IS THERE A NON-SYMBOLIC SNARC-LIKE EFFECT?

Fiona Jane Nemeh

Submitted in total fulfilment of the requirements
of the degree of Doctor of Philosophy

Melbourne School of Psychological Sciences
University of Melbourne

Orchid ID: 0000-0001-9412-1732
September 2019

Abstract

It is claimed that numbers are represented spatially. Evidence for this claim comes from the Spatial-Numerical Association of Response Codes (SNARC) effect. In the symbolic number SNARC effect, faster responses are found for the left/right hand for numerical judgments when a presented target number is smaller/larger than a referent. Yet little research has examined whether a SNARC-like effect is evident for non-symbolic quantities. If a non-symbolic SNARC-like effect can be demonstrated it would suggest that non-symbolic quantity is also represented spatially. This is an important issue conceptually because it may inform claims about the neurocognitive representation of symbolic number and non-symbolic quantity in the brain.

The limited research that has examined the non-symbolic SNARC-like effect has used stimuli that could be translated into symbolic terms or has ignored the potential effects of individual differences by collapsing data across magnitudes during analysis. These methodological issues were addressed in the research reported in this thesis in which the results of five studies are reported to determine whether a SNARC-like effect occurs for non-symbolic stimuli.

In Study 1 adults (n = 28) judged whether the quantity of a target array of dots was smaller or larger than a referent array of dots using either their left or right hand to make judgments. The referent was always 30-dots and judgment reaction times (RTs) to six different target arrays were assessed (i.e., 15, 20, 25, 36, 45 or 60-dots). Participants also completed a visuo-spatial memory (VSWM) and a math ability task. A non-symbolic SNARC-like effect was observed but no relationship between this effect and VSWM or math ability was found.

The rationale of Study 2A was to reduce issues of discrimination and calibration found in Study 1 by reducing task difficult. Thereby increasing the chance of finding a non-symbolic SNARC-like effect for all participants. To investigate the degree to which the non-symbolic SNARC-like effect is affected by the ratio between the referent and target arrays, in Study 2A adults (n = 26) compared target arrays that were either 15 or 60 dots with a referent array of 30 dots (i.e., ratios of 0.5). They also completed VSWM and math tasks. A non-symbolic SNARC-like effect was observed which was related to VSWM. Participants who showed a more pronounced non-symbolic SNARC-like effect showed better VSWM performance.

Study 2B investigated if a more pronounced non-symbolic SNARC-like effect occurs for targets closer in magnitude to the referent. The rationale was to see whether, as in the symbolic SNARC effect, non-symbolic quantity stimuli that result in slower RTs tend to elicit a stronger SNARC-like effect. The referent was 30 dots and the target arrays either 20 or 45 dots (ratios of 0.66).

Adults (*n* = 39) completed the non-symbolic judgment, VSWM, and math tasks. Compared to Study 2A, a more pronounced non-symbolic SNARC-like effect was found.

The symbolic SNARC effect has been found for other response modalities (i.e. eyes, feet). Study 2C (*n* = 18, adults) aimed to determine if the non-symbolic SNARC-like effect transfers to another response modalities (i.e. feet) and the degree to which responses are influenced by the surface area of the dot stimuli, in Study 2C referent and target dot arrays had the same total surface area. A non-symbolic SNARC-like effect was found in this study, which suggests that total dot surface area was not the driver of the SNARC effect found in Experiments 1, 2A, 2B and 2C.

The rationale for Study 3 was to specifically address individual differences in calibration and discrimination for the non-symbolic SNARC-like effect found in Studies 1 and 2. Study 3 (*n* = 87) aimed to investigate the claim that individual differences in the Weber fraction may explain non-symbolic SNARC-like effect judgments using an adaptive staircase to measure the Weber fraction. This Weber threshold was then used calibrate the optimal ratio of referent-to-target for each participant. In the non-symbolic quantity judgment task one of six referents (10, 20, 30, 40, 50 or 60 dots), each being paired with an ‘easy’ (twice threshold) and a ‘hard’ (at threshold) (*n* = 11 – 100 dots) target. However, findings did not support this claim as participants with low and high-performance numerical acuity showed a similar non-symbolic SNARC-like effect.

The findings showed a consistent non-symbolic SNARC-like effect. The pattern of findings suggests the non-symbolic SNARC-like effect is robust as it was shown across all five studies and was resistant to manipulations of ratio, visual cues, and modality of response. Differences in discrimination and calibration were found for numerical judgments suggesting there are individual differences in the spatial representation of non-symbolic quantities. Moreover, individual differences in the spatial representation of non-symbolic quantities may be related to structural differences in the brain. This confirms the importance of the non-symbolic SNARC-like effect for models of numerical cognition providing evidence that non-symbolic quantity is represented spatially from smallest to largest, left to right akin to a symbolic mental number line. This suggests that neurocognitive systems may represent symbolic number and non-symbolic quantity in the brain in a similar manner. This finding has potential implications for education and interventions for those with dyscalculia.

Declaration

This is to certify that

1) The thesis comprises only my own original work towards the degree of Doctor of Philosophy except where indicated in the preface.
2) Due acknowledgement has been made in the text to all other material used.
3) The thesis is fewer than 100,000 words, exclusive of tables, figures, footnotes, references and appendices.

Fiona Nemeh

Preface

For each study, data was collected by the PhD candidate. The manuscript was drafted entirely by the PhD candidate. The manuscript was revised and edited by the PhD candidate and Associate Professor Robert Reeve. All co-authors have agreed to the use of these manuscripts in this thesis and have provided signed copies of the co-author authorization form.

Acknowledgements

First, I would like to thank my supervisors, Associate Professor Robert Reeve and Dr Jason Forte. This work would not have been possible without your expertise, guidance, and support.

Thank-you to my committee members Associate Professor Stefan Bode and Associate Professor Daniel Little, who provided valuable feedback.

There have been several individuals in the department who have generously provided their insight and time, specifically, Dr Judi Humberstone, Dr Fiona Reynolds, and Dr Mark Yates.

Thank-you to my fellow members of the Cognitive and Neuropsychological Development Lab, Jian Chen, Christine Podwysocki, Cindy Chew, and Jacob Paul.

Thank you to Dr Vicki Huang and my PhD writing group. Your company has been of immeasurable value during this journey.

Thank you to my parents and the friends, who have supported me.

Finally, thank you to all the students who participated in the studies.

Publications, conference abstracts, and awards during candidature

Peer reviewed journal articles

Nemeh, F., Humberstone, J., Yates, M. J., & Reeve, R. A. (2018) Non-symbolic magnitudes are represented spatially: Evidence from a non-symbolic SNARC task. *PloS One*.

Conference abstracts

Nemeh, F., Yates, M. J., Loetscher, T., Ma-Wyatt, A., & Nicholls, M. E. R. (April, 2013). Smaller numerosities are associated with the left and larger numerosities with the right: Evidence from a SNARC task. Poster presented at the 40th Australasian Experimental Psychology Conference, Adelaide, Australia.

Nemeh, F., Yates, M. J., Loetscher, T., Ma-Wyatt, A., & Nicholls, M. E. R. (July, 2014). Smaller numerosities are associated with the left and larger numerosities with the right: Evidence from a SNARC task. Poster presented at the 12th International Conference on Cognitive Neuroscience, Brisbane, Australia.

Nemeh, F., & Yates, M. J. (April ,2015). Non-symbolic number SNARC effect is independent of response modality. Poster presented at the 42nd Australasian Experimental Psychology Conference, Sydney, Australia.

Nemeh, F., & Yates, M. J. (November, 2015). Non-symbolic number SNARC effect is independent of response modality. Poster presented at the 5th Australasian Cognitive Neuroscience Society Conference, Auckland, New Zealand.

Nemeh, F., & Yates, M. J. (November, 2015). Non-symbolic number SNARC effect is independent of response modality. Poster presented at the Students of Brain Research Student Symposium, Melbourne, Australia.

Awards

Australian Postgraduate Award, University of Melbourne, 2013

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

Chapter 1: The Research Problem

There is considerable evidence to support the existence of a relationship between number and space. One of the most robust behavioural examples of this relationship is the Spatial-Numerical Association of Response Codes (SNARC) effect (Dehaene, Bossini, & Giraux, 1993). In the SNARC effect, faster responses are found with the left hand for smaller numbers (i.e., 1 or 2) and with the right hand for larger numbers (i.e., 8 or 9). However, research on the SNARC effect has been largely focused on symbolic number (i.e. Arabic numerals). It has been claimed the symbolic SNARC effect shows support for the spatial representation of symbolic number in the form of a mental number line (Dehaene et al., 1993), however the origin of the SNARC effect is not known.

It is claimed that there are two distinct systems for processing non-symbolic quantities (Butterworth, 2010; Dehaene 1997). One system represents small quantities, known as the subitizing system, and another that represents larger quantities, known as the ANS range (Feigenson, Dehaene & Spelke, 2004). Support for this claim is shown by different response signatures for small and large non-symbolic quantities. In the subitizing range (i.e. ~ 1 to 4) responses are quick and accurate (Trick & Pylyshyn, 1994). By contrast, ANS signatures are imprecise (Whalen, Gallistel, & Gelman, 1999) because numerical judgments are based on approximations (Gallistel & Gelman, 1992), and variance or noise increases linearly as a function of non-symbolic quantity (Buckley & Gillman, 1974). That is, slower and less accurate responses are often found when two to-be-compared magnitudes are closer than when they are further apart (Moyer & Landauer, 1967). This suggests that response judgments are affected by the ratio of referent to target (Dehaene, 1992).

It has been suggested that the ANS is the foundation of symbolic number representation (Nieder & Dehaene, 2009) and that its efficacy may be important for math abilities (Halberda, Mazzocco, & Feigenson, 2008; Piazza, 2010; Piazza et al., 2010). Yet surprisingly little is known about the spatial representation of non-symbolic quantity; and previous research that has been conducted has focused on analysing non-symbolic aspects of the ANS by measuring the Weber fraction (Price, Palmer, Batista, & Ansari, 2012). It is suggested herein that a more comprehensive understanding of the relationship between non-symbolic quantity and space is required to gain better understanding of neuro-cognitive representation of quantity as a cognitive basis for the representation of symbolic number.

Several methodological issues need to be addressed when examining the spatial representation of non-symbolic quantity using a non-symbolic quantity judgment task. First, previous research that has examined the non-symbolic SNARC-like effect has used stimuli ranging from 1 to 10 which includes quantities in the subitizing (i.e., 1 – 4; Mandler & Shebo, 1982) and counting range (i.e., 5 – 10). This is problematic as quantities in the subitizing range may be processed automatically, triggering spatial representation of symbolic number. While quantities in the counting range are small enough to allow some participants to count the items (i.e. a set size of 5 dots is converted to verbal symbolic ‘5’).

Second, analysis of the symbolic SNARC effect often consists of data collapsed across ‘smaller’ and ‘larger’ numerical magnitudes. Because of this ‘collapsing’, individual differences in patterns of data for the SNARC effect are often not acknowledged (Beecham, Reeve, & Wilson, 2009). This is problematic since there are known individual differences in acuity of non-symbolic quantity judgments (Chew, Forte, & Reeve, 2016; Halberda & Feigenson, 2008). For example, those with a smaller weber fraction tend to respond to difficult ratios with a high level of accuracy and relatively fast RTs. But those with a larger Weber fraction may be unable to respond above chance level for more difficult ratios.

In sum, tests of the non-symbolic SNARC-like effect involve more complex factors than those used to study the symbolic SNARC effect. An appropriate methodological approach to examine individual differences in the spatial representation of non-symbolic quantity may be important to further knowledge of number-space interactions. Psychophysics methodology may enable the nature of individual differences to be explored or gain further understanding in variance of the non-symbolic SNARC-like effect.

The thesis has three aims. The first is to examine if non-symbolic quantities are represented spatially as evidenced by a non-symbolic SNARC effect. The second is to examine the relationship between the to-be-judged quantity ratios, low level visual cues, and modality of response on the non-symbolic SNARC-like effect. The third is to examine possible reasons for individual differences of the non-symbolic SNARC-like effect. The next chapter (Chapter 2) critically reviews previous research to support the rationale for the research reported in this thesis.

Chapter 2

The purpose of Chapter 2 is to present and defend the claims that form the basis of this thesis. The first and main claim is that the spatial representation of non-symbolic quantities (the magnitude[1] representation of non-symbolic elements) can be explored using a SNARC-like effect. The symbolic SNARC effect is argued as evidence for the spatial mapping of symbolic number. However, the symbolic SNARC effect is experiential and is found only after formal math education and to date no research has specified the origin of this effect. An inherent left-to-right spatial preference in preverbal infants and animals may reflect a right hemisphere dominance drawing attention to left side space, that may later be moderated by cultural artefacts such as rulers, counting on fingers, and reading direction and hence influence the symbolic SNARC effect. Insofar as this is the correct, it would be expected that non-symbolic quantities are spatially represented as evidenced by the non-symbolic SNARC-like effect.

The second claim focuses on the representation of non-symbolic quantity. Many researchers have proposed that two separate systems represent non-symbolic quantity: a precise system for representing small quantity (e.g., 1 to 4~ items), often referred to as the subitizing range; and an approximate number system (ANS) for non-symbolic quantity beyond the 1 to 4 item range (Dehaene, 1997; Inglis & Gilmore, 2014; Mandler & Shebo, 1982; Trick & Pylyshyn, 1994). The ANS extracts magnitude information, independent of visual cues, and is thought to provide a foundation for mapping symbolic number onto non-symbolic quantity. This can be better thought about as symbolic magnitude representation builds on non-symbolic quantity representation. A competing account, sensory integration theory, suggests that visual cues are impossible to remove completely from non-symbolic stimuli. In either case, it would be expected that a non-symbolic SNARC-like effect could be identified. Insofar as the two claims can be substantiated, it would allow the identification of characteristics of the non-symbolic SNARC-like effect to gain better insight into the spatial representation of non-symbolic quantity.

Chapter 2 consists of five sections. The first section will critically examine the claim that number and space are inherently linked. While there is considerable evidence for the relationship between symbolic number and space (Dehaene et al., 1993; Fischer & Shaki, 2014; Hubbard,

[1] The terms ‘quantity’ and ‘magnitude’ are often used interchangeably. However, in this thesis ’quantity’ is only used in association with non-symbolic magnitudes.

Piazza, Pinel, & Dehaene, 2005), there is a lack of research focused on the relationship between non-symbolic quantity and space. It will be argued that non-symbolic quantities are spatially represented. The work of Dehaene et al. will be examined, as will current models of the SNARC effect in order to expand current knowledge and models of the SNARC effect.

The second section of Chapter 2 will investigate non-symbolic quantity processing, the ANS, and its potential link with math ability. There will be a review of estimation of non-symbolic quantity, including a critique of assumptions that underlie estimation. It will be argued that estimation is integral to examining non-symbolic quantity processing for large sets of items. The importance of ratio and calibration for investigating the ANS will be discussed. Current debates in the literature and models of the ANS will be critically reviewed to identify issues with existing methodology and to provide suggestions of how these issues can be addressed in the thesis.

Section 3 will critically review key theories and methodology of the symbolic and the non-symbolic SNARC-like effect. It is argued that the symbolic SNARC effect is evidence for the spatial representation of symbolic number. If this claim is correct, then the non-symbolic SNARC-like effect could be considered evidence for the spatial representation of non-symbolic quantity. Behavioural, neuropsychological, and neuroimaging evidence for the SNARC effect will be reviewed to highlight methodological and theoretical issues in the literature and how these issues will be addressed. A rationale will be provided for quantity comparison being a suitable paradigm to examine the non-symbolic SNARC-like effect.

The fourth section of Chapter 2 will review research that has identified individual differences in numerical cognition. It is argued that although different patterns in data exist, most previous research in numerical cognition has not focussed on individual differences. It is argued that examining individual differences can provide further insight into spatial-numerical associations. Psychophysics methodology for investigating individual differences in the non-symbolic SNARC-like effect will be proposed.

The final section of this Chapter will provide a summary of the argument and the planned research agenda designed to address the issues already identified. It will be argued that greater understanding of the relationship between non-symbolic quantity and space can be advanced by the systematic manipulation of: ratio (i.e., changing the ratio of target to referent and thereby difficulty of magnitude judgments); stimulus arrays (i.e., controlling for low-level visual cues such as total surface area, dot size, density); effector of response (i.e., responding with feet instead of hands); and task demands (i.e., using an adaptive staircase, target set sizes are chosen based on each individual's Weber fraction to find the optimal ratio to test the non-symbolic SNARC-like effect).

To summarise, the main objective of this critical review is to identify issues in experimental design for measurement of the ANS that have followed on from historical theories and methodology but limit claims that can be made about the spatial representation of the ANS. The aims of this chapter are threefold: First, to critically review evidence for the spatial representation of number. Second, to review evidence for the claim that non-symbolic quantity representation occurs in the ANS. Third, to provide a rationale for research questions that motivated the five studies reported in Chapter 3, 4, and 5.

2.1 Numbers and Space

This section reviews evidence for the spatial representation of symbolic number and non-symbolic quantity representation. Three points will be made. First, number and space are inherently linked. Second, the symbolic SNARC effect is evidence for the spatial representation of symbolic magnitude. Third, we know less about the link between non-symbolic quantity and space than symbolic magnitude and space.

Throughout recorded history mathematicians and philosophers have highlighted the importance of number and space (Fischer & Shaki, 2014; Hubbard et al., 2005). Indeed, it has been suggested that the abstract representation of number is mapped onto (more concrete) neural representations of everyday experience such as space (Dehaene, 1997). In contemporary numerical cognition theory, number is thought to be represented by an implicit spatial representation with smaller numbers on the left side and larger numbers on the right side (in so-called Western cultures, at least) (Dehaene et al., 1993). This mapping of number in a left-to-right manner may be influenced by reading direction (or finger counting habits; see Fischer, Kaufmann, & Domahs, 2012). This spatial asymmetry has also been found for time, number, and everyday events (Chatterjee, 2011; Göksun et al., 2013). It has been postulated that a generalised magnitude system may exist in the parietal cortex for abstract concepts such as time, space, and number (ATOM, Walsh, 2003; see also Bueti & Walsh, 2009) and that these differing concepts may be coded abstractly.

An understanding of symbolic number and math is thought to be associated with the relationship between number and space (de Hevia, Vallar, & Girelli, 2008). The ability to map symbolic numbers onto space may be a factor in the successful development of numerical competence (Crollen & Noël, 2017; Gunderson, Ramirez, Beilock, & Levine, 2012; Kurdek & Sinclair, 2001; Lachance & Mazzocco, 2006; but see Cipora, Patro, & Nuerk, 2015), and the spatial coding of number has had a key impact on models of numerical cognition (Dehaene, 1992; Dehaene, Piazza, Pinel, & Cohen, 2003).

Galton (1880a, 1880b) was the first to propose that there may be a unique relationship between number and spatial locations. Galton's two observational reports uncovered a common phenomenon: numbers were often visualised at a particular location in space. However, behavioural evidence supporting Galton's proposal was not reported until Moyer and Landauer's (1967) ground-breaking research nearly a century later. In their study, Moyer and Landauer asked participants to make a numerical judgment about which of two numbers was more (or less) than the other. They found response times were quicker and fewer errors occurred as the numerical distance between the two to-be-judged numbers increased. This 'numerical distance effect' supports Galton's idea of individualised number-space constructs. More specifically Moyer and Landauer hypothesise that numbers are represented spatially in the form of a horizontal 'mental number line' with smaller numbers on the left and larger numbers on the right of this line (Dehaene, 1992; Moyer & Landauer 1967; Restle, 1970).

There is substantial empirical evidence that supports the mental number line hypothesis. This evidence comes from behavioural studies in the normal population (Dehaene et al., 1993; Fischer, Castel, Dodd, & Pratt, 2003; Moyer & Landauer, 1967; Restle, 1970, Schwarz & Keus, 2004); neuropsychological patients (Rossetti et al., 2004; Vuilleumier & Rafal, 1999; Zorzi, Priftis, & Umiltà, 2002, but see Doricchi, Guariglia, Gasparini, & Tomaiuolo, 2005); neural disruption studies (transcranial magnetic stimulation) (Göbel, Calabria, Farnè, & Rossetti, 2006); and functional neuroimaging studies (Gevers, Ratinckx, De Baene, & Fias, 2006; Pesenti, Thioux, Seron, & De Volder, 2000; Pinel, Dehaene, Rivière, & LeBihan, 2001). Taken together this shows strong evidence for the link between symbolic number and space.

2.1.1 Number and Space in the Brain

The possible existence of a specific area for processing number in the brain was first raised in the early 20th century. It was observed that brain damage to the left parietal area led to problems with maths. In Gerstmann's syndrome, lesions are often found on the left angular gyrus (Gerstmann, 1940). Patients with this syndrome often experience issues with number processing and calculation. In Gerstmann's syndrome finger agnosia is common, along with spatial issues such as left-right confusion. More recent studies of patients with lesions in the parietal cortex and activation of areas in the intraparietal sulcus have shown evidence for a relationship between number and space in the brain; specifically, in the horizontal segment of the intraparietal sulcus hIPS, the posterior IPS pIPS, and the superior parietal lobule pSPL (Cipolotti, Butterworth, & Denes, 1991; Zorzi et al., 2002). Hubbard, Piazza, Pinel, and Dehaene (2005) argue that number and space are processed by the same area in the parietal cortex. This claim has been supported by Harvey, Klein, Petridou, and Dumoulin (2013) who showed topographical representation in the human parietal cortex of numerosity and generalised magnitudes of object size (see also Harvey, Fracasso, Petridou, & Dumoulin 2015). These findings show that numbers and space are linked in the brain and are represented in parietal lobes.

2.1.2 Neural Basis of Non-symbolic Number

In Dehaene and Cohen's (1995) triple code model of number processing, non-symbolic numerical quantity processing occurs through the analog magnitude representation route. From an anatomical perspective, when a numeric stimulus is first presented it must be processed by the visual system in the ventral visual pathway. This neural pathway leads to the left occipito-temporal region. In a number comparison task, the parieto-occipito-temporal regions of both hemispheres are involved. Evidence for Dehaene and Cohen's model is provided in single case studies with split brain patients, double dissolutions and neuroimaging studies, specifically fMRI, EEG, and ERP studies (Dehaene, 2011; Dehaene & Cohen, 1995; Skagenholt, Träff, Västfjäll, & Skagerlund, 2018).

In summary, there is much evidence to support a relationship between numbers and space. Behavioural, neuropsychological, and neuroimaging studies show strong evidence for the spatial representation of symbolic number. One of the most popular paradigms used to explore this is the Spatial-Numerical Association of Response Codes effect (SNARC effect) – smaller numbers facilitate left-hand responses and larger numbers facilitate right-hand responses (Dehaene et al., 1993). The SNARC effect will be reviewed in more detail in the next section.

2.1.3 Symbolic Number SNARC Effect

The purpose of this section is to review research investigating the symbolic SNARC effect. It has been claimed that the symbolic SNARC effect is evidence for the spatial representation of symbolic number (Dehaene, 1997; Dehaene et al., 1993; Gevers & Lammertyn, 2005). Insofar as this is correct, the non-symbolic SNARC-like effect may be a good candidate for investigating the spatial representation of non-symbolic quantity. To date, however, most research into the SNARC effect has focused on symbolic number (i.e., Arabic numerals).

In the original SNARC paradigm (symbolic number judgment task, Dehaene, Dupoux & Mehler, 1990), participants judged whether a presented number was smaller or larger than a fixed reference number and indicated their judgment by pressing buttons on the left or right. In one block of trials, ‘smaller’ responses were assigned to the left button and ‘larger’ responses to the right, and for the other block of trials this assignment was reversed. The SNARC effect refers to the finding that ‘smaller’ responses are made more quickly by the left versus the right hand, with the converse effect for ‘larger’ responses. Dehaene, Bossini, and Giraux (1993) conducted a further study to examine this phenomenon. Dehaene and colleagues used a parity judgment task where presented numbers were classified as odd or even. Again, faster responses were found for smaller numbers when with left hand responses and to larger numbers with the right hand. The SNARC effect has been interpreted as evidence for an internal spatial representation of symbolic numbers with smaller numbers on the left and larger numbers on the right.

The SNARC paradigm has been widely used in the field of numerical cognition. The symbolic number SNARC effect has been replicated using different stimuli, including visual number words, auditory number words (Nuerk, Wood, & Willmes, 2005), double-digit numbers (Reynvoet & Brysbaert, 1999), and negative numbers (Shaki & Petrusic, 2005; however, see Nuerk et al., 2004). The symbolic SNARC effect has been found in patients with visual hemi-field neglect (Zorzi et al., 2012), participants who are visually impaired (Crollen, Dormal, Seron, Lepore, & Collingnon, 2013), and in children (Berch, Foley, Hill, & Ryan, 1999).

A range of response measures have been used to examine the symbolic SNARC effect, including pointing (Fischer, 2003), grasping (Andres, Davare, Pesenti, Olivier, & Seron, 2004), eye-movements (Fischer, Warlop, Hill, & Fias, 2004), and foot pedal responses (Müller & Schwarz, 2006), with consistent effects. The symbolic SNARC effect provides evidence that the number-space relationship is automatically triggered, as number magnitude is orthogonal to the parity judgment task (odd or even) (Dehaene et al., 1993, but c.f. Nathan, Shaki, Salti, & Algom, 2009 who argue that the relationship is between space and relative magnitude).

The SNARC effect has also been found with non-numerical stimuli but the results have been less consistent. In one of the earliest studies, Dehaene et al. failed to find a SNARC effect when letter stimuli were classified being at the beginning or the end of the alphabet (Experiment 4, Dehaene et al., 1993). Fischer (2003) also failed to find a SNARC effect for letters using a pointing task. Stimuli consisted of the letters A, B, T, and U. Participants had to classify the letters as a vowel or consonant. Although a SNARC effect was not found for letters using this method, it was found for numbers. Nevertheless, a later study with more statistical power showed a SNARC effect for both letters of the alphabet and months of the year (Gevers, Reynvoet & Fias, 2003). Spatial mapping of other continuous quantity such as physical size (Ren, Nicholls, Ma, & Chen, 2011), pitch (Rusconi, Kwan, Giordano, Umiltà, & Butterworth, 2006), and luminance (Fumarola et al., 2014; Ren et al., 2011) have also been identified.

Overall, there appears to be strong evidence for the symbolic SNARC effect and the spatial representation of symbolic number. However, some research on the symbolic SNARC effect has challenged a direct relationship between number and space: number representation is not always directly related to spatial representation (van Dijck, Abrahamse, Majerus, & Fias, 2013). If this claim is correct, the symbolic SNARC effect may be less to do with a stable spatial representation of number (Marghetis, Kanwal, & Bergen, 2013) and more do to with non-spatial and flexible associations (Fischer, Mills, & Shaki, 2010; Gevers et al., 2010; Santens & Gevers, 2008). Although non-symbolic quantity processing is thought to be critical for numeracy and strong links have been established between numerical magnitude and space, there has been a dearth of research investigating the spatial representation of non-symbolic quantity. The symbolic SNARC paradigm has proven to be a reliable and sensitive task for assessment of spatial representation of symbolic number, and therefore a logical candidate for assessing whether non-symbolic quantity is represented spatially.

2.1.4 Non-symbolic SNARC-like Effect

The aim of this section is to critically review research on the non-symbolic SNARC-like effect. This will identify issues in the literature that will be addressed in this thesis. Another aim of this section is to examine if the non-symbolic SNARC-like effect provides evidence for the spatial representation of non-symbolic number in the form of a left-to-right mental 'number' line.

To our knowledge only a few studies have been conducted on the non-symbolic SNARC-like effect and various paradigms have been used such as parity judgment, magnitude judgments with sequential or pairwise presentation of stimuli, and studies of the attentional SNARC using target detection in both children and adults (Bulf, Cassia, & de Hevia, 2014; Ebersbach, Luwel, & Verschaffel 2014; Lee, Chun, & Cho, 2016; Luccio, Fumarola, Tamburini, & Agostini, 2012; Mitchell, Bull, & Cleland, 2012; Nuerk, et al., 2005; Patro & Haman, 2012; Patro & Shaki, 2016; Zhou, Shen, Li, Li, & Cui, 2016). These studies will be critically reviewed below.

Lee et al. (2016) used a pairwise comparison task with symbolic and non-symbolic quantity in adults to see if task instructions influenced the SNARC effect. The same stimuli were used but the instructions varied. Lee et al. found a standard SNARC effect for the 'choose smaller' condition, and a reversed SNARC effect for the 'choose larger' condition, which was interpreted as showing that the non-symbolic SNARC-like effect was influenced by task instructions. This study shows that like the symbolic SNARC effect, the non-symbolic SNARC-like effect appears to be flexible. However, Lee et al. used stimuli that spanned both the counting (6 – 10) and ANS ranges (11 – 50) with a presentation time of 1000 ms. The use of smaller non-symbolic quantities and longer presentation times may have enabled counting rather than short presentation times that encourage estimation.

Patro and Shaki (2016) also used a pairwise comparison non-symbolic quantity judgment task to investigate the spatial representation of non-symbolic quantity in Polish-speaking and Hebrew-speaking adults These groups were chosen to compare as in the Polish language both text and numbers are read from left to right but in Hebrew, text is read from right to left while numbers are read from left to right. Stimuli consisted of 2 to 10 rectangles shown side by side. Participants had to choose the smaller or larger set. A non-symbolic SNARC-like effect was found, although the effect was smaller in Hebrew participants. This shows that like the symbolic SNARC effect, reading direction may influence the non-symbolic SNARC-like effect.

Nuerk et al. (2005) conducted a series of experiments using a parity judgment task with a range of symbolic and non-symbolic stimuli – 0 to 9. These consisted of auditory number words and visual stimuli consisting of dice dot patterns, Arabic numerals, and number words. Nuerk et al. found a SNARC effect for all stimuli suggesting the SNARC effect is independent of format and modality. However, participants had to perform a parity judgment task and classify familiar dice patterns as odd or even. This may require converting non-symbolic quantity to a symbolic number. Therefore, this task may have unintentionally activated the symbolic number system: the SNARC-like effect for non-symbolic quantities may have been activated by a spatial representation symbolic number, rather than represent a spatial mapping of non-symbolic quantity. Moreover, the stimuli in Nuerk's study spanned the subitizing and counting ranges. It is known that there are differing behavioural RT signatures across these two ranges (Trick & Pylyshyn, 1994).

Mitchell, Bull, & Cleland (2012) explored the non-symbolic SNARC-like effect in adults using an implicit paradigm in which the non-symbolic quantity of stimuli was irrelevant to the task. Arrays consisted of 1 to 9 circles or triangles. Participants made a colour judgment (Experiment 1; Mitchell et al., 2012) or an orientation judgment (Experiment 2; Mitchell et al., 2012) for dot arrays or triangles. A SNARC-like effect was found for orientation judgments only. The authors interpreted this finding as evidence for shared parietal processing of number and orientation judgments. However, it is important to note that stimuli used in this experiment spanned both the subitizing and counting ranges that have differing behaviour signatures. Indeed, in Mitchell et al.'s study a stronger SNARC effect was found in the subitizing range (1 – 4), compared to larger numbers (5 – 9). The authors suggested that non-symbolic quantity representation interferes with responses in the orientation task, but not for the whole range of non-symbolic quantities. Although the focus of this study was not on the spatial representation of non-symbolic quantity it has implications for future studies on the non-symbolic SNARC-like effect. Care should be used when selecting the range of non-symbolic quantities in non-symbolic SNARC-like paradigms.

Luccio, Fumarola, Tamburini, & Agostini (2012) used a detection paradigm to examine an attentional SNARC effect and the spatial representation of non-symbolic quantity. In Luccio et al.'s study, the subitizing and counting ranges were included to see if they were spatially represented in a similar manner to symbolic number. Interestingly, Luccio et al. only found a

SNARC from the counting range and beyond with faster responses found for the left hand with the magnitudes from 7 and 11 and with the right hand with magnitudes from 15 – 23. Luccio et al. claimed that in their study only magnitudes in the counting range seems to be represented spatially. This result contrasts with Mitchell et al.'s (2012) study, who found a non-symbolic SNARC-like effect for magnitudes in the subitizing rather than in the counting range.

Zhou, Shen, Li, Li, & Cui (2016) examined the spatial representation of non-symbolic quantity purely in the ANS range in adults, avoiding the use of stimuli in the subitizing (1 – 4) or counting ranges (5 – 10). Stimuli consisted of the following dot arrays: 11, 14, 17, 20, 23, 26, and 29. In each trial, two of these arrays were displayed sequentially. Participants had to decide if these arrays were the same or different. Zhou et al. found a SNARC-like effect for non-symbolic quantity purely in the ANS range. However, in the analysis Zhou et al. collapsed data across small and large non-symbolic quantities. Yet it is known that individual differences have been found for the symbolic SNARC effect so it is likely that individual differences will be found for the non-symbolic SNARC-like as well.

Other studies have demonstrated that continuous visual properties such as luminance may be spatially represented. Using a luminance discrimination task in which participants had to judge if a target was brighter or darker than a reference, Ren et al. (2011) showed a SNARC-like effect for luminance: that is, research into the non-symbolic SNARC-like effect is further complicated by the influence of low-level visual cues that are also represented spatially. Moreover, Fumarola et al. (2014) found a SNARC-like effect for luminance (in a luminance comparison task where participants judged the colours of stimuli for which luminance were irrelevant to the task). The authors suggested that this spatial-luminance effect was automatic. These studies suggest de Hevia and Spelke's (2009) and Zhou et al.'s (2016) findings may have been affected by participants using other visual cues to make judgments. Bulf et al. (2014) controlled for surface area, but for not contour length or density, and it is possible the SNARC-like effect observed in their study was affected by these variables. This shows some reasonable controls for low-level visual cues should be in place when investigating the non-symbolic SNARC-like effect.

A non-symbolic SNARC-like effect has also been found in children. Patro and Haman (2012) used a non-symbolic quantity judgment task in which pre-schoolers judged which of two displays had larger non-symbolic quantity. Stimuli consisted rectangles ranging from of 2 to 10.

Patro and Haman found a non-symbolic SNARC-like effect. However, again stimuli spanned the subitizing and counting ranges. Ebersbach, Luwel, & Verschaffel (2014) examined the spatial-numerical associations in pre-schoolers using a magnitude judgment comparison task. The reference consisted of 20 blue dots. In the 'smaller' condition, stimuli were 5 or 10 dots; in the 'larger' condition, stimuli were 40 or 80 dots. The total area of the dots was controlled. Ebersbach et al. found a non-symbolic SNARC-like effect in 5- and 6-year-olds prior to the beginning of formal schooling, and claimed their results provide evidence of an innate mapping of number and space, independent of education experiences (e.g., the direction of writing scripts). However, an issue with this claim is that children are exposed to many direction cues by interacting with the world around them and their parents before they begin school, for instance, parents often read to their children (Patro, Nuerk, & Cress, 2016).

Bulf, Cassia, and de Hevia (2014) used the attentional SNARC effect and an eye movement paradigm to investigate the spatial representation of non-symbolic quantity in 8- and 9-month-old preverbal infants. They used stimuli consisting of 2 or 9 dots for the numerosity condition and rainbows of various sizes for the size condition. They found that the presentation of the smaller dot stimuli (i.e., 2 dots) shifted attention to the left (this was indicated by direction of the infant's gaze) and to the right for larger stimuli (i.e., 9 dots). This shift of attention did not occur for the size stimuli, which indicated a SNARC-like effect for non-symbolic quantities in infancy. The authors argue this provides support for an innate mental number line, although they acknowledge this claim should be tested in newborns to completely discount the effect of cultural influences.

As mentioned, another issue with previous non-symbolic SNARC-like studies is that they mostly used non-symbolic stimuli ranging from 1 to 10. This includes stimuli in the subitizing range (i.e., 1 – 4; Kaufman, Lord, Reese, & Volkmann, 1949) and non-symbolic quantities that are in the counting range (i.e., 5 – 10). This is problematic since numbers in the subitizing range maybe processed automatically, triggering spatial representations of symbolic number. Likewise, numbers in the counting range (5 – 10) are small enough for some participants to count the stimuli (i.e., a numerosity of 5 dots is converted to verbal symbolic '5'). Consequently, the SNARC effect found may not be a genuine non-symbolic SNARC-like effect.

In sum, extant research provides mixed support for the spatial representation of non-symbolic quantity. For instance, the non-symbolic quantities used in the studies generally include quantities in the subitizing and counting ranges that these simpler forms of non-symbolic stimuli could be converted to symbolic numbers. Inconsistencies in non-symbolic SNARC-like research methods suggest the findings should be treated with caution. In particular, the spatial representation of larger non-symbolic quantity beyond the counting range remains to be examined. While there is much evidence for a relationship between numbers and space, there is still debate whether number-space mapping is innate or is shaped over time by cultural artefacts. Moreover, to date there is little evidence to date for a non-symbolic SNARC-like effect so more research is needed.

2.2 Accounts of the SNARC Effect

The section will critically review theories of the symbolic SNARC effect to examine the relevance of these claims for the non-symbolic quantity SNARC-like effect.

Although there is strong evidence supporting the symbolic SNARC effect (Dehaene et al., 1993; Fias, 2001; Fischer, 2001, 2003; Fias & Fischer, 2005; Gevers & Lammertyn, 2005; Hubbard et al., 2005), the cognitive mechanisms underlying the symbolic SNARC effect are poorly understood. Three explanations of the symbolic SNARC effect will be reviewed: (1) the mental number line (Dehaene, 1992; Dehaene, 1993; Moyer & Landauer 1967; Restle, 1970); (2) working memory (Fias, van Dijck, & Gevers, 2011); and (3) polarity correspondence principle (Proctor & Cho, 2006). Limitations of these theories will be considered.

2.2.1 The Mental Number Line

In this section, the mental number line account of the symbolic SNARC effect will be considered. It will be shown that there is strong evidence for this account. It will be argued that further research on the non-symbolic SNARC-like effect is needed in order to examine the spatial representation of non-symbolic quantity. Dehaene et al. (1993) were the first to claim that the putative mental number line could account for the symbolic SNARC effect. As previously mentioned, the mental number line describes the symbolic SNARC effect as a spatial congruity between internal mental representation of number and external space with smaller numbers on the left and larger numbers on the right. This suggests that numbers are represented in the brain in a left-to-right orientation (Dehaene, et al., 2003), bilaterally in the intraparietal sulcus (Priftis, Zorzi, Meneghello, Marenzi, & Umiltà, 2006). This proposition later became part of Dehaene's influential neurocognitive triple code model of numerical cognition (Dehaene & Cohen, 1995) and has been the most popular account of the SNARC effect to date (Fischer & Shaki, 2014; Wood, Willmes, Nuerk, & Fischer, 2008).

One explanation of the SNARC effect is that it occurs because of a close link between where a number is situated on a left-to-right mental number line in representational space and the position of the response (Gevers, Santens, Dhooge, Chen, Van den Bossche, Fias, & Verguts, 2010). The dominant mental number line account suggests that the SNARC effect occurs due to the visuospatial coding of magnitude. However, more recently, this claim has been called into question (Gevers et al. 2006; Proctor & Cho, 2006). Santens and Gevers (2008) argue that the SNARC effect does not arise due to visuospatial coding but verbal coding such as ‘small’ and ‘left’ or ‘large’ and ‘right’. It is the similarity of dimensional overlap between these concepts that is at play here. Moreover, the theory of the mental number line does not explain spatial-numerical associations or SNARC effects for infants and animals.

Some critique the mental number line account as a posteriori explanation of the SNARC, begging the question of how many number lines are needed to accommodate negative numbers, fractions, and decimals, as well as natural numbers (Fischer & Shaki, 2014). Even key properties ascribed to the SNARC such as being numerical, spatial, and based on a mental number line have been called into question (Núñez, 2011). Letters of the alphabet (Gevers et al., 2003), pitch (spatial musical association of response codes; SMARC, Ruscioni et al., 2006), and near and far responses independent of left or right movements (Santens & Gevers, 2008) have been shown to be spatially represented. However, perhaps it should be questioned if this is an argument against the SNARC, or whether there might be more spatial effects in cognition more generally, with one author claiming that perhaps any form of magnitude may be represented spatially (spatial quantity association of response codes; SQUARC, Walsh, 2003). As a unique representation of number, the mental number line fails to account for flexible representations of number (i.e., circular representations in the form of a clock face; Bächtold, Baumüller, & Brugger, 1998).

The mental number line account provides strong evidence for the spatial representation of number in long term memory and is the dominant account of the symbolic SNARC effect. Albeit, to date there is little evidence for the spatial representation of non-symbolic quantity. The mental number line theory may explain the spatial representation of non-symbolic quantity as shown by a non-symbolic SNARC-like effect. Given the lack of research in this area, this claim is somewhat speculative. It is known that the mental number line alone cannot account for flexible representations of number. The mental number line also fails to account for spatial-numeric findings with infants and animals. In the next section, the working memory account of the symbolic SNARC effect will be critically evaluated to determine whether it is a more compelling explanation of the non-symbolic SNARC-like effect.

2.2.2 Working Memory

In this section, evidence for the working memory account of the symbolic SNARC effect will be considered. It will be shown that findings are mixed. It will be argued further research on the working memory account of the symbolic SNARC effect is needed.

A challenge to the mental number line account is the proposal that number-space associations are generated in a temporary and flexible working memory, rather than reflecting an enduring, stable relationship (van Dijck & Fias, 2011). This suggests that the symbolic SNARC effect is an ordinal effect instead of a spatial one. A series of studies has provided support for this account (De Belder, Abrahamse, Kerckhof, Fias, & van Dijck, 2015; Ginsburg, van Dijck, Previtali, Fias, & Gevers, 2014; van Dijck, Abrahamse, Acar, Ketels, & Fias, 2014).

Herrera, Macizo and Semenza (2008) used a dual task paradigm to test the influence of working memory on the SNARC effect. Participants had to remember either phonological (non-words) or visuospatial information (Corsi Block Tapping Task) while they completed a magnitude judgement task. In the condition where participants had to remember visuospatial information, no SNARC effect was found. Further support for this was shown by Van Dijck, Abrahamse, Acar, Ketels and Fias (Experiment 2, 2011) who designed an experiment to explore if the symbolic SNARC effect could better be accounted for by the mental number line or working memory theory. Participants memorised four sequential numbers before completing a dot detection task. If the mental number line theory was behind the SNARC effect, a standard response was expected. That is, a quicker response to left side targets after smaller numbers were presented and a faster response to right side targets after larger numbers. Instead, numbers triggered shifts of spatial attention relative to position in the working memory sequence. This provided support for the working memory theory of the SNARC as relative position had more of an influence than magnitude alone.

Moreover, a recent large scale online SNARC study with 604 participants by Gökaydin, Brugger, and Loetscher (2018) showed further support for the working memory account of the symbolic SNARC effect. Findings of this study showed that congruency of trials influenced the SNARC effect. That is response repetitions of incongruent trials (large-left, small-right) resulted in an inversion of the SNARC effect. Taken together, these studies show that working memory may play an important part in spatial numerical associations involved in the SNARC effect and may be constructed on the spot to fit task demands.

The working memory account has also been challenged, with some studies failing to find working memory as a factor (Hoffman, Pigat, & Schiltz, 2014) Hoffman, Pigat, and Schiltz (2014) investigated whether the SNARC effect was related to age, inhibition capacity, working memory, and response speed. Hoffman et al. found that a stronger SNARC effect was found in those with decreased inhibition capacity who were older and had slower RTs. Neither verbal short nor long term memory were related to the SNARC effect.

The working memory account claims that ordinal sequences held in short-term working memory, rather than the spatial representation of number, explain the symbolic SNARC effect. This suggests that number is not represented spatially. However, there is little evidence to support the working memory account of the symbolic SNARC effect and not all studies that have tested this theory have supported this claim. The next section will evaluate the polarity correspondence account as a viable explanation of the non-symbolic SNARC-like effect.

2.2.3 Polarity Correspondence Account

In this section evidence for the polarity correspondence account is considered, and it will be shown that the findings are mixed. It will be argued that limitations of the polarity correspondence account do not allow an overarching explanation of the symbolic SNARC effect. The polarity correspondence account has been proposed to explain the link between number and space (Proctor & Cho, 2006). In tasks that require a binary classification, for example, two alternate forced choice tasks, the stimulus and response options are coded as positive (+) polarity or negative (–) polarity. If the polarities match for stimulus and response, then the choice of response is quicker than if they differ. The coding of stimulus and response options of + and – may reflect a general predisposition for binary coding of cognitive and affective domains (i.e., up + or down –, good + or bad –). Proctor and Cho (2006) claim it is this simple binary coding concept that makes polarity correspondence an elegant account that may explain the SNARC effect. However, one issue is that the SNARC effect is not an all-or-nothing effect but permits degrees (i.e., a strong or weak SNARC effect or a standard or reversed SNARC). The polarity correspondence account cannot explain these individual differences in the SNARC effect.

Proctor and Cho (2006) argued that the SNARC effect can be explained by the polarity correspondence account on par with the mental number line hypothesis. Polarity correspondence differs from the mental number line account that instead of the SNARC being described as a spatial stimulus-response compatibility effect it is likened to an orthogonal stimulus-response compatibility effect. That is, the effect is not spatial – 'large' is coded as a positive polarity and 'small' as a negative polarity. Differences in reaction time for the SNARC effect for congruent and incongruent responses can be explained in terms of the link between lateralised coded responses with left responses as negative and right as positive. In the polarity correspondence account, spatial correspondence for the SNARC effect occurs when these polarities match. However, assignment of the polarity coding for stimulus and response is somewhat arbitrary and it is not clear why a stimulus or response is coded + polarity or – polarity.

Nevertheless, stronger support for the polarity correspondence account is shown by a computational model of the SNARC effect (Gevers, Verguts, Reynvoet, Caessens, & Fias, 2006). The SNARC effect is explained by categorical coding. Digits are coded in categories such as 'large' or 'small' on a graded continuum.

Further support for the polarity correspondence account of the SNARC effect was shown by Bae, Choi, Cho, and Proctor (2009), who used a transfer paradigm to examine the SNARC effect. Participants practised either a magnitude judgment or stimulus-response compatibility task with parallel or orthogonal responses before completing a parity judgment task. They found that for the small-left/large-right magnitude mapping the SNARC was enhanced. For the orthogonal task with small-right/large-left mapping a reversed SNARC was found. In the horizontal condition, a larger SNARC was found for the up-right/down-left mapping than the up-left/down-right mapping. The authors suggested that these findings are consistent with the polarity correspondence principle.

One criticism of polarity correspondence as an overarching explanation of the SNARC effect is that it fails to address the influence of reading direction on the lateralised mapping of number to space. Another difficulty with polarity correspondence is that it does not account for instances where space and number interact for one hand only (Fischer & Shaki, 2014). Like the working memory account of the SNARC effect, polarity correspondence fails to explain differences in RTs for numerical stimuli closer or further from the referent. RTs tend to be faster for easier trials or magnitudes that are further away from the referent (i.e., a referent of 30 dots and a test of 60 dots). RTs tend to be slower for those closer to the referent in magnitude (i.e., a referent of 30 dots and a test of 25 dots). These findings show that the magnitude of the stimulus – not polarity alone – affects RTs (Fischer & Shaki, 2014). Further, in contrast to the mental number line account of the SNARC effect, polarity correspondence does not explain why mapping of dominant stimulus properties onto a non-dominant response interferes with a response (Landy, Jones, & Hummel, 2008). Furthermore, it fails to account for why certain binary dimensions have a dominant association (i.e., which properties or environmental influences increase the value of a dimension; Landy et al., 2008). Although Proctor and Cho (2006) assert that the polarity correspondence account is a viable alternate to explain the symbolic SNARC effect, and the account has been growing in popularity, it may be seen more appropriately as a different form of measurement theory. In short, it seems to be a somewhat atheoretical model that does not add anything further to explain the symbolic SNARC effect. Thus, the polarity correspondence account does not seem to be a viable explanation of the non-symbolic SNARC-like effect.

Overall, there are some limitations to the polarity correspondence account as an overarching explanation of the symbolic SNARC effect. These limitations include the influence of reading direction and left-to-right spatial mapping of symbolic number, an inability to account for the distance effect where targets that are closer in non-symbolic quantity to the reference are responded to more slowly and less accurately. Taken together, this suggests that the polarity correspondence account would not be a good candidate to explain the non-symbolic SNARC-like effect because it does not contain a spatial component.

2.2.4 Summary

Research suggests the mental number line is evidence for the spatial representation of symbolic number (Dehaene et al., 1993). Parity judgment and non-symbolic quantity judgment tasks are believed to tap into the mental number line through the symbolic SNARC effect; however, it is less clear if this is also the case for a non-symbolic SNARC-like effect. The working memory (Fias, van Dijck, & Gevers, 2011) and polarity correspondence (Proctor & Cho,

2006) accounts have less empirical evidence to support them. It is argued that non-symbolic quantity are spatially represented as evidenced by a non-symbolic SNARC-like effect. If this is the case it will provide evidence for the mental number line theory of the non-symbolic SNARC-like effect. Neither polarity correspondence nor the working memory account of the SNARC effect include a spatial basis. In the following section the difference between symbolic magnitude and non-symbolic quantity will be examined. This is important because most of the research on the SNARC effect has been conducted with symbolic number stimuli and there may be key differences between the symbolic and non-symbolic SNARC-like effect.

2.3 Non-symbolic Quantity Processing

Numerical competence includes a broad spectrum of abilities. It is useful to decompose these into more basic foundational skills, such as the ability to correctly identify a number and to distinguish which of two numbers is larger (Dehaene, 1992). As noted, much research has been devoted to understanding the mechanisms that underlie foundational competencies (Piazza, 2010). Nevertheless, the cognitive and neural mechanisms underlying number processes, particularly non-symbolic quantity, are still poorly understood. In the following section the relationship between symbolic and non-symbolic quantity processing and the approximate number system (ANS), and links with math ability, the visual properties of non-symbolic number, and magnitude judgment abilities will be reviewed.

2.3.1 Symbolic Number Processing

Symbolic number processing refers to our ability to identify, compare, and manipulate symbolic numbers (typically Arabic numerals, ‘1’, ‘2’ etc. but also number words presented visually or spoken). Symbolic number processing is thought to be important because it allows a precise representation of number and enables objects to be counted, ranked in order, and identified (Nieder, 2005). The invention of symbolic number only dates back a few thousand years (Ifrah, 2000) and is acquired through cultural experiences such as formal education (Ansari, 2008). Thus, it is unlikely that new brain areas have evolved specifically for processing symbolic number (Dehaene, 2005). Instead, it is more likely that existing brain structures enable symbolic number processing (Gould & Vrba, 1982). Symbolic number processing appears to recruit brain structures dedicated to non-symbolic quantity processing (Dehaene & Cohen, 2007).

2.3.2 Approximate Number System

The purpose of this section is to review evidence for the approximate number system as a core number system for processing and representing of non-symbolic quantity. It is claimed that non-symbolic quantity representation is divided into two separate systems – the subitizing system (for exact representation of up to 4~ objects) and an approximate system for larger quantity (Dehaene, 1997), the latter being the focus of this chapter. A signature of the ANS is its imprecision, larger quantity being represented less accurately than smaller quantity. (Barth, Kanwisher, & Spelke, 2003; Feigenson et al., 2004; Pica, Lemer, Izard, & Dehaene, 2004).

Non-symbolic quantity processing underlies our ability to tell which of two sets of stimuli contains the more elements without counting them (Price, Holloway, Räsänen, Verserinen, & Ansari, 2007). Unlike symbolic number processing, non-symbolic quantity processing seems to be innate, present in both preverbal infants (Xu & Spelke, 2000) and non-human species (Brannon & Terrace, 2000; Meck & Church, 1983; McComb, Packer, & Pusey, 1994; Pepperberg, 2006). However, this system is restricted to approximate representations and basic arithmetic operations (Nieder & Dehaene, 2009).

Research on the ability to judge how many items are in a display was first conducted by Jevons (1871), and subsequently by Barlow (1978) who examined efficiency identifying items in dot patterns. The paradigm involved detecting target items embedded in random dots. The value of d' was computed for each individual to explain accuracy as a function of signal to noise ratio. Barlow found that the most accurate discrimination occurred at 2/3 of the signal to noise ratio, a finding that was interpreted as showing that humans have a limit on the capacity to discriminate non-symbolic quantity.

Number sense was first defined by Dantzig (1954) as the ability to detect if an object has been added or removed from a small group of items without witnessing this directly. However, a key issue with this definition is that researchers differ widely in their use of the term (Gersten, Jordan, & Flojo, 2005). Indeed, across disciplines such as education and mathematical cognition, the term number sense has different meanings (Gersten et al., 2005). Berch (2005) compiled a broad list of numerical concepts from the literature that have been defined as number sense. He noted number sense referred to numerical concepts ranging from having an awareness or intuition, skill, and knowledge, to a conceptual structure or mental number line for these numerical concepts. By contrast, Dehaene (1997) argued there should only be one definition of number sense—the analog representation of quantity. The approximate number system supports other cognitive systems during development and education (Dehaene, 1997). However, some researchers disagree with Dehaene's definition of number sense, asserting that it prejudges the nature of number (see, e.g., Butterworth, Gallistel, & Vallortigara, 2018).

While the ANS is supported by psychophysics and Weber fraction data (Montemayor & Winther, 2016), some aspects of the concept are unclear. While Dehaene (1997) defined the ANS and number sense as different concepts, recent authors have used these terms interchangeably (Sella, Tressoldi, Lucangelli, & Zorzi, 2016; Szűcs & Meyers, 2017). A recent review of the ANS literature suggests the meaning of key terms have shifted over time, with consequent implications for how key concepts should best be tested (Szűcs & Meyers, 2017).

In sum, there is evidence for the ANS being involved in the processing and representation of large non-symbolic quantity. However, its exact role in the acquisition of numerical knowledge is still unclear. The different definitions of the ANS raises the question of exactly what researchers who examine the ANS are testing. Further understanding of the role of the ANS and its relation to mapping of symbolic number will enable better design of educational interventions to address issues in this area.

2.3.3 ANS as a Foundation for Symbolic Number

In this section the claim that the ANS underpins symbolic number representation will be evaluated. It will be argued that there is some evidence for this claim, but that research in this area has shown mixed results. Limitations of research in this area will be discussed. As noted above, Dehaene (2001) proposed the ANS underpins mapping of symbolic number and therefore mathematical ability. Evidence for mapping symbolic onto non-symbolic quantity comes from similar behavioural RT signatures such as the size and distance effect (Moyer & Landauer, 1967; Restle, 1970), and neuroimaging studies showing shared activation of brain areas for symbolic and non-symbolic tasks (Piazza, 2010; Piazza, Pinel, Le Bihan, & Dehaene, 2007; Venkatraman, Ansari, & Chee, 2005).

Non-symbolic quantity representation is claimed to be foundational to more complex mathematic abilities such as calculation. Historically, problems with arithmetic have been attributed to issues with higher-level mathematical processes such as addition or subtraction. As such, interventions focused on improving these higher-level processes. However, recent research has challenged this claim. Basic mathematical processes may act as a foundation for more complex mathematical processes (Butterworth, 2005; Dehaene, 1997; Gallistel & Gelman, 2000). Notably, it has been suggested the ability to tell how many items are in a set may be linked to later achievement in math (Halberda et al., 2008; Piazza et al., 2010; Starr, Libertus, & Brannon, 2013). Education and culture may increase precision of the ANS (Piazza, Pica, Izard, Spelke, & Dehaene, 2013). However, ANS precision is also somewhat malleable and can change depending on individuals' ability to make easy or difficult judgments (Odic, Hock, & Halberda, 2014).

An alternate claim is that aptitude with symbolic number – rather than precision of ANS acuity – improves math ability (Göbel, Watson, Lervåg, & Hulme, 2014; Szűcs, Nobes, Devine, Gabriel, & Gebuis, 2013). However, in earlier stages of development ANS acuity may play a more important role, and some studies have found a relationship between symbolic and non-symbolic representation systems in younger children but not in adults (Fazio, Bailey, Thompson, & Siegler, 2010; Gilmore, Attridge, De Smedt, & Inglis, 2014).

A common claim is that the ANS acuity scaffolds the symbolic number system (Halberda et al., 2008; Piazza et al., 2010). Most previous research focuses on a variable-centred approach for analysis, but a person-centred approach may provide further insight into whether non-symbolic quantity processing scaffolds symbolic number processing. Person-centred approaches enable individual differences in development to be examined (Dowker, 2008; Reeve, Gray, Butterworth, & Paul, 2018). A recent longitudinal study by Chew, Forte, and Reeve (2019) examined non-symbolic quantity and symbolic number comparison in children aged 5 – 6-years over a one-year period. Latent class modelling was used to examine stability/change in four different non-symbolic-symbolic RT and accuracy profile patterns that had different relationships to cognitive and general abilities and were identified in a previous study by Chew et al. (2016). Results of the study showed than most profiles changed over time, suggesting a continued trajectory of normal, delayed, or deficit in development. This raises the possibility that disparities in previous studies may be due to individual differences in developmental pathways. That is, perhaps in some children non-symbolic quantity processing scaffolds symbolic magnitude processing; however, other children show better symbolic magnitude processing ability that is not related to non-symbolic quantity processing ability.

Overall, the claim that non-symbolic quantity is foundational for symbolic number understanding remains unresolved. Some researchers provide support for this claim while others refute it, arguing instead that symbolic number competence is more important for math ability. However, once more there is some inconsistency in research in this area. Some researchers suggest these inconsistent results may be due to individual differences in developmental patterns. These results highlight the need for further research in this area.

2.3.4 Non-symbolic Quantity Judgment Ability as a Measure of the ANS

In this section, the claim that non-symbolic quantity judgment tasks enable a way to test magnitude representation of the ANS will be evaluated. The additional claim that performance in non-symbolic quantity judgment tasks has been linked to math performance will also be examined. Non-symbolic quantity judgment tasks in the form of dot array comparison are claimed to be a measure of the ANS (Dehaene, 2007). In a dot comparison task, two sets of dots are presented either one after the other or simultaneously. Participants decide if a second set of dots presented contains more or less dots than the first set and respond quickly and accurately. Performance is affected by changes in the ratio between the sets of dots. Smaller ratios (numerosities closer in magnitude) are more difficult to discriminate. People with better non-symbolic quantity acuity tend to respond with greater speed and accuracy (Halberda et al., 2008).

2.3.5 Symbolic Number Judgment and Mathematics Ability

In this section the claim that quantity judgment abilities are linked to math ability will be evaluated. Performance of symbolic number comparison tasks has been linked to math performance in children (Bugden & Ansari, 2011; Durand, Hulme, Larkin, & Snowling, 2005; Mundy & Gilmore, 2009). Although some studies have failed to find this link (Lonnemann, Linkersdorfer, Hasselhorn, & Lindberg, 2011; Sasanguie, De Smedt, Defever, & Reynvoet, 2012), overall there is reasonable evidence to support this claim.

