Overcoming overshoot performance limitations of linear systems with reset control ⋆

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Abstract

It is well-known that for a class of minimum-phase relative degree one linear-time-invariant (LTI) systems, with a unit feedback control structure, overshoot necessarily happen if the plant transfer function has poles at origin or unstable poles. This work aims to overcome this overshoot performance limitation (OPL) by using a novel reset controller, which has a generalized first order reset element (GFORE) structure. By tuning parameters of this reset controller carefully, the non-overshoot performance can be ensured. Furthermore, the implementation of the proposed reset controller with a high-pass filter is provided. Parameter tuning guidelines are also provided and, finally, the proposed design is verified with a simulation example.

Key words: Reset controller; performance limitation; non-overshoot; first order reset element

1 Introduction

The concept of reset control was first proposed by Clegg in [1] who introduced the so-called Clegg integrator: a linear integrator and a reset mechanism which resets the state of the integrator to zero when its input and output have opposite sign. This reset mechanism provides a describing function of the Clegg integrator similar to the frequency response of a linear integrator but with only 38.1° phase lag. It was suggested by Clegg that this property could overcome some of the fundamental performance limitations of linear control, but it was not until 1990s that this idea was confirmed by an example in [2]. The Clegg integrator was generalized in [3], where the first order reset elements (FORE) was introduced together with a systematic design procedure. The early work on reset control was summarized in [4].

Since the late 1990s, there has been a renewed interest in reset control systems. Various stability analysis and transient performance improvement results are reported in the literature [5–8], and several experimental applications of reset control such as regulation of exhaust gas recirculation (EGR) valve [9], process control [10] and networked control systems [11] are also presented. The majority of the above-mentioned literature has focused on the stability analysis of the reset control systems as resetting might lead to unstable performance, and the "zero crossing" reset models were used.

In [12], different from FORE, a new reset model was proposed under hybrid systems framework [13], and a clock variable was introduced to avoid the Zeno phenomenon. This model allows flows and jumps on more complicated closed sets, which results in more favourable properties of reset systems [14–19]. For instance, the $\mathcal{L}_2$ stability analysis of reset systems was first presented in [14], exponential stability analysis results with explicit Lyapunov functions and piecewise quadratic Lyapunov functions were given in [17]. Recently, a novel FORE model was proposed in [15], and for this latter a strictly decreasing Lyapunov functions at jumps can be constructed. The performance improvement of reset control systems has been addressed in [18], in [19] $\mathcal{L}_2$ disturbance attenuation performance of linear systems was reported. More analysis and synthesis results for reset control systems can be found in the recent survey paper [20].

Although reset control is well-known for its ability to overcome the performance limitation of linear controller for linear systems, as demonstrated in a few examples

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[2],[21],[22], there is no systematic procedures to design reset laws with a specific objective.

This paper focuses on proposing new reset control laws that can overcome the overshoot performance limitation (OPL) without sacrificing the stability. This is motivated from engineering applications in which overshoot is unacceptable. Classic examples in flight control are the landing and taking off maneuvers, in which an overshoot of the pitch angle can have a disastrous effect. In order to overcome OPL, the tracking error cannot change its sign during the transient response. Hence the reset control law will depend on the tracking error $e(t)$ and its derivative $\dot{e}(t)$. This leads to a novel reset control algorithm which is a generalized form of FORE. The main result of this paper shows that through a careful selection of the parameters, the proposed reset controller can overcome OPL and achieve semi-global practical asymptotic stability. When the derivative of the tracking error is not directly measurable, a high-pass filter is introduced to approximate $\dot{e}(t)$. The tuning guideline of the proposed reset controller, as well as the design trade-off are also provided. The effectiveness of the proposed reset controller is demonstrated through a numerical example.

The rest of this paper is organized as follows. Preliminaries and problem formulation are given in Section 2. A GFORE is proposed in Section 3. Section 4 presents the main results including practical stability and non-overshoot performance, and the implementation of the proposed reset controller. Section 5 presents a systematic parameter tuning procedure. Section 6 provides an illustrative example and Section 7 concludes this paper.

Notation: The set of real (integer) numbers is denoted as $\mathbb{R}$ ($\mathbb{N}$). $\sup\{\mathcal{S}\}$ denotes the supremum of a set $\mathcal{S}$. $|x|$ denotes the Euclidean norm of x. Given a state variable x with jumps, its derivative with respect to time is denoted as $\dot{x}$. At jump instants, we denote the value of state after the jump by x^+, and the value of the state before the jump simply by x. $\mathbb{B}$ denotes the closed unit ball in a Euclidean space of a given dimension. For a set $\mathcal{S} \in \mathbb{R}^n$, denote $|x|_{\mathcal{S}} := \inf_{y\in\mathcal{S}} |x-y|$. $\mathrm{sgn}(\cdot)$ denotes the sign function. $\mathrm{dom}(x)$ denotes the domain of x.

2 Preliminaries and Problem Formulation

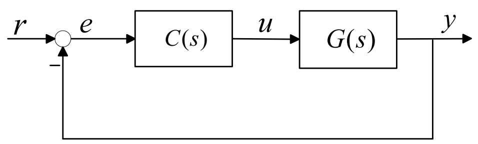

Fig. 1. Unit feedback control structure.

In this work, we are interested in a class of single-input-single-output (SISO) LTI systems with closed-loop structure shown in Figure 1. The plant and controller are represented by transfer functions $G(s)$ and $C(s)$ respectively. u is the control input and y is the plant output. The reference set-point is denoted as r, which is assumed to be constant. Assuming that $C(s)$ can stabilize the loop, the following performance index is introduced to characterize the transient performance of the closed-loop system in Figure 1.

Definition 1 *Define the error $e(t) = r - y(t)$, then, the **steady-state error** e_{ss} of the closed-loop system in Figure 1 is defined as the difference between the reference r and the output $y(t)$ as $t \to \infty$*

$$e_{ss} = r - y(\infty). \tag{1}$$

Remark 1 *In this work, we assume that $e(t_0)$ (t_0 denotes the initial time) is known and it is non-zero. In the context of overcoming overshoot performance limitation, this assumption is very natural. As pointed out in [23], if $e(t_0) = 0$, overshoot always appears.*

Definition 2 *The **overshoot** of the closed-loop system in Figure 1 is defined as the maximum value by which the output exceeds its final set-point value*

$$y_{os} = \sup_{t\geq t_0\geq 0} \{-e(t)\mathrm{sgn}(e(t_0))\}. \tag{2}$$

2.1 Fundamental limitations of linear control

According to [24], the performance of the closed-loop system in Figure 1 will exhibit some fundamental limitations as summarized in the following propositions.

