Author/s:
Khorshidi, HA;Gunawan, I;Ibrahim, Y

Title:
A dynamic unreliability assessment and optimal maintenance strategies for multistate weighted k-out-of-n:F systems

Date:
2016-07-01

Citation:
Khorshidi, H. A., Gunawan, I. & Ibrahim, Y. (2016). A dynamic unreliability assessment and optimal maintenance strategies for multistate weighted k-out-of-n:F systems. Applied Stochastic Models in Business and Industry, 32 (4), pp.485-493. https://doi.org/10.1002/asmb.2173.

Persistent Link:
https://hdl.handle.net/11343/291282

A dynamic unreliability assessment and optimal maintenance strategies for multi-state weighted *k*-out-of-*n*:F systems

Abstract

In this paper, a dynamic evaluation of the multi-state weighted k-out-of-n:F system is presented in an unreliability viewpoint. The expected failure cost of components is used as an unreliability index. Using failure cost provides an opportunity to employ financial concepts in system unreliability estimation. Hence, system unreliability and system cost can be compared easily in order to making decision. The components' probabilities are computed over time to model the dynamic behavior of the system. The whole system has been assessed by recursive algorithm approach. As a result, a bi-objective optimization model can be developed to find optimal decisions on maintenance strategies. Finally, the application of the proposed model is investigated via a transportation system case. Matlab programming is developed for the case, and Genetic Algorithm (GA) is used to solve the optimization model.

Keywords

Multi-state weighted *k*-out-of-*n*:F system; Dynamic assessment; Unreliability evaluation; Bi-objective optimization; Failure cost; Recursive algorithm.

1. Introduction

Improving reliability is an important feature for engineering systems in designing and operation stages. System improvement occurs after having an appropriate analysis of the system. Reliability of a system can be evaluated based on system structure, system state, and the components' reliability index. Then, optimal decisions can be obtained in order to either reliability maximization or cost minimization [1]. In traditional system reliability evaluation, it is assumed that components have two possible states as working and failure. However, practical engineering systems have more states which lie between these two as deteriorated states [2-4]. The deteriorated states can be especially used to define threshold values for preventive maintenance [5]. Many cases such as a multi-engine aircraft, a multi-display cockpit system, a multi-transmitter communication, and an oil supply system with multiple pipelines are formed as *k*-out-of-*n* systems [6, 7]. A *k*-out-of-*n*:F system consists of *n* independent components which work together so that the system fails if and only if at least k components fail [8]. Since components in a system may have different failure behaviors and performance levels, weighted systems are used to define this kind of systems. Therefore, an importance weight can be defined to allocate to each component [9-12]. As a result, a multi-state weighted *k*-out-of-*n*:F system is in state *j* or below if the summation of the allocated weights for failed components is equal to or greater than a threshold value k_j.

A system which does not work properly due to failures is not beneficial, and imposes cost on the stakeholders. Since an available component can generate income for the system [13-15], failure or being in deteriorated states leads to losing the income. As a result, failure has cost that can be considered in system reliability analysis as an unreliability index. In business, everyone would find monetary metrics, such as cost, highly important [16]. Therefore, business administers are more willing to be informed by financial reports rather than technical ones. This cost, which can be called failure cost, varies for each component in different states. When a component works in its perfect functioning state, there is no failure cost. This cost emerges if the component is in either deteriorated or failure states. For instance, consider a manufacturing machine. When the machine is as good as new (in perfect functioning status), it can generate more products. Therefore, more income would be generated by the machine.

However, when the machine works in lower functioning states, its performance decreases. Hence, it is not able to generate the similar quantity of money. This reduction in generated income is interpreted as failure cost.

Most system reliability assessment studies have been done in a non-dynamic situation so that the variation of components' probability distribution through functioning periods is not considered. However, the reliability level of the components/systems changes over time [13, 17]. Therefore, time dimension is to be added in system reliability evaluation and optimization models. Recently, dynamic modelling reliability and maintenance has led to developing many researches. However in these studies, decision variables are just assigned to design time, and functioning periods are not involved in decision making process. In our paper, an unreliability analysis based on expected failure cost is proposed for multi-state weighted *k*-out-of-*n*:F systems by recursive algorithm approach. The probability distribution of the component states is modelled over time to make a dynamic assessment. Also, the expected failure cost is used as an importance weight to denote the different performance levels of components. It provides an opportunity to evaluate system reliability as cost that is helpful to make comparison with other costs of the system. In addition, not only financial concepts are employed but also a bi-objective optimization model can be developed. Some studies have developed bi-objective optimization models to maximize system reliability and system cost simultaneously via using different methods [18-20]. However, the best approach is to develop an appropriate utility function to combine multiple objective functions into a single objective function [21, 22]. In this paper, a utility function is to formulate for a bi-objective system reliability optimization model. In addition, decision variables for optimal maintenance strategies at different periods, both design time and functioning stages, have been considered. Overall, the optimization model can make optimal decisions for the multi-state weighted *k*-out-of-*n*:F system at different stages. Finally, a practical case is provided to illustrate the proposed approach. The case problem is solved by Genetic Algorithm (GA) to find the optimum maintenance strategies.