2.3.6 Non-symbolic Quantity Judgment and Mathematics Ability

In this section evidence will be reviewed to examine the claim that non-symbolic quantity judgment is linked to math ability. Although a link has been found between non-symbolic task performance and math ability in children (Bonny & Lourenco, 2013; Libertus, Feigenson, & Halberda, 2013; Mundy & Gilmore, 2009) and adults (Chen & Li, 2014; Libertus, Odic, & Halberda, 2012; Lyons & Beilock 2011), other studies fail to show a relationship between non-symbolic task performance and math ability (Holloway & Ansari, 2009; Price et al., 2012; Soltesz, Szűcs, & Szűcs, 2010; see De Smedt, Noël, Gilmore, & Ansari, 2013 for a review). These inconsistent finding may reflect differences in measures and tasks.

Lyons, Price, Vaessen, Blomert, and Ansari (2014) conducted a large-scale study (N = 1391) on children in Grades 1-to-6 to investigate which numerical predictors lead to improved math performance. Results showed that children's ability to judge relative magnitude of symbolic number was the strongest predictor of math performance in Grade 6, whereas the ability to judge the relative magnitude of approximate non-symbolic quantities was not a predictor of math performance in any grade. However, findings reveal a different picture: there are individual differences in whether symbolic or non-symbolic tasks relate to math ability (Chew et al., 2016).

To investigate possible reasons for inconsistent findings, Chen and Li (2014) conducted a meta-analysis of non-symbolic number acuity and math performance. They found a small but significant correlation in both cross-sectional and longitudinal data. Chen and Li also conducted a power analysis of previous studies and found most were under-powered.

Overall, there is less support for the claim that non-symbolic quantity processing influences math ability rather than symbolic number processing. However, a more likely possibility is that there is not a direct effect. Instead, the effect may occur through other mediating factors, and requires closer investigation.

2.3.7 Issues with Non-symbolic Quantity Research

In this section research on the ANS will be reviewed to highlight limitations of much non-symbolic quantity research. It will be argued that investigating non-symbolic quantity in the ANS is more complex than researching symbolic number. It is claimed non-symbolic quantity judgment tasks reliably measure acuity of the ANS (De Smedt et al., 2013). However, there are several known issues within non-symbolic quantity research concerning construct validity and ANS measurements.

Smets, Gebuis, Defever, and Reynvoet (2014) administered three different versions of a non-symbolic comparison tasks: change detection, same-different, and standard dot comparison tasks. Acuity measures of accuracy and w for the three tasks were not significantly correlated. Smets et al. argued the lack of validity across ANS tasks may be because they are measuring different aspects of the ANS. Clayton, Gilmore, and Inglis (2015) agreed that ANS research may have built on incorrect assumptions about previous work. Clayton et al. (2015) suggested that study design differences may be responsible for different experimental outcomes for non-symbolic tasks. Non-symbolic tasks differ in visual parameters such as presentation and size of dot arrays, length of array display time, and which performance measures are used (i.e., mean/median RTs, accuracy, Weber fraction, distance effects: Clayton et al. 2015—see also Dietrich, Huber, & Nuerk, 2015, who made similar points.).

Szkudlarek and Brannon (2017) also argued that ANS measures tend to have low reliability, adding to the difficulty of making claims based on research examining ANS acuity and math performance. Some researchers have challenged the utility of investigating the relationship between the ANS and math performance due to known issues of measurement of the ANS. Therefore, it is not clear if the construct of the ANS is being adequately measured (Clayton et al., 2015; Smets et al., 2014). In Clayton et al.'s (2015) study, the same participants were administered two commonly-used sets of ANS stimuli. These stimuli differed in the way researchers had attempted to control for visual confounds. However, no correlation was found between the stimulus set of the two conditions.

Interestingly, DeWind, Adams, Platt, and Brannon (2016) re-analysed Clayton et al.'s (2015) study data using a mathematical stimulus space model that modified the existing logarithmic model of the ANS to incorporate the influence of low-level visual cues on numerical judgments. In DeWind et al.'s model, numerical judgments incorporated the number, size, and spacing of dot stimuli in an array. This enabled DeWind et al. to examine to what degree the Weber fraction contributed to magnitude judgments compared to low-level cues for each individual. De Wind et al. found that low-level cues influenced decisions, but magnitude was the main driver of judgments for most participants. Thus, even when other non-numerical visual cues can be used to discriminate non-symbolic quantity in the ANS, number takes precedence.

Given these design differences, direct comparison between studies is problematic, highlighting the need for novel models to address these issues. Most research on non-symbolic quantity research assumes that it is possible to measure the construct of the ANS. Yet inconsistent results are often found. Perhaps other confounds are influencing research in this area. This proposition will be examined in the next section.

2.3.8 Visual Properties of Non-symbolic Quantity

In this section issues that raise questions about the findings of non-symbolic quantity research will be discussed. The influence of visual properties of non-symbolic quantity stimuli in judgment tasks may undermine the extent of claims about the spatial representation of non-symbolic quantity. Instead, low-level visual properties of stimuli can influence magnitude judgments. De Hevia and Spelke (2009) used a line bisection task to investigate the spatial representation of non-symbolic quantity. In this paradigm, horizontal lines were presented with a dot array at each end that were irrelevant to the task. However, the magnitude of the dot arrays produced a spatial bias towards larger non-symbolic quantity in pre-schoolers, young school children and adults. The authors claimed that the findings supported the spatial representation of non-symbolic quantities in the form of a mental number line: a questionable assertion as the bias was left to right.

De Hevia and Spelke's (2009) study was later criticised by Gebuis and Gevers (2011) who attempted to replicate their results. Gebuis and Gevers claimed that area subtended by the stimuli, not non-symbolic quantity, influenced the results reported in de Hevia and Spelke's study. When Gebuis and Gevers controlled for area subtended by the dots they found a bias towards the numerically smaller magnitude. This finding shows that low-level visual cues can influence the outcome of studies using non-symbolic stimuli, limiting claims that can made about the spatial representation of non-symbolic quantity.

Sensory integration (Gebuis, Cohen Kadosh, & Gevers, 2016, 2017; Gevers, Cohen Kadosh, & Gebuis, 2016) may explain some of the inconsistencies in non-symbolic magnitude research associated with differences in the visual properties of stimuli. As already noted, it is claimed that the ANS extracts the magnitude of numerosities independent of visual cues (Dehaene, 1997). In contrast, the sensory integration model proposes visual cues are impossible to remove completely from non-symbolic stimuli and may contribute to an understanding of number (Gebuis et al., 2016). Non-symbolic quantity stimuli are inherently visual in nature, and magnitude cannot increase without varying at least one visual cue. Nevertheless, many researchers have attempted to control for visual properties of non-symbolic stimuli (Dehaene, 2005; Gebuis & Reynvoet, 2011; Halberda et al., 2008) with varying degrees of success.

Burr and colleagues have conducted several studies examining non-symbolic quantity judgments using psychophysics methods (Anobile, Castaldi, Turi, Tunelli, & Burr, 2016; Anobile, Cicchini, & Burr, 2014; Burr, Anobile, & Arrighi, 2017; Burr, 2017; Burr & Ross, 2008; Burr, Cicchini, & Anobile, 2017; Cicchini, Anobile & Burr, 2016; Ross & Burr, 2010). These studies examined the effects of non-numerical visual cues on numerosity judgments; specifically, texture, density, luminance, and physical size. Burr and colleagues challenged the claim that these visual attributes are prioritised above number. Instead, they found that when multiple visual cues are available, magnitude is prioritised for discrimination in making approximate judgments (Burr et al., 2017). That is, non-symbolic quantities seem to be sensed by vision, not texture alone (Burr & Ross, 2008). However, when density increases and displays become too crowded, stimuli can no longer be discriminated, hence, the shift from the perception of magnitude to texture (Burr et al., 2017). Burr and colleagues' studies demonstrated that in non-symbolic quantity judgments, magnitude seems to take precedence over low-level visual cues. This is important for non-symbolic quantity judgment tasks using stimuli in the ANS range. Although some researchers have questioned the possibility of measuring the construct of the ANS, it seems that this is possible when testing for a non-symbolic SNARC-like effect.

In sum, research has shown that low-level visual cues can influence magnitude judgments with non-symbolic stimuli. However, it is largely agreed that despite these issues it is possible to measure the ANS.

2.3.9 Summary

In summary, non-symbolic quantity is intrinsic to the ANS. The ANS is an approximate system thought to represent non-symbolic quantity on the mental number line. Number sense, or approximate representations, have been associated with the activation of the area of the parietal cortex that has been linked to spatial processing (Spelke & Dehaene, 1999). This makes non-symbolic quantities in the ANS range a good candidate for investigating the spatial representation of non-symbolic quantity in the form of a non-symbolic SNARC-like effect. However, there are complexities to measuring the ANS. There is an assumption that we have the capacity to extract magnitudes from dot arrays separately from visual cues of the stimuli. Yet (unlike paradigms that uniformly use symbolic number stimuli for number comparison tasks) there is no standardized method used to generate non-symbolic quantity stimuli. To counter this issue, visual aspects of the stimuli such as density and dot size are often varied so that they are not always informative of magnitude.

While most research has focused on symbolic number, non-symbolic quantity judgment tasks are claimed to measure the ANS. Accuracy in non-symbolic tasks has been linked to ability in mathematics. However, this is still contentious – issues of construct validity, test-retest reliability, and measurement, the influence of visual properties on numerosity processing, and lack of a consistent study design allowing studies to be compared still persist. More importantly, perhaps, researchers have not yet been able to establish the nature of the core constructs of the ANS they purportedly examine. Another issue – the use of inappropriate statistical models to examine properties of the ANS – will be addressed in the next section.

2.4 Individual Differences

The aim of section 2.4 is to review research that has examined individual differences in symbolic and non-symbolic SNARC-like effects, the ANS, and how these factors may be related to math ability. Although research on the symbolic SNARC effect has been extensive, the focus has been on the general cognitive mechanisms involved, rather than the impact of individual differences in these mechanisms and how they affect performance (Cipora et al. 2015). In this section, it will be argued that it is important to focus on individual differences because group level analyses may mask important differences in effects. The symbolic SNARC effect is often analysed using group measure data in which data is collapsed across small and large number stimuli (Dehaene et al. 1993). RT data for Arabic numerals 1, 2, and 3 are collapsed into a category of 'small' and RT data for symbolic stimuli 7, 8, and 9 are collapsed into a category of 'large'. These two categories are then entered into an ANOVA and analysed at the group level. However, large variability and individual differences in speed and accuracy in magnitude and parity judgment symbolic SNARC tasks are common (Cipora, & Nuerk, 2013). It is suggested psychophysics methods may be a potential tool to identify individual differences in response effects. This methodology will be used in the thesis.

An important matter when examining the SNARC effect for non-symbolic stimuli in the ANS range is that, compared to the SNARC effect for symbolic stimuli, task difficulty must be considered. When making magnitude judgments with non-symbolic stimuli there is greater variability in response due to individual differences in the Weber fraction; that is, task difficulty varies based on an individual's ability to discriminate non-symbolic quantity. It is claimed that individuals with stronger mapping of the ANS and a smaller Weber fraction tend to show faster mean RTs and increased accuracy (Halberda et al., 2008). Compared to data for the symbolic SNARC effect, it is expected that data for the non-symbolic SNARC-like effect will show a broader range of RT and accuracy responses.

Overall, data analysis for the non-symbolic SNARC-like effect has been largely based on analyses designed to test the symbolic SNARC effect that do not take into account individual differences in acuity of non-symbolic quantity abilities. The use of group level analyses does not allow intricacies in data for the non-symbolic SNARC-like effect to be examined. A different analytic approach is necessary when examining the non-symbolic SNARC-like effect for non-symbolic quantity. This issue will be addressed in the thesis.

2.4.1 What is Math Ability and How Does it Relate to Individual Differences?

In this section individual differences will be discussed within the context of math ability. It will be argued that individual differences in the spatial representation of non-symbolic quantity are likely to be found. If so, it would be expected that individual differences will be detectable in the non-symbolic SNARC-like effect. Individuals differ in strategies used to complete math tasks (Dowker, 2005; Vanbinst, Ghesquière, & De Smedt, 2012), as well as speed and accuracy of performance (Ratcliff, Thompson, & McKoon, 2015). Indeed, Dowker (2005; Dowker & Sigley, 2010) proposes math ability consists of different 'abilities' or components that interact. Individuals may show strengths and weaknesses in different math components. Greater variation in math ability is evident in children during development. A child may be good at addition and poor at subtraction. Adults also show individual differences in math ability, albeit to a lesser degree. Psychology, neuroscience, and education literatures support the claim that math ability consists of different components and the importance of understanding patterns of the strengths and weakness of these components in order to design specific interventions that target these weaknesses (Dowker, 2005).

Lefevre, Wells, and Sowinski (2015) highlight the cognitive processes involved in arithmetic and propose four sources of individual difference. First, numerical knowledge involves specific cognitive processes (e.g., discriminating non-symbolic quantities, and subitizing). Second, attentional resources such as working memory or central executive must be engaged to solve arithmetic problems. Third, linguistic knowledge is necessary to learn number words, for counting, numerical rules, and structures. Finally, strategy choice must be learnt as well as rules about when to use specific strategies (Lefevre et al., 2015). These claims highlight a broad range of areas in which individual differences in performance can occur.

Given the wide range of cognitive processes involved in arithmetic, it is not surprising that individual differences in performance are common. Yet the mechanisms behind these individual differences are still poorly understood. It is important to examine these specific mechanisms but also to investigate interactions between these areas as there may be causal factors or correlations between these components. One such example is the spatial representation of non-symbolic quantity and how the interaction between number and space (i.e., the mental number line) may influence math performance. More research is needed on individual differences in the spatial representation of non-symbolic number.

2.4.2 What Factors Influence Individual Differences in Math Ability?

In this section, research on what factors may influence individual differences in non-symbolic quantity processing, representation, and math ability will be reviewed. It will be argued that although there is no single specific factor that influences math ability: instead, a complex set of factors may interact.

Genetic, cognitive, and behavioural factors have been shown to be associated with individual differences in math ability. Dehaene (1997) claimed that humans are born with an innate capacity to process non-symbolic quantity. Butterworth (1999) proposed that individuals are born with a number module that enables non-symbolic quantity to be processed. Adams (2007) supported the claim that genetics influences math ability and proposed that neuro-cognitive and behavioural factors affect ANS acuity, specifically math education. It is worth noting that indigenous cultures without formal schooling tend to show poorer ANS acuity (Gordon, 2004; Pica et al., 2004). Adams' study suggests that although math ability may have a genetic basis, math education seems to have a greater influence on performance. However, there may be many other reasons for these differences. Reeve, Reynolds, Humberstone, and Butterworth (2012) argued that core numerical competences are related to math ability rather than general cognitive abilities. Core numerical competences of enumeration and numerical comparison were examined using cluster analysis within task and across age group. Both dot enumeration and numerical comparison at 6 years of age were stable predictors of math ability at 6, 9.5 and 10 years. However, neither general processing speed nor non-verbal reasoning predicted group membership. Reeve et al. claimed that dot enumeration and number comparison do not reflect general cognitive abilities. Instead, individual differences are related to number specific competencies (Reeve et al., 2012).

Overall, genetic, cognitive and behavioural factors along with formal educational and core number abilities contribute to individual differences in math ability. Individual differences in ANS performance are particularly relevant to testing the non-symbolic SNARC-like effect with non-symbolic quantities within the ANS range and will be discussed in the next section.

2.4.3 Individual Differences in the ANS

In this section the question whether individual differences are found in the ANS will be addressed. The claim that individual differences in the ANS are linked to math performance will be evaluated. Evidence regarding the putative link between the ANS and math performance is mixed. It will be argued that several factors may influence individual differences in ANS and math performance. As already discussed, large individual differences are found for ANS performance. Studies by Halberda and colleagues have found a correlation between individual differences in the ANS and symbolic math ability (Chen & Li, 2014; Halberda & Feigenson, 2008; Halberda et al., 2008). Schneider et al. (2016) agreed with the claim made by Halberda and colleagues that there seems to be a link between math ability and ANS acuity. However, the exact mechanisms behind this relationship are not fully understood.

In summary, there are individual differences in the relationship between the ANS and math ability. Younger children tend to show a stronger link between the ANS and math performance. A possible reason for changing relationships between ANS acuity and symbolic math performance as children get older may be related to the nature of number tasks undertaken also changing. A problem with ANS tasks is that they show low reliability. Moreover, there are known measurement issues of the ANS that call into question whether the ANS construct is being properly measured. There are also individual differences in the strategies used in quantity judgment tasks (i.e., number itself, or low-level visual cues). To date, training studies of the ANS that show improvement in symbolic math are unclear about the causal direction of this relationship, although it is likely that other factors mediate the relationship between ANS acuity and symbolic math performance. Therefore, further research in this area is needed. Perhaps individual differences in the spatial representation of numerical magnitudes as shown by the SNARC effect may provide further insight in this area.

2.4.4 Individual Differences in the Symbolic SNARC Effect

In this section, theories regarding mechanisms behind individual differences in the symbolic SNARC effect will be reviewed together with studies that have found individual differences in the symbolic SNARC effect. It will be argued that most studies of the symbolic SNARC effect have not focused on individual differences. To date, few studies have examined individual differences in the SNARC effect and as previously noted, mechanisms underlying the symbolic SNARC effect are not fully understood, in particular, the degree to which these mechanisms influence the symbolic SNARC effect at the level of the individual.

Wood, Willmes, Nuerk, & Fischer (2008) conducted a meta-analysis of the symbolic SNARC effect and found that the strength of the SNARC effect and spatial representation of number can differ across participants. One of the strongest lines of evidence for individual differences of the symbolic SNARC effect is that not everyone shows a SNARC effect. Beecham, Reeve, and Wilson (2009) found that about a third of participants failed to show a symbolic SNARC effect for both an implicit number task (respond to parity) and an explicit number task (respond to magnitude). Beecham et al. posited three theories as to why these individuals failed to show a SNARC effect: (1) this group do not have a spatial representation of number; (2) the spatial representation of number in these participants does not influence RT in SNARC tasks (i.e., the shape of the representation does not easy map onto key responses); or (3) spatial representation in these individuals may change according to task demands. Beecham et al. suggested level of experience in math may underlie individual differences in the symbolic SNARC effect and including a specific math task may enable a way to further examine this.

Several studies have examined whether math performance explains individual differences in the size of the symbolic SNARC effect. The first study to suggest this link was by Dehaene et al. (1993), who compared individual differences in the SNARC effect for a group of science (mathematically skilled) students compared to a group of literature students (mathematically unskilled). However, Cipora and Nuerk (2013) argued that on closer inspection of Dehaene et al.'s results, a graph showed that the mathematically unskilled group showed a slightly larger

SNARC effect. As no statistically significant effect was reported, the results are only descriptive. Cipora and Nuerk (2013) note that a study by Fischer and Rottmann (2005) replicating Dehaene et al.'s study identified a smaller SNARC effect for the mathematically skilled group. However, the difference did not reach statistical significance.

Cipora and Nuerk (2013) proposed several reasons why previous studies (Dehaene et al., 1993; Fischer & Rottmann, 2005) did not show a consistent relationship: a lack of statistical power due to small sample size, insufficient repetitions for each stimulus, and math ability was inferred from participants' fields of study rather than by measuring individual math ability using a specific math task. Cipora and Nuerk conducted a study to further investigate the claim that mathematically skilled participants show individual differences and therefore group level differences in the SNARC effect compared to less mathematically skilled counterparts. Even though the study included a larger sample size, more repetitions per stimuli, and a math measure, a significant difference was not found for mathematically skilled versus unskilled participants. Nevertheless, Cipora and Nuerk did identify individual differences in the symbolic SNARC effect, with 70% of participants showing a standard SNARC effect in a parity judgment task. Cipora and Nuerk suggest that reasons for individual differences in the symbolic SNARC effect might be related to culture, stimuli used, group skill level, SNARC procedures, or task design. In other words, although there is consistent evidence for individual differences in the symbolic SNARC effect, the reasons why this occurs is less certain.

Hoffman, Mussolin, Martin, Schiltz, & Lappe (2014) also compared individual differences in the symbolic SNARC effect. Hoffman et al. compared the SNARC effect in a math expert group (science, math and engineering students), control group (literature, law and linguistics) and a math difficulties group that consisted of literature, linguistics and law students who reported issues with math but no specific learning disability. Hoffman et al. found that 100% of students in the math difficulties group showed a standard SNARC effect, followed by 89% of those in the control group and 74% of those in the math expert group. The difference between groups was significant. Hoffman et al. argued that individuals with a higher level of math proficiency may have more direct access to numerical representation. This finding demonstrates that there are inconsistent results in the literature regarding math as the underlying factor for individual differences in the symbolic SNARC effect. However, it is not known if a relationship exists between the non-symbolic SNARC-like effect and math ability, or if such a relationship exists, whether it explains individual differences in the non-symbolic SNARC-like effect. These questions are central to this thesis.

Cipora, Patro, and Nuerk (2015) reported six factors that may influence the symbolic SNARC effect. These are: (1) influence of culture, reading and writing direction (Dehaene et al., 1993; Zebian, 2005); (2) spatial demands of the task (i.e., imaging numbers on a clock face, Bächtold et al., 1998); (3) finger counting direction (Fischer, 2008); (4) working memory (i.e., the ordering of numbers held in working memory while doing a symbolic SNARC task; van Dijck & Fias, 2011); (5) the use of words that relate to spatial codes (i.e., respond left if there are more or right if there are less items or vice-versa; Proctor & Cho, 2006); and (6) attentional factors (i.e., a centrally presented number shifting attention to the right or left visual field depending on magnitude). Cipora et al. argued that it is not clear to what degree these factors in isolation or in what combination influence the symbolic SNARC effect; however, they acknowledge the relationship between numerical magnitude and space has been well established. However, another aspect to consider may be how these factors influence an individual's symbolic SNARC effect. Sex and age of participants can also influence the symbolic SNARC effect. Older participants with slower RTs generally show a stronger SNARC effect (Hoffmann et al., 2014; Wood et al., 2008), and males tend to show a stronger SNARC effect than females (Bull, Cleland, & Mitchell, 2013).

Another important aspect to consider when examining individual differences in the symbolic SNARC effect is individual differences in response signatures. Cipora and Wood (2017) acknowledged there can be great variability in accuracy and speed of participants for SNARC tasks. Some participants respond in a similar manner throughout a trial block, while others respond less consistently. As a measure of consistency of response, Cipora and Wood used the standard deviation of response sdR. Four different sdR values – 75, 150, 225, and 300 ms. – were modelled with Monte Carlo simulations using a lognormal distribution. A smaller sdR represented consistent responding whereas a larger sdR represented less consistent responding. One of the key findings of the study was that intra-individual variability in response latency influenced the ability to detect a symbolic SNARC effect. Specifically, the greater the variability in participant reaction time responding, the more difficult it is to detect a symbolic SNARC effect. So, apart from individual differences in SNARC slopes, a range of variability factors affects outcomes. Moreover, Cipora and Wood found that Monte Carlo simulations used in the study suggest that using a minimum of 20 participants and at least 20 trials per condition would increase the statistical power 95%. This study shows that experimental design is important to maximise the chance of finding a SNARC effect.

One of the most consistent and robust influences on the size of the SNARC effect is experimental design, which in turn influences reaction time. Several studies have shown, experiments that elicit slower RTs result in a stronger SNARC effect (Cipora & Nuerk, 2013; Gevers et al., 2006; see Wood et al., 2008, for a meta-analysis).

Viarouge, Hubbard, and McCandliss (2014) examined individual differences in cognitive mechanisms underlying the symbolic SNARC effect over two time-points, two weeks apart. Viarouge et al. examined correlations between number-space associations (measured by the SNARC effect), numerical precision (measured by the distance effect), and visuo-spatial ability (measured by two mental rotation tasks). Multiple regression analysis showed that individual differences in the size of symbolic SNARC effect were significantly related to both visuo-spatial ability (mental rotation) and numerical precision (the distance effect). As the SNARC effect remained relatively stable over time, Viarouge et al. argued that the SNARC effect is related to long-term spatial coding strategies, suggesting that individual differences in the symbolic SNARC effect may be linked to individual differences in the long-term spatial representation of symbolic number (i.e., the mental number line).

Overall, most research on the symbolic SNARC effect has not addressed individual differences. Studies that have examined individual differences in the SNARC effect have identified individual differences in *whether* a SNARC effect is found and the *direction* of the effect. Mechanisms underlying these individual differences and the spatial representation of symbolic number are poorly understood. There is a dearth of research focused on individual differences in the non-symbolic SNARC-like effect: clearly, research is required. It is likely that large individual differences in the non-symbolic SNARC-like effect will be detected because of individual differences in the Weber fraction.

2.4.5 Neuroimaging Evidence for Individual Differences in the SNARC Effect

In this section individual differences in the symbolic SNARC effect will be reviewed with a focus on neuroimaging studies. It will be argued that advances in neuroimaging technology has enabled a better understanding of individual differences in the symbolic SNARC effect. Recent neuroimaging evidence provides further insight into individual differences and the symbolic SNARC effect. Cutini, Scarpa, Scatturin, Dell'acqua, and Zorzi (2012) used functional near-infrared spectroscopy (fNIRS) to examine brain regions activated during a SNARC task. fNIRS is like fMRI but has better temporal accuracy and is less susceptible to movement artefacts. The bilateral intraparietal sulcus (a known region related to magnitude representation) and left angular gyrus (related to verbal number processing) were functionally activated during a SNARC task. Intraparietal sulcus activation was modulated by numerical distance.

Krause, Lindemann, Toni, and Bekkering (2014) conducted a similar study that investigated individual differences in brain structure and the symbolic SNARC effect using MRI. Thirty-three clinically normal adults were tested for the SNARC effect using two different uni-manual paradigms and a parity judgment task. In the first task, response buttons were spatially coded (i.e., there were separate left and right responses with the same hand). The right pointer finger was used for left responses and the third finger for right responses. In the second task only one response button was used with the right thumb; participants were told soft presses were for left responses and hard for right. The force with which the button was pressed was measured. Krause et al. found individual differences in the link between numbers and space. Increased grey matter brain volume around the right pre-cuneus were related to the spatial SNARC. Individual differences in the link between numbers and force were related to increased volume of the angular gyrus. The authors argued their findings provide evidence for multiple mapping of symbolic number in different brain areas. Taken together, the studies by Viarouge et al. (2014), Cutini et al. (2012), and Krause et al. (2014) show behavioural and neuroimaging evidence for individual differences in the spatial representation of symbolic number.

Overall, studies that have examined individual differences in the symbolic SNARC effect using neuroimaging techniques suggest that there are individual differences in the structure of the brain. These structural differences may be related to individual differences in the spatial representation of symbolic number. However, more research is needed in this area.

2.4.6 Individual Differences in the SNARC effect and Issues with Analysis

In this section individual differences in the SNARC effect will be discussed in relation to issues with statistical analysis. It will be argued that some analytical procedures commonly used in SNARC studies do not enable individual differences to be examined. Another key issue when examining individual differences of the SNARC effect is analytic methodology. Fischer (2006) argues that individual-oriented analysis can inform reliability and validity of the SNARC effect, yet individual differences are often not reported. Instead, in the SNARC literature, data is typically reported as being collapsed across conditions. Symbolic SNARC studies commonly use ANOVA analysis where the SNARC effect is examined at the group level (Fischer & Rottman, 2005; Keus, Jenks, & Schwarz, 2005; Schwarz & Müller, 2006) or linear regression (Fias, Brysbaert, Geypens, & dYdewalle, 1996). With ANOVA analysis of the symbolic SNARC effect, data is often collapsed for small (i.e., 1, 2, 3) and large (i.e., 7, 8, 9) conditions so individual differences in the SNARC effect are not easily identifiable. Linear regression analysis of the SNARC effect also involves collapsing data across participants to see if the overall slope when tested with a *t*-test is negative, indicating a group level SNARC effect. Wood, Nuerk, & Willmes (2006) use a correlational analysis called Smallest Space Analysis to examine the effect of individual differences in strategy for the SNARC effect. Wood et al. found reliability in mean RT analyses of the SNARC effect but the SSA showed the SNARC effect was not consistent across participants. In other words, there were individual differences in the symbolic SNARC effect. However, this method has not been popular in the SNARC literature.

In summary, individual differences in the symbolic SNARC effect are commonly found. Many factors, both experimental and individual, seem to influence individual differences in the symbolic SNARC effect. These factors include study design, task demands, number of trials per stimuli, slope size, and an individual’s RTs. Older participants with slower RTs tend to show a stronger SNARC effect. Type of statistical analysis can also influence if a SNARC effect is found at the individual or group level. Behavioural and neuroimaging evidence seem to support individual differences in the symbolic SNARC effect and therefore individual differences in the spatial representation of symbolic number.

2.4.7 Individual Differences and the Approximate Number System

Given the emerging literature on individual differences in the symbolic SNARC effect, it is important to understand how individual differences may potentially influence a non-symbolic SNARC-like effect in the ANS range. Individuals differ in their ability to successfully discriminate which of two briefly-presented sets of dots contains the larger number of dots

(Halberda et al., 2008); however, there has been relatively little research on individual differences and the ANS. Research on individual differences of the ANS will be critically reviewed below.

As mentioned above, the ANS enables us to represent and manipulate quantity (Dehaene, 1997). Gilmore, Attridge, De Smedt, & Inglis (2014) suggested that when we see a set of items, an approximate representation of this quantity is created by the ANS.

Individual differences in ANS precision are seen across development, as indicated by a smaller Weber fraction (i.e., being able to distinguish sets that are closer in non-symbolic quantity) as children grow older. Studies on preverbal infants have shown infants can discriminate a ratio of 1:2 (i.e., 4 vs 8 dots) but not 3:4 (i.e., 8 vs 12 dots; Xu & Spelke, 2000). Adults can discriminate a ratio of 7:8. To calculate the Weber fraction, the difference of the two numbers is calculated, then divided by the smaller number (e.g., 8-7 / 7 = 0.14; see Halberda, 2008). Six-month-olds show a Weber fraction of approximately 1.0, infants at 9-months show a Weber fraction of approximately 0.5 (Xu, 2003), and adults typically show an approximate Weber fraction of 0.14 (Halberda, 2008). These studies show that precision of the ANS improves by adulthood.

It is suggested that individual differences in the ANS occur throughout the lifespan. Halberda, Ly, Wilmer, Naiman, and Germine (2012) conducted a large-scale study that focused on number sense across the lifespan. Halberda et al. assessed performance of an internet sample of 10,000 people, ranging from 11 years to 85 years of age, on the Panamath ANS test. They found ANS precision improves during school age years, perhaps reflecting formal education, peaks at about age 30, and starts to decline thereafter. This suggests individual differences in precision of the ANS may be related to age.

One issue with Halberda et al.'s (2012) study is that no measure of math was used; instead, math ability was self-reported. This limitation was addressed in a second study (see Supporting Information, Halberda et al., 2012) that replicated the first one with a sample of 3000 people, where SAT scores were also collected. Only the math component of the SAT scores correlated with ANS precision. Large individual differences were noted in the ANS in this study, with one in eight adults showing a poorer Weber fraction and RT than a typical 11-year-old. Halberda et al. suggested individual differences in ANS precision may be related to factors other than age. Halberda et al. suggested interventions targeting the ANS may be useful for people of all ages who show poor ANS precision and math performance. This study shows that, although average Weber fractions were calculated for each age range, about 12% of adults showed poor precision of the ANS. This shows that age alone does not explain individual differences in precision of the ANS.

The ANS seems to serve a different function during development. The most supported interpretation of the ANS in development is its role in scaffolding the acquisition of symbolic numbers (Feigenson, Dehaene, & Spelke, 2004). It is claimed that symbolic numbers are mapped onto non-symbolic quantity representations (Ansari, 2008; Butterworth, 2010; Holloway & Ansari, 2009; Mundy & Gilmore, 2009; Piazza, 2010). In adults, the ANS is claimed to be used for estimation (Barth et al., 2003; Dehaene, 1992; Dehaene & Cohen, 1995).

Evidence for this claim comes from the fact that in children a correlation is found between symbolic and non-symbolic comparison ANS tasks and approximate addition of dot arrays (Gilmore et al., 2014). However, in adults ANS tasks are not correlated with symbolic number tasks or with each other (Gebuis & van der Smagt, 2011), even when the same symbolic and non-symbolic quantity stimuli are used (Maloney, Risko, Preston, Ansari, & Fugelsang, 2010). In neuroimaging studies with children, frontal lobe activation progresses to fronto-parietal activation as brain areas become more specialised for number processing (Kaufmann, Kucian, & von Aster, 2014). In adults, non-symbolic tasks activate the left IPS, while symbolic tasks activate the right IPS, a finding that supports the claim that the ANS seems to serve a different function in adults than in children.

Issues with ANS precision have previously been linked to poor math ability, although not consistently. An intergenerational link has been found with similar ANS acuity between parent and child (Braham & Libertus, 2017), suggesting some genetic basis to math ability. Children with dyscalculia (math impairment where cognition is otherwise normal) show poor ANS acuity (Mazzocco, Feigenson, & Halberda, 2011; Piazza et al., 2010). Low SES has also been linked to lower precision of the ANS, with low SES children tending to show poorer ANS acuity and performance at math, perhaps due to lower levels of parental/child engagement with number games and opportunity to learn number words; in other words, these children seem to have less opportunity to practise mapping number representation (Fuhs & McNeil, 2013). However, there are other theories regarding differences in ANS acuity.

One key theory is that education seems to influence individual differences in acuity of the ANS. Evidence for this claim is provided by a study by Piazza, Pica, Izard, Spelke, and Dehaene (2013). Piazza et al. were able to separate the effects of education and age on ANS acuity by testing ANS acuity in both children and adults in an indigenous group in the Amazon – the Mundurucú. The Mundurucú are unique as they have a limited vocabulary to describe number and differing levels of access to math education. Piazza et al. found Mundurucú adults without access to education showed similar Weber fractions ($w = 0.31$) to a group of Italian children with dyscalculia ($w = 0.35$) whom Piazza et al. (2010) tested using the same paradigm. Piazza et al. (2013) argue that dyscalculia may be in part due to poor numerical acquisition during formal education. Greater precision of the ANS was shown in Mundurucú who had accessed formal schooling where numerical concepts of counting and arithmetic were taught, suggesting that education in numeracy increases ANS precision.

Overall, there appears to be individual differences in ANS acuity for adults and children. Some factors that seem to influence precision of the ANS are age, SES, and formal number education. If a non-symbolic SNARC-like effect exists, it is important to examine individual differences of the ANS to begin to understand the mechanisms underlying the non-symbolic SNARC-like effect.

2.4.8 Summary

Our understanding of the symbolic SNARC effect and the spatial representation of symbolic number has been furthered by research employing different paradigms, stimuli and measures. However, surprisingly little research has examined, let alone accounted for, individual differences in the SNARC effect and/or shown a non-symbolic quantity SNARC-like effect using stimuli in the ANS range, hence the research reported in this thesis.

2.5 Summary of Argument and Claims, and Research Agenda

2.5.1 Summary of the Literature Review

Overall, the aim of this thesis is to examine the relationship between number and space. Of specific interest to this thesis is the claim that spatial numerical associations support the mapping of numbers, often conceptualized as a left-to-right mental number line. The link between number and space has been well supported by empirical evidence. It is further claimed the symbolic

SNARC effect is evidence for the spatial representation of symbolic number. However, research on the symbolic SNARC does not specify the origin of this effect, in particular, how this spatial representation of symbolic number occurs. In Chapter 2, it has been argued that while much research has examined the spatial representation of symbolic number, there is a dearth of research on the spatial representation of non-symbolic quantity. This is surprising given that the claim that non-symbolic quantity representation supports symbolic number acquisition (Dehaene, 2001; Nieder & Dehaene, 2009; Pica et al., 2004). Insofar as this claim is correct a non-symbolic SNARC-like effect would provide evidence for non-symbolic quantity representation as a foundation for symbolic number representation.

In Chapter 2 it is argued that researchers who have used a non-symbolic quantity judgment task have focused on outcomes that are not specifically spatial in nature, such as automaticity of processing (Mitchell et al., 2011) or shared format modality (Nuerk et al., 2005). Research that has been conducted on the non-symbolic SNARC-like effect has largely utilized stimuli that can easily be converted to symbolic number.

The second claim that is central to this thesis focuses specifically on the representation of non-symbolic quantity. In Chapter 2 it is argued two separate systems represent non-symbolic quantity: a precise system for representing small quantity (e.g., 1 to 4~ items), often referred to as the subitizing range; and an approximate number system (ANS) for non-symbolic quantities beyond the 1 to 4 item range.

Previous findings claimed to support the non-symbolic SNARC-like effect and the claim that non-symbolic quantity are represented spatially have been undermined by methodological issues. These issues are as follows: First, the use of non-symbolic numerical stimuli that overlap the subitizing and ANS ranges. This is problematic because the subitizing and ANS ranges are separate systems with differing RT signatures. Processing of magnitudes in the subitizing range is precise, quick, and accurate whereas processing of magnitudes in the ANS range is slower and based on approximations. Therefore, it is argued that non-symbolic numerical stimuli that span the subitizing and ANS ranges should be examined separately. This issue will be addressed by only including stimuli in the ANS range.

Second, in previous research there are often measurement issues of the ANS including a lack of standardised stimuli and the influence of visual cues such as luminance, dot size, density and total area. This is problematic because some of these visual cues (i.e., luminance and dot size) have also been shown to be spatially represented. Therefore, it is difficult to attribute previous finding in non-symbolic SNARC-like studies to the spatial representation of non-symbolic quantity alone. This issue will be addressed by attempting to control for luminance, dot size, and surface area.

Third, to date, much research examining the symbolic SNARC effect has used group level analysis such as ANOVA and linear regression thereby collapsing RT data across participants. These statistical methods do not consider potential individual differences of the symbolic SNARC effect and the spatial representation of number. Nevertheless, research suggests that adults may show different patterns in the SNARC effect, but the significance of this is yet to be understood. The issues with symbolic SNARC statistical methods are particularly relevant when examining individual differences of the non-symbolic SNARC-like, as the same analytic methods are generally used. Indeed, these issues may be more relevant for the non-symbolic SNARC-like effect as there is likely more variance in non-symbolic quantity processing due to individual differences in the Weber fraction.

To address this issue an adaptive staircase will be used, which allows a calibration of individual differences in the Weber fraction. Examining the Weber fraction as well as the non-symbolic SNARC-like effect for each individual allows for diverse representation of non-symbolic quantity (i.e., individual patterns of ANS ability and spatial representation). This methodology can complement group level analysis by providing novel insights into cognitive processes that may underpin heterogeneity in the spatial representation of non-symbolic quantity and potentially math performance. Insofar as the non-symbolic SNARC-like effect or Weber fraction are associated with math performance, regardless of patterns of performance, it may suggest that a shared mechanism underlies differences in math performance (thereby supporting a group-level analysis approach). However, if different cognitive markers are related to differences in math performance this would suggest heterogeneity and therefore support the existence of individual differences in cognitive processes that underlie math performance.

2.5.2 Research Agenda

The research agenda taken from the summary of arguments and issues above has three key objectives. First, to determine whether non-symbolic quantity in the ANS range is indeed represented spatially in the form of a non-symbolic SNARC-like effect. Second, if a non-symbolic SNARC-like effect is found, to determine if the pattern of data exhibits a similar Hand by Magnitude interaction effect (i.e. is the shape of the interaction effect of the non-symbolic SNARC-like effect visually similar to that of a typical symbolic SNARC effect?). The third and final aim is to examine whether individual differences occur in the propensity for showing a non-symbolic SNARC-like effect as they do with the symbolic SNARC effect.

The next chapter (Chapter 3) presents a published empirical study that examines the non-symbolic SNARC-like effect. Chapter 4 presents three studies that fine tune characteristics of the non-symbolic SNARC-like effect. Study 2A aims to address issues of discrimination found in Study 1 by employing only two target stimuli with an ‘easy’ referent-to-target ratio’ of 0.5 based on the Weber fraction. Study 2B examines if a more pronounced SNARC-like effect will be found with stimuli that elicit longer processing times. In Study 2B two target stimuli with a ‘easy-ish’ referent-to-target ratio of 0.66 were employed with the potential cueing effect of overall luminance/contrast will be controlled by using an equal number of black and white dots. Study 2C examines if the non-symbolic SNARC-like effect is independent of response modality while controlling for the potential cueing effect of total surface area. Chapter 5 presents a study that investigates individual differences in non-symbolic quantity acuity and the non-symbolic SNARC-like effect. An adaptive staircase will be used to measure the Weber fraction for each individual to calibrate the ratio of referent-to-target for the Non-symbolic quantity judgment task while total surface area, density and display darkness will be controlled using different sized dots in each array and scaling factors. Chapter 6 will discuss findings and implications of the research.

Chapter 3: Study 1

The first study examined two predictions about the spatial representation of non-symbolic quantities using a non-symbolic quantity judgment task to examine the characteristics of the non-symbolic SNARC-like effect.

3.1 Non-symbolic SNARC-like Effect for Non-symbolic Quantities

The first prediction was that a non-symbolic SNARC-like effect would be observed for numerical magnitudes in the ANS range. This prediction was based on findings from the symbolic SNARC task. Specifically, it was predicted smaller non-symbolic quantities would be associated with the left side of space and larger non-symbolic quantities with the right side of space. Specifically, it was expected that faster responses should be observed for the left hand to smaller quantities and the right hand for larger quantities. A hand by quantity interaction was also expected. Insofar as these findings emerged it was argued that it would provide evidence for the spatial representation of non-symbolic quantities purely in the ANS range.

The second prediction concerns the influence of ratio on non-symbolic quantity judgments for stimuli in the ANS range using a within subject design with multiple ratios. Similar to work examining magnitude discrimination using the Weber fraction, it was expected the largest ratio (0.5) would be the easiest to judge. Insofar as this was the case, it was predicted that faster RTs and better accuracy would be found for target arrays of dots further away in numerical magnitude from the referent. Conversely, if ratio was unrelated to the non-symbolic SNARC-like effect, no difference in RTs and accuracy would be observed. Similar to the symbolic SNARC effect findings, it was expected that not all participants would exhibit a non-symbolic SNARC-like effect. It was predicted that participants who failed to show a non-symbolic SNARC-like effect would exhibit poorer visuo-spatial working memory and poorer math performance, compared to participants who showed a non-symbolic SNARC-like effect.

3.2 Nemeh, Humberstone, Yates, and Reeve (2018)

The research in this chapter has been published in *PLOS One* titled ‘Non-symbolic magnitudes are represented spatially: Evidence from a non-symbolic SNARC task.’ Study 1 will be presented in its published format.

PLOS ONE

RESEARCH ARTICLE

Non-symbolic magnitudes are represented spatially: Evidence from a non-symbolic SNARC task

Fiona Nemeh[1], Judi Humberstone[1], Mark J. Yates[2], Robert A. Reeve[1]*

1 Melbourne School of Psychological Sciences, University of Melbourne, Victoria, Australia, **2** Department of Neurological Surgery, Columbia University, New York, New York, United States of America

* r.reeve@unimelb.edu.au

Citation: Nemeh F, Humberstone J, Yates MJ, Reeve RA (2018) Non-symbolic magnitudes are represented spatially: Evidence from a non-symbolic SNARC task. PLoS ONE 13(8): e0203019. https://doi.org/10.1371/journal.pone.0203019

Editor: Mariella Pazzaglia, University of Rome, ITALY

Received: January 3, 2018

Accepted: August 14, 2018

Published: August 30, 2018

Data Availability Statement: All data files are available from the Figshare Digital Repository (10.26188/5b7634eecff73).

Funding: This research was supported by an Australian Government Research Training Program (RTP) Scholarship to Ms Fiona Nemeh. The funders had no role in study design, data collection and analysis, decision to publish, or preparation of the manuscript.

Competing interests: The authors have declared that no competing interests exist.

Abstract

A core proposition in numerical cognition is numbers are represented spatially. Evidence for this proposition comes from the "spatial numerical association of response codes" effect (SNARC) in which faster responses are made by the left/right hand judging whether one of a pair of Arabic digits is smaller/larger than the other. Less is known if a similar SNARC effect exists for non-symbolic magnitudes; and research that has been conducted used stimuli which could be translated into symbolic terms. To overcome this limitation, we employed a referent-to-target judgment paradigm in which a referent dot array ($n = 30$ dots) was follow by a second array of dots (e.g., $n = 45$ or 15 dots)–participants judged if the second array contained fewer or more dots than the referent array. Dot arrays with fewer dots were judged more quickly with the left hand compared to the right hand (i.e., a SNARC effect). Not all participants demonstrated a SNARC effect, however. Neither visuospatial working memory nor math ability was associated with the presence/absence of a non-symbolic SNARC effect. Implications of the non-symbolic SNARC effect for accounts of numerical cognition are discussed.

Introduction

The brain appears to associate numbers with spatial locations: smaller numbers are associated with the left side of space and larger numbers with the right side. The "spatial-numerical association of response codes" (SNARC) illustrates this association: The speed of magnitude judgement differs as a function of the hand used to make the judgement [1]. It has been suggested the SNARC effect activates a cognitive representation of number and occurs because of the association between the spatial code of the side of response and the magnitude of the number [2–5]. With smaller numbers (e.g., 1 or 2), judgments are faster using the left hand, and with larger numbers (e.g., 8 or 9), judgments are faster using the right hand. The SNARC effect is found for visual number words, auditory number words [6], double-digit numbers [7], and negative numbers [8].

The SNARC effect has also been found in non-numerical domains such as letters of the alphabet [9], object size [10–11], pitch height [12–13], loss and gain words [14], and emotional valence [15] (see Macnamara, Keage & Loetscher, for a review [16]). Given non-symbolic magnitude representation may be a more basic form of quantity representation (evident in human infants and animals), which may support symbolic representation, it is important to determine if a SNARC effect exists for non-symbolic quantity information.

The parietal area of the brain appears to represent number in an abstract notation form (i.e., activated by "3", "three", or three objects [17]), which suggests smaller and larger non-symbolic magnitudes (e.g., dot patterns with a smaller or larger numbers of dots) might be associated with spatial locations. Numerosity appears to be mapped topographically in the parietal cortex [18]. The parietal area is also involved in spatial processing [19–22]. Indeed, some findings have been interpreted as showing a spatial-numerical association for non-symbolic quantities using SNARC or SNARC-like paradigms [6, 23–25]. However, caution should be exercised since these findings could be interpreted in other ways.

It is possible that symbolic number representations are activated by non-symbolic stimuli, which implies sub-vocally counting cannot always be discounted. Non-symbolic magnitudes occasionally span the subitizing and approximate number system ranges, which activate different neuro-cognitive systems [26]. The approximate number system—the ANS—allows representations of large approximate non-symbolic magnitudes, whereas the subitizing range focuses on small precise magnitudes (e.g., "1"–"4") (see Feigenson, Dehaene & Spelke [27]). Evidence for this possibility has been found by Mitchell et al., [23] who showed a SNARC-like effect in a task for which non-symbolic magnitude stimuli were irrelevant to the task at hand. Participants judged the colour of dots (Experiment 1) or the orientation of triangles which were displayed as upright or inverted (Experiment 2). They found a stronger SNARC-like effect in the putative subitizing range (1–4), compared to larger numerosities (6–9) for the orientation judgement only.

In a recent study, Zhou et al. [25] used a same/different matching task to investigate the spatial representation of non-symbolic magnitude, size and density, to determine if non-symbolic magnitude is spatially represented, as evidenced by a non-symbolic SNARC effect. In the non-symbolic magnitude and density matching task, two dot arrays were presented sequentially. The first dot array was presented for 200 ms, followed by a blank screen for 1300 ms, followed by the second dot array that remained until a response was made (arrays comprised 11, 14, 17, 20, 23, 26 and 29 dots). In the area matching task, stimuli comprised pentagons differing in size. Findings revealed faster right-hand responses to large non-symbolic magnitudes; however, no difference was found for left hand responses. The authors suggest this was due to the right-hand responses reducing the RT difference between small left and small right data. Neither size nor density affected responses. The authors claimed their findings are consistent with a non-symbolic SNARC effect.

While Zhou et al. [25] overcome potential interpretive limitations of earlier non-symbolic work by excluding quantities in the subitizing range, and by examining the potential influence of visual cues of area and density, the authors employed an independent group design in which each condition was completed by different participants. One potential issue is the design ignores possible individual differences in density and area judgements. Moreover, like other researchers, Zhou et al. collapsed data across different magnitudes (i.e. 'small' consisted of 11, 14 and 17 dots, while 'large' consisted of 23, 26 and 29 dots). Collapsing across data for non-symbolic stimuli may be problematic since there are known individual differences in magnitude processing abilities (see Chew, Forte & Reeve, [28]).

Some cognitive indices (e.g., working memory) are thought to affect the symbolic SNARC effect [29]. Differences in VSWM in children have been linked to poorer math accuracy [30] and the absence of a symbolic SNARC effect in those with comorbid VSWM disability and dyscalculia [31]. It is of interest whether there is a link between a non-symbolic SNARC effect and VSWM.

Not everyone exhibits a symbolic SNARC effect [32–38]. In symbolic SNARC research, data is commonly collapsed over 'small' and 'large" arrays (i.e. 1, 2 and 3 might be deemed small and 7, 8, and 9 large), which may mask individual differences [32]. Moreover, we know little of factors associated with the presence/absence of a SNARC effect. Some studies have shown VSWM is related to non-symbolic magnitude processing [39] and other studies have found a relationship between math ability and symbolic SNARC effects [34, 36]; the relationship between non-symbolic number acuity and math ability is less consistent [40]. Given these findings, it is possible the presence/absence of a non-symbolic effect is related to VSWM and/or math abilities.

The current study

To overcome possible methodological limitations in research that has attempted to identify a non-symbolic SNARC effect, and to determine whether non-symbolic SNARC effects are associated with visuo-spatial working memory and/or math ability, we employed a referent-to-target judgment paradigm, in which stimuli were in the ANS range. Participants were presented a referent dot array for 300 ms followed by a second dot array and judged whether the second array contained fewer or more dots than the referent array. The target judgment arrays (15, 20, 25, 36, 45 and 60 dots) were selected so the ratio between the smallest array (n = 15 dots) and the referent array (n = 30 dots) was the same as the ratio between the largest arrays (n = 60 dots) and the referent array. Similarly, the ratio between arrays containing 20 and 30 dots was the same as arrays containing between 30 and 45 dots, and the ratio between 25 and 30 dots was the same as between 30 and 36 dots. The rationale for the design is, all things being equal, difficulty discriminating two non-symbolic magnitudes reflects the ratio between them (discriminating 9 from 10 dots is as difficult as discriminating 90 from 100 dots; see Mazzocco, Feigenson & Halberda [41]).

The set sizes selected were based on ratios that putatively reflect similar levels of difficulty for pairs of stimuli (i.e., 15 and 60 dots = a ratio of 0.5, 20 and 45 dots = a ratio of 0.66, and 25 and 36 dots = a ratio of 0.83). Most research that has examined the symbolic SNARC effect has focused on distance effects. However, the meaning of "distance" using non-symbolic SNARC stimuli is less obvious. By focusing on ratio, we were able to investigate the effects of the relationship between ratio difficulties and non-symbolic SNARC effects. Like findings from magnitude discrimination studies using the Weber fraction measure, we expected the 0.5 ratio would be easiest to discriminate. We expected participants to respond more quickly to targets more distant from the referent array than targets closer to the referent array. Based on findings from symbolic SNARC research, we hypothesize smaller non-symbolic magnitudes would be associated with the left side of space and larger non-symbolic magnitudes with the right side of space, which, in turn, would be reflected in faster responses made with the left hand for smaller numerical magnitudes and for larger numerical magnitudes using the right hand. We also expected a hand by magnitude interaction. As working hypothesis, we expected the absence of a non-symbolic SNARC effect would be associated with poor VSWM and/or math ability. We used stimuli in the ANS range to avoid the use of sub-vocal strategies.

Method

Participants

Twenty-eight undergraduate psychology students participated for course credit. Participants (10 males, 18 females) were between 17 and 36 years (M = 20.32 years, SD = 3.9) and, as measured by the Edinburgh Handedness Inventory questionnaire (Oldfield), were strongly right handed. While the sample comprised students from a range of cultural backgrounds, all spoke English fluently. All participants had normal or corrected-to-normal vision. The study was conducted in compliance with the approval and requirements of the author's University's Human Ethics Committee, University of Melbourne (# 1237507.2). All participants provided written, informed consent.

Materials and procedure

All participants completed a non-symbolic magnitude judgement, Corsi Blocks Forwards (VSWM), WRAT-4 Math Computational Subtest (Mathematics ability), and the Edinburgh Handedness Inventory (Handedness) tasks. The non-symbolic magnitude judgement task was presented on a 20" computer screen using E-Prime software (version 2.0), with a viewing distance of approximately 50cm at ~ 8 degrees' visual angle. Dot arrays were generated using Microsoft Paint software.

Corsi Blocks-VSWM. The Corsi Blocks VSWM test was administered following Kessels, van Zandvoort, Postma, Kappelle, and de Haan's [42] method. The examiner began by a tapping a two-block sequence. The participant tapped the same sequence back. Sequences gradually increased in the number of blocks tapped and task ended when the participant failed to correctly copy a sequence twice in a row. Block span was calculated as the longest block sequence repeated correctly.

Wide Range Achievement Test -4th Edition (WRAT-4)—math ability. *WRAT-4*, *Math Computational Subtest*–(Wilkinson & Robinson, [43]) comprises 40-items and assesses the capacity for basic mathematic skills though counting, identifying numbers, carrying out basic written math problems such as addition, subtraction, division, multiplication, fractions, decimals and algebra.

Edinburgh Handedness Inventory (Oldfield, [44])—Handedness. This 10-item questionnaire indexes hand preference. Responses range from strong (++) through to less strong (+) and indifferent (+/-).

Non-symbolic magnitude judgement task. Firstly, a blank screen was shown, followed by a grey circle in the centre of the screen for 500 ms. On each trial, participants were presented with two dot displays in sequence (first the referent stimulus for 300ms, followed by the target until response) (see Fig 1 below for the trials sequence). These presentation times were chosen after pilot work showed participants could not enumerate the dots in this time. That is response times indicated they were not enumerating. The first dot display (the referent stimulus) always contained a fixed number of dots (black dots). The second dot display (the target

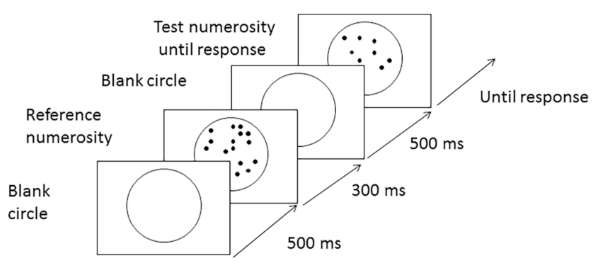

Fig 1. Diagram of procedure.

https://doi.org/10.1371/journal.pone.0203019.g001

stimulus) contained variable numbers of dots (15, 20, 25, 36, 45, or 60 dots). Participants judge whether the second arrays of dots (the target stimulus) contained *fewer* or *more* dots than the referent.

Participants responded by pressing either a left or a right response button, as appropriate. Each participant completed two blocks of 96 trials (192 trials in total). Hand of response was counterbalanced. In Block 1, participants were asked to press the left button if the target stimulus contained *less* dots and the right button if the target contained *more* dots. For Block 2, the instructions for the button response were reversed so that a left button press indicated a *more* response for the target stimulus and a right button press indicated a *less* response for the target stimulus. Participants were asked to switch their hand of response for the second half of the experiment.

Prior to each block of trials, participants completed 20 practice trials. In the practice session, accuracy was shown as a percentage on the screen. Participants with an accuracy of less than 75 percent were required to conduct the practice session again. Participants were given a short break at the halfway point, prior to switching response hand. After the magnitude judgment task, participants completed the WRAT math and VSWM tasks.

Analytic approach

Data analysis in symbolic SNARC studies commonly use ANOVA [2, 6, 45, 46] or linear regression [47] techniques. In ANOVAs a SNARC effect is defined as the interaction between hand and magnitude [1]. The SNARC effect is not contingent on the hand used during motor response selection (Dehaene et al., Exp. 6, [1]), is independent of effector used [48, 49], and appears to be amodal [6]. The SNARC effect has also been defined as the difference between congruent and incongruent response conditions and spatial mapping [50–52]. While ANOVAs test if a SNARC effect is present, and is useful for group-level analysis, it does not identify individual differences in RT latencies or accuracy. One way of overcoming this issue is to examine difference in right-hand RTs and left-hand RTs in a linear regression, in which a SNARC effect is defined as a negative slope.

To test for the presence of a non-symbolic SNARC-like interaction effect, we conducted a 2 × Response Key Position (left or right-side response box) × 2 Direction of the Magnitude/Referent Relationship (smaller or larger than the 30-dot referent array) × 3 Ratios (0.5, 0.66, 0.83) ANOVA with Bonferroni post-hoc contrasts. (It should be noted that we assessed the data distribution characteristics and found the RT distributions to be normal in form, as a consequence of which we did not transform the data, and the findings reported herein are based on the untransformed RT data.) We also examined the influence of the three different ratios (0.5, 0.66 and 0.83) on the non-symbolic SNARC-like effect in separate 2 × Response Key Position (left or right-side response box) × 2 Direction of the Magnitude/Referent Relationship (smaller or larger than the 30-dot referent array) ANOVAs. Incorrect trials were excluded from analysis, as were trials where response times (RTs) of 200ms or less (often considered to be anticipatory responses) and trials which exceeded 1000ms (1000ms was chosen since it was more than 3SDs above the mean RT).

For each participant, twelve median reaction times (RTs) were derived for left and right-hand responses to each of the target stimuli (15, 20, 25, 36, 45 and 60 dots). There were four experimental conditions: (1) left-hand response to target stimuli with smaller non-symbolic magnitude (e.g., a referent containing 30 dots and a target containing 15, 20 or 25 dots); (2) left-hand response to target stimuli with larger non-symbolic magnitude (e.g. a reference containing 30 dots and a target containing 36, 45, or 60 dots; and (3) right-hand response to target stimuli with smaller non-symbolic magnitude; and (4) right-hand response to target stimuli

Table 1. Medians and SE's of RTs and means and SD's of accuracy as a function of magnitude.

