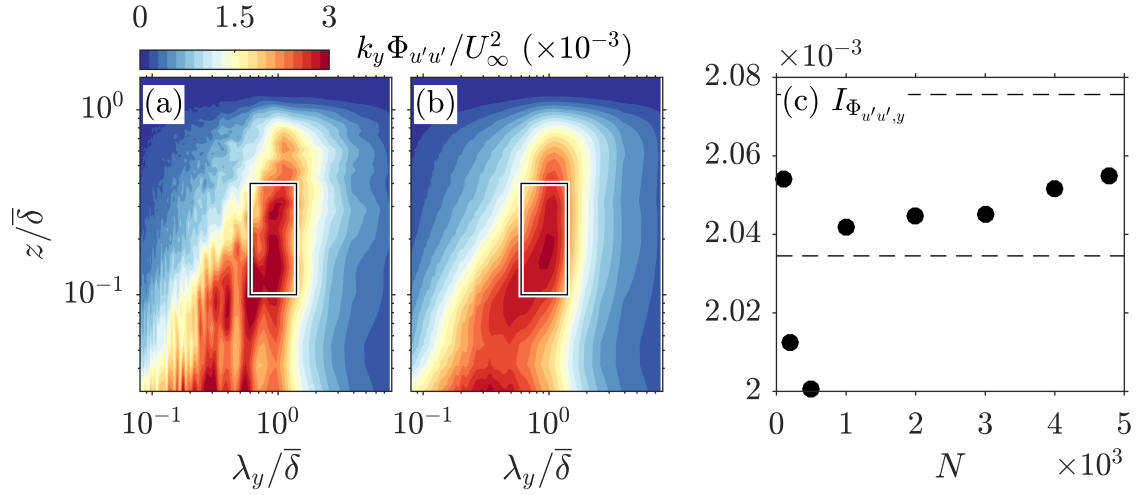


Figure A.8: Contours of premultiplied energy spectra $k_y \Phi_{u'u'}$ of case SR50 ($S/\bar{\delta} = 0.62$). Contours are obtained from SPIV measurements, shown as a function of λ_y and z . In (a), only 100 PIV snapshots are used to compute the energy spectra, while in (b), 4800. (c) Integrated energy spectra $I_{\Phi_{u'u'},y}$ (see equation 7.7) over the area marked by the black boxes in (a,b), as a function of the number of PIV snapshots N . Dashed lines (---) denote the $\pm 1\%$ deviation from the $I_{\Phi_{u'u'},y}$ calculated with all available PIV snapshots ($N = 4800$).

A.4 Energy spectra

Figures A.7(a,b) show the contours of premultiplied energy spectra $k_x \Phi_{u'u'}$ of case SR50 ($S/\bar{\delta} = 0.62$) at the centre of the mean secondary flows, $y/S = 0.5$. This case has the strongest secondary flows and $y/S = 0.5$ is the location of the energetic peak associated with the secondary flows, marked by ① in figure 7.2(e). To measure convergence, energy spectra is computed with various hot-wire sampling time, ranging from 10 to 90 s. In figure A.7(a), $k_x \Phi_{u'u'}$ is computed with only the first 10 s of the hot-wire time series, while in (b), 90 s. Clearly, there is a convergence issue (shown in figure A.7a,b) when the sampling time is too short. However, this issue is masked by integration over λ_x and y . For the type of analysis conducted in this study (see equation 7.2, 7.3 and figure 7.3), the magnitude of integrated energy spectra $I_{\Phi_{u'u'},x}$ over the area described in the black boxes in figure A.7(a,b) (see also equation 7.2, 7.3 and figure 7.2) only varies within $\pm 1\%$ from its value when the full sampling time is used, regardless of the sampling time used for convergence check in figures A.7(c).

Similarly, convergence in premultiplied energy spectra can also be computed from those obtained from SPIV measurements. Figures A.8(a,b) show the contours of premultiplied energy spectra $k_y \Phi_{u'u'}$ of case SR50 ($S/\bar{\delta} = 0.62$). Here, the number of PIV snapshots N used to compute the spectra are varied between 100 to 4800. In figure A.8(a), $k_y \Phi_{u'u'}$ is computed with 100 snapshots, while in (b), $N = 4800$. Similar to the premultiplied spectra $k_x \Phi_{u'u'}$ from the HWA measurements, there is a convergence issue (shown in figure A.8a,b) due to the lack of number of snapshots. The integrated energy spectra $I_{\Phi_{u'u'},y}$ is computed over the area marked by black boxes in figure A.8(a,b), similar to that in figure 7.6 and equation (7.7). Figure A.8(c) shows the magnitude of $I_{\Phi_{u'u'},y}$ computed over a range of N , where the deviation from the magnitude computed using all available PIV snapshots ($N = 4800$) is within $\pm 1\%$ when $N \geq 1000$.

