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Patient-Specific Neural Mass Modelling - Stochastic and Deterministic Methods

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To add at end.
What the paper is about.
Why we are doing it.
THe experiments in the paper.
The results of the experiments.
Big picture.

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1. Introduction

This chapter describes two approaches for forming patient-specific mesoscopic neural mass models; a stochastic and a deterministic method. The aim of these methods is to form a bridge between clinical and computational neuroscience that will facilitate the application of control engineering tools[1] to develop new therapies to neurological conditions, such as epilepsy.

The human brain is arguably the most complicated system known to man. The development of a complete theory for its function is one of the greatest challenges faced by scientists today. To address this challenge, researchers have naturally used both theoretical and experimental frameworks to develop and test hypotheses. Experimental frameworks are typically de-

signed to uncover causal relationships between the systems properties or parameters and its function (reverse engineering the brain). Theoretical studies are typically used in one of two ways. The first is to explain data acquired in an experiment and the second is to predict system behaviour. This work is aimed at developing two more uses for theoretical studies and computational models. The first is to use the models as filters, forming a lens into the physiology of the brain by estimating the states (i.e. neural activity) and the parameters (for instance synaptic strength) of the modelled physiology. The second is to use the model as a tool to develop new therapies, where they can be used to provide feedback to a system that can systematically deliver an intervention (e.g. electrical stimulation) in a robust, controlled manner.

Presently, it is almost half a century since Hodgkin and Huxley shared the Nobel prize (in 1963 with Eccles for Physiology and Medicine) for their influential model that helped to establish the field of theoretical neuroscience. Over this time period, both experimental and theoretical methods have developed considerably. Experimental approaches have been able to isolate function using various forms of manipulation in greater detail. In parallel, theoretical/computational models have been able to explain and provide new hypotheses for an ever increasing assortment of neural phenomena. Through the development of these models a fundamental set of parametric equations has been established that explain neuronal responses to sensory input at varying spatiotemporal scales from small patches of membrane to networks of neural ensembles.

Although the predictive explanatory power of the theoretical models is rather vast when they are tuned to mimic experimental conditions, a major limitation has been in establishing a rigorous method for choosing model parameters in more general situations. For example, over the last decade it has come to light that there is significant variability in neural systems across subjects despite seemingly similar network activity (ref Eve Marder 2006). It has been shown that a target network behaviour may be achieved by a continuum of parameter combinations (Tail Wagging the Dog ref). We expect an analogous situation in disease states and pathological activity such as epileptic seizures. Therefore, for models to be clinically useful they must be subject-specific.

By assimilating experimental or clinical measurements with mathematical models one can infer unmeasured or latent system properties, or parameters, from standard clinical (electrophysiological) recordings. Inference of patient-specific parameters has the potential to revolutionise the treatment

of disease. Typically, one can measure the blood-oxygen level dependent signal through functional Magnetic Resonance Imaging (fMRI) or the electromagnetic fields of the brain through electroencephalography (EEG) or magnetoencephalography (MEG). In clinical neurology, these measurement modalities give a kind or a 'where and when' indication of the presence of a pathology, but provide little information on the causative mechanisms for disease. A classic example of this is in epilepsy management, where the disease is diagnosed using EEG by identifying electrographic seizures. If electrical seizures are recorded with EEG, a medication plan is prescribed. The choice of medication is not based on the data acquired with the EEG, but based on the clinician's experience and the patient's circumstance, then the treatment plan is often modified based on trial and error. This process of searching for the best drug combination is often required and similar electrographic seizures can have fundamentally different mechanisms of initiation (ref JTs paper). This means epilepsy is difficult to treat. Ideally in epilepsy monitoring we would like to image the concentration of ions, the synaptic dynamics, the connectivity strength and structure, or other parameters to better inform a treatment plan by understanding physiological changes that lead to seizures. This is also highly desirable for epileptic seizure prediction which has shown limited progress to date.[2,3]

The signal processing and control theory literature generally provides two approaches to state and parameter estimation of models using measured data (otherwise known in engineering as system identification): stochastic and deterministic. In deterministic approaches[4–6] there is no universal technique that can be applied to a given general nonlinear system. Rather, the synthesis of deterministic approaches is often more specific to the form of the system being estimated. Often deterministic assumptions are initially quite strict when first devising an estimator, however, these assumptions can usually be relaxed in order to deal with noise issues. Stochastic approaches[1,7] on the other hand can be applied more easily to arbitrary systems, however, these approaches typically require that the estimator be initialised close to the true initial condition, which is realistically unknown. Moreover, the convergence of the estimates to the true values is not guaranteed for every trajectory.