2. A brief literature review

Wu and Chen [23] analyzed the reliability of binary weighted *k*-out-of-*n* systems by recursive formulae. Li and Zuo [11] used two methods, universal generating function (UGF) and recursive algorithm, to evaluate multi-state weighted *k*-out-of-*n* system reliability based on two different definitions. They also developed optimization models to find optimal design of the system [24]. A UGF based recursive algorithm has been developed for reliability evaluation of these systems [25]. Eryilmaz [26] has analyzed the lifetime of two different multi-state *k*-out-of-*n* systems. Also, Eryilmaz and Sarikaya [27] presented reliability analysis and modeling on weighted *k*-out-of-*n* systems with two component types. In You and Li [28], redundancy allocation to *k*-out-of-*n* systems is analyzed in terms of system reliability improvement. Khorshidi et al. [29] used the income generated by available components in reliability evaluation of multi-state weighted *k*-out-of-*n* systems by using UGF and recursive algorithm methods. Yamamoto et al. [30] have done studies on calculating multi-state *k*-out-of-*n*:F systems. Yun et al. [31] proposed an economic design for consecutive- *k*-out-of-*n*:F systems with load sharing dependency. Moreover, a multi-state weighted *k*-out-of-*n*:F system is defined and evaluated by UGF method in [9]. A reliability evaluation is proposed for *k*-out-of-*n*:F systems in [32]. As can be seen, the reliability evaluation of *k*-out-of-*n*:F systems has been considered independently, however it needs more concentration.

A dynamic viewpoint makes computational complexity in reliability calculation. Selim et al. [33] presented a dynamic maintenance planning using fuzzy TOPSIS and FMEA. Ge and Yang [34] proposed a reliability analysis for non-repairable systems by dynamic fault trees. Eryilmaz and Xie [35] developed a dynamic evaluation model for three-state *k*-out-of-*n* systems over their lifetime. Coit et al. [36] described a system reliability model for *k*-out-of-*n* systems with component partnership when '*k*' changes dynamically. Faghih-Roohi et al. [37] proposed a dynamic availability assessment of multi-state weighted *k*-out-of-*n* systems by UGF method. Time has been considered in computing the probability of components in different states. They also developed two single-objective optimization models to improve system design.

3. Dynamic system reliability analysis

First of all, the probability distribution of each component through different states and time periods should be calculated in order to determine system availability distribution. For this purpose, the following assumptions have been considered:

- The time to failure for each component is independent.
- The component transits to the nearest lower state when it fails (Figure 1).
- There is only perfect corrective maintenance that changes component's state from complete failure to perfect functioning (Figure 1).
- Each component starts working in perfect functioning state (Figure 2).
- A component can have different failure cost in different states (multi-state component).
- The number of time periods is finite (Figure 2).
- Since the components in *k*-out-of-*n* systems work in parallel, the failure cost of the system is equal to the summation of components' cost.

Each component can transmit from its current state to another state or remain in the same state through time periods with an identified probability (Figure 2). Each component has M+1 possible states that M indicates perfect functioning and 0 denotes complete failure state. Failure and maintenance may cause transition to lower and higher states respectively. The transition probability from state *k* to state *j* for component *i* is denoted by $p_{k,j}^{i}$ which is assumed to follow exponential distribution. Figure 1 shows the state transition of components with transition rates as failure rate (λ) and repair rate (μ). As can be seen, failure modes deteriorate the component from a higher state to the nearest lower one. Likewise, corrective maintenance applies on the component when it is in failure state. Transition probabilities can be calculated by Eq. 1.

$$\begin{cases} p_{j,j-1}^{i} = P_{\lambda_{j,j-1}^{i}}(t < \Delta t) = 1 - e^{-(\lambda_{j,j-1}^{i} \cdot \Delta t)} \\ p_{0M}^{i} = P_{\mu_{0M}^{i}}(t < \Delta t) = 1 - e^{-(\mu_{0M}^{i} \cdot \Delta t)} \\ p_{j,j}^{i} = 1 - \sum_{\substack{w=0 \\ w \neq j}}^{M} p_{j,w}^{i} \end{cases} \tag{1}$$

where $\lambda_{j,j-1}^{i}$ represents the failure rate from state *j* to the nearest lower state (*j-1*)for component *i*, μ_{0M}^{i} represents the repair rate from state *0* to state *M* for component *i*, Δt is the time difference between two subsequent time periods, and *t* follows exponential distribution.

