Model Predictive Control for Lithium-Ion Battery Optimal Charging

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***Abstract*—The charging time and lifetime are important performance for lithium-ion (Li-ion) batteries, but are often competing objectives for charging operations. Model-based charging controls are challenging due to the complicated battery system structure that is composed of nonlinear partial differential equations (PDEs) and exhibits multiple time-scales. This paper proposes a new methodology for battery charging control enabling an optimal trade-off between the charging time and battery state-of-health (SOH). Using recently developed model reduction approaches, a physics-based low-order battery model is first proposed and used to formulate a model-based charging strategy. The optimal fast charging problem is formulated in the framework of tracking model predictive control (MPC). This directly considers the tracking performance for provided SOC and SOH references, and explicitly addresses constraints imposed on input current and battery internal state. The capability of this proposed charging strategy is demonstrated via simulations to be effective in tracking the desirable SOH trajectories. By comparing to the constant-current constant-voltage (CCCV) charging protocol, the MPC-based charging appears promising in terms of both the charging time and SOH. In addition, this obtained charging strategy is practical for real-time implementation.**

***Index Terms*—Lithium-ion battery, battery fast charging, model predictive control, state-of-charge, state-of-health**

I. Introduction

ELECTRIFICATION of vehicle powertrains has been considered as a promising approach to address the international expectation on clean and energy-efficient transportation [1], [2]. However, the high upfront cost, largely contributed by its lithium-ion (Li-ion) battery system, and the long charging time are unavoidable problems for the electric vehicles [3]. Although the battery price is predicted to decrease in the future [4], an immediate solution for cost saving is to increase battery's lifetime. The battery degradation is mainly dependent on charging strategies used [5] and in general the lifetime is a competing factor to the charging time. To charge the battery as fast as possible but without unduly compromising its lifetime, optimal fast charging strategies are needed.

Currently, most Li-ion batteries are charged using constant-current (CC), constant-current constant-voltage (CCCV) or other similar charging protocols [6], [7], [8]. Common to these charging strategies is that the charging profiles are determined by some pre-defined current and voltage limits irrespective of battery's in-situ physical and chemical characteristics. These model-free approaches are *ad hoc* and the solutions are often conservative. Model-based approaches have achieved considerable success in simulation, optimisation, estimation and control [9], and can be good candidates for battery charging control.

To develop model-based algorithms for battery charging, the existing technical challenges arise from the complexity of battery dynamic system and unmeasurable internal states. The dynamics of a Li-ion battery contains electrical, electrochemical, thermal and ageing phenomena over multiple domains (the electrodes and separator) and multiple time-scales. Mathematically, it is governed by a set of coupled nonlinear partial differential equations (PDEs) with singularly perturbed model structure [10]. This model requires extensive computational resources and is intractable for most model-based online applications. Furthermore, state-of-charge (SOC) and state-of-health (SOH) are critical information for battery charging operations, where SOC represents the available capacity stored in a battery and SOH quantifies the degree of battery degradation. However, these two states are functions of battery internal ion concentrations and capacity fade which are unmeasurable [11], [12], [13]. Hence, the development of model-based charging strategies is often fused with model simplifications and state estimation.

Considerable efforts have been devoted to the development of low order battery models over the past decade. These developed models can be roughly classified into equivalent circuit models (ECMs) and physics-based models. Currently, ECMs are extensively employed in industry battery management systems. This type of models, in essence, ignores battery ion diffusion and electrochemical process, but fit the electrical/thermal behaviours by an ideal voltage source, several resistors, and capacitors [14], [15], [16]. The resulted models are relatively simple and easy to parameterise, but their applications are limited by the low accuracy, particularly at a wide range of operating conditions [17]. Furthermore, there is no physical meaning behind model parameters and insight into battery internal information may not be derived [18].

In light of this, reduced order physics-based battery models have gained increasing popularity for state estimation and control. A systematic approach for battery model simplifications was proposed in [10], [19], with libraries of reduced models based on specific assumptions are obtained. Based on simplified electrochemical models, nonlinear observers have been designed for battery SOC and/or SOH estimation [20], [21], [22]. To enable battery state estimation in multiple time-scales, an observer design framework for singularly perturbed

Manuscript received Oct 05, 2016; revised May 11, 2017 and Aug 21, 2017. This work was supported in part by the Australian Research Council through grant FT100100538.

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systems was proposed and demonstrated to be efficient for both the SOC and SOH estimation [11]. These modelling and estimation approaches can be potentially useful for battery optimal fast charging problem.

Model predictive control (MPC) is a model-based control paradigm gaining increased interest in the last 30 years [9]. Its performance in constraint and nonlinearity handling and model-based optimal or sub-optimal control makes it applicable to a wide range of industrial problems [23]. With the computational power fast moving forward, MPC has been introduced in many applications, e.g. vehicle steering system control [24]. In the battery community, MPC may be useful to solve the charge control problem with a battery prediction model and SOC/SOH constraints.

Recently, by using physics-based models and predictive control algorithms, a few attempts have been made towards the development of improved charging strategies. Based on an electrochemical-thermal model with a constant electrolyte concentration, a one-step MPC was presented for fast charging [25]. Similarly, single particle model (SPM) based MPC problems were formulated in [26]. Through incorporating linear input-output models identified from physics-based models, quadratic dynamic matrix control and hybrid MPC algorithm were proposed in [27] and [28], respectively. The referred work aimed at the minimum charging time with some imposed constraints such as on the voltage, temperature, and/or overpotential, and assumed measured battery internal states.