Appendix B

1-D Spanwise energy spectra and co-spectra (from SPIV)

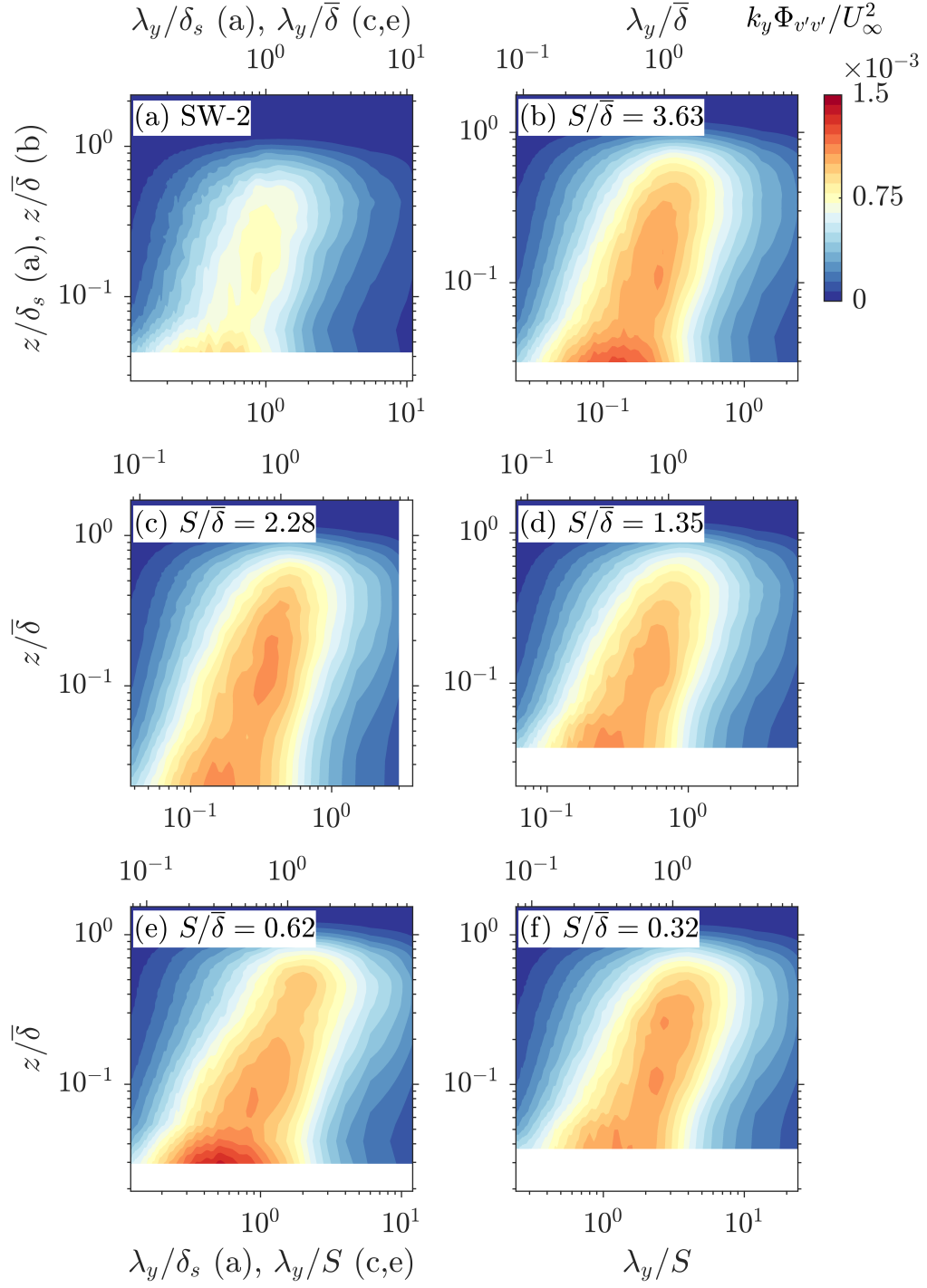


Figure B.1: Contours of premultiplied energy spectra $k_y \Phi_{v'v'}$ as a function of λ_y and z for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Data are obtained from SPIV measurements.