After determining the transition probabilities, the probability distribution of components can be obtained. The probability that component *i* at time *t* is in state *j* which is denoted by $P_j^i(t)$ is calculated as Eq. 2.

$$P_j^i(t) = \sum_{k=0}^{M} P_k^i(t-1) \cdot p_{k,j}^{i}, \qquad i = 1, \dots, n;\ t = 1, \dots, T \tag{2}$$

where *n* is the number of components in the system, T is the number of time periods. The initial conditions for the components' probabilities at design time (t=0) is as $P_M^i(0) = 1$, $P_0^i(0) = \cdots = P_{M-1}^i(0) = 0$. As a result, the summation of all state probabilities at any time for each component is equal to 1 as Eq. 3.

$$\sum_{j=0}^{M} P_j^i(t) = 1\,, \qquad i = 1, \dots, n;\ t = 1, \dots, T \tag{3}$$

Additionally, the probability distribution of the whole system is reached by recursive algorithm. The computed component probabilities are used in recursive formulation. $R_t(i,k_j)$ is the recursive function that shows the probability that the system with *i* components is in state *j* or below which means the total failure cost of the components is equal or more than K_j at time *t*. This probability is computed by Eq. 4.

$$R_t(i,K_j) = \sum_{j=0}^{M} P_j^i(t) \cdot R_t(i-1, K_j - Fc_j^i) \quad (4)$$

where Fc_j^i is failure cost of component *i* in state *j*, $R_t(i-1, K_j - Fc_j^i)$ is an updating element that provides a recursive relationship with the situation of $i-1$ components. This algorithm will continue until the boundary conditions are reached. The boundary conditions are determined in Eq. 5.

$$R_t(i,k) = \begin{cases} 0; & i \le 0 \; and \; k > 0 \\ 1; & i \ge 0 \; and \; k \le 0 \end{cases} \quad (5)$$

$R_t(n,K_j)$ calculates the probability that the whole system with *n* components is in state *j* or below at time *t* when *i* is equal to *n*. To find the probability that the system is in state *j* that is denoted by $R_t^{sys}(n,K_j)$, equation 6 is used to differentiate the probability between K_j and K_{j-1}.

$$R_t^{sys}(n,K_j) = R_t(n,K_j) - R_t(n,K_{j-1}) \quad (6)$$

As a result, the probability distribution of the system can be obtained by recursive algorithm. The present value for the system's expected failure cost would be calculated according to Eq. 7.

$$PV_F = \sum_{j=0}^{M} \sum_{t=0}^{T} R_t^{sys}(n,K_j). K_j/(1+r)^t \quad (7)$$

where *r* is interest rate. Now, the system reliability is assessed by failure cost in a dynamic viewpoint which is helpful to have a comparison view between system cost and system reliability.

4. Optimization model

Improvement strategies can increase the system availability so that they can reduce failure cost of the system. However, applying these strategies is costly. For example, employing more maintenance personnel needs to allocate more budget. Therefore, a balance between system reliability and system cost should be created. An optimization model is useful to reach this goal. In this study, a dynamic assessment has been presented for system unreliability evaluation. In fact, the unreliability value is representing the system reliability which is homogeneous with system cost. Consequently, system reliability and system cost can simply combine into a unified function to develop a bi-objective optimization model. In most reliability evaluation and optimization studies, it is assumed that maintenance actions do not change the degradation trend of the components [5, 38, 39]. However, we consider that applying improvement strategies effects on failure or repair rate that it is formulated as equations 8 and 9.

$$\lambda_{j,j-1}^i(t) = \lambda_{j,j-1}^i(t-1) + \sum_{s=1}^{S} \left(\left(\lambda_{j,j-1}^i(t-1) \cdot a_{j,j-1}^{i,s} - \lambda_{j,j-1}^i(t-1) \right) \cdot m_s(t) \right), \quad t = 1, \dots, T \quad (8)$$

$$\mu_{0M}^i(t) = \mu_{0M}^i(t-1) + \sum_{s=1}^{S} \left(\left(\mu_{0M}^i(t-1) \cdot a_{0M}^{i,s} - \mu_{0M}^i(t-1) \right) \cdot m_s(t) \right), \quad t = 1, \dots, T \quad (9)$$

where $m_s(t)$ is a Boolean variable that its value is 1 if the improvement strategy *s* is applied at time *t*, otherwise it is zero, $a_{k,j}^{i,s}$ is a factor that denotes the effect of strategy *s* on transition rate from *k* to *j* for component *i*, and *S* is the total number of strategies. Also, $\lambda(0)$ and $\mu(0)$ are transition rates at the beginning of the system without applying any improvement plans, while $\lambda(1)$ and $\mu(1)$ are transition rates at the beginning after improvement plans implementation. For example, it can be considered that increasing the allocated resource and manpower for repair by twice leads to shorten the mean time to repair (MTTR) by half. Since repair rate and MTTR have an inverse relationship ($\mu = \frac{1}{MTTR}$), repair rates will be multiplied by 2. Also, the cost related to the improvement strategies is calculated by Eq. 10.

