Relating building space to performance outcomes – A methodology to explore the relationship

1 Introduction

A school as an educational facility has its buildings, which constitute of spaces serving different functional uses. The use of spaces could be for administrative, educational or recreational purposes, and the availability and limitation of a space as well as its quality would have an impact on the user experience in performing different functions. The availability of spaces for educational activities and the specific purposes they are used for, can be considered as dynamic and evolving in reality. Considering that the space used for educational activities is limited by the total built space of a school in general, a school would often be faced with the dilemma of adding space or reorganising space, to ensure better use of it to achieve the required goals of the educational institution. With space representing typically about 20% of costs of an educational institution (Australasian Association of Higher Education Facilities Officers, 2001), its effective use, allocation and reorganisation therefore plays an important role in effectively delivering the educational program.

Designing of schools requires due consideration as the school facilities have been found to make a significant impact on the effectiveness of education received by the students, attributable to several factors including the teacher and student experience in the educational facilities (Schneider & National Clearinghouse for Educational Facilities, 2003). The positive impact of physical environment of schools in improving the quality of education has often been overlooked, however considered increasingly important (Sanoff H, 2001).

Considering that the primary purpose of a school, as an operator of an educational facility is to optimise its space use to maximise the educational performance of students, this paper intends to address the need to relate the use of built space for educational use, specifically focusing on the distribution of the space among several categories of use encompassing different educational activities to the educational outcomes. The paper intends to investigate whether the allocation of space in a building to different use categories, aiming a uniform level of utilisation, would lead to better use of building resources (space) to produce higher level of educational outcomes. The framework has been developed using a primary school set up in Australia as an example. In this instance, the educational performance measured through the National Assessment Plan for Literacy and Numeracy (NAPLAN), a standard assessment scheme available for the Year levels 3, 5, 7 and 9 in Australia, is used as the main indicator to demonstrate the methodology proposed in this paper. The study acknowledges however, that holistic educational outcome that schools are expected to provide to students cover a much broader scope, as for example stated in the Melbourne Declaration on Educational Goals for Young Australians (Ministerial Council on Education, 2008), in addition to the educational parameters tested by NAPLAN.

2 Background

2.1 Review of previous studies

This section provides a critical analysis of the previous research conducted contributing to or complementary to the work presented in this paper.

2.1.1 Review of the impact of schools' physical environment on educational outcome

Design of a school building determines the physical environment of a school and the importance of the physical environment on educational outcome has been emphasized by several researchers. Studies suggest that the learning environment influences the teachers and student performance, in terms of how well teachers teach and how well students learn (Cash; Hines, 1996). While the adverse physical environments are observed to affect the ability of the teachers to deliver a quality program (Schneider & National Clearinghouse for Educational Facilities, 2003), the students have experienced attributes which lead to a more positive attitude towards learning in newer and better facilities (Rudd, Reed, Smith, & National Foundation for Educational, 2008). Direct connections between sensory simulations, learning and physical spaces are suggested by Barrett and Zhang (2009).

A relationship drawn suggesting that quality of buildings and student's standardized achievement results are linked via student mobility, with children occupying low quality buildings and having high rates of mobility reporting lower test scores (Evans, Yoo, & Sipple, 2010). This is endorsed by further studies stating students who have high rates of mobility, studying in low-quality school buildings, perform worse on standardized achievement results (Mahdjoubi & Akplotsyi, 2012).

On the contrary, there are published review works of studies focussing on student performance in terms of both achievement and behaviour, and the condition of the built environment suggests that not all studies performed show a strong relationship exists between performance and the environment (Earthman, Lemasters, & Council of Educational Facility Planners, 1996). Based on the dimensions of the physical environment considered, some showed a weaker relationship (Stewart, 2015)

2.1.2 Review of the impact of design of school buildings to teacher and students performance

Design of spaces of school buildings has found to have a considerable impact in delivering an educational program as well as the student performance. In terms of setting the objectives for designing buildings, Cooper (1982) recognised the user's (i.e. teachers) objectives conflicts with that of the client's. While the educators promote open spaces to facilitate collaboration, teachers seem to prefer the educational spaces to be closed and manageable considering operational realities.

Considering user's expectations as the key parameter in design optimisation of buildings, a multi-criteria fuzzy expert system has been employed to analyse both quantitative and qualitative design parameters by Tabrizi, Al-Hussein, and Inyang (2012).

In terms of student performance, Tanner (2008) analysed four design patterns (movement and circulation, large group meeting places, day lighting and views, and instructional neighbourhoods) relating to school environments, and their relationship to academic achievement of students and found that these are positively correlated.

With school buildings increasingly having shorter life spans, Shabha (1993) emphasizes on the need to adopt to changing requirements and to have loose-fit design approach, where the functional use is not determined and not subject to a static definition. This means that having flexible spaces facilitating built-in flexibility for adaptation is encouraged for educational spaces to reduce the need to rebuild (Shabha, 1993). Another perspective suggests to have 'unfinished' spaces for children to explore their creativity and create meaningful spaces for themselves, rather than designers providing finished spaces, which children do not relate well to (Rigolon, 2011). The impact of physical environment has not necessarily been found out to involve built environments, as there are studies that have proven that the time children spend outdoors have better attention skills (Ulset, Vitaro, Brendgen, Bekkhus, & Borge, 2017).

While the relative significance of the design of the physical environment depends on the functional requirements of a building, adequate day light and thermal comfort have been identified as the two primary requirements for a conducive interior environment (Maitreya, 1979). There are studies that have made close relationships between the effects of classroom lighting to students behaviour and learning in the classroom (Winterbottom & Wilkins, 2009) and perceptions of noise levels to classroom performance (Marchand, Nardi, Reynolds, & Pamoukov, 2014).

Consideration of the impact of both the interior and exterior environments (Ramli, Masri, Zafrullah, Taib, & Hamid, 2012), and the importance of building envelope, building orientation and indoor environment (National Research Council, 2001) in the design process has been highlighted and the physical environment has been related to activities and perceptions of users (Khan & Kotharkar, 2012) in research findings.

Currently, there are guidelines and standards that govern space planning in higher educational facilities such as universities where there are recommendations provided on the optimal spaces for different departments, functional activities and purposes. Recommended levels on usage have also been identified by setting indicators such as room frequency, room occupation and utilisation levels (Australasian Association of Higher Education Facilities Officers, 2001). In Australia, Standards for non-government schools are decided by the Minister of Education and governed by the School Education Act 1999 (Section 159), where Standard 6 specifically discusses facilities of schools (Ministry of Education, 1999). In Australia, the relevant authority responsible for education and/or buildings for education has prepared detailed guidelines on maintaining quality standards of educational facilities. The Building Quality Handbook (Victorian School Building Authority, 2019),Design Guide for Schools (Government Architect New South Wales, 2018) and DECD Design Guidelines (Department for Education and Child Development, 2015) are some of the examples. These guidelines have specifically discussed the construction materials, fixtures, fittings etc., design consideration as educational facilities and specific measures to maintain conducive indoor environments.

However, investigating whether proportional allocation of space during design for different uses in a school or an educational institution has a relationship to educational performance of students has not been researched or explored to suggest a quantitative relationship.

2.1.3 Review of the impact of condition of buildings to student performance

In breaking down the school's physical environment into several subparts, the quality of space in school buildings is given an equal level of importance compared to the quantitative proportions.

With building condition representing an important part of the quality of the physical environment, some studies have reported a positive correlation between the building condition and student performance (Durán-Narucki, 2008; Gunter & Shao, 2016), and there are other studies which state that no relationship or weak relationship exists among them (Picus, Marion, Calvo, & Glenn, 2005). The study by Picus et al. (2005) states that higher quality buildings are unrelated to higher levels of student academic achievement . Contrary to this, there has been a study providing evidence-based identification of educational performance of students studying in buildings of different conditions are affected by those conditions (Council of the Great City, 2014). While there are several other factors influencing student performance other than the physical environment, these relationships suggest that a correlation may exist, though it does not mean causality.

2.2 Research significance

Previous research relating the physical environment of educational facilities to the performance of students have mainly been qualitative analysis or conceptual studies. The studies that have adopted quantitative approaches to investigate whether a relationship exists between them have been focusing on the physical environment relating to building condition or design aspects, and student or teacher performance mainly. Though it is generally known that the school infrastructure affects learning outcomes, consideration of performance and behavioural outcomes into design aspects is identified to be largely theoretical (Fisher, 2001). Considering classrooms as spaces for learning and socialisation influencing psychological development of students, impact of density and privacy has been discussed in research, discussing the absolute space allocation for studying (Rivlin & Weinstein, 1984).

However, how the proportional allocation of built spaces for different educational uses by students relate to educational outcomes is observed to be a novel topic, and a quantitative method/parametric study on such has not been conducted by researchers before.

Considering the built space of a school has limitations for expansion and typically is constant once built, the need to manage built space to optimise its fit for the purpose, (~~i.e~~i.e. to enhance learning outcome of students in a school), become a resource optimisation issue at the operational phase of the buildings. However, at the design phase of a building, knowing such relationships would enable optimum space selection for the intended use by the building designers, improving the quality of the design by leading to better space allocations during the entire life of a building.

This paper develops a theoretical framework to recognise the aggregate effective space use by an average student in conducting educational activities represented by the assessment result of his/her respective educational outcome. The paper hypothesizes that there is a relationship between the aggregate effective space use (AESU) (defined in Section 3.2.1.3) contributing to educational performance, denoted by E~~U~~, and the achieved educational performance (R). The paper establishes methodology for deriving the parameters involved in characterising the relationship and proposes methodology for obtaining optimum values for the proportional space distribution analysing the E~~U~~ and R of a set of sample data representing schools.

While educational performance assessed through NAPLAN is influenced by several other factors, including curriculum, pedagogy (Bakkenes, Vermunt, & Wubbels, 2010) and assessment (Umami, 2018); student competence, learning behaviour and individual learning preferences and methods (Barenberg, Roeder, & Dutke); in this instance, school's physical environment, and particularly the proportional allocation of space use categories has been considered as a factor in isolation, acknowledging that correlation necessarily does not mean causality, ~~i.e~~i.e. the use of the optimum space allocation ratios may not guarantee higher educational outcomes as there are several other influencing factors.

The research contribution of this paper leads providing a foundation to characterise the relationship between space use for educational activities and the educational performance quantitatively. The systematic approach and methodology developed in this research would enhance integration of knowledge on design of physical environment to the intended purpose of buildings. Therefore, this paper would provide useful guidance to educational policy makers for understanding the relationship dynamics between the two and to consider establishing constraints around proportional allocations in setting standards for design of school buildings.

2.3 Aims and scope of this paper

This paper aims to investigate if a relationship exists between the design space allocation for different categories of educational activities and the student performance related to those educational activities. The paper intends to characterise the relationship if it exists. The aims of the paper are to:

- develop a conceptual framework to draw a relationship between the aggregate effective space use to perform educational activities and educational performance.
- approximate the nature of relationship and to establish methodology to validate the approximation and thereby investigating whether a relationship exists
- reengineer the problem to propose preferred proportional space allocation ratios that would result in the optimum aggregate effective space use leading to a condition that the risk of compromise of educational outcomes with inappropriate allocations is minimised.
- elaborate the application of the above methodology using sample data

In developing the conceptual and theoretical frameworks, methodology and the model, key focus is given to schools in Australia, however, the research outcomes can be applied generally to any context.

3 Methodology

3.1 Conceptual framework

The paper presents a methodology to characterise the relationship between the space allocation ratios of different spaces used for educational activities and the educational outcomes as measured through a national assessment scheme, NAPLAN. The conceptual framework stating the methodology in four main steps is presented in Figure 1.

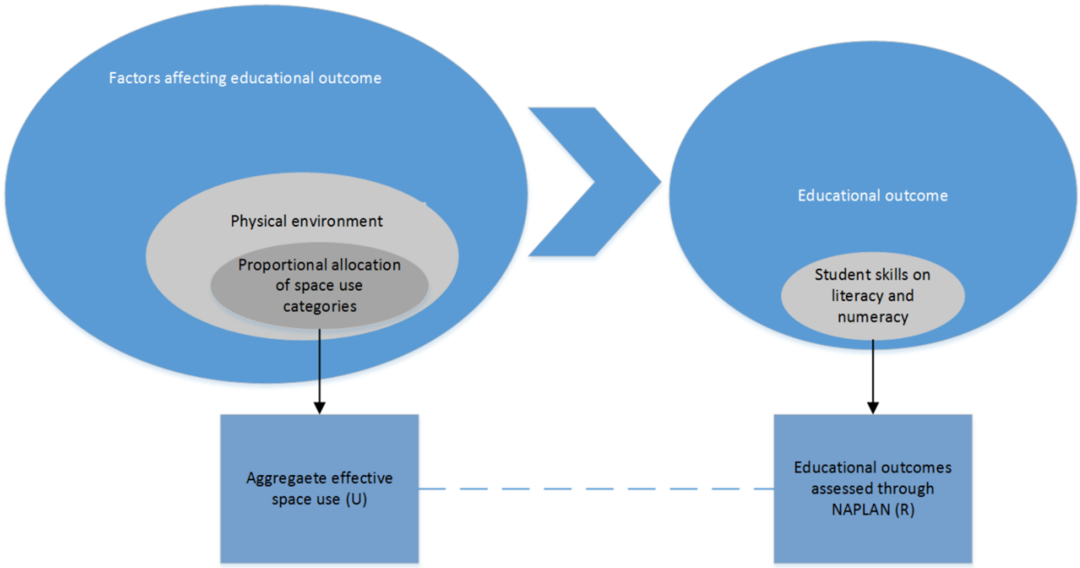
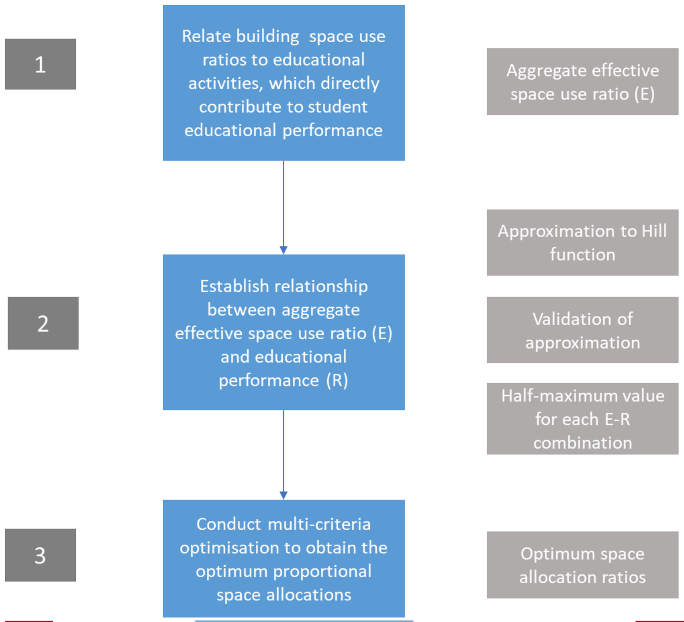


Figure 1: Research framework using proportional allocation of spaces use as a single factor affecting learning outcome and NAPLAN results as a single factor among many indicators to measure educational performance

In order to investigate the relationship between proportional allocations of space use categories in a school, considered as the input, to the assessment outcomes of a school, considered as the output, the link is made through student activity contribution to outcomes in the given spaces. As stated before, among many of the factors which influence the educational outcome, physical environment has been taken in isolation and among many variables present in the physical environment, proportional space allocation is considered as an input variable. Accumulated student activity in the proportionally allocated spaces has been considered as an intermediate variable in order to link building parameters to achievement/outcome parameters, and an indicator referred to as the aggregate effective space use ratio (denoted by E~~U~~ in later sections) is derived for this purpose. Acknowledging that the skills assessed in NAPLAN (literacy, numeracy, spelling, grammar and punctuation and speaking) are a few among many indicators for measuring the educational outcome, R- denoted by NAPLAN results is considered as the dependent variable representing learning outcome. Figure 2 presents the key steps adopted in the research approach. The first step is to relate the building space to relevant educational activities performed in the space. An indicator called aggregate effective space use ratio, E~~U~~ is introduced in this process to establish a relationship between the aggregate effective space use ratio, E~~U~~, and the educational performance measured through NAPLAN, R. Based on the nature of the relationship, it is approximated to follow a pattern of diminishing returns (and hence approximated to follow Hill function). The methodology to validate the approximation and solve the function to obtain the half-maximum value of E~~U~~ relating to each E~~U~~-R combination would be carried out in Step 2. Step 3 involves the use of multi-criteria optimisation to obtain the optimum proportional space allocations that would lead not to compromise of educational outcomes due to the non-availability of space for a given use. Each step is elaborated in detail in Sections 3.2.1 to 3.2.4.



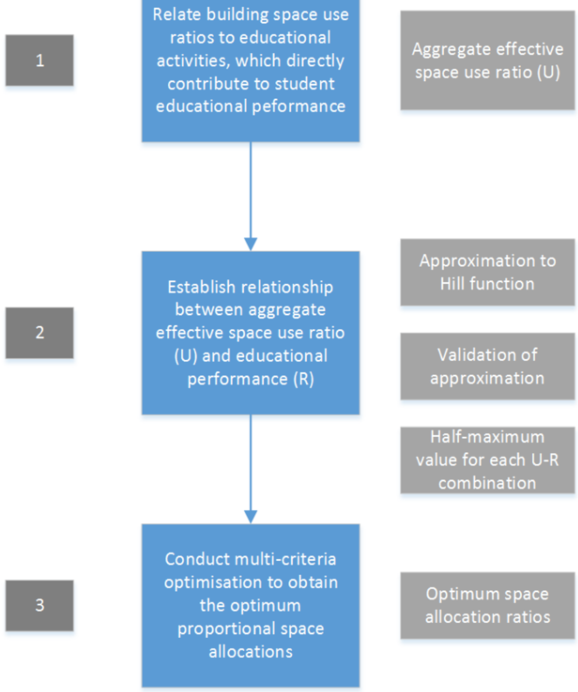


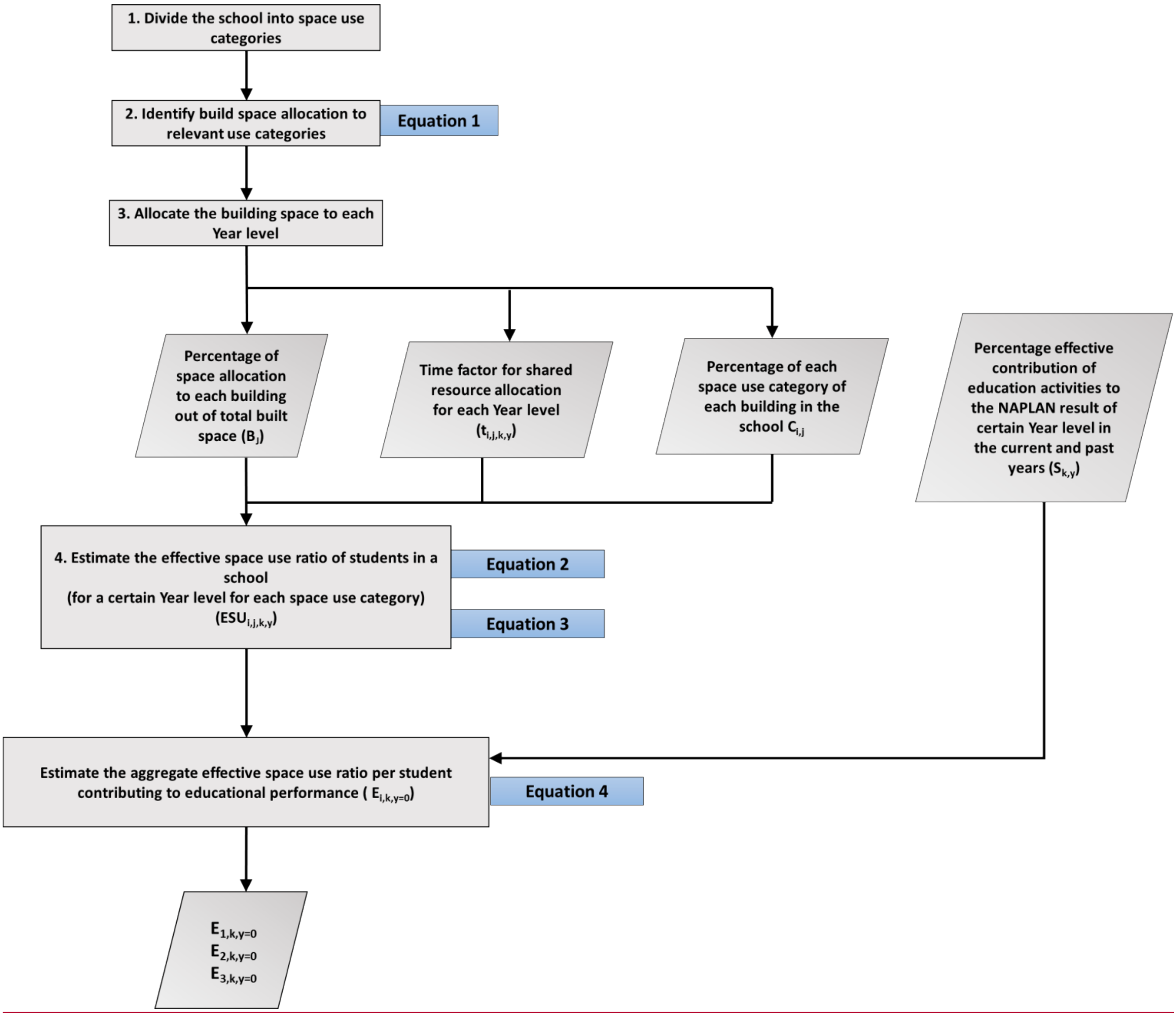
Figure 2: The key steps adopted in the research approach

3.2 Theoretical framework

This section presents the theoretical framework setting out the methodology.

3.2.1 Step 1: Relate building space use ratios to educational activities

The theoretical framework for evaluating aggregate effective space use ratio (E~~U~~) is presented using Figure 2. The first step involves dividing the built space of a school into five categories based on the fundamentals, each of them identified as a space use category (SUC). The SUCs directly contributing to educational performance of students are identified to be SUC 1, SUC 2 and SUC 3. Acknowledging the fact that there would be shared spaces used by students of all Year levels in a school, a time-sharing factor for each Year level is used for common resources to identify the effective space use by a student of a designated Year level for each SUC. As the educational activities of an average student of a certain Year level contributes to the student’s educational performance as measured by NAPLAN at a subsequent Year level, the aggregated (cumulative) effective space use for educational activities (E~~U~~) is estimated by aggregating proportional allocations of space use of each Year level, contributing to their educational performance. The theoretical framework considering the above steps, is further elaborated in Sections 3.2.1.1 to 3.2.1.4.