Ratio	Dot Magnitude	Target to Referent	Left Hand RTs (ms)		Accuracy		Right Hand RTs (ms)		Accuracy	
			Mdn	*SE*	*M*	*SD*	*Mdn*	*SE*	*M*	*SD*
0.5	15	0.5 x Ref	497.50	17.49	0.99	0.03	518.75	17.76	0.99	0.02
0.66	20	1/1.5 x Ref	578.00	17.71	0.95	0.07	559.00	18.20	0.97	0.04
0.83	25	1/1.2 x Ref	641.25	21.85	0.80	0.20	664.50	20.57	0.78	0.20
0.83	36	1.2 x Ref	555.75	23.08	0.93	0.09	522.75	18.24	0.92	0.11
0.66	45	1.5 x Ref	521.25	21.04	0.97	0.05	461.25	17.13	0.97	0.04
0.5	60	2 x Ref	494.00	51.04	1.00	0.01	442.25	15.67	0.99	0.02

https://doi.org/10.1371/journal.pone.0203019.t001

with larger non-symbolic magnitude. Three median RTs from the four experimental conditions were analysed using an ANOVA.

To examine individual differences in the direction of a non-symbolic SNARC-like effect we followed Fias, Brysbaert, Geypens, and d,Ydewalle 's [47] analytic method. The difference between right and left RTs (dRTs) were entered into a linear regression analysis in which the natural logarithm of the non-symbolic magnitude was the independent variable. The linear equation $y = ax + b$ was used to calculate the slope for each individual. β slope coefficients were analysed using a t-test to determine whether slopes were significantly different from 0.

A negative slope indicates a standard SNARC effect (i.e., small non-symbolic magnitudes associated with the left-hand side), and a positive slope would indicate a reverse SNARC effect (small non-symbolic magnitudes associated with the right-hand side).

We also examined the relationship between VSWM, math ability and the non-symbolic SNARC-effect to determine whether these factors were associated with performance on the non-symbolic SNARC task.

Results

The median RTs and accuracy for Magnitude × Hand judgments are shown in Table 1. Table 1 reveals that, as the targets get closer in magnitude to the referent, responses appear to slow and are less accurate.

SNARC effect for non-symbolic numerical magnitudes

Initial analysis showed no error rate interaction between magnitude and hand (F (1, 27) = .95, p = .34, η^2 = .03) (i.e., no evidence of a speed-accuracy trade off: the mean error rate was 7.37%). We conducted a repeated-measures ANOVA to determine whether a SNARC-like effect was evident for non-symbolic numerical magnitudes. In this analysis we used the Greenhouse-Geisser correction of the data since Mauchley's test was significant χ^2 (2) = 15.00, $p <$ .05, which suggests the assumption of sphericity had been violated. The analysis comprised a 2 × Response Key Position (left or right-side response box) × 2 Direction of the Magnitude/Referent Relationship (smaller or larger than the 30-dot referent array) × 3 Ratio (0.5, 0.66, 0.83) ANOVA. The analysis revealed that Hand was not significant (F(1, 27) = 3.15, p = .09). However, Magnitude was significant (F(1,27) = 89.31, $p < .0001$, η^2 = .77) as was Ratio (F(1.39, 37.54) = 94.25, $p < .0001$, η^2 = .78). This finding suggested there was a difference in responses for small and large magnitudes and for differing ratios. We also found a significant interaction between Hand and Magnitude (F(1,27) = 6.29, p = .018, η^2 = .19), which supports the existence of a non-symbolic SNARC-like effect. The Magnitude × Ratio interaction was also significant (F(1.60, 43.05) = 17.20, $p < .0001$, η^2 = .39). However, neither the Hand × Ratio (F(1.65, 44.54)

= 1.58, p = .22) nor the three-way interaction for Hand × Magnitude × Ratio were significant (F(1.86, 50.17) = .48, p = .61).

Since the main effect for ratio was significant, we compared differences in accuracy and RT for the three ratios. The analysis showed that targets further in magnitude from the referent were judged more quickly and more accurately. Bonferroni post-hoc contrasts revealed that targets furthest in magnitude from the referent (a ratio of 0.5) were responded to significantly faster (36 ms faster) than the next ratio closer to the referent (a ratio of 0.66) and 102 ms faster than the ratio closest to the reference (a ratio of 0.83). (all contrasts $p < .05$). This suggests that examining the three ratios separately would be more informative about the nature of the non-symbolic SNARC effect.

Ratio and the SNARC Effect

Three separate 2 x 2 ANOVAs were conducted to assess the relationship between response hand (left hand vs right hand) and non-symbolic magnitude RTs for the three ratios (0.5, 0.66 and 0.83 respectively).

The analysis of the 0.5 ratio showed a Magnitude effect (F(1, 27) = 40.26, p = < .0001, η^2 = .60; and 60 dots were responded to faster than 15 dots. While no effects was found for Hand (F(1, 27) = 1.85, p = . 19), the interaction between Magnitude and Hand was significant (F(1, 27) = 6.141 p = .010, η^2 = .19). This provided support for a non-symbolic SNARC-like effect for the 0.5 ratio. Simple effects analysis showed left hand responses showed no difference for small or large numerosities (F(1,27) = 1.46, p = 0.24), whereas participants responded faster using their right hand (a 38 ms advantage for right hand responses) to larger compared to smaller numerosities (F(1,27) = 9.52, p = 0.01, η^2 = 0.26).

The analysis of the 0.66 ratio showed a Magnitude effect (F(1, 27) = 49.44, $p < .0001$, η^2 = .65), and a Hand effect (F(1, 27) = 8.58, p = .01, η^2 = .24 –the right hand was faster than the left hand), and an interaction between Magnitude and Hand (F(1, 27) = 7.84, p = .01, η^2 = .23). Although, for the 0.66 ratio, the left hand was not faster than the right for smaller numerosities.

The 0.83 ratio analysis showed a Magnitude effect (F(1, 27) = 56.54, $p < .0001$, η^2 = .68), no effect of Hand (F(1, 27) = .12, p = . 73), but a marginally significant interaction between Magnitude and Hand (F(1, 27) = .2.96, p = .10 (two-tailed) or .05 (one-tailed), η^2 = .10).

Following the analytic convention used in symbolic SNARC studies, we collapsed data across small (15, 20, 25) and large (36, 45, 60) magnitudes. Results still showed support for a SNARC-like effect. Hand was not significant (F(1, 27) = 3.17, p = .09). However, Magnitude (F(1,27) = 89.12, $p < .0001$, η^2 = .77) and the Hand × Magnitude Interaction were significant (F(1,27) = 6.30 p = .02, η^2 = .19).

See Fig 2 below for the Hand by Magnitude interaction effect for the three separate ratios (0.5, 0.66 and 0.83). Fig 2 shows the form of the non-symbolic SNARC-like effect differs for each ratio but is similar to the symbolic SNARC effect for the 0.5 and 0.66 ratios.

SNARC, VSWM and Math Abilities

The WRAT standardised score ranged from 87 to 143 with a mean of 113.14. This showed for the sample above average mathematics ability according to WRAT norms. For the Corsi Blocks task, visuospatial span ranged from 5 to 9 with a mean of 6.54. The average test norm for Corsi span is 6. Correlation coefficients for the WRAT, Corsi span, and regression slope are reported in Table 2 below. Table 2 shows that there were no significant correlations with either VSWM (Corsi span task) or math ability (WRAT) and the reaction time task (slope).

To investigate the relationship between VSWM and math ability and the non-symbolic SNARC-like effect, we conducted a 2 × Response Key Position (left or right-side response

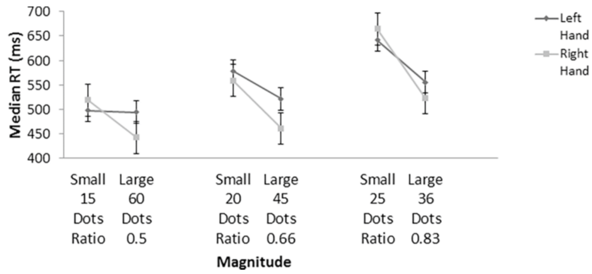

Fig 2. Median RTs comparing 15 vs 60, 20 vs 45 and 25 vs 36 dots to the 30 dot referent as a function of left vs right hand (error bars = standard error).

https://doi.org/10.1371/journal.pone.0203019.g002

box) × 2 Direction of the Magnitude/Referent Relationship (smaller or larger than the 30-dot referent array) × 3 Ratio (0.5, 0.66, and 0.83) repeated measures ANCOVA. The math measure (WRAT standardised score) and VSWM measures (Corsi) were entered as covariates. The interaction effect between Hand and Magnitude was significant ($F(1,27) = 8.08$, p = . 01, $\eta^2 = .23$) suggesting math and VSWM were not significant predictors of the non-symbolic SNARC effect. As the earlier analysis showed that only the 0.5 ratio showed a non-symbolic SNARC effect, we ran a 2 x Response Key Position (left or right-side response box) × 2 Direction of the Magnitude/Referent Relationship (smaller or larger than the 30-dot referent array) repeated measures ANCOVA for this ratio. The math (WRAT standardised score) and VSWM measures (Corsi) were entered as covariates. The original interaction effect for Hand by Magnitude remained significant ($F(1,27) = 6.14$, p = .020, $\eta^2 = .19$). This suggested that for the 0.5 ratio both math and VSWM measures were not significant predictors of the non-symbolic SNARC effect.

Standard and reverse SNARC effects

Inspection of participants' performances showed that a small number of individuals ($n = 8$) exhibited a reverse non-symbolic SNARC effect, with an average regression slope β value of 66.44, indicating a positive slope which significantly differed from 0, ($t(7) = 4.17$, $p = .004$). Participants who exhibited a standard SNARC effect ($n = 19$), had an average regression slope β value of -114.33. This indicated a negative slope which significantly differed from 0, ($t(18) = 5.39$, $p < .0001$). One participant had a regression slope of .00, indicating a flat line and no SNARC effect.

An independent samples t-test confirmed a large difference in β's -180.77 ms for those individuals who exhibited a standard SNARC compared to a reverse SNARC effect. This difference between subgroups was significant ($t(24.22) = 6.81$, $p < .0001$).

Table 2. Correlations between VSWM, math measures and β slope.

	WRAT	Corsi Span	β Slope
WRAT	-		
Corsi Span	0.09	-	
β Slope	0.05	0.07	-

Note: $N = 28$, WRAT = Wide Range Achievement Test

https://doi.org/10.1371/journal.pone.0203019.t002

Standard SNARC/Reverse SNARC, VSWM and math ability

We compared the VSWM and WRAT math measures for individuals who exhibited a standard or a reverse SNARC effect. These analyses revealed no differences in either VSWM or math between the two groups. For the math task, an independent samples t-test showed participants in the standard SNARC subgroup ($M = 114.89$, $SD = 12.73$) scored 5.52 points higher on the WRAT on average than those in the reverse SNARC subgroup ($M = 109.38$, $SD = 18.72$). However, this difference was not significant ($t(25) = .89$, $p = .38$). For the VSWM task, an independent samples t-test showed little difference ($t(25) = .06$, $p = .96$) between participants in the standard SNARC subgroup ($M = 6.47$, $SD = 1.22$) and those in the reverse SNARC subgroup ($M = 6.5$, $SD = 0.93$).

Discussion

This study investigated whether a non-symbolic SNARC effect is evident in large magnitude dot arrays in a referent-to-target judgement paradigm. Three findings are worth noting. First, findings show non-symbolic magnitude is represented spatially, as evidenced by a non-symbolic SNARC effect. Presentation of smaller non-symbolic magnitudes (i.e., 15, 20, and 25) facilitated faster responses using the left hand, and larger non-symbolic magnitudes (i.e., 36, 45 and 60) facilitated faster responses using the right hand, demonstrating non-symbolic magnitude RTs is modulated by response hand. This finding confirmed those of Zhou et al. [25] showing a non-symbolic SNARC effect. In other words, a non-symbolic SNARC effect is evident in the so-called ANS range. A distance effect was also evident: participants were quicker responding to targets further away from the referent dot arrays. For dot arrays closer in magnitude to the referent, responses slowed.

Second, the form of the interaction in the non-symbolic SNARC effect showed a similar pattern to the symbolic SNARC effect (i.e., an interaction effect for non-symbolic magnitude and hand). The difference between left-hand and right-hand RT responses to larger stimuli was more pronounced than left and right-hand responses to smaller stimuli.

Third, only 64% of participants showed a SNARC effect. This finding is similar to those found in symbolic SNARC research, which show just over half of participants exhibited a SNARC effect [32–38]. Indeed, atypical differences in ANS processing abilities more generally are common [53–55].

Given the pattern of findings is similar to symbolic SNARC studies, what factor distinguishes individuals who exhibit a SNARC effect from those who do not? Does the failure to exhibit a SNARC effect, for example, indicate a deficit in number space representation that, in turn, affects numerical processing abilities? There are three aspects to this question (participants assessed herein showed variation in RTs and SNARC slope direction—see the Supplementary Materials section): to what degree are differences in SNARC abilities related to differences in (1) number-space representation, (2) stimuli parameters, and (3) cognitive and/ or number specific abilities. Answers to these questions may help clarify understanding of number-space relationships and the functional value of SNARC data more specifically.

The issue of how best to conceptualize number-space relationships has been raised by Leibovich, Katzin, Harel and Henik [56] who claim a sense of magnitude is the basis of math understanding. Specifically, they argue continuous magnitudes, such as size, area and density, are processed quicker and more accurately than number itself. In fact, they argue it is impossible to separate numerosity from continuous magnitudes in non-symbolic magnitude judgment tasks. Leibovich et al.'s claims are disputed, however. Burr [57], for example, provides evidence showing number strongly influences both area and density, and suggests number is a more fundamental attribute than continuous magnitudes. Similarly, Beran and Parrish [58] argued that although numerical and continuous magnitudes co-vary, continuous magnitudes alone do not influence responses. Indeed, Zhou et al., [25] showed a non-symbolic SNARC effect for non-symbolic numerical magnitudes, but not for continuous magnitudes (i.e., for area and density). These findings suggest it is unlikely a sense of magnitude alone is responsible for differences in non-symbolic SNARC abilities.

ANS theory (Dehaene, [59]) suggests the magnitude of numerosities is extracted independent of visual cues. In contrast, sensory integration theory (Gebuis, Kadosh & Gevers, [60]) proposes visual cues may affect magnitude judgment processes. Indeed, Leibovich et al., [56] argue that magnitude processing is affected by visual cues such as density, luminance and surface area. Since we did not control for low level visual cues, we are unable to draw conclusions about the role of these visual cues in the current study.

Individual differences in SNARC responses may be related to differences in how non-symbolic number is encoded and numerical decisions made. Odic [61] argues that non-symbolic numerical magnitude is encoded with the aid of low level congruent visual cues. For example, if density is manipulated it may influence perception of number because of shared visual cues; that is, numeric and non-numeric cues compete for decision components such as working memory. Odic's arguments are similar to those of Gebuis and Reynvoet [62] who suggest visual cues may affect non-symbolic magnitude abilities. Gebuis et al., [60] argue for a sensory integration theory in which the importance of visual cues is weighted and these weights affect magnitude judgements [63]. Insofar as the impact of visual cues can be demonstrated, they might explain reasons for individual differences in non-symbolic SNARC judgments.

Insofar SNARC abilities are related to magnitude representations more generally, research has found links with magnitude representation, cognitive and math abilities. A study by Cornu, Hornung, Schiltz and Martin [64] discovered spatial skills partially mediated the relationship between number line estimation and arithmetic in kindergarteners. Also, a model with control variables (i.e. age, gender), domain-general skills (i.e. verbal short-term memory and verbal intelligence), domain-specific skills (i.e. counting knowledge, Arabic number knowledge and quantitative knowledge) and spatial skills (i.e. spatial perception, visuo-motor integration and spatial visualisation) was a better predictor of number line estimation and arithmetic four months later than a similar model without spatial skills [64]. Lourenco, Aulet, Ayzenberg, Cheung and Holmes [65], found individual differences in the associations between number, area and duration at 9 months of age predicted similar associations at 3.5 years. This finding shows stable individual differences in number magnitude representations over time. Arguably, non-symbolic numerical magnitude abilities represent a way of acquiring the meaning of Arabic numbers symbols.

A large-scale longitudinal study by Tosto et al., [66] examined individual differences in number sense, its relation to math performance, and other general cognitive measures such as VSWM (Corsi), RTs (speed and accuracy), non-verbal intelligence (Raven's progressive matrices), vocabulary (Mill Hill Vocabulary) and reading comprehension. The aim of the study was to examine which measures were important, at what time during development, how strong the relationship was and to what degree these measures were moderated by other cognitive measures. To do this, Tosto et al. tested the estimation ability of 4,984 16-year-old students using number sense measures (i.e. a dot comparison and number line task). This data was compared to cognitive and achievement data from the same students at ages 7, 9, 10, 12 and 14. Results of the study showed dot comparison and number line tasks correlated with mathematics performance at age 16. However, the strength of the association between dot comparison, number line tasks and earlier math performance varied considerably across development and was moderated by differing cognitive abilities [66]. This shows both domain-general (quantitative) and domain-specific (working memory, spatial) skills are related to magnitude representation and development of math.

Individual differences in the symbolic SNARC effect and number-space mapping may be related to structural differences in the brain. For example, Krause, Lindemann, Toni and Bekkering [67] showed participants with a stronger SNARC effect had increased grey matter in the parietal area of the right precuneus. Amalric and Dehaene [68] showed bilateral activation of brain areas related to numbers and space, but little activation of language areas in expert mathematicians carrying out high level math reasoning. Amalric and Dehaene suggest this finding supports the claim that number-space mapping in early development is related to math achievement. A recent literature review by Myers, Carey and Szucs [69] on the neural correlates of spatial processing and WM in gifted children and adults showed these abilities contributed to being gifted at math.

Indeed, adults with developmental dyscalculia have shown deficits mapping number onto physical space [70]. Some children with visuo-spatial deficits and developmental dyscalculia show a reversed symbolic SNARC effect (suggesting a right-to-left mental number-line) and poorer math performance, compared to matched controls who show a standard SNARC effect [31].

Training in spatial associations improves acuity and math performance [71]. As far as we can ascertain, no studies have investigated the impact of magnitude representation training on changes in SNARC abilities. To more thoroughly investigate the significance of individual differences in SNARC and associated abilities, different analytic methods may be helpful. For example, Chew, Forte and Reeve [28] conducted a latent class analysis of symbolic and non-symbolic magnitude judgments, which revealed distinct and different patterns of judgments associated with math abilities.

Conclusion

The findings confirm a non-symbolic SNARC effect which, in turn, supports the claim that non-symbolic magnitude information is represented mentally in a left-to-right spatial manner. While the focus of the present study was non-symbolic representation, caution should be exercised in drawing parallels between factors that affect differences in non-symbolic and symbolic SNARC effects. It might be not only easier to differentiate 60 and 66 presented as Arabic symbols, compared to presented as dot arrays, but it seems likely different factors might affect the speed of judgments in the two representations [28]. Future researchers would do well to identify those factors that affect individual differences for both symbolic and non-symbolic SNARC judgments.

Supporting information

S1 File. Supplementary information.
(DOCX)

Author Contributions

Conceptualization: Fiona Nemeh, Judi Humberstone, Mark J. Yates, Robert A. Reeve.

Data curation: Judi Humberstone.

Formal analysis: Fiona Nemeh, Judi Humberstone.

Investigation: Fiona Nemeh.

Methodology: Fiona Nemeh, Judi Humberstone, Mark J. Yates, Robert A. Reeve.

Project administration: Fiona Nemeh.

Software: Mark J. Yates.

Supervision: Robert A. Reeve.

Writing – original draft: Fiona Nemeh.

Writing – review & editing: Fiona Nemeh, Robert A. Reeve.

References

1. Dehaene S, Bossini S, Giraux P. The mental representation of parity and number magnitude. Journal of Experimental Psychology: General. 1993; 122(3):371–96. https://doi.org/10.1037/0096-3445.122.3.371
2. Keus IM, Jenks KM, Schwarz W. Psychophysiological evidence that the SNARC effect has its functional locus in a response selection stage. Cognitive Brain Research. 2005; 24(1):48–56. https://doi.org/10.1016/j.cogbrainres.2004.12.005 PMID: 15922157
3. Gevers W, Lammertyn J, Notebaert W, Verguts T, Fias W. Automatic response activation of implicit spatial information: Evidence from the SNARC effect. Acta Psychologica. 2006; 122(3):221–33. https://doi.org/10.1016/j.actpsy.2005.11.004
4. Gevers W, Santens S, Dhooge E, Chen Q, Van den Bossche L, Fias W, et al. Verbal-spatial and visuo-spatial coding of number–space interactions. Journal of Experimental Psychology: General. 2010; 139 (1):180.
5. Fattorini E, Pinto M, Rotondaro F, Doricchi F. Perceiving numbers does not cause automatic shifts of spatial attention. Cortex. 2015; 73:298–316. https://doi.org/10.1016/j.cortex.2015.09.007 PMID: 26520681
6. Nuerk HC, Wood G, Willmes K. The universal SNARC effect—The association between number magnitude and space is amodal. Experimental Psychology. 2005; 52(3):187–94. https://doi.org/10.1027/1618-3169.52.3.187 PMID: 16076066
7. Reynvoet B, Brysbaert M. Single-digit and two-digit Arabic numerals address the same semantic number line. Cognition. 1999; 72(2):191–201. https://doi.org/10.1016/S0010-0277(99)00048-7 PMID: 10553671
8. Shaki S, Petrusic WM. On the mental representation of negative numbers: Context-dependent SNARC effects with comparative judgments. Psychon Bull Rev. 2005; 12(5):931–7. https://doi.org/10.3758/BF03196788 PMID: 16524013
9. Shaki S, Gevers W. Cultural Characteristics Dissociate Magnitude and Ordinal Information Processing. Journal of Cross-Cultural Psychology. 2011; 42(4):639–50. https://doi.org/10.1177/0022022111406100
10. Ren P, Nicholls ME, Ma YY, Chen L. Size matters: non-numerical magnitude affects the spatial coding of response. PLoS One. 2011 Aug 11; 6(8):e23553. https://doi.org/10.1371/journal.pone.0023553 PMID: 21853151
11. Shaki S, Petrusic WM, Leth-Steensen C. SNARC effects with numerical and non-numerical symbolic comparative judgments: Instructional and cultural dependencies. Journal of Experimental Psychology: Human Perception and Performance. 2012; 38(2):515–30. https://doi.org/10.1037/a0026729 PMID: 22288694
12. Rusconi E, Kwan B, Giordano BL, Umiltà C, Butterworth B. Spatial representation of pitch height: the SMARC effect. Cognition. 2006; 99(2):113–29. https://doi.org/10.1016/j.cognition.2005.01.004 PMID: 15925355
13. Nishimura A, Yokosawa K. Effects of laterality and pitch height of an auditory accessory stimulus on horizontal response selection: The Simon effect and the SMARC effect. Psychon Bull Rev. 2009; 16 (4):666–70. https://doi.org/10.3758/PBR.16.4.666 PMID: 19648450
14. Vicario C, Rumiati R. Left-right compatibility in the processing of trading verbs. Frontiers in Behavioral Neuroscience. 2014; 8(16). https://doi.org/10.3389/fnbeh.2014.00016 PMID: 24478662
15. Holmes KJ, Lourenco SF. Common spatial organization of number and emotional expression: A mental magnitude line. Brain and Cognition. 2011; 77(2):315–23. https://doi.org/10.1016/j.bandc.2011.07.002 PMID: 21839568
16. Macnamara A, Keage HAD, Loetscher T. Mapping of non-numerical domains on space: a systematic review and meta-analysis. Exp Brain Res. 2018; 236(2):335–46. https://doi.org/10.1007/s00221-017-5154-6 PMID: 29279982

17. Dehaene S, Piazza M, Pinel P, Cohen L. Three parietal circuits for number processing. Cognitive Neuropsychology. 2003; 20(3–6):487–506. https://doi.org/10.1080/02643290244000239 PMID: 20957581
18. Harvey BM, Klein BP, Petridou N, Dumoulin SO. Topographic Representation of Numerosity in the Human Parietal Cortex. Science. 2013; 341(6150):1123–6. https://doi.org/10.1126/science.1239052 PMID: 24009396
19. Bueti D, Walsh V. The parietal cortex and the representation of time, space, number and other magnitudes. Philosophical Transactions of the Royal Society B: Biological Sciences. 2009; 364(1525):1831–40. https://doi.org/10.1098/rstb.2009.0028 PMID: 19487186
20. Dehaene S, Brannon E, editors. Space, time and number in the brain: Searching for the foundations of mathematical thought. London: Academic Press; 2011.
21. Hubbard EM, Piazza M, Pinel P, Dehaene S. Interactions between number and space in parietal cortex. Nature Reviews Neuroscience. 2005; 6:435. https://doi.org/10.1038/nrn1684 PMID: 15928716
22. Walsh V. A theory of magnitude: common cortical metrics of time, space and quantity. Trends in Cognitive Sciences. 2003; 7(11):483–8. https://doi.org/10.1016/j.tics.2003.09.002 PMID: 14585444
23. Mitchell T, Bull R, Cleland AA. Implicit response-irrelevant number information triggers the SNARC effect: Evidence using a neural overlap paradigm. The Quarterly Journal of Experimental Psychology. 2012; 65(10):1945–61. https://doi.org/10.1080/17470218.2012.673631 PMID: 22524699
24. Patro K, Haman M. The spatial–numerical congruity effect in preschoolers. Journal of Experimental Child Psychology. 2012; 111(3):534–42. https://doi.org/10.1016/j.jecp.2011.09.006 PMID: 22153910
25. Zhou X, Shen C, Li L, Li D, Cui J. Mental Numerosity Line in the Human's Approximate Number System. Experimental psychology. 2016. https://doi.org/10.1027/1618-3169/a000324 PMID: 27404985
26. Hyde DC, Spelke ES. Neural signatures of number processing in human infants: evidence for two core systems underlying numerical cognition. Developmental science. 2011; 14(2):360–71. https://doi.org/10.1111/j.1467-7687.2010.00987.x PMID: 21399717
27. Feigenson L, Dehaene S, Spelke E. Core systems of number. Trends in Cognitive Sciences. 2004; 8(7):307–14. https://doi.org/10.1016/j.tics.2004.05.002 PMID: 15242690
28. Chew CS, Forte JD, Reeve RA. Cognitive factors affecting children's nonsymbolic and symbolic magnitude judgment abilities: A latent profile analysis. Journal of Experimental Child Psychology. 2016; 152:173–91. https://doi.org/10.1016/j.jecp.2016.07.001 PMID: 27560661
29. van Dijck J-P, Fias W. A working memory account for spatial–numerical associations. Cognition. 2011; 119(1):114–9. https://doi.org/10.1016/j.cognition.2010.12.013 PMID: 21262509
30. Crollen V, Noël M-P. Spatial and numerical processing in children with high and low visuospatial abilities. Journal of Experimental Child Psychology. 2015; 132:84–98. https://doi.org/10.1016/j.jecp.2014.12.006 PMID: 25618380
31. Bachot J, Gevers W., Fias W., & Roeyers H. Number sense in children with visuospatial disabilities: Orientation of the mental number line. Psychology science. 2005; 47(1):172. doi: 1854/LU-326765.
32. Beecham R, Reeve RA, Wilson SJ. Spatial Representations Are Specific to Different Domains of Knowledge. PLoS ONE. 2009; 4(5):e5543. https://doi.org/10.1371/journal.pone.0005543 PMID: 19461994
33. Cipora K, Nuerk H-C. Is the SNARC effect related to the level of mathematics? No systematic relationship observed despite more power, more repetitions, and more direct assessment of arithmetic skill. The Quarterly Journal of Experimental Psychology. 2013; 66(10):1974–91. https://doi.org/10.1080/17470218.2013.772215 PMID: 23473520
34. Cipora K, Hohol M, Nuerk H-C, Willmes K, Brożek B, Kucharzyk B, et al. Professional mathematicians differ from controls in their spatial-numerical associations. Psychological Research. 2016; 80(4):710–26. https://doi.org/10.1007/s00426-015-0677-6 PMID: 26063316
35. Fischer MH. The future for SNARC could be stark. . .. Cortex. 2006; 42(8):1066–8. https://doi.org/10.1016/S0010-9452(08)70218-1 PMID: 17209412
36. Hoffmann D, Mussolin C, Martin R, Schiltz C, Lappe M. The Impact of Mathematical Proficiency on the Number-Space Association. PLoS One. 2014; 9(1):e85048. https://doi.org/10.1371/journal.pone.0085048 PMID: 24416338
37. Viarouge A, Hubbard EM, McCandliss BD. The Cognitive Mechanisms of the SNARC Effect: An Individual Differences Approach. PLoS ONE. 2014; 9(4):1–10. https://doi.org/10.1371/journal.pone.0095756 PMID: 24760048
38. Wood G, Willmes K, Nuerk H-K, Fischer MH. On the cognitive link between space and number: A meta-analysis of the SNARC effect. Psychology science quarterly. 2008; 50(4):489.
39. Formoso J, Barreyro JP, Jacubovich S, Injoque-Ricle I. Possible Associations between Subitizing, Estimation and Visuospatial Working Memory (VSWM) in Children. The Spanish Journal of Psychology. 2017; 20. Epub 06/05. https://doi.org/10.1017/sjp.2017.23 PMID: 28578725

40. Chen Q, Li J. Association between individual differences in non-symbolic number acuity and math performance: A meta-analysis. Acta Psychologica. 2014; 148(0):163–72. https://doi.org/10.1016/j.actpsy.2014.01.016 PMID: 24583622
41. Mazzocco MM, Feigenson L, Halberda J. Impaired acuity of the approximate number system underlies mathematical learning disability (dyscalculia). Child development. 2011; 82(4):1224–37. https://doi.org/10.1111/j.1467-8624.2011.01608.x PMID: 21679173
42. Kessels RP, van Zandvoort MJ, Postma A, Kappelle LJ, de Haan EH. The Corsi Block-Tapping Task: standardization and normative data. Applied Neuropsychology. 2000; 7(4):252–8. https://doi.org/10.1207/S15324826AN0704_8 PMID: 11296689
43. Wilkinson GS, Robertson G. Wide Range Achievement Test 4 (WRAT4). Psychological Assessment Resources, Inc, Lutz, FL. 2006.
44. Oldfield RC. The assessment and analysis of handedness: the Edinburgh inventory. Neuropsychologia. 1971; 9(1):97–113. https://doi.org/10.1016/0028-3932(71)90067-4 PMID: 5146491
45. Fischer MH, Rottmann J. Do negative numbers have a place on the mental number line. Psychology Science. 2005; 47(1):22–32.
46. Schwarz W, Muller D. Spatial associations in number-related tasks—A comparison of manual and pedal responses. Experimental Psychology. 2006; 53(1):4–15. https://doi.org/10.1027/1618-3169.53.1.4 PMID: 16610269
47. Fias W, Brysbaert M, Geypens F, d'Ydewalle G. The Importance of Magnitude Information in Numerical Processing: Evidence from the SNARC Effect. Mathematical Cognition. 1996; 2(1):95–110. https://doi.org/10.1080/135467996387552
48. Fischer MH, Warlop N, Hill RL, Fias W. Oculomotor Bias Induced by Number Perception. Experimental Psychology. 2004; 51(2):91–7. https://doi.org/10.1027/1618-3169.51.2.91 PMID: 15114901
49. Schwarz W, Keus IM. Moving the eyes along the mental number line: Comparing SNARC effects with saccadic and manual responses. Perception & Psychophysics. 2004; 66(4):651–64. https://doi.org/10.3758/bf03194909.
50. Fischer MH, Mills RA, Shaki S. How to cook a SNARC: Number placement in text rapidly changes spatial–numerical associations. Brain and Cognition. 2010; 72(3):333–6. https://doi.org/10.1016/j.bandc.2009.10.010 PMID: 19917517
51. Gevers W, Gevers E, Ratinckx W, De Baene W, Fias. Further Evidence that the SNARC Effect is Processed Along a Dual-Route Architecture. Experimental psychology. 2006; 53(1):58–68. https://doi.org/10.1027/1618-3169.53.1.58 PMID: 16610273
52. Gibson L, Maurer D. Development of SNARC and distance effects and their relation to mathematical and visuospatial abilities. Journal of experimental child psychology. 2016; 150:301–13. https://doi.org/10.1016/j.jecp.2016.05.009 PMID: 27376924
53. Dowker A. Individual differences in numerical abilities in preschoolers. Developmental Science. 2008; 11(5):650–4. https://doi.org/10.1111/j.1467-7687.2008.00713.x PMID: 18801119
54. Gray SA, Reeve RA. Preschoolers' Dot Enumeration Abilities Are Markers of Their Arithmetic Competence. PLoS ONE. 2014; 9(4):e94428. https://doi.org/10.1371/journal.pone.0094428 PMID: 24714052
55. Paul JM, Reeve RA. Relationship between single digit addition strategies and working memory reflects general reasoning sophistication. Learning and Instruction. 2016; 42:113–22. https://doi.org/10.1016/j.learninstruc.2016.01.011
56. Leibovich T, Katzin N, Harel M, Henik A. From "sense of number" to "sense of magnitude": The role of continuous magnitudes in numerical cognition. Behavioral and Brain Sciences. 2017; 40. https://doi.org/10.1017/S0140525X16000960 PMID: 27530053
57. Burr DC. Evidence for a number sense. Behavioral and Brain Sciences. 2017; 40. Epub 07/27. https://doi.org/10.1017/S0140525X16002077 PMID: 29342625
58. Beran MJ, Parrish AE. The number sense is neither last resort nor of primary import. Behavioral and Brain Sciences. 2017;40. Epub 07/27. https://doi.org/10.1017/S0140525X16002065 PMID: 29342651
59. Dehaene S. The number sense: How the mind creates mathematics. New York: Oxford University Press; 1997.
60. Gebuis T, Kadosh RC, Gevers W. Sensory-integration system rather than approximate number system underlies numerosity processing: A critical review. Acta Psychologica. 2016; 171:17–35. https://doi.org/10.1016/j.actpsy.2016.09.003 PMID: 27640140
61. Odic D. The contributions of non-numeric dimensions to number encoding, representations, and decision-making factors. Behavioral and Brain Sciences. 2017; 40. Epub 07/27. https://doi.org/10.1017/S0140525X1600220X PMID: 29342632

62. Gebuis T, Reynvoet B. Generating nonsymbolic number stimuli. Behavior Research Methods. 2011; 43 (4):981–6. https://doi.org/10.3758/s13428-011-0097-5 PMID: 21512872

63. Gebuis T, Cohen Kadosh R, Gevers W. Why try saving the ANS? An alternative proposal. Behavioral and Brain Sciences. 2017; 40. Epub 07/27. https://doi.org/10.1017/S0140525X16002107 PMID: 29342626

64. Cornu V, Hornung C, Schiltz C, Martin R. Different aspects of spatial skills and their relation to early mathematics. Poster presented at: Workshop on Domain-General and Domain-Specific Foundations of Numerical and Arithmetic Processing; 2016 Sept 28–30, Tübingen, Germany. doi: 10993/28747

65. Lourenco SF, Aulet LS, Ayzenberg V, Cheung C-N, Holmes KJ. Right idea, wrong magnitude system. Behavioral and Brain Sciences. 2017; 40. Epub 07/27. https://doi.org/10.1017/S0140525X16002156 PMID: 29342631

66. Tosto MG, Petrill SA, Malykh S, Malki K, Haworth C, Mazzocco MM, et al. Number sense and mathematics: Which, when and how? Developmental psychology. 2017; 53(10):1924. https://doi.org/10.1037/dev0000331 PMID: 28758784

67. Krause F, Lindemann O, Toni I, Bekkering H. Different Brains Process Numbers Differently: Structural Bases of Individual Differences in Spatial and Nonspatial Number Representations. J Cogn Neurosci. 2014; 26(4):768–76. https://doi.org/10.1162/jocn_a_00518 PMID: 24188366

68. Amalric M, Dehaene S. Origins of the brain networks for advanced mathematics in expert mathematicians. Proceedings of the National Academy of Sciences. 2016; 113(18):4909–17. https://doi.org/10.1073/pnas.1603205113 PMID: 27071124

69. Myers T, Carey E, Szűcs D. Cognitive and Neural Correlates of Mathematical Giftedness in Adults and Children: A Review. Frontiers in Psychology. 2017; 8(1646). https://doi.org/10.3389/fpsyg.2017.01646 PMID: 29118725

70. Huber S, Sury D, Moeller K, Rubinsten O, Nuerk H-C. A general number-to-space mapping deficit in developmental dyscalculia. Research in developmental disabilities. 2015; 43:32–42. https://doi.org/10.1016/j.ridd.2015.06.003 PMID: 26151441

71. Sella F, Tressoldi P, Lucangeli D, Zorzi M. Training numerical skills with the adaptive videogame "The Number Race": A randomized controlled trial on preschoolers. Trends in Neuroscience and Education. 2016; 5(1):20–9. https://doi.org/10.1016/j.tine.2016.02.002

3.3 Supplementary Information: Individual Participants' dRTs

The regression slope for each participant was examined separately. Figure 3 shows dRTs plotted for each participant who met the 50% correct accuracy threshold for any given non-symbolic quantity. Large individual differences can be seen by the slope of the regression lines. Some participants' regression lines slope downward indicating a negative slope value and a standard SNARC-like effect or upward indicating a positive slope value and a reversed SNARC. A flat line with a slope value of 0 indicates no SNARC-like effect.

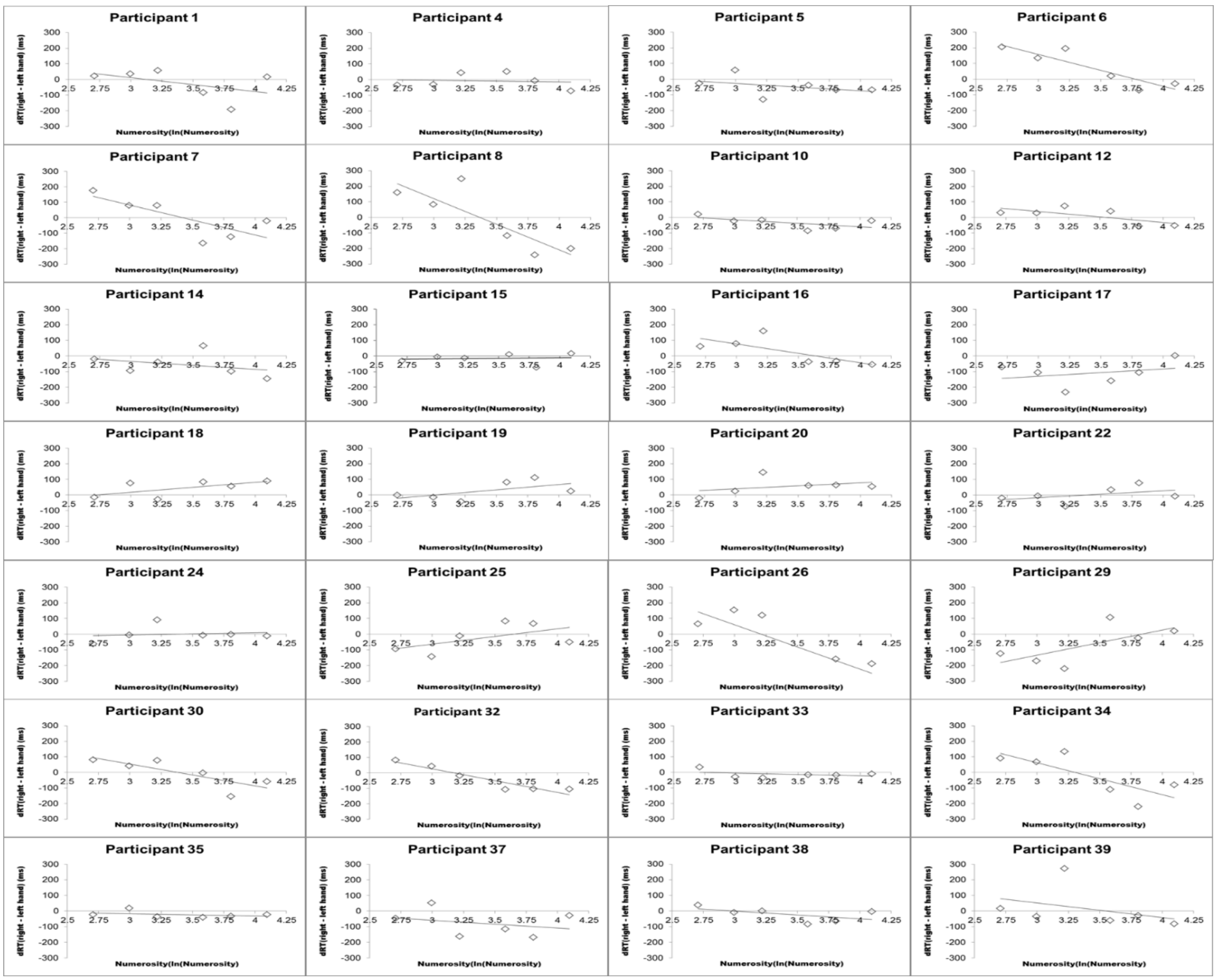

Figure 3. Individual differences of the non-symbolic SNARC-like effect for participants depicting observed data and the regression line, reflecting response time differences for right and left hand as a function of non-symbolic quantity.

Note. The x-axis value for each dot point represents the natural logarithm of the non-symbolic quantity (i.e. 2.70 = ln(15), 3.00= ln(20), 3.22 = ln(25), 3.58 = ln(36), 3.81= ln(45), 4.09 = ln(60). When the gradient of the regression line slopes downward this indicates a non-symbolic SNARC-like effect; an upwards gradient indicates a reversed non-symbolic SNARC-like effect.

Chapter 4: Studies 2A, 2B, and 2C

Chapter 4 will present three studies that examine the relationship between ratio of stimuli, continuous magnitudes, modality of response, and their contributions to the non-symbolic SNARC-like effect. In Chapter 3 the non-symbolic SNARC-like effect was examined using a non-symbolic quantity judgment task. The rationale for using a non-symbolic quantity judgment task was presented in Chapter 2. A brief justification for the first experiment will be given, followed by the aims then details of the study.

Study 1, reported in the last chapter, was designed to examine if non-symbolic quantities were represented spatially, from left-to right side space. The findings from the first study provide some support for the claim that non-symbolic quantities are represented spatially as evidenced by a non-symbolic SNARC-like effect. Furthermore, this was shown for non-symbolic quantities purely in the ANS range. However, Experiment 1 consisted of stimuli with three differing ratios of referent-to-target 0.5 (15 and 60 dots), 0.66 (20 and 45 dots) and 0.83 (25 and 36 dots) so difficulty of numerical judgments varied. The level of difficulty of the numerical judgment increased the closer the target was in numerical distance from the referent. For example, targets that were numerically closer to the referent were the most difficult to judge. Indeed, we found for the 0.83 ratio there seemed to be a calibration issue with some participants unable to respond above chance level (50%). Therefore, several participants were removed from further analysis. For data that remained, wide variance was found for both RT and accuracy. Findings from Study 1 raise the question if a SNARC effect would be found for more participants when reducing difficulty of the task. Compared to the symbolic SNARC, there are several methodological issues when testing for the non-symbolic SNARC which have been described in Chapter 2 but will be reiterated below as they form the rationale for the current study.

4.1 Current Evidence for a Non-symbolic SNARC-like Effect

The first issue is, compared to the many studies showing support for a symbolic SNARC effect, to date, few studies report a non-symbolic SNARC-like effect (Bulf et al., 2014; Eberbach, et al., 2014; Mitchell et al., 2012; Nuerk et al., 2005; Patro & Haman, 2012; Zhou et al., 2016). The non-symbolic SNARC-like effect has not been a strong research focus; and it is unclear why this is the case. However, Mitchell et al., (2012) suggest there may be a publication bias for significant results. Few studies have found a significant non-symbolic SNARC-like effect. There are even less studies that test for a non-symbolic SNARC-like effect and report a null result. For instance, to our knowledge, only two recent papers by Cleland and colleagues (Cleland & Bull, 2018; Cleland, Corsico, White, & Bull, in press) have failed to show a non-symbolic SNARC-like effect for numerical magnitudes using implicit judgments. Cleland and Bull (2018) used a colour judgment task for both symbolic (Arabic digits from 1 – 9 which were blue or green) and non-symbolic stimuli (arrays consisting of 1 - 9 circles which were black, blue or green). A SNARC effect was found for symbolic number but not for non-symbolic stimuli (even when stimuli in the subitizing range were examined separately). Cleland and Bull (2018) argue perhaps non-symbolic quantity representations for implicit judgments do not activate a direct route to a verbal label such as 'more' or 'less'. A verbal label may be necessary for triggering a SNARC effect and an easy route to a verbal label is often found for symbolic SNARC studies (Cleland & Bull, 2018).

Cleland and colleagues (Cleland et al., in press) examined a SNARC effect for non-symbolic stimuli across a series of experiments using implicit judgments. Stimuli consisted of 1 – 9 black triangles. Participants had to make an orientation judgment and low-level visual cues were manipulated. In Experiments 1 and 2, total surface area was held constant but in Experiment 3, total surface area varied and increased in lockstep with numerical magnitude.

In Experiment 1 four different variants of each array were presented to each participant. In Experiment 2 each participant was assigned one of four stimuli to control for low level visual cues (i.e., each participant was presented with only one version of an array). This methodology may have increased variance among participants as they were not viewing the same stimuli. Another potential issue with Cleland and colleagues' (in press) study design was that hand response were counterbalanced across participants. That is, half of the participants responded with their left hand to an upright triangle and right for an inverse triangle. The other half responded with their right hand to an upright triangle and their left hand to an inverse triangle. Inverted shapes may have had longer response times. Experiment 2 was an attempt to replicate Mitchell et al.'s (2012) study, where a non-symbolic SNARC-like effect was found for orientation judgments. However, no SNARC-like effect was found for any of the experiments reported in Cleland et al.'s paper, even when the subitizing range was examined separately Therefore, the non-symbolic SNARC-like effect from Mitchell et al., (2012) was not replicated. Taken together, the studies by Cleland and Bull (2018) and Cleland et al. suggest the non-symbolic SNARC-like effect for non-symbolic quantities may be less robust than the symbolic SNARC effect – at least for implicit judgments. In Study 1 (Chapter 3) explicit non-symbolic quantities and verbal labels were used in a non-symbolic quantity judgment task. Perhaps this is the reason why a non-symbolic SNARC-like effect was found. The current study will use this same methodology to test for a non-symbolic quantity SNARC-like effect.

4.2 SNARC Studies Were Designed for Symbolic Number

The second issue is that the SNARC effect, and paradigms used to identify this behavioural effect, were initially designed for symbolic number. As previously discussed in Chapter 2, there is much evidence to suggest a relationship between number and space. However, most of this evidence is based on the interaction between symbolic number and space. Claims about the mental number line and symbolic SNARC are largely based on symbolic stimuli. Several issues remain when designing studies to examine the non-symbolic SNARC-like effect for non-symbolic quantities.

For instance, the most common paradigm used to examine the symbolic SNARC effect is the parity judgment task (judging if a presented stimulus is odd or even). However, parity judgment relies on exact processing of number and numerical magnitudes in the ANS range are not precise. Therefore, parity judgment tasks arguably are not suited for non-symbolic quantity judgment studies. There are however a few exceptions such as when stimuli presented are in the subitizing range (which can be processed quickly and precisely) or a participant is allowed enough time to enumerate the dots. Another paradigm used to test the symbolic SNARC effect is magnitude comparison. This is suitable to test for a non-symbolic SNARC-like effect and will be used in the current study.

A third issue is that while research has examined non-symbolic quantity judgment abilities (using Weber fraction and the numerical distance measures), the focus has been on acuity rather than the spatial representation of non-symbolic quantities per se. ANS acuity measures such as the Weber fraction ignore reaction time (RT) measures and as such may be affected by a speed accuracy trade off (Park & Starns 2015). The symbolic SNARC effect is determined by using RT measures (mean or median). Accuracy is also considered (speed accuracy trade-off and inverse efficiency RT/accuracy). Both RTs and accuracy will be measured in the current study.

Finally, there are issues of measurement when using non-symbolic quantities particularly in the ANS range. Some questions that arise are, how do you assign numbers to the stimuli and what is the theoretical basis of assigning numbers? What number of dots should be chosen for comparative arrays? There is little previous research regarding these factors.

In Study 1, if the ratio of referent-to-target was too difficult, participants were unable to meet accuracy thresholds and perform above chance level. In the current study this issue was addressed by using an easy to discriminate ratio of 0.5 (15 and 60 dots). This ratio elicited a non-symbolic SNARC-like effect in Study 1.

4.3 Study 2A

In the current study there are two research questions of interest: Are non-symbolic quantities represented spatially and what effect will reducing variance of the target stimuli have on the non-symbolic SNARC-like effect? The most robust finding in number-space interactions is the symbolic SNARC effect (see Gevers & Lammertyn, 2005 for a review) suggesting symbolic number is spatially represented. However, there is still little empirical evidence for non-symbolic quantities (i.e., dot clouds). The aim of Study 2A is to determine whether there is a left-to-right spatial representation of magnitude (for numbers of dots that cannot be exactly enumerated). This will be tested using a SNARC paradigm in the form of non-symbolic quantity judgment tasks.

The rationale for the design of the current study was to reduce task difficulty and maximise the chance of finding a SNARC effect for all participants' variance of stimuli. In the current study, the 0.5 ratio was chosen as this ratio had previously elicited a non-symbolic SNARC-like effect. Indeed, in Study 1 the largest SNARC effect was found for the easiest ratio to process (0.5- 15 and 60 dots) according to the Weber fraction. In the current study, target stimuli remained the same (dot arrays of 15 and 60 dots) and the referent consisted of a dot array containing 30 dots.

Based on previous findings of the symbolic SNARC effect and Study 1, it was predicted that if spatial-numerical associations extend to non-symbolic quantities, then: Faster responses are predicted for 'congruent' stimulus-response pairings (i.e. when 'lesser' responses are assigned to the left hand, and 'more' responses to the right hand) compared to 'incongruent' pairings (lesser/right and more/left). For example, left hand responses to 15 dot arrays should be faster than right hand responses to 15 dot arrays. Similarly, right hand responses to 60 dot arrays should be faster than left hand responses to 60 dot arrays. It is expected that a magnitude by response side interaction should be found.

In Study 1 reported in Chapter 3, neither VSWM nor math ability predicted the non-symbolic SNARC-like effect. Although slight differences were found for the standard and reverse SNARC group for VSWM and math ability, these differences failed to reach significance. In the current study, a more sensitive measure of VSWM – the Corsi block tapping task (backwards span) (Corsi, 1972; Kessels, van Zandvoort, Postma, Kappelle, & de Haan, 2000) was included. Again, we predicted the absence of a SNARC-like effect may be related to poorer VSWM and math ability.

4.3.1 Method

4.3.1.1 Participants

Twenty-six undergraduate students participated for course credit. All met an accuracy threshold of 50% (performing less than at chance). A criterion set in Study 1 for inclusion for further analysis. All participants (13 males, 13 females) were aged between 18 and 46 (*M* = 20.85, *SD* = 5.50) and were strongly right-handed according to the Edinburgh Handedness Inventory questionnaire (Oldfield, 1971; see Appendix A). All participants gave written, informed consent before participating in the study. The study was conducted in accordance with the Declaration of Helsinki and was approved by the Human Research Ethics Committee at the Melbourne School of Psychological Sciences (# 1237507.2). Note, that for all studies in the thesis, the sample sizes were based on participants that were available for recruitment from the first-year psychology student pool. A study by Cipora and Wood, (2017) showed that 20 participants and 20 trials per stimuli are sufficient to detect a SNARC effect. A repeated-measures design was used to increase power. Also, G*Power (Faul, Erdfelder, Lang, & Buchner, 2007) was used to check if a study had sufficient power.

4.3.1.2 Materials, procedure and analytic method

Materials, procedure and analytic method were identical to Study 1 except: (1) the Corsi backward span task (Corsi, 1972; Kessels et al., 2000) was added; (2) only two target stimuli were used (15 and 60 dots); and (3) right minus left-hand RT difference was calculated. The rationale for adding right-left RT difference was because there were only two target stimuli; hence, only two data points were included in the analysis. Unless explicitly stated all ANOVA and t-test results are reported two-tailed. For more in-depth discussion regarding the materials, procedure, and analytic method please refer to Study 1 (Chapter 3).

4.3.2 Results

The median RTs and accuracy for Quantity × Hand judgments are shown in Table 4.1 below. The overall mean RT for the experiment was 411 milliseconds.

Table 4.1

Medians and SE's of RTs and Means and SD's of Accuracy in Study 2A

		Left Hand				Right Hand			
Number of Dots	**Test to Referent**	**RTs (ms)**		**Accuracy**					
		Mdn	***SE***	***M***	***SD***	***Mdn***	***SE***	***M***	***SD***
15	0.5 x Ref	417.92	13.40	0.99	0.01	426.33	16.21	0.99	0.02
60	2 x Ref	406.83	16.02	0.99	0.02	391.33	12.34	0.99	0.01

4.3.2.1 SNARC-like effect for non-symbolic quantities

Initial analysis showed no significant error rate interaction between quantity and hand ($F(1,25) = .02$, $p = .90$, $\eta^2 = .00$) (i.e., no evidence of a speed-accuracy trade off: The mean error rate was 1.06%). We conducted a repeated-measures ANOVA to determine whether a SNARC-like effect was evident for non-symbolic quantities. The analysis comprised a 2 × Response Key Position (left- or right-side response box) × 2 Direction of the Quantity/Referent Relationship (smaller or larger than the 30-dot referent array). The analysis revealed that Hand was not significant (F (1, 25) = .00, $p = .98$). However, Quantity was significant (F (1, 25) = 13.74, $p < .01$, $\eta^2 = .36$). Faster responses were found for larger quantities. We also found a significant interaction between Hand and Quantity (F (1, 25) = 3.96, $p = .03$ (one-tailed), $\eta^2 = .14$), which supports the existence of a non-symbolic SNARC-like effect. See figure 4.1 below for the Hand by Quantity interaction effect. Figure 4.1 below shows the form of the non-symbolic SNARC-like effect is similar to the symbolic SNARC effect.

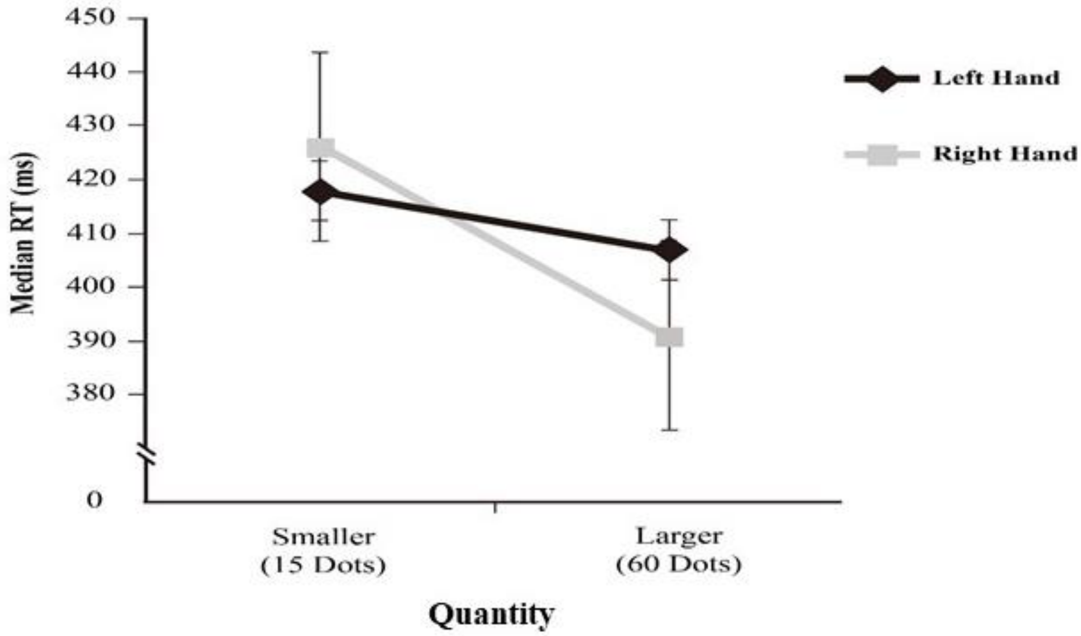

Figure 4.1. Median RTs in Study 2A comparing 15 vs 60 dots to the 30-dot referent as a function of left vs right hand (error bars = standard error).

4.3.2.2 SNARC, VSWM and math abilities

The WRAT[2] standardised score ranged from 98 to 143 with a mean of 118.23. This showed above average mathematics ability for the sample according to WRAT norms. VSWM- Corsi Blocks Forward span ranged from 5 to 8 with a mean of 6.38. Corsi Blocks Backward span ranged from 4 to 8 with a mean of 5.77. The average test norm for Corsi span is 6. The Corsi total score ranged from 72 to 184 with a mean of 113. This placed the sample in the 90th percentile according to test norms. Correlation coefficients for the WRAT, Corsi span, slope and right - left difference RTs are reported in Table 4.2 below. Table 4.2 shows there was a significant correlation between VSWM (Corsi backwards span task/ total score) and RT (right-left difference score). There was also a correlation between math ability (WRAT) and VSWM (Corsi total score). For right minus left difference RTs, participants with a negative score had a higher Corsi backwards span score. So, a larger right-left RT difference was associated with better VSWM memory. There was also a correlation between right-left RT difference and slope.

[2] The Wide Range Achievement Test 4 (WRAT) (Wilkinson & Robertson, 2006).

Table 4.2

Correlations between VSWM, Math Measures, Slope and Right-Left Difference in Study 2A

	WRAT	**Corsi Span (Forwards)**	**Corsi Span (Backwards)**	**Corsi Total**	**Right-Left Difference**	**Slope**
WRAT	-					
Corsi Span F	0.098	-				
Corsi Span B	0.049	0.608**	-			
Corsi Total	0.371*	0.715**	0.734**	-		
R - L Difference	-0.014	0.313	0.467*	0.174	-	
Slope	0.157	-0.234	-0.373*	-0.113	-0.899**	-

Note: * = *p* < .05, ** = *p* <.01. *N*= 26, WRAT = Wide Range Achievement Test, Corsi Span F = Corsi Forwards, Corsi Span B = Corsi Backwards, Corsi Total = Mean total of Corsi Forwards and Corsi Backwards, R - L Difference = RT difference for Right – Left Hand, Slope = Regression slope β value.