To enhance the relevant literature, the present work attempts to explicitly consider battery ageing dynamics in the system model and control objective to address the impact of capacity fade and solid-electrode-interface (SEI) film resistance increase. Constraints are directly imposed on the internal states to avoid battery degradation such as lithium plating. Furthermore, the hypothesis regarding the availability of battery state information is relaxed via online state estimation techniques. Using recently developed model reduction approaches, a physics-based low-order battery model is proposed and used to formulate a health-aware fast charging problem within an MPC framework. The employed control algorithm enables SOC and SOH reference tracking under both the states and input constraints. The primary outcome includes the proposed charging strategy with two benefits: it is able to reduce battery's charging time and increase battery lifetime; and it offers flexibility for consumers to manipulate battery degradation or charge profiles as they desire, in a systematic manner.

The remainder of this paper is organised as follows. In Section II, an MPC-based battery charging strategy is formulated. The proposed charging strategy is implemented and evaluated in Section III. Finally, the presented work and potential future research opportunities are concluded in Section IV.

II. Optimal Charging Strategy

In this section we describe the formulation of model predictive control for optimal charging of a Li-ion battery. The first step in the proposed approach is the specification of the reduced order model to be used in the controller.

TABLE I
NOMENCLATURE

Input:	
I	Applied current
Variables:	
C_e	Lithium-ion concentration in the electrolyte
C_s	Bulk lithium-ion concentration in solid particles
C_{ss}	Lithium-ion concentration at the surface of solid particles
q_s	Concentration flux at the surface of solid particles
R_f	SEI film resistance
Q_{sr}	Capacity fade
SOC	State of charge in the electrode
SOH	State of health for the battery cell
T	Battery temperature
V	Terminal voltage
Parameters and Subscripts:	
C_{e0}	Initial ion concentration in the electrolyte
$Q_{\max}$	The maximum battery capacity
$-$	The negative electrode (Anode)
$+$	The positive electrode (Cathode)

A. Reduced Order Battery Modelling

A Li-ion battery containing coupled electrical, electrochemical, thermal and ageing dynamics via a set of PDEs [10] is used as the starting point for control-oriented modelling and will be considered as a system plant for control implementation in simulations.

Reductions can be systematically conducted in a two-step process. First, since the electrochemical-thermal dynamics are much faster than the degradation process but significantly slower than the electrical dynamics, time-scale separation techniques are used to decouple subsystems. This results in libraries of simplified PDE battery models approximating the initial model.

The second step is to reduce the PDEs to computationally tractable ODE systems using standard methods. This includes polynomial approximations and finite differencing separately for spatial dimension reduction and discretisation. The resulting models arising from these simplifications are detailed in [29], [19], and have been proposed for use in state estimation [11].

Within the reduced order model, the following state vector $x := [C_s^-, q_s^-, C_e^-, T, Q_{sr}]^T$ is defined with input, u, representing the applied current and the notation defined in Table I. The output vector of interest is defined as $z := [SOC, SOH, T, V]^T$. Note that state-of-charge and health are defined as

$$SOC(t) = \frac{C_s^-(t) - C_{s,0\%}^-}{C_{s,100\%}^- - C_{s,0\%}^-} \tag{1a}$$

$$SOH(t) = 1 - \frac{Q_{sr}(t)}{Q_{\max}} \tag{1b}$$

These two states are not directly measurable, and must be estimated online for battery monitoring and state-feedback controls.

Consequently, the control-oriented battery model is summarised as

$$\dot{x}(t) = f^c\left(x(t), u(t)\right) \tag{2a}$$

$$z(t) = g(x(t), u(t)) := \begin{bmatrix} C_1 x(t) + c_1 \\ C_2 x(t) + c_2 \\ C_3 x(t) \\ h(x(t), u(t)) \end{bmatrix} \tag{2b}$$

where f^c, g, h are nonlinear functions, C_1, C_2, C_3 are constant matrices, and c_1, c_2 are constants. A full description of the model structure (2) is provided in [11], [10]. This continuous-time model can be discretised in the temporal domain with Euler method, leading to

$$\begin{aligned} x(k\Delta t + \Delta t) &= x(k\Delta t) + \Delta t \cdot f^c\left(x(k\Delta t), u(k\Delta t)\right) \\ &:= f\left(x(k\Delta t), u(k\Delta t)\right) \end{aligned} \tag{3a}$$

$$z(k\Delta t) = g(x(k\Delta t), u(k\Delta t)) \tag{3b}$$

where Δt is the sampling time and the control is taken as piecewise constant signal. For simplicity, it is defined $x(k) := x(k\Delta t)$ and $u(k) := u(k\Delta t)$.

We now pursue the development of a model-based control approach for battery charging management utilising the reduced order model, beginning with a short overview of the general formulation of the MPC tracking problem.

B. General MPC Tracking Problem Formulation

The formulation of a general MPC tracking problem from [30], [31] is presented in this subsection, before practical modifications to this approach are proposed in the subsequent subsection.

For a given exogenous reference signal, y^r, suppose an admissible steady-state, (x^r, u^r), can be generated for the state and input. The decision variable, $\mathbf{u}_k$, is the sequence of the predicted inputs over the prediction horizon, namely $\mathbf{u}_k = \{u(0), \cdots, u(N-1)\}$. The notation $\|\cdot\|^2$ is interpreted as the squared weighted 2-norm, *i.e.* $\|x\|_Q^2 = x^T Q x$. The open-loop optimisation problem solved at each time step k with x_k as a state measurement is given by

$$\begin{aligned} \mathbf{u}_k^* = \arg\min_{\mathbf{u}_k} \sum_{i=0}^{N-1} & \Big[\|x(i) - x^r(k+i)\|_Q^2 \\ & + \|u(i) - u^r(k+i)\|_R^2 \Big] \\ & + \|x(N) - x^r(k+N)\|_P^2 \end{aligned} \tag{4a}$$

subject to $\forall i \in \{0, \cdots, N-1\}$

$$x(i+1) = A(i)x(i) + B(i)u(i) \tag{4b}$$

$$x(0) = x_k \tag{4c}$$

$$u(i) \in U \subset \mathbb{R}^m \tag{4d}$$

$$x(i) \in X \subset \mathbb{R}^n \tag{4e}$$

$$x(N) \in X_f \subset \mathbb{R}^n \tag{4f}$$

The following paragraphs highlight the constituent parts of the algorithm.