In this chapter we first describe the neural mass model with which we will perform estimation, the Jansen-Rit model.[8] Then we show how two different estimation methods, one stochastic and one deterministic, can perform state and parameter estimation of the Jansen-Rit model to illustrate their usefulness in inferring underlying and unmeasured physiological vari-

ables using only limited physiological measurements. Specifically, we apply a stochastic approach based on the unscented Kalman filter (UKF)[9,10] to estimation of the states and parameters of the Jansen-Rit model using both simulated data and real electrocorticography (ECoG) data of a seizure. Then we apply a deterministic approach, referred to as an adaptive observer,[11,12] to estimation of the states and parameters of the Jansen-Rit model using simulated data. Finally we contrast and compare the stochastic and deterministic approaches in the discussion.

2. Neural Mass Model

To define a standard neural mass model we begin by defining the post-synaptic potential of population n as a result of an input firing rate from population m as their convolution

$$v_n(t) = v_{r,n} + \int_{-\infty}^{t} \frac{\alpha_{mn}(t)}{\tau_{mn}(t)} h_{mn}(t-t') g_m(v_m(t'))\,\mathrm{d}t' \tag{1}$$

$$v_n(t) - v_{r,n} = \int_{-\infty}^{t} \frac{\alpha_{mn}(t)}{\tau_{mn}(t)} h_{mn}(t-t') g_m(v_m(t'))\,\mathrm{d}t' \tag{2}$$

$$\tilde{v}_n(t) = \int_{-\infty}^{t} \frac{\alpha_{mn}(t)}{\tau_{mn}(t)} h_{mn}(t-t') g_m(v_m(t'))\,\mathrm{d}t', \tag{3}$$

where $\alpha_{mn}(t)$ is the gain for the post-synaptic response kernel, denoted by $h_{mn}(t)$, from neural population m to n and $\tau_{mn}(t)$ is the membrane time constant. Typically, $\alpha_{mn}(t)$ and $\tau_{mn}(t)$ are constants (particularly for current based synapses), but for generality we will denote them as time varying quantities. Also, $g_m(v_m(t'))$ describes the input firing rate as a function of the pre-synaptic membrane potential. The resting membrane potential of the post-synaptic population is denoted by $v_{r,n}$, $v_n(t)$ is the post-synaptic membrane potential and $\tilde{v}_n(t)$ is the deviation of the membrane from the resting potential. For the network of neural masses that we are considering in the chapter the index n (post-synaptic) may represent either the pyramidal (p), excitatory interneuron (spiny stellate) (e), or inhibitory interneuron (i) populations.

The post-synaptic response kernel, $h_{mn}(t)$, typically takes one of three different forms - one first order and two second order. The first-order form has an instantaneous rise time and a decay defined by a single time constant.[13] The second order kernels have a finite rise and decay time, with the difference being with one form having separate time constants[14] (bi-exponential) for the rise (synaptic time constant) and decay (membrane

time constant), where the other form is defined using a single time constant variable by[15]

$$h_{mn}(t) = \eta(t)t \exp\left(-\frac{t}{\tau_{mn}(t)}\right) \tag{4}$$

where $\eta(t)$ is the Heaviside step function. This is the form we shall use in this chapter, however the framework holds for other forms.

This convolution can conveniently be written as

$$\mathrm{D}\tilde{v}_n(t) = \frac{\alpha_{mn}(t)}{\tau_{mn}(t)} g_m(v_m(t')), \tag{5}$$

where the linear differential operator, D, is

$$\mathrm{D} = \frac{\mathrm{d}^2}{\mathrm{d}t^2} + \frac{2}{\tau_{mn}(t)}\frac{\mathrm{d}}{\mathrm{d}t} + \frac{1}{\tau_{mn}^2(t)} \tag{6}$$

This allows the dynamics of the neural mass to be described by the differential equation

$$\frac{\mathrm{d}^2\tilde{v}_n(t)}{\mathrm{d}t^2} + \frac{2}{\tau_{mn}(t)}\frac{\mathrm{d}\tilde{v}_n(t)}{\mathrm{d}t} + \frac{1}{\tau_{mn}^2(t)}\tilde{v}_n(t) = \frac{\alpha_{mn}(t)}{\tau_{mn}(t)} g_m(v_m(t)). \tag{7}$$