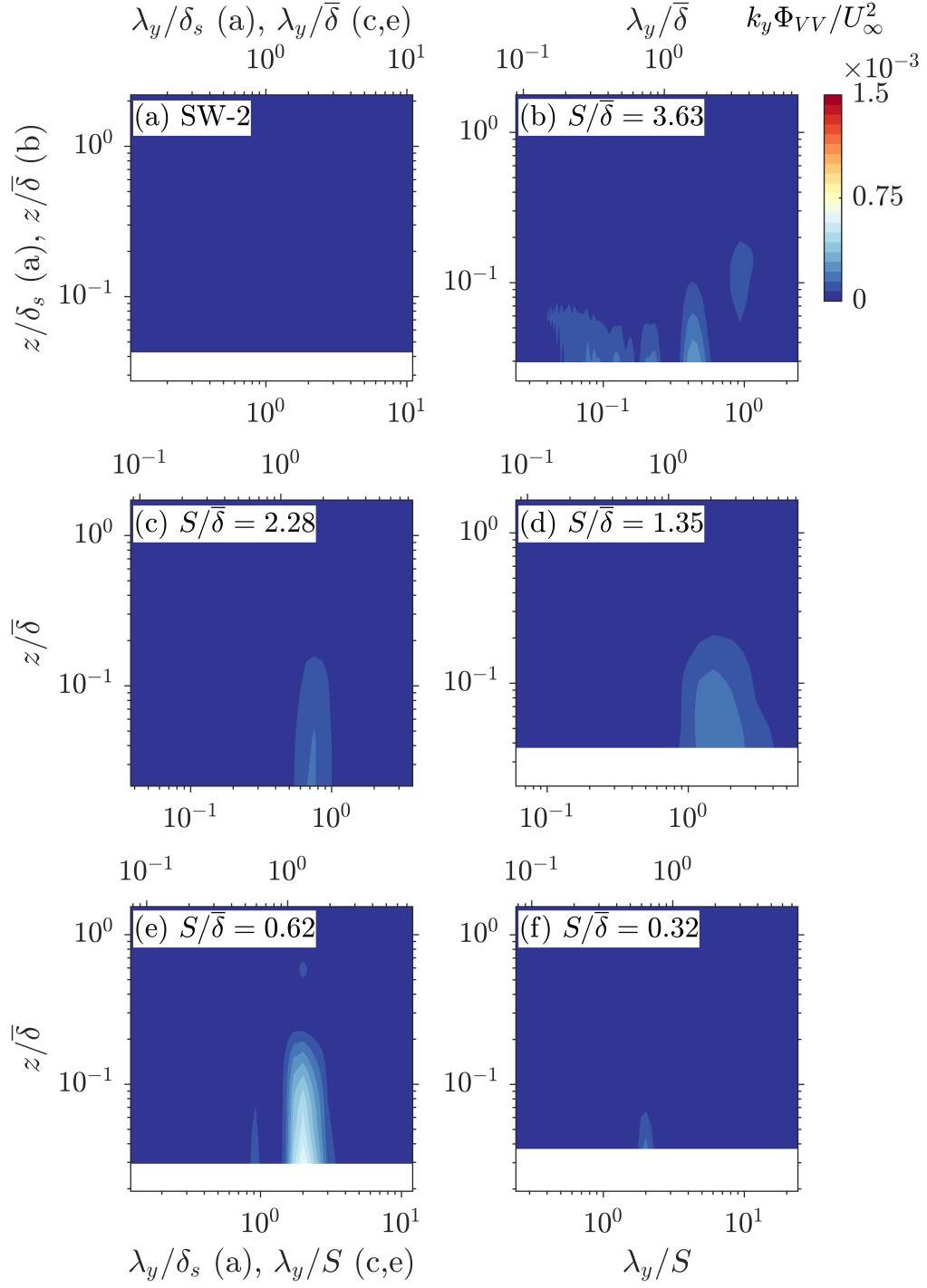


Figure B.2: Contours of premultiplied energy spectra $k_y \Phi_{VV}$ as a function of λ_y and z for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Data are obtained from SPIV measurements.