$$C_m = \sum_{t=1}^{T}\sum_{s=1}^{S}(c_s \cdot m_s(t)/(1+r)^{t-1}) \quad (10)$$

where C_m is the cost of applying improvement strategies on the system, S is the number of improvement strategies, T is the number of time periods, and c_s is the cost of strategy s. Based on these parameters, the optimization model can be developed as Eq. 11.

$$Min\ Z = PV_F + C_m \quad (11)$$

$$m_s(t) = 0\ or\ 1, \quad (s = 1,2,\dots,S;\ t = 1,2,\dots,T)$$

This model finds optimal decisions on applying improvement strategies via minimizing the cost. In fact, it is a bi-objective model that tries to maximize system reliability and minimize system cost simultaneously. System reliability maximization is obtained through system unreliability minimization.

Dynamic models make complex optimization models. Therefore, meta-heuristic techniques are more appropriate for solving this kind of problems. In this paper, GA is used to solve the proposed model. The reasons of selecting GA are as follows. GA is a popular meta-heuristic method for multi-objective problems. It is capable in both problem modeling and finding global optimal solution. Also, large problems can be solved through reasonable time by GA which many researchers have verified. In addition, GA is powerful to solve design optimization models of multi-state weighted *k*-out-of-*n* systems [24, 37, 40, 41].

5. Practical case

In this section, a practical case is used to verify the applicability of the proposed method. The case is a ground ship-rope transportation system to transfer ships between wharf and repair posts. This transportation process consists of 5 tasks. Firstly, ship is transferred from the platform on a traverser by a broaching machine. Then, the ship is moved to the repair post with the traverser via rope transportation system. There are 9 repair posts as destinations. After that, ship is repaired at the repair post. Next, the ship is transported back to the platform via the rope transportation system. Finally, the ship would come back to the water for undocking. The transportation system is composed of three rope broaching machines which work independently. This system was introduced in [42], then it is employed as a numerical case study in [37]. The broaching machines have different capacities in the transportation system. Also, they can work in four capacity levels as fully, high, low and zero which are number as 3, 2, 1, and 0 respectively. Therefore, each machine has four states so that failure cost is imposed when a component is in either deteriorated states or failure state (Table 1). The failure cost of each machine at state 0 (failure), is as the same as generated income when it works with full capacity. For other states, the failure cost can be found relatively. According to the mentioned characteristics, the transportation system is a multi-state weighted *k*-out-of-*n* system.

5.1. Present value analysis of the transportation system

In this section, the present value of the transportation system is investigated based on the expected failure costs. In fact, a dynamic availability assessment would be done for the system based on probability distribution of the broaching machines. The transition graph and transition rates of the machines are depicted in Figure 3 and Table 2 respectively.

The system availability can be defined depending on the maximum expected quantity of the system failure cost (K_j). In this case, the transportation system has two states including available and unavailable which are numbered as 1 and 0 respectively. The system at each time is unavailable if the summation of the expected failure costs of the broaching machines is equal to or greater than 10. Therefore, the cost of an unavailable system is 10 as well

($K_0 = 10$) and the cost of an available system is zero ($K_1 = 0$). Also, the dynamic behavior of the system is modelled by the proposed method in section 2 over 20 functioning periods (T=20). In investment, the investors would like to know how they can return their invested money over a limited time. In this projects, the stakeholders want to know which maintenance decisions provide the lowest cost for the transportation system during 20 operational periods. Each period is equal to a financial year.

The duration between time periods (Δt) is assumed as 1 time unit. Also, the interest rate is considered 0.1 (*r*=0.1) for the system. Matlab programming is provided for evaluating the transportation system. The present value of system unavailability for 20 time periods (PV_F) is calculated as 2.33. Figure 4 shows the expected failure cost that is imposed on the system at each time period. In fact, it is a cash flow for system cost which are caused by the failures. As it is shown, the cost at the first three time periods including the beginning point is zero, because the probability of unavailability for the system is zero. After that, failure cost rises. However, it decreases by applying corrective maintenance. As can be seen, there is a fluctuation that shows dynamic behavior of system availability through time.

5.2. Optimal maintenance strategies

This section aims to decide on improvement plans which will be applied on broaching machines. As it is mentioned, maintenance strategies can improve system availability. It is assumed that the maritime company tends to implement some maintenance strategies. These strategies include either increasing repair personnel and resource or replacement. Increase on repair resources influences repair rates, and replacement effects on failure rates. Also, there are two options for each strategy which are mutually exclusive. Personnel and resource can be increased by twice or one and a half times (1.5 times). Major or minor replacement can be done. As a result, there are four maintenance strategies. As the example mentioned in section 3, double increase in repair personnel and resource will enhance repair rate twice so that the strategy's effect can be quantified by factors as $a^i_{0,3} = 2$ and $a^i_{1,0} = a^i_{2,1} = a^i_{3,2} = 1$. The effect factors in addition to costs of each strategy are given in Table 3.