This second-order ODE can be written as two coupled first-order ODEs by defining

$$z_n(t) = \frac{\mathrm{d}\tilde{v}_n(t)}{\mathrm{d}t}. \tag{8}$$

Recasting the system in this way allows formation of a state-space model in a canonical format. This gives the system

$$\frac{\mathrm{d}\tilde{v}_n(t)}{\mathrm{d}t} = z_n(t) \tag{9}$$

$$\frac{\mathrm{d}z_n(t)}{\mathrm{d}t} = \frac{\alpha_{mn}(t)}{\tau_{mn}(t)} g_m(v_m(t)) - \frac{2}{\tau_{mn}(t)} z_n(t) - \frac{1}{\tau_{mn}^2(t)}\tilde{v}_n(t). \tag{10}$$

There is a sigmoidal relationship between the mean membrane potential and firing rate of each of the populations. This sigmoid nonlinearity may take different forms, for example the cumulative density function (error function) or the logistic / hyperbolic tangent. Typically, the logistic function form is used, which is defined by

$$g\left(\tilde{v}_n(t)\right) = \frac{1}{1 + \exp\left(\varsigma_n(t)\left(v_{0n}(t) - \tilde{v}_n(t)\right)\right)} \tag{11}$$

$$g\left(\tilde{v}_n(t)\right) = \frac{1}{1 + \exp\left(\varsigma_n(t)\left(v_{0n}(t) + v_{rn} - v_n(t)\right)\right)} \tag{12}$$

$$g\left(v_n(t)\right) = \frac{1}{1 + \exp\left(\varsigma_n(t)\left(\tilde{v}_{0n}(t) - v_n(t)\right)\right)} \tag{13}$$

where $\tilde{v}_{0n}(t) = v_{0n}(t) + v_{rn}$. Note that in this formulation we are absorbing the maximal firing rate, which is typically a linear coefficient on the sigmoid, into the PSP gain ($\alpha_{mn}(t)$). This removes a redundant parameter that can not be recovered by estimation methods. The quantities $\varsigma_n(t)$ and $v_{0n}(t)$ describe the slope of the sigmoid (variance of firing thresholds within the populations) and the mean firing threshold, respectively. These quantities are usually considered as constants, but again will be treated as being time varying. The parameter $\tilde{v}_{0n}(t)$ describes the deviation from the resting membrane potential, which becomes our lumped threshold parameter. For ease of notation we can drop the *tilde* remembering the resting membrane potential resides within this term. Figure 1 depicts a standard neural mass.

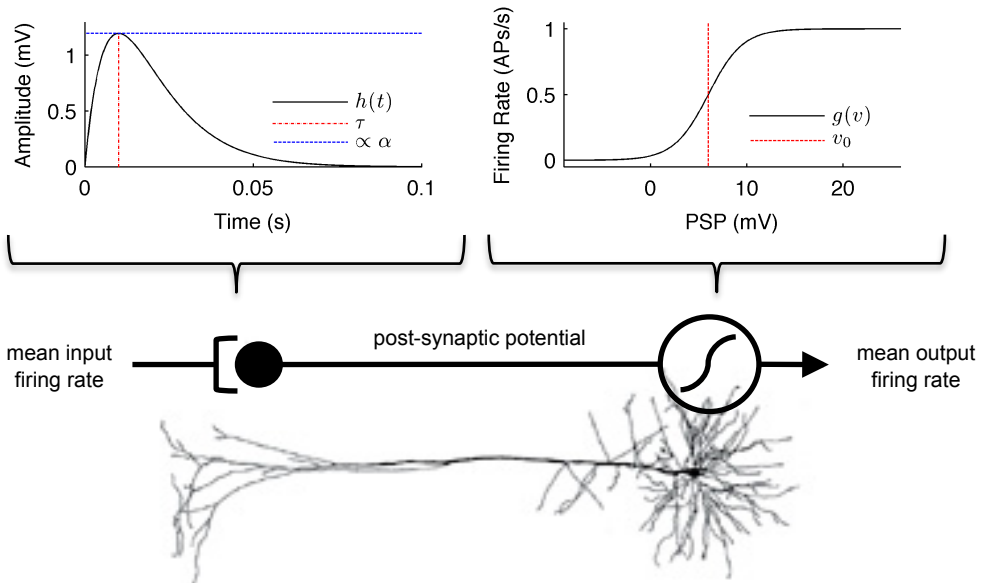

Fig. 1. **Graphical Representation of the Neural Mass Model.** The neural mass model converts an input firing rate to a mean post-synaptic potential by a convolution with the post-synaptic response kernel. The membrane potential is converted to output firing rate by the sigmoidal activation function.