1. Divide the school into space use categories
2. Identify build space allocation to relevant use categories
Equation 1
3. Allocate the building space to each Year level
Percentage of space allocation to each building out of total built space (B_j)
Time factor for shared resource allocation for each Year level ($t_{i,j,k,y}$)
Percentage of each space use category of each building in the school $C_{i,j}$
Percentage effective contribution of education activities to the NAPLAN result of certain Year level in the current and past years ($S_{k,y}$)
4. Estimate the effective space use ratio of students in a school
(for a certain Year level for each space use category)
($ESU_{i,j,k,y}$)
Equation 2
Equation 3
Estimate the aggregate effective space use ratio per student contributing to educational performance ($E_{i,k,y=0}$)
Equation 4
$E_{1,k,y=0}$
$E_{2,k,y=0}$
$E_{3,k,y=0}$

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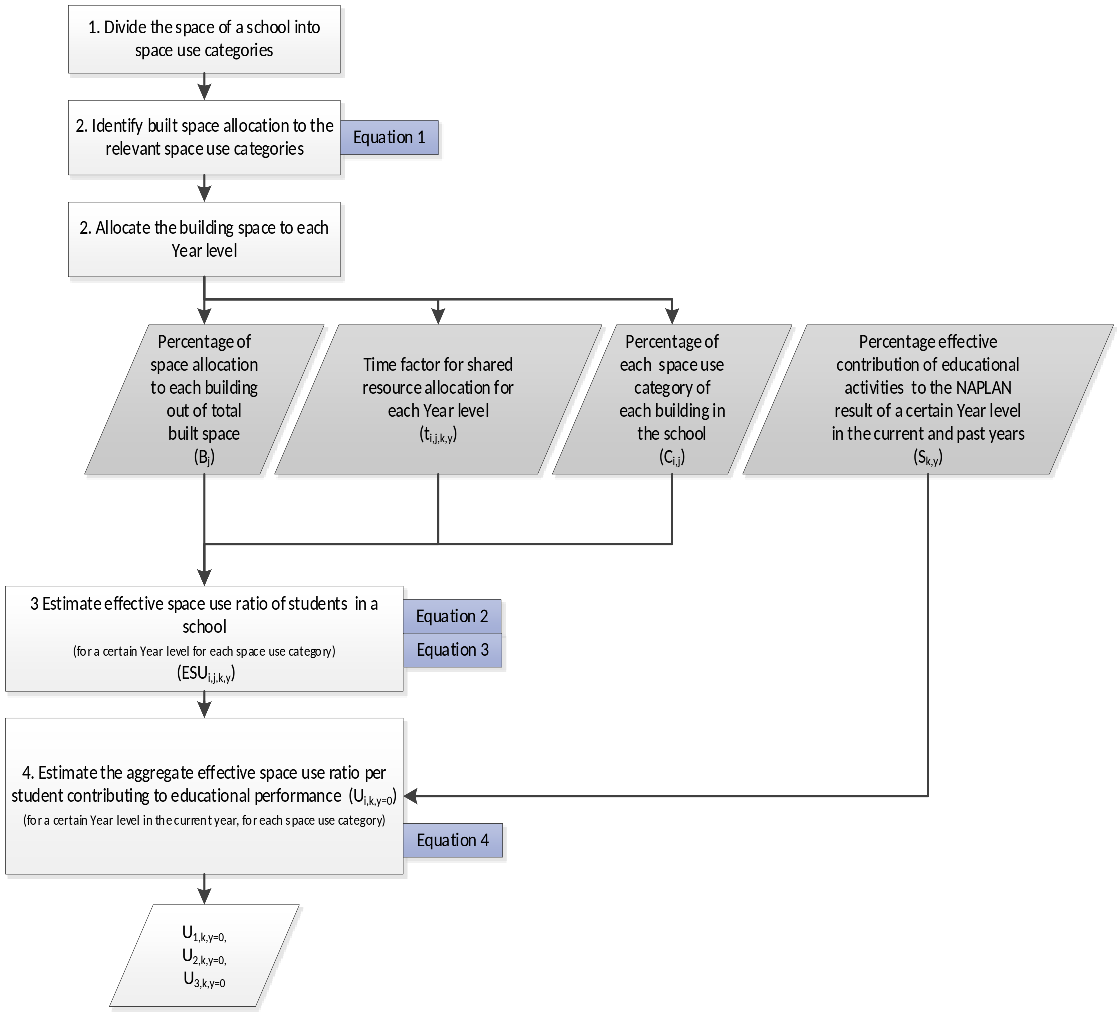


Figure 3: Conceptual framework to evaluate the aggregate effective space use ratio

3.2.1.1 Division of school space to space use categories

A school would consist of different buildings, which would add together to form its entire built space. The space of each school building is divided into five main categories, identified as space use categories (SUC). The ratio of space for each SUC out of total space is referred to as the space use category ratio (SUCR) and termed $C_{i,j}$, where i is the SUC as identified in Table 1 and j is the reference number of the building. This is diagrammatically presented in Figure 4.

Table 1: Building space categories

i	Building space category
1	Classrooms
2	Laboratories and activity rooms
3	Libraries and resource spaces
4	Walkways and circulation spaces
5	Recreation facilities, common spaces and other spaces

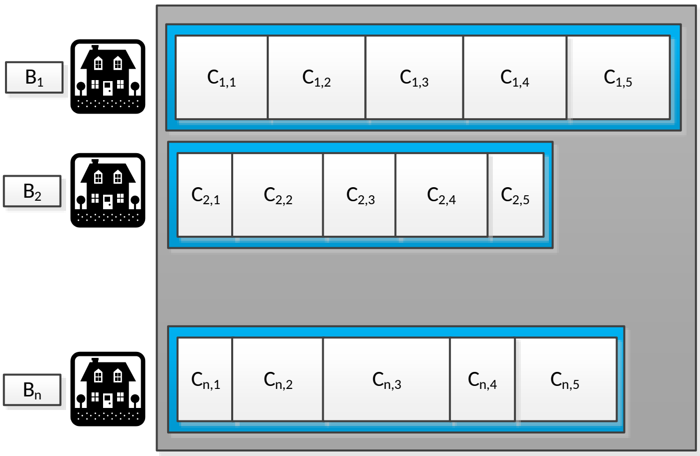


Figure 4: The graphical demonstration of the built space allocation of a school to different space use categories

For the purpose of this study, SUC 1, SUC 2 and SUC 3 are identified as directly relevant to educational activities, which can be measured through the student performance in NAPLAN. The relative importance of these categories relative to each other (e.g. the importance of classroom relative to library spaces) is not analysed in this study. Based on that, the total educational space can be calculated as a ratio of the total built space based on the above definition of SUCs. Social learning spaces, such as break-out rooms, where impromptu/informal discussions happen on a group project is placed under SUC 2.

For each particular building *j* in a school, the sum of the ratios of $C_{i,j}$ is equal to 1, considering that educational space consist of SUCs 1, 2 and 3, which have a direct contribution to the student performance measured through NAPLAN.

$$\sum_{i=1}^{3} C_{i,j} = C_E \qquad (1)$$

It is assumed that recreational facilities and common, and other spaces and circulation spaces denoted by $C_{4,j}$ and $C_{5,j}$, respectively, does not have a direct correlation to the educational outcome achieved through NAPLAN. However, there are some studies indicating that the physical activities and recreation have a correlation to the educational outcome achieved. Extra-curricular activities in general have been found to have associations with the educational outcome. As an example participation in sports have been found to improve educational outcome, particularly numerical skills (Shulruf, 2010), however, there are other studies indicating otherwise. A comprehensive study analysing a sizeable number of associations has revealed that the association is marginally positive or no association exists between the educational outcome and school-based physical activity (Bird, Tripney, & Newman, 2013). For the purpose of ~~simplicity, and~~simplicity and acknowledging that conflicting views exist on this debatable topic, this study has excluded the effect of school-based extra-curricular activities (which consume the space of school buildings) on the educational outcome.

It is also noted that walkways and circulation spaces denoted by SUC5 are usually governed by the building regulations, where there are minimum but not a maximum standard applicable. As only the SUC relating to direct educational outcome measured through NAPLAN are considered for this study, $C_{4,j}$ and $C_{5,j}$ is not considered as a variable. Therefore, in determining the ratios, the total educational space has been made equal to 100%.

3.2.1.2 Allocate the building space to each Year level based on usage

Generally, any given building in a school could have students in the respective Year levels. Therefore, the use of a building will have activities pertaining to different Year levels. It is assumed that the students share SUC_1, SUC_2 and SUC_3 on a time allocation basis, with each year level having an allocation of $t_{i,j,k,y}$, where k is the particular Year 1 to n; i would be the number denoting SUCs 1,2 and 3; j is the building number and y represents the relevant year of data gathering. If data is gathered concerning the current year, y=0. If there are dedicated spaces, then the, $t_{i,j,k,y}$ for that SUC becomes 1 for that Year level.

Therefore, the use of a certain category of SUC by students in a certain Year level of a building, is defined as the effective space use ratio of the school, $ESU_{i,j,k,y}$

$$ESU_{i,k,y} = \sum_{j=1}^{b_n} C_{i,j} * t_{i,j,k,y} * B_j \tag{2}$$

where B_j is percentage of space allocation to each building out of the total built space.

As the focus of the paper is on the design of schools in comparison to determining usage and operational matters that come into effect at the operational stage of schools, the time sharing factor is taken to be a constant and it is assumed that the common spaces are allocated equally among students of each Year level of a particular school. Therefore:

$$t_{i,j,k,y} = 1/k_n \tag{3}$$

where, k_n is the number of Year levels in the particular school under consideration.

3.2.1.3 Aggregation of the effective use of educational space to contributing to educational outcome

When a student makes progress with educational activities in a certain year and moved on to the subsequent Year level, their effective space use contributing to their educational performance accumulates. Therefore, it is considered that a student's education at a certain Year level, k contributes by a percentage of $S_{k,y}$ to the educational assessment results. In elaborating methodology, for simplicity it is assumed that the educational activities of y number of years contribute to the educational outcome measured through NAPLAN for a certain year. Therefore, the aggregated effective space use ratio leading to relevant educational performance measured by NAPLAN for each school, for Years 3,5,7, for each SUC for a school is given by E~~U~~$_{i,k,\ y=0}$,

$$\underline{E}\cancel{U}_{i,k,y=0} = \sum_{y=-m}^{0} ESU_{i,k,y} * S_{k,y} \tag{4}$$

where, m is equal to the number of years of accumulation of educational activities contributing to a particular assessment result. For this study, 2 years of aggregation of activities is assumed to relate to the outcome and hence y=0 (current year) and -1 (previous year) are considered for simplicity. In deriving Equation 4, it is assumed that the absolute space use per student for SUC1, SUC 2 and SUC3 are governed by the ergonomic guidelines in school design and hence maintained within a range in any given school to adhere to the relevant compliance criteria. This study excludes the analysis of absolute space use allocated to a student from its scope and is only concerned with analysing the effect of proportional distribution of space allocation ratios to the educational performance. Therefore, the study assumes that the student numbers have not changed over the year significantly, which could have an impact on shared resource usage and space allocations to each Year level and the spaces provided by the schools are spacious enough for students to pursue their educational activities with convenience, abiding by the ergonomic standards.

3.2.2 Step 2: Establish relationship between aggregate effective space use ratio (E~~U~~) and educational performance (R)

In order to identify how the effective space use ratio relates to the NAPLAN score, the structure of NAPLAN exam is looked at. The NAPLAN test measures the literacy, numeracy, writing, spelling, grammar and punctuation; therefore the NAPLAN result for the respective skills are denoted by $R_{L,k}$, $R_{N,k}$, $R_{W,k}$, $R_{S,k}$ and $R_{G\&P,k}$, where k denotes the Year level NAPLAN score relates to (Year 3,5,7, and 9) and L, N,W,S and G&P denotes the skill levels tested with NAPLAN, namely, literacy, numeracy, writing, spelling and grammar and punctuation. With the aggregate effective space use ratio - *E~~U~~* being identified to relate to the space use to perform educational activities, which is measured through NAPLAN score- *R*, a relationship can be made between *R* and *E~~U~~* so that R is a function of E~~U~~, as in:

$$R_{X,k} = f(\underline{E}\cancel{U}_{1,k,y=0}, \underline{E}\cancel{U}_{2,k,y=0}, \underline{E}\cancel{U}_{3,k,y=0}) \quad (5)$$

where, *X* refer to any of the NAPLAN scores.

3.2.2.1 Identification of the nature of the function

As there are multiple input variables ($\underline{E}\cancel{U}_1$, $\cancel{U}\underline{E}_2$ and $\cancel{U}\underline{E}_3$) relating to multiple output variables, this is identified as a multivariate relationship and graphically demonstrated as in Figure 5.

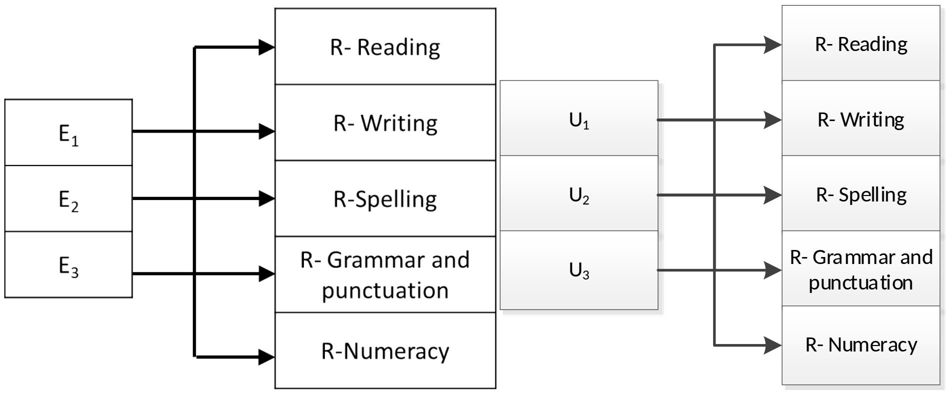


Figure 5: Relationship between E~~U~~ and R in a graphical representation

In the meantime, it is observed that *E~~U~~*'s relationship to *R* follows a pattern of diminishing returns. The reason for the approximation is logically explained as below.

While the increase of aggregate effective space use ratio would contribute to increased educational performance, it can be expected that increase of ratio from low levels to mid/average levels would result in significantly improved educational performance. However, after a certain threshold, increase of input variable has a marginal impact on the output variable, R, and the incremental improvement in R per an incremental increase in E~~U~~ starts to reduce after a certain point.

As an example, for any school the first criteria to check would be whether there is space available for the specific SUCs identified, as an example SUC2 indicates availability of library space. The impact on educational performance is expected to be significant when comparing two schools; one having library space and the other no having library space (significant increase in R for E~~U~~ close to zero). However, if a school has a significant proportion of space allocation to libraries, it can be reasonably considered that the educational performance would not be significantly improved by further increase in library space (diminishing rate of increase of R for higher E~~U~~ values), as the impact on having libraries after a certain threshold would not have an influence on educational performance (Mancell, 2005). This reflects diminishing returns after a threshold value of *E~~U~~*.

3.2.2.2 Hypothesizing and approximating an appropriate function

Approximating that the relationship would follow a pattern of diminishing returns based on the above rationale, the next step would be to characterise the relationship using a function that would behave in a similar manner.

Considering non-linear regression and Hill function for characterisation of a function indicating diminishing returns, Hill function was chosen. Hill function has a typical S-shaped curve, which best describes the nature of the relationship discussed above, and also could be generalised to a half-parabolic function, which is the simplest form of a diminishing returns function. Therefore, it is identified that consideration of Hill function could offer better options to explore intricate features associated with a S-shaped curve.

Hill function, which was originally used for biomedical applications have many other common uses (Likhoshvai & Ratushny, 2007). Figure 6 diagrammatically presents the typical S-shaped curve associated with Hill function. The x axis represents the aggregate effective space use variable $E\cancel{U}$ as a percentage of the maximum possible aggregate effective space use value ($E\cancel{U}_{max}$). Similarly, y-axis of the graph denotes the NAPLAN score as a percentage of the maximum possible NAPLAN score. $E\cancel{U}_{max}$ implies the building design facilitates maximum possible aggregate effective space use possible within the regional design standards of the school. However, achieving maximum aggregate effective space usage may not always be feasible as it would demand capital investment. Also, scenarios where $E\cancel{U}_{max}$ can be achieved for all space categories would be a rare occurrence in real-world due to building space limitations. Therefore, optimising the aggregate effective space usage for delivering optimal educational outcome within the feasible budget and capacity constraints is critical for school building designs.

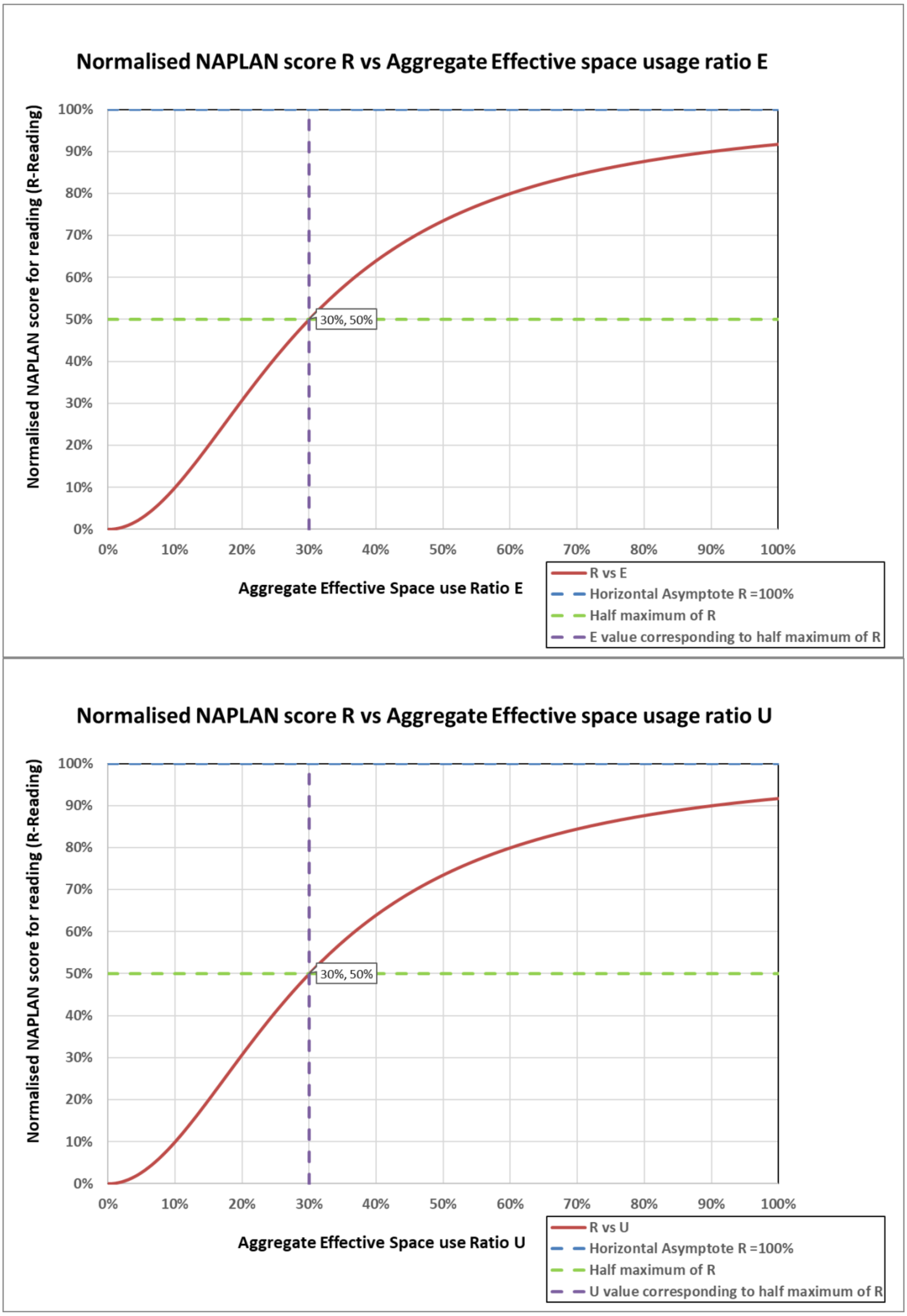


Figure 6: Diagrammatic explanation of Hill function, hypothesized to correlate educational outcome to aggregate effective space use

3.2.2.3 Analysing the characteristics of Hill function and relating with the study outcome

Approximating that the relationship between R and ~~U~~E follows a Hill function based on the rationale described in Section 3.2.2.1, the Hill equation can be written as:

$$\mathrm{R} = \frac{aE\cancel{U}^{n}}{b^{n} + E\cancel{U}^{n}} \qquad (6)$$

where, *n* is the Hill coefficient, *a* is the horizontal asymptote which is the maximum normalised NAPLAN result in this instance and *b* is the half-maximum value of the horizontal asymptote a (value *x* corresponding to the half of the maximum value of R, which is *a*).

As an example, we can take the NAPLAN score for literacy of Year 3 students to relate to $\cancel{U}E_1$ as below.

$$R_{L,3} = \frac{a_{1,3}E\cancel{U}_{1,3}{}^{n_{1,3}}}{b_{1,3}{}^{n_{1,3}} + E\cancel{U}_{1,3}{}^{n_{1,3}}} \qquad (7)$$

Analysing the characteristics of a Hill function based on Equation 6, the following can be observed.

- For values of ~~U~~E close to zero, $\cancel{U}E^n$ in the denominator becomes negligible: $R = a\cancel{U}E^n/b^n$
- For values of E~~U~~ close to infinity, *b* becomes negligible. Therefore: *R= a*
- For values of $E\cancel{U}=b$, $R=a/2$, Therefore b is called the half-maximum value of the horizontal asymptote *a*.
- The function follows a pattern of diminishing returns up to $x=b$, with the gradient of the function increasing up to $x<b$ and decreasing after $x>b$.

With E~~U~~ represented with the *x* axis and *R* represented with the *y* axis as stated above, it is observed that when ~~U~~E is increased from $\cancel{U}E=0$ to $E\cancel{U}=b$, *R* values grow exponentially. This indicates that for ~~U~~E values below $\cancel{U}E=b$, increase of ~~U~~E results in an exponential increase of *R*. However, after $E\cancel{U}=b$, the increase of *R* with the increase of E~~U~~ is occurring with a decreased gradient until ~~U~~E reaches infinity (while *R* reaches *a*). This indicates that value of E~~U~~ above the value of $E\cancel{U}=b$, the increase of *R* decelerates.

Two basic characteristics/rules of the function are identified based on the above:

1. for values of E~~U~~ below $\cancel{U}E=b$ ($E\cancel{U}<b$), the compromise of R is significant for a unit decrease of E~~U~~
2. for values of ~~U~~E greater than $\cancel{U}E=b$ ($E\cancel{U}>b$), the reduction of *R* value becomes less significant for a unit increase of E~~U~~

We can reasonably conclude that the minimum value of E~~U~~ to maintain by any school would be E~~U~~=b, as when E~~U~~ falls below *b*, significant compromise of *R* is observed.

This principle can be applied to SUC1, SUC2 and SUC3 to evaluate the optimum $E\cancel{U}_1$, $E\cancel{U}_2$ and $E\cancel{U}_3$ values relating to the five NAPLAN scores ($R_{L,k}$, $R_{N,k}$, $R_{W,k}$, $R_{S,k}$ and $R_{G\&P,k}$) respectively. Applying Hill function and the above theory to this study, the objective of characterising and solving the relationship can be set to find *n*, *a* and *b* associated with each of the S-curve related to each E~~U~~-R combination. While *n* would determine the shape of the S-curve, *a* would determine the maximum result (R). The most important finding for this study however is the half maximum value of the horizontal asymptote *a*, denoted by *b*, as this would set the minimum value of *E*~~*U*~~ to be maintained for each *E*~~*U*~~$_1$, ~~*U*~~*E*$_2$ and ~~*U*~~*E*$_3$ values for each ~~U~~E-R combination.

3.2.2.4 Process for validating the hypothesis

This section presents the methodology for validating the applicability of the proposed hypothesis with the use of data obtained from schools.

By rearranging the Equation 6, Equation 8 can be obtained.

$$\frac{\left(\frac{R}{a}\right)}{\left(1-\left(\frac{R}{a}\right)\right)} = \left(\frac{E\cancel{U}}{b}\right)^{n} \tag{8}$$

Taking the ~~left hand~~left-hand side (LHS) to be equal to V Equation 8 can be rearranged as:

$$V = E\cancel{U}^{n} * b^{-n} \tag{9}$$

By taking the natural logarithmic (ln) values of both sides

$$\ln(V) = n\ln(E\cancel{U}) - n\ln(b) \tag{10}$$

This corresponds to a linear function equivalent to *y = mx+c*, with *x= ln(E*~~*U*~~*)*, *y= ln(V)*. The gradient, *m* is given by *n* and the intercept, *c* corresponds to *ln(b)*. By plotting *ln(V)* against *ln(*~~*U*~~*E)* the linearity of the relationship can be checked and hence providing proof of the hypothesis. The correlation coefficient relating to the linear plot would determine the fit of the relationship to consider as a strong or weak linear function.