As it is assumed, the maintenance plans can be applied at each time period. Therefore, there are four decision variables for strategies at 20 time periods. In overall, the optimization model has 80 (4×20) decision variables. Hence, there are 2^{80} possible solutions in which the optimal solution should be obtained. According to the case, the bi-objective optimization model has been developed as equations 12-15.

$$Min\ Z = PV_F + C_m \quad (12)$$

$$m_1(t) + m_2(t) \leq 1, \quad (t = 1,2,\dots,20) \quad (13)$$

$$m_3(t) + m_4(t) \leq 1, \quad (t = 1,2,\dots,20) \quad (14)$$

$$m_s(t) = 0\ or\ 1, \quad (s = 1,2,3,4;\ t = 1,2,\dots,20) \quad (15)$$

where equation 12 shows the objective function, constraints 13 and 14 formulate that both strategies 1 and 2 and strategies 3 and 4 are mutually exclusive respectively, and equation 15 denotes that the variables are Boolean and the model is an integer programming.

GA toolbox in Matlab R2014a is used to solve the proposed optimization model. The solution aims to find the best strategy for applying maintenance plans over time which minimizes both expected failure cost of the transportation system over 20 periods and maintenance cost. The population size is set at 100, and the solving process will stop after either 200 generations or 150 stall generations. The results of optimization model are presented in Table 4.

The optimization model suggests that strategy number 4 (minor replacement) should be applied twice at the beginning of periods 3 and 8. There is no decision for applying other strategies. The optimal results show that overall system cost is reduced to 2.0428. This cost consists of strategies' cost (0.6698) and system unreliability

(1.373). As can be seen, system unreliability is decreased. A comparison for the expected failure cost cash flow is presented for the system before applying optimal decisions and after that in Figure 5.

6. Conclusion

This paper provides a recursive analysis on the reliability of the *k*-out-of-*n* system by using failure cost. The variation of components' probabilities and performances over time has been considered to establish a dynamic assessment. The transition probabilities through the operating states over time follow exponential distribution. Also, a model is used to represent multi-state components. In addition, an optimization model is developed to minimize both system unreliability and system cost. The discrete dynamic assessment provides an opportunity to make decisions for maintenance strategies in each time period. The proposed model is programmed by Matlab for a marine transportation system with three broaching machines working as a multi-state *k*-out-of-*n* system over 20 time periods. The optimal solution is found for the transportation system by GA. Also, the optimal situation has been compared with the current situation in terms of the expected failure cost generation as a cash flow.

The proposed model is a general model that can be used for various components. The practitioners just should find the rate with which a component can generate income over time periods. For further research, different failure and repair modes can be considered in the model. Also, the inflation rate could be involved in computing present value of either system unreliability or system cost. Moreover, some constraints for budget, availability, weight, or volume can be added to the optimization model. In addition, other meta-heuristic techniques can be used for solving the proposed model, and the results can be compared.

Acknowledgments

Authors greatly appreciate the editor's and the anonymous reviewer's valuable comments and suggestions that improve this article.

References

[1] Khorshidi, H. A., Gunawan, I. & Ibrahim, M. Y. Investigation on system reliability optimization based on classification of criteria.in *Proc. Industrial Technology (ICIT), 2013 IEEE International Conference on*, 2013.

[2] Lisnianski, A., Frenkel, I. & Ding, Y., *Multi-state system reliability analysis and optimization for engineers and industrial managers,* Springer-Verlag: London, 2010).

[3] Lisnianski, A. & Levitin, G., *Multi-State System Reliability: Assessment, Optimization and Applications,* World Scientific: Singapore, 2003).

[4] Tian, Z., Levitin, G. & Zuo, M. J. A joint reliability-redundancy optimization approach for multi-state series-parallel systems. *Reliability Engineering and System Safety* 94 (10), 1568-1576, 2009.

[5] Liu, B., Xu, Z., Xie, M. & Kuo, W. A value-based preventive maintenance policy for multi-component system with continuously degrading components. *Reliability Engineering & System Safety* 132 83-89, 2014.

[6] Kuo, W. & Zou, M. J., *Optimal Reliability Modeling: Principles and Applications,* John Wiley & Sons: New York, 2003).

[7] Tian, Z., Zuo, M. & Yam, R. Multi-state k-out-of-n system and their performance evaluation. *IIE Transactions (Institute of Industrial Engineers)* 41 (1), 32-44, 2009.