Inspection of participants' performances showed that a small number of individuals (*n* = 9) exhibited a reverse non-symbolic SNARC-like effect, with an average regression slope β value of 30.86, indicating a positive slope which significantly differed from 0, ($t(8) = .4.53$, $p = .002$). Participants who exhibited a standard SNARC effect (*n* = 17), had an average regression slope β value of -52.33. This indicated a negative slope that significantly differed from 0, ($t(16) = 4.33$, $p = .001$). An independent samples *t*-test confirmed a large difference in β's -83.18 ms for those individuals who exhibited a standard SNARC compared to a reverse SNARC effect. This difference between subgroups was significant ($t(23.11) = 5.99$, $p <.0001$).

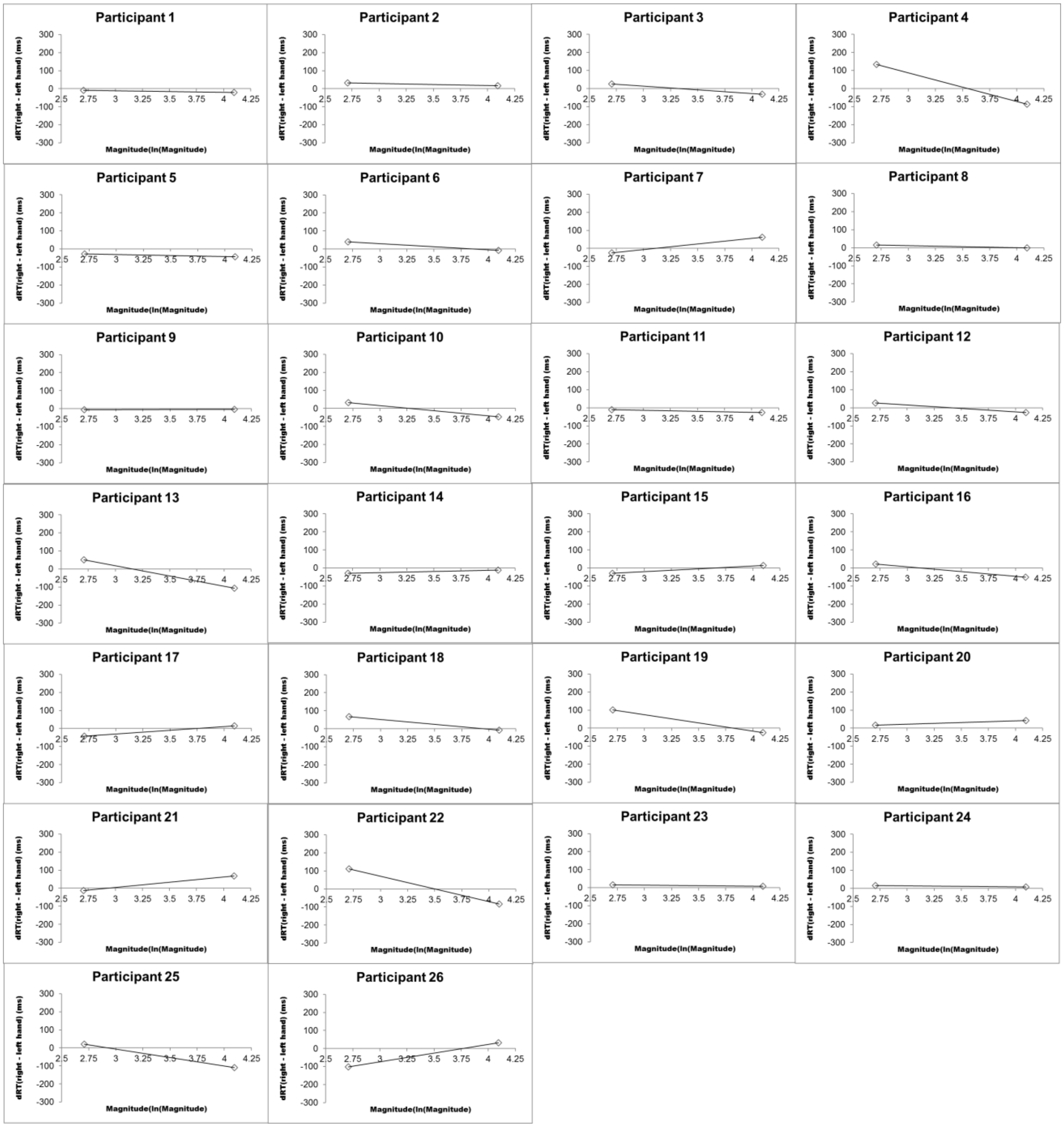

Figure 4.2. Individual differences of the non-symbolic SNARC-like effect for participants depicting observed data and the regression line in Study 2A, reflecting response time differences for right and left hand as a function of non-symbolic quantity.

Note. The x-axis value for each dot point represents the natural logarithm of the non-symbolic magnitude (i.e., 2.70 = ln (15), and 4.09 = ln (60). When the gradient of the regression line slopes downward this indicates a SNARC-like effect. When the gradient of the regression line slopes upwards this indicates a reversed SNARC-like effect.

4.3.2.3 Individual participant dRTs

The regression slope for each participant was examined separately. Figure 4.2 shows dRTs plotted for each participant. Large individual differences can be seen.

4.3.2.4 Standard SNARC/reverse SNARC, VSWM and math ability

We compared the VSWM and WRAT math measures for individuals who exhibited a standard or a reverse SNARC effect. These analyses revealed no differences in either VSWM or math between the two groups. For the math task, an independent samples *t*-test showed participants in the standard SNARC subgroup (M = 116.24, SD = 13.69) scored 5.76 points lower on the WRAT on average than those in the reverse SNARC subgroup (M = 122.00, SD = 13.42). However, this difference was not significant ($t(24) = 1.03$, $p = .776$). For the VSWM task, Corsi forwards span, an independent samples *t*-test showed little difference ($t(24) = .54$, $p = .59$) between participants in the standard SNARC subgroup (M = 6.47, SD = 1.07) and those in the reverse SNARC subgroup (M = 6.22, SD = 1.20). For the VSWM task, Corsi backwards span, an independent samples *t*-test showed little difference ($t(24) = 1.56$, $p = .13$) between participants in the standard SNARC subgroup (M = 6.06, SD = 1.30) and those in the reverse SNARC subgroup (M = 5.22, SD = 1.30). For the VSWM task, Corsi total score, an independent samples *t*-test showed participants in the standard SNARC subgroup (M = 116.74, SD = 33.77) scored 15.91 points higher on the Corsi total score on average than those in the reverse SNARC subgroup (M = 100.83, SD = 46.29). However, this difference was not significant ($t(24) = 1.01$, $p = .33$).

4.3.3 Discussion

The current study was designed to examine if a SNARC-like effect would be found for more participants using an easy judgment based on the Weber fraction. Thereby, reducing previous issues of discrimination and calibration found in Study 1. Again, non-symbolic quantities purely in the ANS range were used. Two target stimuli with the same ratio (0.5 - 15 and 60 dots) to the referent (30 dots) were presented.

There were five main findings. First, the hypothesis that non-symbolic quantity is represented spatially analogous to a left-to-right mental number line was supported. This was evidenced by a non-symbolic SNARC-like effect. With one lower bound (15 dot) and one upper bound (60 dot) array, participants were faster to respond with their left hand to smaller numerical magnitudes than their right hand. Likewise, participants were faster to respond to their right hand to larger numerical magnitudes than with their left hand. Second, the form of the interaction effect for non-symbolic quantities was like that of the symbolic SNARC effect. Third, there appeared to be no issue of calibration. All the participants were able to complete the task with a high degree of accuracy. Fourth, only 65% of participants showed a non-symbolic SNARC-like effect. This was in line with previous symbolic SNARC studies that show not all participants exhibit a SNARC effect (Beecham et al., 2009; Wood et al., 2008). This finding suggests there are individual differences in the spatial representation of non-symbolic quantities. Fifth, a moderate positive relationship was found between RT (Right – Left Difference) and VSWM (Corsi). Participants who showed a larger negative score Right – Left Difference score showed a larger non-symbolic SNARC-like effect and better VSWM performance as measured by the Corsi. Also, a weak positive relationship was found between VSWM (Corsi) and math ability (WRAT standardised score). Participants who showed higher VSWM performance (Corsi total score) performed better in the math task (WRAT). Overall, the findings confirm a non-symbolic SNARC-like effect which provides support for the claim that non-symbolic quantities are represented spatially from smallest to largest, left-to-right.

4.3.4 Conclusion

The current experiment provided evidence for the claim that non-symbolic quantities in the ANS range are represented spatially. A faster response was found with the left hand to smaller numerical magnitudes than the referent and with the right hand for larger numerical magnitudes than the referent. This indicated a non-symbolic SNARC-like effect was found, which was consistent with the findings of Study 1. As only two target stimuli were used which were easy to discriminate based on the Weber fraction, there was no issue of calibration when making numerical judgments. Overall, the findings provide support for the spatial representation of non-symbolic quantities from left-to-right.

4.4 Study 2B

In Study 2A a non-symbolic quantity judgment task triggered a SNARC-like effect. This result was consistent previous studies which found a non-symbolic SNARC-like effect (Mitchell et al., 2011; Nuerk et al., 2005; Zhou et al., 2016). In Study 2A one lower bound (15 dots) and one upper bound (60 dots) target stimuli were used to reduce the amount of variance and address the issue of calibration found in Study 1. The ratio of target to referent was 0.5. However, with the symbolic SNARC effect, stimuli that take longer to processes as indicated by slower RTs tend to elicit a stronger SNARC effect (Wood et al., 2008). Therefore, it was of interest to examine if a stronger SNARC-like effect would be found with stimuli that were more difficult to process based on the Weber fraction. Targets closer to the referent were chosen from stimuli previously used in Study 1. A lower bound target of 20 dots and an upper bound target of 45 dots were compared to a referent of 30 dots. This represented a ratio of 0.66. If luminance was driving the SNARC effect found in Study 1 and 2a, it was expected that no SNARC effect would be found in the current study where luminance was controlled for.

4.4.1 Method

4.4.1.1 Participants

Thirty-nine students participated for course credit. All met an accuracy threshold of 50% (performing less than at chance). A criterion set in Study 1 for inclusion for further analysis. All participants (14 males, 25 females) were aged between 18 and 35 years (*M*=20.74, *SD*= 4.13) were strongly right-handed according to the Edinburgh Handedness Inventory questionnaire (Oldfield, 1971; Appendix A). All participants gave written, informed consent before participating in the study. The study was conducted in accordance with the Declaration of Helsinki and was approved by the Human Research Ethics Committee at the Melbourne School of Psychological Sciences (# 1237507.2).

4.4.1.2 Apparatus, stimuli, design, and procedure

The design, procedure, stimuli and analytic approach were identical to Studies 1 and 2A, except that a ratio was chosen that was slightly more difficult to discriminate according to the Weber fraction than the ration used in Study 2A. A ratio of 0.66 was used. This consisted of a referent stimulus of 30 dots and a target of 20 or 45 dots. To collect more data for each given numerical magnitude more trials (384 in all) were conducted. This consisted of 192 trials with small left and large right and 192 with the opposite hand assignment.

Equal numbers of black and white dots intermixed in the dot display, whereby black dots represented a luminance *increment* from the grey background and white dots a *decrement* from the grey background such that overall mean luminance was balanced at 75 cd/m² with 80% Michelson contrast. That is, the luminance *increment* and *decrement* were offset. Thereby, overall luminance was the same across stimuli. Luminance of the grey background was set at 75 cd/m².

4.4.2 Results

The median RTs and accuracy for Magnitude × Hand judgments are shown in Table 4.3. The overall mean RT for the experiment was 518 milliseconds.

Table 4.3
Medians and SE's of RTs and Means and SD's of Accuracy in Study 2B

		Left Hand				**Right Hand**			
Number of Dots	**Test to Referent**	**RTs (ms)**		**Accuracy**		**RTs (ms)**		**Accuracy**	
		Mdn	***SE***	***M***	***SD***	***Mdn***	***SE***	***M***	***SD***
20	1/1.50 x Ref	530.00	11.97	0.96	0.05	515.00	14.64	0.96	0.04
45	1.5 x Ref	521.50	15.71	0.96	0.03	481.50	12.87	0.97	0.03

4.4.2.1 SNARC-like effect for non-symbolic quantity

Initial analysis showed no significant error rate interaction between quantity and hand ($F(1,38) = 2.43$, $p = .13$) (i.e., no evidence of a speed-accuracy trade off: The mean error rate was 3.5 %). We conducted a repeated-measures ANOVA to determine whether a SNARC-like effect was evident for non-symbolic quantities. The analysis comprised a 2 × Response Key Position (left or right-side response box) × 2 Direction of the Quantity/Referent Relationship (smaller or larger than the 30-dot referent array). The analysis revealed that Hand was not significant (F (1, 38) = 2.58, p =.14). However, Quantity was significant (F (1, 38) = 23.34, $p < .0001$, $\eta^2 = .38$). Faster responses were found for larger quantities. We also found a significant interaction between Hand and Quantity (F (1, 38) = 7.32, p = .01, $\eta^2 = .16$), which supports the existence of a non-symbolic SNARC-like effect. Figure 4.3 shows the form of the non-symbolic SNARC-like effect is similar to the symbolic SNARC effect for larger but not for smaller quantities. To test if a non-symbolic SNARC-like effect was found overall, the combined slope coefficients for all participants were analysed to determine the slope differed from a 0 slope (overall: M = -47.77, *SD* = 108.94), *t*(38) = 2.74, *p* = .001; see figure 4.4 below). This provided further support for a non-symbolic SNARC-like effect.

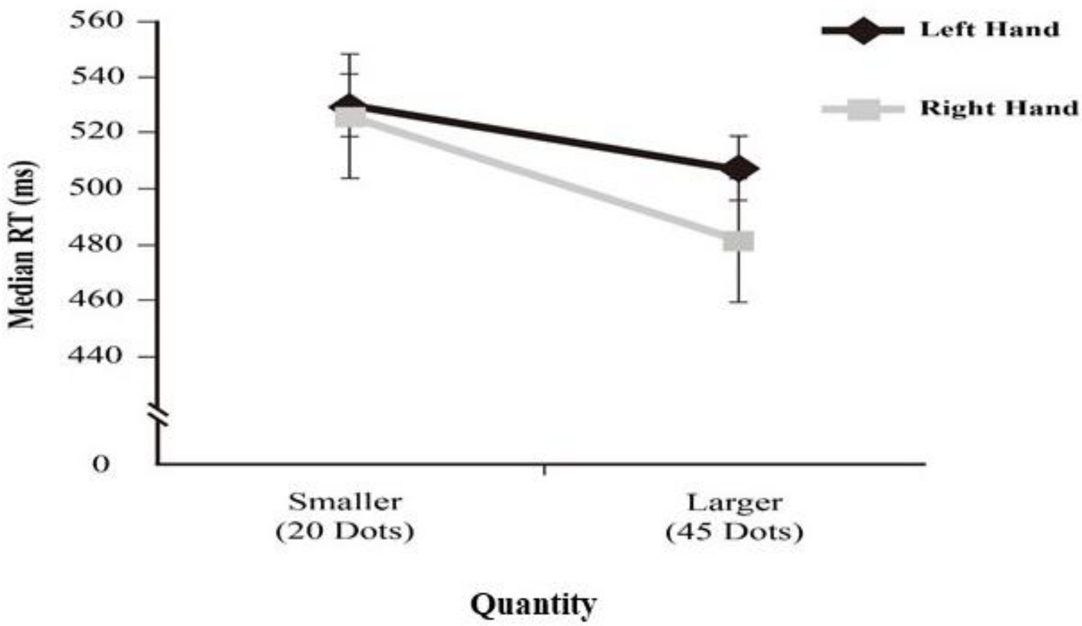

Figure 4.3. Median RTs in Study 2B comparing 20 and 45 dots to the 30-dot referent as a function of left vs right hand (error bars = standard error).

4.4.2.2 SNARC, VSWM, and Math Abilities

The WRAT standardised score ranged from 55 to 143 with a mean of 113.49. This showed above average mathematics ability for the sample according to WRAT norms. For VSWM- Corsi Blocks Forward span ranged from 3 to 8 with a mean of 5.72. Corsi Blocks Backward span ranged from 4 to 7 with a mean of 5.28. The average test norm for Corsi span is 6. The Corsi total score ranged from 47 to 167 with a mean of 90.22. This placed the sample around the 85th percentile according to test norms. Correlation coefficients for the WRAT, Corsi span, slope and right – left difference RTs are reported in Table 4.4 below. Table 4.4 shows a weak positive correlation between math ability (WRAT) and VSWM (Corsi forward span). This shows participants with higher scores on the WRAT math tasks tended to show slightly better VSWM performance on average as measured by the Corsi forward span task. There was also a correlation between slope and right – left RT difference.

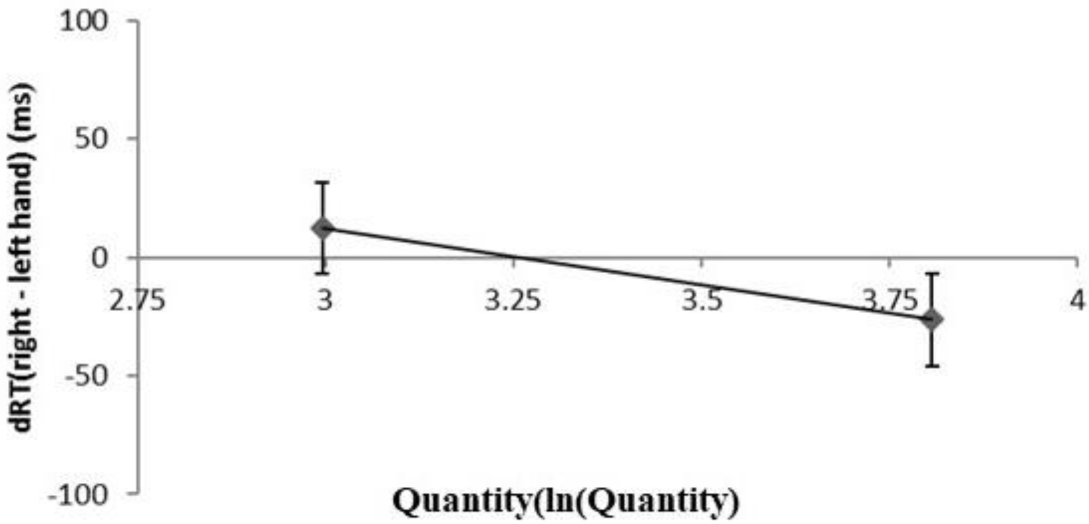

Figure 4.4. Regression line for RT differences between right and left hand in Study 2B.

The x-axis value for each dot point represents the natural logarithm of the non-symbolic magnitude (i.e. 3.00= ln(20), 3.81= ln(45). Positive values show faster left hand than right hand responses.

Inspection of participants' performances showed that a small number of individuals (n = 13) exhibited a reverse non-symbolic SNARC-like effect, with an average regression slope β value of 51.49, indicating a positive slope which significantly differed from 0, ($t(12) = 7.62$, p <.0001). Participants who exhibited a standard SNARC effect (n = 26), had an average regression slope β value of -97.40. This indicated a negative slope which significantly differed from 0, ($t(25) = 4.95$, p <.0001). An independent samples t-test confirmed a large difference in β's -148.89 ms for those individuals who exhibited a standard SNARC compared to a reverse SNARC effect. This difference between subgroups was significant ($t(30.36) = 7.16$, $p < .0001$). Levene's test was significant ($p = .03$). Therefore, equal variances not assumed is reported.

Table 4.4

Correlations between VSWM, Math Measures, Slope and Right- Left Difference in Study 2B

	WRAT	**Corsi Span (Forwards)**	**Corsi Span (Backwards)**	**Corsi Total**	**Right – Left Difference**	**Slope**
WRAT	-					
Corsi Span F	0.35*	-				
Corsi Span B	-0.08	0.46**	-			
Corsi Total	0.10	0.80**	0.85**	-		
R - L Difference	-0.05	0.08	0.09	0.08	-	
Slope	-0.017	-0.055	-0.153	-0.100	-.838**	-

Note: * = $p < .05$, ** = $p <.01$. N= 26, WRAT = Wide Range Achievement Test, Corsi Span F = Corsi Forwards, Corsi Span B = Corsi Backwards, R - L Difference = RT difference for Right – Left Hand, Slope = Regression slope β value.

4.4.2.3 Standard SNARC/Reverse SNARC, VSWM and Math Ability

VSWM and WRAT math measures were compared for individuals who exhibited a standard or a reverse SNARC effect. These analyses revealed a difference in VSWM (Corsi total score) between the two groups. No difference was found between the two groups for the math task. For the math task, an independent samples *t*-test showed participants in the standard SNARC subgroup (M = 115.54, SD = 11.75) scored 6.15 points lower on the WRAT on average than those in the reverse SNARC subgroup (M = 109.38, SD = 22.07). However, this difference was not significant ($t(37) = 1.14$, $p = .26$). For the VSWM task, Corsi forwards span, an independent samples *t*-test showed little difference ($t(37) = 1.54$, $p = .13$ between participants in the standard SNARC subgroup (M = 5.88, SD = .86) and those in the reverse SNARC subgroup (M = 5.38, SD = 1.12). For the VSWM task, Corsi backwards span, an independent samples *t*-test showed little difference ($t(37) = 2.50$, $p = .02$) between participants in the standard SNARC subgroup (M = 5.62, SD = 1.24) and those in the reverse SNARC subgroup (M = 4.62, SD = 1.04). For the VSWM task, Corsi total score, an independent samples *t*-test showed participants in the standard SNARC subgroup (M = 97.40, SD = 30.12) scored 21.56 points higher on the Corsi total score on average than those in the reverse SNARC subgroup (M = 75.85, SD = 26.84). This difference was significant ($t(37) = 2.18$, $p = .04$).

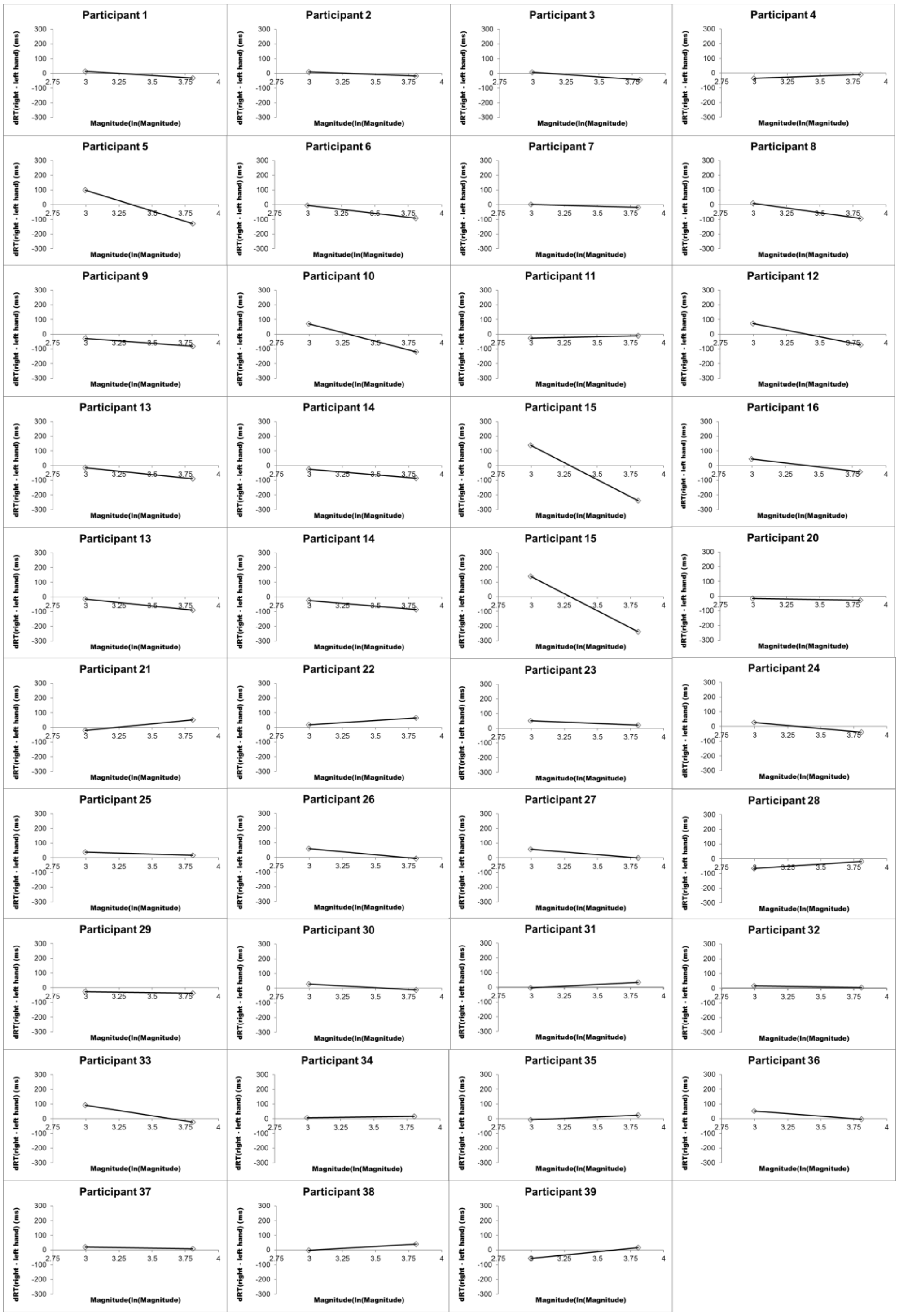

Figure 4.5. Individual differences of the non-symbolic SNARC-like effect for participants in Study 2B depicting observed data and the regression line, reflecting response time differences for right and left hand as a function of non-symbolic quantity.

Note. The x-axis value for each dot point represents the natural logarithm of the non-symbolic quantity (i.e. 3.00 = ln(20), and 3.80 = ln(45). When the gradient of the regression line slopes downward this indicates a SNARC-like effect. When the gradient of the regression line slopes upwards this indicates a reversed SNARC-like effect.

4.4.2.4 Individual participant dRTs

The regression slope for each participant was examined separately. Figure 4.5 above shows dRTs plotted for each participant. Large individual differences can be seen.

4.4.3 Discussion

In the current study, a non-symbolic SNARC-like effect was found using a target-to-referent quantity judgment paradigm and stimuli with a ratio of 0.66 (20 and 45 dots). As predicted, targets larger in quantity than the referent (45 dots) facilitated faster responses for the right hand. However, the presentation of a smaller quantity target than the referent (20 dots) did not facilitate a faster response for the left hand. It is possible this was because 20 dots were not "small" enough to elicit an effect. However, a significant ANOVA interaction between Hand and Quantity and linear regression slope t-test results significantly differ from zero and provide support a non-symbolic SNARC-like effect for Study 2B. This suggested that non-symbolic quantity is represented spatially in a left-to-right manner. Only two target stimuli were used. One lower (20 dots) and one upper bound. (45 dots). These target stimuli were slightly more difficult to discriminate (a ratio of 0.66) based on the Weber fraction compared to Study 2A (a ratio of 0.5). Again, like the pattern of data found in Study 2A, there appeared to be no issue of calibration. In the current experiment, participants were able to discriminate the target magnitude from the referent with a high degree of accuracy. In-line with symbolic SNARC literature (Wood et al., 2008), not all participants showed a non-symbolic SNARC-like effect. Only 67% of participants showed a standard SNARC-like effect as indicated by a negative beta value. The remaining participants showed a reverse SNARC-like effect as indicated by a positive beta value.

4.4.3.1 Was a stronger SNARC-like effect found for target quantities closer to the referent?

As predicted, longer processing times were found for targets closer in non-symbolic quantity to the referent. This resulted in slower overall RTs. In the current experiment, the overall mean RT was 518 ms (0.66 ratio- 20 and 45 dots). This was ~ 100 ms slower than Study 2A (0.5 ratio – 15 and 60 dots), with an overall mean RT of 411 ms, and suggests participants found the 0.66 ratio more difficult to process. As a different group of participants completed each experiment, another possibility is this group of participants were just slower to respond on average. A stronger SNARC-like effect for the current experiment was supported by a larger negative beta value of the regression slope (M = -47.77, p = .001) compared to Study 2A (M = -

32.81, p = .024). However, an independent samples t-test of individual slopes obtained from Study 2A and 2B showed no significant difference $t(63) = 1.04$, p = .303.The non-symbolic SNARC-like effect showed similar response characteristics to the symbolic SNARC effect (Wood et al., 2008). By using stimuli that were more difficult to process based on the Weber fraction, longer processing times were elicited and a stronger SNARC-like effect was found. Another possibility is that by using a mixture of black and white dots participants found the task more difficult and this led to longer reaction times and a stronger SNARC-like effect.

4.4.3.2 VSWM, math ability and RT difference

A relationship between math ability (WRAT) and VSWM (Corsi forward span) was found as indicated by a weak positive correlation. Participants with higher scores on the WRAT math task showed better VSWM performance on average, as measured by the Corsi forward span task. When comparing performance for the standard SNARC and reverse SNARC groups, participants in the standard SNARC group showed better VSWM performance on average than those in the reverse SNARC group as indicated by the Corsi Total Score. This finding is consistent with other research that shows a relationship between VSWM and various aspects of quantity processing (Reeve, Reynolds, Paul & Butterworth, 2018)

4.4.4 Conclusion

The current experiment provided further evidence for the claim that non-symbolic quantities in the ANS range are represented spatially. Faster responses were found with the left hand to smaller quantities than the referent and for the right hand for larger quantities than the referent. This indicated a non-symbolic SNARC-like effect and was consistent with the results of Studies 1 and 2A. Only two target stimuli (20 and 45 dots) were used which were easy to discriminate based on the Weber fraction but more difficult than Study 2A (a ratio of 0.5). There was no issue with calibration when making numerical judgments. In the current experiment, a stronger SNARC-like effect was found for target quantities numerically closer to the referent (ratio of 0.66) compared to Study 2A (ratio of 0.5, 15 and 60 dots). Thereby, a stronger SNARC-like effect was elicited than in Study 2A and the issue of calibration was overcome. Overall, the findings provide support for the spatial representation of non-symbolic quantities from smallest-to-largest and left-to-right.

Study 2B revealed a SNARC-like effect for non-symbolic quantities. However, in Study 2B total surface area co-varied with magnitude. As quantity increased so did total surface area. Therefore, the non-symbolic SNARC-like effect found in Study 2B may have been due to total dot surface area rather than quantity alone. Total surface area can influence quantity judgments (Gebuis & Reynvoet, 2011; Zhou et al., 2016). In Study 2B, because total surface area increased as a function of quantity it was necessary to run a further experiment to try and disentangle these factors

In the symbolic SNARC literature a SNARC effect has been shown for other effectors such as eyes (i.e. saccades) or foot responses. The current study did not examine if a non-symbolic SNARC-like effect occurs for other effectors such as feet. Moreover, the current study did not control for low level cues of total surface area. These low-level visual cues have been found to be represented spatially too. Although results of Studies 1, 2A, and 2B appear to be in line with previous studies (Nuerk et al., 2005) they must be interpreted cautiously as stimulus quantity co-varied with surface area. That is, as quantity increased so did surface area. This confound of total surface area was addressed in the next experiment. In Study 2C, a study is presented that examines a non-symbolic SNARC-like effect for other effectors, namely, foot responses.

4.5 Study 2C

In Studies 1, 2A, and 2B a non-symbolic quantity judgment task triggered a SNARC-like effect, – i.e., when participants were presented with two dot cloud stimuli in sequence and asked to judge whether the second stimulus contained more or lesser dots than the first, faster responses were obtained when ‘less’ responses were assigned to the left hand and ‘more’ responses to the right hand compared to the alternative response assignment. This finding suggests that non-symbolic quantities may also be represented spatially and was consistent with the results of Nuerk, et al. (2005) who found a non-symbolic SNARC-like effect. However, this finding does not explain whether the non-symbolic SNARC-like effect reflects a genuine spatial representation of number or is an over-learned stimulus-response effect. Spatial processing is often thought to have unique “innate” cognitive qualities. These are not new claims, of course (see de Hevia et al., 2014 Gallistel & Gelman, 1992; Gibson & Maurer, 2016; Mandler & Shebo, 1982;). The thrust of these arguments is that “space and/or geometry” have unique cognitive properties (affordances) that contribute to learning/processing of space/quantity relationships.

Below, two key accounts of the SNARC effect will be briefly touched on as these provide a rationale for the current study. These will be followed by the research questions and aims. It has been claimed acquisition of the MNL and thereby the symbolic SNARC effect are influenced by culture, specifically, reading direction (Dehaene & Cohen, 1997). In Western cultures or those that read from left-to-right a standard SNARC effect is found (i.e. small/left, large/right).

Schwarz and Keus (2004) wanted to examine Dehaene and Cohen’s theory (Dehaene & Cohen, 1997) that the symbolic SNARC effect occurs due to a match of the side of response and the internal representation of that number (i.e. the mental number line). This is a spatial-numerical account of the SNARC effect. Schwarz & Keus, (2004) proposed an alternate account of the symbolic SNARC effect. This was termed the manual association hypothesis (Schwarz & Keus, 2004) and posits the symbolic SNARC effect is an overlearned number- hand motor response and is thereby not spatial. For example, when learning to type on a computer keyboard 1-5 is typed with the left hand and 6- 9 with the right hand. Thereby, an automatic association has been learned for number and hand over many repetitions. Schwarz and Keus (2004) argued that if a SNARC effect was found for saccades this would support the spatial account of the SNARC. Across two experiments a SNARC effect was found for both hand responses and saccades providing further evidence for the spatial representation of symbolic number. However, Schwarz and Keus acknowledged that movements of hands and eyes have been shown to be highly correlated (Milner & Goodale, 1995). Also, both hands (writing) and eyes (reading) are involved in written language. So, Schwartz and Keus (2004) were unable to conclude if the SNARC effect for saccades reflected an internal spatial representation of number.

In a later study, Schwarz and Müller (2006) examined if a SNARC effect could be found for foot responses. Schwarz and Müller reasoned that as feet were not involved in written language there was no reason foot responses should show a SNARC effect. This would provide more conclusive evidence the SNARC reflected the spatial representation of symbolic number. A symbolic SNARC effect was found for hand and foot responses providing evidence for the mental number line theory of the SNARC effect rather than an overlearned stimulus-response effect.

4.5.1 The Current Study

As previously mentioned, although initial research on the symbolic SNARC effect has been with manual responses (i.e. hands). Subsequent research by Schwarz and colleagues has demonstrated analogous effects when judgments were indicated via foot pedal responses (Schwarz & Müller, 2006) or leftward/rightward saccades (Schwarz & Keus, 2004), demonstrating that the symbolic SNARC effect is independent of effector modality. Two questions are of interest are: Is the non-symbolic SNARC-like effect specific to hand responses or does it generalise to other response modalities such as feet? And, what is the influence of total surface area on the non-symbolic SNARC-like effect? To examine these questions, foot pedal responses were used instead of hand responses and total surface area of visual stimuli was controlled for.

The current study had two aims. The first was to see if the non-symbolic SNARC-like effect transfers to other effector modalities. Symbolic SNARC studies have shown an effect for foot responses (Schwarz & Müller, 2006). Based on the symbolic SNARC effect, it was predicted faster responses would be found with the left foot to smaller numerical magnitudes and with the right foot to larger numerical magnitudes.

Based on previous findings by Schwarz and Keus (2004) and Schwarz and Müller (2006) we predicted a non-symbolic SNARC-like effect would be found for foot responses. In the current study, in line with Schwarz and Keus (2004) if the manual association hypothesis was true, it was expected that the size of the SNARC effect would be reduced or no SNARC would be found for foot responses as there is no reason foot responses should be associated with numerical magnitudes. This outcome would also support the ontogenetic account of the SNARC effect

(Schwarz & Müller, 2006). Conversely, if the MNL hypothesis was true we would see a similar size non-symbolic SNARC-like effect for foot responses as we had for hand responses in Study 1. This outcome would also show support for the phylogenetic account of the SNARC effect (Fisher, 2003). The rationale on adding three different ratios was to see how participants would respond to these using foot responses.

The second aim was to investigate the effect of total surface area on the non-symbolic SNARC-like effect. To investigate this, total surface area of the dots was controlled so that each differing magnitude had the same total surface area. This was achieved by smaller quantities having larger dot sizes. Hence, the size of the dots increases as quantity decreases. If the SNARC-like effect found in previous experiments was attributed to surface area alone, we predicted that in the current experiment after controlling for total surface area either no SNARC-like effect or a reduced effect would be found. Again, overall luminance/contrast was controlled for.

4.5.2 Method

4.5.2.1 Participants

Eighteen undergraduate students participated in exchange for course credit. The complete data set was excluded for 1 participant who failed to reach an accuracy threshold of 50% (performing less than at chance) for a given magnitude. All the remaining participants (4 males, 14 females) aged between 18 and 31 (M = 21.17, SD = 3.82) were strongly right-handed according to the Edinburgh Handedness Inventory questionnaire (Oldfield, 1971; Appendix A). All participants gave written, informed consent before participating in the study. The study was conducted in accordance with the Declaration of Helsinki and was approved by the Human Research Ethics Committee at the Melbourne School of Psychological Sciences (# 1237507.2)

4.5.2.2 Design, apparatus and procedure

Design, apparatus and procedure and analytic approach were identical to Study 1 with the use of a within subject design with multiple ratios (see p. 48 for more in depth information regarding this), except for, half of the dots in the dots arrays used in the current experiment were black (hue: 160, saturation: 0, luminance: 56), and the other half white (hue: 160, saturation: 0, luminance: 184) to control for overall luminance/contrast of display. Total surface area was controlled for. Note, that the reporting of the experiments in the thesis are not in the chronological order of data collection. The Corsi block tapping task (VSWM and the WRAT-4 (math performance) were added later on as additional factors to consider once we found an initial effect. Therefore, in the current experiment, neither the WRAT-4 (math performance) nor the Corsi block tapping task (VSWM) were administered. Responses were made with left or right foot pedal presses rather than hand responses. See figure 4.6 below for a diagram of the experimental set up.

4.5.2.3 Stimuli

The total area covered by the dot surfaces remained constant across the six quantities (15, 20, 25, 36, 45 and 60). This resulted in a change in dot size across the quantities. See Figure 4.7 below for sample stimuli. In order to achieve this, the number of pixels per individual dot decreased as a function of increasing quantity. For example, the 15 dot stimuli contained 120 pixels per dot whereas the 60 dot stimuli contained 30 pixels per dot. Thus, each image had the same number of pixels, being 1800 in total (see Table 4.5 below).

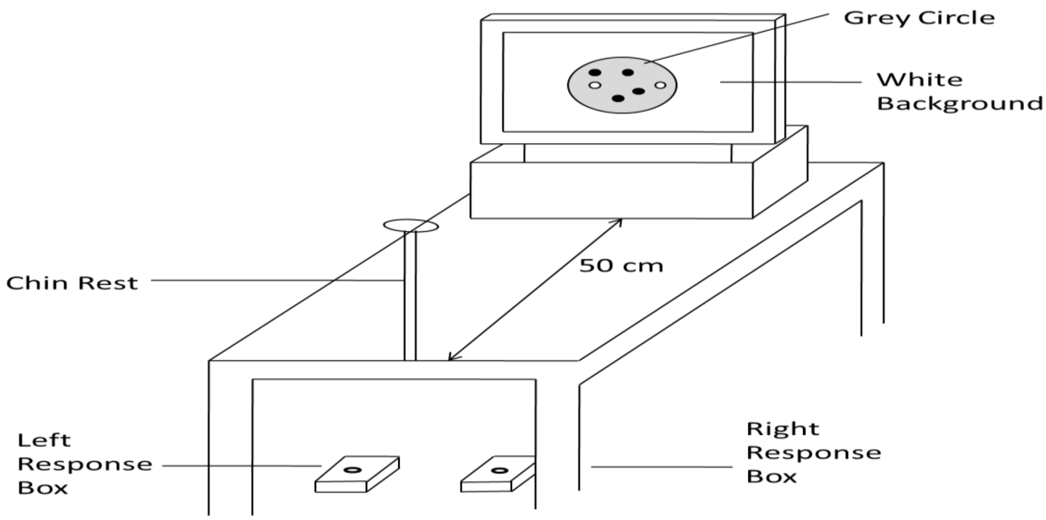

Figure 4.6. Schematic diagram of the apparatus and stimuli in Study 2C. Diagram is not drawn to scale.

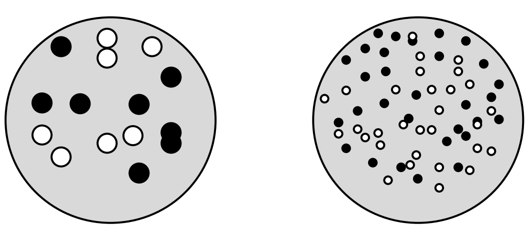

Figure 4.7. Dot pattern stimuli, 15 vs 60 for Study 2C. Stimuli are not drawn to scale.

Table 4.5
Experimental Stimuli for Study 2C

Number of Dots	**Stimuli Type**	**Ratio of Test Quantity to Referent Quantity**	**Total Dot Surface Area (Total Number of Pixels Occupied by Dots)**
15	Test	1/2 x Ref	1800
20	Test	1/1.5 x Ref	1800
25	Test	1/1.2 x Ref	1800
30	Referent	Referent	1800
36	Test	1.2 x Ref	1800
45	Test	1.5 x Ref	1800
60	Test	2 x Ref	1800

4.5.3 Results

The median RTs and accuracy for Quantity × Foot judgments are shown in Table 4.6 below. Table 4.6 reveals that as targets get closer in magnitude to the referent responses tend to slow and are less accurate. The overall mean RT for the experiment was 628 milliseconds.

Table 4.6
Medians and SE's of RTs and Means and SD's of Accuracy as a Function of Quantity

Ratio	**Dot Quantity**	**Target to Referent**	**Left Foot**				**Right Foot**			
			RTs (ms)		**Accuracy**		**RTs (ms)**		**Accuracy**	
			Mdn	***SE***	***M***	***SD***	***Mdn***	***SE***	***M***	***SD***
0.50	15	0.5 x Ref	611.25	14.51	0.99	0.03	593.75	24.76	0.99	0.03
0.66	20	1/1.5 x Ref	645.50	18.01	0.95	0.07	680.75	24.43	0.96	0.07
0.83	25	1/1.2 x Ref	719.75	23.29	0.79	0.13	709.00	29.93	0.87	0.11
0.83	36	1.2 x Ref	654.00	20.05	0.97	0.05	605.50	20.40	0.98	0.03
0.66	45	1.5 x Ref	605.50	21.34	1.00	0.00	563.75	19.75	0.99	0.02
0.50	60	2 x Ref	600.25	20.56	1.00	0.01	560.75	18.62	0.99	0.02

4.5.3.1 SNARC-like effect for non-symbolic quantities

Analysis showed no error rate interaction between quantity and foot (F (1, 17) = .4.56, p = .05) (i.e., no evidence of a speed-accuracy trade-off). The mean error rate was 4%. To examine if a SNARC-like effect was present for non-symbolic quantities, we ran a repeated-measures ANOVA with Greenhouse-Geisser correction. The factors we examined were 2 x Response Key Position (left or right-side response box) × 2 Direction of the Quantity/Referent Relationship (smaller or larger than the 30-dot referent array) × 3 Ratio (0.5, 0.66, 0.83). The ANOVA revealed that for main effects, Foot was significant (F(1, 17) = 4.76, p = .04, η^2 = .22). Quantity was significant (F(1,17) = 19.30, p <.0001, η^2 = .53) as was Ratio (F(1.94, 33.00) = 61.47, p <.0001, η^2 = .78). This suggested there was a difference in responses for small and large quantities and for differing ratios. A significant interaction was found for Foot by Quantity (F(1,17) = 3.85, p = .07 or p = .04 (one-tailed), η^2 = .19). This provided support for a SNARC-like effect. Quantity by Ratio was significant (F(1.69, 28.74) = 6.93, p = .01, η^2 = .29). However, neither Foot by Ratio (F(1.63, 27.77) = 0.21, p = .77) nor the three-way interaction for Foot by Quantity by Ratio were significant (F(1.50, 25.56) = .03, p = .94). See figure 4.8 below for the Foot by Quantity interaction effect for the three separate ratios (0.5, 0.66 and 0.83). Figure 4.8 shows the form of the non-symbolic SNARC-like effect differs for each ratio but is similar to the symbolic SNARC effect for the 0.5 and 0.66 ratios.

This supported the pattern of results shown in Figure 4.8 and suggested the SNARC-like effect differed as a function of Ratio. Since the main effect for Ratio was significant, we investigated the difference in ratios. RTs and accuracy differed for each ratio. Targets that were closer in quantity to the referent were judged more slowly and less accurately.

We conducted post-hoc tests with Bonferroni correction. For main effect of Foot, responses made with the right foot were faster on average (16 ms) than those made with the left foot. For Quantity, targets that were larger in quantity than the referent were responded to more quickly on average (58 ms) than targets than were smaller in quantity than the referent. For Ratio, targets that were furthest in quantity from the referent (a ratio of 0.5) were responded to 29 ms faster than the next ratio closer to the referent (0.66) and 82 ms faster than the ratio closest to the referent (0.83). This result suggested that differences were found for Ratio so separate ANOVAs were run for each ratio.

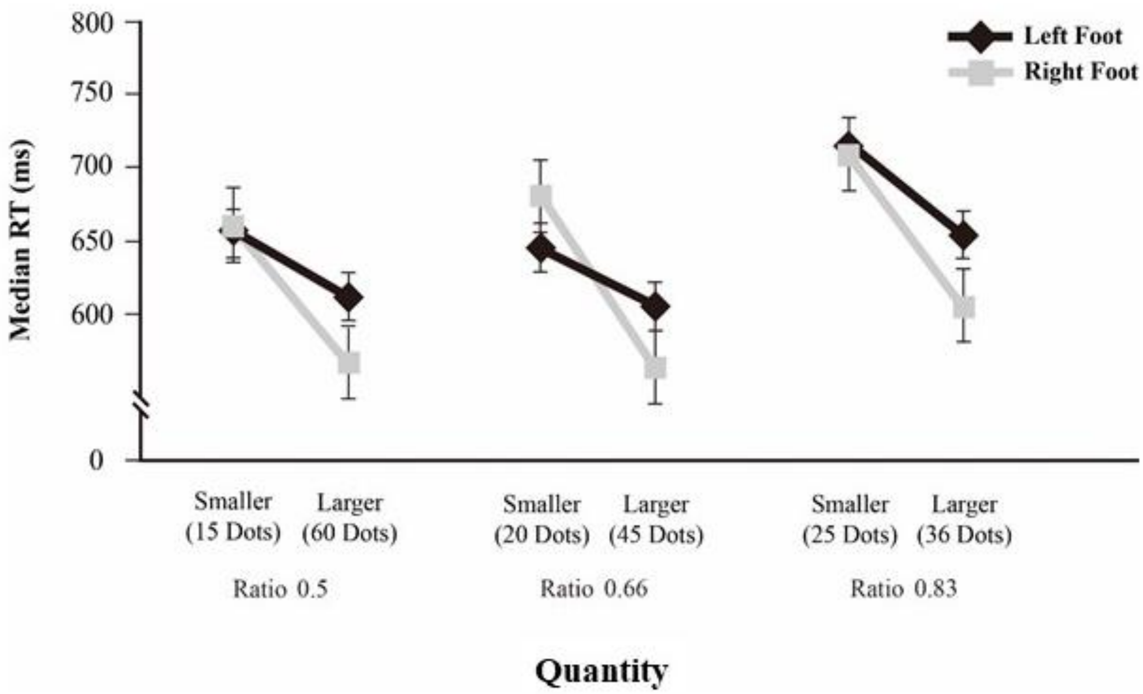

Figure 4.8. Median RTs in Study 2C comparing 15 vs 60, 20 vs 45 and 25 vs 36 dots to the 30-dot referent as a function of left vs right foot (error bars = standard error).

4.5.3.2 Ratio and the SNARC-like effect

Three separate 2 x 2 ANOVAs assessed the relationship between response foot (left foot vs right foot) and non-symbolic quantity RTs for differing Ratios (0.5, 0.66 and 0.83 respectively). The first analysis (0.5 Ratio) showed neither a Quantity effect ($F(1,17) = 3.57$, $p = 0.08$) nor an effect for Foot ($F(1,17) = 4.44$, $p = 0.05$, $\eta^2 = 0.21$) but a marginally significant effect for the Quantity and Foot Interaction ($F(1,17) = 3.26$, $p = .09$ (two-tailed) or $p = 0.05$ (one-tailed), $\eta^2 = 0.16$). Simple effects were tested for the two-way interaction. Left foot responses showed no difference for small or large quantities ($F(1,17) = .04$, $p = 0.85$). By contrast, the right foot was faster to respond (a 44 ms advantage for right foot responses) to larger than smaller quantities ($F(1,17) = 10.40$, $p = 0.01$, $\eta^2 = 0.38$).

The second analysis (0.66 Ratio) showed no Quantity effect ($F(1,17) = 3.57$, $p = 0.08$), nor an effect of Foot– ($F(1,17) = 4.44$, p = 0.05). However, there was a significant interaction effect for Quantity and Foot ($F(1,17) = 3.26$, $p = 0.04$, $\eta^2 = 0.16$). Simple effects were tested for the two-way interaction. Left foot responses showed no difference for small or large quantities ($F(1,17) = .16$, $p = .69$). The right foot was faster to respond (on average, a 43 ms advantage for right foot responses) to larger than smaller quantities ($F(1,17) = 8.94$, $p = 0.01$, $\eta^2 = 0.35$).

The third analysis (0.83 Ratio) showed a Quantity effect (F(1,17) = 16.68 , p = 0.00 , η^2 = 0.50 – 36 dots were responded to faster than 25 dots), but no effect of Foot (F(1,17) = 0.66 , p = .43). The interaction between Quantity and Foot was not significant (F(1,17)= 1.88, p = 0.19 (two-tailed) or 0.1 one-tailed).

Following the convention in symbolic SNARC studies, we collapsed data across small (15, 20, 25) and large (36, 45, 60) quantities. Results still showed support for a non-symbolic SNARC-like effect. Foot was significant (F(1,17)= 4.76 , p = 0.04, η^2 = .22 - the right foot was faster than the left foot) as were Quantity (F(1, 17) = 19.23 , p <. 0001, η^2 = .53 – larger magnitudes were responded to faster than smaller magnitudes) and the Foot by Quantity Interaction was significant (F(1,17)= 3.85, p = . 04, η^2 = .19).

4.5.3.3 Standard SNARC-like / reverse SNARC-like effect

Combined slope coefficients for all participants were analysed to determine whether the slope differed from a 0 slope (overall: M = -49.52, SD = 101.78), t(17) = 2.06, p = .05 (two-tailed) or p = .03 one-tailed) (see Figure 4.9).

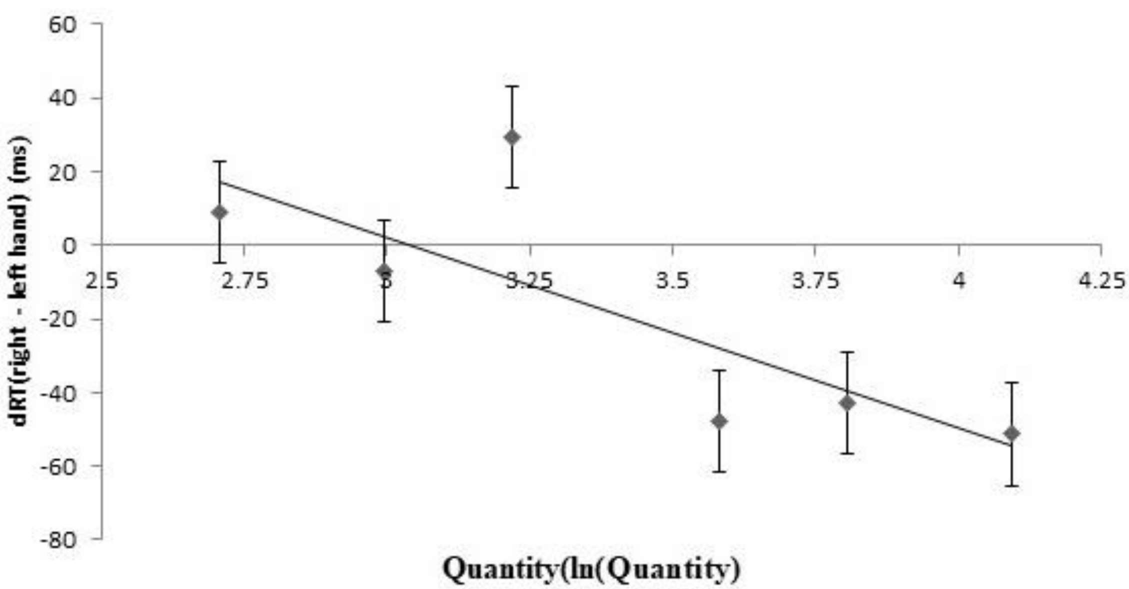

Figure 4.9. Regression line for RT differences between right and left foot in Study 2C.

Note. The x-axis value for each dot point represents the natural logarithm of the non-symbolic quantity (i.e. 2.70 = ln(15), 3.00= ln(20), 3.22 = ln(25), 3.58 = ln(36), 3.81= ln(45), 4.09 = ln(60). Positive values show faster left foot than right foot responses.

To assess subgroup membership, an independent samples *t*-test showed a significant difference for β. Participants in the standard SNARC subgroup $n = 12$ ($M = -106.30$, $SD = 68.32$) showed a large difference in β -170.33 ms compared to those in the reverse SNARC subgroup $n = 6$ ($M = 64.03$, $SD = 41.70$). This difference was significant ($t(16) = 5.56$, $p < .0001$). We split the file and ran a single sample *t*-test for all positive and negative β slopes to see if they differed significantly from 0. We found this was the case for both groups providing support for group membership. The slope for both the standard ($t(11) = 5.39$, $p < .0001$) and reverse SNARC subgroup differed significantly from 0 ($t(5) = 3.76$, $p = .01$).

4.5.3.4 Individual participants' dRTs

The regression slope for each participant was examined separately. Figure 4.10 shows dRTs plotted for each participant who met the 50% correct accuracy threshold for any given non-symbolic quantity. Large individual differences can be seen by the slope of the regression lines. Some participants' regression lines sloped downward, indicating a standard SNARC-like effect. Other participants' regression lines sloped upward indicating, a reversed SNARC-like effect.

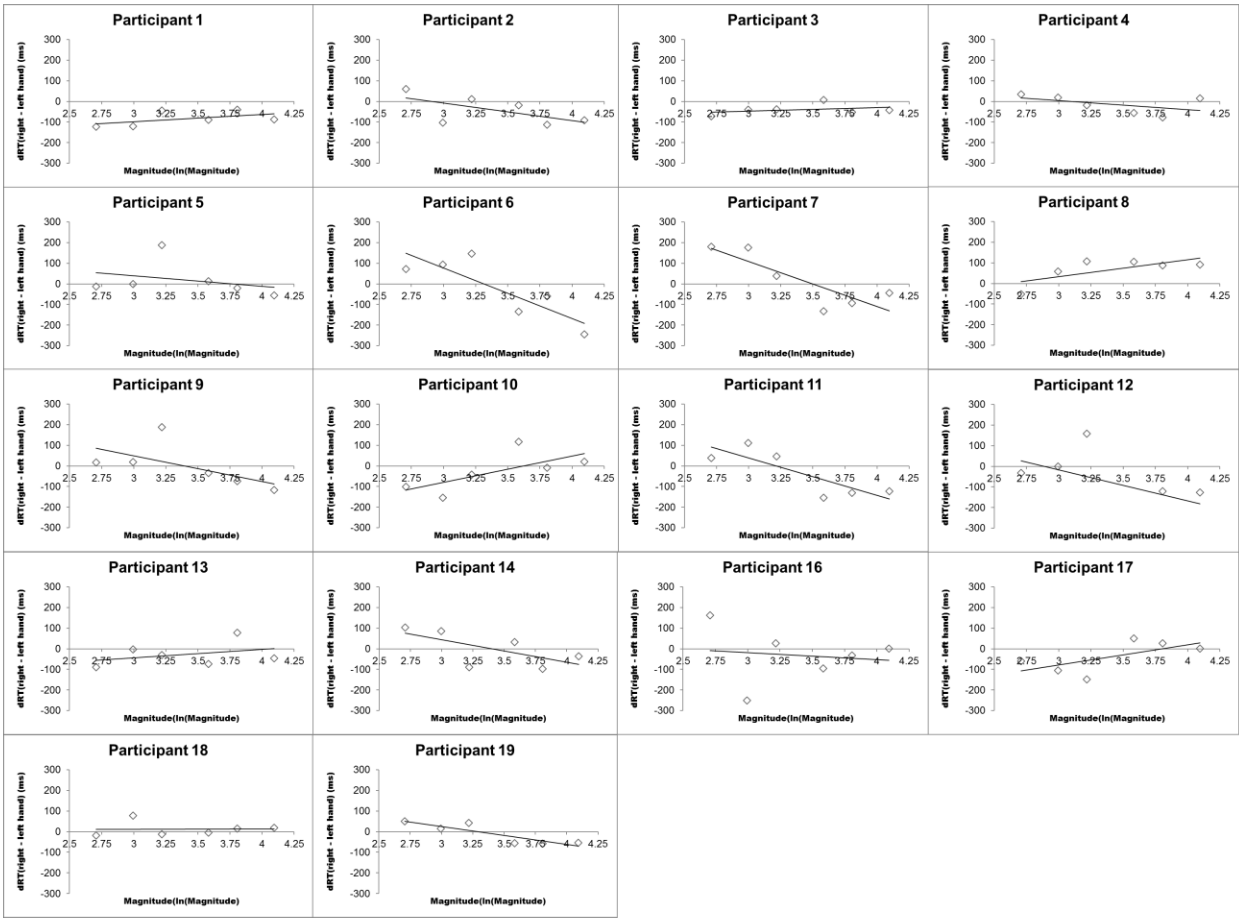

Figure 4.10. Individual differences in Study 2C of the non-symbolic SNARC-like effect for participants showing observed data and the regression line, reflecting response time differences for right and left foot as a function of non-symbolic quantity.

Note. The x-axis value for each dot point represents the natural logarithm of the non-symbolic quantity (i.e. 2.70 = ln(15), 3.00= ln(20), 3.22 = ln(25), 3.58 = ln(36), 3.81= ln(45), 4.09 = ln(60). Positive values show faster left foot than right foot responses. When the gradient of the regression line slopes steeply downward it indicates a SNARC-like effect. When gradient of the regression line slopes upwards it indicates a reversed SNARC-like effect.