Proposition 1 [24] *(Open-Loop Integrators) Suppose that the closed-loop system in Figure 1 is BIBO stable*[1]*, and $r \in \mathbb{R}$ is the reference signal. Then the following statements hold.*

(a) If $\lim_{s\to 0} sG(s)C(s) = c_1$, where c_1 is a non-zero constant, then $\int_0^\infty e(t)dt = \frac{r}{c_1}$.
(b) If $\lim_{s\to 0} s^2G(s)C(s) = c_2$, where c_2 is a non-zero constant, then $\int_0^\infty e(t)dt = 0$.

Moreover, both cases ensure zero steady-state error. ◦

Remark 2 *When there exists a single open-loop integrator, integral equality in item (a) does not imply the existence of the overshoot of the step response y. However, according to [2, Proposition 1], when the rise time $t_r > (2/c_1)$, the step response y necessarily overshoots. When there exist double open-loop integrators, equation (2) and integral equality in item (b) ensure that the step response y overshoots.*

Proposition 2 [24] *(Open-Loop Unstable Real Poles) Suppose that $G(s)$ has a real pole at $s = p$, $p > 0$ and $r \in \mathbb{R}$ is the reference signal. If the closed-loop is BIBO stable, then*

$$\int_0^\infty \exp(-pt)e(t)dt = 0, \int_0^\infty \exp(-pt)y(t)dt = \frac{r}{p}. \tag{3}$$

[1] [24], Section 1.2. BIBO stable means that a bounded input produces a bounded response.

Remark 3 *If the plant $G(s)$ has an unstable real pole, the error e will have a change of sign. This implies that the step response y will overshoot.*

It is worthwhile to highlight that the main focus of this paper is non-overshoot tracking performance. We might have to sacrifice some other performance requirements, such as the zero steady-state error. However, the achievement of non-overshoot tracking performance and arbitrarily small steady-state error is also limitation of linear control.

Corollary 1 *(OPL of Linear Systems) Suppose that the closed-loop system in Figure 1 satisfies the conditions in Proposition 1 or Proposition 2, and that the closed-loop is BIBO stable. Then, for the following two cases:*

(a) There exists an integrator in $G(s)C(s)$ and the rise time $t_r > (2/c_1)$;
(b) There exist integrators in $G(s)C(s)$, or $G(s)$ has a real pole at $s = p$ such that $p > 0$.

For any given $\delta_{ss} > 0$, there does not exist controller $C(s)$ such that $y_{os} = 0$ and $e_{ss} \leq \delta_{ss}$ hold simultaneously.

Proof: From the results of Proposition 1 and Proposition 2, $y_{os} = 0$ cannot be achieved by any LTI controller $C(s)$ in the case of (a) or (b), so, $y_{os} = 0$ and $e_{ss} \leq \delta_{ss}$ cannot hold simultaneously. □

2.2 Minimum-phase relative degree one plant

We assume that the plant $G(s)$ has minimum-phase with a relative degree one. This class of plants can be controlled through controlling a first-order subsystem, because based on the well-known concept of zero dynamics [25, Chapter 4.3], there exists a special state-space representation for $G(s)$ resulting as follows [25]:

$$P : \begin{cases} \dot{z} = A_z z + B_{zy} y, \\ \dot{y} = a_p y + b_p u + C_z z. \end{cases} \tag{4}$$

where $y \in \mathbb{R}$ is the output dynamics, $z \in \mathbb{R}^{(n_p-1)}$ is the zero dynamics, and $u \in \mathbb{R}$ is the control input.

Note that the plant (4) is minimum-phase, hence A_z is Hurwitz, b_p is a non-zero constant and without loss of generality we assume $b_p > 0$. Once $G(s)$ is known, we can obtain $(A_z, B_{zy}, C_z, a_p, b_p)$ [25].

In this paper, we consider the unit feedback control structure with the minimum-phase relative degree one plant (4). The objective is to achieve non-overshoot output tracking performance when the plant possesses integrators or unstable real poles. However, due to the performance limitations presented in Corollary 1, such performance specification cannot be achieved by linear control, but possibly by reset control. The proposed reset control structure is shown in Figure 2 and our objective is to design it to achieve non-overshoot output tracking and overcome the performance limitations presented in Corollary 1.

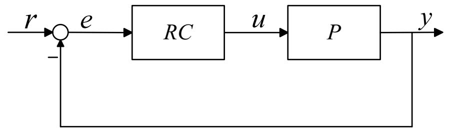

Fig. 2. Closed-loop control system with reset control (RC).

3 Reset Controller Design

Several types of reset controllers exist in literature and majority of them contain a FORE. Thus it is instructive to revisit the main FORE models before introducing the proposed reset controller.

(a) The first one is taken from [3] and is described by the following equations

$$\begin{cases} \dot{x}_r = a_c x_r + b_c u_r, & u_r \neq 0, \\ x_r^+ = 0, & u_r = 0, \end{cases} \tag{5}$$

where a_c and b_c are respectively the pole and input gain of the FORE, and $x_r \in \mathbb{R}$ and $u_r \in \mathbb{R}$ are respectively the state and input.

(b) A second FORE model was proposed in [12], which allows resets on more complicated sets and a tuning parameter ρ is introduced to prevent Zeno behaviour.

$$\left.\begin{aligned} \dot{\tau} &= 1, \\ \dot{x}_r &= a_c x_r + b_c u_r, \end{aligned}\right\} \text{ If } u_r x_r \geq 0 \text{ or } \tau \leq \rho,$$
$$\left.\begin{aligned} \tau^+ &= 0, \\ x_r^+ &= 0, \end{aligned}\right\} \text{ If } u_r x_r \leq 0 \text{ and } \tau \geq \rho. \tag{6}$$

(c) The model (6) is further extended in [15]

$$\left.\begin{aligned} \dot{\tau} &= 1, \\ \dot{x}_r &= a_c x_r + b_c u_r, \end{aligned}\right\} \begin{aligned} &\text{If } \varepsilon u_r^2 + 2u_r x_r \geq 0 \\ &\qquad \text{or } \tau \leq \rho, \end{aligned}$$
$$\left.\begin{aligned} \tau^+ &= 0, \\ x_r^+ &= 0, \end{aligned}\right\} \text{If } \varepsilon u_r^2 + 2u_r x_r \leq 0 \text{ and } \tau \geq \rho. \tag{7}$$

where the tuning parameter $\varepsilon \in \mathbb{R}$ is introduced to tilt the boundary between the flow set and the jump set. The extra design parameter ε provides more flexibility in selecting Lyapunov functions.