Cost function. In (4a), on the right-hand side, the first two terms consist of the stage cost, penalising the deviation between the system input/state and their corresponding future target trajectories. The stage cost is formulated to invoke desired dynamic behaviour throughout the prediction horizon, N, by balancing tracking error against the control effort required to achieve it through appropriate selection of the matrices Q and R. The third term is a terminal cost, used to enforce convergence towards the desired operating condition, and weighted through the matrix P. The three weighting matrices P, Q and R need to satisfy certain conditions regarding the definiteness to ensure stability and feasibility. A detailed introduction of this can be found in [23].

Horizon length. Lengthening the prediction horizon, N, typically increases the guaranteed domain of attraction about the desired equilibrium point, albeit at a computational cost.

Prediction model. (4b) represents the nominal system dynamic model through which the future states can be propagated for prediction. $x(i), u(i)$ are separately the predicted state and input i time step into the future from the current time k. In (4c), $x(0)$ is initialised using the current state.

Constraints. (4d)-(4e) present the constraints applied to the system input and state. Note that output constraints may be mapped to state constraints. The constraint sets X and U are convex and closed subset of $\mathbb{R}^n$ and $\mathbb{R}^m$, respectively. In (4f), a terminal constraint is used to provide a stability guarantee. In the presence of plant-model mismatch and other disturbances, the constrained optimisation problem (4) may become infeasible. It is possible to alleviate this issue by softening some or all of the constraints at the potential cost of constraint adherence.

Implementation of the controller. At each time step k, the optimal input sequence, $\mathbf{u}_k^*$, is obtained by solving the optimisation problem (4). Only the first input is applied to the system, namely, $u_k^* = u^*(0)$, while the remaining inputs namely $\{u^*(1), \cdots, u^*(N-1)\}$ are discarded. This receding horizon approach ensures continual feedback in the close-loop system, thereby providing some inherent robustness in this approach [9].

C. Optimal Battery Charging Problem Formulation

The optimal charging control problem is now formulated by employing the battery model developed in Section II-A and modifying the general MPC algorithm presented in Section II-B. The first stage in the problem definition is the specification of reference trajectories for the state of charge and health.

SOC and SOH references. To ensure reachability of the reference trajectories the following assumption is placed on both reference specifications:

Assumption 1: The change rates of SOC and SOH reference signals with respect to the time satisfy the condition

$$0 \leq \left(\frac{\partial SOC^r}{\partial t}\right)_{\min} \leq \frac{\partial SOC^r}{\partial t} \leq \left(\frac{\partial SOC^r}{\partial t}\right)_{\max} \tag{5}$$

$$\left(\frac{\partial SOH^r}{\partial t}\right)_{\min} \leq \frac{\partial SOH^r}{\partial t} \leq \left(\frac{\partial SOH^r}{\partial t}\right)_{\max} < 0 \tag{6}$$

This assumption avoids nonsensical references being provided to the controller, such as attempting to achieve fast charging with zero state of health degradation within the cell. In Section III, the state of charge reference will be determined through simulation (without consideration of state of health), whilst the state of health reference over a long time scale will be user specified, but satisfying (6).

Battery model. This step describes the development of an appropriate battery model to be used in (4b). To further reduce the computation complexity of model-based control in processors of battery management systems, the nonlinear battery model (3) is successively linearised based on the reference trajectory and previous system input, resulting in

$$x(i+1) = A(i)x(i) + B(i)u(i) + d(i) \tag{7}$$

where $A(i), B(i), d(i)$ are defined as

$$A(i) = \frac{\partial f(x(i), u(i-1))}{\partial x} \tag{8a}$$

$$B(i) = \frac{\partial f(x(i), u(i-1))}{\partial u} \tag{8b}$$

$$d(i) = f(x(i), u(i-1)) - A(i)x(i) - B(i)u(i-1) \tag{8c}$$

It is noted that in (7), N linearisations are required along the prediction horizon to solve the optimisation problem. This may pose computational issues, particularly for a large value of N. To alleviate the computational burden caused by successively calculating the linearisation matrices, the linearised model is assumed to be time-invariant over the horizon. Explicitly, the following further approximation is employed in developing the nominal plant model:

$$A(i) \approx A_k,\; B(i) \approx B_k \tag{9}$$

$$d(i) \approx d_k,\; \forall i \in \{0, \cdots, N-1\} \tag{10}$$

The system outputs of interest for battery charging control include the SOC and SOH degradation with the definition of

$$y(i) := [SOC(i), \Delta SOH(i)]^T \tag{11a}$$

$$\Delta SOH(i) := SOH(i) - SOH(i-1) \tag{11b}$$

By combining (3b) and (11), the output equation for the nominal model can be re-written as

$$y(i) = Cx(i) := [C_1x(i), C_2x(i)]^T \tag{12}$$

The fidelity of the obtained battery model via (7)-(12) will be examined in Section III-A.