4.5.4 Discussion

The current study examined whether the non-symbolic SNARC-like effect is specific to manual (i.e., hand) responses or generalizes to other response modalities (i.e., foot responses) and whether the effect would still occur when visual stimulus properties such as overall luminance/contrast of display and total surface area were controlled for. There are four findings worth noting. First, findings show further support for the theory that non-symbolic quantities are represented spatially from smallest to largest and left to right as evidenced by a non-symbolic SNARC-like effect. Presentation of smaller quantities (i.e. 15, 20 and 25) facilitated faster responses with the left foot, and larger non-symbolic quantities (i.e. 36, 45 and 60) with the right foot. This showed that non-symbolic quantity RT is modulated by response foot and appears to be independent of effector. This finding confirms that of Schwarz and Müller, (2006) who found a SNARC effect for foot responses. However, it is important to note the differences between the current study and that of Schwarz and Müller. The current study used a non-symbolic quantity judgment task, whereas Schwartz and Müller used a symbolic number parity judgment task. Also, Schwarz and Müller (2006) tested both manual (hand) and pedal (foot) responses in the same person. Whereas in the current study only foot responses were examined.

Second, the form of the interaction was like that of the symbolic SNARC effect (i.e. a significant interaction effect was found for quantity and side of response). However, it is important to note that, for smaller quantities, the difference between RTs for the right foot compared to the left foot was minimal (6 milliseconds faster on average for the left). For larger quantities, there was a bigger difference: participants responded 45 milliseconds faster on average for the right foot than for the left foot. This finding supported those of Zhou et al., (2016) who found the non-symbolic SNARC-like effect to be asymmetrical and more pronounced for larger quantities. However, further support for a non-symbolic SNARC-like effect was shown by the regression slope analysis. A significant negative slope indicated a non-symbolic SNARC-like effect.

Third, unlike Nuerk, et al., (2005) here the effect was not due to low level visual cues such as total dot surface area or overall luminance/contrast of display. In the current study, where total dot surface area remained constant as quantity increased a non-symbolic SNARC-like effect was still found for the non-symbolic quantity judgment task. This suggests that quantity alone is sufficient to elicit a SNARC-like effect. While it was not possible to tell from Studies 1, 2A, and 2B whether the observed SNARC-like effect resulted from a spatial representation of quantity or a spatial representation of total surface area of the dots, the results of the current experiment provide support for the first of the two explanations. If total surface area were a driver for the effect in Studies 1, 2A, and 2B a reduced SNARC effect or smaller negative β slope would have been expected in the current experiment, but this was not found. This provides further evidence that total dot surface area was not the driver of the SNARC effect found in Experiments 1, 2A and 2B. However, a caveat for this finding is that it is important to be cautious in making across-study statements/comparisons because of measurement errors/power, different number of trials and sample sizes, and the use of different participants for each experiment.

Fourth, as shown in figure 4.10, clear individual differences were found for RT signatures. Only 67 % of participants showed a SNARC effect. This outcome is like that of Studies 1, 2A, and 2B for the non-symbolic SNARC-like effect, along with previous research on the symbolic SNARC effect where typically, not all participants show a symbolic SNARC effect (Beecham et al., 2009; Cipora & Nuerk, 2013; Viarouge et al., 2014; Wood et al., 2008). The number of participants showing a non-symbolic SNARC-like effect in our studies ranged from to 64 to 67%. Taken together, these results show further evidence for the non-symbolic SNARC-like effect. Indeed, like the symbolic number SNARC effect, the non-symbolic SNARC-like effect may be effector independent.

4.5.4.1 Non-symbolic SNARC-like effect for quantities

The finding of a non-symbolic SNARC-like effect for quantities in the ANS range, adds to previous research with children (Bulf et al., 2014; Patro & Haman, 2012) and adults (Bulf et al., 2014; Mitchell et al., 2012; Nuerk et al., 2003; Zhou et al., 2016) suggesting non-symbolic quantities are represented spatially. This finding provides empirical evidence that non-symbolic quantities seem to be spatially represented in a left to right manner. The finding of a non-symbolic SNARC-like effect for quantities is also consistent with previous research on the SNARC effect for symbolic number (Dehaene et al., 1993; Fias et al., 1996; Wood et al., 2008). It is claimed that the symbolic SNARC effect is evidence for a symbolic mental number line (Dehaene et al., 1993). In the current study it was claimed that this may be the case for non-symbolic quantities. Indeed, this finding suggests that smaller non-symbolic quantities are associated with the left side of space and larger numbers are associated with the right akin to a mental number line. The finding of a non-symbolic SNARC-like effect in the current study provides support for Zhou et al. (2016) and a mental number line for non-symbolic quantities. Gaining understanding into the spatial representation of non-symbolic quantities is important for several math competences such as the processing and quantity representation of non-symbolic quantities, enumeration, being able to conduct arithmetic, and understanding place order of symbolic number. The ability to process non-symbolic quantities may be foundational to symbolic number representation and higher order math.

4.5.4.2 Non-symbolic SNARC-like effect, VSWM, and math ability

In study 2A there was a moderate positive correlation between Right - Left difference RT score and Corsi backwards score. Those with a larger Right - Left difference RT score tended to show on average a higher Corsi backwards score. This suggests that there is a relationship between the non-symbolic SNARC-like effect and VSWM. The findings are consistent with previous research that has shown a relationship between VSWM and numerical/quantity processing (Gibson & Maurer, 2016; Viarouge et al., 2014).

The findings did not provide evidence for a relationship between a non-symbolic SNARC-like effect and math performance. No relationship found between the non-symbolic quantity judgment task (RT right- left difference score) and math ability (WRAT score). Previous research has shown no link between non-symbolic quantity judgment performance and math achievement (Fuhs & McNeil 2013; Holloway & Ansari, 2009; Sasanguie, Van den Bussche, & Reynvoet, 2012; but see Chew, Forte, & Reeve, 2016 who used latent class analysis to show faster and/or more accurate performance for symbolic and non-symbolic quantity judgment correlated with better math performance). These findings suggest that there may not be a direct link between non-symbolic numerical processing and math performance. The finding that there is no direct relationship between non-symbolic quantities and math ability makes intuitive sense as math usually consists of manipulating symbolic number.

Another possible reason why no relationship has been found between non-symbolic numerical processing and math performance in some studies may be a lack of consistency across paradigms. For example, a review by De Smedt, Noël, Gilmore, and Ansari (2014), on symbolic and non-symbolic quantity processing skills and individual differences in math performance, suggests that inconsistencies between non-symbolic quantity judgment and math performance may be due to methodological issues. These include differences regarding the level of control of visual non-numerical cues as well as the types of measures used. A study may report on either reaction time, accuracy, ratio distance effects or *w* (a measure of the Weber fraction).

Similarly, previous research on the relationship between the symbolic SNARC effect and math performance has shown inconsistent results. Some studies have shown science students show a stronger SNARC effect than arts students (Dehaene et al., 1993). Although it is important to note that in the Dehaene et al., (1993) study, math performance was not directly measured. More recent studies have measured math performance using a standardised test with different outcomes. Interestingly, better performance on a math task has been linked to a weaker SNARC effect. For example, a study by Cipora et al., (2015) on professional mathematicians did not show a SNARC effect (Cipora et al., 2015). This suggests that professional mathematicians may have more abstract representation of number, relying less on spatial representation. Another example is a recent study by Kramer, Bressan, and Grassi (2018) which showed those with a stronger SNARC effect (either a standard or a reversed SNARC) showed worse mathematical intelligence as measured by the WAIS-R. It is important to note however, although significant, the negative correlation between the SNARC effect and math intelligence was weak. Kramer et al., claim that in children a stronger SNARC effect appears to be linked to better math performance. However, in adults a stronger SNARC effect may be linked to poorer math performance and may even be counterproductive. This shows that the spatial representation of number may have different roles during development and adulthood.

4.5.4.3 Implications

The ability to judge non-symbolic quantities in the ANS range is thought to be an index of approximate magnitude representation. It seems that the interaction between non-symbolic quantities as evidenced by a non-symbolic SNARC-like effect and space is integral to this representation. One possible reason for this association is that both number and space are represented in the parietal area of the brain (Hubbard, et al., 2005). The finding suggests that understanding the spatial representation of non-symbolic quantities seems to be important for theoretical models of numerical cognition.

There are some limitations to the current study which should be discussed. The first language of participants was not recorded, and Australia where the study was run is a multicultural country. There are a large proportion of international students studying at the University of Melbourne. It is known that reading direction seems to influence the symbolic SNARC effect (Dehaene et al., 1993). Although most countries in the world have languages that read from left to right (Butterworth, 1999) it would be of value to investigate the effect of reading direction on the non-symbolic SNARC-like effect. Also, it has been shown that for the symbolic SNARC effect that the direction of finger counting or finger counting habits modulate spatial-numerical associations (Fischer, 2008). Based on previous research by Fischer (2008) where a relationship was found between the symbolic SNARC effect and finger counting, the potential relationship between finger counting habits and the non-symbolic SNARC-like effect was examined in a pilot study. However, no correlation between finger counting for left-hand or right-hand starters and the non-symbolic SNARC-like effect was found. Therefore, this factor was excluded from further analysis in the thesis. Future research should record reading direction for all participants to see if this factor influences individual differences in the non-symbolic SNARC-like effect. Visual cues such as total envelope area, roundness or density of dot displays were not controlled for. Although previous research has shown it is impossible to control for all visual cues at once (Gebuis & Reynvoet, 2011), it would be of interest to see what effect if any manipulating these other visual cues has.

4.6 Summary of Chapter 4

Chapter 4 presented a series of studies that investigated whether a non-symbolic SNARC-like effect was found for quantities in the ANS range in a referent-to-target non-symbolic quantity judgment task and whether general cognitive abilities (VSWM) and/or math ability predict if a non-symbolic SNARC-like effect is present. The findings show a non-symbolic SNARC-like effect for quantities in the ANS range for a ratio of 0.5 (15 vs 60 dots), as indicated by a significant interaction between quantity and hand of response. Specifically, the presentation of smaller quantities (i.e. 15 dots) facilitated significantly faster responses for the left hand. Likewise, larger quantities (i.e. 60 dots) facilitated significantly faster responses for the right hand. This indicated that the effect of quantity on RT was modulated by response hand. This pattern of data showed a strong similarity to that of the symbolic SNARC effect. ANOVA analysis revealed a SNARC-like effect for magnitudes in the ANS range, as measured by a non-symbolic quantity judgment task. The findings show a SNARC-like effect for large dot array non-symbolic quantities and provide support for the spatial representation of non-symbolic quantities in the ANS range. A moderate positive relationship was found between RT (right – left difference score) and VSWM. Those with a higher Corsi backwards score tended to show a larger Right-Left RT difference score. Symbolic SNARC research report VSWM effects. Therefore, it makes intuitive sense that participants who show a larger Right-Left RT difference score would have a higher VSWM ability. The symbolic SNARC effect is considered to be a behavioural measure of the spatial representation of number (Dehaene et al., 1993). In support of this claim, those with visuospatial working memory deficits tend to show a reduced SNARC effect (Crollen & Noel, 2015) or no SNARC effect (Bachot, et al., 2005). Moreover, a review of evidence from behavioral and neuroimaging studies by de Hevia, et al., (2008) supports the view that visuo-spatial processes and numerical representation are related.

There was also a relationship found between VSWM and math performance. Those who showed a higher level of performance in the VSWM task tended to perform well in the math task. Albeit, this relationship between VSWM and math ability is somewhat complex and may be mediated by other factors (see a recent review by Hawes and Ansari (2020).

The issue of calibration in Chapter 3 where several participants had to be removed as they were unable to achieve an accuracy threshold of 50% appears to have been overcome. That is, all participants were able to do the magnitude judgement task with a high degree of accuracy and were able to distinguish if the second set (target) of dots contained more or lesser dots than the first set (referent). Participants can be surprisingly inaccurate when estimating numerosity. Minturn and Reese (1951) have shown that imprecise estimation of numerosity occurs to a factor of four. In the Minturn and Reese study, the numerosity of a set of 200 dots was estimated to be as low as 50 dots or as high as 700 dots. Participants were more likely to overestimate sets sizes under 20 dots but underestimate arrays of set size 20 and above.

While some overestimate the numerosity of sets, most tend to underestimate set size. A set size as large as 200 dots may be miscalibrated and perceived as 50 dots. Izard & Dehaene (2008) argue that this miscalibration occurs due to an affine scaling of a participant's response grid. Izard and Dehaene (2008) created a theoretical model of calibration of estimation. There are five hypotheses for this model: (1) Numerosity is encoded on the mental number line. (2) The mental number line is compressed and logarithmically scaled. (3) Activation of the mental number line follows a Gaussian distribution (4) Activation of the mental number line is translated into exact number words though a respond grid. (5) This response grid is influence by calibration and can be stretched or compressed accordingly. When no information about numerosity of arrays is given the response grid is spontaneous and responses are not calibrated. However, when a participant is given an example (inducer) and informed how many elements it contains the spontaneous response grid transforms into calibrated response grid and responses increase in accuracy. Thereby, calibration seems to influence discriminability of non-symbolic numerical stimuli.

However, in the next study 2B, to increase task difficulty a ratio of 0.66 was chosen (20 vs 45 dots). It was of interest to see if a non-symbolic SNARC-like effect could be captured using only these stimuli. Thereby reducing the calibration issue but increasing processing time. Again, to see if a stronger non-symbolic SNARC-like effect was found. In Study 2B a SNARC-like effect was found for a more difficult ratio of 0.66 (i.e. 20 and 45 dots) where overall luminance/contrast of display was controlled for. In Study 2C a non-symbolic SNARC-like effect was found for foot responses where total surface area was controlled for. Taken together, the results for Studies 2A, 2B and 2C provide further evidence for the non-symbolic SNARC-like effect across a range of stimuli and effectors while controlling for visual cues of overall luminance/contrast of display and total surface area. These results provide support for the claim that non-symbolic quantities seem to be spatially represented from smallest-to-largest and left-to-right. In each study, individual differences were found for the non-symbolic SNARC-like effect. In Studies 1 and 2C some participants were not able to judge the 0.83 ratio (25 and 36 dots) above chance level. This finding points to a calibration model of the non-symbolic SNARC-like effect. That is, to elicit a SNARC-like effect participants must be able to discriminate between stimuli (i.e. which is lesser or greater?). This ability varies based on an individual's Weber fraction. This finding points to the need to further investigate these individual differences.

Although Studies 2A, 2B, and 2C confirm a general finding of a non-symbolic SNARC-like effect findings of Study 2 leave unanswered questions about individual differences and variability of the SNARC effect. Where does the non-symbolic SNARC-like effect occur in this calibration model? Is it possible to refine the model of the non-symbolic SNARC-like effect? Study 1 and 2 showed a possible difficulty in existing models of the non-symbolic SNARC-like effect due to a lack of calibration. These earlier studies highlighted individual differences in the non-symbolic SNARC-like effect but did not specifically address variability in the SNARC effect. This points to the need for a broader model of the non-symbolic SNARC-like effect which includes calibration for individual differences. Variability of the non-symbolic SNARC-like effect will be examined in Chapter 5.

Chapter 5: Study 3

In the research reported earlier in this thesis four studies investigated a non-symbolic SNARC-like effect using differing ratios, modality of response and manipulation of low-level visual cues. A robust non-symbolic SNARC-like effect was consistently found across all these studies. However, there was much variability with distinct individual differences in the direction and gradient of linear regression SNARC slopes (see Figure 3 in the supplementary information for Chapter 3 and Figures 4.2, 4.5 and 4.10 in Chapter 4). Most participants showed a standard SNARC effect (faster responses for a small/left and large/right). Others showed a reversed SNARC effect (faster responses for a large/left and small/right). A minority showed no SNARC effect at all. However, the reasons for this variability were not addressed in these studies.

Individuals differ in the precision with which they represent the approximate numerical magnitude of items in a visual set (Halberda et al., 2008). During magnitude comparison tasks, for example, most people show a ratio effect (i.e., slower RTs and more errors for smaller ratios); nevertheless, individual differences in performance varies widely (Halberda, Ly, Wilmer, Naiman & Germine, 2012). And individual differences have been found in the symbolic SNARC effect and the spatial representation of symbolic number (Beecham, et al., 2009; Cipora et al., 2016; Wood et al., 2008). However, most previous studies that have used aggregated data methods (i.e., employ analytic methods that focus on mean or median data) have not always found a relationship between the symbolic SNARC effect and the Weber fraction (a measure of the ANS). This is an important issue since there are known construct and discriminant validity issues associated with 'measuring' the ANS, which might account for the inconsistent findings reported in previous studies (see Clayton et al., 2015; Dietrich et al., 2015).

In previously published research (see also Chapter 3 in this thesis) we described individual differences in response characteristics for a non-symbolic quantity judgment task (Nemeh et al., 2018). However, based on the set sizes selected (a referent of 30 and a target of 15, 20, 25, 36, 45 or 60), which were based on the ratio of referent to target (easy ratio- 0.5, moderate ratio - 0.66, hard ratio- 0.83) issues of discrimination and calibration remained. Numerical judgments increased in difficulty when targets are closer in quantity to the referent. Several participants were removed from analysis as they did not achieve an accuracy threshold of 50% for any given numerical magnitude. In other words, the ratio of referent to target seems to influence numerical judgments. Presentation of smaller ratios result in slower reaction times and less accuracy (Halberda et al., 2008). This supports the claim that non-symbolic numerical judgments follow

Weber's law. Individuals with smaller Weber fractions show better numerical acuity and find it easier to differentiate between numerical stimuli when ratios are smaller. This, in turn, suggests that individual differences in the ability to make non-symbolic numerical judgments (the Weber fraction) should be considered as a possible explanatory factor in non-symbolic SNARC-like judgments.

In the current study this suggestion is investigated using an adaptive staircase method to find the threshold of numerical acuity (Weber fraction). This will enable the calibration of ratio of referent to target stimuli for each individual participant in the non-symbolic quantity judgment task. Variable ratios at threshold and twice threshold will be used. The rationale for this is that by only presenting targets at a participant's threshold should increase discrimination and encourage the use of non-symbolic quantity to make judgments rather than low-level cues.

In sum, the present study investigated the relationship between the non-symbolic SNARC-like effect and the Weber fraction to gain further understanding of the nature of individual differences; specifically, with respect to the processing and spatial representation of non-symbolic quantities in the ANS range. An in-depth overview of the theoretical and methodological issues associated with measurement of the ANS was discussed in Chapter 2. Unlike the symbolic number SNARC effect it is difficult to ensure non-symbolic numerical judgments are made on non-symbolic quantity alone rather than other criterion such as low-level visual cues.

5.1 Measuring Individual Differences in the SNARC Effect

Several analytic issues regarding individual differences in the SNARC effect need to be resolved Analysis of the symbolic SNARC effect typically involves ANOVA where a SNARC effect is indicated by an interaction for hand of response and number and/or linear regression where the difference for right – left reaction times (dRTs) are entered into a regression. These dRTs are regressed onto the log of each numerical magnitude. Then the beta coefficient of the regression slope is calculated. Negative beta coefficients indicate a standard symbolic SNARC effect. However, ANOVA and linear regression do not enable a more fine-grained analysis of individual difference in the SNARC effect as data is collapsed across participants. Alternatively, psychophysics methods enable detection of near-threshold numerical judgments to stimuli and enable individual responses to be examined separately to find the best fitting model for the data.

5.2 The Current Study

The aim of the present study is to address issues associated with discrimination and calibration of the ANS and determine if differences in the Weber fraction can account for these differences in the non-symbolic quantity SNARC-like effect. It is possible individual differences in the Weber fraction would result in different RT and accuracy signatures. We used an adaptive staircase method, which is often used to measure the threshold of response or ability to discriminate a stimulus (Dixon & Mood, 1948). It works in the following manner: when a participant gets a numerical judgment correct several times in a row, the task judgment becomes incrementally more difficult. When a participant gets a numerical judgment incorrect, the task becomes incrementally easier. The advantages of adaptive staircase methodology are: (1) reduced response bias compared to when a preselected stimulus range is used, and (2) data may be collected before determining the appropriate range of stimulus values for test condition (Nachmias, 2006). The later point is particularly relevant to the current study as the adaptive staircase enabled test stimulus values for the non-symbolic quantity judgment task to be individually chosen. This is key when measuring the non-symbolic SNARC-like effect as there are known individual differences in ANS acuity.

The non-symbolic quantity judgment task comprising a referent-to-target non-symbolic quantity judgment task with six base set sizes (10, 20, 30, 40, 50 or 60) and targets ranging from (11 to 100), which were selected based on the Weber threshold for each individual. As the ratio of referent to target is important for non-symbolic numerical SNARC judgments (Zhou et al., 2016) each referent was paired with an ‘easy’ and ‘hard’ target. These ratios were based on the Weber fraction for each individual. The low ratio was at threshold of the Weber fraction and the high ratio at twice threshold.

In sum, while individual differences have been consistently found for the non-symbolic quantity SNARC-like effect, it is not clear what accounts for these differences. The research questions of interest were: (1) Does the Weber fraction account for individual differences in the non-symbolic SNARC-like effect and the spatial representation of non-symbolic quantity? (2) How does ratio (low/high) presentation order (referent or target displayed first) or response mapping (small/left large/right or vice-versa) affect RT and accuracy for non-symbolic SNARC-like judgments? (3) How can we design dot arrays to encourage participants to use numerical magnitude to make judgments rather than low level visual cues? To address these research questions, an adaptive staircase design measured each participant’s threshold of response (i.e.

Weber fraction). This Weber fraction was used to select target stimuli for each participant considering individual discrimination of stimuli and calibration of responses to test the non-symbolic SNARC-like effect. Presentation order of referent and target varied; this was necessary because the target was always larger than the referent. Thereby, presentation order varied so that there would equal amounts of correct right hand and left-hand responses. This design enabled manipulation of order independent of quantity which is confounded in many other SNARC studies. Participants could get a correct response regardless of quantity and order of stimuli. Different ratios of referent-to target were used, and dot stimuli were designed to decrease participant's likelihood of using other criterion such as density, dot size or roundness of display to make quantity judgments.

It was predicted that individual differences in the Weber fraction as measured by an adaptive staircase would be related to individual differences in the mapping of right- and left-hand responses to 'smaller' and 'larger' non-symbolic quantity stimuli (i.e. the non-symbolic SNARC-like effect). However, if differences in the non-symbolic SNARC-like effect were not found for participants with differing Weber fractions, it would suggest that individual differences in the Weber fraction do not account for individual differences in the non-symbolic SNARC-like effect and the spatial representation of non-symbolic quantities and that parameters other than non-symbolic quantity may be influencing judgments.

5.3 Method

5.3.1 Participants

Eighty-seven undergraduate students participated in return for course credit. All participants (19 males, 68 females) were aged between 17 and 24 (*M* = 18.79, *SD* = 1.13), had normal or corrected-to-normal vision and were right-handed. All participants gave written, informed consent before participating in the study. The study was conducted in accordance with the Declaration of Helsinki and was approved by the Human Research Ethics Committee at the Melbourne School of Psychological Sciences (# 1237507.2).

5.3.2 Apparatus

A Dell desktop computer with MATLAB software and Psychophysics toolbox (Brainard, 1997; Pelli, 1997) were used to generate and present dot stimuli. Dot arrays were presented on a 22-inch Sony Trinitron Multiscan 420 CRT monitor with a 1280 x 1024 pixels spatial resolution and refresh rate of 120 Hz. Participants sat approximately 50 cm from the monitor and a chinrest was used to control viewing distance. Response boxes consisted of two separate gaming mechanical number pads with 1kHz sampling or pooling to capture left- and right-hand responses.

5.3.3 Materials and Procedure

Testing was conducted in a quiet, well-lit room. All participants completed an adaptive staircase to measure the Weber fraction and a non-symbolic quantity judgment task. The Weber task was always completed first as the participant's Weber fraction was used to select the target stimuli for the non-symbolic quantity judgment task. The adaptive staircase and non-symbolic quantity judgment task were presented on a CRT monitor using MATLAB software. Each dot array appeared for 200 ms to decrease the likelihood that participants were able to count the dots. Participants who did not score above 50% averaged across set sizes for each condition (LR/RL and easy/hard) were excluded from further analysis. The use of this criterion resulted in 19 participants being excluded which left a sample of 68 participants.

5.3.4 Stimulus Design

Dot arrays were created for set sizes *n* = 10-100. As quantity of set sizes increased, scaling of visual cues within the dot display decreased the usefulness of low-level visual cues to make quantity judgments. We used a base stimulus radius of 226 pixels and 100 randomly generated examples of set sizes 10 to 100. Dots were generated using the following equations:

We first calculated the set size scaling factor (S_N) where N is the set size.

$$S_N = \sqrt{\log_{10} N}$$

We calculated the set size radius for a given set size (r_N) where r is the radius in pixels (226).

$$r_N = S_N r$$

We calculated the total dot area for a given set size (D_N) where S_N is the set size scaling factor and r_N is the set size radius.

$D_N = \frac{S_N(2\pi r_N)^2}{20}$We randomly calculated each set of dot radii for a given set size (R_N) where S_N is the set size scaling factor, and $W_{N[0,1]}$ is N random samples with equal probability from 0 to 1. We then linearly scaled those dot radii such that the total dot area equalled D_N.

$$R_N = S_N\left(\left(\frac{\sin^{-1}(2W_{N[0,1]} - 1)}{\pi}\right) + 0.5\right) + 1$$

We randomly determined each dot position using polar coordinates. Polar coordinates of a P(C_D, A_D) was computed where r_N is the set size radius, and $W_{[0,1]}$ is a random number with equal probability from 0 to 1; A dot was valid if the distance to the other dot in each pair was greater than the sum of the dot pair diameters divided by the sqrt(2).

$$C_D = r_N\left|\frac{2\sin^{-1}(2W_{[0,1]} - 1)}{\pi}\right|, \; A_D = 2\pi W_{[0,1]}$$

See Figure 5.1 below.

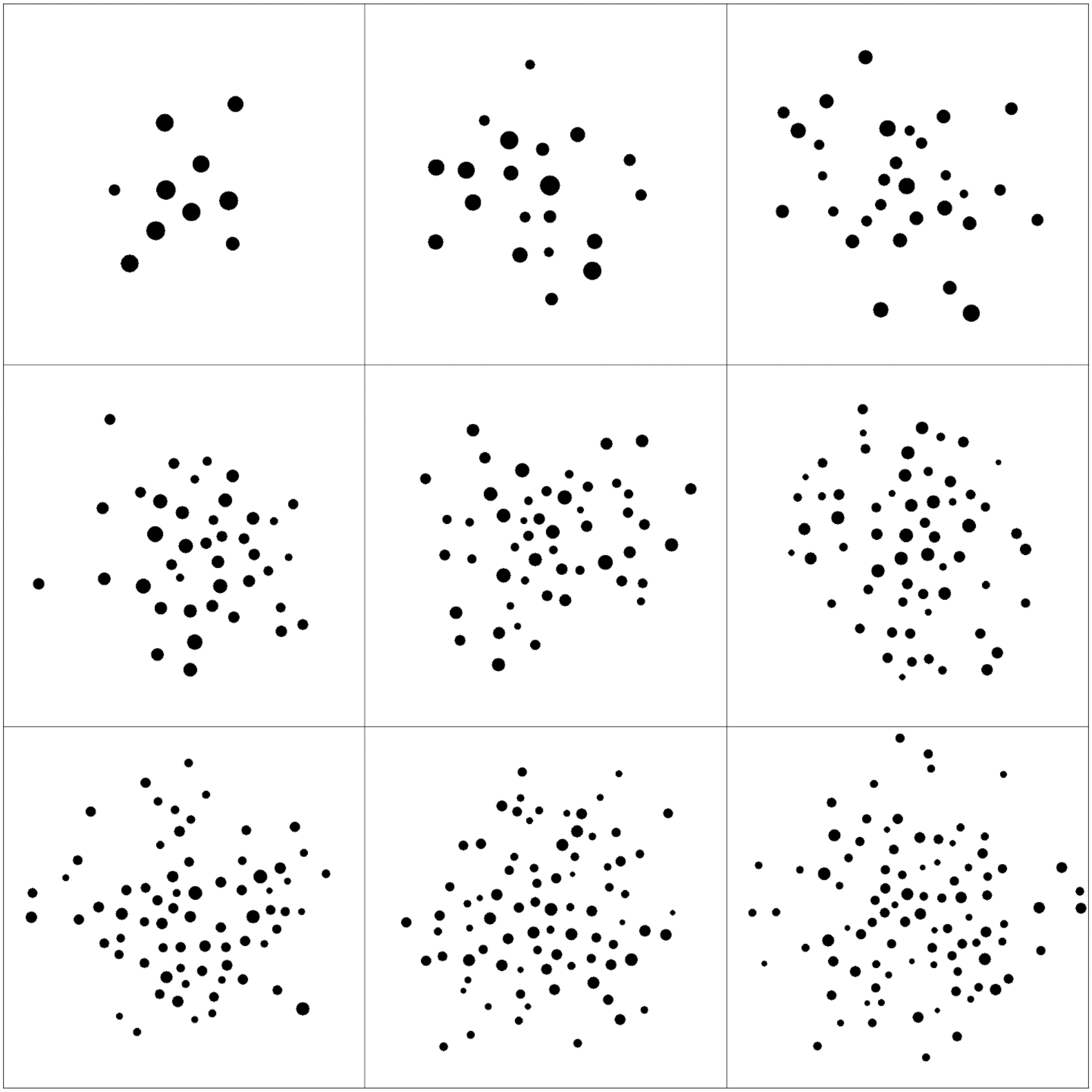

Figure 5.1. Examples of dot stimuli for Study 3.

Note. Stimuli consisted of N dots for base set sizes (10, 20, 3, 40, 50 and 60) and targets (which ranged from 11-100). Dot arrays were black presented on a white background. The perimeter of the dot array was not perfectly rounded with a constant radius that could be used to judge area. The area of dots was scaled so it was constant. This decreased the usefulness of density of a reliable cue. Neither aperture, density not largest dot size increased monotonically with numerical magnitude. Total black of display was set to be ~ ¼ of total aperture area. Thereby, the darkness of the display was not a useful cue.

5.3.5 Adaptive Staircase

The staircase followed a ‘1 up 2 down’ rule that targets the 70.7% level on the psychometric function (Levitt, 1971). The staircase went up one step in magnitude if wrong, down one if two responses in a row were correct. Number adjusted in log space.

There was a baseline of 30. The staircase started at 1.77 log units (set size 60). The initial step size = 0.176 log units; There were 30 trials in staircase. Actual order (if the second set of dots was less/more) was randomised.

Instructions for the non-symbolic judgment task were as follows:

‘We want to see if you can tell when two sets of dots differ in number.

Place your left pointer finger on the BLUE key.

Place your right pointer finger on the RED key.

Two set of dots will appear on the screen, one after the other.

If there are LESS dots in the second set than the first, press the LEFT key.

If there are MORE dots in the second set than the first, press the RIGHT key.

Give your answer as soon as you can!

Press the left key when you are ready to begin...’

Each trial sequence began with the first stimulus, which was displayed for a duration of 200ms. Then the screen was blank for 1400ms, followed by the second stimulus, also displayed for 200ms. Then screen went black again while responding. The screen remained blank for a variable time of 1.5 – 2 seconds until the next trial.

After a reversal of the staircase, the step size reduced to 0.6 of the current step size. The Weber threshold was calculated as the *geometric* mean of last 10 values. We recorded accuracy of response to assess the Weber threshold.

5.3.6 Non-Symbolic Quantity Judgment Task

In the non-symbolic quantity judgment task, the Weber threshold was used to select target stimuli for each individual. Participants were presented with two sequential dot arrays for a duration of 200ms each, consisting of one of six base set sizes (10, 20, 30, 40, 50 or 60 dots) paired with a target (n = 11 – 100 dots) with a low and high ratio of base set size to target. However, this was not a fixed ratio and was based on individual’s Weber threshold. If a participant had a Weber threshold of 0.2 base set size of 10 would be paired with a target of set size 12 in the low ratio condition and set size 14 in the high ratio condition. Presentation order of referent (base set size) and target varied. In the S/T condition - the referent (base set size) appeared at time 1 and the target at time 2. In the T/S condition - the target appeared at time 1 and the referent (base set size) at time 2. There were two response mapping conditions (left-to-right and right-to-left). In the SNARC congruent mapping response condition, participants were asked to press the left key if the second set of dots contained lesser dots than the first set and the right key if the second set of dots contained more dots than the first set. In the SNARC incongruent mapping condition participants were asked to press the left key if the second set contained more dots than the first and the right key if the second set contained lesser dots than the first set. Participants were told to respond as quickly and accurately as possible. Trials were randomised in blocks of 60 trials (Ratio- Low/High, Presentation Order- Referent or Target first). Participants completed 8 blocks (n = 480 trials overall) with a self-timed rest between blocks. We recorded accuracy and reaction time to measure the non-symbolic SNARC-like effect.

5.3.7 Analytic Approach

The overarching aim of the analyses was to compare the non-symbolic SNARC effect and the Weber fraction to see if differing levels of non-symbolic numerical acuity could explain individual differences in the SNARC effect. The first step was to check the manipulation for difficulty by examining the proportion of correct responses for low and high ratios of referent-to-target.

To determine if there was a non-symbolic SNARC effect we plotted right minus left-hand RTs (dRTs) for low and high ratios across mapping response condition (left-to-right and right-to-left). Then we pooled RT data across low and high ratios to see if there was a difference for left-to-right and right-to-left response conditions. We then compared correct RTs based on performance rather than participant ability for left-to-right and right-to-left response mappings.

Then data for correct and incorrect responses RTs for all participants was plotted to determine if a non-symbolic SNARC-like effect was found regardless of task performance. Finally, to check if the non-symbolic SNARC-like effect was based on base set sizes or magnitude of test responses we plotted RT data as a function of test magnitude and binned the data in increments of 10.

The data was not linear, and the distribution of the data was positively skewed, thereby linear regression (the typical SNARC analysis used to examine individual differences) was deemed inappropriate for analysis. Conversely, bootstrap methodology does not require the assumption of normally distributed data to be met (Field, 2009).

To address the last six steps, we used a nested bootstrapping methodology (Efron & Tibshirani, 1993) to produce 95% confidence intervals. A distribution of group mean data was produced through the calculation of the mean of 10,000 samples of participant data (with replacement) using resampled SNARC data (with replacement) for each participant.

To determine if there was a difference for mapping condition (i.e. a non-symbolic SNARC effect) for the low versus high ratio data/ low versus high performance groups (correct responses) low versus high performance groups (all participants correct and incorrect responses) and base set size versus test magnitude we performed a series of permutation t-tests for each of these groups which compared response mapping direction base set size across base set sizes. A paired-samples permutation t-test enabled the 95% confidence interval of the t-statistic for each individual's dataset to be calculated through the use of a bootstrapping procedure. The t-value was recomputed 10,000 times while randomly assigning the data to each group to obtain the distribution of t-statistics. If the t-value of the data falls outside the 95% confidence interval of the bootstrap t-distribution this allows us to conclude that it is likely that the data differ for the group and/or left-to-right and the right-to-left mapping condition.

There were 24 experimental conditions 2 x Ratio of referent to target: (High Ratio – twice threshold or Low Ratio – at threshold by 2 x Order of Presentation (referent or target presented first) by 6 x base set size (10, 20, 30, 40, 50, 60 dots). For each participant, 24 median RTs were derived for right and left-hand responses for each of the six referents. Incorrect trials were excluded from further analysis along with trials with RTs of 200ms or less (considered anticipatory responses) and 3507.8 ms which was greater than 3 SDs (SD = 896.11 ms) from the overall mean (M = 819.49 ms) of the experiment.

Inglis and Gilmore (2014) modelled the ratio of two to be compared magnitudes of an individual's expected accuracy for differing values of w. Inglis and Gilmore (2014) found as accuracy reaches 0.5, w asymptotically reaches infinity. Inglis and Gilmore argue that only participants whose responses follow Weber-Fechner law and have an accuracy of above 0.5 should be included for analysis. Therefore, following Inglis and Gilmore (2014), an inclusion criterion of a Weber fraction of 0.10 to 0.33 was deemed necessary for further analysis.

5.4 Results

Overall, the data is consistent with a non-symbolic quantity SNARC-like effect. For each participant we calculated the Weber fraction using an adaptive staircase and collected the data for left-to-right and right-to-left response conditions in the non-symbolic quantity judgment task. However, a similar pattern of results was found for those with low and high numerical discrimination performance. This suggests that individual differences in non-symbolic quantity acuity (the Weber fraction) do not account for non-symbolic quantity SNARC-like judgments for this data.

5.4.1 Difficulty Manipulation

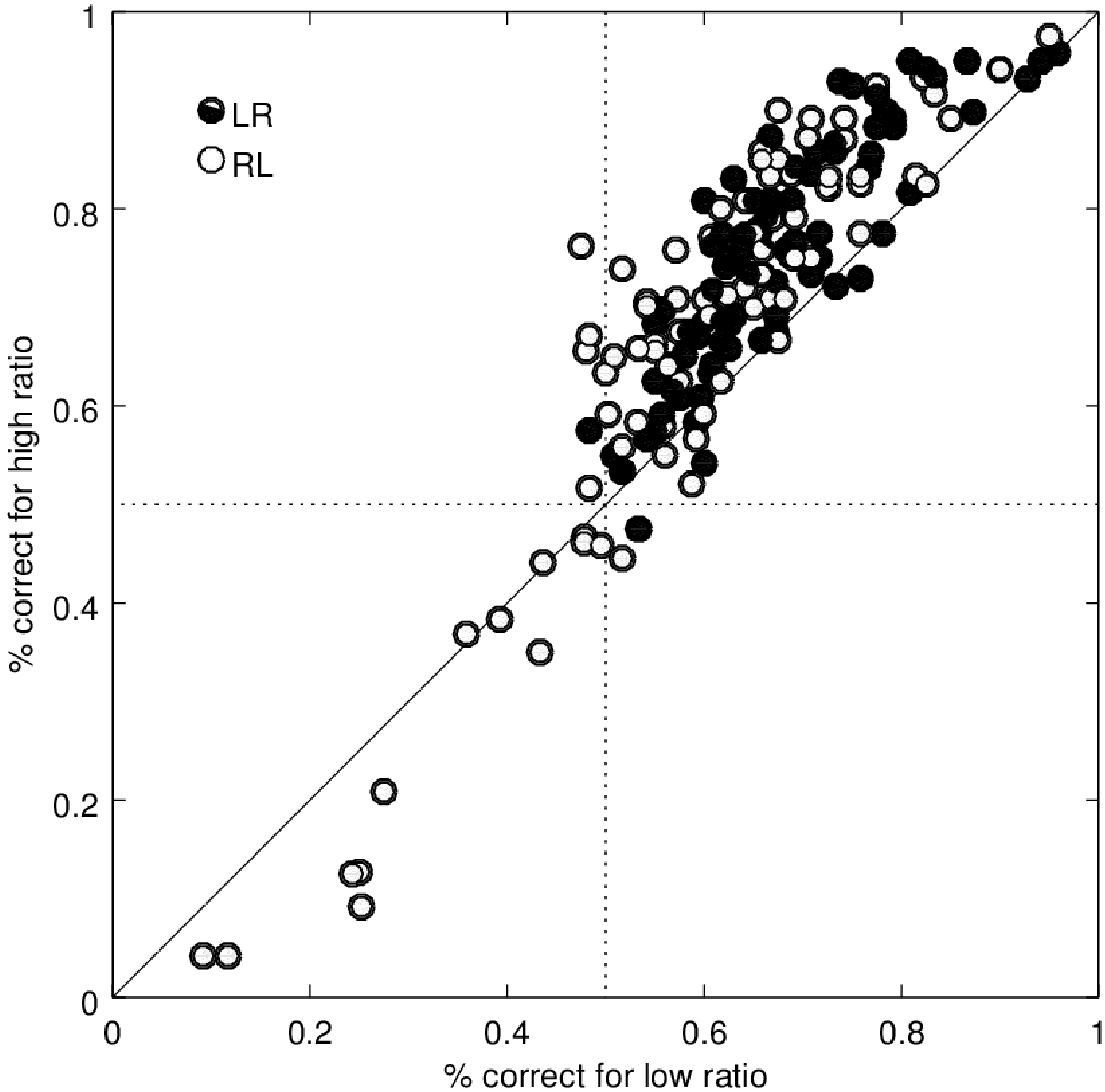

Figure 5.2. Scatterplot of proportion correct as a function of hard versus easy manipulation and left-to right and right-to-left mapping response for all participants in Study 3.

Note. The x-axis is the difficult condition. The y-axis is the easy condition. The black solid circular data points denote left-to-right mapping responses. The hollow circular data points denote right-to-left mapping responses. The diagonal line represents a veridical linear mapping to highlight accuracy over the 50% correct criterion. The dashed lines create four quadrants for responses.

Figure 5.2 shows the proportion of correct responses for the hard versus easy condition and left-to-right and right-to-left response mapping in the non-symbolic quantity judgment task. The data shows that we did not successfully manipulate performance in the easy/hard conditions.

Although the majority of participants performed better in the easy condition, there were a few who showed a lower proportion of correct responses for the easy than the hard manipulation. If the manipulation had been successful it was expected for the easy condition participants would have shown around 100% correct responses. For the hard condition we expected around 71% correct responses. We expected the 1 up 2 down staircase to have converged on 71%

A small number of people performed below chance. These data were all for the RL condition and is consistent with a failure to switch hand use in the RL condition. Performance was used to exclude some participants. Participants who performed below 0.5 proportion correct (below chance level or failure to use correct hand to respond) were excluded from further analysis. Of the original 87 participants. 19 were removed for averaging below 0.5 correct responses, leaving an analysis sample size of 68 participants.

5.4.2 Ratio and the Non-Symbolic SNARC-like Effect

The next step was to determine if there was a difference for left-to-right and right-to-left mapping response conditions across base set sizes for low and high ratios. We wanted to see if the pattern of results showed a non-symbolic SNARC-like effect and if performance differed for each ratio. Positive dRT values indicate faster left hand than right hand responses. Negative dRT values indicate faster right hand than left hand responses. In Figure 5.3 below the data points represent group averages of individual data for left-to-right and right-to-left response mapping for the non-symbolic quantity judgment task. The solid circular data points denote the high ratio and the hollow circular data points the low ratio. Figure 5.3 below shows the group mean of the *t*-score for each set size for the non-symbolic quantity judgment task.

To determine if there was a difference for mapping condition (i.e. a non-symbolic SNARC-like effect) for the low and high ratio data we conducted a permutation *t*-test for each base set size which compared response mapping direction. Twelve comparisons were conducted separately, one for each set size (10, 20, 30, 40, 50 and 60 dots) for the left-to-right and the right-to-left mapping conditions for both low and high ratios. If the *t*-value of the data falls outside the 95% confidence interval of the bootstrap *t*-distribution this allows us to conclude that it is likely that the data differ for the left-to-right and the right-to-left mapping condition. The trend in the data confirmed as statistically significant by *t*-tests suggests that responses differed across left-to-right and the right-to-left response mapping for base set size (10, 30, 40, 50 and 60). The exception was for set size 20 where responses were similar for each response mapping condition and did not fall outside the 95% confidence interval.

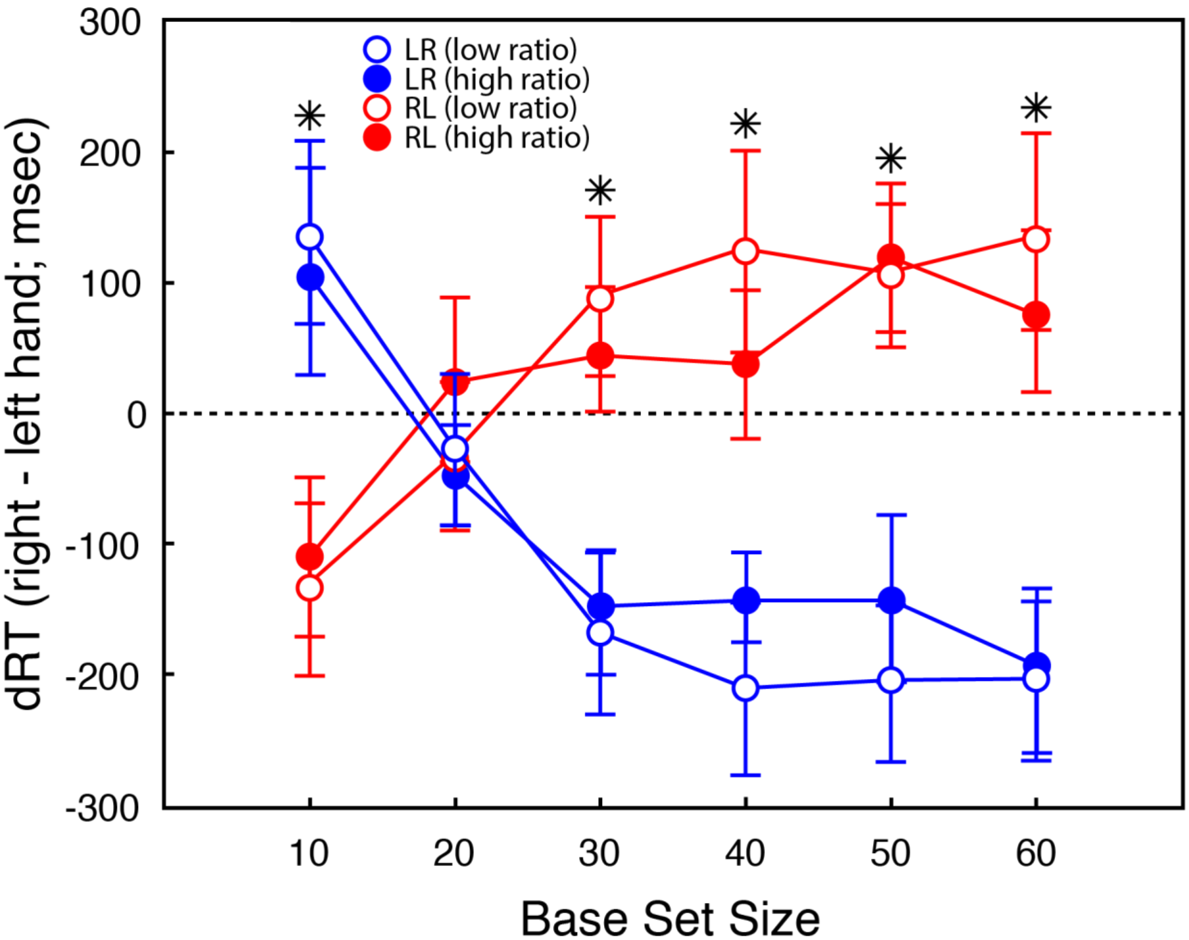

Figure 5.3. dRT in milliseconds for the mean group average plotted as a function of base set size for low and high ratios in Study 3.

Note: The red lines depict low ratio (hollow circular data points) and high ratio (solid circular data points) for right to left mapping responses. The blue lines depict low ratio (hollow circular data points) and high ratio (solid circular data points) for left to right mapping responses. There were no significant differences between low and high ratios for any set size or between response conditions. '+' denotes a significant difference for the high ratio between left-to-right and right-to-left response conditions for set sizes 10, 30, 40, 50 and 60. '×' denotes a significant difference for the low ratio between left-to-right and right-to-left response conditions for set sizes 10, 30, 40, 50 and 60. If both are significant, the overlapping '+' and '×' make a symbol resembling an eight spoked asterisk '*'. Error bars represent 95% confidence intervals of the bootstrapped mean.

The trend of the SNARC effect and *t*-scores are good evidence for a non-symbolic SNARC-like effect with faster responses for smaller base numbers (10) with the left hand and larger base numbers (30, 40, 50 and 60) with the right hand. There was a cross-over around base 20.

We found no evidence that the pattern of the SNARC-like effect was different across low and high ratio conditions. The trajectory of the SNARC-like effect for left is less is similar for low and high ratio stimuli, and the trajectory is similar for right is left low and high ratio stimuli. This may indicate that the difficulty manipulation was unsuccessful or that performance has little impact on the SNARC-like effect we report here. We therefore investigated how the SNARC-like effect changes with set size, collapsed across the different test ratios.

5.4.3 Non-symbolic SNARC-like Effect for Pooled Data across Low and High Ratios

The third step was to examine response mapping when the data was pooled across low and high ratios. Figure 5.4 below shows the group mean of the *t*-score for each set size for the non-symbolic quantity judgment task. To determine if there was a difference for mapping condition (i.e. a non-symbolic SNARC-like effect) for the pooled data we conducted a paired-samples permutation *t*-test for each base set size which compared response mapping direction. Six comparisons were conducted separately, one for each set size (10, 20, 30, 40, 50 and 60 dots) for the left-to-right and the right-to-left mapping conditions. The trend in the data confirmed as statistically significant by *t*-tests indicates that responses differed for left-to-right and the right-to-left response mapping providing evidence for a non-symbolic SNARC-like effect. The exception was for set size 20 where responses were similar for each response mapping condition and did not fall outside the 95% confidence interval.

Like Figure 5.3, Figure 5.4 suggests that the relative speed of responses for left and right hand depends on base set size values (referent) rather than a running average of the referent. This was indicated by different dRTs for each base set size.

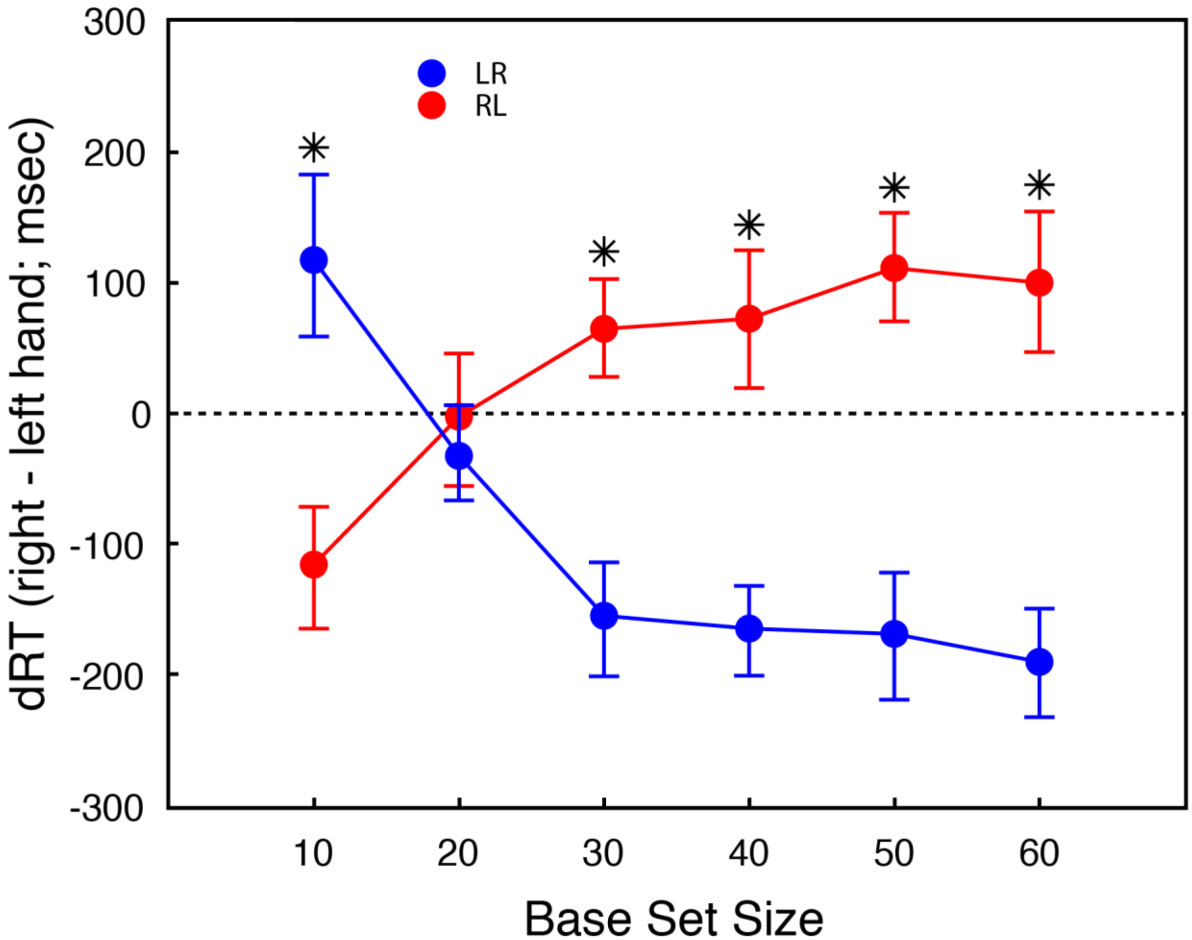

Figure 5.4. dRTs in milliseconds plotted as a function of referent (i.e., base set size of 10, 20, 30, 40, 50, or 60) and response mapping where data is collapsed across low and high ratios for Study 3.

Note: The blue line depicts for left to right mapping responses (LR). The red line depicts right-to-left mapping (RL). The asterisk indicates a significant difference between response conditions (left-to-right from right-to-left) at 0.5 significance level which was based on the bootstrapped *t*-statistic of a paired-sample *t*-test. Error bars represent 95% confidence intervals of the bootstrapped mean.

5.4.4 Non-symbolic SNARC-like Effect and Performance Level (Correct Responses)

The fourth step was to determine if participant's performance differed regardless of numerical ability. Figure 5.5 shows the group mean average of the *t*-score for each set size for the non-symbolic quantity judgment task for low and high-performance groups.

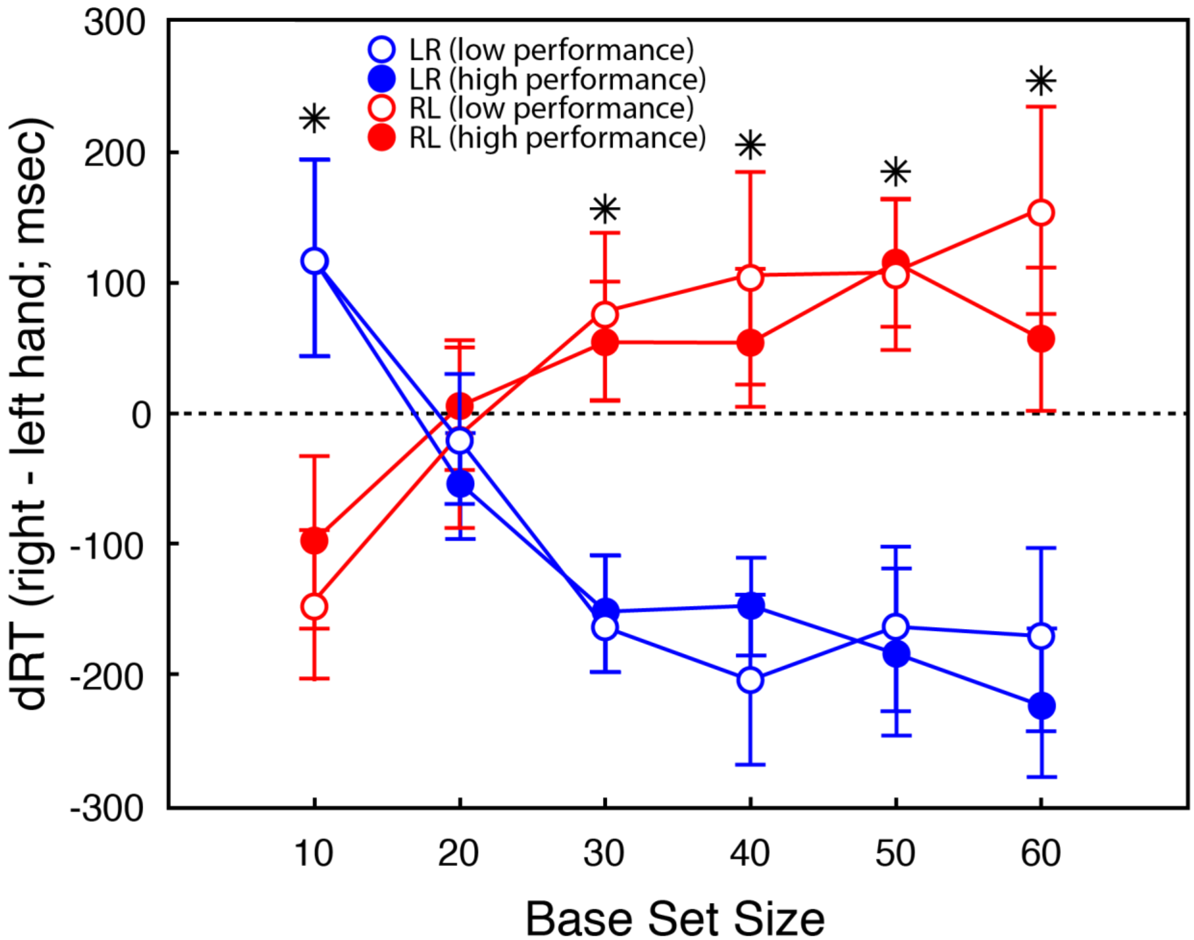

Figure 5.5. dRTs in milliseconds for hand response direction (correct responses) plotted as a function of base set size (i.e., 10, 20, 30, 40, 50, or 60) for low and high performance in Study 3.

Note: The red lines depict right-to-left mapping (RL) for low performance (red hollow circular data points) and high-performance groups (red solid circular data points). The blue lines depict left-to-right (LR) mapping for low performance (blue hollow circular data points) and high-performance groups (blue solid circular data points). An asterisk indicates a significant difference between response conditions (left-to-right from right-to-left; low and high performance groups) at 0.5 significance level which was based on the bootstrapped t-statistic of a paired-sample *t*-test. Error bars represent 95% confidence intervals of the bootstrapped mean.

In the previous analysis (see Figures 5.3 and 5.4), data included participants with over 50% of correct responses for each base set size (referent). For participants performing just above 50% their data will contain a large number of correct guesses. In other words, it is possible the magnitude of the SNARC thus reported is diminished by random guesses. In order to address this potential issue a median split was conducted on ‘high’ versus ‘low’ performance. The median of the data was 70 (remarkably close to the expected threshold from the Weber threshold task). Data from low and high ratio for each individual could fall into either one or two performance categories depending on whether the accuracy for a given ratio was above or below 70%. This means the data was spilt according to performance rather than the person. Data below the median of 70 was placed into a low-performance category. Data above the median of 70 was placed in a high-performance category.

To determine if there was a difference for mapping condition (i.e. a non-symbolic SNARC-like effect) for the on high versus low performance data we conducted a paired-samples permutation *t*-test for each base set size which compared response mapping direction. Twelve comparisons were conducted separately, one for each set size (10, 20, 30, 40, 50 and 60 dots) for the left-to-right and the right-to-left mapping conditions for each performance group. The trend in the data confirmed as statistically significant by *t*-tests suggests that there was a difference for mapping conditions for each performance group (i.e. a non-symbolic SNARC-like effect). However, the data suggests that there was little difference between the low and high-performance groups. This could indicate that the magnitude of the SNARC does not depend on the being able to get the task correct. If this were the case, then calculating SNARC using all responses rather than just correct ones should have little impact in the magnitude of the SNARC.