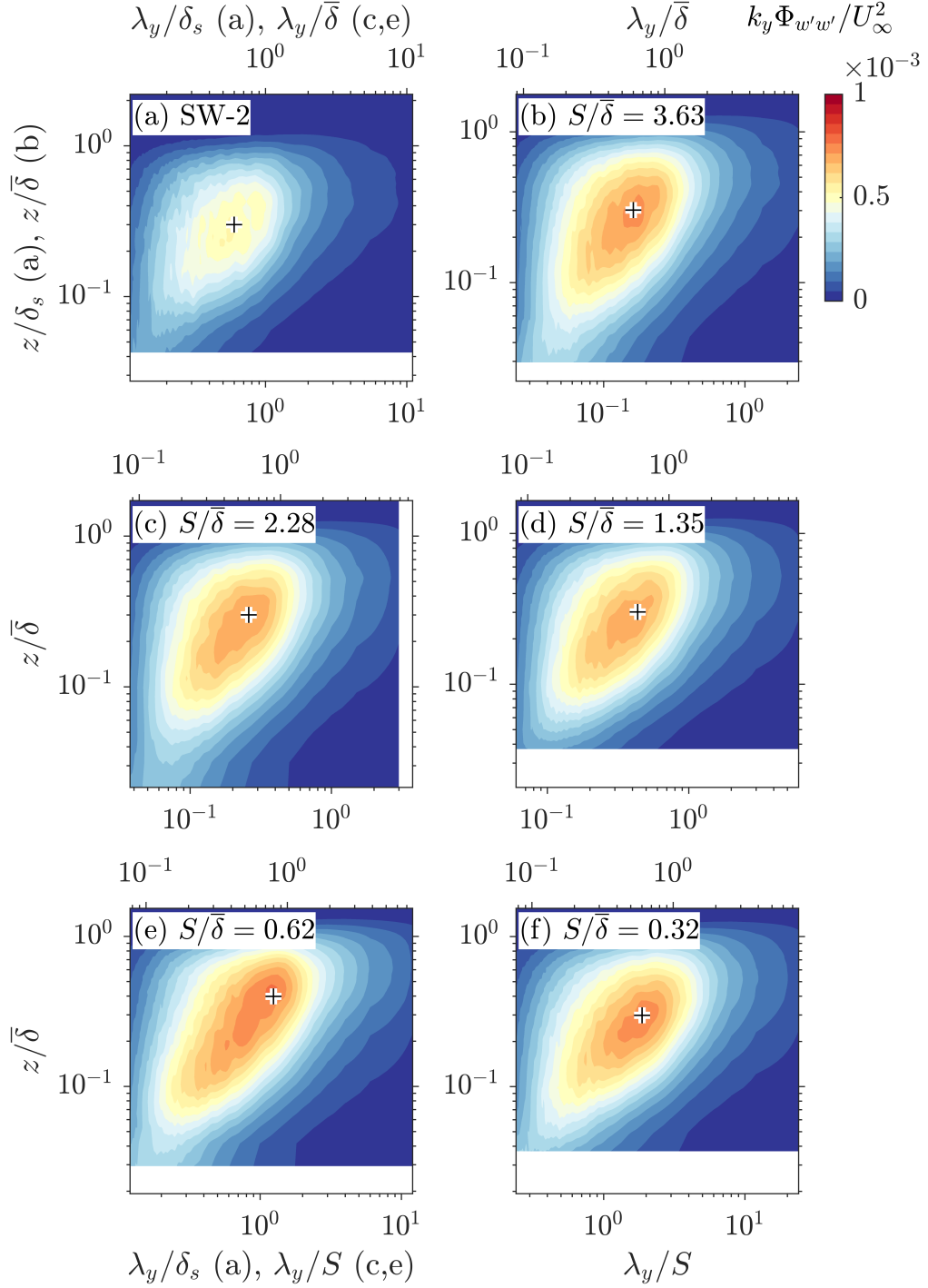


Figure B.3: Contours of premultiplied energy spectra $k_y \Phi_{w'w'}$ as a function of λ_y and z for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Data are obtained from SPIV measurements. '+' marks the peak of energy spectra in the outer layer.