While *ln(*~~*U*~~*E)* can be calculated by the data obtained from schools (in accordance with the data presented in Figure 2), in order to evaluate *ln(V)* values, a need to be determined as per Equation 7. Horizontal asymptote, *a*, for each ~~U~~E-R combination need to be obtained by plotting *R* against ~~*U*~~*E* first for the estimation of *V* and hence *ln(V)*.

3.2.2.5 Solving the function

As mentioned in Section 3.2.2.3, the objective of characterising the function is to identify *a*, *n* and *b* values.

R against ~~*U*~~*E* is plotted first to identify the maximum value of R, which is considered as the horizontal asymptote for each Hill curve. *R* in this instance would be the absolute NAPLAN score normalised to the maximum score nationally. Therefore, *a* would be the highest *R*. Using *a* in Equation 7, *V* values can be estimated to apply in Equation 9. By creating a plot of Equation 9, *n* and *b* values can be estimated by identifying the slope (*m*) and the intercept (*c*) of the trend line, considering its linearity. The slope, *m* corresponds to *n* while:

$$b = e^{-\left(\frac{c}{n}\right)} \tag{10}$$

where, *c* corresponds to the intercept of the function.

Adopting this method would enable identification of b values corresponding to each ~~U~~E-R combination considering the five NAPLAN scores and ~~U~~E_1, E~~U~~$_2$ and E~~U~~$_3$ values. This would result in 15 values of b, stated in a matrix as below, related to one assessment of NAPLAN.

$$b = \begin{pmatrix} \mathrm{b}_{1,R} & \mathrm{b}_{1,W} & \mathrm{b}_{1,S} & \mathrm{b}_{1,S\&P} & \mathrm{b}_{1,N} \\ \mathrm{b}_{2,R} & \mathrm{b}_{2,W} & \mathrm{b}_{2,S} & \mathrm{b}_{2,S\&P} & \mathrm{b}_{2,N} \\ \mathrm{b}_{3,R} & \mathrm{b}_{3,W} & \mathrm{b}_{3,S} & \mathrm{b}_{3,S\&P} & \mathrm{b}_{3,N} \end{pmatrix} \tag{11}$$

Each of this *b* value represents the minimum ~~*U*~~*E* value to maintain for each of the *R* values (representing the specific skills assessed by NAPLAN). As this presents 15 conditions to abide by in determining the optimum space use ratios of a school, an optimisation problem is required to find out the optimum values in order to fulfil the most relevant requirements stipulated by each E~~U~~-R combination.

3.2.3 Step 3: Conduct multi-criteria optimisation

The objective of the study is to identify the optimum space allocation ratios for ~~U~~E_1, ~~U~~E_2, and E~~U~~$_3$ so that the identified values are above the half-maximum value of b for each E~~U~~-R combination. As per the above discussed hypothesis that would indicate that the ratios of space allocation for SUC1, SUC 2 and SUC 3 would not result in significant compromise of NAPLAN scores.

Let us call this optimum values x_1, x_2 and x_3. ~~If x~~If x_1, x_2 and x_3 are such that they are positioned after each *b* in the S-curve for each E~~U~~-R combination, we can introduce another variable called *p* where $p_i = x_i - b_i$. If p_i is a positive value for all E~~U~~-R combinations, it is considered as an optimum result. Equally, larger all the p_i ~~values, larger~~values, larger the sum of p_i for all positive p_i values, the more optimum *R* is, which is desirable in terms of educational outcome. At the same time, a smaller all the p_i values and a smaller the sum of p_i for all negative p_i values, close the x_i is to the *b* value in the S-curve, which is considered second best if p_i is negative.

Based on the above, the objectives for the problem to identify the optimum x_1, x_2 and x_3 values are:

- Primary objective (Priority 1): to have the maximum number of p_i values>0 out of 15 values
- Secondary objective 1 (Priority 2): to maximise the sum of p_i values for positive p_i values
- Secondary objective 1 (Priority 2): to minimise the sum of p_i values for negative p_i values

One important thing to note is that the sum of x_i values is equal to 1, which acts as a linear constraint to the problem and x_i values are constrained to be between 0 and 1, which act as non-linear constraints to the problem. This is due to the fact that the education space is considered as 100% and referring to Equation 4, E~~U~~ becomes equivalent to C_i with the use of percentage values for aggregation.

Considering the above three objectives and constraints, multi-criteria optimisation using genetic algorithm is carried out to identify x_1, x_2 and x_3 values for this problem.

4 Application, demonstration and results

This section demonstrates the application of the process and shows sample results when the methodology is applied. A parametric study is carried out to demonstrate the variation in results, where a sample data set is applied using this methodology.

4.1 Step 1: Relate building space use ratios to educational activities

4.1.1 Application and demonstration

For this demonstration, a hypothetical primary school, which conducts classes up to Year 5, has been considered; ~~therefore~~therefore, the students are expected to sit NAPLAN examination at Year 3 and Year 5.

Considering a sample data set relating to the current context of Australian primary schools, the space allocation ratios for each respective building for the current year and the previous year are presented in Table 2, together with the ratio of space of each building.

Tables 3 provides the data on the estimated contribution of each Year level to the educational performance measured at a subsequent year level and the equality assumption on usage among all Year levels. As this paper is mainly concerned with finding ratios of building SUCs to provide inputs for design, the factors concerning usage has been maintained constant. Therefore, it is assumed that common SUCs are used equally among all Year levels, resulting in 20% usage per each level for a primary school consisting of Year 1,2,3,4 and 5. The assumed percentage contribution of each Year level to the performance indication by NAPLAN is stated in Table 4. Table 5 provides the current statistics for NAPLAN as extracted from National Report for 2016, pertaining to 2016 for each of the five skills measured (Australian Curriculum Assessment and Reporting Authority, 2016).

Table 2: C_i and B_j values for the sample school for the current and previous year

Building	Building space allocation - Current year					Building space allocation - Previous year					Pecentage of building area as a total built space (B_j)
	Y=0 (Current year)					Y=-1 (Previous year)					
	C_1	C_2	C_3	C_4	C_5	C_1	C_2	C_3	C_4	C_5	
1	5%	20%	30%	4%	41%	5%	10%	30%	4%	51%	24%
2	20%	50%	13%	5%	12%	10%	50%	10%	8%	22%	13%
3	10%	27%	25%	10%	28%	10%	34%	34%	10%	12%	11%
4	45%	22%	16%	5%	12%	45%	23%	23%	5%	4%	33%
5	10%	45%	23%	10%	12%	10%	54%	23%	10%	3%	20%

Table 3: Effective contribution to educational performance measured through NAPLAN by each Year level

Primary school	Effective contributin to NAPLAN score	Effective space usage - Current year	Effective space usage - Previous year
Year 1	0%	20%	20%
Year 2	30%	20%	20%
Year 3	70%	20%	20%
Year 4	30%	20%	20%
Year 5	70%	20%	20%

Table 4: NAPLAN indicators for Year 3 in 2016

Year 3	Mean	Standard deviation
Reading	425.6	85.6
Writing	420.7	62.0
Spelling	419.8	85.8
Grammar	435.8	89.9
Numerac	402.0	73.4

4.1.2 Results

Methodology stipulated in Sections 3.2.1.1 to 3.2.1.3 has been performed and the resulting E~~U~~ values are mentioned in Table 5.

Table 5: Aggregate space use ratios for SUC1, SUC2 and SUC3 presented as $E\cancel{U}_1$, $E\cancel{U}_2$ and $E\cancel{U}_3$

Year level	U1	U2	U3
Year 3	4.21%	6.06%	4.44%

Year Level	E_1	E_2	E_3
Year 3	4.21%	6.06%	4.44%

4.2 Step 2: Establish relationship between aggregate effective space use ratio (E~~U~~) and educational performance (R)

4.2.1 Application and demonstration

Demonstration of this part of the methodology would require obtaining E~~U~~ and R values for a number of schools and plotting them against each other as a starting step. The R value concerned here the NAPLAN score of a public primary school normalised to the maximum NAPLAN score for that particular Year level, as a percentage.

By plotting R against ~~U~~E, a can be determined first as stated in Section 3.2.2.4. Then validation of the function to follow a Hill function can be done using the methodology stated in Section 3.2.2.5. By identifying the characteristics of the linear trend line, n and b values can be evaluated accordingly.

4.2.2 Results

As this section demonstrates the application of methodology with the absence of a complete data set, it is assumed that the data set follows the Hill function and the parameters obtained with the plotting of R against E~~U~~ have been varied to discuss the effect of such and to interpret them. The method for validating ~~this assumptions~~this assumption is stated in Section 3.2.2.4.

The parameters subject to change here are *a*- the horizontal asymptote, *b*- the half-maximum value and *n*- the Hill coefficient. Table 6 shows the parameters varied of Hill function as shown in Figures 7, 8 and 9. The figures in this section show plots of different S-curves associated with Hill function with the variations in *n* and *b*. *a* is estimated as the normalised NAPLAN score percentage based on the highest NAPLAN score for 2016 obtained from Table 4.

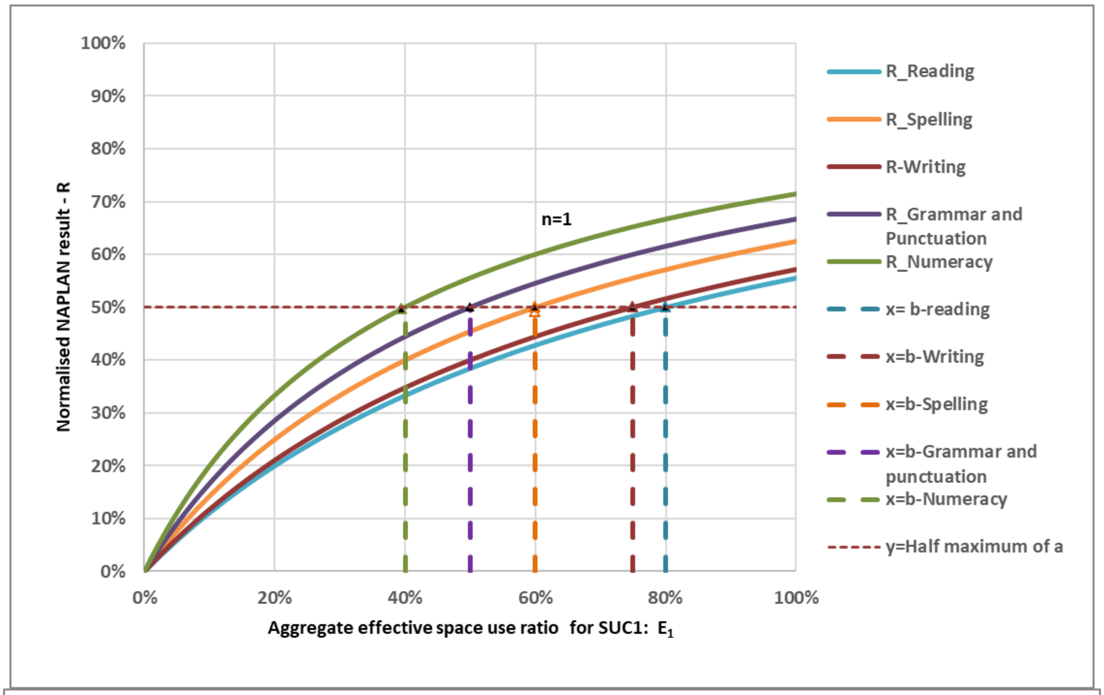
Figures 7-12 show that n, the Hill coefficient and b- the half-maximum value determines the shape of the Hill function. While *b* has been predetermined in this instance with the absence of data, the objective of creating a linear plot as mentioned in Section 3.2.2.5 would be to finally obtain the b value for each E~~U~~-R combination. In order to carry out the optimisation problem, which is the next step, a set of *b* values has been used as below. This is done considering the fact that the majority of the space would consist of classrooms (E~~U~~$_1$), and resource and activity spaces (E~~U~~$_2$ and ~~U~~E$_3$) would be relatively low compared to that with maximum sharing occurring with them. With approximated values for b for all three ~~U~~E categories and five NAPLAN scores measuring different skill levels, the following matrix has been obtained.

Table 6: Parameters of Hill function in the Figures 7~~6~~-12~~1~~

Figure	E_1	R_Reading	R-Writing	R_Spelling	R_Grammar and Punctuation	R_Numeracy
Figure 7	a	100%	100%	100%	100%	100%
	b	0.8	0.75	0.6	0.5	0.4
	n	1	1	1	1	1
Figure 8	a	100%	100%	100%	100%	100%

	b	0.8	0.75	0.6	0.5	0.4
	n	3	3	3	3	3
Figure	E_2	R_Reading	R-Writing	R_Spelling	R_Grammar and Punctuation	R_Numeracy
Figure 9	a	100%	100%	100%	100%	100%
	b	0.05	0.1	0.15	0.2	0.3
	n	1	1	1	1	1
Figure 10	a	100%	100%	100%	100%	100%
	b	0.05	0.1	0.15	0.2	0.3
	n	3	3	3	3	3
Figure	E_3	R_Reading	R-Writing	R_Spelling	R_Grammar and Punctuation	R_Numeracy
Figure 11	a	100%	100%	100%	100%	100%
	b	0.15	0.15	0.25	0.3	0.3
	n	1	1	1	1	1
Figure 12	a	100%	100%	100%	100%	100%
	b	0.15	0.15	0.25	0.3	0.3
	n	3	3	3	3	3

Figure	U_1	R_Reading	R-Writing	R_Spelling	R_Grammar and Punctuation	R_Numeracy
Figure 7	a	100%	100%	100%	100%	100%
	b	0.8	0.75	0.6	0.5	0.4
	n	1	1	1	1	1
Figure 8	a	100%	100%	100%	100%	100%
	b	0.8	0.75	0.6	0.5	0.4
	n	2	2	2	2	2
Figure 9	a	100%	100%	100%	100%	100%
	b	0.8	0.75	0.6	0.5	0.4
	n	4	4	4	4	4
Figure	U_2	R_Reading	R-Writing	R_Spelling	R_Grammar and Punctuation	R_Numeracy
Figure 10	a	100%	100%	100%	100%	100%
	b	0.05	0.1	0.15	0.2	0.3
	n	1	1	1	1	1
Figure 11	a	100%	100%	100%	100%	100%
	b	0.05	0.1	0.15	0.2	0.3
	n	2	2	2	2	2
Figure 12	a	100%	100%	100%	100%	100%
	b	0.05	0.1	0.15	0.2	0.3
	n	4	4	4	4	4



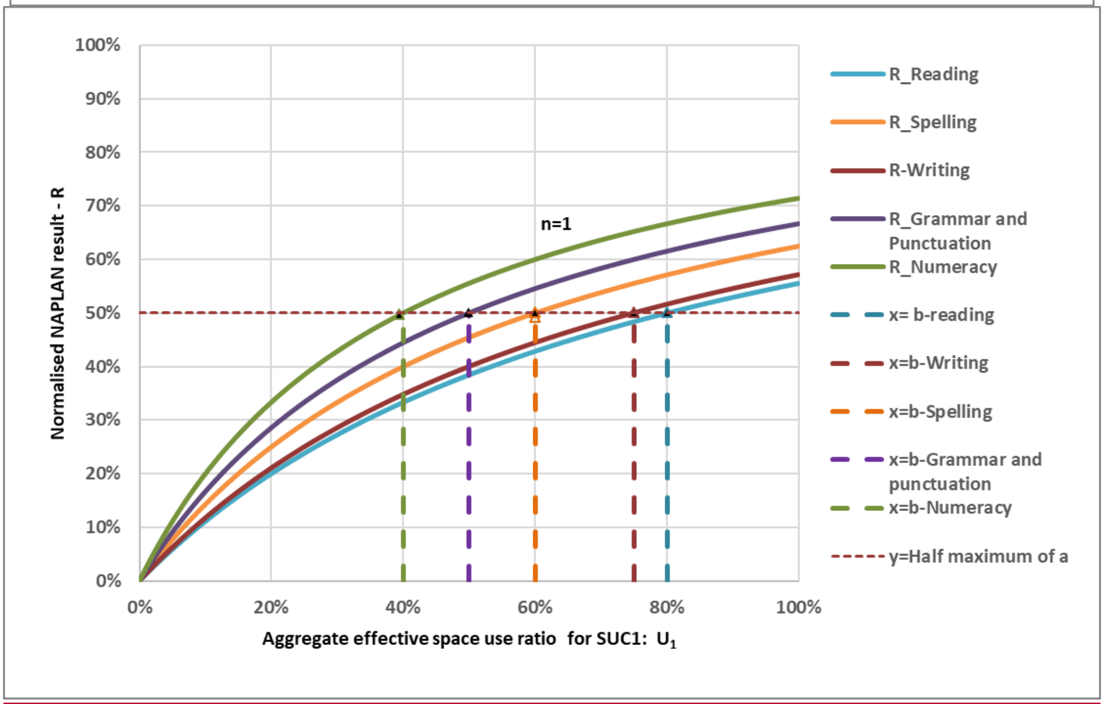
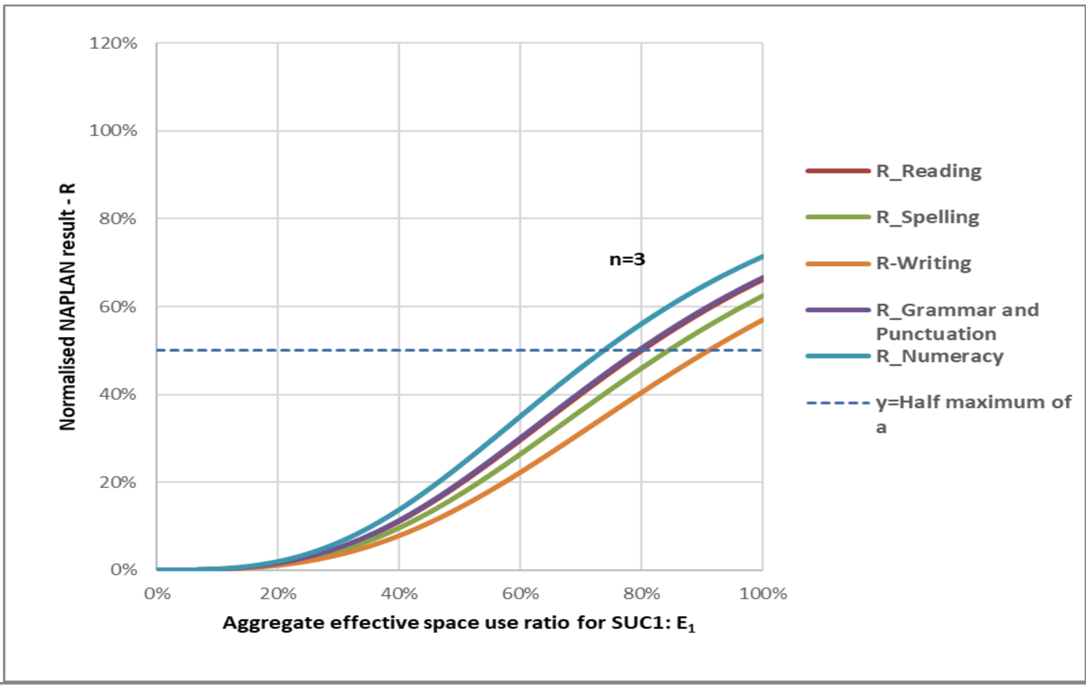


Figure 7: Normalised NAPLAN scores against ~~U~~E$_1$ with n=1



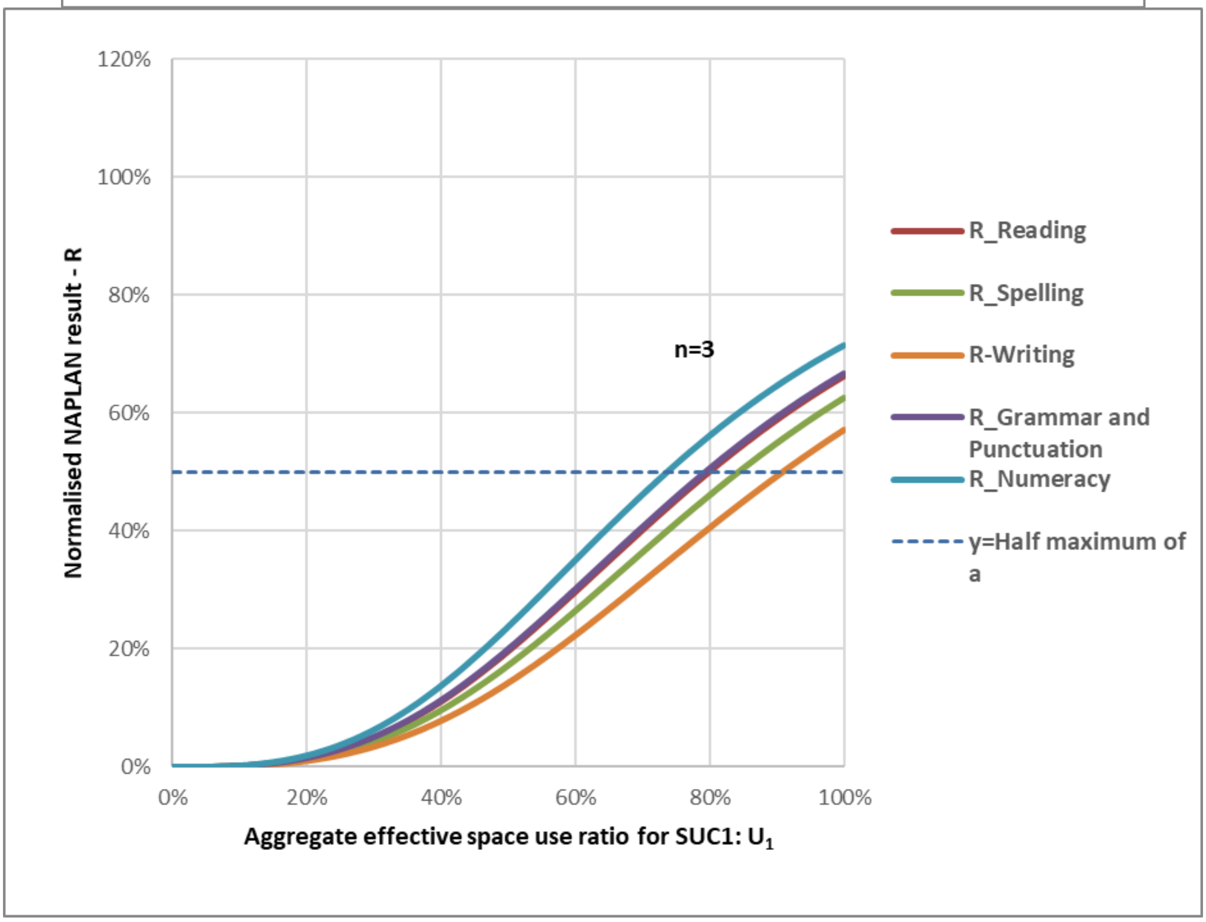
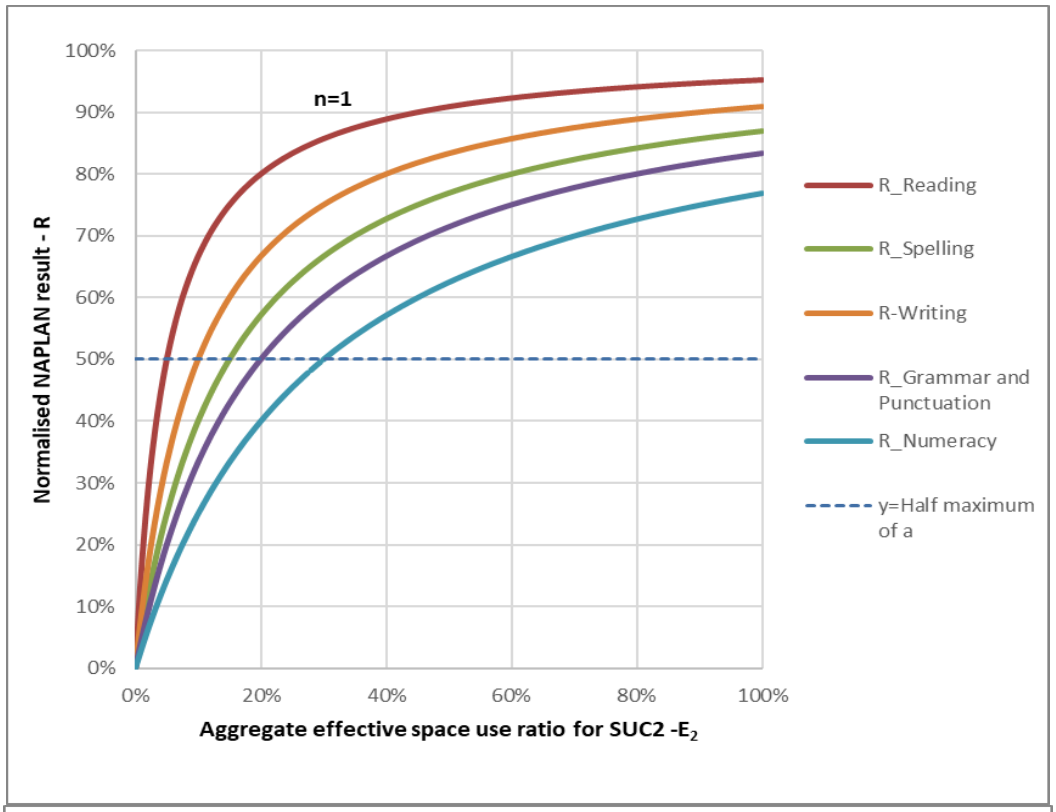