5.4.5 Non-symbolic SNARC-like Effect and Performance Level (All Responses)

The fifth step was to examine correct and incorrect responses for all participants to determine if a non-symbolic SNARC-like effect was found regardless of whether responses were correct. Figure 5.6 below shows the group mean average of the *t*-score for each set size for the non-symbolic quantity judgment task.

To determine if there was a difference for mapping condition (i.e. a non-symbolic SNARC-like effect) for the high versus low performance data we conducted a paired-samples permutation *t*-test for each base set size which compared response mapping direction. Twelve comparisons were conducted separately, one for each set size (10, 20, 30, 40, 50 and 60 dots) for the left-to-right and the right-to-left mapping conditions for each performance group.

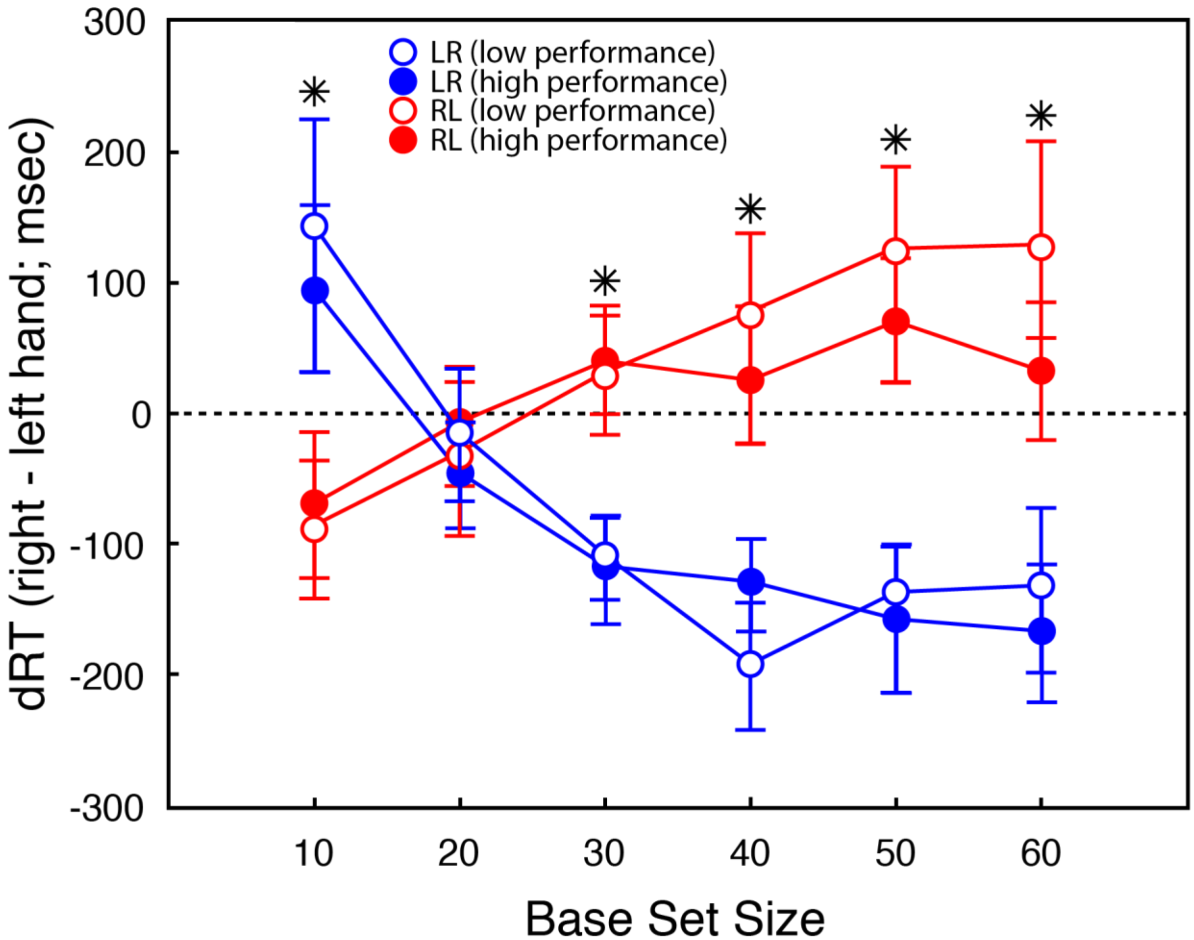

Figure 5.6. dRTs in milliseconds for all participants for response mapping conditions (left-to-right/right-to-left) for all responses plotted as a function of base set size (i.e., 10, 20, 30, 40, 50, or 60) for low and high performance in Study 3.

Note: The red lines depict right-to-left mapping (RL) for low performance (red hollow circular data points) and high performance (red solid circular data points). The blue lines depict left-to-right mapping (LR) for low performance (blue hollow circular data points) and high performance (blue solid circular data points). An asterisk indicates a significant difference between response conditions (left-to-right from right-to-left; low and high performance groups) at 0.5 significance level which was based on the bootstrapped t-statistic of a paired-sample *t*-test. Error bars represent 95% confidence intervals of the bootstrapped mean.

The trend in the data confirmed as statistically significant by *t*-tests for Figure 5.6 suggests that there was a difference across the mapping conditions for each performance group providing evidence for a non-symbolic SNARC-like effect. This data suggests that a non-symbolic SNARC-like effect was found regardless of how well people performed in the task. In other words, using the dot stimuli to make a numerically correct computation is not necessary to produce the SNARC-like effect. The question is then: what is driving the SNARC-like effect if not the computation? One possibility is simply the magnitude of the dots themselves. Perhaps passively viewing non-symbolic stimuli would be sufficient to drive the mechanisms that produce the non-symbolic SNARC-like effect we see here. However, participants were not entirely passive, as the task required an active response. Another possibility is that as stimuli were presented at threshold participants were able to do the task but found it difficult to swap response from trial to trial, particularly if there were several trials in a row where the right or left button was the correct response. A further possibility is there may have been an initial processing of stimuli that has not gone through to conscious awareness. Participants may have a predisposition to acting in this manner. However, these suggestions are somewhat speculative as we are not entirely sure what caused this pattern in the data.

In Figures 5.3, 5.4, 5.5, and 5.6, there was a similar pattern of results with a cross-over function of zero at around set size 20. This may indicate that there is something about 20 which marks a transition from small to large non-symbolic quantity. However, while we have presented data according to the base set size, the magnitude of the test was always larger than the base and depending on the Weber threshold for an individual could be much higher than the base. Therefore, it would be important to see how the SNARC-like effect changes as a function of the test magnitude rather than the base magnitude.

5.4.6 Non-symbolic SNARC-like effect for Base and Test Data

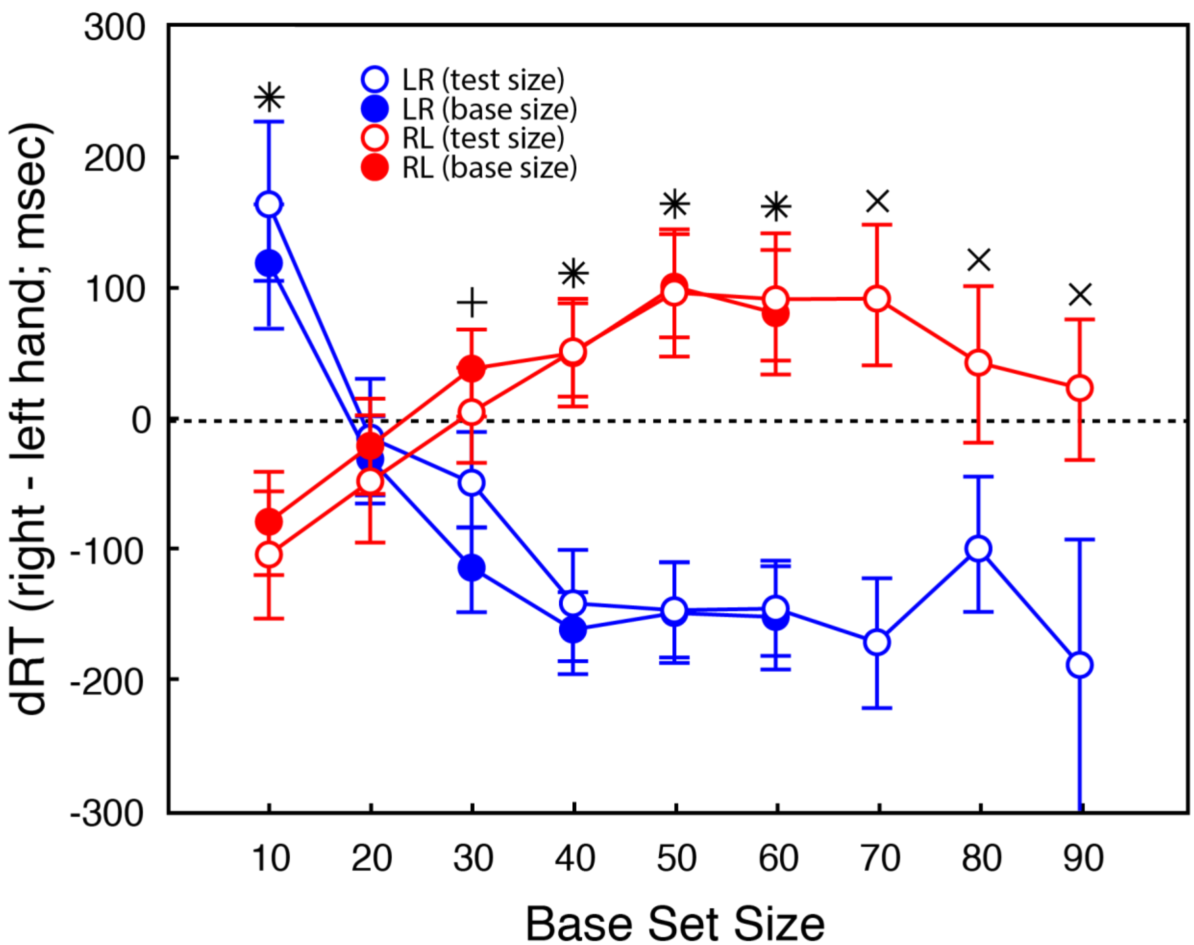

Figure 5.7. dRTs in milliseconds for hand response direction (correct responses only) plotted as a function of base (i.e. 10, 20, 30, 40, 50 or 60) and test set size binned as decades (10, 20, 30, 40, 50, 60, 70, 80 and 90).

Note. The red lines depict right-to-left mapping (RL) for base (red hollow circular data points) and test responses (red filled circular data points). The blue lines depict left-to-right mapping (LR) for base (blue hollow circular data points) and test responses (blue filled circular data points). The '+' or '×' indicate a significant difference between response conditions at 0.5 significance level which was based on the bootstrapped t-statistic of a paired-sample *t*-test. The '+' above a given set size indicates for base set sizes that the left-to-right response condition differs from the right-to-left response condition from the 95th percentile value. The '×' denotes a significant difference for between left-to-right and right-to-left response conditions for target magnitude. If both are significant, the overlapped symbol appears as an asterisk. Error bars represent 95% confidence intervals of the bootstrapped mean. A smoothing of lines was achieved using a Gaussian function.

To determine if there was a difference for mapping condition (i.e. a non-symbolic SNARC-like effect) for judgments made on base set size versus target magnitude we performed a series of paired-samples permutation *t*-tests for each base set size which compared response mapping direction and for each target magnitude binned in increments of 10. Eighteen comparisons were conducted separately, one for each base set size (10, 20, 30, 40, 50 and 60 dots) and for each target magnitude binned in 10s (10, 20, 30, 40, 50, 60, 70, 80, 90 dots) for the left-to-right and the right-to-left mapping conditions for each performance group. The data for 100 dots was not included as there was too few data points. The trend in the data confirmed as statistically significant by *t*-tests for Figure 5.7 suggests there was a difference across the mapping conditions for judgments made on base set size and target providing evidence for a non-symbolic SNARC-like effect. However, this data suggests that a non-symbolic SNARC-like effect was found, and that effect seemed to be driven by base set size rather than target magnitude.

Overall, the findings suggest that individual differences in the Weber fraction were not related to individual differences in the non-symbolic SNARC-like effect. There was no difference in the non-symbolic SNARC-like effect for participants with low or high numerical acuity or for correct or incorrect responses. This suggests that the Weber fraction cannot account for individual differences in the non-symbolic SNARC-like effect.

5.5 Discussion

We examined non-symbolic SNARC-like effect judgments for participants with low and high numerical acuity ability. The aim was to examine if the Weber fraction may account for individual differences in non-symbolic SNARC-like judgments. It was predicted that individual differences in the Weber fraction as measured by an adaptive staircase would be related to individual differences in the mapping of right- and left-hand responses to ‘smaller’ and ‘larger’ numerical magnitude stimuli (i.e. the non-symbolic SNARC-like effect). A brief discussion of the findings for Study 3 will be undertaken here, followed by a more comprehensive discussion in the in Chapter 6.

There were five findings of note: First, a non-symbolic SNARC-like effect was consistently found across differing ratios (low and high), presentation order (referent or test presented first) Weber fractions (low and high numerical ability), Performance level and correct and incorrect responses with a similar form to the symbolic SNARC effect. Second, for the smaller ratio slower RTs were found and a more pronounced non-symbolic SNARC-like effect than for the larger ratio. Third, judgments seemed to be made on non-symbolic quantity of base set size rather than target. Fourth, there were distinct individual differences in RT and accuracy of responses for the Weber and SNARC task.

A non-symbolic SNARC-like effect was found for non-symbolic quantities in the ANS range which supports and extends the symbolic SNARC effect Dehaene et al., (1993) and supports previous research that has shown a non-symbolic SNARC-like effect (Mitchell et al., 2012; Nuerk et al., 2005; Patro & Haman, 2012; Zhou et al., 2016). Findings showed there was a left-to-right response mapping bias**.** The biggest effect of non-symbolic quantity was found for SNARC congruent trials when left is lesser, right is more. A lesser effect was found for SNARC incongruent trials when left is more, right is lesser. Perhaps for incongruent trials participants had to consciously think about swapping (or inhibiting) natural response mapping so slower responses were made with the left and right hand. This finding supports Gevers et al., (2006) who conducted an EEG study on the symbolic SNARC effect. Gevers et al., (2006) and showed that for the incongruent SNARC condition a dip in the lateralised readiness potential (an EEG component that occurs just before preparation of motor response) was found towards preparation of the incorrect response but not for the congruent SNARC condition.

Findings for RT data showed a more pronounced non-symbolic SNARC-like effect for variable low ratios (at participant threshold) than high ratios (twice participant threshold). It is likely based on the Weber fraction participants found the low ratio harder to judge than the high ratio. This was evidenced by slower RTs for the low ratio and a more pronounced SNARC effect. This supports Wood et al., (2008) who found tasks that increased processing time of numerical stimuli show a more pronounced symbolic SNARC effect.

5.5.1 Calibration and Discrimination of Responses

It is possible that participants who showed poor levels of accuracy for the Weber fraction and SNARC tasks had issues of calibration for numerical stimuli. No feedback was given during the experiment, so participants were not able to anchor responses on a correct example or recalibrate and correct responses. This supports Burgess and Barlow (1983) who argue that when participants make consistent errors when judging the number of dots in an array, this is due to an issue of calibration of the stimuli.

5.5.2 The Effect of Set Size on Discrimination

It is possible participants use different strategies to discriminate numerical magnitude based on set size. For smaller set sizes such as base 10 participants might try a different strategy to enumerate the dots such as groupitising and adding. Using visual statistics, smaller groups of elements are perceived separately. These groups can be subitized and the total number of dots quickly enumerated. This supports Burgess and Barlow (1983) who argue that for set sizes under 10 dots another mechanism such as subitizing may be activated.

For larger set sizes findings showed a similar pattern of RT responses for judgments of base numbers 30, 40, 50 and 60. A possible reason for this is that participants were not able to distinguish these base set sizes from each other. Instead, they may have been making the judgment based on if the target was more or lesser than the referent. This supports Burgess and Barlow (1983) who showed length of viewing time of stimuli did not seem to influence accuracy for 20 to and 50 dots. Burgess and Barlow claim that over set size 20 observers are unable to discriminate set sizes exactly and are only able to tell if they are ‘more’ or ‘less’ than each other. Evidence for this claim was provided by Nieder and Merten (2007) who examined numerosities 1 – 30 and found there were less neurons tuned to larger numerosities. This may be due to the nature of representation of numerical magnitude in the brain.

There seemed to be something significant about set sizes containing 20 or more dots as there was a cross over point of 0 for function lines for response mapping condition. Most symbolic SNARC studies use small magnitude stimuli (i.e. 1 – 9). However, a few studies have examined the SNARC effect with larger magnitudes. Findings support Zhou, Chen, Chen, and Dong (2008) who used symbolic stimuli ranging from 12 to 98 and found a similar SNARC cross over point of zero between 12 and 22 dots. Findings also support Zhou et al., (2016) who found a non-symbolic SNARC-like effect with a cross over point of 0 for arrays of 17 dots.

5.5.3 Individual Differences in the Weber Fraction

Findings showed individual differences for numerical acuity ability as evidenced by the Weber fraction (M = 0.27, SD = 0.32) 13 participants showed a very low Weber fraction (i.e. < 0.1), 16 showed a high Weber fraction (i.e. > .33), and 57 participants showed a moderate Weber fraction (i.e. between 0.1 and 0.33). It is worth noting that Ross (2003) found the average Weber fraction for base numbers ranging from 8 to 60 was 0.25. There was a slight decline in the Weber fraction for base numbers below 16 and above 32. van Oeffelen and Vos (1982) found a Weber fraction of 0.11. However, low-level visual cues such as density and luminance were confounded with numerosity. Piazza et al., (2004) found a Weber fraction of 0.17 when controlling for low level visual cues. Also, feedback was given in the van Oeffelen & Vos (1982) and Piazza et al., (2004) studies which could explain why a lower Weber threshold was found. Izard et al., (2004) found an average Weber threshold of 0.22. This showed that a similar Weber fraction was found for this sample of participants and that it is likely the adaptive staircase was a good method to measure the Weber fraction.

However, although we found individual differences in the Weber fraction, we did not find evidence these differences could account for the non-symbolic SNARC-like effect. Findings showed a similar non-symbolic SNARC-like effect for low ability and high ability groups. This suggests parameters other than non-symbolic quantity may be influencing judgments.

5.5.4 Individual Differences and Strategies for Non-symbolic Quantity Judgments

An interesting finding was that participants showed a robust SNARC effect regardless of whether their responses were correct or incorrect. A possible explanation for this finding is the SNARC effect was based on passive viewing of low-level visual cues instead of non-symbolic quantity. Passive viewing of numerical stimuli from EEG and fMRI studies elicits a response in the brain (Izard et al., 2009). Gebuis and Reynvoet (2013) examined passive and active viewing of non-symbolic quantity using EEG and found no effects of quantity in the passive or active conditions. Although, when data was grouped according to low level visual cues strong effects of visual cues were found in parietal and occipital areas of the brain. Differing levels of activation were not found for different non-symbolic quantities. Gebuis and Reynvoet (2013) claim that this finding shows evidence for general cognitive processing rather than processing of non-symbolic quantity itself (also see Gebuis & Reynvoet, 2012).

It is possible for the range of set sizes different strategies were used to make judgments. Unlike the symbolic SNARC effect where judgments are made on symbolic number, which is precise, judgments for the non-symbolic SNARC-like effect in the ANS range can involve different strategies. In the statistical learning model, participants change strategy until they find the most effective strategy for the task. It is possible, participants were making judgments based on low-level visual cues such as density or total surface area rather than numerosity. Although an attempt was made to control for some low-level visual cues by manipulating these cues so that they were less indicative of numerical magnitude, it is not possible to rule these cues out completely. It is possible participants with larger Weber fractions (poorer numerical acuity) were unable to discriminate sets of dots using non-symbolic quantity so were more likely to use low-level visual cues.

It is claimed that the symbolic SNARC effect directly activates the mental number line (Dehaene et al., 1993). However, for the non-symbolic SNARC-like effect in the ANS ranges an approximation of non-symbolic quantity may have activated the mental number line indirectly through the ANS. Findings suggest another possibility - the polarity correspondence account of the SNARC effect where verbal coding of ‘more’ and ‘lesser’ activate spatial coding responses of left and right. Findings showed faster responses with the left hand to smaller base set sizes (i.e. referent 10) and with the right hand to larger base set sizes (i.e. referents, 30, 40, 50 and 60).

Previous research on the non-symbolic SNARC-like effect has shown that an explicit numerical judgment of ‘more’ or ‘less’ seems to be necessary to activate the effect (Cleland & Bull, 2018; Cleland et al., in press).

5.5.5 Limitations and Implications for Further Research

Participants showed individual differences for accuracy and RT (see Appendix C and Appendix D). However, some of these participants with similar response signatures could be grouped together. Similar to the symbolic SNARC effect it is likely only some of the participants in the current study showed a non-symbolic SNARC-like effect and that these individuals were driving a group level non-symbolic SNARC-like effect. A recent study by Cipora et al. (in press) examined individual differences in the symbolic SNARC effect using parametric and bootstrapping methods. Results of the study suggest that SNARC effect at the group level was driven by a subset of individuals. Future research should examine mechanisms of the non-symbolic SNARC-like effect using other statistical methods such as latent class modelling to develop profiles of shared patterns of individual differences in the signature of accuracy and RT response measures. Latent class analysis that enables participants with similar responses to be clustered together in subgroups. This may enable a way to disentangle participants who are using non-symbolic quantity to make judgments from those using low-level visual cues. However, latent class requires large sample sizes so a larger sample would be needed for this analysis.

5.5.6 Conclusion

It has been proposed that individual differences in the Weber fraction may explain non-symbolic SNARC-like effect judgments. We examined this claim using an adaptive staircase to measure the Weber fraction for each individual. However, the findings did not support this claim as participants who performed well showed a similar non-symbolic quantity SNARC-like effect to those who performed poorly.

Chapter 6: General Discussion

6.1 Rationale and Research Questions

The aim of the research reported in this thesis was to examine if non-symbolic quantity in the ANS range is represented spatially from smallest-to-largest, left-to-right as evidenced by a non-symbolic SNARC-like effect. The motivation for the research was twofold. First, it has been claimed that the non-symbolic SNARC-like effect could be used to examine the spatial representation of non-symbolic quantity. The symbolic SNARC effect is considered evidence for the spatial representation of non-symbolic quantity (Dehaene et al., 1993); yet the origin of the symbolic SNARC effect is not known. While there is some evidence for a non-symbolic SNARC-like effect (Bulf, et al., 2014; Ebersbach, et al., 2014; Lee, et al., 2016; Luccio, et al., 2012; Mitchell, et al., 2012; Nuerk, et al., 2005; Patro & Haman, 2012; Patro & Shaki, 2016; Zhou et al., 2016), interpreting previous research into the non-symbolic SNARC-like effect is problematic because of methodological issues. Specifically, most previous studies used stimuli that overlap the subitizing and ANS ranges. It is proposed that these systems have differing RT and accuracy signatures, and cognitive functions (Dehaene, 1997; Mandler & Shebo, 1982). Further, the relationship between non-symbolic quantity and space has often been assessed using symbolic SNARC effect methodology with group level analyses, such as ANOVA and linear regression. These analytic methods collapse data across participants and/or magnitudes (Dehaene et al., 1993; Keus et al., 2005; Zhou et al., 2016). It is possible that relationships may differ for non-symbolic quantity representation compared to symbolic number representation. Processing of symbolic number is precise and does not depend on the ratio of referent-to-target. In contrast, processing of non-symbolic quantity is approximate and is dependent on individual differences in numerical acuity and the Weber fraction. We used relative ratios since the meaning of distance effects using non-symbolic distance stimuli is unclear. Relative ratios are claimed to be more robust indicators of non-symbolic discrimination abilities (Libertus & Brannon, 2010).

The second motivation was to better understand the representation of non-symbolic quantity. It has been proposed that non-symbolic quantity is represented by two systems: a precise system for small quantities (i.e. ~1 – 4 elements) known as the subitizing range, and an approximate system for larger quantities (i.e. > 4 elements) known as the ANS range (Dehaene, 1997; Inglis & Gilmore, 2014; Mandler & Shebo, 1982; Trick & Pylyshyn, 1994).

It has been proposed that the ANS can be measured independently of low-level visual cues and is used as foundation for mapping of symbolic number onto non-symbolic quantity representations (Dehaene, 1997). However, a recent proposition has challenged this claim. Sensory integration theory proposes instead that the influence of low-level visual cues cannot be eliminated from processing of non-symbolic quantity and therefore non-symbolic quantity judgments (Gebuis et al., 2016; Gevers et al., 2016). Previous research has highlighted the difficulty in measuring the ANS, since visual cues such as density, luminance and dot size have been shown to affect spatial representation (Fumarola et al., 2014; Ren et al., 2011 Szűcs et al., 2013). Insofar as these two claims can be substantiated, it was predicted that a non-symbolic SNARC-like effect would be found.

The research was designed to investigate these three issues. Study 1 (Chapter 3) examined if non-symbolic quantity in the ANS range was represented spatially in the form of a non-symbolic SNARC-like effect and if this effect is related to domain general cognitive processes such as visuospatial working memory and/or math ability. Study 2 (Chapter 4) examined characteristics of the non-symbolic SNARC-like effect for differing quantities, when low-level visual cues such as total surface area and size of dot stimuli were manipulated and if other response modes such as foot responses elicited a SNARC-like effect. Finally, Study 3 (Chapter 5) aimed to investigate whether individual differences in the Weber fraction could account for non-symbolic SNARC-like effect judgments. The purpose of this chapter is to review key findings of each study, followed by a discussion of broader implications of these findings for non-symbolic quantity-space accounts, and finally, to suggest directions for future research.

6.2 Key Research Findings

The research reported in this thesis examined the spatial representation of non-symbolic quantity in the ANS range as evidenced by a non-symbolic SNARC-like effect. The key findings were as follows. First, a non-symbolic SNARC-like effect was found consistently, providing support for the spatial representation of non-symbolic quantity. Study 1 provided evidence for the non-symbolic SNARC-like effect: faster responses were found with the left hand for smaller non-symbolic quantities than the referent – a display of 30 dots, and with the right hand for larger non-symbolic quantities than the referent. This suggests non-numerical quantity in the ANS range is represented spatially. The findings of Study 2A, which employed only two target stimuli with the ‘easy’ referent-to-target ratio of 0.5, provided further support for a non-symbolic SNARC-like effect. Study 2B, in which again only two target stimuli with the ‘easy-ish’ referent-to-target ratio of 0.66 but where the potential cueing effect of display darkness was controlled by using an equal number of black and white dots, resulted in longer processing times, slower RTs and a more pronounced non-symbolic SNARC-like effect. Interestingly, the form of the non-symbolic SNARC-like effect was similar to the symbolic SNARC effect for larger but not for smaller non-symbolic quantity. The combined analysis of slope coefficients for all participants provided additional support for a non-symbolic SNARC-like effect. Study 2C, where participants responded with their feet rather than their hands, also supported the occurrence of the non-symbolic SNARC-like effect independently of response modality. Study 3, in which an adaptive staircase was used to measure the Weber fraction for each individual to calibrate the ratio of referent-to-target for the non-symbolic quantity judgment task, but where total surface area, density and display darkness were controlled using different sized dots in each array and scaling factors provided additional and compelling support for the existence of a non-symbolic SNARC-like effect.

Second, similar to a distance effect, the ratio of referent-to-target seems to influence both reaction time and accuracy of responses. As the results of Studies 1, 2B, 2C, and 3 demonstrate, targets closer in magnitude to the referent are processed more slowly and less accurately. This finding suggests that the non-symbolic quantity judgment task was activating the ANS.

Third, as is evident from the findings of Studies 1 and 2, not all participants showed a non-symbolic SNARC-like effect. Some participants displayed a reversed non-symbolic SNARC-like effect, while one participant in Study 1 did not show any SNARC-like effect. Taken together, these findings indicate the existence of individual differences in the spatial representation of non-symbolic quantity.

Fourth, as the findings of Study 3 demonstrate, calibration appears to be a factor in the design of paradigms testing the non-symbolic SNARC-like effect as some ratios of referent-to-target may be too difficult for individuals with poorer non-symbolic quantity acuity and a larger Weber fraction.

Fifth, models of the non-symbolic SNARC-like effect appear robust to manipulation of visual cues such as total surface area, density, roundness and darkness of display, and occur for other modalities of response such as feet (Studies 2 and 3). However, these findings should be treated with caution as it is not known the extent to which these low-level parameters may have mediated judgments.

The key research findings demonstrate that the relationship between non-symbolic quantity and space is heterogeneous with wide variance in non-symbolic quantity processing found between individuals. This may be due to individual differences in the Weber fraction, use of different strategies, such as making judgements on one or several low-level visual cues, instead of non-symbolic quantity alone. These findings extend current models of the symbolic SNARC effect and non-symbolic SNARC-like effect for non-symbolic quantity in the ANS range. This suggests that the relationship between non-symbolic quantity and space has further complexity to account for than that of symbolic number and space. In the following section, theoretical and practical implications of the research are discussed.

6.3 Implications of the Research Findings

The purpose of this section is to discuss broader implications of the findings in relation to previous research. First, findings are discussed in relation to existing models of the symbolic and non-symbolic SNARC-like effect, theories of the ANS, and individual differences in the spatial representation of non-symbolic quantity. Second, theoretical implications of the findings for the non-symbolic SNARC-like effect will be discussed. Finally, practical implications of the findings for education will be discussed.

6.3.1 The Non-Symbolic SNARC-like Effect

In Chapter 2 it was argued that (1) number is associated with space as evidenced by the SNARC effect, however most research examining this relationship has focused on symbolic number; and (2) previous research on the non-symbolic SNARC-like effect has methodological issues due to the use of stimuli spanning both the subitizing and the ANS ranges even though the ranges are thought to be two separate number representational systems with differing response characteristics (Dehaene, 1997; Mandler & Shebo, 1982). In this research, this issue was addressed by using referent-to-target paradigm with non-symbolic quantity stimuli purely in the ANS range. The findings presented in this thesis indicate a non-symbolic SNARC-like effect was found for non-symbolic quantity.

All the studies reported in this thesis avoid the interpretive limitations of previous studies (Bulf et al., 2014; Nuerk et al., 2005; Mitchell et al., 2011; Patro & Haman, 2012; but c.f. Zhou et al., 2016) in that stimuli did not span both the subitizing (1 - 4 ~ items), counting (> 4 ~ items), and ANS ranges (>10 ~ items)[3]. In Studies 1 and 2 the brief presentation of dot arrays purely in the ANS range (15 – 60 items) and in Study 3 (11 – 100 items) meant non-symbolic quantity could not readily be converted to symbolic number. Moreover, neither enumeration nor subvocalizing could be used to make non-symbolic quantity judgments

[3] See Chapter 2, section(s) …

6.3.1.1 Is the SNARC effect confined to symbolic number?

A key difference between the symbolic SNARC and the non-symbolic SNARC-like effects is there seems to be a right-hand bias for the non-symbolic SNARC-like effect. This right-hand bias was robust over Studies 1 and 2 and supports the work of Zhou et al., (2016). A possible explanation of this finding is that we used strongly right-handed participants[4]. Participants may therefore have been faster to respond with the right hand to smaller quantities, thus minimising the effect for the left hand (i.e. less of a difference was found for smaller quantities). It is possible that as only stimuli in the ANS range were used, a stronger SNARC effect was found with the right hand to large stimuli. This differs from the majority of symbolic SNARC studies that use stimuli from 1 – 10. Perhaps using stimuli with larger quantity coupled with a stronger effect for the right hand washed out the effect for the left hand.

Study 2C shows that the non-symbolic SNARC-like effect is not specific to manual responses but generalises to other response modalities. Faster responses were found for quantities smaller than the referent with the left foot and for magnitudes larger than the referent with the right foot. A symbolic SNARC effect has been found for other effectors such as eyes (i.e., saccades) and feet. This finding supports the findings of Schwarz and Müller (2006), who found a symbolic SNARC effect for foot responses and Schwarz and Keus (2004) who found a symbolic SNARC effect with eyes (saccades). Dehaene et al., (1993) claim that the symbolic SNARC effect with hand responses is related to reading and writing direction. This claim was challenged by Schwarz and Müller, whose symbolic SNARC effect for foot responses suggested that the SNARC effect is not just an overlearned stimulus-response sensorimotor effect as responding left or right with feet is not related to reading or writing. Instead it may be related to an internal spatial representation of number. In support of Schwarz and Müller, the results of Study 2C suggest that the non-symbolic SNARC-like effect is evidence for the spatial representation of non-symbolic quantity.

6.3.1.2 Spatial representation of non-symbolic quantity

The studies contained in this thesis provide strong support for the claim that non-symbolic quantity is represented spatially. In Studies 1, 2 and 3, a non-symbolic SNARC-like effect was found, supporting previous SNARC research that has shown evidence for the spatial representation of symbolic number (Dehaene et al., 1993; Wood et al., 2008) and non-symbolic

[4] Participants' handedness was assessed using the Edinburgh Handedness Inventory (Oldfield, 1971).

quantity (Bulf et al., 2014; Nuerk et al., 2005; Patro & Haman, 2012, Mitchell et al., 2011; Zhou et al., 2016).

The spatial representation of number is important as it is regarded as providing a more concrete way for an abstract concept such as number to be visualized, stored, accessed, and manipulated in the brain (Dehaene et al., 1993)[5]. In a mathematical context, space is thought to enable coding of numerical information, support place order of numbers and allow comparison and computation of numbers. Our findings show the importance of acknowledging the complexity of the relationship between spatial and non-symbolic quantity processing. Recently, Fias and Bonato (2018) have argued that math ability and spatial processing do not co-exist in a simple one-to-one relationship. There are nuances and complexities within this relationship: certain spatial processes may be linked to certain non-symbolic quantity processes but not others. For example, a number of studies have shown that those with better spatial ability tend to perform better on math tasks such as number estimation (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007), counting (Kyttälä, Aunio, Lehto, Van Luit, & Hautamaki, 2003) and overall math performance (Meyer, Salimpoor, Wu, Geary, & Menon, 2010). Cheng and Mix (2014) have shown that mental rotation training improved math performance. Also, specific tasks such as 3D mental rotation have been used to measure individual differences in spatial visualisation (Viarouge et al., 2014). Hawes and Ansari (2020) define spatial visualisation as being able to generate, maintain, recall, and manipulate visual-spatial images.

Hawes and Ansari (2020) reviewed evidence from neuroscience, psychology and education and argue that specific spatial processes such as spatial visualisation may facilitate numerical reasoning skills such as number sense and operations. Hawes and Ansari (2020) further argue that the relationship between number and space is complex and propose four accounts to explain spatial visualisation skills and numerical competencies. Hawes and Ansari (2020) argue that these accounts work in tandem to produce spatial numerical associations. In the working memory account, VSWM and spatial visualisation tasks such as mental rotation are strongly related to basic measures of numerical competency such as numerical comparison, arithmetic and number line estimation (Kaufman, 2007; Kyttälä, et al., 2003; Mix et al., 2016). VSWM may mediate the relationship between spatial visualisation and math.

[5] See Chapter 2 section …

The influence of visual cues on non-symbolic quantity judgments in the ANS range

The ANS account (Dehaene, 1997) claims that quantity is extracted independently of visual cues. In contrast, the sensory integration account (Gebuis et al., 2016; Gevers et al, 2016) suggests that visual cues influence quantity judgments. Although we did manipulate some visual cues such as luminance in Study 2A and 2B as well as dot size (as quantity increased, dot size decreased) and total surface area in Study 2C, and used dots of differing sizes in one array, controlling for total surface are, roundness of display and density of dot in Study 3 we did not strictly control for these visual cues. The interpretation of findings, although consistent, are open to interpretation. Our study design did not enable us to determine if participants were using low-level visual cues to make judgments (see e.g., Gebuis & Reynvoet, 2011; 2012). Nor did our design enable us to conduct post-hoc analysis to examine the influence of visual cues on non-symbolic quantity judgments (see Gebuis & Reynvoet, 2011). This would have required several arrays to be created that each control only one visual cue. These arrays would then be presented in interleaved trials which could be grouped according to control of each visual cue for analysis. So, we cannot draw firm conclusions on the role of visual cues in this thesis. Some researchers have used varied dot sizes within the same display to control for total surface area and remove dot size as a reliable cue (Zhou et al., 2016). Others have created specific program protocols to stringently control low-level visual cues (Gebuis & Reynvoet, 2011). However, low-level visual cues occur naturally in our environment and can be used to discriminate quantity. Perhaps in differing situations where low-level cues are more relevant to quantity judgments the weight of these visual cues can be varied according to task demands. However, research has shown that number can influence both area and density, suggesting number is more fundamental than these continuous magnitudes (Burr, 2017). Also, despite number and magnitude co-varying, continuous magnitudes alone do not influence responses (Beran & Parrish, 2017). For example, Zhou et al., (2016) found a non-symbolic SNARC-like effect for non-symbolic quantity but not for continuous magnitudes such as area or density. This suggests that continuous magnitudes alone are not responsible for the non-symbolic SNARC-like effect found in Studies 1, 2, and 3.

6.3.1.3 The non-symbolic SNARC-like effect: Ratio and the ANS

In Chapter 2 it was argued that a non-symbolic SNARC-like effect with stimuli in the ANS range could activate the ANS. It was further claimed that the closer a ratio is to 1 the slower RTs and the higher error rates would be for accuracy (ratio effect). The findings of Studies 1 and 2 showed clear ratio effects for RT and accuracy measures conforming to Weber’s law: longer RTs and lower accuracy were observed as ratio and difficulty increased. This supports the work of

Patro and Haman, (2012) who found ratio effects for the non-symbolic SNARC like effect. This finding suggests that the ANS was indeed activated to complete the non-symbolic quantity judgment task, and that the referent-to-target paradigm used in Studies 1 and 2 was a good measure of the construct of the ANS. Although Study 3 showed a difference for low and high ratios, this difference was not significant. The ANS account posits that ratio effects occur due to overlap in tuning curves of number specific neurons in the brain (Piazza et al., 2004).

However, it is important to note that in Study 1, several participants were excluded for not being able to differentiate the 0.83 ratio above chance level (50%). This suggests that the 0.83 ratio was too hard to discriminate for some participants. It has been found that people who have better non-symbolic quantity acuity tend to respond with greater speed and accuracy for magnitude judgments (Halberda et al., 2008). That is, task difficulty is affected by changes in the ratio between set sizes of dot arrays (Lyons, Nuerk, & Ansari, 2015), with smaller ratios being harder to distinguish from each other. Individual differences in ratio effects for non-symbolic quantity and distance effects for symbolic number may also correlate with more complex mathematical components (e.g., multiplication) (Lyons et al., 2015). Buckley and Gillman (1974) have shown a size and distance effect for non-symbolic numerical magnitudes and symbolic number. Similarly, Temple and Posner (1998) found distance effects for both adults and children. Taken together, these studies suggest that the ratio effect and processing of stimuli the in ANS range may show some similarities to the distance effect for symbolic number processing. The outcome of Studies 1, 2, and 3 suggest that individual differences in non-symbolic quantity discrimination are important to consider for the non-symbolic SNARC-like effect. In the following section individual differences in the numerical cognition and the spatial representation of non-symbolic quantity will be discussed.

6.3.1.4 Individual differences in numerical cognition

In Chapter 2 it was claimed that (1) individuals show markedly different Weber fraction indices in non-symbolic quantity tasks; and (2) individual differences are important to consider when utilising stimuli in the ANS range as people have differing levels of acuity when judging approximate quantity. Yet, the mechanisms behind these individual differences in non-symbolic quantity processing and the spatial representation of non-symbolic quantity and how they relate to math ability and domain general processes such as VSWM are still poorly understood. In Study 1 and 2 reported here, individual differences were found in the size and direction of the non-symbolic SNARC-like effect as well as the ability to discriminate dot arrays. This finding supports the existence of individual differences in the SNARC-like effect and the spatial representation of non-symbolic quantity. However, although Study 1 and 2 showed marked variability in SNARC responses this was not the focus of these studies. This pointed to the need to specifically address the potential mechanisms behind these individual differences. This was addressed in Study 3 where individual differences in the ability to make non-symbolic quantity judgments (the Weber fraction) were examined to see if this could account for non-symbolic quantity judgments. An adaptive staircase method was used to measure the threshold of non-symbolic quantity acuity (Weber fraction). This enabled the calibration of ratio of referent to target for each individual in the non-symbolic quantity judgment task. We used variable ratios at threshold and twice threshold because if targets were at or above a participant's threshold discrimination should increase, encouraging judgments based on non-symbolic quantity rather than low-level cues. However, the findings did not support this claim as participants with a high level of performance showed a similar SNARC-like effect to those who performed poorly. This suggested that the non-symbolic SNARC-like effect for some participants may reflect the spatial representation of low-level cues. For example, some participants may have used low-level visual cues such as density, total surface area or dot size to make judgments. As previously mentioned, a SNARC-like effect has been found for low-level cues such as luminance (Fumarola et al., 2014), dot size (Ren et al., 2011) as well as other non-numerical domains (see Macnamara, et al., 2018 for a recent review). This suggests these low-level cues may be represented spatially too. Therefore, the SNARC-like effect for some participants may have be based on the spatial representation of these low-level cues rather than quantity.

Non-symbolic SNARC-like effect and math ability

In Chapter 2 it was claimed that non-symbolic quantity representation may be the foundation for more complex mathematics and the ability to tell how many items are in a set, may be linked to later achievement in math. However, this claim was not supported. In Studies 1 and 2 we did not find a direct relationship between the non-symbolic SNARC-like effect and math performance as measured by the WRAT math computation task. This finding contrasts with that of Halberda et al., (2008) who found a link between ANS acuity and math performance. However, the non-symbolic SNARC-like effect and the Weber fraction are not directly related and measure differing aspects of non-symbolic quantity processing. The Weber fraction is related to numerical acuity and the non-symbolic SNARC-like effect is related to the spatial representation of non-symbolic quantity. The results of Studies 1 and 2 support previous research that has shown that processing of symbolic number is more likely to correlate with math performance than non-symbolic quantity (De Smedt & Gilmore, 2011; Holloway & Ansari, 2009, see Chen & Li, 2014 for a meta-analysis). We did not directly measure math ability in Study 3.

Non-symbolic *SNARC-like effect and VSWM*

In Study 2A a moderate positive relationship was found between right – left hand difference RT and VSWM. Participants with better VSWM performance showed a more pronounced non-symbolic SNARC-like effect. This suggests the non-symbolic SNARC-like effect has a spatial component, and supports Viarouge et al., (2014) who found a relationship between the symbolic SNARC effect and VSWM using a mental rotation task. Moreover, using a dual task paradigm, Hererra, Macizo, and Semenza (2008) found that when items were held in VSWM the symbolic SNARC effect disappeared. Additionally, children with impaired VSWM do not show a symbolic SNARC effect (Bachot, Gevers, Fias, & Roeyers, 2005). However, in Studies 1 and 2B, individual differences in the non-symbolic SNARC-like effect did not correlate with VSWM. Taken together, these findings provide partial support for the claim that VSWM is related to individual differences in the non-symbolic SNARC-like effect and constitute further evidence for the spatial representation of non-symbolic quantity.

6.3.1.5 Implications of findings for theoretical accounts of the SNARC effect

Several accounts of the symbolic SNARC effect have been proposed: the mental number line (Dehaene et al., 1993); polarity coding (Proctor & Cho, 2006); and the working memory account (van Dijck & Fias, 2011). The mental number line has been described as a spatial representation account of the SNARC effect, polarity coding as a stimulus-response, correspondence account and working memory as a strategic account. Findings from Study 1, 2 and 3 show support for the claim that the mental number line account of the symbolic SNARC effect can be extended to include non-symbolic quantity.

Dehaene et al. (1993) claimed that the symbolic SNARC effect reflects activation of a left-to-right number line. Dehaene further discussed this claim within the framework of the triple-code model of numerical cognition (Dehaene, 1992). According to Dehaene et al., the symbolic SNARC effect occurs due to an automatic Arabic numeral-to-analog transcoding process. Although the mental number line account of the symbolic SNARC effect is intuitive, deeming it the only framework to explain mechanisms of the SNARC effect is problematic. Mechanisms underlying the symbolic SNARC effect may well be more complex and go beyond automatic transcoding. For instance, an issue with the mental number line account of the SNARC effect is it does not explain some aspects of the SNARC literature such as the flexible spatial representation of number (see e.g., Bächtold et al., 1998). Strategic accounts of the symbolic SNARC effect such as the working memory account claim that the SNARC occurs as result of activations of visuo-spatial representations in short-term memory (Fias et al., 2011). These short-term spatial representations are built during a task as opposed to activation of spatial representation of number in long-term memory. A recent study by Cleland et al., (in press) failed to find a non-symbolic SNARC-like effect across four experiments and argue that this provides evidence for the working memory account of the SNARC effect as non-symbolic, non-canonical stimuli are difficult to conceptualise in an ordinal sequence. However, we did not test the working memory account of the SNARC directly in Studies 1, 2, and 3, so cannot draw further conclusions about this theory.

The non-symbolic SNARC-like effect found in Studies 1, 2, and 3 can also be explained, as an extension of the polarity correspondence account (Proctor & Cho, 2006). As mentioned in Chapter 2, the polarity correspondence account can be used to explain speeded binary decisions such as those required for numerical judgments. In Studies 1, 2, and 3, non-symbolic quantity judgments of target stimuli were made using verbal coding labels of ‘more’ or ‘less’ than the referent. It has been argued that the symbolic SNARC effect is an example of polarity correspondence theory (Proctor & Cho, 2006). For instance, in a target-to referent non-symbolic quantity judgment task for the symbolic SNARC effect, stimulus and response categories could be coded as lesser (polarity -1) more (polarity +1), left (polarity -1) and right (polarity +1). In this example lesser/left and more/right have the same polarity so a non-symbolic SNARC-like effect would be found.

In Chapter 2 it was claimed that the symbolic SNARC effect taps into the mental number line and demonstrates that symbolic number is spatially represented (Dehaene et al., 1993). Therefore, based on previous claims of the symbolic SNARC effect, a further claim can be made; that a non-symbolic SNARC-like effect provides evidence that non-symbolic quantity is also represented spatially. This extends the theory of the mental number line for symbolic number to include non-symbolic quantity. However, the scope of this thesis does not provide a way to differentiate if a non-symbolic SNARC-like effect suggests that the mental number line is amodal with shared representation for symbolic number and non-symbolic quantity. Or it may be that there are separate mental number lines for symbolic number and for non-symbolic quantity (Weis, Nuerk, & Lachmann, 2018). It is possible that the findings can be explained with the working memory and polarity account. However, as we did not explicitly test the working memory or polarity account of the SNARC. we are unable to make any claims based on these accounts of the SNARC effect. Therefore, the findings are inconclusive regarding other theoretical accounts of the SNARC effect.

6.3.1.6 Theoretical implications: Summary

The findings of this thesis suggest that testing the spatial representation of non-symbolic quantity may be more complex than testing the spatial representation for symbolic number. There are key differences in numerical processing for symbolic number and non-symbolic quantity judgments. The difference between non-symbolic quantity and symbolic number is that the processing of non-symbolic quantities in the ANS are approximate and noisy, whereas the processing of symbolic number is precise. The symbolic SNARC effect is claimed to be based on automatic processing of numerical stimuli (Wood et al., 2008) whereas in the non-symbolic SNARC-like effect this claim is less certain. Previous studies of the non-symbolic SNARC-like effect suggest that an explicit judgment is required to activate a SNARC effect (Cleland & Bull, 2018; Cleland et al., in press). Mitchell et al., (2012) found stimuli in the subitizing range elicited a more pronounced SNARC-like effect perhaps as this is a precise activation of non-symbolic quantity.

A robust non-symbolic SNARC-like effect was found across all five studies, supporting the claim that non-symbolic quantity is represented spatially. However, the interpretation of findings, although consistent, are open to interpretation. Nevertheless, these findings have implications for existing models of the SNARC effect by extending this to include non-symbolic quantity in the ANS range and showing a non-symbolic SNARC-like effect using foot responses. Similarities between the symbolic SNARC and non-symbolic SNARC-like effects suggests that there is a shared neurocognitive basis for the spatial representation of symbolic number and non-symbolic quantity in the brain.

6.3.2 Practical Implications of the Research Findings

Our finding that non-symbolic quantity is spatially represented as evidenced by a non-symbolic SNARC-like effect may have implications for the design of education curriculum and appropriate interventions for children with dyscalculia. However, implications of the current research for dyscalculia are of a speculative nature and await research data. The research reported in this thesis only tested the spatial representation of non-symbolic quantity in normal adults: we can only speculate about children with dyscalculia. However, we know that children with dyscalculia fail to show a SNARC effect for symbolic number (von Aster & Shalev, 2007) and tend to show less accuracy in number line tasks (marking the position of a target number on a line) than controls (Geary, et al., 2008). Children with dyscalculia also tend to have issues with visuospatial memory (Rotzer et al., 2009). These studies suggest that children with dyscalculia may have problems with the spatial representation of symbolic number. Training programs that reinforce the mental number line have had some success for both normal children and those with dyscalculia (Fischer et al., 2012; Kucian et al., 2011). Dyscalculia is complex in that it cannot be attributed solely to one cognitive deficit, genetic abnormality, or damage to a specific neural pathway or region of the brain (Kaufmann et al., 2013; Rubinsten, 2015; Skagerlund & Träff, 2016). From a neurocognitive perspective, number processing activates the IPS sulci and posterior parietal gyri (Nieder, 2005). Yet, there is no current consensus on whether the dyscalculia is related to issues with the ANS or symbolic number brain circuitry (Butterworth, 2010; Cappelletti et al., 2014; Ceulemans, Titeca, Loeys, Hoppenbrouwers, Rousseau, & Desoete, 2014; Cohen Kadosh, Bien, & Sack, 2012; Dehaene et al., 2003; Vogel, Grabner, Schneider, Siegler, & Ansari, 2013). Nevertheless, what is known is that together subitising and the ANS enable understanding of abstract number concepts such as cardinality and ordinality, supporting numerical representation in the brain (Jacob & Nieder, 2008).

Although there is some evidence of a neural basis and genetic contribution to dyscalculia, environmental factors are also important (Butterworth, 2010). For example, the issue may lie with numerical acquisition during schooling. Calculation involves the activation of both complex brain circuitry and the use individual strategies; it is an acquired skill. Children with dyscalculia can fail to understand cardinality (the abstract concept of number itself) or ordinality (place principal or ordering of number). Without an understanding of these concepts, calculation can elude these children (Rapin, 2016). Innate visual sensitivity to small quantities of items (subitising) combines with the ANS and enables the ability to detect differences in the number of items shown in a display, the precision of which reduces as the number of items increases (Feigenson et al., 2004; Mandler & Shebo, 1982). This innate spatial ability has been purported to be analogous to being able to tell the difference between stimulus intensity and magnitude for other sensory modalities (Dehaene, 2003). Early intervention in dyscalculia is critical: the earlier issues are identified, the earlier remediation can take place (Butterworth, 2010).

Those with dyscalculia often show poor ANS acuity (Mazzocco et al., 2011; Piazza et al., 2010). Issues with ANS precision have been linked to poor math ability but not consistently. Similarly, spatial skills have been shown to be predictive of later math ability (Gunderson, Ramirez, Beilock, & Levine, 2012) and are critical for the ANS, comparison (Cantlon, Platt, &

Brannon, 2009) and visualising the mental number line (DeWolf, Grounds, Bassok, Holyoak, 2014).

An EEG training study by Grabner and De Smedt (2012) showed that after just two days of training on multiplication and visual-spatial tasks, increased accuracy and quicker responses were found to both trained and novel multiplication and visual-spatial stimuli. This improvement was related to increased theta brain waves (3 – 6 Hz) in the occipital and parietal regions of the brain. These theta oscillations were interpreted as evidence for practice enhanced learning of spatial and numerical skills. The study by Grabner and De Smedt (2012) demonstrates that even brief spatial-numerical training can lead to a positive outcome.

This thesis supported the important but under-researched role of individual differences in the processing (ratio effect) and spatial representation (non-symbolic SNARC-like effect) of non-symbolic quantity. In a series of studies, a non-symbolic SNARC-like effect was found. Clear individual differences in the ANS were shown, providing further understanding of the mechanisms underlying the non-symbolic SNARC-like effect. There was considerable heterogeneity of responses, with participants differing markedly in their spatial representation and ability to process non-symbolic quantity. The findings support the use of person-centred rather than variable-centred analysis (Dowker, 2005). A better understanding of individual differences in the spatial representation of non-symbolic quantity would help design more appropriate math curriculums by targeting more specific interventions for those with math difficulty.

The findings of this research highlight the importance of the spatial representation of non-symbolic numerical quantity and taking into account individual differences in grasping abstract numerical concepts such as cardinality, ordinality. Both the spatial representation of non-symbolic quantity in the form of a mental number line as well as individual differences in the different components that together form competency with math should be taken into account when designing education curriculum and interventions for those with dyscalculia or math disability where cognition is otherwise normal. Training of ANS acuity and spatial-numerical may enable those with dyscalculia to progress their math ability. Competency with numerical concepts is often needed for career success or just being able to navigate activities of daily life such as shopping and telling the time. Therefore, more research should focus on examining how the brain processes number.

6.4 Future Research Agenda

This section will focus on implications of the thesis findings for future research into the spatial representation of non-symbolic quantity. By considering individual differences of the non-symbolic SNARC-like effect, the studies reported in this thesis took into account SNARC and ANS cognitive perspectives in a more comprehensive manner than previous studies of the spatial representation of non-symbolic quantity.

The findings also show individual differences in calibration for the non-symbolic quantity judgment task. Ratio of test-to-referent influenced the speed and accuracy of responding. Individual differences in non-symbolic quantity abilities such as the Weber fraction point to the possibility some participants found ratios where the target was numerically closer to the referent more difficult than others. It is crucial to examine these relationships both at the group and individual level: without accounting for individual differences in numerical acuity for stimuli in the ANS range subtle differences in the data may be missed. Similarly, the effect of ratio on non-symbolic quantity judgments is unclear if data is collapsed across targets of differing non-symbolic quantities: the data should be also examined separately for each individual ratio.

Regardless of the important contributions of this thesis, the findings are limited by sample size. In all studies we had small sample sizes (*N*) and large individual differences in RT and accuracy of response. The spatial representation of non-symbolic quantity does not appear to be continuous like the spatial representation of symbolic number. Exploring subgroups based on performance signatures may permit closer examination of individual differences in spatial representation of non-symbolic quantity. Typical symbolic SNARC analyses such as ANOVA and linear regression that focus on group level outcomes do not allow in-depth analyses into individual differences. Latent profile analysis allows a fine-grain analysis of individual differences. Scores on continuous variables are used to identify discrete classes of similar individuals. Members of these subgroups are homogenous and tend to exhibit similar response patterns and on a latent variable (Vermunt & Magidson, 2005). Latent profile analysis could be used to identify possible subgroup spatial representation performance of the non-symbolic SNARC-like effect. Individual differences in non-symbolic quantity processing abilities and the spatial representation of non-symbolic quantity may be of great importance during development but its role is less clear for adults.

Although a non-symbolic SNARC-like effect was found for stimuli entirely in the ANS range, we must exercise caution and cannot attribute this result to non-symbolic quantity alone.

We did not control for all low-level visual cues such as dot size, luminance, texture density and total surface area in the same display or in different trials for the same participant. Visual cues such as luminance, dot size and total surface area can be used to make non-symbolic quantity judgments and have been shown to be spatially represented and show ratio effects. The results of Studies 1, 2, and 3 showed a non-symbolic SNARC-like effect suggesting that non-symbolic quantity is represented spatially, though it is unclear if this was indeed the case for all participants or whether one or more low-level visual cues was being represented spatially. Another explanation for Study 1 is that the non-symbolic SNARC-like effect we found was not based on quantity magnitude but instead on luminance (see e.g., Fumarola et al., 2014). However, in two of our studies[6] we controlled for luminance by using stimuli with mixed contrast polarity (i.e., numbers of black and white dots). A non-symbolic SNARC-like effect was still found, evidence that the non-symbolic SNARC-like effect we found was not solely based on a judgment of luminance.

Another explanation for the results of Study 1 is that the non-symbolic SNARC-like effect may have been due to total surface area, which co-varied with magnitude. However, in Study 2C we controlled for total surface area by creating targets that had the same number of pixels regardless of set size and the pattern of results from Study 2C was similar to Study 1. In Study 3, dots of differing sizes were presented in the same display supporting Zhou et al., (2016) who used differing dot sizes in the same display and still found a SNARC-like effect. Findings from Study 1, 2A, 2B are similar to that of Zhou et al., (2016). This suggests participants made judgments based on quantity rather than total surface area. Moreover, a study by Ren et al., (2011) has shown a SNARC-like effect for dot size. Dot that are smaller in size were mapped to the left side of space. Dots that were larger in size were mapped to the right side of space. That is, faster responses were found in the Ren et al., study with the right hand to smaller size dots and with the right hand to larger size dots. Yet in Study 2C physically larger dots that were smaller in non-symbolic quantity were responded to more quickly with the left hand and physically smaller dots that were larger in non-symbolic quantity were responded to more quickly with the right hand. This suggests that in Study 2C judgments were made on the non-symbolic quantity of arrays rather than dot size.

Studies 2A, 2B, 2C, and 3 explored the effect of different visual cues on the non-symbolic SNARC-like effect. However, we cannot rule out the influence of visual cues on the non-

[6] Studies 2B and 2C.

symbolic SNARC-like effect, nor can we control for all visual cues simultaneously. However, given that across the studies reported here the same phenomenon was observed whether visual parameters were controlled or free to vary. This suggests the non-symbolic SNARC-like effect is robust across these controls.

Recent studies of the ANS have shown that some of these results are questionable because of construct validity (Dietrich et al., 2015; Szkudlarek & Brannon, 2017). Further development of good measures of ANS acuity may be able to be used as a screening tool for children with math disability. However, until construct validity issues are properly addressed, an appropriate screening tool can neither be developed nor implemented. Researchers have argued about these controls (i.e. Gebuis & Reynvoet, 2011; Ross & Burr, 2010). Yet it is questionable if research on the ANS is just based on these visual controls. Even if you could control for all visual cues completely would no non-symbolic SNARC-like effect would be found? In everyday life numerosity co-varies with visual cues. Nonetheless, future research could benefit from further examining the role of visual cues for the non-symbolic SNARC-like effect to see the degree to which these cues influence non-symbolic SNARC-like judgments.