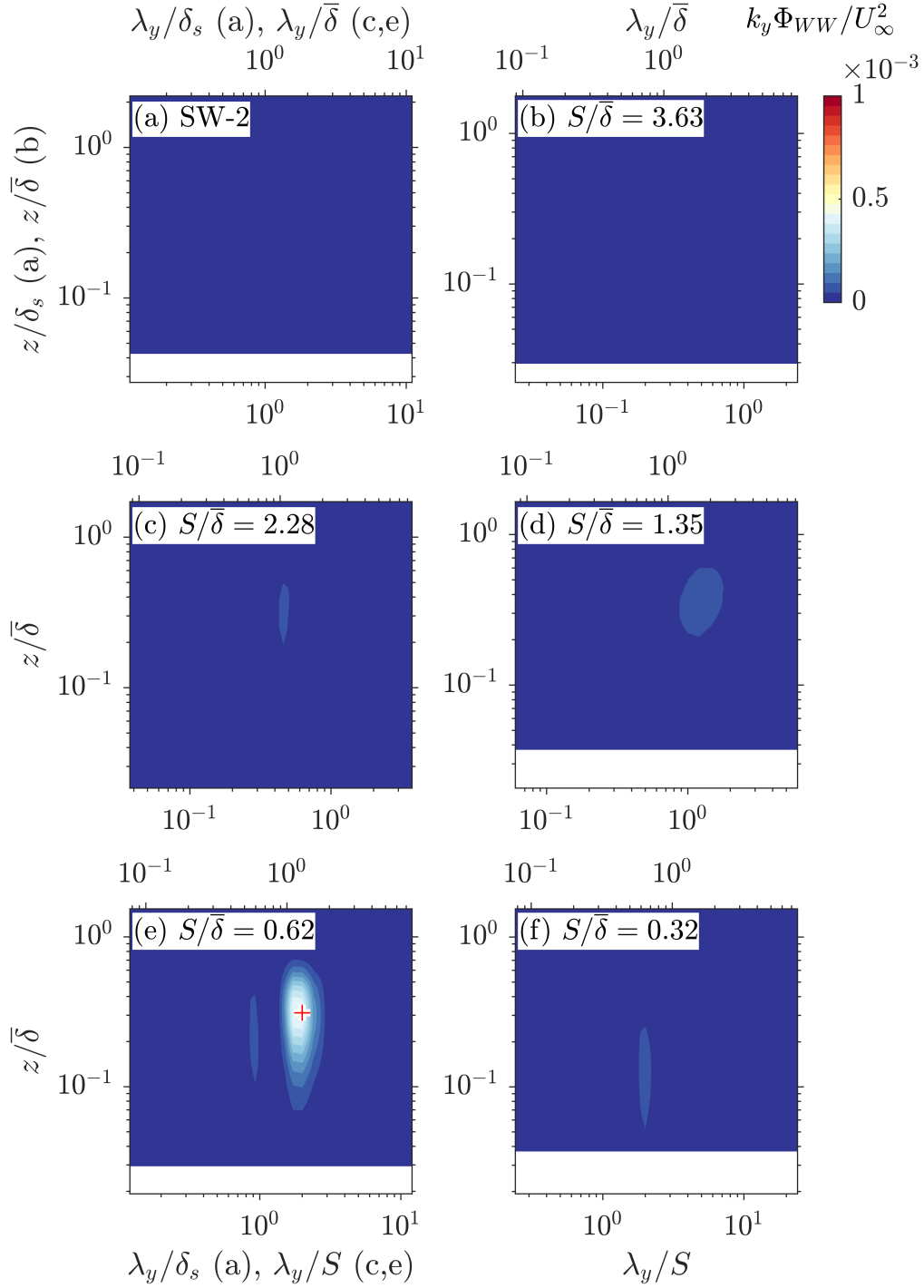


Figure B.4: Contours of premultiplied energy spectra $k_y \Phi_{WW}$ as a function of λ_y and z for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Data are obtained from SPIV measurements. ‘+’ marks the peak of energy spectra in the outer layer related to the spanwise wavelengths shown in red arrows in figure 7.8.