Note that only stabilization or set-point tracking were considered with the above FOREs. In order to achieve non-overshoot performance, a new FORE model is presented next. The non-overshoot performance requires that the tracking error cannot change sign [23]. Thus, besides the tracking error $e(t)$, its derivative information is also needed to design an appropriate reset rule. This new reset controller is a generalization of the FORE and we named it *GFORE*. More precisely, unlike the FORE that possesses a single input, the GFORE input includes two signals: (v_1, v_2). These signals are defined as:

$$v_1 = e, \quad v_2 = \dot{e} \tag{8}$$

where e is the tracking error ($e = r - y$) and $\dot{e}$ its derivative. The flow set $\mathcal{F}$ and the jump set $\mathcal{J}$ are defined as follows

$$\begin{aligned}\mathcal{F} &:= \{(x_r, v_1, v_2) \in \mathbb{R}\times\mathbb{R}\times\mathbb{R} | \varepsilon v_1^2 - 2v_1v_2 \geq 0\},\\ \mathcal{J} &:= \{(x_r, v_1, v_2) \in \mathbb{R}\times\mathbb{R}\times\mathbb{R} | \varepsilon v_1^2 - 2v_1v_2 \leq 0\}.\end{aligned} \tag{9}$$

Similar to (7), the parameter $\varepsilon > 0$ is a small constant that is used to tilt the boundary between the flow set and the jump set. This leads to the following GFORE,

$$\begin{aligned}&\left.\begin{aligned}\dot{\tau} &= 1\\ \dot{x}_r &= a_c x_r + b_c v_1\end{aligned}\right\} \text{If } (x_r, v_1, v_2) \in \mathcal{F} \text{ or } \tau \leq \rho,\\ &\left.\begin{aligned}\tau^+ &= 0\\ x_r^+ &= \frac{1}{b_p}(v_2 - \alpha) + x_r\end{aligned}\right\} \begin{aligned}&\text{If } (x_r, v_1, v_2) \in \mathcal{J}\\ &\text{and } \tau \geq \rho,\end{aligned}\end{aligned} \tag{10}$$

where b_p is the input gain from (4). In the GFORE (10), $(a_c, b_c, \varepsilon, \rho, \alpha)$ are tuning parameters and the output is x_r. The role of $(a_c,\ b_c)$ is similar to that in (7): a_c is the pole and b_c is the input gain of the GFORE. The parameter α plays an important role in achieving non-overshoot performance, and it is used to change the sign of $\dot{e}(t)$ at jumps. From this part over, and without loss of generality, we assume the following.

Assumption 1: The initial condition $v_1(t_0) > 0$.

Remark 4 *As discussed in Remark 1, the initial error* $e(t_0)$ *must be known, and we set the parameter* α *to have the same sign as this initial error.*

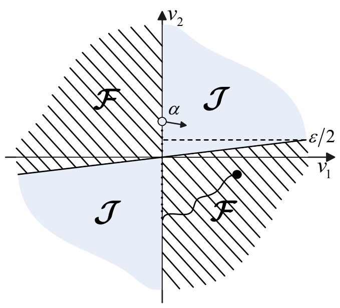

Fig. 3. Flow and jump sets.

Figure 3 shows the flow and jump sets defined in (9). A sample trajectory starting from the flow set (black dot) is given to show the dynamic behavior.

Remark 5 *The GFORE* (10) *uses a different choice of* v_1 *and* v_2 *than other FOREs. In particular, letting* $v_1 = e, v_2 = -b_p x_r, \alpha = 0$ *and* $\tilde{\varepsilon} = \varepsilon/b_p$, *we have* $x_r^+ = 0$ *and* $\varepsilon v_1^2 - 2v_1v_2 \geq 0 \Leftrightarrow \tilde{\varepsilon}e^2 + 2ex_r \geq 0$. *In this case, the GFORE* (10) *is the same as the FORE* (7). *If we further let* $\varepsilon = 0$, *the GFORE* (10) *is the same as the FORE* (6). *This shows that* (10) *is a generalized FORE and the* α *parameter provides extra freedom to overcome OPL.*

4 Main Results

4.1 Stability and non-overshoot performance

The plant (4) and GFORE (10) are interconnected with $u = x_r$, $e = r - y$ and (8). The arising closed-loop system can be written in the following form

$$\begin{aligned}&\left.\begin{aligned}\dot{\tau} &= 1\\ \dot{x} &= Ax + Br\end{aligned}\right\} \text{If } x \in \mathcal{F} \text{ or } \tau \leq \rho,\\ &\left.\begin{aligned}\tau^+ &= 0\\ x^+ &= A_r x + B_r\alpha\end{aligned}\right\} \text{If } x \in \mathcal{J} \text{ and } \tau \geq \rho,\\ &\quad y = Cx\end{aligned} \tag{11}$$

where $x := [z^T\ y\ x_r]^T \in \mathbb{R}^n$, $C = [0\ 1\ 0]$ and

$$\mathcal{F} = \{x \in \mathbb{R}^n | (x - E_r)^T M (x - E_r) - 2a_p(x - E_r)^T E_r \geq 0\}$$
$$\mathcal{J} = \{x \in \mathbb{R}^n | (x - E_r)^T M (x - E_r) - 2a_p(x - E_r)^T E_r \leq 0\}$$

$$A = \begin{bmatrix} A_z & B_{zy} & 0\\ C_z & a_p & b_p\\ 0 & -b_c & a_c\end{bmatrix}, B = \begin{bmatrix}0\\0\\b_c\end{bmatrix}, A_r = \begin{bmatrix} I_{n-2} & 0 & 0\\ 0 & 1 & 0\\ -\frac{C_z}{b_p} & -\frac{a_p}{b_p} & 0\end{bmatrix},$$

$$B_r = \begin{bmatrix}0\\0\\-\frac{1}{b_p}\end{bmatrix}, E_r = \begin{bmatrix}0\\r\\0\end{bmatrix}, M = \begin{bmatrix}0 & -C_z^T & 0\\ -C_z & \varepsilon - 2a_p & -b_p\\ 0 & -b_p & 0\end{bmatrix}.$$

The reset controller design consists of two steps: (1) Design parameters $(a_c, b_c, \varepsilon, \rho^{*}$ [2]$)$ such that for any $\rho \in (0, \rho^*]$, the closed-loop system with $\alpha = 0, r = 0$ is exponentially stable; (2) Design parameter α such that the non-overshoot tracking performance can be ensured for sufficiently small $\rho \in (0, \rho^*]$.

The next proposition (similar to [15, Theorem 3]) shows that with $\alpha = 0, r = 0$ and appropriate (a_c, b_c), exponential stability of the closed-loop system conditionally to hierarchically small (ε, ρ) [3] can be achieved.