This neural mass maps from a mean pre-synaptic firing rate to a post-synaptic mean membrane potential. The terms that are usually considered parameters of the model include ζ **is not defined** $\zeta(t)$, $\alpha(t)$, $v_0(t)$, and $\varsigma(t)$. These can be set to model different neural populations, such as pyrami-

dal neurons, spiny stellate cells, and fast and slow inhibitory interneurons ($GABA_a$ and $GABA_b$). The neural populations can then be configured to represent the circuitry of a cortical column and networks of cortical columns. Contributions in this regard have been made by **Lopes Da Silva et al., Jansen and Rit, Wendling et al., Friston et al., and others**.

The parameters of the neural masses not only define the population type, but also the behaviour the model exhibits. For example, for certain parameter combination we get a model of a cortical column **that** will generate alpha type activity and for another set of parameters we get another model **does it have a name or a reference?** that will exhibit epileptic behaviour. Therefore, we consider this neural mass as a family of models, which we define as

$$\dot{\mathbf{x}}(t) = f_\theta\left(\mathbf{x}(t), \varepsilon(t)\right) \tag{14}$$

$$y(t) = \mathbf{C}\mathbf{x}(t) + e(t), \tag{15}$$

where $\mathbf{x}(t) \in \mathbb{R}^{n_x}$ is a state vector representing the postsynaptic membrane potentials and their derivatives, n_x is the number of states, ε represents system noise, which may be unmodeled inputs and model inaccuracies. The function $f_\theta(\cdot)$ describes the dynamics where $\theta \in \mathbb{R}^{n_\theta}$ determines the mass type and the behaviour it exhibits. The EEG is denoted by $y(t)$, $\mathbf{C}$ is the observation matrix, and $e(t)$ is the observation noise. An example of the Jansen and Rit model of a cortical column is shown in Figure 2.

3. Stochastic Estimation

In this section we shall switch to discrete time notation, indicated by using the **superscript** subscript t to index the samples. Our goal is to estimate $\mathbf{x}^t$ given knowledge of the biophysics of the mass action of the brain and the noisy iEEG measurements. In other words, we want to find

$$\hat{\mathbf{x}}^{t+} = \mathbb{E}\left[\mathbf{x}^t | y^1, y^2, \ldots, y^t\right], \tag{16}$$

which is known as the *a posteriori* state estimate. This can be found by optimally combining the *a priori* state estimate, which is defined as

$$\hat{\mathbf{x}}^{t-} = \mathbb{E}\left[\mathbf{x}^t | y^1, y^2, \ldots, y^{t-1}\right], \tag{17}$$

with the noisy measurements. The *a priori* an estimate of the same quantity as the *a posteriori* estimate, but with only using the prediction from the model and not using the information from the most recent observation at time t. The way in which the *a priori* estimate and the noisy observation is

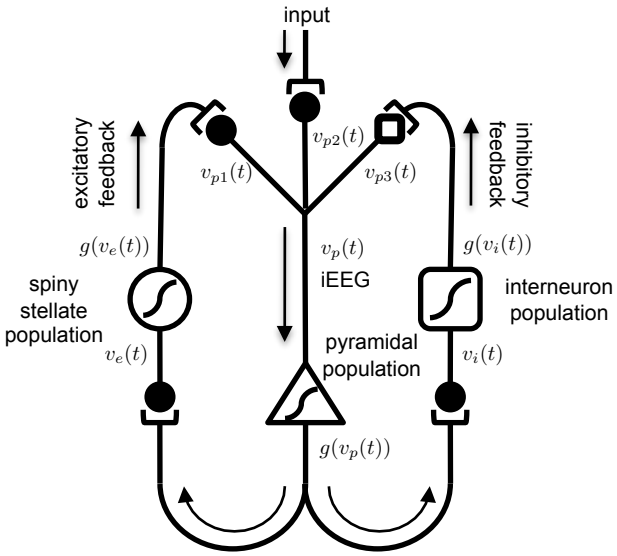

Fig. 2. **Model of a Cortical Column.** The model shows three interconnected neural masses, which are pyramidal neurons, excitatory spiny stellate cells, and inhibitory interneurons. The specific subtype of neural population is defined by the parameters that describe the post-synaptic response kernels.

combined is based on the level of confidence we have in our measurement (the noise level) and the inaccuracy of our model (the disturbance level).