[8] Levitin, G., *The Universal Generating Function in reliability analysis and optimization,* Springer-Verlag: London, 2005).

[9] Ding, Y., Zio, E., Li, Y., Cheng, L. & Wu, Q. Definition of multi-state weighted k-out-of-n: F systems. *International Journal of Performability Engineering* 8 (2), 217-219, 2012.

[10] Ding, Y., Zuo, M. J., Tian, Z. & Li, W. The hierarchical weighted multi-state k-out-of-n system model and its application for infrastructure management. *IEEE Transactions on Reliability* 59 (3), 593-603, 2010.

[11] Li, W. & Zuo, M. J. Reliability evaluation of multi-state weighted k-out-of-n systems. *Reliability Engineering and System Safety* 93 (1), 161-168, 2008.
[12] Wang, Y., Li, L., Huang, S. & Chang, Q. Reliability and covariance estimation of weighted k-out-of-n multi-state systems. *European Journal of Operational Research* 221 (1), 138-147, 2012.
[13] Hamadani, A. Z. & Khorshidi, H. A. System reliability optimization using time value of money. *International Journal of Advanced Manufacturing Technology* 66 (1-4), 97-106, 2013.
[14] Marais, K. B. & Saleh, J. H. Beyond its cost, the value of maintenance: An analytical framework for capturing its net present value. *Reliability Engineering and System Safety* 94 (2), 644-657, 2009.
[15] Saleh, J. H. & Marais, K. Reliability: How much is it worth? Beyond its estimation or prediction, the (net) present value of reliability. *Reliability Engineering and System Safety* 91 (6), 665-673, 2006.
[16] Rhee, S. J. & Ishii, K. Using cost based FMEA to enhance reliability and serviceability. *Advanced Engineering Informatics* 17 (3-4), 179-188, 2003.
[17] Kołowrocki, K. & Kwiatuszewska-Sarnecka, B. Reliability and risk analysis of large systems with ageing components. *Reliability Engineering & System Safety* 93 (12), 1821-1829, 2008.
[18] Abouei Ardakan, M., Zeinal Hamadani, A. & Alinaghian, M. Optimizing bi-objective redundancy allocation problem with a mixed redundancy strategy. *ISA Transactions* 55 116-128, 2015.
[19] Lins, I. D. & Droguett, E. L. Redundancy allocation problems considering systems with imperfect repairs using multi-objective genetic algorithms and discrete event simulation. *Simulation Modelling Practice and Theory* 19 (1), 362-381, 2011.
[20] Zoulfaghari, H., Zeinal Hamadani, A. & Abouei Ardakan, M. Bi-objective redundancy allocation problem for a system with mixed repairable and non-repairable components. *ISA Transactions* 53 (1), 17-24, 2014.
[21] Beji, N. & Chabchoub, H. The goal programming model to solve the series-parallel systems reliability allocation. *International Journal of Data Analysis and Information Systems* 1 (2), 79-87, 2009.
[22] Li, Z., Liao, H. & Coit, D. W. A two-stage approach for multi-objective decision making with applications to system reliability optimization. *Reliability Engineering & System Safety* 94 (10), 1585-1592, 2009.
[23] Wu, J.-S. & Chen, R.-J. An Algorithm for computing the reliability of weighted-k-out-of-n systems. *IEEE Transactions on Reliability* 43 (2), 327-328, 1994.
[24] Li, W. & Zuo, M. J. Optimal design of multi-state weighted k-out-of-n systems based on component design. *Reliability Engineering and System Safety* 93 (11), 1673-1681, 2008.
[25] Yuan, Y. & Ding, Y. Universal generating function based recursive algorithms for reliability evaluation of multi-state weighted k-out-of-n systems. *The 11th International Probabilistic Safety Assessment and Management Conference and the Annual European Safety and Reliability Conference*, Helsinki, Finland, 946-954, 2012.
[26] Eryilmaz, S. Lifetime of multistate k-out-of-n systems. *Quality and Reliability Engineering International* 30 (7), 1015-1022, 2014.
[27] Eryilmaz, S. & Sarikaya, K. Modeling and analysis of weighted-k-out-of-n:G system consisting of two different types of components. *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability* 228 (3), 265-271, 2014.
[28] You, Y. & Li, X. On allocating redundancies to k -out-of- n reliability systems. *Applied Stochastic Models in Business and Industry* 30 (3), 361-371, 2014.
[29] Khorshidi, H. A., Gunawan, I. & Ibrahim, M. Y. On Reliability Evaluation of Multistate Weighted k-out-of-n System Using Present Value. *Engineering Economist* 60 (1), 22-39, 2015.
[30] Yamamoto, H., Akiba, T., Yamaguchi, T. & Nagatsuka, H. An evaluating algorithm for system state distributions of generalized multi-state k-out-of-n:F systems. *Journal of Japan Industrial Management Association* 61 (6), 347-354, 2011.
[31] Yun, W. Y., Kim, G. R. & Yamamoto, H. Economic design of a load-sharing consecutive k-out-of-n:F system. *IIE Transactions (Institute of Industrial Engineers)* 44 (1), 55-67, 2012.
[32] Khorshidi, H. A., Gunawan, I. & Ibrahim, M. Y. Financial view and profitability evaluation on multistate weighted k-out-of-n:F system reliability. *International Journal of Reliability and Safety* 8 (2-4), 284-298, 2014.