Cost function. Using the reachability condition of the reference signals (Assumption 1), it follows from [30] that the MPC cost function can be reformulated as an output tracking problem. Furthermore, given the state of charge is simply the integral of the system input (the charging current) and the (5) has been applied, then the input weight vector can be set as $R := 0$. Note that $R = 0$ will not impact on the stability of the closed-loop system [23]. This leaves the stage cost as containing two competing objectives of maximising charge rate whilst simultaneously minimising change in state of health. The balance between these two factors may be captured by the use of a weight, w, in the output penalty matrix:

$$Q := \begin{bmatrix} 1 & 0 \\ 0 & w \end{bmatrix} \tag{13}$$

The following assumption is now placed on w:

Assumption 2: There exists an optimal w^* such that minimising the stage cost over the horizon using the P, Q, and R matrices defined above achieves the permitted state of health degradation specified by the reference trajectory, SOH^r.
In general, the optimal value w^* is not known *a priori*, and must be obtained using some form of online algorithms. The online update law for w can be given in the following general form, with the error feedback taken as the SOH tracking error in the slow time scale, j, *i.e.*:

$$w_{j+1} = W\left(w_j, SOH^r(j) - SOH(j)\right) \tag{14}$$

where $W(\cdot)$ is a monotonic function ensuring w converges w^*. If w is linearly related to $(SOH^r - SOH)$, then the integral control can be employed here, of which the convergence properties are well established. Similar approaches have been proposed in other application areas such as hybrid vehicle powertrain control [32].

The matrix P is calculated from the following discrete Lyapunov equation

$$(A_k + B_kK)^T P_k (A_k + B_kK) - P_k = -(Q + K^TRK) \tag{15}$$

where K is a stabilising control gain such that $(A_k + B_kK)$ is Schur. Since A_k and B_k are time varying, this makes P_k time varying, unlike in [30].

Input and state constraints. Input constraints are specified for charging (defined as a negative input current) up to some physically allowable limit, *i.e.*:

$$-I_{\max} \leq u(i) \leq 0 \tag{16}$$

The only constraints that need to be applied to the battery internal states reflect the need to avoid permanent damage to the battery chemistry. Whilst there is no universally accepted model to capture all phenomena such as solid-electrolyte film growth, lithium plating and electrolyte degradation, it is known that maintaining certain key states within acceptable ranges can avoid these problems [17], [25]. The following state constraints are considered in the charging problem:

$$C_{ss,\min} \leq C_{ss}^{\pm}(i) \leq C_{ss,\max} \tag{17a}$$

$$C_{e,\min} \leq C_{e}^{\pm}(i) \leq C_{e,\max} \tag{17b}$$

$$T(i) \leq T_{\max} \tag{17c}$$

or in the compact form for $i = \{1, \cdots, N-1\}$:

$$Mx(i) \leq c_0 \tag{18}$$

Note that the terminal constraint set in (4f) can potentially also be represented in this compact form. In alternative linear-time-varying (LTV)-MPC implementations, such as [24], the terminal constraint set is replaced with point constraint, $X_f = \mathbf{0}$, in order to demonstrate provable asymptotic stability. The inherent stability of the battery dynamics (*i.e.* SOC and SOH are naturally restricted to the range of $[0, 1]$) potentially alleviates the need to consider this terminal constraint, whilst guaranteeing overall system stability. Consequently, the terminal constraint is relaxed in this case, with stability checked only via case studies in Section III. Given the terminal constraint

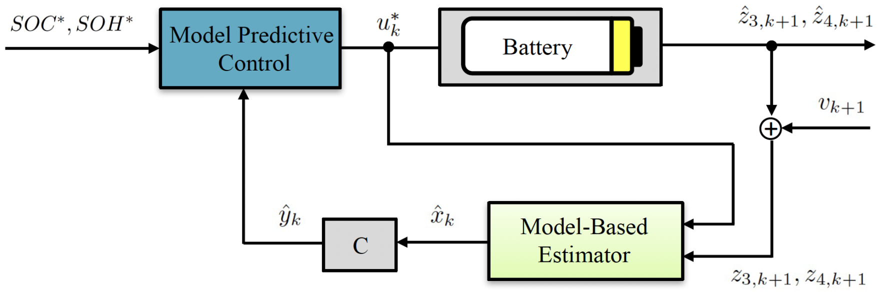

Fig. 1. Model predictive control (MPC) algorithm for battery charging.

is relaxed, there is no auxiliary positive invariant set. It is true that there is no theoretical guarantee that the input constraints (16) can be always satisfied by $u = Kx$ with the stabilising control gain K in (15). Indeed, there is a practical way to deal with this, and that is to check via simulation studies for any possible state within the operating domain.

Given the uncertainty of the impact associated with explicit violation of these constraints, it is proposed to implement them as soft constraints by introducing a vector of slack variables, s. This will alleviate any infeasibility issues in the solution of the optimisation problem, at the cost of increasing the number of decision variables in the optimisation problem.

Under this relaxation, an additional term $s^T\Gamma s$ is added to the stage cost, and the state constraints are replaced by:

$$Mx(i) \le c_0 + s \tag{19}$$

$$s \ge 0 \tag{20}$$

D. State Estimation

The current battery state, $x(0)$, is required to initialise the prediction model for solving the optimisation problem. A model-based estimation algorithm developed in [11] can be adopted here to estimate the required states, $\hat{x}$, and the output, $\hat{y}$. Based on the model (3), the estimator is formulated in the following

$$\hat{x}_{k+1} = f(\hat{x}_k, u_k) + L_k \cdot [z_{k+1} - \hat{z}_{k+1}] \tag{21a}$$

$$\hat{z}_{k+1} = g(f(\hat{x}_k, u_k), u_k) \tag{21b}$$

$$\hat{y}_{k+1} = C\hat{x}_{k+1} \tag{21c}$$

The details including calculation of the estimator gain L_k can be found in [11]. As shown therein, this estimator is effective in obtaining SOC and SOH despite a range of common errors due to model order reduction, linearisation and measurement noise. It is worth mentioning that the separation principle for estimator and controller design does not hold in general for nonlinear systems. In this case, its effectiveness will be shown via simulations in Section III.