Figure 8: Normalised NAPLAN scores against E~~U~~$_1$ with n=3



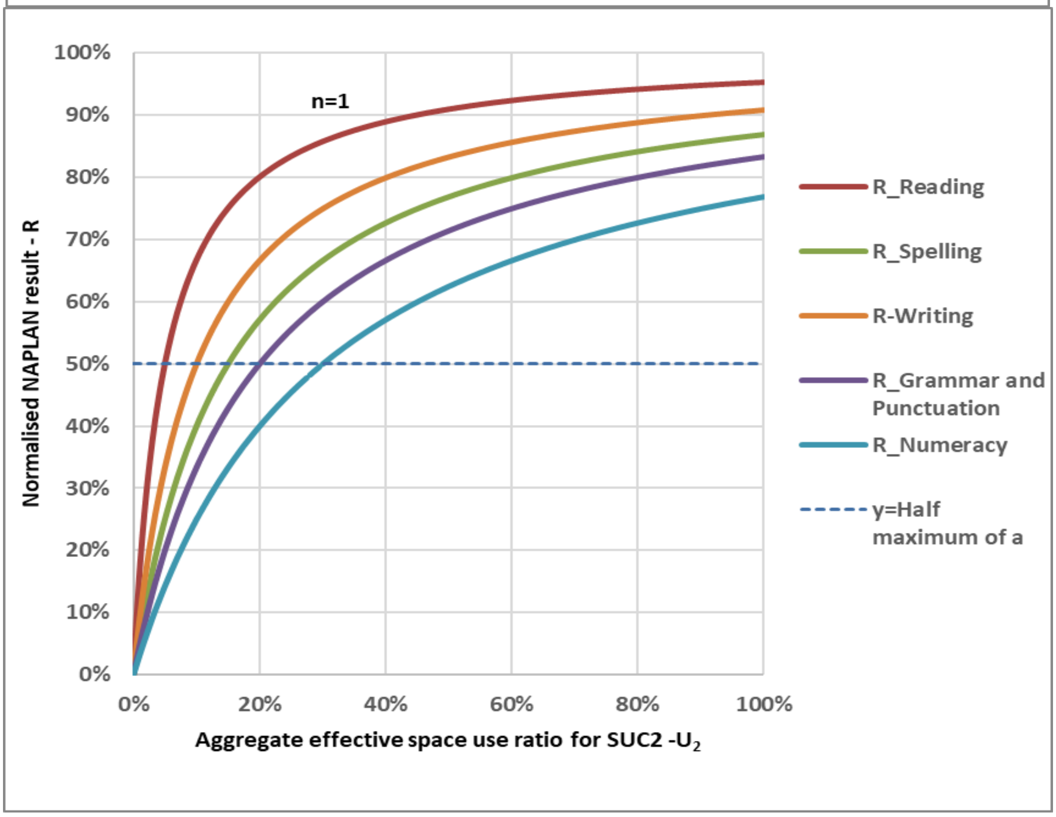
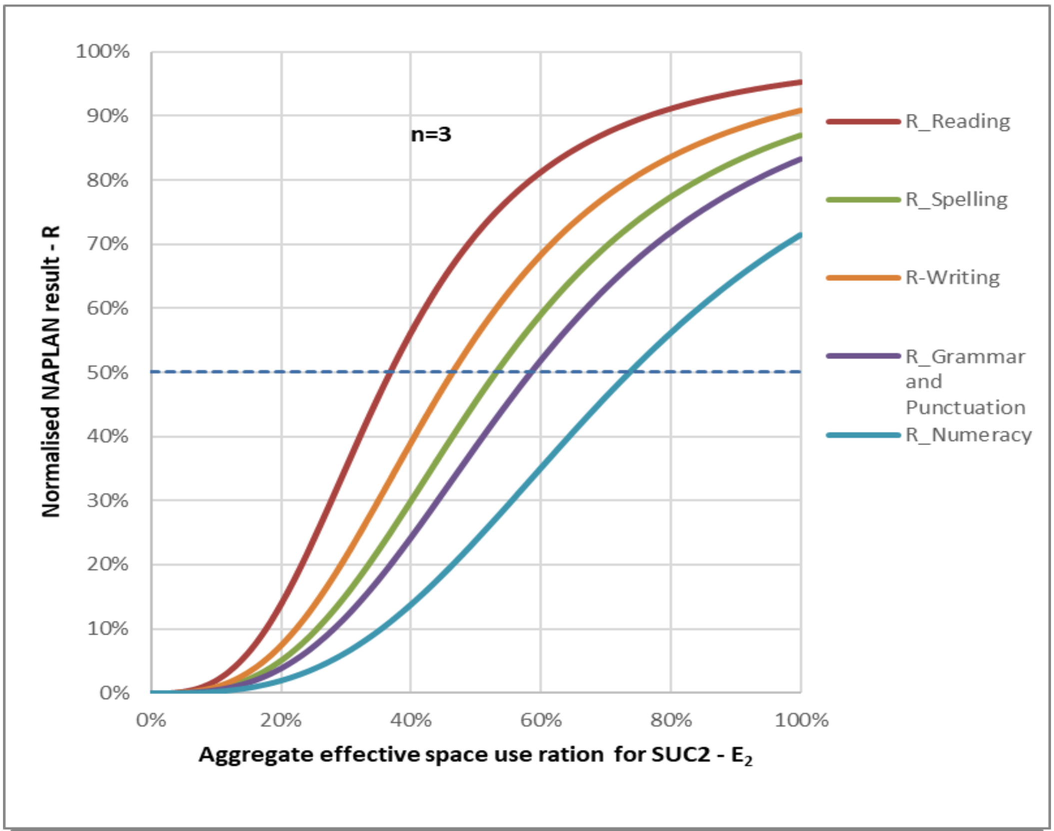


Figure 9: Normalised NAPLAN scores against E~~U~~$_2$ with n=1



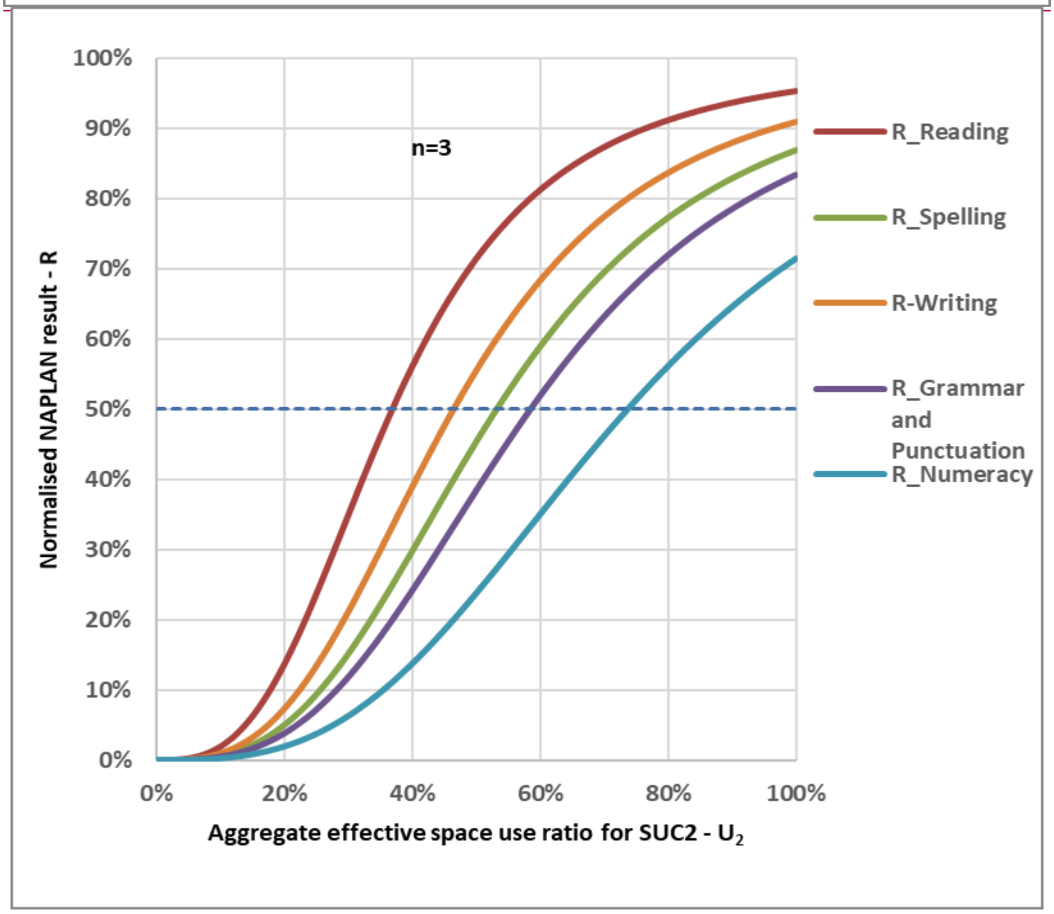
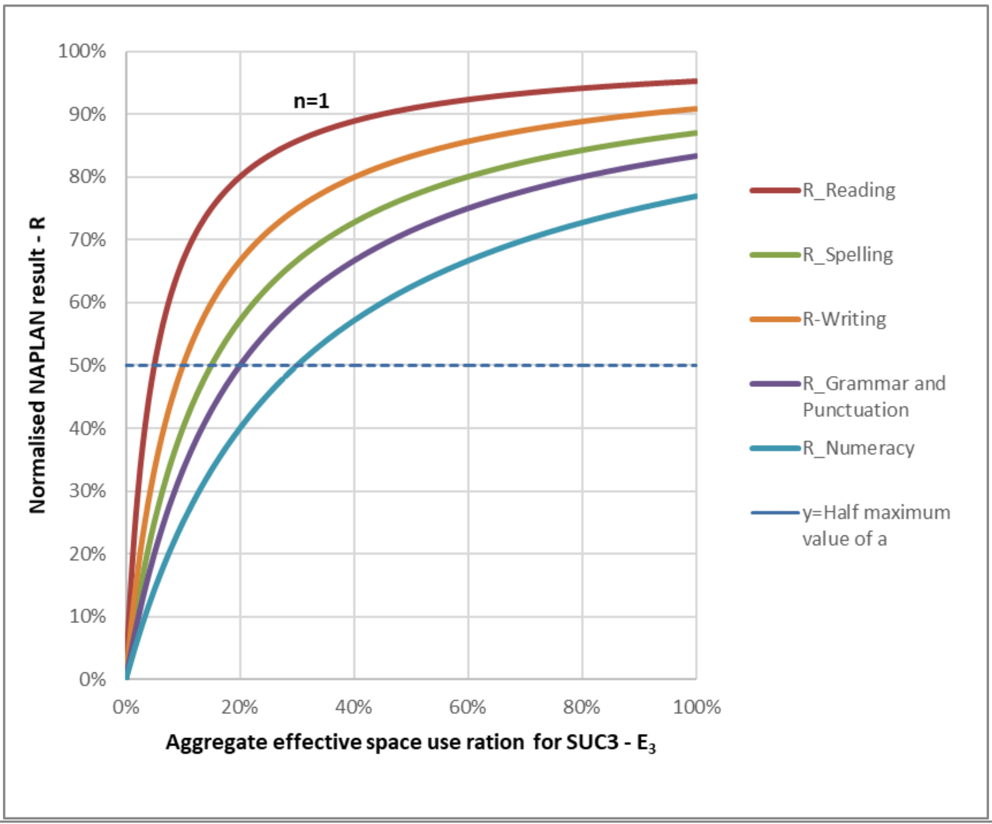


Figure 10: Normalised NAPLAN scores against E~~U~~$_2$ with n=3



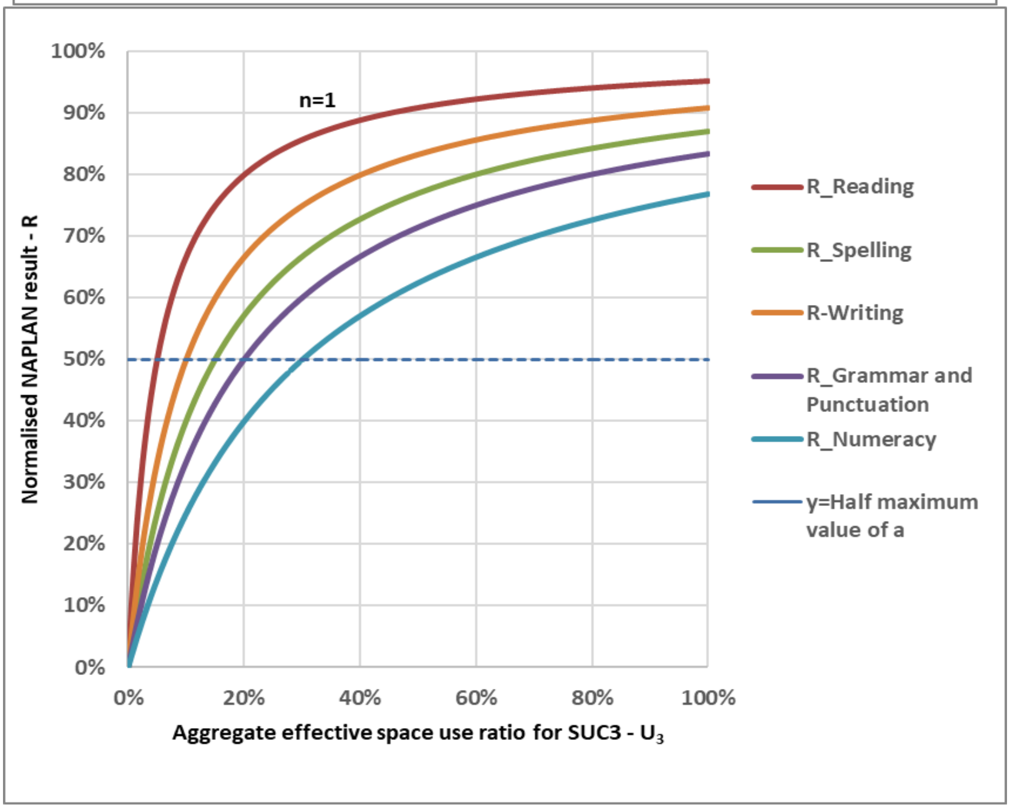
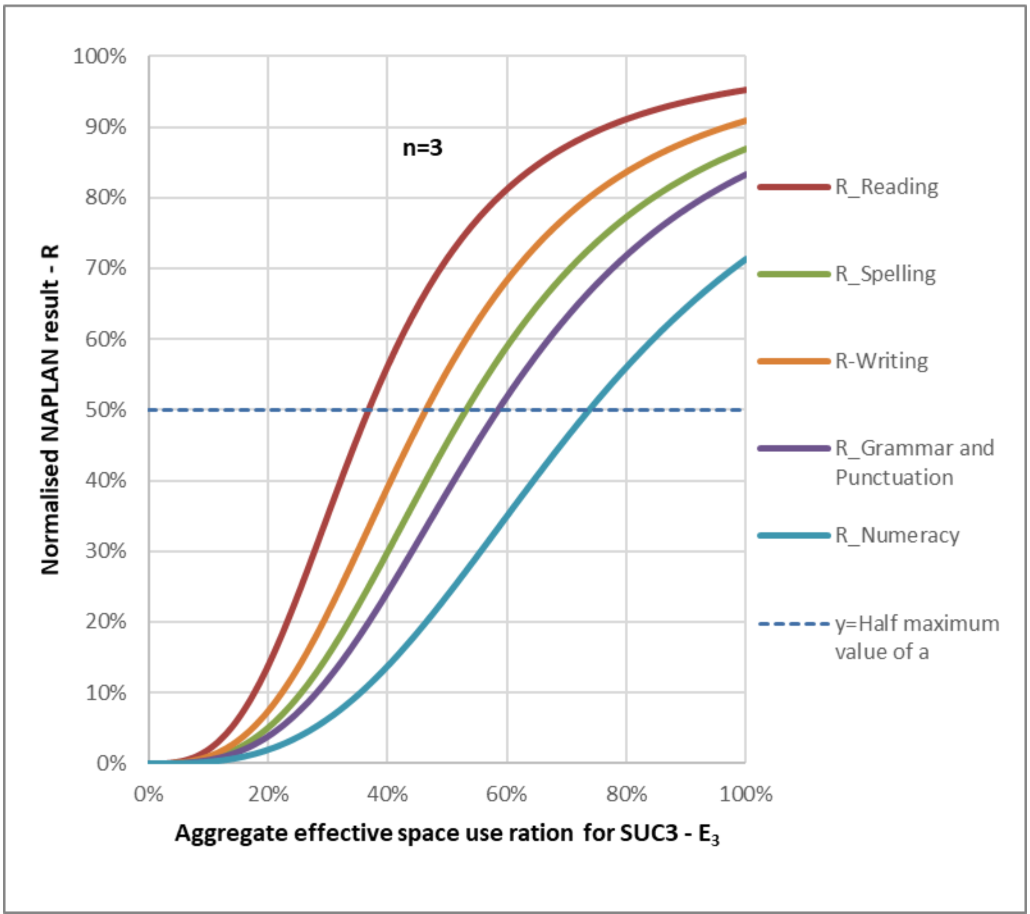


Figure 11: Normalised NAPLAN scores against E~~U~~$_3$ with n=1


n=3
Normalised NAPLAN result - R
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
0%
20%
40%
60%
80%
100%
Aggregate effective space use ration for SUC3 - E3
R_Reading
R_Spelling
R-Writing
R_Grammar and Punctuation
R_Numeracy
y=Half maximum value of a

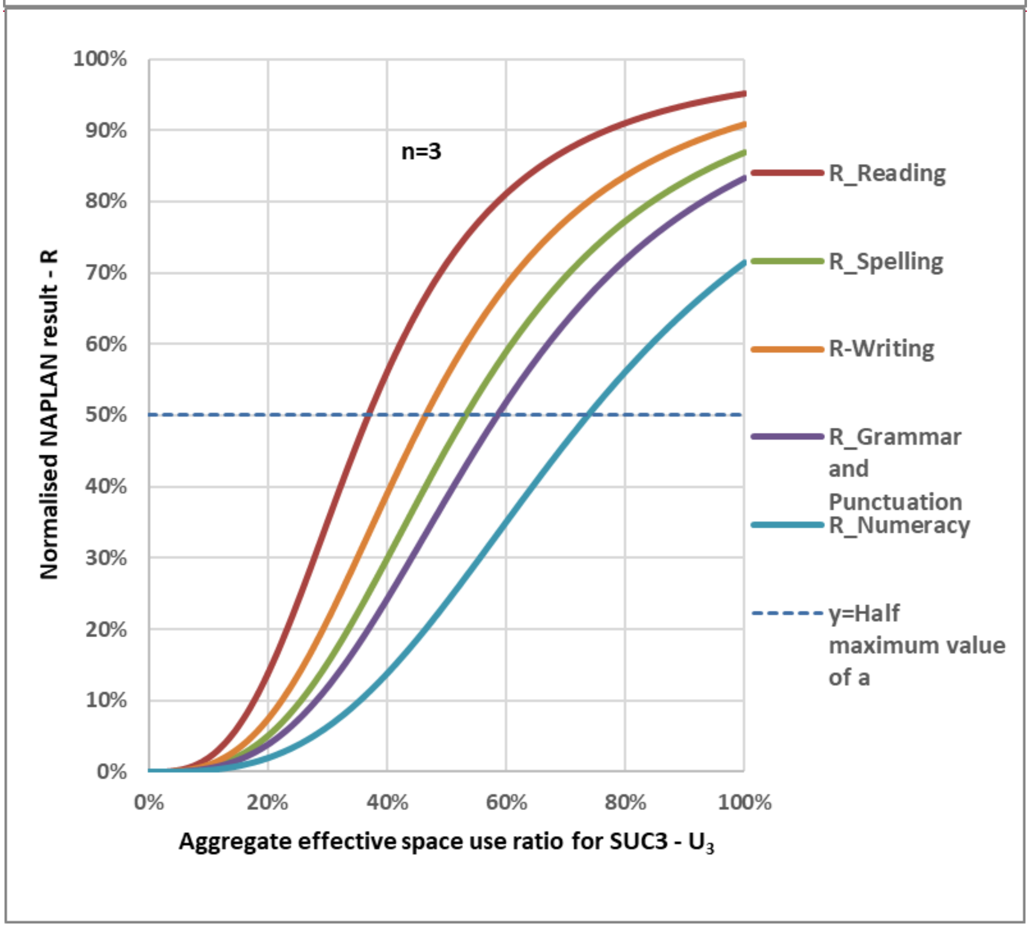

n=3
Normalised NAPLAN result - R
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
0%
20%
40%
60%
80%
100%
Aggregate effective space use ratio for SUC3 - U3
R_Reading
R_Spelling
R-Writing
R_Grammar and Punctuation
R_Numeracy
y=Half maximum value of a

Figure 12: Normalised NAPLAN scores against E~~U~~$_3$ with n=3

4.3 Step 3: Conduct multi-criteria optimisation

4.3.1 Application and demonstration

The next step is to run an optimisation problem with the b values obtained for each E~~U~~-R combination. Genetic algorithm based multi-criteria optimisation was chosen, and optimisation was carried using 'gamultiobj' function in MATLAB with the criteria mentioned in Section 3.2.3. This function uses Pareto optimisation to evaluate objective functions constrained with equality and inequality relations along with lower and upper bounds. The function returns a set of Pareto points and objective function values which defines the points in the Pareto front. The objective for the 'gamultiobj' function was to find the combination of x_1, x_2 and x_3 which met the primary and secondary objectives defined in section 3.2.3. With the use of a data set obtained from several schools, b values can be calculated for each ~~U~~E-R combination.