[33] Selim, H., Yunusoglu, M. G. & Yilmaz Balaman, S. A dynamic maintenance planning framework based on fuzzy TOPSIS and FMEA: Application in an international food company. *Quality and Reliability Engineering International* 2015.
[34] Ge, D. & Yang, Y. Reliability analysis of non-repairable systems modeled by dynamic fault trees with priority AND gates. *Applied Stochastic Models in Business and Industry* 2015.
[35] Eryilmaz, S. & Xie, M. Dynamic modeling of general three-state k-out-of-n:G systems: Permanent-based computational results. *Journal of Computational and Applied Mathematics* 272 97-106, 2014.
[36] Coit, D. W., Chatwattanasiri, N., Wattanapongsakorn, N. & Konak, A. Dynamic k-out-of-n system reliability with component partnership. *Reliability Engineering & System Safety* 138 82-92, 2015.
[37] Faghih-Roohi, S., Xie, M., Ng, K. M. & Yam, R. C. M. Dynamic availability assessment and optimal component design of multi-state weighted k-out-of-n systems. *Reliability Engineering & System Safety* 123 57-62, 2014.
[38] Rasmekomen, N. & Parlikad, A. K. Maintenance optimization for asset systems with dependent performance degradation. *IEEE Transactions on Reliability* 62 (2), 362-367, 2013.
[39] Wang, S. & Yu, J. An effective heuristic for flexible job-shop scheduling problem with maintenance activities. *Computers and Industrial Engineering* 59 (3), 436-447, 2010.
[40] Konak, A., Coit, D. W. & Smith, A. E. Multi-objective optimization using genetic algorithms: A tutorial. *Reliability Engineering and System Safety* 91 (9), 992-1007, 2006.
[41] Sooktip, T., Wattanapongsakorn, N., Coit, D. W. & Chatwattanasiri, N. Multi-objective optimization for k-out-of-n redundancy allocation problem. *International Conference on Quality, Reliability, Risk, Maintenance, and Safety Engineering, ICQR2MSE*, Chengdu, China, 1050-1054, 2012.
[42] Blokus-Roszkowska, A. & Kolowrocki, K., Reliability and risk analysis of a ground ship-rope transporter in variable operation conditions. *Reliability, Risk, and Safety, Three Volume Set.* CRC Press, 2009).