E. Overall Problem Formulation

Based on the analysis in Sections 2.1-2.4, the overall problem formulation for battery health-conscious fast charging

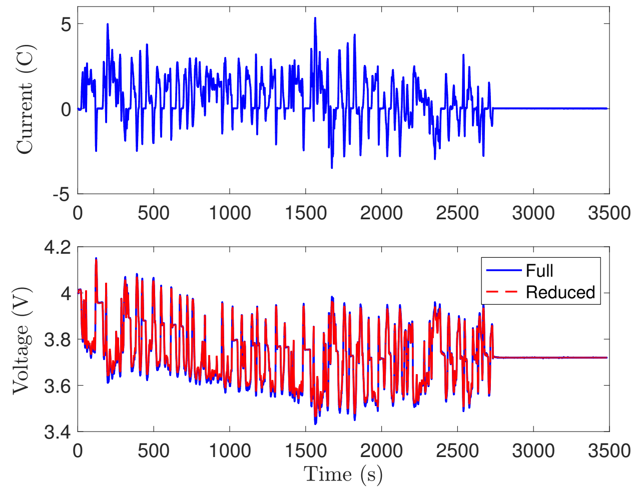

Fig. 2. Validation of the proposed model under the UDDS test.

is summarised as

$$\mathbf{u}_k^* = \arg\min_{\mathbf{u}_k, s} \sum_{i=0}^{N-1} \|\hat{y}(i) - y^r(k+i)\|_Q^2 + \|s\|_\Gamma^2 + \|\hat{y}(N) - y^r(k+N)\|_{P_k}^2 \tag{22a}$$

subject to $\forall i \in \{0, \cdots, N-1\}$

$$\hat{x}(i+1) = A_k\hat{x}(i) + B_k u(i) + \hat{d}_k \tag{22b}$$

$$\hat{y}(i) = C\hat{x}(i) \tag{22c}$$

$$\hat{x}(0) = \hat{x}_k \tag{22d}$$

$$-I_{\max} \le u(i) \le 0 \tag{22e}$$

$$M\hat{x}(i) \le c_0 + s \tag{22f}$$

$$s \ge 0 \tag{22g}$$

Furthermore, this proposed control algorithm is illustrated in Fig. 1. For given references, SOC^r and SOH^r, and state information from the estimator, $\hat{x}_k$, the designed MPC controller calculates the optimal input current, $\mathbf{u}_k^*$, at each time step k. The inherent robustness of receding horizon control will reject small disturbances arising from the output estimation, $\hat{y}$, rather than measurements.

III. Results And Discussion

The proposed charging strategy is implemented on a Li-ion battery in this section. As extensive experimental testing that

TABLE II
ACCURACY OF THE REDUCED ODE MODEL RELATIVE TO THE INITIAL PDE MODEL

C-rate	0.5C	1C	2C	4C	UDDS
RMSE (mV)	11.2	17.9	24.3	35.4	15.1

includes the state of health results can be prohibitively expensive and time-consuming, high-fidelity simulations are undertaken instead to demonstrate the key ideas in the paper. The battery plant is represented by a high-fidelity model described by a set of coupled PDE equations across the electrochemical, thermal, and ageing domains [10]. The implementation of the PDEs utilises the high-resolution simulator described in [33], with appropriate augmentation for the battery ageing states. The model is parameterised using Li-ion parameters taken from [5], [34], and the simulations are carried out using MATLAB R2012b and YALMIP [35].

A. Model Validation

To evaluate the accuracy of the obtained linearised battery model relative to the original PDE system, as a first step, comprehensive simulations are conducted over constant current tests and Urban Dynamometer Driving Schedule (UDDS) test. Specifically, for constant current tests, operations under 0.5, 1, 2, and 4 C-rates are investigated, respectively. The UDDS test is related to electric vehicle applications, in which the maximum current has been artificially scaled to 5C, as a way to test the model validity under harsh operating conditions. The root-mean-square (RMS) error in the prediction of terminal voltage is used to infer the model accuracy over the five tests. The simulation results are summarised in Table II and Fig. 2.

Whilst the prediction errors increase at higher charging rates, the RMS error remains below 35mV (corresponding to about 1% error on a normalised scale) for all the constant current tests, even though the charging rate is fixed as large as 4C. This indicates a good agreement between the developed reduced order model and its benchmark. For the UDDS test, it is illustrated that the trajectories of the two models closely match each other over the whole operation process with the RMS error as 15.1mV.

The simplified model is also validated against experimental data in the fast time-scale. The $LiFePO_4$/graphite battery cells with the nominal capacity as 2.3Ah are selected for this purpose. The UDDS test is performed, where the input current has the same C-rate profile as shown in Fig. 2 and the battery ambient temperature is controlled by a thermal chamber. Parameter identification can be conducted with techniques such as Fisher information and genetic algorithm [36]. For the considered specific battery, parameters are adopted from [36], [37]. The comparison results with respect to the output voltage are illustrated in Fig. 3. This again demonstrates that the utilised model has a high fidelity in predicting battery voltage characteristics.

B. Explicit Problem Formulation

In this section, the remaining parameters required in the problem formulation are obtained. These include the additional weighting matrices, constraints and reference trajectories.

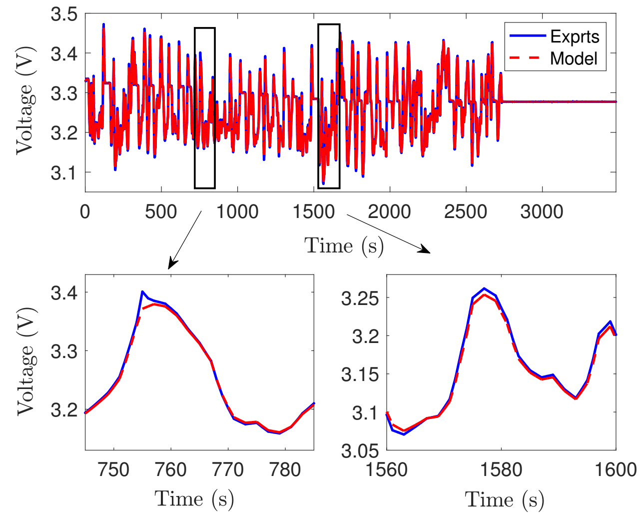

Fig. 3. Validation of the proposed model under an experimental test. The bottom are zoom-in figures from the top one.