Proposition 3 *Let* $\alpha = 0$ *and* $r = 0$. *For any Hurwitz matrix* A_z *in* (4), *there exits a parameter pair* (a_c, b_c) *and a positive constant* ε^* *such that for each* $\varepsilon \in (0, \varepsilon^*]$ *there exists* ρ^* *such that for any* $\rho \in (0, \rho^*]$, *the solutions of the reduced system*

$$\begin{aligned}&\left.\begin{aligned}\dot{\tau} &= 1\\ \dot{x} &= Ax\end{aligned}\right\} \text{If } x^T M x \geq 0 \text{ or } \tau \leq \rho,\\ &\left.\begin{aligned}\tau^+ &= 0\\ x^+ &= A_r x\end{aligned}\right\} \text{If } x^T M x \leq 0 \text{ and } \tau \geq \rho.\end{aligned} \tag{12}$$

satisfy the following inequality:

$$|x(t,j)| \leq m|x(t_0,0)| \exp(-lt),\ \forall (t,j) \in \mathrm{dom}(x). \tag{13}$$

[2] ρ^* is a design parameter that denotes an upper bound for ρ. How to choose ρ^* is discussed in Proposition 3.

[3] For the definition of "exponentially stable conditionally to hierarchically small (ε, ρ)", see [15, Definition 1].

where $x(t_0,0)$ is defined on hybrid time domain[4] *, and (m,l) are positive constants. That is the origin of the system* (12) *is exponentially stable conditionally to hierarchically small* (ε,ρ).

Next semi-globally practically asymptotically stable (SGP-AS) properties are provided for the hybrid system in (11).

Definition 3 *[27] A compact set $\mathcal{A}$ is said to be SGP-AS for the closed-loop system* (11) *as $\alpha\to 0$, if there exists $\beta\in\mathcal{KL}$ and for each $\mu>0$ and $\nu>0$ there exists $\alpha^*>0$ such that, for each $\alpha\in(0,\alpha^*]$, each solution x of* (11) *that satisfies $|x(t_0,0)|_{\mathcal{A}}\le\mu$ also satisfies*

$$|x(t,j)|_{\mathcal{A}}\le\beta(|x(t_0,0)|_{\mathcal{A}},t)+\nu,\ \forall(t,j)\in\mathrm{dom}(x).$$

In this work, the compact set (or attractor) $\mathcal{A}$ has the following form, which is obtained by letting $Ax+Br=0$ and noting that the corresponding tracking error e is 0.

$$\mathcal{A}:=\left\{x\in\mathbb{R}^n|x=\left[-(A_z^{-1}B_{zy}r)^T\ r\ 0\right]^T\right\}\tag{14}$$

What follows is the first main result of the paper.

Theorem 1 (SGP-AS) *Consider the closed-loop system* (11)*. Let $r\in\mathbb{R}$. Suppose that the compact set $\mathcal{A}$ defined in* (14) *is exponentially stable for the reduced system* (12) *with respect to $\beta\in\mathcal{KL}$. Then, the compact set $\mathcal{A}$ is SGP-AS as $\alpha\to 0$.*

Define $\lambda=\left[0_{1\times(n-2)}\ 1\ 0\right]^T$, we have $\lambda^Tx\le r\Leftrightarrow v_1\ge 0\Leftrightarrow r-y\ge 0$. The next theorem states that the non-overshoot output performance can be achieved with the proposed reset controller.

Theorem 2 (Non-overshoot performance) *Consider the closed-loop system* (11)*. Suppose that Assumption 1 holds, and the parameters $(a_c,b_c,\varepsilon,\rho^*)$ are such that the reduced system* (12) *is exponentially stable with respect to $\beta\in\mathcal{KL}$. Then, there exists $\alpha^*>0$ such that for each $\alpha\in(0,\alpha^*]$, there exists sufficiently small $\rho\in(0,\rho^*]$ such that $\mathcal{A}$ is SGP-AS as $\alpha\to 0$ and the following set*

$$\mathcal{S}_v:=\{x\in\mathbb{R}^n|\lambda^Tx\le r\}\tag{15}$$

is positively invariant[5] *. That is the output response y tracks the reference r without overshoot.*

Remark 6 *It is known from Proposition 3 that the reduced system* (12) *is exponentially stable for any $\rho\in(0,\rho^*]$, where ρ^* is an upper bound for ρ. Hence, once ρ^* is known, the parameter ρ can be arbitrarily adjusted in the range $\rho\in(0,\rho^*]$ while preserving exponential stability. This property provides an extra degree of freedom that can be used to achieve non-overshoot performance as shown in Theorem 2.*

In many engineering applications, the differential signal $v_2=\dot e=\dot v_1$ is usually not available for feedback, however, a high-pass filter (approximate differentiator) can be used to estimate $\dot e$ as presented in the next section.

4.2 Implementation with high-pass filter

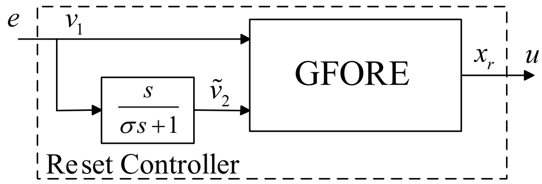

Fig. 4. Implementation of the reset controller with a GFORE (10) and a high-pass filter (16).

A high-pass filer $\frac{s}{\sigma s+1}$ (see Figure 4) is used to estimate the signal v_2. The estimate $\tilde v_2$ satisfies the following relation:

$$\sigma\dot{\tilde v}_2=-\tilde v_2+\dot v_1=-\tilde v_2+v_2\tag{16}$$

where $\sigma>0$ tunes the filter bandwidth. Replacing v_2 with $\tilde v_2$, we have the following new flow set and jump set

$$\begin{aligned}\tilde{\mathcal{F}}&:=\{(x_r,v_1,\tilde v_2)\in\mathbb{R}\times\mathbb{R}\times\mathbb{R}|\varepsilon v_1^2-2v_1\tilde v_2\ge 0\},\\ \tilde{\mathcal{J}}&:=\{(x_r,v_1,\tilde v_2)\in\mathbb{R}\times\mathbb{R}\times\mathbb{R}|\varepsilon v_1^2-2v_1\tilde v_2\le 0\}.\end{aligned}\tag{17}$$