Now we can define the *a posteriori* and *a priori* state estimate error covariances as

$$P^{t-} = \mathbb{E}\left[(\mathbf{x}^t - \hat{\mathbf{x}}^{t-})(\mathbf{x}^t - \hat{\mathbf{x}}^{t-})^\top\right] \tag{18}$$

$$P^{t+} = \mathbb{E}\left[(\mathbf{x}^t - \hat{\mathbf{x}}^{t+})(\mathbf{x}^t - \hat{\mathbf{x}}^{t+})^\top\right], \tag{19}$$

respectively. For linear systems, the famous Kalman filter[16] provides the solution to the estimation problem. However, for the nonlinear neural mass model the integration that is required for solving the expectations has no closed-form solution. Therefore, an approximate solution is required for efficient filtering (of high dimensional systems). An appropriate method for this approximation is the unscented transform, which leads to the unscented Kalman filter.[17]

The unscented transform (UT) is a method for approximating the statistics of a random variable that undergoes a nonlinear transformation. Consider transforming an n-dimensional random variable, $\mathbf{x}$ (state vector), through a nonlinear function, $\mathbf{x}^{t+1} = f(\mathbf{x}^t)$. Assume $\mathbf{x}^t \sim \mathcal{N}(\bar{\mathbf{x}}^t, \mathbf{P}_{\mathbf{x}^t})$.

To map the statistics through the nonlinear transform, we first define the so-called $(2n+1)$ sigma vectors, which form the matrix $\mathcal{X}$ as

$$\mathcal{X}_0^t = \bar{\mathbf{x}}^t \tag{20}$$

$$\mathcal{X}_i^t = \bar{\mathbf{x}}^t + \left(\sqrt{(n+\lambda)\,\mathbf{P}_{\mathbf{x}}}\right)_i \quad i = 1, \ldots, n \tag{21}$$

$$\mathcal{X}_i^t = \bar{\mathbf{x}}^t - \left(\sqrt{(n+\lambda)\,\mathbf{P}_{\mathbf{x}}}\right)_{i-n} \quad i = n+1, \ldots, 2n, \tag{22}$$

where the quantity $\lambda = \alpha^2\,(n+\kappa) - n$ (usually a small positive value e.g. $1 \geq \alpha \geq 10^{-4}$) is a scaling parameter. The constant α determines the spread of the sigma points around the mean, $\bar{\mathbf{x}}^t$. The constant κ is a secondary scaling parameter, which is usually set to 0 or $3-n$. $\left(\sqrt{(n+\lambda)\,\mathbf{P}_{\mathbf{x}^t}}\right)_i$ is the i^{th} column of the matrix square root (e.g. lower triangle Cholesky factorisation). The sigma vectors are propagated through the nonlinear system

$$\mathcal{X}_i^{t+1} = f\left(\mathcal{X}_i^t\right) \quad i = 1, \ldots, 2n, \tag{23}$$

and the mean and covariance for the transformed variable, $\mathbf{y}$, are approximated by

$$\bar{\mathbf{x}}^{t+1} \approx \sum_{i=0}^{2n} W_i^{(m)} \mathcal{X}_i^{t+1} \tag{24}$$

$$\mathbf{P}_{\mathbf{x}^{t+1}} \approx \sum_{i=0}^{2n} W_i^{(c)} \left(\mathcal{X}_i^{t+1} - \bar{\mathbf{x}}^{t+1}\right)\left(\mathcal{X}_i^{t+1} - \bar{\mathbf{x}}^{t+1}\right)^\top \tag{25}$$

where the weights, W_i are

$$W_0^{(m)} = \frac{\lambda}{n+\lambda} \tag{26}$$

$$W_0^{(c)} = \frac{\lambda}{n+\lambda} + \left(1 - \alpha^2 + \beta\right) \tag{27}$$

$$W_i^{(m)} = \frac{1}{2\,(n+\lambda)} \quad i = 1, \ldots, 2n, \tag{28}$$

and β is a variable that includes prior knowledge of the distribution of $\mathbf{x}$. By using the UT to propagate the state estimates and errors through time and the standard Kalman filter update equations (since the observation equation is modelled as being linear) the filter is formed and the states can be estimated. The UT captures the properties of the nonlinear transform of a Gaussian random variable to 3^{rd} order Taylor series expansion for all nonlinearities **lets discuss** (and higher with the appropriate choice of α and β).