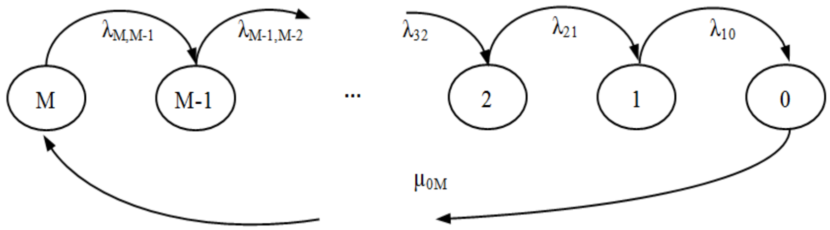

Figure 3. State transition in a multi-state component

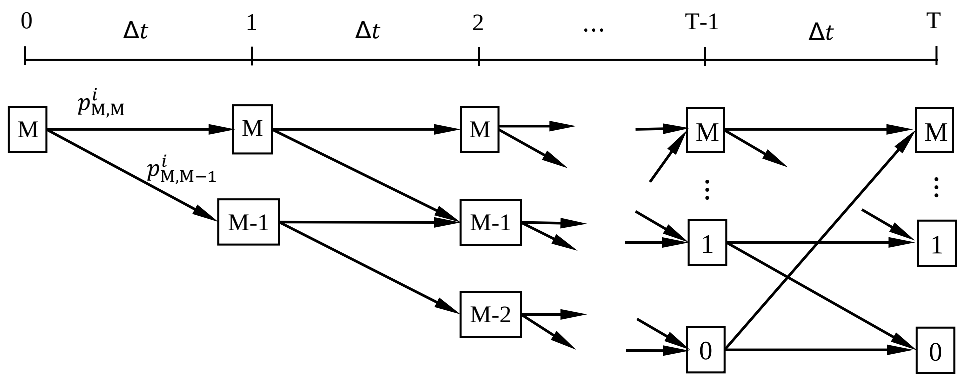

Figure 2. State distribution of a component during time periods

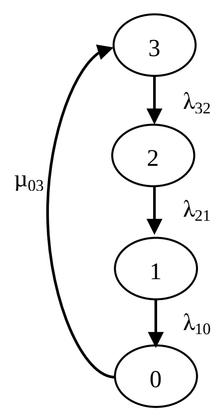

Figure 1. Model of the state transitions of broaching machines in dynamic system

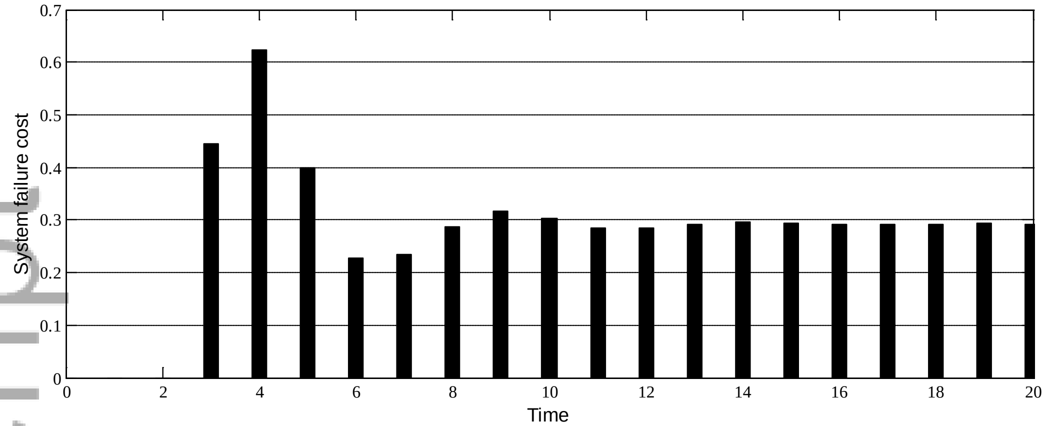

Figure 4. The cash flow of the system expected failure cost

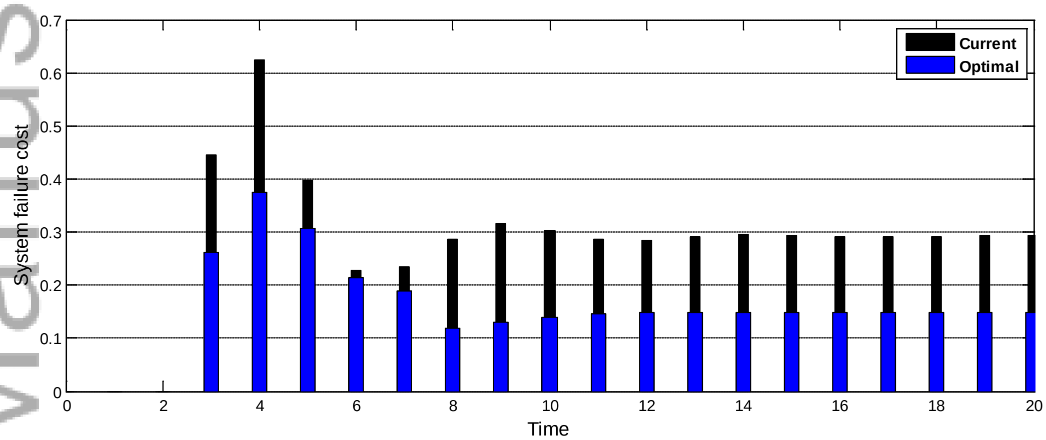

Figure 5. The expected system failure cost cash flow for current and optimal situation

Table 1. Failure cost distribution of components

Machine | State	0	1	2	3
1	4	2	1	0
2	4.4	2	1.6	0
3	5	2	1	0

Table 2. Transition data

Machine	$\lambda_{1,0}$	$\lambda_{2,1}$	$\lambda_{3,2}$	$\mu_{0,3}$
1	0.3	0.9	2	4.2
2	0.2	0.8	1.8	7.2
3	0.5	1.2	2.2	5.4

Table 3. Maintenance strategies' effect factors and cost

Strategy	s	$a_{1,0}^{i}$	$a_{2,1}^{i}$	$a_{3,2}^{i}$	$a_{0,3}^{i}$	Cost
Double resource	1	1	1	1	2	1
1.5 times resource	2	1	1	1	1.5	0.5
Major replacement	3	0.5	0.5	0.5	1	1.2
Minor replacement	4	0.75	0.75	0.75	1	0.5

Table 4. Optimization results

Optimal values	Z	PV_F	C_m
	2.0428	1.373	0.6698
Optimal decisions	Strategy	Periods	
	1	-	
	2	-	
	3	-	
	4	3, 8	