Then, the reset controller GFORE (10) and the high-pass filter (16) can be represented as

$$\begin{aligned}&\left.\begin{aligned}\dot\tau&=1\\ \dot x_r&=a_cx_r+b_cv_1\\ \sigma\dot{\tilde v}_2&=-\tilde v_2+\dot v_1\end{aligned}\right\}\text{ If }(x_r,v_1,\tilde v_2)\in\tilde{\mathcal{F}}\text{ or }\tau\le\rho,\\ &\left.\begin{aligned}\tau^+&=0\\ x_r^+&=\frac{1}{b_p}(\tilde v_2-\alpha)+x_r\\ \tilde v_2^+&=\tilde v_2\end{aligned}\right\}\begin{aligned}&\text{If }(x_r,v_1,\tilde v_2)\in\tilde{\mathcal{J}}\\ &\text{and }\tau\ge\rho.\end{aligned}\end{aligned}\tag{18}$$

The set of tuning parameters for (18) is $\{a_c,b_c,\varepsilon,\alpha,\rho,\sigma\}$. The closed-loop system consisting of (4) and (18) can be written in the following form

$$\begin{aligned}&\left.\begin{aligned}\dot\tau&=1\\ \dot x&=Ax+Br\\ \sigma\dot{\tilde v}_2&=-\tilde v_2+L_2x\end{aligned}\right\}\text{ If }(x,\tilde v_2)\in\tilde{\mathcal{F}}\text{ or }\tau\le\rho,\\ &\left.\begin{aligned}\tau^+&=0\\ x^+&=A_rx+B_r\alpha\\ \tilde v_2^+&=\tilde v_2\end{aligned}\right\}\text{ If }(x,\tilde v_2)\in\tilde{\mathcal{J}}\text{ and }\tau\ge\rho.\end{aligned}\tag{19}$$

[4] A *hybrid time domain* is a subset of $\mathbb{R}_{\ge 0}\times\mathbb{N}$, and it is the union of finitely/infinitely many intervals of the form $[t_j,t_{j+1}]\times\{j\}$, with the last one possibly of the form $[t_j,\infty)\times\{j\}$. More details about hybrid systems can be found in [15],[26].

[5] A set $\mathcal{S}\subset\mathbb{R}^n$ is *positively invariant* if for all $x(t_0,0)\in\mathcal{S}$, $x(t,j)\in\mathcal{S}$ for all $(t,j)\in\mathrm{dom}(x)$.

where $L_2 = \begin{bmatrix} -C_z & -a_p & -b_p \end{bmatrix}$ and A, B, A_r, B_r are defined in (11).

Remark 7 *Note that when σ is very small, system (19) can be considered as a singularly perturbed hybrid system [28,29], with a constant disturbance on the jump map.*

The singularly perturbed hybrid system (19) has an extra parameter σ compared to (11). Definition 3 can be easily generalized for (19) showing that the compact set $\mathcal{A}$ is SGP-AS as $(\alpha, \sigma) \to 0$. The next theorem shows that though the signal $\tilde{v}_2$ is obtained physically, the SGP-AS and non-overshoot performance can still be achieved.

Theorem 3 (stability and non-overshoot performance) *Consider the closed-loop system (19). Assume that Assumption 1 holds and the parameters $(a_c, b_c, \varepsilon, \rho^*)$ are such that the compact set $\mathcal{A}$ is exponentially stable for the reduced system (12) with respect to $\beta \in \mathcal{KL}$. Then, for each $\mu > 0$ and $\nu > 0$, there exists $\alpha^* > 0$ such that for each $\alpha \in (0, \alpha^*]$, there exists sufficiently small $\rho \in (0, \rho^*]$ such that for ρ, there exists $\sigma^* > 0$ such that for any $\sigma \in (0, \sigma^*]$, we have*

(1) The solution x of the closed-loop system (19) with $|x(t_0, 0)|_{\mathcal{A}} \leq \mu$ satisfies the following inequality:

$$|x(t,j)|_{\mathcal{A}} \leq \beta(|x(t_0,0)|_{\mathcal{A}}, t) + \nu, \ \forall (t,j) \in \mathrm{dom}(x).$$

(2) The following set is positively invariant.

$$\mathcal{S}_v := \{x \in \mathbb{R}^n | \lambda^T x \leq r\} \tag{20}$$

The above result implies that the output response y tracks the reference r without overshoot and the steady-state error e_{ss} can be made arbitrarily small. We conclude that the limitations of linear feedback control in Corollary 1 can be overcome by the proposed reset control design.

Remark 8 *Theorem 3 shows that by carefully selecting the parameters α, ρ, σ, the attractor $\mathcal{A}$ is SGP-AS for (19). It should be noted that SGP-AS indicates that the steady-state error is non-zero, more precisely, the non-overshoot performance is achieved at the cost of non-zero steady-state error. Hence, there is a performance trade-off between zero steady-state error and non-overshoot.*

Remark 9 *The role of α is to ensure non-overshoot performance. When the reset condition (17) is satisfied and $v_1 > 0$, if α is set to be zero in the jump dynamics, it leads to $v_2^+ = \dot{v}_1^+ = v_2 - \tilde{v}_2$. That is, v_1 decreases right after jump if $v_2 - \tilde{v}_2 < 0$. However, because $v_1 > 0$, v_1 cannot cross 0 immediately, and we can tune parameters σ, ρ such that when v_1 converges to a small neighbourhood of 0 the reset condition is satisfied, α is then used to guarantee that v_1 moves away from 0 right after jump and overshoot does not occur.*

Based on this idea, a nonlinear mapping $s(\cdot)$ can be used to provide a possibility to reduce the steady-state error, and the reset conditions and the jump dynamics of x_r in (18) are modified as follows

$$x_r^+ = \frac{1}{b_p}(\tilde{v}_2 - \alpha) + x_r$$
$$\text{If } \varepsilon v_1^2 - 2v_1\tilde{v}_2 \geq 0 \text{ and } \tau \leq \rho \text{ and } 0 \leq v_1 \leq s(\alpha), \tag{21a}$$

$$x_r^+ = \frac{1}{b_p}\tilde{v}_2 + x_r$$
$$\text{If } \varepsilon v_1^2 - 2v_1\tilde{v}_2 \leq 0 \text{ and } \tau \geq \rho \text{ and } v_1 > s(\alpha). \tag{21b}$$

where $s(\cdot)$ is class $\mathcal{K}$ function. If the x_r dynamics in (18) is substituted for (21), it has the potential to achieve a smaller steady-state error with appropriate $s(\alpha)$ (the role of $s(\alpha)$ is to avoid overshoot), because with the jump dynamics in (18), the error v_1 moves away from zero each time right after jump, however, with the jump dynamics (21), only when $0 \leq v_1 \leq s(\alpha)$, the error v_1 moves away from zero right after jump.

5 Parameter Tuning

The following procedure presents the tuning guidelines for the selection of parameters $\{a_c, b_c, \varepsilon, \alpha, \rho, \sigma\}$. Step 1 and step 2 are taken from [15], while the last two steps are novel and present the selection of parameters α, ρ, σ.