An alternative way of describing the model in equation 14 is

$$\dot{\mathbf{x}}(t) = f\left(\mathbf{x}(t), \theta(t), \varepsilon(t)\right) \tag{29}$$

$$\dot{\theta}(t) = 0 + \varphi(t), \tag{30}$$

which is an augmented state-space model. By augmenting the state vector with parameters that are to be estimated we can perform state and parameter estimation simultaneously. The disturbance term, $\varphi(t)$, essentially adds a small amount of uncertainty to the trivial dynamics. This allows the Kalman filter to provide small corrections to the model predictions with each arrival of new data.[18]

3.1. *Results*

3.2. *Synthetic Data*

In this section we demonstrate how the stochastic estimation frame is able to estimate the neural mass model parameters when their values are initialized 60% above and below their actual values. The data **the** was used in this simulation was generated using the parameters described in Jansen and Rit et al. (xxxx) for generating alpha activity.

3.3. *ECoG Data*

Describe the data here.

4. Deterministic Estimation: Adaptive Observer

This section describes the adaptive observer considered here.[19] Proofs that demonstrate **that is extra I guess** that the observer's parameter**s** and state estimates converge to the true values for the Jansen-Rit model are provided in.[19] Here adequate details and equations for the adaptive observer are given so that one can implement it easily. The adaptive observer considered here has been designed for systems which have the following state space form

$$\begin{aligned} \dot{x}_0 &= A_0 x_0 + \phi_0(y)\theta \\ \dot{x}_1 &= A_1 x_1 + \phi_1(x_0, u)\theta \\ y &= C_1 x_1, \end{aligned} \tag{31}$$

where $x_0 \in \mathbb{R}^{n_0}$, $x_1 \in \mathbb{R}^{n_1}$ are the states, $\theta \in \mathbb{R}^p$ is a vector of constant and unknown parameters, $y \in \mathbb{R}^{n_y}$ is the measurement and $u \in \mathbb{R}^{n_u}$ is the input.

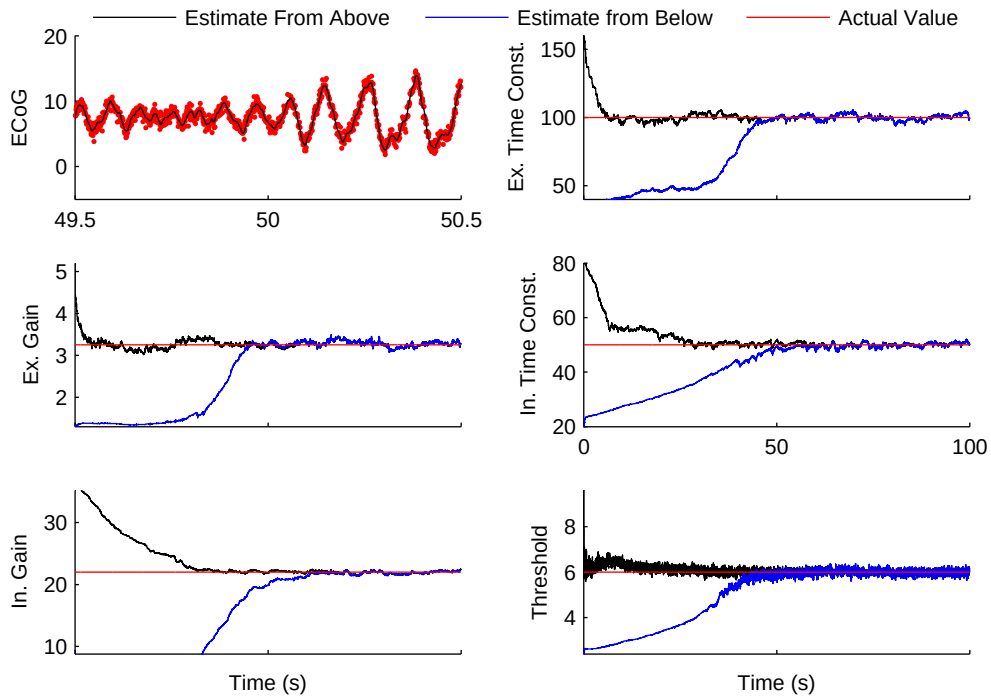

Fig. 3. **Estimation Results from Synthetic Data.** a) An example of the synthetic data that was used, illustrating the level of noise add to the model output. Subplots b) to f) show the estimated parameters from when the initial guess was 60% above and below the actual value. b) Estimate of the sigmoid threshold parameter, $v_0(t)$. c) Estimate of the excitatory gain parameter. d) Estimate of the excitatory synaptic time constant. e) Estimate of the Inhibitory gain parameter. f) Estimate of the inhibitory time constant.