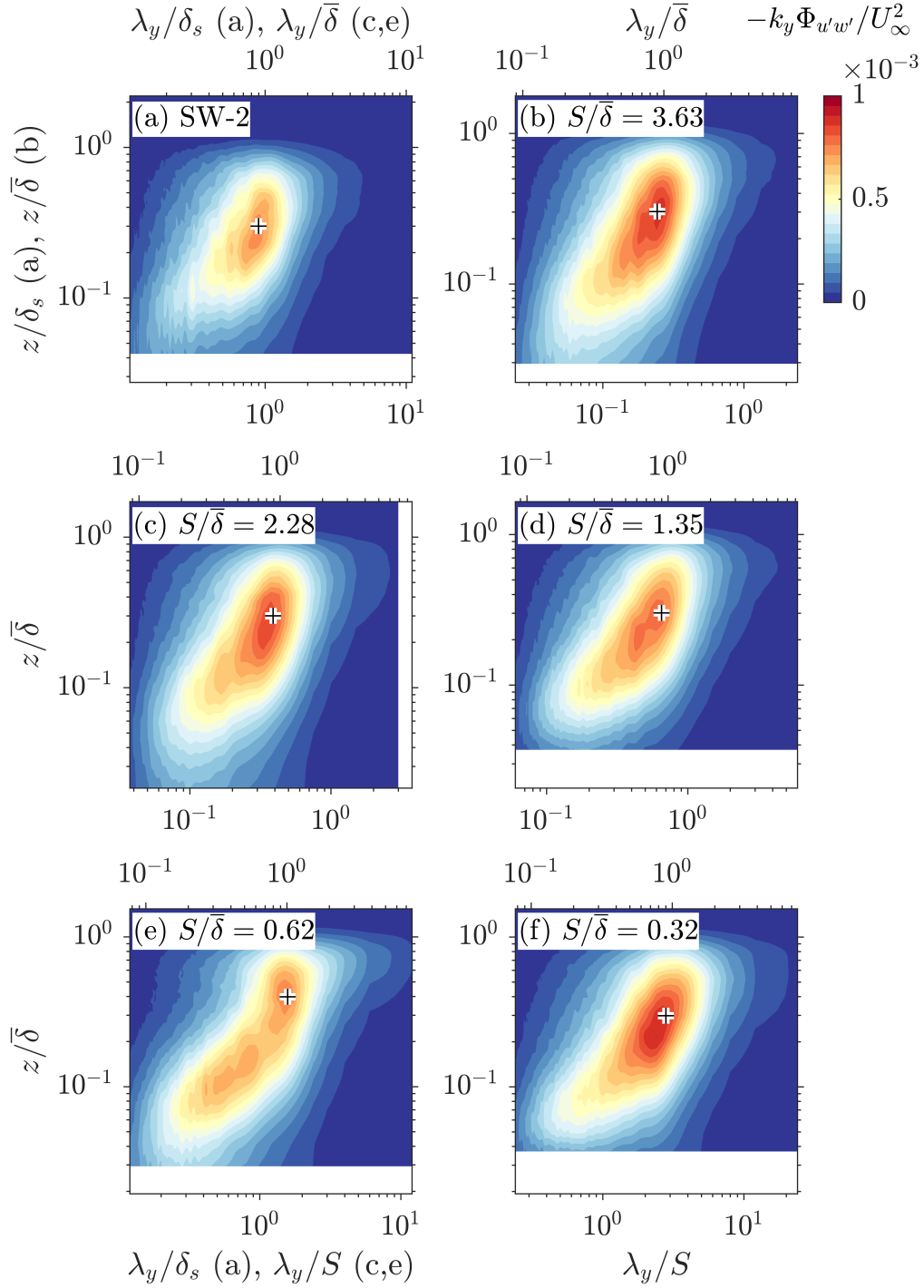


Figure B.5: Contours of premultiplied energy spectra $-k_y \Phi_{u'w'}$ as a function of λ_y and z for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Data are obtained from SPIV measurements. '+' marks the peak of energy spectra in the outer layer.