Step 1: If the plant (4) includes one or more integrators (respectively, does not include integrators), select (a_c, b_c) (respectively, $a_c = 0, b_c$) such that at least one of the following conditions holds.

1). There exist $\gamma_z > 0, \gamma_v > 0$ and symmetric positive definite matrices $P_z \in \mathbb{R}^{(n-2)\times(n-2)}, P_s \in \mathbb{R}^{2\times 2}$ such that the following conditions are satisfied.

$$\begin{bmatrix} P_z A_z + A_z^T P_z + I & -P_z B_{zy} \\ * & -\gamma_z^2 \end{bmatrix} < 0 \tag{22a}$$

$$\begin{bmatrix} P_s \tilde{A}_s + \tilde{A}_s^T P_s + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} & P_s \tilde{B}_s \\ * & -\gamma_v^2 \end{bmatrix} < 0 \tag{22b}$$

$$\gamma_z \gamma_v ||a_c C_z - C_z A_z|| < 1 \tag{22c}$$

where A_z, B_{zy}, C_z are from (4), $\tilde{B}_s = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$

$$\tilde{A}_s = \begin{bmatrix} 0 & 1 \\ -b_p b_c - a_p a_c + C_z B_{zy} & a_p + a_c \end{bmatrix} \tag{23}$$

2). There exist $\gamma_z > 0, P_z > 0$ such that (22a) is satisfied, and a sufficiently large b_c such that $2\sqrt{b_p b_c + a_p a_c - C_z B_{zy}} + a_p + a_c > 0$ is satisfied.

Step 2: Select $\varepsilon > 0$ and the corresponding ρ^* such that for any $\rho \in (0, \rho^*]$, the stability result Proposition 3 holds.

Step 3: Select $\alpha > 0$ such that SGP-AS in Theorem 3 holds.

Step 4: Select sufficiently small ρ and small enough σ^* such that for any $\sigma \in (0, \sigma^*]$, the non-overshoot performance is achieved.

Remark 10 *Inequality (22b) is always feasible if $\tilde{A}_s$ is Hurwitz, and by selecting an appropriate pair (a_c, b_c), inequalities (22a)-(22c), or the conditions in 2) of Step 1 can be guaranteed.*

Remark 11 *Theorem 3 has shown that if the reduced system (12) is exponentially stable with parameters $(a_c, b_c, \varepsilon, \rho^*)$, theoretically, for a given initial condition, it is always possible to select α and a sufficiently small ρ and σ to achieve non-overshoot performance. However, in practical applications, it is usually hard to find these parameters, and trial and error method might be used.*

Moreover, the design procedure is based on parametric models, which may be difficult to obtain in real applications. In the future, we will focus on extending the proposed technique to non-parametric models.

6 An Illustrative Example

Consider the following minimum-phase relative degree one plant

$$G(s) = \frac{(s+1)(s^2+s+1)}{s(s-2)(s^2-2s+5)}, \tag{24}$$

which can be represented in the state space form (4) as

$$\begin{aligned} \dot{z} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -2 \end{bmatrix} z + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} y, \\ \dot{y} &= 6y + u - \begin{bmatrix} 6 & 1 & 19 \end{bmatrix} z. \end{aligned} \tag{25}$$

The plant includes an integrator, an unstable real pole and unstable complex conjugate poles. Therefore, the fundamental limitations reported in Corollary 1 are satisfied.

Suppose that the set-point reference $r = 1$. Then, we select the parameters of reset controller (18) by the tuning procedure in Section 5. First, we choose $a_c = -10, b_c = 81$ to place the eigenvalues of $\tilde{A}_s$ at $(-2 \pm 6\mathrm{i})$, such that the conditions in 2) of Step 1 are satisfied. For zero initial conditions, the tuning parameters are selected as $\varepsilon = 0.1, \alpha = 0.1, \rho = 0.0005, \sigma = 0.001$. Simulation results are shown in Figure 5, where we can appreciate the non-overshoot performance and non-zero steady-state error.

We compare our findings with the FOREs in [15] and [17], tuned with a_c, b_c, ε as in this work and $\lambda_r = -5, k = 81$ respectively. Note that the FOREs in these literature

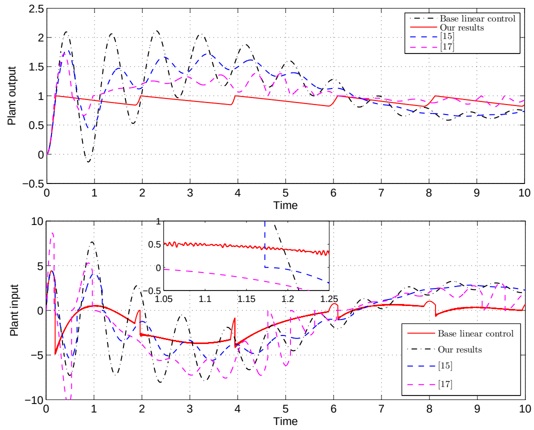

Fig. 5. Top: plant output $y(t)$. Bottom: plant input $u(t)$.

are parameterized, and the simulation is given with only one group of parameters due to space limitation. Other state trajectories are possible by selecting different parameters, but the non-overshoot performance cannot be achieved because the derivative of the error $e(t)$ was not considered in [15],[17]. From the comparison results, the non-overshoot output performance is achieved with the proposed reset controller, and it is explicitly shown that the OPLs of linear systems presented in Corollary 1 are overcome.

In order to reduce the steady-state error, while preserving the non-overshoot performance, the modified controller jump dynamics (21) are substituted into reset controller (18) with $s(\alpha) = 0.1\alpha$, while the other controller parameters remain unchanged. The simulation results are shown in Figure 6, where a much smaller steady-state error is achieved compared to Figure 5.

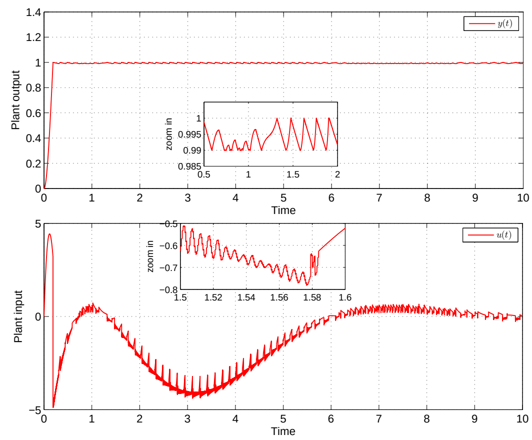

Fig. 6. Top: plant output $y(t)$. Bottom: plant input $u(t)$.