Variations of *b* values referring to Equation 11 is presented in Table 7. Basically, as $b_{1, R}$, $b_{1, W}$, $b_{1, N}$, $b_{1, G\&P}$ and $b_{1, S}$ relate to x_1, which specifies the aggregate effective space use ratio for SUC1, it is considered to vary from 30% to 80% respectively. Similarly, the limits of *b* values for SUC2 and SUC3 (pertaining to the second and third rows of the matrix mentioned in Equation 11) are as mentioned in Table 7. The population size of the multi-criteria optimisation algorithm was by default set as 50 (as the number of objective functions was less than 5) implying that the MATLAB function will first populate 50 datasets of x_1, x_2 and x_3 which satisfies the objective functions. The function will then use a genetic algorithm to try and evaluate a set of *x* values such that the solutions of objective functions are better than its preceding results. The process is iterated multiple times until any stopping criteria are satisfied. The stopping criteria are selected such that the geometric average relative variation of *x* values is lesser than 1% for each iteration. Since the *x* values are ideally considered to be greater than b values, the limits for *x* and *b* values are set to be the same. By default, the number of solutions in the Pareto front is assigned as 35% of the size of the population.

Table 7: Assumed dataset of b values considered for multi-criteria optimisation

Relevant b values	Lower limit	Upper limit
$b_{1, R}$, $b_{1, W}$, $b_{1, N}$, $b_{1, G\&P}$, $b_{1, S}$	30%	80%
$b_{2, R}$, $b_{2, W}$, $b_{2, N}$, $b_{2, G\&P}$, $b_{2, S}$	10%	35%
$b_{3, R}$, $b_{3, W}$, $b_{3, N}$, $b_{3, G\&P}$, $b_{3, S}$	10%	35%

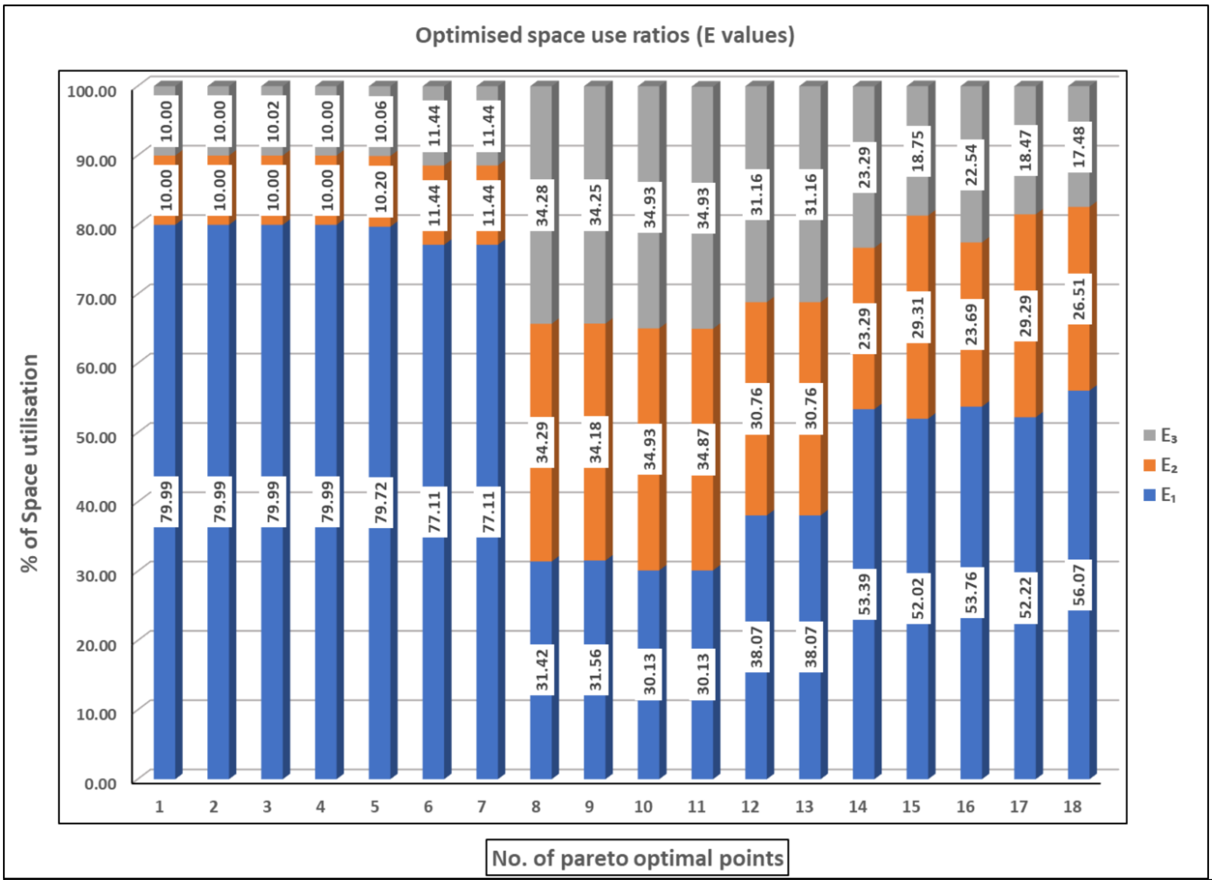
4.3.2 Results

After running multi-criteria optimisation in MATLAB, the x_1, x_2 and x_3 values are obtained as presented in Table 8. Detailed analysis of the aggregate effective space use ratio suggests that, with the educational space being limited to 100%, and with the use of other ratios which add up as percentages, the aggregate effective space use ratio would simplify to the space use ratios for each SUC. It is observed that the optimisation process has been able to converge to a set of Pareto points and objective function values in the Pareto front, satisfying all the primary and secondary objectives as shown in Table 8. Although the optimisation results returned 18 optimal datasets, the primary objective has the highest priority, it can be used to determine the best possible values for ~~U~~E$_1$, ~~U~~E$_2$ and ~~U~~E$_3$. Thus, the maximum non-zero b values were obtained or 17th and 18th datasets in Table 8 out of which 18th dataset had the highest positive p-value. The E~~U~~ values thus obtained would be the ideal values for the corresponding E~~U~~ values beyond which the gain in R-value with increasing E~~U~~ starts to decrease. Hence, given the set of b values assumed In Table 7, the optimum E~~U~~$_1$, E~~U~~$_2$ and E~~U~~$_3$ values would be 56.07%, 26.51% and 17.48% respectively.

Table 8: Results from multi-criteria optimization using the genetic algorithm.

Sl no.	Optimal values for E~~U~~			Optimal values of primary and secondary objectives		
	E~~U~~$_1$ (%)	E~~U~~$_2$ (%)	E~~U~~$_3$ (%)	Number of non-zero b values	Sum of positive p values	Sum of negative p values
1	79.99	10.00	10.00	5	1.50	-1.90
2	79.99	10.00	10.00	5	2.00	-1.10
3	79.99	10.00	10.02	5	1.90	-1.80
4	79.99	10.00	10.00	6	2.00	-1.70
5	79.72	10.20	10.06	7	1.90	-1.00
6	77.11	11.44	11.44	7	1.60	-1.20
7	77.11	11.44	11.44	8	1.70	-0.80
8	31.42	34.29	34.28	9	1.30	-1.30
9	31.56	34.18	34.25	9	1.50	-1.10
10	30.13	34.93	34.93	10	1.10	-1.40
11	30.13	34.87	34.93	10	1.50	-1.00
12	38.07	30.76	31.16	11	1.40	-0.70
13	38.07	30.76	31.16	12	1.20	-0.50
14	53.39	23.29	23.29	12	1.20	-0.20
15	52.02	29.31	18.75	13	0.90	-0.30
16	53.76	23.69	22.54	13	1.10	-0.10
17	52.22	29.29	18.47	14	1.00	-0.10

18	56.07	26.51	17.48	14	1.30	-0.10



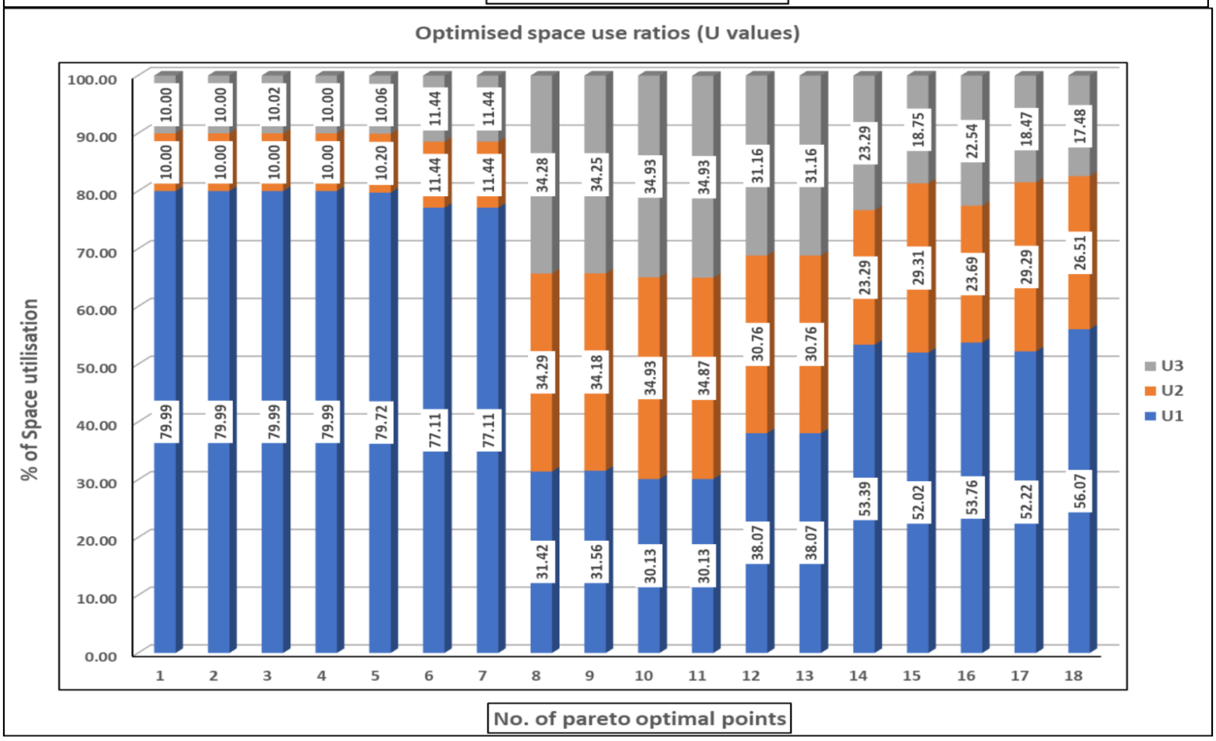


Fig13: Solutions from multi-objective optimisation using the genetic algorithm.

5 Summary and Conclusions

This paper conceptualises how the space distribution in an educational setting, particularly considering public schools, relate to the educational performance of students in a school. The educational performance used in this instance is specified as the student educational outcome measured through NAPLAN. After identifying ~~how the~~how the educational performances measured through NAPLAN and the space use ratios for each space use categories (SUC) can be correlated (does not necessarily mean causality), the paper demonstrates a methodology to obtain optimum space use ratios which represent the optimum educational performance.

The outcome of this research is broadly to indicate that the intended use of a building, and hence the indicators which measure its effectiveness can be correlated to the design parameters quantitatively. This provides some guidance to the designers on the constraints; and at the same time, for educational policy makers the framework provides a foundation to integrate other aspects which lead to higher educational outcome and are constrained in terms of resources.

Summarised below are some of the key outcomes.

1. To relate the space use to the educational performance, an indicator called aggregate space use ratio (E~~U~~) is introduced. ~~U~~E is defined for specific SUCs belonging to the educational space of a school and is a ratio indicating the aggregated space use by an average student to achieve educational performance measured through NAPLAN. The conceptual framework, theoretical framework and methodology for evaluating E~~U~~ concerning different SUCs have been developed and demonstrated in this paper.
2. The relationship between E~~U~~ for each SUC and the educational performance measured through NAPLAN (R) for each skill level is observed to be a multivariate function. Further evaluation of the relationship presents that E~~U~~ for each SUC and R for each skill level would have the characteristics of a function with diminishing returns. Considering the fact when the space allocated to a certain SUC is relatively small (close to zero), having extra space for a certain space use category (SUC) makes a significant difference to the educational performance (R), while having extra space when there is already a significant amount of space for a SUC does not make a significant difference, the diminishing value relationship is approximated to follow a Hill function.
3. Based on that approximation, it is observed that the aggregate space use ratio (~~U~~E) needs to be maintained above the value corresponding to the half-maximum of the horizontal asymptote (a) denoted by b, if a significant compromise of R ~~need~~needs to be avoided. While

the ~~U~~E is optimum close to the horizontal asymptote (a), E~~U~~ values above the half-maximum (b) values are considered to have less compromise compared to the ones below.

4. The theoretical framework leading to validating a data set to check whether the approximation follows Hill function and the methodology for solving the Hill function to obtain the characteristics are elaborated in this paper. The key characteristics to define the function are: a – the horizontal asymptote, n- the Hill coefficient and b – the half max relevant to the horizontal asymptote.
5. Having 15 E~~U~~-R combinations for each SUC related to education and each different ~~skills~~skill assessed by NAPLAN, multiple b values are obtained with the solving of the Hill function. In order to obtain the suggested E~~U~~ value which meets with the requirement for most of E~~U~~-R combinations, multi-criteria optimisation with genetic algorithm is performed.

The framework and methodology suggested in this paper can be used to evaluate an optimum space distribution between different SUC in order to obtain values which represent the highest educational performance of students. The framework has been developed considering a primary school and yet applicable for a school setting in general. The method follows a logical path and presents a systematic approach to correlate space use ratios as a single indicator influencing the physical environment affecting student activity and hence effectiveness of their performance. While the methodology needs to be validated using a data set, once validated, it can be universally employed by educational policy makers to integrate with other factors which influence student performance. For building designers, architects, regulators and policy makers, the research presented in this paper provides a framework and methodology as to how to incorporate the purpose of design, represented by one factor in this instance, into the ~~decision making~~decision-making process as a possible constraint.

5.1 Limitations

The relationship between the space use ratios used in designing the school building have been considered in isolation in determining the relationship between the space use ratios and the educational outcome assessed through NAPLAN. However it is acknowledged that correlation does not mean causality, i.e. optimum design of school buildings will not necessarily lead to student scoring high marks in NAPLAN, as there are a number of other significant influencing factors involved NAPLAN is used as an indicator to determine the student performance and hence the effectiveness of their educational outcome as quantitative results are available for a comparison, though the authors acknowledge that the student achievement in NAPLAN is a single aspect reflecting the holistic development occurs in a school.

Appendix

Nomenclature

NAPLAN - National Assessment Plan for Literacy and Numeracy.

R – Normalised NAPLAN score for a skill level with respect to national average.

AESU – Aggregate Effective Space Utilisation.

SUC – Space Use Category.

SUCR – Space Use Category Ratio.

ESU – Effective Space Utilisation.

E~~U~~ – Aggregate effective space use ratio.

C_{ij} – space use category ratio for building category I in j building.

B_j – Building number in the school.

C_E – Sum of space use category ratios of all building categories contributing to NAPLAN score.

$t_{i,j,k,y}$ – time allocation basis for a category for a particular year.

k_n – number of year levels in the particular school under consideration.

$S_{k,y}$ – percentage contribution of year level k to the NAPLAN score.

E~~U~~$_{max}$ – maximum possible aggregate effective space utilisation for a space category with the resource and regulation constraints.

a - Horizontal asymptote signifying maximum possible NAPLAN score for the particular category and skill level.

b – Aggregate effective space use ratio at half maximum of NAPLAN score for the particular category and skill level.

n – hill function coefficient for a particular category.

Acknowledgements

The authors would like to thank the Australian Research Council's Linkage grant # LP160100259 titled: Value in Operations project "innovative procurement theories to optimize education per cost of school" for the support and funding for this study.

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Highlights:

- Proposes how to find a relationship between buildings' intended use and its design
- Building design is related to its design and intended use to produce outcomes
- Systematic methodology to find relationship dynamics and characteristics

Relating building space to performance outcomes – A methodology to explore the relationship

1 Introduction

A school as an educational facility has its buildings, which constitute of spaces serving different functional uses. The use of spaces could be for administrative, educational or recreational purposes, and the availability and limitation of a space as well as its quality would have an impact on the user experience in performing different functions. The availability of spaces for educational activities and the specific purposes they are used for, can be considered as dynamic and evolving in reality. Considering that the space used for educational activities is limited by the total built space of a school in general, a school would often be faced with the dilemma of adding space or reorganising space, to ensure better use of it to achieve the required goals of the educational institution. With space representing typically about 20% of costs of an educational institution (Australasian Association of Higher Education Facilities Officers, 2001), its effective use, allocation and reorganisation therefore plays an important role in effectively delivering the educational program.

Designing of schools requires due consideration as the school facilities have been found to make a significant impact on the effectiveness of education received by the students, attributable to several factors including the teacher and student experience in the educational facilities (Schneider & National Clearinghouse for Educational Facilities, 2003). The positive impact of physical environment of schools in improving the quality of education has often been overlooked, however considered increasingly important (Sanoff H, 2001).

Considering that the primary purpose of a school, as an operator of an educational facility is to optimise its space use to maximise the educational performance of students, this paper intends to address the need to relate the use of built space for educational use, specifically focusing on the distribution of the space among several categories of use encompassing different educational activities to the educational outcomes. The paper intends to investigate whether the allocation of space in a building to different use categories, aiming a uniform level of utilisation, would lead to better use of building resources (space) to produce higher level of educational outcomes. The framework has been developed using a primary school set up in Australia as an example. In this instance, the educational performance measured through the National Assessment Plan for Literacy and Numeracy (NAPLAN), a standard assessment scheme available for the Year levels 3, 5, 7 and 9 in Australia, is used as the main indicator to demonstrate the methodology proposed in this paper. The study acknowledges however, that holistic educational outcome that schools are expected to provide to students cover a much broader scope, as for example stated in the Melbourne Declaration on Educational Goals for Young Australians (Ministerial Council on Education, 2008), in addition to the educational parameters tested by NAPLAN.

2 Background

2.1 Review of previous studies

This section provides a critical analysis of the previous research conducted contributing to or complementary to the work presented in this paper.

2.1.1 Review of the impact of schools' physical environment on educational outcome

Design of a school building determines the physical environment of a school and the importance of the physical environment on educational outcome has been emphasized by several researchers. Studies suggest that the learning environment influences the teachers and student performance, in terms of how well teachers teach and how well students learn (Cash; Hines, 1996). While the adverse physical environments are observed to affect the ability of the teachers to deliver a quality program (Schneider & National Clearinghouse for Educational Facilities, 2003), the students have experienced attributes which lead to a more positive attitude towards learning in newer and better facilities (Rudd, Reed, Smith, & National Foundation for Educational, 2008). Direct connections between sensory simulations, learning and physical spaces are suggested by Barrett and Zhang (2009).

A relationship drawn suggesting that quality of buildings and student's standardized achievement results are linked via student mobility, with children occupying low quality buildings and having high rates of mobility reporting lower test scores (Evans, Yoo, & Sipple, 2010). This is endorsed by further studies stating students who have high rates of mobility, studying in low-quality school buildings, perform worse on standardized achievement results (Mahdjoubi & Akplotsyi, 2012).

On the contrary, there are published review works of studies focussing on student performance in terms of both achievement and behaviour, and the condition of the built environment suggests that not all studies performed show a strong relationship exists between performance and the environment (Earthman, Lemasters, & Council of Educational Facility Planners, 1996). Based on the dimensions of the physical environment considered, some showed a weaker relationship (Stewart, 2015)

2.1.2 Review of the impact of design of school buildings to teacher and students performance

Design of spaces of school buildings has found to have a considerable impact in delivering an educational program as well as the student performance. In terms of setting the objectives for designing buildings, Cooper (1982) recognised the user's (i.e. teachers) objectives conflicts with that of the client's. While the educators promote open spaces to facilitate collaboration, teachers seem to prefer the educational spaces to be closed and manageable considering operational realities.

Considering user's expectations as the key parameter in design optimisation of buildings, a multi-criteria fuzzy expert system has been employed to analyse both quantitative and qualitative design parameters by Tabrizi, Al-Hussein, and Inyang (2012).

In terms of student performance, Tanner (2008) analysed four design patterns (movement and circulation, large group meeting places, day lighting and views, and instructional neighbourhoods) relating to school environments, and their relationship to academic achievement of students and found that these are positively correlated.

With school buildings increasingly having shorter life spans, Shabha (1993) emphasizes on the need to adopt to changing requirements and to have loose-fit design approach, where the functional use is not determined and not subject to a static definition. This means that having flexible spaces facilitating built-in flexibility for adaptation is encouraged for educational spaces to reduce the need to rebuild (Shabha, 1993). Another perspective suggests to have 'unfinished' spaces for children to explore their creativity and create meaningful spaces for themselves, rather than designers providing finished spaces, which children do not relate well to (Rigolon, 2011). The impact of physical environment has not necessarily been found out to involve built environments, as there are studies that have proven that the time children spend outdoors have better attention skills (Ulset, Vitaro, Brendgen, Bekkhus, & Borge, 2017).

While the relative significance of the design of the physical environment depends on the functional requirements of a building, adequate day light and thermal comfort have been identified as the two primary requirements for a conducive interior environment (Maitreya, 1979). There are studies that have made close relationships between the effects of classroom lighting to students behaviour and learning in the classroom (Winterbottom & Wilkins, 2009) and perceptions of noise levels to classroom performance (Marchand, Nardi, Reynolds, & Pamoukov, 2014).

Consideration of the impact of both the interior and exterior environments (Ramli, Masri, Zafrullah, Taib, & Hamid, 2012), and the importance of building envelope, building orientation and indoor environment (National Research Council, 2001) in the design process has been highlighted and the physical environment has been related to activities and perceptions of users (Khan & Kotharkar, 2012) in research findings.

Currently, there are guidelines and standards that govern space planning in higher educational facilities such as universities where there are recommendations provided on the optimal spaces for different departments, functional activities and purposes. Recommended levels on usage have also been identified by setting indicators such as room frequency, room occupation and utilisation levels (Australasian Association of Higher Education Facilities Officers, 2001). In Australia, Standards for non-government schools are decided by the Minister of Education and governed by the School Education Act 1999 (Section 159), where Standard 6 specifically discusses facilities of schools (Ministry of Education, 1999). In Australia, the relevant authority responsible for education and/or buildings for education has prepared detailed guidelines on maintaining quality standards of educational facilities. The Building Quality Handbook (Victorian School Building Authority, 2019),Design Guide for Schools (Government Architect New South Wales, 2018) and DECD Design Guidelines (Department for Education and Child Development, 2015) are some of the examples. These guidelines have specifically discussed the construction materials, fixtures, fittings etc., design consideration as educational facilities and specific measures to maintain conducive indoor environments.

However, investigating whether proportional allocation of space during design for different uses in a school or an educational institution has a relationship to educational performance of students has not been researched or explored to suggest a quantitative relationship.

2.1.3 Review of the impact of condition of buildings to student performance

In breaking down the school's physical environment into several subparts, the quality of space in school buildings is given an equal level of importance compared to the quantitative proportions.

With building condition representing an important part of the quality of the physical environment, some studies have reported a positive correlation between the building condition and student performance (Durán-Narucki, 2008; Gunter & Shao, 2016), and there are other studies which state that no relationship or weak relationship exists among them (Picus, Marion, Calvo, & Glenn, 2005). The study by Picus et al. (2005) states that higher quality buildings are unrelated to higher levels of student academic achievement . Contrary to this, there has been a study providing evidence-based identification of educational performance of students studying in buildings of different conditions are affected by those conditions (Council of the Great City, 2014). While there are several other factors influencing student performance other than the physical environment, these relationships suggest that a correlation may exist, though it does not mean causality.