The Jansen-Rit model[8] can be expressed in the state-space form of equation 31 by taking the states to be[a] $x_0 = (x_{01}, x_{02}) \in \mathbb{R}^2$ and $x_1 = (x_{11}, \ldots, x_{14}) \in \mathbb{R}^4$, where x_{01}, x_{11}, x_{13} are membrane potential contributions of the pyramidal neurons, the excitatory and inhibitory interneurons respectively, and x_{02}, x_{12}, x_{14} are their respective derivatives. The measured output (EEG) is $y \in \mathbb{R}$, $u \in \mathbb{R}$ is the excitatory input from neighbouring columns which is assumed to be known and $\theta = (\theta_A, \theta_B) \in \Theta \subseteq \mathbb{R}^2$ is a vector of *unknown* parameters where θ_A and θ_B represent the synaptic gains of the excitatory and inhibitory neuronal populations respectively.

[a]According to the notation of,[8] $x_0 = (y_0, y_3)$ and $x_1 = (y_1, y_4, y_2, y_5)$.

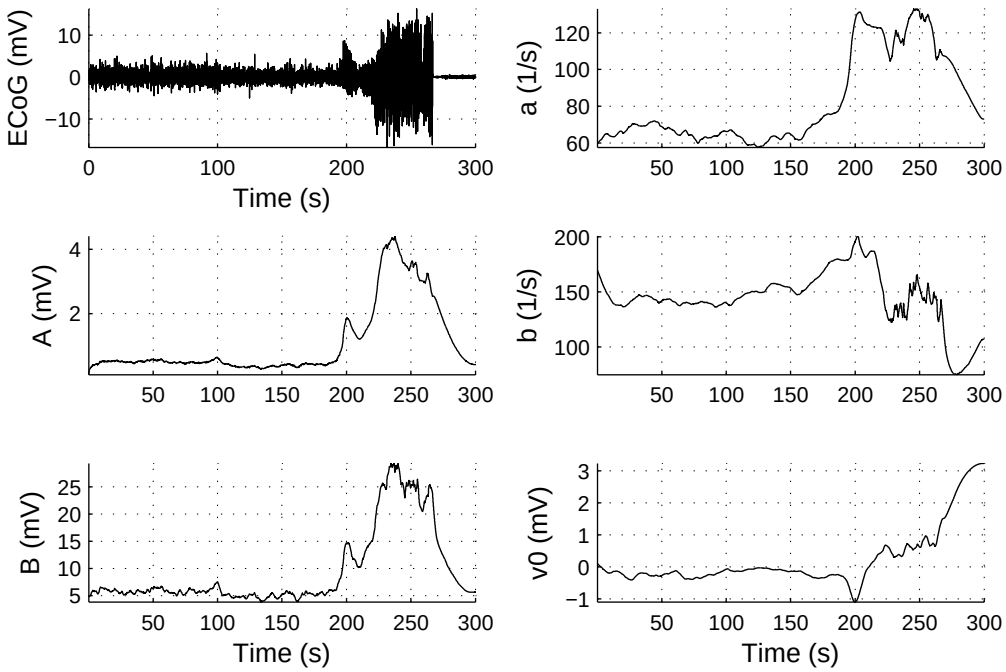

Fig. 4. **Estimation Results from ECoG Data.** a) The ECoG data from a focal channel recorded from a grid array. b) Estimate of the excitatory synaptic time constant. c) Estimate of the excitatory gain parameter. d) Estimate of the inhibitory time constant.. e) Estimate of the Inhibitory gain parameter. f) Estimate of the sigmoid threshold parameter, $v_0(t)$

The matrices in (31) are defined as:

$$C_1 = \begin{pmatrix} 1 & 0 & -1 & 0 \end{pmatrix}, \; A_0 = A_a, \; A_1 = \mathrm{diag}(A_a, A_b),$$

$$\text{where } A_a = \begin{pmatrix} 0 & 1 \\ -a^2 & -2a \end{pmatrix}, \; A_b = \begin{pmatrix} 0 & 1 \\ -b^2 & -2b \end{pmatrix}.$$

The post-synaptic response time constant parameters $a, b \in \mathbb{R}_{\geq 0}$ are assumed to be known. It has to be noted that the matrices A_0 and A_1 are