7 Conclusion

This paper investigates the problem of overcoming overshoot performance limitations of linear feedback control using reset control. We focus on a class of minimum-phase relative degree one LTI plants under unit feedback control structure. A novel GFORE is proposed with the goal of overcoming the OPLs of linear systems presented in Corollary 1. Systematic reset control design procedures and an implementation of the proposed reset controller with a high-pass filter are given. Finally, an illustrative example is presented to verify the effectiveness of the proposed generalized reset rule.

Appendix

Proof of Proposition 3: Consider the closed-loop system (12). Define $\tilde{x} = [z^T\ v_1\ v_2]^T$ and recalling that $v_1 = -y, v_2 = \dot{v}_1$, we have

$$x = \begin{bmatrix} z \\ y \\ x_r \end{bmatrix} = \begin{bmatrix} I_{n-2} & 0 & 0 \\ 0 & -1 & 0 \\ -\frac{C_z}{b_p} & \frac{a_p}{b_p} & -\frac{1}{b_p} \end{bmatrix} \begin{bmatrix} z \\ v_1 \\ v_2 \end{bmatrix} = T\tilde{x}, \tag{26}$$

where the matrix $T \in \mathbb{R}^{n\times n}$ is invertible. Then, the transformed system can be written as

$$\left.\begin{aligned} \dot{\tau} &= 1 \\ \dot{\tilde{x}} &= \tilde{A}\tilde{x} \end{aligned}\right\} \text{ If } \tilde{x}^T\tilde{M}\tilde{x} \geq 0 \text{ or } \tau \leq \rho, \tag{27a}$$

$$\left.\begin{aligned} \tau^+ &= 0 \\ \tilde{x}^+ &= \tilde{A}_r\tilde{x} \end{aligned}\right\} \text{ If } \tilde{x}^T\tilde{M}\tilde{x} \leq 0 \text{ and } \tau \geq \rho, \tag{27b}$$

where

$$\tilde{A} = \begin{bmatrix} A_z & -B_{zy} & 0 \\ 0 & 0 & 1 \\ a_cC_z - C_zA_z & -b_pb_c - a_pa_c + C_zB_{zy} & a_p + a_c \end{bmatrix}$$

$$\tilde{A}_r = \begin{bmatrix} I_{n-2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tilde{M} = \begin{bmatrix} 0_{n-2} & 0 & 0 \\ 0 & \varepsilon & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

The closed-loop system (27) can be viewed as the interconnection of two subsystems as follows

(1) A stable linear subsystem of the form

$$\dot{z} = A_zz - B_{zy}[1\ 0]\tilde{x}_s = A_zz + B_{zy}y \tag{28}$$

(2) A planar reset control system of the form

$$\left.\begin{aligned} \dot{\tau} &= 1 \\ \dot{\tilde{x}}_s &= \tilde{A}_s\tilde{x}_s + \tilde{B}_s\tilde{d}_s \end{aligned}\right\} \text{ If } \tilde{x}_s^T\tilde{M}_s\tilde{x}_s \geq 0 \text{ or } \tau \leq \rho,$$
$$\left.\begin{aligned} \tau^+ &= 0 \\ \tilde{x}_s^+ &= \tilde{A}_{sr}\tilde{x}_s \end{aligned}\right\} \text{ If } \tilde{x}_s^T\tilde{M}_s\tilde{x}_s \leq 0 \text{ and } \tau \geq \rho, \tag{29}$$

where $\tilde{x}_s = \begin{bmatrix} v_1 & v_2 \end{bmatrix}^T, \tilde{d}_s = (a_cC_z - C_zA_z)z$, $\tilde{A}_s, \tilde{B}_s$ are defined in (23) and $\tilde{A}_{sr} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \tilde{M}_s = \begin{bmatrix} \varepsilon & -1 \\ -1 & 0 \end{bmatrix}$.

Note that the system (27) has the same form as [15, (14)] with different system matrices. Hence, the following proof follows from the proof of [15, Theorem 3]. To show exponential stability of the x-subsystem in reduced system (12), we use the following lemma which is similar as the results of [15, Theorem 2], so its proof is omitted due to space limitation.

Lemma 1 *Consider the closed-loop system* (27). *Let A_z be Hurwitz and $b_p > 0$. Then, the following statements are true.*

(1) If there exist (a_c, b_c), $\gamma_z > 0, \gamma_v > 0$ and symmetric positive definite matrices P_z, P_s such that the conditions in (22) *are satisfied, then, the origin of the x dynamics is exponentially stable conditionally to hierarchically small (ε, ρ) .*

(2) If there exists $\gamma_z > 0$ and positive definite matrix P_z such that the condition (22a) *is satisfied, then, the origin of the x dynamics is exponentially stable conditionally to large b_c and hierarchically small (ε, ρ).*

Because A_z is Hurwitz, there exists P_z such that (22a) is satisfied with a finite $\gamma_z > 0$. Consider a pair of parameters (a_c, b_c), if the conditions in item 1) of Lemma 1 are not satisfied, we can always pick sufficiently large b_c such that the conditions in item 2) of Lemma 1 are satisfied, then, we can conclude that there exist a pair of parameters (a_c, b_c) and positive constant ε^* such that for each $\varepsilon \in (0, \varepsilon^*]$ there exists ρ^* such that for any $\rho \in (0, \rho^*]$, the solutions of the system (12) satisfy

$$|x(t, j)| \leq \beta(|x(t_0, 0)|, t),\ \forall(t, j) \in \text{dom}(x)$$

with $\beta(s, t) = ms\exp(-lt)$ for a positive pair (m, l). In other words, there exist (a_c, b_c) such that the origin of x dynamics is exponentially stable conditionally to hierarchically small (ε, ρ). □

Proof of Theorem 1: Perform the change of coordinate $x \mapsto \hat{x} := x - x^*$ for the closed-loop system (11), where x^* is a vector that satisfies $Ax^* + Br = 0, Cx^* = r$ where $C = [0_{1\times(n-2)}\ 1\ 0]$. Then, the arising dynamics results

$$\left.\begin{aligned} \dot{\tau} &= 1 \\ \dot{\hat{x}} &= A\hat{x} \end{aligned}\right\} \text{ If } \hat{x}^TM\hat{x} \geq 0 \text{ or } \tau \leq \rho,$$
$$\left.\begin{aligned} \tau^+ &= 0 \\ \hat{x}^+ &= A_r\hat{x} + B_r\alpha \end{aligned}\right\} \text{ If } \hat{x}^TM\hat{x} \leq 0 \text{ and } \tau \geq \rho, \tag{30}$$

which coincides with the system (12) with $r = 0$, and an extra constant disturbance proportional to α acting on the jump equation. Then, based on Proposition 3, the origin of $\hat{x}$ dynamics is exponentially stable conditionally to hierarchically small (ε, ρ) when $\alpha = 0$. From $\hat{x} = x - x^*$, we deduce that the compact set $\mathcal{A} = \{x \in \mathbb{R}^n | x = x^* = [-(A_z^{-1}B_{zy}r)^T\ r\ 0]^T\}$ is exponentially stable and the following inequality holds

$$|x(t, j)|_{\mathcal{A}} \leq \beta(|x(t_0, 0)|_{\mathcal{A}}, t), \forall(t, j) \in \text{dom}(x). \tag{31}$$

