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Technology-supported classrooms: New opportunities for communication and development of mathematical understanding

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Abstract.

This chapter provides an overview of some themes which have emerged over two decades of Bärbel Barzel's work related to the teaching and learning of school mathematics with technology. The themes which are discussed include technology supporting mathematical communication, technology supporting cognitive activities and technology supporting an open classroom. Overall, the focus is on the potential for technology-supported classrooms to promote students' understanding in secondary school mathematics. Four papers are used to illustrate Barzel's contribution.

Introduction: Technology provides opportunities

Bärbel Barzel has produced a large corpus of work, predominantly written in German, focussed on research into teaching and learning with technology. Barzel has a clear vision for the implementation of technology into school mathematics, predicated on the notion that there is the opportunity to deepen students' mathematical understanding through meaningful integration of technology.

Teaching and learning with technology has been the subject of much research over the last three decades (see for example Berry, Monaghan, Kronfellner & Kutzler, 1997; Ball, Drijvers, Ladel, Siller, Tabach & Vale, 2018; Blume & Heid, 2008; Hoyles & Lagrange, 2010), with many studies investigating the potential offered through access to technology. Technology offers opportunities of several different types. Pierce and Stacey (2010) classified the range of pedagogical opportunities made possible through access to mathematics analysis software (e.g. a TI-Nspire). For example, they identified the capacities to *change classroom dynamics,* to *build metacognition and overview*, to *explore regularity and variation* and to *link representations* as opportunities afforded by technology. These and other opportunities highlight the potential for technology to impact the way that students do mathematics. For example, students may have a choice between using pen-and-paper or an inbuilt solve feature to solve an equation, capitalising on the 'explosion of methods' observed by Artigue (2002). These pedagogical opportunities also highlight the potential of technology to impact the way that classrooms operate, including the types of discussions and the roles of the teachers and students in the classroom. A major contribution of Barzel's work is that she has carefully researched the ways in which these opportunities can be translated into realities, in the classrooms of a wide spectrum of teachers and students. Examples of this are discussed below.

Barzel's work has investigated a vision of classrooms with a strong social constructivist underpinning and a strong focus on students' self-regulated learning, where students have significant control over their learning paths. Many of her studies support students to work in groups, supported by a teacher and with expected access to technology. There is a clear message that technology has the potential to increase the range of approaches available to both teachers and students. Therefore we might anticipate more diversity and exploration in the approaches used by students. Barzel's work investigates the possibilities across a range of settings from early learning of algebra in lower secondary school to senior secondary mathematics, thus providing an overview of the potential for technology to transform teaching and learning across the secondary school years.

Technology supports mathematical communication

A key aspect of Barzel's research studies is the importance placed on the use of a social constructivist approach for developing students' understanding. Mathematics is viewed as being developed through communication fostered by rich tasks and technology displays (Ball & Barzel, 2018; Barzel, 2007). The study of communication in mathematics education is not new, with researchers such as Yackel (2002) and Yackel and Cobb (1996) highlighting the importance of communication as students solved problems and justified their solutions, thus contributing to the development of productive socio-mathematical norms in the classroom. What is new in the work of Barzel and colleagues is consideration of the role of technology and the communication fostered through the new types of tasks enabled by technology. With technology providing instant feedback in the form of displays of graphs, solutions of equations, dynamic representations, output of symbolic solutions, etc., students have the opportunity to develop mathematical understanding through discussion of such displays.

Consideration of technology displays can promote inquiry learning (Fuglestad, 2009) and this is supported by the findings of Barzel and Möller (2001). They conducted a study where a group of 15 and 16 year-old students investigated power functions using a discovery approach with technology. One task showed students a screendump of a picture, made of graphs of functions, including the window settings. Students were asked the open question "Which functions create the following picture and how do they connect?" (p. 3). To solve the problem students had to use knowledge of the graph of a given function, say $f(x)$ and know how to reflect the function around an axis, etc. Technology could be used to explore the mathematics here, through enabling students to enter functions and observe if they were correct and it also supported communication as students worked in groups to find the functions. The very positive findings from this early study revealed an enthusiastic response from students to working in this new way in the classroom and to having more control over the path of their learning, and a desire by teachers to transfer the new method of working and its technology to other topics. For the researchers, the study underlined the need to be able to provide teachers with well-researched 'reassuring support' for these changes.