2.2 Research significance

Previous research relating the physical environment of educational facilities to the performance of students have mainly been qualitative analysis or conceptual studies. The studies that have adopted quantitative approaches to investigate whether a relationship exists between them have been focusing on the physical environment relating to building condition or design aspects, and student or teacher performance mainly. Though it is generally known that the school infrastructure affects learning outcomes, consideration of performance and behavioural outcomes into design aspects is identified to be largely theoretical (Fisher, 2001). Considering classrooms as spaces for learning and socialisation influencing psychological development of students, impact of density and privacy has been discussed in research, discussing the absolute space allocation for studying (Rivlin & Weinstein, 1984).

However, how the proportional allocation of built spaces for different educational uses by students relate to educational outcomes is observed to be a novel topic, and a quantitative method/parametric study on such has not been conducted by researchers before.

Considering the built space of a school has limitations for expansion and typically is constant once built, the need to manage built space to optimise its fit for the purpose, (i.e. to enhance learning outcome of students in a school), become a resource optimisation issue at the operational phase of the buildings. However, at the design phase of a building, knowing such relationships would enable optimum space selection for the intended use by the building designers, improving the quality of the design by leading to better space allocations during the entire life of a building.

This paper develops a theoretical framework to recognise the aggregate effective space use by an average student in conducting educational activities represented by the assessment result of his/her respective educational outcome. The paper hypothesizes that there is a relationship between the aggregate effective space use (AESU) (defined in Section 3.2.1.3) contributing to educational performance, denoted by E, and the achieved educational performance (R). The paper establishes methodology for deriving the parameters involved in characterising the relationship and proposes methodology for obtaining optimum values for the proportional space distribution analysing the E and R of a set of sample data representing schools.

While educational performance assessed through NAPLAN is influenced by several other factors, including curriculum, pedagogy (Bakkenes, Vermunt, & Wubbels, 2010) and assessment (Umami, 2018); student competence, learning behaviour and individual learning preferences and methods (Barenberg, Roeder, & Dutke); in this instance, school's physical environment, and particularly the proportional allocation of space use categories has been considered as a factor in isolation, acknowledging that correlation necessarily does not mean causality, i.e. the use of the optimum space allocation ratios may not guarantee higher educational outcomes as there are several other influencing factors.

The research contribution of this paper leads providing a foundation to characterise the relationship between space use for educational activities and the educational performance quantitatively. The systematic approach and methodology developed in this research would enhance integration of knowledge on design of physical environment to the intended purpose of buildings. Therefore, this paper would provide useful guidance to educational policy makers for understanding the relationship dynamics between the two and to consider establishing constraints around proportional allocations in setting standards for design of school buildings.

2.3 Aims and scope of this paper

This paper aims to investigate if a relationship exists between the design space allocation for different categories of educational activities and the student performance related to those educational activities. The paper intends to characterise the relationship if it exists. The aims of the paper are to:

- develop a conceptual framework to draw a relationship between the aggregate effective space use to perform educational activities and educational performance.
- approximate the nature of relationship and to establish methodology to validate the approximation and thereby investigating whether a relationship exists
- reengineer the problem to propose preferred proportional space allocation ratios that would result in the optimum aggregate effective space use leading to a condition that the risk of compromise of educational outcomes with inappropriate allocations is minimised.
- elaborate the application of the above methodology using sample data

In developing the conceptual and theoretical frameworks, methodology and the model, key focus is given to schools in Australia, however, the research outcomes can be applied generally to any context.

3 Methodology

3.1 Conceptual framework

The paper presents a methodology to characterise the relationship between the space allocation ratios of different spaces used for educational activities and the educational outcomes as measured through a national assessment scheme, NAPLAN. The conceptual framework stating the methodology in four main steps is presented in Figure 1.

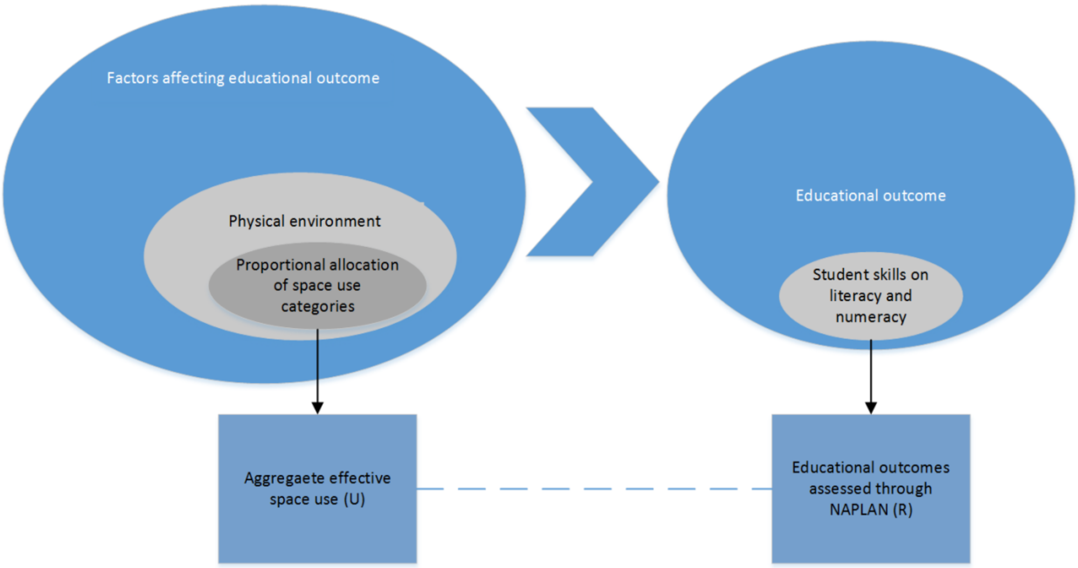

Figure 1: Research framework using proportional allocation of spaces use as a single factor affecting learning outcome and NAPLAN results as a single factor among many indicators to measure educational performance

In order to investigate the relationship between proportional allocations of space use categories in a school, considered as the input, to the assessment outcomes of a school, considered as the output, the link is made through student activity contribution to outcomes in the given spaces. As stated before, among many of the factors which influence the educational outcome, physical environment has been taken in isolation and among many variables present in the physical environment, proportional space allocation is considered as an input variable. Accumulated student activity in the proportionally allocated spaces has been considered as an intermediate variable in order to link building parameters to achievement/outcome parameters, and an indicator referred to as the aggregate effective space use ratio (denoted by E in later sections) is derived for this purpose. Acknowledging that the skills assessed in NAPLAN (literacy, numeracy, spelling, grammar and punctuation and speaking) are a few among many indicators for measuring the educational outcome, R- denoted by NAPLAN results is considered as the dependent variable representing learning outcome. Figure 2 presents the key steps adopted in the research approach. The first step is to relate the building space to relevant educational activities performed in the space. An indicator called aggregate effective space use ratio, E is introduced in this process to establish a relationship between the aggregate effective space use ratio, E, and the educational performance measured through NAPLAN, R. Based on the nature of the relationship, it is approximated to follow a pattern of diminishing returns (and hence approximated to follow Hill function). The methodology to validate the approximation and solve the function to obtain the half-maximum value of E relating to each E-R combination would be carried out in Step 2. Step 3 involves the use of multi-criteria optimisation to obtain the optimum proportional space allocations that would lead not to compromise of educational outcomes due to the non-availability of space for a given use. Each step is elaborated in detail in Sections 3.2.1 to 3.2.4.

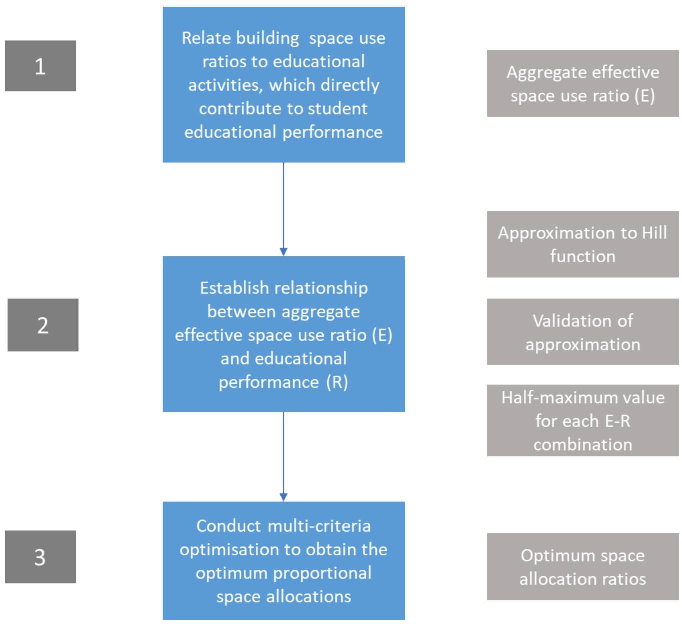

Figure 2: The key steps adopted in the research approach

3.2 Theoretical framework

This section presents the theoretical framework setting out the methodology.

3.2.1 Step 1: Relate building space use ratios to educational activities

The theoretical framework for evaluating aggregate effective space use ratio (E) is presented using Figure 2. The first step involves dividing the built space of a school into five categories based on the fundamentals, each of them identified as a space use category (SUC). The SUCs directly contributing to educational performance of students are identified to be SUC 1, SUC 2 and SUC 3. Acknowledging the fact that there would be shared spaces used by students of all Year levels in a school, a time-sharing factor for each Year level is used for common resources to identify the effective space use by a student of a designated Year level for each SUC. As the educational activities of an average student of a certain Year level contributes to the student's educational performance as measured by NAPLAN at a subsequent Year level, the aggregated (cumulative) effective space use for educational activities (E) is estimated by aggregating proportional allocations of space use of each Year level, contributing to their educational performance. The theoretical framework considering the above steps, is further elaborated in Sections 3.2.1.1 to 3.2.1.4.

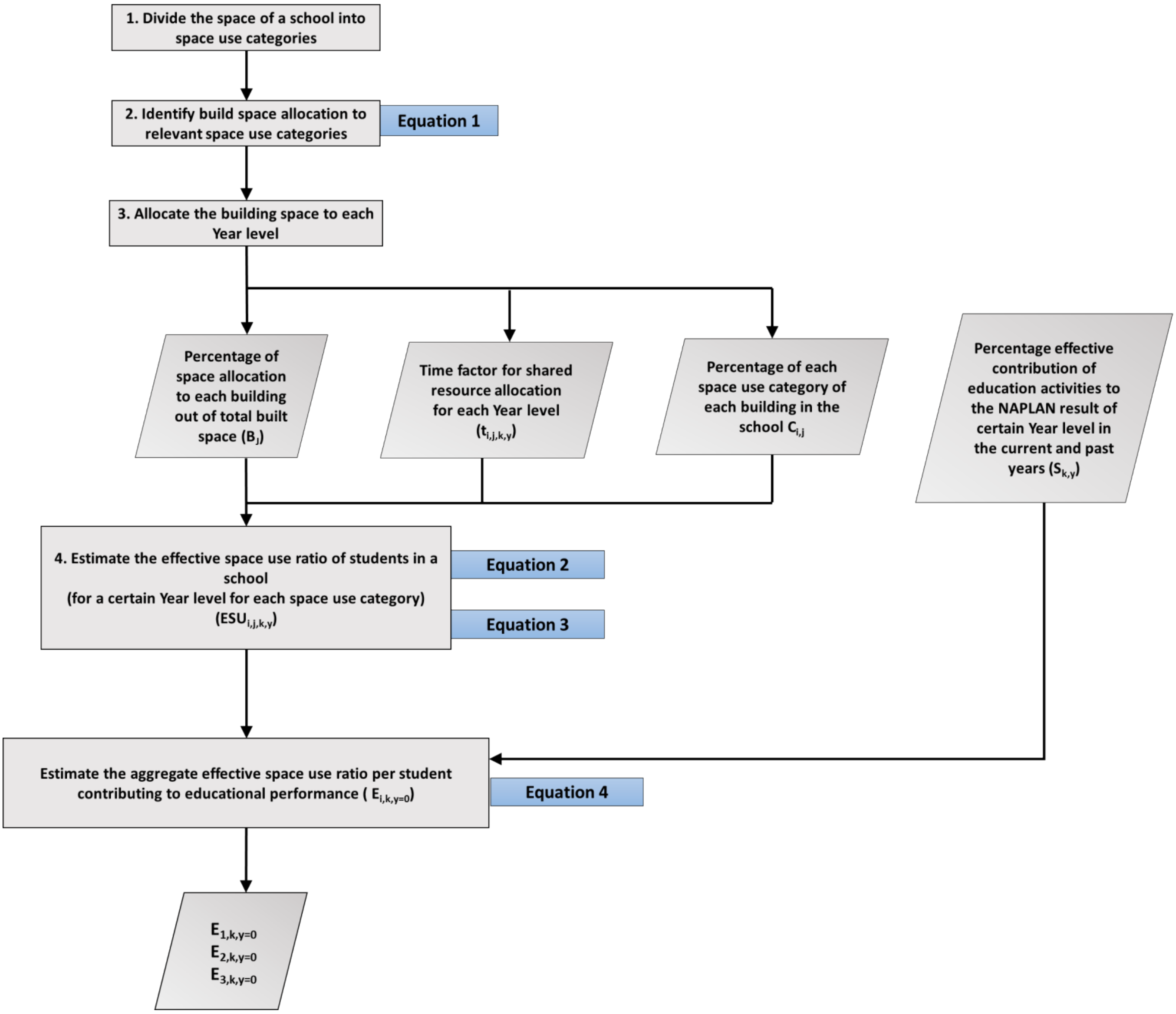

Figure 3: Conceptual framework to evaluate the aggregate effective space use ratio

3.2.1.1 Division of school space to space use categories

A school would consist of different buildings, which would add together to form its entire built space. The space of each school building is divided into five main categories, identified as space use categories (SUC). The ratio of space for each SUC out of total space is referred to as the space use category ratio (SUCR) and termed $C_{i,j}$, where i is the SUC as identified in Table 1 and j is the reference number of the building. This is diagrammatically presented in Figure 4.

Table 1: Building space categories

i	Building space category
1	Classrooms
2	Laboratories and activity rooms
3	Libraries and resource spaces
4	Walkways and circulation spaces
5	Recreation facilities, common spaces and other spaces

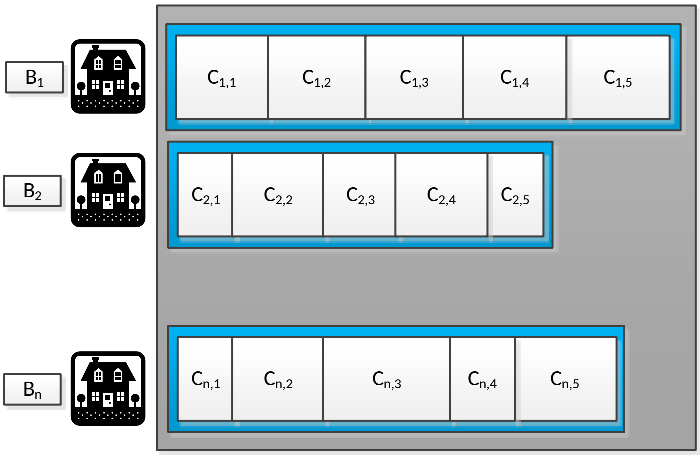

Figure 4: The graphical demonstration of the built space allocation of a school to different space use categories

For the purpose of this study, SUC 1, SUC 2 and SUC 3 are identified as directly relevant to educational activities, which can be measured through the student performance in NAPLAN. The relative importance of these categories relative to each other (e.g. the importance of classroom relative to library spaces) is not analysed in this study. Based on that, the total educational space can be calculated as a ratio of the total built space based on the above definition of SUCs. Social learning spaces, such as break-out rooms, where impromptu/informal discussions happen on a group project is placed under SUC 2.

For each particular building *j* in a school, the sum of the ratios of $C_{i,j}$ is equal to 1, considering that educational space consist of SUCs 1, 2 and 3, which have a direct contribution to the student performance measured through NAPLAN.

$$\sum_{i=1}^{3} C_{i,j} = C_E \quad (1)$$

It is assumed that recreational facilities and common, and other spaces and circulation spaces denoted by $C_{4,j}$ and $C_{5,j}$, respectively, does not have a direct correlation to the educational outcome achieved through NAPLAN. However, there are some studies indicating that the physical activities and recreation have a correlation to the educational outcome achieved. Extra-curricular activities in general have been found to have associations with the educational outcome. As an example participation in sports have been found to improve educational outcome, particularly numerical skills (Shulruf, 2010), however, there are other studies indicating otherwise. A comprehensive study analysing a sizeable number of associations has revealed that the association is marginally positive or no association exists between the educational outcome and school-based physical activity (Bird, Tripney, & Newman, 2013). For the purpose of simplicity and acknowledging that conflicting views exist on this debatable topic, this study has excluded the effect of school-based extra-curricular activities (which consume the space of school buildings) on the educational outcome.

It is also noted that walkways and circulation spaces denoted by SUC5 are usually governed by the building regulations, where there are minimum but not a maximum standard applicable. As only the SUC relating to direct educational outcome measured through NAPLAN are considered for this study, $C_{4,j}$ and $C_{5,j}$ is not considered as a variable. Therefore, in determining the ratios, the total educational space has been made equal to 100%.

3.2.1.2 Allocate the building space to each Year level based on usage

Generally, any given building in a school could have students in the respective Year levels. Therefore, the use of a building will have activities pertaining to different Year levels. It is assumed that the students share SUC_1, SUC_2 and SUC_3 on a time allocation basis, with each year level having an allocation of $t_{i,j,k,y}$, where k is the particular Year 1 to n; i would be the number denoting SUCs 1,2 and 3; j is the building number and y represents the relevant year of data gathering. If data is gathered concerning the current year, y=0. If there are dedicated spaces, then the, $t_{i,j,k,y}$ for that SUC becomes 1 for that Year level.

Therefore, the use of a certain category of SUC by students in a certain Year level of a building, is defined as the effective space use ratio of the school, $ESU_{i,j,k,y}$

$$ESU_{i,k,y} = \sum_{j=1}^{b_n} C_{i,j} * t_{i,j,k,y} * B_j \quad (2)$$

where B_j is percentage of space allocation to each building out of the total built space.

As the focus of the paper is on the design of schools in comparison to determining usage and operational matters that come into effect at the operational stage of schools, the time sharing factor is taken to be a constant and it is assumed that the common spaces are allocated equally among students of each Year level of a particular school. Therefore:

$$t_{i,j,k,y} = 1/k_n \quad (3)$$

where, k_n is the number of Year levels in the particular school under consideration.

3.2.1.3 Aggregation of the effective use of educational space to contributing to educational outcome

When a student makes progress with educational activities in a certain year and moved on to the subsequent Year level, their effective space use contributing to their educational performance accumulates. Therefore, it is considered that a student's education at a certain Year level, k contributes by a percentage of $S_{k,y}$ to the educational assessment results. In elaborating methodology, for simplicity it is assumed that the educational activities of y number of years contribute to the educational outcome measured through NAPLAN for a certain year. Therefore, the aggregated effective space use ratio leading to relevant educational performance measured by NAPLAN for each school, for Years 3,5,7, for each SUC for a school is given by $E_{i,k,\ y=0}$,

$$E_{i,k,y=0} = \sum_{y=-m}^{0} ESU_{i,k,y} * S_{k,y} \tag{4}$$

where, m is equal to the number of years of accumulation of educational activities contributing to a particular assessment result. For this study, 2 years of aggregation of activities is assumed to relate to the outcome and hence y=0 (current year) and -1 (previous year) are considered for simplicity. In deriving Equation 4, it is assumed that the absolute space use per student for SUC1, SUC 2 and SUC3 are governed by the ergonomic guidelines in school design and hence maintained within a range in any given school to adhere to the relevant compliance criteria. This study excludes the analysis of absolute space use allocated to a student from its scope and is only concerned with analysing the effect of proportional distribution of space allocation ratios to the educational performance. Therefore, the study assumes that the student numbers have not changed over the year significantly, which could have an impact on shared resource usage and space allocations to each Year level and the spaces provided by the schools are spacious enough for students to pursue their educational activities with convenience, abiding by the ergonomic standards.

3.2.2 Step 2: Establish relationship between aggregate effective space use ratio (E) and educational performance (R)

In order to identify how the effective space use ratio relates to the NAPLAN score, the structure of NAPLAN exam is looked at. The NAPLAN test measures the literacy, numeracy, writing, spelling, grammar and punctuation; therefore the NAPLAN result for the respective skills are denoted by $R_{L,k}$, $R_{N,k,}$ $R_{W,k}$, $R_{S,k}$ and $R_{G\&P,k,}$ where k denotes the Year level NAPLAN score relates to (Year 3,5,7, and 9) and L, N,W,S and G&P denotes the skill levels tested with NAPLAN, namely, literacy, numeracy, writing, spelling and grammar and punctuation. With the aggregate effective space use ratio - *E* being identified to relate to the space use to perform educational activities, which is measured through NAPLAN score- *R*, a relationship can be made between *R* and *E* so that R is a function of E, as in:

$$R_{X,k} = f(E_{1,k,y=0}, E_{2,k,y=0}, E_{3,k,y=0}) \tag{5}$$

where, *X* refer to any of the NAPLAN scores.

3.2.2.1 Identification of the nature of the function

As there are multiple input variables (E_1,E_2 and E_3) relating to multiple output variables, this is identified as a multivariate relationship and graphically demonstrated as in Figure 5.

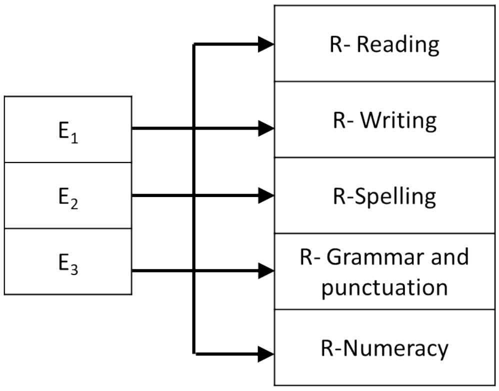

Figure 5: Relationship between E and R in a graphical representation

In the meantime, it is observed that *E*'s relationship to *R* follows a pattern of diminishing returns. The reason for the approximation is logically explained as below.

While the increase of aggregate effective space use ratio would contribute to increased educational performance, it can be expected that increase of ratio from low levels to mid/average levels would result in significantly improved educational performance. However, after a certain threshold, increase of input variable has a marginal impact on the output variable, R, and the incremental improvement in R per an incremental increase in E starts to reduce after a certain point.

As an example, for any school the first criteria to check would be whether there is space available for the specific SUCs identified, as an example SUC2 indicates availability of library space. The impact on educational performance is expected to be significant when comparing two schools; one having library space and the other no having library space (significant increase in R for E close to zero). However, if a school has a significant proportion of space allocation to libraries, it can be reasonably considered that the educational performance would not be significantly improved by further increase in library space (diminishing rate of increase of R for higher E values), as the impact on having libraries after a certain threshold would not have an influence on educational performance (Mancell, 2005). This reflects diminishing returns after a threshold value of *E*.

3.2.2.2 Hypothesizing and approximating an appropriate function

Approximating that the relationship would follow a pattern of diminishing returns based on the above rationale, the next step would be to characterise the relationship using a function that would behave in a similar manner.

Considering non-linear regression and Hill function for characterisation of a function indicating diminishing returns, Hill function was chosen. Hill function has a typical S-shaped curve, which best describes the nature of the relationship discussed above, and also could be generalised to a half-parabolic function, which is the simplest form of a diminishing returns function. Therefore, it is identified that consideration of Hill function could offer better options to explore intricate features associated with a S-shaped curve.