When $\alpha \neq 0$, $B_r\alpha$ can be viewed as a constant disturbance, and (30) can be thought as an inflated system of the form

$$\begin{aligned} &\left.\begin{aligned} \dot{\tau} &= 1 \\ \dot{\hat{x}} &= A\hat{x} \end{aligned}\right\} \text{ If } \hat{x}^T M\hat{x} \geq 0 \text{ or } \tau \leq \rho, \\ &\left.\begin{aligned} \tau^+ &= 0 \\ \hat{x}^+ &\in A_r\hat{x} + B_r\,|\alpha|\,\mathbb{B} \end{aligned}\right\} \text{ If } \hat{x}^T M\hat{x} \leq 0 \text{ and } \tau \geq \rho. \end{aligned} \tag{32}$$

The constant disturbance leads to practical asymptotical stability, in particular, according to [26, Theorem 17], for each $\mu > 0, \nu > 0$ there exists $\alpha^* > 0$ such that for any $\alpha \in (0, \alpha^*]$, the solutions of the closed-loop system (32) with $|x(t_0, 0)|_{\mathcal{A}} \leq \mu$ satisfy

$$|x(t,j)|_{\mathcal{A}} \leq \beta(|x(t_0,0)|_{\mathcal{A}}, t) + \nu, \forall (t,j) \in \operatorname{dom}(x), \tag{33}$$

which implies that the compact set $\mathcal{A}$ is SGP-AS as $\alpha \to 0$ according to Definition 3. $\square$

Proof of Theorem 2: The SGP-AS implies that the steady-state error e_{ss} can be made arbitrarily small. It is known from Remark 6 that ρ can be further adjusted in the range $(0, \rho^*]$ while preserving stability. Next, consider $\alpha > 0$ and $v_1(t_0, 0) > 0$ and the set $\mathcal{S}_v = \{x \in \mathbb{R}^n | \lambda^T x \leq r\}$ which is equivalent to $\mathcal{S}_v = \{x \in \mathbb{R}^n | v_1 \geq 0\}$. Note that the output overshoots if v_1 changes sign. We show next that v_1 does not cross zero by selecting sufficiently small $\rho \in (0, \rho^*]$.

Because $v_1(t_0, 0) > 0$, there are two possible cases:

Case 1: v_1 never reaches zero, and because the system is SGP-AS, v_1 converges to a neighbourhood of the origin without zero-crossing.

Case 2: Suppose that T_0 is the first time instant when v_1 is zero, that is $v_1(T_0, j) = 0$ for some $j \in \mathbb{N}$.

(a) If there is no jumps during $t \in [t_0, T_0)$, then, there exists a $\rho_1 = T_0 - t_0 > 0$ such that for each $\rho \in (0, \rho_1]$ the reset map of (10) is activated when $v_1 = 0$, which implies that the *jump occurs* when $v_1(T_0, j) = 0$.

(b) If there are jumps during $t \in [t_0, T_0)$, assume that $t = T_{-1}$ is the time associated to the last jump. From the reset map of (10) and (4), it can be derived that $v_2(T_{-1}, j) = \alpha$. Then, according to $v_2 = \dot{v}_1$, it implies that v_1 will increase after this jump and escaping from the origin. Then, because $\alpha > 0$ is constant, and the base system ((11) without jumps) is linear, it will take v_1 finite time to return to the origin again, which implies that the existence of a sufficiently small ρ_2 such that $T_0 - T_{-1} \geq \rho_2$. Then, for any $\rho \in (0, \rho_2]$ the reset map of (10) is activated when $v_1 = 0$, which implies that the *jump occurs* when $v_1(T_0, j) = 0$.

In both cases (a) and (b), the jump occurs when $v_1(T_0, j) = 0$, and it can be derived that $v_2(T_0, j + 1) = \alpha > 0$, which implies that v_1 will increase after jump and has no zero-crossing at T_0. When $t > T_0$, it follows from the above analysis that there exists sufficiently small $\rho_3 > 0$ such that for any $\rho \in (0, \rho_3]$, the jump occurs each time when $v_1 = 0$ and $v_2^+ = \alpha > 0$ is satisfied at jumps.

Based on the above results, we can conclude that if $\rho \leq \min\{\rho_1, \rho_2, \rho_3, \rho^*\}$,the reset occurs whenever $v_1 = 0$. Hence, it can be seen that v_1 has no zero-crossing for $\forall t \geq 0$, that is the set $\mathcal{S}_v$ is positively invariant, which also implies that the output response y tracks the reference r without overshoot. $\square$

Proof of Theorem 3: We present a sketch of the proof due to space limitations, and the complete proof can be obtained following the proofs of Theorem 1, Theorem 2 and the singular perturbed hybrid systems results in [28],[29]. Performing the change of coordinate $x \mapsto \hat{x} := x - x^*$ for (19), where x^* is a vector that satisfies $Ax^* + Br = 0, Cx^* = r$. We obtain a system that can be regarded as a *singularly perturbed hybrid system* with sufficiently small σ. The *reduced system* is (12) and he *boundary layer system* is given by

$$\left.\begin{aligned} &\dot{\tau} = 0,\ \dot{\hat{x}} = 0 \\ &\dot{\tilde{v}}_2 = -\tilde{v}_2 + L_2\hat{x} \end{aligned}\right\} \begin{aligned} &\hat{x}^T M\hat{x} + 2L_1\hat{x}(L_2\hat{x} - \tilde{v}_2) \geq 0 \\ &\qquad\qquad \text{or } \tau \leq \rho. \end{aligned}$$

Then, according to the temporal regularization property [15], it can be shown that an inflated version of (12) can be obtained, and the solutions to the singularly perturbed hybrid system are contained in the solutions of the inflated system. Then, according to [26, Theorem 17], the SGP-AS of the inflated system implies the SGP-AS of the $\hat{x}$ dynamics. Moreover, it follows from the proof of Theorem 2 that the non-overshoot property can still be achieved with sufficiently small σ. $\square$

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