Communication is a key theme in the work of Barzel, with a focus on having students discuss technology inputs and outputs (i.e. displays) to promote mathematical understanding. To elaborate on the different ways that technology can promote collaboration, Ball and Barzel (2018) discuss communication *through*, *with* and *of* technology. 'Communication through technology' describes situations where technology is the medium used for communication, such as when students in different locations use a social network for collaborative problem solving or where data is collected from students within a classroom and displayed to the class to promote discussion. 'Communication with technology' is the communication needed to produce mathematical results, for example, entering correct syntax into a CAS to solve an equation; in this case the student needs to move from mathematical conventions to machine conventions to get the technology to produce the required result. Finally, 'communication of technology displays' is where a technology display prompts communication. A dynamic geometry worksheet prompting students to discuss a conjecture is one example; having many symbolic or numeric results produced quickly to enable mathematical exploration is another. When students use technology displays as a prompt for mathematical discussion, there is also the potential to build conceptual understanding, for example, in the use of an applet to promote understanding of the base ten place value system. The classification of communication *through*, *with* and *of* technology is useful for thinking about the different roles that technology can play in mediating classroom communication, but these are often intertwined, as students are going to need to enter syntax (i.e. 'communicate *with* technology'), in order to produce a result which can prompt 'communication *of* technology' and this result might be displayed using a data projector (i.e. 'communication *through* technology'). Ball and Barzel highlight the importance of considering the different types of communication so that teachers can consider the affordances of any given technology.

Communication to foster conceptual understanding and metacognition can be promoted by teachers in their choice of tasks. Barzel (2007) describes the types of tasks included in the material developed in her research project. One focus was the inclusion of open-ended tasks where a range of representations was used to motivate students to discuss the findings. Discussion to deepen mathematical understanding was also supported by explicit comments in teaching material. Examples such as "What are the benefits and problems of different representations" (Barzel, 2007, p. 78) and "Which functions create the following picture and how do they connect?" (Barzel & Möller, 2001, p. 3) drive communication of mathematical ideas. In the second example, students were provided with a screen capture of several functions graphed on one set of axes, as well as the window settings for the screen display. In this case ‘communication *of* technology displays’ prompted students to discuss key features of functions. Each group of students presented their findings as a poster, so they had to agree on what to present about the key features of the functions. In this way, the activity supported mathematical communication, both verbal and in written form. Students presented two types of posters; the first type contained a list of functions and the second contained assumptions made by students, as well as any questions. In the study the students and teachers gave positive feedback about the use of the social constructivist approach.

Over a number of studies, Barzel and colleagues have cleverly crafted classroom materials to encourage students’ communication about the mathematics produced by technology. Barzel (2007) provides one example, where students are presented with 13 cards (each showing a graph) and students are asked “Which belong together?” (p. 82)? Students need to find three sets of graphs from within the thirteen cards, where each set has a function, as well as its derivative and second derivative of the function. Barzel reports two students working on this task where “communication becomes more intensive and incorrect ideas are mutually corrected” (p.82). One good design feature of this task is that there is more than one set of cards which “belong together”, so students have to provide reasoning for their choices of cards which make each of the three sets. They also need to decide which graphs do not fit into any of the sets. This design of rich tasks has been a key feature of Barzel’s studies. The tasks provide researchers, professional development providers and teachers with access to novel tasks that capitalize on availability of technology to promote mathematical understanding. Careful reporting of both the tasks used and the focus on promotion of students’ communication, provides real insight into task features that can promote technology as a mediator of mathematical work.

Technology promotes cognitive activities

Zbiek, Heid, Blume & Dick (2007) highlighted the overarching role that technology can play in mediating the relationships between the student, teacher, mathematical activity and curriculum content. This idea of technology mediating the work in the mathematics classroom is central in the work of Barzel and colleagues. In a classroom where a social constructivist perspective drives development of teaching materials and where teaching materials require student discussion and the provision of reasoning to explain findings or displays from technology, the technology takes a mediating role by facilitating communication. The ability to use technology to foster cognitive activities such as discussing, explaining and structuring (Ball & Barzel, 2018) can motivate student communication about mathematics. An example of a task used by Barzel (2007) to promote cognitive discussions is the use of a motion detector to display a graph of a student walking or where a student has to walk a given graph. Analysis of graphs produced by a student walking or trying to replicate a given distance-time graph can promote cognitive discussions which help students to develop conceptual understanding of the notion of a derivative.