Hill function, which was originally used for biomedical applications have many other common uses (Likhoshvai & Ratushny, 2007). Figure 6 diagrammatically presents the typical S-shaped curve associated with Hill function. The x axis represents the aggregate effective space use variable E as a percentage of the maximum possible aggregate effective space use value (E_{max}). Similarly, y-axis of the graph denotes the NAPLAN score as a percentage of the maximum possible NAPLAN score. E_{max} implies the building design facilitates maximum possible aggregate effective space use possible within the regional design standards of the school. However, achieving maximum aggregate effective space usage may not always be feasible as it would demand capital investment. Also, scenarios where E_{max} can be achieved for all space categories would be a rare occurrence in real-world due to building space limitations. Therefore, optimising the aggregate effective space usage for delivering optimal educational outcome within the feasible budget and capacity constraints is critical for school building designs.

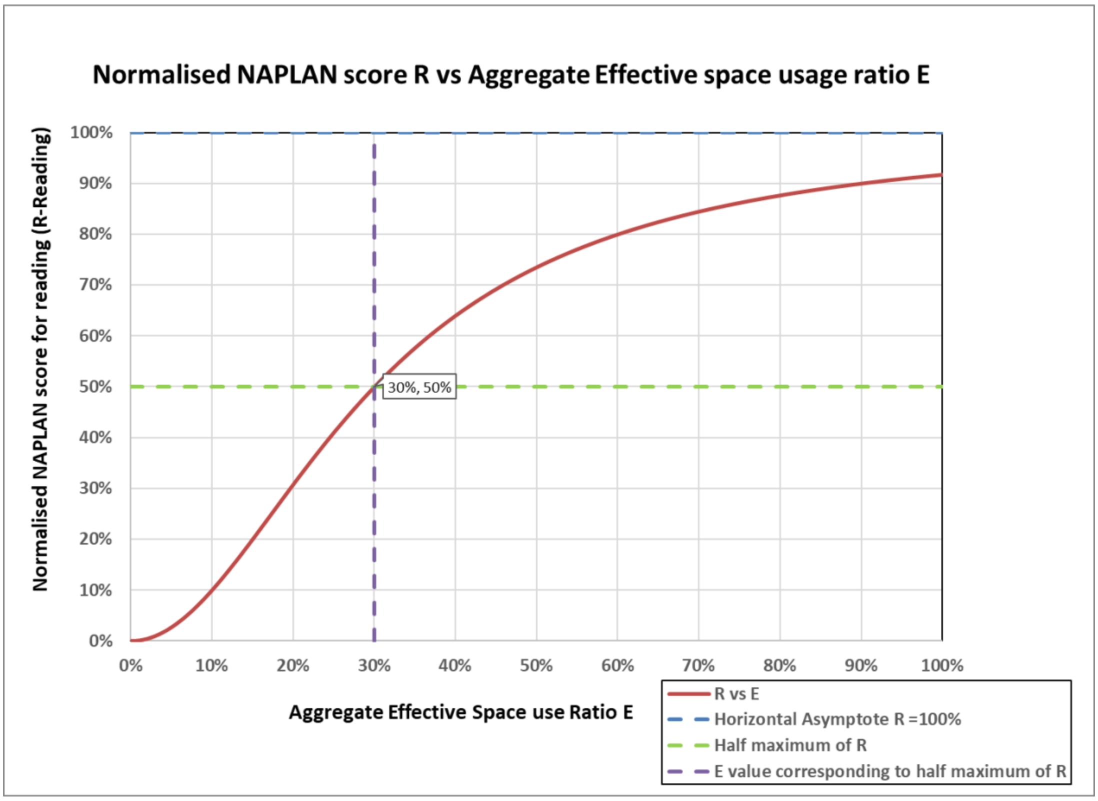

Figure 6: Diagrammatic explanation of Hill function, hypothesized to correlate educational outcome to aggregate effective space use

3.2.2.3 Analysing the characteristics of Hill function and relating with the study outcome

Approximating that the relationship between R and E follows a Hill function based on the rationale described in Section 3.2.2.1, the Hill equation can be written as:

$$\mathrm{R} = \frac{aE^n}{b^n + E^n} \tag{6}$$

where, *n* is the Hill coefficient, *a* is the horizontal asymptote which is the maximum normalised NAPLAN result in this instance and *b* is the half-maximum value of the horizontal asymptote a (value *x* corresponding to the half of the maximum value of R, which is *a*).

As an example, we can take the NAPLAN score for literacy of Year 3 students to relate to E_1 as below.

$$R_{L,3} = \frac{a_{1,3}{E_{1,3}}^{n_{1,3}}}{{b_{1,3}}^{n_{1,3}} + {E_{1,3}}^{n_{1,3}}} \tag{7}$$

Analysing the characteristics of a Hill function based on Equation 6, the following can be observed.

- For values of *E* close to zero, E^n in the denominator becomes negligible: $R = aE^n/b^n$
- For values of *E* close to infinity, *b* becomes negligible. Therefore: $R= a$
- For values of $E=b$, $R=a/2$, Therefore b is called the half-maximum value of the horizontal asymptote *a*.
- The function follows a pattern of diminishing returns up to $x=b$, with the gradient of the function increasing up to $x<b$ and decreasing after $x>b$.

With *E* represented with the *x* axis and *R* represented with the *y* axis as stated above, it is observed that when *E* is increased from $E=0$ to $E=b$, *R* values grow exponentially. This indicates that for *E* values below $E=b$, increase of *E* results in an exponential increase of *R*. However, after $E=b$, the increase of *R* with the increase of *E* is occurring with a decreased gradient until *E* reaches infinity (while *R* reaches *a*). This indicates that value of E above the value of $E=b$, the increase of *R* decelerates.

Two basic characteristics/rules of the function are identified based on the above:

1. for values of *E* below $E=b$ ($E<b$), the compromise of R is significant for a unit decrease of *E*
2. for values of *E* greater than $E=b$ ($E>b$), the reduction of *R* value becomes less significant for a unit increase of *E*

We can reasonably conclude that the minimum value of *E* to maintain by any school would be *E*=b, as when *E* falls below *b*, significant compromise of *R* is observed.

This principle can be applied to SUC1, SUC2 and SUC3 to evaluate the optimum E_1, E_2 and E_3 values relating to the five NAPLAN scores ($R_{L,k}$, $R_{N,k}$, $R_{W,k}$, $R_{S,k}$ and $R_{G\&P,k}$) respectively. Applying Hill function and the above theory to this study, the objective of characterising and solving the relationship can be set to find *n*, *a* and *b* associated with each of the S-curve related to each E-R combination. While *n* would determine the shape of the S-curve, *a* would determine the maximum result (R). The most important finding for this study however is the half maximum value of the horizontal asymptote *a*, denoted by b, as this would set the minimum value of E to be maintained for each E_1, E_2 and E_3 values for each E-R combination.

3.2.2.4 Process for validating the hypothesis

This section presents the methodology for validating the applicability of the proposed hypothesis with the use of data obtained from schools.

By rearranging the Equation 6, Equation 8 can be obtained.

$$\frac{\left(\frac{R}{a}\right)}{\left(1-\left(\frac{R}{a}\right)\right)} = \left(\frac{E}{b}\right)^n \tag{8}$$

Taking the left-hand side (LHS) to be equal to V Equation 8 can be rearranged as:

$$V = E^n * b^{-n} \tag{9}$$

By taking the natural logarithmic (ln) values of both sides

$$\ln(V) = n\ln(E) - n\ln(b) \tag{10}$$

This corresponds to a linear function equivalent to $y = mx+c$, with $x= ln(E)$, $y= ln(V)$. The gradient, m is given by n and the intercept, c corresponds to $ln(b)$. By plotting $ln(V)$ against $ln(E)$ the linearity of the relationship can be checked and hence providing proof of the hypothesis. The correlation coefficient relating to the linear plot would determine the fit of the relationship to consider as a strong or weak linear function.

While $ln(E)$ can be calculated by the data obtained from schools (in accordance with the data presented in Figure 2), in order to evaluate $ln(V)$ values, a need to be determined as per Equation 7. Horizontal asymptote, a, for each E-R combination need to be obtained by plotting R against E first for the estimation of V and hence $ln(V)$.

3.2.2.5 Solving the function

As mentioned in Section 3.2.2.3, the objective of characterising the function is to identify a, n and b values.

R against E is plotted first to identify the maximum value of R, which is considered as the horizontal asymptote for each Hill curve. R in this instance would be the absolute NAPLAN score normalised to the maximum score nationally. Therefore, a would be the highest R. Using a in Equation 7, V values can be estimated to apply in Equation 9. By creating a plot of Equation 9, n and b values can be estimated by identifying the slope (m) and the intercept (c) of the trend line, considering its linearity. The slope, m corresponds to n while:

$$b = e^{-\left(\frac{c}{n}\right)} \tag{10}$$

where, c corresponds to the intercept of the function.

Adopting this method would enable identification of b values corresponding to each E-R combination considering the five NAPLAN scores and E_1, E_2 and E_3 values. This would result in 15 values of b, stated in a matrix as below, related to one assessment of NAPLAN.

$$b = \begin{pmatrix} \mathrm{b}_{1,R} & \mathrm{b}_{1,W} & \mathrm{b}_{1,S} & \mathrm{b}_{1,S\&P} & \mathrm{b}_{1,N} \\ \mathrm{b}_{2,R} & \mathrm{b}_{2,W} & \mathrm{b}_{2,S} & \mathrm{b}_{2,S\&P} & \mathrm{b}_{2,N} \\ \mathrm{b}_{3,R} & \mathrm{b}_{3,W} & \mathrm{b}_{3,S} & \mathrm{b}_{3,S\&P} & \mathrm{b}_{3,N} \end{pmatrix} \tag{11}$$

Each of this *b* value represents the minimum *E* value to maintain for each of the *R* values (representing the specific skills assessed by NAPLAN). As this presents 15 conditions to abide by in determining the optimum space use ratios of a school, an optimisation problem is required to find out the optimum values in order to fulfil the most relevant requirements stipulated by each E-R combination.

3.2.3 Step 3: Conduct multi-criteria optimisation

The objective of the study is to identify the optimum space allocation ratios for E_1, E_2, and E_3 so that the identified values are above the half-maximum value of b for each E-R combination. As per the above discussed hypothesis that would indicate that the ratios of space allocation for SUC1, SUC 2 and SUC 3 would not result in significant compromise of NAPLAN scores.

Let us call this optimum values x_1, x_2 and x_3. If x_1, x_2 and x_3 are such that they are positioned after each *b* in the S-curve for each E-R combination, we can introduce another variable called *p* where $p_i = x_i - b_i$.

If p_i is a positive value for all E-R combinations, it is considered as an optimum result. Equally, larger all the p_i values, larger the sum of p_i for all positive p_i values, the more optimum *R* is, which is desirable in terms of educational outcome. At the same time, a smaller all the p_i values and a smaller the sum of p_i for all negative p_i values, close the x_i is to the *b* value in the S-curve, which is considered second best if p_i is negative.

Based on the above, the objectives for the problem to identify the optimum x_1, x_2 and x_3 values are:

- Primary objective (Priority 1): to have the maximum number of p_i values>0 out of 15 values
- Secondary objective 1 (Priority 2): to maximise the sum of p_i values for positive p_i values
- Secondary objective 1 (Priority 2): to minimise the sum of p_i values for negative p_i values

One important thing to note is that the sum of x_i values is equal to 1, which acts as a linear constraint to the problem and x_i values are constrained to be between 0 and 1, which act as non-linear constraints to the problem. This is due to the fact that the education space is considered as 100% and referring to Equation 4, E becomes equivalent to C_i with the use of percentage values for aggregation.

Considering the above three objectives and constraints, multi-criteria optimisation using genetic algorithm is carried out to identify x_1, x_2 and x_3 values for this problem.

4 Application, demonstration and results

This section demonstrates the application of the process and shows sample results when the methodology is applied. A parametric study is carried out to demonstrate the variation in results, where a sample data set is applied using this methodology.

4.1 Step 1: Relate building space use ratios to educational activities

4.1.1 Application and demonstration

For this demonstration, a hypothetical primary school, which conducts classes up to Year 5, has been considered; therefore, the students are expected to sit NAPLAN examination at Year 3 and Year 5. Considering a sample data set relating to the current context of Australian primary schools, the space allocation ratios for each respective building for the current year and the previous year are presented in Table 2, together with the ratio of space of each building.

Tables 3 provides the data on the estimated contribution of each Year level to the educational performance measured at a subsequent year level and the equality assumption on usage among all Year levels. As this paper is mainly concerned with finding ratios of building SUCs to provide inputs for design, the factors concerning usage has been maintained constant. Therefore, it is assumed that common SUCs are used equally among all Year levels, resulting in 20% usage per each level for a primary school consisting of Year 1,2,3,4 and 5. The assumed percentage contribution of each Year level to the performance indication by NAPLAN is stated in Table 4. Table 5 provides the current statistics for NAPLAN as extracted from National Report for 2016, pertaining to 2016 for each of the five skills measured (Australian Curriculum Assessment and Reporting Authority, 2016).

Table 2: C_i and B_j values for the sample school for the current and previous year

Building	Building space allocation - Current year					Building space allocation - Previous year					Pecentage of building area as a total built space (B_j)
	Y=0 (Current year)					Y=-1 (Previous year)					
	C_1	C_2	C_3	C_4	C_5	C_1	C_2	C_3	C_4	C_5	
1	5%	20%	30%	4%	41%	5%	10%	30%	4%	51%	24%
2	20%	50%	13%	5%	12%	10%	50%	10%	8%	22%	13%
3	10%	27%	25%	10%	28%	10%	34%	34%	10%	12%	11%
4	45%	22%	16%	5%	12%	45%	23%	23%	5%	4%	33%
5	10%	45%	23%	10%	12%	10%	54%	23%	10%	3%	20%

Table 3: Effective contribution to educational performance measured through NAPLAN by each Year level

Primary school	Effective contributin to NAPLAN score	Effective space usage - Current year	Effective space usage - Previous year
Year 1	0%	20%	20%
Year 2	30%	20%	20%
Year 3	70%	20%	20%
Year 4	30%	20%	20%
Year 5	70%	20%	20%

Table 4: NAPLAN indicators for Year 3 in 2016

Year 3	Mean	Standard deviation
Reading	425.6	85.6
Writing	420.7	62.0
Spelling	419.8	85.8
Grammar	435.8	89.9
Numerac	402.0	73.4

4.1.2 Results

Methodology stipulated in Sections 3.2.1.1 to 3.2.1.3 has been performed and the resulting E values are mentioned in Table 5.

Table 5: Aggregate space use ratios for SUC1, SUC2 and SUC3 presented as E_1, E_2 and E_3

Year Level	E_1	E_2	E_3
Year 3	**4.21%**	**6.06%**	**4.44%**

4.2 Step 2: Establish relationship between aggregate effective space use ratio (E) and educational performance (R)

4.2.1 Application and demonstration

Demonstration of this part of the methodology would require obtaining E and R values for a number of schools and plotting them against each other as a starting step. The R value concerned here the NAPLAN score of a public primary school normalised to the maximum NAPLAN score for that particular Year level, as a percentage.

By plotting R against E, a can be determined first as stated in Section 3.2.2.4. Then validation of the function to follow a Hill function can be done using the methodology stated in Section 3.2.2.5. By identifying the characteristics of the linear trend line, n and b values can be evaluated accordingly.

4.2.2 Results

As this section demonstrates the application of methodology with the absence of a complete data set, it is assumed that the data set follows the Hill function and the parameters obtained with the plotting of R against E have been varied to discuss the effect of such and to interpret them. The method for validating this assumption is stated in Section 3.2.2.4.

The parameters subject to change here are *a*- the horizontal asymptote, *b*- the half-maximum value and *n*- the Hill coefficient. Table 6 shows the parameters varied of Hill function as shown in Figures 7, 8 and 9. The figures in this section show plots of different S-curves associated with Hill function with the variations in *n* and *b*. *a* is estimated as the normalised NAPLAN score percentage based on the highest NAPLAN score for 2016 obtained from Table 4.

Figures 7-12 show that n, the Hill coefficient and b- the half-maximum value determines the shape of the Hill function. While *b* has been predetermined in this instance with the absence of data, the objective of creating a linear plot as mentioned in Section 3.2.2.5 would be to finally obtain the b value for each E-R combination. In order to carry out the optimisation problem, which is the next step, a set of *b* values has been used as below. This is done considering the fact that the majority of the space would consist of classrooms (E_1), and resource and activity spaces (E_2 and E_3) would be relatively low compared to that with maximum sharing occurring with them. With approximated values for b for all three E categories and five NAPLAN scores measuring different skill levels, the following matrix has been obtained.

Table 6: Parameters of Hill function in the Figures 7-12

Figure	E_1	R_Reading	R-Writing	R_Spelling	R_Grammar and Punctuation	R_Numeracy
Figure 7	a	100%	100%	100%	100%	100%
	b	0.8	0.75	0.6	0.5	0.4
	n	1	1	1	1	1
Figure 8	a	100%	100%	100%	100%	100%
	b	0.8	0.75	0.6	0.5	0.4
	n	3	3	3	3	3
Figure	E_2	R_Reading	R-Writing	R_Spelling	R_Grammar and Punctuation	R_Numeracy
Figure 9	a	100%	100%	100%	100%	100%
	b	0.05	0.1	0.15	0.2	0.3
	n	1	1	1	1	1
Figure 10	a	100%	100%	100%	100%	100%
	b	0.05	0.1	0.15	0.2	0.3
	n	3	3	3	3	3

Figure	E_3	R_Reading	R-Writing	R_Spelling	R_Grammar and Punctuation	R_Numeracy
Figure 11	a	100%	100%	100%	100%	100%
	b	0.15	0.15	0.25	0.3	0.3
	n	1	1	1	1	1
Figure 12	a	100%	100%	100%	100%	100%
	b	0.15	0.15	0.25	0.3	0.3
	n	3	3	3	3	3