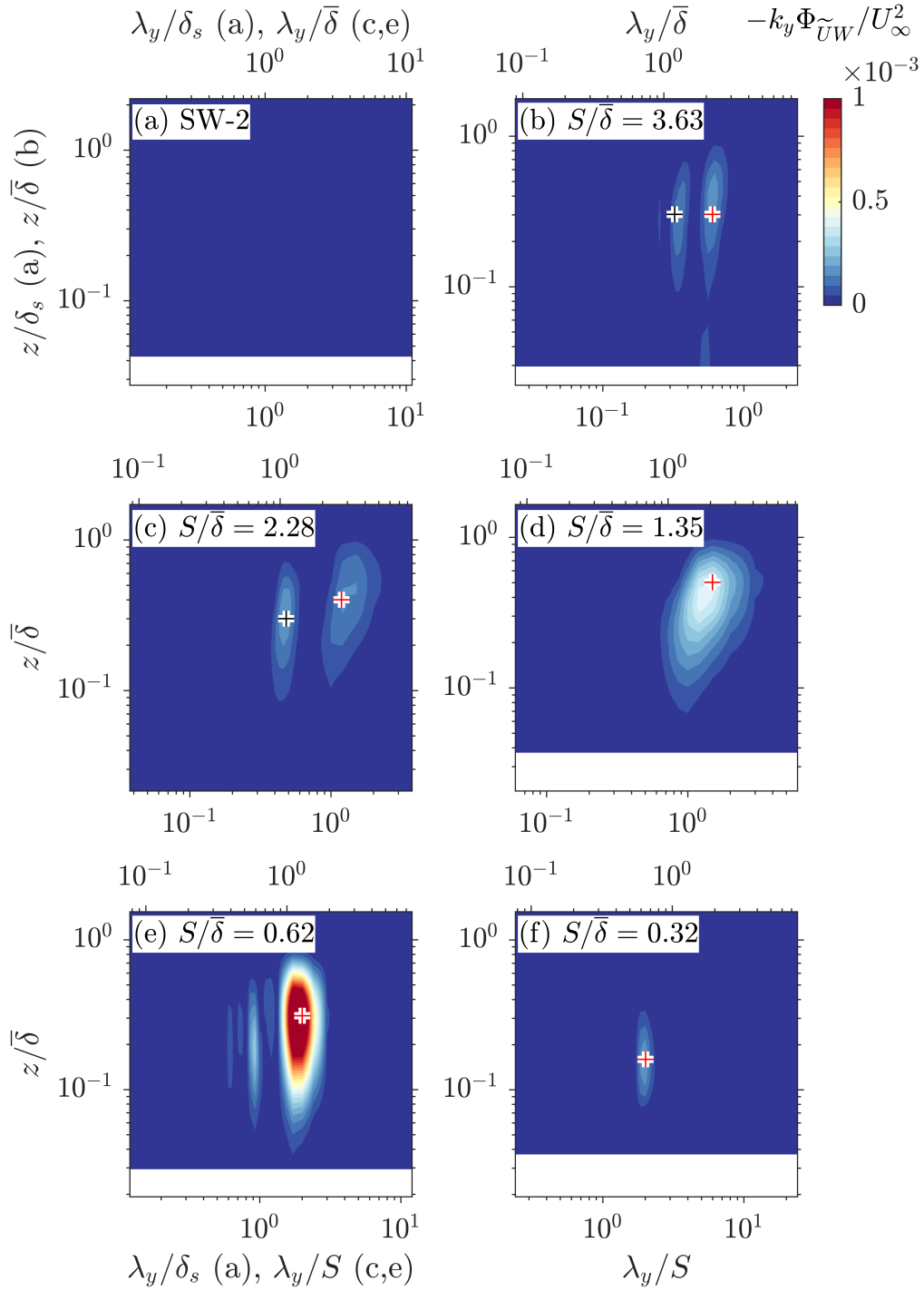


Figure B.6: Contours of premultiplied energy spectra $-k_y \Phi_{\tilde{U}W}$ as a function of λ_y and z for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Data are obtained from SPIV measurements. ‘+’ and ‘+’ mark the peaks of energy spectra in the outer layer related to the spanwise wavelengths shown in black and red arrows in figure 7.8.

Appendix C

2-D Energy spectra and co-spectra

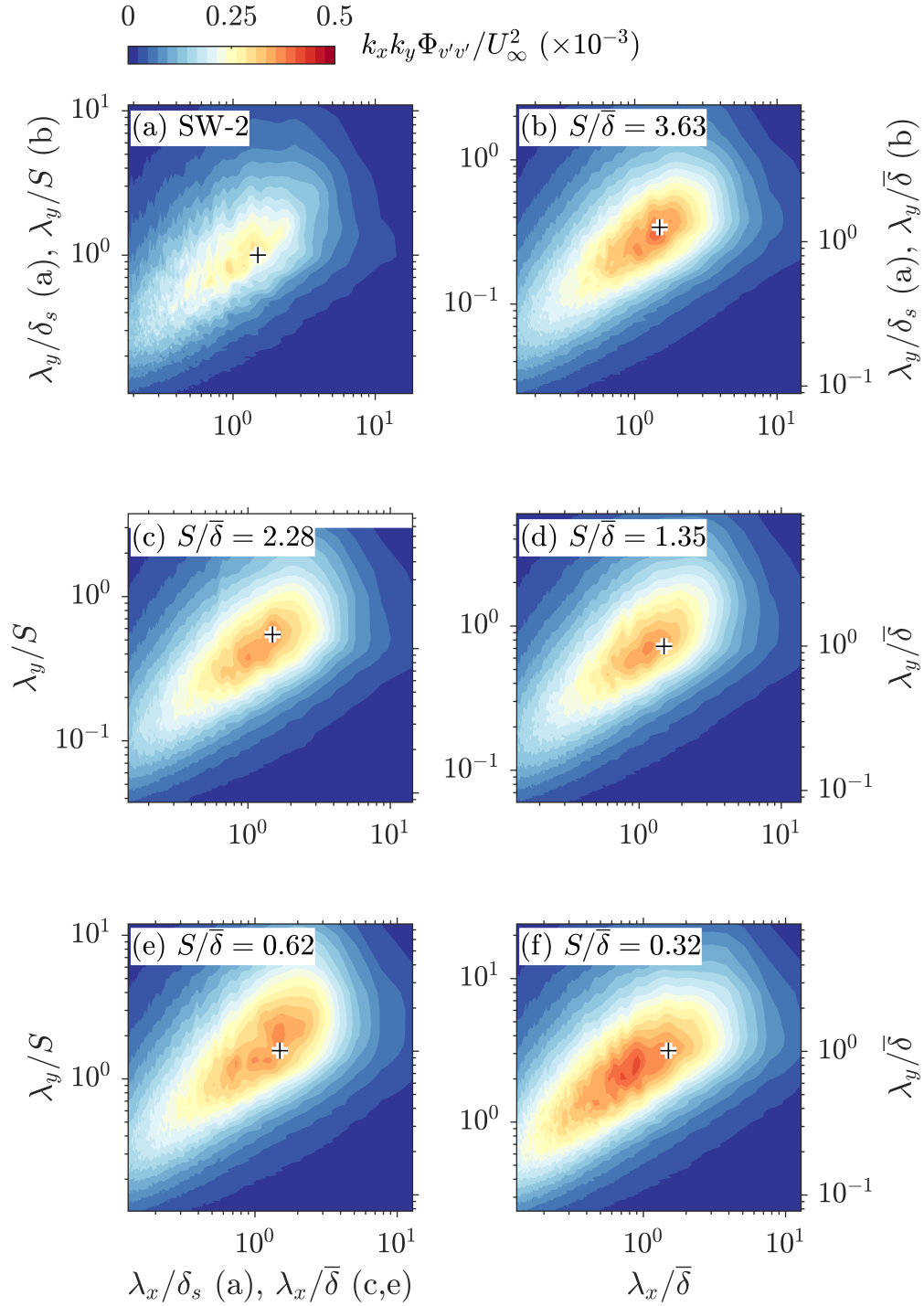


Figure C.1: Contours of premultiplied energy spectra $k_x k_y \Phi_{v'v'}$ as a function of λ_x and λ_y for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Data are obtained from WPPIV measurements. '+' marks the peak of energy spectra in the outer layer.