Apart from Study 2A, the findings did not identify a consistent relationship between other cognitive factors, such as VSWM or math ability, and the spatial representation of non-symbolic quantity. Further research is required to examine the relationship between VSWM and non-symbolic quantity. However, it is important to note that the correlation between non-symbolic quantity processing and math performance is quite low and less consistent than that of symbolic number processing and math performance.

There is currently stronger evidence for an association between non-symbolic quantity and space in the form of a SNARC-like effect for younger children (Bulf et al., 2016; de Hevia, Izard, Coubart, Spelke & Streri et al., 2014; Ebersbach et al., 2014; Patro & Haman, 2012) and animals such as chimpanzees, (Adachi, 2014) rhesus monkeys, (Drucker & Brannon, 2014) and day old chicks, (Rugani, Vallortigara, Priftis, & Regolin, 2015) than in older children (Berch et al., 1999) and adults (Mitchell et al., 2012; Zhou et al., 2016). A possible explanation for this difference is put forward by Cleland, et al., (in press) who propose that infants may be predisposed to associate non-symbolic quantities and space (de Hevia et al., 2014). However, repeated exposure to symbolic number may shift this association from non-symbolic quantity to symbolic number. Thereby, factors such as reading direction and finger counting may influence this spatial-numerical association for symbolic number. It is less clear how these factors may be related to the non-symbolic SNARC-like effect, but future studies should examine this in more detail.

In terms of the relationship between the non-symbolic SNARC-like effect and math, Cipora, He and Nuerk (in press) acknowledge the complexity of the relationship between the symbolic SNARC effect and math skill. Cipora et al., reviewed studies that examine the relationship between the SNARC effect and math skill. They acknowledge null results have been found across studies as well as a difference of the direction of the relationship between the SNARC effect and math for adults and children (i.e. larger SNARC is linked to between math performance in children but the opposite is generally found in adults).

To address this issue Cipora et al., (in press) propose a multi-route model framework of a SNARC-math skill relationship where several other factors should be considered under differing conditions. For instance, in children differing pathways such as automatic magnitude and/or ordinality processing should be considered. Whereas in adults, flexibility in representation and inhibition of interference may be more important. Cipora et al., suggest the proposed pathways may act in different directions and cancel each other out and recommend considering domain general and domain specific processes cognitive processes that may underlie the relationship between the symbolic SNARC effect and math skill. Some of the known factors to date that seem to influence spatial numerical associations are age, experience, general cognitive abilities, learning and practise of math as well as culture (Cipora et al., in press).

Little is known about what factors influence the non-symbolic SNARC-like effect. It is likely that although some factors may be similar to the symbolic SNARC effect, there may be others that differ or are more important for non-symbolic quantity processing. More research is needed in this area. Future research should examine the specific factors such as VSWM and which components of numerical processing and math that influence the non-symbolic SNARC-like effect. Future research should also examine profiles for standard and reverse non-symbolic SNARC and see if there is a correlation between individual differences in the non-symbolic SNARC, the weber fraction and math ability. It would also be of interest to test the symbolic and non- symbolic SNARC effect in the same participant as well as including stimuli in the subitising range.

In sum, inferences that can be made from findings in this thesis are limited by the small sample size. Individual differences in the spatial representation of non-symbolic quantity can be further elucidated using other statistical methods such as latent class modelling. This would enable different profiles of non-symbolic quantity representation to be identified. Nonetheless, this research provides a theoretical framework of the non-symbolic SNARC-like effect to further guide research into the spatial representation of non-symbolic quantity. Based on our findings, we would like to propose the benefit of a calibration framework to test the non-symbolic SNARC-like effect where individual differences in the Weber fraction are used to select ratios of referent and target stimuli. Specifically, the research highlights the importance of considering individual differences in the non-symbolic SNARC-like effect. Furthermore, this research indicates important areas for further examination, comprising of a more in-depth investigation into mechanisms that drive individual differences of the non-symbolic SNARC-like effect, and spatial representation of non-symbolic quantity in the ANS range, by considering ratio of referent-to-target stimuli, differing levels of non-symbolic quantity acuity and the use of low-level cues and strategies when designing tasks to test the non-symbolic SNARC-like effect.

6.5 Conclusion

This thesis had three aims. The first was to examine if non-symbolic quantities are represented spatially as evidenced by a non-symbolic SNARC effect. The second was to examine the relationship between the to-be-judged quantity ratios, low level visual cues, and modality of response on the non-symbolic SNARC-like effect. The third was to examine possible reasons for individual differences of the non-symbolic SNARC-like effect. Several methodological limitations were identified that have undermined previous research in this area. First, non-symbolic quantity has been used across the subitizing and ANS ranges. Yet it has been shown that these are distinct representation systems with differing RT and accuracy signatures. Second, there are known measurement issues of the ANS. For example, low-level visual cues such as luminance, surface area, and dot size can influence non-symbolic quantity processing. The degree of influence of these visual cues is problematic. Previous research has shown that luminance and physical size of dots are also represented spatially. This means that previous spatial representation of non-symbolic quantity cannot be attributed fully to magnitude. Third, there is a convention toward group-level analysis in the symbolic SNARC literature (i.e. ANOVA and linear regression). The issue with this form of analysis is that it fails to consider the potential of individual differences in RT and accuracy performance. However, it has been shown that there are individual differences in the symbolic SNARC effect and the spatial representation of number, but the significance those of individual differences is still poorly understood. Group level analyses have also typically been used in research of the non-symbolic SNARC-like effect. Nevertheless, the need to acknowledge and investigate these individual differences is perhaps of even greater importance for models of the non-symbolic SNARC-like effect as there is an increase of variance and known individual differences in non-symbolic quantity processing in the form of individual differences in the Weber fraction.

The key contribution of this thesis has been to support the claim that non-symbolic quantity in the ANS range is represented spatially from smallest to largest, left to right. This was demonstrated by a robust non-symbolic SNARC-like effect across all five studies that was unaffected by manipulations of ratio, visual cues or response modality. There were distinct individual differences in discrimination of non-symbolic quantity, suggesting that calibration of the ratio of referent-to-test is an important factor of design of paradigms of the non-symbolic SNARC-like effect. Large variance of the non-symbolic SNARC-like effect was found suggesting there are individual differences in the spatial representation of non-symbolic quantity. The form of the non-symbolic SNARC-like effect was similar to the symbolic SNARC effect. This finding has implications for neurocognitive models of numerical cognition and suggests non-symbolic quantity and symbolic number are represented in a similar manner in the brain. Further understanding of the spatial representation of non-symbolic quantity, especially using analytic techniques that permit the detection and investigation of individual differences in non-symbolic quantity representation, may help us gain better insight into how the brain processes non-symbolic quantity and the cognitive representation of number.

References

Adachi, I. (2014). Spontaneous spatial mapping of learned sequence in chimpanzees: evidence for a SNARC-like effect. *PLoS One, 9*(3). doi: 10.1371/journal.pone.0090373

Adams, J. W. (2007). Individual differences in mathematical ability: Genetic, cognitive and behavioural factors. *Journal of Research in Special Education Needs, 7*(2), 97-103. https://doi.org/10.1111/j.1471-3802.2007.00085.x

Andres, M., Davare, M., Pesenti, M., Olivier, E., & Seron, X. (2004). Number magnitude and grip aperture interaction. *NeuroReport, 15*(18), 2773-2777.

Anobile, G., Castaldi, E., Turi, M., Tinelli, F., & Burr, D. C. (2016). Numerosity but not texture-density discrimination correlates with math ability in children. *Developmental Psychology, 52*(8), 1206-1216. https://dx.doi.org/10.1037%2Fdev0000155

Anobile, G., Cicchini, G. M., & Burr, D. C. (2014). Separate mechanisms for perception of numerosity and density. *Psychological Science, 25*(1), 265-270. https://doi.org/10.1177%2F0956797613501520

Ansari, D. (2008). Effects of development and enculturation on number representation in the brain. *Nature Reviews Neuroscience, 9*, 278-291. https://doi:10.1038/nrn2334

Bachot, J., Gevers, W., Fias, W., & Roeyers H. (2005). Number sense in children with visuospatial disabilities: Orientation of the mental number line. *Psychology Science, 47*(1), 172. doi: 1854/LU-326765.

Bächtold, D., Baumüller, M., & Brugger, P. (1998). Stimulus-response compatibility in representational space. *Neuropsychologia, 36*(8), 731-735. https://doi.org/10.1016/S0028-3932(98)00002-5

Bae, G. Y., Choi, J. M., Cho, Y. S., & Proctor, R. W. (2009). Transfer of magnitude and spatial mappings to the SNARC effect for parity judgments. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 35*(6), 1506-1521. http://dx.doi.org/10.1037/a0017257

Barlow, H. B. (1978). The efficiency of detecting changes of density in random dot patterns. *Vision Research, 18*(6), 637-650. https://doi.org/10.1016/0042-6989(78)90143-8

Barth, H., Kanwisher, N., & Spelke, E. (2003). The construction of large number representations in adults. *Cognition, 86*(3), 201-221. https://doi.org/10.1016/S0010-0277(02)00178-6

Beecham, R, Reeve, R. A., & Wilson, S. J. (2009). Spatial representations are specific to different domains of knowledge. *PLoS ONE, 4*(5), e5543. https://doi.org/10.1371/journal.pone.0005543 PMID: 19461994

Beran, M. J., & Parrish, A. E. (2017). The number sense is neither last resort nor of primary import. *Behavioral and Brain Sciences, 40*. Epub 07/27. https://doi.org/10.1017/S0140525X16002065 PMID: 29342651

Berch, D. B. (2005). Making sense of number sense: Implications for children with mathematical disabilities. *Journal of Learning Disabilities, 38*(4), 333-339. https://doi.org/10.1177%2F00222194050380040901

Berch, D.B., Foley, E.J., Hill, R.J., & McDonough-Ryan, P.M. (1999). Extracting parity and magnitude from Arabic numerals: Developmental changes in number processing and mental representation. *Journal of Experimental Child Psychology, 74*(4), 286–308. doi:10.1006/jecp.1999.2518

Bonny, J. W., & Lourenco, S. F. (2013). The approximate number system and its relation to early math achievement: Evidence from the preschool years. *Journal of Experimental Child Psychology, 114*(3), 375- 388. doi: 10.1 016/j.jecp.2012.09.015

Braham, E. J., & Libertus, M. E. (2017). Intergenerational associations in numerical approximation and mathematical abilities. *Developmental Science, 20*(5), e12436. doi: 10.1111/desc.12436

Brainard, D. H. (1997). The psychophysics toolbox. *Spatial Vision, 10*, 433–436. doi: 10.1163/156856897x00357

Brannon, E. M., & Terrace, H. S. (1998). Ordering of the numerosities 1 to 9 by monkeys. *Science, 282*(5389), 746-749. https://doi.org/10.1126/science.282.5389.746

Buckley, P. B., & Gillman, C. B. (1974). Comparisons of digits and dot patterns. *Journal of Experimental Psychology, 103*(6), 1131-1136. http://dx.doi.org/10.1037/h0037361

Bueti, D., & Walsh, V. (2009). The parietal cortex and the representation of time, space, number and other magnitudes. *Philosophical Transactions of the Royal Society B: Biological Sciences, 364*(1525), 1831-1840. https://doi.org/10.1098/rstb.2009.0028 PMID: 19487186

Bugden, S., & Ansari, D. (2011). Individual differences in children's mathematical competence are related to the intentional but not automatic processing of Arabic numerals. *Cognition, 118*(1), 32-44. https://doi.org/10.1016/j.cognition.2010.09.005

Bulf, H., Cassia, V. M., & de Hevia, M. D. (2014). Are numbers, size and brightness equally efficient in orienting visual attention? Evidence from an eye-tracking study. *PLoS ONE 9*(6), e99499. doi: 10.1371/journal.pone.0099499

Bull, R., Cleland, A. A., & Mitchell, T. (2013). Sex differences in the spatial representation of number. *Journal of Experimental Psychology: General, 142*(1), 181-192. http://dx.doi.org/10.1037/a0028387

Burgess, A., & Barlow, H. B. (1983). The precision of numerosity discrimination in arrays of random dots. *Vision Research, 23*(8), 811-820. https://doi.org/10.1016/0042-6989(83)90204-3

Burr, D. C. (2017). Evidence for a number sense. *Behavioral and Brain Sciences, 40*, Epub 07/27. https://doi.org/10.1017/S0140525X16002077 PMID: 29342625

Burr, D. C., & Ross, J. (2008). A visual sense of number. *Current Biology, 18*(6), 425-428. https://doi.org/10.1016/j.cub.2008.02.052

Burr, D. C., Anobile, G., & Arrighi, R. (2017). Psychophysical evidence for the number sense. *Philosophical Transactions of the Royal Society B: Biological Sciences 373*, 20170045. https://doi.org/10.1098/rstb.2017.0045

Burr, D. C., Cicchini, G., & Anobile, G. (2017). Spontaneous perception of numerosity in humans. *Journal of Vision, 17*(10), 744. https://doi:10.1167/17.10.744

Butterworth, B. (1999). *The mathematical brain*. London: Macmillan.

Butterworth, B. (2005). The development of arithmetical abilities. *Journal of Child Psychology and Psychiatry, 46*(1), 3–18. https://doi:10.1111/j.1469-7610.2004.00374.x

Butterworth, B. (2010). Foundational numerical capacities and the origins of dyscalculia. *Trends in Cognitive Sciences, 14*, 534-541. https://doi:10.1016/j.tics.2010.09.007

Butterworth, B., Gallistel, C. R., & Vallortigara, G. (2018). Introduction: The origins of numerical abilities. *Philosophical Transactions of the Royal Society B: 273*(1740). https://doi.org/10.1098/rstb.2016.0507

Cantlon, J. F., Libertus, M. E., Pinel, P., Dehaene, S., Brannon, E. M., & Pelphrey, K. A. (2009). The neural development of an abstract concept of number. *Journal of cognitive neuroscience, 21*(11), 2217-2229. https://doi: 10.1162/jocn.2008.21159

Cappelletti, M., Chamberlain, R., Freeman, E. D., Kanai, R., Butterworth, B., Price, C. J., & Rees, G. (2014). Commonalities for numerical and continuous quantity skills at temporo-parietal junction. *Journal of Cognitive Neuroscience*, *26*(5), 986-999. https://doi.org/10.1162/jocn_a_00546

Ceulemans, A., Titeca, D., Loeys, T., Hoppenbrouwers, K., Rousseau, S., & Desoete, A. (2014). Enumeration of small and large numerosities in adolescents with mathematical learning disorders. *Research in developmental disabilities*, *35*(1), 27-35. https://doi.org/10.1016/j.ridd.2013.10.018

Chatterjee, A. (2011). Directional asymmetries in cognition: What is left to write about? In T. Schubert & A. Maass (Eds.), *Spatial dimensions of social thought* (pp. 189-210), Boston, MA: Walter de Gruyter.

Chen, Q., & Li, J. (2014). Association between individual differences in non-symbolic number acuity and math performance: A meta-analysis. *Acta Psychologica*, *148*(0), 163-172. https://doi.org/10.1016/j.actpsy.2014.01.016 PMID: 24583622

Cheng, Y. L., & Mix, K. S. (2014). Spatial training improves children's mathematics ability. *Journal of Cognition and Development*, *15*(1), 2-11. https://doi.org/10.1080/15248372.2012.725186

Chew, C. S., Forte, J. D., & Reeve, R. A. (2016). Cognitive factors affecting children's nonsymbolic and symbolic magnitude judgment abilities: A latent profile analysis. *Journal of Experimental Child Psychology, 152*, 173-191. https://doi.org/10.1016/j.jecp.2016.07.001 PMID: 27560661

Chew, C. S., Forte, J. D., & Reeve, R. A. (2019). Implications of change/stability patterns in children's non-symbolic and symbolic magnitude judgment abilities over one year: A latent transition analysis. *Frontiers in Psychology, 10*, 441. https://doi.org/10.3389/fpsyg.2019.00441

Cicchini, G. M., Anobile, G., & Burr, D. C. (2016). *Spontaneous perception of numerosity in humans. Nature Communications, 7*, 12536. doi:10.1038/ncomms12536

Cipolotti, L., Butterworth, B., & Denes, G. (1991). A specific deficit for numbers in a case of dense acalculia. *Brain, 114*(6), 2619–2637.

Cipora et al., (in press). A minority pulls the sample mean: on the individual prevalence of robust group-level cognitive phenomena – the instance of the SNARC effect can't get journal name yet

Cipora, K., He, Y., & Nuerk, H. C. (in press). The SNARC effect and math skill: why should they be related? A model framework. can't get journal name yet

Cipora, K., Hohol, M., Nuerk, H-C., Willmes, K., Brożek, B., Kucharzyk, B., et al. (2016). Professional mathematicians differ from controls in their spatial-numerical associations. *Psychological Research, 80*(4), 710-726. https://doi.org/10.1007/s00426-015-0677-6 PMID: 26063316

Cipora, K., & Nuerk, H-C. (2013). Is the SNARC effect related to the level of mathematics? No systematic relationship observed despite more power, more repetitions, and more direct assessment of arithmetic skill. *The Quarterly Journal of Experimental Psychology, 66*(10), 1974-1991 doi:10.1080/17470218.2013.772215

Cipora, K., & Wood, G. (2017). Finding the SNARC instead of hunting it: A 20∗20 Monte Carlo investigation. *Frontiers in Psychology, 8*(July), 1–11. https://doi.org/10.3389/fpsyg.2017.01194

Cipora, K., Hohol, M., Nuerk, H-C., Willmes, K., Brożek, B., Kucharzyk, B, et al. (2016). Professional mathematicians differ from controls in their spatial-numerical associations. *Psychological Research, 80*(4), 710-726. https://doi.org/10.1007/s00426-015-0677-6 PMID: 26063316

Cipora, K., Patro, K., & Nuerk, H.-C. (2015). Are spatial–numerical associations a cornerstone for arithmetic learning? The lack of genuine correlations suggests: No. *Mind, Brain, & Education, 9*(4), 190–207.

Clayton, S., Gilmore, C., & Inglis, M. (2015). Dot comparison stimuli are not all alike: The effect of different visual controls on ANS measurement. *Acta Psychologica, 161*, 177-184. https://doi.org/10.1016/j.actpsy.2015.09.007

Cleland, A. A., & Bull, R. (2019). Automaticity of access to numerical magnitude and its spatial associations: The role of task and number representation. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 45*(2), 333. http://dx.doi.org/10.1037/xlm0000590

Cleland, A. A., Corsico, K., White, K., & Bull, R. (in press). Non-symbolic numerosities do not automatically activate spatial-numerical associations: Evidence from the SNARC effect.

Cohen Kadosh, R., Bien, N., & Sack, A. T. (2012). Automatic and intentional number processing both rely on intact right parietal cortex: a combined FMRI and neuronavigated TMS study. *Frontiers in human neuroscience, 6*, 2. https://doi.org/10.3389/fnhum.2012.00002

Cornu, V., Hornung, C., Schiltz, C., & Martin, R. (2016, September). *Different aspects of spatial skills and their relation to early mathematics*. Poster presented at: Workshop on Domain-General and Domain-Specific Foundations of Numerical and Arithmetic Processing; 2016 Sept 28–30, Tübingen, Germany. doi: 10993/28747

Corsi, P. M. (1972). *Human memory and the medial temporal region of the brain*. Unpublished doctoral dissertation, McGill University, Montreal, Canada.

Crollen, V., & Noël, M-P. (2015). Spatial and numerical processing in children with high and low visuospatial abilities. *Journal of Experimental Child Psychology, 132*, 84–98. https://doi.org/10.1016/j.jecp.2014.12.006 PMID: 25618380

Crollen, V., Dormal, G., Seron, X., Lepore, F., & Collignon, O. (2013). Embodied numbers: The role of vision in the development of number-space interactions. *Cortex, 49*(1), 276–283.

Cutini, S., Scarpa, F., Scatturin, P., Dell'Acqua, R., & Zorzi, M. (2014). Number-space 461 interactions in the human parietal cortex: Enlightening the SNARC effect with functional 462 near-infrared spectroscopy. *Cerebral Cortex, 24*(2), 44-51. https://doi.org/10.1093/cercor/bhs321

Dantzig, T. (1954). *Number: The language of science*. New York: MacMillan.

De Belder, M., Abrahamse, E., Kerckhof, M., Fias, W., & van Dijck, J.-P. (2015). Serial position markers in space: Visuospatial priming of serial order working memory retrieval. *PloS ONE, 10*, e0116469. doi: 10.1371/journal.pone.0116469

de Hevia, M. D., Izard, V., Coubart, A., Spelke, E. S., & Streri, A. (2014). Representations of space, time, and number in neonates. *Proceedings of the National Academy of Sciences, 111*(13), 4809-4813. https://doi.org/10.1073/pnas.1323628111

de Hevia, M. D., & Spelke, E. S. (2009). Spontaneous mapping of number and space in adults and young children. *Cognition, 110*(2), 198–207. doi: 10.1016/j.cognition.2008.11.003

de Hevia, M. D., Vallar, G., & Girelli, L. (2008). Visualizing numbers in the mind's eye: The role of visuo-spatial processes in numerical abilities. *Neuroscience & Biobehavioral Reviews, 32*(8), 1361–1372. doi: 10.1016/j.neubiorev.2008.05.015

De Smedt, B., & Gilmore, C. K. (2011). Defective number module or impaired access? Numerical magnitude processing in first graders with mathematical difficulties. *Journal of experimental child psychology, 108*(2), 278-292. https://doi.org/10.1016/j.jecp.2010.09.003

De Smedt, B., Noël, M.-P., Gilmore, C., & Ansari, D. (2013). How do symbolic and nonsymbolic numerical magnitude processing skills relate to individual differences in children's mathematical skills? A review of evidence from brain and behavior. *Trends in Neuroscience and Education, 2*(2), 48–55. https://doi:10.1016/j.tine.2013.06.001

Dehaene, S., & Brannon, E. (Eds.) (2011). *Space, time and number in the brain: Searching for the foundations of mathematical thought*. London: Academic Press; 2011.

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition, 44*(1-2), 1–42. https://doi:10.1016/0010-0277(92)90049-N

Dehaene, S. (1997). *The number sense: How the mind creates mathematics*. New York: Oxford University Press.

Dehaene, S. (2003). The neural basis of the Weber-Fechner law: A logarithmic mental number line. *Trends in Cognitive Sciences, 7*(4), 145-147. https://doi:10.1016/S1364-6613(03)00055-X

Dehaene, S. (2005). Evolution of human cortical circuits for reading and arithmetic: The “neuronal recycling” hypothesis. In S. Dehaene, J. Duhamel, M. Hauser, & G. Rizzolattie (Eds.), *From monkey brain to human brain*. Cambridge, MA: MIT Press.

Dehaene, S. Subtracting pigeons: Logarithmic or linear? (2001). *Psychological Science. 12*(3), 244-6

Dehaene, S., & Cohen, L. (1995). Towards an anatomical and functional model of number processing. *Mathematical Cognition, 1*(1), 83-120.

Dehaene, S., & Cohen, L. (1997). Cerebral pathways for calculation: Double dissociation between rote verbal and quantitative knowledge of arithmetic. *Cortex, 33*(2), 219–250. doi: 10.1016/S0010-9452(08)70002-9

Dehaene, S., & Cohen, L. (2007). Cultural recycling of cortical maps. *Neuron, 56*(2), 384-398. doi:10.1016/j.neuron.2007.10.004

Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: General, 122*(3), 371-396. https://doi.org/10.1037/0096-3445.122.3.371

Dehaene, S., Dupoux, E., & Mehler, J. (1990). Is numerical comparison digital? Analogical and symbolic effects in two-digit number comparison. *Journal of Experimental Psychology: Human Perception and Performance, 16*(3), 626-641. doi:10.1037/0096-1523.16.3.626

Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology, 20*(3–6), 487–506. https://doi.org/10.1080/02643290244000239 PMID: 20957581

DeWind, N. K., Adams, G. K., Platt, M. L., & Brannon, E. M. (2015). Modeling the approximate number system to quantify the contribution of visual stimulus features. *Cognition, 142*, 247- 265. doi:10.1016/j.cognition.2015.05.016

DeWolf, M., Grounds, M. A., Bassok, M., & Holyoak, K. J. (2014). Magnitude comparison with different types of rational numbers. Journal of Experimental Psychology: Human Perception and Performance, 40(1), 71. https://doi.org/10.1037/a0032916

Dietrich, J. F., Huber, S., & Nuerk, H.-C. (2015). Methodological aspects to be considered when measuring the approximate number system (ANS): A research review. *Frontiers in Psychology, 6*, 295. https://doi.org/10.3389/fpsyg.2015.00295

Dixon, W. J., & Mood, A. M. (1948). A method for obtaining and analyzing sensitivity data. *Journal of the American Statistical Association, 43*(241), 109-126.

Doricchi, F., Guariglia, P., Gasparini, .M, & Tomaiuolo, F. (2005). Dissociation between physical and mental number line bisection in right hemisphere brain damage. *Nature Neuroscience, 8*(12), 1663-1665. doi:10.1038/nn1563

Dowker, A. (2005). Individual differences in arithmetic: Implications for psychology, neuroscience and education. Hove: Psychology Press

Dowker, A. (2008). Individual differences in numerical abilities in preschoolers. *Developmental Science, 11*(5), 650-654. https://doi.org/10.1111/j.1467-7687.2008.00713.x

Dowker, A., & Sigley, G. (2010). Targeted interventions for children with arithmetical difficulties. In BJEP Monograph Series II, Number 7- *Understanding number development and difficulties* (Vol. 65, No. 81, pp. 65-81). British Psychological Society. https://doi.org/10.1111/j.1467-7687.2008.00713.x PMID: 18801119

Drucker, C. B., & Brannon, E. M. (2014). Rhesus monkeys (Macaca mulatta) map number onto space. Cognition, 132(1), 57-67. https://doi.org/10.1016/j.cognition.2014.03.011

Durand, M., Hulme, C., Larkin, R., & Snowling, M. (2005). The cognitive foundations of reading and arithmetic skills in 7- to 10-year-olds. *Journal of Experimental Child Psychology, 91*(2), 113-136. https://doi.org/10.1016/j.jecp.2005.01.003

Ebersbach, M., Luwel, K., Frick, A., Onghena, P., & Verschaffel, L. (2008). The relationship between the shape of the mental number line and familiarity with numbers in 5- to 9-year old children: Evidence for a segmented linear model. *Journal of Experimental Child Psychology, 99*(1), 1–17. https://doi.org/10.1027/1618-3169/a000250

Efron, B., & Tibshirani, R. (1993). *An introduction to the bootstrap*. New York: Chapman & Hall.

Fattorini, E., Pinto, M., Rotondaro, F., & Doricchi, F. (2015). Perceiving numbers does not cause automatic shifts of spatial attention. *Cortex*, *73*, 98–316. https://doi.org/10.1016/j.cortex.2015.09.007 PMID: 26520681

Faul, F., Erdfelder, E., Lang, A.-G., & Buchner, A. (2007). G*Power 3: A flexible statistical power analysis program for the social, behavioral, and biomedical sciences. *Behavior Research Methods*, *39*, 175-191. https://doi.org/10.3758/BF03193146

Fazio, L. K., Bailey, D. H., Thompson, C. A., & Siegler, R. S. (2014). Relations of different types of numerical magnitude representations to each other and to mathematics achievement. *Journal of Experimental Child Psychology*, *123*, 53–72. doi: 10.1016/j. jecp.2014.01.013

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8*(7), 307–314. https://doi.org/10.1016/j.tics.2004.05.002 PMID: 15242690

Fias, W., & Fischer, M. H. (2005). Spatial representation of numbers. In J. Campbell (Ed.), *Handbook of mathematical cognition* (pp. 43-54). Hove: Psychology Press.

Fias, W., (2001). Two routes for the processing of verbal numbers: Evidence from the SNARC effect. *Psychological Research, 65*(4), 250-259. https://doi.org/10.1007/s004260100065

Fias, W., Brysbaert, M., Geypens, F., & d'Ydewalle, G. (1996). The importance of magnitude information in numerical processing: Evidence from the SNARC effect. *Mathematical Cognition, 2*(1), 95–110. https://doi. org/10.1080/135467996387552

Fias, W., van Dijck, J. F., & Gevers, W. (2011). How number is associated with space? The role of working memory. In S. Dehaene & E. Brannon (Eds.), *Space, time and number in the brain: Searching for the foundations of mathematical thought* (pp. 133 - 148). Burlington, NY: Elsevier/Academic Press.

Fischer, M. H. (2006). The future for SNARC could be stark. *Cortex, 42*(8), 1066–8. https://doi.org/10. 1016/S0010-9452(08)70218-1 PMID: 17209412

Fischer, M. H. (2001). Cognition in the bisection task. *Trends in Cognitive Sciences, 5*(11), 460-462. doi: 10.1016/S1364-6613(00)01790-3

Fischer, M. H. (2003). Cognitive representation of negative numbers. *Psychological Science, 14*(3), 278-282. doi: 10.1111/1467-9280.03435

Fischer, M. H. (2008). Finger counting habits modulate spatial-numerical associations. *Cortex, 44*(4), 386–392. doi: 10.1016/j.cortex.2007.08.004

Fischer, M. H., & Rottmann, J. (2005). Do negative numbers have a place on the mental number line? *Psychology Science, 47*(1), 22–32.

Fischer, M. H., & Shaki, S. (2014). Spatial associations in numerical cognition: From single digits to arithmetic. *The Quarterly Journal of Experimental Psychology, 67*(8), 1461–1483. doi.org/10.1080/17470218.2014.927515

Fischer, M. H., Castel, A. D., Dodd, M. D., & Pratt, J. (2003). Perceiving numbers causes spatial shifts of attention. *Nature Neuroscience, 6*(6), 555–556. doi: 10.1038/nn1066

Fischer, M. H., Kaufmann, L., & Domahs, F. (2012). Finger counting and numerical cognition. *Frontiers in Psychology, 3,* 108. https://doi.org/10.3389/fpsyg.2012.00108

Fischer, M. H., Mills, R. A., & Shaki, S. (2010). How to cook a SNARC: Number placement in text rapidly changes spatial–numerical associations. *Brain and Cognition, 72*(3), 333–336. https://doi.org/10.1016/j.bandc.2009.10.010 PMID: 19917517

Fischer, M. H., Warlop, N., Hill, R. L., & Fias, W. (2004). Oculomotor bias induced by number perception. *Experimental Psychology, 51*(2), 91–7. https://doi.org/10.1027/1618-3169.51.2.91 PMID: 15114901

Formoso, J., Barreyro, J. P., Jacubovich, S., & Injoque-Ricle, I. (2017). Possible associations between subitizing, estimation and visuospatial working memory (VSWM) in children. *The Spanish Journal of Psychology, 20*, Epub 06/05. https://doi.org/10.1017/sjp.2017.23 PMID: 28578725

Fuhs, M. W., & McNeil, N. M. (2013). ANS acuity and mathematics ability in preschoolers from low-income homes: Contributions of inhibitory control. *Developmental Science, 16*(1), 136-148. https://doi.org/10.1111/desc.12013

Fumarola, A., Prpic, V., Da Pos, O., Murgia, M., Umiltà, C., & Agostini, T. (2014). Automatic spatial association for luminance. *Attention, Perception, & Psychophysics, 76*(3), 759-765. https://doi.org/10.3758/s13414-013-0614-y

Gallistel, C. R., & Gelman, I. (2000). Non-verbal numerical cognition: From reals to integers. *Trends in Cognitive Sciences, 4*(2), 59-65. http://dx.doi.org/10.1016/S1364-6613(99)01424-2

Gallistel, C., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition, 44*(1–2), 43–74. http://dx.doi.org/10.1016/0010-0277(92)90050-R

Galton, F. (1880a). Statistics of mental imagery. *Mind, 5*(19), 301-318.

Galton, F. (1880b). Visualised numerals. *Nature, 21,* 252-256. https://doi.org/10.1038/021252a0

Geary, D. C., Hoard, M. K., Nugent, L., & Byrd-Craven, J. (2008). Development of number line representations in children with mathematical learning disability. *Developmental Neuropsychology, 33*(3), 277–299. doi:10.1080/87565640801982361

Gebuis, T., & Gevers, W. (2011). Numerosities and space; Indeed a cognitive illusion! A reply to de Hevia and Spelke (2009). *Cognition, 121*(2), 248-252; discussion 253-245. doi: S0010-0277(10)00221-0[pii]10.1016/j.cognition.2010.09.008

Gebuis, T., & Reynvoet, B. (2011). . Generating nonsymbolic number stimuli. *Behavior Research Methods, 43*(4), 981-986. https://doi.org/10.3758/s13428-011-0097-5 PMID: 21512872

Gebuis, T., & Reynvoet, B. (2012). The interplay between nonsymbolic number and its continuous visual properties. *Journal of Experimental Psychology: General*, *141*(4), 642. http://dx.doi.org/10.1037/a0026218

Gebuis, T., & Reynvoet, B. (2013). The neural mechanisms underlying passive and active processing of numerosity. *NeuroImage, 70,* 301-307. https://doi.org/10.1016/j.neuroimage.2012.12.048

Gebuis, T., & van der Smagt, M. J. (2011) False approximations of the approximate number system? *PLoS ONE 6*(10):e25405 https://. doi: 10.1371/journal.pone.0025405

Gebuis, T., Cohen Kadosh R.., & Gevers, W. (2016). Sensory-integration system rather than approximate number system underlies numerosity processing: A critical review. *Acta Psychologica, 171,* 17–35. https://doi.org/10.1016/j.actpsy.2016.09.003 PMID: 27640140

Gebuis, T., Cohen Kadosh, R., & Gevers, W. (2017). Why try saving the ANS? An alternative proposal. *Behavioral and Brain Sciences, 40,* Epub 07/27. https://doi.org/10.1017/S0140525X16002107 PMID: 29342626

Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities, 38*(4), 293–304. https://doi.org/10.1177/00222194050380040301

Gerstmann, J. (1940). Syndrome of finger agnosia: Disorientation for right and left, agraphia and acalculia. *Archives of Neurology and Psychiatry, 44*(2), 398-408. https://doi:10.1001/archneurpsyc.1940.02280080158009

Gevers, W., Gevers, E., Ratinckx, W., De Baene, W., & Fias, W. (2006). Further evidence that the SNARC effect is processed along a dual-route architecture. *Experimental Psychology, 53*(1), 58–68. https://doi.org/ 10.1027/1618-3169.53.1.58 PMID: 16610273

Gevers, W., Lammertyn, J., Notebaert, W., Verguts, T., & Fias, W. (2006). Automatic response activation of implicit spatial information: Evidence from the SNARC effect. *Acta Psychologica, 122*(3), 221–33. https://doi.org/10.1016/j.actpsy.2005.11.004

Gevers, W., & Lammertyn, J. (2005). The hunt for SNARC. *Psychology Science, 47*(1), 10–21.

Gevers, W., Cohen Kadosh, R., & Gebuis, T. (2016). The sensory integration theory: An alternative to the Approximate Number System. In A. Henik (Ed.), *Continuous issues in numerical cognition*. Elsevier

Gevers, W., Ratinckx, E., De Baene, W., & Fias, W. (2006). Further evidence that the SNARC effect is processed along a dual-route architecture: Evidence from the lateralized readiness potential. *Experimental Psychology, 53*(1), 58-68. . https://doi.org/10.1027/1618-3169.53.1.58 PMID: 16610273

Gevers, W., Reynvoet, B., & Fias, W. (2003). The mental representation of ordinal sequences is spatially organized. *Cognition, 87*(3), B87-B95. doi:10.1016/S0010-0277(02)00234-2

Gevers, W., Santens, S., Dhooge, E., Chen, Q., Van den Bossche, L., Fias, W., & Verguts, T. (2010). Verbal-spatial and visuospatial coding of number–space interactions. *Journal of Experimental Psychology: General*, *139*(1), 180-190. http://dx.doi.org/10.1037/a0017688

Gevers, W., Verguts, T., Reynvoet, B., Caessens, B., & Fias, W. (2006). Numbers and space: a computational model of the SNARC effect. *Journal of Experimental Psychology: Human Perception and Performance, 32*(1), 32–44.

Gibson, L., & Maurer, D. (2016). Development of SNARC and distance effects and their relation to mathematical and visuospatial abilities. *Journal of Experimental Child Psychology, 150,* 301–13. https://doi.org/ 10.1016/j.jecp.2016.05.009 PMID: 27376924

Gilmore, C., Attridge, N., De Smedt, B., & Inglis, M. (2014). Measuring the approximate number system in children: Exploring the relationships among different tasks. *Learning and Individual Differences, 29,* 50-58. doi:10.1016/j.lindif.2013.10.004

Ginsburg, V., van Dijck, J.-P., Previtali, P., Fias, W., & Gevers, W. (2014). The impact of verbal working memory on number–space associations. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 40*(4), 976-986. http://dx.doi.org/10.1037/a0036378

Göbel, S. M., Calabria, M., Farnè, A., & Rossetti, Y. (2006). Parietal rTMS distorts the mental number line: Simulating 'spatial' neglect in healthy subjects. *Neuropsychologia, 44*(5), 860-868. https://doi.org/10.1016/j.neuropsychologia.2005.09.007

Göbel, S. M., Watson, S. E., Lervåg, A., & Hulme, C. (2014). Children's arithmetic development: It is number knowledge, not the approximate number sense, that counts. *Psychological Science, 25*(3), 789–798. https://doi.org/10.1177/0956797613516471

Gökaydin, D., Brugger, P., & Loetscher, T. (2018). Sequential effects in SNARC. *Scientific reports (Nature Publisher Group), 8,* 1-13. http://dx.doi.org.ezp.lib.unimelb.edu.au/10.1038/s41598-018-29337-2.

Göksun, T., Woods, A. J., Chatterjee, A., Zelonis, S., Glass, L., & Smith, S. E. (2013). Elementary school children's attentional biases in physical and numerical space. *European Journal of Developmental Psychology, 10*(4), 433-448.

Gordon, P. (2004). Numerical cognition without words: Evidence from Amazonia. *Science, 306*(5695), 496–499. doi:10.1126/science.1094492

Gould, S. J., & Vrba, E. S. (1982). Exaptation: A missing term in the science of form. *Paleobiology, 8*(1), 4-15. https://doi.org/10.1017/S0094837300004310

Grabner, R. H., & De Smedt, B. (2012). Oscillatory EEG correlates of arithmetic strategies: a training study. *Frontiers in psychology, 3,* 428. https://doi.org/10.3389/fpsyg.2012.00428

Gray, S. A., & Reeve, R. A. (2014). Preschoolers' dot enumeration abilities are markers of their arithmetic competence. *PLoS ONE, 9*(4), e94428. https://doi.org/10.1371/journal.pone.0094428 PMID: 24714052

Guillaume, M., & Van Rinsveld, A. (2018). Comparing numerical comparison tasks: A meta-analysis of the variability of the Weber Fraction relative to the generation algorithm. *Frontiers in Psychology, 9,* 1694. https://doi.org/10.3389/fpsyg.2018.01694

Gunderson, E. A., Ramirez, G., Levine, S.C., & Beilock, S. L. (2012). The role of parents and teachers in the development of gender-related math attitudes. *Sex Roles, 66*(3-4). 153-166. https://doi.org/10.1007/s11199-011-9996-2

Halberda, J., & Feigenson, L. (2008). Developmental change in the acuity of the "number sense": The approximate number system in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental Psychology, 44*(5), 1457-1465. http://dx.doi.org/10.1037/a0012682

Halberda, J., Ly, R., Wilmer, J. B., Naiman, D. Q., & Germine, L. (2012). Number sense across the lifespan as revealed by a massive internet-based sample. *Proceedings of the National Academy of Sciences, 109*(28) 11116-11120. https://doi.org/10.1073/pnas.1200196109

Halberda, J., Mazzocco, M. M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature, 455,* 665–668. doi: 10.1038/nature07246

Harvey, B. M., Fracasso, A., Petridou. N., & Dumoulin, S. O. (2015). Parietal object size maps and generalized quantity. *Proceedings of the National Academy of Sciences*, *112*(44), 13525-13530. doi:10.1073/pnas.1515414112

Harvey, B. M., Klein, B. P., Petridou, N., & Dumoulin, S. O. (2013). Topographic representation of numerosity in the human parietal cortex. *Science, 341*(6150), 1123–6. https://doi.org/10.1126/science.1239052 PMID: 24009396

Hawes, Z., & Ansari, D. (2020). What explains the relationship between spatial and mathematical skills? A review of evidence from brain and behavior. *Psychonomic Bulletin & Review*, 1-18. https://doi.org/10.3758/s13423-019-01694-7

Herrera, A., Macizo, P., & Semenza, C. (2008). The role of working memory in the association between number magnitude and space. *Acta Psychologica, 126*(2), 225-237. https://doi.org/10.1016/j.actpsy.2008.01.002

Hoffman, D., Pigat, D., & Schiltz, C. (2014). The impact of inhibition capacities and age on number–space associations. *Cognitive Processing, 15*(3), 329-342. https://doi.org/10.1007/s10339-014-0601-9

Hoffmann, D., Mussolin, C., Martin, R., Schiltz, C., & Lappe, M. (2014). The impact of mathematical proficiency on the number-space association. *PLoS One, 9*(1), e85048. https://doi.org/10.1371/journal.pone.0085048 PMID: 24416338

Holloway, I. D., & Ansari, D. (2009). Mapping numerical magnitudes onto symbols: The numerical distance effect and individual differences in children’s mathematics achievement. *Journal of Experimental Child Psychology, 103*(1), 17–29. https://doi:10.1016/j.jecp.2008.04.001

Holmes, K. J., & Lourenco, S. F. (2011). Common spatial organization of number and emotional expression: A mental magnitude line. *Brain and Cognition, 77*(2), 315–23. https://doi.org/10.1016/j.bandc.2011.07.002 PMID: 21839568

Hubbard, E. M., Piazza, M., Pinel, P., & Dehaene, S. (2005). Interactions between number and space in parietal cortex. *Nature Reviews Neuroscience, 6*, 435. https://doi.org/10.1038/nrn1684 PMID: 15928716

Huber, S., Sury, D., Moeller, K., Rubinsten, O., & Nuerk, H-C. (2015). A general number-to-space mapping deficit in developmental dyscalculia. *Research in Developmental Disabilities, 43*, 32–42. https://doi.org/10.1016/j.ridd.2015.06.003 PMID: 26151441

Hyde, D. C., & Spelke, E. S. (2011). Neural signatures of number processing in human infants: evidence for two core systems underlying numerical cognition. *Developmental Science, 14*(2), 360–71. https://doi.org/10.1111/j.1467-7687.2010.00987.x PMID: 21399717

Ifrah, G. (2000). *The universal history of numbers: From prehistory to the invention of the computer*. New York: Wiley.

Inglis, M., & Gilmore, C. (2014). Indexing the approximate number system. *Acta Psychologica, 145*, 147-155. https://doi.org/10.1016/j.actpsy.2013.11.009

Izard, V., & Dehaene, S. (2008). Calibrating the mental number line. *Cognition*, *106*(3), 1221-1247. https://doi.org/10.1016/j.cognition.2007.06.004

Izard, V., Sann, C., Spelke, E. S., & Streri, A. (2009). Newborn infants perceive abstract numbers. *Proceedings of the National Academy of Sciences, 106*(25), 10382–10385. doi: 10.1073/ pnas.0812142106

Jacob, S. N., & Nieder, A. (2008). The ABC of cardinal and ordinal number representations. *Trends in cognitive sciences*, *12*(2), 41-43. https://doi.org/10.1016/j.tics.2007.11.006

Jevons, W. S. (1870). On the mechanical performance of logical inference. *Philosophical Transactions of the Royal Society of London*, *160*, 497-518. https://doi.org/10.1098/rstl.1870.0022

Kaufman, S. B. (2007). Sex differences in mental rotation and spatial visualization ability: Can they be accounted for by differences in working memory capacity? *Intelligence, 35,* 211–223. https://doi.org/10.1016/j.intell.2006.07.009

Kaufman, E. L., Lord, M. W., Reese, T. W., & Volkmann, J. (1949). The discrimination of visual number. *The American Journal of Psychology, 62*(4), 498-525. doi:10.2307/1418556

Kaufmann, L., Kucian, K., & von Aster, M. (2014). Development of the numerical brain. In R. Cohen Kadosh & A. Dowker (Eds.), *The Oxford handbook of numerical cognition*. Oxford: Oxford University Press. doi:101093/oxfordhb/9780199642342.013.008

Kaufmann, L., Mazzocco, M. M., Dowker, A., von Aster, M., Goebel, S., Grabner, R., ... & Rubinsten, O. (2013). Dyscalculia from a developmental and differential perspective. *Frontiers in psychology*, *4*, 516. https://doi.org/10.3389/fpsyg.2013.00516

Kessels, R. P, van Zandvoort, M. J., Postma, A., Kappelle, L. J., & de Haan, E. H. (2000). The Corsi Block-Tapping Task: Standardization and normative data. *Applied Neuropsychology*, *7*(4), 252–8. https://doi.org/10.1207/S15324826AN0704_8

Keus, I. M., Jenks, K. M., & Schwarz, W. Psychophysiological evidence that the SNARC effect has its functional locus in a response selection stage. *Cognitive Brain Research, 24*(1), 48–56. https://doi.org/10.1016/j.cogbrainres.2004.12.005 PMID: 15922157

Kramer, P., Bressan, P., & Grassi, M. (2018). The SNARC effect is associated with worse mathematical intelligence and poorer time estimation. *Royal Society Open Science, 5*(8), 172362. https://doi.org/10.1098/rsos.172362

Krause, F., Lindemann, O., Toni, I., & Bekkering, H. (2014). Different brains process numbers differently: Structural bases of individual differences in spatial and nonspatial number representations. *Journal of Cognitive Neuroscience, 26*(4), 768–76. https://doi.org/10.1162/jocn_a_00518 PMID: 24188366

Kucian, K., Grond, U., Rotzer, S., Henzi, B., Schönmann, C., Plangger, F., ... & von Aster, M. (2011). Mental number line training in children with developmental dyscalculia. *NeuroImage, 57*(3), 782-795. https://doi.org/10.1016/j.neuroimage.2011.01.070

Kucian, K., Kaufmann, L., & von Aster, M. (2014). Brain correlates of numerical disabilities. In R. Cohen Kadosh & A. Dowker (Eds.), *The Oxford handbook of numerical cognition* (pp. 732-744). Oxford: Oxford University Press. doi:10.1093/oxfordhb/9780199642342.013.009

Kurdek, L. A., & Sinclair, R. J. (2001). Predicting reading and mathematics achievement in fourth-grade children from kindergarten readiness scores. *Journal of Educational Psychology, 93*(3), 451-455. http://dx.doi.org/10.1037/0022-0663.93.3.451

Kyttälä, M., Aunio, P., Lehto, J. E., Van Luit, J., & Hautamäki, J. (2003). Visuospatial working memory and early numeracy. *Educational and Child Psychology*, *20*(3), 65-76.

Lachance, J. A., & Mazzocco, M. M. M. (2006). A longitudinal analysis of sex differences in math and spatial skills in primary school age children. *Learning and Individual Differences, 16*(3), 195-216. https://doi.org/10.1016/j.lindif.2005.12.001

Landy, D.H., Jones, E.L., Hummel, J.E., 2008. Why spatial-numeric associations aren't evidence for a mental number line. *Proceedings of the Annual Meeting of the Cognitive Science Society, 30*(30).

Lee, D., Chun, J., & Cho, S. (2016). The instructional dependency of SNARC effects reveals flexibility of the space-magnitude association of nonsymbolic and symbolic magnitudes. *Perception, 45*(5), 552–567. https://doi.org/10.1177/0301006616629027

Lefevre, J.-A., Wells, E., & Sowinski, C. (2014). Individual differences in basic arithmetical processes in children and adults. In R. Cohen Kadosh & A. Dowker (Eds.), *The Oxford handbook of numerical cognition* (pp. 1-17). Oxford: Oxford University Press doi:10.1093/oxfordhb/9780199642342.013.005.

Leibovich, T., Katzin, N., Harel, M., & Henik, A. (2017). From "sense of number" to "sense of magnitude": The role of continuous magnitudes in numerical cognition. *Behavioral and Brain Sciences, 40*. https://doi. org/10.1017/S0140525X16000960 PMID: 27530053

Levitt, H. C. C. H. (1971). Transformed up-down methods in psychoacoustics. *The Journal of the Acoustical society of America*, 49(*2B*), 467-477. https://doi.org/10.1121/1.1912375

Libertus, M. E., & Brannon, E. M. (2010). Stable individual differences in number discrimination in infancy. *Developmental science*, *13(*6), 900-906. https://doi.org/10.1111/j.1467-7687.2009.00948.x

Libertus, M. E., Feigenson, L., & Halberda, J. (2013). Is approximate number precision a stable predictor of math ability? *Learning and Individual. Differences, 25*, 126–133. doi:10.1016/j.lindif.2013.02.001

Libertus, M. E., Odic, D., & Halberda, J. (2012). Intuitive sense of number correlates with math scores on college-entrance examination. *Acta Psychologica, 141*(3), 373-379. doi:10.1016/j.actpsy.2012.09.009

Lonnemann, J., Linkersdörfer, J., Hasselhorn, M., & Lindberg, S. (2011). Symbolic and nonsymbolic distance effects in children and their connection with arithmetic skills. *Journal of Neurolinguistics, 24*(5), 583–591. https://doi.org/10.1016/j.jneuroling.2011.02.004

Lourenco, S. F., Aulet, L. S., Ayzenberg, V., Cheung, C-N., & Holmes, K.J. (2017). Right idea, wrong magnitude system. *Behavioral and Brain Sciences, 40*. Epub 07/27. https://doi.org/10.1017/S0140525X16002156 PMID: 29342631

Luccio, R.., Fumarola, A., Tamburini, G., & Agostini, T. (2012). *The spatial representation of non-symbolic numerical quantities*. Proceedings of Fechner Day, Ottawa, ON, Canada, 18–21 October 2012; pp. 321–327.

Lyons, I. M., & Beilock, S. L. (2011). Numerical ordering ability mediates the relation between number-sense and arithmetic competence. *Cognition, 121*(2), 256-261. https://doi.org/10.1016/j.cognition.2011.07.009

Lyons, I. M., Nuerk, H.-C., & Ansari, D. (2015). Rethinking the implications of numerical ratio effects for understanding the development of representational precision and numerical processing across formats. *Journal of Experimental Psychology: General, 144*(5), 1021-1035. http://dx.doi.org/10.1037/xge0000094

Lyons, I. M., Price, G. R., Vaessen, A., Blomert, L., & Ansari, D. (2014), Numerical predictors of arithmetic success in grades 1–6. *Developmental Science, 17*(5), 714-726. doi:10.1111/desc.12152

Macnamara, A., Keage, H. A. D., & Loetscher, T. (2018). Mapping of non-numerical domains on space: a systematic review and meta-analysis. *Experimental Brain Research, 236*(2), 335–46. https://doi.org/10.1007/s00221-017- 5154-6 PMID: 29279982

Maloney, E. A., Risko, E. F., Preston, F., Ansari, D., & Fugelsang, J. (2010). Challenging the reliability and validity of cognitive measures: The case of the numerical distance effect. *Acta Psychologica, 134*(2), 154-161. doi:http://dx.doi.org/10.1016/j.actpsy.2010.01.006

Mandler, G., & Shebo, B. J. (1982). Subitizing: An analysis of its component processes. *Journal of Experimental Psychology: General, 111*(1), 1-22. http://dx.doi.org/10.1037/0096-3445.111.1.1

Marghetis, T., Kanwal, J., & Bergen, B. (2013). P*lacing numbers in behavioral space: Activity-specific interactions between number and space with a single response button*. In Proceedings of the Annual Meeting of the Cognitive Science Society (Vol. 35, No. 35). No Doi

Mazzocco, M. M., Feigenson, L., & Halberda, J. (2011). Impaired acuity of the approximate number system underlies mathematical learning disability (dyscalculia). *Child Development, 82*(4), 1224–37. https://doi.org/10.1111/j.1467-8624.2011.01608.x PMID: 21679173

McComb K., Packer, C., & Pusey, A. (1994). Roaring and numerical assessment in contests between groups of female lions, Panthera leo. *Animal Behaviour*, *47*(2), 379-387. https://doi.org/10.1006/anbe.1994.1052

Meck, W. H., & Church, R. M. (1983). A mode control model of counting and timing processes. *Journal of Experimental Psychology: Animal Behavior Processes, 9*(3), 320-334. http://dx.doi.org/10.1037/0097-7403.9.3.320

Meyer, M. L., Salimpoor, V. N., Wu, S. S., Geary, D. C., & Menon, V. (2010). Differential contribution of specific working memory components to mathematics achievement in 2nd and 3rd graders. *Learning and Individual Differences*, *20*(2), 101-109.

Milner, D., & Goodale, M. (2006). *The visual brain in action* (Vol. 27). OUP Oxford.

Minturn, A. L., & Reese, T. W. (1951). The effect of differential reinforcement on the discrimination of visual number. *The Journal of Psychology*, *31*(2), 201-231.

Mitchell, T., Bull, R., & Cleland, A. A. (2012). Implicit response-irrelevant number information triggers the SNARC effect: Evidence using a neural overlap paradigm. *The Quarterly Journal of Experimental Psychology, 65*(10), 1945-1961. https://doi.org/10.1080/17470218.2012.673631 PMID: 22524699

Mix, K. S., Levine, S. C., Cheng, Y. L., Young, C., Hambrick, D. Z., Ping, R., & Konstantopoulos, S. (2016). Separate but correlated: The latent structure of space and mathematics across development. *Journal of Experimental Psychology: General*, *145*(9), 1206-1227. https://doi.org/10.1037/xge0000182

Montemayor, C., & Winther, R. G. (2015). Review of '*Space, time, and number in the brain'*. *Mathematical Intelligencer, 37*(2), 93-98.