Barzel’s studies often have a broad focus, such as the use of real problems, communication and cognitive activities. For example, the study by Zeller and Barzel (2010) looked at whether the use of CAS (TI-Nspire) was beneficial for students learning algebra in early secondary school, with a focus on both symbol sense (Arcavi, 1994) and the syntactical skills required to work with algebra. Syntactical skills were needed to communicate mathematics, whereas symbol sense related to students’ conceptual understanding of algebra and the ability to use algebra in a flexible way. In the study the teachers were provided with teaching material, a common approach in the work of Barzel. The teaching material involved exploration of matchstick patterns and real-life problems, with students having to develop familiarity with different representations. Discovery was the cognitive activity here. Students could choose when and how to use technology. Because they had some technical skill and familiarity with their technology devices, they did not have to learn a new technology alongside learning new algebra. The study included control groups (who had TI-Nspire non-CAS calculators containing graphing, dynamic geometry and spreadsheet capabilities) and one experimental group (who had TI-Nspire CAS calculators, with all features of the TI-Nspire non-CAS calculators plus symbolic capabilities). The groups used a similar teaching sequence, which enabled investigation of any benefit in using a CAS calculator (i.e, having access to a technology with symbolic capabilities) in development of early algebraic understanding. The research focussed on the influence of CAS on students’ perceptions of algebra, students’ use of algebra compared to other representations and the influence on cognitive activities and conceptual understanding. Students’ mathematical work was found to be influenced by the technology they had access to, with non-CAS students avoiding algebra. It was hypothesised that the non-CAS students might have more difficulty in making connections between arithmetic and algebra as the calculator did not work with algebra and thus “algebra takes a special position” (Zeller & Barzel, 2010, p. 782); and indeed, it was found that CAS supported a move from arithmetic to algebra. This could be a good argument for using CAS calculators, rather than graphics calculators for introductory algebra. In this study the CAS students recognised that algebraic expressions could be treated in flexible ways; the commands evident in the CAS supported development of an understanding that algebraic expressions can be dealt with in many different ways. CAS students accepted an algebraic expression as useful when working on a problem. The available technology impacted both students' perceptions of algebra as well as their view of the relationship of algebra to arithmetic. If the type of technology impacts a student's understanding of, and inclination to use, algebra then there is an argument for teachers to choose technologies that support the use of algebra.

Empirical studies such as this prompt reflection on choices about use of technology in classrooms. This study has important implications for teachers who are inclined to avoid technology with symbolic capabilities. The study showed that there is some advantage in allowing discovery learning with CAS. However, the advantage relies on using tasks that help students make these gains. One key feature of Barzel's studies is the provision of carefully constructed teacher material that has been empirically shown to do this. There are definite implications for curriculum writers, teachers and continuing professional development providers in the findings of this work. Barzel has taken up this challenge to impact on practice through decades of teacher professional development and production of materials for teachers, including textbooks.

Technology supports an open classroom

Barzel's studies implement a philosophy that students can develop mathematical understanding through exploration of targeted teaching materials and discussion with peers, with a high degree of self-regulation. This has been a consistent approach, with Barzel (2001) highlighting the benefits of having students explore connections between graphs and functions with technology. One key element of the empirical studies of Barzel and colleagues has been the design of teaching materials, incorporating real life problems and exploration, to be used in an open classroom situation. The focus of an 'open classroom' is on students working through material in groups, socially constructing mathematical meaning through communication and collaboration. This contrasts with a classroom where teacher exposition is the main approach used to develop students' understanding. Paramount to success of an open classroom approach is the availability of good material which can promote development of key mathematical ideas, as well as a teacher with the pedagogical content knowledge necessary to ensure that the discussion, both within groups and as a class, fosters development of the key mathematical ideas.

An important contribution of Barzel and colleagues is their investigation of open classrooms with technology, focussing on communication as a key element of development of understanding. Of note is the size of some studies. For example, Barzel (2007) reported a study with 45 classes and approximately 12000 students. In this study students worked through a 'learning workshop', consisting of a set of modules, in groups of 4 to 6 for approximately six weeks. The modules were designed so that students could complete the modules in any order. There were points in the learning workshop that focussed on comparison of different representations and the advantages or disadvantages of particular representations. However, having technology meant that students could access different representations at different times, rather than be constrained to a particular sequencing of tasks. The tasks were open-ended, promoted discussion, considered different representations to cater for different student preferences for learning, included a focus on experimentation and were relevant to students' lives. The learning workshop provided an organisational structure for the teacher to use to integrate technology into teaching and learning, thereby supporting orchestration in the sense of Trouche (2004). Having pre-prepared material where students work in groups promotes a classroom where communication is important in developing understanding. Barzel has been instrumental in providing leadership in continuing professional development in Germany, enabling her research findings to have direct impact on what is happening in mathematics classrooms across the country.

Conclusion

This chapter highlights the important contribution of Barzel to the work on technology in secondary school mathematics. In particular her innovative work, developing and using teaching material in an open-classroom situation, provides a way of conceptualising teaching and learning with technology where students take a very active role in their learning. Barzel has made a significant contribution to considerations about technology-active tasks, through research projects, textbooks and professional development materials. All materials include examples and problems that prompt students to make connections and resolve cognitive conflict, underpinned by a social constructivist approach. Barzel's large classroom studies provide findings which are transferable to many different teaching situations. This issue of 'scaling up' is an important one for educational systems aiming to integrate technology into mathematics education and Barzel's work provides both teaching materials and guidance on classroom organisation supported by empirical evidence. The foci on transferring findings from research into resources for teachers and on supporting teacher professional growth highlights the potential for researchers to influence what occurs in classrooms, with the goal of making students' mathematical experiences richer. Barzel's commitment to producing textbooks and leading professional development has ensured that there is a synergy between research and the work of teachers. Our wish is for more publications in English, as the papers already in English have fuelled our desire to read more about her ground-breaking work.

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