To generate a battery SOC reference, the fastest possible charging strategy is considered. This is determined by ignoring any SOH penalty, *i.e.* setting $w = 0$ in (13). In this case, the weighting matrix penalising non-zero slack variables is chosen as $\Gamma = 10^5 \times I$. The prediction horizon is considered to be $N = 10$.

The next step is to determine the stabilising control gain, K, for the system which must satisfy the condition

$$|\lambda(A_k + B_k\bar{K})| < 1 \tag{23}$$

where $\lambda(X)$ is taken to be the maximum eigenvalue of X. This could be potentially achieved by solving an optimisation problem for all A_k and B_k of the form:

$$\min_{\bar{K}} \left|\lambda(A_k + B_k\bar{K})\right| \tag{24}$$

This would result in a single $\bar{K}$ that can be used in (15). Alternatively, it may be possible to identify $\bar{K}$ satisfying (23) using a heuristic approach. The latter approach leads to the selection $\bar{K} = -[1.22 \times 10^{-5}, 7.63 \times 10^{-6}, -7.30 \times 10^{-12}, 7.34\times10^{-10}, 1.35\times10^{-21}]$, which was found to satisfy (23) for all A_k, B_k in the five tests described in Section III-A.

Once K is established, the solution for P_k may be obtained from (15) at each iteration.

The setup for (17) will depend on battery cells and their responding applications. For the sake of demonstrating the proposed charging strategy, we choose the upper and lower bounds for the surface concentration as $C_{ss,\max} = 0.95C_{s,\max}$ and $C_{ss,\min} = 0.05C_{s,\max}$, where the parameter $C_{s,\max}$ represents the maximum possible solid-phase Li-ion concentration. The aggressive cases namely $C_{ss,\max} = C_{s,\max}$ and $C_{ss,\min} = 0$ are not used here to protect the battery in consideration of state errors due to modelling, estimation and disturbance. Similarly, $C_{e,\min} = 0.1C_{e0}$ and $C_{e,\max} = 2C_{e0}$ are set up for the electrolyte state, where the parameter C_{e0} is the quasi-steady-state electrolyte concentration. The maximum temperature is $T_{\max} = 40^o$. This is a reasonable value to prevent over-heating. For the input constraint, $I_{\max} = 10$C, corresponding to 6 minutes to fully charge a battery.

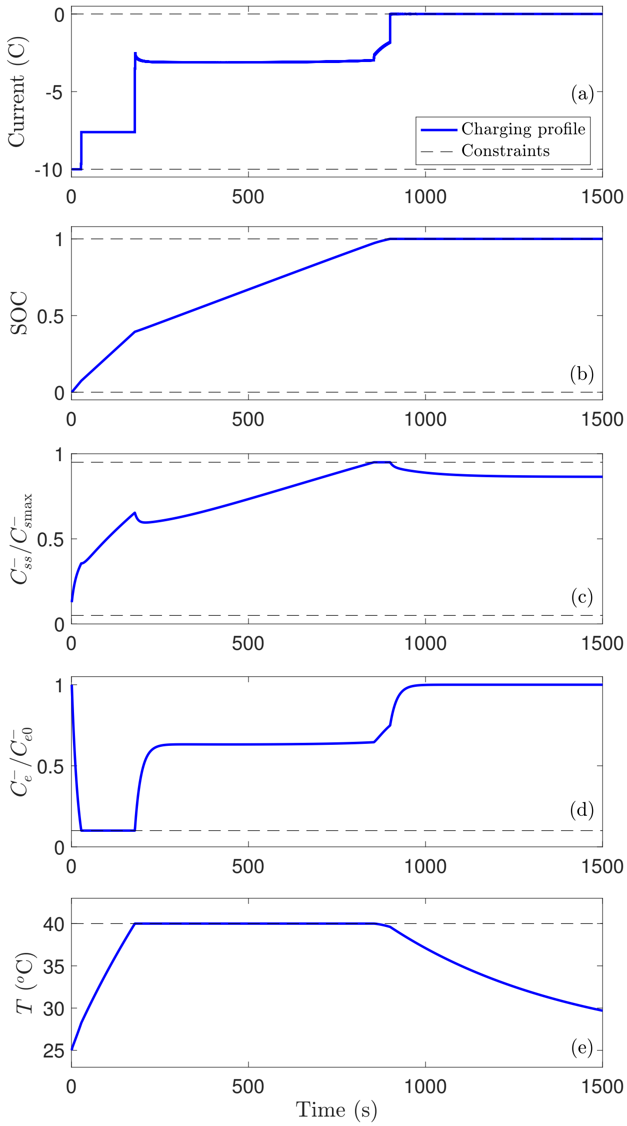

Fig. 4. Battery SOC reference generated by setting $w = 0$. (a) Input current. (b) State of charge. (c) Normalised surface concentration. (d) Normalised electrolyte concentration. (e) Temperature.

The resulting fastest possible charging trajectory is illustrated in Fig. 4. As might reasonably be expected, the largest charging rates are present when no state constraints are active. Furthermore, there does not appear to be any significant constraint violations due to the large penalty matrix Γ. Subsequently, the developed trajectories are chosen as reasonable for adoption as y_1^r in (22a) when implementing the reference tracking MPC algorithm.

The reference for state of health degradation, y_2^r, is taken to be specified by the user as satisfying Assumption 1, with different choices to be investigated in Subsection III-C.

Assumption 2 requires an optimal w^* exists to deliver the desired balance between fast charging and state of health degradation, whilst this optimal value can potentially be determined through selection of an iterative law (14).