Hurwitz. The nonlinear terms in (31) are given by:

$$\phi_0(y) = \begin{pmatrix} 0 & 0 \\ aS(y) & 0 \end{pmatrix},$$
$$\phi_1(x_0,u) = \begin{pmatrix} 0 & 0 \\ ac_2S(c_1x_{01})+au & 0 \\ 0 & 0 \\ 0 & bc_4S(c_3x_{01}) \end{pmatrix}.$$

The connectivity parameters $c_1, c_2, c_3, c_4 \in \mathbb{R}_{\geq 0}$ are assumed to be known parameters and S denotes the sigmoid function, for $v \in \mathbb{R}$: $S(v) = \frac{2e_0}{1+e^{r(v_0-v)}}$ with known constants for the sigmoid rate-function $e_0, v_0, r \in \mathbb{R}_{\geq 0}$.

For ease of notation, we write (31) in the following form:

$$\begin{aligned} \dot{x} &= Ax + \phi(y,u,x)\theta \\ y &= Cx, \end{aligned} \tag{32}$$

where $x = (x_0, x_1)$, $A = \mathrm{diag}(A_0, A_1)$, $C = (0, C_1)$ and $\phi = (\phi_0, \phi_1)$. The nonlinear terms $\phi_0 : \mathbb{R} \to \mathbb{R}^{n_0} \times \mathbb{R}^p$ and $\phi_1 : \mathbb{R}^{n_0} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_1} \times \mathbb{R}^p$ are globally Lipschitz and bounded. The matrices A_0 and A_1 are Hurwitz. We consider the following adaptive observer for system (32):

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + \phi(y,u,\hat{x})\hat{\theta} + \Gamma(y-\hat{y}) \\ \hat{y} &= C\hat{x} \\ \dot{\hat{\theta}} &= \bar{\Gamma}(y-\hat{y}) \\ \dot{\Upsilon} &= A\Upsilon + \Delta\phi(y,u,\hat{x}) \qquad \text{with } \Upsilon(0) = 0 \\ \dot{P} &= dP - dP\Upsilon^{\mathrm{T}}C^{\mathrm{T}}C\Upsilon P \qquad \text{with } P(0) = P(0)^T > 0, \end{aligned} \tag{33}$$

where $\Gamma = \Delta^{-1}\Upsilon\bar{\Gamma}$, $\bar{\Gamma} = P\Upsilon^{\mathrm{T}}C^{\mathrm{T}}$ and $\Delta = \mathrm{diag}(\mathbb{I}_{n_0}, \frac{1}{d}\mathbb{I}_{n_1})$ with $d > 0$ a design parameter. The vector $\hat{x}$ denotes the estimate of x and $\hat{\theta}$ the estimate of θ. The variable $\Upsilon \in \mathbb{R}^{(n_0+n_1)\times p}$ is initialized to $\Upsilon(0) = 0$ in order to simplify the technical statements. Our result also applies when it is not the case. An essential assumption for our design to work is the condition stated below.

Assumption: For any signals $u, y, \hat{x}$ that belong to $\mathcal{L}_\infty$, there exist $a_1, a_2 \in \mathbb{R}_{\geq 0}$, $T \in \mathbb{R}_{\geq 0}$ such that the solution to:

$$\dot{\Upsilon} = A\Upsilon + \Delta\phi(y,u,\hat{x}) \text{ with } \Upsilon(0) = 0, \tag{34}$$

satisfies for all $t \geq 0$:

$$a_1\mathbb{I}_2 \leq \int_t^{t+T} \Upsilon^{\mathrm{T}}(\tau)C^{\mathrm{T}}C\Upsilon(\tau)d\tau \leq a_2\mathbb{I}_2. \tag{35}$$

The condition 35 is known as the persistency of excitation of the signal $C\Upsilon(t)$ and is a well-known condition in identification and adaptive control

literature (see[20]). This is a sufficient condition for the identification of the parameter θ using the adaptive observer proposed here and it is similar to the condition used in (A4) of[21] and (A3) in.[22] Inequality 35 is hard to verify analytically in general. Nevertheless, we have observed in simulations that this condition is satisfied for the model we consider under the simulation conditions stated in Section **??**. A unique solution exists for all time, for all the ordinary differential equations in 32 and 33.

4.1. *Results*

5. Discussion

Will describe the pros and cons of the two approaches. Adaptive observer is guarenteed to converge. Stochastic method deals with uncertainty and model error implicitly.

Will discuss how the ability to reconstruct models should limit the complexity of the computational models, or drive better/higher fidelity data acquisition.

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