Moyer, R. S., & Landauer, T. K. (1967). Time required for judgements of numerical inequality. *Nature, 215*, 1519–1520. doi: 10.1038/2151519a0

Müller, D., & Schwarz, W. (2007). Is there an internal association of numbers to hands? The task set influences the nature of the SNARC effect. *Memory & Cognition, 35*(5), 1151-1161. https://doi.org/10.3758/BF03193485

Mundy, E. & Gilmore, C. K. (2009). Children's mapping between symbolic and nonsymbolic representations of number. *Journal of Experimental Child Psychology, 103*(4), 490-502. https://doi:10.1016/j.jecp.2009.02.003

Myers, T., Carey, E., & Szűcs, D. (2017). Cognitive and neural correlates of mathematical giftedness in adults and children: A review. *Frontiers in Psychology, 8*(1646). https://doi.org/10.3389/fpsyg.2017.01646 PMID: 29118725

Nachmias, J. (2006). The role of virtual standards in visual discrimination. *Vision Research, 46*(15), 2456-2464. https://doi.org/10.1016/j.visres.2006.01.029

Nathan, M. B., Shaki, S., Salti, M., & Algom, D. (2009). Numbers and space: Associations and dissociations. *Psychonomic Bulletin & Review, 16*(3), 578–582. https://doi.org/10.3758/PBR.16.3.578

Nemeh, F., Humberstone, J., Yates, M. J., & Reeve, R. A. (2018). Non-symbolic magnitudes are represented spatially: Evidence from a non-symbolic SNARC task. *PloS One, 13*, e0203019. https://doi.org/10.1371/journal.pone.0203019

Nieder, A. (2005). Counting on neurons: the neurobiology of numerical competence. *Nature Reviews. Neuroscience, 6*, 177–190. https://doi:10.1038/nrn1626

Nieder, A., & Dehaene, S. (2009). Representation of number in the brain. *Annual Review of Neuroscience, 32*, 185–208. https://doi:10.1146/annurev.neuro.051508.135550

Nieder, A., & Merten, K. (2007). A labeled-line code for small and large numerosities in the monkey prefrontal cortex. *Journal of Neuroscience, 27*(22), 5986-5993. https://doi.org/10.1523/JNEUROSCI.1056-07.2007

Nishimura, A., & Yokosawa, K. (2009). Effects of laterality and pitch height of an auditory accessory stimulus on horizontal response selection: The Simon effect and the SMARC effect. *Psychonomic Bulletin Review, 16*(4), 666–70. https://doi.org/10.3758/PBR.16.4.666 PMID: 19648450

Nuerk, H. C., Iversen, W., & Willmes, K. (2004). Notational modulation of the SNARC and the MARC (linguistic markedness of response codes) effect. *The Quarterly Journal of Experimental Psychology Section A, 57*(5), 835-863. https://doi: 10.1080/02724980343000512

Nuerk, H. C., Wood, G., & Willmes, K. (2005). The universal SNARC effect: The association between number magnitude and space is amodal. *Experimental Psychology, 52*(3), 187–94. https://doi.org/10.1027/1618-3169.52.3.187 PMID: 16076066

Núñez, R. E. (2011). No innate number line in the human brain. *Journal of Cross-Cultural Psychology, 42*(4), 651-668. doi: 10.1177/0022022111406097

Odic, D. The contributions of non-numeric dimensions to number encoding, representations, and decision-making factors. (2017). *Behavioral and Brain Sciences, 40*, Epub 07/27. https://doi.org/10.1017/S0140525X1600220X

Odic, D., Hock, H., & Halberda, J. (2014). Hysteresis affects approximate number discrimination in young children. *Journal of Experimental Psychology: General, 143*(1), 255-265. http://dx.doi.org/10.1037/a0030825

Oldfield, R.C. (1971). The assessment and analysis of handedness: The Edinburgh Inventory. *Neuropsychologia, 9*(1), 97–113. https://doi.org/10.1016/0028-3932(71)90067-4 PMID: 5146491

Park, J., & Starns, J. J. (2015). The approximate number system acuity redefined: A diffusion model approach. *Frontiers in Psychology, 6*, 1955. https://doi.org/10.3389/fpsyg.2015.01955

Patro, K., & Haman, M. (2012). The spatial–numerical congruity effect in preschoolers. *Journal of Experimental Child Psychology*, *111*(3), 534-542. https://doi.org/10.1016/j.jecp.2011.09.006 PMID: 22153910

Patro, K., & Shaki, S. (2016). SNARC for numerosities is modulated by comparative instruction (and resembles some non-numerical effects). *Cognitive Processing, 17*(2), 127-137. https://doi.org/10.1007/s10339-015-0745-2

Patro, K., Nuerk, H.-C., & Cress, U. (2016). Mental number line in the preliterate brain: The role of early directional experiences. *Child Development Perspectives, 10*(3), 172-177. https://doi.org/10.1111/cdep.12179

Paul, J. M., & Reeve, R. A. (2016). Relationship between single digit addition strategies and working memory reflects general reasoning sophistication. *Learning and Instruction*, *42*, 113–22. https://doi.org/10.1016/j.learninstruc.2016.01.011

Pelli, D. G. (1997). The VideoToolbox software for visual psychophysics: Transforming numbers into movies. *Spatial Vision*, *10*(4), 437–442. doi: 10.1163/156856897x00366

Pepperberg, I. M., (2006). Cognitive and communicative abilities of Grey parrots. *Applied Animal Behaviour Science*, *100*(1-2), 77-86. https://doi.org/10.1016/j.applanim.2006.04.005

Pesenti, M., Thioux, M., Seron, X., & De Volder, A, (2000). Neuroanatomical substrates of Arabic number processing, numerical comparison, and simple addition: A PET study. *Journal of Cognitive Neuroscience*, *12*(3), 461-479. https://doi.org/10.1162/089892900562273

Piazza, M. (2010). Neurocognitive start-up tools for symbolic number representations. *Trends in Cognitive Sciences*, *14*(12), 542-551. https://doi: 10.1016/j.tics.2010.09.008

Piazza, M., & Izard, V. (2009). How humans count: Numerosity and the parietal cortex. *Neuroscientist 15*(3), 261–273. https://doi: 10.1177/1073858409333073

Piazza, M., Facoetti, A., Trussardi, A. N., Berteletti, I., Conte, S., Lucangeli, D., Dehaene, S., & Zorzi, M. (2010). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition*, *116*(1), 33-41. https://doi: 0.1016/j.cognition.2010.03.012

Piazza, M., Izard, V., Pinel, P., Le Bihan, D., & Dehaene, S. (2004). Tuning curves for approximate numerosity in the human intraparietal sulcus. *Neuron*, *44*(3), 547-555. https://doi.org/10.1016/j.neuron.2004.10.014

Piazza, M., Pica, P., Izard, V., Spelke, E. S., & Dehaene, S. (2013). Education enhances the acuity of the nonverbal approximate number system. *Psychological Science*, *24*(6), 1037–1043. https://doi.org/10.1177/0956797612464057

Piazza, M., Pinel, P., Le Bihan, D., & Dehaene, S. (2007), A magnitude code common to numerosities and number symbols in human intraparietal cortex. *Neuron*, *53*(2), 293-305. https://doi.org/10.1016/j.neuron.2006.11.022

Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group. *Science*, *306*(5695), 499-503. https://doi: 0.1126/science.110208

Pinel, P., Dehaene, S., Rivière, D., & Le Bihan, D. (2001). Modulation of parietal activation by semantic distance in a number comparison task. *NeuroImage*, *14*(5), 1013-1026. https://doi.org/10.1006/nimg.2001.0913

Price, G. R., Holloway, I., Räsänen, P., Vesterinen, M., & Ansari, D. (2007). Impaired parietal magnitude processing in developmental dyscalculia. *Current Biology*, *17*(24), R1042-R1043. https://doi.org/10.1016/j.cub.2007.10.013

Price, G. R., Palmer, D., Battista, C., & Ansari, D. (2012). Nonsymbolic numerical magnitude comparison: reliability and validity of different task variants and outcome measures, and their relationship to arithmetic achievement in adults. *Acta Psychologia*, *140*(1), 50–57. doi: 10.1016/j.actpsy.2012.02.008

Priftis, K., Zorzi, M., Meneghello, M., Marenzi, R., & Umiltà, C. (2006). Explicit versus implicit processing of representational space in neglect: Dissociations in accessing the mental number line. *Journal of Cognitive Neuroscience*, *18*(4), 680-688. https://doi.org/10.1162/jocn.2006.18.4.680

Proctor, R. W., & Cho, Y. S. (2006). Polarity correspondence: A general principle for performance of speeded binary classification tasks. *Psychological Bulletin*, *132*(3), 416-442. http://dx.doi.org/10.1037/0033-2909.132.3.416

Rapin, I. (2016). Dyscalculia and the calculating brain. *Pediatric neurology*, *61*, 11-20. https://doi.org/10.1016/j.pediatrneurol.2016.02.007

Ratcliff, R., Thompson, C. A., & McKoon, G. (2015). Modeling individual differences in response time and accuracy in numeracy. *Cognition, 137*, 115-136. https://doi.org/10.1016/j.cognition.2014.12.004

Reeve, R., Gray, S., Butterworth, B., & Paul, J. (2018). Variability in single digit addition problem-solving speed over time identifies typical, delay and deficit math pathways. *Frontiers in Psychology, 9*, 1498. doi: 10.3389/FPSYG.2018.01498

Reeve, R., Reynolds, F., Humberstone, J., & Butterworth, B. (2012). Stability and change in markers of core numerical competencies. *Journal of Experimental Psychology: General, 141*(4), 649–666. doi:10.1037/a0027520

Reeve, R. A., Reynolds, F., Paul, J., & Butterworth, B. L. (2018). Culture-independent prerequisites for early arithmetic. *Psychological science*, *29*(9), 1383-1392. https://doi org.ezp.lib.unimelb.edu.au/ 10.1177/0956797618769893

Ren, P., Nicholls, M. E., Ma, Y. Y., & Chen, L. (2011). Size matters: Non-numerical magnitude affects the spatial coding of response. *PLoS One*, *6*(8), e23553. https://doi.org/10.1371/journal.pone.0023553PMID: 21853151

Restle, F. (1970). Speed of adding and comparing numbers. *Journal of Experimental Psychology, 83*(2, Pt.1), 274-278. http://dx.doi.org/10.1037/h0028573

Reynvoet, B., & Brysbaert, M. (1999). Single-digit and two-digit Arabic numerals address the same semantic number line. *Cognition, 72*(2), 191–201. https://doi.org/10.1016/S0010-0277(99)00048-7 PMID:10553671

Ross, J. (2003). Visual discrimination of number without counting. *Perception, 32*(7), 867-870. https://doi.org/10.1068/p5029

Ross, J., & Burr, D. C. (2010). Vision senses number directly. *Journal of Vision, 10*(2), 10. doi:10.1167/10.2.10

Rossetti, Y., Jacquin-Courtois, S., Rode, G., Ota, H., Michel, C., & Boisson, D. (2004). Does action make the link between number and space representation? Visuo-manual adaptation improves number bisection in unilateral neglect. *Psychological Science, 15*(6), 426-430. doi: 10.1111/j.0956-7976.2004.00696.x

Rotzer, S., Loenneker, T., Kucian, K., Martin, E., Klaver, P., & Von Aster, M. (2009). Dysfunctional neural network of spatial working memory contributes to developmental dyscalculia. *Neuropsychologia*, *47*(13), 2859-2865. https://doi.org/10.1016/j.neuropsychologia.2009.06.009

Rubinsten, O. (2015). Link between cognitive neuroscience and education: the case of clinical assessment of developmental dyscalculia. *Frontiers in human neuroscience*, *9*, 304. https://doi.org/10.3389/fnhum.2015.00304

Rugani, R., Vallortigara, G., Priftis, K., & Regolin, L. (2015). Number-space mapping in the newborn chick resembles humans' mental number line. Science, 347(6221), 534-536. doi: 10.1126/science.aaa1379

Rusconi, E., Kwan, B., Giordano, B. L., Umiltà, C., & Butterworth, B. (2006). Spatial representation of pitch height: The SMARC effect. *Cognition, 99*(2), 113–29. https://doi.org/10.1016/j.cognition.2005.01.004

Santens, S., & Gevers, W. (2008). The SNARC effect does not imply a mental number line. *Cognition, 108*(1), 263-270. https://doi.org/10.1016/j.cognition.2008.01.002

Sasanguie, D., De Smedt, B., Defever, E., & Reynvoet, B. (2012). Association between basic numerical abilities and mathematics achievement. *British Journal of Developmental Psychology, 30*(2), 344–357. doi: 10.1111/j.2044-835X.2011.02048.x

Sasanguie, D., Van den Bussche, E., & Reynvoet, B. (2012). Predictors for mathematics achievement? Evidence from a longitudinal study. *Mind, Brain, and Education, 6*(3), 119–128. doi: 10.1111/j.1751-228X.2012.01147.x

Schneider, M., Beeres, K., Coban, L., Merz, S., Schmidt, S., Stricker, J., et al. (2016). Associations of non-symbolic and symbolic numerical magnitude processing with mathematical competence: A meta-analysis. *Developmental Science, 20*(3), e12372. doi: 10.1111/desc.12372

Schwarz, W., & Keus, I. M. (2004). Moving the eyes along the mental number line: Comparing SNARC effects with saccadic and manual responses. *Perception & Psychophysics, 66*(4), 651–64. https://doi.org/10.3758/bf03194909.

Schwarz, W., & Müller, D. (2006). Spatial associations in number-related tasks: A comparison of manual and pedal responses. *Experimental Psychology, 53*(1), 4–15. https://doi.org/10.1027/1618-3169.53.1.4 PMID: 16610269

Sella, F., Tressoldi, P., Lucangeli, D., & Zorzi, M. (2016). Training numerical skills with the adaptive videogame "The Number Race": A randomized controlled trial on preschoolers. *Trends in Neuroscience and Education*, *5*(1), 20–9. https://doi.org/10.1016/j.tine.2016.02.002

Shaki, S., & Gevers, W. (2011). Cultural characteristics dissociate magnitude and ordinal information processing. *Journal of Cross-Cultural Psychology, 42*(4), 639–50. https://doi.org/10.1177/0022022111406100

Shaki, S., & Petrusic, W. M. (2005). On the mental representation of negative numbers: Context-dependent SNARC effects with comparative judgments. *Psychonomic Bulletin Review, 12*(5), 931-937. https://doi.org/10.3758/BF03196788 PMID: 16524013

Shaki, S., Petrusic, W. M., & Leth-Steensen, C. (2012), SNARC effects with numerical and non-numerical symbolic comparative judgments: Instructional and cultural dependencies. *Journal of Experimental Psychology: Human Perception and Performance, 38*(2), 515–30. https://doi.org/10.1037/a0026729 PMID: 22288694

Skagenholt, M., Träff, U., Västfjäll, D., & Skagerlund, K. (2018). Examining the triple code model in numerical cognition: An fMRI study. *PLoS ONE, 13*(6): e0199247. https://doi.org/10.1371/journal.pone.0199247

Skagerlund, K., & Träff, U. (2016). Number processing and heterogeneity of developmental dyscalculia: Subtypes with different cognitive profiles and deficits. *Journal of learning disabilities*, *49*(1), 36-50. https://doi.org/10.1177/0022219414522707

Smets, K., Gebuis, T., Defever, E., & Reynvoet, B. (2014). Concurrent validity of approximate number sense tasks in adults and children. *Acta Psychologica, 150*, 120-128. https://doi.org/10.1016/j.actpsy.2014.05.001

Soltész, F., Szűcs, D., & Szűcs, L. (2010). Relationships between magnitude representation, counting and memory in 4- to 7-year-old children: A developmental study. *Behavioral and Brain Functions, 6*, 13. https://doi.org/10.1186/1744-9081-6-13

Spelke. E., & Dehaene, S. (1999). Biological foundations of numerical thinking. *Trends in Cognitive Sciences*, *3*, 365–366. https://doi-org.ezp.lib.unimelb.edu.au/10.1016/S1364-6613(99)01385-6

Starr, A., Libertus, M. E., & Brannon, E. M. (2013). Infant number sense predicts later math abilities. *Proceedings of the National Academy of Sciences, 110*(45), 18116-18120. doi:10.1073/pnas.1302751110

Szkudlarek, E., & Brannon, E. M. (2017) Does the approximate number system serve as a foundation for symbolic mathematics? *Language Learning and Development, 13*(2), 171-190. doi:10.1080/15475441.2016.1263573

Szűcs, D., & Myers, T. (2017). A critical analysis of design, facts, bias and inference in the approximate number system training literature: A systematic review. *Trends in Neuroscience and Education, 6*, 187-203. doi: 10.1016/j.tine.2016.11.002

Szűcs, D., Nobes, A., Devine, A., Gabriel, F. C., & Gebuis, T. (2013). Visual stimulus parameters seriously compromise the measurement of approximate number system acuity and comparative effects between adults and children. *Frontiers in Psychology, 4*(44). doi:10.3389/fpsyg.2013.00444

Temple, E., & Posner, M. I. (1998). Brain mechanisms of quantity are similar in 5-year-old children and adults. *Proceedings of the National Academy of Sciences*, *95*(13), 7836-7841. https://doi.org/10.1073/pnas.95.13.7836

Tosto, M. G., Petrill, S. A., Malykh, S., Malki, K., Haworth, C., Mazzocco, M. M., et al. (2017). Number sense and mathematics: Which, when and how? *Developmental Psychology, 53*(10), 1924. https://doi.org/10.1037/ dev0000331 PMID: 28758784

Trick, L. M., & Pylyshyn, Z. W. (1994). Why are small and large numbers enumerated differently? A limited-capacity preattentive stage in vision. *Psychological Review, 101*(1), 80-102. doi:10.1037/0033-295X.101.1.80

van Dijck, J.-P., Abrahamse, E. L., Acar, F., Ketels, B., & Fias, W, (2014). A working memory account of the interaction between numbers and spatial attention, *The Quarterly Journal of Experimental Psychology, 67*(8), 1500-1513, doi:10.1080/17470218.2014.903984

van Dijck, J.-P., Abrahamse, E. L., Majerus, S., & Fias, W. (2013). Spatial attention interacts with serial-order retrieval from verbal working memory. *Psychological Science, 24*(9), 1854–1859. https://doi.org/10.1177/0956797613479610

van Dijck, J-P., & Fias, W. (2011). A working memory account for spatial–numerical associations. *Cognition, 119*(1), 114–9. https://doi.org/10.1016/j.cognition.2010.12.013 PMID: 21262509

Van Oeffelen, M. P., & Vos, P. G. (1982). A probabilistic model for the discrimination of visual number. *Perception & Psychophysics, 32*(2), 163-170. https://doi.org/10.3758/BF03204275

Vanbinst, K., Ghesquière, P., & De Smedt, B. (2012). Numerical magnitude representations and individual differences in children's arithmetic strategy use. *Mind, Brain, and Education, 6*(3), 129-136. https://doi.org/10.1111/j.1751-228X.2012.01148.x

Venkatraman, V., Ansari, D., & Chee, M. W. L. (2005). Neural correlates of symbolic and non-symbolic arithmetic. *Neuropsychologia, 43*(5), 744-753. https://doi.org/10.1016/j.neuropsychologia.2004.08.005

Vermunt, J. K., & Magidson, J. (2005). *Technical guide for Latent GOLD 4.0: Basic and advanced*. Belmont Massachusetts: Statistical Innovations Inc.

Viarouge, A., Hubbard, E. M., & McCandliss, B. D. (2014). The cognitive mechanisms of the SNARC effect: An individual differences approach. *PLoS ONE, 9*(4), 1–10. https://doi.org/10.1371/journal.pone.0095756 PMID: 24760048

Vicario, C., & Rumiati, R. (2014). Left-right compatibility in the processing of trading verbs. *Frontiers in Behavioral Neuroscience, 8*(16). https://doi.org/10.3389/fnbeh.2014.00016 PMID: 24478662

Vogel, S. E., Grabner, R. H., Schneider, M., Siegler, R. S., & Ansari, D. (2013). Overlapping and distinct brain regions involved in estimating the spatial position of numerical and non-numerical magnitudes: an fMRI study. *Neuropsychologia, 51*(5), 979-989. https://doi.org/10.1016/j.neuropsychologia.2013.02.001

von Aster, M., & Shalev, R. (2007). Number development and developmental dyscalculia. *Developmental Medicine & Child Neurology, 49*(11), 868–873. doi: 10.1111/j.1469-8749.2007.00868.x

Vuilleumier, P., & Rafal, R. (1999). "Both" means more than "two": Localizing and counting in patients with visuospatial neglect. *Nature Neuroscience, 2*(9), 783-784. 783-784. doi:10.1038/12150

Walsh, V. (2003). A theory of magnitude: Common cortical metrics of time, space and quantity. *Trends in Cognitive Sciences, 7*(11), 483–8. https://doi.org/10.1016/j.tics.2003.09.002 PMID: 14585444

Weis, T., Nuerk, H.-C., & Lachmann, T. (2018). Attention allows the SNARC effect to operate on multiple number lines. *Scientific Reports, 8*(1), 13778. https://doi.org/10.1038/s41598-018-32174-y

Whalen, J., Gallistel, C. R., & Gelman, R. (1999). Nonverbal counting in humans: The psychophysics of number representation. *Psychological Science, 10*(2), 130–137. https://doi.org/10.1111/1467-9280.00120

Wilkinson, G. S., & Robertson, G. (2006). *Wide Range Achievement Test 4 (WRAT4)*. Lutz, FL: Psychological Assessment Resources, Inc.

Wood, G., Nuerk, H.-C., & Willmes, K. (2006). Variability of the SNARC effect: systematic interindividual differences or just random error? *Cortex, 42*(8), 1119–1123.

Wood, G., Willmes, K., Nuerk, H.-K., & Fischer, M. H. (2008). On the cognitive link between space and number: A meta-analysis of the SNARC effect. *Psychology Science Quarterly, 50*(4), 489.

Xu, F. (2003). Numerosity discrimination in infants: Evidence for two systems of representations. *Cognition, 89*(1), B15-B25. https://doi.org/10.1016/S0010-0277(03)00050-7

Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition, 74*(1), B1-B11. https://doi.org/10.1016/S0010-0277(99)00066-9

Zebian, S. (2005). Linkages between number concepts, spatial thinking, and directionality of writing: The SNARC effect and the reverse SNARC effect in English and Arabic monoliterates, biliterates, and illiterate Arabic speakers. *Journal of Cognition and Culture, 5*(1-2), 165–190.

Zhou, X., Chen, C., Chen, L., & Dong, Q. (2008). Holistic or compositional representation of two-digit numbers? Evidence from the distance, magnitude, and SNARC effects in a number-matching task. *Cognition, 106*(3), 1525-1536. https://doi.org/10.1016/j.cognition.2007.06.003

Zhou, X., Shen, C., Li, L., Li, D., & Cui, J. (2016). Mental numerosity line in the human's approximate number system. *Experimental Psychology, 63*(3). https://doi.org/10.1027/1618-3169/a000324 PMID: 27404985

Zorzi, M., Barbiero, C., Facoetti, A., Lonciari, I., Carrozzi, M., . . . Ziegler, J. C. (2012). Wider letter spacing helps dyslexic readers. *Proceedings of the National Academy of Sciences*, *109*(28) 11455-11459. doi:10.1073/pnas.1205566109

Zorzi, M., Priftis, K., & Umiltà, C. (2002). Brain damage: neglect disrupts the mental number line. *Nature, 417*, 138–139. doi: 10.1038/ 417138a

Appendix A: Edinburgh Handedness Inventory

Name:

Age:

Sex:

Please indicate your preferences in the use of hands in the following activities *by putting + in the appropriate column*. Where the preference is so strong that you would never try to use the other hand, unless absolutely forced to, *put* ++. If in any case you are really indifferent, *put + in both columns*.

Some of the activities listed below require the use of both hands. In these cases, the part of the task, or object, for which hand preference is wanted is indicated in brackets. Please try and answer all of the questions, and only leave a blank if you have no experience at all with the object or task.

	Left	Right
1. Writing		
2. Drawing		
3. Throwing		
4. Scissors		
5. Toothbrush		
6. Knife (without fork)		
7. Spoon		
8. Broom (upper hand)		
9. Striking Match (match)		
10. Opening box (lid)		
i. Which foot do you prefer to kick with?		
ii. Which eye do you use when using only one?		

Appendix B: Individual RT plots for all participants in Study 3

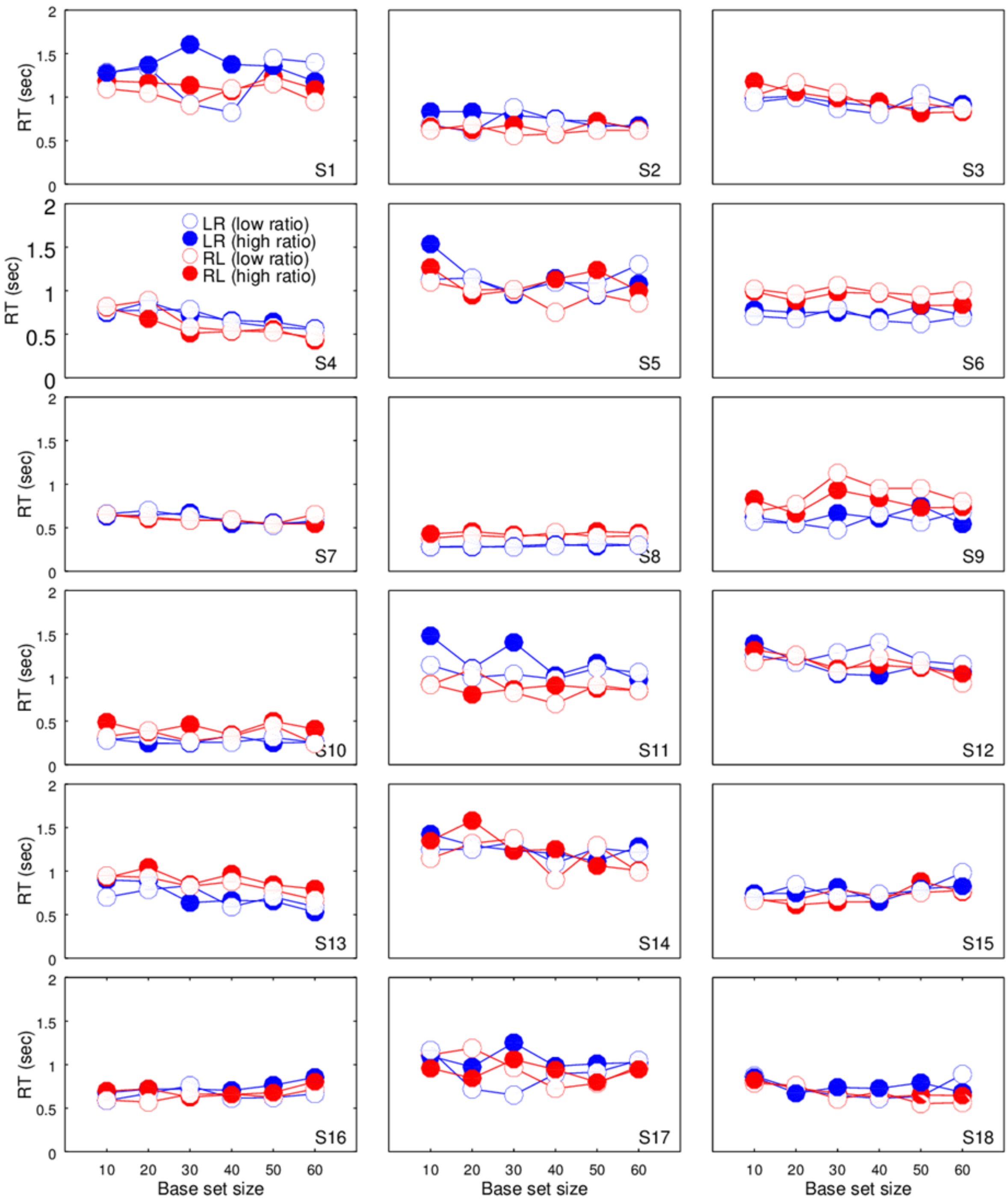

Figure 1. RT in seconds as a function of base set size for low and high ratio. For participants S1 – S18. LR denotes left-to-right mapping response. RL denotes right to left mapping.

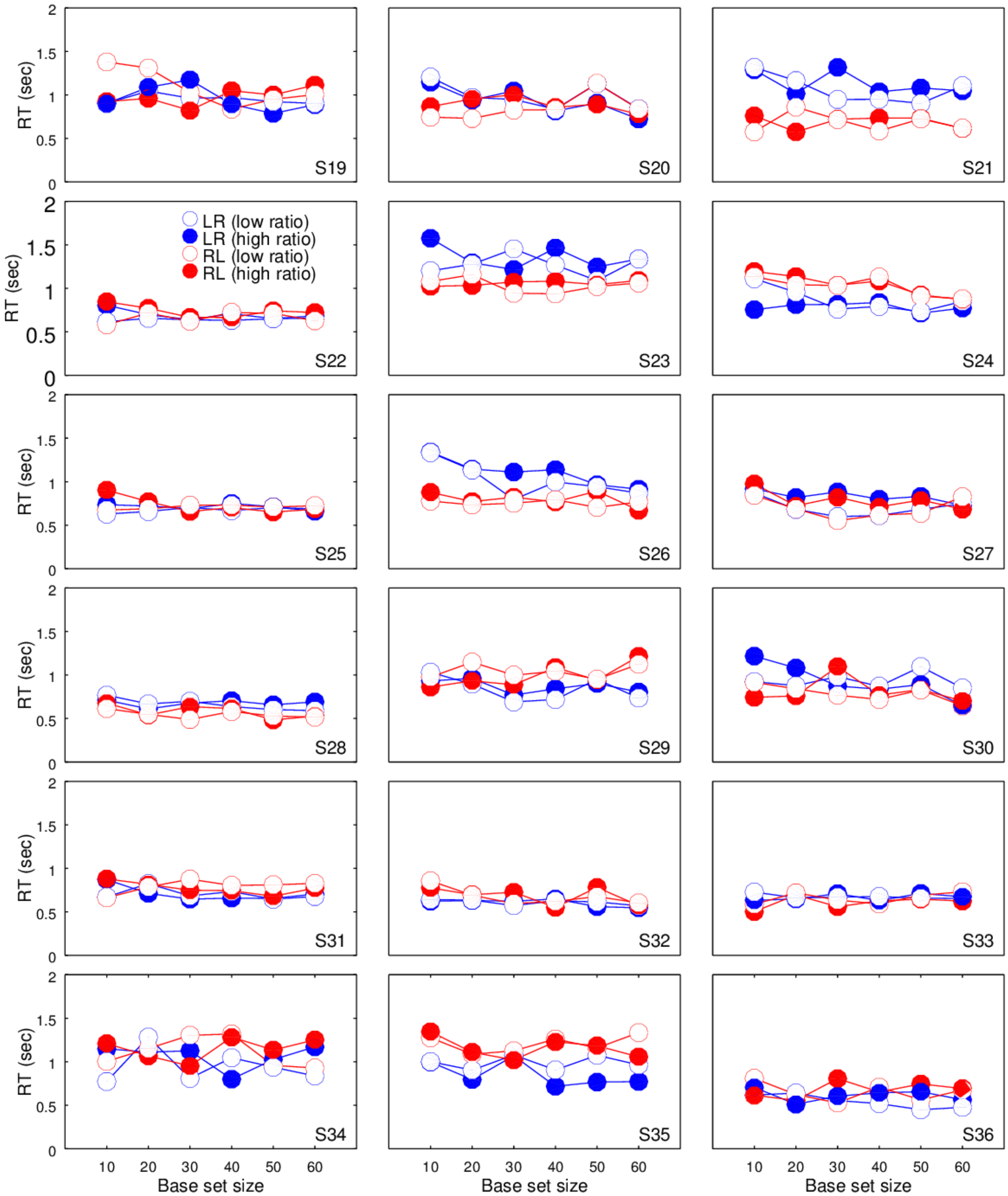

Figure 2. RT in seconds as a function of base set size for low and high ratio. For participants S19 – S36. LR denotes left-to-right mapping response. RL denotes right to left mapping.

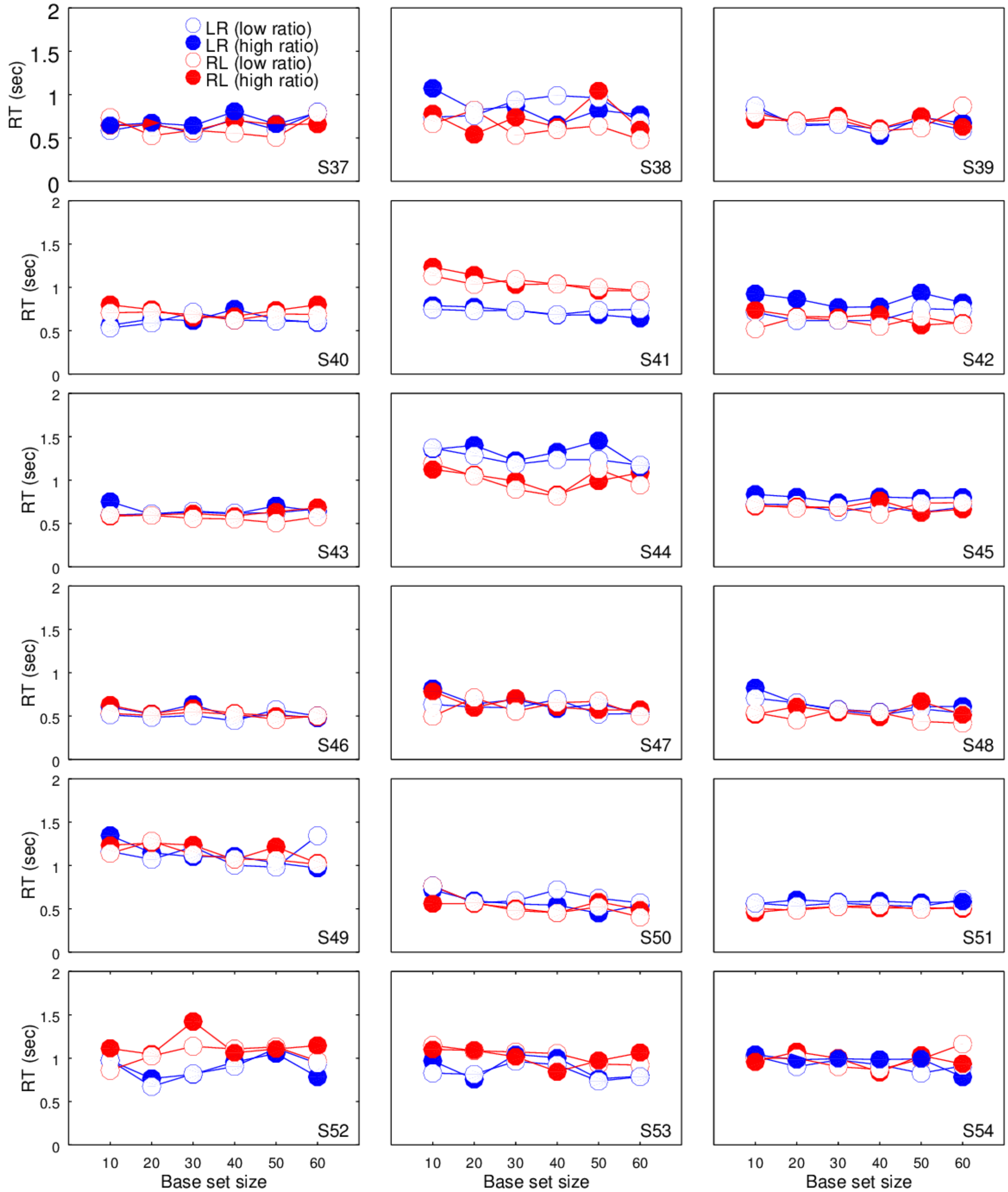

Figure 3. RT in seconds as a function of base set size for low and high ratio. For participants S37 – S54. LR denotes left-to-right mapping response. RL denotes right to left mapping.

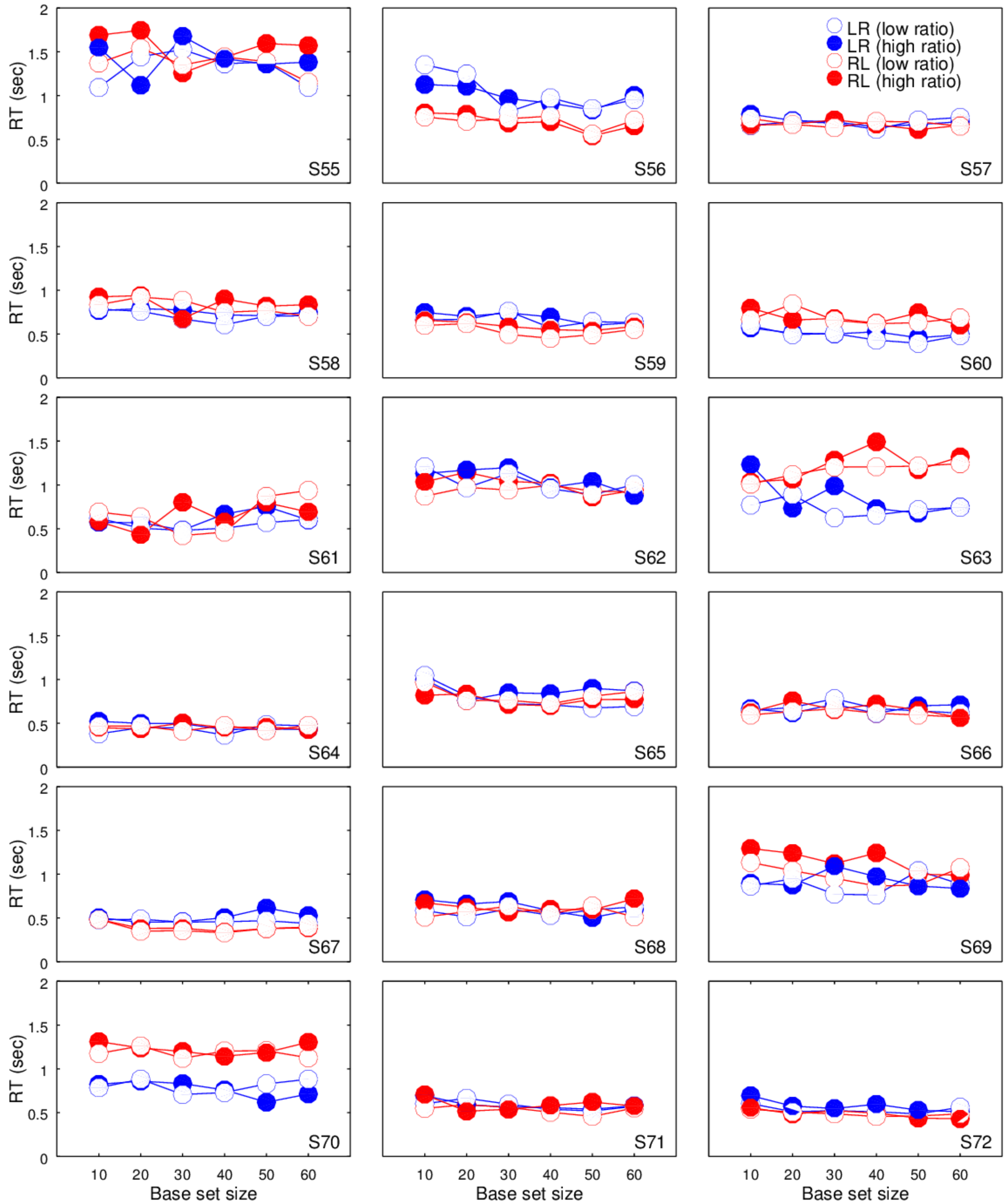

Figure 4. RT in seconds as a function of base set size for low and high ratio. For participants S37 – S54. LR denotes left-to-right mapping response. RL denotes right to left mapping.

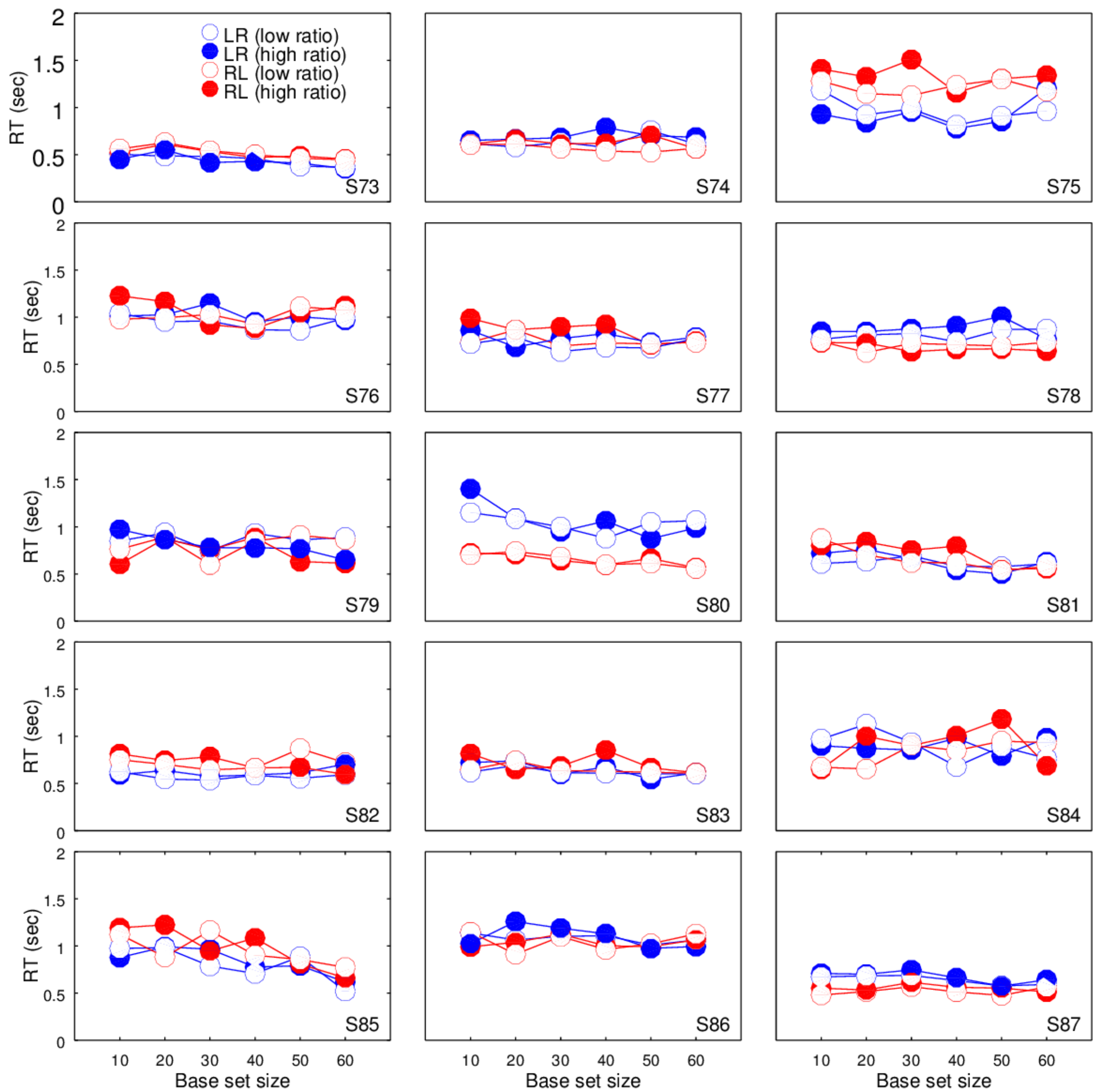

Figure 5. RT in seconds as a function of base set size for low and high ratio. For participants S73 – S87. LR denotes left-to-right mapping response. RL denotes right to left mapping.

Appendix C: Individual Accuracy plots for all participants in Study 3

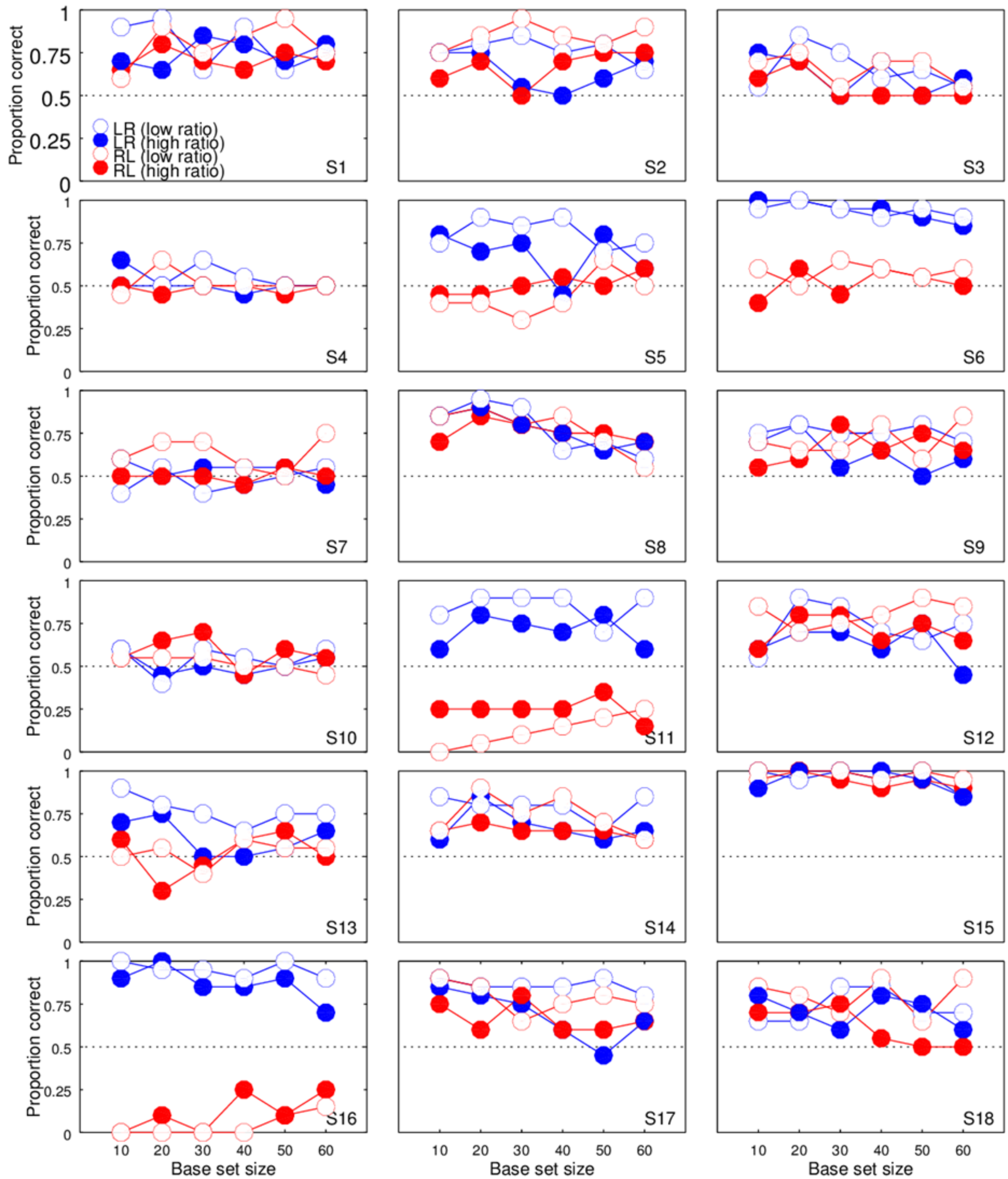

Figure 1. Proportion correct as a function of base set size for low and high ratio. For participants S1 – S18. LR denotes left-to-right mapping response. RL denotes right to left mapping.

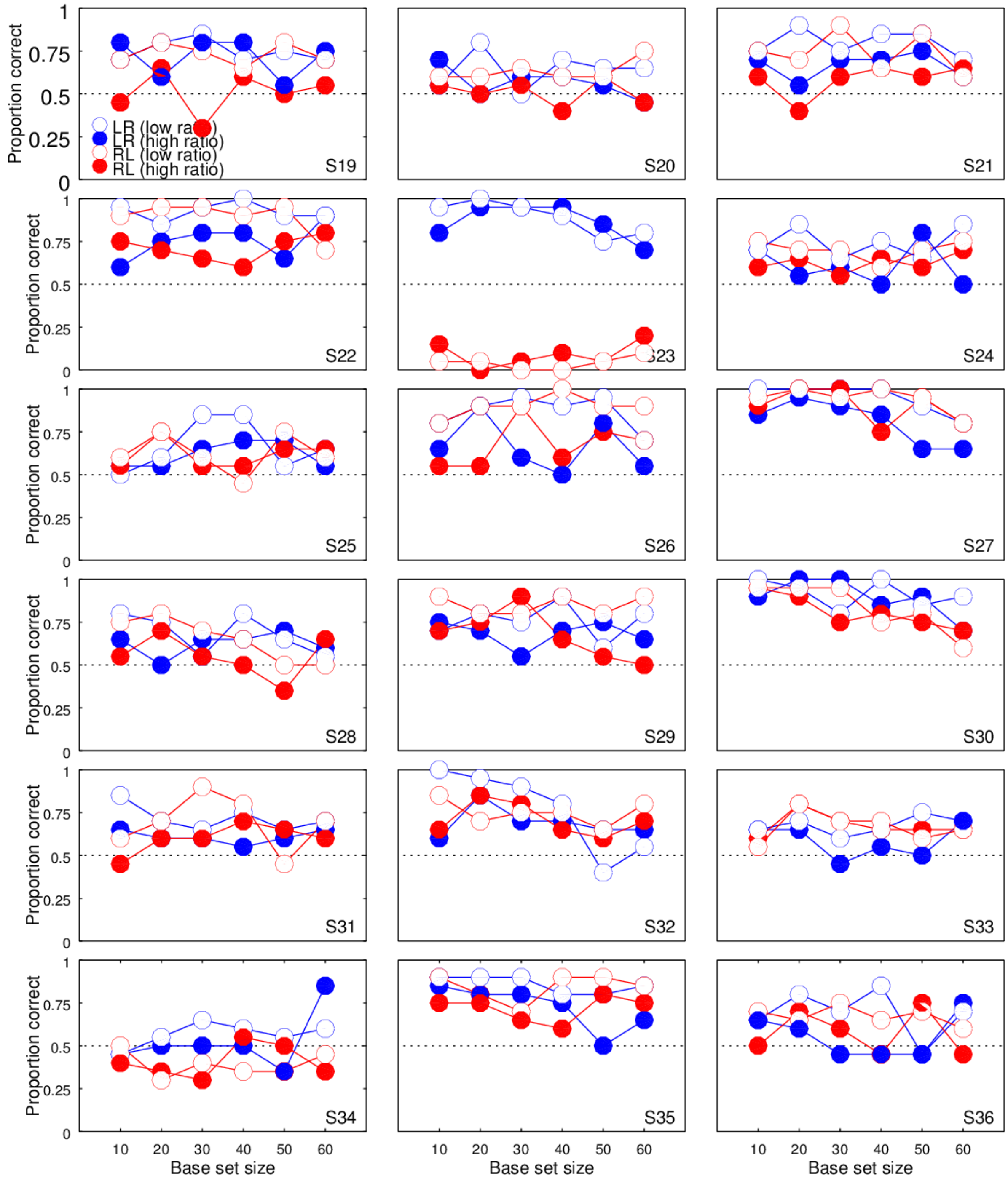

Figure 2. Proportion correct as a function of base set size for low and high ratio. For participants S19 – S36. LR denotes left-to-right mapping response. RL denotes right to left mapping.

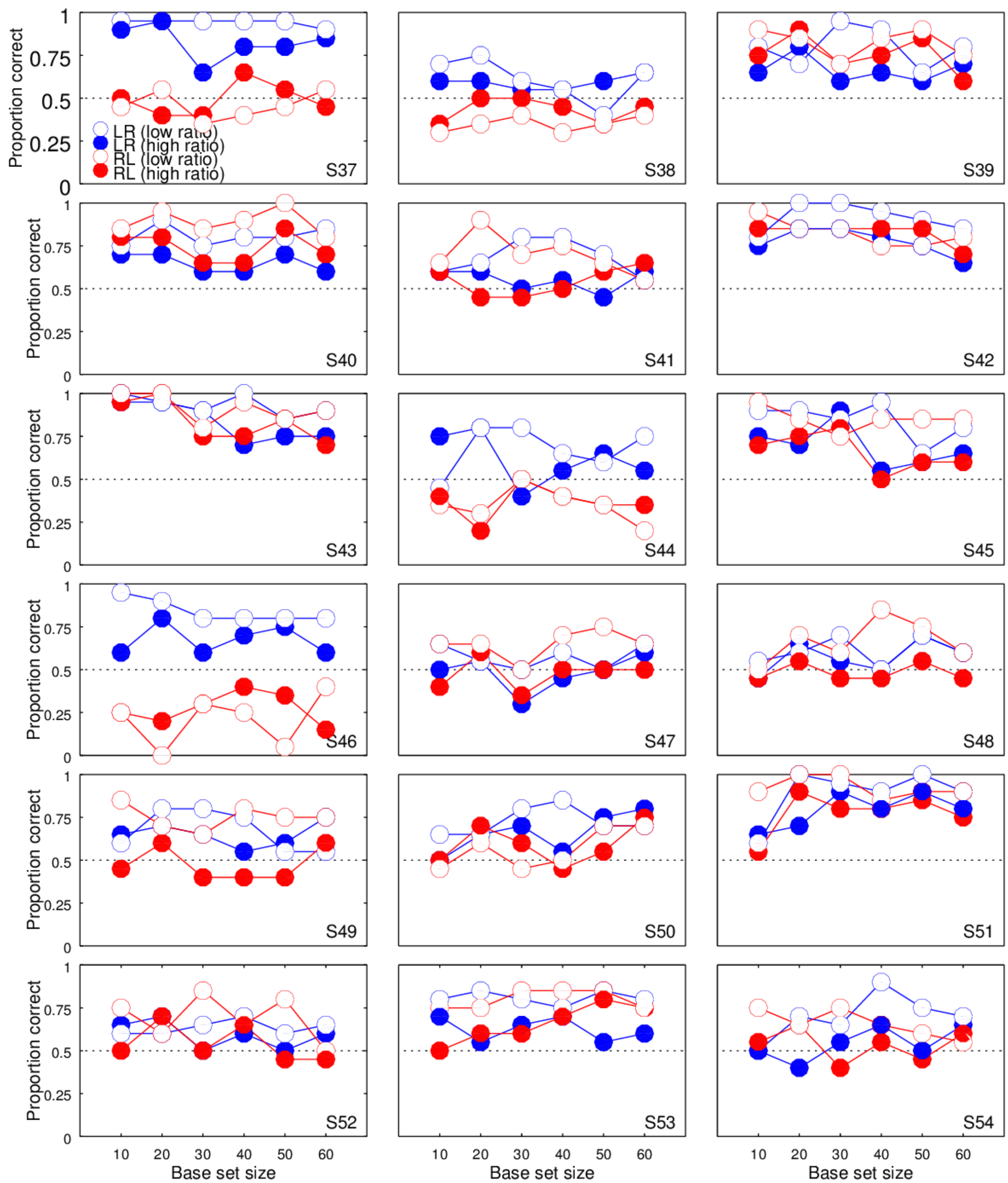

Figure 3. Proportion correct as a function of base set size for low and high ratio. For participants S37 – S54. LR denotes left-to-right mapping response. RL denotes right to left mapping

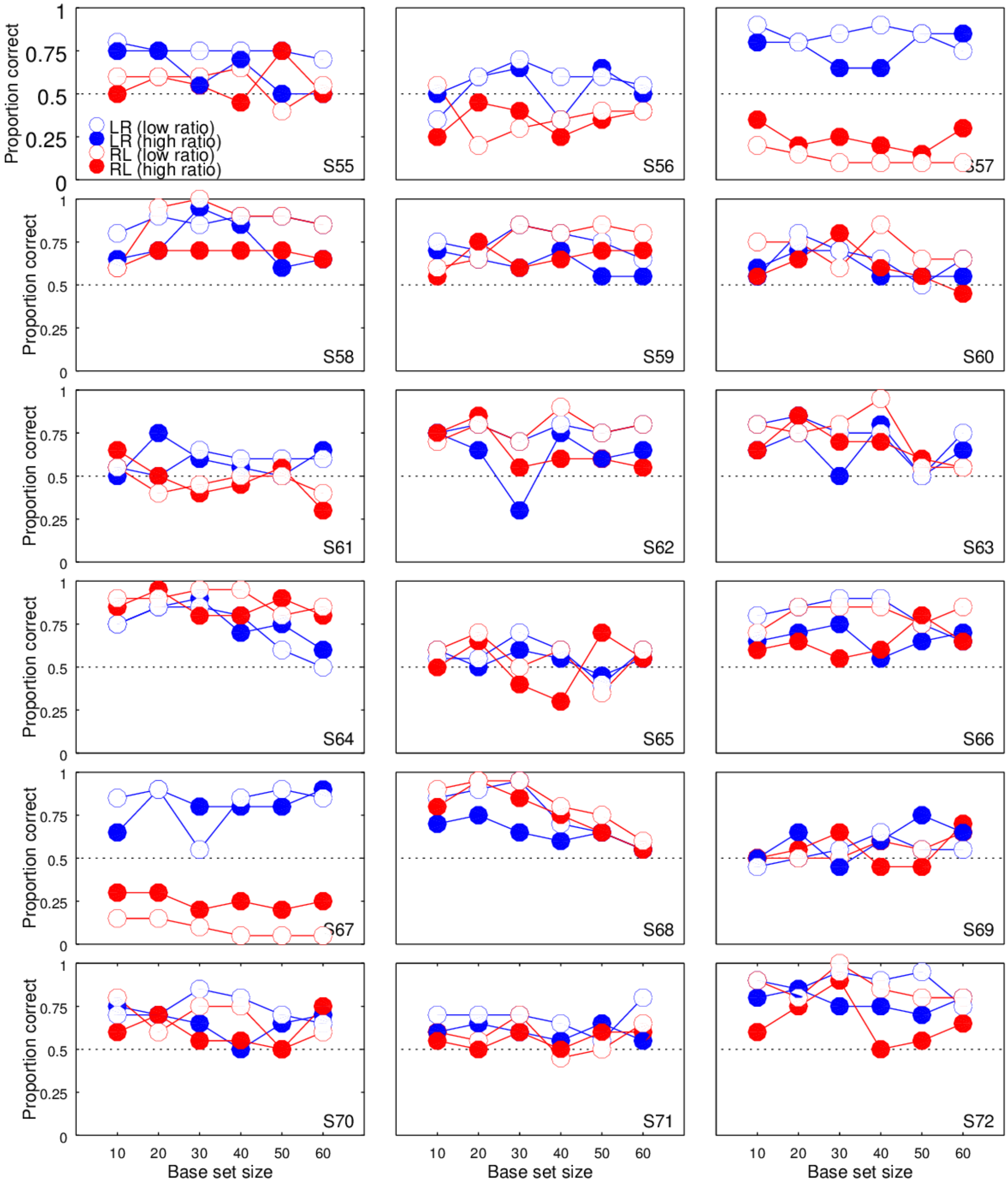

Figure 4. Proportion correct as a function of base set size for low and high ratio. For participants S55 – S72. LR denotes left-to-right mapping response. RL denotes right to left mapping.

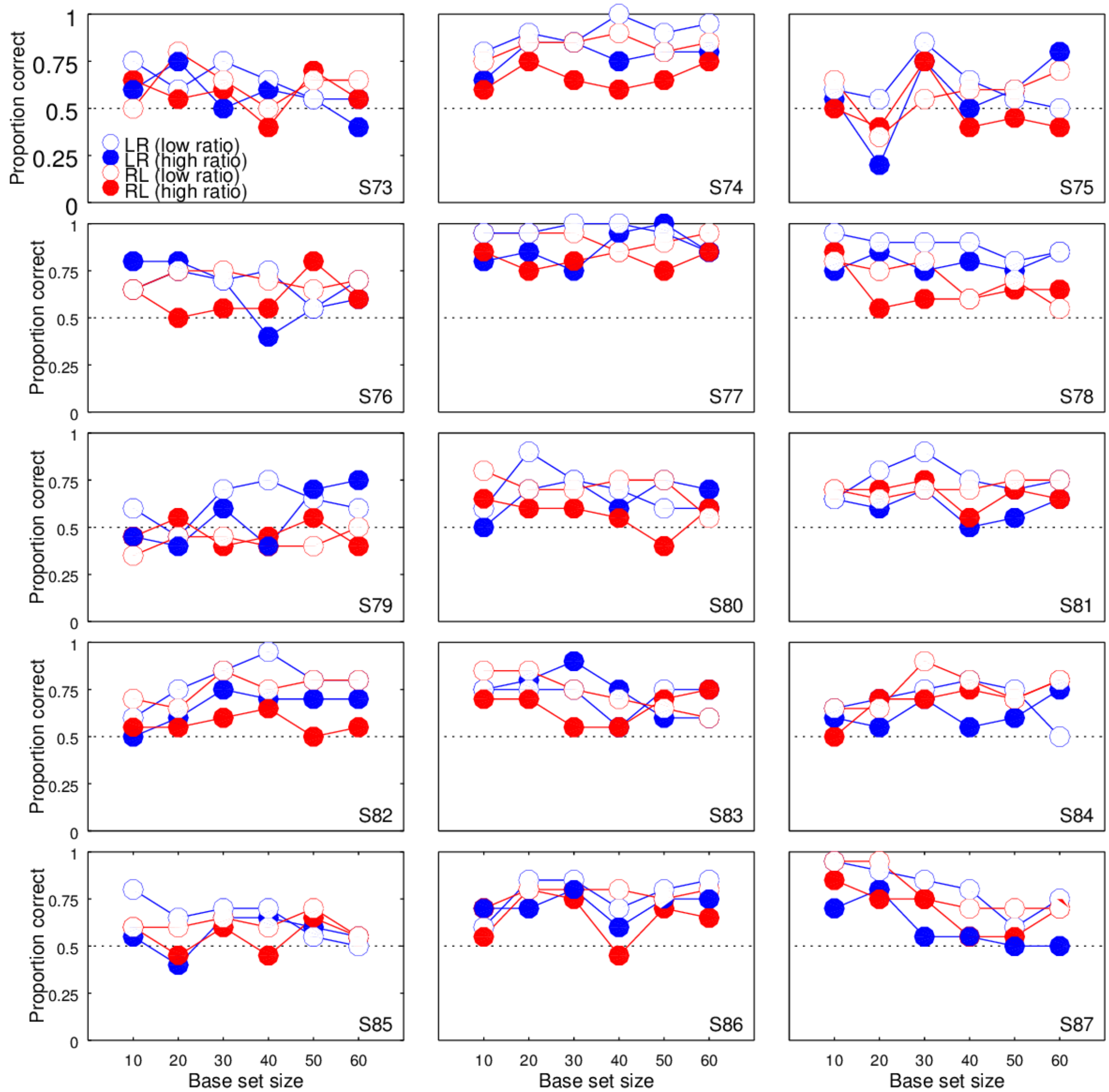

Figure 5. Proportion correct as a function of base set size for low and high ratio. For participants S73 – S87. LR denotes left-to-right mapping response. RL denotes right to left mapping.