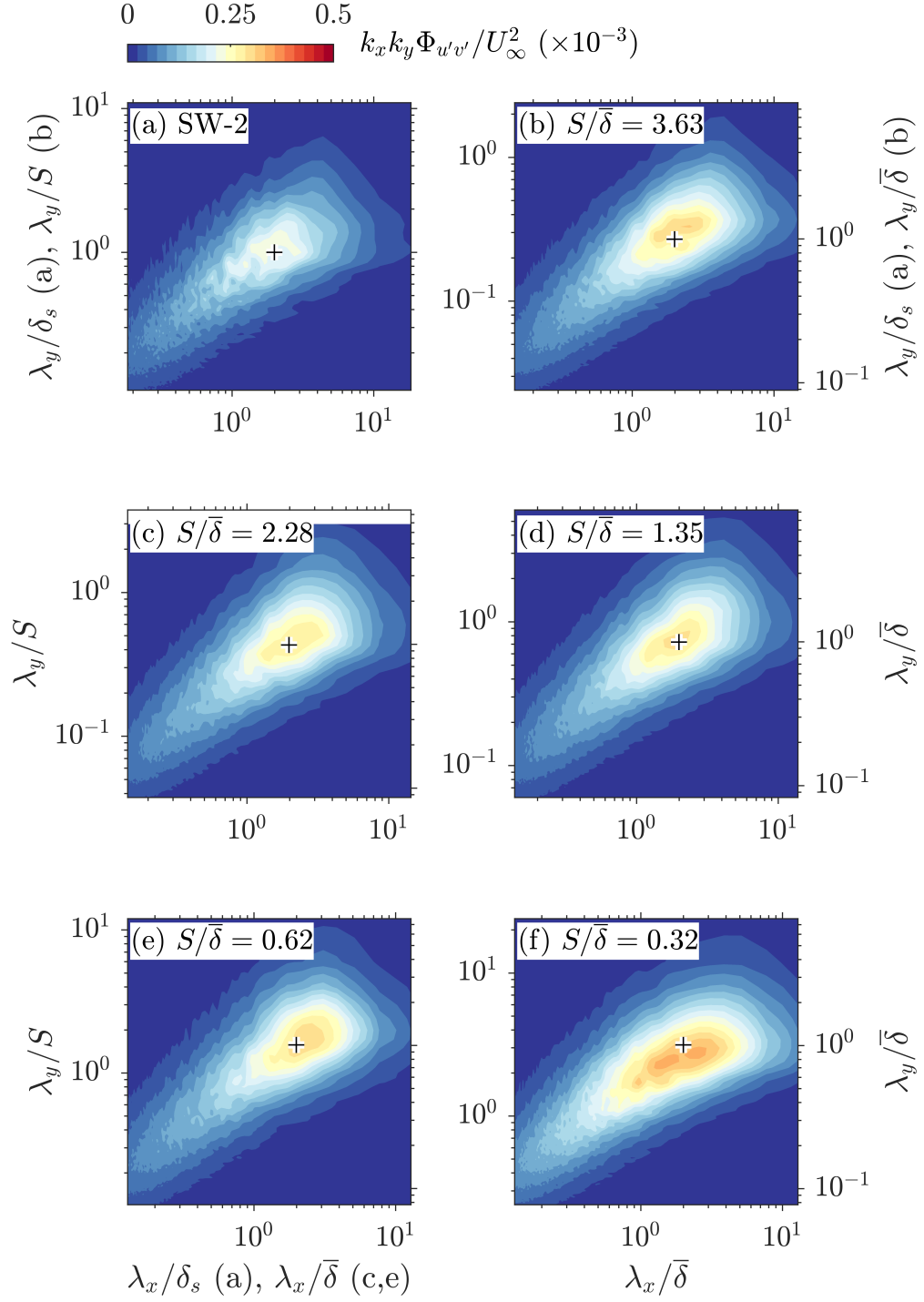


Figure C.2: Contours of premultiplied energy spectra $k_x k_y \Phi_{w'w'}$ as a function of λ_x and λ_y for (a) reference case SW-2 and SR cases: (b) SR250-2, (c) SR160, (d) SR100, (e) SR50, (f) SR25. Data are obtained from WPPIV measurements. '+' marks the peak of energy spectra in the outer layer.