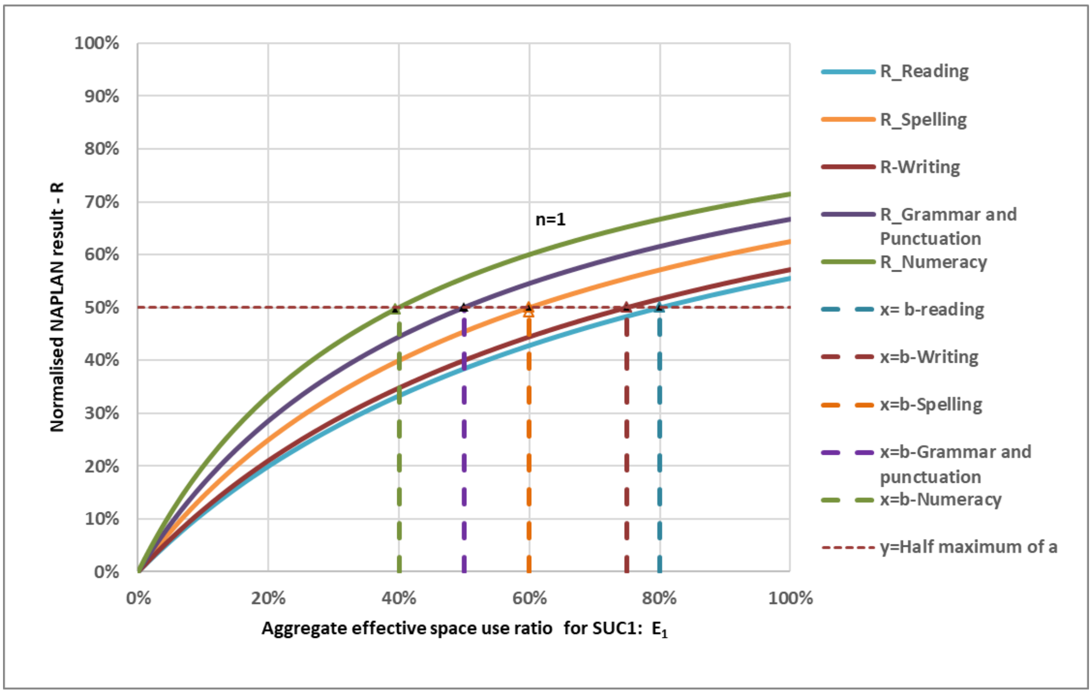

Figure 7: Normalised NAPLAN scores against E_1 with n=1

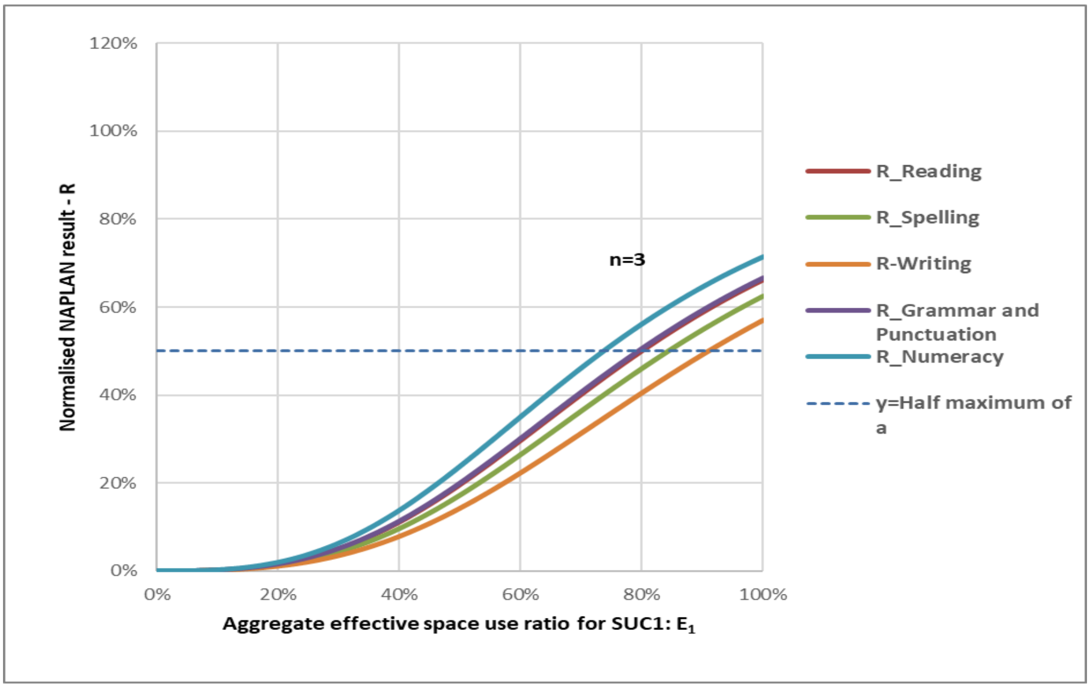

Figure 8: Normalised NAPLAN scores against E_1 with n=3

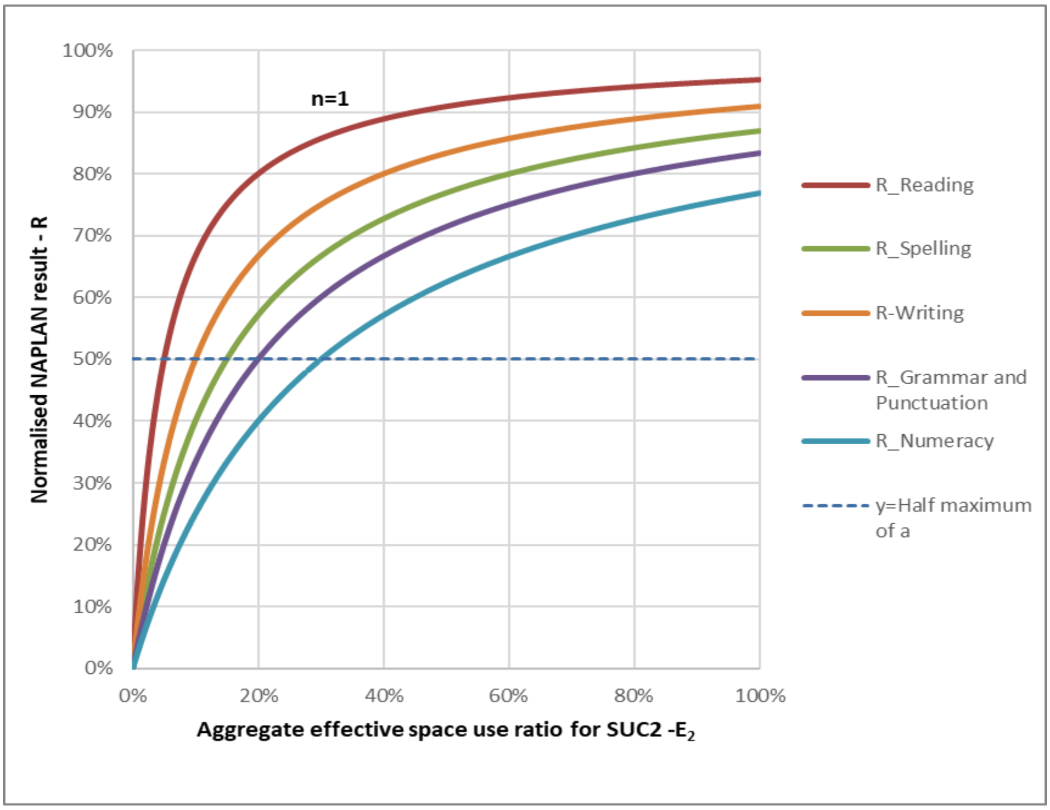

Figure 9: Normalised NAPLAN scores against E_2 with n=1

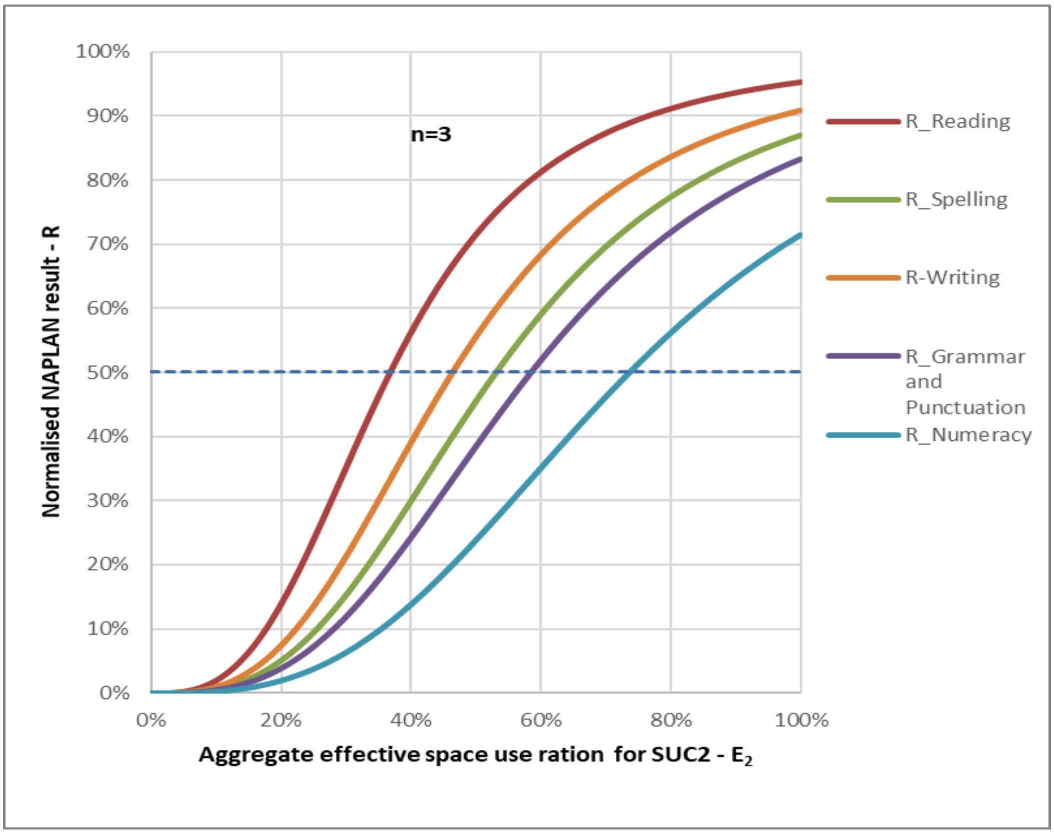

Figure 10: Normalised NAPLAN scores against E_2 with n=3

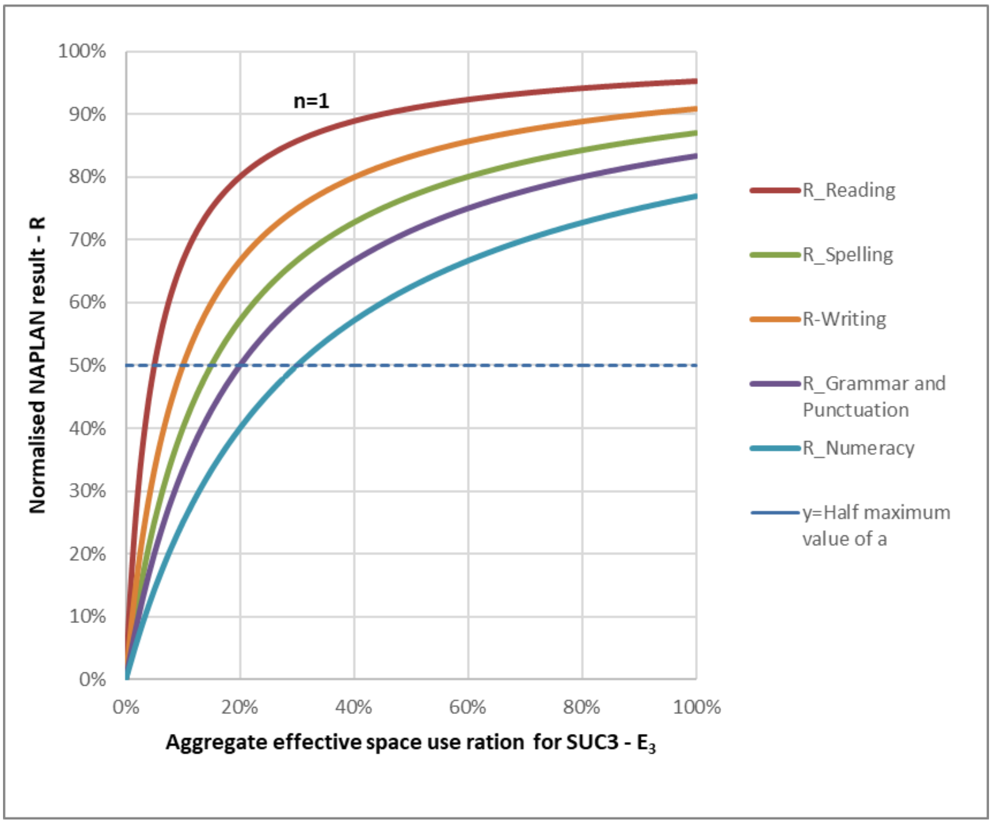

Figure 11: Normalised NAPLAN scores against E_3 with n=1

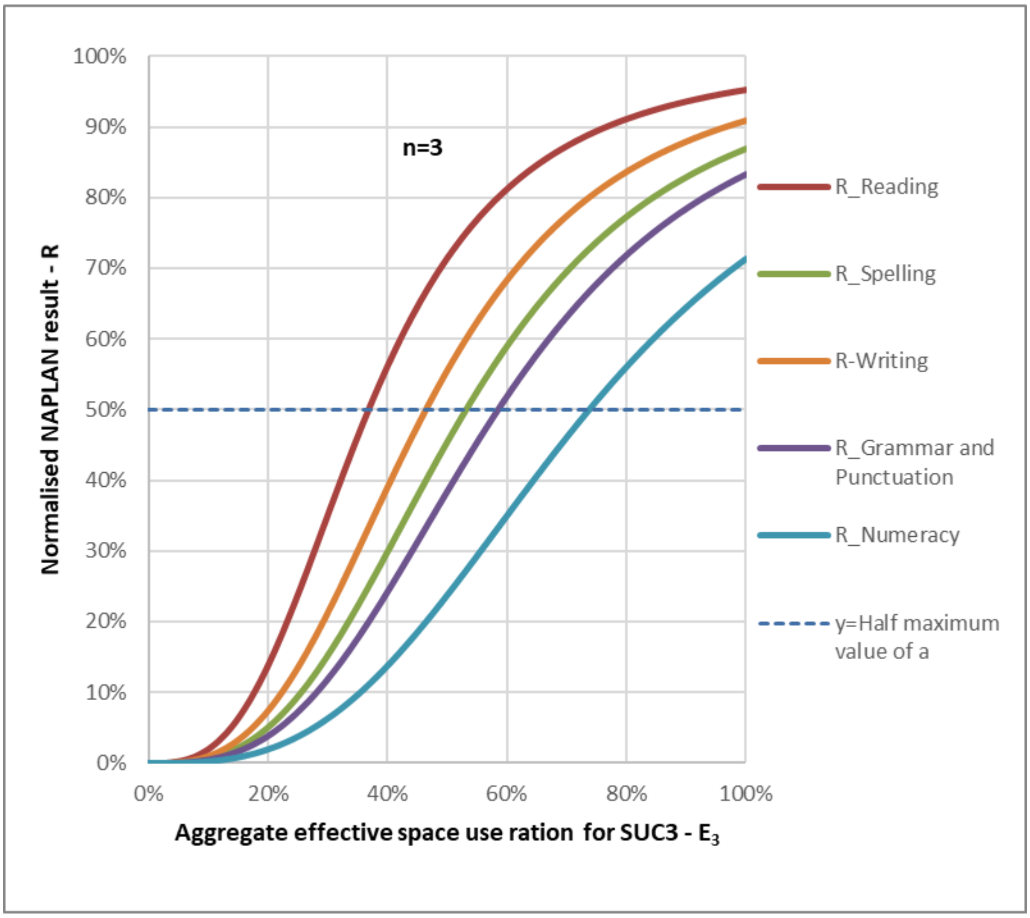

Figure 12: Normalised NAPLAN scores against E_3 with n=3

4.3 Step 3: Conduct multi-criteria optimisation

4.3.1 Application and demonstration

The next step is to run an optimisation problem with the b values obtained for each E-R combination. Genetic algorithm based multi-criteria optimisation was chosen, and optimisation was carried using 'gamultiobj' function in MATLAB with the criteria mentioned in Section 3.2.3. This function uses Pareto optimisation to evaluate objective functions constrained with equality and inequality relations along with lower and upper bounds. The function returns a set of Pareto points and objective function values which defines the points in the Pareto front. The objective for the 'gamultiobj' function was to find the combination of x_1, x_2 and x_3 which met the primary and secondary objectives defined in section 3.2.3. With the use of a data set obtained from several schools, b values can be calculated for each E-R combination.

Variations of *b* values referring to Equation 11 is presented in Table 7. Basically, as $b_{1,\,R}$, $b_{1,\,W}$, $b_{1,\,N}$, $b_{1,\,G\&P}$ and $b_{1,\,S}$ relate to x_1, which specifies the aggregate effective space use ratio for SUC1, it is considered to vary from 30% to 80% respectively. Similarly, the limits of *b* values for SUC2 and SUC3 (pertaining to the second and third rows of the matrix mentioned in Equation 11) are as mentioned in Table 7. The population size of the multi-criteria optimisation algorithm was by default set as 50 (as the number of objective functions was less than 5) implying that the MATLAB function will first populate 50 datasets of x_1, x_2 and x_3 which satisfies the objective functions. The function will then use a genetic algorithm to try and evaluate a set of *x* values such that the solutions of objective functions are better than its preceding results. The process is iterated multiple times until any stopping criteria are satisfied. The stopping criteria are selected such that the geometric average relative variation of *x* values is lesser than 1% for each iteration. Since the *x* values are ideally considered to be greater than b values, the limits for *x* and *b* values are set to be the same. By default, the number of solutions in the Pareto front is assigned as 35% of the size of the population.

Table 7: Assumed dataset of b values considered for multi-criteria optimisation

Relevant b values	Lower limit	Upper limit
$b_{1,\,R}$, $b_{1,\,W}$, $b_{1,\,N}$, $b_{1,\,G\&P}$, $b_{1,\,S}$	30%	80%
$b_{2,\,R}$, $b_{2,\,W}$, $b_{2,\,N}$, $b_{2,\,G\&P}$, $b_{2,\,S}$	10%	35%
$b_{3,\,R}$, $b_{3,\,W}$, $b_{3,\,N}$, $b_{3,\,G\&P}$, $b_{3,\,S}$	10%	35%

4.3.2 Results

After running multi-criteria optimisation in MATLAB, the x_1, x_2 and x_3 values are obtained as presented in Table 8. Detailed analysis of the aggregate effective space use ratio suggests that, with the educational space being limited to 100%, and with the use of other ratios which add up as percentages, the aggregate effective space use ratio would simplify to the space use ratios for each SUC. It is observed that the optimisation process has been able to converge to a set of Pareto points and objective function values in the Pareto front, satisfying all the primary and secondary objectives as shown in Table 8. Although the optimisation results returned 18 optimal datasets, the primary objective has the highest priority, it can be used to determine the best possible values for E_1, E_2 and E_3. Thus, the maximum non-zero b values were obtained or 17^{th} and 18^{th} datasets in Table 8 out of which 18^{th} dataset had the highest positive p-value. The E values thus obtained would be the ideal values for the corresponding E values beyond which the gain in R-value with increasing E starts to decrease. Hence, given the set of b values assumed In Table 7, the optimum E_1, E_2 and E_3 values would be 56.07%, 26.51% and 17.48% respectively.

Table 8: Results from multi-criteria optimization using the genetic algorithm.

Sl no.	Optimal values for E			Optimal values of primary and secondary objectives		
	E_1 (%)	E_2 (%)	E_3 (%)	Number of non-zero b values	Sum of positive p values	Sum of negative p values
1	79.99	10.00	10.00	5	1.50	-1.90
2	79.99	10.00	10.00	5	2.00	-1.10
3	79.99	10.00	10.02	5	1.90	-1.80
4	79.99	10.00	10.00	6	2.00	-1.70
5	79.72	10.20	10.06	7	1.90	-1.00
6	77.11	11.44	11.44	7	1.60	-1.20
7	77.11	11.44	11.44	8	1.70	-0.80
8	31.42	34.29	34.28	9	1.30	-1.30
9	31.56	34.18	34.25	9	1.50	-1.10
10	30.13	34.93	34.93	10	1.10	-1.40
11	30.13	34.87	34.93	10	1.50	-1.00
12	38.07	30.76	31.16	11	1.40	-0.70
13	38.07	30.76	31.16	12	1.20	-0.50
14	53.39	23.29	23.29	12	1.20	-0.20
15	52.02	29.31	18.75	13	0.90	-0.30
16	53.76	23.69	22.54	13	1.10	-0.10
17	52.22	29.29	18.47	14	1.00	-0.10
18	56.07	26.51	17.48	14	1.30	-0.10

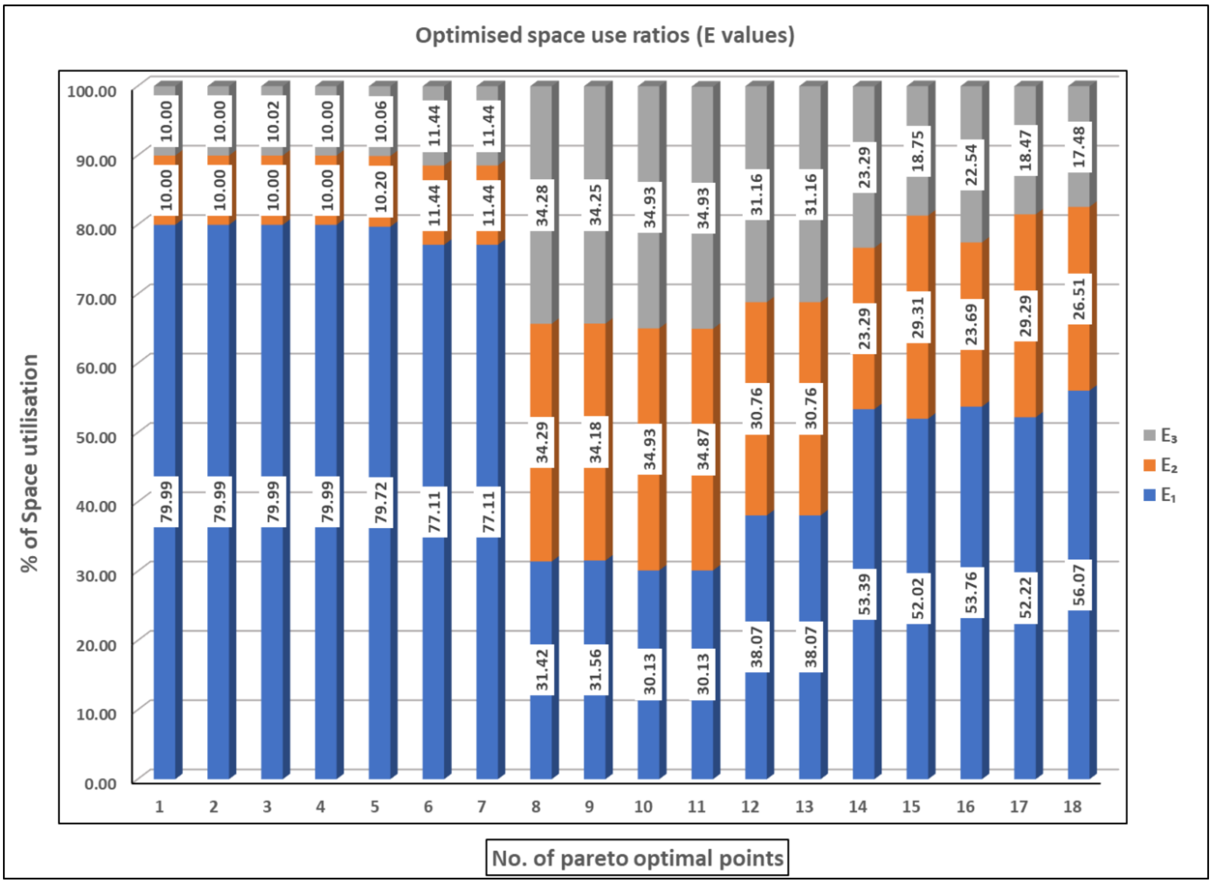

Fig13: Solutions from multi-objective optimisation using the genetic algorithm.

5 Summary and Conclusions

This paper conceptualises how the space distribution in an educational setting, particularly considering public schools, relate to the educational performance of students in a school. The educational performance used in this instance is specified as the student educational outcome measured through NAPLAN. After identifying how the educational performances measured through NAPLAN and the space use ratios for each space use categories (SUC) can be correlated (does not necessarily mean causality), the paper demonstrates a methodology to obtain optimum space use ratios which represent the optimum educational performance.

The outcome of this research is broadly to indicate that the intended use of a building, and hence the indicators which measure its effectiveness can be correlated to the design parameters quantitatively. This provides some guidance to the designers on the constraints; and at the same time, for educational policy makers the framework provides a foundation to integrate other aspects which lead to higher educational outcome and are constrained in terms of resources.

Summarised below are some of the key outcomes.

1. To relate the space use to the educational performance, an indicator called aggregate space use ratio (E) is introduced. E is defined for specific SUCs belonging to the educational space of

a school and is a ratio indicating the aggregated space use by an average student to achieve educational performance measured through NAPLAN. The conceptual framework, theoretical framework and methodology for evaluating E concerning different SUCs have been developed and demonstrated in this paper.

2. The relationship between E for each SUC and the educational performance measured through NAPLAN (R) for each skill level is observed to be a multivariate function. Further evaluation of the relationship presents that E for each SUC and R for each skill level would have the characteristics of a function with diminishing returns. Considering the fact when the space allocated to a certain SUC is relatively small (close to zero), having extra space for a certain space use category (SUC) makes a significant difference to the educational performance (R), while having extra space when there is already a significant amount of space for a SUC does not make a significant difference, the diminishing value relationship is approximated to follow a Hill function.
3. Based on that approximation, it is observed that the aggregate space use ratio (E) needs to be maintained above the value corresponding to the half-maximum of the horizontal asymptote (a) denoted by b, if a significant compromise of R needs to be avoided. While the E is optimum close to the horizontal asymptote (a), E values above the half-maximum (b) values are considered to have less compromise compared to the ones below.
4. The theoretical framework leading to validating a data set to check whether the approximation follows Hill function and the methodology for solving the Hill function to obtain the characteristics are elaborated in this paper. The key characteristics to define the function are: a – the horizontal asymptote, n- the Hill coefficient and b – the half max relevant to the horizontal asymptote.
5. Having 15 E-R combinations for each SUC related to education and each different skill assessed by NAPLAN, multiple b values are obtained with the solving of the Hill function. In order to obtain the suggested E value which meets with the requirement for most of E-R combinations, multi-criteria optimisation with genetic algorithm is performed.

The framework and methodology suggested in this paper can be used to evaluate an optimum space distribution between different SUC in order to obtain values which represent the highest educational performance of students. The framework has been developed considering a primary school and yet applicable for a school setting in general. The method follows a logical path and presents a systematic approach to correlate space use ratios as a single indicator influencing the physical environment affecting student activity and hence effectiveness of their performance. While the methodology needs to be validated using a data set, once validated, it can be universally employed by educational policy makers to integrate with other factors which influence student performance. For building designers, architects, regulators and policy makers, the research presented in this paper provides a framework and methodology as to how to incorporate the purpose of design, represented by one factor in this instance, into the decision-making process as a possible constraint.

5.1 Limitations

The relationship between the space use ratios used in designing the school building have been considered in isolation in determining the relationship between the space use ratios and the educational outcome assessed through NAPLAN. However it is acknowledged that correlation does not mean causality, i.e. optimum design of school buildings will not necessarily lead to student scoring high marks in NAPLAN, as there are a number of other significant influencing factors involved NAPLAN is used as an indicator to determine the student performance and hence the effectiveness of their educational outcome as quantitative results are available for a comparison, though the authors acknowledge that the student achievement in NAPLAN is a single aspect reflecting the holistic development occurs in a school.

Appendix

Nomenclature

NAPLAN - National Assessment Plan for Literacy and Numeracy.

R – Normalised NAPLAN score for a skill level with respect to national average.

AESU – Aggregate Effective Space Utilisation.

SUC – Space Use Category.

SUCR – Space Use Category Ratio.

ESU – Effective Space Utilisation.

E – Aggregate effective space use ratio.

C_{ij} – space use category ratio for building category I in j building.

B_j – Building number in the school.

C_E – Sum of space use category ratios of all building categories contributing to NAPLAN score.

$t_{i,j,k,y}$ – time allocation basis for a category for a particular year.

k_n – number of year levels in the particular school under consideration.

$S_{k,y}$ – percentage contribution of year level k to the NAPLAN score.

E_{max} – maximum possible aggregate effective space utilisation for a space category with the resource and regulation constraints.

a - Horizontal asymptote signifying maximum possible NAPLAN score for the particular category and skill level.

b – Aggregate effective space use ratio at half maximum of NAPLAN score for the particular category and skill level.

n – hill function coefficient for a particular category.

Acknowledgements

The authors would like to thank the Australian Research Council's Linkage grant # LP160100259 titled: Value in Operations project "innovative procurement theories to optimize education per cost of school" for the support and funding for this study.

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