To first test the validity of Assumption 2, a value for w is chosen from the set $\{0.001, 0.01, 0.1, \ 1, 10, 100, 1000\}$ and simulation over one charging event is performed with the control strategy (22). The charging completion times and

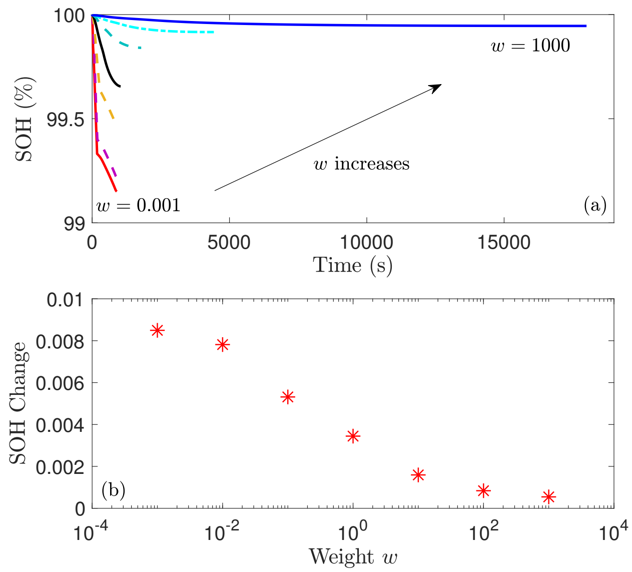

Fig. 5. SOH changes under the proposed MPC-based charging strategy for different values of w.

SOH evolution profiles derived from the obtained charging strategies are illustrated in Fig. 5. It can be seen that as w increases from 0.001 to 1000, the change in state of health is diminished at the expense of charging time. Whilst this trend is expected, thus validating Assumption 2, the nonlinear change in w to achieve a linear change in SOH is potentially problematic.

To deal with this circumstance, the update of w_j must be carefully chosen to ensure the required range of possible values can be covered. Consequently, the following two-step update is proposed, and necessitates the introduction of a new intermediate variable, α_j:

$$\alpha_{j+1} = \alpha_j + K_I \cdot (SOH^r(j) - SOH(j)) \tag{25a}$$

$$w_{j+1} = 10^{\alpha_{j+1}} \tag{25b}$$

where, the integral control gain is chosen as $K_I = 250$, and $\alpha_0 = 1.2$.

C. Simulation results for the proposed strategy

The proposed strategy (22) with the specific aspects introduced in Subsections III-A-III-B can be now implemented.

The computational requirement of the proposed charging strategy is first studied. The sampling time for the fast electrochemical and thermal dynamics is chosen to be 1 second. The simulations are conducted on a desk computer with 3.4GHz processor and Gurobi is used as the optimizer. The time solving the optimisation problem (22) is less than 10 milliseconds. This means that the computation time may not be an issue for real-time implementation of the proposed charging strategy.

1) Results in fast time-scale. As an initial comparison, the proposed method with the fixed weight $w = 1$ is compared to the well-known CCCV strategies [8]. The first CCCV charging profile, denoted CCCV1, is specified to have a current limit of 5C and a voltage limit of 4.2V. Comparing the proposed MPC strategy and CCCV1 over a single charging cycle is displayed in Fig. 6.

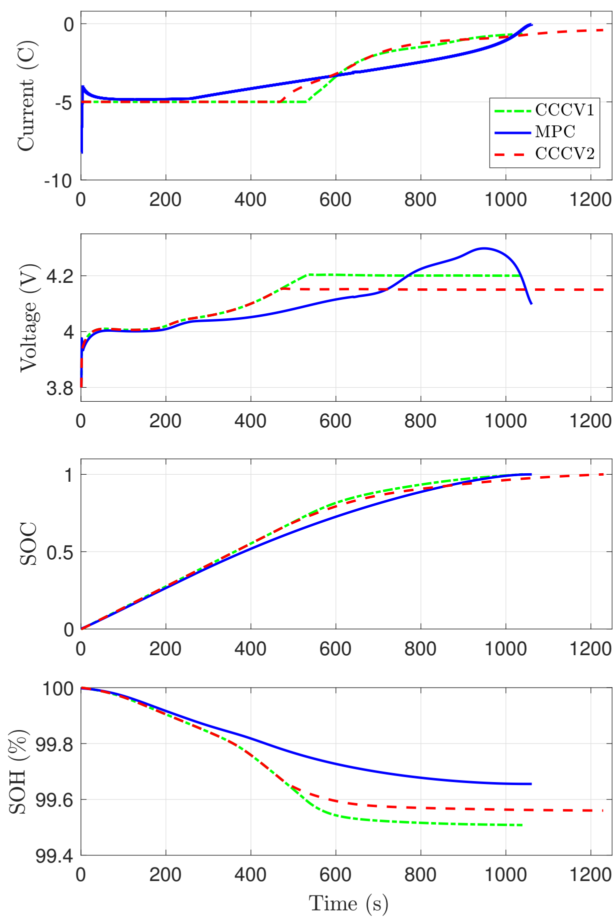

Fig. 6. Comparison of the proposed charging strategy with the industry standard charging methods, CCCV1 and CCCV2.

It can be seen that the CCCV1 (green dot-dash lines in Fig. 6) takes similar charging time as the MPC based approach to accomplish the same charging task. However, its resulting average degradation rate is 47% larger in comparison to the proposed approach, where the average degradation rate is defined as the SOH change over the entire charging operation divided by the total charging time. The improved relative performance of the proposed strategy can be attributed to the incorporation of a state of health model within the approach. This allows a more aggressive charging approach when it will not unduly impact SOH. The decrease in voltage under the MPC approach at 950 seconds is due to the related decrease in charging current, as might intuitively be expected.

Given the voltage limit in a CCCV charging will impact the degradation rate during operations [8], a second CCCV charging profile, denoted CCCV2, with a voltage limit of 4.15V is considered for further comparison. The charging results arising from the use of CCCV2 are also provided in Fig. 6 (red dash lines). These depict a reduction in state of health degradation as might reasonably be expected from the reduced voltage limit. However, the charging time is no longer equivalent to the proposed MPC strategy, reinforcing the benefits of utilising model based information in the charging approach.

In addition, it can be observed from the voltage evolution profiles in Fig. 6 that the end voltages for these charging strategies do not coincide. This is because of the presence of different terminal charging currents and ageing dynamics within the three algorithms.

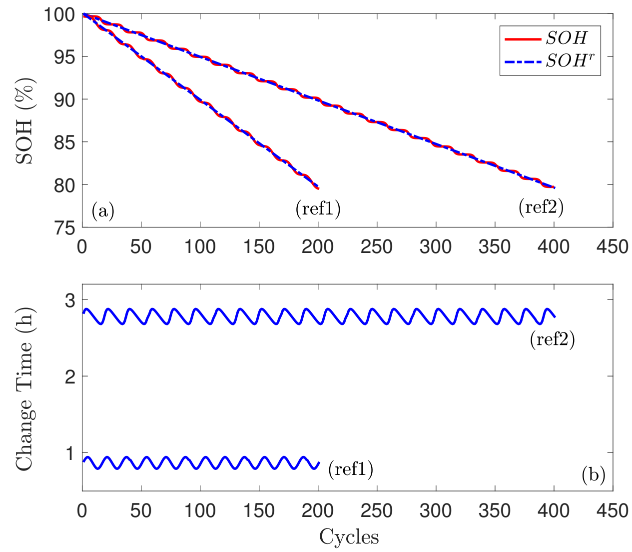

Fig. 7. Tracking performance of two given battery SOH evolution references and their corresponding charging times.

2) Results in slow time-scale. To demonstrate the ability of the proposed approach to track a reference SOH trajectory over a large number of cycles, the adaptive weight law (25) is now activated and simulations are performed over a longer time period. In the first instances, the battery lifetime (taken to be when the SOH degrades to 80% of its maximum value) is arbitrarily taken to be 200 and 400 cycles respectively, with constant associated degradation rates. Note that the lifetime of lithium-ion batteries can range from several hundred to several thousand cycles, depending on cell chemistries and operating conditions. In simulation studies, in addition to charging profiles, the lifetime is also dependent on battery ageing parameters. The parameters used in this work, adopted from [5], are related to accelerated degradation. However, this may better depict the advantages of the proposed approach, given it represents a more challenging situation. The simulation results are depicted in Fig. 7.

It may be observed that the SOH in both cases tracks the reference trajectories with only a slight oscillation about zero tracking error. If desired, further tuning of the algorithm (25) and the gain K_I may lead to reductions in these fluctuations, although they are not too severe in this case. The effect on charging time is also presented in Fig. 7(b), where it can be observed that the effect of doubling time almost three times as long.

To further demonstrate the utility of the proposed approach, an additional scenario is now considered whereby the battery is initially allowed to be charged quite aggressively, over a period of 100 cycles, before switching to a more conservative charging mode. The results depicted in Fig. 8 demonstrate successful tracking in this scenario, which is not as easily handled by existing CCCV approaches. Furthermore, it highlights the potential for "second-life" uses of batteries enabled by appropriate control actions.

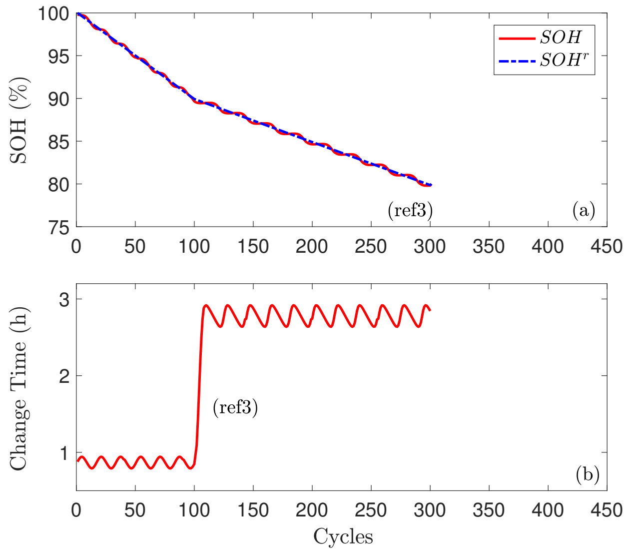

Fig. 8. Tracking performance of the third given SOH reference and its corresponding charging time.

IV. Conclusion

In this paper, a new charging algorithm has been proposed for a Li-ion battery to balance the competing objectives of battery lifetime and charging time. Under the proposed algorithm, the user can specify the balance through a desired SOH evolution profile.

The novelty of this work arises from model based control algorithm for battery fast charging. Specifically, an optimal charging control problem has been formulated for SOC and SOH reference tracking based on the MPC algorithm, which makes use of an internal battery model and explicitly handles operating constraints.

The capability of the proposed approach has been demonstrated by incorporating a multi-time-scale estimation algorithm in high-fidelity simulations. Significant improvement was observed in terms of charging time and state of health preservation relative to the industry standard charging algorithms.

Future potential work includes a condensed QP (quadratic program) formulation for the proposed algorithm to save the implementation time. Also, research opportunities exist in experimental validation of the MPC-based charging strategy across a large number of cells with potentially different model accuracy, and extension to battery pack management.

Acknowledgment

The authors would like to thank Dr. Rohan Shekhar and Mr. Gokul Sankar from The University of Melbourne for valuable discussions on the control problem formulation.

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